Lab 1 Preparation model answer:

①

- Steel ball:

$$M\ddot{y} = Mg - F = Mg - \frac{i^2}{y^2}$$

- Electromagnet:

$$u = L\frac{di}{dt} + Ri$$

- $x = \begin{bmatrix} y \\ \dot{y} \\ i \end{bmatrix}$, $\dot{x} = f(x,u) = \begin{bmatrix} \dot{y} \\ \frac{-1}{M}\frac{i^2}{y^2} + g \\ \frac{-R}{L}i + \frac{1}{L}u \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \frac{-i^2}{y^2} + 9.8 \\ -3i + u \end{bmatrix}$, $h(x,u) = y$.

② $f(x^*, u^*) = 0 \Rightarrow \dot{y}^* = 0$, $\frac{-i^{*2}}{y^{*2}} + 9.8 = 0 \Rightarrow i^* = \sqrt{9.8}\, y^*$

$-3i^* + u^* = 0 \Rightarrow u^* = 3i^* = 3\sqrt{9.8}\, y^*$.

$(x^*, u^*) = \left([y^* \quad 0 \quad \sqrt{9.8}\, y^*]^T, 3\sqrt{9.8}\, y^* \right)$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2\times 9.8}{y^*} & 0 & \frac{-2\sqrt{9.8}}{y^*} \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$C = [1 \quad 0 \quad 0]$, $D = 0$

③ $y^* = 1$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 19.6 & 0 & -6.26 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0], \quad D = 0$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$= [1 \quad 0 \quad 0] \begin{bmatrix} s & -1 & 0 \\ -19.6 & s & 6.26 \\ 0 & 0 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= [1 \quad 0 \quad 0] \begin{bmatrix} \frac{-6.26}{(s+3)(s^2-19.6)} \\ \\ \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{-6.26}{(s+3)(s^2-19.6)}$$

$G(s)$ has no zeros, it has poles at $-3, \pm 4.4272$.

④ $\mathcal{L}^{-1}\{G(s)\} = \mathcal{L}^{-1}\left\{ \frac{0.59}{s+3} - \frac{0.495}{s+4.4272} - \frac{0.095}{s-4.4272} \right\}$

$$g(t) = \left(0.59\, e^{-3t} - 0.495\, e^{-4.4t} - 0.095\, e^{4.4t}\right) \mathbf{1}(t)$$