



Figure 2: Block diagram of the closed-loop system

1. The transfer function of the plant is $P(s) = \frac{a}{s+b}$, it has one pole at $s = -b$. Since $b \neq 0$. The plant is type zero.

$$\frac{V(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)}, \quad E(s) = R(s) - V(s) = R(s) \left[1 - \frac{C(s)P(s)}{1 + C(s)P(s)} \right]$$

$$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1 + C(s)P(s)} = \frac{T_I s(s+b)}{T_I s(s+b) + a k (T_I s + 1)} = \frac{T_I s(s+b)}{T_I s^2 + (b T_I + a k T_I) s + a k}$$

$$\frac{V(s)}{D(s)} = \frac{P(s)}{1 + C(s)P(s)} \Rightarrow \frac{E(s)}{D(s)} = \frac{-P(s)}{1 + C(s)P(s)} = \frac{-a T_I s}{T_I s(s+b) + a k (T_I s + 1)} = \frac{-a T_I s}{T_I s^2 + (b T_I + a k T_I) s + a k}$$

$$\textcircled{2} \quad R(s) = \frac{\bar{V}}{s}, \quad D(s) = \frac{\bar{d}}{s}.$$

using superposition

$$E(s) = \frac{T_I \cancel{s}(s+b)}{T_I s^2 + (bT_I + aKT_I)s + aK} \frac{\bar{V}}{\cancel{s}} + \frac{-aT_I \cancel{s}}{T_I s^2 + (bT_I + aKT_I)s + aK} \frac{\bar{d}}{\cancel{s}}$$

$\textcircled{3}$ Assuming $\lim_{t \rightarrow \infty} e(t)$ exists (The closed loop is BIBO stable).

Then we can apply F.V.T.

$$\begin{aligned} \lim_{s \rightarrow 0} sE(s) &= \lim_{s \rightarrow 0} s \left(\frac{T_I \bar{V} (s+b) - aT_I \bar{d}}{T_I s^2 + (bT_I + aKT_I)s + aK} \right) \\ &= 0. \end{aligned}$$