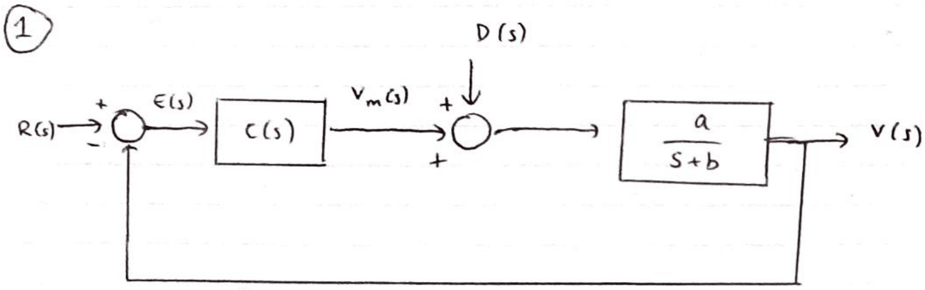
Lab 3 Prep

Shadman. Kaif

(1)



$$C(s) = \frac{V_m(s)}{E(s)} = K\left(1 + \frac{1}{T_1 s}\right) \quad ; \; K, T > 0$$

$$E_R(s) = R(s) \cdot \frac{1}{1 + C(s) \cdot \frac{a}{s+b}}$$

$$= R(s) \cdot \frac{1}{1 + \left(K + \frac{K}{T_1 s}\right) \cdot \frac{a}{s+b}}$$

$$= R(s) \cdot \frac{1}{1 + \frac{KT_1 s + K}{T_1 s} \cdot \frac{a}{s+b}}$$

$$= R(s) \cdot \frac{1}{\frac{(T_1 s)(s+b) + a(KT_1 s + K)}{(T_1 s)(s+b)}}$$

$$\boxed{E_R(s) = \frac{R(s)\,(s+b)(T_1 s)}{(s+b)(T_1 s) + Ka(T_1 s + 1)}}$$

$$-E_D(s) = D(s) \cdot \frac{\frac{a}{s+b}}{1 + C(s) \cdot \frac{a}{s+b}}$$

$$= D(s) \cdot \frac{\frac{a}{(s+b)}}{\frac{(T_1 s)(s+b) + a(KT_1 s + K)}{(T_1 s)(s+b)}}$$

$$= D(s) \cdot \frac{a}{s+b} \times \frac{(T_1 s)(s+b)}{(T_1 s)(s+b) + a(KT_1 s + K)}$$

$$= D(s) \cdot \frac{(T_1 s)a}{(T_1 s)(s+b) + Ka(T_1 s + 1)}$$

$$\boxed{E_D(s) = -\frac{D(s)(T_1 s)a}{(T_1 s)(s+b) + Ka(T_1 s + 1)}}$$

Since transfer function is asked for in the question, we can rewrite $E_R(s)$ and $E_D(s)$.

$$\frac{E_R(s)}{R(s)} = \frac{(s+b)(T_1 s)}{(s+b)(T_1 s)+Ka(T_1 s+1)} \qquad -\frac{E_D(s)}{D(s)} = \frac{a(T_1 s)}{(s+b)(T_1 s)+Ka(T_1 s+1)}$$

② $D(s) = \frac{\bar{d}}{s}$, $R(s) = \frac{\bar{v}}{s}$

$$E(s) = E_R(s) + E_D(s)$$

$$E(s) = R(s) \cdot \frac{(T_1 s)(s+b)}{(T_1 s)(s+b) + Ka(T_1 s+1)} - D(s) \cdot \frac{a(T_1 s)}{(T_1 s)(s+b) + Ka(T_1 s+1)}$$

$$\boxed{E(s) = \frac{\bar{v}}{s} \cdot \frac{(T_1 s)(s+b)}{(T_1 s)(s+b) + Ka(T_1 s+1)} - \frac{\bar{d}}{s} \frac{a(T_1 s)}{(T_1 s)(s+b) + Ka(T_1 s+1)}}$$

③ Apply FVT!

$$e(\infty) = \lim_{x \to \infty} e(t) = \lim_{s \to 0} s\,E(s)$$

$$= s\left[\frac{\bar{v}}{\cancel{s}} \cdot \frac{T_1 \cancel{(s)}(s+b)}{(T_1 s)(s+b) + Ka(T_1 s+1)} - \frac{\bar{d}}{\cancel{s}} \frac{a T_1 \cancel{(s)}}{(T_1 s)(s+b) + Ka(T_1 s+1)}\right]$$

$$= s\left[\bar{v} \cdot \frac{T_1(s+b)}{(T_1 s)(s+b) + Ka(T_1 s+1)} - \bar{d} \frac{a T_1}{(T_1 s)(s+b) + Ka(T_1 s+1)}\right]$$

$$= (0)(E(0))$$

$$= 0$$

$\therefore$ It is proved that a PI controller can meet SPEC1.