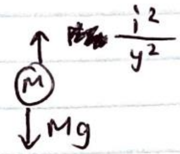
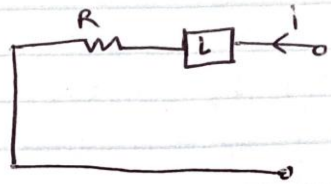
Question 1:

ECE311 Lab 1 Preperation

① FBD: $y \downarrow$



System 1



System 2

$g = 9.8\,ms$
$M = 1\,kg$
$R = 3\,\Omega$
$L = 1\,H$

System 1:

$$\Sigma F = Ma = M\ddot{y}$$

$$-\frac{i^2}{y^2} + Mg = M\ddot{y}$$

System 2:

$$-u + V_L + V_R = 0$$

$$-u + L\frac{di}{dt} + iR = 0$$

$$L\frac{di}{dt} + Ri = u$$

Subs. $M = 1\,kg$, $g = 9.8\,m/s^2$, $R = 3\,\Omega$, $L = 1\,H$

System 1:

$$-\frac{i^2}{y^2} + 9.8 = \ddot{y}$$

System 2:

$$\frac{di}{dt} + 3i = u$$

State Vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ i \end{bmatrix}$

State Space Model

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \ddot{y} = 9.8 - \frac{i^2}{y^2} = 9.8 - \frac{x_3^2}{x_1^2} \\ \dot{x}_3 = \dot{i} = u - 3x_3 \\ y = x_1 \end{cases}$$

Question 2:

(2)
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 9.8 - \frac{x_3^2}{x_1^2} \\ \dot{x}_1 = u - 3x_3 \end{cases}$$

For equilibrium, $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$

$x_2^* = 0$

$9.8 - \frac{x_3^{*2}}{x_1^{*2}} = 0$

$\frac{x_3^{*2}}{x_1^{*2}} = 9.8$

$x_3^* = 3.13x_1^*$

$u^* - 3x_3^* = 0$

$u^* = 3x_3^*$

$u^* = 3(3.13)x_1^*$

$u^* = 9.39x_1^*$

$$(x^*, u^*) = \left(\begin{bmatrix} x_1^* \\ 0 \\ 3.13x_1^* \end{bmatrix}, 9.39x_1^* \right)$$

$$\partial_x f = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_3}{\partial x_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2x_3^2}{x_1^3} & 0 & -\frac{2x_3}{x_1^2} \\ 0 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ \frac{2(3.13x_1^*)^2}{x_1^{*3}} & 0 & -\frac{2(3.13x_1^*)}{x_1^{*2}} \\ 0 & 0 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 19.6x_1^{*-1} & 0 & -6.26x_1^{*-1} \\ 0 & 0 & -3 \end{bmatrix}$$

$$\partial_u f = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = B$$

$$\partial_x h = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} = C$$

$$\partial_u h = 0$$

Question 2 Continued:

$$\partial x = x - x^*$$

$$= \begin{bmatrix} x_1 - x_1^* \\ x_2 \\ x_3 - 3.13 x_1^* \end{bmatrix}$$

$$\partial u = u - u^*$$

$$= u - 9.39 x_1^*$$

Linearization is:

$$\frac{d}{dt}(dx) = \begin{bmatrix} 0 & 1 & 0 \\ 19.6 x_1^{*-1} & 0 & -6.26 x_1^{*-1} \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$dy = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

where $dx = \begin{bmatrix} x_1 - x_1^* \\ x_2 \\ x_3 - 3.13 x_1^* \end{bmatrix}$ and $du = u - 9.39 x_1^*$

Question 3:

③ Setting $x_1^* = y^* = 1$,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 19.6 & 0 & -6.26 \\ 0 & 0 & -3 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 \\ -19.6 & s & +6.26 \\ 0 & 0 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{s}{s^2-1.96} & \frac{1}{s^2-19.6} & \frac{-6.26}{(s+3)(s^2-19.6)} \\ \frac{19.6}{s^2-19.6} & \frac{s}{s^2-19.6} & \frac{-6.26s}{(s+3)(s^2-19.6)} \\ 0 & 0 & (s+3)^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{s^2-1.96} & \frac{1}{s^2-19.6} & \frac{-6.26}{(s+3)(s^2-19.6)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{-6.26}{(s+3)(s^2-19.6)}$$

$$= \frac{-6.26}{(s+3)(s+\sqrt{19.6})(s-\sqrt{19.6})}$$

There aren't any zeros, but the poles are:

$-\sqrt{19.6}$, -3, $\sqrt{19.6}$

Question 4:

(4) $$G(s) = \frac{-6.26}{(s+3)(s+\sqrt{19.6})(s-\sqrt{19.6})}$$

$$= \frac{-6.26}{(s+3)(s+4.43)(s-4.43)}$$

$$g(t) = \mathcal{L}^{-1}\left\{\frac{-6.26}{(s+3)(s+4.43)(s-4.43)}\right\}$$

Use Residue Theorem

$$g(t) = \left[\left.\frac{-6.26e^{st}}{(s+4.43)(s-4.43)}\right|_{s=-3} + \left.\frac{-6.26e^{st}}{(s+3)(s+4.43)}\right|_{s=4.43} + \left.\frac{-6.26e^{st}}{(s+3)(s-4.43)}\right|_{s=-4.43}\right] \cdot 1(t)$$

$$= \left[+0.59e^{-3t} - 0.095e^{4.43t} - 0.49e^{-4.43t}\right] \cdot 1(t)$$

Plot of g(t):