

Name: _____

Student ID: _____

If last 3 digits of your student ID mod 3 = m and student ID mod 5 = n , then you need to solve the m -th problem from the first group and the n -th problem from the second group.

a) Design pushdown automata (PDA) for the following: (6)

0. $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } i=k\}$
1. The set of all strings with twice as many 0's as 1's.
2. $\{a^n b^n c^m d^m \mid n, m \geq 1\} \cup \{a^n b^m c^m d^n \mid n, m \geq 1\}$

b) Design Turing machines to perform the following operations (the leftmost bit is LSB) using the simulator provided (for manual and more details, see <https://schaetztc.github.io/tursi/>) (8)

0. Increment (add 1). The tape initially contains wc , where w is a binary number. Your TM should add 1 to w and write the sum to the right of the c .
1. Decrement (subtract 1). The tape initially contains wc , where w is a binary number. Your TM should subtract 1 from w and write the result to the right of the c .
2. Logical XOR. The tape initially contains $w_1 c w_2 c$, where w_1 and w_2 are binary numbers. Your TM should compute logical XOR of w_1 and w_2 and write the result to the right of the second c .
3. Logical AND. The tape initially contains $w_1 c w_2 c$, where w_1 and w_2 are binary numbers. Your TM should compute logical AND of w_1 and w_2 and write the result to the right of the second c .
4. Logical OR. The tape initially contains $w_1 c w_2 c$, where w_1 and w are binary numbers. Your TM should compute logical OR of w_1 and w_2 and write the result to the right of the second c .

c) Show that the halting problem, i.e. given a pair (M, w) , deciding whether the Turing machine M halts when given input w , is RE but not recursive. (6)

Submission instructions:

Submit the *.tm file and the solutions to the problems in a zipped folder, named using your student ID, through moodle.

Deadline: 15/09/2023 (Friday) 11:59pm