Cavendish Experiment:

Measuring the Gravitational Constant

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The purpose of this experiment was to determine the gravitational constant by measuring the attractive force between a small and large mass. The gravitational force can be calculated by using $F = G\frac{m_1 m_2}{b^2}$ where m_1 and m_2 are the two masses, b is the distance between the center of the two masses and G is the gravitational constant. This was accomplished by using a torsion balance and taking readings of the deflection and oscillation of the apparatus. The deflection and oscillation were measured by reflecting a laser off of the center lever to a meter stick were measurements were observed. G was calculated using a derived equation

$$G = \pi^2 \Delta S b^2 \left(\frac{d^2 + \frac{2}{5}r^2}{T^2 m_1 L d} \right) \tag{1}$$

Where ΔS is the difference of the settled left and right deflection, b is the distance from the center of small mass to center of large mass, d is the length of the lever arm from the center to the small mass on the end, r is the radius of the small mass, T is the period of oscillation of the lever arm, m_1 is the large mass, and L is the distance from the meter stick to the mirror on the torsion balance.

I. EXPERIMENTAL PROCEDURE AND APPARATUS

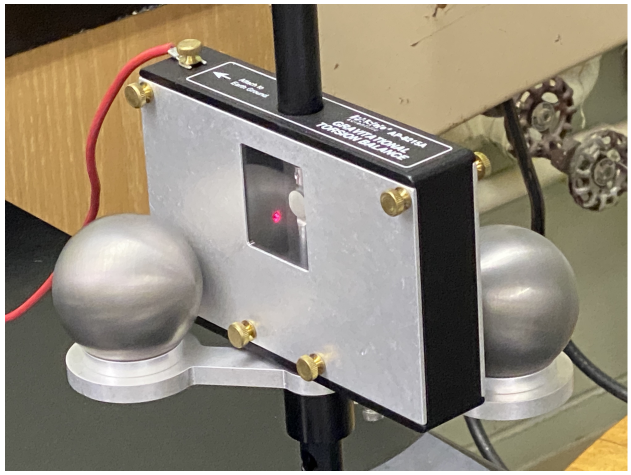

FIG. 1. Gravitational Torsion Balance in left position.

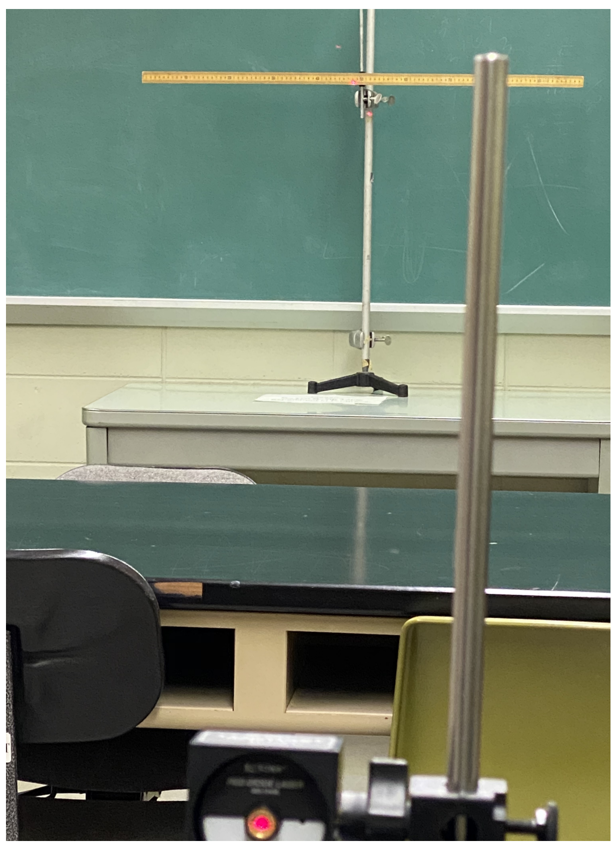

FIG. 2. View from Torsion Balance to ruler.

This experiment was conducted using a Gravitational Torsion Balance which can be seen in Fig. 1. Inside of the box of the balance is a beam attached by a beryllium copper ribbon that has two small weights on its ends. The gravitational force between the small and large weights deflects the beam and mirror. The weights of the device were moved from the left to the center to the right. After each change in position it took several hours for the beam to settle on the meter stick. When the beam settled the position on the meter stick was recorded. These measurements were used to determine ΔS. Next the balance position was changed and the beam oscillated with a stopwatch the position on the meter stick and the time were recorded. Finally the distance from the balance to the meter stick was recorded as can be seen in Fig. 2

II. DATA AND ANALYSIS

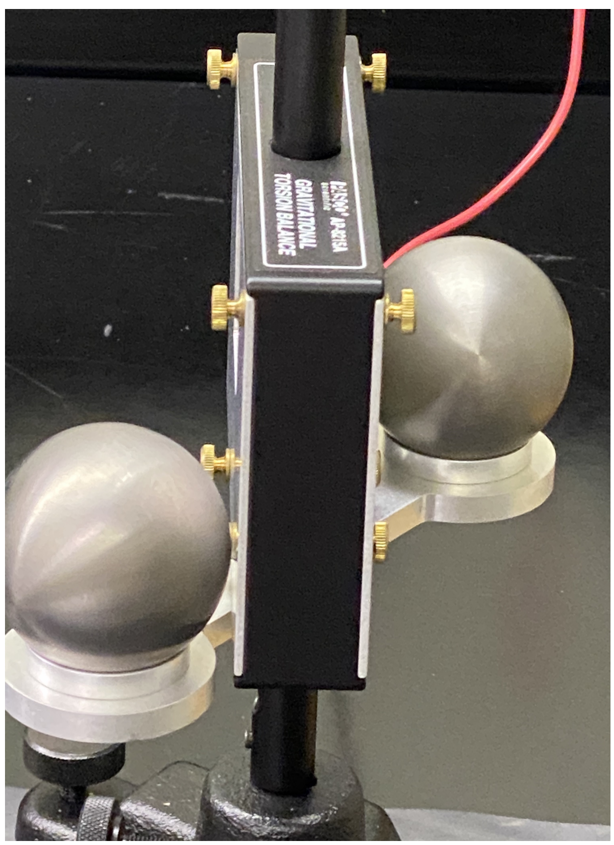

FIG. 3. Gravitational Torsion Balance in right position.

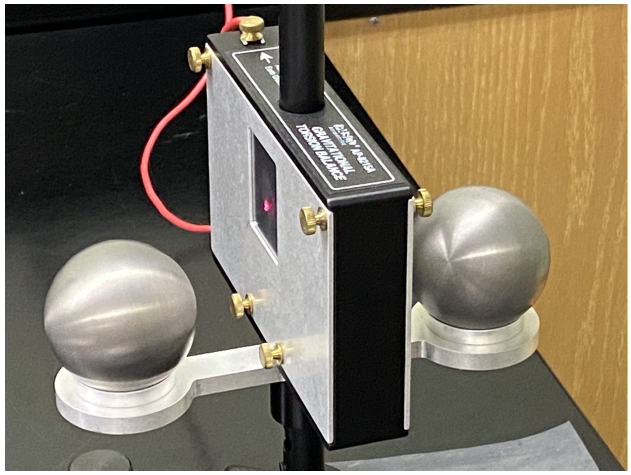

FIG. 4. Gravitational Torsion Balance in zero position.

TABLE I. Below are the values recorded when the beam had settled. The left right and zero positions can be seen in Fig. 1, Fig. 3, and Fig. 4

Position	Measurement (cm)	$\Delta S/2$ (cm)
zero	48.2 ± 0.2	
left	39.9 ± 0.2	8.3 ± 0.4
zero	48.2 ± 0.2	8.3 ± 0.4
right	54.9 ± 0.2	6.7 ± 0.4
zero	48.0 ± 0.2	6.9 ± 0.4
left	40.2 ± 0.2	7.8 ± 0.4
right	55.8 ± 0.2	7.8 ± 0.4
zero	48.2 ± 0.2	7.6 ± 0.4
right	55.2 ± 0.2	7.0 ± 0.4
left	40.2 ± 0.2	7.5 ± 0.4
zero	48.1 ± 0.2	7.9 ± 0.4

A. Calculating ΔS

$\Delta S/2$ was calculated with $\Delta S/2 = Measurement_i - Measurement_{i-1}$

$\Delta S/2$ mean $= (7.6 \pm 0.18)\, cm$

$\Delta S/2$ standard deviation $= 0.56cm$

$\Delta S = (15.2 \pm 0.35)\, cm$

The uncertainty in $\Delta S/2$ was calculated using $\frac{s}{\sqrt{N}}$ where s is the standard deviation of $\Delta S/2$ and N is the number of measurements

B. Finding the period

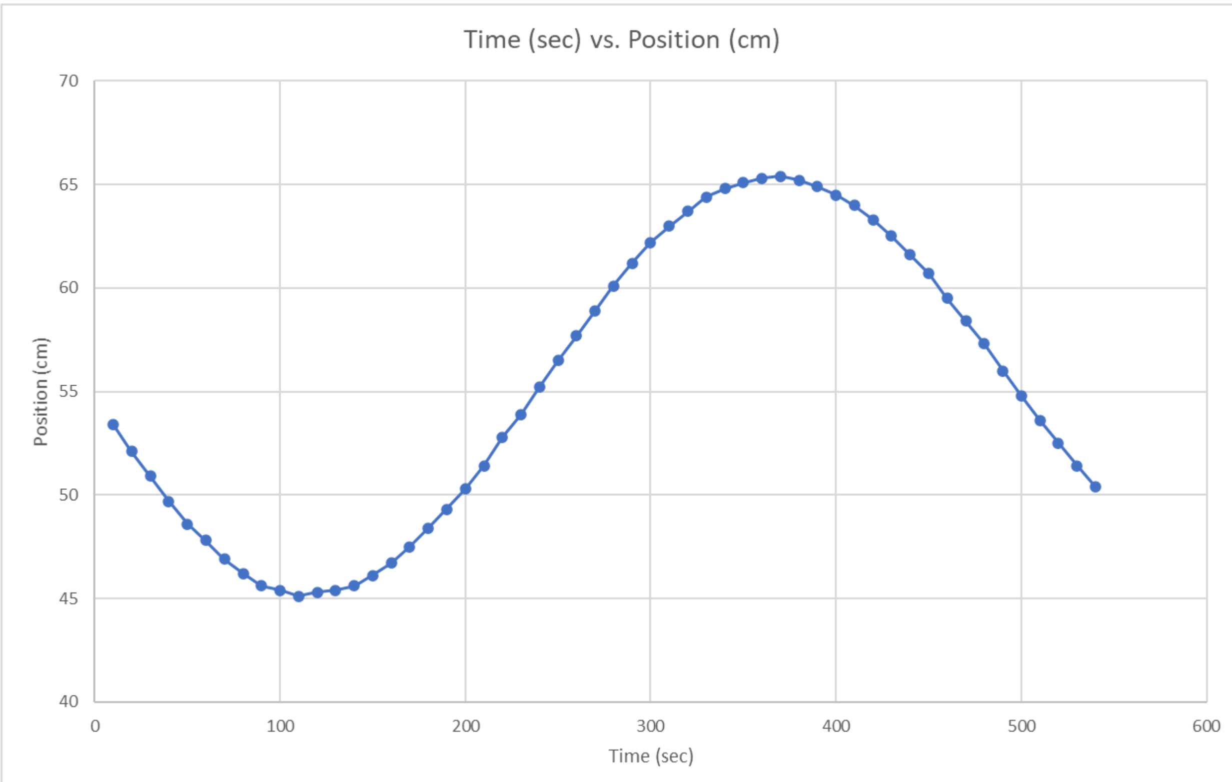

FIG. 5. Position on the ruler versus time.

The graph in Fig. 5 shows the oscillation in the reflected beam before it settles. The start and end period times were found by graphing the position on the ruler versus the time and finding two the two closest repeating points heading in the same direction. The uncertainty for period start and end time was determined by how much the time would vary given the $\pm$ 0.2cm uncertainty in the position measurements. The data used for the graph in Fig. 5 can be seen in full in appendix A table II

Period Start Time $= (10 \pm 5)$ s

Period End Time $= (510 \pm 5)$ s

T $= (500 \pm 10)$ s

C. Calculation of G

r $= (0.00955 \pm 0.00001)\, m$

d $= (0.050 \pm 0.001)\, m$

b $= (0.0422 \pm 0.001)\, m$

$m_1 = (1.5 \pm 0.1)\, kg$

L $= (5.95 \pm 0.02)\, m$

$\Delta S = (0.152 \pm 0.004)\, m$

T $= (500 \pm 10)\, s$

Using equation (1) we get the following result:

$G = (6.1 \pm .28){\times}10^{-11}\ m^3 kg^{-1} s^{-2}$

The accepted value of G is $6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

So we have a percent difference of 10% between the experimental and accepted value.

To determine the uncertainty in the experimental value of G the equation shown in (2) was used. During the derivation of this equation most covariance error was ignored because the above measurements are independent. Also during the deviation the as a final step negligible uncertainties were removed this included the uncertainty in r, d, b, and m_1

$$\sigma_G = \sqrt{\frac{\sigma_{\Delta S}^2}{\Delta S^2} + \sigma_T^2 \left(\frac{2}{T}\right)^2 + \sigma_L^2 \left(\frac{1}{L}\right)^2} \tag{2}$$

D. Corrected value of G

Since the equation in (1) doesn't account for the force of gravity between the small and large masses on opposite sides of the Torsion Balance a correction factor will be applied to our previously calculated G. The equation in (3) shows how the factor is calculated and (4) where G_0 is the corrected value of G, show how it is applied to G.

$$\beta = \frac{b^3}{(b^2 + 4d^2)^{\frac{3}{2}}} \tag{3}$$

$$G_0 = \frac{G}{(1 - \beta)} \tag{4}$$

using equation (3) we get:

$\beta = (0.059 \pm 0.005)$

And with equation (4) applied:

$G_0 = (6.4 \pm .3)\times 10^{-11}\ m^3 kg^{-1} s^{-2}$

This gives a smaller percent difference of 4% between the corrected experimental and accepted value.

The uncertainty values for β and G_0 were calculated using equations (5) and (6) respectively. Again covariance was ignored because the values effecting β and G_0 were independent.

$$\sigma_\beta = \beta\sqrt{\sigma_b^2\left(\frac{12d^2}{b(b^2+4d^2)}\right)^2 + \sigma_d^2\left(\frac{12d}{b^2+4d^2}\right)^2} \tag{5}$$

$$\sigma_{G_0} = G_0\sqrt{\sigma_G^2\left(\frac{1}{G}\right)^2 + \sigma_\beta^2\left(\frac{1}{1-\beta}\right)^2} \tag{6}$$

III. RESULTS AND CONCLUSIONS

$G = (6.1 \pm .28)\times 10^{-11}\ m^3 kg^{-1} s^{-2}$

$G_0 = (6.4 \pm .3)\times 10^{-11}\ m^3 kg^{-1} s^{-2}$

The accepted value of G is $6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

There is a percent difference of 10% between the experimental G and the accepted value, and a smaller percent difference of 4% between the corrected experimental G_0 and accepted value.

Overall this experiment resulted in a value for G that falls within the expected accuracy that was stated in the lab handout 5%, where as our corrected value was within 4%. The accepted value of G was also within the range of uncertainty of the corrected value G_0. By far the larges source of error for this experiment was the measurement of the period T = (500 ± 10) s it had an uncertainty of $\pm$ 10s. This was due to the difficulty in reading the moving beam on the meter stick. The laser was fairly dispersed by the time it reached the meter stick making it difficult to determine which mark it was pointing at. This in addition with a moving point resulted in the largest uncertainty for the period time. This might have mitigated by adding a vertical slit between the laser and the meter stick. Adding a slit could have made it easier to more accurately read the beams position on the meter stick. With a less uncertain position measurement the period uncertainty would have been reduced. This is because the uncertainty for period start and end time was determined by how much the time would vary given the $\pm$ 0.2cm uncertainty in the position measurements.

Appendix A: Tables

TABLE II. Below are the values recorded as the beam oscillated

Time (sec)	Position (cm)	Time (sec)	Position (cm)
10$\pm$ 0.5	**53.4** $\pm$ 0.2	280 $\pm$ 0.5	60.1 $\pm$ 0.2
20 $\pm$ 0.5	52.1 $\pm$ 0.2	290 $\pm$ 0.5	61.2 $\pm$ 0.2
30 $\pm$ 0.5	50.9 $\pm$ 0.2	300 $\pm$ 0.5	62.2 $\pm$ 0.2
40 $\pm$ 0.5	49.7 $\pm$ 0.2	310 $\pm$ 0.5	63.0 $\pm$ 0.2
50 $\pm$ 0.5	48.6 $\pm$ 0.2	320 $\pm$ 0.5	63.7 $\pm$ 0.2
60 $\pm$ 0.5	47.8 $\pm$ 0.2	330 $\pm$ 0.5	64.4 $\pm$ 0.2
70 $\pm$ 0.5	46.9 $\pm$ 0.2	340 $\pm$ 0.5	64.8 $\pm$ 0.2
80 $\pm$ 0.5	46.2 $\pm$ 0.2	350 $\pm$ 0.5	65.1 $\pm$ 0.2
90 $\pm$ 0.5	45.6 $\pm$ 0.2	360 $\pm$ 0.5	65.3 $\pm$ 0.2
100 $\pm$ 0.5	45.4 $\pm$ 0.2	370 $\pm$ 0.5	65.4 $\pm$ 0.2
110 $\pm$ 0.5	45.1 $\pm$ 0.2	380 $\pm$ 0.5	65.2 $\pm$ 0.2
120 $\pm$ 0.5	45.3 $\pm$ 0.2	390 $\pm$ 0.5	64.9 $\pm$ 0.2
130 $\pm$ 0.5	45.4 $\pm$ 0.2	400 $\pm$ 0.5	64.5 $\pm$ 0.2
140 $\pm$ 0.5	45.6 $\pm$ 0.2	410 $\pm$ 0.5	64.0 $\pm$ 0.2
150 $\pm$ 0.5	46.1 $\pm$ 0.2	420 $\pm$ 0.5	63.3 $\pm$ 0.2
160 $\pm$ 0.5	46.7 $\pm$ 0.2	430 $\pm$ 0.5	62.5 $\pm$ 0.2
170 $\pm$ 0.5	47.5 $\pm$ 0.2	440 $\pm$ 0.5	61.6 $\pm$ 0.2
180 $\pm$ 0.5	48.4 $\pm$ 0.2	450 $\pm$ 0.5	60.7 $\pm$ 0.2
190 $\pm$ 0.5	49.3 $\pm$ 0.2	460 $\pm$ 0.5	59.5 $\pm$ 0.2
200 $\pm$ 0.5	50.3 $\pm$ 0.2	470 $\pm$ 0.5	58.4 $\pm$ 0.2
210 $\pm$ 0.5	51.4 $\pm$ 0.2	480 $\pm$ 0.5	57.3 $\pm$ 0.2
220 $\pm$ 0.5	52.8 $\pm$ 0.2	490 $\pm$ 0.5	56.0 $\pm$ 0.2
230 $\pm$ 0.5	53.9 $\pm$ 0.2	500 $\pm$ 0.5	54.8 $\pm$ 0.2
240 $\pm$ 0.5	55.2 $\pm$ 0.2	**510** $\pm$ 0.5	**53.6**$\pm$ 0.2
250 $\pm$ 0.5	56.5 $\pm$ 0.2	520 $\pm$ 0.5	52.5 $\pm$ 0.2
260 $\pm$ 0.5	57.7 $\pm$ 0.2	530 $\pm$ 0.5	51.4 $\pm$ 0.2
270 $\pm$ 0.5	58.9 $\pm$ 0.2	540 $\pm$ 0.5	50.4 $\pm$ 0.2