In[∘]:= $x[\xi] := \dfrac{c}{H0\,\text{Sqrt}\left[\Omega m\,(1+\xi)^3 + \Omega\Lambda + (1-\Omega m-\Omega\Lambda)\,(1+\xi)^2\right]}$ `(*use ξ until we integrate the function, then switch to z*)`

In[∘]:= $\text{seriesExpansion} = x[0] + \xi\, x'[0] + \dfrac{\xi^2}{2}\, x''[0]$ `(*expansion of the stuff inside the integral*)`

Out[∘]=

$$\frac{c}{H0}+\frac{1}{2}\xi^2\left(\frac{3\,c\,(3\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))^2}{4\,H0}-\frac{c\,(6\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))}{2\,H0}\right)-\frac{c\,\xi\,(3\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))}{2\,H0}$$

In[∘]:=
```
xSeries[z_] := Integrate[seriesExpansion, ξ] /. {ξ → z}; Print["Series Expansion for χ: ", xSeries[z]](*integrated series, so this is the series we will be using for X. switch variables to z*)
```

Series Expansion for χ: $\dfrac{c\,z}{H0}-\dfrac{3\,c\,z^2\,\Omega m}{4\,H0}+\dfrac{c\,z^3\,(3\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))^2}{8\,H0}-\dfrac{c\,z^3\,(6\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))}{12\,H0}-\dfrac{c\,z^2\,(1-\Omega m-\Omega\Lambda)}{2\,H0}$

In[∘]:=
```
dH0[z_] := D[xSeries[z], H0]; Print["∂χ/∂H0 = ", dH0[z]](*write out our three partial derivatives*)
dΩm[z_] := D[xSeries[z], Ωm]; Print["∂χ/∂Ωm = ", dΩm[z]]
dΩΛ[z_] := D[xSeries[z], ΩΛ];
Print["∂χ/∂ΩΛ = ", dΩΛ[z]]
```

$$\frac{\partial\chi}{\partial H0} = -\frac{c\,z}{H0^2}+\frac{3\,c\,z^2\,\Omega m}{4\,H0^2}-\frac{c\,z^3\,(3\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))^2}{8\,H0^2}+\frac{c\,z^3\,(6\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))}{12\,H0^2}+\frac{c\,z^2\,(1-\Omega m-\Omega\Lambda)}{2\,H0^2}$$

$$\frac{\partial\chi}{\partial \Omega m} = -\frac{c\,z^2}{4\,H0}-\frac{c\,z^3}{3\,H0}+\frac{c\,z^3\,(3\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))}{4\,H0}$$

$$\frac{\partial\chi}{\partial \Omega\Lambda} = \frac{c\,z^2}{2\,H0}+\frac{c\,z^3}{6\,H0}-\frac{c\,z^3\,(3\,\Omega m+2\,(1-\Omega m-\Omega\Lambda))}{2\,H0}$$

In[∘]:=
```
FisherM[z_] := {{dH0[z] × dH0[z], dH0[z] × dΩm[z], dH0[z] × dΩΛ[z]},
                {dΩm[z] × dH0[z], dΩm[z] × dΩm[z], dΩm[z] × dΩΛ[z]},
                {dΩΛ[z] × dH0[z], dΩΛ[z] × dΩm[z], dΩΛ[z] × dΩΛ[z]}}
```

In[∘]:=
```
Simplify[FisherM[z]] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
\frac{c^2 z^2 \left(24-6 z (2+\Omega m-2 \Omega\Lambda)+z^2 \left(8+4 \Omega m+3 \Omega m^2-20 \Omega\Lambda-12 \Omega m \Omega\Lambda+12 \Omega\Lambda^2\right)\right)^2}{576 H0^4} & -\frac{c^2 z^3 (-3+z (2+3 \Omega m-6 \Omega\Lambda)) \left(24-6 z (2+\Omega m-2 \Omega\Lambda)+z^2 \left(8+4 \Omega m+3 \Omega m^2-20 \Omega\Lambda-12 \Omega m \Omega\Lambda+12 \Omega\Lambda^2\right)\right)}{288 H0^3} & \frac{c^2 z^3 (-3+z (5+3 \Omega m-6 \Omega\Lambda)) \left(24-6 z (2+\Omega m-2 \Omega\Lambda)+z^2 \left(8+4 \Omega m+3 \Omega m^2-20 \Omega\Lambda-12 \Omega m \Omega\Lambda+12 \Omega\Lambda^2\right)\right)}{144 H0^3} \\
-\frac{c^2 z^3 (-3+z (2+3 \Omega m-6 \Omega\Lambda)) \left(24-6 z (2+\Omega m-2 \Omega\Lambda)+z^2 \left(8+4 \Omega m+3 \Omega m^2-20 \Omega\Lambda-12 \Omega m \Omega\Lambda+12 \Omega\Lambda^2\right)\right)}{288 H0^3} & \frac{c^2 z^4 (-3+z (2+3 \Omega m-6 \Omega\Lambda))^2}{144 H0^2} & -\frac{c^2 z^4 (-3+z (2+3 \Omega m-6 \Omega\Lambda)) (-3+z (5+3 \Omega m-6 \Omega\Lambda))}{72 H0^2} \\
\frac{c^2 z^3 (-3+z (5+3 \Omega m-6 \Omega\Lambda)) \left(24-6 z (2+\Omega m-2 \Omega\Lambda)+z^2 \left(8+4 \Omega m+3 \Omega m^2-20 \Omega\Lambda-12 \Omega m \Omega\Lambda+12 \Omega\Lambda^2\right)\right)}{144 H0^3} & -\frac{c^2 z^4 (-3+z (2+3 \Omega m-6 \Omega\Lambda)) (-3+z (5+3 \Omega m-6 \Omega\Lambda))}{72 H0^2} & \frac{c^2 z^4 (-3+z (5+3 \Omega m-6 \Omega\Lambda))^2}{36 H0^2}
\end{pmatrix}$$

In[∘]:=
```
vars = {H0 → 70, Ωm → 0.3, ΩΛ → 0.7, c → 1}; (*assign values to all the variables*)
```

In[∘]:=
```
fisherSum = FisherM[0.01]/(0.01 xSeries[0.01])^2 + FisherM[0.1]/(0.01 xSeries[0.1])^2 + FisherM[0.2]/(0.01 xSeries[0.2])^2 + FisherM[0.3]/(0.01 xSeries[0.3])^2 /. vars; fisherSum // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 8.16327 & 25.3626 & -40.0625 \\ 25.3626 & 123.075 & -188.545 \\ -40.0625 & -188.545 & 290.053 \end{pmatrix}$$

In[∘]:=
```
CovarianceM = Inverse[fisherSum]; CovarianceM // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0.577383 & 0.762929 & 0.57568 \\ 0.762929 & 2.95275 & 2.02477 \\ 0.57568 & 2.02477 & 1.39913 \end{pmatrix}$$

In[∘]:=
```
Print["H0 = ", Around[H0 /. vars, Sqrt[CovarianceM[[1, 1]]]]]
Print["Ωm = ", Around[Ωm /. vars, Sqrt[CovarianceM[[2, 2]]]]]
Print["ΩΛ = ", Around[ΩΛ /. vars, Sqrt[CovarianceM[[3, 3]]]]]
```

H0 = 70.0 ± 0.8

Ωm = 0.3 ± 1.7

ΩΛ = 0.7 ± 1.2