In[61]:= `vars := {ξ3 → 1.202, η → 6.1 × 10^{-10}, T0 → 2.3 × 10^{-4}, EI → 13.6, Me → 510 999}`

In[70]:= `x = Solve[`$\frac{1 - Xe}{Xe^2}$` == f, Xe]` `(*solve this quadratic formula*)`

Out[70]=

$$\left\{\left\{Xe \to \frac{-1 - \sqrt{1 + 4f}}{2f}\right\}, \left\{Xe \to \frac{-1 + \sqrt{1 + 4f}}{2f}\right\}\right\}$$

In[72]:= `sol = x[[2]] /. f →` $\frac{2\,ξ3}{π^2}$ `η` $\left(\frac{2\,π\,T0\,(1+z)}{Me}\right)^{\frac{3}{2}}$ `Exp[`$\frac{EI}{T0\,(1+z)}$`]` `/. vars`

`(*plug in f(z) and variables to the positive root of our solution*)`

Out[72]=

$$\left\{Xe \to \frac{2.23756 \times 10^{22}\,e^{-\frac{59130.4}{1+z}}\left(-1 + \sqrt{1 + 8.93832 \times 10^{-23}\,e^{\frac{59130.4}{1+z}}\,(1+z)^{3/2}}\right)}{(1+z)^{3/2}}\right\}$$

In[73]:= `X = Xe /. sol`

Out[73]=

$$\frac{2.23756 \times 10^{22}\,e^{-\frac{59130.4}{1+z}}\left(-1 + \sqrt{1 + 8.93832 \times 10^{-23}\,e^{\frac{59130.4}{1+z}}\,(1+z)^{3/2}}\right)}{(1+z)^{3/2}}$$
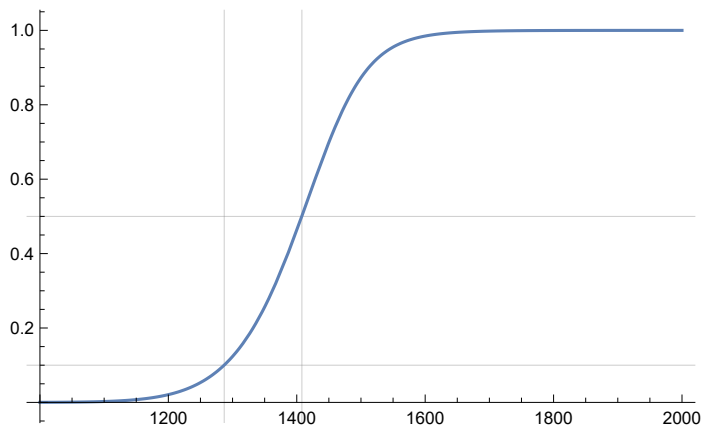
In[67]:= `X[1500]`

Out[67]=

$$\frac{2.23756 \times 10^{22}\,e^{-\frac{59130.4}{1+z}}\left(-1 + \sqrt{1 + 8.93832 \times 10^{-23}\,e^{\frac{59130.4}{1+z}}\,(1+z)^{3/2}}\right)}{(1+z)^{3/2}}[1500]$$

In[125]:=

`Plot[X, {z, 1000, 2000},`
  `GridLines → {{1287, 1408}, {0.1, 0.5}}]`

Out[125]=

In[126]:=

X /. z → 1287 (*find redshift where ionization fraction is 0.1*)

Out[126]=

0.100248

In[127]:=

X /. z → 1408 (*find redshift where ionization fraction is 0.1*)

Out[127]=

0.50106

In[130]:=

X /. z → 267 (*value I got for part 5 of problem 1*)

Out[130]=

3.92476×10^{-39}