

Chapter Name: Divide And Conquer

For problems 1-5, you should do at least the following things:

1. Describe your algorithm in natural language AND pseudo-code;
 2. Prove the correctness of your algorithm;
 3. Analyse the complexity of your algorithm.
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1. Question Number 1

You are interested in analyzing some hard-to-obtain data from two separate databases. Each database contains n numerical values, so there are $2n$ values total and you may assume that no two values are the same. You'd like to determine the median of this set of $2n$ values, which we will define here to be the n th smallest value. However, the only way you can access these values is through queries to the databases. In a single query, you can specify a value k to one of the two databases, and the chosen database will return the k th smallest value that it contains. Since queries are expensive, you would like to compute the median using as few queries as possible. Give an algorithm that finds the median value using at most $O(\log n)$ queries.

2. Question Number 2

Given any 10 points, p_1, p_2, \dots, p_{10} , on a two-dimensional Euclidean plane, please write an algorithm to find the distance between the closest pair of points.

- (a) Using a brute-force algorithm to solve this problem, analyze the time complexity of your implemented brute-force algorithm and explain why the algorithm's time complexity is $O(n^2)$, where n is the number of points.
- (b) Propose an improved algorithm to solve this problem with a time complexity better than the brute-force algorithm. Describe the algorithm's idea and analyze its time complexity.

3. Question Number 3

Given an integer n , where $100 < n < 10000$, please design an efficient algorithm to calculate 3^n , with a time complexity not exceeding $O(n)$.

- (a) Implement a naive calculation method to compute 3^n and analyze the time complexity of the naive calculation method.
- (b) Propose an improved algorithm to calculate 3^n with a time complexity not exceeding $O(n)$. Describe the algorithm's concept and analyze its time complexity.

4. Question Number 4

Given a binary tree T , please give an $O(n)$ algorithm to invert binary tree. For example below, inverting the left binary tree, we get the right binary tree.

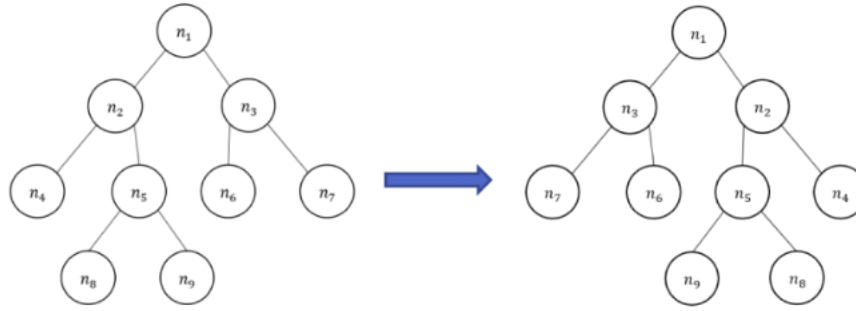


Figure 1: Binary Tree

5. Question Number 5

There are N rooms in a prison, one for each prisoner, and there are M religions, and each prisoner will follow one of them. If the prisoners in the adjacent room are of the same religion, escape may occur. Please give an $O(n)$ algorithm to find out how many states escape can occur. For example, there are 3 rooms and 2 kinds of religions, then 6 different states escape will occur.