

Chapter Name: Network flow

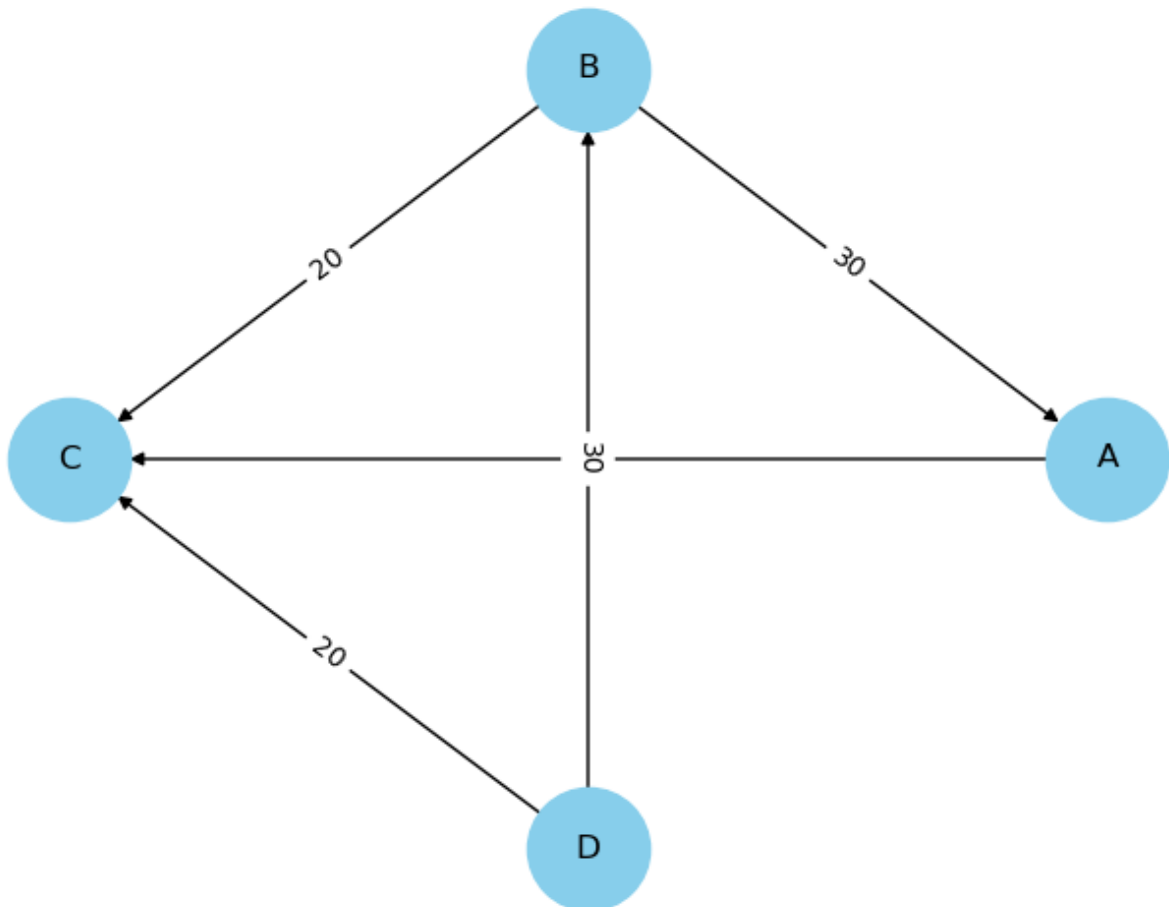
For problems 1-5, you should do at least the following things:

1. You only need to model the following questions, without the need for solving them or providing specific solving algorithms, and there's no need to discuss time complexity and space complexity.

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1. Question1

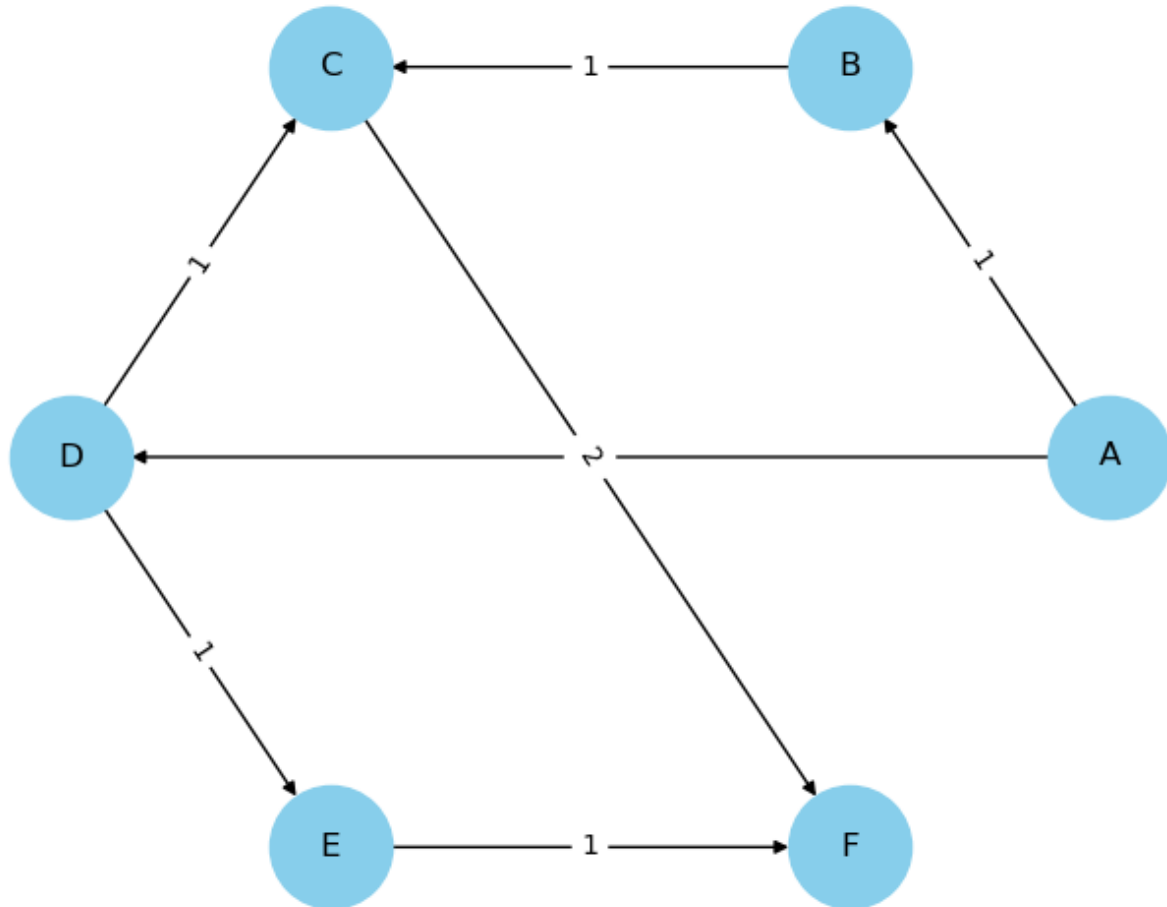
Given a network graph, along with its source and sink, find out its maximum flow. Source vertex is 4 and sink vertex is 3. Please give pseudo code and time complexity.



2. Question2

When the earthquake occurred, students at a Middle School were in class. As soon as the earthquake happened, the teachers immediately led 7 students to escape. The entire school can be abstractly represented as a directed graph with 6 vertices and 7 edges. Vertex A represents the classroom, and vertex F is the safe zone. Each edge can only accommodate a certain number of students;

exceeding this number would cause the structure to collapse. Due to the large number of students, the principal decided to evacuate the students in several batches. Only after all the students in the first batch have safely evacuated can the second batch start escaping from vertex A. Now, please help the principal calculate how many students can be evacuated in each batch at most, and how many batches are needed to evacuate all 7 students. Please give pseudo code and time complexity.



### 3. Question3

The school is on holiday... some students have gone home, and some have old good friends coming to visit, so accommodation is a problem.

For example, A and B are both students at the school. A wants to go home, and C comes to see B. C and A do not know each other. We assume that each person can only sleep in the bed of someone he or she directly knows. Then one solution is for B to sleep in A's bed and C to sleep in B's bed. The actual situation may be very complicated. Some people may know many students at the school, and not all students at the school may know each other.

We know that there are a total of 3 people, and know whether each of them is a student of this school, and also know whether each student of this school goes home. Ask if there is a plan so that all our students who are not going home and others who come to visit them have a place to stay.

First line 3 numbers, the i-th number represents the i Whether the individual is a student (0 means no,1 means yes). The next line 3 numbers, the i-th number represents the whether person i returns home ( 0 means not going home, 1 means go home). Note that if the i person is not a student, so the number in this position is a random number, and you should ignore it after reading it in).

Next 3 lines each 3 numbers, The number in row i and column j indicates whether i and j know each other. (1 means knowing, 0 means don't know. The value of row i and column i is 0, but obviously I can still sleep in my own bed), the relationship of acquaintance is mutual. Please give pseudo code and time complexity.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

#### 4. Question4

During World War II, the British Royal Air Force recruited a large number of foreign pilots from occupied countries. Each aircraft dispatched by the Royal Air Force needed to be manned by two pilots who could cooperate in terms of flying skills and language, one of whom was a British pilot and the other a foreign pilot. Among the many pilots, each foreign pilot was able to work well with several British pilots.

There are a total of 10 pilots, among which there are 5 foreign pilots and 5 British pilots. The foreign pilots are numbered from 1 to 5, while the British pilots are numbered from 6 to 10. Given the compatibility between certain foreign pilots and British pilots, find the best pairing scheme for pilots, so that the Royal Air Force can dispatch the maximum number of aircraft at one time.

Each line contains two integers u, v, indicating that the foreign pilot u can cooperate with the British pilot v. Please give pseudo code and time complexity.

$$\begin{bmatrix} 5 & 10 \\ 1 & 7 \\ 1 & 8 \\ 2 & 6 \\ 2 & 9 \\ 2 & 10 \\ 3 & 7 \\ 3 & 8 \\ 4 & 7 \\ 4 & 8 \\ 5 & 10 \end{bmatrix}$$

#### 5. Question5

For a matrix filled with 0 and 1, you know the sum of every row and column. You are asked to give such a matrix which satisfies the conditions.