THE UNIVERSITY OF BRITISH COLUMBIA

School of Engineering
Okanagan Campus

ENGR 544, Life Cycle Assessment and Management

School of Engineering, Faculty of Applied Science

The University of British Columbia (Okanagan)

How Time and Interest Affect Money

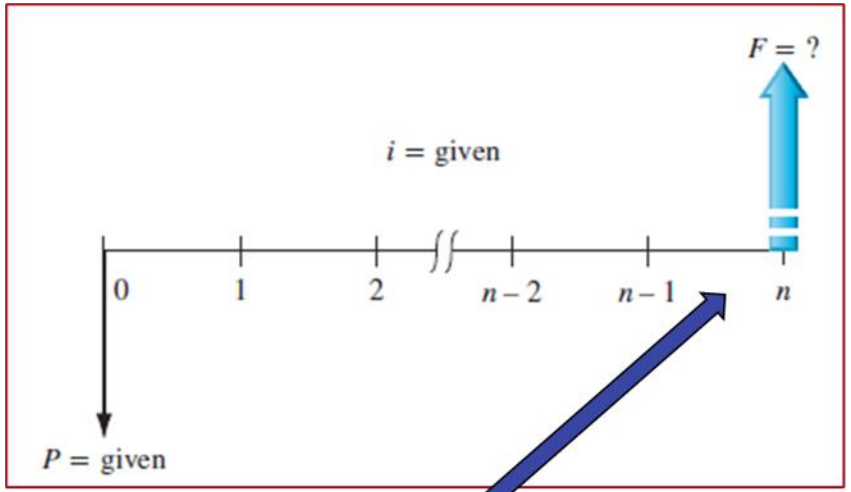
1. *F/P* and *P/F* Factors
2. *P/A* and *A/P* Factors
3. *F/A* and *A/F* Factors
4. Factor Values

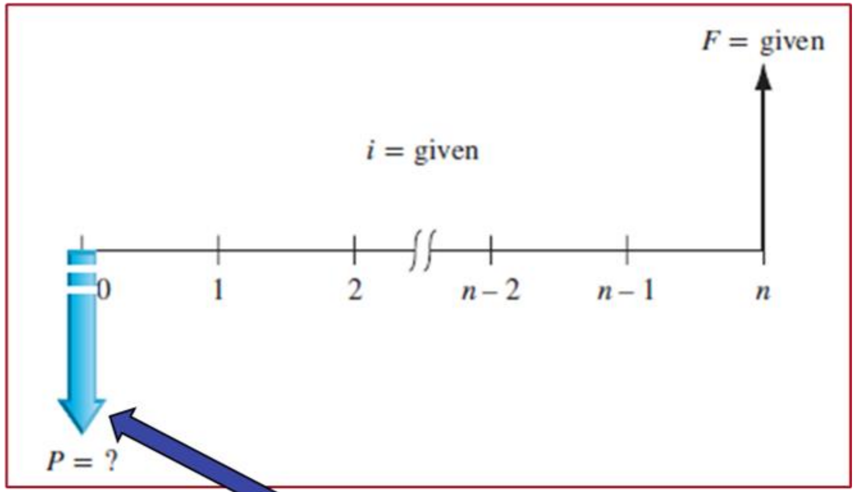
F/*P* and *P*/*F* Factors

Single Amount Factors (*F/P* and *P/F*)

Single payment factors involve only *P* and *F*

Cash flow diagrams show amounts in opposite directions





$F = P(1 + i)^n$ **Factor formulas** $P = F[1/(1 + i)^n]$

- **Terms in parentheses or brackets are called *factors*. Values are in tables for *i* and *n* values**

Factors are represented in *standard notation format such as* (*F/P,i,n*), where letter to left of slash is what is sought; letter to right represents what is given

Spreadsheets Functions for *F / P* and *P / F* Factors

Future value *F* is displayed using FV function:

$$= \mathrm{FV}(i\%, n,, P)$$

Present value *P* is calculated using PV function:

$$= \mathrm{PV}(i\%, n,, F)$$

Notes: 1. Double commas in each function indicate that no *A* series is present

2. Spreadsheet functions display opposite sign on result. Place minus sign (−) immediately prior to function name to maintain same sign as entries, e.g., $\boxed{= -\mathrm{FV}(i\%, n,, P)}$

TABLE 2–1 *F/P* and *P/F* Factors: Notation and Equations

Factor: Notation	Factor: Name	Find/Given	Standard Notation Equation	Equation with Factor Formula	Spreadsheet Function
$(F/P,i,n)$	Single-payment compound amount	F/P	$F = P(F/P,i,n)$	$F = P(1+i)^n$	$= FV(i\%,n,,P)$
$(P/F,i,n)$	Single-payment present worth	P/F	$P = F(P/F,i,n)$	$P = F(1+i)^{-n}$	$= PV(i\%,n,,F)$

Example

$\boxed{(P/F,5\%,10)}$

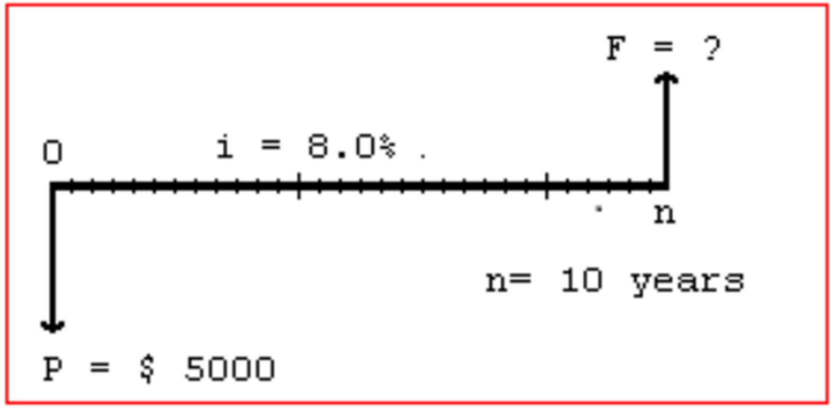
$$
\boxed{\begin{aligned}(P/F,5\%,10) &= \frac{1}{(1+i)^n} \\ &= \frac{1}{(1.05)^{10}} \\ &= \frac{1}{1.6289} = 0.6139\end{aligned}}
$$

5% TABLE 10 Discrete Cash Flow: Compound Interest Factors 5%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0500	0.9524	1.00000	1.0000	1.05000	0.9524		
2	1.1025	0.9070	0.48780	2.0500	0.53780	1.8594	0.9070	0.4878
3	1.1576	0.8638	0.31721	3.1525	0.36721	2.7232	2.6347	0.9675
4	1.2155	0.8227	0.23201	4.3101	0.28201	3.5460	5.1028	1.4391
5	1.2763	0.7835	0.18097	5.5256	0.23097	4.3295	8.2369	1.9025
6	1.3401	0.7462	0.14702	6.8019	0.19702	5.0757	11.9680	2.3579
7	1.4071	0.7107	0.12282	8.1420	0.17282	5.7864	16.2321	2.8052
8	1.4775	0.6768	0.10472	9.5491	0.15472	6.4632	20.9700	3.2445
9	1.5513	0.6446	0.09069	11.0266	0.14069	7.1078	26.1268	3.6758
10	1.6289	$\boxed{0.6139}$	0.07950	12.5779	0.12950	7.7217	31.6520	4.0991

Example: Finding Future Value

You deposit $5000 into an investment account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:



Solution

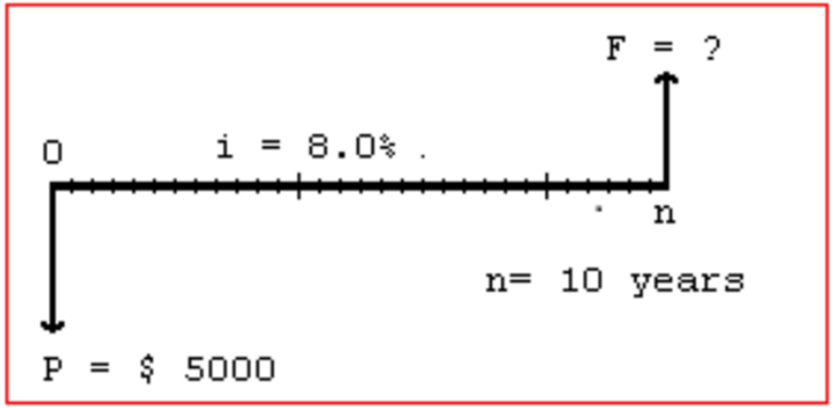
$$F = P(F/P, i, n)$$
$$= 5000(F/P, 8\%, 10)$$
$$= 5000(2.1589)$$
$$= \$10{,}794.50$$

$$\boxed{(1+i)^n = (1+0.08)^{10} = 2.1589}$$

8%	TABLE 13 Discrete Cash Flow: Compound Interest Factors						8%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0800	0.9259	1.00000	1.0000	1.08000	0.9259		
2	1.1664	0.8573	0.48077	2.0800	0.56077	1.7833	0.8573	0.4808
3	1.2597	0.7938	0.30803	3.2464	0.38803	2.5771	2.4450	0.9487
4	1.3605	0.7350	0.22192	4.5061	0.30192	3.3121	4.6501	1.4040
5	1.4693	0.6806	0.17046	5.8666	0.25046	3.9927	7.3724	1.8465
6	1.5869	0.6302	0.13632	7.3359	0.21632	4.6229	10.5233	2.2763
7	1.7138	0.5835	0.11207	8.9228	0.19207	5.2064	14.0242	2.6937
8	1.8509	0.5403	0.09401	10.6366	0.17401	5.7466	17.8061	3.0985
9	1.9990	0.5002	0.08008	12.4876	0.16008	6.2469	21.8081	3.4910
10	2.1589	0.4632	0.06903	14.4866	0.14903	6.7101	25.9768	3.8713

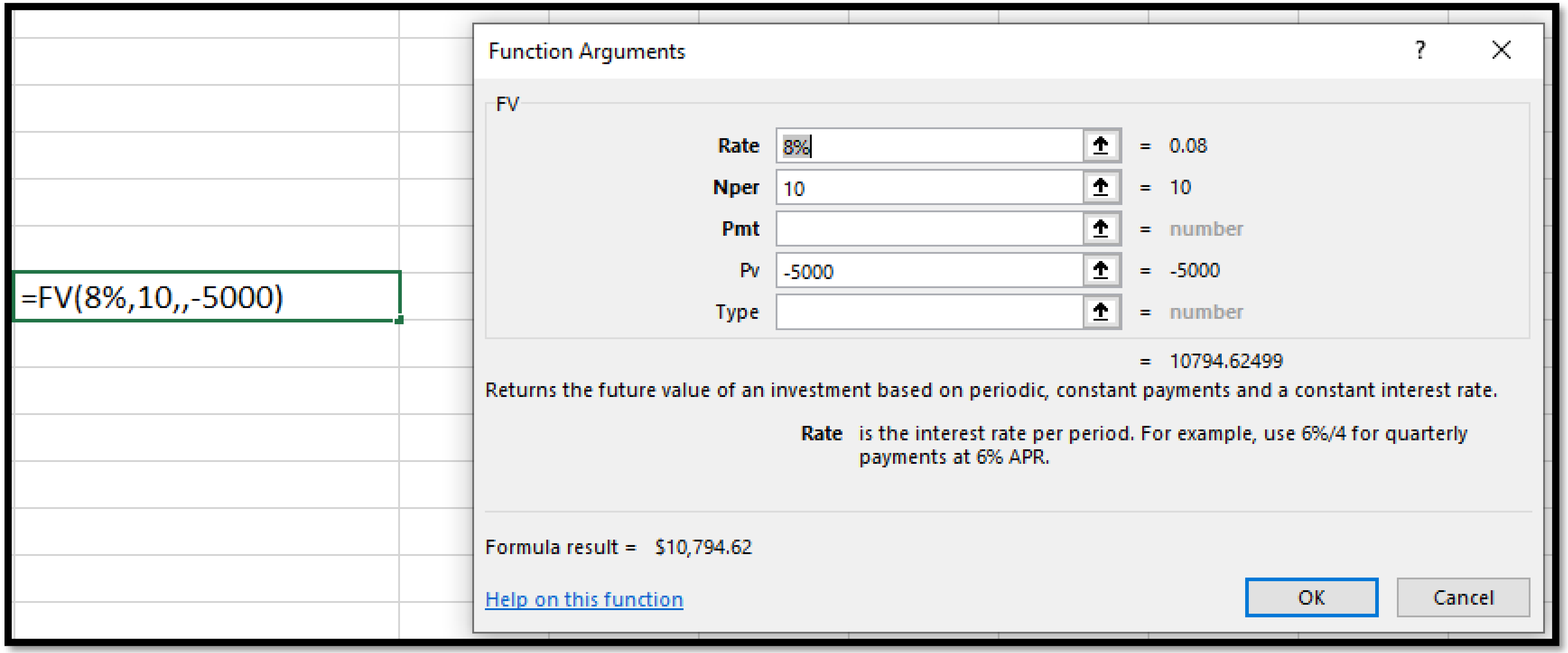
Example: Finding Future Value

You deposit $5000 into an investment account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:



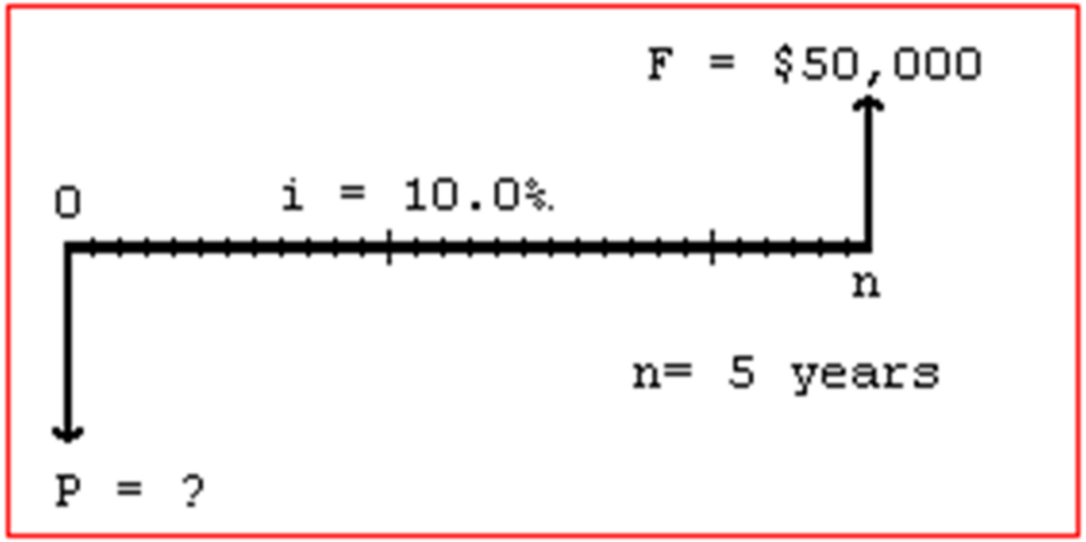
Solution

Spreadsheet := FV(8%,10,,–5000) displays $10,794.62



Example: Finding Present Value

A small company wants to make a single deposit now to have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is closest to:



Class Participation 22: Finding Present Value

A small company wants to make a single deposit now to have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is closest to:

P/A and *A/P* Factors

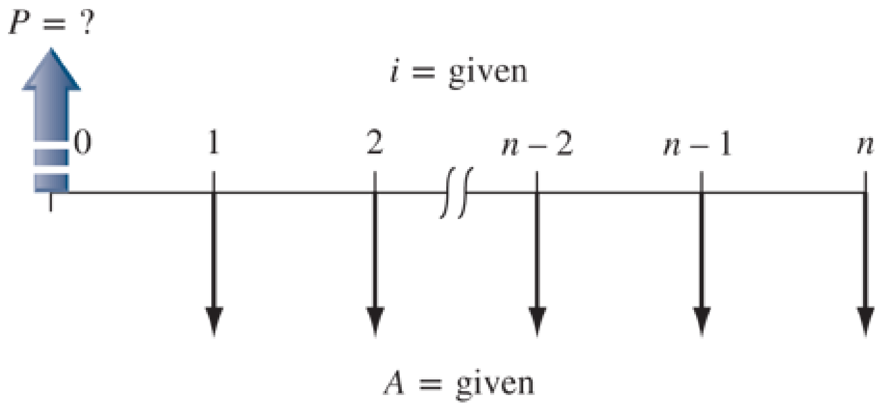
Uniform Series Involving P/A and A/P Factors

Uniform series factors that involve an ***A* series and *P*** assume that:

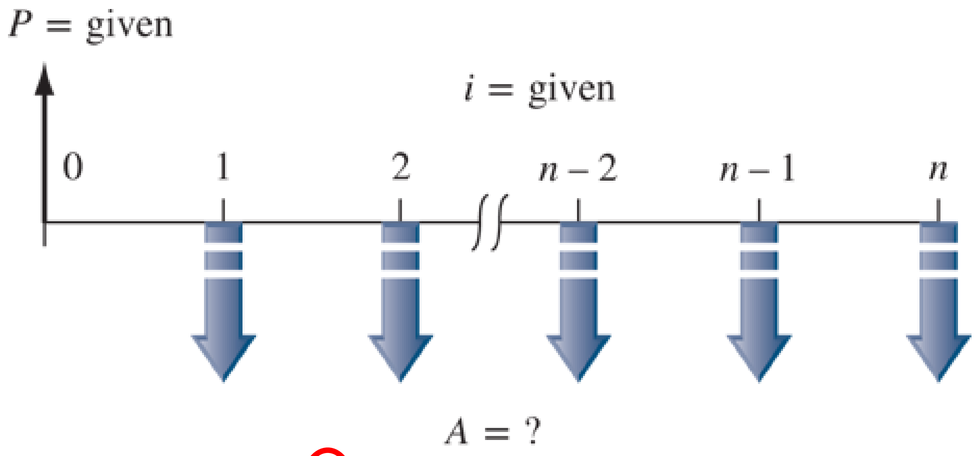
(1) Cash flows occur in ***consecutive*** periods starting ***one year after P***

(2) Cash flow amount is ***same*** in each period

The equivalent present worth *P* of a uniform series *A* of end-of-period cash flows (investments) is shown in the following figures. An expression for the present worth can be determined by considering each **A value as a future worth *F***, calculating its present worth with the *P/F* factor, and summing the results.



(*a*) *P*, given a uniform series *A*.



(*b*) *A*, given a present worth *P*.

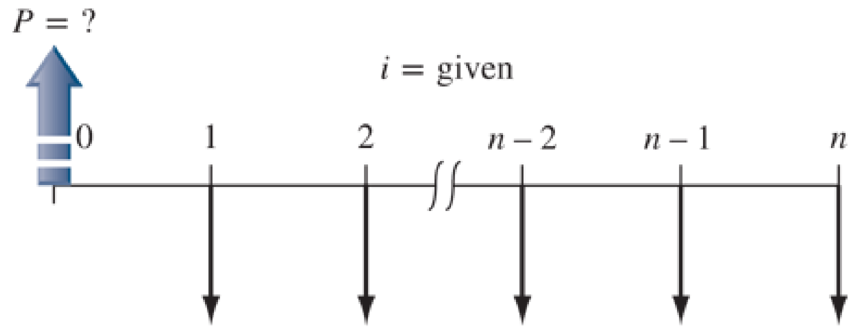
?

IMPORTANT: *P* is always one period *<u>ahead</u>* of first *A* value

Uniform Series Involving P/A and A/P Factors

$$P = A\left[\frac{1}{(1+i)^1}\right] + A\left[\frac{1}{(1+i)^2}\right] + A\left[\frac{1}{(1+i)^3}\right] + \dots$$
$$+A\left[\frac{1}{(1+i)^{n-1}}\right] + A\left[\frac{1}{(1+i)^n}\right]$$

$$P = A\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \dots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right]$$



(a) P, given a uniform series A.

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad i \neq 0$$

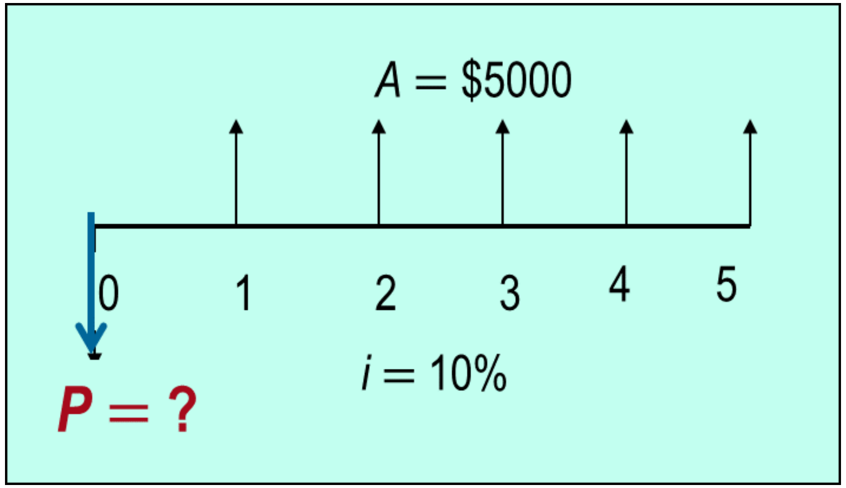
$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

TABLE 2–2 *P/A* and *A/P* Factors: Notation and Equations

Factor Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Spreadsheet Function
$(P/A,i,n)$	Uniform series present worth	P/A	$\frac{(1+i)^n - 1}{i(1+i)^n}$	$P = A\,(P/A,i,n)$	$= \mathrm{PV}\,(i\%,n,A)$
$(A/P,i,n)$	Capital recovery	A/P	$\frac{i(1+i)^n}{(1+i)^n - 1}$	$A = P\,(A/P,i,n)$	$= \mathrm{PMT}\,(i\%,n,P)$

Example: Uniform Series Involving *P/A*

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, the company would earn an extra $5000 per year. At an interest rate of 10% per year, how much could the company **afford to spend *now*** to just break even over a 5 year project period?



Solution

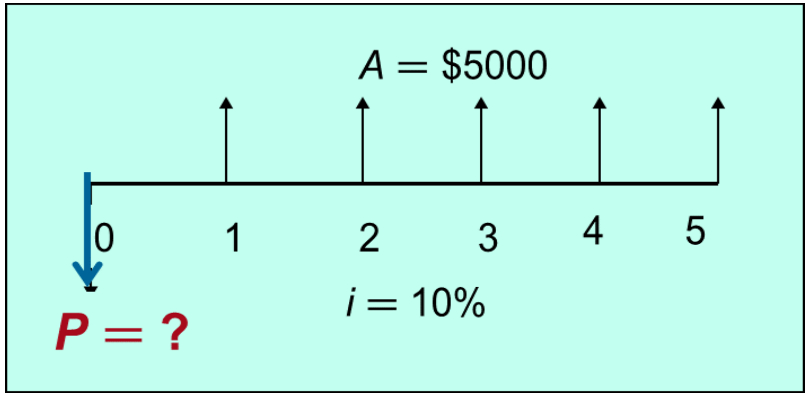
$$P = 5000(P/A,10\%,5)$$
$$= 5000(3.7908)$$
$$= \$18,954$$

10%	TABLE 15 Discrete Cash Flow: Compound Interest Factors							10%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.1000	0.9091	1.00000	1.0000	1.10000	0.9091		
2	1.2100	0.8264	0.47619	2.1000	0.57619	1.7355	0.8264	0.4762
3	1.3310	0.7513	0.30211	3.3100	0.40211	2.4869	2.3291	0.9366
4	1.4641	0.6830	0.21547	4.6410	0.31547	3.1699	4.3781	1.3812
5	1.6105	0.6209	0.16380	6.1051	0.26380	$\boxed{3.7908}$	6.8618	1.8101

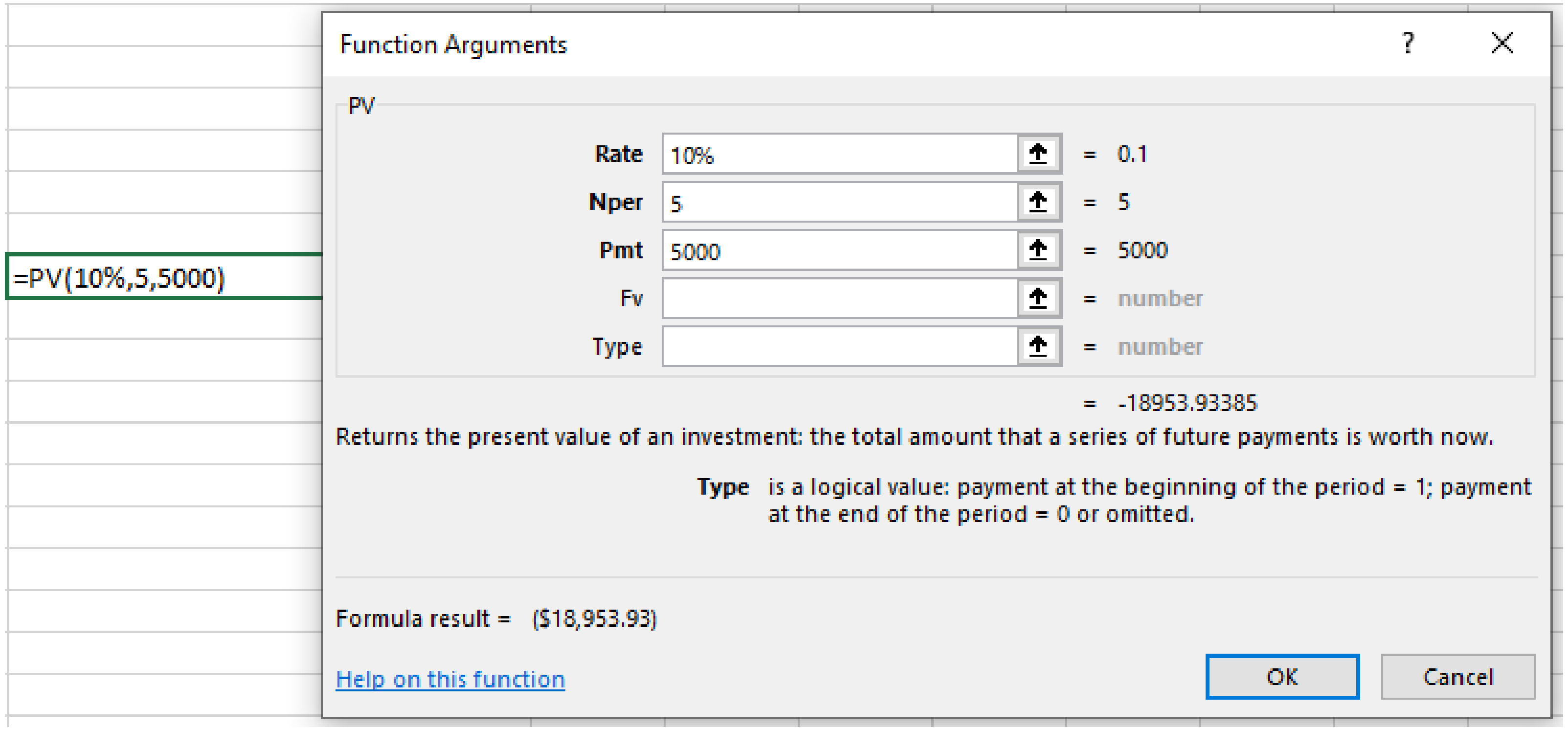
$$(P/A10\%,5) = \frac{(1+i)^n - 1}{i(1+i)^n} = \frac{(1.10)^5 - 1}{0.10(1.10)^5} = 3.7908$$

Example: Uniform Series Involving *P/A*

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, the company would earn an extra $5000 per year. At an interest rate of 10% per year, how much could the company **afford to spend *now*** to just break even over a 5 year project period?



Spreadsheet: = PV(10%, 5, 5000) displays – $18,953.93



Class Participation 23: Uniform Series Involving P/A

How much money should you be willing to pay now for a guaranteed \$600 per year for 9 years starting next year, at a rate of return of 8% per year?

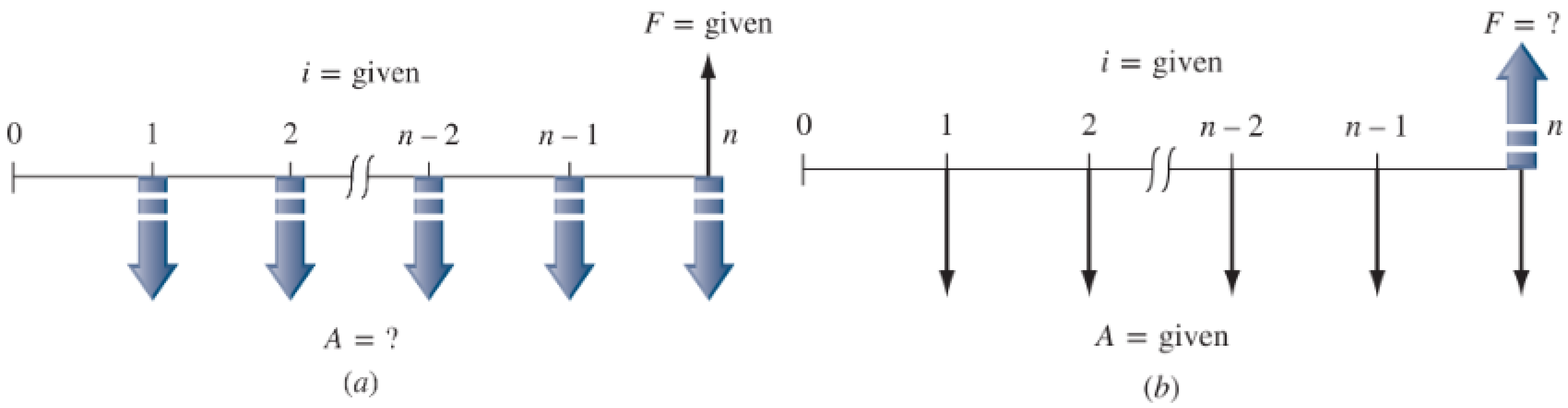
F/A and A/F Factors

Uniform Series Involving *F*/*A* and *A*/*F* Factors

Uniform series factors involving an ***A* series** **and *F*** assume that:

(1) Cash flows occur in ***consecutive*** periods ending in ***same*** period as *F*

(2) Cash flow amount is ***same*** in each period



Cash flow diagrams to (*a*) find *A*, given *F*, and (*b*) find *F*, given *A*.

IMPORTANT: *F* always occurs in *same* period as last *A*

Uniform Series Involving *F*/*A* and *A*/*F* Factors

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

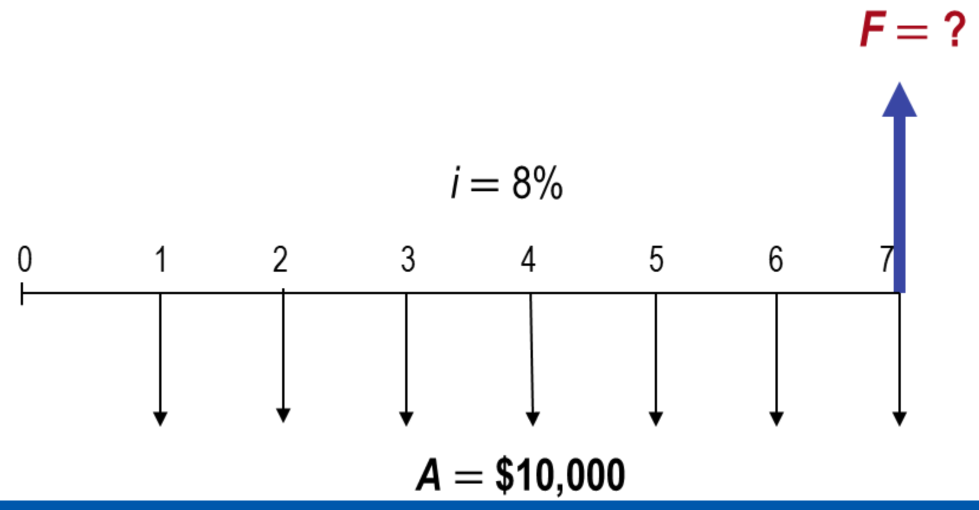
$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

TABLE 2–3 *F/A* and *A/F* Factors: Notation and Equations

Factor Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Spreadsheet Functions
$(F/A,i,n)$	Uniform series compound amount	F/A	$\frac{(1+i)^n - 1}{i}$	$F = A\,(F/A,i,n)$	$= \mathrm{FV}\,(i\%,n,A)$
$(A/F,i,n)$	Sinking fund	A/F	$\frac{i}{(1+i)^n - 1}$	$A = F\,(A/F,i,n)$	$= \mathrm{PMT}\,(i\%,n,F)$

Example: Uniform Series Involving *F*/*A* (1)

An industrial engineer made a modification to a chip manufacturing process that will save the employer $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?



Solution

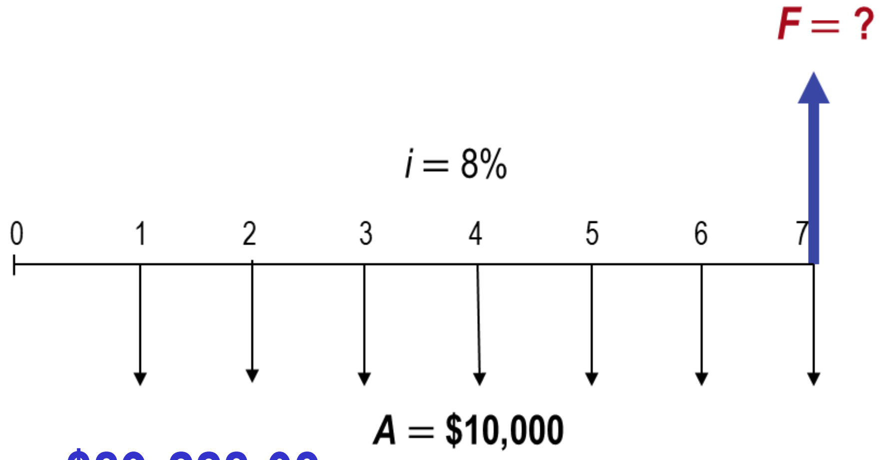
$$F = 10{,}000(F/A,8\%,7)$$
$$= 10{,}000(8.9228)$$
$$= \$89{,}228$$

$$(F/A8\%,7) = \frac{(1+i)^n - 1}{i} = \frac{(1.08)^7 - 1}{0.08} = 8.9228$$

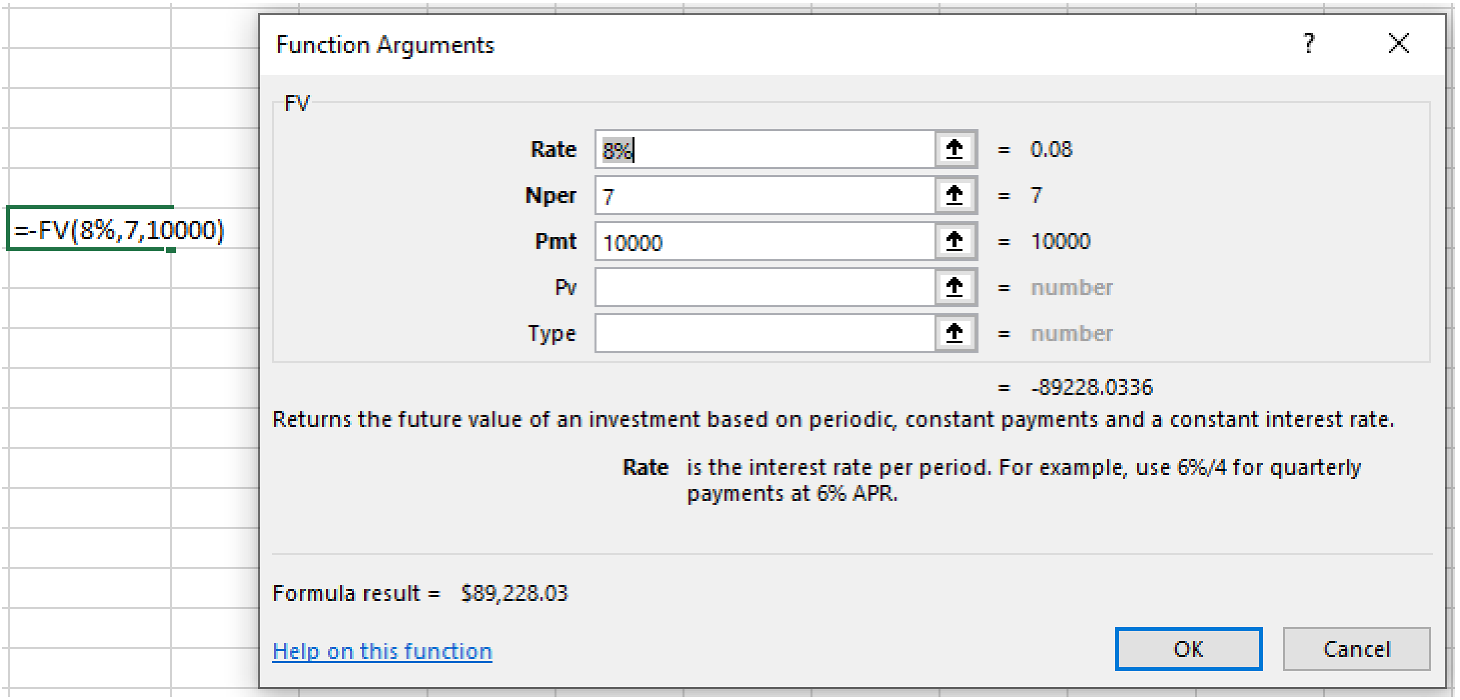
8%	TABLE 13 Discrete Cash Flow: Compound Interest Factors							8%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0800	0.9259	1.00000	1.0000	1.08000	0.9259		
2	1.1664	0.8573	0.48077	2.0800	0.56077	1.7833	0.8573	0.4808
3	1.2597	0.7938	0.30803	3.2464	0.38803	2.5771	2.4450	0.9487
4	1.3605	0.7350	0.22192	4.5061	0.30192	3.3121	4.6501	1.4040
5	1.4693	0.6806	0.17046	5.8666	0.25046	3.9927	7.3724	1.8465
6	1.5869	0.6302	0.13632	7.3359	0.21632	4.6229	10.5233	2.2763
7	1.7138	0.5835	0.11207	8.9228	0.19207	5.2064	14.0242	2.6937
8	1.8509	0.5403	0.09401	10.6366	0.17401	5.7466	17.8061	3.0985
9	1.9990	0.5002	0.08008	12.4876	0.16008	6.2469	21.8081	3.4910
10	2.1589	0.4632	0.06903	14.4866	0.14903	6.7101	25.9768	3.8713

Example: Uniform Series Involving F/A (1)

An industrial engineer made a modification to a chip manufacturing process that will save the employer \$10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

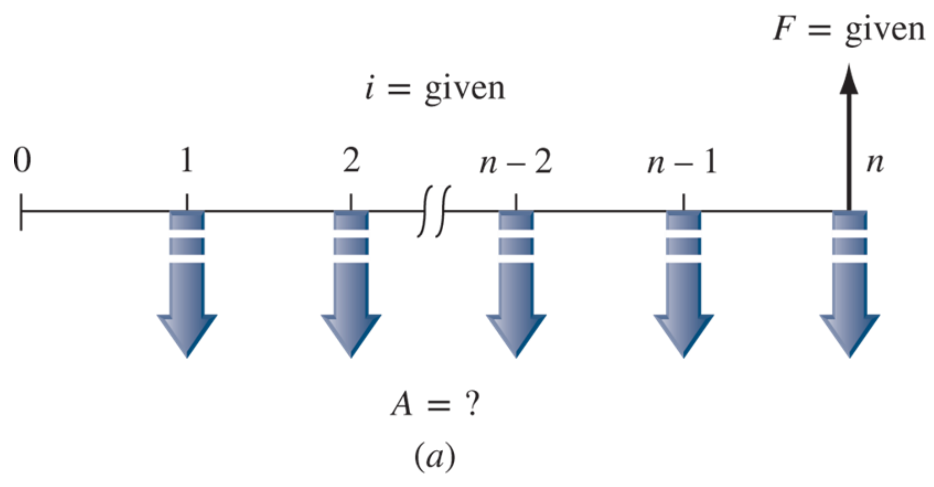


Spreadsheet : $= -FV(8\%, 7, 10000)$ displays \$89,228.03



Example: Uniform Series Involving F/A (2)

A new graduate has a position with a salary of \$125,000 per year. There is a plan to purchase a condo in 5 years using a down payment of \$50,000. At an anticipated 7% per year rate of return, how much must be invested in years **1 through 5** to have the \$50,000?



(a)

Solution

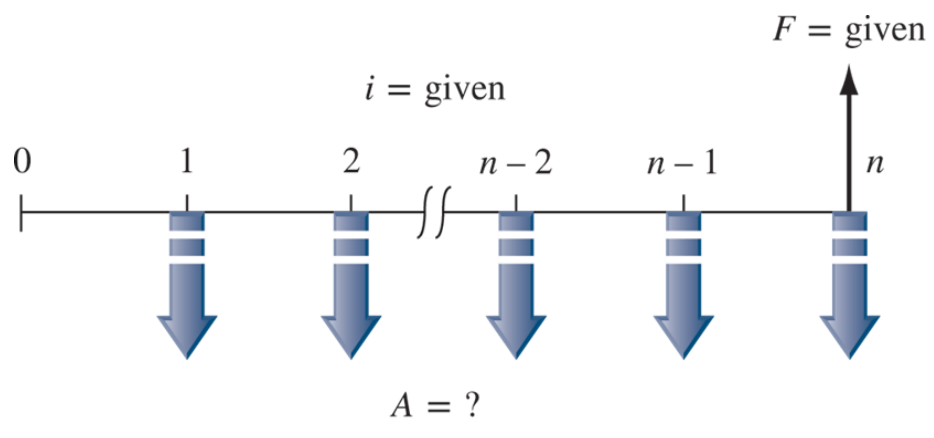
$$A = 50{,}000(A/F, 7\%, 5)$$
$$= 50{,}000(0.17389)$$
$$= \$8{,}694.50 \text{ per year}$$

7%	TABLE 12 Discrete Cash Flow: Compound Interest Factors							7%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0700	0.9346	1.00000	1.0000	1.07000	0.9346		
2	1.1449	0.8734	0.48309	2.0700	0.55309	1.8080	0.8734	0.4831
3	1.2250	0.8163	0.31105	3.2149	0.38105	2.6243	2.5060	0.9549
4	1.3108	0.7629	0.22523	4.4399	0.29523	3.3872	4.7947	1.4155
5	1.4026	0.7130	0.17389	5.7507	0.24389	4.1002	7.6467	1.8650

$$(A/F\,7\%, 5) = \frac{i}{(1+i)^n - 1} = \frac{0.07}{(1.07)^5 - 1} = 0.17389$$

Example: Uniform Series Involving F/A (2)

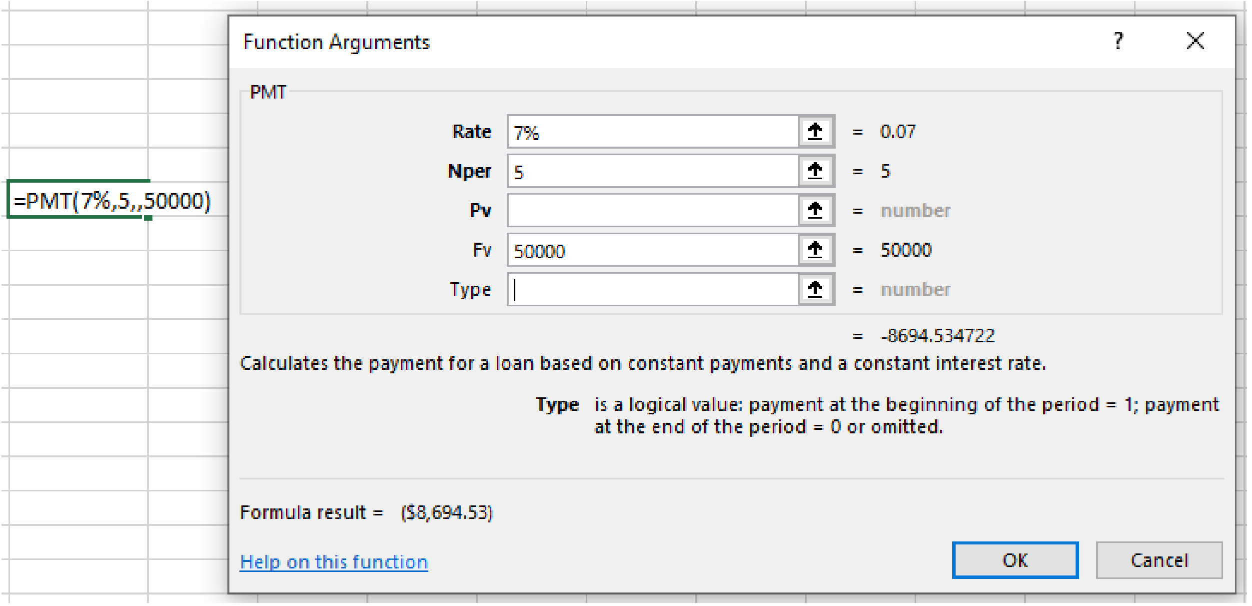
A new graduate has a position with a salary of $125,000 per year. There is a plan to purchase a condo in 5 years using a down payment of $50,000. At an anticipated 7% per year rate of return, how much must be invested in years **1 through 5** to have the $50,000?



(a)

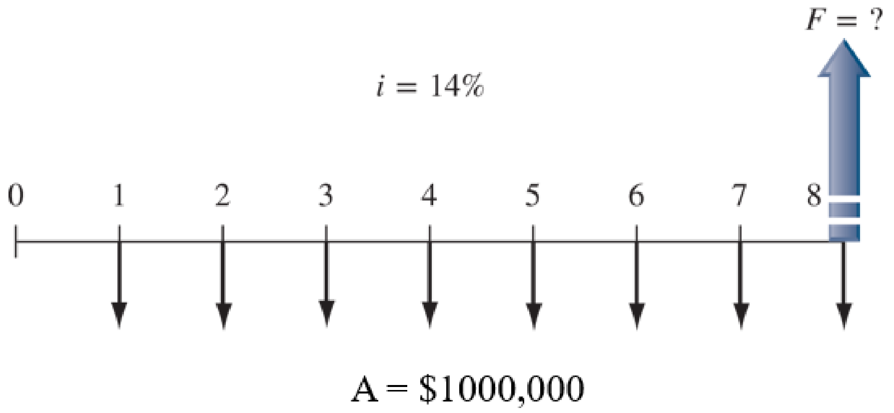
Solution

Spreadsheet: = PMT(7%,5,,50000) displays – $8,694.53



Class Participation 24: Uniform Series Involving *F/A*

A vice president of Tesla Inc. an American electric vehicle (EV) and clean energy company, wants to know the equivalent future worth of a $1 million capital investment each year for 8 years, starting 1 year from now. Tesla capital earns at a rate of 14% per year.



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