ENGR580 Sample Project

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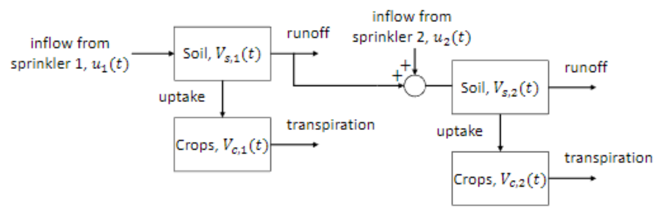
***Abstract*—ENGR580 Sample Project**

***Keywords*—sample, project**

I. System Representation

A. State Space Representation

A state space representation of the compartmental system is designed, figure 1.



The system can be presented as, (Handout#2, 2.1):

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$where\ A = \begin{bmatrix} -Q_r - Q_u & 0 & 0 & 0 \\ Q_u & -Q_t & 0 & 0 \\ \frac{1}{4}Q_r & 0 & -Q_r - Q_u & 0 \\ 0 & 0 & Q_u & -Q_t \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$C = [0.2Q_r \quad 0 \quad 0.015Q_r \quad 0]$$
$$D = [0 \quad 0]$$

B. System limitations

This system is only a rough estimation of the system, where the water in the soil and crop and not actually measured. Also, for a real system the flow of water is positive, e.g. the variables in x are no smaller than zero. Other influences like precipitation are not considered in this model. Moreover, the real system is not restricted linear as explained in the linear model. Possible improvements can be covert the system in a green house, so that the environment for the system is well controlled, so that the estimation can be more precise. Alternatively, we can divide the system into many smaller subsets, and present each subset as a linear system. In this way, the system can more precisely present the non-uniformity of the system (Handout#2, 2.2).

The eigenvalues are on the diagonal of the matrix since it is a lower regular matrix. The eigenvalues are $[-Q_r - Q_u, -Q_r - Q_{u,} \ -Q_{t,} \ -Q_t]$, or in numerical values [-0.0094, -0.0094, -0.0042, -0.0042]. (Handout#2, 2.3)

C. Stability

Since all the eigenvalues for this LTI are smaller than zeros, the system is both internally stable and exponentially stable.

The Jordan form for the matrix A is: (Handout#2, 2.4)

$$A = \begin{bmatrix} -Q_t & 0 & 0 & 0 \\ 0 & -Q_r - Q_u & 1 & 0 \\ 0 & 0 & -Q_r - Q_u & 0 \\ 0 & 0 & 0 & -Q_t \end{bmatrix}$$

or numerically

$$\begin{bmatrix} -0.0042 & 0 & 0 & 0 \\ 0 & -0.0094 & 1 & 0 \\ 0 & 0 & -0.0094 & 0 \\ 0 & 0 & 0 & -0.0042 \end{bmatrix}$$

Let $P = \int_0^\infty e^{A^T t} Q e^{At} dt, then\ A^T P + P A = -Q$. Given all eigenvalues of A are negative, the P is finite. And because of the symmetric definition of P, P is symmetric. Therefore, the system is stable in sense of Lyapunov. (Handout#2, 2.5)

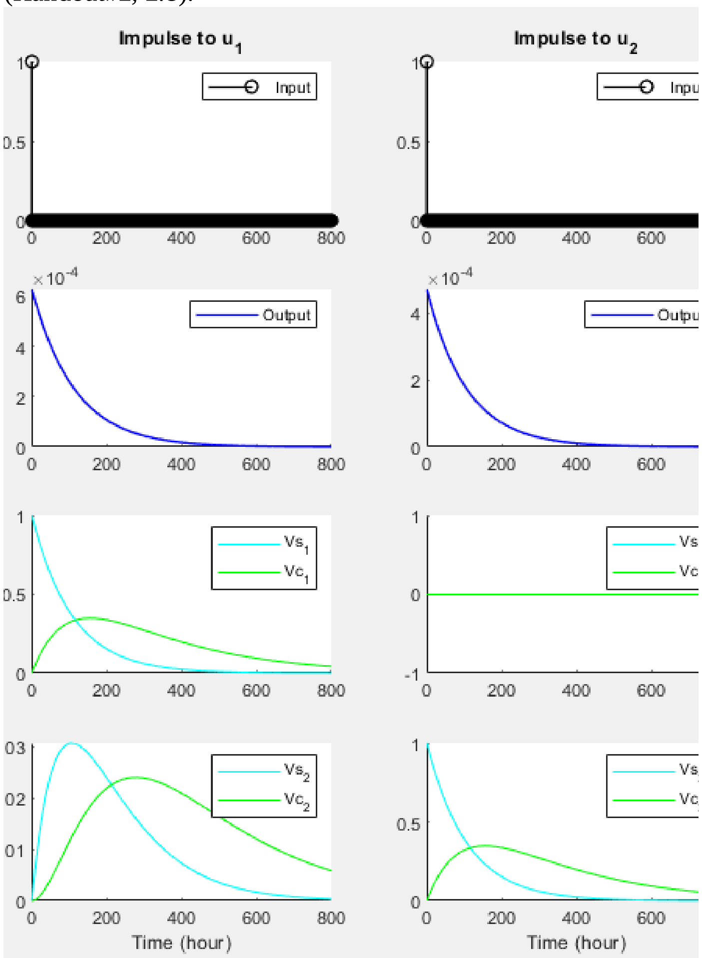
Because all the eigenvalues of the system are negative, the poles of transfer function will be negative. Thus, the system is BIBO stable. (Handout#2, 2.6)

D. Equilibrium

Set $\dot{x} = Ax + Bu = 0, and\ x = [60000,90000,60000,90000]^T, u = [565, 517.9]$ (Handout#2, 2.7)

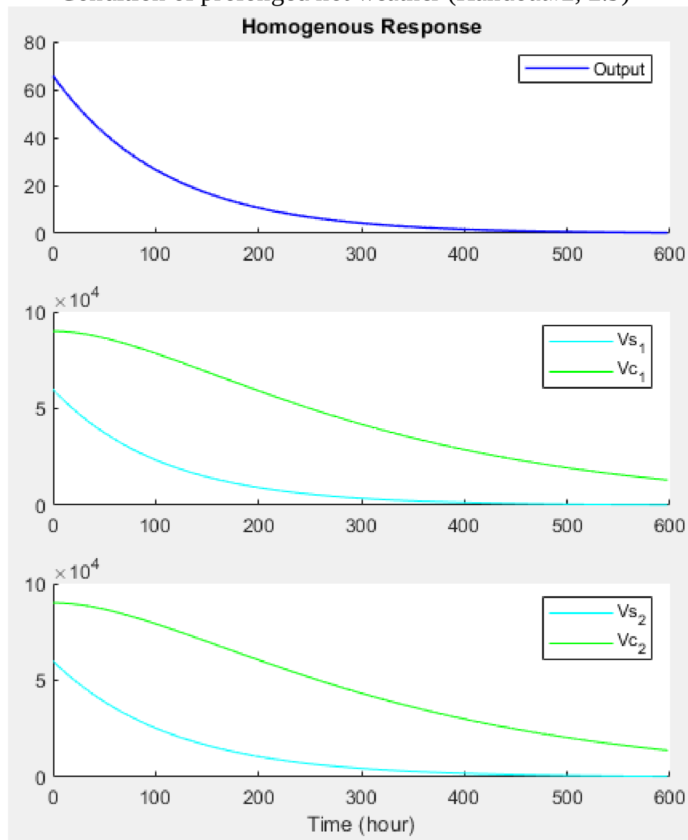
E. *Impulse Response*

Impulse response from Sprinkler 1 and Sprinkler 2 (Handout#2, 2.8):



F. *Homogeneous Response*

Condition of prolonged hot weather (Handout#2, 2.9)

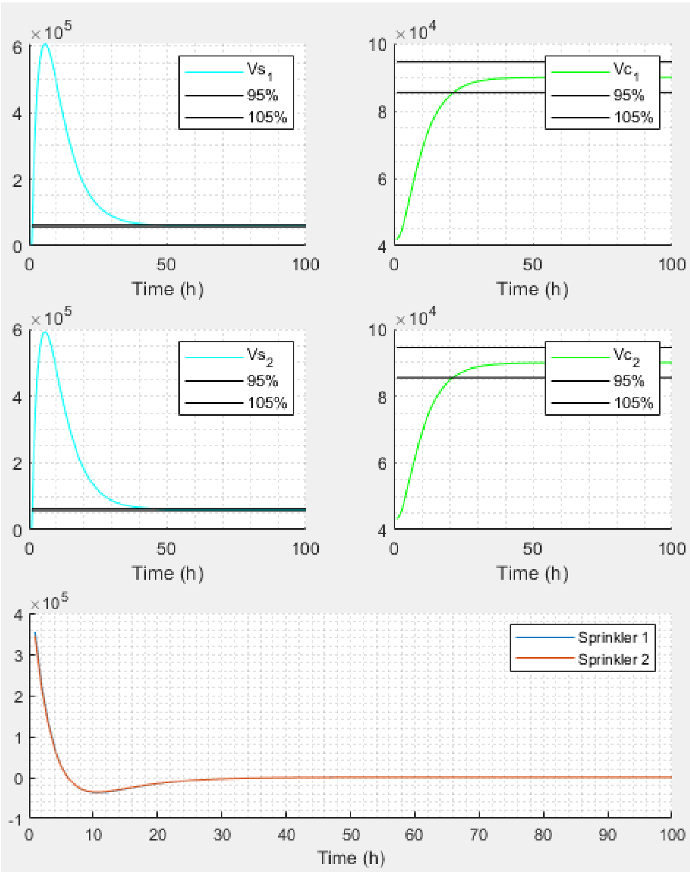


G. *Controlability*

The controllability matrix is C=[B AB A2B A3B], which is rank 4. So, the system is controllable. (Handout#3, 1.1)

H. *Feedback controller*

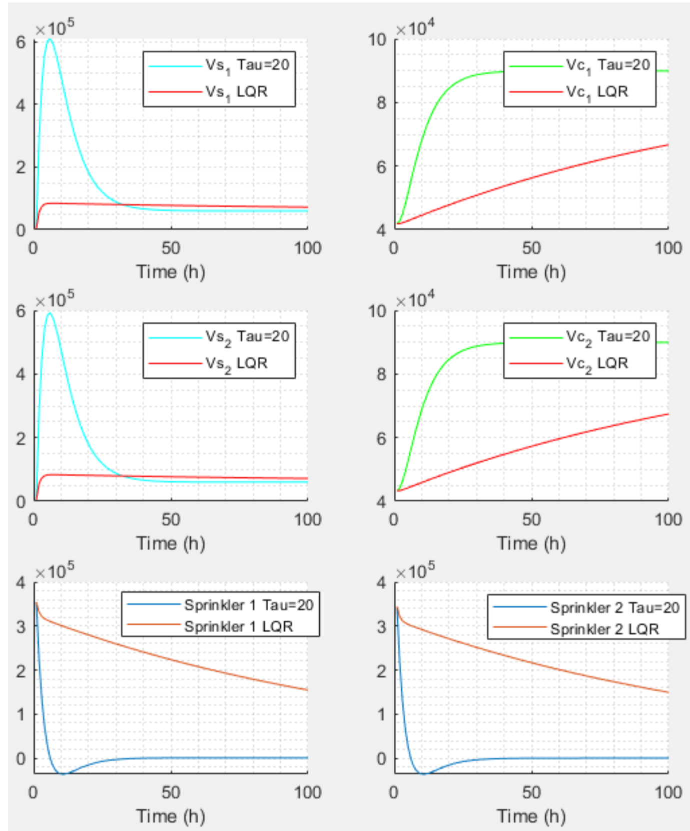
Feedback controller is designed for the system where the stable time (±5%) is within 48h. The initial condition was selected as it dried without water input for 300 hours from equilibrium state. (Handout#3, 1.2). The water in Vs1 was flooded 10 times the equilibrium value. The land is now a pool and crops flow on top.



An LQR controller was designed where Q and R has equal weight.

The eigenvalues are [-1±0.0004i, -0.0075, -0.0075]. The absolute values of eigenvalues are much smaller than the previous designed controller, the stable time is much longer. (Handout#3, 1.3)

I. *Broken Actuator*



If sprinkler 2 is broken, the B matrix would be $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. (Handout#3, 1.4)

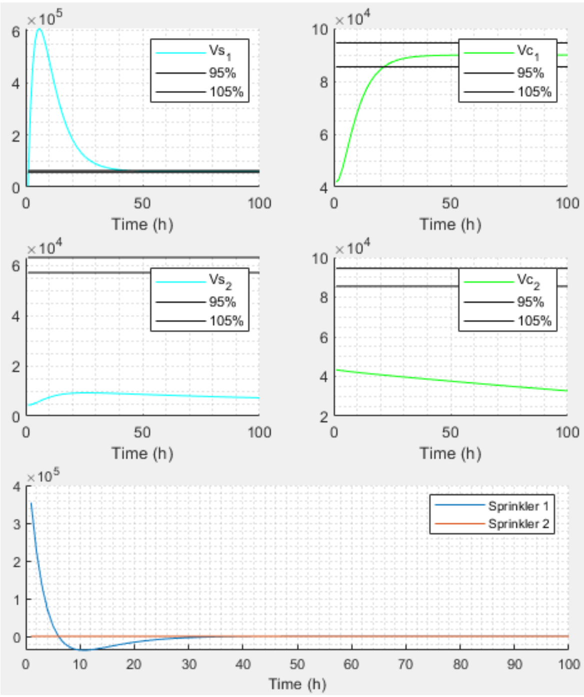
The rank of controllability matrix becomes 3, the system is not controllable. The matrix V that converting A to Jordan form can convert broken B into

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$where\ A = \begin{bmatrix} -Q_t & 0 & 0 & 0 \\ 0 & -Q_r - Q_u & 1 & 0 \\ 0 & 0 & -Q_r - Q_u & 0 \\ 0 & 0 & 0 & -Q_t \end{bmatrix}$$

The last row of A is not controlled. But since $-Q_t < 0$, the system is still stabilizable. (Handout#3, 1.5)

Simulation if sprinkler 2 is broken: (Handout#3, 1.6)



It is impossible to design a controller to use only one sprinkle to spray two lands. When we investigate the input required at equilibrium, $u = [565, 517.9]$. the input of the second system is the same as the first around 565L, yet within which only 47L, less than 10%, is supplied form the first system. Therefore, it is impossible to use this flow to fully cover the need of the second system without harming the crops in the first system. Repairing the sprinkler 2 is a better solution. (Handout#3, 1.6).

J. *Observability*

The observability matrix has rank 2, so the system is not observable. (Handout#4, 1.1)

Two more sensors is required so that the system can be observable. Basically, the water in crops in the two system need to be measured. (Handout#4, 1.2)

The Kalman decomposition of the system is:

$$A = \begin{bmatrix} -0.0094 & -0.0008 & 0 & 0 \\ 0 & -0.0094 & 0 & 0 \\ -0.0058 & 0.7071 & -0.0042 & 0 \\ 0.7071 & 0.0058 & 0 & -0.0042 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0016 & -1.4142 \\ -1.4142 & 0.0016 \\ 0^{+} & 0^{+} \\ 0^{-} & 0^{+} \end{bmatrix}$$

$$C = \begin{bmatrix} -0.0003 & 0.0004 & 0 & 0 \end{bmatrix}$$

The last 2 columns are unobservable, where $0^{\pm}$ in B indicates a very small number. (Handout#4, 1.3)