

10.1

- b) None of the obtained filters are linear phase.
- c) We can observe that different filters have different parameters to characterize them. Hence, based on requirement we can choose one of these filters. In a general sense, we usually go with Butterworth or equiripple.
- f) We know that, cut off for the filters is designed in the way that it lets f_1 to pass through and attenuates frequency f_2 , here we can see that $f_1 = 500\text{Hz}$, $f_2 = 3\text{kHz}$ & $f_c = 2\text{kHz}$ as $f_1 < f_c < f_2$ holds. Hence output only depends upon $\sin 2\pi f_1 t$ & independent of $\sin 2\pi f_2 t$.

10.2a) Given that, $H(z) = b_0 (1 - e^{j\omega_0} z^{-1})(1 - e^{-j\omega_0} z^{-1})$

$$\Rightarrow H(z) = b_0 (1 - z^{-1}(e^{j\omega_0} + e^{-j\omega_0}) + z^{-2})$$

We know that, $\omega_0 = \pi$ & $H(1) = 1$

$$\Rightarrow H(1) = b_0 (1 - (2\cos\omega_0)z^{-1} + z^{-2})$$

$$\Rightarrow b_0 (2 - 2\cos\omega_0) = 1$$

$$\Rightarrow b_0 = \frac{1}{2(1 - \cos\omega_0)}$$

$$\Rightarrow b_0 = \frac{2 + \sqrt{2}}{6} = 1.7071$$

b) Given that $H(z) = \frac{b_0(1 - e^{j\omega_0} z^{-1})(1 - e^{-j\omega_0} z^{-1})}{1 + (1 - \alpha_0^2) z^{-1}(1 - \alpha_0^2 z^{-1})}$

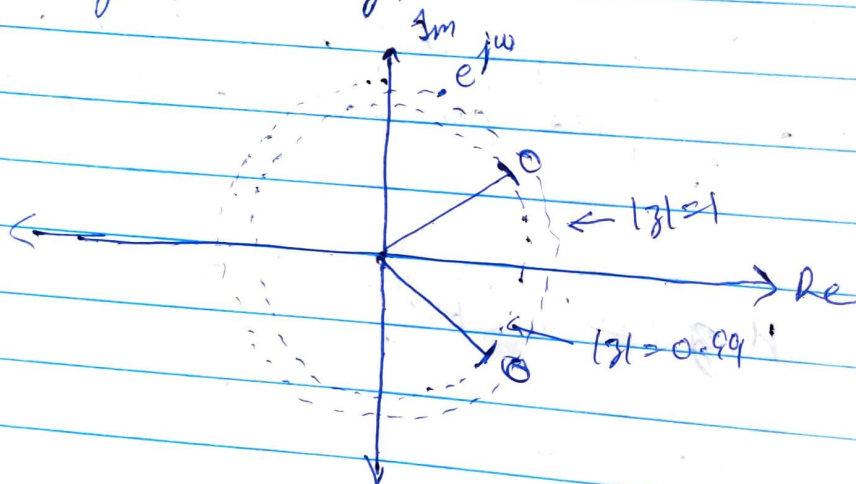
\Rightarrow Given that $H(1) = 1$, $\omega_0 = \frac{\pi}{4}$ & $\alpha_0 = 0.99$

$\Rightarrow H(1) = \frac{b_0(1 - 2\cos\omega_0 + 1)}{1 - 2\alpha_0\cos\omega_0 + \alpha_0^2}$

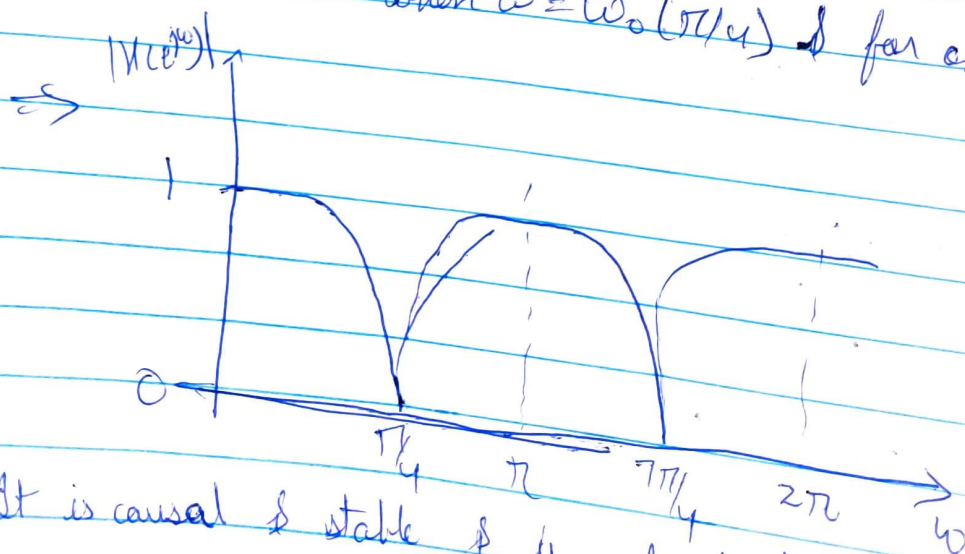
$\Rightarrow b_0 = \frac{1 + \alpha_0^2 - 2\alpha_0\cos\omega_0}{2(1 - \cos\omega_0)}$

$\Rightarrow b_0 = 0.9170$

By visualizing geometrically,



$\Rightarrow H(e^{j\omega}) = 0$ when $\omega = \omega_0(\pi/4)$ & for other values it ≈ 1



It is causal & stable & is a band stop filter that attenuates freq $\approx \pi/4$

c) On reducing α , the phase plot becomes non linear i.e. it moves away from ideal behaviour.
On changing α from 0.99 to 0.5, the ideal nature of filter is lost.

e) For the given sound, given samples are 73113 & sampling rate is 8192 Hz.
 \Rightarrow duration of signal is 8.9249 seconds.
When we add the sinusoid with freq 1024 Hz, a sharp beep distortion is obtained.

$$\text{Now, } F = \frac{f_c}{f_s} \Rightarrow \frac{1024}{8192} = \frac{1}{8} \text{ Hz}$$

$$\& \Rightarrow \omega_F = 2\pi F = \frac{\pi}{4}$$

This is equal to cut off freq. for above notch filter, hence the distortion is removed.