HW7 - Z-transforms

Wednesday, April 13, 2022 12:13 AM

HW 7

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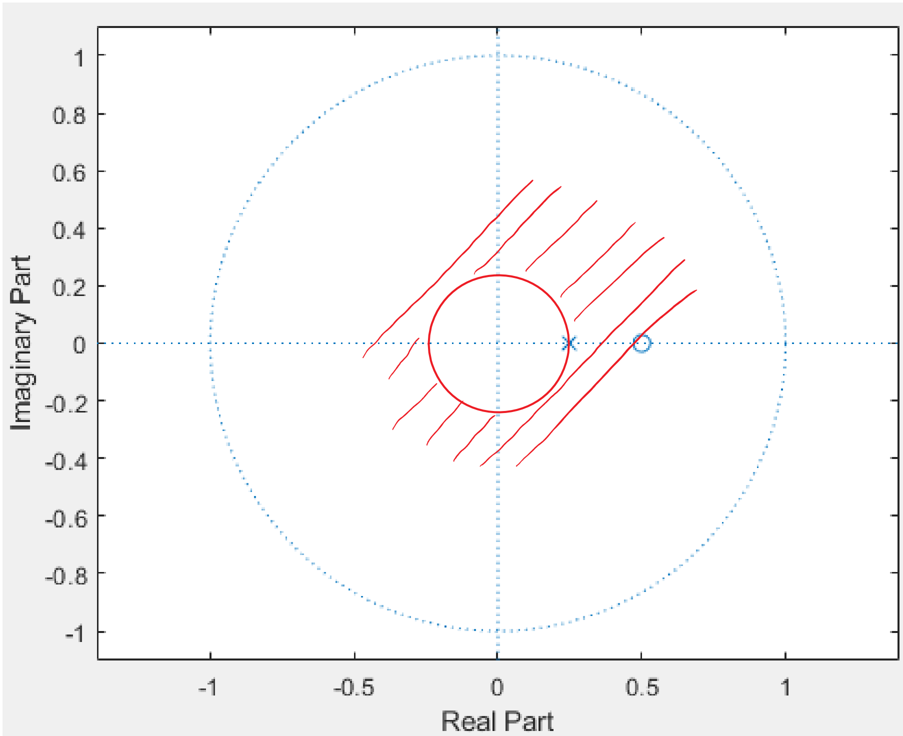
Problem 4-1

A system has input $x[n] = \frac{1}{2}^n u[n]$ and output $y[n] = \frac{1}{4}^n u[n-1]$.

(a) Plot $H(z)$, including region of convergence.

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \qquad y(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-1}}$$



$|z| > \frac{1}{4}$

(b) Find $h[n]$.

$$h[n] = \mathcal{Z}^{-1}(H(z)) = 2\delta[n] - \frac{1}{4}^{n}$$

(c) Find the difference equation that relates the output to the input.

$$y[n] - \frac{1}{4}y[n-1] = x[n] - \frac{1}{2}x[n-1]$$

Problem 4-2

For each of the systems shown in **Figure 4.33**, the ROC is not specified. Check (√) for each of the following statements that is *always* true.

Statement	a	b	c	d	e	f
The system is FIR.			✓			✓
The system is IIR.	✓	✓		✓	✓	
The system is (or could be) causal.	✓	✓		✓		✓
The system is (or could be) stable.	✓	✓	✓	✓	✓	✓
If the system is causal, it is stable.		✓		✓		✓
If the system is stable, it is causal.				✓		✓

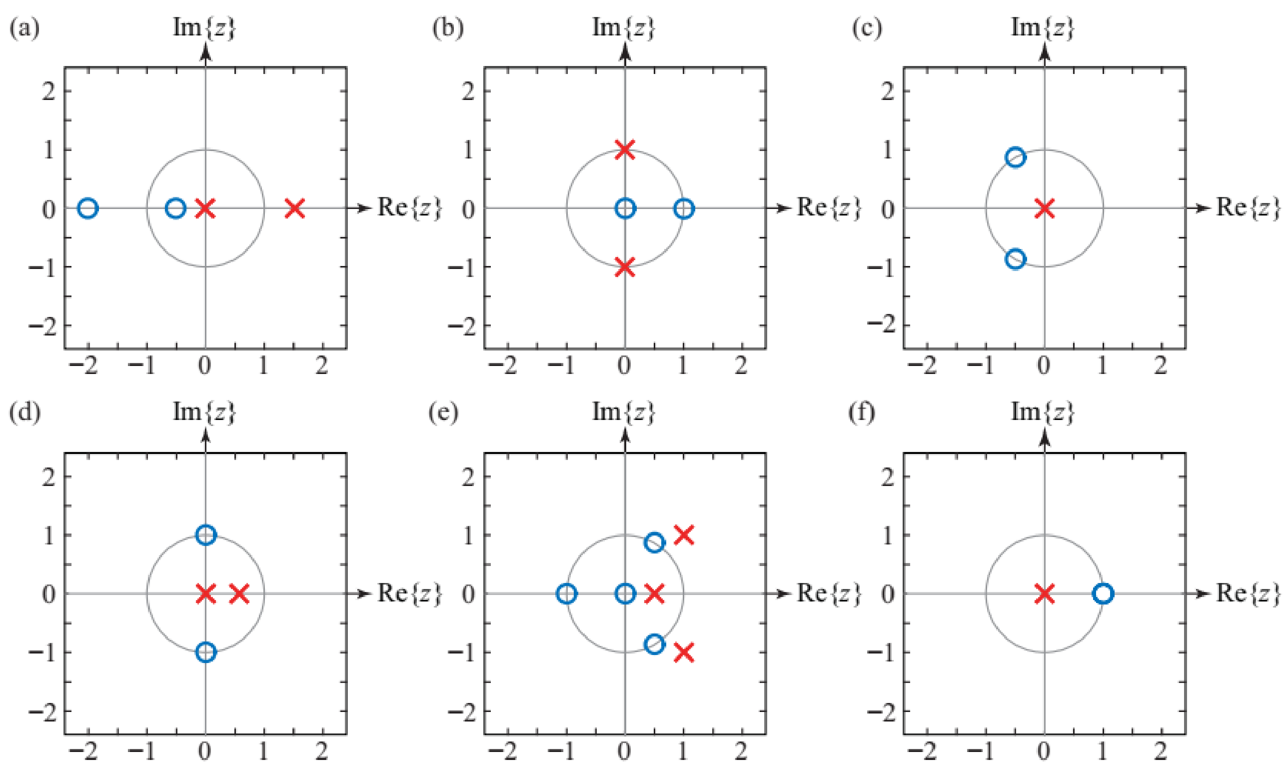


Figure 4.33

Problem 4-3

The stable system shown in **Figure 4.34** has $h_1[n] = \frac{1}{6}^n u[n]$, $h_2[n] = \frac{1}{6}\delta[n-1]$ and $h_3[n] = \frac{2}{3}^n u[n]$.

(a) Find $H(z)$, including the region of convergence.

(b) Find the difference equation that relates the output $y[n]$ to the input $x[n]$.

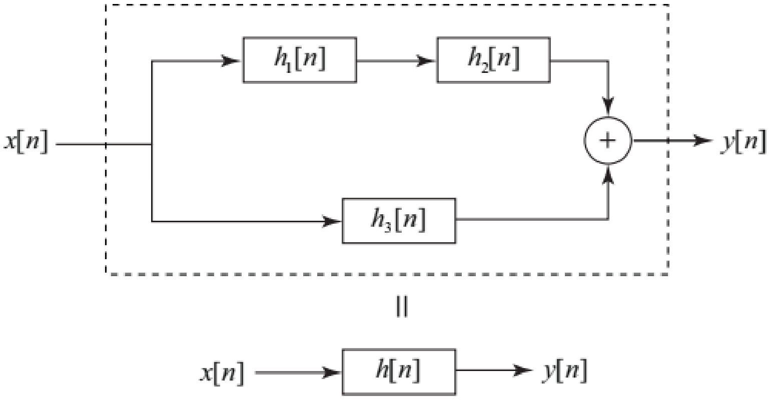


Figure 4.34

$$H_1(z) = \frac{1}{1 - \frac{1}{6}z^{-1}}$$

$$H_2(z) = \frac{z^{-1}}{6}$$

$$H_3(z) = \frac{1}{1 - \frac{2}{3}z^{-1}}$$

$$|z| > |\alpha|$$

$$H_1(z) \cdot H_2(z) = \frac{z^{-1}}{(1 - \frac{1}{6}z^{-1})6}$$

$$+ H_3(z) = \frac{z^{-1}}{(1 - \frac{1}{6}z^{-1})6} + \frac{1}{1 - \frac{2}{3}z^{-1}} \qquad |z| > \frac{2}{3}$$

$$= -\frac{2(z+3)(z-3)}{(3 - 2z^{-1})(6 - z^{-1})} \qquad |z| > \frac{2}{3}$$

$$= -\frac{18 - 2z^{-2}}{18 - 15z^{-1} + 2z^{-2}}$$

$$18y[n] - 15y[n-1] + 2y[n-2] = -18x[n] - 2x[n-2]$$

Problem 4-5

When the input to a system is $x[n] = (1 - \frac{1}{2}^n)u[n]$, the output of the system is $y[n] = \frac{1}{4}^n u[n]$.

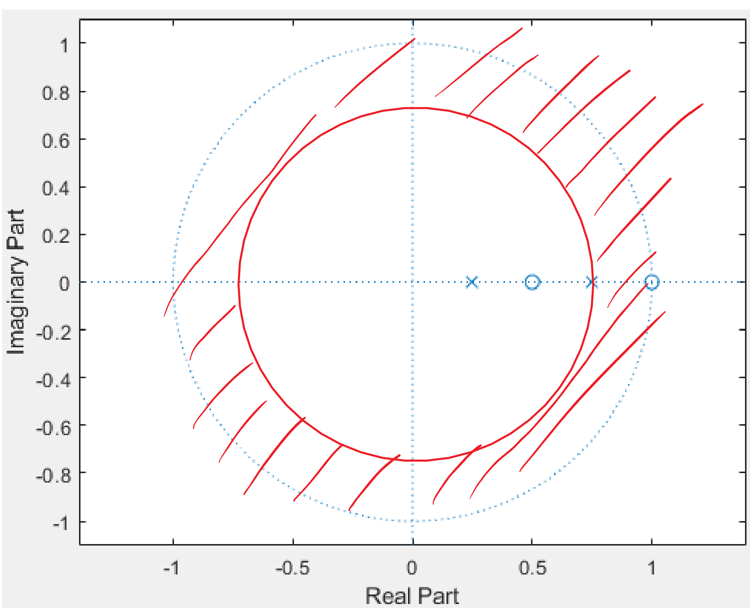
(a) Plot the pole-zero plot for this system, with ROC.

$$X(z) = \frac{1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-1} + \frac{z}{z-\frac{1}{2}} = \frac{2z^2 - \frac{3}{2}z}{(z-1)(z-\frac{1}{2})}$$

$$= \frac{2-\frac{3}{2}z^{-1}}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}$$

$$Y(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$H(z) = \quad X(z) \quad \frac{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}{(2-\frac{3}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = \frac{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}{2 - 2z^{-1} + \frac{3}{8}z^{-2}}$$



$|z| > \frac{3}{2}$

(b) Find the LCCDE of this system.

$$2y[n] - 2y[n-1] + \frac{3}{8}y[n-2] = x[n] - \frac{3}{8}x[n-1] + \frac{1}{2}x[n-2]$$

(c) Find the impulse response $h[n]$.

$$h[n] = Z^{-1}(H(z)) = \frac{4}{3}\delta[n] + \left(-.0833\left(\frac{3}{4}\right)^n - \frac{3}{4}\left(\frac{1}{4}\right)^n\right)u[n]$$

Problem 4-6

Given the LCCDE $y[n] - \frac{1}{4}y[n-2] = x[n] - \frac{1}{2}x[n-2]$,

(a) find $h[n]$.

$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{1 - \frac{1}{4}z^{-2}}$$

$$h[n] = 2\delta[n] + \left(-\frac{1}{2}\left(\frac{1}{2}\right)^n \quad -\frac{1}{2}\left(-\frac{1}{2}\right)^n\right)u[n]$$

(b) find the input $x[n]$ that makes the output $y[n] = \frac{1}{2}^n u[n]$.

$$X(z) \cdot \frac{1 - \frac{1}{2}z^{-2}}{1 - \frac{1}{4}z^{-2}} = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$(\quad)\left(1 - \frac{1}{2}z^{-1}\right) = \left(1 - \frac{1}{4}z^{-2}\right)$$

$$\frac{1 - \frac{1}{4}z^{-2}}{1 - \frac{1}{2}z^{-1}} = 1 + \frac{1}{2}z^{-1}$$

$$X(z) = \frac{1 - \frac{1}{2}z^{-2}}{1 + \frac{1}{2}z^{-1}}$$

$$x[n] = 2\delta[n] - \delta[n] - \left(-\tfrac{1}{2}\right)^n u[n]$$

$$= \delta[n] - \left(-\tfrac{1}{2}\right)^n u[n]$$

Problem 4-7

Given a system with

$$H(z) = \frac{2 + 2z^{-2}}{1 - 1.5z^{-1} - z^{-2}},$$

(a) find $h[n]$ given that the system is causal.

$$-2\delta[n] + 2 \cdot 2^n u[n] + 2 \cdot \left(-\tfrac{1}{2}\right)^n u[n]$$

(b) find $h[n]$ given that the system is stable.

$$-2\delta[n] + 2 \cdot 2^n u[-n-1] + 2 \cdot \left(-\tfrac{1}{2}\right)^n u[n]$$

Problem 4-8

A stable system is shown in **Figure 4.35**, comprising a cascade of two discrete-time filters such that $z[n] = x[n]$.

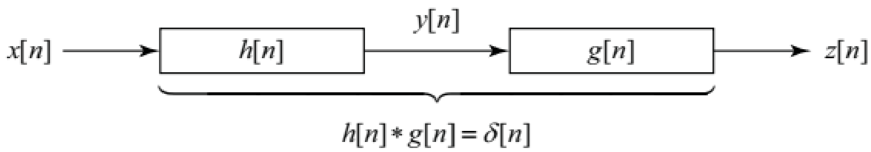


Figure 4.35

The first filter has unknown impulse response $h[n]$. The second filter is defined by the difference equation

$$z[n] = y[n] - \tfrac{3}{4}y[n-1] + \tfrac{1}{8}y[n-2].$$

Find $h[n]$.

$$G(z) = 1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}$$

$$H(z) = \frac{1}{G(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{2}{2 - \frac{1}{2}} + \frac{-1}{2 - \frac{1}{4}}$$

$$h[n] = 2 \cdot \left(\tfrac{1}{2}\right)^n u[n] - \left(\tfrac{1}{4}\right)^n u[n]$$

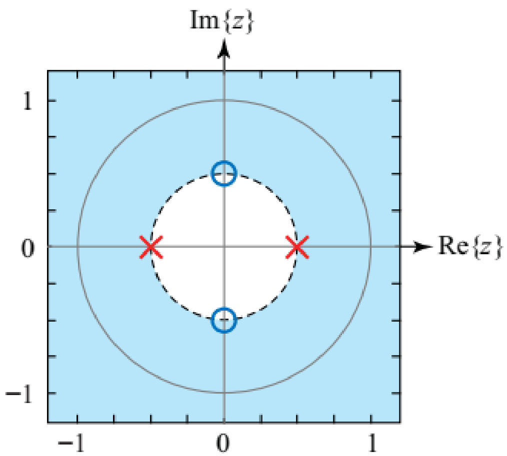
DON'T DO Q9

Problem 4-9 DON'T DO Q9

A stable system comprises a cascade of two discrete-time filters, as shown in **Figure 4.35**. The first filter, with impulse response $h[n]$, is defined by the pole-zero plot for $H(z)$ shown in **Figure 4.36**, along with the information that $h[0] = 2$.

(a) Find $h[n]$.

(b) Find the difference equation of a second system, characterized by $g[n]$, such that $z[n] = x[n]$.



DON'T DO Q9

DON'T DO Q9

DON'T DO Q9

Figure 4.36

Problem 4-10

Repeat Problem 4-9 with $H(z)$ shown in **Figure 4.37**.

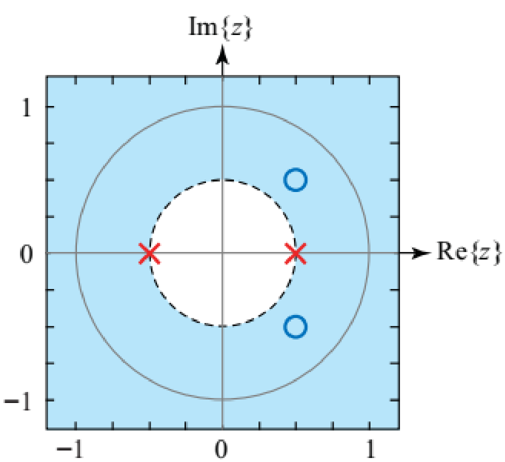


Figure 4.37

$$H(z) = A \cdot \frac{\left(z - \frac{1}{2} + j\frac{1}{2}\right)\left(z - \frac{1}{2} - j\frac{1}{2}\right)}{\left(z - \frac{1}{2}\right)\left(z + \frac{1}{2}\right)}$$

$$= A \cdot \frac{z^2 - z + \frac{1}{2}}{z^2 - \frac{1}{4}} = A \cdot \frac{1 - z^{-1} + \frac{1}{2} z^{-2}}{1 - \frac{1}{4} z^{-2}}$$

$$h[0] = \lim \frac{1 - z^{-1} + \frac{1}{2} z^{-2}}{\quad} \rightarrow A \cdot \frac{1 - 0 - 0}{\quad} = 2 \quad \therefore A = 2$$

$$h[0] = \lim_{z\to\infty} \frac{1 - z^{-1} + \frac{1}{2}z^{-2}}{1 - \frac{1}{4}z^{-2}} \rightarrow A \cdot \frac{1-0-0}{1-0} = 2 \quad \therefore A = 2$$

$H(z) =$

$$\frac{2\left(z^2 - z + \frac{1}{2}\right)}{z^2 - \frac{1}{4}} = \frac{2z^2 - 2z + 1}{z^2 - \frac{1}{4}} = -8z - 4 + \frac{10}{z - \frac{1}{4}}$$

$$h[n] = -8\delta[n] - 4\delta[n-1] + 10 \cdot \left(\tfrac{1}{4}\right)^n u[n]$$

$$G(z) = \frac{1}{H(z)} = \frac{2z^2 - 2z + 1}{z^2 - \frac{1}{4}} = \frac{Z(z)}{Y(z)}$$

$$\frac{2 - 2z^{-1} + z^{-2}}{1 - \frac{1}{4}z^{-2}}$$

$$z[n] - \frac{1}{4}z[n-2] = 2y[n] - 2y[n-1] + y[n-2]$$

Problem 4-23

Given a system $H(z)$, whose pole-zero plot is shown in **Figure 4.41**, find how many responses of the given type exist.

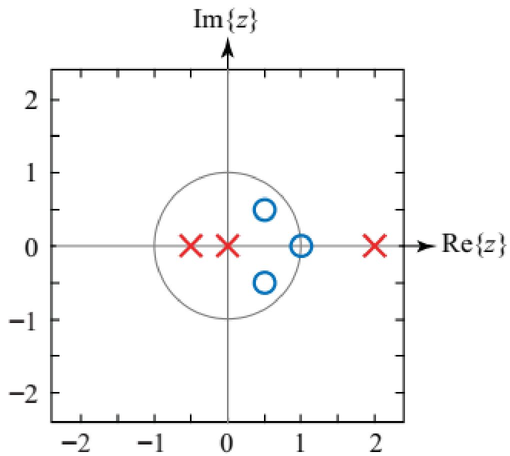


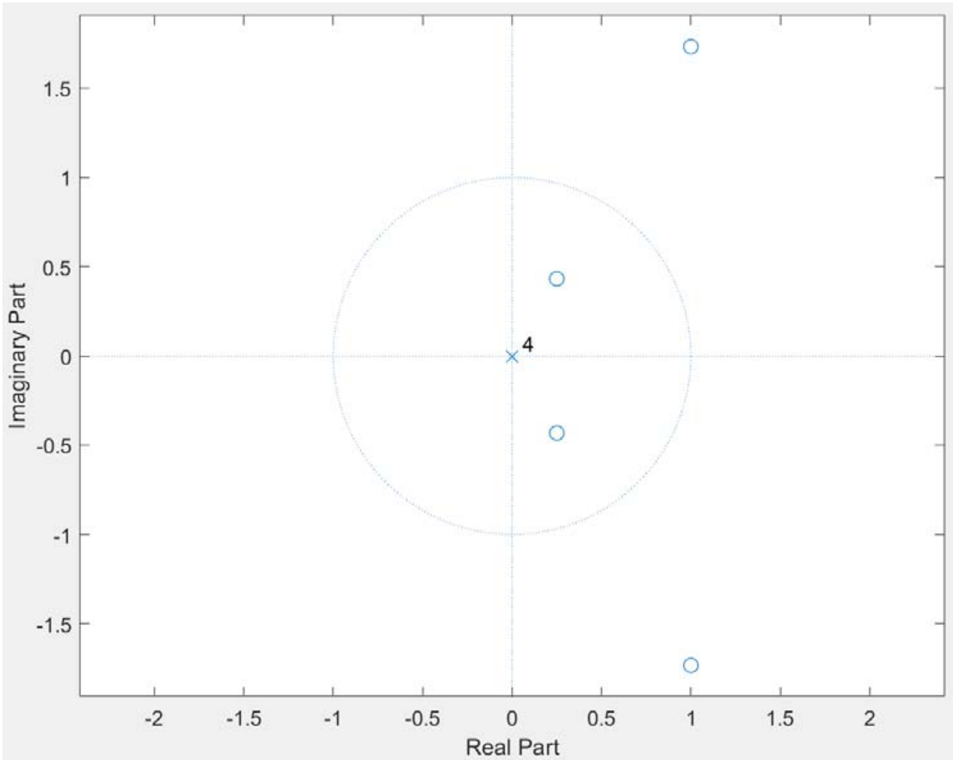
Figure 4.41

Right-sided	Left-sided	Two-sided	Stable	Non-stable	How many?
X			X		0
X				X	1
	X		X		0
	X			X	1
		X	X		1
		X		X	1

Problem 4-28

Given $h[n] = \delta[n] - \frac{5}{2}\delta[n-1] + \frac{21}{4}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-4]$, find $H(z)$ and plot the pole-zero plot.

$$H(z) = 1 - \frac{5}{2}z^{-1} + \frac{21}{4}z^{-2} - \frac{5}{2}z^{-3} + z^{-4}$$



1.5
1
0.5
0
-0.5
-1
-1.5
Imaginary Part
4
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Real Part