ECE 3640 - Discrete-Time Signals and Systems
FINAL EXAM - SPRING 2022

Name: Zach Wilcox	Due: Friday, 29 April 2022 at 12 AM (midnight).

Instructions:

1. You may use: your text book, your homework, your notes, or resources available from the course web page.
2. Do not talk to anyone about this exam or get help from any source on this exam.
3. Write your answers in the spaces provided.
4. By signing in the name box above, you verify that you have complied with these instructions.

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1. The continuous-time signal $x(t) = \exp(j2\pi F_0 t)$, where $F_0 = 29655$ Hz, is sampled at a rate of $F_s = 900$ samples per second. What is the frequency $f_0 \in [-\frac{1}{2}, +\frac{1}{2})$ (in cycles/sample) of the sampled sequence $x[n]$?

$f_0 =$ −0.05 cycles/sample

2. A continuous-time signal $x(t)$ is reconstructed from a discrete-time sequence $x[n] = \cos(2\pi f_0 n)$, where $f_0 =$ 0.0833 cycles/sample, assuming a reconstruction rate of $F_s = 72,000$ sample/second. What is the frequency F_0 (in Hz) of the reconstructed signal?

$F_0 =$ 5,997.6 Hz

3. What is the minimum sampling rate F_s (in samples/second) for signal $x(t) = \frac{\sin(21352t)}{t}$ if aliasing is to be avoided?

$F_s =$ 6797 samples/second

4. A signal $x(t)$ with highest frequency 7832 Hz is going to be sampled at a rate of 8820 samples/second. To avoid aliasing, the signal $x(t)$ is filtered prior to sampling through an ideal low pass filter having frequency response

$$H(F) = \begin{cases} 1, & |F| \le F_0, \\ 0, & |F| > F_0. \end{cases}$$

What is the largest frequency F_0 (in Hz) that will prevent aliasing in the sampled signal?

$F_0 =$ 4410 Hz

5. The sample rate of a real discrete-time signal $x[n]$ with highest frequency 0.4 cycles/samples to be adjusted by a rational factor by processing as shown below.

$$x[n] \longrightarrow \boxed{\uparrow U} \longrightarrow \boxed{\text{LPF}} \longrightarrow \boxed{\downarrow D} \longrightarrow y[n]$$

What upsampling and downsampling factors U and D will produce a sampled signal that occupies the full bandwidth from $-1/2$ to $+1/2$? Use the smallest possible values of U and D.

$U =$ 4

$D =$ 5

P1:

$F_0 = \frac{29655 \text{ cycles}}{1 \text{ sec}}$ $\qquad \frac{F_0}{F_s} = 32.95 \frac{\text{cycles}}{\text{sample}}$

$F_s = \frac{900 \text{ samples}}{1 \text{ sec}}$ $\qquad -33 = -0.05$

P2:

$F_0 = ? \frac{\text{cycles}}{1 \text{ sec}}$

$f_0 = \frac{.0833 \text{ cycles}}{\text{sample}}$ $\qquad f_0 \cdot F_s = 5997.6 \text{ Hz}$

$F_s = \frac{72000 \text{ samples}}{1 \text{ sec}}$

P3:

$\frac{21352}{2\pi} = 3398.28$

$\times \quad 2$

6796.55

$\therefore F_s \geq 6796.55$

$F_s = 6797$

P4:

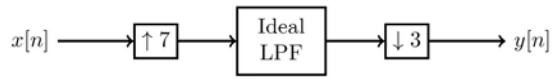
$F_{max} \leq \frac{F_s}{2} \therefore 4410$

P5:

$.4 \frac{D}{U} = .5$

$\frac{D}{U} = 1.25 = \frac{5}{4}$

6. If an ideal low pass filter is used in the system below, what is the cut-off frequency $f_{\text{cut-off}}$ (in cycles/sample) for the filter?



$f_{\text{cut-off}} =$ 0.0714 cycles/sample

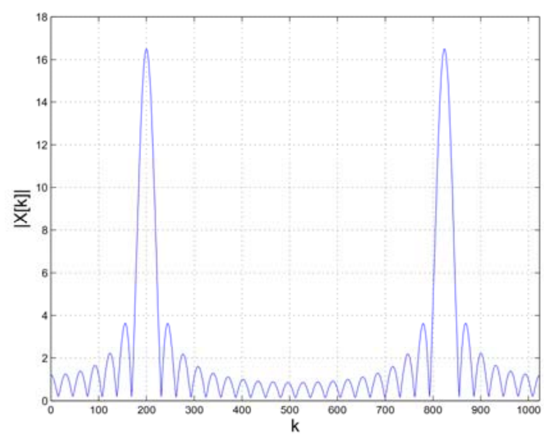
7. If the signal $x[n]$ is sampled at $F_{s,\text{input}} = 8000$ sample/second, what is the sample rate $F_{s,\text{output}}$ (in samples/second) of $y[n]$?

$F_{s,\text{output}} =$ 18666.667 samples/second

8. If the signal $x[n] = \cos(2\pi 0.38 n)$ is processed as shown below, then $y[n] = \cos(2\pi f_0 n)$. What is f_0 (in cycles/sample)?

$f_0 =$ .1629 cycles/sample

9. Aliasing is avoided when a real continuous-time signal $x(t)$ is sampled at the rate $F_s = 36000$ samples/second producing the discrete-time signal $x[n]$. A rectangular window of length $N = 32$ samples is applied to the sampled sequence $x[n]$ and the result is zero padded to length 1024. A 1024-point FFT is computed on the zero padded sequence and the magnitude $|X[k]|$ of the result is plotted in the figure below.



What is the frequency F_0 (in Hz) of the input signal $x(t)$?

$F_0 =$ 180 Hz

P6: $f = \min\left(\frac{1}{2 \cdot 7}, \frac{1}{2 \cdot 3}\right)$

$= \frac{1}{14} = .0714$

10. The signal $x(t) = \exp(j2\pi 35000t) + \exp(j2\pi 37000t)$ is sampled at a rate $F_s = 9000$ samples/second and an FFT is applied to $N = 16384$ samples of the signal to compute the DFT $X[k]$. In what bins k_1 and k_2 of the DFT do we expect to see peaks?

$k_1 =$ 63715.5

$k_2 =$ 67356

11. In Matlab, N-point circular convolution may be computed efficiently by `ifft(fft(x,N).*fft(h,N))`. Suppose this method is used to perform linear convolution of the signal $x[n], n = 0, 1, \cdots, 79999$ with the filter impulse response $h[n], n = 0, 1, \cdots, 80$. What is the smallest value of `N` that causes circular convolution to compute linear convolution. (Hint, this is a zero padding question.)

$N =$ 80,080

12. A finite length $L = 100$ sequence $x[n]$ has DTFT given by $X(f) = \sqrt{1 + (f/0.1)^4}$ for $-0.5 \leq f < 0.5$. A length $N = 800$ point DFT $X[k]$ is computed on $x[n]$. What are $X[197]$ and $X[711]$?

$X[197] =$ 6.146

$X[711] =$ 78.994

13. A 1024-point DFT is used to analyze a signal $x[n]$ after applying a window $w[n]$ of length $N \leq 1024$. Which of the following windows is most likely able to resolve two equal amplitude sinusoids with nearly equal frequencies? (circle one)

(a) 256-point rectangular window
(b) 512-point rectangular window
(c) 1024-point rectangular window
(d) 256-point Hann window
(e) 512-point Hann window
(f) 1024-point Hann window

14. An 1024-point DFT is used to analyze a signal $x[n]$ after applying a window $w[n]$ of length $N \leq 1024$. Which of the following windows is most likely able to resolve a small amplitude sinusoid in the presence of a large amplitude sinusoid? (circle one)

(a) 256-point rectangular window
(b) 512-point rectangular window
(c) 1024-point rectangular window
(d) 256-point Hann window
(e) 512-point Hann window
(f) 1024-point Hann window

15. The everlasting sinusoid $x[n] = 7\cos(2\pi(1/3)n + 9.7\pi)$ is input to a system with transfer function $H(z) = 1/(1 + 0.7z^{-1} + 0.9z^{-2})$. What is the output $y[n]$?

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + .7z^{-1} + .9z^{-2}} \qquad z = e^{j\frac{2\pi}{3}}$$

$$35000\left(\frac{16384}{9000}\right) = 63715.5$$

$$37000\left(\frac{16384}{9000}\right) = 67356$$

$$X\left(\frac{k}{N}\right)$$

$$= X\left(\frac{197}{800}\right) =$$

$$X\left(\frac{711}{800}\right) =$$

$y[n] = A\cos(2\pi f_0 n + \phi)$

$A =$ 3.779

$f_0 =$ 1/3

$\phi =$ -40.99

16. The everlasting complex exponential $x[n] = \exp(j2\pi(1/5)n)$ is input to an LTI system described by the difference equation $y[n] = 0.9\exp(j2\pi/5)y[n-1] + x[n]$. What is the output $y[n]$?

$y[n] = A\exp(j[2\pi f_0 n + \phi])$

$A =$ 10

$f_0 =$ 1/5

$\phi =$ 0

17. Let $H(z) = 1/(1 - z^{-1} - z^{-2})$. Let $h[n]$ be the impulse response of a causal realization of this system. What is $h[8]$?

$h[8] =$ 34

18. Let $H(z) = 1/(1 - z^{-1} - z^{-2})$. Let $h[n]$ be the impulse response of a stable realization of this system. What is $h[8]$?

$h[8] =$ 34

19. Let $H(z) = 1/(1 - z^{-1} - z^{-2})$. Let $h[n]$ be the impulse response of an anti-causal realization of this system. What is $h[-8]$?

$h[-8] =$ 13

20. Let

$$H(z) = \frac{5 + 3z^{-1} + 2z^{-2} + 5z^{-3}}{1 + 6z^{-1} + 3z^{-2} + 2z^{-3} + 5z^{-4} + z^{-5}}. \tag{1}$$

How many different regions of convergence does this $H(z)$ have?

Number of regions = 5

21. For $H(z)$ defined in the previous problem, how many of the regions of convergence lead to a stable $h[n]$.

Number of regions = 2

$$H(z) = \frac{1}{1 - .9e^{j\frac{2\pi}{5}} z^{-1}}$$

$$\left|H\left(\tfrac{2\pi}{5}\right)\right| = \left|\frac{1}{1 - .9(1)}\right| = \frac{1}{.1} = 10$$

$$\angle H\left(\tfrac{2\pi}{5}\right) = 0$$

22. Find the impulse response $h[n]$ corresponding to the causal system described by the difference equation

$$y[n] = 0.9863y[n-1] - 0.4494y[n-2] + 0.1093x[n] + 0.1942x[n-1] + 0.1093x[n-2].$$

Express your answer in the form $Ap^n \cos(\omega n + \varphi)u[n]$.

$h[n] =$ $.1093 + .188^n \cos(n - \frac{3}{4})u[n]$

23. For the system in problem 22, find the output $y[n]$ when the input is $x[n] = 0.8^n u[n]$. Assume the system is initially at rest. Where possible combine complex conjugate pole pairs to form $\cos(\omega n + \varphi)u[n]$ terms.

$y[n] =$ $\left[.8^n + .56(.67)^n \cos(\frac{3n}{4} - .99)\right]u[n]$

24. Using the Kaiser window method, design a linear phase low pass filter that meets the following specification.

- Pass band: $0 \leq f \leq f_p = 0.2$ cycles/sample, maximum ripple $A_p = 0.05$ dB
- Stop band: $f_s = 0.25 \leq f \leq 0.5$ cycles/sample, maximum ripple $A_s = 55$ dB

(a) What is the minimum filter length L that meets the specification?
(b) What are the filter order $M = L - 1$ and group delay $g = M/2$?
(c) What type (I, II, III, IV) of filter did you use?

$\beta = 5.1$
$M = 66$ $g = 33$
$L = 67$ type II

Attach a magnitude frequency response plot of the filter you designed. Include a "zoom" of the entire pass band to show that your design meets the specification in that band. Do the same for the stop band. Plot the impulse response.

25. Using the Kaiser window method, design a linear phase band stop filter that meets the following specification.

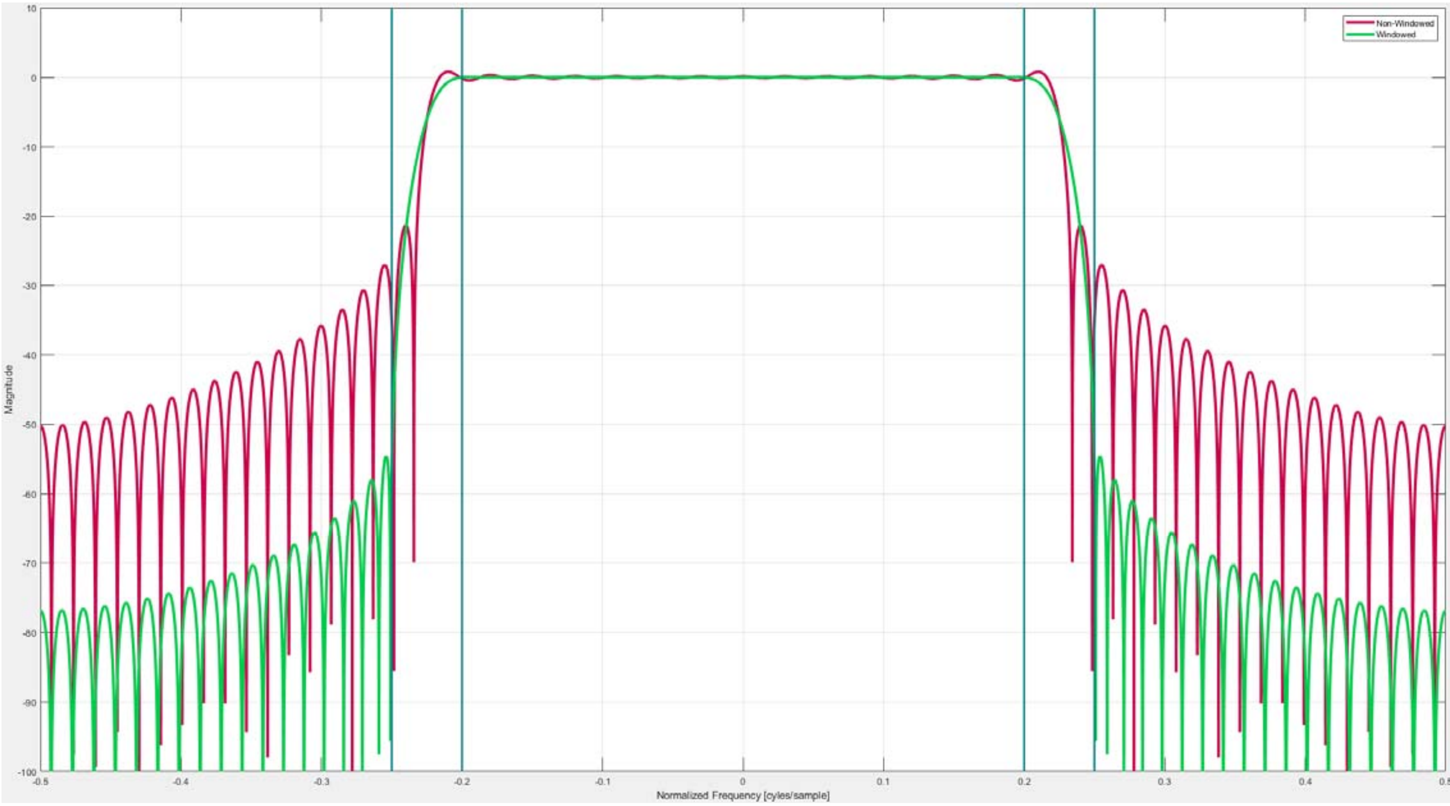
- Pass band 1: $0 \leq f \leq f_{p,1} = 0.1$ cycles/sample, maximum ripple $A_p = 0.05$ dB
- Stop band: $f_{s,1} = 0.15 \leq f \leq f_{s,2} = 0.3$ cycles/sample, maximum ripple $A_s = 75$ dB
- Pass band 2: $f_{p,2} = 0.4 \leq f \leq 0.5$ cycles/sample, maximum ripple $A_p = 0.05$ dB

(a) What is the minimum filter length L that meets the specification?
(b) What are the filter order $M = L - 1$ and group delay $g = M/2$?
(c) What type (I, II, III, IV) of filter did you use?

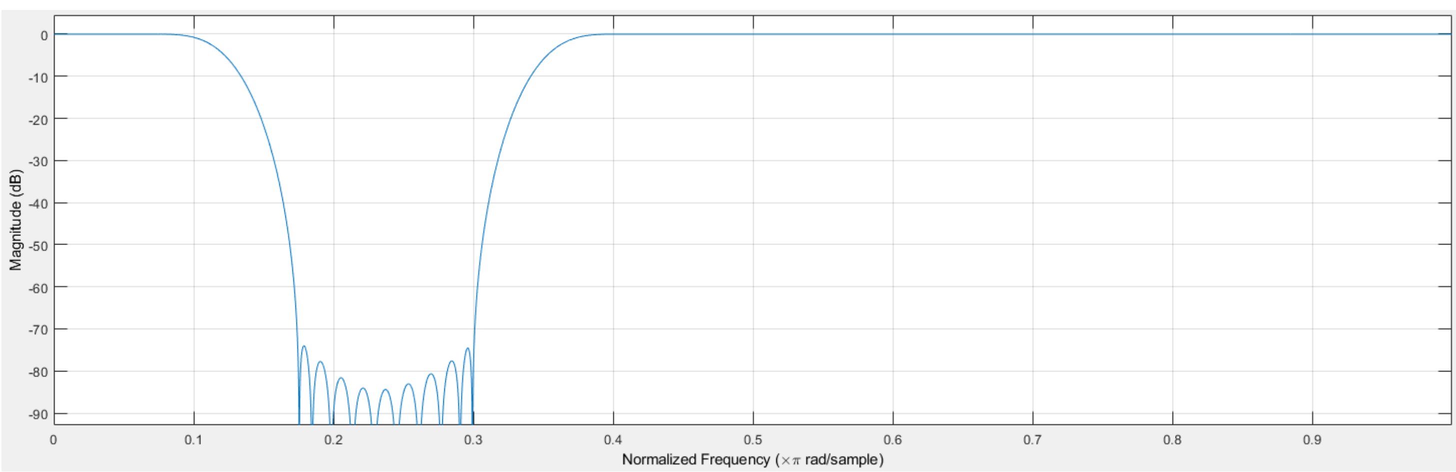
$\beta = 7.3$
$M = 94$ $g = 47$
$L = 95$ type II

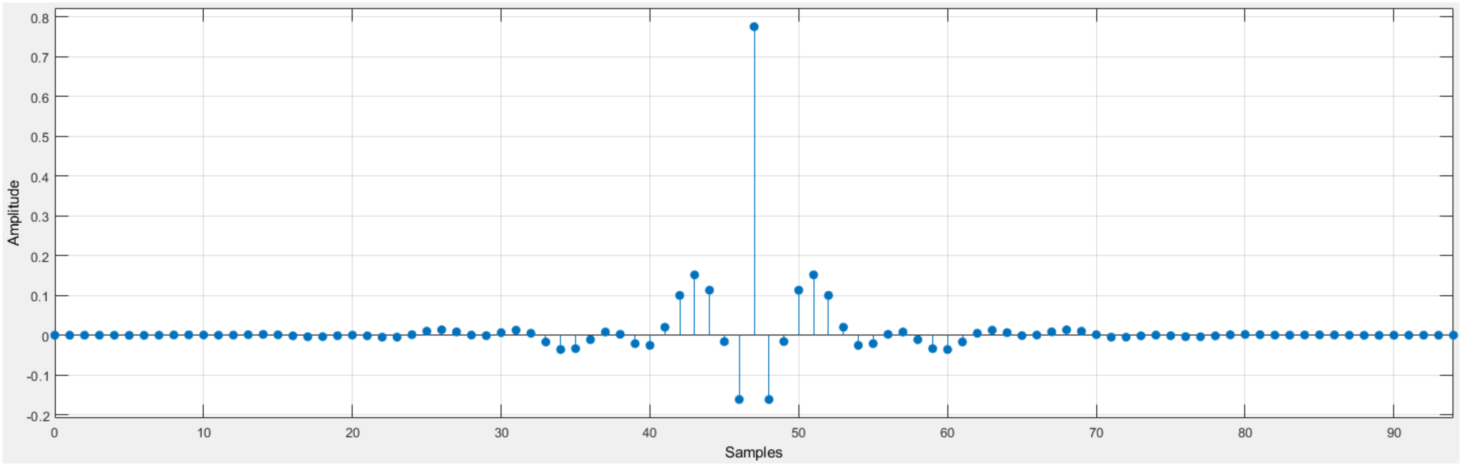
Attach a magnitude frequency response plot of the filter you designed. Include a "zoom" of the entire pass band to show that your design meets the specification in that band. Do the same for the stop band. Plot the impulse response.

Q24


Non-Windowed
Windowed
Magnitude
Normalized Frequency [cyles/sample]

Q25


Magnitude (dB)
Normalized Frequency (×π rad/sample)


Amplitude
Samples
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
10
20
30
40
50
60
70
80
90

$$H(z) = \frac{.1093 + .1942z^{-1} + .1093z^{-2}}{1 - .9863z^{-1} + .4494z^{-2}}$$

$$= .1093 + \frac{.151 - .2303j}{z-(.49+.45j)} + \frac{.151 + .2303j}{z-(.49-.45j)}$$

$$= .1093 + \frac{.28e^{j.99}}{z - .67e^{j.75}} + \frac{.28e^{j.99}}{z - .67e^{-j.75}}$$

$$= .1093\delta[n] + .188^n \cos(n - \tfrac{3}{4})u[n]$$

$$\frac{.1093z^2 + .1942z^1 + .1093}{z^2 - .9863z^1 + .4494} \cdot \frac{z}{z-.8}$$

$$\frac{.1093z^3 + .1942z^2 + .1093z}{z^3 - 1.77z^2 + 1.24z - .36}$$

$$.8^n + .28e^{j.99}(.67e^{j.75})^n + .28e^{j.99}(.67e^{-j.75})^n$$

$$\left[.8^n + .56(.67)^n \cos(\tfrac{3n}{4} - .99)\right]u[n]$$