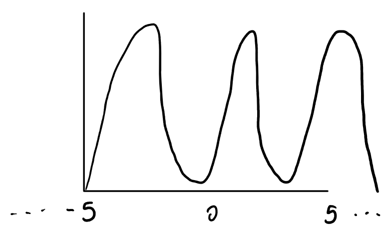
HW2

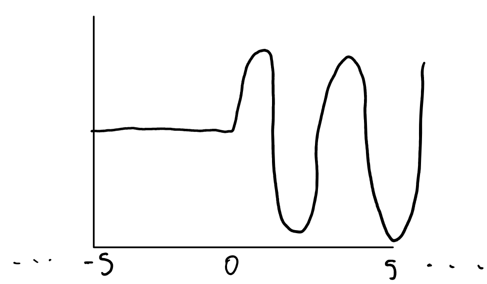
Tuesday, January 11, 2022 12:17 AM

1. Define and sketch the three types of DT CE signals: everlasting, causal, and finite (windowed).

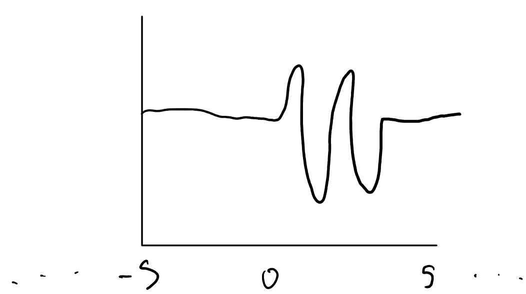
everlasting: continues from $-\infty$ to ∞



Causal: 0 for negative times, active for positive times



finite: active only for a short duration, 0 as time goes toward $-\infty$ or ∞



2. When a CT CE with frequency $F = 440$ Hz is sampled at $\frac{1}{T} = 8000$ samples/second, what is the frequency f of the resulting DT CE signal?

$$\frac{440}{8000} = .055$$

3. If a CT CE is reconstructed from DT CE with frequency $f = 0.26257$ cycles/sample using a sample rate of $\frac{1}{T} = 6$ Giga samples/second, what is the resulting frequency F in Hertz?

$.26257 \cdot 6G = 1575420$ Hz

4. What is the angular frequency ω associated with the cyclic frequency $f = 0.26257$ cycles/sample?

$.26257 * 2\pi = 1.65$ rad/sample

5. Let $x(t) = e^{jt}$ and $x[n] = e^{jn}$.

(a) Explain why $x(t)$ is periodic but $x[n]$ is not.

CT CE signals are periodic for all frequencies, but DT CE are periodic only for rational frequencies.

(b) What are the frequencies of $x(t)$ and $x[n]$?

$$f_{x(t)} = \frac{1}{2\pi} = .159$$

(c) What is the period of $x(t)$?

$$\frac{1}{f_{x(t)}} = 2\pi$$

6. Let $x[n] = e^{j2\pi fn}$ where $f = \frac{213}{355}$.

(a) Explain why $x[n]$ is periodic.

f is a rational number

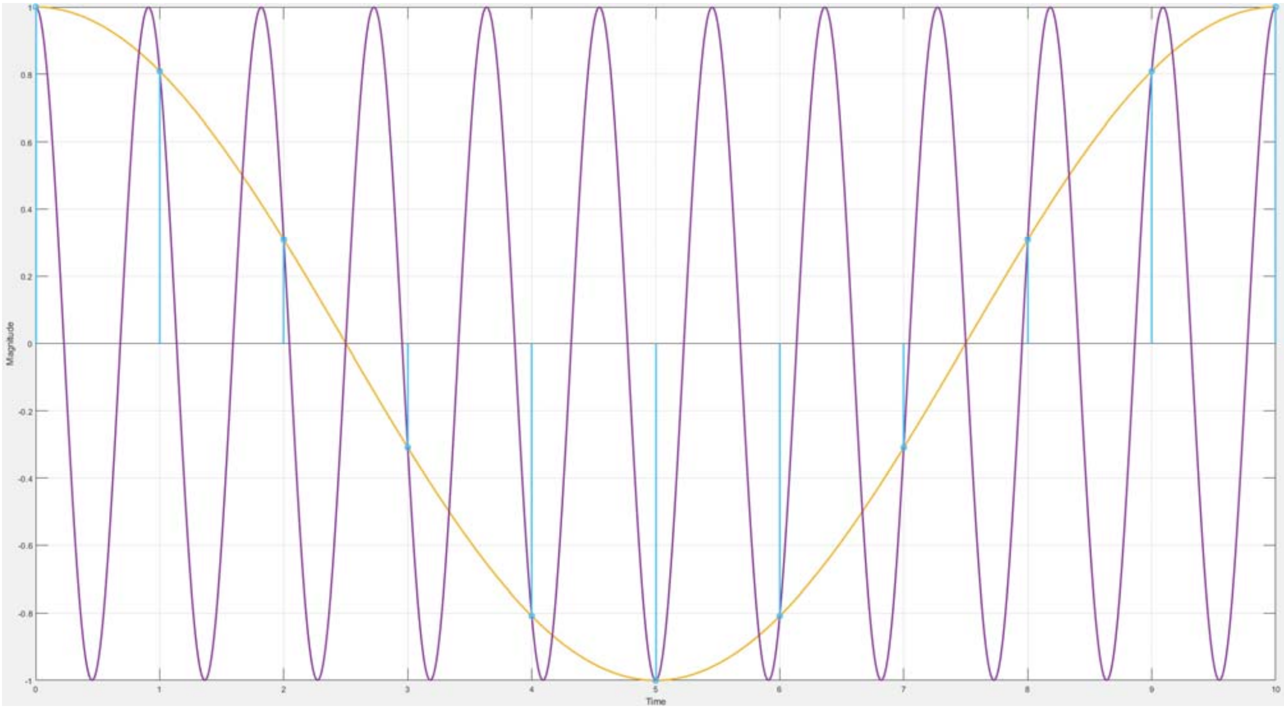
(b) What is the period of $x[n]$?

$$\frac{1}{f} = \frac{355}{213}$$

7. Do the following in Matlab.

(a) Plot $e^{j2\pi 0.1t}$ and $e^{j2\pi 1.1t}$ for $0 \leq t \leq 10$ on the same axis. (Hint: Use `t=[0:0.01:10];` to generate the time samples.)

(b) On the same axis add $e^{j2\pi 0.1n}$ and $e^{j2\pi 1.1n}$ as stem plots. (Hint: Use Matlab's `stem` function instead of the `plot` function. Use `n=[0:10];` to generate the time samples.)



(c) Explain why $F = 0.1$ Hz and $F = 1.1$ Hz give different CT CE signals while $f = 0.1$ cycles/sample and $f = 1.1$ cycles/sample gave the same DT CE sequence.

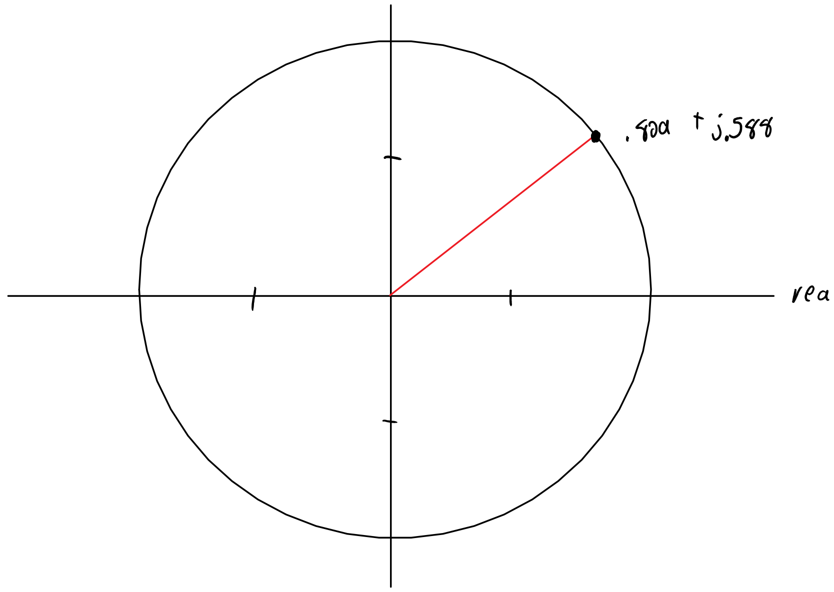
CT signals are unique for every frequency, but due to aliasing, DT signals intersect at the same point for frequencies offset by an integer value, so the resulting sequences given by f = .1 and f = 1.1 are the same.

(d) Draw the unit circle on the complex plane. Show the point $e^{j2\pi 0.1} = e^{j2\pi 1.1}$ and use the fact $(e^{j2\pi 0.1})^n = e^{j2\pi 0.1n} = (e^{j2\pi 1.1})^n = e^{j2\pi 1.1n}$ to explain frequency aliasing which is that $e^{j2\pi fn} = e^{j2\pi(f+k)n}$ for all n, where $k \in \mathbb{Z}$.

$\sin(.628) = \sin(6.91) = .588j$

$\cos(.628) = \cos(6.91) = .809$

Imag



As demonstrated above, each integer value added to the initial frequency makes a complete rotation around the circle, resulting in the same value as the previous one. This is true for all whole integer values added.

8. Find a frequency alias for $f = 37.8$ cycles/sample in the fundamental interval assuming:

(a) $0 \leq f < 1$ is the fundamental interval

$f = .8$

(b) $-\frac{1}{2} \leq f < \frac{1}{2}$ is the fundamental interval

$f = -.2$

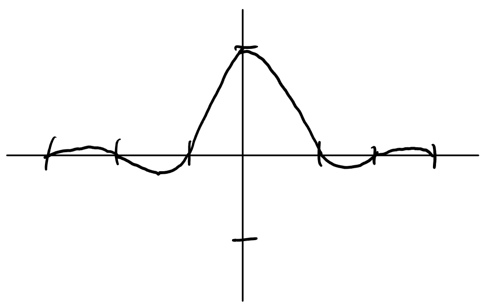
9. Why is $f = \pm\frac{1}{2}$ cycles/sample the highest frequency in discrete time?

Due to aliasing, any frequency that increases above 1/2 or decreases below -1/2 start to repeat a previous frequency which is lower than the extremes found at +-1/2.

10. Which is a higher frequency $f = 26.4$ cycles/sample or 38.9 cycles/sample? (Hint: Compare their aliased frequencies.)

$.4 \; / \; -.1$ $\qquad$ $f = 26.4$ is higher

11. Explain why $\frac{\sin(\pi n)}{\pi n} = \delta[n]$. Include a sketch in your explanation.



at $n=0$, the output is 1,
but at any integer $\neq 0$,
the output is 0.

12. Prove by integration that $e^{j2\pi 10t}$ and $e^{j2\pi 30t}$ are orthogonal over the time interval $0 \leq t < 0.2$ seconds.

$$\int_0^{.2} e^{j2\pi(10-30)t} \, dt$$

$$= \frac{j e^{-40j\pi t}}{40\pi} \Bigg|_0^{.2} = 0$$

13. Prove by summation that $e^{j\frac{2\pi 2n}{9}}$ and $e^{j\frac{2\pi 4n}{9}}$ are orthogonal over $0 \le n < 9$.

$$\sum_{n=0}^{8} e^{j2\pi\left(\frac{2}{9}-\frac{4}{9}\right)n}$$

$$= 0$$

14. Suppose the everlasting DT CE sequence

$$x[n] = e^{j2\pi fn}, \quad -\infty < n < \infty$$

is applied to a DT LTI system with frequency response

$$H(f) = \frac{\sin(3\pi f)}{\sin(\pi f)} e^{-j2\pi f}.$$

What is the resulting output signal $y[n]$ if

(a) $f = \frac{2}{3}$

$$\sum_{n=-\infty}^{\infty} e^{j2\pi\left(\frac{2}{3}\right)n} \quad \frac{\sin\left(3\pi\left(\frac{2}{3}\right)\right)}{\sin\left(\pi\left(\frac{2}{3}\right)\right)} e^{-j2\pi\left(\frac{2}{3}\right)}$$

$$= 0$$

(b) $f = \frac{1}{2}$

$$\sum_{n=-\infty}^{\infty} e^{j2\pi(\frac{1}{2})n} \quad \frac{\sin(3\pi(\frac{1}{2}))}{\sin(\pi(\frac{1}{2}))} e^{-j2\pi(\frac{1}{2})}$$

$$= 1$$

Code used:

```
% HW2 7a
T = [0:0.01:10];
S = exp(-1i*2*pi*.1*T);
plot(T,S,'LineWidth',2);
hold on;
plot(T,exp(-1i*2*pi*1.1*T),'LineWidth',2);
%hold off;
grid on;
xlabel('Time');
ylabel('Magnitude');
orient landscape;


% HW2 7b
n = [0:10];
S = exp(-1i*2*pi*.1*n);
stem(n,S,'LineWidth',2);
stem(n,exp(-1i*2*pi*1.1*n),'LineWidth',2);
hold off;
grid on;
orient landscape;


theta = [0:0.01:2*pi];
f = exp(1i*theta);
plot(real(f),imag(f));
```