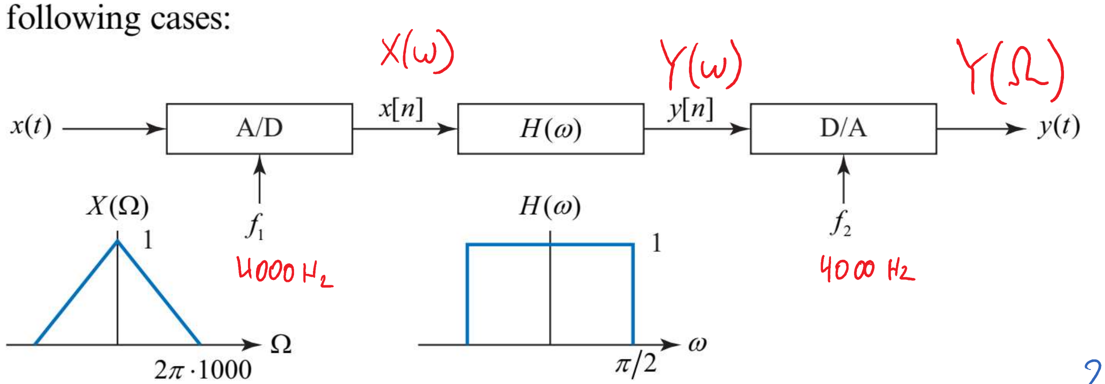
# HW5

Thursday, February 24, 2022 2:12 AM

## Problem 6-1

The discrete-time filtering system shown in **Figure 6.68** comprises an A/D converter sampling at rate $f_1$, a discrete-time filter with frequency response $H(\omega)$ and an ideal D/A converter reconstructing at rate $f_2$. Ideal means that the converter contains an ideal lowpass reconstruction filter with a bandwidth of $\pi f_2$ and a gain of $1/f_2$. The spectrum of the input, $X(\Omega)$, is shown in **Figure 6.68**. Provide a fully labeled sketch of $X(\omega)$, $Y(\omega)$ and $Y(\Omega)$ for each of the following cases:




$\Omega_s = 2\pi f_s.$

$\Omega_s \triangleq 2\pi/T$

$\Omega = \omega/T:$

**Figure 6.68**

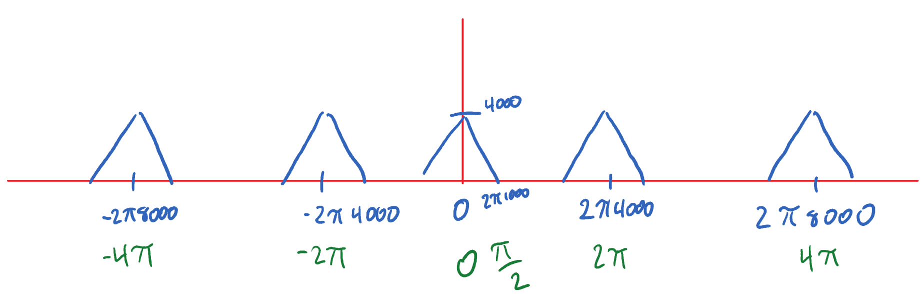
$2\pi \cdot 1000$

$\Omega = 2\pi \cdot 2000$

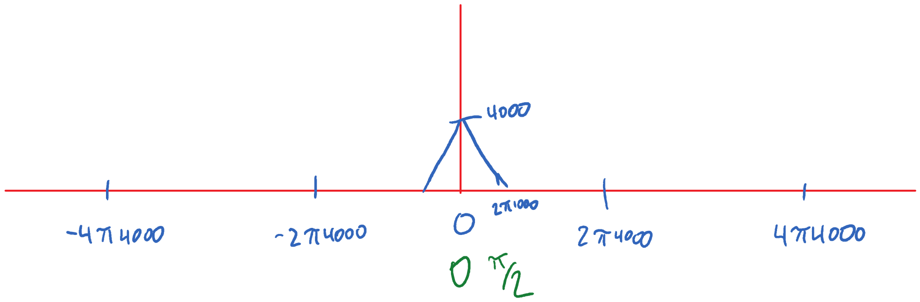
(a) $f_1 = f_2 = 4000$ Hz.

$\Omega_s = 2\pi f_s = 2\pi \cdot 4000$

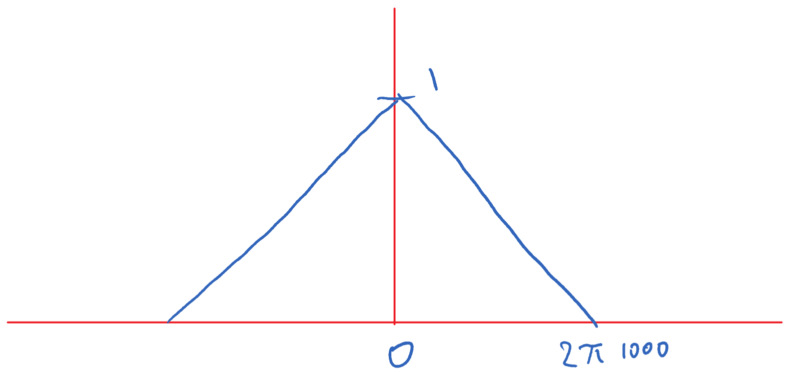
$X(\omega)$




$Y(\omega)$

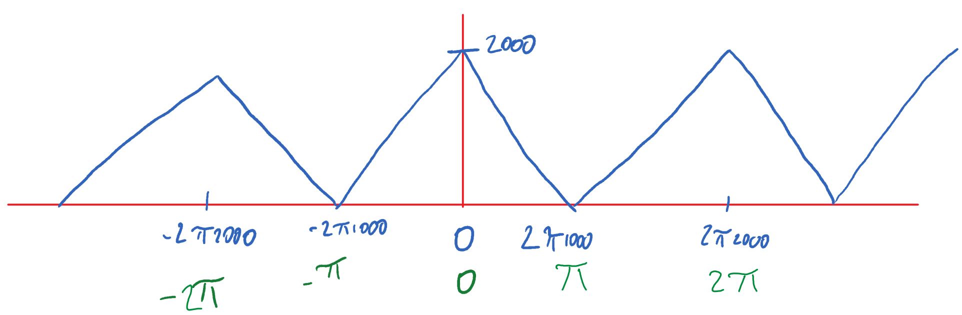



$Y(\Omega)$

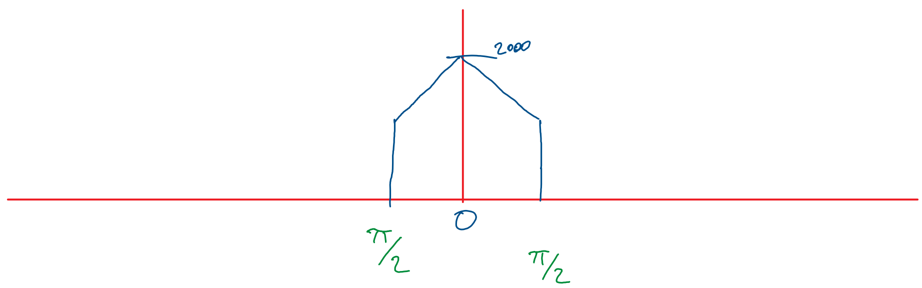



(b) $f_1 = f_2 = 2000$ Hz.

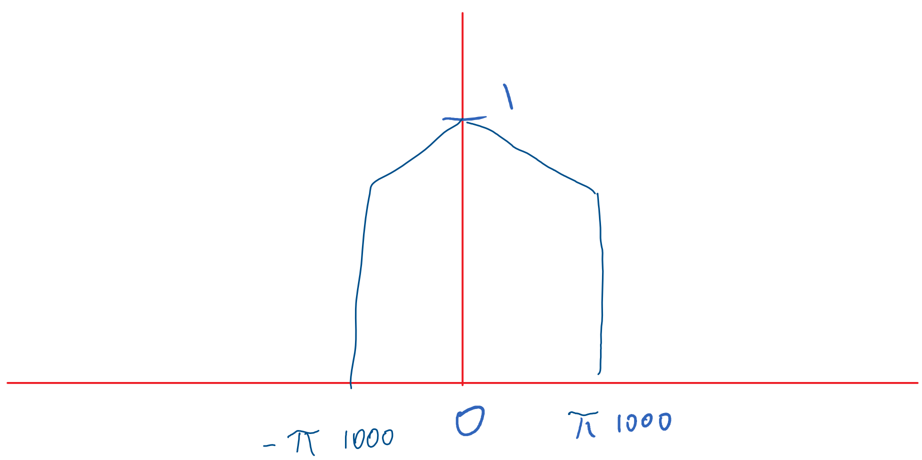
$X(\omega)$




$Y(\omega)$

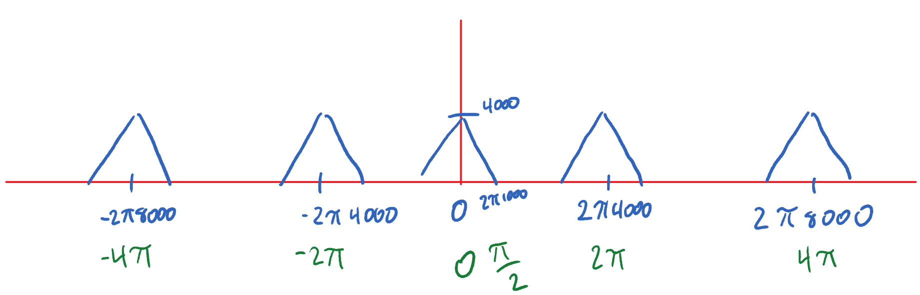



Y(Ω)

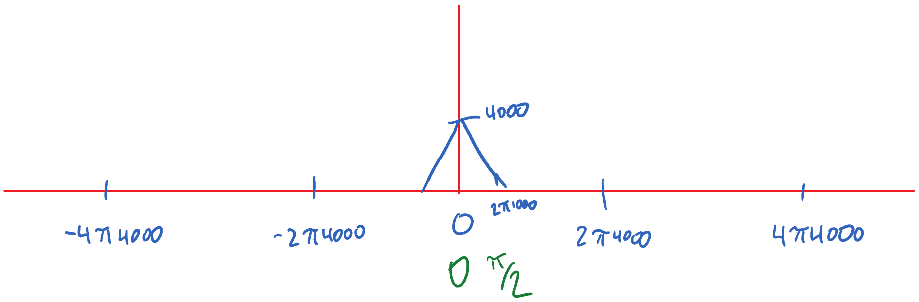



(c) $f_1 = 4000$ Hz, $f_2 = 2000$ Hz.

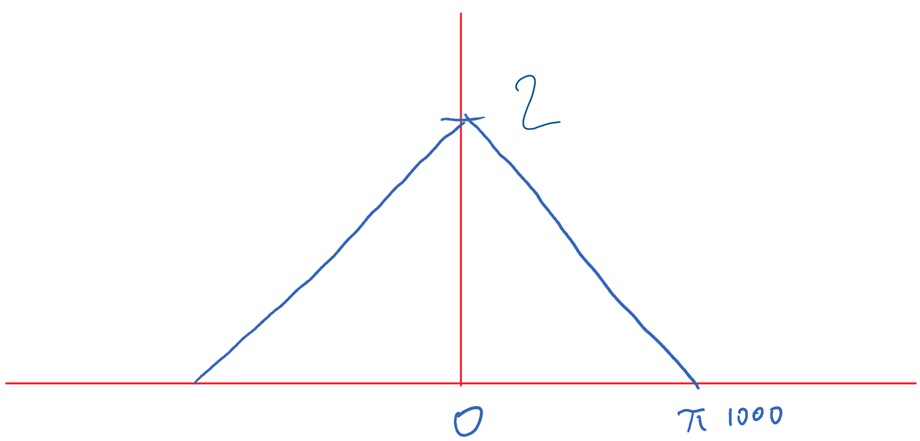
X(ω)




Y(ω)

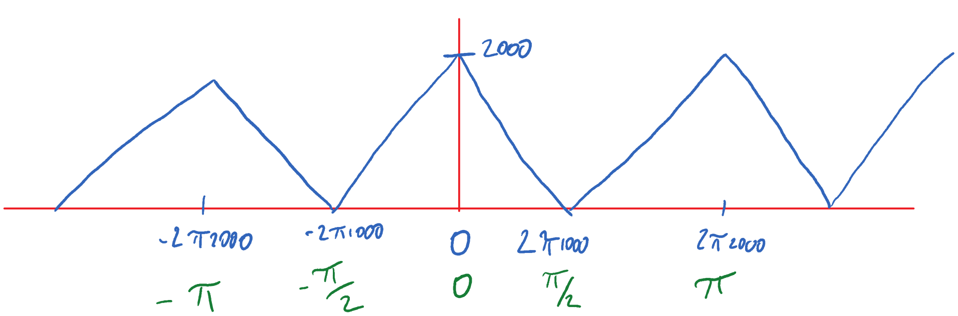


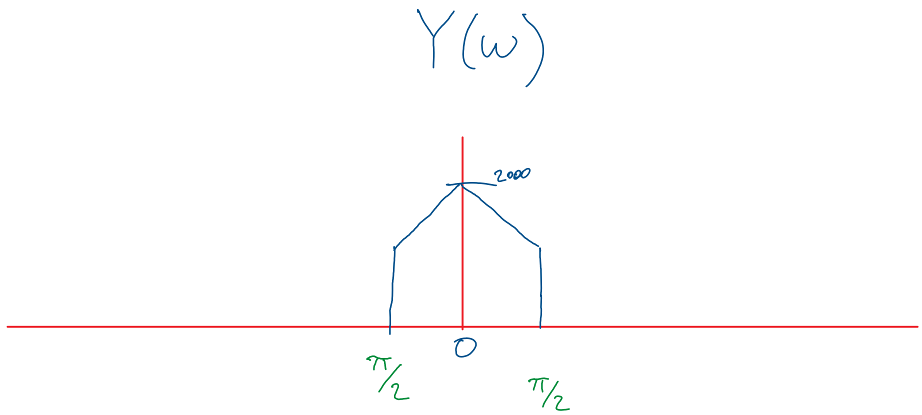

$Y(\Omega)$

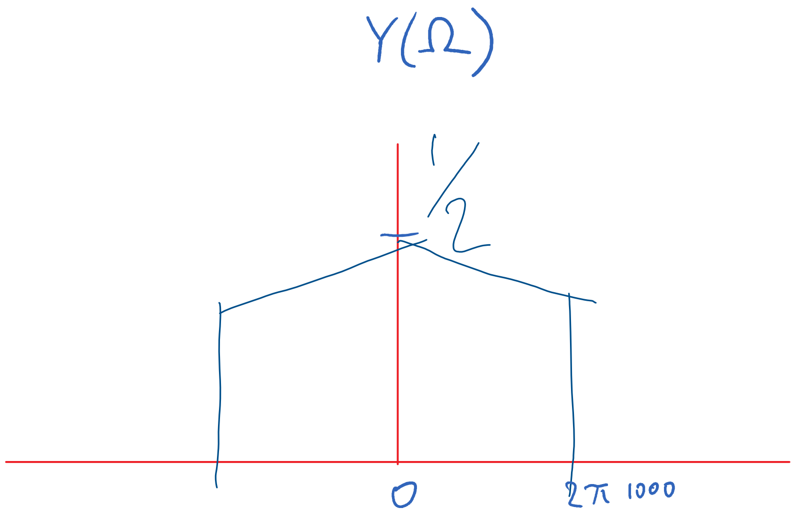



(d) $f_1 = 2000$ Hz, $f_2 = 4000$ Hz.

$X(\omega)$





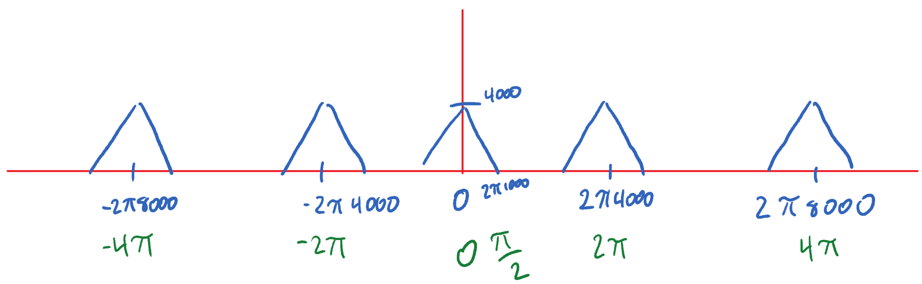




## Problem 6-2

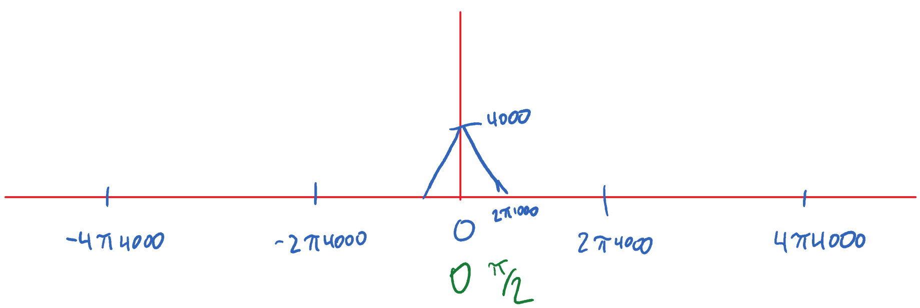
Given the discrete-time filtering system of **Figure 6.68** with $x(t) = \cos 2\pi \cdot 1000t$, provide a fully labeled sketch of $X(\omega)$, $Y(\omega)$ and $Y(\Omega)$ and find $y(t)$ for each of the following cases:

(a) $f_1 = f_2 = 4000\text{Hz}$.

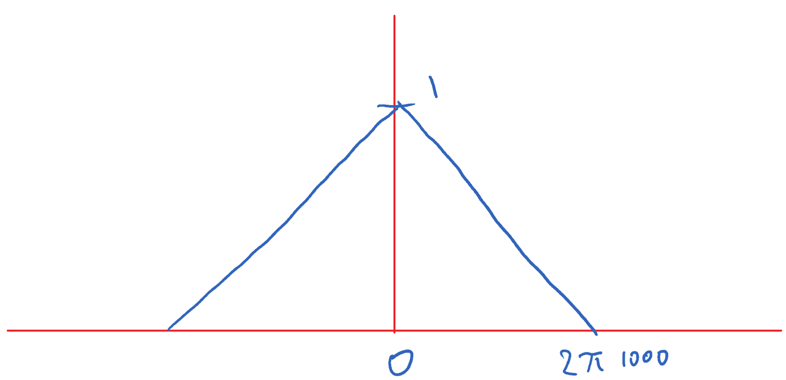
$X(\omega)$




$Y(\omega)$



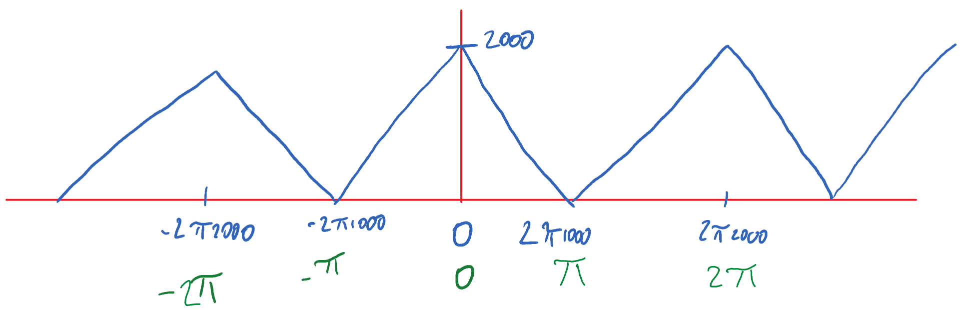

$Y(\Omega)$



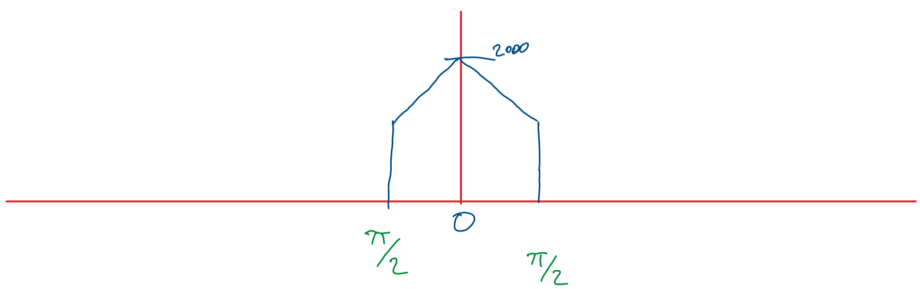

$$y(t) = \frac{1000\pi}{\pi} \operatorname{sinc}(1000\pi t)$$

(b) $f_1 = f_2 = 2000$ Hz.

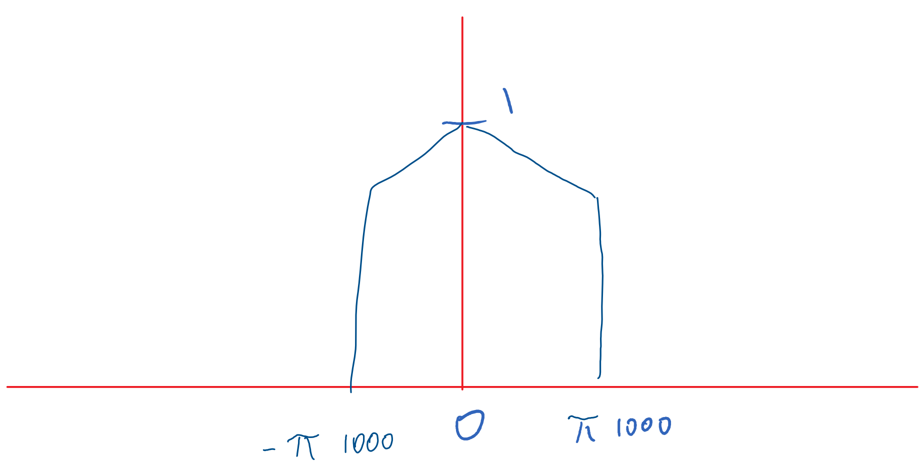
$X(\omega)$




$Y(\omega)$



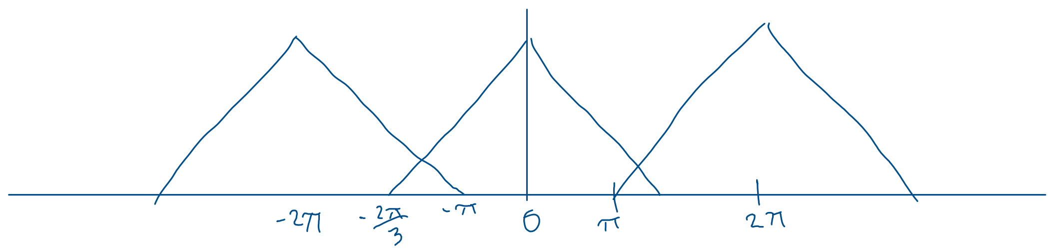

$Y(\Omega)$



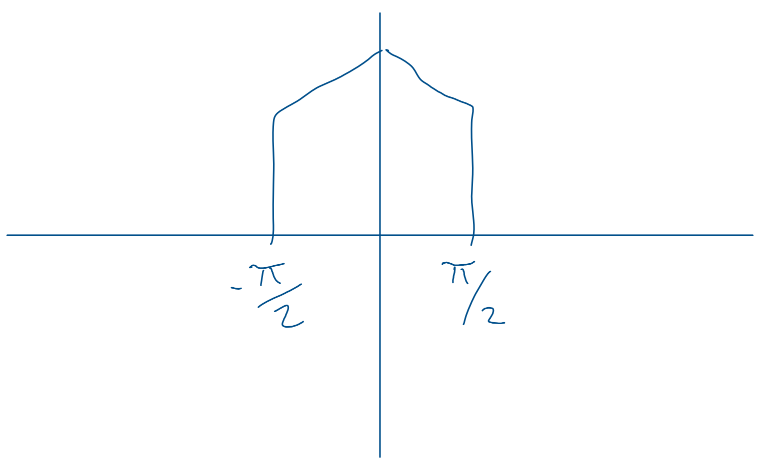
$$y(t) = \frac{1000\pi}{\pi} \operatorname{Sinc}(1000\pi t) \cdot 1000 \operatorname{Sinc}(1000\pi t)$$

(c) $f_1 = f_2 = 1333$ Hz

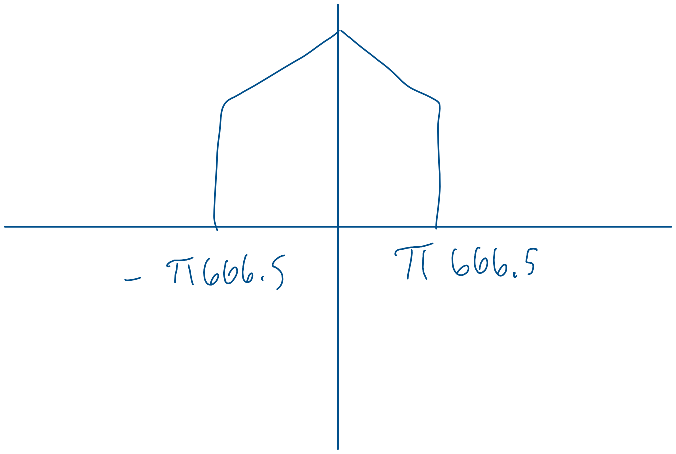
$X(\omega)$




$Y(\omega)$



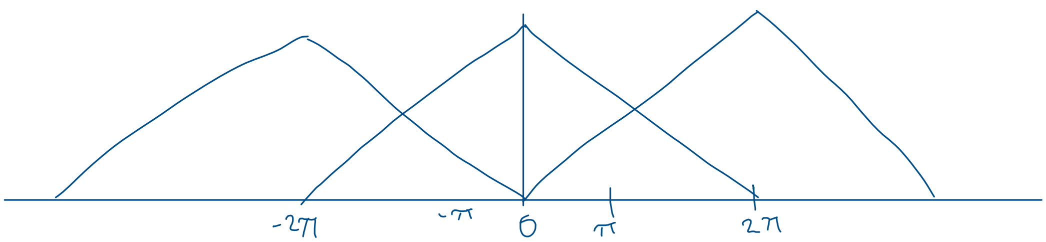

$Y(\Omega)$



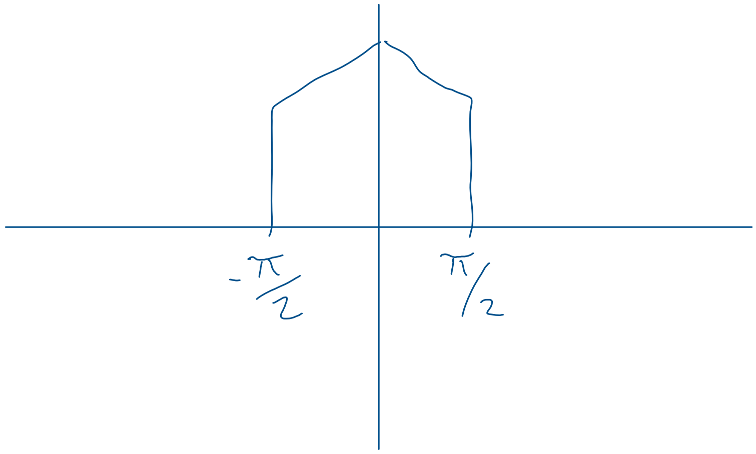

$$y(t) = 666.5 \,\text{sinc}(666.5t) \cdot 1000 \,\text{sinc}(1000t)$$

(d) $f_1 = f_2 = 1000$ Hz.

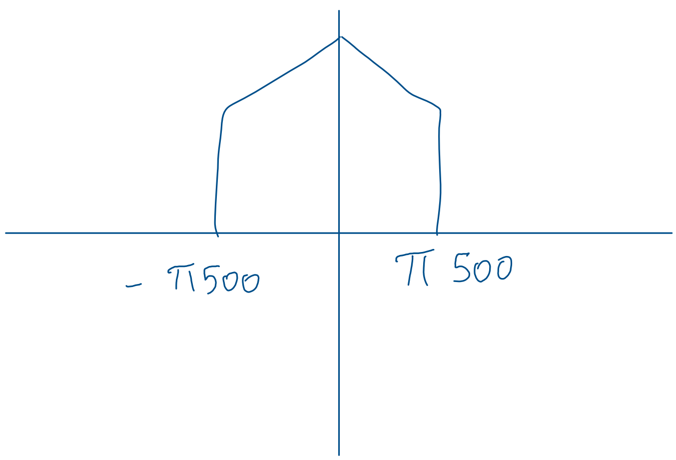
$X(\omega)$




$Y(\omega)$




$Y(\Omega)$




$$y(t) = \frac{500\pi}{\pi}\,\mathrm{sinc}(500t)\cdot 1000\,\mathrm{sinc}(1000t)$$

## Problem 6-5

The discrete-time filtering system shown in **Figure 6.70** comprises an A/D converter sampling at rate $f_1$, a discrete-time filter with frequency response $H(\omega)$, a resampler that resamples at rate $D{:}U$ and an ideal D/A converter at rate $f_2$. "Ideal" means that the converter contains an ideal lowpass reconstruction filter with a bandwidth of $\pi f_2$ and a gain of $1/f_2$. Assume that the resampler is ideal (upsample by padding $y[n]$ with $U-1$ zeros, discrete-time filter with gain of $U$ and bandwidth of $\pi/\max(U, D)$, downsample at $D$, tossing $D-1$ points). The spectrum of the input, $X(\Omega)$, is shown in the lower panel of the figure. For each of the following parts, plot the spectra $X(\omega)$, $Y(\omega)$, $Z(\omega)$ and $Z(\Omega)$.

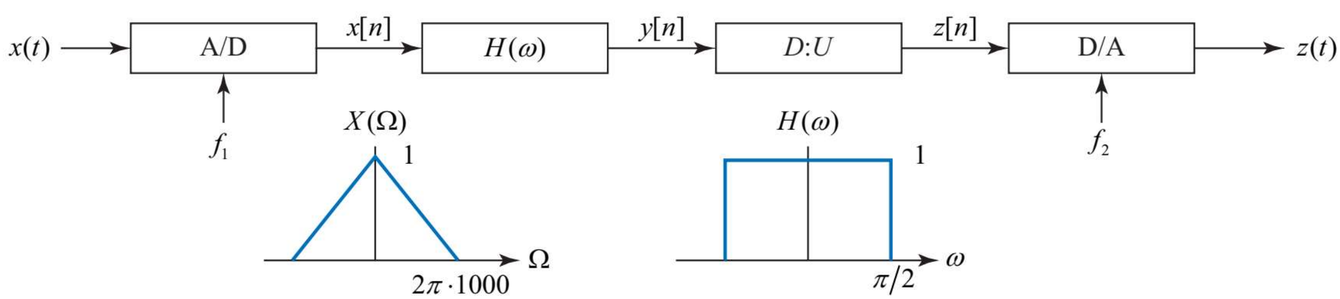
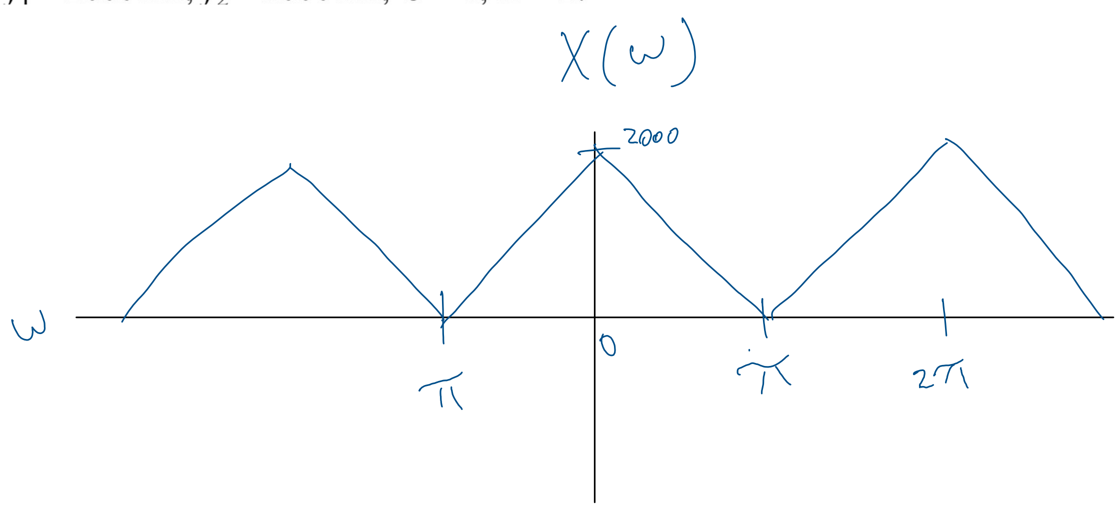
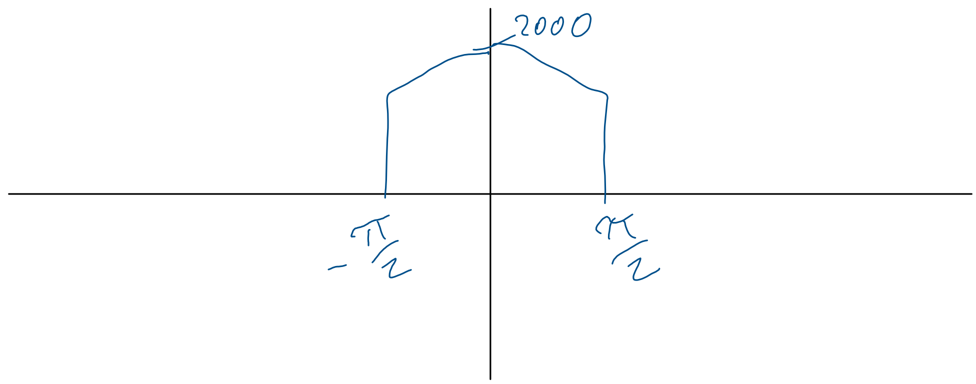



**Figure 6.70**

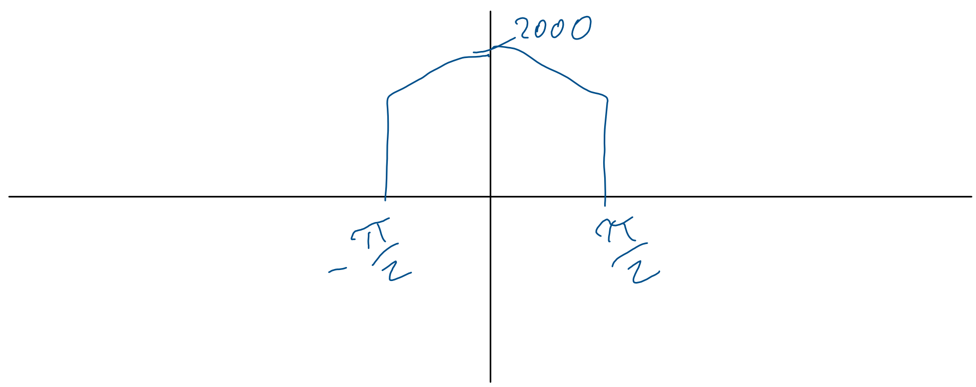
(a) $f_1 = 2000$ Hz, $f_2 = 1000$ Hz, $U = 1$, $D = 2$.



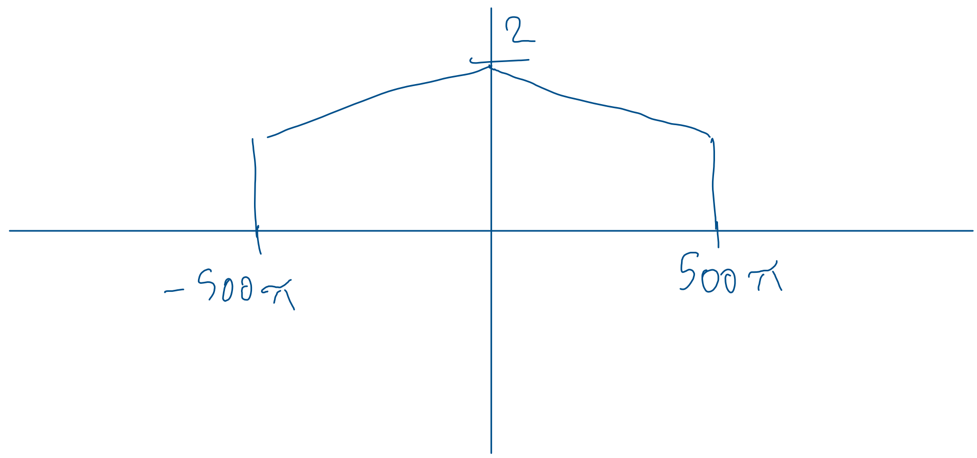
$Y(\omega)$.



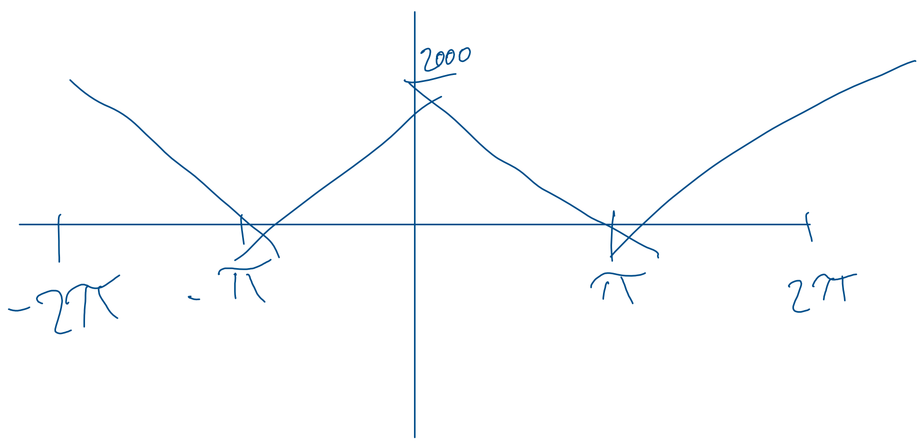

$Z(f)$



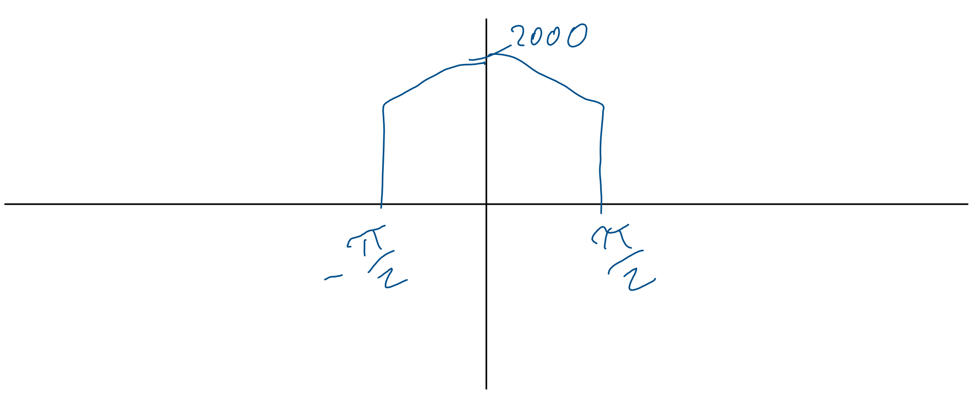

$Z(\Omega)$



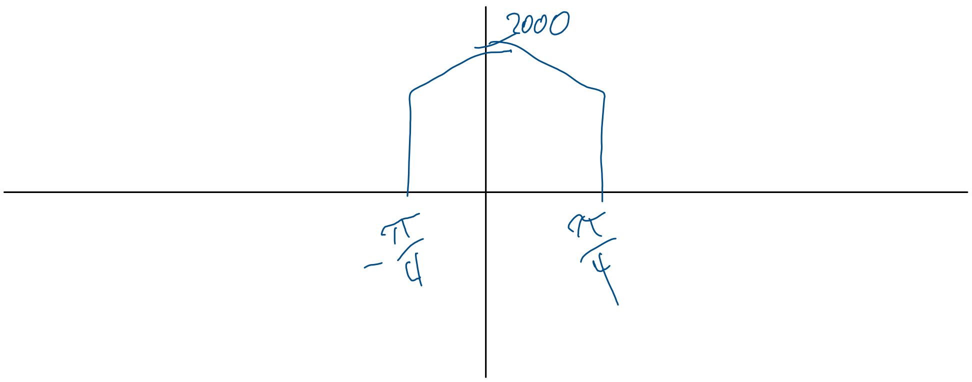

(b) $f_1 = 2000\text{ Hz}, f_2 = 4000\text{ Hz},\ U = 2,\ D = 1$



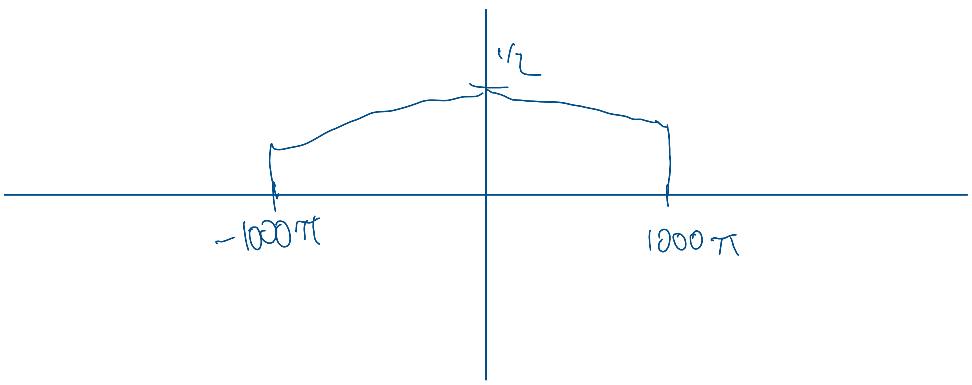

$Y(\omega)$.



$Z(\omega)$

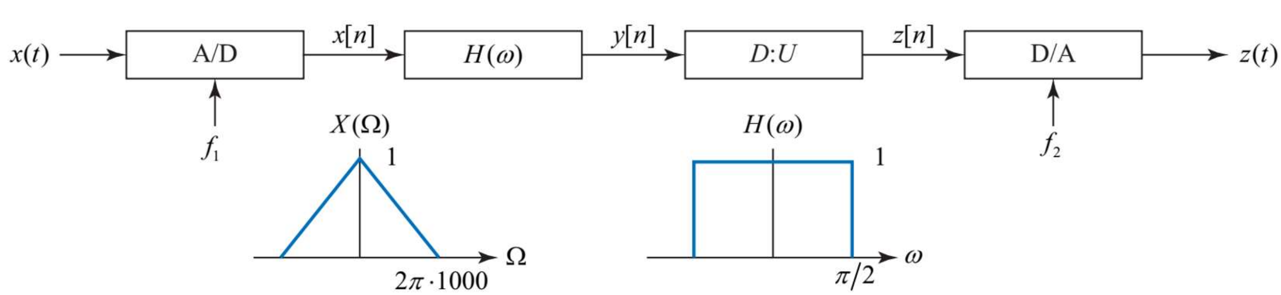



$Z(\Omega)$




## Problem 6-6

The discrete-time filtering system shown in **Figure 6.70** comprises an A/D converter sampling at rate $f_1 = 6000$ Hz, a filter with frequency response $H(\omega)$, as shown in the figure, a 2 : 1 downsampler and an ideal D/A converter reconstructing at rate $f_2 = 3000$ Hz. The input is $x(t) = 1 + \cos(2\pi \cdot 1000t) + \cos(2\pi \cdot 2000t)$. Provide a fully labeled sketch of $X(\omega)$, $Y(\omega)$, $Z(\omega)$ and $Z(\Omega)$.

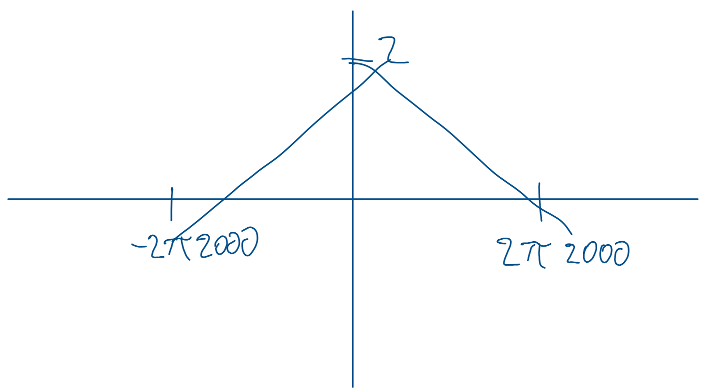


$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

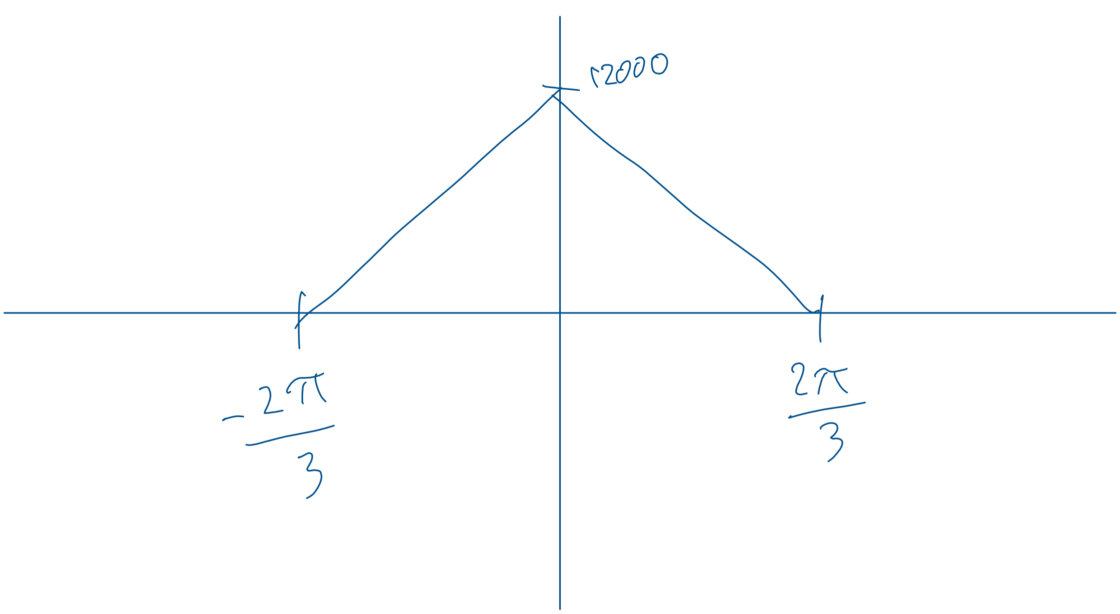
$$2\cos\left(\frac{2\pi 1000t + 2\pi 2000t}{2}\right)\cos\left(\frac{2\pi 1000t - 2\pi 2000t}{2}\right)$$

$$2\cos\left(2\pi 1500t\right)\cos\left(2\pi 500t\right)$$

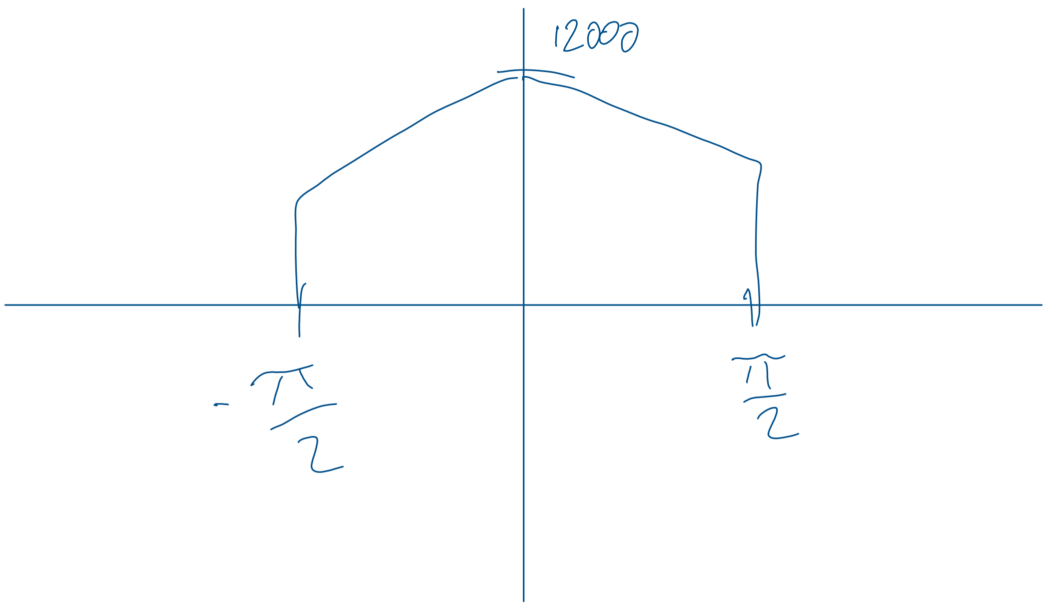
$X(\Omega)$



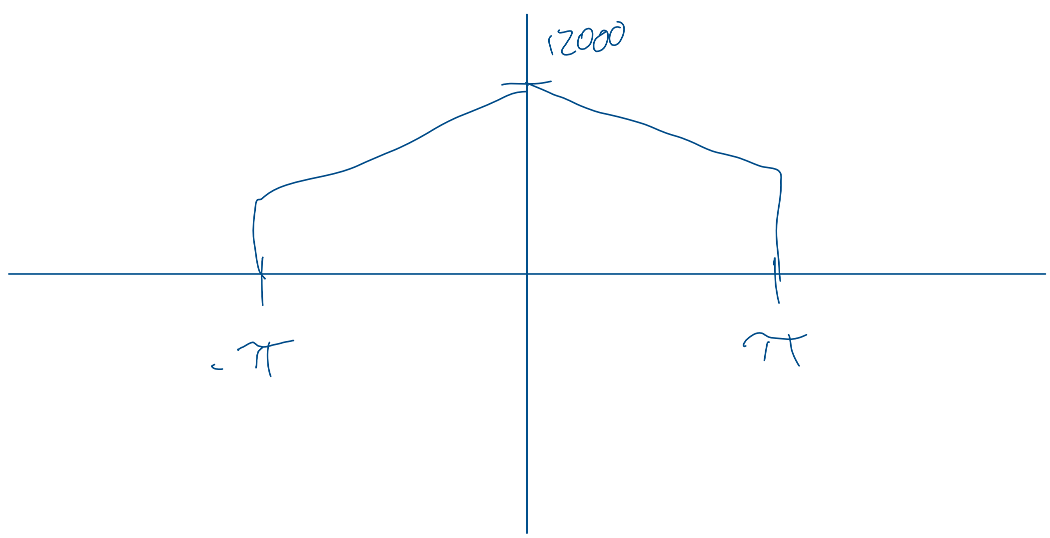

$X(\omega)$



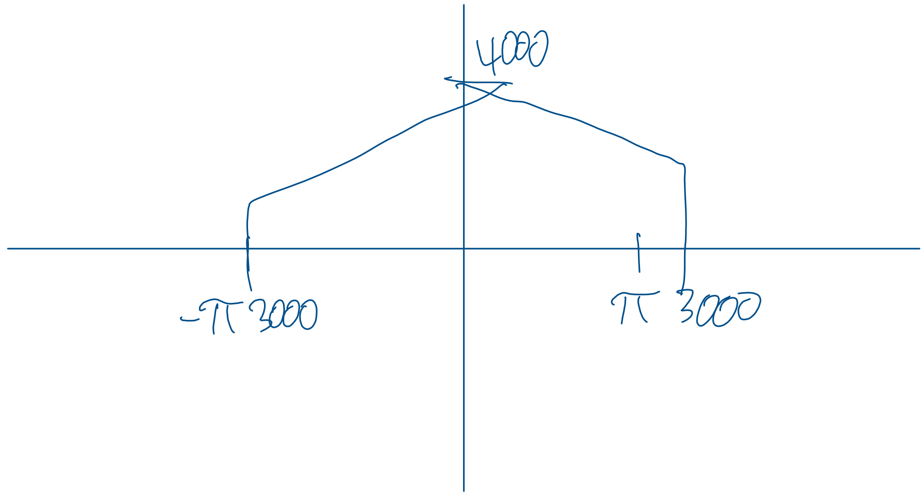
Y(ω)




Z(ω)



$Z(\Omega)$




## Problem 6-8

A discrete-time filtering system comprises an A/D converter sampling at rate $f_1$, decimation by a factor of $D$ such that $y[n] = x[Dn]$, a discrete-time filter with frequency response $H(\omega)$ and bandwidth $3\pi/4$ and an ideal D/A converter operating at rate $f_2$, as shown in **Figure 6.72**. The "ideal" D/A converter contains an ideal lowpass reconstruction filter with a bandwidth of $\pi f_2$ and a gain of $1/f_2$. The input to the system is $x(t) = \cos(2\pi \cdot 1000t)$. For each of the following parts, make a detailed, accurate sketch of $X(\omega)$, $Y(\omega)$, $Z(\omega)$ and $Z(\Omega)$ and find $z(t)$.

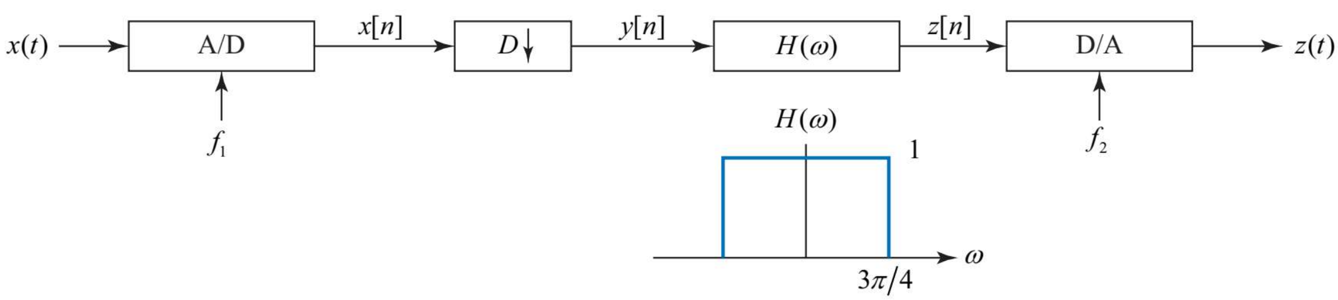
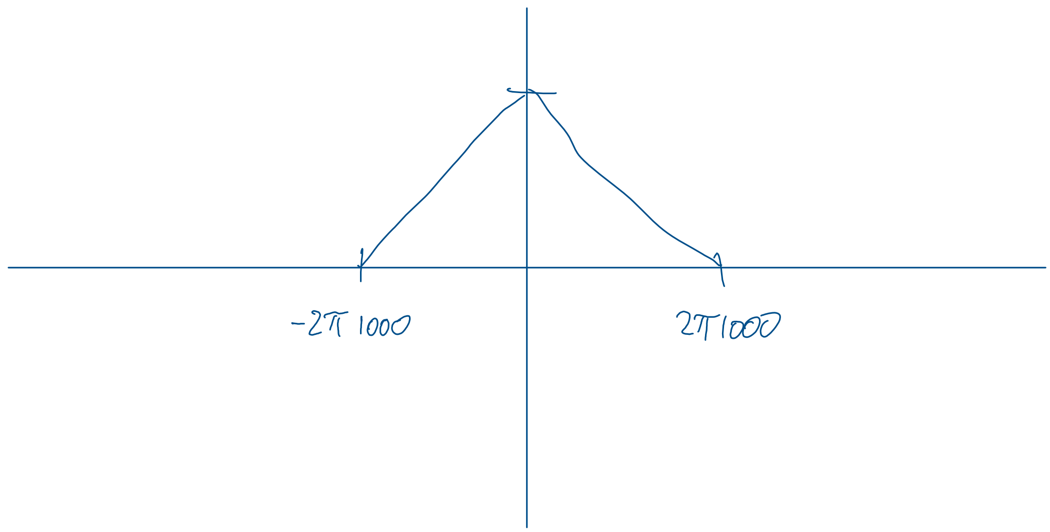
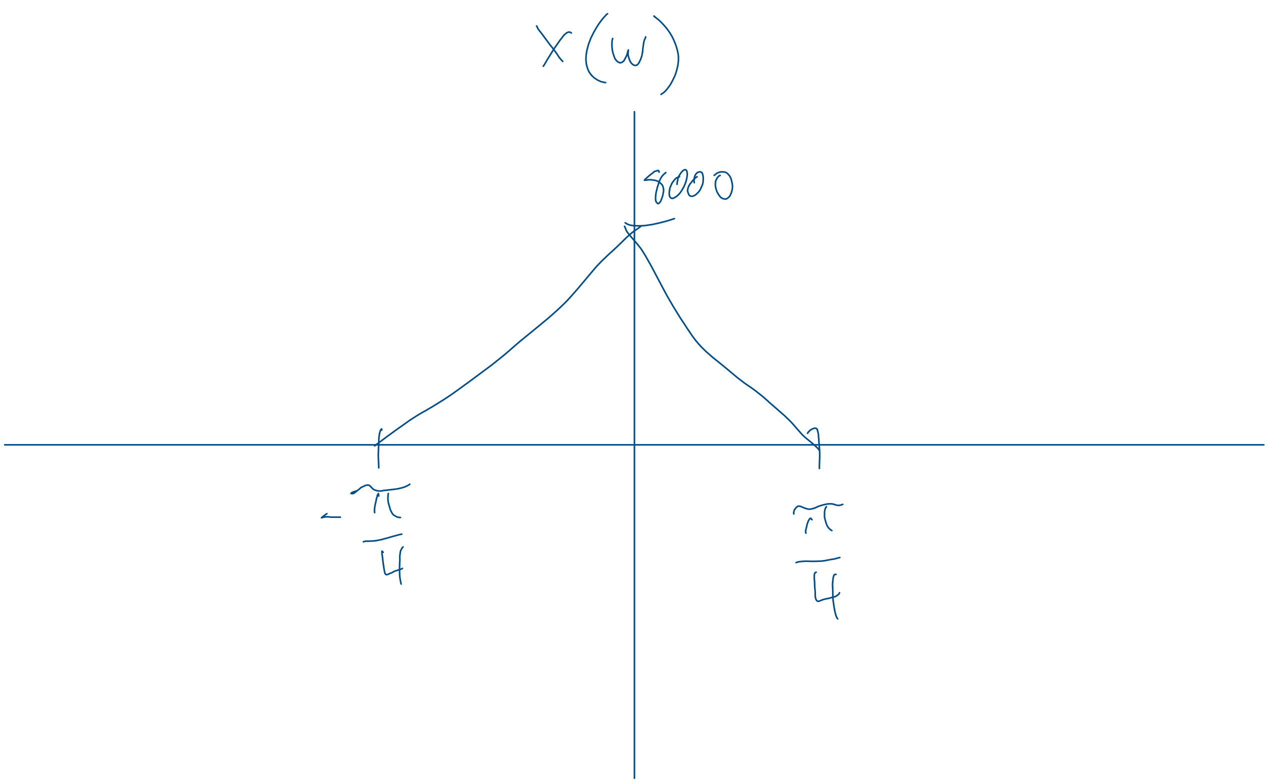



**Figure 6.72**

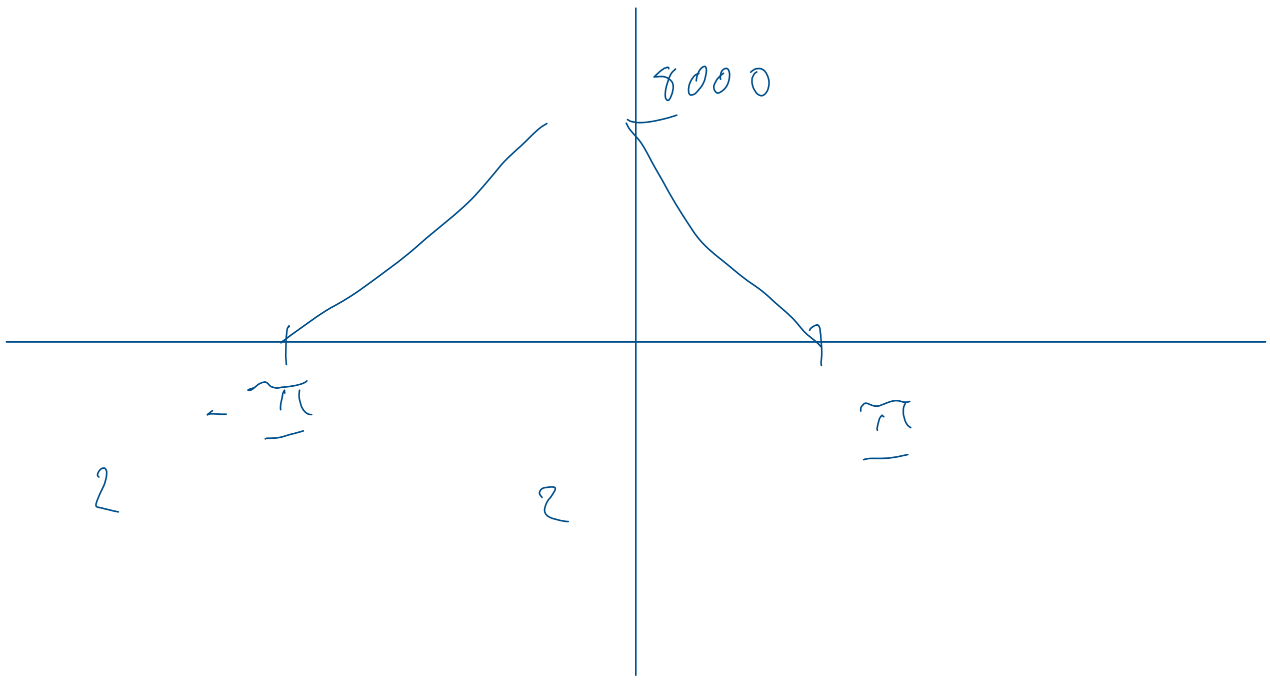
(a) $f_1 = 8$ kHz, $f_2 = 4$ kHz, $D = 2$.

$X(\Omega)$

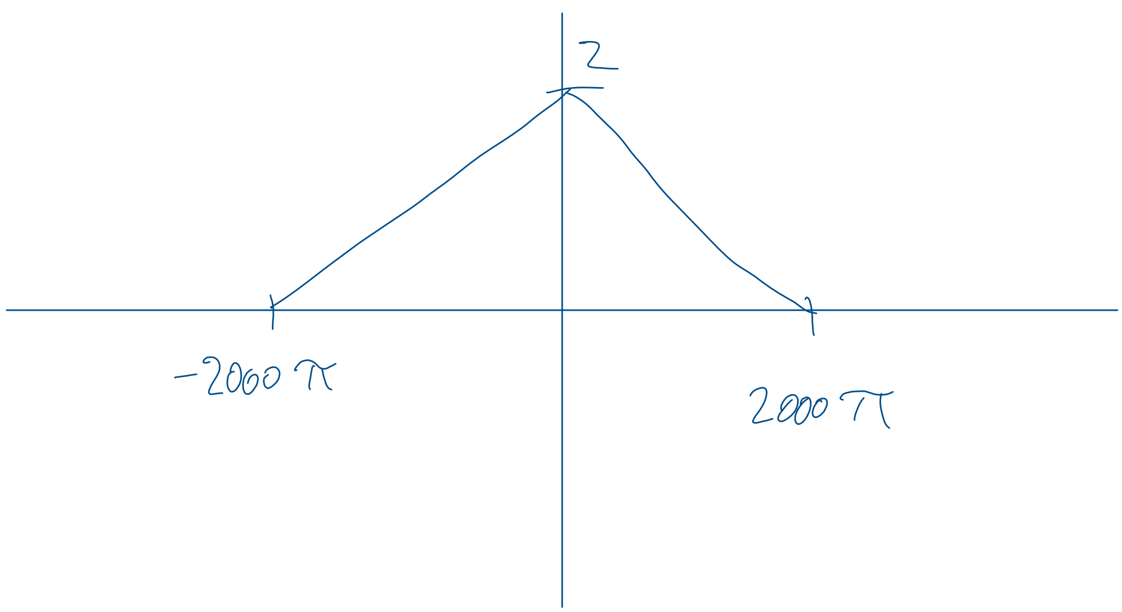



$$Y(\omega) = 2(\omega)$$

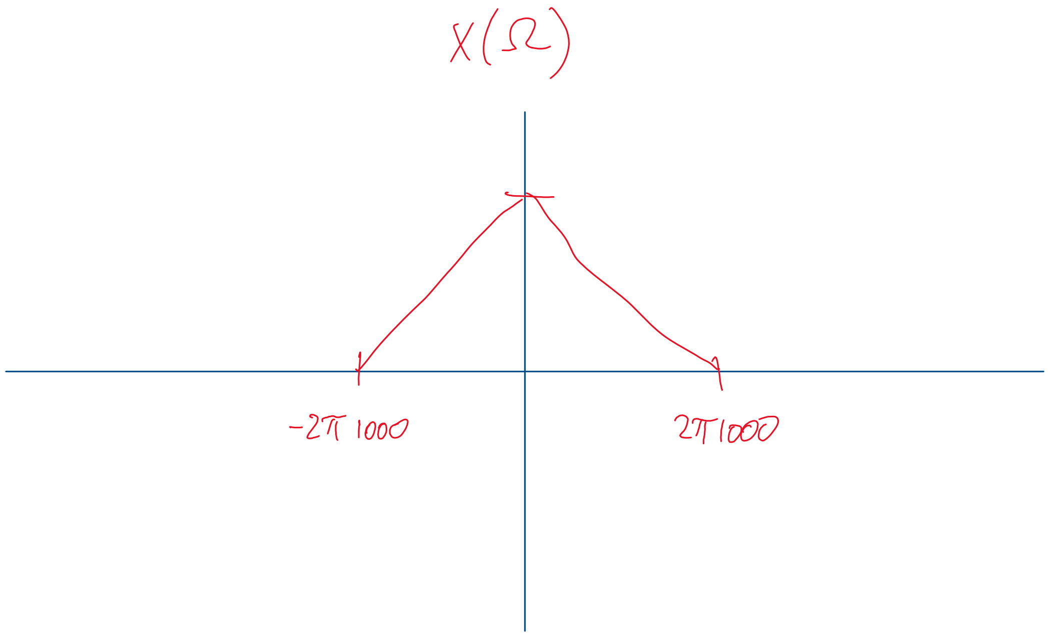



$Z(\Omega)$

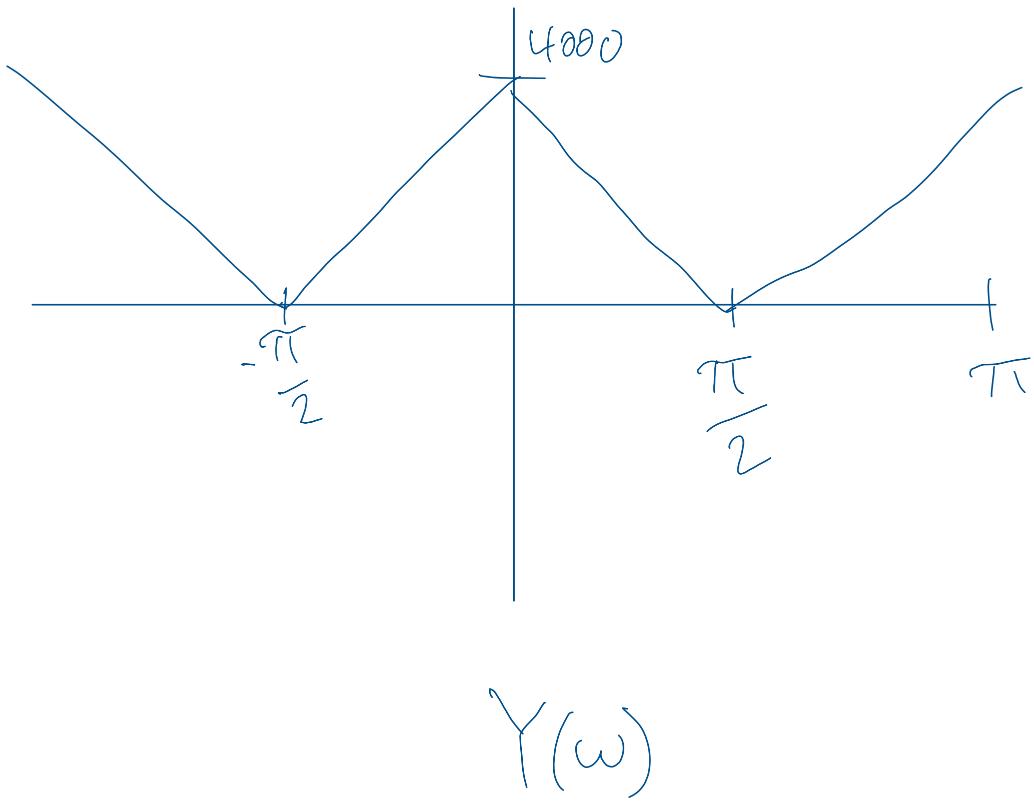


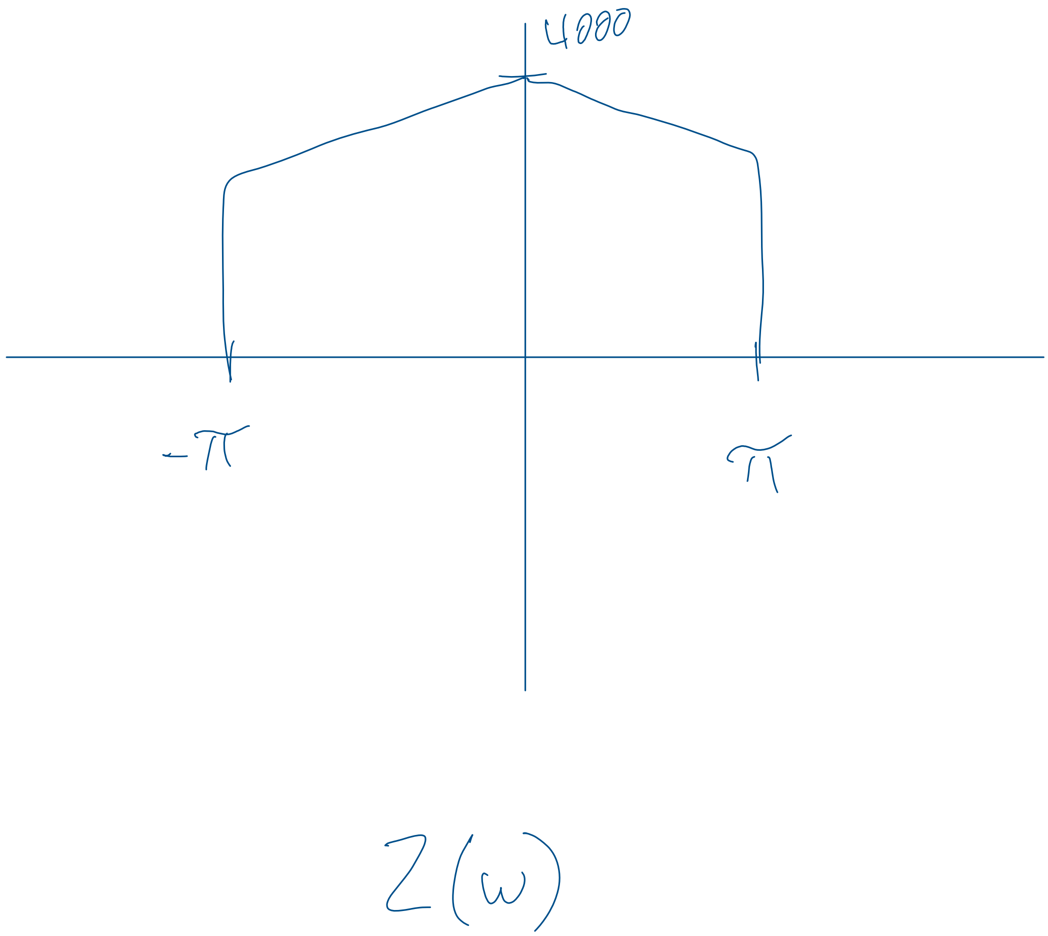
$Z(t) = 2\cos(2\pi \cdot 1000t)$

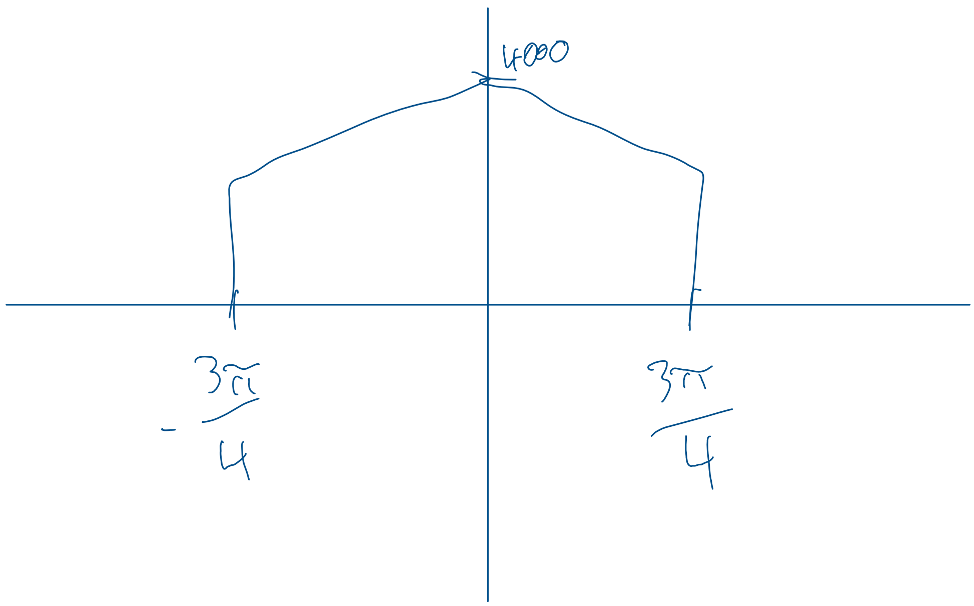
(b) $f_1 = 4$ kHz, $f_2 = 2/3$ kHz, $D = 6$.



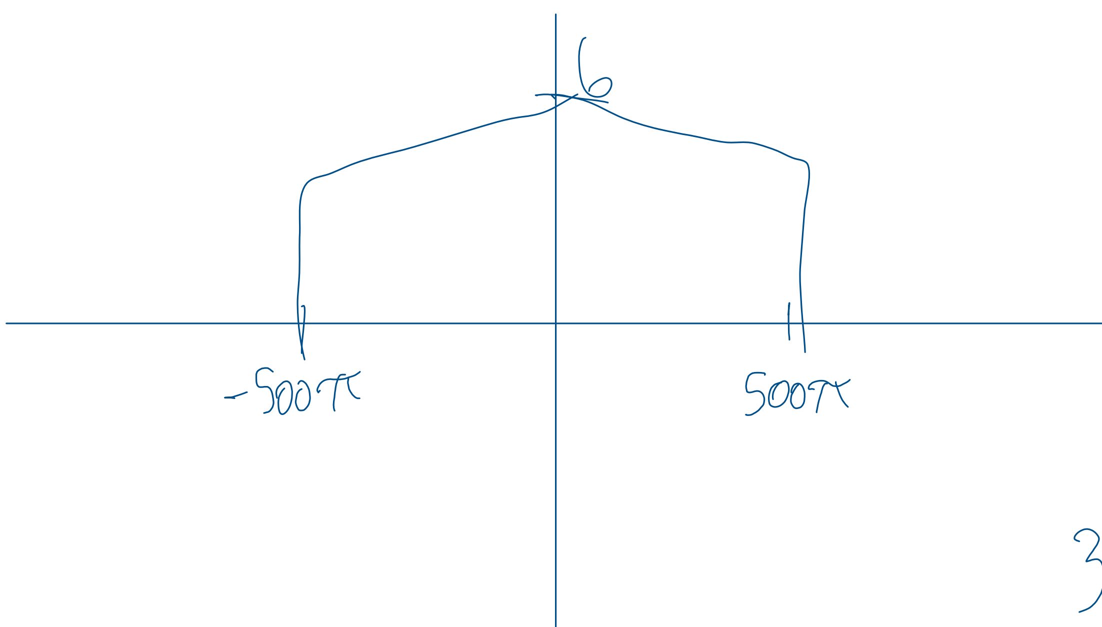

$X(\omega)$


4000
$-\pi/2$
$\pi/2$
$\pi$
$Y(\omega)$


4000
$-\pi$
$\pi$
$Z(\omega)$




$Z(\Omega)$




$3\pi \cdot \frac{2}{3}L$

$2L\pi$

$$Z(t) = 6 \cdot \cos(2\pi \cdot 1000\tau \cdot)(\delta(-500\pi) - \delta(500\pi))$$

## Problem 6-11

A discrete-time demodulator shown in **Figure 6.74** comprises an A/D converter sampling at rate $f_1$, a discrete-time filter with response $H(\omega)$ and an ideal D/A converter reconstructing at rate $f_2$. The A/D converter has no anti-aliasing filter in it, so frequencies in the input greater than $2f_1$ are not attenuated. Given that the analog signal has an input spectrum, $X(\Omega)$, shown in the figure, with $\Omega_b = 2\pi \cdot 2\ \text{kHz}$ and $\Omega_c = 2\pi \cdot 10\ \text{kHz}$, design a system such that the output spectrum of the analog signal is $Y(\Omega)$.

➤**Hint:** Think "undersampling."

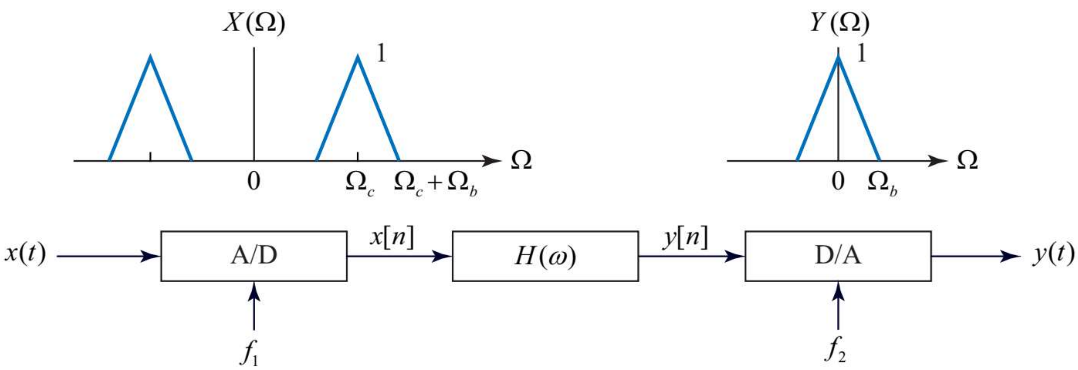
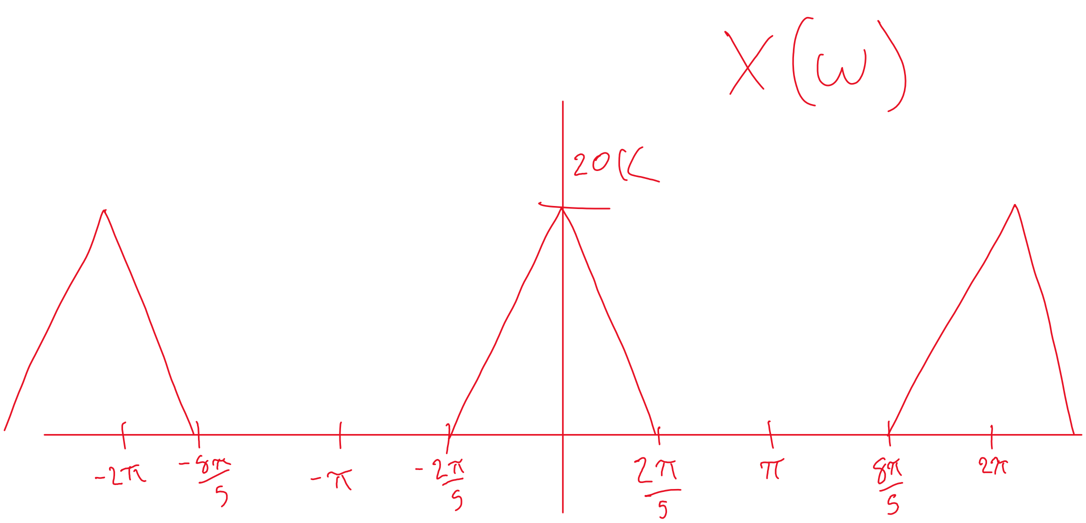
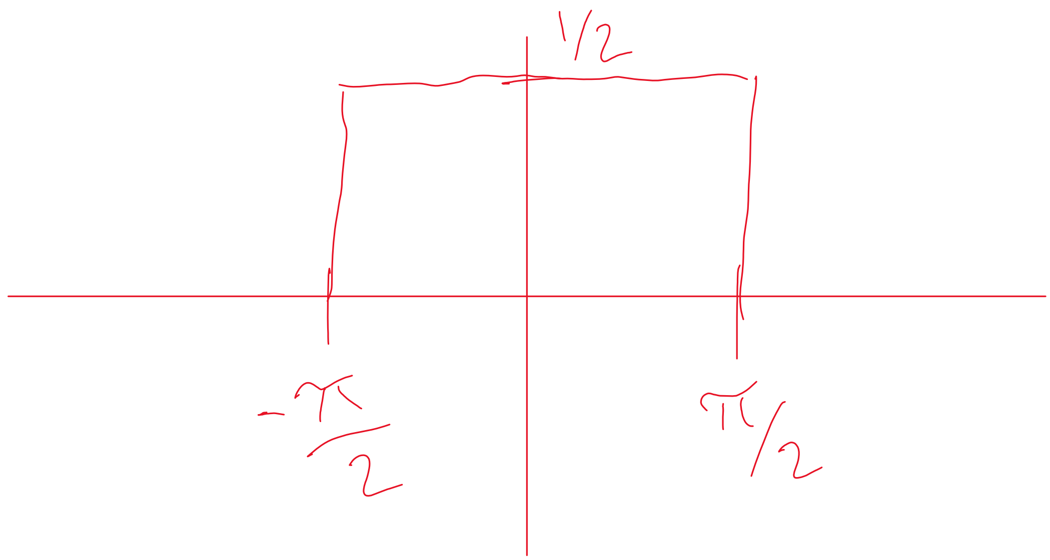



**Figure 6.74**

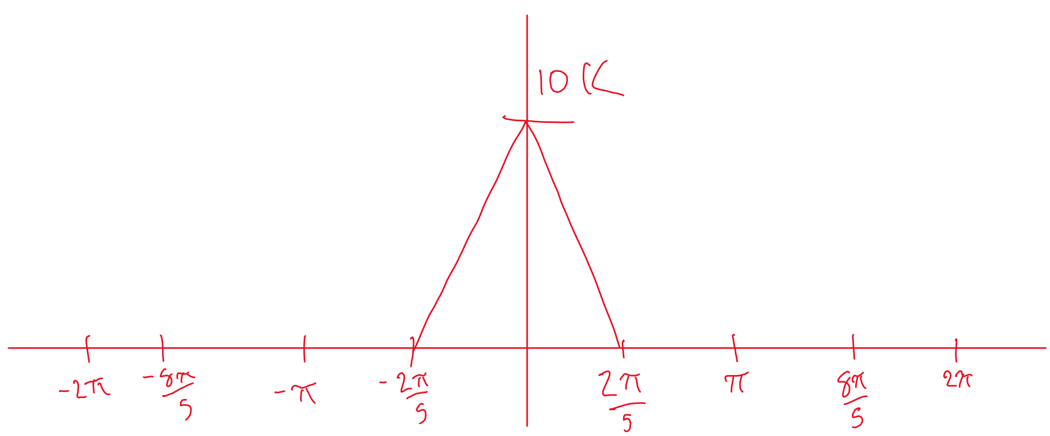
$f_1 = 10K\ Hz$



$H(\omega)$




$Y(\omega)$



$f_2 = 10\,KHz$

$Y(\Omega)$