# ECE 3640 - Discrete-Time Signals and Systems
# Midterm 1 - Spring 2022

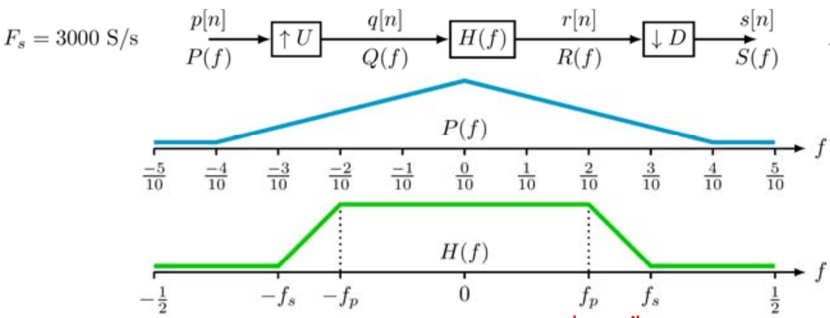
| Name: Zach Wilcox | Due: Friday, xx March 2022 at 11 PM on Canvas. |
|---|---|

**Instructions:**

1. Allowed resources: Your text book, your homework, your notes, the course web site, and Matlab.
2. Do not talk to anyone about this exam or get help from any source on this exam.
3. Write your answers in the spaces provided. Draw a box around your answers.
4. By signing in the name box above, you verify that you have complied with these instructions.

---

1. Consider the sample rate conversion system depicted below comprised by a $U$-fold exapander, a low pass filter $H(f)$, and a $D$-fold compressor.




$F = f F_s$

$\frac{U}{D} F_s = F_{out}$

$\frac{F_{out}}{F_s} = \frac{4k}{3k} = \frac{4}{3}$

$1/10$ $\left(\frac{1}{4} - \frac{1}{10}\right)$

This problem is to choose $U$ and $D$ and design the low pass filter $H(f)$ by selecting $f_p$ and $f_s$ to perform the sample rate conversion from $F_s = 3000$ samples/second at the input to $F_s = 4000$ samples/second at the output. Use the smallest integer factors $U$ and $D$ to perform the conversion. Choose $f_p$ and $f_s$ giving $H(f)$ the widest possible transition band. Write the parameters you choose in the space below.

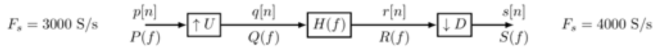
$U = 4$

$D = 3$

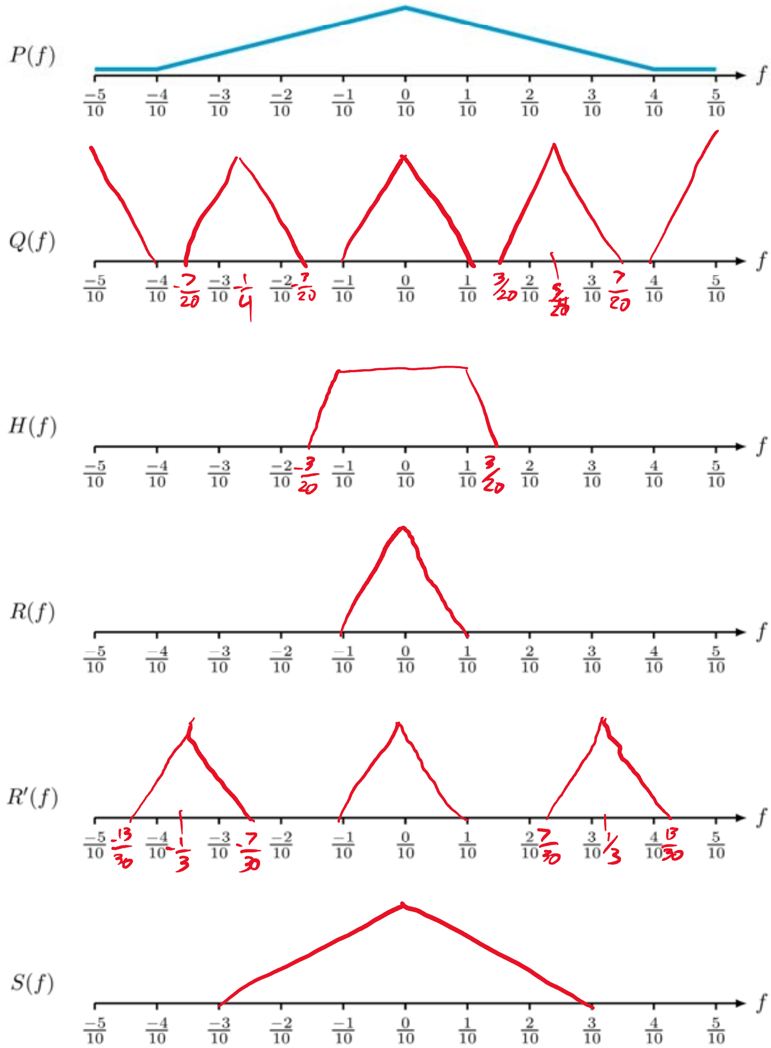
$f_p = 1/10$

$f_s = 3/20$

$$f_s = \frac{1 - f_m}{U} = \frac{1 - 4/10}{4} = 3/20$$




Sketch the spectra $Q(f), H(f), R(f), S(f)$ on the axes below. An extra axis has been provided to sketch $R'(f)$, the aliased version of $R(f)$. Label important frequencies in each plot. Important frequencies are the start, end, and center frequency of each "image" or "replica". Only the frequencies matter. Don't worry about the amplitudes in these plots.

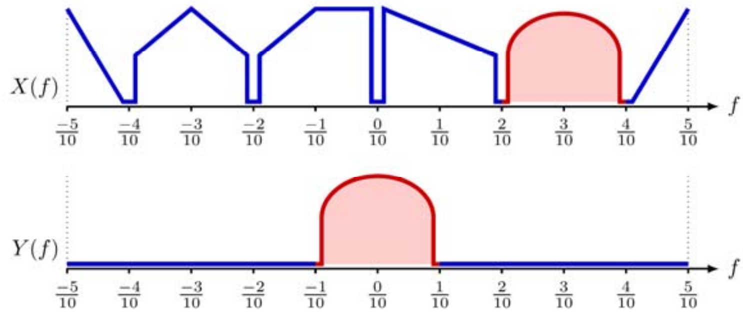



$\frac{f}{4}$

$\frac{1}{3} \pm \frac{1}{10}$

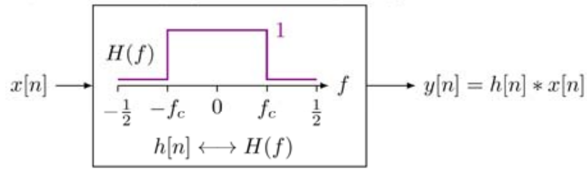
$3f$

2. Consider the signal $x[n]$ having a spectrum $X(f)$ shown in the figure below. Your job is to isolate at baseband the red-shaded component occupying $\frac{2}{10} \le f \le \frac{4}{10}$ cycles/sample. The resulting signal $y[n]$ has the spectrum $Y(f)$ shown below.

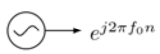



To accomplish this task, you are allowed to use the three different parts as shown below. Using these parts, design a system to process $x[n]$ into $y[n]$. Illustrate your design by drawing a block diagram. On the block diagram indicate the configuration parameters of each part used. On a piece of graph paper (such as the one provided for HW 5), neatly sketch the spectrum of the signal at the input(s) and output for each part in your diagram being careful to label the frequency axes and indicate any important frequencies.

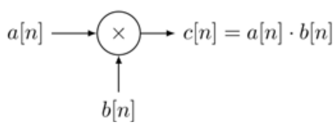
part name: low pass filter
parameter: $f_c$ (filter cutoff frequency)

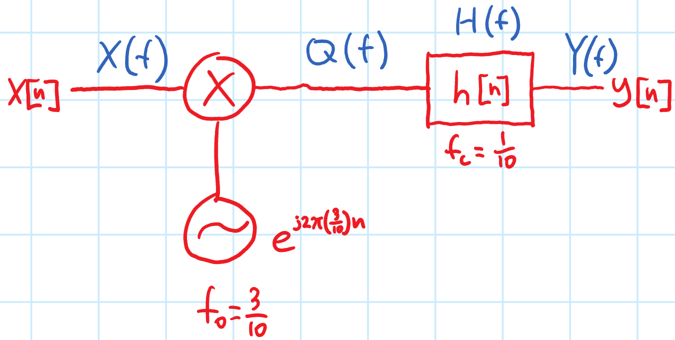



part name: oscillator
parameter: $f_0$ (oscillator frequency)

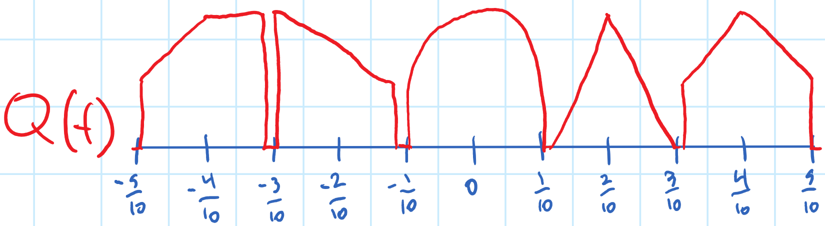



part name: mixer
parameter: none

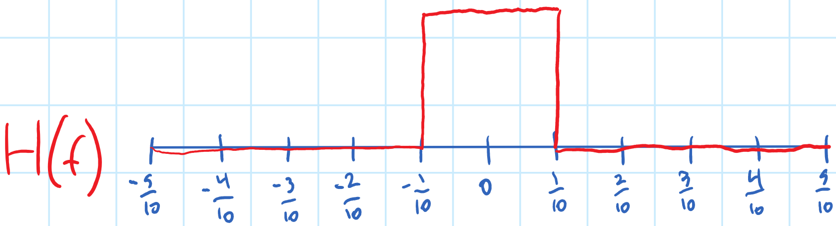




x[n]
X(f)
Q(f)
H(f)
h[n]
Y(f)
y[n]
$f_c = \frac{1}{10}$
$e^{j2\pi(\frac{3}{10})n}$
$f_0 = \frac{3}{10}$

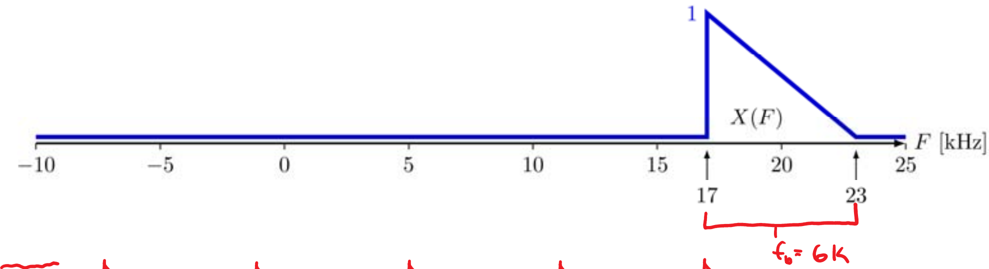



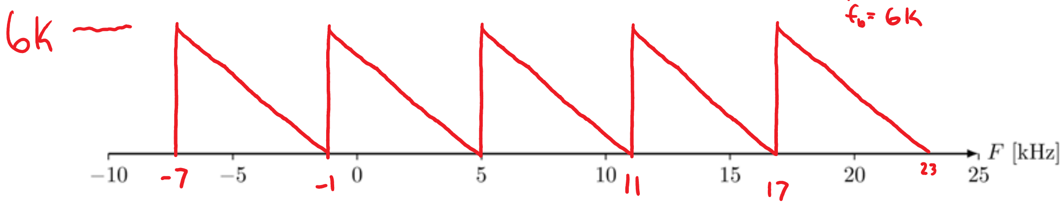
Q(f)
$-\frac{5}{10}$
$-\frac{4}{10}$
$-\frac{3}{10}$
$-\frac{2}{10}$
$-\frac{1}{10}$
0
$\frac{1}{10}$
$\frac{2}{10}$
$\frac{3}{10}$
$\frac{4}{10}$
$\frac{5}{10}$

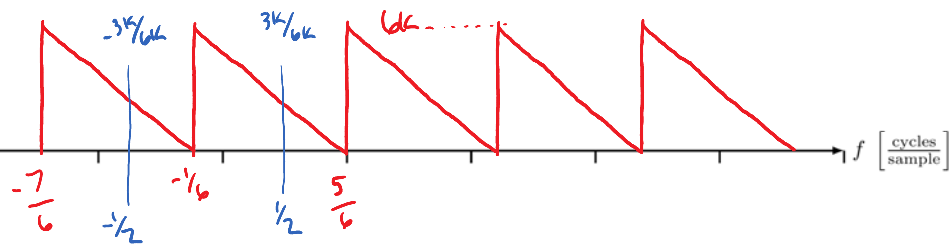



H(f)
$-\frac{5}{10}$
$-\frac{4}{10}$
$-\frac{3}{10}$
$-\frac{2}{10}$
$-\frac{1}{10}$
0
$\frac{1}{10}$
$\frac{2}{10}$
$\frac{3}{10}$
$\frac{4}{10}$
$\frac{5}{10}$

3. Use the principles of sampling to find the minimum sample rate $F_s$ that avoids aliasing when the complex-valued signal $x(t)$ is sampled. The CTFT $X(F)$ of $x(t)$ is pictured below. Note that $X(F) = 0$ at all frequencies not included in the picture. (Hint: The highest frequency in $x(t)$ is 23 kHz. Choosing $F_s > 2 \cdot 23 = 46$ kHz does avoid aliasing, but aliasing can be avoided at lower sample rates as well. This question asks you to find the minimum.)



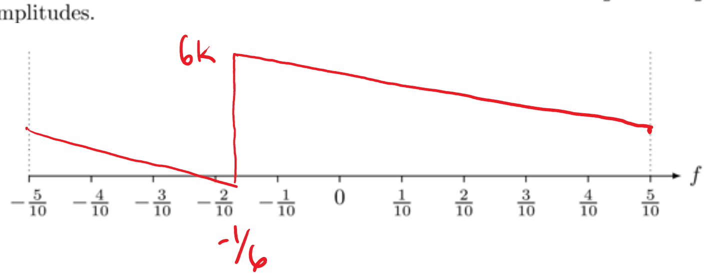

On the axis in the middle, sketch the result of applying the aliasing formula, and on the bottom axis sketch the result of applying the sampling formula and label the frequency axis. In these sketches, use the minimum sampling rate. What is the minimum sampling rate?

$$\text{aliasing formula:} \quad F_s \sum_{k=-\infty}^{\infty} X(F - kF_s) \tag{1}$$

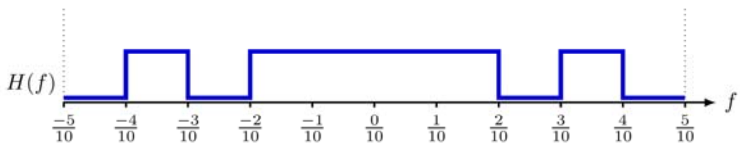
$F = fF_s$

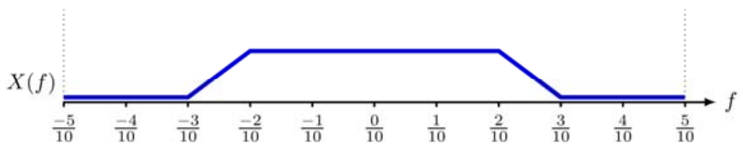
$$\text{sampling formula:} \quad F_s \sum_{k=-\infty}^{\infty} X([f - k]F_s) \tag{2}$$

On the axis below, sketch the spectrum of the sampled signal for frequencies $-\frac{1}{2} \le f < \frac{1}{2}$. Label important frequencies and amplitudes.



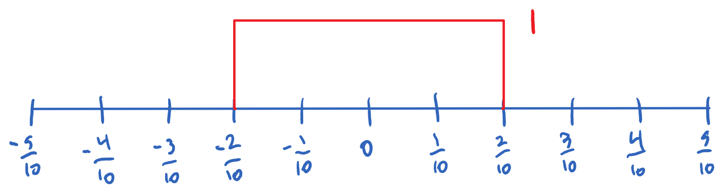
4. Consider a filter with frequency response $H(f)$ having multiple pass bands as shown below. Let $h[n]$ be the impulse response of this filter. Also consider the signal $x[n]$ having the spectrum $X(f)$ shown below. Calculate $y[n] = h[n] * x[n]$. Assume that the peak magnitudes in $H(f)$ and $X(f)$ are 1.




$y[n] = h[n] * x[n] \Leftrightarrow Y(f) = H(f)X(f)$

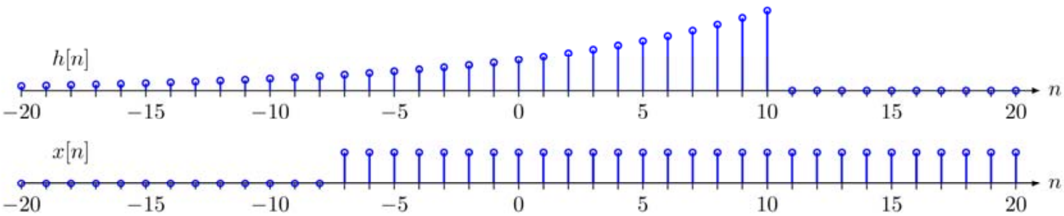
$Y(f)$




$Y(f) = u[f + \frac{2}{10}] - u[f - \frac{2}{10}]$

$y[n] = \int_{-\frac{1}{2}}^{\frac{1}{2}} Y(f) e^{j2\pi fn} = .4 e^{j2\pi fn}$

5. Consider a non-causal system with impulse response $h[n] = a^n u[10-n]$ which his plotted below for $a = 1.1$. Let the input to this system be $x[n] = u[n+7]$.




Convolution evaluates the sum

n= -10

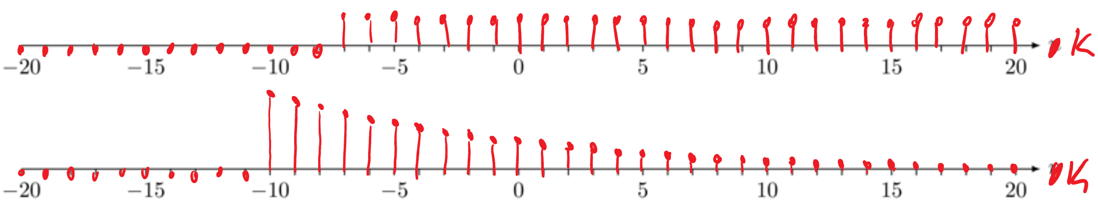
$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

h -10 - -20

+

h -10 - -10

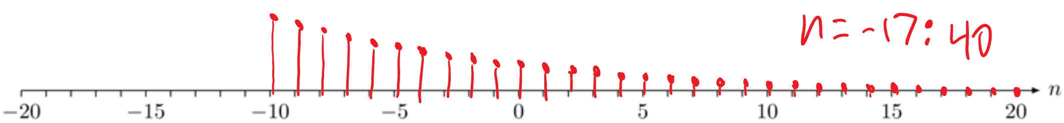
for each $n$. On the axes below, sketch $x[k]$ and $h[-k]$.



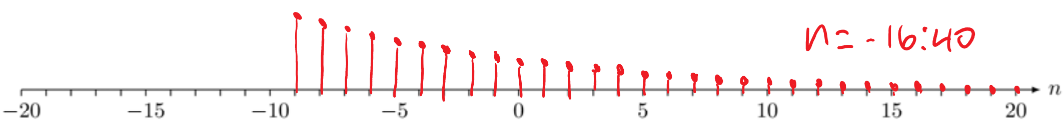

Considering $x[k]$ and $h[n-k]$, how many different overlap cases are there? Sketch $h[n-k]$ on the axes below for each of the cases and indicate the range of $n$ values for which each case holds.

4 cases

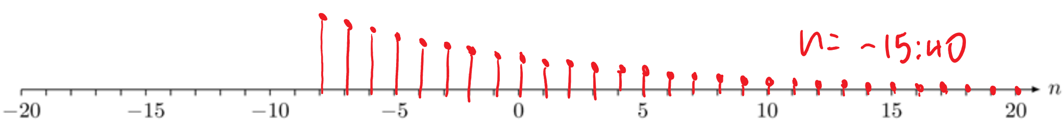
n=-17:40



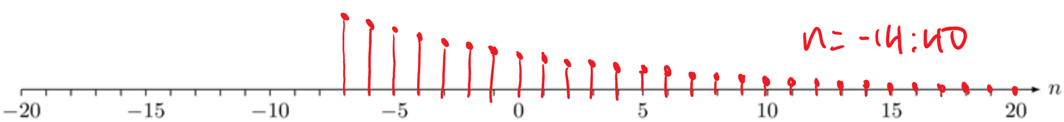
n=-16:40

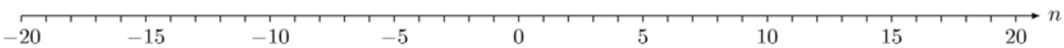


n= -15:40



n= -14:40







$M_1 = -20$, $M_2 = 7$ -7:20

For each of the distinct cases identified above, what are the simplified limits on the convolution sum? Put your answers on the sums below and include the time limits from above.

$$y[n] = \sum_{k=\boxed{-7}}^{\boxed{20}} x[k]h[n-k] = \sum_{k=\boxed{-20}}^{\boxed{7}} a^{n-k}, \qquad \boxed{n=0}$$

$y[0] = 70.15$

$$\sum_{k=n-M_2}^{n-M_1}$$

$$y[n] = \sum_{k=\boxed{-7}}^{\boxed{20}} x[k]h[n-k] = \sum_{k=\boxed{-19}}^{\boxed{7}} a^{n-k}, \qquad \boxed{n=1}$$

$y[1] = 69.76$

$N_1 + M_2 \le n \le M_1 + N_2$

$$y[n] = \sum_{k=\boxed{-7}}^{\boxed{20}} x[k]h[n-k] = \sum_{k=\boxed{-18}}^{\boxed{7}} a^{n-k}, \qquad \boxed{n=2}$$

$y[2] = 69.34$

$$y[n] = \sum_{k=\boxed{-7}}^{\boxed{20}} x[k]h[n-k] = \sum_{k=\boxed{-17}}^{\boxed{7}} a^{n-k}, \qquad \boxed{n=3}$$

$y[3] = 68.87$

$$y[n] = \sum_{k=\boxed{\phantom{0}}}^{\boxed{\phantom{0}}} x[k]h[n-k] = \sum_{k=\boxed{\phantom{0}}}^{\boxed{\phantom{0}}} a^{n-k}, \qquad \boxed{\phantom{n=0}}$$

Calculate the value of the output $y[n]$ at time $n = -2$ and the value at time $n = 5$.

6. The everlasting complex exponential signal $x[n] = e^{j2\pi 0.2n}$ is input to a LTI system having frequency response given by

$$H(f) = \frac{2 - 1.5371e^{-j2\pi f}}{1 - 1.5371e^{-j2\pi f} + 0.9025e^{-j4\pi f}}. \qquad \frac{Y(f)}{X(f)}$$

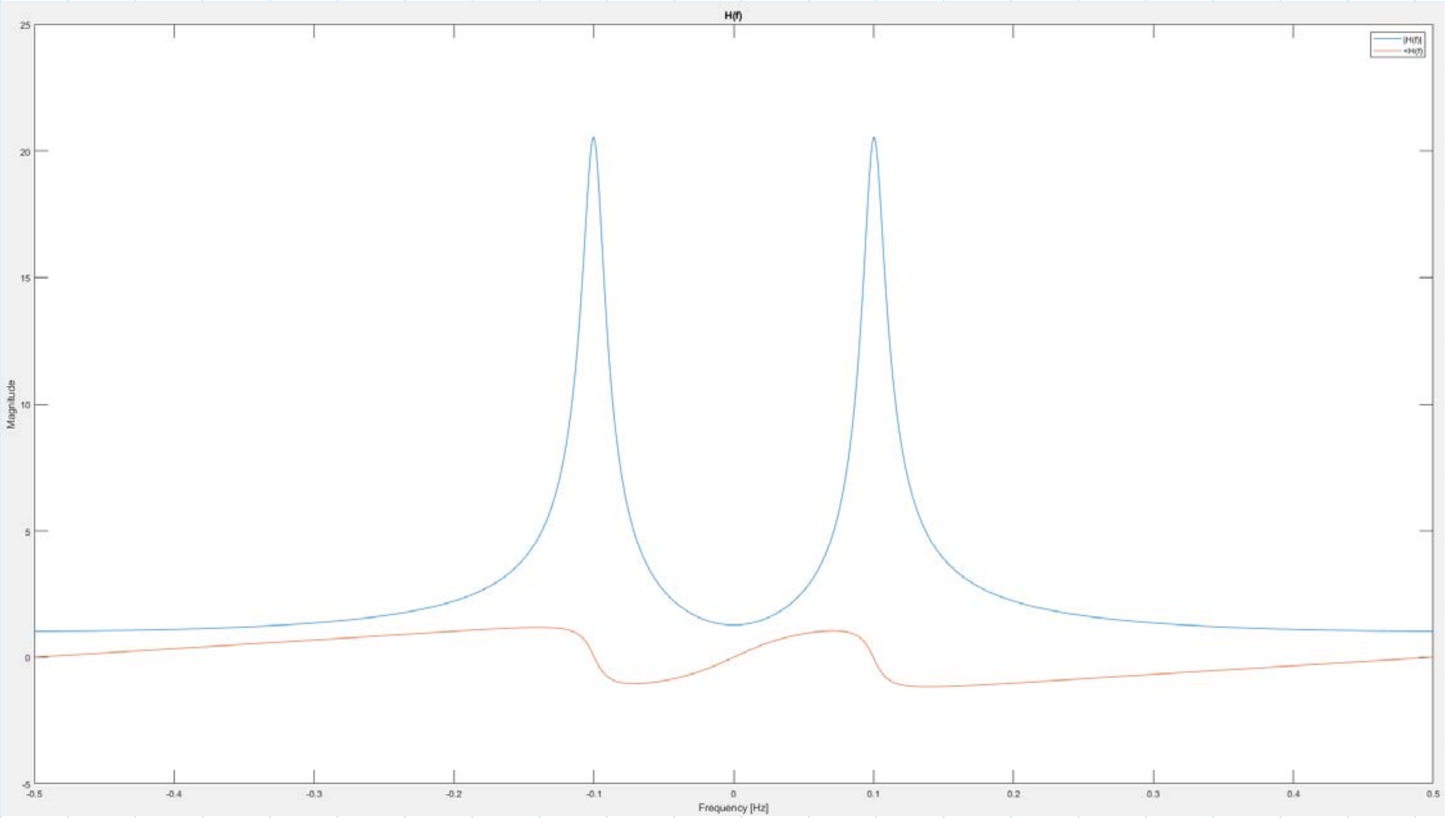
Write an expression for the output signal $y[n]$.

$y[n] = H(f)x[n] = H(f)e^{j2\pi f n}, \quad f = .2$

$$y[n] = \frac{2 - 1.5371e^{-j2\pi f}}{1 - 1.5371e^{-j2\pi f} + .9025e^{-j4\pi f}} e^{j2\pi(.2)n} \rightarrow 2.23e^{j1.02}$$

Plot the magnitude $|H(f)|$ and phase $\angle H(f)$ in Matlab for `f=[-0.5:0.001:0.5]`. (Include your code and plots on a separate page.) Given the symmetry of $H(f)$, what can you say about the impulse response $h[n]$? Given the continuity of $H(f)$, what can you say about the impulse response $h[n]$? (Is it absolutely summable, square summable (energy), or mean-square summable (power)?)

h[n] is also even, and absolutely summable




```
f = [-0.5:0.001:0.5];
H = @(f) (2 - 1.5371*exp(-1i*2*pi*f)) ./ (1 - 1.5371*exp(-1i*2*pi*f) + .9025*exp(-1i*4*pi*f));
Y = @(f) (2 - 1.5371*exp(-1i*2*pi*f));
X = @(f) (1 - 1.5371*exp(-1i*2*pi*f) + .9025*exp(-1i*4*pi*f));
plot(f,abs(H(f))); hold on;
plot(f,angle(H(f))); hold off;
title('H(f)');
legend('|H(f)|','<H(f)');
xlabel('Frequency [Hz]');
ylabel('Magnitude');
shg;
```

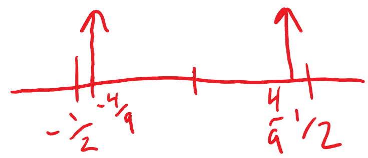
7. What are the frequencies and periods of the following signals. Give all frequencies in the interval $-\frac{1}{2} \le f < \frac{1}{2}$.

(a) $x[n] = \cos\left(2\pi \frac{430}{45} n\right), \quad -\infty < n < \infty$

$T = \frac{9}{86} \quad f = \frac{430}{45}$

$= \frac{1}{2} e^{j2\pi \frac{430}{45} n} + \frac{1}{2} e^{-j2\pi \frac{430}{45} n} \Rightarrow \delta\left(f - \frac{430}{45} n - k\right)$

$f = \frac{4}{9}, -\frac{4}{9}$




(b) $x[n] = \sin\left(2\pi \sqrt{\frac{430}{45}} n + \frac{4\pi}{3}\right), \quad -\infty < n < \infty$

$T = \frac{3}{\sqrt{86}} \quad f = \frac{\sqrt{86}}{3}$

$= -\frac{1}{2}\left(e^{j\frac{\pi}{6} - \frac{2}{3} j\sqrt{86}\pi n} - e^{-j\frac{\pi}{6} + \frac{2}{3} j\sqrt{86}\pi n}\right)$

$f = .09, -.09$