HW6

Monday, March 28, 2022 8:30 AM

Problem 10-1

Two 8-point sequences, $x[n]$ and $h[n]$, are shown in **Figure 10.19**.

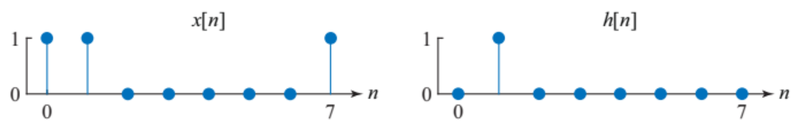


Figure 10.19

(a) Find the 8-point DFT of $x[n]$,

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N},$$

by brute-force computation.

```
3.0000    2.4142    1.0000    -0.4142    -1.0000    -0.4142    1.0000    2.4142
```

3, 2.4, 1, -.4, -1, -.4, 1, 2.4

(b) Find $X[k]$ by recognizing that it can be obtained by sampling the transform of $x[n] = \delta[n+1] + \delta[n] + \delta[n-1]$ at eight points.

$$e^{j2\pi f_0} + 1 + e^{-j2\pi f_0} = 1 + 2\cos(2\pi f_0)$$

$$1 + 2\cos(0) = 3$$

$$1 + 2\cos(2\pi \tfrac{1}{8}) \approx 2.4$$

$$1 + 2\cos(2\pi \tfrac{2}{8}) \approx 1$$

$$\vdots$$

$$1 + 2\cos(2\pi \tfrac{7}{8}) \approx 2.4$$

(c) Find the sequence $y[n]$ defined as the inverse DFT of $Y[k] = X[k]H[k]$. You *do not* have to compute $Y[k]$ to solve this problem.

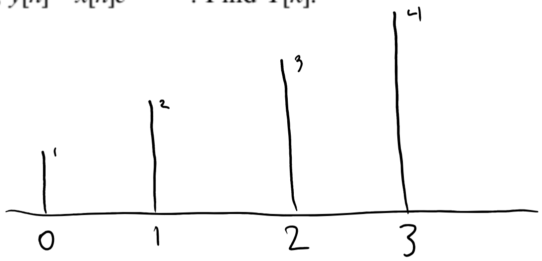
$$y[n] = x[n] \circledast h[n] =$$

1, 1, 1, 0, 0, 0, 0, 0,

Problem 10-2

(Note: second delta[n-2] term should be delta[n-3])

A sequence $x[n]$ of length $N=4$ has DFT $X[k]=\delta[k]+2\delta[k-1]+3\delta[k-2]+4\delta[k-3]$. A new sequence is created, $y[n]=x[n]e^{-j1.5\pi n}$. Find $Y[k]$.



$x[n] = 2.5 \quad -\frac{1}{2}-j\frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2}+j\frac{1}{2}$

$y[n] = 2.5 \quad \frac{1}{2}-j\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}+j\frac{1}{2}$

$Y[k] = 4 \quad 1 \quad 2 \quad 3$

Problem 10-3

Given $x[n]=[1\ 2\ 3\ 4\ 5\ 0\ 0\ 6]$, find the circular shift $y[n]=x[(n-n_0)_8]$ for the following values of n_0:

(a) $n_0=2$.

0 6 1 2 3 4 5 0

(b) $n_0=-2$.

3 4 5 0 0 6 1 2

(c) $n_0=1068$.

5 0 0 6 1 2 3 4

Problem 10-4

Find the circular convolution $y[n]=x[n]\circledast h[n]$ of the following two sequences,

$x[n]=[1\ 3\ 0\ 2]$
$h[n]=[1\ 1\ 0\ 1]$,

given

(a) $N=8$.

1 4 3 3 5 0 2 0

(b) $N = 6$.

3 4 3 3 5 0

(c) $N = 4$.

6 4 5 3

Problem 10-6

Use the multiplication property of the DFT to compute the product of the following two sequences,

$$x[n] = [2 \;\; 1 \;\; 0 \;\; 1]$$
$$w[n] = [1 \;\; -1 \;\; 1 \;\; -1].$$

$X[n]\,w[n] = X[k] \circledast W[k]$

(a) Find the 4-point DFTs $X[k]$ and $W[k]$.

$X[k] = 4 \quad 2 \quad 0 \quad 2$

$W[k] = 0 \quad 0 \quad 4 \quad 0$

(b) Compute

$$Y[k] = \frac{1}{N} X[k] \circledast W[k].$$

$\frac{1}{4}\left[0 \quad 8 \quad 16 \quad 8\right]$

$= 0 \quad 2 \quad 4 \quad 2$

(c) Find the IDFT $y[n]$, and verify that the result matches the multiplication of $x[n]$ and $w[n]$ in the time domain.

$$y[k] = \frac{1}{N} \sum_{n=0}^{N-1} Y[n] \, e^{j2\pi k \frac{n}{N}}$$

$$y[n] = 2 \quad -1 \quad 0 \quad -1$$

$$\begin{array}{r} 2 \\ \times\ 1 \\ \hline 2 \end{array} ✓ \qquad \begin{array}{r} 1 \\ \times\ -1 \\ \hline -1 \end{array} ✓ \qquad \begin{array}{r} 0 \\ \times\ 1 \\ \hline 0 \end{array} ✓ \qquad \begin{array}{r} 1 \\ \times\ -1 \\ \hline -1 \end{array} ✓$$

Problem 10-7

Two 4-point sequences, $x[n]$ and $h[n]$, are shown in **Figure 10.20**.

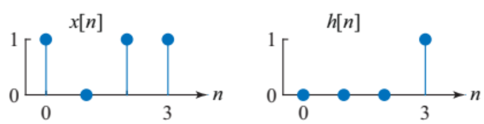


Figure 10.20

(a) Find $X[k]$ and $H[k]$, the 4-point DFTs of $x[n]$ and $h[n]$.

$$X[k] = \sum_{n=0}^{3} x[n] \, e^{-j2\pi k \frac{n}{N}} , \qquad H[k] = \sum_{n=0}^{3} h[n] \, e^{-j2\pi k \frac{n}{N}}$$

$$X[k] = 3 \quad j \quad 1 \quad -j$$

$$H[k] = 1 \quad j \quad -1 \quad -j$$

(b) The sequence $y[n]$ is defined as the inverse DFT of $Y[k] = X[k]H[k]$. Find $y[n]$ by computing $Y[k]$ and taking the inverse transform.

$$Y[k] = X[k]\,H[k] = 3 \quad -1 \quad -1 \quad -1$$

$$y[n] \ \frac{1}{4} \sum_{k=0}^{3} Y[k] \, e^{j2\pi n \frac{k}{N}}$$

$$= 0 \quad 1 \quad 1 \quad 1$$

(c) Find $y[n]$ by periodic convolution (e.g., linear convolution followed by time-domain aliasing) and show that the results are the same.

$x[n] * h[n]$

$= 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1$ but $7 > 4$, so

$x[n] \circledast h[n]$

$$
\begin{array}{rccccccccccccccc}
= & 0 & 0 & 0 & 1 & 0 & 1 & 1 & & & & & & & & \\
+ & & & & & 0 & 0 & 0 & 1 & 0 & 1 & 1 & & & & \\
+ & & & & & \downarrow & \downarrow & \downarrow & \downarrow & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
\hline
 & 0 & 0 & 0 & 1 & \boxed{0} & \boxed{1} & \boxed{1} & \boxed{1} & 0 & 1 & 1 & 1 & 0 & 1 & 1
\end{array}
$$

Problem 10-12

For each of the sequences $x[n]$ shown in **Figure 10.22**, find and plot the magnitude and phase of the DFT, $|X[k]|$ and $\measuredangle X[k]$.

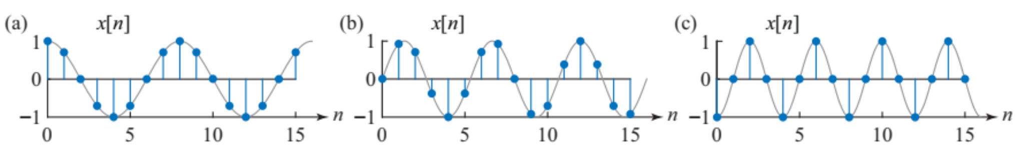
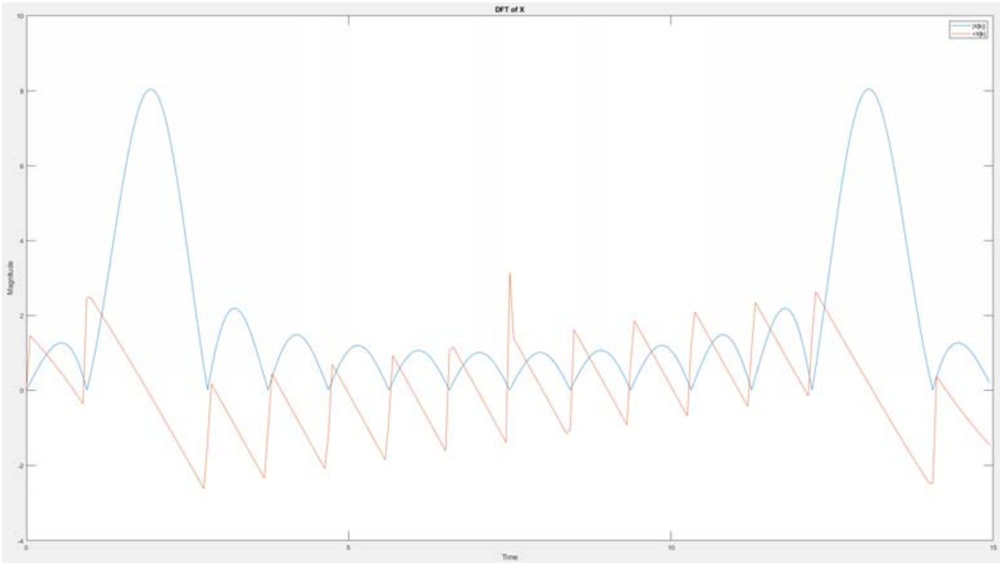
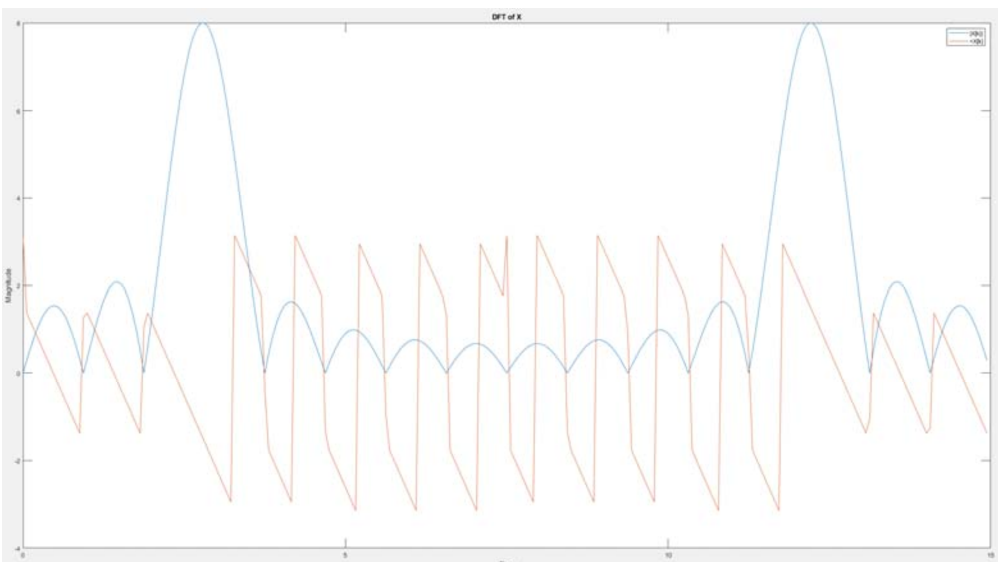


Figure 10.22

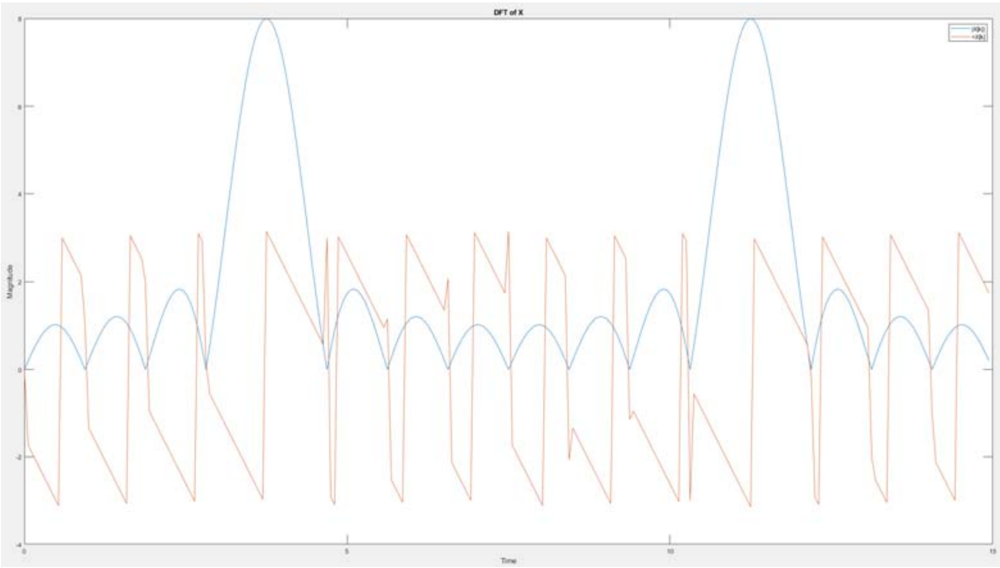
a: $y[n] = \cos\left(\frac{2\pi}{8} x\right)$



b: $\sin\left(\frac{3\pi}{8}x\right)$



C: $-\cos\left(\frac{\pi}{2}x\right)$



Problem 10-15

A signal $x[n] = [1\ 2\ 3\ 0\ 3\ 2]$ has DFT $X[k] = [11\ 0 - 4\ 3 - 4\ 0]$. Find $y[n]$, the sequence corresponding to the DFT $Y[k] = [3 - 4\ 0\ 11\ 0 - 4]$.

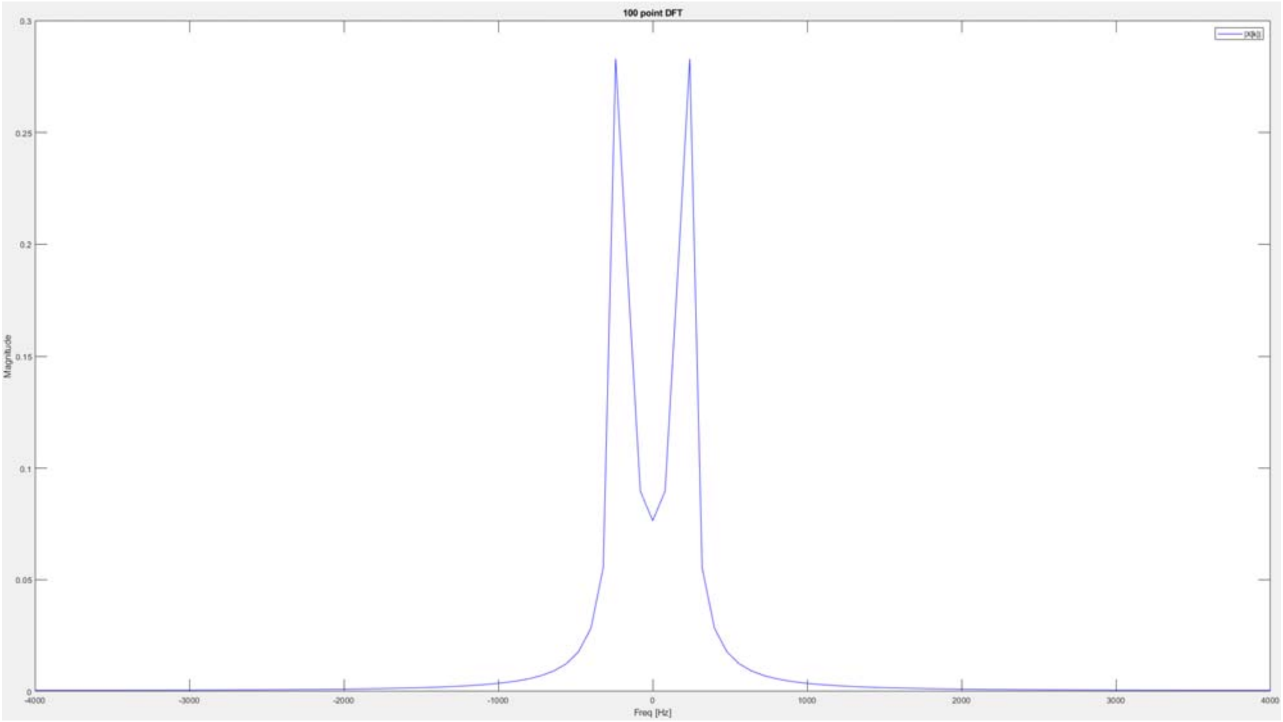
$$y[n] = \frac{1}{6} \sum_{k=0}^{5} Y[k] e^{j2\pi n \frac{k}{6}}$$

$$y[n] = 1 \quad -2 \quad 3 \quad 0 \quad 3 \quad -2$$

- The purpose of this problem is to derive or recall and use formulas to compute the frequency vector for a true scaled frequency axis in a spectral plot. Both even and odd length cases are considered.

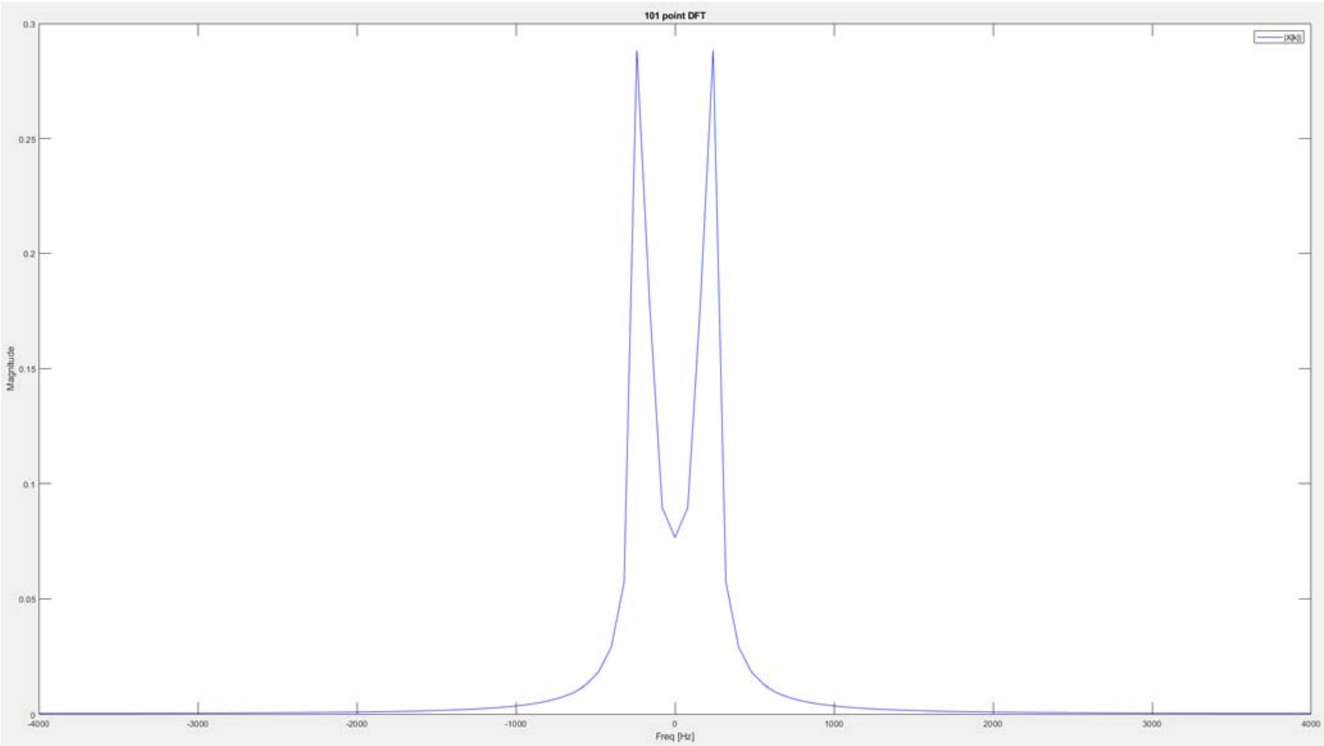
  Let $x_c(t) = \cos(2\pi 220 t)$ be sampled at $F_s = 8000$ samples/second leading to $x[n]$. By windowing, extract a length $L = 100$ set of samples.

a) Use the FFT function in Matlab to compute the $N = 100$ point DFT. Plot the magnitude of the DFT on a frequency axis scaled to show frequencies in Hertz with DC (zero frequency) in the center of the frequency axis. Turn in your plot and Matlab code.




```
Fs = 8000;
L = 100;
t = [0:1/Fs:L/Fs];
xt = cos(2*pi*220*t);
x  = xt.*t;

freq = Fs*(0:L/2)/L;

y = fft(x,100);

y = y(1:L/2+1);

plot(freq, abs(y), 'b'); hold on;
plot(-freq, abs(y), 'b'); hold off;
title('100 point DFT');
legend('|X[k]|');
xlabel('Freq [Hz]');
ylabel('Magnitude');
shg;
```

b) Use the FFT function in Matlab to compute the $N = 101$ point DFT. Plot the magnitude of the DFT on a frequency axis scaled to show frequencies in Hertz with DC (zero frequency) in the center of the frequency axis. Turn in your plot and Matlab code.




```
Fs = 8000;
L = 100;
t = [0:1/Fs:L/Fs];
xt = cos(2*pi*220*t);
x  = xt.*t;

freq = Fs*(0:L/2)/L;

y = fft(x,101);

y = y(1:L/2+1);

plot(freq, abs(y), 'b'); hold on;
plot(-freq, abs(y), 'b'); hold off;
title('101 point DFT');
legend('|X[k]|');
xlabel('Freq [Hz]');
ylabel('Magnitude');
shg;
```

- This problem explores the use of the FFT for performing linear convolution of two sequences.

  Let $x[n] = 1, n = 0, 1, 2, \cdots, 9$ be a length $L = 10$ sequence and let $h[n] = 1, n = 0, 1, 2, 3, 4$ be a length $M = 5$ sequence.

a) Using Matlab's conv function, compute the linear convolution $y[n] = x[n] * h[n]$. b) Let K be the length of the convolution result $y[n]$. For general L and M, what is the length K of $y[n]$ in terms of L and M? c) For the specific values of L and M in this problem, what is the length K of $y[n]$? d) The convolution property of the DFT may be stated as

$$\begin{array}{rcl} x[n] & \stackrel{\text{DFT}}{\longleftrightarrow} & X[k] \\ h[n] & \longleftrightarrow & H[k] \\ y[n] & \longleftrightarrow & Y[k] \\ y[n] = x[n] \circledast_N h[n] & \longleftrightarrow & Y[k] = X[k]H[k] \end{array}$$

where $x[n] \circledast_N h[n]$ denotes N-point circular convolution

$$y[n] = x[n] \circledast_N h[n] = \sum_{k=0}^{N-1} x[k]h[(n-k)_N],$$

where $(n-k)_N$ denotes $n-k$ modulo N. What condition on N guarantees that the circular convolution is equal to the linear convolution? e) For the values of L and M in this problem, what is the minimum value of N for linear-circular convolution equivalence? f) Using Matlab code, show how to use the FFT to perform fast convolution of $x[n]$ and $h[n]$. Rather than using the minimum value of N. Let the transform length be the smallest power of 2 greater than the minimum N. What is this N? Turn in your code and a plot of $y[n]$. Make sure that the result computed using the FFT matches the result obtained using conv.

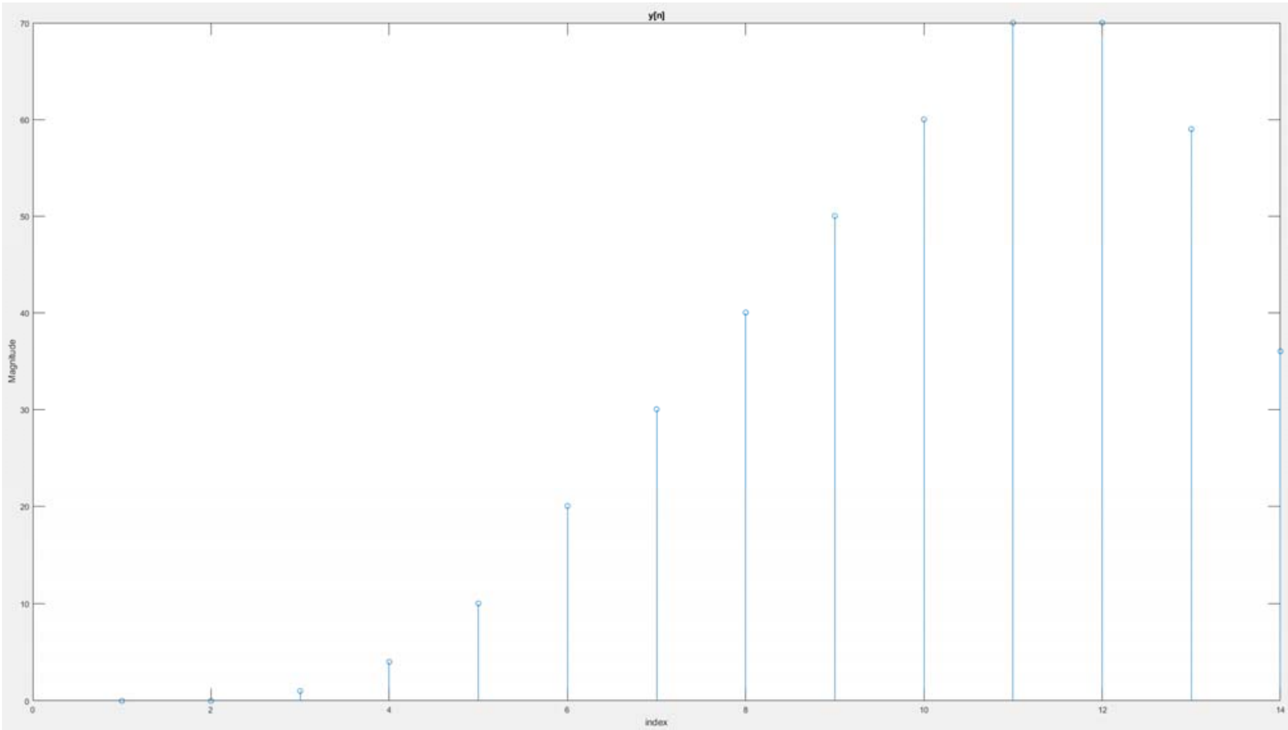
b: $L + M - 1$

c: 14

d: $N \geq L + M - 1$

e: 14

f: $N = 2^4 = 16$




```
clear x; clear h;
x = [0:9]; h = [0:4];% M = 5;

y = conv(x,h)
cy = cconv(x,h,14);
K = length(y);


fftconv = ifft(fft(x,2^4).*fft(h,2^4))

stem(y);
title('y[n]');
xlabel('index');
ylabel('Magnitude');
```

```
y =

     0     0     1     4    10    20    30    40    50    60    70    70    59    36


fftconv =

         0         0    1.0000    4.0000   10.0000   20.0000   30.0000   40.0000   50.0000   60.0000   70.0000   70.0000   59.0000   36.0000    0.0000         0
```