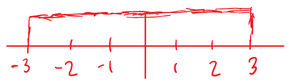
HW3 - LTI Systems

Tuesday, January 25, 2022 10:52 PM

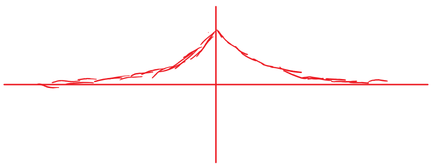
1. Let the impulse response $h[n]$ be given by the piece-wise defined sequence



$$h[n] = \begin{cases} 1, & -3 \le n \le 3, \\ 0, & \text{otherwise} \end{cases}$$

Convolve $h[n]$ with the following input signals. In each case, write the convolution result $y[n] = h[n] * x[n]$ as a piece-wise defined sequence.

(a) $x_1[n] = a^{|n|}$, where $|a| < 1$



$$y[n] = \sum_{k=n-3}^{n+3} a^{|k|} h[n-k], \quad -\infty < n < \infty$$

(b) $x_2[n] = a^n u[n]$, where $|a| < 1$

$$y[n] = \begin{cases} \sum_{k=n-3}^{n+3} a^{k} h[n-k], & 0 \le n \\ 0, & n < 0 \end{cases}$$

(c) $x_3[n] = a^n(u[n] - u[n-10])$

$$y[n] = \begin{cases} \sum_{k=n-3}^{n+3} a^{k} h[n-k], & 0 \le n \le 16 \\ 0, & n < 0 \text{ OR } n > 16 \end{cases}$$

(d) $x_4[n] = x_2[n] + 3x_2[n-10]$ (hint: use linearity and time invariance)

$$y_4[n] = y_2[n] + 3y_2[n-10]$$

$$a^n u[n] + 3a^{n-10} u[n-10]$$

$$y[n] = \begin{cases} \sum_{k=n-3}^{n+3} a^{k} h[n-k], & 0 \le n \le 7 \\ \sum_{k=n-3}^{n+3} a^{k} \cdot 3a^{n-10} h[n-k], & 7 < n < 13 \\ 0 & n < 0, \end{cases}$$

(e) $x_5[n] = e^{j2\pi f n}$ (everlasting complex exponential) and evaluate the output when $f = \frac{3}{7}$ and when $f = \frac{5}{14}$.

$$y[n] = \sum_{k=n-3}^{n+3} e^{j2\pi f n} h[n-k], \quad -\infty < n < \infty$$

$$\sum_{k=n-3}^{n+3} e^{j2\pi \frac{3}{7} n} h[n-k]$$

$$\sum_{k=n-3}^{n+3} e^{j2\pi \frac{5}{14} n} h[n-k]$$

(f) $x_6[n] = e^{j2\pi f n} u[n]$ (causal complex exponential) and evaluate the output when $f = \frac{3}{7}$

$$y[n] = \sum_{k=n-3}^{n+3} e^{j2\pi f n} h[n-k], \quad 0 \leq n < \infty$$

$$\sum_{k=n-3}^{n+3} e^{j2\pi \frac{3}{7} n} h[n-k]$$

(g) Is the system with impulse response $h[n]$ causal?

no, System is non-causal

(non-zero values for $h[-n]$ exist)

(h) Is the system with impulse response $h[n]$ stable?

yes, $\sum_{-\infty}^{\infty} |h[n]| < \infty$ ✓

2. Let $y[n] = x[n] * x^*[-n]$. Give an interpretation of $y[0]$.

$x[0] = a + jb$
$x^*[0] = a - jb$

$y[0] = x[0] * x^*[0] = a^2 + b^2$

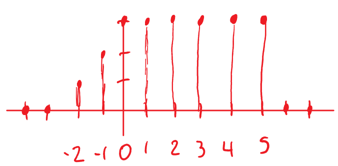
3. Let $h[n]$ be given by

$$
\begin{aligned}
h[n] &= 0, \quad n \leq -3 \\
h[-2] &= 1 \\
h[-1] &= 2 \\
h[n] &= 3, \quad n = 0, 1, 2, 3, 4, 5 \\
h[n] &= 0, \quad n \geq 6
\end{aligned}
$$

Let $x[n] = 0.9^n u[n]$. Do the following:

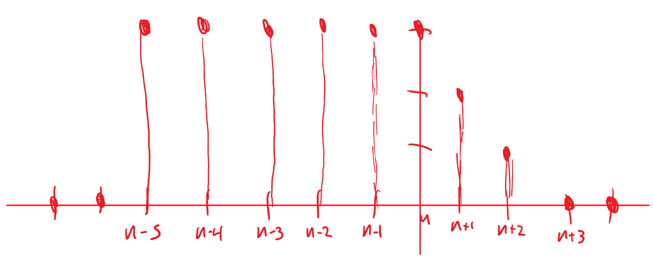
(a) Sketch $h[-k]$ on the k axis

$h[k] =$

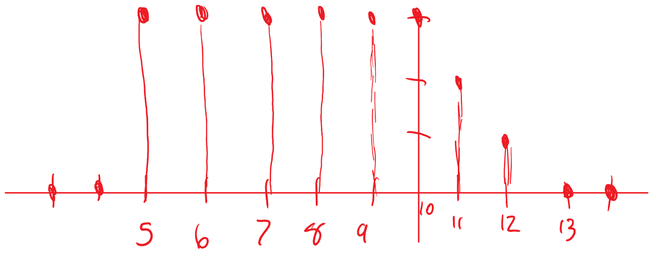


$h[-k] =$

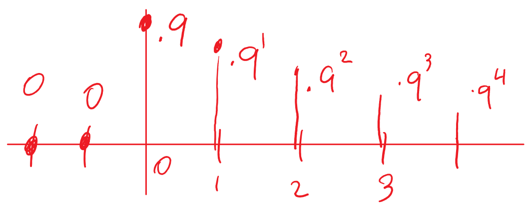
(b) Sketch $h[n-k]$ on the k axis



(c) Sketch $h[10-k]$ on the k axis



(d) Sketch $x[k]$ on the k axis



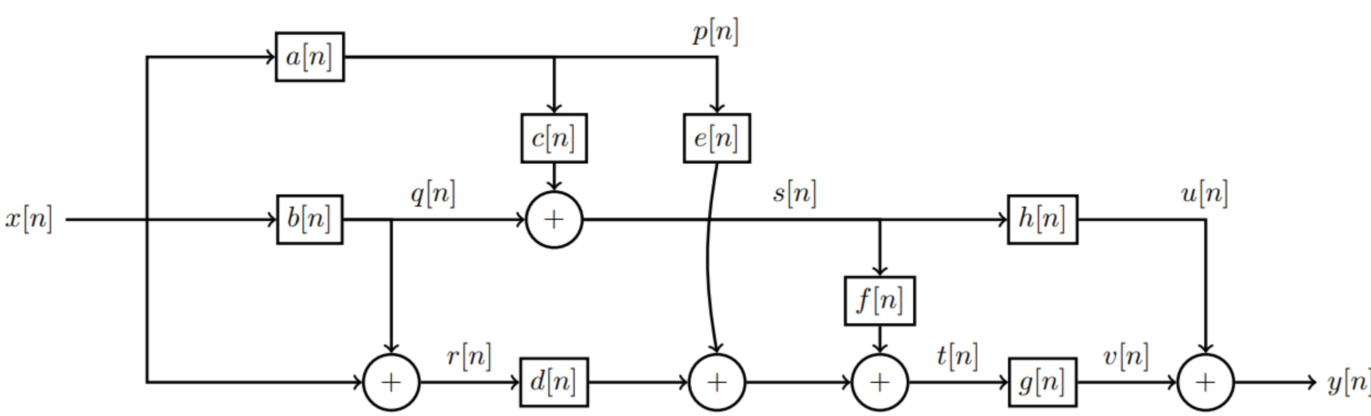
(e) Compute the value of $y[10]$.

$$y[10] = \sum_{k=-2}^{17} x[k]\, h[10-k]$$

$$= 9.217$$

In your sketches, include the region $k = -2, -1, 0, \cdots, 15$.

4. Consider the cascade interconnection of LTI systems show below. Find an expression for the impulse response of an equivalent system.



- $d[n]\ g[n]\ x[n]$
- $b[n]\ d[n]\ g[n]\ x[n]$

$b[n]\ f[n]\ g[n]\ x[n]$

$b[n]\,h[n]\,x[n]$

$a[n]\,c[n]\,h[n]\,x[n]$

$a[n]\,c[n]\,f[n]\,g[n]\,x[n]$

$+\;a[n]\,e[n]\,g[n]\,x[n]$

$= d[n]\,g[n]\left((1+b[n])+f[n]\right) + b[n]\,h[n]$

$+\;a[n]\left(c[n]\,h[n] + c[n]\,f[n]\,g[n] + e[n]\,g[n]\right)$

5. Let $h[0], h[1], \cdots, h[L-1]$ be a length L impulse response and let $x[0], x[1], \cdots, x[M-1]$ be a length $M > L$ input sequence. Let $y[0], y[1], \cdots, y[N-1]$ be the length N convolution result.

(a) What is the length N of the output $y[n]$ in terms of L and M?

$N = L+M-1$

(b) How many samples of the output $y[n]$ (and which ones) are starting transients?

$M-L$ Samples $\quad y[0] \ldots y[M-L]$

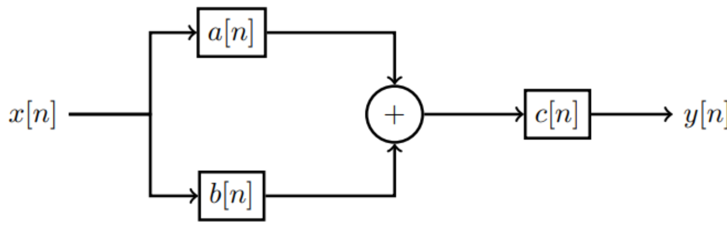
(c) How many samples of the output $y[n]$ (and which ones) are ending transients?

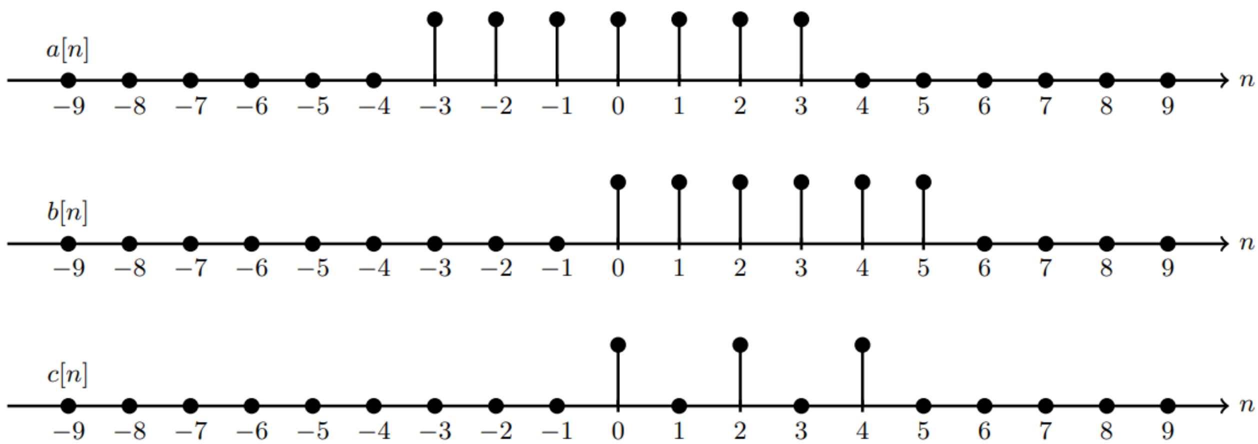
$M-L$ Samples $\quad y[(N-1)-(M-L)] \ldots y[N-1]$

(d) How many samples of the output $y[n]$ (and which ones) are valid output samples.

$N-2(M-L)$, $\quad y[M-L+1] \ldots y[(N-2)-(M-L)]$

6. Sketch the impulse response for the equivalent system.





$$h[n] = (a[n] + b[n]) * c[n]$$

$a[n] + b[n]$