

$$(1 + x + x^2 + x^3 + x^4 + \dots + x^n) \% \text{MOD}$$

$$1 \leq x, N \leq 10^9, \text{MOD} = 10^9 + 7$$

$$\left[1 + \frac{x(x^n - 1)}{x - 1} \right] \% \text{MOD}$$

$$(A \div B) \% M = A \% M \div B \% M$$
~~$$\frac{A}{B} \% M \neq \frac{A \% M}{B \% M}$$~~

$$f(x, n) \rightarrow (1 + x + x^2 + x^3 + \dots + x^n) \% \text{MOD}$$

$$f(oh) \rightarrow$$

$$a^8 = (a^4)^2$$

$$\log_2 N$$

$$f(x) \rightarrow (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7) \% \text{MOD}$$

$$= 1 \cdot (1 + x + x^2 + x^3) + x^4 (1 + x + x^2 + x^3)$$

$$\dots \dots \dots (1 + x + x^2 + x^3) \% \text{MOD}$$

$$= (1+x^4)(1+x+x^2+x^3) \pmod{409}$$

fermat's little

$$\boxed{a^p \equiv a \pmod{p} \quad \text{given } p \text{ is prime}}$$

$$\boxed{a^{p-1} \equiv 1 \pmod{p}}$$

$$4^6 \pmod{7} = 1$$

$$a^{p-1} \equiv 1 \pmod{p}$$

$$\frac{a}{b} \pmod{M}$$

$$\Rightarrow (a \cdot b^{-1}) \pmod{M}$$

$$\Rightarrow \frac{a^{p-1}}{a} \equiv \frac{1}{a} \pmod{p}$$

$$\Rightarrow (a \pmod{M} \times \boxed{b^{-1} \pmod{M}})$$

$$\Rightarrow a^{-1} \cdot a^{p-1} \equiv a^{-1} \pmod{p}$$

$$\Rightarrow a^{p-2} \equiv a^{-1} \pmod{p}$$

$$\frac{1}{b} \pmod{M} \mid \frac{a^x}{b^y} \pmod{M}$$

$$\Rightarrow a^x \pmod{M} \times b^{-y} \pmod{M}$$

$$\Rightarrow a^x \pmod{M} \times (b^{(-1) \cdot y}) \pmod{M}$$



$$\Rightarrow a^x \pmod{M} \times (\boxed{b^y})^{-1} \pmod{M}$$

$$\Rightarrow a^x \pmod{M} \times (b^{-1})^y \pmod{M}$$

$$\frac{a}{b} \pmod{M}$$

$$(a \pmod{M} \times b^{M-2} \pmod{M}) \pmod{M}$$

$$\boxed{a \pmod{M} \times b^{-1} \pmod{M}}$$

$$a^{xy} = (a^x)^y$$

$$1 \dots 2 \dots 2^3 \dots 2^m \pmod{M}$$

$$\log_2 N$$

$$1+x+x^2+x^3+\dots x^n \quad \text{v. 3}$$

$$\log_2 N$$

$$\frac{x^{n+1}-1}{x-1} \quad \text{v. 17}$$

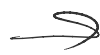
$$a \frac{n^{k+1}-1}{n-1}$$

$$\rightarrow \log_2 17$$

$$x=2, n=1$$

$$1+2=3$$

$$N_{CR}$$



$$\frac{20}{5} \text{ v. 3} = 2$$

$$\frac{a}{b} \text{ v. 17}$$

$$\frac{20}{6} \text{ v. 5}$$

$$\frac{20 \times 3}{6} \text{ v. 9}$$

$$\sqrt{-1}$$

$$\frac{20}{6} \text{ v. 7}$$

$$= 20 \text{ v. 7} \times 6^5 \text{ v. 7}$$

$$= 6 \times 6$$

$$\frac{10 \times 20 \times 3}{6} \text{ v. 7}$$

$$= 10 \text{ v. 7}$$

$$= 3$$

$$= 6 \times 6$$

$$= \textcircled{36} \% 7$$

$$= 1$$

$$\begin{array}{l} = 3 \\ \hline \textcircled{\frac{30}{6} \% 7} \times 3 \% 7 \end{array}$$

$$= 1 \times 3$$

$$= 3$$

$${}^N C_R = \frac{N!}{R! (N-R)!}$$

$$10^5$$

$${}^N C_R \% 10^9 + 7$$

$$N, R \leq 10^5$$

$$\textcircled{\sqrt{N}}$$

$$Q \sqrt{N}$$

$$0 \rightarrow 59$$

$$\frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \frac{N}{7}$$

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59

~~50~~ ~~51~~ ~~52~~ 53 ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ 59

$$H[4] = 1$$

$$H[6] = 1$$

$$7.1$$

$$7.2$$

$$7.3$$

$$7.4$$

$$7.5$$

$$7.6$$

$$\rightarrow 7.7$$

$$\frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \frac{N}{7} + \frac{N}{11} + \dots + \frac{N}{\sqrt{N}}$$

$$N \left[\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \dots + \frac{1}{\sqrt{N}} \right]$$

$$\log_2$$

$$\log_2$$

$$2.7...$$

Harmonic Series

$$H(N) \gg 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots + \frac{1}{N} \approx \ln N + \gamma$$

$$< N / \ln N$$

$$\approx N \log \log N$$

$$\approx N$$

$$10^7 \quad \log 10^7$$

$$= 25$$

$$\log 25$$

$$\approx 6$$