

**Proposition:** Let  $E$  be an unsorted set of  $n$  segments that are the edges of a convex polygon. Describe an  $O(n \log n)$  algorithm that computes from  $E$  a list containing all vertices of the polygon, sorted in clockwise order.

*Proof.* Let  $E = \{e_1, \dots, e_n\}$  defined as  $e_i = p_i q_i$  where  $p_i$  and  $q_i$  are the end points of the line segments and  $p_i < q_i$  using lexicographic ordering. Now define a ordering on the elements of  $E$  by  $e_i < e_j$  if and only if  $p_i(x) < p_j(x)$  or  $p_i(x) = p_j(x)$  and  $q_i(y) < q_j(y)$ .

Algorithm Vertices of polygon of edges Input:  $E$  Output: a ordered list of vertices of the polygon of  $E$  is clockwise order

1. Order  $E$  by the relation above so that  $E = \{S_1, \dots, S_n\}$ .
2. Let  $\mathcal{L}_U$  be a set and let  $p_1, q_1$  from  $S_1$  be put in to it.
3. keep =  $q_1$
4. For  $i = 1$  to  $n$
5.   if  $p_i = \text{keep}$ , then add  $q_i$  in to  $\mathcal{L}_U$
6.       keep =  $q_i$ .
7. let  $\mathcal{L}_L$  be the lower vertices and put keep in to it from the last loop
8. for  $i = n$  to 1
9.   if  $q_i = \text{keep}$
10.       then add  $p_i$  to  $\mathcal{L}_L$ , keep =  $p_i$ .
11. Delete the last and first points in  $\mathcal{L}_L$  and we can take the order list as  $\mathcal{L}_U \cup \mathcal{L}_L$ .

