Proposition: The Convex hull of a set S in the plane is defined to be the intersection of all convex sets that contains S. For the convex hull of a set of points it was indicated that the convex hull is the convex set with smallest perimeter. We want to show that these are equivalent definitions.
(a) Prove that the intersection of two convex sets is agian convex.
(b) Prove that the smallests perimeter polygon $\mathcal{P}$ containing a set of points P is convex.
(c) Prove that any convex set containing the set of points P contains the smallest perimeter polygon $\mathcal{P}$.

Proof. (a) is easy
(b), suppose $\mathcal{P}$ is the smallests preiter polygon containing a set of poitns P. Suppose for a contradiction it is not convex, then there rae two vertices p, q such that the line from p to q is not in $\mathbb{P}$. But that means we can find a point on that line and add it to the polygon to get a even smaller perimeter, since the line is always shorter distance then the additino of the two sides from the triangle by the triangle inequality.
(c) Let C be any convex set containing the points P. Let $\mathcal{P}$ be the smallests perimeter polygon that conains P. We can take the intersection of C with $\mathcal{P}$ and take the collectin of vertices of $\mathcal{P}$ contained in C and the intersection points(there are a finite number of them since we are intersecting it with a polygon) and make a polygon that all of the points P (since both sets did). By minimality we must have $\mathcal{P}$ is contained in C.

Problem 1.2

Proposition: problem 1.2 in book

Proof. Let P be a set of points. Let $\mathcal{P}$ be a convex polygon that with vertices from P and contains all points of P. Let $\widetilde{\mathcal{P}}$ be the smallest premeter polygon that contains P. Therefore we have

$$\mathcal{P} \subseteq \widetilde{\mathcal{P}}.$$

Now since $\widetilde{\mathcal{P}}$ is the smallest priemeter polygon that contains P which is also convex. Then it must contain $\mathcal{P}$ as well since it is convex and all of its vertices are in P. Hence they are the same. THen it is clear that it is the intersection of all convex sets that contain P since

$$\mathcal{P} \subseteq \cap C_\alpha \subseteq \widetilde{\mathcal{P}}.$$