problem 1.8

Proposition: We shall develop an algorithm based on another paradigm, namely divide-and-conquer.
(a) Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be two disjoint convex polygons with n vertices in total. Give an $O(n)$ time algorthm that computes the convex hull of $\mathcal{P}_1 \cup \mathcal{P}_2$.
(b) Us ethe algorithm from part (a) to develop an $O(n \log n)$ time divide-and-conquer algorithm to compute the convex hull of a set of n points in the plane.

Proof. (a) Let $\mathcal{P}_1 = \{x_1, \ldots, x_k\}$ and $\mathcal{P}_2 = \{x_{k+1}, \ldots, x_n\}$. Algorithm:

1. find the vertex in $\mathcal{P}_1$ that is the smallest in terms of lexo, and do the same for $\mathcal{P}_2$ this is O(n) since we need to go through it once
2. List=$\{x_1, x_2\}$ set the fist two points as x_1, x_2
3. set $index_1 = 3$, $index_2 = 1$. These are the current vertices we are at for each convex polygon
4. $j = 3$; this is the current place where we need to fill in
5. flip index $i = 2$; this is the ability to flip between which polygon we are testing, since we need to flip between them, otherwise we would just complete one of the convex polygons right in the beginning with out getting anywhere
6. while($index_1 \neq$ last index for $\mathcal{P}_1$ and $index_2 \neq$ last index for $\mathcal{P}_2$)
7. add x_{index_i} in to $list$ at the jth spot
8. $index_i++$.
9. if(check if $list[j-2], list[j-1], list[j]$ turn left)
10. delete $list[j-1]$ and $index_j--$ that corresponded to this vertex that we deleted, which is $i + 1 mod 2$
11. $j--$
12. else
13. $j++$
14. flip $i = i + 1 \ mod 2$.
15. the list that we have will have all the vertices to the new convex polygon in clockwise order.