Hidden Nonlocality

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Bell's Inequalities, Nonlocality and Entangled States

A bell inequality is described by a set of real numbers $\gamma(a,b,x,y) \in R$ and a local bound L $\in R$ such that, if $p_L(ab|xy)$ are the probabilities of a Bell local strategy, it holds that

$$\sum_{abxy} \gamma(a,b,x,y) p_L(ab|xy) \leq L.$$

The **violation any Bell inequality** (e.g., the CHSH inequality) **is sufficient to prove the nonlocality of a system**. While **all entangled pure states violate some Bell inequality** [5], this is **not always true for mixed states**. Some entangled states, such as Werner's state, appear local and do not violate Bell's inequalities after a single ideal measurement [6]. However, it is known that all bipartite entangled states display some hidden nonlocality [11].

Werner's state for $d \geq 5$, does not violate Bell inequalities directly but can after sequential measurements, represented by:

$$W = \frac{1}{d^2}\left(\frac{1}{d}I^{d\times d} + 2\sum_{i<j;i,j=1}^{d} |S_{ij}\rangle\langle S_{ij}|\right) \text{ with } |S_{ij}\rangle = \frac{1}{\sqrt{2}}\left(|i\rangle_1|j\rangle_2 - |j\rangle_2|i\rangle_1\right).$$

The **two-qubit Werner state**, remains local, considering arbitrary POVMs, after any local filtering.

$$\rho_W(\alpha) = \alpha|\phi^+\rangle\langle\phi^+| + (1-\alpha)\frac{I}{4} \text{ with } |\phi^+\rangle = (|00\rangle + |11\rangle)\frac{1}{\sqrt{2}} \text{ and } \alpha \leq 0.3656.$$

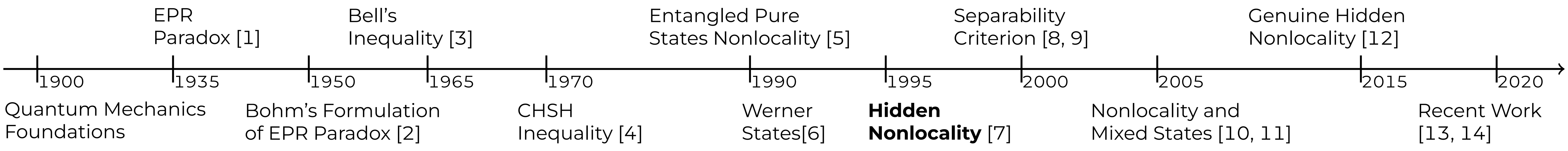

Figure 1. Timeline of key developments in the study of hidden nonlocality and related concepts in quantum mechanics.

Local Filtering

A **local filtering** is a four-outcome measurement that originates from the local two-outcome measurements $A = \{F_A, \overline{F}_A\}$ for Alice and $B = \{F_B, \overline{F}_B\}$ for Bob. Explicitly, it **corresponds to the global measurement**

$$F = \{F_A \otimes F_B, F_A \otimes \overline{F}_B, \overline{F}_A \otimes F_B, \overline{F}_A \otimes \overline{F}_B\}.$$

This technique is **particularly useful for mixed states**, where single ideal measurements may not reveal nonlocality. Popescu in [7] shows that applying local operations can "filter" the state, making its nonlocal properties more apparent.

For example, in a Werner state, local filtering can be applied using projectors P and Q defined on smaller subspaces:

$$P = |1\rangle_{11}\langle 1| + |2\rangle_{11}\langle 2| \quad \text{and} \quad Q = |1\rangle_{22}\langle 1| + |2\rangle_{22}\langle 2|.$$

The filtered state after Alice applies P and Bob applies Q followed by measurement A or A' on Alice's side and B or B' on Bob's side is:

$$W' = \frac{(P\otimes Q)W(P^\dagger \otimes Q^\dagger)}{Tr((P\otimes Q)W(P^\dagger \otimes Q^\dagger))} = \frac{2d}{2d+4}\left(\frac{1}{2d}I^{2\times 2} + |S_{12}\rangle\langle S_{12}|\right).$$

Here, Alice randomly chooses one of the operators A, and A' to measure (after measurement of P). Similarly, Bob randomly chooses between B, and B'. The nondegenerate part (pertaining to the eigenvalues 1, -1) of the A, A', B, B' operators are chosen to maximize the violation of the Clauser, Horne, Shimony and Holt (CHSH) inequality [4] for the singlet state $|S_{12}\rangle$. In this new state, **Bell inequalities can be violated**. For $d \geq 5$, the filtered Werner state W' results in

$$\text{Tr}(W'(AB + AB' + A'B - A'B')) = \frac{2d}{2d+4} \cdot 2\sqrt{2} \geq 2.$$

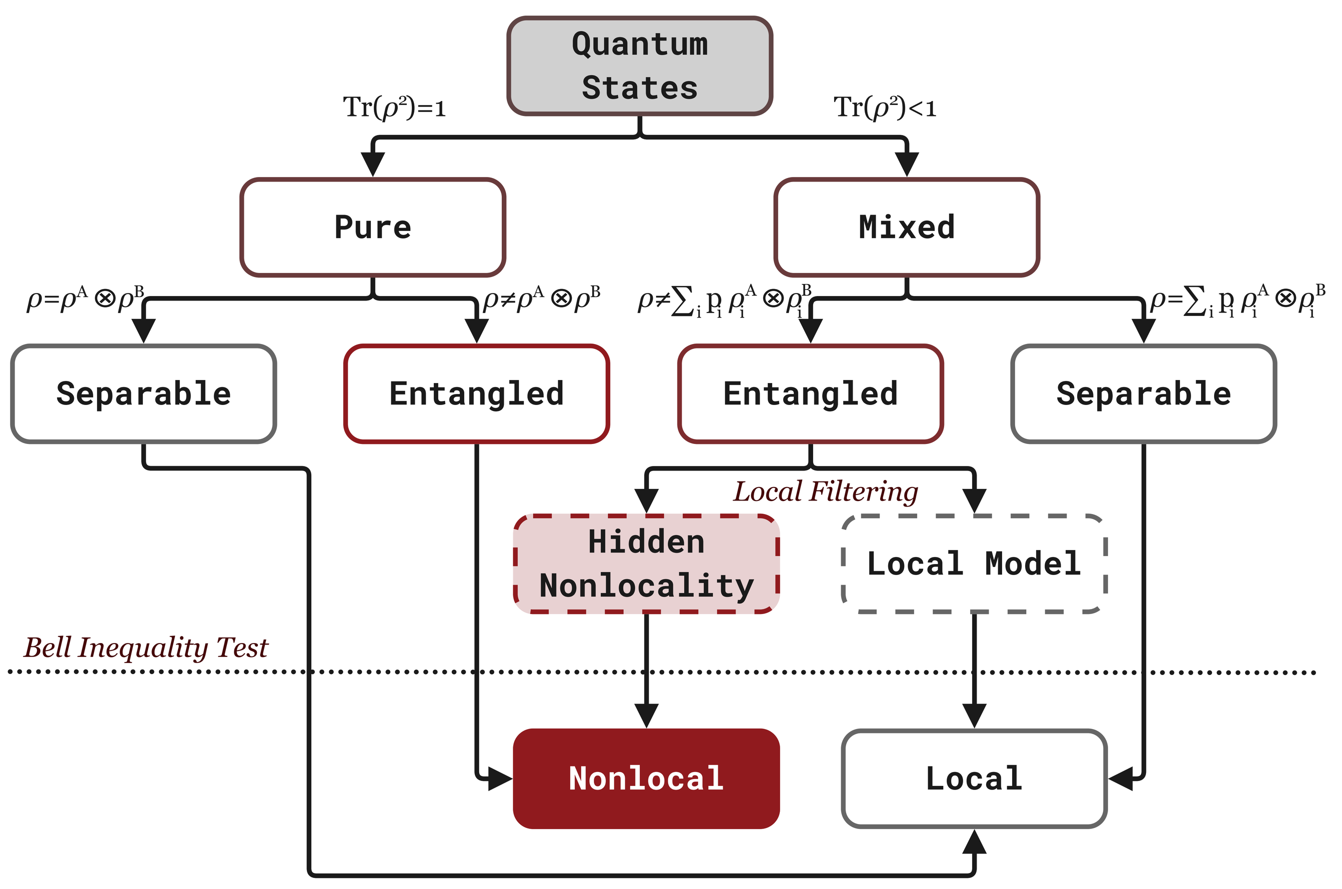

Figure 2. Classification of quantum states highlighting the distinction between pure and mixed states.

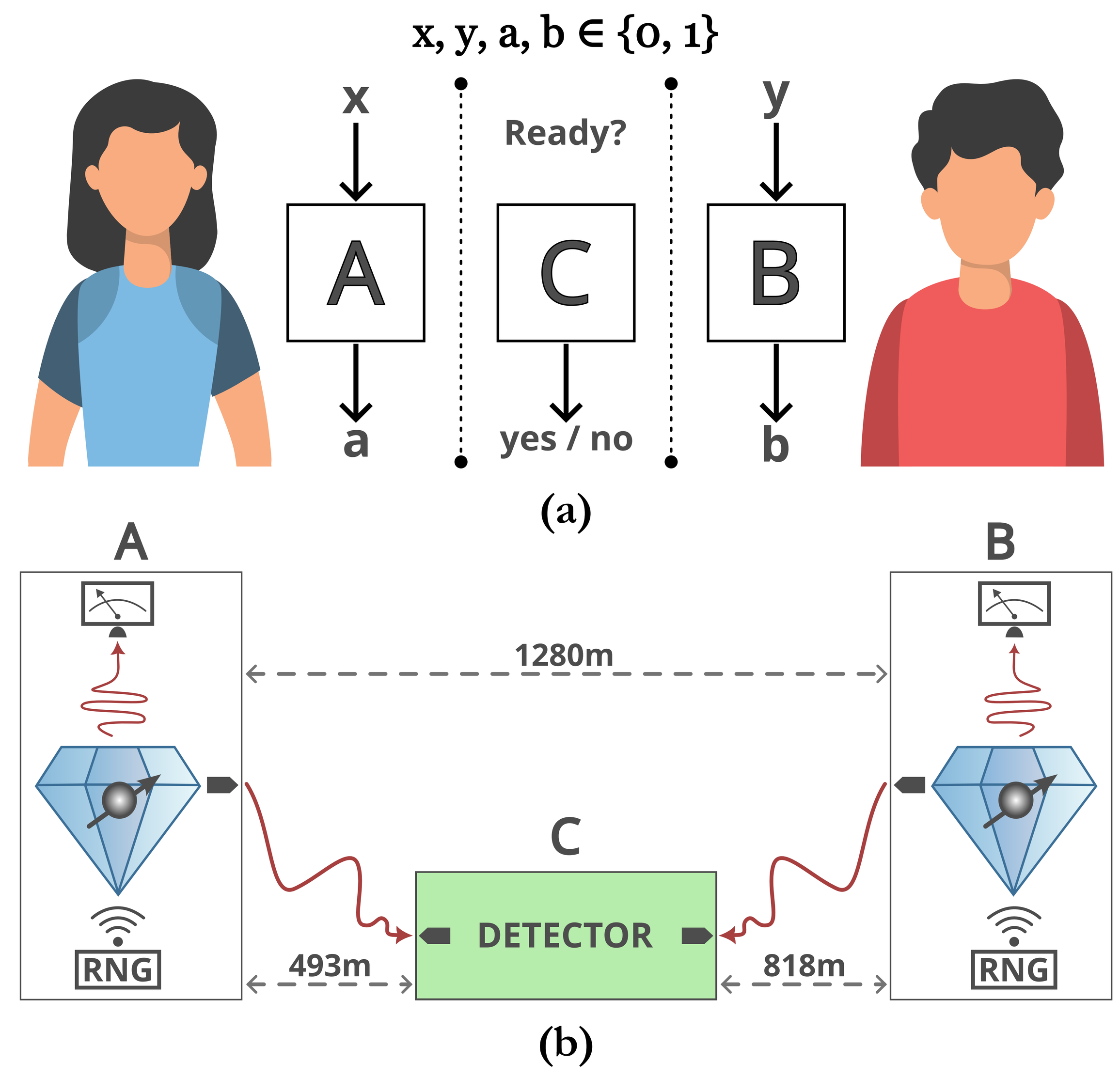

Figure 3. (a) Conceptual Bell Test Framework with Event-Ready Scheme, (b) Loophole-Free Bell Test Setup. Experimental setup from 2015 for a Bell test using electron spins in nitrogen vacancy centers in diamond, achieving loophole-free entanglement verification over 1.28 kilometers [15].

Revealing Hidden Nonlocality

"Hidden nonlocality" refers to the **nonlocal properties of an entangled mixed state** that are not observable through standard Bell-type measurements but **become apparent after specific local operations** or filtering processes. Popescu's paper illustrates this by showing that local filtering applied to Werner states reveals their nonlocal nature.

Popescu's results highlight that an entangled mixed state's nonlocal properties might only be uncovered through specific sequences of projective measurements, **leaving the exploration of POVMs as an open question**. Recent work by Hirsch et al. [12] has demonstrated the existence of **genuine hidden nonlocality**, where certain entangled states show nonlocality only after a sequence of measurements, despite appearing local under general measurements (POVMs).

To ensure a true test of nonlocality without loopholes, Popescu's setting requires Bob's random choice of measurements to occur after Alice's measurement on P is completed. Historically, Bell test experiments faced the **"detection loophole"** (where not all entangled particles are detected, potentially skewing results) and the **"locality loophole"** (where communication between measurement devices could influence results). Landmark experiments by Hensen et al. in [15] **closed these loopholes** by ensuring high detection efficiency, spatial separation of devices, and independent, fast switching of measurement settings, providing **definitive evidence of quantum nonlocality**.

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