Date: ______ no: ______

1 improved high-order model free adaptive control

IHO MFAC

2- model free adaptive control

no model → so what

3- principle of symmetric similarity
is to find the similarities between current states and old time samples states to build estimation for the future

4- the paper compares between
1- IHOMFAC 2- $\boxed{MFAC}$

5- Problem formulation :-

$y(t+1) = \boxed{f(y(t)}\ \boxed{y(t-1)})$

assumptions :-

the partial derivative of $f(\cdot)$ with respect to the (n_y+2)th variable is continuous for all time with finite excetions

Conclusion :-

1- ~~y~~ let $\dot{x}$ is the change of states with respect to time

$\dot{x} = f(x, u)$:- x is the system state

u is the input

-lets say u input = $g(x, \theta)$
where g is the function of the control policy

1- what is pseudo-partial derivative equations

2-

$y(t+1) =$ لازم تحول Φ

$f(y(t) + y(t-1), y(t-2), u(t), u(t-1), u(t-2))$

$\Delta y(t+1) = \boxed{\phi(t)}\ \boxed{\Delta u(t)}$

ϕ is a pseudo partial derivative

1 improved high-order model free adaptive control

IHO MFAC

2- model free adaptive control

no model → so what

3- principle of symmetric similarity

: to find the similarities between current states and old time samples states to build estimation for the future

4 the paper compares between

1- IHOMFAC 2- MFAC

5- Problem formulation :-

$y(t+1) = f(y(t), y(t-1))$

assumptions :-

the partial derivative of $f()$ with respect to the (n_y+2)th variable is continuous for all time with finite excetions

Conclusion :-

1- let $\dot{x}$ is the change of states with respect to time

$\dot{x} = f(x, u)$: x is the system state

u is the input

- lets say u input $= g(x, \theta)$ where g is the function of the control policy

1- what is pseudo-partial derivative equations

2-

$y(t+1) = f(y(t), y(t-1), y(t-2), u(t), u(t-1), u(t-2))$

$\Delta y(t+1) = \boxed{\phi(t)}\,\boxed{\Delta u(t)}$

ϕ is a pseudo partial derivative

Date: ________ no: ________

First step:-

(1-) I would like to be specific so I will start with 3rd order ~~general~~ nonlinear system.

$$\rightarrow y(t+1) = f(y(t), y(t-1), y(t-2), u(t); u(t-1), u(t-2)).$$

(2-) ~~the function~~ $y(t+1)$ is the output in the next time sample

(3) $f(y(t); u\,t\,\text{-----})$ is a function depends on the previous outputs and the current output and

* the previous inputs and the current input and this is the concept of (principle of symmetric similarity) wich is mentioned in the paper introduction and abstract.

(4) we will assume that the function $f(\cdot)$ or $f(y(t), u(t))$ ~~has continuous partial derivative for the [illegible] any time sample except some cases~~ has continuous partial derevative ~~[illegible]~~ with respet th the (n_g+2) ~~[illegible]~~ variable for all time with some exceptions.
this will assure to allow us to use parameter estimation or real time parameter estimation

Date: ______ no: ______

because the derivative is continuous in other meaning :-

the function that describes the rate of change will be smooth and continuous in the specific domain without a jump in the values suddenly or having a discontinuous ~~thing~~ curve.

this will allow us to have the partial derivative to do real time parameter estimation or optimization.

(5) we will assume 2nd assumption :- which is that the system satisfies Lipschitz condition (شرط ليبشيتز) (for all t with finite exceptions).

Lipsshitz condition :- change produced $\leq$ b* change introduced :- wher b is a positive constant

its means that the output ~~math~~ is bouded ~~and~~ (realistic) ~~becaus~~ depend in on the law of energy convertion is refered in the Remark 1.

$$|y(t_1+1) - y(t_2+1)| \leq b\,|u(t_1) - u(t_2)|$$

then $\Delta y(t+1) = \phi(t)\, \Delta u(t)$

~~The cost~~

~~the~~ the $\phi(t)$ is called pseudo partial derivative (PPD) and must be $\leq b$ according to Lipschitz condition

6 - designing controller $\Rightarrow$ model free ~~parameter~~ adaptive. OR MFAC

(the cost ~~function~~ function)

$$J(u(t), \alpha_{t,i}) = |y^*(t+1) - y(t+1)|^2$$

$$+ \lambda \left| u(t) - \sum_{i=1}^{L} \alpha_{t,i} \, u(t-i) \right|^2$$

$u(t) - \sum_{i=1}^{L} \alpha_{t,i} \, u(t-i) \Rightarrow$ the sum of product between alpha coefficients and the past control inputs

note $\rightarrow$ because we want to ~~punish the~~ have a specific pattern among the input signal and $\boxed{\text{this is a constraint}}$

$\boxed{\lambda}$ is a trade off parameter to control the ratio between fitting process and regularization process now substitute 3 in 4

since $y(t+1) - y_t = \phi\,(u_t - u(t-1))$

$\therefore\ y(t+1) = \phi\, u(t) - \phi\, u(t-1) + y_t$

and eq4 :- $J(u(t), \alpha_{t,i}) = |y^*(t+1) - y(t+1)|^2 + \lambda \left|u(t) - \sum_{i=1}^{t} \alpha_{t,i}\, u(t-i)\right|^2$

$$\therefore\ = |y(t+1) - \phi u(t) + \phi u(t-1) - y(t)|^2 + \lambda \left|u(t) - \sum_{i=1}^{t} \alpha_{t,i}\, u(t-i)\right|^2$$

now take the partial derivative $\frac{\partial}{\partial u(t)}$ and get the minimum :-

$$\frac{1}{2}\,\frac{\partial J(u(t), \alpha_{t,i})}{\partial u(t)} = 0$$

proving eq 5

$$= \left(y^*(t+1) - \phi u(t) + \phi u(t-1) - y(t)\right)\cdot(-\phi) + \lambda\left(u(t) - \sum_{i=1}^{t} \alpha_{t,i}\, u(t-i)\right)\cdot(1)$$

$$\therefore\ -\phi\, y^*(t+1) + \phi^2 u(t) - \phi^2 u(t-1) + \phi\, y(t) + \lambda u(t) - \lambda \sum_{i=1}^{t} \alpha_{i,t}\, u(t-i)$$

$$\therefore\ (\phi^2 + \lambda)\, u(t) = \phi^2 u(t-1) + \lambda \sum_{i=1}^{t} \alpha_{t,i}\, u(t-i) + \rho\,\phi\,(y^*(t+1) - y(t))$$

added factore

Date: no:

now divide by $(\phi^2 + \lambda)$

$$u(t) = \frac{\phi(t)^2}{\phi(t)^2 + \lambda} u(t-1) + \frac{\lambda}{\lambda + \phi(t)^2} \sum_{i=1}^{L} \alpha_{i,i} \, u(t-1)$$

$$+ \frac{\rho \, \phi(t)}{\lambda + \phi^2(t)} \left(y^*(t+1) - y(t)\right)$$

eq 5 proved.

PPD estimation algorithm

→ eq 7 and 8

$$J(\hat{\phi}(t)) = \left|\underbrace{y(t) - y(t-1)}_{\Delta y_{actual}(t)} - \overbrace{\hat{\phi}(t)\,\Delta u(t-1)}^{\Delta y^*(t)}\right|^2$$

$$+ \mu \left|\hat{\phi}(t) - \hat{\phi}(t-1)\right|^2$$

weighting → μ ; regularization term → $\left|\hat{\phi}(t) - \hat{\phi}(t-1)\right|^2$

taking derivative = 0

$$0 = \left(\Delta y(t) - \hat{\phi}(t)\,\Delta u(t-1)\right)\left(-\Delta u(t-1)\right)$$
$$+ \mu \left(\hat{\phi}(t) - \hat{\phi}(t-1)\right) \cdot (1)$$

then :-

$$(\mu + \Delta u(t-1)^2)\,\hat{\phi}(t) = \mu\,\hat{\phi}(t-1) + \Delta y(t) \cdot \Delta u(t-1)$$

then :-

$$\hat{\phi}(t) = \frac{\mu\,\phi(t-1)}{\mu + \Delta u(t-1)^2} + \frac{\Delta y(t-1) \cdot \Delta u(t-1)}{\mu + \Delta u(t-1)^2}$$

Date: ______ no: ______

then:- $\hat{\phi}(t) = \frac{\mu\,\phi(t-1)}{\mu + \Delta u(t-1)^2} + \frac{\Delta y(t-1)\cdot \Delta u(t-1)}{\mu + \Delta u(t-1)^2} + \hat{\phi}(t-1) - \hat{\phi}(t-1)$

$\therefore \; -\hat{\phi}(t-1) = -\left(\frac{\mu + \Delta u(t-1)^2}{\mu + \Delta u(t-1)^2}\right)\hat{\phi}(t-1)$

$\therefore \; \hat{\phi}(t) = \hat{\phi}(t-1) + \frac{-\Delta u(t-1)^2}{\mu + \Delta u(t-1)^2} + \frac{\Delta y(t-1)\cdot\Delta u(t-1)}{\mu + \Delta u(t-1)^2}$

common factor $\therefore \; \hat{\phi}(t) = \hat{\phi}(t-1) + \left(\frac{\eta\,\Delta u(t-1)}{\mu + \Delta u(t-1)^2}\right)\Big(\Delta y(t-1) - \Delta u(t-1)\cdot\hat{\phi}(t-1)\Big)$

eq 7 is proved as well as 8

note :- η is a wighting factor.

now $\boxed{\text{proving equation (10)}}$

as we proved eq (5)

1- take partial derivatve of cost function and get the optimal solution by equalizing to zero.

$0 = \left[y^*(t+1) - \phi(t)\,u(t) + \phi(t)\,u(t-1) - y(t)\right]\cdot(-\phi)$
$+ \lambda\left(u(t) - \sum_{i=1}^{t} \alpha_{t,i}\, u(t-1)\right)$, (1)

$\therefore \; -\phi(t)\,y^*(t+1) + \phi^2(t)\,u(t) - \phi^2(t)\,u(t-1) + \phi(t)\,y(t) + \lambda u(t) - \lambda \sum_{i=1}^{t} \alpha u(t-i)$

$\therefore \; (\phi^2(t) + \lambda)\,u(t) = \phi^2(t)\,u(t-1) + \lambda \sum_{i=1}^{t} \alpha_{t,i}\, u(t-i)$
$+ \rho\,\phi(t)\,(y^*(t+1) - y(t))$

added factor

ROXI

Date: ______ no: ______

now divide by $(\phi^2 + \lambda)$

$$u(t) = \frac{\phi(t)^2}{\phi(t)^2 + \lambda} u(t-1) + \frac{\lambda}{\lambda + \phi(t)^2} \sum_{i=1}^{t} \alpha_{t,i} u(t-i)$$

$$+ \frac{\rho\,\phi(t)}{\lambda + \phi^2(t)} \underbrace{(y^*(t+1) - y(t))}_{e(t)}$$

$$u(t) = \frac{\phi(t)^2}{\phi(t)^2 + \lambda} u(t-1) + \frac{\lambda}{\lambda + \phi(t)^2} \sum_{i=1}^{l} \alpha_{t,i} u(t-i)$$

$$+ \frac{\rho\,\phi(t)}{\lambda + \phi^2(t)} \cdot (e(t))$$

equation 10 proved.

our cost function for the Improved high order MFAC.

$$J(\hat{\phi}(t), \beta_{t,i}) = |y(t) - y(t-1) - \hat{\phi}\,\Delta u(t-1)|^2$$
$$+ \mu\,|\hat{\phi}(t) - \sum_{i=1}^{t} \beta_{t,i}\,\hat{\phi}(t-i)|^2$$

equation (11)

$\beta_{i,t}$ is a weigt factor

eq (B)

and $\hat{\phi}(t) = \sum_{i=1}^{l} \beta_{t,i}\,\hat{\phi}(t-i) + \frac{\eta\,\Delta u(t-1)}{\mu + \Delta u(t-1)^2}$

$\cdot \left(\Delta y(t) - \Delta u(t-1) \sum_{i=1}^{l} \beta_{t,i}\,\hat{\phi}(t-i)\right)$ lets prove it

Date: no:

Proving → equation (P3)

taking derivative = 0

$$0 = \left(\Delta y(t) - \Delta u(t-1) \cdot \sum_{i=1}^{l} B \hat{\phi}(t-1)\right)\left(-\Delta u(t-1)\right)$$

$$+ \mu \left(\hat{\phi}(t) - \sum_{i=0}^{l} B_{t,i} \phi(t-i)\right) \cdot (1)$$

then :- $(\mu + \Delta u(t-1)^2)\, \hat{\phi}(t) = \mu \sum_{i=0}^{l} B_{t,i} \hat{\phi}(t-i) +$

$\Delta y(t) \cdot \Delta u(t-1)$

then :-

$$\hat{\phi}(t) = \frac{\mu \sum_{i=0}^{l} B_{t,i} \phi(t-i)}{\mu + \Delta u(t-1)^2} + \frac{\Delta y(t-1)\, \Delta u(t-1)}{\mu + \Delta u(t-1)^2}$$

same like we did in equation (7)
we will add and subtract

$$\hat{\phi}(t) = \sum_{i=0}^{l} B_{t,i} \hat{\phi}(t-i) + \frac{\eta \, \Delta u(t-1)^2}{\mu + \Delta u(t-1)^2} \left(\Delta y(t-1) - \Delta u(t-1) \cdot \sum_{i=1}^{l} B_{t,i} \hat{\phi}(t-i)\right)$$

done

Date: ________ no: ________

now proving 15
it is same as 10

as we proved eq.(5)

1- take partial derivative of cost function and get the optimal solution by equalizing to zero.

$$0 = \left(y^*(t+1) - \phi(t)\,u(t) + \phi(t)\,u(t-1) - y(t)\right).(-\phi)$$
$$+ \lambda\left(u(t) - \sum_{i=1}^{t} \alpha_{t,i}\, u(t-1)\right).(1)$$

$$\therefore -\phi(t)\,y^*(t+1) + \phi^2(t)\,u(t) - \phi^2(t)\,u(t-1) + \phi(t)\,y(t) + \lambda u(t) - \lambda \sum_{i=1}^{t} \alpha\, u(t-i)$$

$$\therefore (\phi^2(t) + \lambda)\,u(t) = \phi^2(t)\,u(t-1) + \lambda \sum_{i=1}^{t} \alpha_{t,i}\, u(t-i)$$
$$+ \rho\,\phi(t)\,(y^*(t+1) - y(t))$$

added factor

Date: ________ no: ________

now divide by $(\phi^2 + \lambda)$

$$u(t) = \frac{\phi(t)^2}{\phi(t)^2 + \lambda} u(t-1) + \frac{\lambda}{\lambda + \phi(t)^2} \sum_{i=1}^{L} \alpha_{t,i}\, u(t-i)$$

$$+ \frac{\rho\, \phi(t)}{\lambda + \phi^2(t)} \underbrace{(y^*(t+1) - y(t))}_{e(t)}$$

~~equatio~~

$$u(t) = \frac{\phi(t)^2}{\phi(t)^2 + \lambda} u(t-1) + \frac{\lambda}{\lambda + \phi(t)^2} \sum_{i=1}^{L} \alpha_{t,i}\, u(t-i)$$

$$+ \frac{\rho\, \phi(t)}{\lambda + \phi^2(t)} \cdot (e(t))$$

equation 15 proved.

Scanned with CamScanner