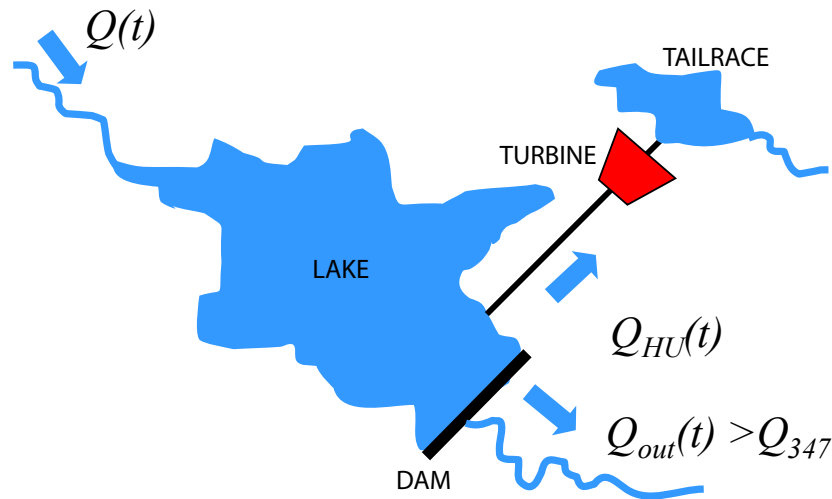


# Assignment: Plan of a multipurpose reservoir

## Water resources engineering ENV-424



### Introduction

This assignment will resume and practically apply the concept that we saw during theoretical classes. Students are requested to apply a series of coding strategies to solve a problem concerning a multipurpose reservoir to be planned. The project should be preferentially carried out in groups of three people each and should be handed in through Moodle in a single submission no later than January 2nd, 2023 at noon. Students are free to carry out the project in Matlab (.mlx) or Python (.ipynb), provided that their submission consists in a Notebook containing all figures and scripts. The main notebook (the one that calls all other functions and where all results are provided) should be named "main".

### Synopsis

A reservoir is used for hydroelectrical energy production and flood prevention. You should evaluate the feasibility of improving the flood control operations of an existing reservoir of a hydropower plant. A larger fraction of the storage needs to be reserved for flood control and cannot be used for hydroelectric generation. The downstream part of the river should be protected from floods larger than  $Q_{lim}$ . The larger the volume for hydroelectric use, the higher the energy production. The larger the volume for flood control, the more efficiently floods can be attenuated. The two uses (hydroelectric and flood control) are therefore in competition. The goal of the assignment is to estimate the trade-off between money earned with energy production and probability that the released flow  $Q_{out}$  exceeds  $Q_{lim}$ , as a function of the volume reserved for flood control. Note that the discharge used for hydropower generation is then released into a different river system.

### Tasks

1. Develop a continuous lumped hydrological model for the drainage basin of the reservoir to transform rainfall into discharge (inflow in the reservoir).
2. Fit the hydrological model using as training set the available time series of precipitation and discharge.
3. By using a Poisson pulse model, generate a 100-year-long rainfall time series with the same statistical properties as the observed rainfall.
4. Transform generated rainfall into a generated time series of discharge by means of the fitted hydrological model.
5. Simulate the reservoir routing and the flood control operations for different maximum levels for hydroelectric use.
6. Evaluate the average annual energy produced and the flooding probability.

### Input data

- Six-year-long series of measured rainfall and discharge (files `P.txt`, `Q_obs.txt`);
- Monthly mean series of temperature and crop factor (files `temperature.txt` and `kc.txt`);
- Parameters for the hydrological model: wilting point  $s_w$ , soil moisture for maximum plant transpiration  $s_1$ , porosity  $n$ , base flow  $Q_b$ , mean superficial residence time ( $t_{sup}$ ), area of the basin  $A$  and latitude (see Table 3);
- Relation between area of reservoir lake and level of the reservoir (file `area_rating_curve.txt`);
- Reservoir parameters: discharge coefficients for sluice gate and spillway ( $C_{q,sl}$ ,  $C_{q,sp}$ ) spillway length and height ( $L$ ,  $p$ ) (see Table 3);
- The design discharge of the turbine  $Q_T$  (see Table 3);
- Power plant parameters: pipe diameter, length and sand equivalent roughness ( $D$ ,  $L_p$ ,  $k_s$ ), turbine efficiency  $\eta$ , altitude gap between the bottom of the reservoir and the tailrace  $\Delta h$ , minimum level for hydropower production (see Table 3).

### Procedure

**Hydrological model.** Implement the hydrological model on the 6-year-long series of precipitation and discharge.

1. Compute the potential evapotranspiration  $ET_0$  via the Thornthwaite method.
2. Attribute arbitrary values to the parameters  $K_{sat}$ ,  $c$ ,  $t_{sub}$ ,  $z$ .
3. Use an Euler explicit scheme to integrate the system of equations at hourly scale (see Table 2).
4. Test the mass balance of your system in order to ensure a correct implementation.

5. Evaluate the goodness of fit of the chosen parameter set by calculating the Nash-Sutcliffe index  $NS$ .

$$NS = 1 - \frac{\sum_{t=t_0}^{t_{end}} [Q_{obs}(t) - Q_{mod}(t)]^2}{\sum_{t=t_0}^{t_{end}} [Q_{obs}(t) - \overline{Q_{obs}}]^2};$$

where  $Q_{obs}$  and  $Q_{mod}$  are the measured and modelled discharges, respectively.  $\overline{Q_{obs}}$  is the average measured discharge.

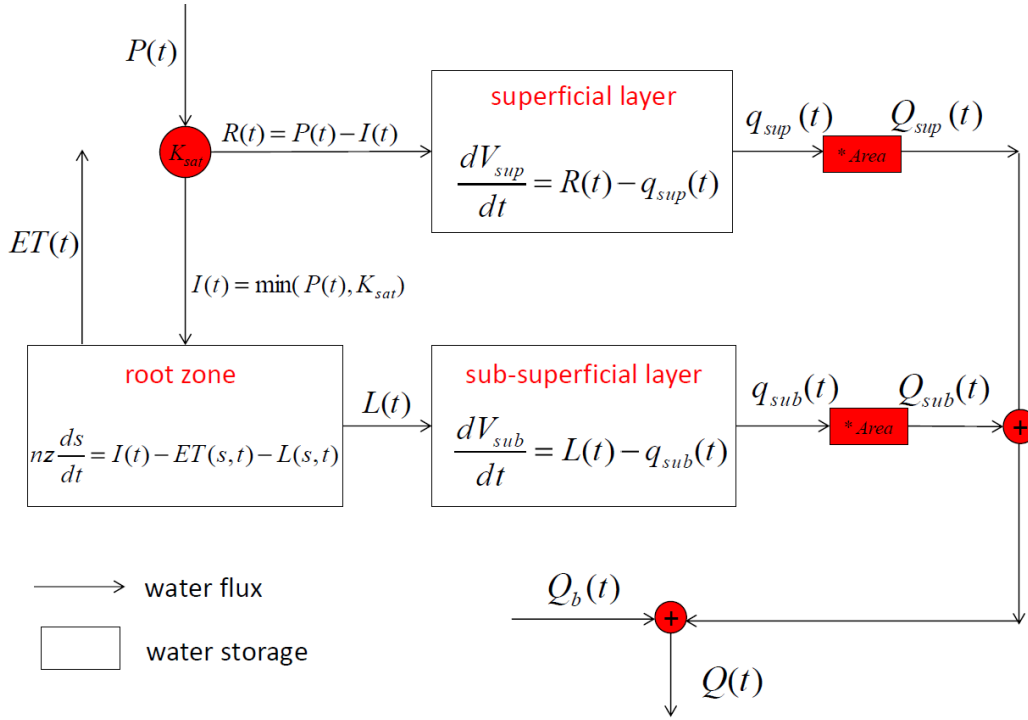
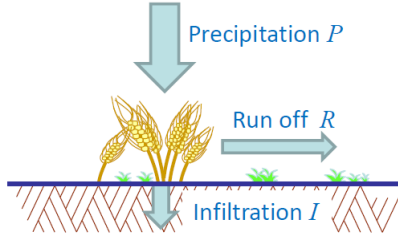


Figure 1: Scheme of the hydrological model. Boxes indicate water storages, arrows indicate water fluxes.

Variable	Symbol	Unit
Precipitation	$P$	$[LT^{-1}]$
Infiltration	$I$	$[LT^{-1}]$
Runoff	$R$	$[LT^{-1}]$
Soil moisture	$s$	$[-]$
Evapotranspiration	$ET$	$[LT^{-1}]$
Leaching	$L$	$[LT^{-1}]$
Superficial storage	$V_{sup}$	$[L]$
Superficial specific discharge	$q_{sup}$	$[LT^{-1}]$
Superficial discharge	$Q_{sup}$	$[L^3T^{-1}]$
Sub-superficial storage	$V_{sub}$	$[L]$
Sub-superficial specific discharge	$q_{sub}$	$[LT^{-1}]$
Sub-superficial discharge	$Q_{sub}$	$[L^3T^{-1}]$

Table 1: List of variables for the hydrological model

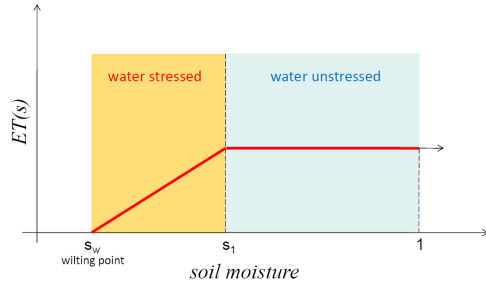


### Runoff generation (Horton mechanism):

$$P(t) = R(t) + I(t);$$

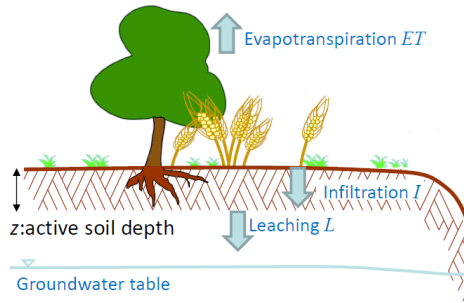
$$I(t) = \min(P(t), K_{sat});$$

$$R(t) = P(t) - I(t);$$



### Evapotranspiration computation:

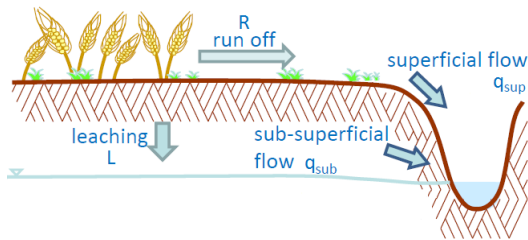
$$ET(t) = \begin{cases} 0, & \text{if } 0 \leq s(t) \leq s_w, \\ k_c ET_0 \cdot \frac{s(t) - s_w}{s_1 - s_w}, & \text{if } s_w < s(t) \leq s_1. \\ k_c ET_0, & \text{if } s_1 < s(t) \leq 1. \end{cases}$$



### Soil moisture dynamics:

$$nz \frac{ds}{dt} = I(t) - ET(s, t) - L(s, t);$$

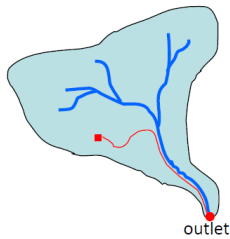
$$L(t) = K(s) = K_{sat} \cdot s(t)^c;$$



### Linear reservoir scheme:

$$\frac{dV_{sup}}{dt} = R(t) - q_{sup}(t); \quad q_{sup}(t) = t_{sup}^{-1} V_{sup}(t);$$

$$\frac{dV_{sub}}{dt} = L(t) - q_{sub}(t); \quad q_{sub}(t) = t_{sub}^{-1} V_{sub}(t).$$



### Discharge computation:

$$Q_{sup}(t) = \mathcal{A} \cdot q_{sup}(t);$$

$$Q_{sub}(t) = \mathcal{A} \cdot q_{sub}(t);$$

$$Q(t) = Q_{sup}(t) + Q_{sub}(t) + Q_b.$$

Table 2: System of equations constituting the hydrological model.

**Model calibration.** Find the set of parameters  $\{K_{sat}, c, t_{sub}, z\}$  that maximizes the Nash-Sutcliffe index. Use a simulated annealing strategy in order to find the parameter set that maximizes  $NS$ .

1. Define a functional form for the temperature

$$T_{SA}(i) = \exp(-c_r \cdot i)$$

where  $i$  counts the iterations of the calibration procedure, while  $c_r$  is a cooling rate (see Table 3).

2. Attribute arbitrary values to the parameters  $K_{sat}, c, t_{sub}, z$ . Run the hydrological model and evaluate  $NS_{old}$ .
3. Select a new parameter set by drawing from a truncated normal distribution centered in the previous parameter set (use function `TruncNormRnd.m`).
4. Run the hydrological model with the new parameter set and evaluate  $NS_{new}$ .
5. If  $NS_{new} > NS_{old}$ , accept the new parameter set.
6. Otherwise, accept the new parameter set with probability

$$\exp\left(\frac{NS_{new} - NS_{old}}{T_{SA}(i)}\right)$$

7. Repeat from 3 until convergence. A good fitting should achieve  $NS \geq 0.87$ .

**Simulated rainfall sequence.** Generate 100 years of rainfall (at daily timescale) with the parameters (mean daily rainfall depth  $\alpha$  and rainfall frequency  $\lambda$ ) estimated from the observed precipitation.

1. Up-scale the given hourly rainfall series to average daily rainfall. The resulting time series should have one value for each day with mm/day as unit.
2. Evaluate the parameters  $\alpha$  (mean precipitation intensity) and  $\lambda$  (frequency of rainfall events). Account for seasonality by computing different values of the parameters for each month.
3. Generate the 100-year-long rainfall series. For any generic day of month  $m$ :
  - a rainfall event occurs with probability  $\lambda(m)\Delta t$ , where  $\Delta t = 1$  day.
  - If a rainfall events occurs, the rainfall depth is extracted from an exponential distribution with mean  $\alpha(m)$ . Use the inverse transformation method to generate exponentially distributed random variables.

Points 2 and 3 are repeated for all the days and months of the generation period.

4. Use the function `downscaling.m` to down-scale the daily generated rainfall to hourly rainfall.

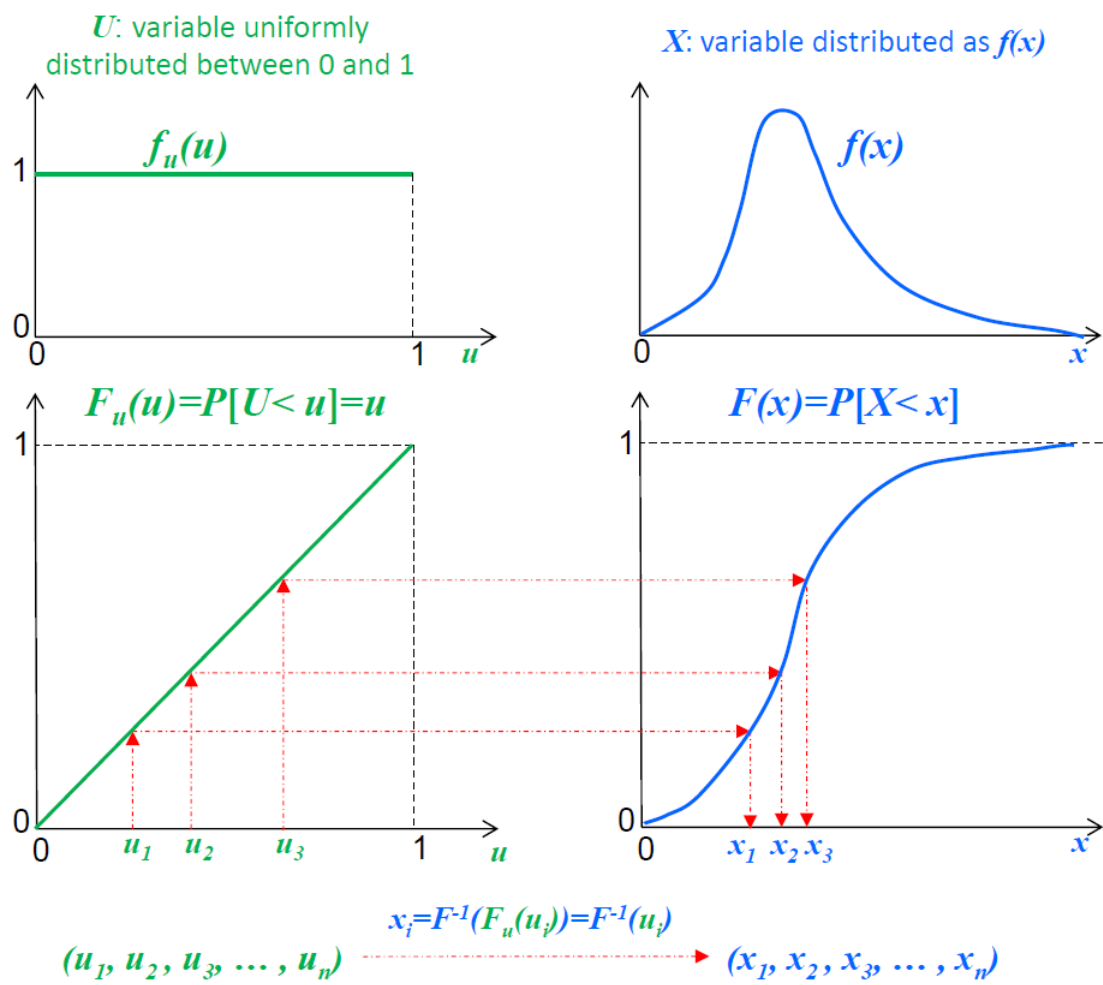


Figure 2: Inverse transformation method.

**Simulated discharge sequence.** Run the calibrated hydrological model forced by the generated rainfall and obtain the discharge time series that will constitute the input of the reservoir.

**Reservoir routing and flood control** Implement the flood control practice and the reservoir routing. Assume that the power plant works only during peak hours (from 06:00 to 22:00, 16 hours). If at the beginning of the day (midnight) the level in the reservoir is above the minimum level for hydroelectrical use, the plant will work during that day, otherwise it will not. When the turbine is working,  $Q_{HU}$  is always equal to the turbine design discharge  $Q_T$ . To compute the net head of the turbine, account for frictional head losses along the pipe, entrance head loss (half of the kinematic term) and exit head loss (one kinematic term). The efficiency of the turbine  $\eta_T$  can be assumed as constant.

1. Compute the minimum flow target  $Q_{347}$  (discharge that is exceeded 95% of the time) from the 100-year-long generated input discharge time series.
2. Starting from the area rating curve, compute the volume rating curve (use trapezoidal approximation).
3. Assume a value for the maximum level for hydroelectric use (e.g. 15 m) and compute the corresponding volume.
4. For each time step of integration ( $\Delta t = 1$  hour):
  - Evaluate the level of the reservoir (use the function `level_volume` which is faster than `interp1`).
  - Compute the discharge  $Q_{HU}$  routed to the power plant.  $Q_{HU} = Q_T$  during peak hours if the power plant is working in that particular day, otherwise  $Q_{HU} = 0$ .
  - Compute the sluice opening area  $A$ . The opening of the sluice gate is operated so that:
    - the discharge through the gate  $Q_g(t)$  is larger than the minimum flow  $Q_{347}$  and lower than  $Q_{lim}$ ;
    - the level is kept, if possible (i.e. if the gate discharge is within the aforementioned limits), at the maximum level for hydroelectrical use;
    - if during floods the maximum level for hydroelectrical use is exceeded, the reservoir is emptied as quick as possible.

The above practice can be implemented as follows. At each time step, the opening of the sluice gate is computed so that volume at the end of the time step equals (if possible, i.e. if the gate discharge is within the aforementioned limits) the volume corresponding to the maximum level for hydroelectrical use ( $V_{max,HU}$ ).

$$Q_g(t) = \max \begin{cases} Q_{347} \\ \min \left\{ \frac{V(t) + [Q(t) - Q_{HU}(t)] \cdot \Delta t - V_{max,HU}}{\Delta t}, Q_{lim} \right\} \end{cases} \quad (1)$$

where  $V$  is the volume stored in the reservoir.

- Compute the total output discharge  $Q_{out}$ :

$$Q_{out} = \begin{cases} C_{q,sl} A \sqrt{2gl} & \text{if } l \leq p, \\ C_{q,sl} A \sqrt{2gl} + C_{q,sp} L \sqrt{2g(l-p)^3}, & \text{if } l > p. \end{cases} \quad (2)$$

where  $l$  is the level in the reservoir (with respect to the empty pool),  $p$  and  $L$  are the level of the spillway crest and the spillway length, respectively;  $C_{q,sl}$  and  $C_{q,sp}$  are the discharge coefficients for sluice gate and spillway, respectively;  $g$  is the gravity acceleration.

- Integrate the storage equation:

$$\frac{dV(t)}{dt} = Q(t) - Q_{out}(l(V(t))) - Q_{HU}(t), \quad (3)$$

via an Euler explicit scheme. Other fluxes (e.g. evaporation from the lake, input of rainfall into the lake, deep percolation) are negligible.

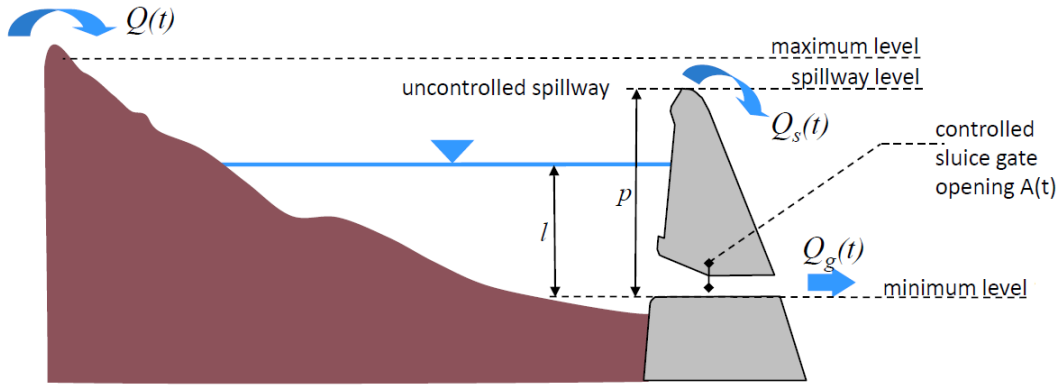


Figure 3: Cross-section of the reservoir

**Energy production and flooding probability.** Repeat the flood control practice and the routing of the reservoir for different values (from 9 to 19 m, with step size equal to or lower than 1 m) of maximum level for hydroelectrical use  $l_{max,HU}$ . Compute the average annual energy production in [GWh] and the probability that the released discharge  $Q_{out}$  is larger than  $Q_{lim}$  (for numerical reasons, the condition should read:  $Q_{out} > (Q_{lim} + 1)$ ) as a function of  $l_{max,HU}$ . Use the same sequence of generated discharge (100-year-long) to simulate the reservoir routing for different values of  $l_{max,HU}$ .

### Deliverables (should clearly appear in your main notebook)

1. Report the best fit parameters for the hydrological model.
2. Plot the time series (6-year-long) of precipitation in [mm/h], run off in [mm/h], infiltration in [mm/h], soil moisture, leakage in [mm/h] and evapotranspiration in [mm/h] for the calibrated hydrological model forced by the given precipitation).
3. Plot the time series of observed discharge and of the discharge as simulated by the hydrological model with the best fit parameters.



4. Plot Markov chains of the calibrated parameters and sequences of values of  $NS$  and  $T_{SA}$  as a function of the number of iterations. Only plot accepted parameter sets.
5. Plot the comparison between the statistics (mean, standard deviation,  $\alpha$  and  $\lambda$ ) of the observed and generated precipitation.
6. Plot the time series (100-year-long) of generated precipitation in [mm/h], run off in [mm/h], infiltration in [mm/h], soil moisture, leakage in [mm/h] and evapotranspiration in [mm/h] for the calibrated hydrological model forced by the generated precipitation).
7. Plot the area and the volume rating curves for the given levels.
8. Report the minimum flow (discharge that is equalled or exceeded 95% of the time) of the generated discharge and plot the discharge duration curves.
9. For a maximum level for hydroelectric use of 15 m, plot the time series (100 years long) of input discharge, output discharge, volume within the reservoir and level.
10. Plot the average annual energy production in [GWh] and the probability that  $Q_{out}$  exceeds  $Q_{lim}$  for different values of the maximum level for hydroelectrical use.
11. Plot the Pareto front of average annual energy production in [GWh] vs the probability that  $Q_{out}$  exceeds  $Q_{lim}$  computed from different values of the maximum level for hydroelectrical use.

### Required files to submit

- Main notebook with all required figures and outputs listed. Elaboration should be inserted as text. Any assumption should be clearly stated, along with a description of the procedure that you implemented to execute the different parts.
- Copy of the notebook in a PDF and HTML format.
- All auxiliary functions and file that were provided.
- All additional complementary functions that you created in support of the main notebook.

### Notes

- The assignment will receive a grade that is based on several factors. The following criteria will be taken into account:
  1. Correctness of results: results should be correct and the whole procedure rightfully implemented;
  2. Notebook and script readability: your codes must be fully understood both from a hydraulic and a computational point of view. Make use of text and equations in the notebook to explain how you solved the problem;
  3. Minimal results. The minimal results that are asked should be present in the notebook. However, feel free to show any additional result (in form of figure and numbers) that you deem interesting or important.
  4. Units: units should be shown throughout your computations and in all figures/outputs that you decide to plot/print.

5. All codes should not report errors and run completely. If you use specific packages, please specify that.

This assignment, as a whole, counts 30% towards the final grade.

- All groups must submit their own files through Moodle. Please add before your files the preamble GroupX\_, where X is your group letter. You do not need to add this prefix to files and data that were submitted by us.

Table 3: List of parameters

Parameter	Symbol	Value	Unit
<i>Hydrological model: to be calibrated</i>			
Hydraulic conductivity for saturated soil	$K_{sat}$	-	[LT <sup>-1</sup> ]
Exponent for power-law relation $L(s)$	$c$	-	[-]
Root zone depth	$z$	-	[L]
Mean sub-superficial residence time	$t_{sub}$	-	[T]
<i>Hydrological model: other</i>			
Wilting point	$s_w$	0.25	-
Soil moisture stress threshold	$s_1$	0.4	-
Porosity	$n$	0.3	-
Baseflow	$Q_b$	7	m <sup>3</sup> /s
Mean superficial residence time	$t_{sup}$	22	h
Catchment area	$\mathcal{A}$	4000	km <sup>2</sup>
Latitude	$\phi$	38	°
<i>Calibration of the hydrological model</i>			
Cooling rate	$c_r$	1/1200	-
<i>Reservoir</i>			
Discharge coefficient for sluice gate	$C_{q,sl}$	0.6	-
Discharge coefficient for spillway	$C_{q,sp}$	0.7	-
Length of the spillway	$L_{spill}$	140	m
Level of the spillway crest	$p$	19	m
<i>River</i>			
Discharge limit for floods	$Q_{lim}$	100	m <sup>3</sup> /s
<i>Power plant</i>			
Discharge to the turbine	$Q_T$	35	m <sup>3</sup> /s
Pipe diameter	$D$	3.3	m
Pipe length	$L_p$	1200	m
Sand equivalent roughness	$k_s$	0.5	mm
Turbine efficiency	$\eta$	0.73	-
Altitude gap between the bottom of the reservoir and the tailrace	$\Delta z$	75	m
Minimum level for hydroelectric production	$l_{min,HU}$	7	m
Total working hours (6h00 to 21h59)	—	16	h