

- ① Binomial distribution - determines number of successes in fixed number of attempts. In each attempt has probability of success (p), and probability of failure ($q=1-p$)

example: coin flipped 7 times - what prob of gaining 3 heads?

$n=7 \quad p=\frac{1}{2} \quad k=3$

~~$P(X=k) = nCk \cdot p^k \cdot q^{n-k}$~~

$P(X=k) = nCk \cdot p^k \cdot q^{n-k} = P$

- Geometric distribution - determines number of attempts until first success obtained in this sequence - each has p

x - number of attempts
 p - prob of success

$P(X=k) = (1-p)^{k-1} \cdot p$

example: person rolls six-sided die until get a six, what is the prob that they need to roll the die 3 times?

$P(X=3) = \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} \approx 0.1494$

- ② Classical - $P(A) = \frac{\text{Favorable results}}{\text{Total amount of results}}$ \Rightarrow it's measure of likelihood of event happening

Requirements -

- the outcomes must be equally likely
- the outcomes must be countable

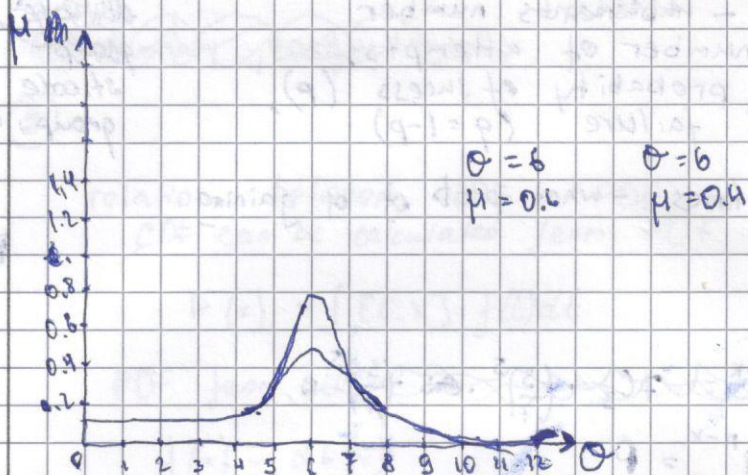
Statistical -

Statistical - prob measure of the degree of expected event happening. stands on ~~known~~ data well-known, data.

event A - $P(A) = \frac{p(A|D) \cdot p(D)}{p(D)}$

- requirements - data knowledge about event
- requires model specification

③ Plot



Effects

④

CLT

- let x_1, x_2, \dots, x_n be identically distributed, random vars
- their mean would be μ
- their variance σ^2

$n \rightarrow \infty$ - the distribution of sample mean

$$\frac{(x_1 + x_2 + \dots + x_n)}{n} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

CLT - states that the sample mean

- CLT is fundamental result in probability theory
- CLT is often used to approximate sample mean, when its size is large

⑤

gamma distribution

- often used to model the amount of time until an event happens

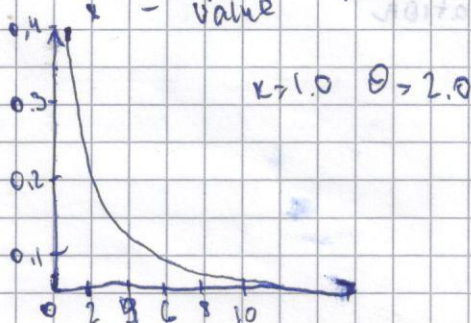
$$f(x) = \left(\frac{x^{k-1}}{\Gamma(k)} \right) \cdot e^{-\frac{x}{\theta}}$$

k - shape param

θ - scale param

$\Gamma(k)$ - gamma function

x - value



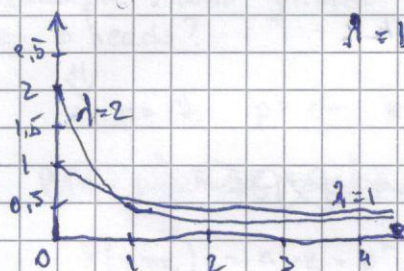
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exponential distribution

= time between events, used to estimate time until failure

$$f(x) = \lambda e^{-\lambda x}$$

x - time λ - rate



⑤

Prior prob

= prob of event or hypothesis happening before new data obtained.

Posterior prob

= prob of event or hypothesis after new data is obtained.

Bayesrule

$$P(H|D) = \frac{P(D|H) \cdot P(H)}{P(D)}$$

$P(H|D)$ - posterior prob (with new data)

$P(D|H)$ - likelihood

$P(H)$ - prior prob (initial data)

$P(D)$ - marginal prob (inspecting data regardless of event)

⑦

For large sample is constructed using CLT

the confidence interval for μ is:

1) - calc \bar{x} - sample mean, s - standart deviation

2) - calc SE - standart error of \bar{x}

3) - calc z' - critical value

$$z' = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \quad \Phi - \text{cumulative distribution}$$

4) construct confidence interval

$$\bar{x} \pm (z' \cdot SE)$$

width of interval depends:

- sample size (n) - size increases \rightarrow width of interval decreases
- (α) - confidence level - (a) higher \rightarrow interval became wider

Direction

4. Summary, large samples

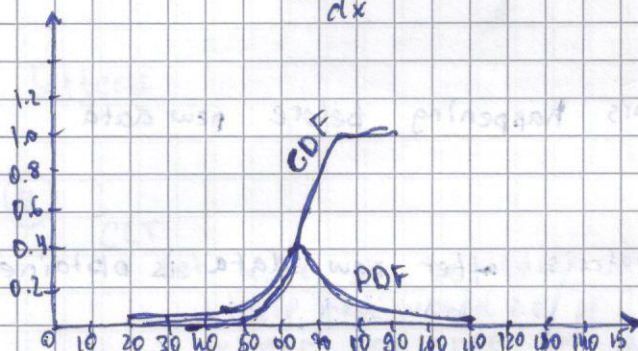
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relation between CDF and PDF
CDF can be calculated from PDF with:

$$F(x) = \int_{[0, x]} f(t) dt$$

PDF from CDF:

$$f(x) = \frac{dF(x)}{dx}$$



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random event

- situation or result ~~of unpredictable~~ ^{which cannot be} predicted
- may not happen at all
- result not determined by anything

example: rolling die, drawing card from deck, flipping coin

random variable:

- math interpretation of random event
- it's a function which assigns value to each possible outcome of event

example: result of flipping coin can be represented as x , $x=0$ - coin lands on heads, $x=1$ - coin lands on tails

differences

- event is uncertain situation, variable is math interpretation of that outcome

additional
continue on next
pages...

- ⑩ Inconsistent events - cannot occur at one moment. \Rightarrow if one event occurs, the other cannot

$$A: P(A) = 0.5$$
$$B: P(B) = 0.3$$



Aleksander Furjev
st. code 81582
group: 4202 BDA

$$P(A \cap B) = 0$$

Independent: ^{can} occur at one moment and are independent

$$A: P(A) = 0.4$$
$$B: P(B) = 0.2$$



$$P(A \cap B) = P(A) \cdot P(B) = 0.08$$

difference

- inconsistent - cannot happen simultaneously
- independent - can happen simultaneously

⑪

- 1) determine H_0 - null hypothesis
 H_1 - alternative hypothesis

- 2) choose significance level (α)

- 3) calc test statistic

t-test example

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s / \sqrt{n}}$$

- 4) determine critical interval - range of values where we reject H_0

⑫

formulation:

x_1, x_2, \dots, x_n - sequence of random vars
determine mean and variance are equal to λ
Let $S = x_1 + x_2 + \dots + x_n$ be sum of the first n vars

theorem states that $\lim_{n \rightarrow \infty} P(S \leq k) = P(\text{Poisson}(\lambda) \leq k)$

~~$P(\text{Poisson}(\lambda) \leq k)$~~ - prob that poisson varz with λ param

Proof:

steph - 1) S - characteristic