Calculus and Analytic Geometry (Thomas) Notes

Steven Labalme

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Contents

Chapter 8

Hyperbolic Functions

8.1 Introduction

6/24: • **Hyperbolic functions**: Certain combinations of e^x and e^{-x} that are used to solve certain engineering problems (the hanging cable) and are useful in connection with differential equations.

8.2 Definitions and Identities

- Let

$$\cosh u = \frac{1}{2}(e^u + e^{-u}) \qquad \sinh u = \frac{1}{2}(e^u - e^{-u})$$

  - These combinations of exponentials occur sufficiently frequently that we give a special name to them.
  - Although the names may seem random, $\sinh u$ and $\cosh u$ do share many analogous properties with $\sin u$ and $\cos u$.
  - Pronounced to rhyme with "gosh you" and as "cinch you," respectively.
- Like $x = \cos u$ and $y = \sin u$ are associated with the point (x, y) on the unit circle $x^2 + y^2 = 1$, $x = \cosh u$ and $y = \sinh u$ are associated with the point (x, y) on the unit hyperbola $x^2 - y^2 = 1$.
  - Note that $x = \cosh u$ and $y = \sinh u$ are associated with the *right-hand* branch of the unit hyperbola.
  - Also note that sine and cosine are sometimes referred to as the **circular functions**.
- Analogous to sine and cosine, we have the identity

$$\cosh^2 u - \sinh^2 u = 1$$

- We define the remaining hyperbolic trig functions as would be expected.

$$\tanh u = \frac{\sinh u}{\cosh u} = \frac{e^u - e^{-u}}{e^u + e^{-u}} \qquad \operatorname{sech} u = \frac{1}{\cosh u} = \frac{2}{e^u + e^{-u}}$$
$$\coth u = \frac{\cosh u}{\sinh u} = \frac{e^u + e^{-u}}{e^u - e^{-u}} \qquad \operatorname{csch} u = \frac{1}{\sinh u} = \frac{2}{e^u - e^{-u}}$$

- Since $\cosh u + \sinh u = e^u$, we can replace any combination of exponentials with hyperbolic sines and cosines and vice versa.
- Note that the hyperbolic functions are *not* periodic.
  - This does mean, though, that they have more easily defined properties at infinity.
- "Practically all the circular trigonometric identities have hyperbolic analogies" (Thomas, 1972, p. 267).

References

Thomas, G. B., Jr. (1972). *Calculus and analytic geometry* (fourth). Addison-Wesley Publishing Company.