4 Harmonic Oscillators II and the Hydrogen Atom

10/27: 1. The $J = 0$ to $J = 1$ transition for carbon monoxide ($^{12}C^{16}O$) occurs at 1.153×10^5 MHz.

   (a) Calculate the value of the bond length in carbon monoxide.

   (b) Predict the $J = 1$ to $J = 2$ transition for carbon monoxide.

2. The harmonic oscillator has a finite zero-point energy because of the uncertainty relation. In contrast, the lowest possible energy for the 2D rigid rotor is zero.

   (a) For the ground state of the 2D rigid rotor, what is the expectation value of the angular momentum, and what is the uncertainty ΔL_z in the expectation value? Recall that

   $$(\Delta L_z)^2 = \langle \hat{L}_z^2 \rangle - \langle \hat{L}_z \rangle^2$$

   (b) In words, describe the uncertainty in position.

   (c) Using your answers to (a) and (b), explain briefly why the 2D rigid rotor can have a vanishing zero-point energy and yet still remain consistent with the uncertainty relation.

3. For the ground state of the hydrogen atom, compute

   (a) The *average* distance from the nucleus for finding the electron.

   (b) The *most probable* distance from the nucleus for finding the electron.

   (c) Repeat the calculation for the second excited state ($n = 3$ and $l = 0$) and compare your results with the ground state.

4. Using non-relativistic quantum mechanics, compute the ratio of the ground-state energy of hydrogen to that of atomic tritium.

5. The Hamiltonian operator for a hydrogen atom in a magnetic field where the field is in the z-direction is given by

   $$\hat{H} = \hat{H}_0 + \frac{\beta_B B_z}{\hbar} \hat{L}_z$$

   where $\hat{H}_0$ is the Hamiltonian operator in the absence of the magnetic field, B_z is the z-component of the magnetic field, and β_B is a constant called the Bohr magneton.

   (a) Show that the wave functions of the Schrödinger equation for a hydrogen atom in a magnetic field are the same as those for the hydrogen atom in the absence of the field.

   (b) Show that the energy associated with the wave function $\psi_{n,l,m}$ is

   $$E = E_n^{(0)} + \beta_B B_z m$$

   where $E_n^{(0)}$ is the energy in the absence of the field and m is the magnetic quantum number.