## Chapter 28

## Rate Laws

## 28.1 Definitions and Methods of Determination

4/8: • Consider a general chemical equation

$$\nu_A A + \nu_B B \longrightarrow \nu_Y Y + \nu_Z Z$$

• The extent of the reaction via the progress variable  $\xi$  is

$$n_A(t) = n_A(0) - \nu_A \xi(t)$$
  $n_Y(t) = n_Y(0) + \nu_Y \xi(t)$ 

• The rate of change (moles/second) is

$$\frac{\mathrm{d}n_A}{\mathrm{d}t} = -\nu_A \frac{\mathrm{d}\xi}{\mathrm{d}t} \qquad \qquad \frac{\mathrm{d}n_Y}{\mathrm{d}t} = \nu_Y \frac{\mathrm{d}\xi}{\mathrm{d}t}$$

- Deriving the rate of reaction for a gas-based chemical reaction.
  - Time-dependent concentration changes

$$\frac{1}{V}\frac{\mathrm{d}n_A}{\mathrm{d}t} = \frac{\mathrm{d}[A]}{\mathrm{d}t} = -\frac{\nu_A}{V}\frac{\mathrm{d}\xi}{\mathrm{d}t} \qquad \qquad \frac{1}{V}\frac{\mathrm{d}n_Y}{\mathrm{d}t} = \frac{\mathrm{d}[Y]}{\mathrm{d}t} = -\frac{\nu_Y}{V}\frac{\mathrm{d}\xi}{\mathrm{d}t}$$

- The rate (or speed) of reaction, also known as the differential rate law, is

$$v(t) = -\frac{1}{\nu_A} \frac{\mathrm{d}[A]}{\mathrm{d}t} = -\frac{1}{\nu_B} \frac{\mathrm{d}[B]}{\mathrm{d}t} = \frac{1}{\nu_Y} \frac{\mathrm{d}[Y]}{\mathrm{d}t} = \frac{1}{\nu_Z} \frac{\mathrm{d}[Z]}{\mathrm{d}t} = \frac{1}{V} \frac{\mathrm{d}\xi}{\mathrm{d}t}$$

- All terms are positive.
- Rate laws with a constant k are of the form

$$v(t) = k[A]^{m_A}[B]^{m_B}$$

- The exponents are known as **orders**.
- The overall order reaction is  $\sum m_i$ .
- The orders and overall order of the reaction depends on the fundamental reaction steps and the reaction mechanism.

 $\bullet\,$  For example, for the reaction  $2\,\mathrm{NO}_{(\mathrm{g})} + \mathrm{O}_{2(\mathrm{g})} \longrightarrow 2\,\mathrm{NO}_{2(\mathrm{g})},$  we have

$$v(t) = -\frac{1}{2}\frac{\mathrm{d[NO]}}{\mathrm{d}t} = -\frac{\mathrm{d[O_2]}}{\mathrm{d}t} = -\frac{1}{2}\frac{\mathrm{d[NO_2]}}{\mathrm{d}t}$$

- It follows that  $v(t) = k[NO]^2[O_2]$ .
- This is a rare elementary reaction that proceeds with the kinetics illustrated by the equation.
- Rate laws must be determined by experiment.
  - Multi-step reactions may have more complex rate law expressions.
  - Oftentimes, 1/2 exponents indicate more complicated mechanisms.
  - For example, even an equation as simple looking as  $H_2 + Br_2 \longrightarrow 2 HBr$  has rate law

$$v(t) = \frac{k'[H_2][Br_2]^{1/2}}{1 + k''[HBr][Br_2]^{-1}}$$

- Determining rate laws.
  - Method of isolation.
    - Put in a large initial excess of A so that it's concentration doesn't change that much; essentially incorporates  $[A]^{m_A}$  into k for determination of the order of B.
    - We can then do the same thing the other way around.
  - Method of initial rates.
    - We approximate

$$v = -\frac{\mathrm{d}[A]}{\nu_A \, \mathrm{d}t} \approx -\frac{\Delta[A]}{\nu_A \Delta t} = k[A]^{m_A} [B]^{m_B}$$

■ Consider two different initial values of [B], which we'll call  $[B_1]$ ,  $[B_2]$ . Then

$$v_1 = -\frac{1}{\nu_A} \left( \frac{\Delta[A]}{\Delta t} \right)_1 = k[A]_0^{m_A} [B]_1^{m_B} \qquad v_2 = -\frac{1}{\nu_A} \left( \frac{\Delta[A]}{\Delta t} \right)_2 = k[A]_0^{m_A} [B]_2^{m_B}$$

■ Take the logarithm and solve for  $m_B$ .

$$m_B = \frac{\ln(v_1/v_2)}{\ln([B]_1/[B]_2)}$$

• Does an example problem.