

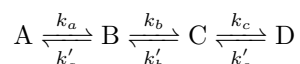
Week 5

Catalysis

5.1 Midterm Review and Intro to Catalysts

4/27: • Example problem 1: Steady-state approximation.

– Let



Suppose $[A]$ is maintained at a fixed value and the product D is removed from the reaction as it is formed. Find the rate at which the product is formed in terms of $[A]$.

- By hypothesis, we have that at all times t , $[A] = [A]_0$ and $[D] = 0$.
- The hypotheses also imply that we can apply the steady-state approximation to both B and C .
- Thus, we have that

$$\begin{aligned}\frac{d[C]}{dt} &= 0 = k_b[B] - k_c[C] - k'_b[C] \\ [B] &= \frac{k'_b + k_c}{k_b}[C]\end{aligned}$$

so that

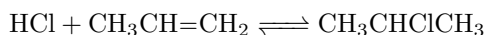
$$\begin{aligned}\frac{d[B]}{dt} &= k_a[A] - k_b[B] - k'_a[B] + k'_b[C] \\ 0 &= k_a[A] - k_b \cdot \frac{k'_b + k_c}{k_b}[C] - k'_a \cdot \frac{k'_b + k_c}{k_b}[C] + k'_b[C] \\ [C] &= \frac{k_a k_b}{k_b k_c + k'_a k'_b + k'_a k_c}[A]\end{aligned}$$

and therefore

$$\begin{aligned}\frac{d[D]}{dt} &= k_c[C] - k'_c \cdot 0 \\ &= \frac{k_a k_b k_c}{k_b k_c + k'_a k'_b + k'_a k_c}[A]\end{aligned}$$

• Example problem 2.

– Consider the reaction



which proceeds by the mechanism

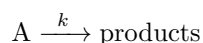
1. $\text{HCl} + \text{HCl} \rightleftharpoons (\text{HCl})_2$ (equilibrium constant K_1).
 2. $\text{HCl} + \text{CH}_3\text{CH}=\text{CH}_2 \rightleftharpoons \text{complex}$ (equilibrium constant K_2).
 3. $(\text{HCl})_2 + \text{complex} \rightleftharpoons \text{CH}_3\text{CHClCH}_3 + \text{HCl} + \text{HCl}$ (equilibrium constant K_3).
- The equilibrium constants for the two pre-equilibria are

$$K_1 = \frac{[(\text{HCl})_2]_{\text{eq}} c^\circ}{[\text{HCl}]_{\text{eq}}^2} \qquad K_2 = \frac{[\text{complex}]_{\text{eq}} c^\circ}{[\text{HCl}]_{\text{eq}} [\text{CH}_3\text{CH}=\text{CH}_2]_{\text{eq}}}$$

- We can divide the mass-action expression for K_1 by $(c^\circ)^2$ to get each concentration over c° within its exponent.
- The rate of product formation is

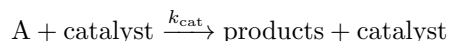
$$\begin{aligned} v &= \frac{d[\text{CH}_3\text{CHClCH}_3]}{dt} \\ &= k_r [(\text{HCl})_2] [\text{complex}] \\ &\approx k_r [(\text{HCl})_2]_{\text{eq}} [\text{complex}]_{\text{eq}} \\ &= k_r \cdot \frac{K_1 [\text{HCl}]_{\text{eq}}^2}{c^\circ} \cdot \frac{K_2 [\text{HCl}]_{\text{eq}} [\text{CH}_3\text{CH}=\text{CH}_2]_{\text{eq}}}{c^\circ} \\ &= \frac{k_r K_1 K_2}{(c^\circ)^2} [\text{HCl}]_{\text{eq}}^3 [\text{CH}_3\text{CH}=\text{CH}_2]_{\text{eq}} \end{aligned}$$

- There's a key assumption with the steady state and something about being able to apply the equilibrium concentration of the intermediate as the steady-state quantity.
 - This question wants to let you know that an equilibrium constant like K_1 might indicate a steady-state approximation.
- Note: Mind the positive and negative signs when constructing differential rate laws!
 - The midterm will be posted this Friday (April 29) and will be available until the following Friday (May 6). There will be a timed 2 hour period to take it.
 - **Catalyst:** A substance that participates in the chemical reaction but is not consumed in the process.
 - A catalyst affects the mechanism and activation energy of a chemical reaction.
 - A catalyst can give rise to a reaction path with a negligible activation barrier.
 - The exothermicity or endothermicity of the chemical reaction is not altered by the presence of a catalyst.
 - **Homogeneous catalysis:** Catalysis in which the catalyst is in the same phase as the reactants and products.
 - **Heterogeneous catalysis:** Catalysis in which the catalyst is in a different phase from the reactants and products.
 - Imagine that initially, we have the reaction



where k is the observed rate constant.

- When a catalyst is introduced into solution, this mechanism continues, but we now also have the new reaction pathway



- If each of these competing reactions is an elementary process, then

$$-\frac{d[A]}{dt} = k[A] + k_{\text{cat}}[A][\text{catalyst}]$$

- In most cases, catalysts enhance reaction rates by many orders of magnitude, and therefore only the rate law for the catalyzed reaction need be considered in analyzing experimental data.
- Reviews the Nobel Prizes in 2020 and 2021 (for CRISPR and asymmetric organocatalysis, respectively).
- An example of homogeneous catalysis.
 - Consider the reaction

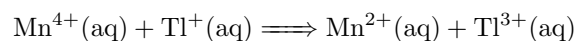


- In the absence of a catalyst,

$$v = k[\text{Tl}^{+}][\text{Ce}^{4+}]^2$$

and the mechanism is a termolecular elementary reaction.

- However, with Mn^{2+} as the catalyst, we have the mechanism



where the step with k_{cat} is the rate-determining step.

- Thus, for this mechanism, we have that

$$v = k_{\text{cat}}[\text{Ce}^{4+}][\text{Mn}^{2+}]$$

- The overall rate law for this reaction is therefore

$$v = k[\text{Tl}^{+}][\text{Ce}^{4+}]^2 + k_{\text{cat}}[\text{Ce}^{4+}][\text{Mn}^{2+}]$$

5.2 Enzymatic Catalysis

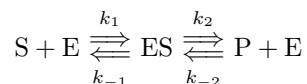
4/27:

- Midterm questions:
 - First 10 are T/F. He will test key concepts by making statements that are either true or false.
 - We should expect to spend no more than 30 minutes out of our 2 hours on these.
 - 4 calculation problems.
 - First- and second-order reactions.
 - Collisions.
 - A reaction mechanism problem.
 - Use calculators, do online searches, and use the textbook.
 - Do not talk to your classmates.
 - The midterm will become available Friday at noon.
- **Enzyme:** A protein molecule that catalyzes a specific biochemical reaction.
 - For example, hexokinase converts glucose and ATP to glucose 6-phosphate, ADP, and H^{+} .
- **Substrate:** The reactant molecule acted upon by an enzyme.

- **Active site:** The region of the enzyme where the substrate reacts.
- **Lock-and-key model:** The active site and substrate have complementary three-dimensional structures and dock without the need for major atomic rearrangements.
- **Induced fit model:** Binding of the substrate induces a conformation change in the active site. The substrate fits well in the active site after the conformational change has taken place.
- The Michaelis-Menten mechanism is a reaction mechanism for enzyme catalysis.
- Intuition.
 - Imagine we have a solution of enzymes and substrate molecules.
 - Limiting factors of an enzymatically catalyzed reaction.
 - The enzyme-substrate affinity.
 - The turnover number.
 - If the substrate concentration is low (i.e., $[S]_0 \ll [E]_0$) and the enzyme-substrate affinity is strong (but not so strong that the enzyme-substrate complex is energetically favorable), then we expect $v_{\text{initial}} \propto [S]_0$ because we'd think that all of the substrate will immediately be absorbed and transformed.
 - If the substrate concentration is large (i.e., $[S]_0 \gg [E]_0$) and the enzyme-substrate affinity is strong, then we expect $v_{\text{initial}} \propto [E]_0$ and, importantly, $v_{\text{initial}} \not\propto [S]_0$.
- Mathematical derivation.
 - Experimental studies reveal that the rate law for many enzyme-catalyzed reactions has the form

$$-\frac{d[S]}{dt} = \frac{k[S]}{K_m + [S]}$$

- This is the final goal of the derivation.
 - The mechanism is



- Thus,

$$\begin{aligned} -\frac{d[S]}{dt} &= k_1[E][S] - k_{-1}[ES] \\ -\frac{d[ES]}{dt} &= (k_2 + k_{-1})[ES] - k_1[E][S] - k_{-2}[E][P] \\ \frac{d[P]}{dt} &= k_2[ES] - k_{-1}[E][P] \end{aligned}$$

- Note that

$$[E]_0 = [ES] + [E]$$

- Plugging that equation into the rate law for the enzyme-substrate complex and applying the steady-state approximation yields

$$\begin{aligned} -\frac{d[ES]}{dt} = 0 &= [ES](k_1[S] + k_{-1} + k_2 + k_{-2}[P]) - k_1[S][E]_0 - k_{-2}[P][E]_0 \\ [ES] &= \frac{k_1[S] + k_{-2}[P]}{k_1[S] + k_{-2}[P] + k_{-1} + k_2} [E]_0 \end{aligned}$$

- Substituting this and the original expression for $[E]_0$ into the rate law for the substrate yields

$$v = -\frac{d[S]}{dt} = \frac{k_1 k_2 [S] + k_{-1} k_{-2} [P]}{k_1 [S] + k_{-2} [P] + k_{-1} + k_2} [E]_0$$

- If the experimental measurements of the reaction rate are taken during the time period when only a small percentage (1-3%) of the substrate is converted to product, then

$$[S] \approx [S]_0$$

and

$$[P] \approx 0$$

- Using this approximation simplifies the above rate law to

$$v = -\frac{d[S]}{dt} = \frac{k_1 k_2 [S]_0 [E]_0}{k_1 [S]_0 + k_{-1} + k_2} = \frac{k_2 [S]_0 [E]_0}{K_m + [S]_0}$$

where $K_m = (k_{-1} + k_2)/k_1$ is the **Michaelis constant**.

- The Michaelis constant tells you the ratio of dissociation of the enzyme-substrate complex to the formation of the enzyme-substrate complex. In other words, it provides information on the enzyme-substrate affinity.
- Note that k_{-2} is not present in the denominator of the Michaelis constant because for a good enzyme, k_{-2} should be very small.
- The unit of K_m should be concentration.
- When $K_m = [S]_0$, $v = v_{\max}/2$
- An enzyme-catalyzed reaction is first order in the substrate at low substrate concentrations ($K_m \gg [S]_0$) and then becomes zero order in the substrate at high substrate concentrations ($K_m \ll [S]_0$).
- Thus, at low substrate concentrations, the above equation holds, but at high substrate concentrations,

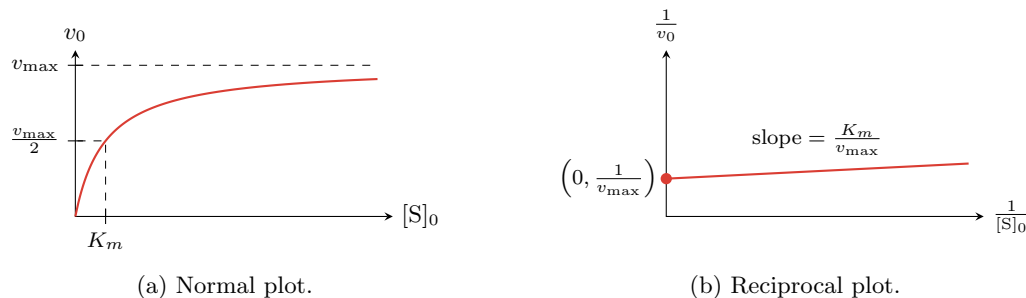
$$-\frac{d[S]}{dt} = k_2 [E]_0 \qquad v_{\max} = k_2 [E]_0$$

resulting in the **Lineweaver-Burk plot**, canonically represented by the second of the two equivalent forms below.

$$v = \frac{v_{\max}}{1 + K_m/[S]_0} \qquad \frac{1}{v} = \frac{1}{v_{\max}} + \frac{K_m}{v_{\max}} \frac{1}{[S]_0}$$

5.3 Measuring Catalytic Efficiency and Correcting Collision Theory

- 4/29:
- Everything on the midterm comes from Tian's lecture notes. Thus, he recommends we go through them before taking the midterm.
 - The T/F will be 20-30% of the grade.
 - There will be some integration on the calculation problems, but they'll be pretty easy. All formulas that will appear have been covered in class.
 - The midterm will cover up to Monday's class (this week).
 - Consider the substrate concentration $[S]_0$ vs. the initial rate v_0 .
 - Normal plot (Figure 5.1a).
 - As $[S] \rightarrow \infty$, v_0 approaches an asymptote line defined by $v_{\max} = k_2 [E]_0$.

Figure 5.1: Plotting v_0 vs. $[S]_0$.

- At the beginning (low $[S]$), the paradigm is almost linear (thus, the reaction is first order wrt. substrate concentration here).
- Note that we make use of the assumptions that $[P] \approx 0$ and $[S] \approx [S]_0$.
- Where we have $v_{\max}/2$ on the v_0 -axis, we have K_m (the Michaelis constant) on the $[S]_0$ axis.
- Reciprocal plot (Figure 5.1b).
 - Note that a plot of v_0 vs. $[S]_0$ is nonlinear but a plot of $1/v_0$ vs. $1/[S]_0$ is linear.
 - This reciprocal plot (the Lineweaver-Burk plot) gives v_{\max} (via the y -intercept) and K_m via this information and the slope.
 - Note that

$$\begin{aligned}
 \frac{1}{v_0} &\propto \frac{K_m}{v_{\max}} \frac{1}{[S]_0} \\
 &= \frac{(k_{-1} + k_2)/k_1}{k_2[E]_0} \frac{1}{[S]_0} \\
 &= \frac{k_{-1} + k_2}{k_1 k_2} \frac{1}{[E]_0 [S]_0}
 \end{aligned}$$

- Evaluating the performance of a catalyst.

$$\frac{k_{-1} + k_2}{k_1 k_2} = \frac{K_m}{v_{\max}} \cdot [E]_0$$

- K_m is relevant to the enzyme-substrate affinity.
- v_{\max} tells us about conversion from the ES with a focus on the second elementary step.
- An alternate form of the Lineweaver-Burk plot is

$$\frac{1}{v_0} = \frac{1}{v_{\max}} + \frac{k_2 + k_{-1}}{k_1 k_2} \frac{1}{[E]_0 [S]_0}$$

- Regrouping the terms, we have

$$\frac{1}{v_0} = \frac{1}{v_{\max}} + \frac{k_2 + k_{-1}}{k_2} \frac{1}{k_1 [S]_0 [E]_0} = \frac{1}{v_{\max}} + \frac{k_2 + k_{-1}}{k_2} \frac{1}{v_{f1}}$$

where v_{f1} is the forward reaction rate for the first elementary step.

- **Turnover number:** The number of catalytic cycles that each active site undergoes per unit time.
Given by

$$\text{TON} = \frac{v_{\max}}{n[E]_0} = \frac{k_2[E]_0}{n[E]_0} = \frac{k_2}{n}$$

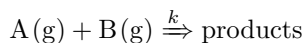
- Indicates how fast the ES complex proceeds to E + P.

- k_2/n is the number of active sites per enzyme.

- **Catalytic efficiency:** The following quantity. *Given by*

$$\frac{\text{TON}}{K_m}$$

- Consider the reaction



- The rate of the general bimolecular elementary gas-phase reaction is

$$v = -\frac{d[\text{A}]}{dt} = k[\text{A}][\text{B}]$$

- Using the naïve assumption that every collision between the hard spheres A and B yields products,

$$v = Z_{\text{AB}} = \sigma_{\text{AB}} \langle u_r \rangle \rho_{\text{A}} \rho_{\text{B}}$$

- Moreover,

$$k = \sigma_{\text{AB}} \langle u_r \rangle$$

- Unfortunately, this is not accurate. We make our first improvement to collision theory by taking into account the dependence of the reaction rate on the relative speed, or energy, of the collision. Thus, we average over all possible collision speeds.

$$k = \int_0^\infty du_r f(u_r) k(u_r) = \int_0^\infty du_r u_r f(u_r) \sigma_r(u_r)$$

- Since $f(u_r)$ is the distribution of relative speeds in the gas sample, we have that

$$u_r f(u_r) du_r = \left(\frac{\mu}{k_{\text{B}} T} \right)^{3/2} \left(\frac{2}{\pi} \right)^{1/2} u_r^3 e^{-\mu u_r^2 / 2 k_{\text{B}} T} du_r$$

- To compare this with the traditional Arrhenius form of k , we need to change the dependent variable from u_r to E , which we can do via

$$E_r = \frac{1}{2} \mu u_r^2 \quad u_r = \sqrt{\frac{2E_r}{\mu}} \quad du_r = \sqrt{\frac{1}{2\mu E_r}} dE_r \quad E_a = k_{\text{B}} T^2 \frac{d \ln k}{dT}$$

5.4 Office Hours (Tian)

- Can Problem 28-46 be done purely with integrated forms of the Arrhenius equation?
 - We can use either case depending on how the question was asked, and often it's an instance of which would be easier to use. We should always think both ways.
- Ask about the Problem 29-24 derivation from class final steps as well as molecularity.
 - You have a 2 and a squared. The exponent comes from the fact that it's bimolecular. The coefficient comes from the fact that we're talking about the rate of change of that substance.
 - In particular, if $2\text{C} \xrightarrow{k_1} \text{D}$, then

$$v = k_1 [\text{C}]^2$$

- But recall that v is the *rate of reaction*, a specifically defined quantity. Indeed, we know that

$$\begin{aligned} v &= -\frac{1}{\nu_C} \frac{d[C]}{dt} = \frac{1}{\nu_D} \frac{d[D]}{dt} \\ &= -\frac{1}{2} \frac{d[C]}{dt} = \frac{1}{1} \frac{d[D]}{dt} \end{aligned}$$

so that this step's contribution to $d[C]/dt$ is

$$\begin{aligned} -\frac{1}{2} \frac{d[C]}{dt} &= k_1[C]^2 \\ \frac{d[C]}{dt} &= -2k_1[C]^2 \end{aligned}$$

- Eyring equation and thermodynamics questions: So what we need to know and be able to work with are the Eyring equation and you said “the dimensional analysis?” I’m still unclear on what $\langle u_{ac} \rangle$ and ν_c are.
 - ν_c is frequency. It’s units are reciprocal seconds. It’s 1 over the time it takes to cross the barrier. Length divided by time gives the rate of crossing the barrier. And rate depends on reaction coordinate.
 - This is very hard to visualize along a typical, complicated reaction coordinate. However, if we think about an S_N2 for instance, we can picture δ as some tolerance around the equatorial ligands being perfectly coplanar with the central atom, $1/\nu_c$ as the time from when the molecules begin reacting (nucleophilic attack) to when the leaving group has fully left, and thus $\langle u_{ac} \rangle$ as the average speed with which the activated complex crosses the barrier top, i.e., is formed, moved, and dissociates.
 - The conclusion is more important than the derivation. There’s a lot of assumptions that are not as relevant to the eventual result. The conclusion being that we can express the pre-exponential factor can be expressed in terms of molecular quantities from statistical mechanics.
 - TST is imperfect; some textbooks use P° instead of c° since this is a gas-phase reaction.
 - $\Delta^\ddagger PV = \Delta^\ddagger n RT$ hails from the ideal gas law. $\Delta^\ddagger n$ is the change in the number of molecules from the reactants to the activated complex (which is always unimolecular!). $1 - \Delta n = \text{molecularity}$. We need $\Delta^\ddagger U^\circ$. To get an expression for it, we invoke enthalpy (H) which brings in the change in PV going to the transition state, which is related to the change in the number of moles of gas!
- What does the double vs. single arrow signify?
 - Elementary reactions vs. reactions that might not be.
- The differences between $F(u)$, $f(u_x)$, and $h(u)$.
 - $f(u_x)$ is the Gaussian distribution of velocity components.
 - $F(u)$ is the M-B distribution.
 - $h(u_x, u_y, u_z)$ is the product $f(u_x)f(u_y)f(u_z)$ and describes the fraction of molecules not just moving linearly in one dimension or the other, but in any direction. Graphically, we’d need four dimensions, but we can visualize it either as equipotential 2-spheres or, using the color model in 3D, as intensity getting brighter with a Gaussian distribution cubed as you approach the origin.
- Midterm stuff.
 - The midterm will have two problems on rate laws (one of which might concern the Eyring equation), one mechanism question (how the approximations provided lead into the final rate law), and one collision theory question (the original collision theory stuff, not the more recent stuff).

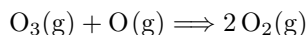
- For each T/F problem, there will be two sentences. You need to identify all of the mistakes in the sentences. Answer if it's true or false, and explain. One point for T/F; two points for your explanation. If the sentence is true, a few words might be helpful, too.
- We'll need to show some decent steps to recreate the work. Clear doubt from the TA's mind.
- If there is a mistake in a problem, just do your best to solve it and flag it for him.
- Fast equilibrium vs. steady state?
 - Detailed balance only applies to elementary steps?

5.5 Chapter 29: Chemical Kinetics II — Reaction Mechanisms

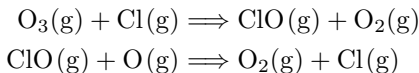
From McQuarrie and Simon (1997).

- 5/1:
- Methods of increasing the rate of reaction.
 1. Increase the temperature.
 - Drawback: Reactions in solution are constrained to the temperature range between the melting and boiling point of the solvent.
 2. Enable the reaction to proceed by an alternate mechanism having a lower activation energy.
 - **Catalyst**: A substance that participates in a chemical reaction but is not consumed in the process.
 - By participating, a catalyst provides an alternate mechanism.
 - The trick is to construct this alternate mechanism such that it has a lower activation barrier.
 - McQuarrie and Simon (1997) defines **homogeneous** and **heterogeneous** catalysis.
 - Because a catalyst is not consumed or otherwise chemically altered (in a net sense), the thermodynamics of the reaction do not change when one is present.
 - The exponential form of the Arrhenius equation implies that even small changes in activation energy can lead to substantial changes in reaction rate.
 - Note that since a catalyzed reaction has multiple mechanisms, we say it proceeds along reaction *coordinates*, plural.
 - McQuarrie and Simon (1997) derives the rate law for a unimolecular uncatalyzed, bimolecular catalyzed reaction.
 - McQuarrie and Simon (1997) gives the ionic homogeneous catalysis example.
 - An example of heterogeneous catalysis.
 - Consider the reaction
$$3 \text{H}_2(\text{g}) + \text{N}_2(\text{g}) \longrightarrow 2 \text{NH}_3(\text{g})$$
 - The activation barrier is roughly 940 kJ/mol.
 - In the presence of an iron surface, however, the activation barrier drops over an order of magnitude to 80 kJ/mol.
 - The mechanism will be discussed in depth in Chapter 31, along with other heterogeneous surface-catalyzed gas-phase reactions.
 - An example where both types of catalysis play a role.
 - Consider the destruction of ozone in the stratosphere.

- Naturally, this reaction occurs via



- In the presence of chlorine atoms, however, the following mechanism becomes available.

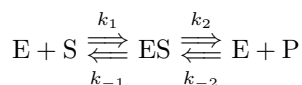


- This is an example of homogeneous catalysis.

- Over time, however, chlorine atoms get bound up in the reservoir molecules $\text{HCl}(\text{g})$ and $\text{ClONO}_2(\text{g})$.
- Nevertheless, the surface of polar stratospheric clouds catalyzes the reaction between these two molecules to liberate a molecule of diatomic chlorine gas (which can then be homolytically dissociated by sunlight to regenerate the gaseous monoatomic chlorine catalyst) and a molecule of $\text{HNO}_3(\text{g})$.

- Since the surface of the clouds are of a different phase from the gas, this is an example of heterogeneous catalysis.

- McQuarrie and Simon (1997) defines **enzymes**, **substrates**, and **active sites**.
- **Michaelis-Menten mechanism:** A simple mechanism that accounts for the rate law commonly observed for enzyme-catalyzed reactions. *Given by*



- Proposed by Leonor Michaelis and Maude Menten in 1913.
- This reaction sees an initial buildup period of ES, followed by a period during which $[\text{ES}]$ is relatively constant. Thus, we may apply the SS approximation to it.
- High substrate concentrations in the Michaelis-Menten mechanism.
 - At high substrate concentrations, essentially all of the enzymes are tied up with substrate, so adding more substrate doesn't do anything and the reaction is zero-order in substrate concentration.
 - Assuming that all enzyme is tied up in substrate means that $[\text{ES}] \approx [\text{E}]_0$ and the rate of SM consumption is essentially equal to the rate of product formation, which is just proportional to the dissociation of the reacted enzyme-substrate complex. Mathematically,

$$\begin{aligned}-\frac{d[\text{S}]}{dt} &= k_2[\text{E}]_0 \\ v_{\max} &= k_2[\text{E}]_0\end{aligned}$$

- Note that we introduce the v_{\max} terminology because it is under these conditions (excess substrate) that the reaction rate is at its maximum.

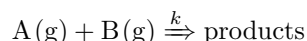
- **Turnover number:** The maximum rate divided by the concentration of enzyme active sites.
 - By definition, the turnover is the maximum number of substrate molecules that can be converted into product molecules per unit time by an enzyme molecule.
 - Note that the concentration of enzyme *active sites* is not necessarily equal to $[\text{E}]$ because some enzymes have more than one active site.
- For an enzyme having only one active site, the turnover number is given by $v_{\max}/[\text{E}]_0 = k_2$.

5.6 Chapter 30: Gas-Phase Reaction Dynamics

From McQuarrie and Simon (1997).

5/18:

- Goals of the chapter.
 - Describe bimolecular gas-phase reactions, some of the simplest naturally occurring elementary kinetic processes.
 - Analyze the reaction $F(g) + D_2(g) \Rightarrow DF(g) + D(g)$, an exothermic cousin of the hydrogen exchange reaction $H_A + H_B-H_C \Rightarrow H_A-H_B + H_C$.
- Naïve hard-sphere collision theory.
 - Consider the following general bimolecular elementary gas-phase reaction.



- The rate of reaction is given by

$$v = -\frac{d[A]}{dt} = k[A][B]$$

- Recall that when we derived this equation in Chapter 29, we assumed that every collision between molecules of A and B is chemically active and thus related the rate of reaction to the collision frequency per unit volume Z_{AB} via

$$v = Z_{AB} = \sigma_{AB} \langle u_r \rangle \rho_A \rho_B$$

where σ_{AB} is the hard-sphere collision cross section of A and B molecules (see Figure 2.1), $\langle u_r \rangle$ is the average relative speed of a colliding pair of A and B molecules, and ρ_A, ρ_B are the respective number densities.

- Note that $\sigma_{AB} = \pi d_{AB}^2$ where d_{AB} is the sum of the radii of the two colliding spheres.
- Since we are assuming that every collision is successful, we have that every one of the Z_{AB} collisions happening every second in every cubic meter of volume consumes a reactant molecule. In other words, Z_{AB} gives the change in molecular concentration of A per unit time, so the statement $v = -d[A]/dt = Z_{AB}$ is justified.
- Identifying $[A] \sim \rho_A$ and $[B] \sim \rho_B$ gives the rate constant as

$$k = \sigma_{AB} \langle u_r \rangle$$

- To convert k from the standard SI units of $\text{m}^3 \text{molecule}^{-1} \text{s}^{-1}$ to the more conventional units of $\text{dm}^3 \text{mol}^{-1} \text{s}^{-1}$, we can multiply the above expression by $(1000 \text{ dm}^3 \text{ m}^{-3})(N_A \text{ mol}^{-1})$.
- Problems with naïve hard-sphere collision theory.
 - The calculated rate constants are often significantly larger than experimental rate constants.
 - Since $\langle u_r \rangle \propto T^{1/2}$, it predicts $k \propto T^{1/2}$ instead of the experimental Arrhenius dependence of $k \propto e^{1/T}$.
- Two underlying assumptions of naïve hard-sphere collision theory.
 - Each pair of reactants approaches one another with a relative speed of $\langle u_r \rangle$.
 - Reality: Pairs of reactant molecules approach each other with a (M-B) distribution of speeds.
 - Every collision is chemically reactive, regardless of speed or energy.
 - Reality: Since the valence electrons of the two molecules repel one another, a reaction will not occur unless the relative speed is sufficient to overcome this repulsive force.

- Developing a model of collision theory that accounts for differing speeds.
 - We consider each speed u_r with which molecules can collide to give rise to a different rate constant $k(u_r)$ and corresponding **reaction cross section** $\sigma_r(u_r)$. These quantities are related via

$$k(u_r) = u_r \sigma_r(u_r)$$

- It makes sense to continue using this relationship since $k = \sigma_{AB} \langle u_r \rangle$ relies on the assumption that all molecules travel at the same speed $\langle u_r \rangle$, and $k(u_r) = u_r \sigma_r(u_r)$ relies on the assumption that all molecules travel at the same speed u_r .
 - To calculate the observed rate constant, we must average all $k(u_r)$'s over the speed distribution. We can do this with

$$k = \int_0^\infty du_r f(u_r) k(u_r) = \int_0^\infty du_r u_r f(u_r) \sigma_r(u_r)$$

- As per Chapter 27,

$$\begin{aligned} u_r f(u_r) du_r &= 4\pi \left(\frac{\mu}{2\pi k_B T} \right)^{3/2} u_r^3 e^{-\mu u_r^2 / 2k_B T} du_r \\ &= \left(\frac{\mu}{k_B T} \right)^{3/2} \left(\frac{2}{\pi} \right)^{1/2} u_r^3 e^{-\mu u_r^2 / 2k_B T} du_r \end{aligned}$$

- We now encounter the question of what our threshold relative molecular collision speed is. In fact, we have never discussed such a quantity. However, we have spent plenty of time developing a theory of the threshold energy E_a , the Arrhenius activation energy. As such, a change of variables from speed to energy is in order. In particular, since

$$E_r = \frac{1}{2} \mu u_r^2$$

we have that

$$u_r = \left(\frac{2E_r}{\mu} \right)^{1/2} \quad du_r = \left(\frac{1}{2\mu E_r} \right)^{1/2} dE_r$$

- Making these substitutions yields

$$u_r f(u_r) du_r = \left(\frac{2}{k_B T} \right)^{3/2} \left(\frac{1}{\mu \pi} \right)^{1/2} E_r e^{-E_r / k_B T} dE_r$$

- We may now easily define the reaction cross section $\sigma_r(E_r)$ by the following simple model, where E_0 is the threshold energy.

$$\sigma_r(E_r) = \begin{cases} 0 & E_r < E_0 \\ \sigma_{AB} & E_r \geq E_0 \end{cases}$$

- Resubstituting into the original expression for k gives

$$\begin{aligned} k &= \left(\frac{2}{k_B T} \right)^{3/2} \left(\frac{1}{\mu \pi} \right)^{1/2} \int_0^\infty dE_r E_r e^{-E_r / k_B T} \sigma_r(E_r) \\ &= \left(\frac{2}{k_B T} \right)^{3/2} \left(\frac{1}{\mu \pi} \right)^{1/2} \int_{E_0}^\infty dE_r E_r e^{-E_r / k_B T} \sigma_{AB} \\ &= \left(\frac{8k_B T}{\mu \pi} \right)^{1/2} \sigma_{AB} e^{-E_0 / k_B T} \left(1 + \frac{E_0}{k_B T} \right) \\ &= \langle u_r \rangle \sigma_{AB} e^{-E_0 / k_B T} \left(1 + \frac{E_0}{k_B T} \right) \end{aligned}$$

- Using this model, if we reverse engineer E_0 from experimental data on the other parameters, we can often get an answer on the same order of magnitude as E_a !