# Week 5

# Magnetic Resonance Spectroscopy

## 5.1 Lecture 8: Magnetic Resonance Spectroscopy

1/31:
- Refer to Chapter 14 of McQuarrie and Simon [1].

- There is both NMR (nuclear magnetic resonance) and ESR (electron spin resonance) or EPR (electron paramagnetic resonance).

  - The last two are the same thing.

- Two fields: The static magnetic field, and the probing *electro*magnetic field.

- Derivation of quantized angular momentum.

  - In molecules, there is a multiplicity/degeneracy of states that grows as $2J+1$. They have quantized angular momentum.
  - So is the orbital angular momentum of electrons!
  - Putting atoms into a magnetic field *creates* the anisotropy necessary for discussing the $z$-component (or any coordinate component) of angular momentum.
  - Spin angular momentum: Just means that the objects (e.g., electrons and nuclei) have a property that looks a lot like spin and/or angular momentum.
  - We say that each nucleon has a spin of 1/2. Protons and neutrons add separately.
    - ■ Even number of protons and neutrons? Spin 0.
    - ■ Mixed even/odd? We have actual nuclear spin.
      - ➢ We need a nucleus like this to detect!
    - ■ Odd/odd? We have 0 spin again.
  - We focus on spin 1/2 particles. These have two degenerate energy states that split in a magnetic field.

- Classical picture of spin angular momentum.

  - Picture a charged particle with angular momentum. The circulating charge produced a magnetic field which aligns along the direction of the angular momentum.
  - Indeed, a spinning charged particle behaves like a dipole.
  - $\gamma = 2mc$ is the **gyromagnetic ratio**.

- Quantum spin angular momentum.

  - Basically the same thing; we just rephrase everything from before in the language of operators.

- **Zeeman effect**: Two energy levels split with increasing $B$.

- **Larmor frequency**: The frequency $\nu = \gamma B/2\pi$ in the radio frequency range that induces a shift.

- Typical operating conditions.

  - NMR vs. ESR: NMR has a stronger magnetic field, longer EM excitation radio waves, and significantly lower gyromagnetic ratios.

  - There is only a tiny difference between nuclear state occupation at room temperature; hence supercooling to get something detectable.

- What is the magnetic dipole doing in the magnetic field?

  - No constraints on the $x$ and $y$ components of $I$.

  - A dipole in a field experiences a torque.

    - Dipole precesses around $B$ at the Larmor frequency $\nu$.

  - FT-NMR spectrometers use pulsed rf fields to synchronize and detect the procession of spins.

- Lots of good extension material on NMR; also worth rewatching at some point!

## 5.2   Lecture 9: Magnetic Resonance Spectroscopy 2

2/2:
- Summary of last time.

  - The quantity that we're measuring is spin angular momentum $\bar{I}$, which is a vector quantity.

  $$|\bar{I}| = \hbar\sqrt{I(I+1)}$$

  where $I = 1/2$ is the nuclear spin quantum number.

  - The other quantity of concern is the projection $I_z = m_I \hbar$ where $m_i = \pm 1/2$.

  - In a magnetic field, we break degeneracy, getting $E(m_I) = -\gamma_N \hbar m_I B$ and $\Delta E = -\gamma_N \hbar B$.

  - Electromagnetic resonance is achieved when the frequency $\nu$ of incident radiation satisfies $h\nu = \Delta E$.

- The interest in chemistry: Chemical shift.

  - There are small variations in the frequency for different types of protons depending on the surrounding electron density.

  - Measured frequency depends on effective magnetic field.

    - Shielding: The influence of electrons around the nucleus on the effective magnetic field.

    - The effective field is smaller than the applied field.

  - Shielding *decreases* the splitting (this is why nearby highly polar groups lead to large shifts, while alkanes have small shifts).

- Measuring the chemical shift.

  - We measure the difference in the Larmor frequency relative to a standard (TMS).

  - Shielding is a small effect (on the order of $10^{-6}$, so we use ppm $\delta$).

  - Example: 1 ppm at $500\,\text{MHz}$ is $\nu = 500\,\text{Hz}$, which is tiny (on the order of microjoules).

- Chemical shift charts (from OChem) are included in the slides.

- FT-NMR spectrometers.

  - How do we make these measurements?

  - NMR spectrometers are almost all working in FT mode these days.

    – They use pulsed radiofrequency (r.f.) fields and detect the precession of spins.

    – Precession of one spin in a magnetic field occurs at the Larmor frequency.

    – Applying an excitation field creates a superposition of $m_s = \pm 1/2$ states. The net dipole is now perpendicular, and precesses that way (i.e., in the $xy$ plane) in a mathematically describable fashion.

    – Putting your superconducting coil along the $x$- or $y$-axis allows you to detect changes.

- **Magnetization**: The macroscopic alignment of magnetic dipoles $\bar{M} = \sum \bar{\mu}$.

    – At equilibrium, $\bar{M}$ aligns along $\bar{B}$.

    – An rf field rotates magnetization to $x$: This is a **$\pi/2$-pulse** or a **90°-pulse**.

    – We detect precessing magnetization and return to equilibrium during the **free-induction decay**. An FT of the decay then generates our spectrum.

- Relaxation mechanism.

    – Spin state lifetime ($T_1$).

       ■ "Spin-lattice" or "longitudinal" relaxation.

       ■ Recovery of the magnetization along $z$.

       ■ A molecular property.

       ■ Transfer of energy to the environment.

       ■ The return of magnetization to equilibrium has a characteristic time constant $T_1$ which appears in the time vs. relaxation plot $1 - \mathrm{e}^{-t/T_1}$.

    – Dephasing ($T_2$).

       ■ "Transverse" relaxation.

       ■ Loss of magnetization in the $xy$-plane of many different sources.

       ■ We have the loss described by $\mathrm{e}^{-t/T_2}$.

    – These two processes are not independent.

- There are numerous types of NMR experiments.

    – In our lab, we just scratch the surface.

    – We use a population inversion to measure $T_1$.

       ■ **$\pi$-pulse** to invert magnetization.

       ■ Wait for relaxation.

       ■ Read out following another **$\pi/2$-pulse**.

    – Two-dimensional spectroscopy.

       ■ Heteronuclear single quantum coherence spectroscopy (HSQC).

       ■ Excite $^1$H; transfer its magnetization to $^{13}$C, which is nice because $^{13}$C is hard to excite on its own.

       ■ Transfer back to $^1$H and detect.

       ■ Tells us which protons transfer magnetization.

## 5.3   Lab 3: NMR

**Lab Manual**

- Modern NMR is performed in the time-domain, and frequency-domain spectra are obtained via Fourier Transformation of the data.

- Goal: Apply FT-NMR to investigate relaxation processes in liquids.

- – Use an NMR technique called inversion recovery to determine the spin-lattice relaxation times $T_1$ for $^{13}$C in n-hexanol, hexanoic acid, hexylamine, hexane thiol, or bromohexane.

- NMR basics (misc. notes).

  - – Review from Chapter 14 of McQuarrie and Simon [1].

  - – Definitions of the **Zeeman Hamiltonian** and **Zeeman levels** are new.

  - – $\Delta E = -\gamma \hbar B_0$ gives only a static picture of nuclear interaction with the magnetic field. In reality, a single nucleus remains in a certain state no longer than a time $T_1$ on average.

  - – A nuclear spin exposed to a magnetic field $B_0$ tends to "align" with said field. It's not so much aligning, though, as it is acting like a gyroscope in a gravitational field; indeed, it will precess about the direction of $B_0$ with a characteristic frequency $\omega$.

  - – To rigorously treat the populations of the ground and excited spin states, we consider an **ensemble** of nuclei and the resultant thermodynamic argument.

  - – The **total magnetization** takes the form of a vector aligned in the direction of $B_0$ (which is Z; there is no net magnetization in the XY-plane).

  - – An alternating transmitter field $B_1$ "tips" the magnet from its equilibrium position by driving the Larmor procession. The angle $\alpha$ through which **M** is tipped depends, via the following relation, on the strength of $B_1$ and the length of time $t_p$ during which $B_1$ is applied.

$$\alpha = -\gamma B_1 t_p$$

  - – Immediately following this process, the transverse components $M_X, M_Y$ decay to zero with a time constant $T_2$, and the longitudinal component $M_Z$ is restored to its equilibrium value with time constant $T_1$.

  - – As with general time constants (which tend to be exponential factors), it will take much longer than $T_1$ for equilibrium to be "restored."

- **Zeeman Hamiltonian**: The Hamiltonian operator relevant to nuclear magnetic spins. *Denoted by* $\mathcal{H}_Z$. *Given by*

$$\mathcal{H}_Z = -\mu \cdot B_0 = -B_0 I \gamma \hbar$$

- **Zeeman levels**: The $2I + 1$ energy levels $E(m) = -B_0 \gamma \hbar m$, where $m = -I, -I+1, \dots, I$.

- **Spin-lattice relaxation time**: The average time for which a single nucleus remains in a certain state. *Denoted by* $T_1$.

- **Spin-spin relaxation time**: A type of interaction that occurs between different spins. *Denoted by* $T_2$.

- **Larmor frequency**: The characteristic frequency of a given nucleus at which it processes in a magnetic field. *Denoted by* $\omega$.

- **Total magnetization**: The vector sum of all the magnetic moments. *Denoted by* **M**.

- FT-NMR (pulsed NMR).

  - – Experiment: A strong transmitter field $B_1$ is applied for a short time $t_p$.

  - – Specifically, since the transverse magnetization has its maximum when $\alpha = 90°$, we want to choose $t_p, B_1$ such that $\gamma B_1 \cdot t_p = 90°$. This is called a **90° pulse**. $t_p$ is on the order of a few microseconds in general.

  - – After the pulse, the relaxation contains spectroscopic information: The transverse magnetization will decay exponentially to zero due to spin relaxation in a manner that can be picked up by the receiver as a **free induction decay**.

– Differing transmitter and Larmor frequencies: $M_Y$ and $B_1$ interfere, yielding a sine wave with exponentially decreasing amplitude:

$$M_Y(t) = M_{Y_0} \cos[(\omega_0 - \omega_1)t] e^{-t/T_2}$$

  ■ $M_{Y_0}$ is the transverse magnetization immediately after the pulse. It is given by

$$M_{Y_0} = M_{Z_0} \cos\alpha = M_{Z_0} \cos(\gamma B_1 t_p)$$

  where $M_{Z_0}$ is the equilibrium magnetization in the Z-direction.

– In general, a molecule will have multiple chemical environments, each with its own Larmor frequency. All of these will affect the FID, so we will need to use an FT to separate the different contributing frequencies.

– For more on pulsed NMR, see Bloch [2].

- **90° pulse**: An application of a strong transmitter field $B_1$ over a time $t_p$ sufficiently short such that $-\gamma B_1 t_p = \alpha = 90°$.

- **Free induction decay**: A decay that occurs in the absence of an RF-field. *Also known as* **FID**.

- Chemical shifts and shielding.

  – Review from Chapter 14 of McQuarrie and Simon [1].

- $^1$H vs. $^{13}$C.

- Signal processing considerations.

  – Real + imaginary parts of the FT data.

- Locking.

  – Making sure that the magnetic field is what the machine thinks it is.

  – This is like calibration, and we use a substance with a single sharp known NMR line (typically deuterium $^2$H) to do the locking.

- Spin decoupling.

  – Using a third magnetic field $B_2$ to resonate the nuclei to be decoupled.

  – We will use this technique to spin-decouple $^1$H and $^{13}$C, greatly simplifying the latter spectrum.

- Spin-lattice relaxation.

  – $T_1$ is the "lifetime" of the first-order rate process that returns the magnetization to the Boltzmann equilibrium along the +Z-axis.

  – The **spin-lattice relaxation rate** depends on the strength of intramolecular interactions and molecular motion.

  – Molecular mobility can be quantified by a **correlation time**.

  – Mechanisms involved in relaxation: Dipolar coupling, quadrupolar coupling, paramagnetic interaction, scalar coupling, chemical shift anisotropy, and spin rotation.

    ■ Protonated carbons: Dipole-dipole interactions with the attached protons are overwhelmingly dominant.

- **Spin-lattice relaxation rate**: The reciprocal of the spin-lattice relaxation time $T_1$. *Given by*

$$\frac{1}{T_1}$$

- **Correlation time**: The time it takes for a molecule (or a molecular fragment) to reorient by a unit angle. *Denoted by $\tau_C$.*

- Dipole-dipole coupling.

  - Depends on the strength of dipolar coupling, orientation of the interacting nuclei, distance between the interacting nuclei, and molecular motion.
  - Relaxation rate for a protonated carbon.

  $$\frac{1}{T_1} = n\left(\frac{\mu_0}{4\pi}\right)^2 \frac{\hbar^2 \gamma_C^2 \gamma_H^2}{r_{CH}^6} \tau_C$$

    - $\mu_0$ is the permeability of a vacuum.
    - $\gamma_i$ is the gyromagnetic ratio of atom $i$ ($i = {}^{13}\text{C}, {}^1\text{H}$).
    - $n$ is the number of bonded hydrogens.
    - $r_{CH}$ is the average $C-H$ bond distance.
  - The above equation only applies in the **extreme narrowing limit**.

- **Extreme narrowing limit**: The case where $1/\tau_C$ is much greater than the resonance frequency.

  - Holds in liquids of low viscosity.
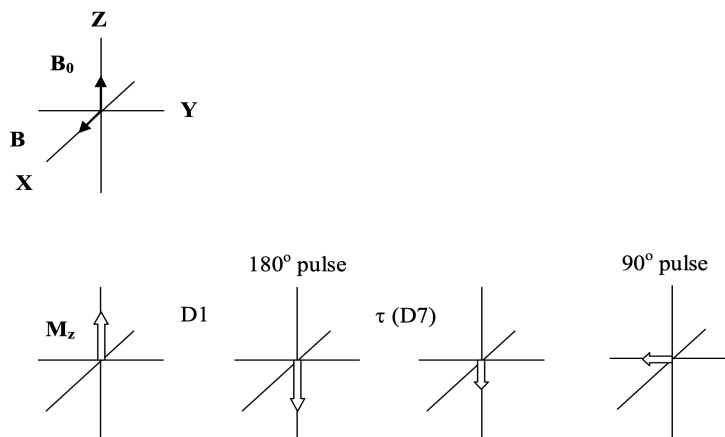
- Inversion recovery technique.



Figure 5.1: NMR inversion recovery.

  - Use a multi-pulse NMR technique: Delay $\rightarrow$ 180° pulse $\rightarrow$ delay $\tau$ $\rightarrow$ 90° pulse $\rightarrow$ acquisition (FID).
  - This has the effect depicted in Figure 5.1.
  - The decay of magnetization in the +Z-direction is

  $$\frac{\mathrm{d}M_Z}{\mathrm{d}t} = -\frac{M_Z - M_{Z_0}}{T_1}$$

  - Integrating the above yields

  $$M_Z = M_0(1 - 2\mathrm{e}^{-\tau/T_1})$$

  which can be used to calculate $T_1$.
  - Implications of the above equation.
    - After a delay of $T_1$, 63% of the magnetization is recovered along the +Z-axis.
    - To recover 99% of the magnetization, a delay of at least $5T_1$ is needed.