CHEM 30100 (Advanced Inorganic Chemistry I) Notes

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Weeks

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Week 1

A Rigorous Definition of Symmetry

1.1 Symmetry: Symmetry Elements and Operations

9/28:
- Dr. Anna Wuttig (AH-nuh WUH-tig).
  - Teaches exclusively on the blackboard.
  - Will record lectures, however; if there is a technical error, she will upload last year's lecture.
- Syllabus.
  - PSets graded on completion, not accuracy.
  - Two exams: One on the first half of the course; one on the second half of the course.
    - Cumulativeness: You'll need to understand the first half to do the second half, but there won't be questions specifically targeted to first-half material.
  - No final.
  - Participation. Showing up to class and working in groups.
- Chris, Dan, Amy, Matt, Jintong, Yibin, Ben, Sara, Ryan, Joe, Owen, Isabella, Pierce are the people.
  - People come from a diversity of chemistry subfields (physical, inorganic, organic, materials, biological).
- Every day will have a handout that we will write on (in pencil).
- Study the learning objectives!
- (Local) symmetry of a molecule helps us predict and describe bonding, spectroscopic properties, and reactivity.
  - We describe symmetry with group theory.
- **Symmetry operation**: An operation which moves a molecule into a new orientation equivalent to its original one (geometrically indistinguishable).
  - Symmetry operations that can be applied to an object always form a **group**.
- **Symmetry element**: A point, line, or plane about which a symmetry operation is applied.
- Symmetry operations.
  1. Identity operation (E): Do nothing; null operation.
  2. Reflection through a plane (σ): Subdivided into...

  - σ_d: dihedral mirror planes, which contain the principle C_n axis and bisect the angles formed between adjacent C_2 axes;
  - σ_h: horizontal mirror planes, in which the mirror plane is perpendicular to the principal C_n axis;
  - σ_v: vertical mirror planes, which contain the C_n axis and are not dihedral mirror planes.

3. Rotation about an axis (C_n): A clockwise[1] rotation about the C_n axis.
4. Improper rotation (S_n): A two-step symmetry operation consisting of a C_n followed by a σ that is perpendicular to C_n (i.e., σ_h).
5. Inversion (i): Take any point with coordinates (x, y, z) to $(-x, -y, -z)$.

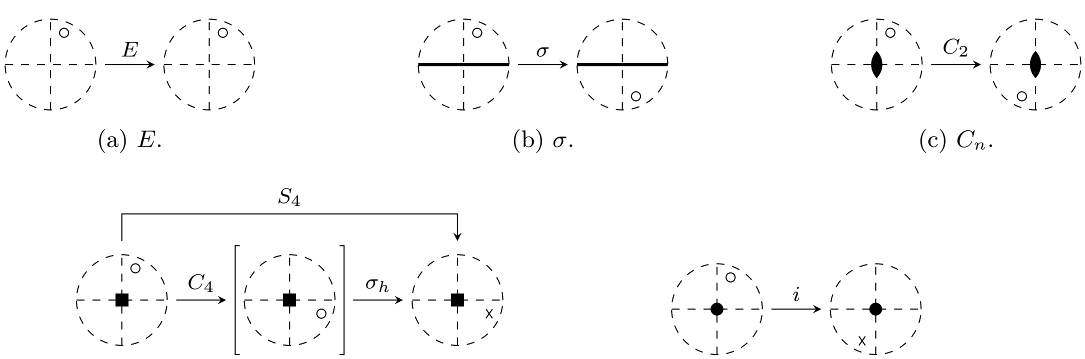
- To describe the operations, we'll introduce **stereographic projections**.

(a) Rotations. (b) Improper rotations.

Table 1.1: Symbols for stereographic projections.

  - We have a working area (the plane of the page is the xy-plane). It is useful to draw quadrants.
  - We describe a general point which experiences our symmetry operation.
    - When the point reflects through the working area, we denote the image with an "X" instead of a circle.
  - We need a gear symbol in the middle for rotations and improper rotations (see Table 1.1).
    - Must stereographic projections be drawn one at a time because it seems that the squares should not be in a reflection?
    - No — the symbols are to help us and should be included somewhere, but there are no hard-and-fast rules.

- Stereographic projections for each of the five elementary symmetry operations.



(a) E. (b) σ. (c) C_n. (d) S_n. (e) i.

Figure 1.1: Stereographic projections of the elementary symmetry operations.

- Principal C_n axis: The C_n axis for which n is the highest.
  - In a stereographic projection, the C_n axis is the one that is perpendicular to the working area (goes in/out of the page).

[1]Really?

- Example: Give the symmetry elements of NH_3.
  - C_3 axis, 3 σ_v mirror planes (denoted σ_v, σ_v', and σ_v'').
  - The symmetry operations are E, C_3, C_3^2, σ_v, σ_v', and σ_v''. These operations form the C_{3v} point group.
- Direct products of symmetry operations: $YX = Z$ means "operation X is carried out first and then operation Y," giving the same net effect as would the carrying out of the single operation Z.
  - If $YX = XY = Z$, then the two operations Y and X commute.
- What is the direct product of C_2 and σ_h?
  - $\sigma_h C_2 = S_2 = i$. They do commute.
- Do C_4 and $\sigma_{x,z}$ commute? Take the plane of this page as xy.
  - They do not (determine by drawing out both sets of stereographic projections).
- Don't get careless, Steven. This is easy, but it's also easy to make easy mistakes.
- New symmetry operations *of your group* are generated by taking the direct product of two.

1.2 Point Groups

9/30:

- The symmetry operations that apply to a given molecule collectively possess the properties of a mathematical **group**.
- **Group**: A set of symmetry operations that satisfy the following conditions.
  - *Closure*: All binary products must be in the group, i.e., the product of any two operators must also be a member of the group.
  - *Identity*: Must contain an identity, i.e., E must be part of the group.
  - *Inverse*: All elements must have an inverse in the group, and they must commute with their inverse.
  - *Associativity*: The associative law $(A \cdot B) \cdot C = A \cdot (B \cdot C)$ must hold.
- **Abelian** (group): A group in which all direct products commute.
  - Not all groups are Abelian.
- Question: Do C_3 and σ_v form a group?
  - No: No identity (for example).
  - Wuttig draws out a stereographic projection for $C_3 \cdot \sigma_v$ and overlays the first and last picture, showing that $C_3 \cdot \sigma_v$ is a reflection over a new mirror plane σ_v'.
  - C_3 and σ_v do **generate** the set of operations $E, C_3, C_3^2, \sigma_v, \sigma_v', \sigma_v''$, which collectively form the **point group $\boldsymbol{C_{3v}}$**.
- To prove something on a pset or exam, it's probably a good idea to do it in terms of stereographic projections!
- **Point group**: A group such that at least one point in space is invariant to all operations in the group.
- **Group order**: The number of symmetry operations in the group. *Given by $\boldsymbol{h}$.*
- Table activity: Finding E, principal C_n, σ, $C_2 \perp C_n$, C_n position relative to σ (collinear or perpendicular), and i for various point groups.

  - These properties are the ones that distinguish each point group from every other point group.

- Notes on the pedagogy: Animations and/or tangible models should be used to discuss this stuff. PowerPoint slides are definitely the way to go — far more tangible tools; blackboard should be a supplement. It is key to be careful what you say (*element* and *operation* must be consistently used). Dr. Wuttig is skipping a lot of key points (like naming point groups).
- Developing a flow chart that distinguishes between D_{nh}, D_{nd}, D_n, C_{nh}, C_{nv}, C_n, and S_n.

Week 2

Introduction to Representation Theory

2.1 Matrix Representations of Symmetry Operations

10/3:
- Tools for identifying symmetry elements.
  - Chem 3D (visualization).
  - Otterbein University symmetry gallery (examples of molecules that satisfy all of the point groups).
- Gives examples of molecules that satisfy the high-symmetry point groups.
  - $C_{\infty v}$: CO.
  - $D_{\infty h}$: CO_2.
  - T_d: CH_4.
  - T_h: $[Co(NO_2)_6]^{3+}$.
    - T_h is T_d with σ_h symmetry.
  - O_h: $[Co(NH_3)_6]^{3+}$
  - I_h: N/a.
    - 120 symmetry elements in total; we will not be asked to identify all of these!
  - K_h: N/a.
    - Symmetry of the sphere.
  - T, O, I are subgroups of T_h, O_h, I_h, respectively, and only have proper (not improper) rotations. These are very rare point groups. An example of a molecule in the T point group is $[Ca(THF)_6]^{2+}$.
- Learn T, O, I from Otterbein University example and ask questions!
- Low symmetry: C_1, C_i, C_s.
- The mirror plane in a C_s molecule is denoted by σ (no subscript).
- **Vector**: A series of numbers which we write in a row or a column.
- **Matrix**: Any rectangular array of numbers set between two brackets.
- Basics of matrix multiplication: $A \cdot \vec{x} = \vec{y}$ given in terms of matrix multiplication, e.g., if A is $n \times m$ and $\vec{x} \in \mathbb{R}^m$, then

$$y_i = \sum_{j=1}^{m} a_{ij} x_j$$

  for $i = 1, \dots, n$.

- Matrix representations:
  - E: What matrix A satisfies $A \cdot \vec{x} = \vec{x}$ for all $\vec{x}$? The 3×3 matrix I does.
  - i: What matrix A satisfies $A \cdot \vec{x} = -\vec{x}$ for all $\vec{x}$? The 3×3 matrix $-I$ does.
  - σ_{xy}: What matrix A flips the sign of the z-coordinate of $\vec{x}$? The 3×3 matrix $\text{diag}(1, 1, -1)$ does.
  - C_2: What matrix A flips the sign of the x, y-coordinates of $\vec{x}$? The 3×3 matrix $\text{diag}(-1, -1, 1)$ does.
  - C_3: Consider a C_{3v} molecule.

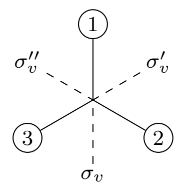


Figure 2.1: C_3 matrix representation setup.

Instead of describing a rotation in $\mathbb{R}^3$ using radians, we can think of a rotation as a permutation of the numbered atoms. So in this example,

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{C_3} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

- We will only be asked for matrix representations of very simple things, e.g., these or 90° or 180° turns.
- The above matrices form a mathematical group, which obeys the same multiplication table as the operations.
  - For example,

$$\underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{C_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\sigma_h} = \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{i}$$

- The matrix representations given above are not the "simplest" way of describing these symmetry operations.
  - The simplest way is using the **character**.
  - We find the character using a **similarity transformation** to take our matrix representations to block-diagonalized forms and then compute the characters of the blocks from there.
  - Recall that analogous blocks multiply in a block-diagonal matrix.
- **Character** (of a symmetry operation): The trace (sum of the diagonal elements) of the matrix representation of that operation. *Denoted by* $\boldsymbol{\chi}$.
- **Similarity transformation** (matrix): The matrix which, when conjugated with a matrix representation of a symmetry operation, yields the block-diagonalized form of that matrix. *Denoted by* $\boldsymbol{R}$.
  - We don't need to know how to compute these.

- Similarity transformation example: The C_3 matrix representation given above is not block diagonal, but there exists a matrix R (that we don't have to know how to find) such that

$$RC_3R^{-1} = \left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{array}\right]$$

  - The characters of the blocks of the above matrix are 1 and -1, respectively. The character of the overall matrix is still 0.