

## Week 5

# Exam and Intro to MO Theory

### 5.1 First Exam Review

10/17:

- Arrive by 9:25 AM on Wednesday.
- Probably four problems.
  - Won't be doing IRRs for  $T_d$  point groups or anything really complicated like that.
- All character tables we need to solve the questions will be provided.
- The questions are independent; if we can't solve problem 1, we'll still be able to take a shot at problem 2.
- Question: Assigning Cartesian coordinates or  $R_{xyz}$  to the Mulliken symbols.
  - Example: Assign  $R_z$  and  $x^2 - y^2$  in  $D_4$ .
  - Draw the  $d_{x^2-y^2}$  orbital and  $R_z$  vector in 3D space.
  - Subject the drawings to each symmetry operation.
    - For  $E, C_4, C_2, C'_2, C''_2$ ,  $d_{x^2-y^2}$  gets sent to itself, it's inverse, itself, itself, and it's inverse. Thus,  $\Gamma = (1, -1, 1, 1, -1) = B_1$ .
    - Likewise, we have for  $R_z$  that  $\Gamma = (1, 1, 1, -1, -1) = A_2$ .
    - $\Gamma_{p_y} = (1, 0, \dots)$ . Once you get to  $\chi(C_4) = 0$ , you know that it must be  $E$ . For  $C'_2$ ,  $p_x \mapsto 1$  and  $p_y \mapsto -1$  or vice versa, so we sum these two to get 0. Since  $p_x, p_y$  both have  $\chi(C_4) = 0$ , they must be degenerate and equal to  $E$ .
  - On Wednesday, you don't need to draw out every image, but you need some justification, e.g., "I did a 90° turn."
- Memorize Mulliken symbol rules!
- Question: Problem C on PSet 2.
  - Ignore all of the text about qubits.
  - First step: Figure out what the basis is (it's the outer plane vectors, i.e., the arrows).
  - Point group:  $D_{4h}$ .
  - We need to find  $\Gamma_{\text{out-of-plane}}$ , a reducible representation! We're finding a RR for all of the arrows at once (not projecting one into the others), and then decomposing it into IRRs and hoping that one of them is  $a_{2u}$ .
  - We get

$D_{4h}$	$E$	$2C_4$	$C_2$	$2C'_2$	$2C''_2$	$i$	$2S_4$	$\sigma_h$	$2\sigma_v$	$2\sigma_d$
$\Gamma$	4	0	0	-2	0	0	0	-4	2	0

- We can reduce this down to

$$\Gamma = a_{2u} + b_{2u} + e_g$$

to identify  $a_{2u}$ .

- Similarly, in  $C_{4v}$ , we can reduce  $\Gamma = (4, 0, 0, 2, 0) = a_1 + b_1 + e$  to identify  $a_1$ .

- Review PSets and answers! I keep missing easy things. Really break down a procedure to attack the different types of problems.
- Question: Assign symmetries based off of IR spectra.

- Example: Vibrational spectroscopy has played a role in supporting the structure of the ion shown below. Raman spectroscopy of the tetramethylammonium salt of this ion shows a single absorption in the region expected for I=O stretching vibrations at  $789\text{ cm}^{-1}$ . Is a single Raman band consistent with the proposed trans orientation of the oxygen atoms? Rationalize your answer.
- Approach: In  $D_{5h}$ , create an IRR with the two I=O bond vectors as your basis. Thus,

$D_{5h}$	$E$	$2C_5$	$2C_5^2$	$5C'_2$	$\sigma_h$	$2S_5$	$2S_5^3$	$5\sigma_v$
$\Gamma$	2	2	2	0	0	0	0	2

- We have that  $\Gamma = a'_1 + a''_2$ . Since only one of these ( $a'_1$ ) is Raman active, a single Raman band is consistent with this structure.

- Question: Finding vibrational modes.

- Example: What are the normal modes of *trans*- $\text{N}_2\text{F}_2$ .
- Not linear, so we expect  $3N - 6 = 6$  normal modes to deal with.
- We begin doing the Cartesian displacement method.

$C_{2h}$	$E$	$C_2(z)$	$i$	$\sigma_h$
$\Gamma_{\text{unm}}$	4	0	0	4
$\Gamma_{xyz}$	3	–	–	1
$\Gamma_{3N}$	12	0	0	4

- Note that we use dashes in two entries of  $\Gamma_{xyz}$  because since there are zeroes above and we are multiplying, it does not matter what these values are (they will end up being zero in the direct product, regardless).
- We can reduce to  $\Gamma_{3N} = 4A_g + 2B_g + 2A_u + 4B_u$ . Subtracting out  $\Gamma_{\text{rot}} = A_g + 2B_g$  and  $\Gamma_{xyz} = A_u + 2B_u$ , we end up with  $\Gamma_{\text{vib}} = 3A_g + A_u + 2B_u$ .
- Let's go in depth. The spectroscopic activities are:
  - $A_u + 2B_u$  are IR active.
  - $3A_g$  is Raman active.
- Let's go even more in depth.
  - $A_u$  is the only vibration that has  $\chi(\sigma_h) = -1$ , i.e., it is the only one that vibrates out-of-plane.
  - If  $\chi(\sigma_h) = +1$ , then the vibration is symmetric with respect to  $\sigma_h$ . This must mean that the molecules are entirely confined to the plane. If  $\chi(\sigma_h) = -1$ , then we have an out of plane vibration (e.g., F atoms going above and below the plane in equal and opposite amounts — there is a motion that can be inverted). If one F atom goes twice as far, this is probably  $E$  (think about what would be required for the projection operator).
  - If you get pictures of normal modes (hint hint!!), you can retroactively get Mulliken symbols by observing the symmetry with respect to the observations.

## 5.2 Midterm Thoughts

- Generating a RR based on a basis set tells us the symmetry of something (vibrations, stretches, orbitals, ...). Projecting out an IRR on a basis gives us a specific example of something (an actual molecular orbital, vibrational mode, ...).
- Determining the depolarization ratio. Determine which Mulliken symbols are the *most* symmetric, i.e., symmetric to the most elements. For example,  $A_2$  is more symmetric than  $B_2$  because it is antisymmetric to 1 element instead of 2 elements.