The Knot Book

Notes

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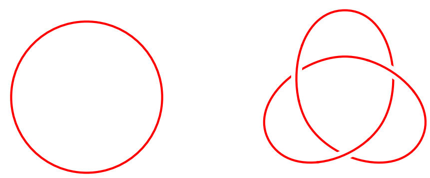
1 Introduction

1.1 Introduction

- **Knot**: "A knotted loop of string, except that we think of the string as having no thickness, its cross-section being a single point" (2).
- Do not distinguish between a 'nice, even' knot and one that has been deformed through space.
- **Unknot**: "The simplest knot of all. . . the unknoted circle" (2). *Also known as* **trivial knot**. See Figure 1a.
- **Trefoil knot**: "The next simplest knot" (2). See Figure 1b.



(a) Trivial knot.

(b) Trefoil knot.

Figure 1: Projections of the two simplest knots.

- **Projection**: A picture of a knot, such as those in Figure 1.
  - The same knots can have multiple projections (as they are deformed in space).
- **Crossings**: The places in a projection where a knot crosses itself.
  - The trefoil knot in Figure 1b is a three-crossing knot because it crosses itself 3 times.
  - Any one-crossing knot is trivial.
  - *Exercise 1.2*: Any two-crossing knot must be trivial because the simplest nontrivial knot is the trefoil knot, which has three crossings.
- Atoms were originally thought to be tangles (knots) in the ether of the universe, but when chemists moved on, mathematicians took up knot theory. In the 1980s, biochemists began to see applications of knot theory in their research (see Section **??**).
- **Topology**: "The study of the properties of geometric objects that are preserved under deformations" (6).
  - Knot theory is a subfield of topology (see Section **??**).

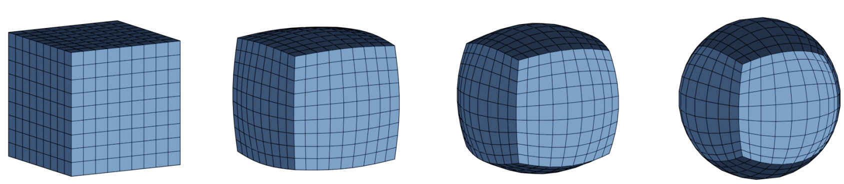


Figure 2: Deformation of a cube into a sphere.

- Any knot can have a projection with as many crossings as desired.

- **Alternating knot**: "A knot with a projection that has crossings that alternate between over and under as one travels around the knot in a fixed direction" (7).
  - The trefoil is such a knot.
- *Exercise 1.7**: By changing some of the crossings from over to under or vice versa, any projection of a knot can be made into a projection of the unknot[1]. See Figure 3.

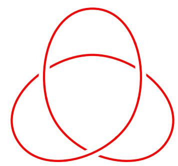


Figure 3: A projection of the unknot evoking the trefoil knot.

1.2 Composition of Knots

- **Composition** (of two knots): "A new knot obtained by removing a small arc from two knot projections and then connecting the four endpoints by two new arcs" (7).
  - If two knots are designated J and K, then their composition is denoted $J\#K$.
  - Do not overlap the projections and choose two arcs that are on the outside to avoid new crossings.
  - Make sure that the new arcs do not cross any of the the original knot projections or each other.
- **Composite knot**: A knot that "can be expressed as the composition of two knots, neither of which is the trivial knot" (8).
  - This definition is analogous to composite integers, where an integer is composite if it is the product of positive integers, neither of which is 1.
  - Similarly, if we compose any knot with the unknot, we get the same knot back.
- **Factor knots**: "The knots that make up the composite knot" (8).
- **Prime knot**: "A knot [that] is not the composition of any two nontrivial knots" (9).
- The unknot, trefoil knot, and figure-eight knots are all prime (see Section **??**).
  - The unknot is not composite for the same reason that 1 is not the product of two integers greater than 1.

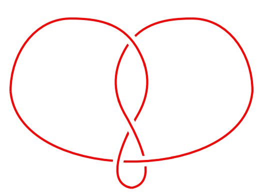


Figure 4: The figure-eight knot.

- Similar to integers, "a composite knot factors into a unique set of prime knots" (10).

[1]How can I *show* something? How can I do these proofs? What kind of logic solves one of these?

- *Exercise 1.8*: Using the appendix table, identify the factor knots that make up the composite knot in Figure 5.

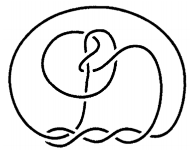
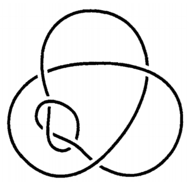


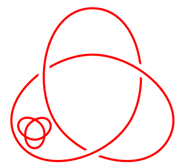
Figure 5: The composite knot.

  – Figure out when knot cord arrives.

- *Exercise 1.9*: Show that the knot in Figure 6 is composite.



(a) The composite knot.



(b) Factors.

Figure 6: Factorization of a 'double trefoil.'