ONE-TO-ONE and ONTO LINEAR TRANSFORMATIONS

- $T:V \to W$ **is called one-to-one if** T **maps distinct vectors in** V **to distinct vectors in** W **.**
- T **is called onto if** $range(T) = W$ **.**
  - $T:V \to W$ **is one-to-one if for all vectors in** V **,** $u \neq v$ **implies that** $T(u) \neq T(v)$ **and** $T(u) = T(v)$ **implies that** $u = v$ **.**

EXAMPLES:

$$T:\mathbb{R}^2 \to \mathbb{R}^3 : T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ x-y \\ 0 \end{bmatrix}$$

- T **is one-to-one because Let** $T\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = T\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$**. Then** $\begin{bmatrix} 2x_1 \\ x_1-y_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2x_2 \\ x_2-y_2 \\ 0 \end{bmatrix}$**, which implies** $2x_1 = 2x_2$ **and** $x_1 - y_1 = x_2 - y_2$ **so** $x_1 = x_2$ **and** $y_1 = y_2$ **.**
- **T is NOT onto because its range is not all of** $\mathbb{R}^3$**. There is no vector such that** $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$**.**

$$D:P_3 \to P_2 : D(p(x)) = p'(x)$$

- **D is not one-to-one because distinct polynomials in** P_3 **can have the same derivative.**
- **D is onto because** $range(D) = P_2$ **.**

$T : V \to W$ **is one-to one if and only if** $\ker(T) = \ 0$

$T\begin{bmatrix} a \\ b \end{bmatrix} = a + (a+b)x$ **is one-to-one because** $\ker(T) = \ 0$ **since if** $T(A) = 0$**, then** $a = 0$ **and** $a + b = 0$ **(therefore** $b = 0$**).** $T\begin{bmatrix} a \\ b \end{bmatrix} = a + (a+b)x$ **is also onto because if** $\ker(T) = 0$ **then it is full rank:**

$$rank(T) = \dim(\mathbb{R}^2) - nullity(T) = 2 - 0 = 2$$