1) $\int_{-\pi}^{\pi} \sin(nt)dt = \left[-\frac{1}{n}\cos(nt)\right]_{-\pi}^{\pi} = \frac{1}{n}\left(-\cos(n\pi)+\cos(n\pi)\right) = 0, n \neq 0$

2) $\int_{-\pi}^{\pi} \cos(nt)dt = \left[\frac{1}{n}\sin(nt)\right]_{-\pi}^{\pi} = 0, n \neq 0$

3) $\int_{-\pi}^{\pi} f(t)dt = \frac{1}{2}\int_{-\pi}^{\pi} a_0 dt + \sum_{n=1}^{\infty}\left(\int_{-\pi}^{\pi} a_n \cos(nt)dt + \int_{-\pi}^{\pi} b_n \sin(nt)dt\right) = \frac{1}{2}\left[a_0 t\right]_{-\pi}^{\pi} + \sum_{n=1}^{\infty}(0+0)$

$\therefore \int_{-\pi}^{\pi} f(t)dt = \frac{1}{2}(2a_0\pi) \to a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)dt \to \frac{a_0}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)dt$

4) $\int_{-\pi}^{\pi} \sin(nt)\cos(mt)dt = \frac{1}{2}\left(\int_{-\pi}^{\pi} \sin(n+m)tdt + \int_{-\pi}^{\pi} \sin(n-m)tdt\right) = \frac{1}{2}(0+0) = 0$ using the results from problems 1 and 2 since if n is an integer and m is an integer then $n+m$ and $n-m$ are also integers.

5) $\int_{-\pi}^{\pi} \cos(nt)\cos(mt)dt = \frac{1}{2}\left(\int_{-\pi}^{\pi} \cos(n+m)tdt + \int_{-\pi}^{\pi} \cos(n-m)tdt\right) = \frac{1}{2}(0+0) = 0$ when $n \neq m$

6) $\int_{-\pi}^{\pi} \cos^2(nt)dt = \frac{1}{2}\int_{-\pi}^{\pi} \left(1+\cos(2n)t\right) dt = \frac{1}{2}\left[t + \frac{1}{2n}\sin(2n)t\right]_{-\pi}^{\pi} = \pi$ when $n \neq 0$

7) $\pi - (-\pi) = 2\pi$

8) a) $\int_{-\pi}^{\pi} \sin(nt)\sin(mt)dt = 0$

b) $\int_{-\pi}^{\pi} \sin^2(nt)dt = \frac{1}{2}\int_{-\pi}^{\pi} \left(1-\cos(2nt)\right) dt = \pi$

c) $\int_{-\pi}^{\pi} \sin(nt)\sin(mt)dt = 0$

For integers m, n:

$$\int_{-\pi}^{\pi} \sin(nt)\cos(mt)dt = 0$$

$$\int_{-\pi}^{\pi} \cos(nt)\cos(mt)dt = \left\{\begin{matrix} 0, n \neq m \\ \pi, n = m \neq 0 \\ 2\pi, n = m = 0 \end{matrix}\right\}$$

$$\int_{-\pi}^{\pi} \sin(nt)\sin(mt)dt = \left\{\begin{matrix} 0, n \neq m, n = m = 0 \\ \pi, n = m \end{matrix}\right\}$$