Let $A$ be an $n \times n$ matrix. Let $T : V \to W$ be a linear transformation whose transformation matrix $T$ is $A$.

**THE FOLLOWING ARE ALL EQUIVALENT STATEMENTS:**

- $A$ is invertible.
- $Ax = b$ has a unique solution for every $b$ in $\mathbb{R}^n$.
- $Ax = 0$ has only the trivial solution.
- The reduced row echelon form of $A$ is $I$.
- $A$ is a product of elementary matrices.
- rank $(A)$ = $n$.
- nullity ( $A$ )=0.
- The column vectors of $A$ are linearly independent.
- The column vectors of $A$ span $\mathbb{R}^n$.
- The column vectors of $A$ form a basis for $\mathbb{R}^n$.
- The row vectors of $A$ are linearly independent.
- The row vectors of $A$ span $\mathbb{R}^n$.
- The row vectors of $A$ form a basis for $\mathbb{R}^n$.
- $|A| \neq 0$
- 0 is not an eigenvalue of $A$.
- 0 is not a singular value of $A$.
- $T$ is invertible.
- $T$ is one-to-one and onto.
- ker $(T) = 0$
- range $(T) = W$