Script 10

Compactness

10.1 Journal

2/23: **Definition 10.1.** We say that a function $f : A \to \mathbb{R}$ is **bounded** if $f(A)$ is a bounded subset of $\mathbb{R}$. We say that f is **bounded above** if $f(A)$ is bounded above and that f is **bounded below** if $f(A)$ is bounded below.

If $f : A \to \mathbb{R}$ is bounded above, we say that f **attains** (its least upper bound) if there is some $a \in A$ such that $f(a) = \sup f(A)$. Similarly, if $f : A \to \mathbb{R}$ is bounded below, we say that f **attains** (its greatest lower bound) if there is some $a \in A$ such that $f(a) = \inf f(A)$.