Chapter 9

Advanced Spectral Theory

10/22:

- Let $p(z) = \sum_{i=0}^{n} a_i z^i$ be a polynomial. Let A be an $n \times n$ matrix. We let $p(A) = \sum_{i=0}^{n} a_i A^i$.
- Theorem: If A is an $n \times n$ and $p(\lambda) = \det(A - \lambda I)$, then $p(A) = 0$.
  - We know that $p(\lambda) = a(z - \lambda_1) \cdots (z - \lambda_n)$ where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues.
  - Thus $p(A) = a(A - \lambda_1 I) \cdots (A - \lambda_n I)$.
  - If you are in $\mathbb{R}^n$ and have this property, you can factorize your matrix.
  - Thus, $p(A)\mathbf{x} = \mathbf{0}$ since $\mathbf{x}$ can be decomposed into a linear combination of eigenvectors of A, which will be taken to 0 one by one by the terms of $p(A)$.
- $\sigma(B) = \{\text{eigenvalues of } B\}$ is known as the **spectrum** of B.
- If p is an arbitrary polynomial and A is $n \times n$, then μ is an eigenvalue of $p(A)$ if and only if $\mu = p(\lambda)$ where λ is an eigenvalue of A. In essence, $\sigma(p(A)) = p(\sigma(A))$.
- Chapter 9 will not be on the exam. We don't have to know the generalization to infinite dimensional spaces.