Chapter 3

Determinants

9/29:

- The determinant, geometrically, is the volume of the object (in $\mathbb{R}^3$) you get when you take linear combinations of the vectors.
- In 2D:
  - Let v_1, v_2 be two vectors. Put tail to tail and forming a parallelogram, the determinant of the matrix (v_1, v_2) is the area of said parallelogram.
  - Linearity 1: $D(av_1, v_2, \ldots, v_n) = aD(v_1, \ldots, v_n)$ is the same as saying that if you stretch one vector by a, you scale up the area by that much, too.
  - Linearity 2: $D(v_1, \ldots, v_{k+} + v_{k-}, \ldots, v_n) = D(-) + D(+)$.
  - Antisymmetry: $D(v_1, \ldots, v_k, \ldots, v_j, \ldots, v_n) = -D(v_1, \ldots, v_j, \ldots, v_k, \ldots, v_n)$. Interchanging columns flips the sign of the determinant.
  - Basis: $D(e_1, \ldots, e_n) = 1$.
- Determinant: Denoted by $D(v_1, \ldots, v_n)$, where $(v_1, \ldots, v_n)$ is an $n \times n$ matrix.