

## Chapter 4

# Continuity

### 4.1 Notes

- 11/8:
- Consider a function  $f : X \rightarrow Y$  whose domain and codomain are, respectively, the metric spaces  $(X, d_X)$  and  $(Y, d_Y)$ .
  - **Limit** (of  $f$  at  $p$ ): A point  $q \in Y$  such that for all  $\epsilon > 0$ , there exists  $\delta$  such that  $d_X(x, p) < \delta$  implies  $d_Y(q, f(x)) < \epsilon$ , where  $p$  is a limit point of  $X$  (otherwise,  $x \not\rightarrow p$ ).
  - **Continuous** (function  $f$  at  $p$ ): A function  $f$  such that  $\lim_{x \rightarrow p} f(x) = f(p)$ .
  - $f$  is continuous on  $X$  if it is continuous at every  $p \in X$ .
  - **Uniformly continuous** (function  $f$ ): A function  $f$  such that for every  $\epsilon > 0$ , there exists a  $\delta > 0$  such that  $d_X(x, y) < \delta$  implies  $d_Y(f(x), f(y)) < \epsilon$  for all  $x, y \in X$ .