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Momentum Bonus

Task

Given the following two equations, which describe the conservation of momentum and kinetic energy, respectively, before and after an elastic collision, derive formulas for the final velocity of each object (v_{1f} and v_{2f}) in terms of only these object's initial velocities and masses (not each other's final velocities).

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad (1)$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (2)$$

Solution

Solve first for one of the variables (say, v_{1f}) and then for the other variable using an identical procedure (v_{2f} , if v_{1f} is solved for first).

Treat equations 1 and 2 as a two-variable system of equations, where the two variables are v_{1f} and v_{2f} . Solve the "system" by substitution i.e. solve one equation (say, equation 1) for v_{2f} and plug the result into equation 2. This will create a third equation (equation 3) without v_{2f} . Finally, solve equation 3 algebraically for v_{1f} to give the first of the two formulas (equation 4). This procedure can be carried out as follows.

$$\begin{aligned} m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \\ m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f} &= m_2 v_{2f} \\ v_{2f} &= \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2} \end{aligned}$$