Chapter 18

Thermal Properties of Matter

18.1 Macroscopic Derivation of the Ideal Gas Law

8/19:

- Atoms have a nucleus (composed of protons and neutrons) orbited by electrons.
- **Atomic number**: The number of protons. *Denoted by* $\boldsymbol{Z}$.
- **Atomic mass**: Essentially the number of protons plus the number of neutrons. *Denoted by* $\boldsymbol{A}$, $\boldsymbol{\mu}$. *Units* amu.
  - $1\,\text{amu} = \frac{1}{12}m({}^{12}_{6}\text{C})$.
- **Avogadro's number**: The number of molecules per mole of a substance, i.e., 6.02×10^{23}. *Denoted by* $\boldsymbol{N_A}$.
  - N_A carbon-12 atoms weighs $12\,\text{g}$.
  - We define $1\,\text{mol} = N_A$ of something.
- **Boyle's Law**: The product of the pressure and volume of a gas is a constant (that depends on the gas at hand).
- **Ideal gas law**: The relation
$$pV = nRT$$
relating the pressure, volume, number of moles, and temperature of a dilute gas to a constant (that is not specific to any particular gas).
- **Universal gas constant**: The constant $8.314\,\frac{\text{J}}{\text{mol}\,\text{K}}$. *Denoted by* $\boldsymbol{R}$.
- Thus, we can think of temperature as being a reflection of a few macroscopic properties of gasses (e.g., pressure, volume, and number of moles).

18.2 Microscopic Derivation of the Ideal Gas Law

- Covers the derivation of the ideal gas law from KMT, as described Chapter 5 of Labalme (2020).
- Important addition:
$$p = \frac{1}{3}\rho\bar{v}^2$$
where p is pressure, ρ is density, and $\bar{v}$ is the average velocity of the molecules.
  - This relates a macroscopic and a microscopic quantity.

  - Thus, we can relate the average speed of the molecules to the measurable pressure via

$$v_{\text{rms}} = \sqrt{\frac{3p}{\rho}}$$

- We can calculate from the above equation that the root mean square velocity of hydrogen gas at STP is about $1\,800\,\text{m/s}$. For nitrogen gas, it's about $450\,\text{m/s}$.
- **Boltzmann constant**: The quotient of the universal gas constant and Avogadro's constant, having value $1.38 \times 10^{-23}\,\text{J/K}$. *Denoted by* k.
- It follows that

$$\overline{KE}_{\text{molecule}} = \frac{3}{2}kT$$

- Additionally, we have that

$$v_{\text{rms}} = \sqrt{\frac{3kT}{\mu}}$$

- This property can be taken advantage of for **diffusion separation of isotopes**.
- How to separate ^{238}U from ^{235}U:
  - Create UF_6, a gas.
  - The lighter molecules will effuse slightly faster out of a box with a hole.
  - If you apply the cycle over and over again, you will enrich it a little bit each time.
  - Eventually, you will have a large proportion of $^{235}\text{UF}_6$, from which the ^{235}U can be extracted.