

# PHYS 13300 (Waves, Optics, and Heat) Notes

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# Chapter 15

## Mechanical Waves

### 15.1 Course Information

- 8/5:
- HW 1 will be posted after class. Due Monday at 10 AM.
  - 2 labs, 2 days each.
    - Department policy is that you have to do all the labs to pass the class.
  - First meeting with lab TA will be on Monday at 2:30, 3:30, or 4:30.
    - Email Dr. Gazes for later timeslot.
  - HW accounts for 85% of your grade because it helps with the tests.
  - Quiz assignment that you print out, write on, and then scan and upload.
  - Office hours (Gazes): 5:30-7:00. TA office hours to be posted soon.
  - Wants us to learn the material, not compete with each other.
    - Expects collaboration on the homework, but wants us to write up our own answers.

### 15.2 Wave Basics

- **Wave:** A disturbance that propagates (carrying energy).
- **Mechanical** (wave): A wave in a medium that has an equilibrium.
  - Air, for instance, is in equilibrium when its pressure/density is everywhere equal. But you can create a disturbance by making a high-pressure region somewhere in space. This disturbance then propagates.
  - When a slinky compression wave is created, what's propagating isn't the slinky — no coil can move past another. What's moving is the *high-density region*.
- **Compression:** The high-density region of a wave.
- **Rarefaction:** The low-density region of a wave.
- **Longitudinal** (wave): A wave where the disturbance is parallel to the propagation of the wave.
  - Example: Compression wave in a slinky; air (sound).
- **Transverse** (wave): A wave where the disturbance is perpendicular to the propagation of the wave.

- Example: A string tied to the wall where you shake one end; waves at the beach (the water is going up and down but the wave is moving toward the beach).
- A charge  $q$  creates an electric field. If  $q$  moves at a constant velocity  $v$ , it will create a magnetic field. If you make the charge accelerate with acceleration  $a$ , it will produce an **electromagnetic wave**.
- **Electromagnetic (wave)**: A wave that does not require a medium to move in.
  - A medium is physical; made up of matter. The electric and magnetic fields in which an electromagnetic wave moves are not media — they can contain energy, but not in the same way a physical medium can.
- **Wavefunction**: A mathematical function that represents the behavior of a wave.
  - $y(x, t)$  represents a one-dimensional wave,  $x$  being position and  $t$  being time.
  - $y$  represents the magnitude of the disturbance.
    - Example: The density of slinky links in a longitudinal wave; the displacement of a transverse wave from the  $x$ -axis, taken to be equilibrium.
- **Wave speed**: The velocity with which the wave propagates. *Denoted by  $v$ .*
  - NOT, for example, the speed with which the string moves up and down in a transverse wave.
- If we let the  $xy$ -axes be the standard ones, we can also define  $x'y'$ -axes that move with the wave with velocity  $v$ .

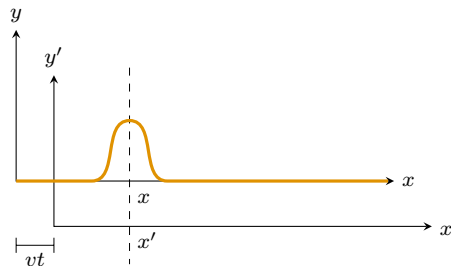


Figure 15.1: Axes that eliminate the effect of time.

- In the  $x'y'$ -axes, the wave isn't moving.
- From Figure 15.1, we can see that  $x = x' + vt$ .
- Additionally, we can express (shape of) the wave as  $y' = f(x')$ .
- Thus,  $y = f(x - vt)$  represents a wave propagating in the  $+x$  direction.
- Similarly,  $y = f(x + vt)$  represents a wave propagating in the  $-x$  direction.
- When two waves collide (or we otherwise have to deal with more than one wave in the same medium), we apply the **superposition principle**.
- **Superposition principle**: If  $y_1, y_2, \dots$  are individual wavefunctions, the total disturbance  $y$  is given by  $y(x, t) = y_1(x, t) + y_2(x, t) + \dots$ .
- **Constructive interference**: When two waves in the same medium add to produce a bigger wave.
- **Destructive interference**: When two waves in the same medium cancel parts of each other out.
  - Difference between a medium at equilibrium and a medium with two waves destructively interfering (at the instant the waves collide, the medium looks as if it's at equilibrium):

- The energy of the wave is contained in the kinetic energy of the individual particles of the medium moving up and down.
- As such, even when we don't see a visible wave, those particles still have a velocity vector that is containing the energy. It's like the *position* gets back to equilibrium for a moment, but the *velocity*, where the kinetic energy is contained, is most definitely not at equilibrium.
- In PHYS 13100, we used  $F = ma$  to analyze a block of mass  $m$  oscillating on a spring, solving

$$F = ma$$

$$-kx = m \cdot \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} \cdot x = 0$$

to describe its dynamics.

- Creating an analogy to  $F = ma$  for wave motion (deriving the wave equation).

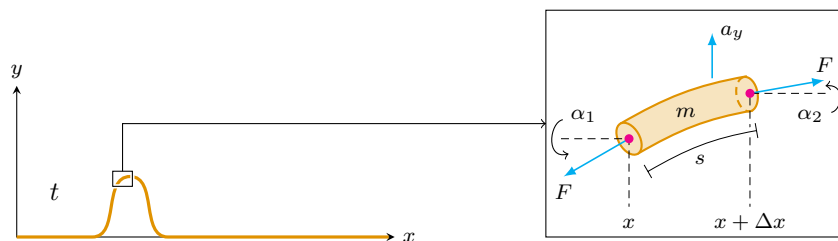


Figure 15.2: Deriving the wave equation.

- $F$  is a tension force.
- We know for the sliver of the string in Figure 15.2,  $F_y = ma_y$ .
- From our FBD, we have that  $F_y = F \sin \alpha_2 - F \sin \alpha_1$ .
- Since the string segment is short, assume  $\alpha_1 = \alpha_2$ . Let's also ignore gravity since  $F \gg F_g$ : it doesn't matter in what position you play an instrument, relative to the Earth's surface, does it?
- For small values of  $\alpha$  (we assume our string is taut),  $\sin \alpha \approx \tan \alpha = \frac{\partial y}{\partial x}$ .
- Thus,  $F_y = F(\frac{\partial y}{\partial x} \big|_{x+\Delta x} - \frac{\partial y}{\partial x} \big|_x)$ .
- Additionally,  $m = Ms$ , where  $M$  is the linear mass density and  $s$  is the arc length of the string segment. Furthermore, since  $\alpha$ 's are small in taut strings,  $\Delta s \approx \Delta x$ , so  $m \approx M\Delta x$ .
- Lastly, observe that  $a_y = \partial^2 y / \partial t^2$ .
- Therefore,  $F = ma$  becomes

$$F \left( \frac{\partial y}{\partial x} \bigg|_{x+\Delta x} - \frac{\partial y}{\partial x} \bigg|_x \right) = M\Delta x \cdot \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\frac{\partial y}{\partial x} \big|_{x+\Delta x} - \frac{\partial y}{\partial x} \big|_x}{\Delta x} = \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}$$

from which we can take limits as follows:

$$\lim_{\Delta x \rightarrow 0} \frac{\frac{\partial y}{\partial x} \big|_{x+\Delta x} - \frac{\partial y}{\partial x} \big|_x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}}$$

- **Wave equation:** The final result above.
  - Holds for a 1D wave on a string.
- Tie a piece of string to a wall and shake the free end like a harmonic oscillator. This creates a **harmonic** wave that propagates towards the wall.
- **Harmonic** (wave): A wave produced by a disturbance changing like a harmonic oscillator.
  - The wavefunction for a harmonic wave is sinusoidal, propagates like a wave (i.e., like  $f(x - vt)$ ), and needs to have a constant  $k$  to make the dimensional argument of sine dimensionless:  $y(x, t) = A \sin(k[x - vt])$ .
- **Amplitude:** The constant  $A$  in the wavefunction of a harmonic wave.
- **Wavenumber:** The constant  $k$  in the wavefunction of a harmonic wave. *Units are  $\text{m}^{-1}$ .*
- **Wavelength:** The *distance* over which wave motion repeats *for a fixed time  $t$* . Denoted by  $\lambda$ .
  - Mathematically, the existence of the wavelength implies that  $y(x, t) = y(x + \lambda, t)$ .
  - But for a harmonic wave, this implies that  $A \sin(k[x - vt]) = A \sin(k[(x + \lambda) - vt])$ , meaning that  $k\lambda = 2\pi$ .
  - Thus, we know that the wave number  $k = \frac{2\pi}{\lambda}$ .
- **Period:** The *time* over which wave motion repeats *for a fixed point  $x$* . Denoted by  $T$ .
  - Similarly,  $y(x, t) = y(x, t + T)$ .
  - For a harmonic wave,  $A \sin(k[x - vt]) = A \sin(k[x - v(t + T)])$ , meaning that  $kvT = 2\pi$ .
  - Thus, we know that the wave speed  $v = \frac{2\pi}{k} \cdot \frac{1}{T} = \lambda f$ , where  $f$  is the frequency of the wave, for simple harmonic motion.
  - Alternately, if we let  $\omega = 2\pi f$  be the angular frequency, then  $v = \frac{\omega}{k}$ .
- It follows that for a harmonic wave,

$$\begin{aligned} y(x, t) &= A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right] \\ &= A \sin[kx - \omega t]^{[1]} \end{aligned}$$

- To account for cosine and other waves that “start” at different parts, we include a **phase constant**  $\phi$ :

$$y(x, t) = A \sin[kx - \omega t + \phi]$$

- To check that the above is in fact a wave, we must feed it into the wave equation:

$$\begin{aligned} \frac{\partial^2}{\partial x^2} (A \sin[kx - \omega t + \phi]) &= \frac{M}{F} \cdot \frac{\partial^2}{\partial t^2} (A \sin[kx - \omega t + \phi]) \\ -Ak^2 \sin[kx - \omega t + \phi] &= \frac{-A\omega^2 M}{F} \sin[kx - \omega t + \phi] \\ k^2 &= \frac{M\omega^2}{F} \\ \frac{\omega}{k} &= \sqrt{\frac{F}{M}} \end{aligned}$$

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<sup>1</sup>Dr. Gazes prefers this form, but both are correct and can be used.

- It follows since  $v = \frac{\omega}{k}$  that  $v = \sqrt{F/M}$ .
- We originally found this speed/force/mass relationship to be true for a harmonic wave, but this shows that it is true for *any* wave.
- **General 1D wave equation:** Making the modification from above, the following equation.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}$$

### 15.3 Office Hours (Gazes)

- How does proving that  $v = \sqrt{F/M}$  with a harmonic wavefunction prove that this relation holds for *all* waves?
  - Applies to any wave in a string. If you have a shape that doesn't look like a harmonic wave, you can construct it out of harmonic waves (Fourier math). The superposition principle allows us to add these waves.
- Importance of reading the textbook?
  - To be used as we wish.
  - We *could* read it instead of coming to lecture.
  - Think of it as something to consult as needed; i.e., for clarification.
  - Some people read it before class.
  - He will talk about some things in class that aren't in the textbook, and vice versa. If the textbook talks about it and he doesn't, you aren't responsible for knowing it.