PHYS 13300 (Waves, Optics, and Heat) Notes

Steven Labalme

August 22, 2021

Contents

List of Figures

Chapter 15

Mechanical Waves

15.1 Course Information

8/5:

- HW 1 will be posted after class. Due Monday at 10 AM.
- 2 labs, 2 days each.
  - Department policy is that you have to do all the labs to pass the class.
- First meeting with lab TA will be on Monday at 2:30, 3:30, or 4:30.
  - Email Dr. Gazes for later timeslot.
- HW accounts for 85% of your grade because it helps with the tests.
- Quiz assignment that you print out, write on, and then scan and upload.
- Office hours (Gazes): 5:30-7:00. TA office hours to be posted soon.
- Wants us to learn the material, not compete with each other.
  - Expects collaboration on the homework, but wants us to write up our own answers.

15.2 Wave Basics

- **Wave**: A disturbance that propagates (carrying energy).
- **Mechanical** (wave): A wave in a medium that has an equilibrium.
  - Air, for instance, is in equilibrium when its pressure/density is everywhere equal. But you can create a disturbance by making a high-pressure region somewhere in space. This disturbance then propagates.
  - When a slinky compression wave is created, what's propagating isn't the slinky — no coil can move past another. What's moving is the *high-density region*.
- **Compression**: The high-density region of a wave.
- **Rarefaction**: The low-density region of a wave.
- **Longitudinal** (wave): A wave where the disturbance is parallel to the propagation of the wave.
  - Example: Compression wave in a slinky; air (sound).
- **Transverse** (wave): A wave where the disturbance is perpendicular to the propagation of the wave.

- Example: A string tied to the wall where you shake one end; waves at the beach (the water is going up and down but the wave is moving toward the beach).

- A charge q creates an electric field. If q moves at a constant velocity v, it will create a magnetic field. If you make the charge accelerate with acceleration a, it will produce an **electromagnetic wave**.

- **Electromagnetic** (wave): A wave that does not require a medium to move in.
  - A medium is physical; made up of matter. The electric and magnetic fields in which an electromagnetic wave moves are not media — they can contain energy, but not in the same way a physical medium can.

- **Wavefunction**: A mathematical function that represents the behavior of a wave.
  - $y(x, t)$ represents a one-dimensional wave, x being position and t being time.
  - y represents the magnitude of the disturbance.
    - Example: The density of slinky links in a longitudinal wave; the displacement of a transverse wave from the x-axis, taken to be equilibrium.

- **Wave speed**: The velocity with which the wave propagates. *Denoted by $\boldsymbol{v}$.*
  - NOT, for example, the speed with which the string moves up and down in a transverse wave.

- If we let the xy-axes be the standard ones, we can also define $x'y'$-axes that move with the wave with velocity v.

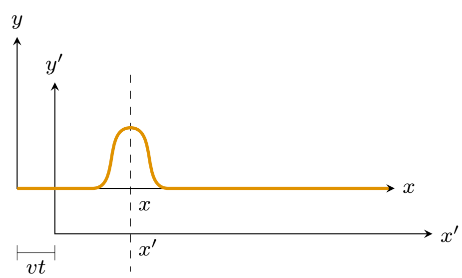


Figure 15.1: Axes that eliminate the effect of time.

  - In the $x'y'$-axes, the wave isn't moving.
  - From Figure 15.1, we can see that $x = x' + vt$.
  - Additionally, we can express (shape of) the wave as $y' = f(x')$.
  - Thus, $y = f(x - vt)$ represents a wave propagating in the $+x$ direction.
  - Similarly, $y = f(x + vt)$ represents a wave propagating in the $-x$ direction.

- When two waves collide (or we otherwise have to deal with more than one wave in the same medium), we apply the **superposition principle**.

- **Superposition principle**: If $y_1, y_2, \ldots$ are individual wavefunctions, the total disturbance y is given by $y(x, t) = y_1(x, t) + y_2(x, t) + \cdots$.

- **Constructive interference**: When two waves in the same medium add to produce a bigger wave.

- **Destructive interference**: When two waves in the same medium cancel parts of each other out.
  - Difference between a medium at equilibrium and a medium with two waves destructively interfering (at the instant the waves collide, the medium looks as if it's at equilibrium):

- The energy of the wave is contained in the kinetic energy of the individual particles of the medium moving up and down.
- As such, even when we don't see a visible wave, those particles still have a velocity vector that is containing the energy. It's like the *position* gets back to equilibrium for a moment, but the *velocity*, where the kinetic energy is contained, is most definitely not at equilibrium.

- In PHYS 13100, we used $F = ma$ to analyze a block of mass m oscillating on a spring, solving

$$F = ma$$
$$-kx = m \cdot \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{k}{m} \cdot x = 0$$

to describe its dynamics.

- Creating an analogy to $F = ma$ for wave motion (deriving the wave equation).

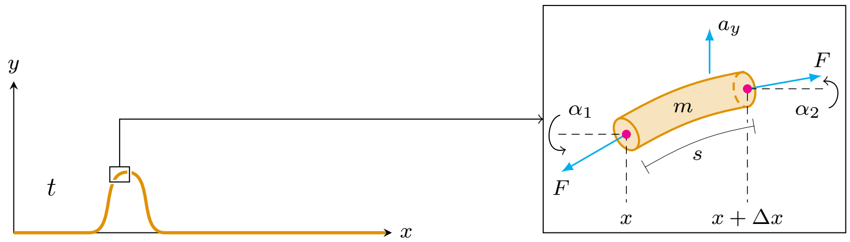


Figure 15.2: Deriving the wave equation.

- F is a tension force.
- We know for the sliver of the string in Figure 15.2, $F_y = ma_y$.
- From our FBD, we have that $F_y = F \sin\alpha_2 - F \sin\alpha_1$.
- Since the string segment is short, assume $\alpha_1 = \alpha_2$. Let's also ignore gravity since $F >> F_g$: it doesn't matter in what position you play an instrument, relative to the Earth's surface, does it?
- For small values of α (we assume our string is taut), $\sin\alpha \approx \tan\alpha = \frac{\partial y}{\partial x}$.
- Thus, $F_y = F(\partial y/\partial x\,|_{x+\Delta x} - \partial y/\partial x\,|_x)$.
- Additionally, $m = Ms$, where M is the linear mass density and s is the arc length of the string segment. Furthermore, since α's are small in taut strings, $\Delta s \approx \Delta x$, so $m \approx M\Delta x$.
- Lastly, observe that $a_y = \partial^2 y/\partial t^2$.
- Therefore, $F = ma$ becomes

$$F\left(\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_{x}\right) = M\Delta x \cdot \frac{\partial^2 y}{\partial t^2}$$
$$\frac{\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_{x}}{\Delta x} = \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}$$

from which we can take limits as follows:

$$\lim_{\Delta x \to 0} \frac{\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}$$
$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}}$$

- **Wave equation**: The final result above.
  - Holds for a 1D wave on a string.
- Tie a piece of string to a wall and shake the free end like a harmonic oscillator. This creates a **harmonic** wave that propagates towards the wall.
- **Harmonic** (wave): A wave produced by a disturbance changing like a harmonic oscillator.
  - The wavefunction for a harmonic wave is sinusoidal, propagates like a wave (i.e., like $f(x - vt)$), and needs to have a constant k to make the dimensional argument of sine dimensionless: $y(x,t) = A\sin(k[x - vt])$.
- **Amplitude**: The constant A in the wavefunction of a harmonic wave.
- **Wavenumber**: The constant k in the wavefunction of a harmonic wave. *Units are* m^{-1}.
- **Wavelength**: The *distance* over which wave motion repeats *for a fixed time* t. *Denoted by* $\boldsymbol{\lambda}$.
  - Mathematically, the existence of the wavelength implies that $y(x,t) = y(x+\lambda,t)$.
  - But for a harmonic wave, this implies that $A\sin(k[x - vt]) = A\sin(k[(x+\lambda) - vt])$, meaning that $k\lambda = 2\pi$.
  - Thus, we know that the wave number $k = \frac{2\pi}{\lambda}$.
- **Period**: The *time* over which wave motion repeats *for a fixed point* x. *Denoted by* $\boldsymbol{T}$.
  - Similarly, $y(x,t) = y(x,t+T)$.
  - For a harmonic wave, $A\sin(k[x - vt]) = A\sin(k[x - v(t+T)])$, meaning that $kvT = 2\pi$.
  - Thus, we know that the wave speed $v = \frac{2\pi}{k} \cdot \frac{1}{T} = \lambda f$, where f is the frequency of the wave, for simple harmonic motion.
  - Alternately, if we let $\omega = 2\pi f$ be the angular frequency, then $v = \frac{\omega}{k}$.
- It follows that for a harmonic wave,

$$\begin{aligned} y(x,t) &= A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \\ &= A\sin[kx - \omega t] \end{aligned}$$ [1]

- To account for cosine and other waves that "start" at different parts, we include a **phase constant** ϕ:

$$y(x,t) = A\sin[kx - \omega t + \phi]$$

- To check that the above is in fact a wave, we must feed it into the wave equation:

$$\begin{aligned} \frac{\partial^2}{\partial x^2}(A\sin[kx - \omega t + \phi]) &= \frac{M}{F} \cdot \frac{\partial^2}{\partial t^2}(A\sin[kx - \omega t + \phi]) \\ -Ak^2\sin[kx - \omega t + \phi] &= \frac{-A\omega^2 M}{F}\sin[kx - \omega t + \phi] \\ k^2 &= \frac{M\omega^2}{F} \\ \frac{\omega}{k} &= \sqrt{\frac{F}{M}} \end{aligned}$$

[1]Dr. Gazes prefers this form, but both are correct and can be used.

  - It follows since $v = \frac{\omega}{k}$ that $v = \sqrt{F/M}$.
- We originally found this speed/force/mass relationship to be true for a harmonic wave, but this shows that it is true for *any* wave.
- **General 1D wave equation**: Making the modification from above, the following equation.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}$$

15.3 Office Hours (Gazes)

- How does proving that $v = \sqrt{F/M}$ with a harmonic wavefunction prove that this relation holds for *all* waves?
  - Applies to any wave in a string. If you have a shape that doesn't look like a harmonic wave, you can construct it out of harmonic waves (Fourier math). The superposition principle allows us to add these waves.
- Importance of reading the textbook?
  - To be used as we wish.
  - We *could* read it instead of coming to lecture.
  - Think of it as something to consult as needed; i.e., for clarification.
  - Some people read it before class.
  - He will talk about some things in class that aren't in the textbook, and vice versa. If the textbook talks about it and he doesn't, you aren't responsible for knowing it.

15.4 Wave Dynamics

8/6:
- TA office hours on Wednesday and Sunday; 2 timeslots on both days.
- Dr. Gazes will post lab sections this afternoon.
- **Transverse velocity**: The speed at which a fixed point in the medium through which a transverse wave travels moves up and down. *Given by*

$$v_t = \frac{\partial y}{\partial t}$$

  - For a harmonic wave, $v_t = \omega A \cos(kx - \omega t + \phi)$.
- **Transverse acceleration**: The acceleration of a fixed point in the medium through which a transverse wave travels. *Given by*

$$a_t = \frac{\partial v_t}{\partial t}$$

  - For a harmonic wave, $a_t = -\omega^2 A \sin(kx - \omega t + \phi)$.
- When a point achieves its maximum positive displacement $y = +A$, it has $v_t = 0$ and $a_t = -\omega^2 A$.
  - Similarly, at $y = -A$, it still has $v_t = 0$, but it also has $a_t = \omega^2 A$.
  - When a point has zero displacement ($y = 0$), it has $v_t = \pm\omega A$ and $a_t = 0$.
- y and a_t are 180° out of phase with each other.
- y and v_t are 90° out of phase with each other.

- **Power**: The rate at which a wave carries energy. *Given by*

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t} = \vec{F} \cdot \vec{v}$$

- Wave energy:
  - Kinetic: $K = \frac{1}{2}mv_t^2$ for each little sliver of the string.
    - Thus, since $v = \omega A = 2\pi f A$, we have that $K \propto \omega^2, f^2, A^2$.
    - Additionally, since $P = \vec{F} \cdot \vec{v} = F \cdot (2\pi f A)$, we have that $P \propto v, f^2, A^2$.
  - Places where the string crosses the equilibrium axis have maximum stretching, i.e., potential energy.
- When you shake a string attached to a wall, the power P_{hand} exerted by your hand is given by

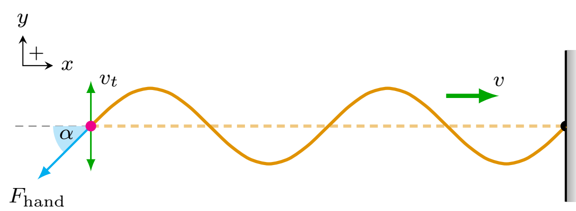


Figure 15.3: Power of a wave.

$$\begin{aligned}
P_{\text{hand}} &= \vec{F}_{\text{hand}} \cdot \vec{v} \\
&= F_{\text{hand},y} v_t \\
&= (F \cdot -\sin\alpha) \cdot \left(\frac{\partial y}{\partial t}\right) \\
&\approx (-F \tan\alpha) \cdot \left(\frac{\partial y}{\partial t}\right) \\
&= \left(-F \cdot \frac{\partial y}{\partial x}\right) \cdot \left(\frac{\partial y}{\partial t}\right) \\
&= (-F \cdot kA\cos(kx - \omega t + \phi)) \cdot (-\omega A \cos(kx - \omega t + \phi)) \\
&= Fk\omega A^2 \cos^2(kx - \omega t + \phi) \\
&= Mv^2 k\omega A^2 \cos^2(kx - \omega t + \phi) \\
&= Mv\omega^2 A^2 \cos^2(kx - \omega t + \phi)
\end{aligned}$$

  - Thus, since the average value of $\cos^2(x) = \frac{1}{2}$, the average power $\bar{P}$ of a wave on a string is given by

$$\bar{P} = \frac{1}{2} Mv\omega^2 A^2$$

    - Increasing the amplitude of a wave increases the power of the wave without changing the frequency or wave speed.
    - This is what radio stations do to boost the power of their broadcast (since they can't change the speed of light and changing the frequency would change their channel).
- **Compound string**: Two pieces of string (of differing composition) attached together.
- When an incident wave encounters a change of medium, it both transmits *and* reflects in parts.
  - The "knot" moving up and down is the source of the transmitted and reflected waves.

- Compound string analysis:

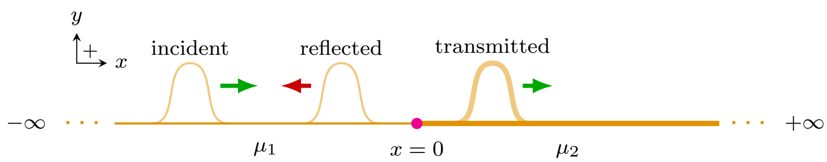


Figure 15.4: Compound string waves.

  - General wave equations for the incident wave (y_i), the transmitted wave (y_t), and the reflected wave (y_r):

$$\begin{aligned} y_i(x,t) &= A_i\cos(k_1x \pm \omega_1 t)\\ y_t(x,t) &= A_t\cos(k_2x \pm \omega_2 t)\\ y_r(x,t) &= A_r\cos(k_3x \pm \omega_3 t) \end{aligned}$$

  - According to the coordinate system in Figure 15.4, we choose $-,-,+$ from top to bottom for our wave equations.
  - $\omega_1 = \omega_2 = \omega_3$ because the frequency of the incident wave will be the frequency with which the knot moves.
  - $k = \frac{\omega}{v} = \omega\sqrt{M/F}$ varies because while ω and the tension force are the same (the latter because otherwise the knot would be accelerating), the linear mass density varies.
    - However, since the incident and reflected waves move in the same medium, $k_1 = k_3$.
  - Boundary conditions:
    1. String doesn't break, so y is continuous at $x = 0$.
    2. String has no kinks (because then you would have a point of zero mass with an unbalanced force on it, leading to an infinite acceleration, which is impossible), so $\partial y/\partial x$ is continuous at $x = 0$.
  - Thus, since

$$y = \begin{cases} y_i + y_r & x < 0\\ y_t & x > 0 \end{cases}$$

boundary condition 1 implies that $y_i(0,t) + y_r(0,t) = y_t(0,t)$ for all t. Consequently,

$$\begin{aligned} A_i\cos(k_1(0) - \omega t) + A_r\cos(k_1(0) + \omega t) &= A_t\cos(k_2(0) - \omega t)\\ A_i\cos(-\omega t) + A_r\cos(-\omega t) &= A_t\cos(-\omega t)\\ A_i + A_r &= A_t \end{aligned}$$

  - Additionally, boundary condition 2 implies that $\partial y_i/\partial x\,\big|_{x=0} + \partial y_r/\partial x\,\big|_{x=0} = \partial y_t/\partial x\,\big|_{x=0}$ for all t. Consequently,

$$\begin{aligned} \frac{\partial}{\partial x}(A_i\cos(k_1x - \omega t))\bigg|_{x=0} + \frac{\partial}{\partial x}(A_r\cos(k_1x + \omega t))\bigg|_{x=0} &= \frac{\partial}{\partial x}(A_t\cos(k_2x - \omega t))\bigg|_{x=0}\\ -A_ik_1\sin(k_1x - \omega t)\bigg|_{x=0} + -A_rk_1\sin(k_1x + \omega t)\bigg|_{x=0} &= -A_tk_2\sin(k_2x - \omega t)\bigg|_{x=0}\\ -A_ik_1\sin(-\omega t) - A_rk_1\sin(\omega t) &= -A_tk_2\sin(-\omega t)\\ -A_ik_1\sin(-\omega t) + A_rk_1\sin(-\omega t) &= -A_tk_2\sin(-\omega t)\\ -A_ik_1 + A_rk_1 &= -A_tk_2\\ k_1(A_i - A_r) &= k_2A_t \end{aligned}$$

  - It follows by solving like a system of equations that

$$\frac{A_r}{A_i} = \frac{k_1 - k_2}{k_1 + k_2} \qquad\qquad \frac{A_t}{A_i} = \frac{2k_1}{k_1 + k_2}$$

  - This combined with the fact that $k_1 \propto \sqrt{\mu_1}$ and $k_2 \propto \sqrt{\mu_2}$ implies that

$$\frac{A_r}{A_i} = \frac{\sqrt{\mu_1} - \sqrt{\mu_2}}{\sqrt{\mu_1} + \sqrt{\mu_2}} \qquad\qquad \frac{A_t}{A_i} = \frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}}$$

- Let's run a few checks on some special cases.
  - Let $\mu_1 = \mu_2$, i.e., the compound string is a uniform string. Then $A_r/A_i = 0$ and $A_t/A_i = 1$, as we would expect.
  - Let $\mu_1 << \mu_2$, i.e., one string is tied to an immovable wall. Then $A_r/A_i \to -1$ and $A_t/A_i \to 0$, as we would expect by Newton's third law.
  - Let $\mu_1 >> \mu_2$. Then $A_r/A_i \to 1$ and $A_t/A_i \to 2$.
- Suppose you have a string tied between two walls.

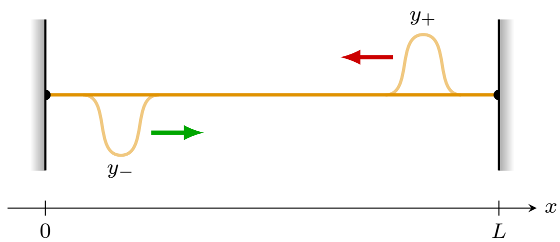


Figure 15.5: A string tied between two walls.

  - If you send a wave y_+ in the $-x$-direction, it will be reflected and inverted in its entirety at the left wall into the wave y_-.
  - This yields a total wavefunction

$$\begin{aligned}
y &= y_+ + y_- \\
&= A\cos(kx + \omega t) - A\cos(kx - \omega t) \\
&= A[\cos(kx + \omega t) - \cos(kx - \omega t)] \\
&= 2A\sin\left(\frac{(kx + \omega t) + (kx - \omega t)}{2}\right)\sin\left(\frac{(kx + \omega t) - (kx + \omega t)}{2}\right) \\
&= 2A\sin(kx)\sin(\omega t)
\end{aligned}$$

  - Boundary conditions:
    1. $y(0, t) = 0$ for all t.
    2. $y(L, t) = 0$ for all t.
  - From the second boundary condition, we know that we must have $\sin(kL) = 0$, i.e., $kL = n\pi$ for some $n \in \mathbb{N}$ (the wavenumber cannot be negative or zero by definition).
  - Thus, $k_n = \frac{n\pi}{L}$.
  - It follows since $k = \frac{2\pi}{\lambda}$ that $L = \frac{n}{2} \cdot \lambda$.
- More specifically, if $L = \frac{n}{2} \cdot \lambda$ for some $n \in \mathbb{N}$, then we will have a **standing wave**.
- **Node**: A point in the medium of a standing wave with amplitude zero.

- **Antinode**: A point in the medium of a standing wave with maximum amplitude.
- Frequency of standing waves:

(a) $n = 1$. (b) $n = 2$.

Figure 15.6: Fundamental harmonic frequencies.

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \sqrt{\frac{F}{M}} \cdot \frac{n}{2L} \end{aligned}$$

  - When $n = 1$, we call $f_1 = \frac{1}{2L}\sqrt{F/M}$ the **first fundamental harmonic frequency**.
  - When $n = 2$, we call $f_1 = 2f_1$ the **second fundamental harmonic frequency**.
  - Similarly, $f_n = nf_1$ for the $\boldsymbol{n}^{\textbf{th}}$ **fundamental harmonic frequency**.
- Different instruments have different **overtones** (combinations of harmonics).
- We have some lab stuff to do before Monday.
- If you vibrate a string at a certain frequency, you can build up energy in the wave. Otherwise, you will just have all sorts of dissonant destructive interference. Think about pushing a swing — you have to push it at the right time to build up a big amplitude.

15.5 Chapter 15: Mechanical Waves

From Young and Freedman (2019).

8/10:

- An alternate way of deriving the speed of a harmonic wave:
  - The wave speed is equivalent to the speed we must move at in the x-direction to stay at the same "wave part," be that a crest, a trough, or anywhere in between.
  - At every similar wave part, y-displacement from the x-axis is equal; in other words, $A\sin(kx - \omega t) =$ constant.
  - But this implies that $kx - \omega t =$ constant.
  - Taking partial derivatives wrt time of the above equation, we get

$$\begin{aligned} k \cdot \frac{\partial x}{\partial t} - \omega &= 0 \\ v &= \frac{\omega}{k} \end{aligned}$$

    as desired.
- The second partial derivative of the harmonic wave function wrt x yields the **curvature** of the string.
- An alternate way of deriving the general 1D wave equation (for a harmonic wave):

- Take second partial derivatives in both variables:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx-\omega t) \qquad \frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 a\cos(kx-\omega t)$$

- Notice that

$$\frac{\partial^2 y(x,t)/\partial t^2}{\partial^2 y(x,t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2$$

- The above can be rearranged to yield

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

- Applying a constant, perpendicular force to the end of a string does not cause the end of the string to accelerate upwards with constant acceleration, but rather "the effect of the force F_y is to set successively more and more mass in motion" (Young & Freedman, 2019, p. 475).
  - From the impulse-momentum theorem, the transverse impulse up to time t must be mirrored by the change in transverse momentum, i.e., $F_y t = mv_y$.
  - Since the wave moves with constant velocity, the amount of mass set into motion varies with time t, so v_y does not have to change.
  - Extends this to derive $v = \sqrt{F/M}$ for a triangular pulse, but notes that the equation is valid for any pulse since every pulse is a series of pulses with different values of v_y.
- The principle of superposition doesn't hold for mediums that don't obey Hooke's law, i.e., are not linear.
- We call a traveling wave that to distinguish it from a standing wave.
- The fact that the nodes of a standing wave $y = 2A\sin kx\sin\omega t$ don't move follows from the fact that nodes are found where $y = 0$, i.e., where $\sin kx = 0$, and the solutions to the latter equation don't depend on time.
- **Fundamental frequency**: The smallest possible frequency that can produce a standing wave on a string.
- **Harmonic**: One of the fundamental harmonic frequencies.
- f_2 is the second harmonic, or the first **overtone**.
  - Similarly, f_3 is the third harmonic, or the second overtone.
- **Normal mode**: A motion in which all particles of an oscillating system move sinusoidally with the same frequency.
- **Harmonic content**: The extent to which frequencies higher than the fundamental are present.
- We can write that $f_1 = \frac{1}{2L}\sqrt{F/M}$.
  - With respect to string instruments, this implies that tighter strings yield higher frequencies, and heavier strings have lower frequencies!

Chapter 16

Sound and Hearing

16.1 Intro to Sound Waves

8/9:

- First quiz this Friday.
  - There to get you ready for the midterm.
  - Starts at 10:00 AM.
  - 30 minutes for the quiz plus 20 minutes to scan and upload $\Rightarrow$ due at 10:50 AM.
  - Send Dr. Gazes an email if you have technical issues.
  - Study:
    - HW 1-2.
    - Chapter 15-16, and parts of 33.
    - No questions on homework material to which we don't have the solutions.
- **Standing wave**: A wave with nodes and antinodes that do not move.
- Consider an air-filled pipe of length L with a piston at one end and being open at the other end.

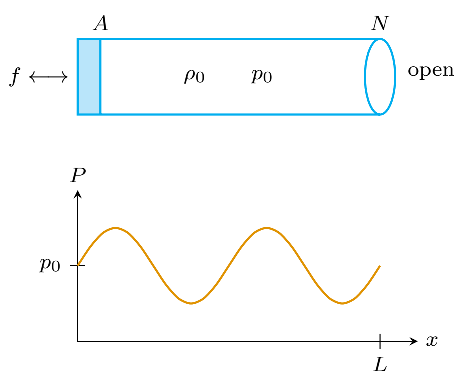


Figure 16.1: An air-filled pipe.

  - The air in it has density ρ_0 and pressure p_0.
  - Reviews compression and rarefaction.
  - Creating a plot of pressure vs. x-distance yields a transverse pressure wave.
  - We consider the pressure at the end to be essentially "clamped" at atmospheric pressure p_0.
    - Thus, the wave gets reflected at the end of the pipe.

- **Sound wave**: A wave propagating in a material.
- Speed of sound:
  - For a wave on a string, $v = \sqrt{F/M}$.
  - Tension is how hard a sliver of string is being pulled on by its neighbors. Mass density is inertial; it tells us how much a sliver of string resists being moved by its neighbor.
  - Thus, for a sound wave, we should have something kind of like $v = \sqrt{p_0/\rho_0}$.
  - In fact, adjusting for some other factors, we get (at STP)

$$v_{\text{sound}} = \sqrt{\frac{1.4p_0}{\rho_0}}$$

- Let $\delta p = p - p_0$.

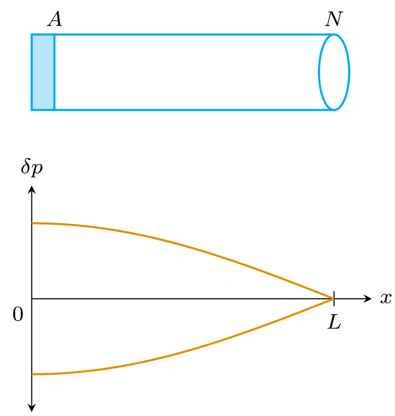


Figure 16.2: Standing waves in an air-filled pipe.

  - Then some ways to get a standing wave are $L = \frac{\lambda}{4}, \frac{\lambda}{4} + \frac{\lambda}{2}, \frac{\lambda}{4} + \lambda, \dots$.
  - Thus, standing waves are given by $L = \frac{\lambda}{4} + m \cdot \frac{\lambda}{2}$, where $m \in \mathbb{N} \cup \{0\}$.
- When you have a node for pressure, you have an antinode for displacement and vice versa.
- It follows from the fact that $L = \frac{n\lambda}{4}$ for $n \in 2\mathbb{N} + 1$ that $L = \frac{n}{4} \cdot \frac{v_{\text{sound}}}{f}$ for $n \in 2\mathbb{N} + 1$.
  - Thus, $f_n = \frac{n}{4} \cdot \frac{v_{\text{sound}}}{L}$.
  - f_1 is again the fundamental frequency, and $f_n = nf_1$, but only where $n \in 2\mathbb{N} + 1$.
- Hissing air has all kinds of frequencies. When you blow it over the opening of a bottle, only the frequencies with large amplitudes will produce standing waves.
  - When you partially fill the bottle, decreasing the length of the tube of air, higher frequencies are selected for.
- When you blow air past a tube that is open at both ends, you have pressure nodes (denoted by N_p) at both ends and you can get all sorts of standing waves in between.

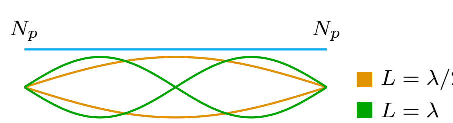


Figure 16.3: Standing waves in an uncapped pipe.

  - Here, we have $L = n \cdot \frac{\lambda}{2}$, where $n \in \mathbb{N}$.
  - Open-open pipes are just like a string clamped at both ends.

16.2 Sound Waves in More Dimensions

- 2001: A Space Odyssey starts with a 16 Hz sound.
- Sound in 1D vs. sound in 3D.

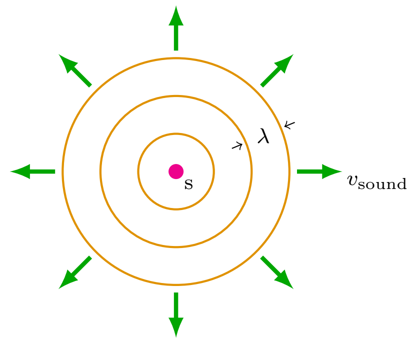


Figure 16.4: Sound waves in 3D.

- **Wavelength** (3D): The distance between the crests of adjacent waves.
- How much energy is captured by your ear depends on the **intensity**.
- **Intensity**: The average power per unit area. *Denoted by* $\boldsymbol{I}$. *Units* $\mathrm{W/m^2}$.
  - In 3D, $I = \frac{\bar{P}}{4\pi r^2}$.
  - Thus, the power at your ear is given by $P_{\text{ear}} = IA_{\text{ear}}$.
- **Threshold intensity**: The lowest intensity that can still be heard. *Denoted by* $\boldsymbol{I_0}$.
  - For humans, $I_0 \approx 1 \times 10^{-12}\,\mathrm{W/m^2}$.
- **Sound intensity level**: The following quantity. *Units* dB.

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

  - $\beta(\text{whisper}) \approx 20\,\text{dB}$.
  - $\beta(\text{NYC Subway}) \approx 100\,\text{dB}$.
    - 1×10^8 times the intensity of a whisper!
  - $\beta(\text{ears hurt}) \approx 120\,\text{dB}$.

16.3 Sound Wave Phenomena

- Speakers at varying distances from one's ear:

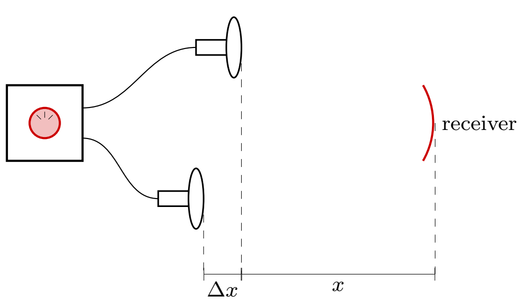


Figure 16.5: Speakers at varying distances from one's ear.

- $y = y_1 + y_2 = A\cos(kx - \omega t) + A\cos(k[x + \Delta x] - \omega t)$.
- If $\Delta x = 0$, then
$$y = 2A\cos(kx - \omega t)$$
- If $\Delta x = \frac{\lambda}{2}$, then
$$\begin{aligned} y &= A\left[\cos(kx - \omega t) + \cos\left(kx + k \cdot \frac{\lambda}{2} - \omega t\right)\right] \\ &= A\left[\cos(kx - \omega t) + \cos\left(kx + \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} - \omega t\right)\right] \\ &= A[\cos(kx - \omega t) + \cos(kx - \omega t + \pi)] \\ &= A[\cos(kx - \omega t) - \cos(kx - \omega t)] \\ &= 0 \end{aligned}$$
  so you get total cancellation/destructive interference.
- Similarly, you can electronically delay the signal. If $\Delta t = \frac{T}{2}$, then the waves cancel. This is the principle behind noise-cancelling headphones.
- Δx is called the **path length difference**.

• Sound waves of slightly varying frequency:

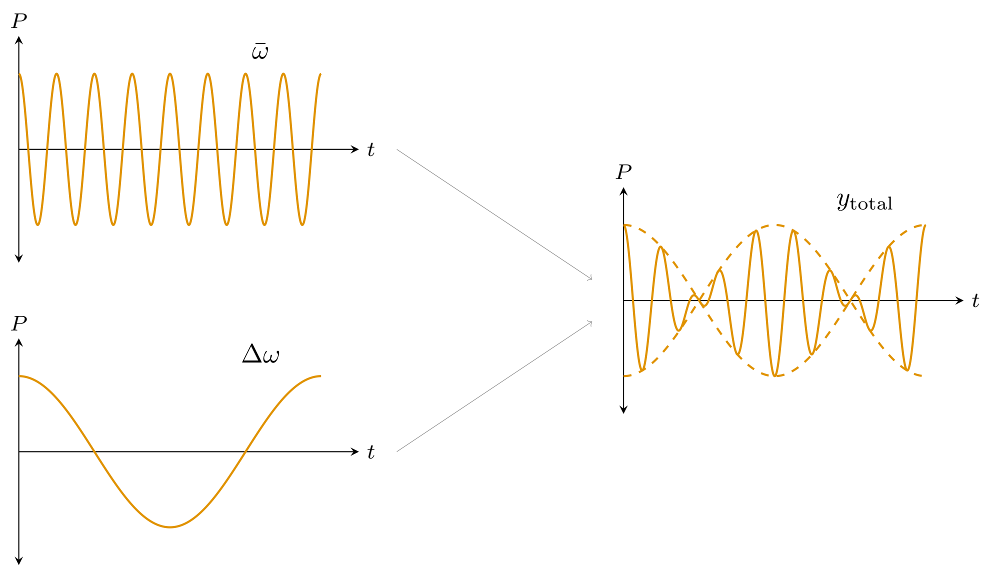


Figure 16.6: Sound waves of slightly varying frequencies.

- Consider frequencies f_1, f_2 where $\Delta f << f_1, f_2$.
- Since $k = 2\pi f / v_{\text{sound}}$ and $\omega = 2\pi f$ (i.e., both quantities depend on frequency), we have that
$$\begin{aligned} y &= y_1 + y_2 \\ &= A[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)] \end{aligned}$$
  Suppose $x = 0$.
$$\begin{aligned} &= A[\cos(\omega_1 t) + \cos(\omega_2 t)] \\ &= 2A\cos\left(\frac{\omega_1 + \omega_2}{2} \cdot t\right)\cos\left(\frac{\omega_1 - \omega_2}{2} \cdot t\right) \\ &= 2A\cos(\bar{\omega} t)\cos\left(\frac{\Delta\omega}{2} \cdot t\right) \end{aligned}$$

  - Since $\bar{\omega} >> \Delta\omega$, y looks like the end result in Figure 16.6.
  - Thus, we will hear $\bar{\omega}$, but there will be silences interspersed.
    - These nodes are called **beats**, and $f_{\text{beat}} = f_1 - f_2$.
- Suppose we have a source s producing a sound of frequency f. An observer o runs toward the source at speed v_o.
  - Thus, the observer is being hit by wavefronts moving, relative to them, at speed $v + v_o$. Thus, since $v = \lambda f$, the frequency f' that the observer hears is given by

$$f' = \frac{v + v_o}{\lambda} = \frac{v + v_o}{v} \cdot f > f$$

- **Doppler Effect**: The change in frequency produced by the speed of the observer relative to the source. *Also known as* **Doppler Shift**.
  - Also happens when the source moves toward the observer. In this case, though, λ varies: With respect to the source, waves are being emitted at the same frequency, but they're only moving away from the source at speed $v - v_s$. Thus, $\lambda' = v - v_s/f$, so $f' = \frac{v}{v-v_s} \cdot f$.

Chapter 17

Temperature and Heat

17.1 Thermodynamics Terminology and Fundamentals

8/19:
- **System**: A collection of objects.
  - We will focus on systems comprised of gas molecules.
- **Internal energy**: The energy (of such a system) associated with the microscopic motion of the molecules. *Denoted by* $\boldsymbol{E}_{\text{int}}$.
- Systems of molecules conserve momentum within themselves, like any system, but they can also interact with the rest of the world by exchanging energy.
- **Heat**: Energy exchanged between hot and cold systems. *Denoted by* $\boldsymbol{Q}$.
- **Temperature**: A measure of E_{int}. *Denoted by* $\boldsymbol{T}$.
- Heat flows from a hot system to a cold system until thermal equilibrium is reached.
  - Thermal equilibrium implies equal temperatures.
- **Zeroth Law of Thermodynamics**: If $T_A = T_B$ and $T_B = T_C$, then $T_A = T_C$.
  - You can think of B as a thermometer — if it reads the same for two different systems, those systems have the same temperature.
- To measure temperature, we need a thermometric property.
  - One example of a thermometric property is the dependence of the volume of mercury on temperature.
    - Celsius stuck a column of mercury in ice water and called it 0°. Similarly, he called a column of mercury in hot steam 100°.
    - Fahrenheit used iced brine (salt water) for 0° and sheep's blood for 100°.
  - Alternatively, we could measure the **pressure** of a gas, for instance.
- **Pressure**: The quotient of force and area. *Units* Pa.
  - $1\,\text{Pa} = 1\,\text{N/m}^2$.
  - $1\,\text{atm} = 1.01 \times 10^5\,\text{Pa} \approx 14\,\text{lb/in}^2$.
- **Absolute temperature**: The temperature defined by

$$T = 273 \cdot \frac{P}{P_{1\,\text{atm}}}$$

  where P is the pressure of a gas at temperature T.

- **Absolute zero**: The temperature when the pressure of a gas is 0.
- Note that

$$T_C = T - 273^\circ \qquad\qquad T_F = \frac{9}{5}T_C + 32$$

- Equations of motion relate kinematic quantities, such as time, position, velocity, and acceleration.
- Equations of state relate thermodynamic properties of a system, such as pressure, volume, temperature, and moles.

Chapter 18

Thermal Properties of Matter

18.1 Macroscopic Derivation of the Ideal Gas Law

8/19:

- Atoms have a nucleus (composed of protons and neutrons) orbited by electrons.
- **Atomic number**: The number of protons. *Denoted by* $\boldsymbol{Z}$.
- **Atomic mass**: Essentially the number of protons plus the number of neutrons. *Denoted by* $\boldsymbol{A}$, $\boldsymbol{\mu}$. *Units* amu.
  - $1\,\text{amu} = \frac{1}{12}m({}^{12}_{6}\text{C})$.
- **Avogadro's number**: The number of molecules per mole of a substance, i.e., 6.02×10^{23}. *Denoted by* $\boldsymbol{N_A}$.
  - N_A carbon-12 atoms weighs $12\,\text{g}$.
  - We define $1\,\text{mol} = N_A$ of something.
- **Boyle's Law**: The product of the pressure and volume of a gas is a constant (that depends on the gas at hand).
- **Ideal gas law**: The relation
$$pV = nRT$$
relating the pressure, volume, number of moles, and temperature of a dilute gas to a constant (that is not specific to any particular gas).
- **Universal gas constant**: The constant $8.314\,\frac{\text{J}}{\text{mol K}}$. *Denoted by* $\boldsymbol{R}$.
- Thus, we can think of temperature as being a reflection of a few macroscopic properties of gasses (e.g., pressure, volume, and number of moles).

18.2 Microscopic Derivation of the Ideal Gas Law

- Covers the derivation of the ideal gas law from KMT, as described Chapter 5 of Labalme (2020).
- Important addition:
$$p = \frac{1}{3}\rho\bar{v}^2$$
where p is pressure, ρ is density, and $\bar{v}$ is the average velocity of the molecules.
  - This relates a macroscopic and a microscopic quantity.

- Thus, we can relate the average speed of the molecules to the measurable pressure via

$$v_{\text{rms}} = \sqrt{\frac{3p}{\rho}}$$

- We can calculate from the above equation that the root mean square velocity of hydrogen gas at STP is about 1 800 m/s. For nitrogen gas, it's about 450 m/s.
- **Boltzmann constant**: The quotient of the universal gas constant and Avogadro's constant, having value 1.38×10^{-23} J/K. *Denoted by* k.
- It follows that

$$\overline{KE}_{\text{molecule}} = \frac{3}{2}kT$$

- Additionally, we have that

$$v_{\text{rms}} = \sqrt{\frac{3kT}{\mu}}$$

- This property can be taken advantage of for **diffusion separation of isotopes**.
- How to separate ^{238}U from ^{235}U:
  - Create UF_6, a gas.
  - The lighter molecules will effuse slightly faster out of a box with a hole.
  - If you apply the cycle over and over again, you will enrich it a little bit each time.
  - Eventually, you will have a large proportion of $^{235}UF_6$, from which the ^{235}U can be extracted.

18.3 Thermodynamic Work

8/20:

- Final on Wednesday.
  - Posted at 10 AM CT, due on Canvas at 12:20 PM CT.
  - 2 hours.
  - Quantitative questions on HW 1-5 material, qualitative questions on Monday/Tuesday lecture material.
  - All chapters.
  - Will weight our grades with more/less emphasis on the final and take the higher of the two.
- Assuming that gas molecules have no internal structure with which to store energy (i.e., they can only store kinetic energy of motion), we have that

$$\begin{aligned}
E_{\text{int}} &= nN_A\overline{KE}_{\text{molecule}}\\
&= \frac{3}{2}n(N_Ak)T\\
&= \frac{3}{2}nRT
\end{aligned}$$

  - This implies that temperature alone determines the internal energy of a gas.
  - This is a good approximation for monoatomic gasses, but we may need other formulas for gasses with more complex molecular structures.
- **Heat bath**: A bath that maintains a constant temperature.
  - Lake Michigan is a good example: Adding or removing heat from it will not significantly affect its temperature.

- Consider a container in a heat bath of temperature T.
  - The container is made of conducting walls and filled with gas of pressure p. It is also capped by a piston with cross-sectional area A, pushing down on the gas with force F.
  - If the piston moves up by a tiny distance $\mathrm{d}x$, then the work $\mathrm{d}W$ exerted by the gas on the piston is given by

$$\begin{aligned}\mathrm{d}W &= F \cdot \mathrm{d}x \\ &= pA\,\mathrm{d}x \\ &= p\,\mathrm{d}V\end{aligned}$$

  - Similarly, the work exerted by the piston on the gas is given by $\mathrm{d}W = -p\,\mathrm{d}V$.
  - Thus, pushing down on the piston raises the internal energy of the gas. But since the container is in a heat bath, temperature stays the same, i.e., internal energy stays the same. Consequently, work being done on the gas by the piston must cause heat to flow out of the gas.
  - If the piston moves from a to b, then

$$\begin{aligned}W &= \int_a^b -p\,\mathrm{d}V \\ &= -nRT\int_a^b \frac{\mathrm{d}V}{V} \\ &= -nRT\ln\left(\frac{V_b}{V_a}\right)\end{aligned}$$

    - Do remember that this equation only holds in isothermal conditions.
  - If we have an expansion, then $V_b > V_a$, so $W < 0$.
    - Similarly, if we have a compression, then $V_b < V_a$, so $W > 0$.
- Convention: We will typically talk about work *done on the gas* and heat *added to the gas*.
- **Isothermal process**: A process during which temperature is held constant.
- **Isotherm**: A line of constant temperature in a pressure vs. volume graph.
- **Isochoric process**: A process during which volume is held constant.
- **Isobaric process**: A process during which pressure is held constant.
- Changes in pressure and volume are pathway independent.
- Consider an insulated container.
  - As before, though, the container is filled with gas of pressure p. It is also capped by a piston with cross-sectional area A, pushing down on the gas with force F.
  - If you compress the gas by pushing the piston down, you're doing positive work on the gas and E_{int} increases.
  - Temperature increases so much, actually, that you can ignite a gasoline-soaked cotton ball in a fire piston.
  - In the piston's frame, a gas molecule colliding with a moving piston leaves with the same velocity it came in with.
  - In the lab's frame, a colliding gas molecule leaves the collision with extra velocity (think of a collision between a very light and a very heavy object).
- **Adiabatic process**: A process with no transfer of heat.

Chapter 19

The First Law of Thermodynamics

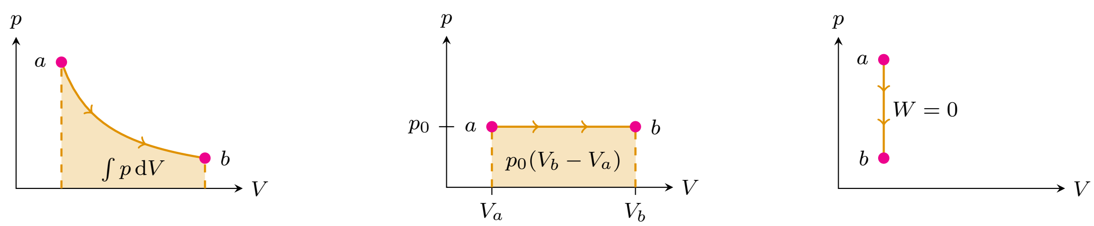
19.1 The First Law and Heat Capacity

8/20: • By the conservation of energy, $\Delta E_{\text{int}} = Q + W$. This implies the following.

- **First Law of Thermodynamics**: The following formula, which is analogous to conservation of energy.

$$\mathrm{d}E_{\text{int}} = \mathrm{d}Q - p\,\mathrm{d}V$$

- The work done *by* the gas is the area under the curve defined by the path between two pressure-volume conditions.



(a) Isothermic process. (b) Isobaric process. (c) Isochoric process.

Figure 19.1: Work as a geometric integral of a pressure/volume graph.

  - Thus, for an isothermic process, $W = -\int p\,\mathrm{d}V$.
  - For an isobaric process, $W = -P(V_b - V_a)$.
  - For an isochoric process, $W = 0$.

- Two processes of getting from condition a to condition b:

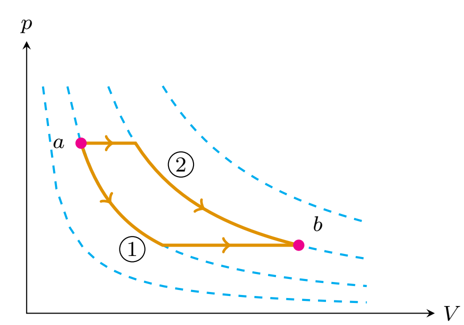


Figure 19.2: Alternate paths.

- We know from the first law that ΔE_{int} is the same for both properties.
- However, from the above, we know that W is not the same for both (different areas under the curve).
- Thus, by the first law, heat cannot be the same (it must be offset to compensate).

- **Molar heat capacity** (of a gas at constant volume): Defined by the following.

$$C_V = \frac{1}{n}\frac{\mathrm{d}Q_V}{\mathrm{d}T}$$

  - Tells you how much heat you have to add to raise the temperature a certain amount.
- Additionally, since $\mathrm{d}W = 0$ at constant volume, $\mathrm{d}E_{\text{int}} = \mathrm{d}Q_V$.
  - Thus, $C_V = 1/n \cdot \mathrm{d}E_{\text{int}}/\mathrm{d}T$.
  - Consequently, since $E_{\text{int}} = \frac{3}{2}nRT$, we have that $\mathrm{d}E_{\text{int}}/\mathrm{d}T = \frac{3}{2}nR$.
  - Therefore, at constant volume, $C_V = \frac{3}{2}R$.
- **Molar heat capacity** (of a gas at constant pressure): Defined by the following.

$$C_p = \frac{1}{n}\frac{\mathrm{d}Q_p}{\mathrm{d}T}$$

  - For the same $\mathrm{d}T$, $\mathrm{d}Q_p \neq \mathrm{d}Q_V$.
- We know that $C_p > C_V$.
  - This is principally because of the difference between the equations $\mathrm{d}E_{\text{int}} = \mathrm{d}Q$, pertaining to an isochoric process, and $\mathrm{d}E_{\text{int}} + \mathrm{d}W = \mathrm{d}Q$, pertaining to an isobaric process.
  - In the former, we can see that the change in internal energy (which is directly related to the temperature for an ideal gas) is directly reflected by the amount of heat flowing into the system.
  - However, in the latter, we raise the temperature by the same amount, so internal energy increases by the same amount. However, the gas must expand to maintain its constant pressure. Thus, some work is done. Thus, more heat must flow into the system to account for the work done *and* maintain the same temperature.
- For some $\mathrm{d}T$ (i.e., some $\mathrm{d}E_{\text{int}}$).
  - Heat added at constant volume: $\mathrm{d}E_{\text{int}} = \mathrm{d}Q_V = nC_V\,\mathrm{d}T$.
  - Heat added at constant pressure: $\mathrm{d}E_{\text{int}} = \mathrm{d}Q_p - p\,\mathrm{d}V = nC_p\,\mathrm{d}T - p\,\mathrm{d}V$.
  - The amount of heat you add in both cases will be different, but the change in internal energy will be the same.
  - Thus,

$$\begin{aligned}
nC_V\,\mathrm{d}T &= nC_p\,\mathrm{d}T - p\,\mathrm{d}V \\
C_p - C_V &= \frac{p}{n}\frac{\mathrm{d}V}{\mathrm{d}T} \\
&= \frac{p}{n}\cdot\frac{nR}{p} \\
&= R \\
C_p &= R + \frac{3}{2}R \\
&= \frac{5}{2}R
\end{aligned}$$

19.2 Storing Energy in Bonds

- We know that

$$\begin{aligned}\frac{3}{2}kT &= \overline{KE}_{\text{molecule}}\\ &= \frac{1}{2}\mu\bar{v}^2\\ &= \frac{1}{2}\mu\bar{v}_x^2 + \frac{1}{2}\mu\bar{v}_y^2 + \frac{1}{2}\mu\bar{v}_z^2\end{aligned}$$

- **Equipartition of energy theorem**: Every **degree of freedom** has associated with it, on average, $\frac{1}{2}kT$ of energy.
  - Can be derived, but we're not expected to know this.
- **Degree of freedom**: Any direction that can be expressed as the time derivative of a coordinate, squared.
- For example, translational kinetic energy has three degrees of freedom: v_x^2, v_y^2, and v_z^2.
  - Notice that since each has $\frac{1}{2}kT$ of associated energy, the total translational kinetic energy has $\frac{3}{2}kT$ of energy, as desired.
- Rotational kinetic energy has three degrees of freedom:

$$\begin{aligned}K_{\text{rot}} &= \frac{1}{2}I\omega^2\\ &= \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2\end{aligned}$$

  - For a diatomic molecule, it's symmetry implies that K_{rot} actually equals just $\frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$, if its axis is aligned with the x-axis.
- Thus, adding up these five degrees of freedom, we have that for each diatomic molecule, $E_{\text{int}} = \frac{5}{2}kT$.
  - This implies that for a mole of diatomic molecules, $E_{\text{int}} = \frac{5}{2}RT$, making $C_V = \frac{5}{2}R$ and $C_P = \frac{7}{2}R$.
- Factoring in vibrational energy, if we let x be the separation between atoms, we have x^2 and v_x^2, corresponding to potential- and kinetic-energy degrees of freedom.
  - This yields $C_V = \frac{7}{2}R$ and $C_P = \frac{9}{2}R$.
- For hydrogen, we have changes in the molar heat capacity as a function of temperature.
  - This can be explained by the increase in degrees of freedom at higher temperatures (first only translational, then rotational gets added in, then vibrational).
  - Why? Quantum physics — more excited states become available at higher temperatures.
- If D is the number of degrees of freedom of a molecule, then

$$C_V = D \cdot \frac{1}{2}R$$

  - On the other hand, it is always true that $C_P = C_V + R$.
  - Similarly, $E_{\text{int}} = D \cdot \frac{1}{2}nRT$.

19.3 Thermal Expansion

- Consider a potential energy vs. bond stretch graph.
  - Since you can't push atoms too close together, the graph will be asymmetric.
  - Indeed, at higher energies, the bond will *stretch* more, but it will not shrink that much more.
  - Thus, the average separation increases, and the gas expands.
- **Coefficient of linear expansion**: The following quantity, where L is the length of something that's expanding.

$$\alpha = \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}T}$$

  - This formula essentially tells us how much something expands per unit increase in temperature: $\mathrm{d}L = \alpha L\,\mathrm{d}T$.
- If we have a circular metal disk with a hole, raising its temperature will enlarge the whole thing (both the outer radius and inner radius increase).

Chapter 32

Electromagnetic Waves

32.1 Creating EM Waves

8/10:

- Quizzes and tests are open notebook, open notes, open textbook.
  - You can use a TI-84 type calculator, but nothing fancier.
- Reviews that a charge at rest generates an electric field and that a charge moving with constant velocity v generates a magnetic field in addition to its electric field.
  - Relativity says that you can't tell whether a charge is moving relative to you or whether you're moving relative to the charge, so a charge at rest and a moving charge have identical field lines, when appropriate frames of reference are taken.
- However, when a charge accelerates for a brief time, kinks will be generated in the field lines that correspond to exactly what was going on during the acceleration.

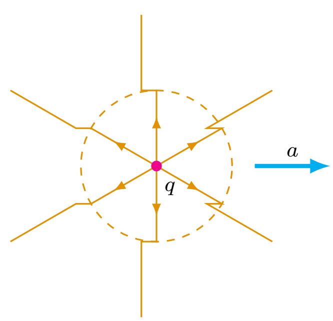


Figure 32.1: An accelerating charge.

  - The field lines are radial before and after the acceleration, but not radial during it (these are the kinks).
  - The kinks are perpendicular to the field lines, and generate a transverse electric field.
  - The transverse electric field then propagates at the speed of light.

32.2 Defining EM Waves

- **Electromagnetic wave**: The propagation of the transverse electric field.
- According to Faraday's Law, changing magnetic fields induce changing electric fields.

- According to Ampere's Law, currents induce magnetic fields.
  - Maxwell adjusts this.
  - You can have currents that are real, but also currents that are not technically currents.

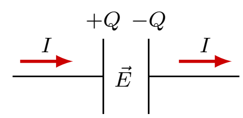


Figure 32.2: Generating a displacement current.

    - For example, between the plates of a charging capacitor, the changing electric field still produces a magnetic field.
    - This **displacement current** is induced by the changing electric flux $\mathrm{d}\Phi_E/\mathrm{d}t$, which is the root cause of the magnetic field.
    - Mathematically, $I_{\text{displacement}} = \epsilon_0 \cdot \mathrm{d}\Phi_E/\mathrm{d}t$.
  - Thus, according to the Maxwell-Ampere law, changing electric fields induce changing magnetic fields.
- As the transverse electric field approaches you, the changing magnetic field induces a planar displacement current going in one direction at the front end and the opposite direction at the other end.

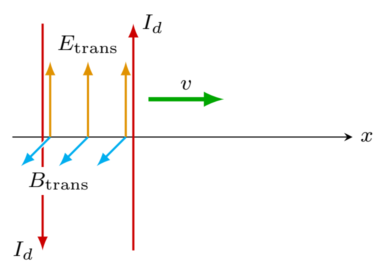


Figure 32.3: An idealized wave with a planar wavefront.

  - As the front passes you, a magnetic field is induced, too, by the Maxwell-Ampere law.
  - The two interchanging pulses create a self-sustaining wave.
- Essentially, Maxwell's conclusion is that changing transverse electric fields and changing transverse magnetic fields induce each other.
  - The speed that this occurs at is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

  - To derive this speed, Maxwell used his observation that in one dimension, the transverse electric field obeys the equation, $\frac{\partial^2 E_{\text{trans}}}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_{\text{trans}}}{\partial t^2}$.
  - By comparison with the wave equation, $\mu_0 \epsilon_0 = \frac{1}{v^2}$, which can be solved for the above.

32.3 EM Waves in the World

- Accelerating charges are commonly seen in

1. Orbiting electrons in an atom;
2. LC circuits.

- For an LC circuit,
$$f_{\text{EM wave}} = f_{\text{LC}} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$$
- Homemade circuits can make frequencies between 1×10^4 - $1 \times 10^{11}\,\text{s}^{-1}$, and thus since $c = f\lambda$, wavelengths between 1×10^4 - 1×10^{-3} m (waves from kilometers to millimeters in length).
- Orbiting electrons give frequencies between 1×10^{11} - $1 \times 10^{18}\,\text{s}^{-1}$, and wavelengths between 1×10^{-3} - 1×10^{-10} m (waves from millimeters down to angstroms in length).
  - Thus, we can cover 14 orders of magnitude in total.
  - Visible is only 4000-7000 angstroms!

Chapter 33

The Nature and Propagation of Light

33.1 Light Wave Terminology and Basics

8/10:

- **Geometric optics**: The study of situations in which EM radiation interacts with objects (possibly with holes) such that $\lambda <<$ size of obstacles, holes.
- **Physical optics**: The study of situations in which EM radiation interacts with objects (possibly with holes) such that $\lambda \approx$ size of obstacles, holes.
- **Ray**: An imaginary line, perpendicular to the wave fronts, that indicates the direction of propagation.
- Any time a wave hits a medium, you get reflection and transmission.
- **Huygens principle**: All points on a wavefront act as point sources of spherical wavelets. After a time Δt, the new position of the wavefronts is the surface of tangency of the wavelets.

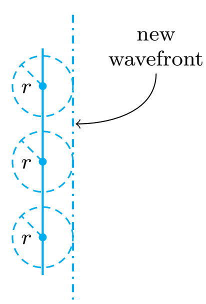
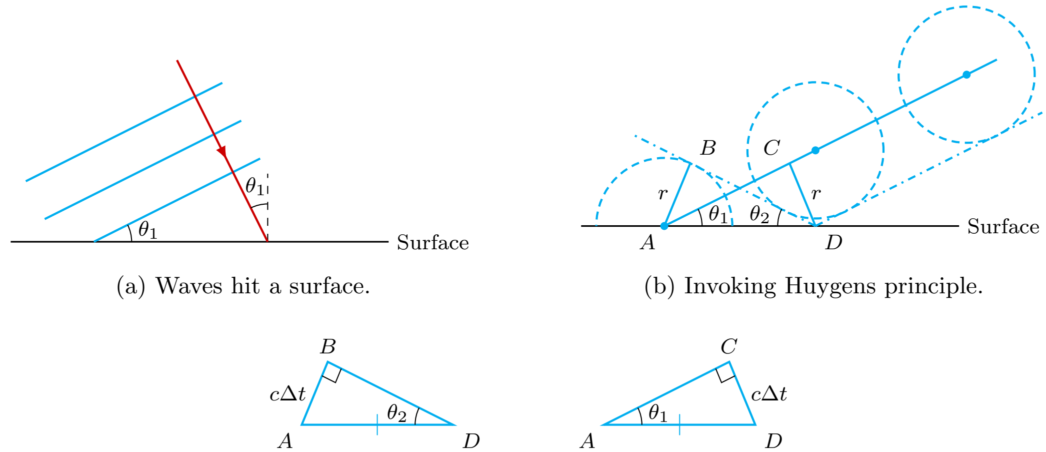


Figure 33.1: Huygens principle.

  - The radius r of the wavelets, in terms of the time Δt from their creation, is $r = c\Delta t$.
- Light rays hitting a surface (see Figure 33.2).
  - Since the triangles containing the **angle of incidence** and the **angle of reflection** ($\triangle ACD$ and $\triangle ABD$, respectively) share $\overline{AD}$, r, and a right angle (see Figure 33.2c), we have that they are identical.
  - Thus, $\theta_1 = \theta_2$.
  - Since light rays have a constant phase offset (specifically, 90°) from light waves, it follows that light rays also reflect off of surfaces with their original angle of incidence.
- **Angle of incidence**: The angle with which wavefronts hit a surface, or the angle a light ray makes with a normal to a surface.



(a) Waves hit a surface.

(b) Invoking Huygens principle.

(c) Proving identical triangles.

Figure 33.2: Properties of reflecting waves.

  - The quantity θ_1 in Figure 33.2.

- **Angle of reflection**: The angle with which the reflected wavefront intersects with a surface.
  - The quantity θ_2 in Figure 33.2b.

- **Law of reflection**: The principle that $\theta_1 = \theta_2$.

33.2 Reflection

- Corner reflector:

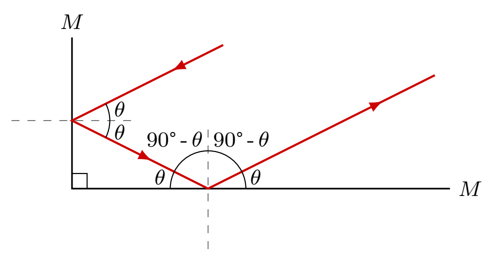


Figure 33.3: Corner reflector.

  - The law of reflection implies that the ray entering at an angle θ will exit at the same angle, just displaced a bit.

- If you don't want radar to detect planes:
  - Eliminate right angles (they're corner reflectors).
  - Make the surface something that absorbs radiation (hi tech ceramic).

- Mirrors create images; tie to projective geometry.
  - Seeing an object in the mirror is equivalent to seeing it as far behind the mirror as the thing is in front of the mirror.
  - This is **ray tracing**!

- **Virtual image**: An image that is created by light rays that don't really exist, i.e., projected light rays.
- Mirrors don't reverse left and right; they reverse front and back.

33.3 Refraction

- Light waves and glass.
  - Light waves enter glass, shake the atoms therein, and then those atoms emit their own light.
  - However, the light emitted by the atoms goes in all directions.
  - When you sum up all the secondary sources in a horrible integral, you end up with an effective wave propagating to the right, but at a speed less than the speed of light.
- **Index of refraction**: The quotient of the speed of light in a vacuum and the speed of light in a particular medium. *Denoted by* $\boldsymbol{n}$.

$$n = \frac{c}{v}$$

  - Since light can never travel faster than the speed of light, $n \geq 1$.
  - Some common n values:
    - $n_{\text{water}} \approx 1.33$.
    - $n_{\text{glass}} \approx 1.5$.
    - $n_{\text{diamond}} \approx 2.5$.
    - $n_{\text{air}} \approx 1.003 \approx 1$.
- In materials, v changes and f remains constant, so λ changes.
  - $c = f\lambda$ and $v = f\lambda'$ imply that

$$\lambda' = \lambda \cdot \frac{v}{c} = \frac{\lambda}{n}$$

- In a surface, the wavefront gets bent.

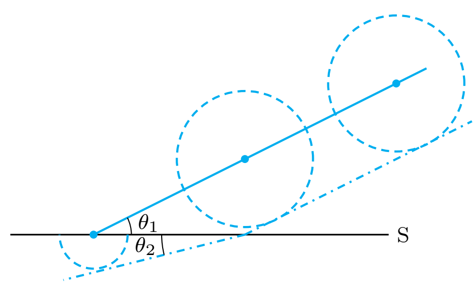


Figure 33.4: Properties of refracting waves.

  - Similarly, the ray gets bent.
  - When looking from air into a different surface, $\theta_2 < \theta_1$.
  - Explains why when you reach for something in water, it appears closer and in a different spot — you're reaching for the virtual image!
- **Angle of refraction**: The angle with which the refracted wavefront intersects with a surface.
  - The quantity θ_2 in Figure 33.4.
- **Snell's law**: The formula $n_1 \sin\theta_1 = n_2 \sin\theta_2$. *Also known as* **law of refraction**.

- **Critical angle**: The angle of incidence such that light will be refracted at 90°.
  - If $\theta_{\text{inc}} > \theta_{\text{crit}}$, you only get reflection!
  - From Snell's Law,
$$\theta_{\text{crit}} = \sin^{-1}(n_2/n_1)$$
- **Total Internal Reflection**: Conditions such that all light is reflected and none is refracted. *Also known as* **TIR**.
  - Only a possibility when the light wave is moving to a medium with a lower index of refraction.
- If you have a glass rod and you put light in at one end at an angle greater than the critical angle, it will be trapped and can only come out at the other end.
  - Total internal reflection allows us to redirect light however we want.
  - Light "flowing" through this construction is analogous to water flowing through a pipe.
  - As long as the angle we bend the light pipe at isn't too sharp, it will stay trapped in the light pipe.
  - In a well-designed light pipe, you will loose very little intensity.
  - This is the principal behind fiber optics.
  - You lose current in a wire due to resistance, but you don't lose much intensity in a light pipe.

33.4 Office Hours (Pandey)

8/11:

- Can you explain the upwards and downwards displacement currents in Figure 32.3?
  - Not really.

33.5 Dispersion

8/12:

- The way that atoms shake in a material with light passing through it depends on the frequency of the light.
- Thus, $n = n(f)$.
  - For example, $n_{\text{blue}} \neq n_{\text{red}}$.
- In general, n increases as f increases (or λ decreases).
  - Thus, $n_{\text{blue}} > n_{\text{red}}$, for example.
- Thus, if you have light made up of all kinds of different colors, every color will travel through the material at a different speed.

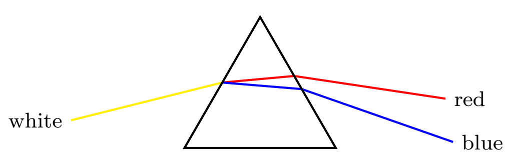


Figure 33.5: A prism dispersing light.

  - This is the principle behind a **prism**.
- **Dispersion**: The dependence of the index of refraction on the frequency of light.

- When light passes through water droplets in the sky, it gets refracted by different amounts, depending on the color of the light. This is what creates a rainbow!
- **Fermat's Principle**: Light follows the path that takes the least time.
- Light traveling between A and B via a mirror.
  - In this case, least time implies shortest distance.
  - And indeed, the path with the shortest distance via the mirror is the one where $\theta_1 = \theta_2$, as can be proven with optimization and calculus.
- Light traveling between A and B via a change of medium.
  - In this case, shortest distance *does not* imply least time.
  - If $n_1 < n_2$, you can travel faster in n_1 then you can in n_2, so you want to minimize the time you spend in n_2 without going too far out of your way.
  - It follows that the path with the least time will be the one given by Snell's law.
- Why does light take the path of least time?
  - Fermat says, "because it does."
  - Quantum physics has a more bizarre reason for this.
- How does light know which path to take?
  - It doesn't — it tries all paths and only one succeeds.

33.6 Polarization

8/13:
- Midterm on Tuesday.
  - 80 minutes, more questions.
  - HW 1-3.
  - Chapter 15-16, 33-34.
- **Polarized** (wave) **in the *xy*-plane**: A wave where both the medium and displacement are entirely contained in the xy-plane. *Also known as* **vertically polarized** (wave).
- If you shake a string up and down, you will create a polarized wave in the xy-plane.
- If you shake electrons up and down, you will create a vertically polarized EM wave, with $\vec{E}_{\text{trans}}$ entirely contained in the vertical plane.
  - If you face said wave parallel to the propagation, you only see movement up and down.
- A metal comb running vertically blocks vertically polarized microwaves since the microwaves lose energy creating currents in the metal.
  - Works with microwaves since they're long, but wouldn't with visible light.
  - However, a polaroid does work with the visible spectrum.
- A polaroid filter is composed of long, stretched out organic molecules analogous to the teeth on the metal comb.
  - If the **transmission axis** is vertical, all vertical waves get through.
  - If it is horizontal, they all get blocked.

- If it is rotated at some angle ϕ from the vertical, decompose the radial vector into parallel and perpendicular components.
  - The perpendicular component is totally blocked, and the parallel component is totally transmitted.
  - It follows that $\vec{E} = \vec{E}_{\|} = \vec{E}_{\text{polarized}} \cos\phi$, so the wave amplitude of vertically polarized light decreases by $\cos\phi$.

- Since $I \propto A^2$, we get the following.
- **Law of Malus**: The following formula, where I_2 is the intensity of light that gets past the polaroid filter and I_1 is the initial intensity.

$$I_2 = I_1 \cos^2\phi$$

- For unpolarized light, $I_{\text{trans}} = I_{\text{unpol}} \cos^2\phi$, but for light at every angle ϕ.
  - Thus, if we average $\cos^2\phi$ over all ϕ, we get that

$$I_{\text{trans}} = \frac{1}{2} I_{\text{unpol}}$$

  - Note that the above result is for a **perfect filter**. In reality, stacking filters causes some additional loss of intensity.
- If you put a two filters on top of each other at perpendicular angles, it will entirely eliminate the intensity.
  - At some angle ϕ in between, you'll have a variable loss of intensity.
- If you put a third filter at an angle between two perpendicular filters, you'll regain some intensity.

Chapter 34

Geometric Optics

34.1 Mirrors

8/12:

- Take what we know about reflection and refraction and apply it to mirrors and lenses.
- Bathroom mirror: A flat piece of glass with a shiny background behind it.
- Spherical mirror: Some part of a reflective sphere.
- Relating the **radius of curvature** and the distance to the **focal point**, or the **focal length**, of a concave spherical mirror.

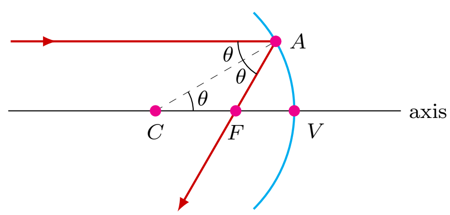


Figure 34.1: Spherical mirror analysis.

  - For a distant source s, we can approximate the rays as parallel.
  - Consider one specific ray.
  - Drawing a normal to the surface of the spherical mirror, this normal will be a radius passing through the **center of curvature**.
  - It follows by the law of reflection and the alternate interior angle theorem that all θ are equal.
  - This makes $\triangle CFA$ isosceles.
  - Now assume **paraxial rays**.
  - Then $\triangle CFA$ converges to a line segment (a radius) with midpoint F.
  - Therefore,
$$f = \frac{R}{2}$$
    for a concave spherical mirror.

- **Radius of curvature**: The radius of the sphere into which the spherical mirror would fit. *Denoted by* $\boldsymbol{R}$.
- **Focal point**: The point where parallel rays converge, after reflecting off of a curved mirror. *Denoted by* $\boldsymbol{F}$.

- **Focal length**: The distance from the focal point to the mirror. *Denoted by* $\boldsymbol{f}$.
- **Center of curvature**: The center of the sphere into which the spherical mirror would fit. *Denoted by* $\boldsymbol{C}$.
- **Paraxial ray**: A ray that is close to the mirror axis.
- Alternatively, if you look at a convex mirror, it appears (via ray tracing) that all rays of light originated from the focal point.
  - The image in this type of mirror will be a virtual image.
  - In this case, we say that $f = -R/2$.
- A spherical mirror does not focus all rays to a *single* point — the rays only go to *approximately* the same point.
  - The farther the rays get from being paraxial, the more they diverge from the focal point.
- To have a true focal point, we need a parabolic mirror.
- Unfortunately, parabolic mirrors are hard to make.
- **Ray tracing**: Take a few **principal rays** and see where the image forms.

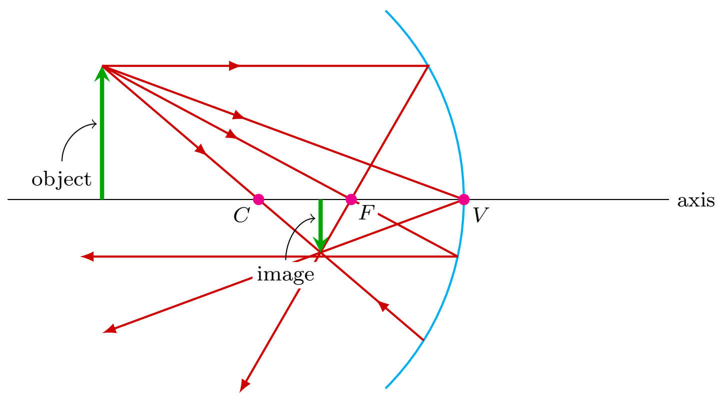


Figure 34.2: Ray tracing a concave mirror.

- **Principal ray**: A ray with well-understood geometry.
- **Focal ray**: A ray that goes through the focal point.
  - Will be reflected out as a parallel ray.
- **Central ray**: A ray that hits the center of the mirror.
  - Will be reflected out with the same incident angle relative to the axis.
- **Radial ray**: A ray that passes through the center of curvature.
  - It follows a *radius* of the sphere of curvature.
  - Will be reflected such that it heads right back to where it started.
- Calculating the location of the image in a spherical mirror.
  - The image in a spherical mirror will be an inverted, **real** image.

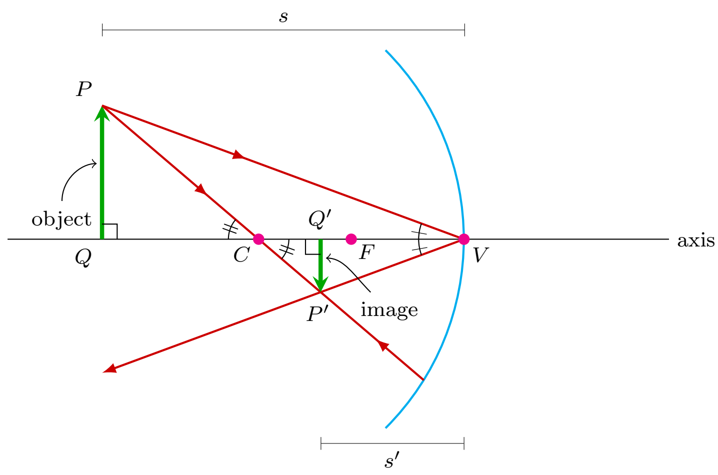


Figure 34.3: Deriving the mirror equation.

- Two principal rays define where the image is.
- Considering the image of the top point in our object, choose to analyze a central ray and a radial ray.
- Then $\triangle PVQ$ and $\triangle P'VQ'$ are similar, and $\triangle PCQ$ and $\triangle P'CQ'$ are similar.
- It follows that

$$\frac{\overline{PQ}}{\overline{P'Q'}} = \frac{\overline{QV}}{\overline{Q'V}} = \frac{s}{s'} \qquad\qquad \frac{\overline{PQ}}{\overline{P'Q'}} = \frac{\overline{QC}}{\overline{Q'C}} = \frac{s-R}{R-s'}$$

- Thus, we have the following, which can be solved for the mirror equation.

$$\frac{s}{s'} = \frac{s-R}{R-s'}$$

- **Real image**: An image that could be substituted for an image on a screen in real space.
- **Object distance**: The distance from an object to a spherical mirror. *Denoted by* $\boldsymbol{s}$.
- **Image distance**: The distance from an object's image to the mirror. *Denoted by* $\boldsymbol{s'}$.
- **Mirror equation**: The formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- **Lateral magnification**: The ratio of the distance from the object to the mirror and the distance from the image to the mirror. *Denoted by* $\boldsymbol{m}$.
  - Mathematically,

$$m = \frac{-s'}{s}$$

  - If $m < 0$, the image is inverted.
  - If $m > 0$, the image is upright.
  - Sign convention: s, s' are positive in front of the mirror and negative behind the mirror.
- Check the above equations on the example of a plane mirror.
  - For a plane mirror, $R = \infty$, so $\frac{1}{f} = 0$.

- It follows by the mirror equation that $s' = -s$.
- Therefore, $m = 1$, as desired.

8/13: • Ray tracing a convex mirror.

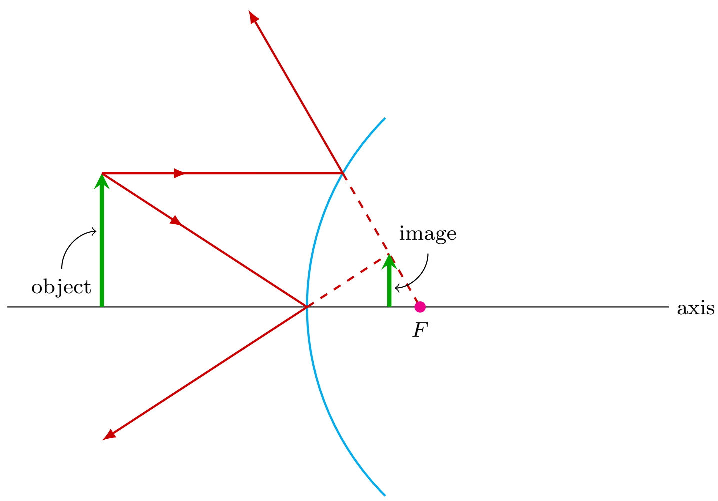


Figure 34.4: Ray tracing a convex mirror.

- Upright, smaller, virtual image.

34.2 Lenses

8/12: • **Double convex** (lens): A lens with two convex exterior surfaces. *Also known as* **convex** (lens), **converging** (lens), **positive** (lens).

- **Double concave** (lens): A lens with two concave exterior surfaces. *Also known as* **concave** (lens), **diverging** (lens), **negative** (lens).
- **Plano convex** (lens): A lens with one concave exterior surface and one flat exterior surface.

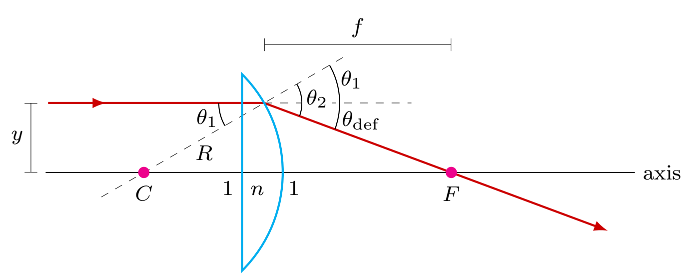


Figure 34.5: Plano convex lens analysis.

- Clearly, $\theta_{\text{def}} = \theta_2 - \theta_1$ and $\sin\theta_1 = y/R$.
- If this is a **thin lens**, then we can invoke the small angle approximation and say that $\theta_1 \approx y/R$.
- By Snell's Law, $\sin\theta_2 = n\sin\theta_1$.
- Invoking the SAA again, we have that $\theta_2 = n\theta_1$.

- It follows that

$$\begin{aligned}\theta_{\text{def}} &= \theta_2 - \theta_1\\ &= n\theta_1 - \theta_1\\ &= (n-1)\theta_1\\ &= (n-1)\cdot\frac{y}{r}\end{aligned}$$

- Therefore, since $\sin\theta_{\text{def}} = \frac{y}{f}$ (i.e., with the SAA $\theta_{\text{def}} = y/f$), we have that

$$\begin{aligned}f &= \frac{y}{\theta_{\text{def}}}\\ &= \frac{y}{(n-1)\cdot y/R}\\ &= \frac{R}{n-1}\end{aligned}$$

- More commonly, we express this with

$$\frac{1}{f} = (n-1)\cdot\frac{1}{R}$$

- **Thin lens**: A lens where θ's are small.
- If you substitute a convex lens for the plano convex lens, everything gets doubled.
  - Importantly, we now have
$$\frac{1}{f} = (n-1)\cdot\frac{2}{R}$$
- We can also generalize a convex lens to a lens with two convex sides of varying radii of curvature on its two sides.
- **Lens maker's equation**: The following formula.
$$\frac{1}{f} = (n-1)\cdot\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
  - Notice that if, for instance, the first side is planar, then $R_1 = \infty$ and the $1/R_1$ term disappears.
  - Sign convention: R_1, R_2 are positive for convex lenses and negative for concave lenses.
- Remember that knowing the focal length of a lens allows us to ray trace.

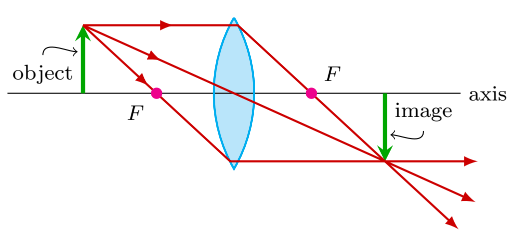


Figure 34.6: Convex lens rays.

  - Central rays in thin lenses have negligible lateral displacement.
  - An image through a convex lens will be an inverted real image.

8/13: - Ray tracing a concave lens.

  - Upright, smaller, virtual image.
- Note that the lens maker's equation used by Young and Freedman (2019) is $1/f = (n-1)(1/R_1 - 1/R_2)$, along with a complicated sign convention.
  - The textbook uses this formula because it follows from the study of thick lenses, with which many older textbooks start.
- A concave mirror and a plano convex lens with a mirrored background image the same way.
  - A convex lens mirrors the same way as both, but with the image on the other side of the lens as opposed to the same side of the apparatus.
- Therefore, the lens equation is also analogously

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

  - However, this equation comes with the sign convention that s is positive in front of the lens and negative behind the lens, and s' is positive behind the lens and negative in front of the lens.
- Similarly,

$$m = \frac{-s'}{s}$$

  for lenses, with the new sign convention.
- Note that the focal length is positive for convex lenses and negative for concave lenses.
- Just like mirrors can image images from other mirrors, lenses can image images from other lenses.
  - You do it the same way, too — one at a time.
- Example: Suppose that you have two identical thin lenses, 15 cm apart, with focal length 10 cm each. Place an object 15 cm to the left of the left lens. How does it image?

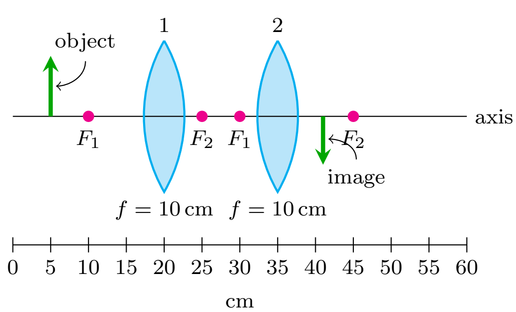


Figure 34.7: Imaging two lenses.

  - Image location:
    - For lens 1, we have $f = 10$ cm and $s = 15$ cm, so $s' = 30$ cm by the lens equation.
    - For lens 2, we still have $f = 10$ cm but now we have have $s = -15$ cm, so $s' = 6$ cm by the lens equation.
  - Magnification:

$$\begin{aligned} m_{\text{total}} &= m_1 m_2 \\ &= \left(-\frac{30\,\text{cm}}{15\,\text{cm}}\right)\left(-\frac{6\,\text{cm}}{-15\,\text{cm}}\right) \\ &= -\frac{4}{5} \end{aligned}$$

  - Thus, the total image is inverted, 4/5 times the size of the original, and 6 cm to the right of the rightmost lens.
- Mirrors and lenses have limitations.
  - Thus, it does make sense to use multiple mirrors/lenses in some circumstances.
  - Compensating for the paraxial approximation: When you need a sharper focus in good cameras.
  - Compensating for the dependence of n on frequency: Have any single lens do less work.

34.3 Chapter 34: Geometric Optics

From Young and Freedman (2019).

8/16:
- **Point object**: A theoretical object consisting only of a zero-dimensional point.
- **Extended object**: An object that is not a point object.
- **Diffuse** (surface): A surface which scatters incoming rays in uncorrelated directions.
- **Virtual image**: An image for which the outgoing rays do not pass through the image point.
- **Real image**: An image for which the outgoing rays do pass through the image point.
- "When the center of curvature C is on the same side as the outgoing light, the radius of curvature is positive; otherwise, it is negative" (Young & Freedman, 2019, p. 1112).
- **Lateral magnification**: The ratio of image height to object height.
- **Erect** (image): An image that is oriented in the same direction as the object.
  - Such as when an image arrow points in the same direction as the object arrow.
- **Inverted** (image): An image that is oriented in the opposite direction relative to the object.
  - Such as when an image arrow points in the opposite direction as the object arrow.
- **Spherical aberration**: The smearing out of the image "point" from a zero-dimensional point due to the approximations made in deriving the mirror equation.
- If you remove part of a spherical mirror, a full image is still formed, just a dimmer one. This is because light rays from any object point reflect off of *all* parts of the mirror to converge to the image point. However, without all of the mirror, fewer light rays will be reflected, but no paths will be entirely eliminated.
- Measure s and s' to where the mirror intersects the optic axis.
- **Virtual focal point**: The focal point of a convex mirror, lying behind it.
- "When the object distance s is positive, a convex mirror *always* forms an erect, virtual, reduced, reversed image" (Young & Freedman, 2019, p. 1120).

Chapter 35

Interference

35.1 Measuring Interference in Radio and Light Waves

8/16:
- Midterm tomorrow.
  - 80 minutes long.
  - Probably 5 problems.
  - Released at 10:00 AM CT.
  - 20 extra minutes for downloading/uploading.
  - Due at 11:40 AM CT.
  - Class Resumes at 11:50 CT.
  - Covers HW 1-3, Chapter 15-16, 33-34.
  - If there's anything that the textbook covers that he doesn't, you're not responsible for it.
  - Review the homeworks, his class examples, and the worked examples in the textbook.
- It's hard to measure interference analogous to that associated with Figure 16.5 for light waves because their wavelength is so small, so let's look first at radio waves.
- If we have an LC circuit with megahertz frequencies, the wavelength is a few centimeters.
  - Hertz (1880s): If you set up radio waves between two chunks of grounded metal, you can set up a standing wave between them.
- Are radio waves and light both EM waves?

Radio Waves	**Light Waves**
Speed c	Speed c
Reflection	Reflection
Refraction	Refraction
Polarized	Polarized
Interference	Interference???

- We measure interference of visible light with the double slit experiment.
  - Huygen's principle tells us that the two slits with parallel light waves coming in from behind behave as point sources of light.
  - Let r_1 be the distance from slit 1 to a point P on the screen and let r_2 be the distance from slit 2 to P.

- Define $\Delta r = r_2 - r_1$.
- In 3D, $y(r,t) = A\cos(kr - \omega t)$.
- At point P,

$$\begin{aligned} y &= y_1(r_1,t) + y_2(r_2,t) \\ &= y_1(r,t) + y_2(r+\Delta r,t) \\ &= A\cos(kr-\omega t) + A\cos(k[r+\Delta r]-\omega t) \\ &= A\cos(kr-\omega t) + A\cos(kr-\omega t+\phi) \end{aligned}$$

- Clearly, $\phi = k\Delta r$. If $\phi = 2\pi n$ for $n \in \mathbb{Z}$, we get constructive interference. If $\phi = \pi + 2\pi n$ for $n \in \mathbb{Z}$, we get destructive interference.
- Thus, if we want the waves to perfectly add, we require that $\Delta r = n\lambda$ for some $n \in \mathbb{Z}$, and if we want the waves to cancel, we require that $\Delta r = \lambda/2 + n\lambda$ for some $n \in \mathbb{Z}$.
- We can succinctly sum up this idea with

$$\frac{\phi}{2\pi} = \frac{\Delta r}{\lambda}$$

- **Central maximum**: The bright spot in the middle of the screen of the double slit experiment.
- **Lateral maxima**: All noncentral bright spots on the screen.
- **First lateral maximum**: The bright spot directly to the right of the central maximum facing the screen.
- If the screen is far away compared to slit separation d, then the rays of light are virtually parallel.

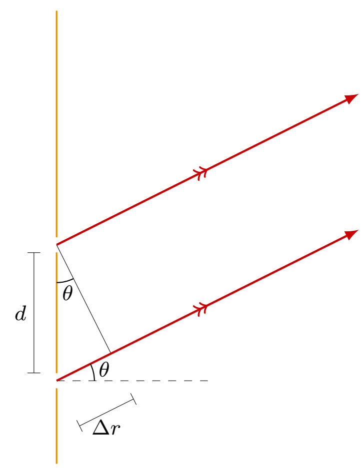


Figure 35.1: Double slit experiment Δr derivation.

- If we let θ be the angle between the rays and a normal to the slitted surface, then

$$\Delta r = d\sin\theta$$

  is a very good approximation.
- It follows that $d\sin\theta = n\lambda$ yields maxima and $d\sin\theta = (n+1/2)\lambda$ yield minima.
- Thus we can measure the wavelength of light with the following:

$$d\sin\theta_{\text{1st lateral maximum}} = \lambda$$

- If we want to measure the intensity on the surface of the screen a distance r_0 from the slits (more specifically, the center of the slits), then we can do the following.

$$\begin{aligned} E(r,t) &= A\left[\cos\left(k\left[r_0+\frac{\Delta r}{2}\right]-\omega t\right)+\cos\left(k\left[r_0-\frac{\Delta r}{2}\right]-\omega t\right)\right] \\ &= A\left[\cos\left(kr_0-\omega t+k\cdot\frac{\Delta r}{2}\right)+\cos\left(kr_0-\omega t-k\cdot\frac{\Delta r}{2}\right)\right] \\ &= 2A\cos(kr_0-\omega t)\cos\left(\frac{k\Delta r}{2}\right) \\ &= 2A\cos(kr_0-\omega t)\cos\left(\frac{k}{2}\cdot d\sin\theta\right) \end{aligned}$$

  - x will be constant, and ωt will flicker so fast that we don't notice it changing. The other cosine term matters, though:
  - Since $I \propto E^2$, $I(\theta) \propto \cos^2(kd\sin\theta/2) = \cos^2(\pi d\sin\theta/\lambda)$.
  - Invoking the SAA for small θ, it follows that

$$I(\theta) \propto I_{\max}\cos^2(\pi d\theta/\lambda)$$

- In the double slit experiment, the two slits behave as two **coherent** sources of light.

35.2 Thin Films

- Shining light on a bubble.
  - A bubble is a thin film of soapy water with air on both sides.
  - If it has thickness t, when a wave of light impinges on it, some will be reflected off of the outside of the film, and some will be transmitted before being reflected off of the inside of the film (and some will continue to be transmitted).
  - Thinking back to a compound string, $\mu_1 > \mu_2$ implies no flip of the reflected wave and $\mu_1 < \mu_2$ implies a flip.
  - Analogously, if air has IOR n_1 and soapy water has IOR n_2, $n_1 < n_2$ implies a flip and $n_1 > n_2$ implies no flip.
  - It follows that when a ray of light enters the thin film, the reflection will be flipped, and when a ray of light exits the thin film, the reflection will not be flipped.
  - For perpendicular rays, approximate $\Delta r = 2t$ between the two reflected rays.
  - Thus, accounting for the flip (which adds an extra half-wavelength of Δr), $2t = m\lambda$ for $m \in \mathbb{Z}$ will imply *destructive* interference. For constructive interference, though, we have to take into account the distortion of the wavelength by the IOR of the soapy water ($\lambda' = \lambda/n_2$): $2tn_2 = (m+1/2)\lambda$.
  - But this will vary for different wavelengths, so different colors will be emphasized at different parts of the bubble.
  - If the bubble is very thin ($t << \lambda$ where λ is the wavelength of light), we'll only retain the flip, meaning that we get destructive interference and a black bubble.
- Oil also forms a thin film on water.

35.3 Multiple Slits

- Three slits:
  - Assume consistent slit separation d.

  - If $\Delta r = r_2 - r_1 = r_3 - r_2 = m\lambda$, then all three add constructively.
    - No matter how many adjacent slits you have, if $d\sin\theta = m\lambda$, then you get a maximum.
  - $d\sin\theta = m\lambda$ does *not* imply a minimum for three slits (if two cancel, you still have the third).
  - So what is it? How do we get all three cosines in the following equation to sum to zero? Let $\alpha = kr_0 - \omega t$. Then

$$E = A\left[\cos(\alpha) + \cos(\alpha+\phi) + \cos(\alpha+2\phi)\right]$$

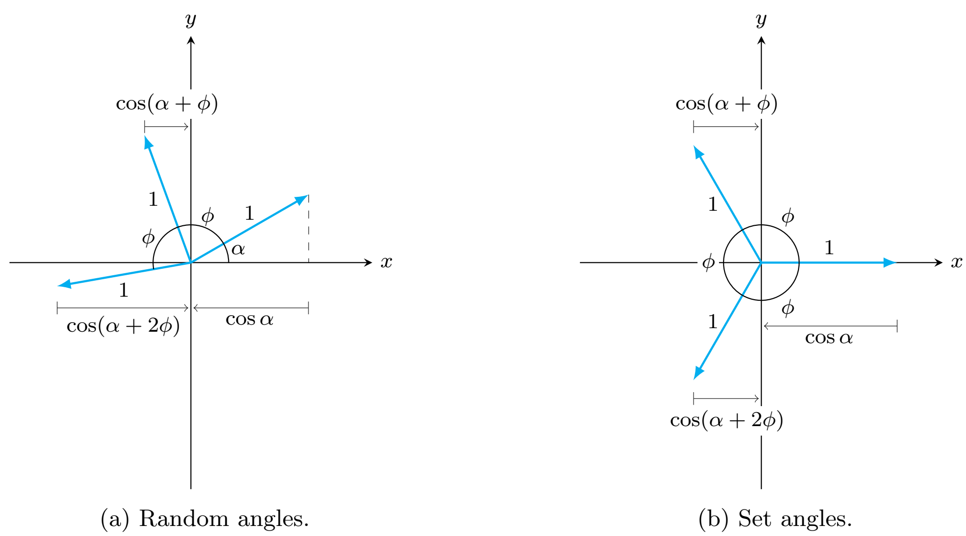


Figure 35.2: Phaser diagram for three cosines.

    - We can use a **phaser diagram**.
    - From Figure 35.2, we can see that if $\phi = 120°, 240°, \dots$, then we get cancellation. In other words, we must have $\Delta r = n\lambda/3$ for $\lambda \in \mathbb{Z}$.

- N slits.
  - Maxima at $d\sin\theta = m\lambda$ for $m \in \mathbb{Z}$.
  - Minima at $d\sin\theta = m\lambda/N$ for $m \in \mathbb{Z}$.
  - Thus, the bands given by $I(\theta)$ will become spikes at with increasingly greater separation.
  - Sharp bright spots are a good thing because they help us achieve greater accuracy in measuring the wavelength of light.
- Example: Na gas gives off yellow light ($\lambda \approx 6\,000$ Å).
  - More specifically, it gives off light at $5\,890$ Å and $5\,896$ Å, so we should have two interference patterns, but very difficult to tell apart.
  - For N slits, $\theta_{\text{1st min}} = \sin^{-1}(\lambda/dN) \approx \lambda/dN$.
  - Thus, the width of the maxima is approximately λ/dN, so for more slits, we get sharper maxima.
  - Considering the **separation angle**, if $\Delta\theta_{\text{sep}} > \theta_{\text{width of max}}$, we can tell them apart.
  - So to tell whether Na is giving off one wavelength or two, we'll have $d\sin\theta_1 = m\lambda_1$ and $d\sin\theta_2 = m\lambda_2$. Subtracting, we get

$$\begin{aligned}
d(\sin\theta_1 - \sin\theta_2) &= m(\lambda_1 - \lambda_2)\\
d(\theta_1 - \theta_2) &= m(\lambda_1 - \lambda_2)\\
d\Delta\theta_{\text{sep}} &= m\Delta\lambda\\
\Delta\theta_{\text{sep}} &\approx \frac{m\Delta\lambda}{d}
\end{aligned}$$

- Thus, we can barely resolve the two wavelengths if $m\Delta\lambda/d = \lambda/Nd$, or if

$$\frac{\lambda}{\Delta\lambda} = mN$$

- Thus, for the sodium doublet, we need $mN \geq 1000$.
- m is the **order of the maximum**.

35.4 Office Hours (Gazes)

- Whatever side of the lens the object is on is the $+s$ and $-s'$ side.
- Diverging lenses:
  - Suppose you have a wide beam of light and want to make it more narrow. You could shine it through a smaller hole, but then you would lose intensity. Better: Shine it through a converging lens to compress it and then a diverging lens to make rays parallel again.
  - Diverging lenses put images farther away; opposite of converging lenses.
- Two sides of a lens have a *combined* focal length. Thus, rays *do* get refracted by both sides; we just do both refractions at the same size.
  - This is part of the thin lens approximation.
- Thin lenses also imply that the two focal points are equidistant from the lens.
  - Again, a single lens has a *combined* focal length, not two focal lengths.
- **Birefringent materials**: Have a preferred optical axis.
  - There are IORs that depend not just on wavelength, but on polarization.
  - Calcite, for example, is such a material.
  - Plastic is not birefringent, but if you try to break it, the resultant lucite is birefringent.
  - Only the stressed parts of plastic are birefringent. Thus, if you put plastic between two perpendicular polaroid filters, you can image it, and the most stressed parts (most birefringent) will show up!
- Figure 32.3:
  - Imagine moving the charge briefly up and back down to the same original place, creating a transverse wave pulse.
  - Going from no electric field to an electric field is a big change in electric flux; thus, you get a displacement current.
  - Opposite at the back end.
  - With two sheets of current and Ampere's law, you can show that you have a strong magnetic field in between the sheets but no magnetic field on either side.
  - Analogous to no electric field outside a capacitor's plate, but a strong one in between.

Chapter 36

Diffraction

36.1 Single Slit Diffraction

8/17:

- Shining light through only one slit still yields an interference pattern.
- We explain this with **diffraction**.
- Finding the location of minima on the screen:

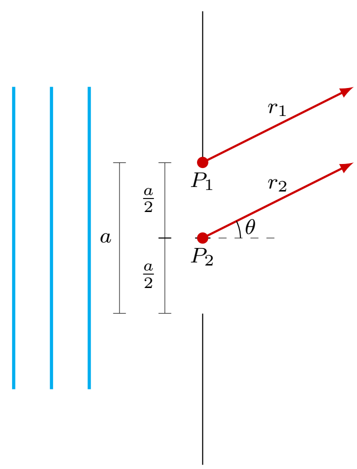


Figure 36.1: Finding diffraction minima.

  - Let the one slit have width a.
  - Only the part of the wavefront that aligns with the slit will pass through. However, according to Huygen's principle, when the light wave reaches the slit, it will act like infinitely many point sources of light all along the length of the slit.
  - Consider two specific rays r_1 and r_2 emanating from the slit in same direction, one at the top and one in the middle. We know that if they are oriented at an angle that makes $\Delta r = \lambda/2$, then they cancel out.
  - Generalizing, if any two rays satisfy $\frac{a}{2}\sin\theta = \lambda/2$ (i.e., satisfy $a\sin\theta = \lambda$), then they will cancel.
  - Indeed, every θ satisfying $a\sin\theta = \lambda$ will cancel: Consider all the rays originating from every point in the slit that point in the θ-direction, and notice that for any point in the slit, there will be a point $a/2$ units away from it; the rays from these two points will cancel. Thus, every ray is associated with another ray that cancels it out, guaranteeing that θ is an interference minimum.
  - Note that if θ yields an interference minimum, then θ satisfying $a\sin\theta = m\lambda$ where $m \in \mathbb{N}$ will yield interference minima.

36.2 Intensity of Single Slit Diffraction

- Like before, $\theta = 0°$ gives a **central diffraction maximum**.
- Finding the intensity maxima in general:

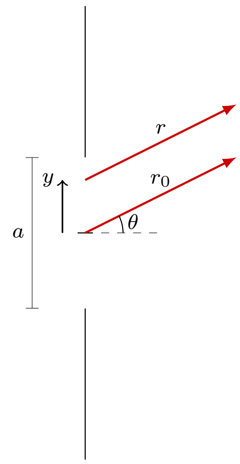


Figure 36.2: Finding diffraction maxima.

  - To find the intensity maxima, we derive an equation for the intensity in general as a function of θ.
  - To do so, we sum up every infinitesimal contribution of all the points along the slit with an integral, as follows.
  - Since $\Delta r = y \sin\theta$ (see Figure 36.2), the wave function for the electric field wave along the arbitrary ray r a distance y from the central ray is given by
$$\begin{aligned}\cos(kr - \omega t) &= \cos(k[r_0 + \Delta r] - \omega t)\\ &= \cos\left(kr_0 - \omega t + \frac{2\pi}{\lambda} \cdot y \sin\theta\right)\end{aligned}$$
  - It follows that the electric field E at some point P on the screen is given by
$$\begin{aligned}E &= A \int_{-a/2}^{a/2} \cos\left(kr_0 - \omega t + \frac{2\pi \sin\theta}{\lambda} \cdot y\right) \mathrm{d}y\\ &= \frac{C}{\sin\theta} \cos(kr_0 - \omega t) \sin\left(\frac{\pi a}{\lambda} \sin\theta\right)\end{aligned}$$
where C represents a bunch of constants.
  - Thus, since $I \propto E^2$,
$$I \propto \frac{\sin^2\left(\frac{\pi a}{\lambda} \sin\theta\right)}{\sin^2\theta}$$
  - Additionally, if we define $\alpha = \frac{\pi a}{\lambda} \sin\theta$, then
$$I = I_{\text{max}} \frac{\sin^2\alpha}{\alpha^2}$$
  - Notice that $\theta \to 0$ implies $\alpha \to 0$ implies $\sin(\alpha)/\alpha \to 1$ implies $I \to I_{\text{max}}$, as expected.
  - Furthermore, since $\sin\alpha$ is bounded but α is not, $\sin^2(\alpha)/\alpha^2$ yields a graph of maxima that drop off in intensity as $\alpha \to \pm\infty$.
- **Diffraction**: Bending of a light wave as it goes through a small slit.
  - As slit width a decreases, minima spread out.

36.3 Combining Interference and Diffraction

8/19:

- Every place you have a diffraction minimum, the wave that gets to the screen has 0 amplitude.
  - If you have a point P that's a diffraction minimum of both slits, interference doesn't matter — you're going to have no intensity at P.
- Slits S_1 and S_2 have the same diffraction pattern, just shifted by d.
  - But if the diffraction pattern is large relative to d, as it usually is, we can neglect the shift.
- Total intensity:

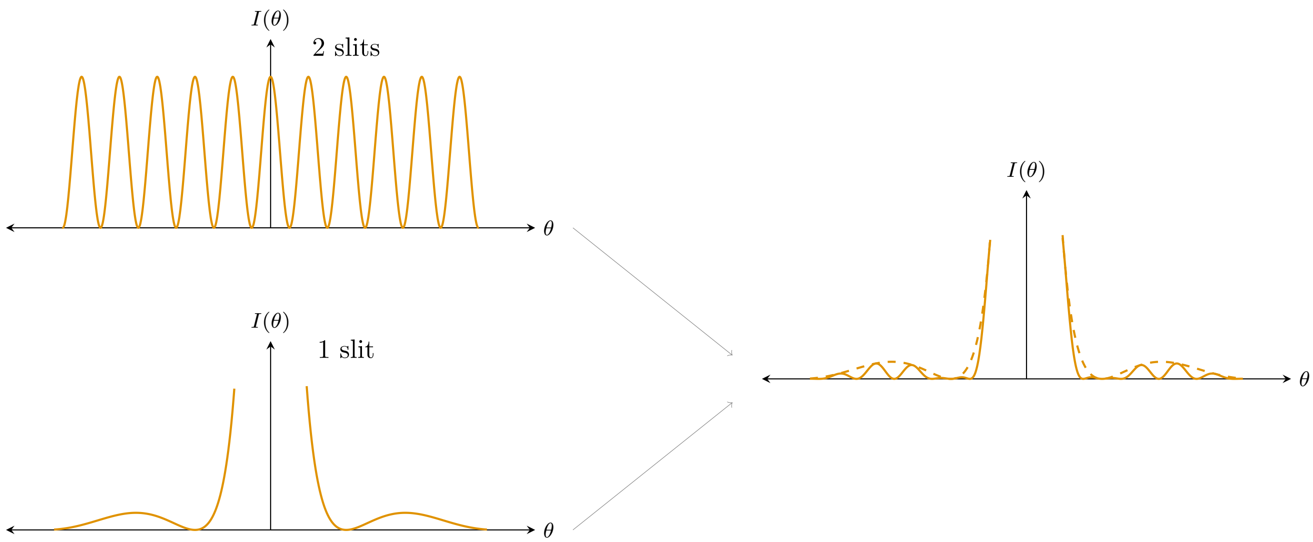


Figure 36.3: Intensity considering both interference and diffraction.

  - We have that $I(\theta) = I(\theta)_{2 \text{ slits' interference}} \times$ diffraction envelope.

36.4 Circular Hole

- Consider light passing through a circular hole of diameter a.
  - This yields concentric rings of intensity separated by nodes, i.e., a slit diffraction pattern that accounts for slits of every angle added on top of each other.
  - As a gets smaller, the central diffraction maximum gets bigger.
- **Slit**: A one-dimensional opening.
- **Aperture**: A two-dimensional opening.
- We can no longer use $a\sin\theta = m\lambda$; we have to consider what happens with the extra dimensions.
  - When we redo the calculation in two dimension, we get

$$a\sin\theta = 1.22m\lambda$$

    for $m \in \mathbb{Z}$.
  - If θ's are small, then $\theta_{\text{1st min}} \approx 1.22\lambda/a$.

  - Recall that $\theta_{\text{1st min}} = \theta_{\frac{1}{2}\text{ width of central max}}$, so we can use this formula to estimate the width of the central maximum.
- As light passes through your pupil (an aperture), it undergoes diffraction and gets bigger before impinging on your retina.

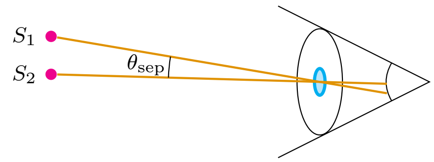


Figure 36.4: Distinguishing sources of light.

  - Thus, to be able to distinguish two sources of light, we require $\theta_{\text{sep}} \geq \theta_{\frac{1}{2}\text{ width of central max}}$.
  - For this reason, bigger telescopes are used not only to collect more light but also to minimize the effects of diffraction.
- **Rayleigh criterion**: The condition for distinguishing sources of light, given by

$$\theta_{\text{sep}} \geq 1.22\lambda/a$$

- Pinhole camera.
  - For a sharp focus, you want a small pinhole, improving the geometry.
  - But if you make it too small, diffraction will come into play.

References

Labalme, S. (2020). *AP chemistry notes* (Accessed 19 August 2021.). https://github.com/shadypuck/APChemNotes/blob/master/main.pdf

Young, H. D., & Freedman, R. A. (2019). *University physics with modern physics* (Fifteenth). Pearson Education.