Chapter 16

Sound and Hearing

16.1 Intro to Sound Waves

8/9:

- First quiz this Friday.
  - There to get you ready for the midterm.
  - Starts at 10:00 AM.
  - 30 minutes for the quiz plus 20 minutes to scan and upload $\Rightarrow$ due at 10:50 AM.
  - Send Dr. Gazes an email if you have technical issues.
  - Study:
    - HW 1-2.
    - Chapter 15-16, and parts of 33.
    - No questions on homework material to which we don't have the solutions.
- **Standing wave**: A wave with nodes and antinodes that do not move.
- Consider an air-filled pipe of length L with a piston at one end and being open at the other end.

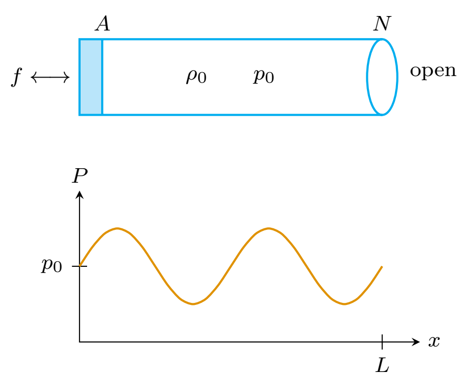

Figure 16.1: An air-filled pipe.

  - The air in it has density ρ_0 and pressure p_0.
  - Reviews compression and rarefaction.
  - Creating a plot of pressure vs. x-distance yields a transverse pressure wave.
  - We consider the pressure at the end to be essentially "clamped" at atmospheric pressure p_0.
    - Thus, the wave gets reflected at the end of the pipe.

- **Sound wave**: A wave propagating in a material.
- Speed of sound:
  - For a wave on a string, $v = \sqrt{F/M}$.
  - Tension is how hard a sliver of string is being pulled on by its neighbors. Mass density is inertial; it tells us how much a sliver of string resists being moved by its neighbor.
  - Thus, for a sound wave, we should have something kind of like $v = \sqrt{p_0/\rho_0}$.
  - In fact, adjusting for some other factors, we get (at STP)

$$v_{\text{sound}} = \sqrt{\frac{1.4p_0}{\rho_0}}$$

- Let $\delta p = p - p_0$.

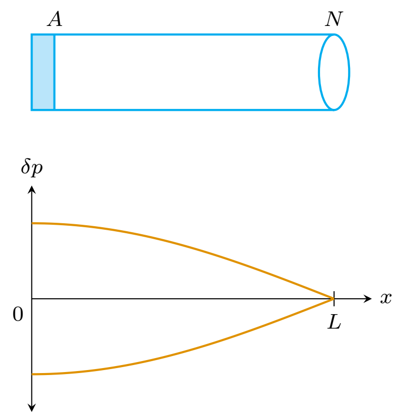

Figure 16.2: Standing waves in an air-filled pipe.

  - Then some ways to get a standing wave are $L = \frac{\lambda}{4}, \frac{\lambda}{4} + \frac{\lambda}{2}, \frac{\lambda}{4} + \lambda, \dots$.
  - Thus, standing waves are given by $L = \frac{\lambda}{4} + m \cdot \frac{\lambda}{2}$, where $m \in \mathbb{N} \cup \{0\}$.
- When you have a node for pressure, you have an antinode for displacement and vice versa.
- It follows from the fact that $L = \frac{n\lambda}{4}$ for $n \in 2\mathbb{N} + 1$ that $L = \frac{n}{4} \cdot \frac{v_{\text{sound}}}{f}$ for $n \in 2\mathbb{N} + 1$.
  - Thus, $f_n = \frac{n}{4} \cdot \frac{v_{\text{sound}}}{L}$.
  - f_1 is again the fundamental frequency, and $f_n = nf_1$, but only where $n \in 2\mathbb{N} + 1$.
- Hissing air has all kinds of frequencies. When you blow it over the opening of a bottle, only the frequencies with large amplitudes will produce standing waves.
  - When you partially fill the bottle, decreasing the length of the tube of air, higher frequencies are selected for.
- When you blow air past a tube that is open at both ends, you have pressure nodes (denoted by N_p) at both ends and you can get all sorts of standing waves in between.

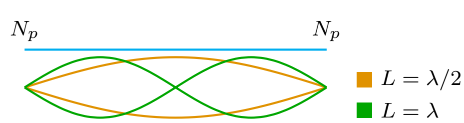

Figure 16.3: Standing waves in an uncapped pipe.

  - Here, we have $L = n \cdot \frac{\lambda}{2}$, where $n \in \mathbb{N}$.
  - Open-open pipes are just like a string clamped at both ends.

16.2 Sound Waves in More Dimensions

- 2001: A Space Odyssey starts with a 16 Hz sound.
- Sound in 1D vs. sound in 3D.

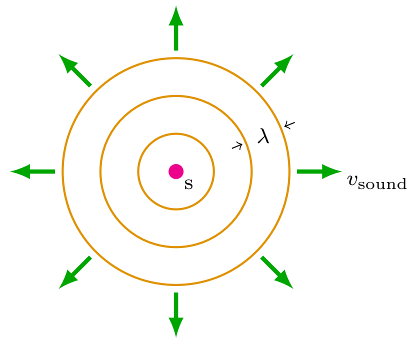

Figure 16.4: Sound waves in 3D.

- **Wavelength** (3D): The distance between the crests of adjacent waves.
- How much energy is captured by your ear depends on the **intensity**.
- **Intensity**: The average power per unit area. *Denoted by* $\boldsymbol{I}$. *Units* W/m^2.
  - In 3D, $I = \frac{\bar{P}}{4\pi r^2}$.
  - Thus, the power at your ear is given by $P_{\text{ear}} = IA_{\text{ear}}$.
- **Threshold intensity**: The lowest intensity that can still be heard. *Denoted by* $\boldsymbol{I_0}$.
  - For humans, $I_0 \approx 1 \times 10^{-12}\,\text{W/m}^2$.
- **Sound intensity level**: The following quantity. *Units* dB.

$$\beta = 10 \log \left(\frac{I}{I_0} \right)$$

  - $\beta(\text{whisper}) \approx 20\,\text{dB}$.
  - $\beta(\text{NYC Subway}) \approx 100\,\text{dB}$.
    - 1×10^8 times the intensity of a whisper!
  - $\beta(\text{ears hurt}) \approx 120\,\text{dB}$.

16.3 Sound Wave Phenomena

- Speakers at varying distances from one's ear:

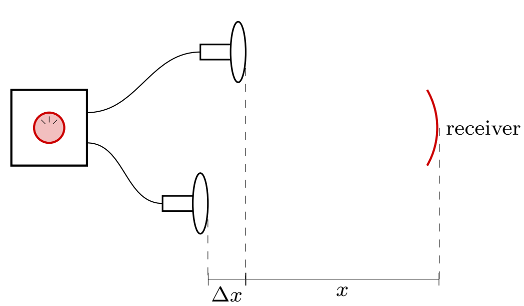

Figure 16.5: Speakers at varying distances from one's ear.

- $y = y_1 + y_2 = A\cos(kx - \omega t) + A\cos(k[x + \Delta x] - \omega t)$.
- If $\Delta x = 0$, then
$$y = 2A\cos(kx - \omega t)$$
- If $\Delta x = \frac{\lambda}{2}$, then
$$\begin{aligned} y &= A\left[\cos(kx - \omega t) + \cos\left(kx + k\cdot\frac{\lambda}{2} - \omega t\right)\right] \\ &= A\left[\cos(kx - \omega t) + \cos\left(kx + \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} - \omega t\right)\right] \\ &= A[\cos(kx - \omega t) + \cos(kx - \omega t + \pi)] \\ &= A[\cos(kx - \omega t) - \cos(kx - \omega t)] \\ &= 0 \end{aligned}$$
so you get total cancellation/destructive interference.
- Similarly, you can electronically delay the signal. If $\Delta t = \frac{T}{2}$, then the waves cancel. This is the principle behind noise-cancelling headphones.
- Δx is called the **path length difference**.

- Sound waves of slightly varying frequency:

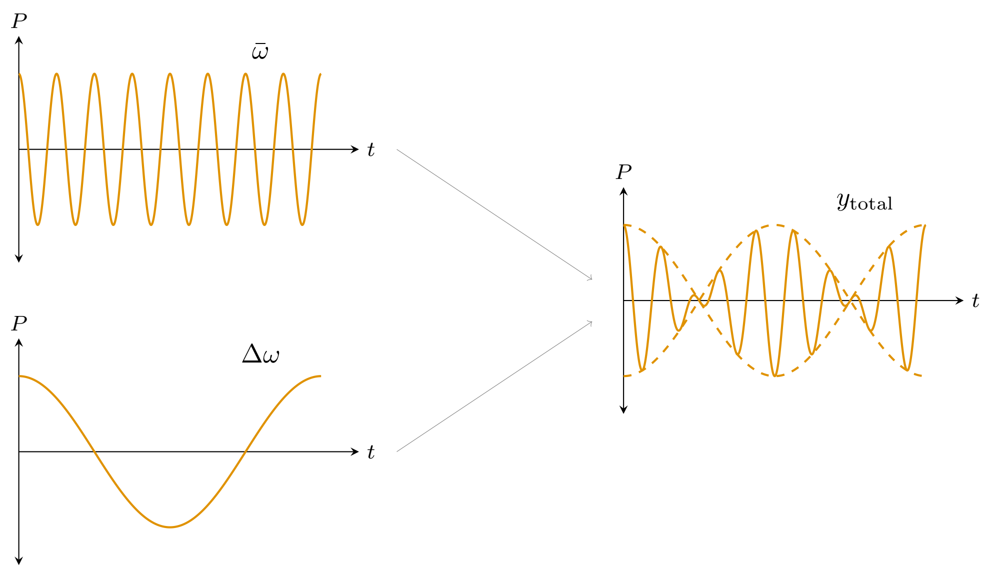

Figure 16.6: Sound waves of slightly varying frequencies.

- Consider frequencies f_1, f_2 where $\Delta f << f_1, f_2$.
- Since $k = 2\pi f/v_{\text{sound}}$ and $\omega = 2\pi f$ (i.e., both quantities depend on frequency), we have that
$$\begin{aligned} y &= y_1 + y_2 \\ &= A[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)] \end{aligned}$$
Suppose $x = 0$.
$$\begin{aligned} &= A[\cos(\omega_1 t) + \cos(\omega_2 t)] \\ &= 2A\cos\left(\frac{\omega_1 + \omega_2}{2}\cdot t\right)\cos\left(\frac{\omega_1 - \omega_2}{2}\cdot t\right) \\ &= 2A\cos(\bar{\omega}t)\cos\left(\frac{\Delta\omega}{2}\cdot t\right) \end{aligned}$$

  - Since $\bar{\omega} >> \Delta\omega$, y looks like the end result in Figure 16.6.
  - Thus, we will hear $\bar{\omega}$, but there will be silences interspersed.
    - These nodes are called **beats**, and $f_{\text{beat}} = f_1 - f_2$.
- Suppose we have a source s producing a sound of frequency f. An observer o runs toward the source at speed v_o.
  - Thus, the observer is being hit by wavefronts moving, relative to them, at speed $v + v_o$. Thus, since $v = \lambda f$, the frequency f' that the observer hears is given by

$$f' = \frac{v + v_o}{\lambda} = \frac{v + v_o}{v} \cdot f > f$$

- **Doppler Effect**: The change in frequency produced by the speed of the observer relative to the source. *Also known as* **Doppler Shift**.
  - Also happens when the source moves toward the observer. In this case, though, λ varies: With respect to the source, waves are being emitted at the same frequency, but they're only moving away from the source at speed $v - v_s$. Thus, $\lambda' = v - v_s/f$, so $f' = \frac{v}{v-v_s} \cdot f$.