Chapter 35

Interference

35.1 Measuring Interference in Radio and Light Waves

8/16: • Midterm tomorrow.
  - 80 minutes long.
  - Probably 5 problems.
  - Released at 10:00 AM CT.
  - 20 extra minutes for downloading/uploading.
  - Due at 11:40 AM CT.
  - Class Resumes at 11:50 CT.
  - Covers HW 1-3, Chapter 15-16, 33-34.
  - If there's anything that the textbook covers that he doesn't, you're not responsible for it.
  - Review the homeworks, his class examples, and the worked examples in the textbook.
- It's hard to measure interference analogous to that associated with Figure 16.5 for light waves because their wavelength is so small, so let's look first at radio waves.
- If we have an LC circuit with megahertz frequencies, the wavelength is a few centimeters.
  - Hertz (1880s): If you set up radio waves between two chunks of grounded metal, you can set up a standing wave between them.
- Are radio waves and light both EM waves?

Radio Waves	**Light Waves**
Speed c	Speed c
Reflection	Reflection
Refraction	Refraction
Polarized	Polarized
Interference	Interference???

- We measure interference of visible light with the double slit experiment.
  - Huygen's principle tells us that the two slits with parallel light waves coming in from behind behave as point sources of light.
  - Let r_1 be the distance from slit 1 to a point P on the screen and let r_2 be the distance from slit 2 to P.

- Define $\Delta r = r_2 - r_1$.
- In 3D, $y(r,t) = A\cos(kr - \omega t)$.
- At point P,

$$\begin{aligned} y &= y_1(r_1,t) + y_2(r_2,t) \\ &= y_1(r,t) + y_2(r+\Delta r,t) \\ &= A\cos(kr-\omega t) + A\cos(k[r+\Delta r]-\omega t) \\ &= A\cos(kr-\omega t) + A\cos(kr-\omega t+\phi) \end{aligned}$$

- Clearly, $\phi = k\Delta r$. If $\phi = 2\pi n$ for $n \in \mathbb{Z}$, we get constructive interference. If $\phi = \pi + 2\pi n$ for $n \in \mathbb{Z}$, we get destructive interference.
- Thus, if we want the waves to perfectly add, we require that $\Delta r = n\lambda$ for some $n \in \mathbb{Z}$, and if we want the waves to cancel, we require that $\Delta r = \lambda/2 + n\lambda$ for some $n \in \mathbb{Z}$.
- We can succinctly sum up this idea with

$$\frac{\phi}{2\pi} = \frac{\Delta r}{\lambda}$$

- **Central maximum**: The bright spot in the middle of the screen of the double slit experiment.
- **Lateral maxima**: All noncentral bright spots on the screen.
- **First lateral maximum**: The bright spot directly to the right of the central maximum facing the screen.
- If the screen is far away compared to slit separation d, then the rays of light are virtually parallel.

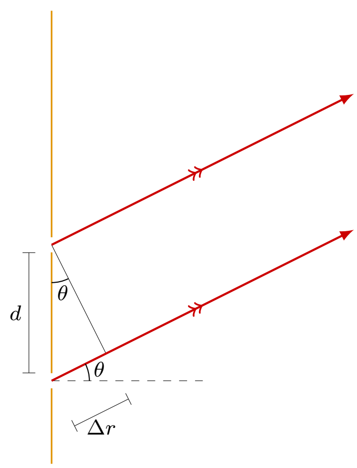

Figure 35.1: Double slit experiment Δr derivation.

- If we let θ be the angle between the rays and a normal to the slitted surface, then

$$\Delta r = d\sin\theta$$

is a very good approximation.
- It follows that $d\sin\theta = n\lambda$ yields maxima and $d\sin\theta = (n+1/2)\lambda$ yield minima.
- Thus we can measure the wavelength of light with the following:

$$d\sin\theta_{\text{1st lateral maximum}} = \lambda$$

- If we want to measure the intensity on the surface of the screen a distance r_0 from the slits (more specifically, the center of the slits), then we can do the following.

$$\begin{aligned}
E(r,t) &= A\left[\cos\left(k\left[r_0+\frac{\Delta r}{2}\right]-\omega t\right)+\cos\left(k\left[r_0-\frac{\Delta r}{2}\right]-\omega t\right)\right] \\
&= A\left[\cos\left(kr_0-\omega t+k\cdot\frac{\Delta r}{2}\right)+\cos\left(kr_0-\omega t-k\cdot\frac{\Delta r}{2}\right)\right] \\
&= 2A\cos(kr_0-\omega t)\cos\left(\frac{k\Delta r}{2}\right) \\
&= 2A\cos(kr_0-\omega t)\cos\left(\frac{k}{2}\cdot d\sin\theta\right)
\end{aligned}$$

  - x will be constant, and ωt will flicker so fast that we don't notice it changing. The other cosine term matters, though:
  - Since $I \propto E^2$, $I(\theta) \propto \cos^2(kd\sin\theta/2) = \cos^2(\pi d\sin\theta/\lambda)$.
  - Invoking the SAA for small θ, it follows that

$$I(\theta) \propto I_{\max}\cos^2(\pi d\theta/\lambda)$$

- In the double slit experiment, the two slits behave as two **coherent** sources of light.

35.2 Thin Films

- Shining light on a bubble.
  - A bubble is a thin film of soapy water with air on both sides.
  - If it has thickness t, when a wave of light impinges on it, some will be reflected off of the outside of the film, and some will be transmitted before being reflected off of the inside of the film (and some will continue to be transmitted).
  - Thinking back to a compound string, $\mu_1 > \mu_2$ implies no flip of the reflected wave and $\mu_1 < \mu_2$ implies a flip.
  - Analogously, if air has IOR n_1 and soapy water has IOR n_2, $n_1 < n_2$ implies a flip and $n_1 > n_2$ implies no flip.
  - It follows that when a ray of light enters the thin film, the reflection will be flipped, and when a ray of light exits the thin film, the reflection will not be flipped.
  - For perpendicular rays, approximate $\Delta r = 2t$ between the two reflected rays.
  - Thus, accounting for the flip (which adds an extra half-wavelength of Δr), $2t = m\lambda$ for $m \in \mathbb{Z}$ will imply *destructive* interference. For constructive interference, though, we have to take into account the distortion of the wavelength by the IOR of the soapy water ($\lambda' = \lambda/n_2$): $2tn_2 = (m+1/2)\lambda$.
  - But this will vary for different wavelengths, so different colors will be emphasized at different parts of the bubble.
  - If the bubble is very thin ($t << \lambda$ where λ is the wavelength of light), we'll only retain the flip, meaning that we get destructive interference and a black bubble.
- Oil also forms a thin film on water.

35.3 Multiple Slits

- Three slits:
  - Assume consistent slit separation d.

  - If $\Delta r = r_2 - r_1 = r - 3 - r_2 = m\lambda$, then all three add constructively.
    - No matter how many adjacent slits you have, if $d\sin\theta = m\lambda$, then you get a maximum.
  - $d\sin\theta = m\lambda$ does *not* imply a minimum for three slits (if two cancel, you still have the third).
  - So what is it? How do we get all three cosines in the following equation to sum to zero? Let $\alpha = kr_0 - \omega t$. Then

$$E = A\left[\cos(\alpha) + \cos(\alpha + \phi) + \cos(\alpha + 2\phi)\right]$$

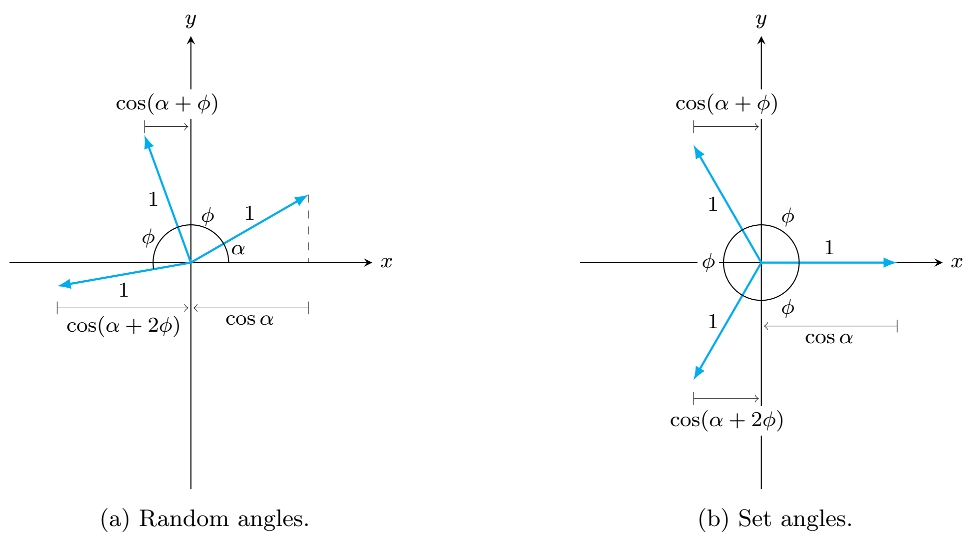

Figure 35.2: Phaser diagram for three cosines.

    - We can use a **phaser diagram**.
    - From Figure 35.2, we can see that if $\phi = 120°, 240°, \dots$, then we get cancellation. In other words, we must have $\Delta r = n\lambda/3$ for $\lambda \in \mathbb{Z}$.

- N slits.
  - Maxima at $d\sin\theta = m\lambda$ for $m \in \mathbb{Z}$.
  - Minima at $d\sin\theta = m\lambda/N$ for $m \in \mathbb{Z}$.
  - Thus, the bands given by $I(\theta)$ will become spikes at with increasingly greater separation.
  - Sharp bright spots are a good thing because they help us achieve greater accuracy in measuring the wavelength of light.
- Example: Na gas gives off yellow light ($\lambda \approx 6\,000$ Å).
  - More specifically, it gives off light at $5\,890$ Å and $5\,896$ Å, so we should have two interference patterns, but very difficult to tell apart.
  - For N slits, $\theta_{\text{1st min}} = \sin^{-1}(\lambda/dN) \approx \lambda/dN$.
  - Thus, the width of the maxima is approximately λ/dN, so for more slits, we get sharper maxima.
  - Considering the **separation angle**, if $\Delta\theta_{\text{sep}} > \theta_{\text{width of max}}$, we can tell them apart.
  - So to tell whether Na is giving off one wavelength or two, we'll have $d\sin\theta_1 = m\lambda_1$ and $d\sin\theta_2 = m\lambda_2$. Subtracting, we get

$$\begin{aligned} d(\sin\theta_1 - \sin\theta_2) &= m(\lambda_1 - \lambda_2) \\ d(\theta_1 - \theta_2) &= m(\lambda_1 - \lambda_2) \\ d\Delta\theta_{\text{sep}} &= m\Delta\lambda \\ \Delta\theta_{\text{sep}} &\approx \frac{m\Delta\lambda}{d} \end{aligned}$$

- Thus, we can barely resolve the two wavelengths if $m\Delta\lambda/d = \lambda/Nd$, or if

$$\frac{\lambda}{\Delta\lambda} = mN$$

- Thus, for the sodium doublet, we need $mN \geq 1000$.
- m is the **order of the maximum**.