Chapter 40

Quantum Mechanics I: Wave Functions

40.1 The Wave Equation

8/24:

- **Quantum wave**: The wave-like nature of an electron.
- But waves must satisfy a wave equation.
  - The classical one didn't work.
  - In 1925, Schrödinger determined that in one dimension, an electron moving in a potential V (the nucleus-electron Coulombic attraction) satisfies

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

    - This wave equation wasn't derived in an analogous method to Figure 15.2, but rather was constructed from conservation of energy.
  - This wave function has both real and imaginary parts with the inclusion of $i = \sqrt{-1}$.

40.2 Electrons in the Double Slit Experiment

- Electrons exhibit both diffraction and interference in the double slit experiment.
  - $d\sin\theta = m\lambda$ applies when λ is the deBroglie wavelength.
  - Even with only one electron being emitted at a time, you get interference (further confirms wave-like nature of electrons): $\Psi_{\text{electron}} = \Psi_{\text{slit 1}} + \Psi_{\text{slit 2}}$.
  - Observing which slit an electron goes through removes the interference pattern.
- $P \propto \Psi^2$ is independent of time, so you have static charge distributions.

40.3 Office Hours (Gazes)

- Quantifying optical roughness?
- What are C_V and C_p? Are they specific to certain gasses?
- What is an adiabatic process? Is it a straight conversion from temperature (internal) energy to work? I find this highly unintuitive.

  - Heat flow is held constant, so we have the process happening in an insulating container.
  - Temperature, pressure, and volume are all changing, hence the isotherm-crossing curve in Figures 20.1 and 20.3.
  - Running a fire piston backwards does cause the interior to get colder.
  - Releasing air from a compressed air can is also an adiabatic process (pressure decreases, volume effectively gets much bigger as it enters the room, and can gets cold).
- Additive entropies derivation.

40.4 Final Review Sheet

2/1/24:

- A wavefunction propagates via $f(x-vt)$.
- Derive the wave equation via free-body diagramming an infinitesimal segment of the rope.

$$\frac{\partial^2 y}{\partial x^2} = \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}$$

- Convert it into the following form by substituting a generic sinusoidal wave $y(x,t) = A\sin(kx-\omega t)$ into the above wave equation and recalling that $v = \omega/k$.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}$$

- Transverse velocity & acceleration.
- Average power of a wave on a string.

$$\bar{P} = \frac{1}{2} M v \omega^2 A^2$$

  - Derive with $P = \vec{F} \cdot \vec{v}$. Recall that the average value of $\cos^2(x)$ is $1/2$.
- Compound string, boundary conditions, reflection and transmission (like in quantum).
  - Limiting case is standing waves.
- Speed of sound in terms of the medium at STP.

$$v_{\text{sound}} = \sqrt{\frac{1.4p_0}{\rho_0}}$$

  - p_0 is the pressure of the air; ρ_0 is the density of the air.
- Intensity: Power per unit area.

$$I = \frac{\bar{P}}{4\pi r^2}$$

- Constructive and destructive interference of soundwaves.
- Beating for sound waves of very similar frequencies.
- For the Doppler effect, recall that $v = \lambda f$ and then think about how v changes.
  - When an observer runs toward the source at v_o, the wavefronts hit them at velocity $v + v_o$, so they hear them at frequency

$$f' = \frac{v+v_o}{\lambda} = \frac{v+v_o}{v} \cdot f > f$$

  - If the source is moving toward the observer at speed v_s, then the wavefronts still hit the observer at the same speed v and they are still emitted at the same frequency f, but the spacing λ between them changes. Relative to the source, the waves move away more slowly, so

$$\lambda' = \frac{v - v_s}{f}$$

    which implies that

$$f' = \frac{v}{\lambda'} = \frac{v}{v - v_s} \cdot f$$

  - The tricky part here is the special relativity I have to use and keep straight in my head.

- A things about ideal gases.
  - Recall that

$$KE = \frac{3}{2}RT = \frac{1}{2}M\bar{v}^2 \qquad\qquad v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

  - Let p be pressure, ρ be density, and $\bar{v}$ be the average velocity of the molecules. Then

$$p = \frac{1}{3}\rho\bar{v}^2$$

    - This yields

$$v_{\text{rms}} = \sqrt{\frac{3\rho}{p}}$$

    - I can relate this to the density form of the ideal gas law:

$$\begin{aligned} PV &= nRT \\ \frac{PM}{RT} &= \frac{nM}{V} = \rho \\ \frac{1}{3}\bar{v}^2 = \frac{RT}{M} &= \frac{P}{\rho} \end{aligned}$$

- In isothermal conditions,

$$W = -nRT \ln \frac{V_b}{V_a}$$

- Molar heat capacities at constant volume (C_V) vs. constant pressure (C_P):

$$C_V = \frac{1}{n}\frac{\mathrm{d}Q_V}{\mathrm{d}T} \qquad\qquad C_P = \frac{1}{n}\frac{\mathrm{d}Q_P}{\mathrm{d}T}$$

  - These quantities are related by considering that

$$\mathrm{d}E_{\text{int}} = \mathrm{d}Q_V = nC_V\,\mathrm{d}T \qquad \mathrm{d}E_{\text{int}} \qquad = \mathrm{d}Q_P - p\,\mathrm{d}V = nC_P\,\mathrm{d}T - p\,\mathrm{d}V$$

    and hence, algebraically,

$$C_P - C_V = R \qquad\qquad C_V = \frac{1}{n}\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{3}{2}nRT\right) = \frac{3}{2}R$$

  - Don't forget the equipartition of energy theorem (every DOF [translational, rotational, vibrational] contributes $kT/2$ of energy).

- Coefficient of linear expansion.

$$\alpha = \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}T}$$

- Adiabatic process: $\mathrm{d}Q = 0$.
- Ratio of specific heats: $\gamma = C_P/C_V$.
  - Varies for different kinds of gases (e.g., monoatomic, diatomic with rotation, diatomic with rotation and vibration).
- For an adiabatic process,
$$TV^{\lambda-1} = \text{constant}$$
  - Derive this by noting that $\mathrm{d}Q = 0$ and using transitivity between $\mathrm{d}E_{\text{int}} = -P\,\mathrm{d}V = nC_V\,\mathrm{d}T$ where $P = nRT/V$.
- $W_{\text{by engine}} = Q_{\text{in}} - Q_{\text{out}}$ per cycle.
- Huygens principle: Each part of a wavefront acts as a point source of spherical wavelets.
  - Theoretical basis for angle of incidence being equal to angle of reflection.
- Index of refraction: $n = c/v$.
  - n is a function of frequency, hence why blue light gets diffracted more than red light in a prism.
- IOR plus Huygens principle allows us to derive the bending of incoming light waves.
- Snell's law:
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
  - Implies a critical angle such that for $\theta_{\text{inc}} > \theta_{\text{crit}}$, you only get reflection; no refraction.
- Fermat's principle: Light follows the path that takes the least time.
- Law of Malus: The following formula, where I_2 is the intensity of light that gets past the polaroid filter and I_1 is the initial intensity.
$$I_2 = I_1 \cos^2\phi$$
  - Averaging $\cos^2\phi$ again, we get
$$I_{\text{trans}} = \frac{1}{2} I_{\text{unpol}}$$
- For a spherical mirror, the focal length $f = R/2$.
  - Ray tracing to find where the image of an object is.
- Mirror equation.
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
- Lateral magnification.
$$m = -\frac{-s'}{s}$$
- Plano-convex lens.
$$\frac{1}{f} = (n-1)\cdot\frac{1}{R}$$
- Lens maker's equation.
$$\frac{1}{f} = (n-1)\cdot\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
- Double slit experiment Δr derivation.
$$E(r,t) = 2A\cos(kr_0 - \omega t)\cos\left(\frac{k}{2}\cdot d\sin\theta\right)$$

- Diffraction: If $\alpha = (\pi a/\lambda)\sin\theta$, then
$$I = I_{\max}\frac{\sin^2\alpha}{\alpha^2}$$
- Rayleigh criterion: The condition for distinguishing sources of light, given by
$$\theta_{\text{sep}} \geq \frac{1.22\lambda}{a}$$