Chapter 34

Geometric Optics

34.1 Mirrors

8/12: • Take what we know about reflection and refraction and apply it to mirrors and lenses.

- Bathroom mirror: A flat piece of glass with a shiny background behind it.
- Spherical mirror: Some part of a reflective sphere.
- Relating the **radius of curvature** and the distance to the **focal point**, or the **focal length**, of a concave spherical mirror.

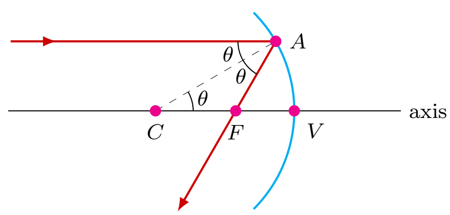

Figure 34.1: Spherical mirror analysis.

  - For a distant source s, we can approximate the rays as parallel.
  - Consider one specific ray.
  - Drawing a normal to the surface of the spherical mirror, this normal will be a radius passing through the **center of curvature**.
  - It follows by the law of reflection and the alternate interior angle theorem that all θ are equal.
  - This makes $\triangle CFA$ isosceles.
  - Now assume **paraxial rays**.
  - Then $\triangle CFA$ converges to a line segment (a radius) with midpoint F.
  - Therefore,
    $$f = \frac{R}{2}$$
    for a concave spherical mirror.
- **Radius of curvature**: The radius of the sphere into which the spherical mirror would fit. *Denoted by* $\boldsymbol{R}$.
- **Focal point**: The point where parallel rays converge, after reflecting off of a curved mirror. *Denoted by* $\boldsymbol{F}$.

- **Focal length**: The distance from the focal point to the mirror. *Denoted by* $\boldsymbol{f}$.
- **Center of curvature**: The center of the sphere into which the spherical mirror would fit. *Denoted by* $\boldsymbol{C}$.
- **Paraxial ray**: A ray that is close to the mirror axis.
- Alternatively, if you look at a convex mirror, it appears (via ray tracing) that all rays of light originated from the focal point.
  - The image in this type of mirror will be a virtual image.
  - In this case, we say that $f = -R/2$.
- A spherical mirror does not focus all rays to a *single* point — the rays only go to *approximately* the same point.
  - The farther the rays get from being paraxial, the more they diverge from the focal point.
- To have a true focal point, we need a parabolic mirror.
- Unfortunately, parabolic mirrors are hard to make.
- **Ray tracing**: Take a few **principal rays** and see where the image forms.

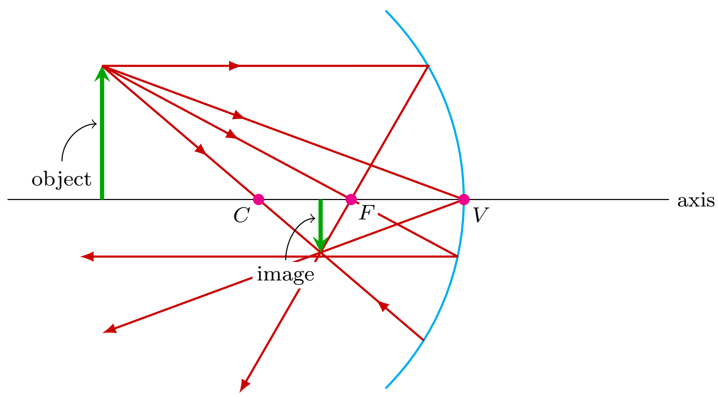

Figure 34.2: Ray tracing.

- **Principal ray**: A ray with well-understood geometry.
- **Focal ray**: A ray that goes through the focal point.
  - Will be reflected out as a parallel ray.
- **Central ray**: A ray that hits the center of the mirror.
  - Will be reflected out with the same incident angle relative to the axis.
- **Radial ray**: A ray that passes through the center of curvature.
  - It follows a *radius* of the sphere of curvature.
  - Will be reflected such that it heads right back to where it started.
- Calculating the location of the image in a spherical mirror.
  - The image in a spherical mirror will be an inverted, **real** image.

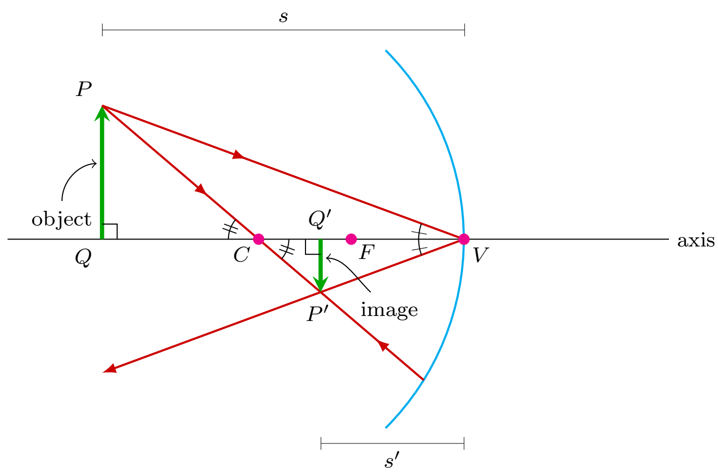

Figure 34.3: Deriving the mirror equation.

- Two principal rays define where the image is.
- Considering the image of the top point in our object, choose to analyze a central ray and a radial ray.
- Then $\triangle PVQ$ and $\triangle P'VQ'$ are similar, and $\triangle PCQ$ and $\triangle P'CQ'$ are similar.
- It follows that

$$\frac{\overline{PQ}}{\overline{P'Q'}} = \frac{\overline{QV}}{\overline{Q'V}} = \frac{s}{s'} \qquad\qquad \frac{\overline{PQ}}{\overline{P'Q'}} = \frac{\overline{QC}}{\overline{Q'C}} = \frac{s-R}{R-s'}$$

- Thus, we have the following, which can be solved for the mirror equation.

$$\frac{s}{s'} = \frac{s-R}{R-s'}$$

- **Real image**: An image that could be substituted for an image on a screen in real space.
- **Object distance**: The distance from an object to a spherical mirror. *Denoted by* $\boldsymbol{s}$.
- **Image distance**: The distance from an object's image to the mirror. *Denoted by* $\boldsymbol{s'}$.
- **Mirror equation**: The formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- **Lateral magnification**: The ratio of the distance from the object to the mirror and the distance from the image to the mirror. *Denoted by* $\boldsymbol{m}$.
  - Mathematically,

$$m = \frac{-s'}{s}$$

  - If $m < 0$, the image is inverted.
  - If $m > 0$, the image is upright.
  - Sign convention: s, s' are positive in front of the mirror and negative behind the mirror.
- Check the above equations on the example of a plane mirror.
  - For a plane mirror, $R = \infty$, so $\frac{1}{f} = 0$.
  - It follows by the mirror equation that $s' = -s$.
  - Therefore, $m = 1$, as desired.

34.2 Lenses

- **Double convex** (lens): A lens with two convex exterior surfaces. *Also known as* **convex** (lens), **converging** (lens), **positive** (lens).
- **Double concave** (lens): A lens with two concave exterior surfaces. *Also known as* **concave** (lens), **diverging** (lens), **negative** (lens).
- **Plano convex** (lens): A lens with one concave exterior surface and one flat exterior surface.

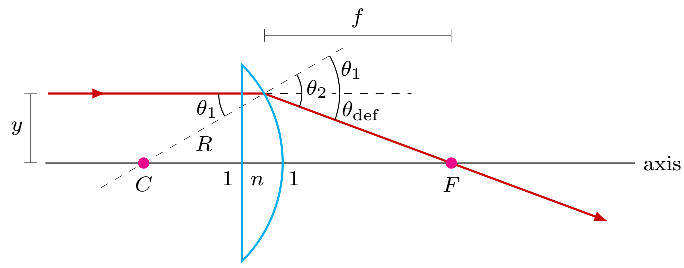

Figure 34.4: Plano convex lens analysis.

  - Clearly, $\theta_{\text{def}} = \theta_2 - \theta_1$ and $\sin\theta_1 = y/R$.
  - If this is a **thin lens**, then we can invoke the small angle approximation and say that $\theta_1 \approx y/R$.
  - By Snell's Law, $\sin\theta_2 = n\sin\theta_1$.
  - Invoking the SAA again, we have that $\theta_2 = n\theta_1$.
  - It follows that

$$\begin{aligned}\theta_{\text{def}} &= \theta_2 - \theta_1\\ &= n\theta_1 - \theta_1\\ &= (n-1)\theta_1\\ &= (n-1)\cdot\frac{y}{r}\end{aligned}$$

  - Therefore, since $\sin\theta_{\text{def}} = \frac{y}{f}$ (i.e., with the SAA $\theta_{\text{def}} = y/f$), we have that

$$\begin{aligned}f &= \frac{y}{\theta_{\text{def}}}\\ &= \frac{y}{(n-1)\cdot y/R}\\ &= \frac{R}{n-1}\end{aligned}$$

  - More commonly, we express this with

$$\frac{1}{f} = (n-1)\cdot\frac{1}{R}$$

- **Thin lens**: A lens where θ's are small.
- If you substitute a convex lens for the plano convex lens, everything gets doubled.
  - Importantly, we now have

$$\frac{1}{f} = (n-1)\cdot\frac{2}{R}$$

- We can also generalize a convex lens to a lens with two convex sides of varying radii of curvature on its two sides.
- **Lens maker's equation**: The following formula.

$$\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

  - Notice that if, for instance, the first side is planar, then $R_1 = \infty$ and the $1/R_1$ term disappears.
  - Sign convention: R_1, R_2 are positive for convex lenses and negative for concave lenses.
- Remember that knowing the focal length of a lens allows us to ray trace.

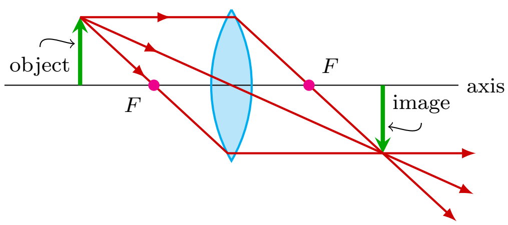

Figure 34.5: Convex lens rays.

  - Central rays in thin lenses have negligible lateral displacement.
  - An image through a convex lens will be an inverted real image.