Chapter 34

Geometric Optics

34.1 Mirrors

8/12:

- Take what we know about reflection and refraction and apply it to mirrors and lenses.
- Bathroom mirror: A flat piece of glass with a shiny background behind it.
- Spherical mirror: Some part of a reflective sphere.
- Relating the **radius of curvature** and the distance to the **focal point**, or the **focal length**, of a concave spherical mirror.

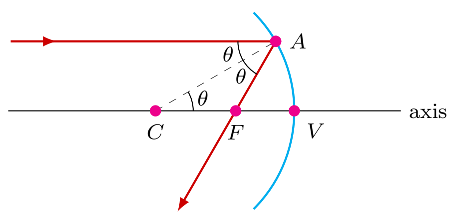

Figure 34.1: Spherical mirror analysis.

  - For a distant source s, we can approximate the rays as parallel.
  - Consider one specific ray.
  - Drawing a normal to the surface of the spherical mirror, this normal will be a radius passing through the **center of curvature**.
  - It follows by the law of reflection and the alternate interior angle theorem that all θ are equal.
  - This makes $\triangle CFA$ isosceles.
  - Now assume **paraxial rays**.
  - Then $\triangle CFA$ converges to a line segment (a radius) with midpoint F.
  - Therefore,

    $$f = \frac{R}{2}$$

    for a concave spherical mirror.
- **Radius of curvature**: The radius of the sphere into which the spherical mirror would fit. *Denoted by* $\boldsymbol{R}$.
- **Focal point**: The point where parallel rays converge, after reflecting off of a curved mirror. *Denoted by* $\boldsymbol{F}$.

- **Focal length**: The distance from the focal point to the mirror. *Denoted by* $\boldsymbol{f}$.
- **Center of curvature**: The center of the sphere into which the spherical mirror would fit. *Denoted by* $\boldsymbol{C}$.
- **Paraxial ray**: A ray that is close to the mirror axis.
- Alternatively, if you look at a convex mirror, it appears (via ray tracing) that all rays of light originated from the focal point.
  - The image in this type of mirror will be a virtual image.
  - In this case, we say that $f = -R/2$.
- A spherical mirror does not focus all rays to a *single* point — the rays only go to *approximately* the same point.
  - The farther the rays get from being paraxial, the more they diverge from the focal point.
- To have a true focal point, we need a parabolic mirror.
- Unfortunately, parabolic mirrors are hard to make.
- **Ray tracing**: Take a few **principal rays** and see where the image forms.

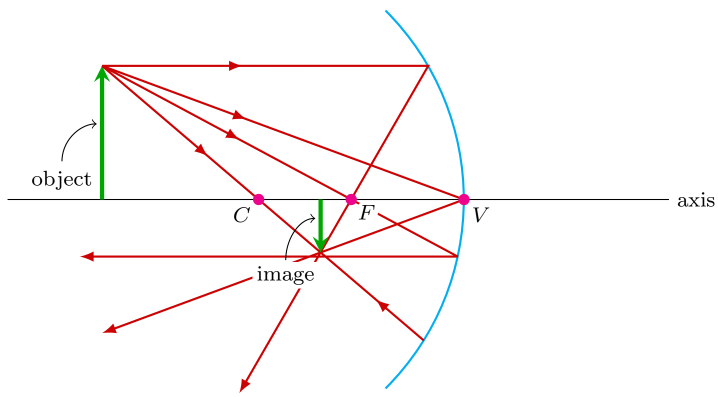

Figure 34.2: Ray tracing a concave mirror.

- **Principal ray**: A ray with well-understood geometry.
- **Focal ray**: A ray that goes through the focal point.
  - Will be reflected out as a parallel ray.
- **Central ray**: A ray that hits the center of the mirror.
  - Will be reflected out with the same incident angle relative to the axis.
- **Radial ray**: A ray that passes through the center of curvature.
  - It follows a *radius* of the sphere of curvature.
  - Will be reflected such that it heads right back to where it started.
- Calculating the location of the image in a spherical mirror.
  - The image in a spherical mirror will be an inverted, **real** image.

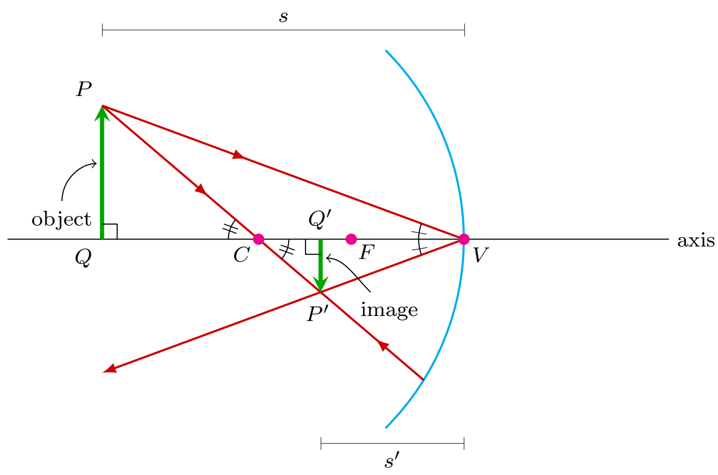

Figure 34.3: Deriving the mirror equation.

- Two principal rays define where the image is.
- Considering the image of the top point in our object, choose to analyze a central ray and a radial ray.
- Then $\triangle PVQ$ and $\triangle P'VQ'$ are similar, and $\triangle PCQ$ and $\triangle P'CQ'$ are similar.
- It follows that

$$\frac{\overline{PQ}}{\overline{P'Q'}} = \frac{\overline{QV}}{\overline{Q'V}} = \frac{s}{s'} \qquad\qquad \frac{\overline{PQ}}{\overline{P'Q'}} = \frac{\overline{QC}}{\overline{Q'C}} = \frac{s-R}{R-s'}$$

- Thus, we have the following, which can be solved for the mirror equation.

$$\frac{s}{s'} = \frac{s-R}{R-s'}$$

- **Real image**: An image that could be substituted for an image on a screen in real space.
- **Object distance**: The distance from an object to a spherical mirror. *Denoted by* $\boldsymbol{s}$.
- **Image distance**: The distance from an object's image to the mirror. *Denoted by* $\boldsymbol{s'}$.
- **Mirror equation**: The formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- **Lateral magnification**: The ratio of the distance from the object to the mirror and the distance from the image to the mirror. *Denoted by* $\boldsymbol{m}$.
  - Mathematically,

$$m = \frac{-s'}{s}$$

  - If $m < 0$, the image is inverted.
  - If $m > 0$, the image is upright.
  - Sign convention: s, s' are positive in front of the mirror and negative behind the mirror.
- Check the above equations on the example of a plane mirror.
  - For a plane mirror, $R = \infty$, so $\frac{1}{f} = 0$.

- It follows by the mirror equation that $s' = -s$.
- Therefore, $m = 1$, as desired.

8/13: • Ray tracing a convex mirror.

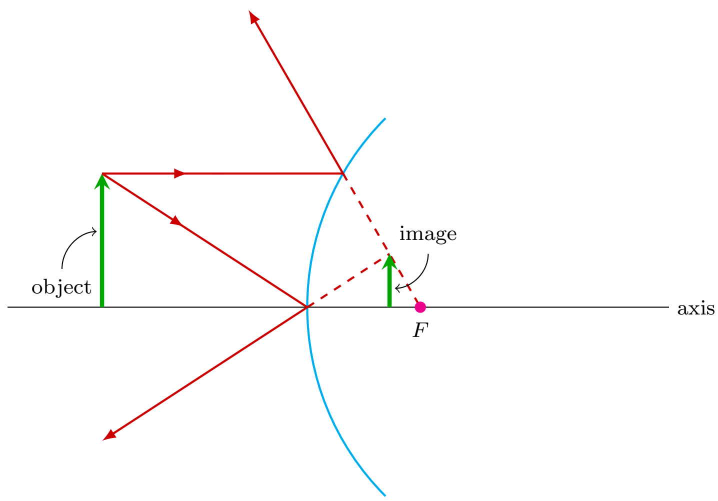

Figure 34.4: Ray tracing a convex mirror.

- Upright, smaller, virtual image.

34.2 Lenses

8/12: • **Double convex** (lens): A lens with two convex exterior surfaces. *Also known as* **convex** (lens), **converging** (lens), **positive** (lens).

- **Double concave** (lens): A lens with two concave exterior surfaces. *Also known as* **concave** (lens), **diverging** (lens), **negative** (lens).
- **Plano convex** (lens): A lens with one concave exterior surface and one flat exterior surface.

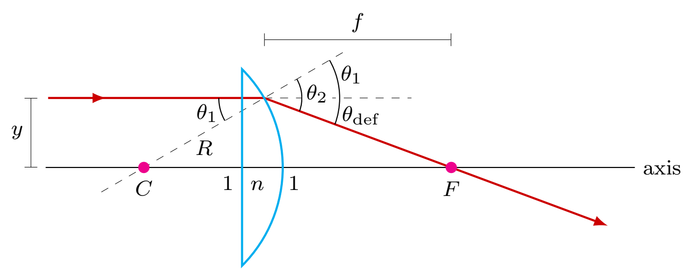

Figure 34.5: Plano convex lens analysis.

- Clearly, $\theta_{\text{def}} = \theta_2 - \theta_1$ and $\sin\theta_1 = y/R$.
- If this is a **thin lens**, then we can invoke the small angle approximation and say that $\theta_1 \approx y/R$.
- By Snell's Law, $\sin\theta_2 = n\sin\theta_1$.
- Invoking the SAA again, we have that $\theta_2 = n\theta_1$.

- It follows that

$$
\begin{aligned}
\theta_{\text{def}} &= \theta_2 - \theta_1 \\
&= n\theta_1 - \theta_1 \\
&= (n-1)\theta_1 \\
&= (n-1) \cdot \frac{y}{r}
\end{aligned}
$$

- Therefore, since $\sin\theta_{\text{def}} = \frac{y}{f}$ (i.e., with the SAA $\theta_{\text{def}} = y/f$), we have that

$$
\begin{aligned}
f &= \frac{y}{\theta_{\text{def}}} \\
&= \frac{y}{(n-1) \cdot y/R} \\
&= \frac{R}{n-1}
\end{aligned}
$$

- More commonly, we express this with

$$\frac{1}{f} = (n-1) \cdot \frac{1}{R}$$

- **Thin lens**: A lens where θ's are small.
- If you substitute a convex lens for the plano convex lens, everything gets doubled.
  - Importantly, we now have
$$\frac{1}{f} = (n-1) \cdot \frac{2}{R}$$
- We can also generalize a convex lens to a lens with two convex sides of varying radii of curvature on its two sides.
- **Lens maker's equation**: The following formula.

$$\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

  - Notice that if, for instance, the first side is planar, then $R_1 = \infty$ and the $1/R_1$ term disappears.
  - Sign convention: R_1, R_2 are positive for convex lenses and negative for concave lenses.
- Remember that knowing the focal length of a lens allows us to ray trace.

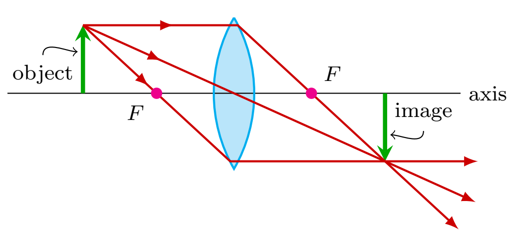

Figure 34.6: Convex lens rays.

  - Central rays in thin lenses have negligible lateral displacement.
  - An image through a convex lens will be an inverted real image.

8/13: • Ray tracing a concave lens.

    - Upright, smaller, virtual image.

- Note that the lens maker's equation used by Young and Freedman (2019) is $1/f = (n-1)(1/R_1 - 1/R_2)$, along with a complicated sign convention.
    - The textbook uses this formula because it follows from the study of thick lenses, with which many older textbooks start.
- A concave mirror and a plano convex lens with a mirrored background image the same way.
    - A convex lens mirrors the same way as both, but with the image on the other side of the lens as opposed to the same side of the apparatus.
- Therefore, the lens equation is also analogously

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

    - However, this equation comes with the sign convention that s is positive in front of the lens and negative behind the lens, and s' is positive behind the lens and negative in front of the lens.
- Similarly,

$$m = \frac{-s'}{s}$$

  for lenses, with the new sign convention.
- Note that the focal length is positive for convex lenses and negative for concave lenses.
- Just like mirrors can image images from other mirrors, lenses can image images from other lenses.
    - You do it the same way, too — one at a time.
- Example: Suppose that you have two identical thin lenses, 15 cm apart, with focal length 10 cm each. Place an object 15 cm to the left of the left lens. How does it image?

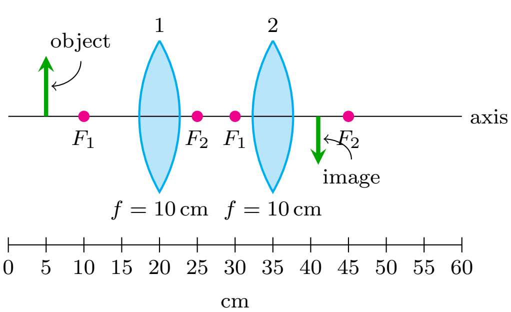

Figure 34.7: Imaging two lenses.

    - Image location:
        - For lens 1, we have $f = 10\,\text{cm}$ and $s = 15\,\text{cm}$, so $s' = 30\,\text{cm}$ by the lens equation.
        - For lens 2, we still have $f = 10\,\text{cm}$ but now we have have $s = -15\,\text{cm}$, so $s' = 6\,\text{cm}$ by the lens equation.
    - Magnification:

$$\begin{aligned} m_{\text{total}} &= m_1 m_2 \\ &= \left(-\frac{30\,\text{cm}}{15\,\text{cm}}\right)\left(-\frac{6\,\text{cm}}{-15\,\text{cm}}\right) \\ &= -\frac{4}{5} \end{aligned}$$

  - Thus, the total image is inverted, 4/5 times the size of the original, and 6 cm to the right of the rightmost lens.

- Mirrors and lenses have limitations.
  - Thus, it does make sense to use multiple mirrors/lenses in some circumstances.
  - Compensating for the paraxial approximation: When you need a sharper focus in good cameras.
  - Compensating for the dependence of n on frequency: Have any single lens do less work.