

# Chapter 36

## Diffraction

### 36.1 Single Slit Diffraction

- 8/17:
- Shining light through only one slit still yields an interference pattern.
  - We explain this with **diffraction**.
  - Finding the location of minima on the screen:

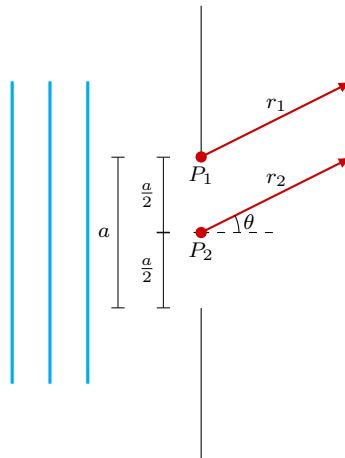


Figure 36.1: Finding diffraction minima.

- Let the one slit have width  $a$ .
- Only the part of the wavefront that aligns with the slit will pass through. However, according to Huygen's principle, when the light wave reaches the slit, it will act like infinitely many point sources of light all along the length of the slit.
- Consider two specific rays  $r_1$  and  $r_2$  emanating from the slit in same direction, one at the top and one in the middle. We know that if they are oriented at an angle that makes  $\Delta r = \lambda/2$ , then they cancel out.
- Generalizing, if any two rays satisfy  $\frac{a}{2} \sin \theta = \lambda/2$  (i.e., satisfy  $a \sin \theta = \lambda$ ), then they will cancel.
- Indeed, every  $\theta$  satisfying  $a \sin \theta = \lambda$  will cancel: Consider all the rays originating from every point in the slit that point in the  $\theta$ -direction, and notice that for any point in the slit, there will be a point  $a/2$  units away from it; the rays from these two points will cancel. Thus, every ray is associated with another ray that cancels it out, guaranteeing that  $\theta$  is an interference minimum.
- Note that if  $\theta$  yields an interference minimum, then  $\theta$  satisfying  $a \sin \theta = m\lambda$  where  $m \in \mathbb{N}$  will yield interference minima.

## 36.2 Intensity of Single Slit Diffraction

- Like before,  $\theta = 0^\circ$  gives a **central diffraction maximum**.
- Finding the intensity maxima in general:

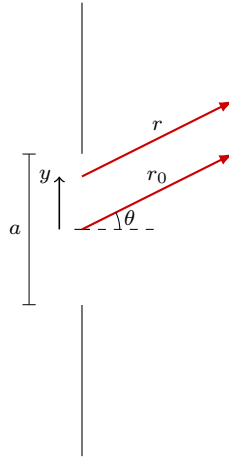


Figure 36.2: Finding diffraction maxima.

- To find the intensity maxima, we derive an equation for the intensity in general as a function of  $\theta$ .
- To do so, we sum up every infinitesimal contribution of all the points along the slit with an integral, as follows.
- Since  $\Delta r = y \sin \theta$  (see Figure 36.2), the wave function for the electric field wave along the arbitrary ray  $r$  a distance  $y$  from the central ray is given by

$$\begin{aligned}\cos(kr - \omega t) &= \cos(k[r_0 + \Delta r] - \omega t) \\ &= \cos\left(kr_0 - \omega t + \frac{2\pi}{\lambda} \cdot y \sin \theta\right)\end{aligned}$$

- It follows that the electric field  $E$  at some point  $P$  on the screen is given by

$$\begin{aligned}E &= A \int_{-a/2}^{a/2} \cos\left(kr_0 - \omega t + \frac{2\pi \sin \theta}{\lambda} \cdot y\right) dy \\ &= \frac{C}{\sin \theta} \cos(kr_0 - \omega t) \sin\left(\frac{\pi a}{\lambda} \sin \theta\right)\end{aligned}$$

where  $C$  represents a bunch of constants.

- Thus, since  $I \propto E^2$ ,

$$I \propto \frac{\sin^2\left(\frac{\pi a}{\lambda} \sin \theta\right)}{\sin^2 \theta}$$

- Additionally, if we define  $\alpha = \frac{\pi a}{\lambda} \sin \theta$ , then

$$I = I_{\max} \frac{\sin^2 \alpha}{\alpha^2}$$

- Notice that  $\theta \rightarrow 0$  implies  $\alpha \rightarrow 0$  implies  $\sin(\alpha)/\alpha \rightarrow 1$  implies  $I \rightarrow I_{\max}$ , as expected.
- Furthermore, since  $\sin \alpha$  is bounded but  $\alpha$  is not,  $\sin^2(\alpha)/\alpha^2$  yields a graph of maxima that drop off in intensity as  $\alpha \rightarrow \pm\infty$ .

- **Diffraction:** Bending of a light wave as it goes through a small slit.

- As slit width  $a$  decreases, minima spread out.

### 36.3 Combining Interference and Diffraction

- 8/19:
- Every place you have a diffraction minimum, the wave that gets to the screen has 0 amplitude.
    - If you have a point  $P$  that's a diffraction minimum of both slits, interference doesn't matter — you're going to have no intensity at  $P$ .
  - Slits  $S_1$  and  $S_2$  have the same diffraction pattern, just shifted by  $d$ .
    - But if the diffraction pattern is large relative to  $d$ , as it usually is, we can neglect the shift.
  - Total intensity:

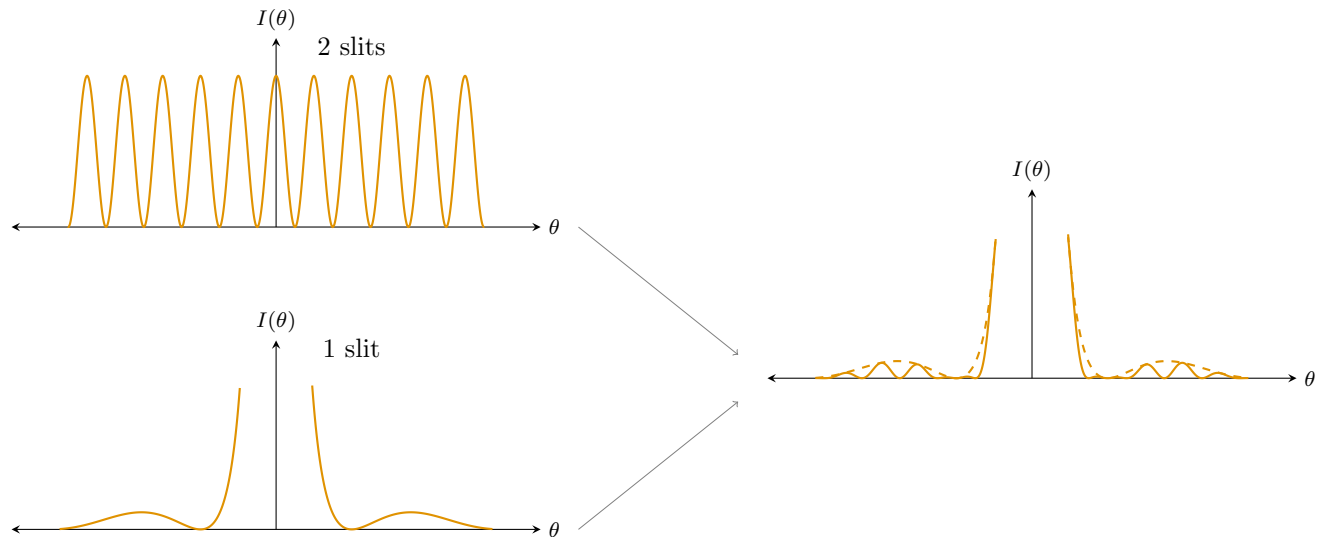


Figure 36.3: Intensity considering both interference and diffraction.

- We have that  $I(\theta) = I(\theta)_{2 \text{ slits' interference}} \times \text{diffraction envelope}$ .

### 36.4 Circular Hole

- Consider light passing through a circular hole of diameter  $a$ .
  - This yields concentric rings of intensity separated by nodes, i.e., a slit diffraction pattern that accounts for slits of every angle added on top of each other.
  - As  $a$  gets smaller, the central diffraction maximum gets bigger.
- **Slit:** A one-dimensional opening.
- **Aperture:** A two-dimensional opening.
- We can no longer use  $a \sin \theta = m\lambda$ ; we have to consider what happens with the extra dimensions.
  - When we redo the calculation in two dimension, we get
 
$$a \sin \theta = 1.22m\lambda$$
 for  $m \in \mathbb{Z}$ .
    - If  $\theta$ 's are small, then  $\theta_{1\text{st min}} \approx 1.22\lambda/a$ .

- Recall that  $\theta_{1\text{st min}} = \theta_{\frac{1}{2} \text{ width of central max}}$ , so we can use this formula to estimate the width of the central maximum.
- As light passes through your pupil (an aperture), it undergoes diffraction and gets bigger before impinging on your retina.

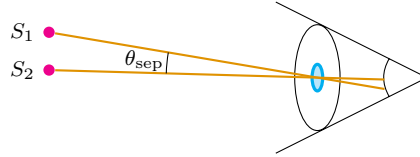


Figure 36.4: Distinguishing sources of light.

- Thus, to be able to distinguish two sources of light, we require  $\theta_{\text{sep}} \geq \theta_{\frac{1}{2} \text{ width of central max}}$ .
- For this reason, bigger telescopes are used not only to collect more light but also to minimize the effects of diffraction.
- **Rayleigh criterion:** The condition for distinguishing sources of light, given by

$$\theta_{\text{sep}} \geq 1.22\lambda/a$$

- Pinhole camera.
  - For a sharp focus, you want a small pinhole, improving the geometry.
  - But if you make it too small, diffraction will come into play.