Chapter 36

Diffraction

36.1 Single Slit Diffraction

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- Shining light through only one slit still yields an interference pattern.
- We explain this with **diffraction**.
- Finding the location of minima on the screen:

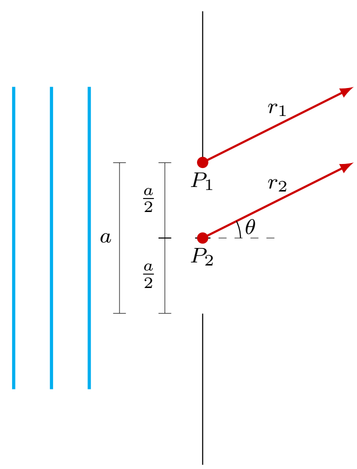

Figure 36.1: Finding diffraction minima.

- Let the one slit have width a.
- Only the part of the wavefront that aligns with the slit will pass through. However, according to Huygen's principle, when the light wave reaches the slit, it will act like infinitely many point sources of light all along the length of the slit.
- Consider two specific rays r_1 and r_2 emanating from the slit in same direction, one at the top and one in the middle. We know that if they are oriented at an angle that makes $\Delta r = \lambda/2$, then they cancel out.
- Generalizing, if any two rays satisfy $\frac{a}{2}\sin\theta = \lambda/2$ (i.e., satisfy $a\sin\theta = \lambda$), then they will cancel.
- Indeed, every θ satisfying $a\sin\theta = \lambda$ will cancel: Consider all the rays originating from every point in the slit that point in the θ-direction, and notice that for any point in the slit, there will be a point $a/2$ units away from it; the rays from these two points will cancel. Thus, every ray is associated with another ray that cancels it out, guaranteeing that θ is an interference minimum.
- Note that if θ yields an interference minimum, then θ satisfying $a\sin\theta = m\lambda$ where $m \in \mathbb{N}$ will yield interference minima.

36.2 Intensity of Single Slit Diffraction

- Like before, $\theta = 0°$ gives a **central diffraction maximum**.
- Finding the intensity maxima in general:

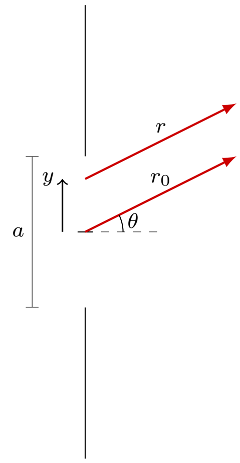

Figure 36.2: Finding diffraction maxima.

  - To find the intensity maxima, we derive an equation for the intensity in general as a function of θ.
  - To do so, we sum up every infinitesimal contribution of all the points along the slit with an integral, as follows.
  - Since $\Delta r = y \sin\theta$ (see Figure 36.2), the wave function for the electric field wave along the arbitrary ray r a distance y from the central ray is given by
$$\begin{aligned}\cos(kr - \omega t) &= \cos(k[r_0 + \Delta r] - \omega t)\\ &= \cos\left(kr_0 - \omega t + \frac{2\pi}{\lambda} \cdot y\sin\theta\right)\end{aligned}$$
  - It follows that the electric field E at some point P on the screen is given by
$$\begin{aligned}E &= A\int_{-a/2}^{a/2} \cos\left(kr_0 - \omega t + \frac{2\pi\sin\theta}{\lambda} \cdot y\right) \mathrm{d}y\\ &= \frac{C}{\sin\theta}\cos(kr_0 - \omega t)\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)\end{aligned}$$
where C represents a bunch of constants.
  - Thus, since $I \propto E^2$,
$$I \propto \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\sin^2\theta}$$
  - Additionally, if we define $\alpha = \frac{\pi a}{\lambda}\sin\theta$, then
$$I = I_{\max}\frac{\sin^2\alpha}{\alpha^2}$$
  - Notice that $\theta \to 0$ implies $\alpha \to 0$ implies $\sin(\alpha)/\alpha \to 1$ implies $I \to I_{\max}$, as expected.
  - Furthermore, since $\sin\alpha$ is bounded but α is not, $\sin^2(\alpha)/\alpha^2$ yields a graph of maxima that drop off in intensity as $\alpha \to \pm\infty$.
- **Diffraction**: Bending of a light wave as it goes through a small slit.
  - As slit width a decreases, minima spread out.