Chapter 15

Mechanical Waves

15.1 Course Information

8/5:

- HW 1 will be posted after class. Due Monday at 10 AM.
- 2 labs, 2 days each.
  - Department policy is that you have to do all the labs to pass the class.
- First meeting with lab TA will be on Monday at 2:30, 3:30, or 4:30.
  - Email Dr. Gazes for later timeslot.
- HW accounts for 85% of your grade because it helps with the tests.
- Quiz assignment that you print out, write on, and then scan and upload.
- Office hours (Gazes): 5:30-7:00. TA office hours to be posted soon.
- Wants us to learn the material, not compete with each other.
  - Expects collaboration on the homework, but wants us to write up our own answers.

15.2 Wave Basics

- **Wave**: A disturbance that propagates (carrying energy).
- **Mechanical** (wave): A wave in a medium that has an equilibrium.
  - Air, for instance, is in equilibrium when its pressure/density is everywhere equal. But you can create a disturbance by making a high-pressure region somewhere in space. This disturbance then propagates.
  - When a slinky compression wave is created, what's propagating isn't the slinky — no coil can move past another. What's moving is the *high-density region*.
- **Compression**: The high-density region of a wave.
- **Rarefaction**: The low-density region of a wave.
- **Longitudinal** (wave): A wave where the disturbance is parallel to the propagation of the wave.
  - Example: Compression wave in a slinky; air (sound).
- **Transverse** (wave): A wave where the disturbance is perpendicular to the propagation of the wave.

  - Example: A string tied to the wall where you shake one end; waves at the beach (the water is going up and down but the wave is moving toward the beach).

- A charge q creates an electric field. If q moves at a constant velocity v, it will create a magnetic field. If you make the charge accelerate with acceleration a, it will produce an **electromagnetic wave**.

- **Electromagnetic** (wave): A wave that does not require a medium to move in.

  - A medium is physical; made up of matter. The electric and magnetic fields in which an electromagnetic wave moves are not media — they can contain energy, but not in the same way a physical medium can.

- **Wavefunction**: A mathematical function that represents the behavior of a wave.

  - $y(x,t)$ represents a one-dimensional wave, x being position and t being time.
  - y represents the magnitude of the disturbance.
    - Example: The density of slinky links in a longitudinal wave; the displacement of a transverse wave from the x-axis, taken to be equilibrium.

- **Wave speed**: The velocity with which the wave propagates. *Denoted by* $\boldsymbol{v}$.

  - NOT, for example, the speed with which the string moves up and down in a transverse wave.

- If we let the xy-axes be the standard ones, we can also define $x'y'$-axes that move with the wave with velocity v.

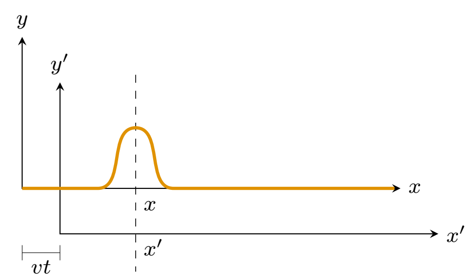

Figure 15.1: Axes that eliminate the effect of time.

  - In the $x'y'$-axes, the wave isn't moving.
  - From Figure 15.1, we can see that $x = x' + vt$.
  - Additionally, we can express (shape of) the wave as $y' = f(x')$.
  - Thus, $y = f(x - vt)$ represents a wave propagating in the $+x$ direction.
  - Similarly, $y = f(x + vt)$ represents a wave propagating in the $-x$ direction.

- When two waves collide (or we otherwise have to deal with more than one wave in the same medium), we apply the **superposition principle**.

- **Superposition principle**: If $y_1, y_2, \ldots$ are individual wavefunctions, the total disturbance y is given by $y(x,t) = y_1(x,t) + y_2(x,t) + \cdots$.

- **Constructive interference**: When two waves in the same medium add to produce a bigger wave.

- **Destructive interference**: When two waves in the same medium cancel parts of each other out.

  - Difference between a medium at equilibrium and a medium with two waves destructively interfering (at the instant the waves collide, the medium looks as if it's at equilibrium):

- The energy of the wave is contained in the kinetic energy of the individual particles of the medium moving up and down.
- As such, even when we don't see a visible wave, those particles still have a velocity vector that is containing the energy. It's like the *position* gets back to equilibrium for a moment, but the *velocity*, where the kinetic energy is contained, is most definitely not at equilibrium.

- In PHYS 13100, we used $F = ma$ to analyze a block of mass m oscillating on a spring, solving

$$F = ma$$
$$-kx = m \cdot \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{k}{m} \cdot x = 0$$

to describe its dynamics.

- Creating an analogy to $F = ma$ for wave motion (deriving the wave equation).

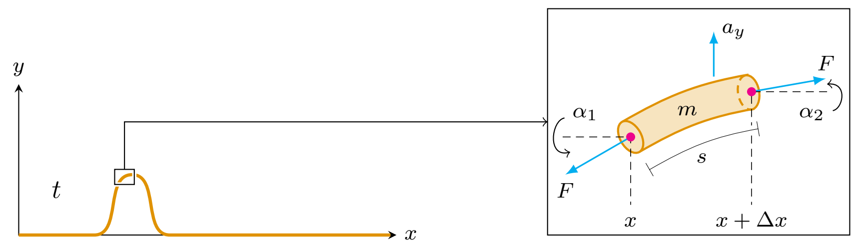

Figure 15.2: Deriving the wave equation.

  - F is a tension force.
  - We know for the sliver of the string in Figure 15.2, $F_y = ma_y$.
  - From our FBD, we have that $F_y = F\sin\alpha_2 - F\sin\alpha_1$.
  - Since the string segment is short, assume $\alpha_1 = \alpha_2$. Let's also ignore gravity since $F >> F_g$: it doesn't matter in what position you play an instrument, relative to the Earth's surface, does it?
  - For small values of α (we assume our string is taut), $\sin\alpha \approx \tan\alpha = \frac{\partial y}{\partial x}$.
  - Thus, $F_y = F(\partial y/\partial x\,|_{x+\Delta x} - \partial y/\partial x\,|_x)$.
  - Additionally, $m = Ms$, where M is the linear mass density and s is the arc length of the string segment. Furthermore, since α's are small in taut strings, $\Delta s \approx \Delta x$, so $m \approx M\Delta x$.
  - Lastly, observe that $a_y = \partial^2 y/\partial t^2$.
  - Therefore, $F = ma$ becomes

$$F\left(\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_x\right) = M\Delta x \cdot \frac{\partial^2 y}{\partial t^2}$$
$$\frac{\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_x}{\Delta x} = \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}$$

from which we can take limits as follows:

$$\lim_{\Delta x \to 0} \frac{\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_x}{\Delta x} = \lim_{\Delta x \to 0} \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}$$
$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{M}{F} \cdot \frac{\partial^2 y}{\partial t^2}}$$

- **Wave equation**: The final result above.
  - Holds for a 1D wave on a string.
- Tie a piece of string to a wall and shake the free end like a harmonic oscillator. This creates a **harmonic** wave that propagates towards the wall.
- **Harmonic** (wave): A wave produced by a disturbance changing like a harmonic oscillator.
  - The wavefunction for a harmonic wave is sinusoidal, propagates like a wave (i.e., like $f(x-vt)$), and needs to have a constant k to make the dimensional argument of sine dimensionless: $y(x,t) = A\sin(k[x-vt])$.
- **Amplitude**: The constant A in the wavefunction of a harmonic wave.
- **Wavenumber**: The constant k in the wavefunction of a harmonic wave. *Units are* m^{-1}.
- **Wavelength**: The *distance* over which wave motion repeats *for a fixed time* t. *Denoted by* $\boldsymbol{\lambda}$.
  - Mathematically, the existence of the wavelength implies that $y(x,t) = y(x+\lambda,t)$.
  - But for a harmonic wave, this implies that $A\sin(k[x-vt]) = A\sin(k[(x+\lambda)-vt])$, meaning that $k\lambda = 2\pi$.
  - Thus, we know that the wave number $k = \frac{2\pi}{\lambda}$.
- **Period**: The *time* over which wave motion repeats *for a fixed point* x. *Denoted by* $\boldsymbol{T}$.
  - Similarly, $y(x,t) = y(x,t+T)$.
  - For a harmonic wave, $A\sin(k[x-vt]) = A\sin(k[x-v(t+T)])$, meaning that $kvT = 2\pi$.
  - Thus, we know that the wave speed $v = \frac{2\pi}{k}\cdot\frac{1}{T} = \lambda f$, where f is the frequency of the wave, for simple harmonic motion.
  - Alternately, if we let $\omega = 2\pi f$ be the angular frequency, then $v = \frac{\omega}{k}$.
- It follows that for a harmonic wave,

$$\begin{aligned} y(x,t) &= A\sin\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right] \\ &= A\sin[kx-\omega t]^{[1]} \end{aligned}$$

- To account for cosine and other waves that "start" at different parts, we include a **phase constant** ϕ:

$$y(x,t) = A\sin[kx-\omega t+\phi]$$

- To check that the above is in fact a wave, we must feed it into the wave equation:

$$\begin{aligned} \frac{\partial^2}{\partial x^2}(A\sin[kx-\omega t+\phi]) &= \frac{M}{F}\cdot\frac{\partial^2}{\partial t^2}(A\sin[kx-\omega t+\phi]) \\ -Ak^2\sin[kx-\omega t+\phi] &= \frac{-A\omega^2 M}{F}\sin[kx-\omega t+\phi] \\ k^2 &= \frac{M\omega^2}{F} \\ \frac{\omega}{k} &= \sqrt{\frac{F}{M}} \end{aligned}$$

[1] Dr. Gazes prefers this form, but both are correct and can be used.

  - It follows since $v = \frac{\omega}{k}$ that $v = \sqrt{F/M}$.
- We originally found this speed/force/mass relationship to be true for a harmonic wave, but this shows that it is true for *any* wave.
- **General 1D wave equation**: Making the modification from above, the following equation.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}$$

15.3 Office Hours (Gazes)

- How does proving that $v = \sqrt{F/M}$ with a harmonic wavefunction prove that this relation holds for *all* waves?