Chapter 20

The Second Law of Thermodynamics

20.1   Thermodynamic Cycles

- Examples: Engines and pumps.

  - In a thermodynamic cycle, the energy doesn't change, so $W_{\text{on gas}} = -Q$, i.e., $W_{\text{by gas}} = Q_{\text{net}}$.

  - Thus, $W_{\text{by engine}} = Q_{\text{in}} - Q_{\text{out}}$ per cycle.

  - **Otto cycle**: The thermodynamic cycle used in car engines. *Also known as* **auto cycle**.
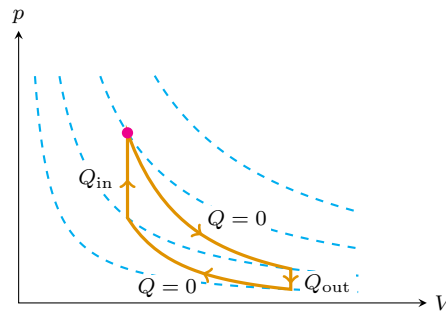

Figure 20.1: Otto cycle.

  - Two adiabatic and two isochoric processes.

- **Stirling engine cycle**: The thermodynamic cycle used in steam engines.

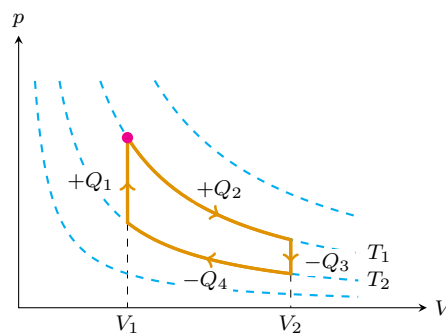

Figure 20.2: Stirling engine cycle.

– Two isothermal and two isochoric processes.

– For the isothermal processes, $\Delta E_{\text{int}} = 0$, so $Q = W_{\text{by gas}} = nRT \ln(V_f/V_i)$.

– For the isochoric process, $Q = nC_V \Delta T$.

– $Q_1 + Q_3 = nC_V(T_1 - T_2) + nC_V(T_2 - T_1) = 0$, so we only have to worry about Q_2 and Q_4 to determine the work done by the gas:

$$W_{\text{by gas}} = Q_2 + Q_4$$
$$= nR \ln\left(\frac{V_2}{V_1}\right) \cdot (T_1 - T_2)$$

– To get more work, you want the **compression factor** to be as big as possible.

– You also want the difference between the two temperatures to be as big as possible.

– The engine is powered until $T_1 = T_2$, because at that point you can't exchange heat.

• **Compression factor**: The ratio of the final volume to the initial volume, i.e., V_2/V_1.

• **Efficiency**: The quotient of the work an engine does and the heat you put in per cycle. *Denoted by e. Given by*

$$e = \frac{W_{\text{engine}}}{Q_{\text{in}}}$$
$$= \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}}$$
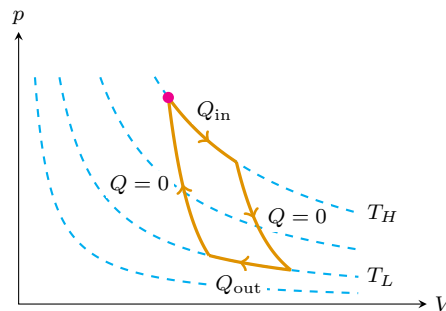$$= 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}}$$

• **Carnot cycle**.

Figure 20.3: Carnot cycle.

– Two isothermal and two adiabatic processes.

– For this engine, $e = 1 - Q_{\text{out}}/Q_{\text{in}} = 1 - T_L/T_H$.

■ Thus, as the temperatures approach each other, efficiency approaches 1.

■ But, you cannot run an engine when the heat bath temperatures are equal to each other.

– Therefore, the Carnot cycle is the most efficient engine you can build, but it's just not physically viable.

■ A car run on the Carnot cycle would have great mileage but wouldn't get you very far.

– Conclusion: The Carnot cycle is interesting, but it does not yield a particularly useful engine.

20.2   The Second Law

- **Second Law of Thermodynamics**: It is not possible to remove heat at a high temperature and convert it entirely to work done by the engine. In other words, some heat is always exhausted to the low temperature.

- Maximizing engine efficiency:

  - You want to maximize the temperature difference.
  - You could technically run an engine between air temperature and ice, but it's hard to lug around a bunch of ice.
  - Additionally, you can't have T_H be too high because hotter engines emit nastier exhaust (specifically, exhaust that contributes more to acid rain).
  - Recall that the area on a pV-graph encompassed by the cycle is equal to the work done by the gas.

- If you run a heat engine backwards, you get a **heat pump**.

- **Heat pump**: A device that removes heat from low temperature sinks and exhausts it into high temperature sinks.

  - This is an air conditioner.
  - The second law of thermodynamics asserts that you can't remove heat at a low temperature and move it to a high temperature without doing work on the gas. Thus, you have to plug in the air conditioner and provide power — heat won't magically flow from cold to hot.

20.3   Statistical Mechanics

- Free expansion.

  - Imagine you have gas on one side of a thermally insulated container separated from the rest of the container by a partition.
  - If you remove the partition, the gas will expand out to fill the entire container.
  - However, the gas is not working against a force ($p = 0$) and heat is not flowing in ($Q = 0$).
  - Therefore, $dE_{\text{int}} = dQ - p\,dV = 0 - 0\,dV = 0$.
  - From this, we can conclude that $\Delta T = 0$.
  - Note that this is an **irreversible process**.

- **Irreversible process**: A process that looks like it could occur on a microscopic level but could not on a macroscopic level.

  - Imagine a video of the milk molecules moving in a cup of coffee (which would look normal forwards and backwards) vs. a video of milk spreading out in a cup of coffee (which most certainly would not look normal backwards).

- **Macrostate**: A set of thermal variables (n, p, V, T) defining a system at an instant in time.

- **Microstate**: A way of arranging the molecules in a system that produces a macrostate.

  - Multiple microstates can correspond to the same macrostate.

- Consider a box with 10 molecules inside.

  - Each molecules has a 50/50 chance of being on the left *or* right side of the box.
  - We want to achieve the macrostate defined by 10 molecules on the left side and 0 on the right.

- ■ There is only 1 corresponding microstate (namely that just described).
  - – We want to achieve the macrostate defined by 3 molecules on the left side and 7 on the right.
    - ■ There are $\frac{10!}{7!3!} = 120$ corresponding microstates.
  - – For the even 5/5 split, there are $\binom{10}{5} = 252$ corresponding microstates.
- If there are N molecules and we want m on the left and $N - m$ on the right, the number of microstates that will realize this macrostate is $\frac{N!}{m!(N-m)!}$.
  - – This corresponds to a binomial distribution.
- **Fundamental Assumption of Statistical Physics**: All microstates of a system are equally probable.
- **Number of microstates of a macrostate**: *Denoted by* $\mathbf{\Omega_{macro}}$.
- It follows from the fundamental assumption that

$$P_{\text{macro}} \propto \Omega_{\text{macro}}$$

- Relating this back to our example, since $P_{m,N-m} \propto \Omega m, N - m$ we have that $(5,5)$ is approximately twice as likely as $(3,7)$.
- If we have N_A of molecules, it is going to be much more likely that the molecules are evenly (or almost evenly) distributed on both sides of the box than anything else.
  - – $\Omega_{N_A,0} = 1$ still, but $\Omega_{N_A/2,N_A/2} = \frac{N_A!}{(N_A!)^2} \approx 10^{N_A/3} \approx 10^{2\times10^{23}}$, which is a huge number.
- If you make N and m continuous rather than integer quantities, the binomial distribution becomes a Gaussian distribution.
  - – Thus, Gaussian distributions describe the probability distribution corresponding to the number of molecules on the left and the right.
  - – The standard deviation of the Gaussian distribution of N_A macrostates is $\sigma \approx \sqrt{N_A}$.
  - – Thus, the fraction of the width of the central peak to all states is $\sigma/N_A \approx 10^{-12}$.
  - – Therefore, the width of the spike is approximately one-trillionth the width of the whole distribution, meaning that although you won't typically see a perfect 50/50 split, you won't see very large fluctuations (even 49/51 is very unlikely).
- It's not that you *can't* have all of the air molecules move to one side of the room; it's that you *won't* have this happen.

20.4   Office Hours (Gazes)

- Preference for sine vs. cosine when describing harmonic waves?
  - – No preference.
  - – Gazes tends to start with sines, but it really doesn't matter; there is no canonical preference.
- We say that the open end of a pipe is a node and we often say that closed ends are antinodes. However, on the midterm, there was a question with a tuning fork at one end of a pipe with water at the bottom, and we were supposed to infer that the open end with the tuning fork was an antinode and the end with water was a node. As such, I'm wondering if there's any unambiguous way we can identify open and closed ends of pipes.
  - – We do assume the end with the tuning fork to be a pressure node and the water to be a pressure antinode.

- It's a *pressure* node at the end of the tube, but a displacement antinode.
- Some transmission does occur at the end of a tube.
- Flaring the ends of instruments cuts down on reflection and increases transmission.

- How do we know that even rays with not so well understood geometry converge on the image?

- A computer can just brute force the problem and come to this conclusion.

- When refracting an object through multiple lenses, the side the *original* object is on is always the $+s$ and $-s'$ side, even when the first image may be on the other side of the next lens?

- Yes.
- Because the rays just continue on through the second lens. Those *rays* act as the object and effectively determine the sign.

- Plano-convex lens on top of a mirror?

- You can make the contraption image the same way as a concave mirror.
- Allows analogy of concave mirrors to regular lenses.
- Also helps define our sign convention: flips what defines positive image distance.

- Quantifying optical roughness?

- What are C_V and C_p? Are they specific to certain gasses?

- What is an adiabatic process? Is it a straight conversion from temperature (internal) energy to work? I find this highly unintuitive.

- When do we make use of vibration? In Problem 19.43, shouldn't we have needed to use $\frac{7}{2}nR\Delta T$ for every degree Kelvin above $1\,000\,\text{K}$?

- Didn't quite cross over the threshold.
- Won't have to account for this on the final.
- Everything is monoatomic on the final!