PHYS 13300 (Waves, Optics, and Heat) Problem Sets

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Contents

1 Mechanical Waves

8/9: 1) Young and Freedman (2019): Problem 15.9.

Which of the following wave functions satisfies the wave equation?

(a) $y(x,t) = A\cos(kx + \omega t)$.
(b) $y(x,t) = A\sin(kx + \omega t)$.
(c) $y(x,t) = A(\cos kx + \cos \omega t)$.
(d) For the wave of part (b), write the equations for the transverse velocity and transverse acceleration of a particle at point x.

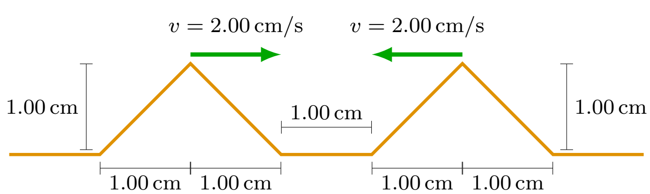
2) Young and Freedman (2019): Problem 15.26.

A fellow student with a mathematical bent tells you that the wave function of a traveling wave on a thin rope is $y(x,t) = (2.30\,\text{mm})\cos[(6.98\,\text{rad/m})x + (742\,\text{rad/s})t]$. Being more practical, you measure the rope to have a length of 1.35 m and a mass of 0.003 38 kg. You are then asked to determine:

(a) Amplitude.
(b) Frequency.
(c) Wavelength.
(d) Wave speed.
(e) Direction the wave is traveling.
(f) Tension in the rope.
(g) Average power transmitted by the wave.

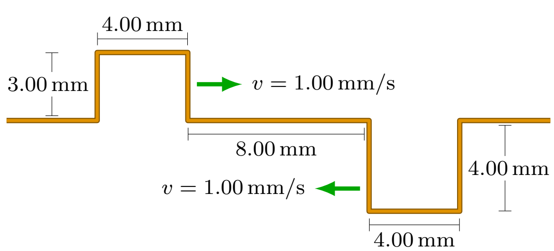
3) Young and Freedman (2019): Problem 15.30.

Interference of Triangular Pulses. Two triangular wave pulses are traveling toward each other on a stretched string as shown below. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at $t = 0$. Sketch the shape of the string at $t = 0.250\,\text{s}$, $t = 0.500\,\text{s}$, $t = 0.750\,\text{s}$, $t = 1.000\,\text{s}$, and $t = 1.250\,\text{s}$.



4) Young and Freedman (2019): Problem 15.32.

Interference of Rectangular Pulses. The below figure shows two rectangular wave pulses on a stretched string traveling toward each other. Each pulse is traveling with a speed of 1.00 mm/s and has the height and width shown in the figure. If the leading edges of the pulses are 8.00 mm apart at $t = 0$, sketch the shape of the string at $t = 4.00\,\text{s}$, $t = 6.00\,\text{s}$, and $t = 10.0\,\text{s}$.



5) Young and Freedman (2019): Problem 15.34.

   Adjacent antinodes of a standing wave on a string are 15.0 cm apart. A particle at an antinode oscillates in simple harmonic motion with amplitude 0.850 cm and period 0.0750 s. The string lies along the $+x$-axis and is fixed at $x = 0$.

   (a) How far apart are the adjacent nodes?

   (b) What are the wavelength, amplitude, and speed of the two traveling waves that form this pattern?

   (c) Find the maximum and minimum transverse speeds of a point at an antinode.

   (d) What is the shortest distance along the string between a node and an antinode?

6) Young and Freedman (2019): Problem 15.64.

   A strong string of mass 3.00 g and length 2.20 m is tied to supports at each end and is vibrating in its fundamental mode. The maximum transverse speed of a point at the middle of the string is 9.00 m/s. The tension in the string is 330 N.

   (a) What is the amplitude of the standing wave at its antinode?

   (b) What is the magnitude of the maximum transverse acceleration of a point at the antinode?

7) A harmonic wave travels down a string in the $+x$ direction. At position $x = 0$ and time $t = 0$, the following is observed: the displacement of the string is $+1.0$ cm, the transverse velocity is -2.0 cm/s, and the transverse acceleration is $-4.0\,\text{cm/s}^2$.

   (a) What is the frequency of the wave?

   (b) What is the amplitude of the wave?

8) A long, uniform rope of length L hangs vertically. The only tension in the rope is that produced by its own weight.

   (a) Show that, as a function of the distance y from the lower end of the rope, the speed of a transverse wave pulse on the rope is $\sqrt{gy}$.

   (b) How much time does it take for a wave pulse to travel from one end of the rope to the other?

9) Using continuity conditions on a string, we derived the relative amplitudes for transmitted and reflected waves at a boundary. Show that the average power of the *transmitted* wave plus the average power of the *reflected* wave is equal to the average power of the *incident* wave. (Otherwise, energy would not be conserved.)

2 Sound and Light

8/12: 1) Young and Freedman (2019): Problem 15.25.

A jet plane at takeoff can produce sound of intensity $10.0\,\mathrm{W/m^2}$ at $30.0\,\mathrm{m}$ away. But you prefer the tranquil sound of normal conversation, which is $1.0\,\mu\mathrm{W/m^2}$. Assume that the plane behaves like a point source of sound.

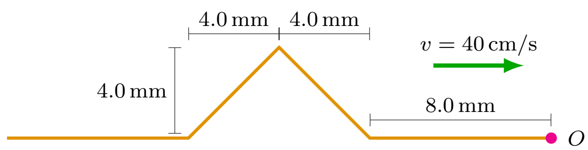
(a) What is the closest distance you should live from the airport runway to preserve your peace of mind?

(b) What intensity from the jet does your friend experience if she lives twice as far from the runway as you do?

(c) What power of sound does the jet produce at takeoff?

2) Young and Freedman (2019): Problem 15.28.

Reflection. A wave pulse on a string has the dimensions shown in the following figure at $t = 0$. The wave speed is $40\,\mathrm{cm/s}$.



(a) If point O is a fixed end, draw the total wave on the string at $t = 15\,\mathrm{ms}$, $20\,\mathrm{ms}$, $25\,\mathrm{ms}$, $30\,\mathrm{ms}$, $35\,\mathrm{ms}$, $40\,\mathrm{ms}$, and $45\,\mathrm{ms}$.

(b) Repeat part (a) for the case in which point O is a free end.

3) Young and Freedman (2019): Problem 16.24.

The fundamental frequency of a pipe that is open at both ends is $524\,\mathrm{Hz}$.

(a) How long is the pipe?

If one end is now closed, find the new fundamental's

(b) Wavelength;

(c) Frequency.

4) Young and Freedman (2019): Problem 16.35.

Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is $12.0\,\mathrm{m}$ to the right of speaker A. The frequency of the waves emitted by each speaker is $688\,\mathrm{Hz}$. You are standing between the speakers, along the line connecting them, and are at a point of constructive interference. How far must you walk toward speaker B to move to a point of destructive interference?

5) Young and Freedman (2019): Problem 16.50.

The siren of a fire engine that is driving northward at $30.0\,\mathrm{m/s}$ emits a sound of frequency $2\,000\,\mathrm{Hz}$. A truck in front of this fire engine is moving northward at $20.0\,\mathrm{m/s}$.

(a) What is the frequency of the siren's sound that the fire engine driver hears reflected from the back of the truck?

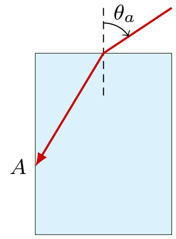
(b) What wavelength would this driver measure for these reflected sound waves?

6) Young and Freedman (2019): Problem 16.62.

A bat flies toward a wall, emitting a steady sound of frequency $1.70\,\mathrm{kHz}$. This bat hears its own sound plus the sound reflected by the wall. How fast should the bat fly in order to hear a beat frequency of $8.00\,\mathrm{Hz}$?

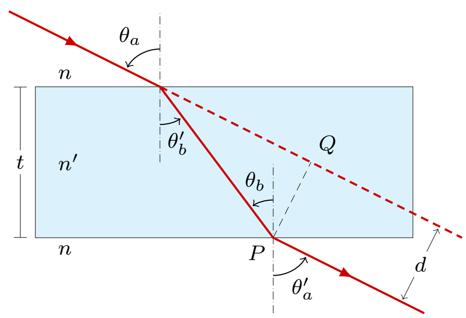
7) Young and Freedman (2019): Problem 33.39.

   A ray of light is incident in air on a block of a transparent solid whose index of refraction is n. If $n = 1.38$, what is the *largest* angle of incidence θ_a for which total internal reflection will occur at the vertical face (point A in the below figure)?



8) Young and Freedman (2019): Problem 33.52.

   Light is incident in air at an angle θ_a on the upper surface of a transparent plate, the surfaces of the plate being plane and parallel to each other.



   (a) Prove that $\theta_a = \theta'_a$.

   (b) Show that this is true for any number of different parallel plates.

   (c) Prove that the lateral displacement d of the emergent beam is given by the relationship

$$d = t \cdot \frac{\sin(\theta_a - \theta'_b)}{\cos \theta'_b}$$

   where t is the thickness of the plate.

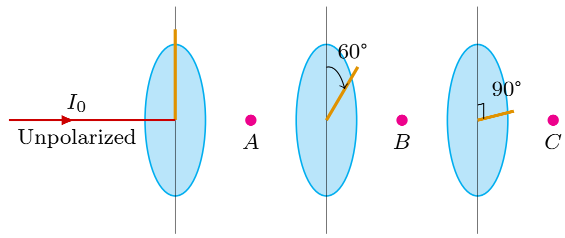
   (d) A ray of light is incident at an angle of 66.0° on one surface of a glass plate 2.40 cm thick with an index of refraction of 1.80. The medium on either side of the plate is air. Find the lateral displacement between the incident and emergent waves.

9) An airplane has a defective speedometer. In order to figure out how fast the plane is flying, the pilot (an aspiring opera singer) leans out the window and sings "middle C" (a note of frequency 262 Hz) at a mountain looming ahead. The echo off the mountain is heard by the pilot as "middle A" (frequency 440 Hz). How fast is the plane flying?

3 Mirrors, Lenses, and Polarization

8/16: 1) Young and Freedman (2019): Problem 33.29.

A beam of unpolarized light of intensity I_0 passes through a series of ideal polarizing filters with their polarizing axes turned to various angles as shown in the following figure.

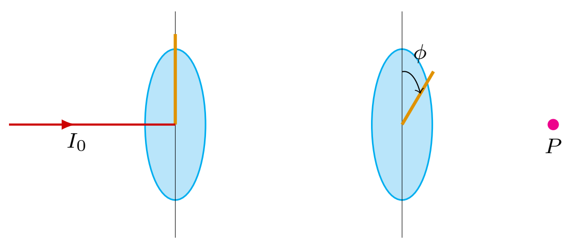


(a) What is the light intensity (in terms of I_0) at points A, B, and C?

(b) If we remove the middle filter, what will be the light intensity at point C?

2) Young and Freedman (2019): Problem 33.30.

Light of original intensity I_0 passes through two ideal polarizing filters having their polarizing axes oriented as shown in the following figure. You want to adjust the angle ϕ so that the intensity at point P is equal to $I_0/10$.



(a) If the original light is unpolarized, what should ϕ be?

(b) If the original light is linearly polarized in the same direction as the polarizing axis of the first polarizer the light reaches, what should ϕ be?

3) Young and Freedman (2019): Problem 34.40.

A converging lens with a focal length of 12.0 cm forms a virtual image 8.00 mm tall, 17.0 cm to the right of the lens. Determine the position and size of the object. Is the image erect or inverted? Are the object and image on the same side or opposite sides of the lens? Draw a principal-ray diagram for this situation.

4) Young and Freedman (2019): Problem 34.43.

Combination of Lenses I. A 1.20 cm-tall object is 50.0 cm to the left of a converging lens of focal length 40.0 cm. A second converging lens, this one having a focal length of 60.0 cm, is located 300.0 cm to the right of the first lens along the same optic axis.

(a) Find the location and height of the image (call it I_1) formed by the lens with a focal length of 40.0 cm.

(b) I_1 is now the object of the second lens. Find the location and height of the image produced by the second lens. This is the final image produced by the combination of lenses.

5) Young and Freedman (2019): Problem 34.69.

You are in your car driving on a highway at 25 m/s when you glance in the passenger-side mirror (a convex mirror with radius of curvature 150 cm) and notice a truck approaching. If the image of the truck is approaching the vertex of the mirror at a speed of 1.9 m/s when the truck is 2.0 m from the mirror, what is the speed of the truck relative to the highway?

6) Young and Freedman (2019): Problem 34.77.

   (a) You want to use a lens with a focal length of 35.0 cm to produce a real image of an object, with the height of the image twice the height of the object. What kind of lens do you need and where should the object be placed?

   (b) Suppose you want a virtual image of the same object, with the same magnification — what kind of lens do you need, and where should the object be placed?

7) Young and Freedman (2019): Problem 34.86.

   An object is placed 22.0 cm from a screen.

   (a) At what two points between object and screen may a converging lens with a 3.00 cm focal length be placed to obtain an image on the screen?

   (b) What is the magnification of the image for each position of the lens?

8) Young and Freedman (2019): Problem 34.90.

   Two Lenses in Contact.

   (a) Prove that when two thin lenses with focal lengths f_1 and f_2 are placed *in contact*, the focal length of the combination is given by the relationship

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

   (b) A converging meniscus lens has an index of refraction of 1.55 and radii of curvature for its surfaces of magnitudes 4.50 cm and 9.00 cm. The concave surface is placed upward and filled with carbon tetrachloride (CCl_4), which has $n = 1.46$. What is the focal length of the CCl_4-glass combination.

9) We start out with vertically polarized light. This light then passes through a stack of N sheets of Polaroid. The first sheet has its transmission axis oriented $90^\circ/N$ from the vertical. Subsequent sheets are oriented such that each is rotated by $90^\circ/N$ relative to the one before it, so that the last sheet is oriented horizontally. Show that in the limit as $N \to \infty$, there will be *no* loss of intensity through the stack.

4 Interference and Diffraction

8/19: 1) Young and Freedman (2019): Problem 35.9.

Two slits spaced 0.450 mm apart are place 75.0 cm from a screen. What is the distance between the second and third dark lines of the interference pattern on the screen when the slits are illuminated with coherent light with a wavelength of 500 nm?

2) Young and Freedman (2019): Problem 35.20.

Two slits spaced 0.072 0 mm apart are 0.800 m from a screen. Coherent light of wavelength λ passes through the two slits. In their interference pattern on the screen, the distance from the center of the central maximum to the first minimum is 3.00 mm. If the intensity at the peak of the central maximum is 0.060 0 $\mathrm{W/m^2}$, what is the intensity at points on the screen that are

(a) 2.00 mm from the center of the central maximum;

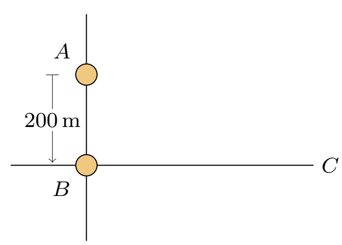
(b) 1.50 mm from the center of the central maximum?

3) Young and Freedman (2019): Problem 35.39.

Suppose you illuminate two thin slits by monochromatic coherent light in air and find that they produce their first interference *minima* at $\pm 35.20^\circ$ on either side of the central bright spot. You then immerse these slits in a transparent liquid and illuminate them with the same light. Now you find that the first minima occur at $\pm 19.46^\circ$ instead. What is the index of refraction of this liquid?

4) Young and Freedman (2019): Problem 35.41.

Two radio antennae radiating in phase are located at points A and B, 200 m apart (see the following figure). The radio waves have a frequency of 5.80 MHz. A radio receiver is moved out from point B along a line perpendicular to the line connecting A and B (line BC shown in the following figure). At what distances from B will there be *destructive* interference? (Note: The distance of the receiver from the sources is not large in comparison to the separation of the sources.)



5) Young and Freedman (2019): Problem 35.45.

White light reflects at normal incidence from the top and bottom surfaces of a glass plate ($n = 1.52$). There is air above and below the plate. Constructive interference is observed for light whose wavelength in air is 477.0 nm. What is the thickness of the plate if the next longer wavelength for which there is constructive interference is 540.6 nm?

6) Young and Freedman (2019): Problem 36.1.

Monochromatic light from a distant source is incident on a slit 0.750 mm wide. On a screen 2.00 m away, the distance from the central maximum of the diffraction pattern to the first minimum is measured to be 1.35 mm. Calculate the wavelength of the light.

7) Young and Freedman (2019): Problem 36.14.

   Monochromatic light of wavelength $\lambda = 620\,\text{nm}$ from a distant source passes through a slit $0.450\,\text{mm}$ wide. The diffraction pattern is observed on a screen $3.00\,\text{m}$ from the slit. In terms of the intensity I_0 at the peak of the central maximum, what is the intensity of the light at the screen the following distances from the center of the central maximum:

   (a) $1.00\,\text{mm}$.
   (b) $3.00\,\text{mm}$.
   (c) $5.00\,\text{mm}$.

8) Young and Freedman (2019): Problem 36.26.

   Monochromatic light is at normal incidence on a plane transmission grating. The first-order maximum in the interference pattern is at an angle of 11.3°. What is the angular position of the fourth-order maximum?

9) In an experiment, light of wavelength λ falls on three slits evenly spaced a distance d apart, and an interference pattern is observed on a distant screen. If the intensity at 0° with two adjacent slits open (and the third slit covered) is I_2, what is the intensity at 0° when all three slits are open, I_3?

5 Thermodynamics

8/23: 1) Young and Freedman (2019): Problem 36.21.

An interference pattern is produced by light of wavelength 580 nm from a distant source incident on two identical parallel slits separated by a distance (between centers) of 0.530 mm.

(a) If the slits are very narrow, what would be the angular positions of the first-order and second-order, two-slit interference maxima?

(b) Let the slits have width 0.320 mm. In terms of the intensity I_0 at the center of the central maximum, what is the intensity at each of the angular positions in part (a)?

2) Young and Freedman (2019): Problem 36.44.

Observing Jupiter. You are asked to design a space telescope for Earth orbit. When Jupiter is 5.93×10^8 km away (its closest approach to Earth), the telescope is to resolve, by Rayleigh's criterion, features on Jupiter that are 250 km apart. What minimum-diameter mirror is required? Assume a wavelength of 500 nm.

3) Young and Freedman (2019): Problem 18.6.

You have several identical balloons. You experimentally determine that a balloon will break if its volume exceeds 0.900 L. The pressure of the gas inside the balloon equals air pressure (1.00 atm).

(a) If the air inside the balloon is at a constant 22.0 °C and behaves as an ideal gas, what mass of air can you blow into one of the balloons before it bursts?

(b) Repeat part (a) if the gas is helium rather than air.

4) Young and Freedman (2019): Problem 18.21.

Modern vacuum pumps make it easy to attain pressures of the order of 1×10^{-13} atm in the laboratory. Consider a volume of air and treat the air as an ideal gas.

(a) At a pressure of 9.00×10^{-14} atm and an ordinary temperature of 300.0 K, how many molecules are present in a volume of 1.00 cm^3?

(b) How many molecules would be present at the same temperature but at 1.00 atm instead?

5) Young and Freedman (2019): Problem 18.38.

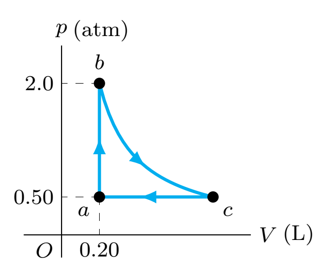
Perfectly rigid containers each hold n moles of an ideal gas, one being hydrogen (H_2) and the other being neon (Ne). If it takes 300 J of heat to increase the temperature of the hydrogen by 2.50 °C, by how many degrees will the same amount of heat raise the temperature of the neon?

6) Young and Freedman (2019): Problem 19.10.

Five moles of an ideal monatomic gas with an initial temperature of 127 °C expand and, in the process, absorb 1 500 J of heat and do 2 100 J of work. What is the final temperature of the gas?

7) Young and Freedman (2019): Problem 19.43.

The following figure shows a pV-diagram for 0.004 0 mol of *ideal* H_2 gas. The temperature of the gas does not change during segment bc.



(a) What volume does this gas occupy at point c?

(b) Find the temperature of the gas at points a, b, and c.

(c) How much heat went into or out of the gas during segments ab, ca, and bc? Indicate whether the heat has gone into or out of the gas.

(d) Find the change in the internal energy of this hydrogen during segments ab, bc, and ca. Indicate whether the internal energy increased or decreased during each segment.

8) You hear the weather report on the radio. However, the announcer forgets to say what scale is being used: Celsius or Fahrenheit. If it doesn't matter, how cold is it outside?

9) A pinhole camera can produce a surprisingly sharp image. The key is using a small hole so only a narrow bundle of rays is allowed through. However, if the pinhole is too small, then diffraction will limit the sharpness of the image. The optimum pinhole size is one that makes the fuzziness due to bundle size comparable to the fuzziness due to diffraction. Assume the distance from pinhole to screen is 1 foot, and the wavelength of the light is 5 500 Å. What is the optimum size of the pinhole?

References

Young, H. D., & Freedman, R. A. (2019). *University physics with modern physics* (Fifteenth). Pearson Education.