PHYS 18500 (Intermediate Mechanics) Problem Sets

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Contents

1 Linear Motion

10/6: **1.** One particle of mass m is subject to force

$$F = \begin{cases} -b & x > 0 \\ b & x < 0 \end{cases}$$

A second particle is subject to force $F = -kx$.

A) Find the potential energy functions for each force. (1 pt)

B) Find the trajectory $x(t)$ for each particle during the first period, assuming it is released at the origin at $t = 0$ at velocity $v > 0$. Describe the motion of each particle, and sketch each trajectory $x(t)$. Solve for the period and the points x^* where each particle is stationary. (6 pts)

C) Solve for v such that the trajectories have the same period. Which particle travels further? Given this v, how many times do the two particles' trajectories cross during one period? (3 pts)

2. The potential energy of a particle of mass m is

$$V(x) = E((\mu_1 x + a)(\mu_2 x - b))^2$$

where $E > 0$ is a constant with units of energy, and $\mu_1, \mu_2, a, b > 0$.

A) Sketch the potential energy function. Identify and label the locations of any minima. (3 pts)

B) Write expressions for the potential energy a distance δx from each minimum, up to second order in δx. (2 pts)

C) For each minimum, what condition should δx fulfill for this approximation to be valid? (i.e., δx should be small compared to what length scale?) (3 pts)

D) For each minimum, use your approximate potential energy function to specify the trajectory $x(t)$ of a particle of mass m released from rest a distance δx away from the minimum. (2 pts)

3. Kibble and Berkshire (2004), Q2.13. A particle falling under gravity is subject to a retarding force proportional to its velocity.

A) Find its position as a function of time, if it starts from rest. (7 pts)

B) Show that it will eventually reach a terminal velocity, and solve for this velocity. (3 pts)

4. Suppose we have an oscillator with negative damping described by

$$m\ddot{x} + \lambda\dot{x} + kx = 0$$

where $\lambda < 0$ and $k > 0$.

A) Solve for $x(t)$ for the particle, if it begins at velocity v at the origin. (4 pts)

B) Describe the behavior of the particle. Under what conditions does it oscillate? Sketch the possible trajectories. (4 pts)

C) In which case does the particle gain energy the fastest for large times? Explain. (2 pts)

5. Kibble and Berkshire (2004), Q2.25. For an oscillator under periodic force $F(t) = F_1 \cos(\omega_1 t)\ldots$

A) Calculate the **power** (defined as the rate at which the force does work). (4 pts)

B) Show that the **average power** (defined as the time average over a complete cycle) is $P = m\omega_1^2 a_1^2/\gamma$, and hence verify that it is equal to the average rate at which energy is dissipated against the resistive force. (3 pts)

C) Show that the power P — as a function of ω_1 — is at a maximum at $\omega_1 = \omega_0$. Also find the values of ω_1 for which it has half its maximum value. (3 pts)

6. Kibble and Berkshire (2004), Q2.32. Find the Green's function of an oscillator in the case $\gamma > \omega_0$. Use it to solve the problem of an oscillator that is initially in equilibrium, and is subjected from $t = 0$ to a force increasing linearly with time via $F = ct$.

7. How long did you spend on this problem set?

References

Kibble, T. W. B., & Berkshire, F. H. (2004). *Classical mechanics* (Fifth). Imperial College Press.