Chapter 1

Introduction

1.1 Introduction; Principle of Relativity; Newton's Laws

9/27:

- Course logistics to start.
  - Prof: Elizabeth Jerison, GCIS E231, OH M 4-5:30, (ejerison@uchicago.edu).
  - Discussion sections start *next week* on W 4:30-5:20; we'll receive additional information.
  - Problem session by TAs: Th 4-7pm, location TBA.
  - HW due Fridays at 11:30am on Canvas.
    - Write names of anyone you work with at the bottom of the page.
    - Optional makeup PSet at the end of the quarter to drop lowest grade.
  - Solutions posted Monday.
    - Thus, late assignments accepted up until Monday.
  - Midterm: 11/1/23, 4:30-5:15 *or* 4:30-6:00.
    - She dislikes 45 minute exams, so there is the option to take a longer exam.
    - 45 min exam will be *half* the 90 minute exam and scored for full credit.
    - There may be conflict makeup times, too.
  - More syllabus stuff on Canvas; we can email or stop at OH if we have questions.
- Course material overview.
  - Review Newtonian mechanics.
  - Lagrangian mechanics.
    - Same laws of physics, but easier to generalize to a broader class of problems, which makes it more powerful in a broader class of problems.
    - An equivalent formulation.
  - Hamiltonian mechanics.
    - Symmetries of the Hamiltonian give rise to previous courses' conservation laws.
  - Post-Thanksgiving break: Intro to dynamical systems, nonlinear systems.
    - No closed-form analytical solutions, but you can still put a lot of constraints on behavior from a geometric perspective.
  - Introduce Lagrangian pretty quickly; do it more formally in November.
- Brief note about "Physics."
- **Physics**: Extract math to govern matter.

- Three stages.
  1. Make observations; see quantitative patterns.
  2. Formulate hypotheses (mathematical models).
  3. Test + iterate.
- **Law**: A well-tested hypothesis. *Also known as* **principle**.
- By necessity, the very confusing and engaging process of creating this knowledge is often given short shrift, and we are only presented in class with the very successful hypotheses.
- The subject of mechanics.
  - We have N particles with positions $\vec{r}_1, \dots, \vec{r}_N$ at $t = t_0$, and we want to predict their positions at all future times.
  - The exploration of this problem is fundamental to mechanics and, in many cases, all physics.
- Notation.
  - Tries to stick with the textbook.
  - Cartesian unit vectors: $\hat{\imath} = (1,0,0)$, $\hat{\jmath} = (0,1,0)$, and $\hat{k} = (0,0,1)$.
  - Position: $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.
  - Velocity: $\dot{\vec{r}} = \mathrm{d}r/\mathrm{d}t = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$.
  - Velocity: $\ddot{\vec{r}} = \mathrm{d}^2r/\mathrm{d}t^2 = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$.
  - Momentum: $\vec{p} = m\vec{v}$.
- Principle of relativity.
- Galileo's relativity principle.
  - Updated by Einstein via special relativity, but that's outside the scope of this course.
  - Relies on the principle that space is **homogeneous** and **isotropic**.[1] Additionally, time is homogeneous.
  - There are **inertial reference frames**, which move at a constant velocity relative to one another.
  - All accelerations and particle interactions are the same in any inertial reference frame, i.e., $\vec{r} = \vec{r}\,' + \vec{v}t$ and $t = t'$; this is a **Galilean transformation**.
  - Note 1: It could be different!
    - Aristotle thought that there was an absolute center to the universe (in the center of the Earth) and that the laws of physics varied with distance from that point. However, we have no empirical evidence to support this claim.
  - Note 2: This breaks down as $\|\vec{v}\| \to c$.
    - However, we can use Lorentz transformation to recover laws of mechanics, but this is special relativity.
  - Note 3: Conservation laws arise directly from relativity.
- **Homogeneous**: No special direction.
- **Isotropic**: No absolute position.
- Newtonian mechanics.
  - If we know what to call the force $\vec{V}_i$ on particle i, then we know the future positions via $\vec{F}_i = m_i\vec{a}_i$ (Newton's second law).

[1] I.e., affine.

- The fact that forces and acceleration are only related through a scalar mass is quite nontrivial!
- This law gives us equations of motion (EOM), which allow us to solve for what's going to happen to our particle.
- EOMs:
$$\ddot{\vec{r}} = \frac{\vec{F}_i(\vec{r}_1,\dots,\vec{r}_N,\dot{\vec{r}}_1,\dots,\dot{\vec{r}}_N,t)}{m}$$
  - This is a series of 2nd order ODEs for position of i, $\vec{r}_i(t)$.
  - Solvable if we have 2 initial conditions: $\vec{r}(t=0)$ and $\dot{\vec{r}}(t=0)$.
- Newton's third law:
$$\vec{F}_i = \sum_{j=1}^{N} \vec{F}_{ij}$$
where $\vec{F}_{ij}$ is the force on i due to j.
  - $\vec{F}_{ij}$ depends on $\vec{r}_i$, $\vec{r}_j$, $\vec{v}_i$, and $\vec{v}_j$.
  - In fact, it depends on $(\vec{r}_i - \vec{r}_j)$ and $(\vec{v}_i - \vec{v}_j)$.
  - Also, $\vec{F}_{ij} = -\vec{F}_{ji}$.
  - Again, it could have been different; it's just that no one has ever found a force that depends on three bodies.

- **Force**: Something that generates an acceleration.
- Physical phenomena that aren't mechanical?
  - Most people would say that there are constraints, e.g., electricity, speed of light.
- Consequence #1 of Newton's Laws: Conservation of momentum.
  - Suppose we have 2 bodies.
  - From the third then second law,
$$\begin{aligned}\vec{F}_i &= -\vec{F}_j\\ m_1\vec{a}_1 &= -m_2\vec{a}_2\end{aligned}$$
  - It follows by adding $m_2\vec{a}_2$ to both sides and integrating that the total momentum in the system is constant.
- Consequence #2 of Newton's Laws: Mass is additive.
  - Suppose we have 3 bodies.
  - From consecutive applications of the third law,
$$\begin{aligned}m_1\vec{a}_1 &= \vec{F}_{12} + \vec{F}_{13}\\ m_2\vec{a}_2 &= \vec{F}_{21} + \vec{F}_{23}\\ m_3\vec{a}_3 &= \vec{F}_{31} + \vec{F}_{32}\end{aligned}$$
  - Since $\vec{F}_{ij} = -\vec{F}_{ji}$, adding the three equations above causes the right side to cancel, yielding
$$m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 = 0$$
  - If we stick 2 & 3 together to create a composite particle 4 with $\vec{a}_4 := \vec{a}_2 = \vec{a}_3$, then
$$\begin{aligned}m_1\vec{a}_1 + (m_2+m_3)\vec{a}_4 &= 0\\ m_1\vec{a}_1 + m_4\vec{a}_4 &= 0\end{aligned}$$

  - Thus, by setting the two equations above equal to each other and simplifying, we obtain

$$m_4 = m_2 + m_3$$

  - This is summarized as the **principle of mass additivity**.

- **Principle of mass additivity**: The mass of a composite object is the sum of the masses of its elementary components.
  - Another very simple but very fundamental concept.