8 Final Exam Review

12/4: **1.** Kibble and Berkshire (2004), Q7.2. Where is the center of mass of the Sun-Jupiter system? (The mass ratio is $M_S/M_J = 1047$. The semi-major axis of Jupiter's orbit is 5.20 AU, where $1\,\text{AU} = 1.50 \times 10^8$ km is the semi-major axis of the Earth's orbit.) Through what angle does the Sun's position — as seen from the Earth — oscillate because of the gravitational attraction of Jupiter?

2. Kibble and Berkshire (2004), Q9.6.

A) A simple pendulum supported by a light rigid rod of length l is released from rest with the rod horizontal. Find the reaction at the pivot as a function of the angle of inclination.

B) For the cube of Problem 9.1 (a uniform solid cube of edge length $2a$ suspended from a horizontal axis along one edge), find the horizontal and vertical components of the reaction on the axis as a function of its angular position. Compare your answer with the corresponding expressions for the equivalent simple pendulum.

3. Kibble and Berkshire (2004), Q9.16. A uniformly charged sphere is spinning freely with angular velocity $\vec{\omega}$ in a uniform magnetic field $\vec{B}$. Taking the z-axis in the direction of $\vec{\omega}$, and $\vec{B}$ in the xz-plane, write down the moment about the center of the magnetic force on a particle at $\vec{r}$. Evaluate the total moment of the magnetic force on the sphere, and show that it is equal to $(q/2M)\vec{J} \times \vec{B}$, where q and M are the total charge and mass, respectively. Hence show that the axis will precess around the direction of the magnetic field with precessional angular velocity equal to the Larmor frequency

$$\omega_{\text{L}} = \frac{qB}{2M}$$

What difference would it make if the charge distribution were spherically symmetric, but non-uniform?

4. Kibble and Berkshire (2004), Q12.5. Consider a simple pendulum of mass m and length l, hanging in a trolley of mass M running on smooth horizontal rails. The pendulum swings in a plane parallel to the rails. Use the position x of the trolley and the angle of inclination θ of the pendulum as generalized coordinates. Find the Hamiltonian of this pendulum. Show that x is ignorable. To what symmetry does this correspond?

5. A bead of mass m is on a circular hoop of radius R, oriented vertically (i.e., with its radius lined up with $\hat{k}$). The hoop is rotating at constant rate ω about $\hat{k}$.

A) Find the Hamiltonian for the system.

B) Find Hamilton's equations of motion.

C) Find and classify the fixed points of the system for all values of $\omega > 0$. For what value of ω does a bifurcation occur?

D) Draw a bifurcation diagram using the parameter $\gamma = R\omega^2/g$, i.e., draw a plot in the γ-θ plane where solid lines represent stable fixed points, and dashed lines represent unstable fixed points. (Hint: This is called a **pitchfork bifurcation**.) Sketch an example trajectory $\theta(t)$ if $\omega(t)$ is being slowly turned up via

$$\omega(t) = \sqrt{\frac{gat}{R}}$$

where $a \ll \sqrt{g/l}$.