A Unified Model for Spatial and Temporal Information

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Many applications of spatial information systems require not just spatial data handling but a unified approach to space and time. This paper begins by motivating this requirement with some examples, continues by identifying some of the key issues in this area and then discusses a unified generic model for information which is referenced to two spatial dimensions and two temporal dimensions (database and event times).

1. INTRODUCTION

Developments in hardware, database technology and graphics during the last decade have made possible the development of systems which are capable of powerfully supporting the handling of spatially referenced data. Such capabilities open up a very wide range of applications, from decision support for local area planning to global environmental management. The arguments in this paper are built upon the premise that for such applications to be properly handled, systems must support not only spatial analysis but a unified model of spatiotemporal information.

Much information which is referenced to space is also referenced to time. Indeed, space, time and process are closely interconnected. In current systems, the temporal dimension often plays a subordinate role, temporal variation being represented by a series of static snapshots. For such systems to truly come of age, the temporal and spatial domains must be unified. This paper describes work done on the modelling of information which references a unified spatiotemporal dimensionality. The paradigm adopted is object-based (in contrast to the field-based approach, where the information space is viewed as a set of variational fields over a spatiotemporal plenum) in the sense that the 'information space' is viewed as being populated with independently existing, self-contained objects, each encapsulating state and behaviour.

Traditionally, databases have held information about the state of the world which is independent of time and space. However, in the last decade there has been considerable research on general temporal databases (for a recent survey, see Soo, 1991) and another large body of work on spatial or geographic databases (see Guenther and Buchmann, 1990; Maguire *et al.*, 1991). Unification of the two branches of activity, allowing management of spatially and temporally referenced information in the same system, is relatively novel (for a recent list of papers, see Al-Taha *et al.*, 1993).

The paper is structured to progress from concrete motivating examples to a general theory. In the following section, three examples are presented which show some of the possibilities for a spatiotemporal system. A model is then presented which fuses ideas on purely spatial modelling using simplicial complexes with temporal modelling involving two orthogonal dimensions representing database and event time. The latter stages of the work discuss a query algebra, similar in some respects to relational algebra.

2. MOTIVATING EXAMPLES

2.1. Administrative areas

Administrative regions reference areal objects which represent the areal extent of the regions. Assume, as in the UK, that every 10 years, when the new census is taken, the boundaries may have been changed. Census and other statistics often relate to these regions. In order to conduct longitudinal studies, it is necessary to relate statistics to the correct historical versions of the regions to which the data relate. Thus, what is required is an information system containing historical spatial data on the regions. Transactional information, while useful, is not essential here due to the long periods between update.

Figure 1 shows a highly simplified picture of change of regional structure over time. In the census year 1971, the district Dis_1 is divided into two regions Reg_1 and Reg_2. In 1981, the spatial extent of Reg_1 is reduced and that of Reg_2 is correspondingly enlarged. In 1991, the spatial extent of the entire district is increased and the spatial extent of Reg_2 is reduced to accommodate a new region, Reg_3.

Suppose that there exist various non-spatial datasets referenced to the administrative areas. For example, suppose that data exists on population totals referenced

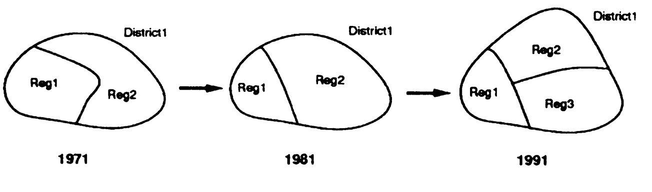

FIGURE 1. Change of regional structure through time.

to regions and mortality figures referenced to districts. The following questions are examples of those that may then be posed:

1. What variation has there been in the population density of Reg_2 between 1971 and 1991?
2. What variation has there been in the morbidity ratio (ratio of deaths to population) of Reg_2 between 1971 and 1991?

The characteristics exemplified here are of information referenced to complex spatial patterns which suffer discrete change with respect to time. A simplifying feature is that the temporal component is essentially unidimensional, in the sense that we are concerned mainly here with discrete changes in the real world being modelled in the information system, rather than the changes to the information system itself, i.e. we are dealing with an *historical* database. The temporal dimension is also linear, in that neither future nor past branching nor any cyclicity is admitted. In fact, these simplifications are too stringent for some of the systems under current consideration. For example, we may wish to hold information on projections of future boundaries under consideration by the UK Boundary Commission. Also, we may want to be able to retroactively change a boundary without deleting the earlier version. Such extensions to the basic '2-D space + 1-D time' are considered in the following examples.

2.2. Road networks information

This example was constructed by the author (Worboys, 1993) to motivate clearly the need for a unified model of two-dimensional space and two-dimensional time. Figure 2 shows three stages in the development of a bypass round a town. The diagrams indicate the information which is available to be input into the database.

In 1993, there exists information on the spatial configuration of an existing road through the town along with the spatial configuration of a bypass road *abc* whose construction is projected for the following year. In 1994, there exists information that the bypass is constructed in 1994, but with a different spatial configuration *adefc* to the 1993 projection. (Maybe the road had to be re-routed around a conservation area at point *b*.) In 1995, a revision has been made. Further information has been received which indicates that some parts of the bypass (*efc* shown by a hatched line) were not built until 1995 itself. The type of query envisaged to a system containing this kind of information is exemplified by:

> 'Retrieve the spatial configuration of the bypass, as it was forecast in 1993' (query occurred in 1995).

It is clear that, unlike the preceding example, there are two temporal dimensions operative in this example. Later sections will show how these dimensions are modelled.

2.3. Land ownership

The final example shows some of the requirements that land information systems have concerning land ownership. Information related to land ownership comprises spatial, temporal, legal and other components. Ownership is affected by contracts, death and inheritance, legal proceedings, fire, etc. The example we use is a much modified version of one given by Al-Taha (1992). Figure 3 shows the spatial and ownership variation in a fictitious land area through some decades of this century. The chronology is as follows.

Year 1908 (original records). Information is held on a street, a land parcel owned by Jeff (parcel 1), a land parcel owned by Jane (parcel 2), further parcels and buildings.

Year 1920. Jane has incorporated parcel number 3 into her ownership, now named parcel number 5.

Year 1938. Parcel 4 has been enlarged and a school has been built on it.

Year 1958. Jane's house has been destroyed by fire and Jane has died. Jack now owns the buildings and land in parcel 1.

Year 1960. The council gives notice of its intention to build a path through parcel 5 in 1962 so as to give better access to the school.

Year 1962. The building of the path is postponed until 1964.

Year 1964. Jack has built an extension which intrudes partly into parcel 5. The council has built the path through parcel 5.

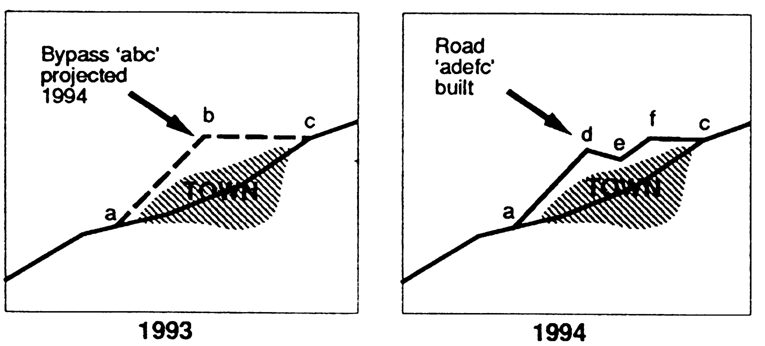

FIGURE 2. Bypass development.

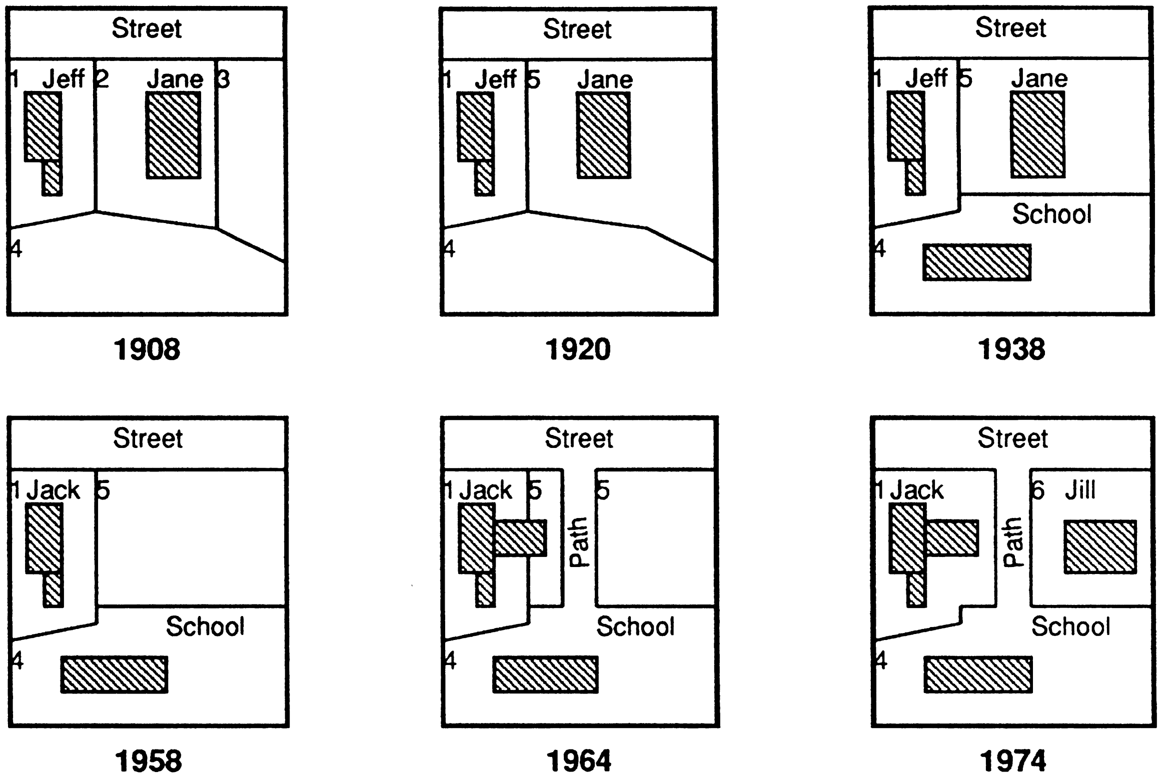

FIGURE 3. Spatiotemporal variation of land ownership.

Year 1974. Jack has incorporated part of parcel 5 (by adverse possession) into his ownership of parcel 1. Jill has taken possession of land parcel 6 and built a house upon it.

This example contains event and database temporality. For example, in 1962 (database time) information is received that the path, originally forecast in 1960 (database time) to be built in 1962 (event time) is postponed until 1964 (event time). Many of the event times are unclear from the information provided, for example, the exact year in which Jane's house was burned down is unknown.

3. TEMPORAL AND SPATIAL DATABASES

The above examples motivate the requirement for a model which handles spatial configurations existing and varying in at least two temporal dimensions. This section discusses the work on temporal and spatial systems which are separately germane to this paper.

3.1. Temporal databases

The information system is required to handle at least two separate components of time.

- *Database time*, exemplified here in the land ownership example by the years 1908, 1920,...,1974; and in the road network example, by the three diagrams, 1993, 1994 and 1995. This is the time when transactions actually take place with the information system.
- *Event time*, when the events actually occur in the application. In the administrative areas example, all the times are event times; in the road network example, the event times are shown inside the frames of the diagrams.

3.1.1. Previous work on temporal databases

There is now a large body of research on temporal information systems. Such systems can be subdivided into four types: *static*, *historic*, *rollback* and *bitemporal* (Snodgrass and Ahn, 1985; Snodgrass, 1992). Each type supports zero to two temporal dimensions. *Static* systems support neither database nor event time. *Historic* systems support only event time. Such a system would be required for the administrative areas example. *Rollback* systems support only database time and *bitemporal* systems (sometimes termed just *temporal*) support both database and event times. Table 1 shows the temporal dimension(s) which are supported in each case. Rollback systems store and manage the transaction history of the

TABLE 1. Temporal dimensions supported by system types

	No support for transaction time	*Support for transaction time*
No support for event time	static	rollback
Support for event time	historic	bitemporal

information system. Historical systems handle the data relating when events happened in the real world. An early paper on historic databases is Clifford and Tansel (1985). More recently, algebras have been developed for querying historic databases based on the *conceptual structure* data model (Tuzhilin and Clifford, 1990; Gabbay and McBrien, 1991). Bitemporal systems combine both the handling of the transaction history and the event times. Queries can be made both on the occurrence of transactions relating to events and also on the occurrence of the events themselves. For work in the area of bitemporal systems, see Ariav (1986) and Snodgrass (1987), for example.

3.1.2. Bitemporal elements

Bitemporal references may be expressed using *bitemporal elements* (*BTE*) (Snodgrass, 1992). Event times and database times are measured along two orthogonal axes. A BTE is the union of a finite set of Cartesian products of intervals of database and event time. Let T_D and T_E be the domains of database and event *chronons*, respectively. Assume that the domain T_E contains elements $-\infty$ and ∞, representing the indefinite past (or initial state) and future (or final state), respectively.

Definition 1 (bitemporal element). A *bitemporal element* or *BTE* is defined to be the union of a finite set of Cartesian products of intervals of the form $I_D \times I_E$, where I_D is an interval of database time and I_E is an interval of event time.

The semantics expressed by a BTE T are that $(t_D, t_E) \in T$ if, and only if, at time t_D there is information in the database that the object bitemporally referenced by T exists as event time t_E.

To illustrate the concept, Figure 4 shows the BTE which is to be associated with the portion of the bypass, labelled *ef* in Figure 2. In database time 1993, there is no event time in which the spatial configuration *ef* exists, since the possibility of having to go around point *b* has not yet arisen. In database time 1994, object *ef* exists from event time 1994 into the indefinite future, since a transaction has taken place providing information about the completion of this section of the bypass. In database time 1995, object *ef* exists from event time 1995 into the indefinite future, since a transaction has occurred which has revised the completion of this section of the bypass to event time 1995.

We need additional constructs for later work on the unified spatiotemporal model. It may be the case that one object exists at all database-event times that a second object exists. This may be tested by examining their respective BTEs. The relation defined by $T \leqslant T'$ iff BTE T is a subset of BTE T', expresses this idea. Clearly, the relation $\leqslant$ is a partial ordering of BTEs.

A second idea required later is the observation that any of the usual Boolean set-theoretic operations constructed from set union, set intersection and set difference, when applied to two BTEs will result in another BTE, since the resulting set can be expressed as the union of a finite number of Cartesian products of intervals. Any binary operation on BTEs which returns a BTE will be called a *β-operation.*

3.2. Spatial databases

Spatial databases are the subject of growing attention amongst computer scientists, as this Special Issue of the *Computer Journal* indicates. Work has proceeded in various areas, notably spatial data structures and access methods (see, e.g. Guenther, 1988), spatial data models (Egenhofer *et al.*, 1989; Worboys, 1992) and spatial query languages (Egenhofer, 1989). An overview of research issues for computer science in this area is given in Guenther and Buchmann (1990).

The work presented here builds on the model developed in Worboys (1992) based upon combinatorial topology. Spatial objects, assumed to be embedded in Euclidean two-space, are represented as simplicial complexes. A *simplex* is either a single point, finite straight line segment or triangular area. A *simplicial complex* is a collection of non-overlapping simplexes, such that if a simplex belongs to the complex then so do all its component simplexes. A simplicial complex is uniquely determined by its maximal component simplexes. An instance taken from the land ownership example is given in Figure 5, which shows the school (as an areal feature)

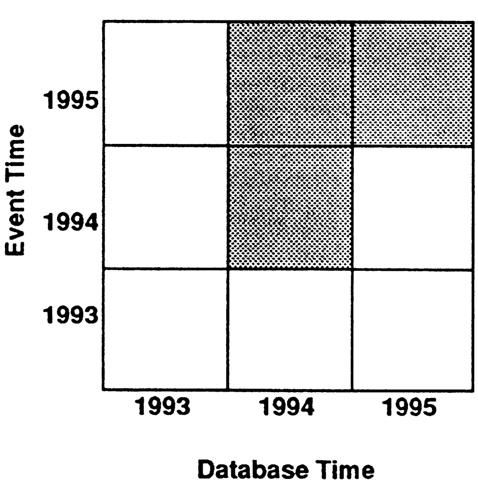

FIGURE 4. Example of a bitemporal element.

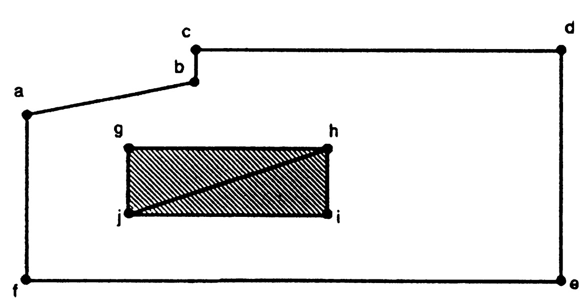

FIGURE 5. Simplicial complex $\{ab, bc, cd, de, ef, fa, ghj, hij\}$.

and the boundary of parcel number 4 (as a linear feature) as they were in 1938. Further details may be found in the author's earlier work cited above or in a text on combinatorial topology.

4. MODEL FOR SPATIO-BITEMPORAL INFORMATION

A *spatio-bitemporal object* is a unified object which has both spatial and bitemporal extents. In order to represent such objects, we attach BTEs as labels to components of simplicial complexes.

4.1. ST-simplexes

Intuitively, an ST-simplex is an elemental spatial object (simplex) to which is attached a bitemporal reference. More formally:

Definition 2 (ST-simplex). An *ST-simplex* is an ordered pair $\langle S, T\rangle$, where S is a simplex and T is a BTE.

The idea is that an ST-simplex is a spatio-bitemporal object expressing the fact that a basic spatial configuration exists over a given range of database-event times. Spatial and bitemporal projection functions are defined as follows:

Definition 3 (projection operators). Let the ST-simplex $R = \langle S, T\rangle$. Then, $\pi^s(R) = S$ and $\pi^t(R) = T$.

4.2. ST-complexes

The structure which represents a bitemporally-referenced spatial configuration is an *ST-complex*. An ST-complex is a collection of ST-simplexes, subject to some constraints. Firstly, the spatial projections of the constituent ST-simplexes must all be distinct. In other words, the same spatial simplex cannot occur more than once in an ST-complex with a different BTE. Secondly, the spatial projections of the constituent ST-simplexes must themselves form a spatial simplicial complex. Thirdly, any face of a spatial simplex occurring as a component in the ST-complex must have at least as much temporal referencing as its parent. For example, this condition ensures that the end-nodes of a line segment are always extant when the line-segment itself is extant. (A line segment cannot exist without its end-nodes.) These conditions may be formally expressed in the following definition:

Definition 4 (ST-complex). An *ST-complex*, C, is a finite set of ST-simplexes satisfying the properties:

1. The spatial projections of ST-simplexes in C are pairwise disjoint. Taken together, they form a spatial simplicial complex.
2. $\forall\, R, R' \in C \mid \pi^s(R)$ is a face of $\pi^s(R')$ implies that $\pi^t(R) \geqslant \pi^t(R')$.

Figure 6 shows the representation of the spatiotemporal extent of the boundary of land parcel 4 as an ST-complex. (Note that this does not represent the areal properties of the land parcel, but only the linear properties of its boundary. For the areal properties, the parcel would be represented using triangular simplexes.) Figure 6 also contains information about the entry of the path.

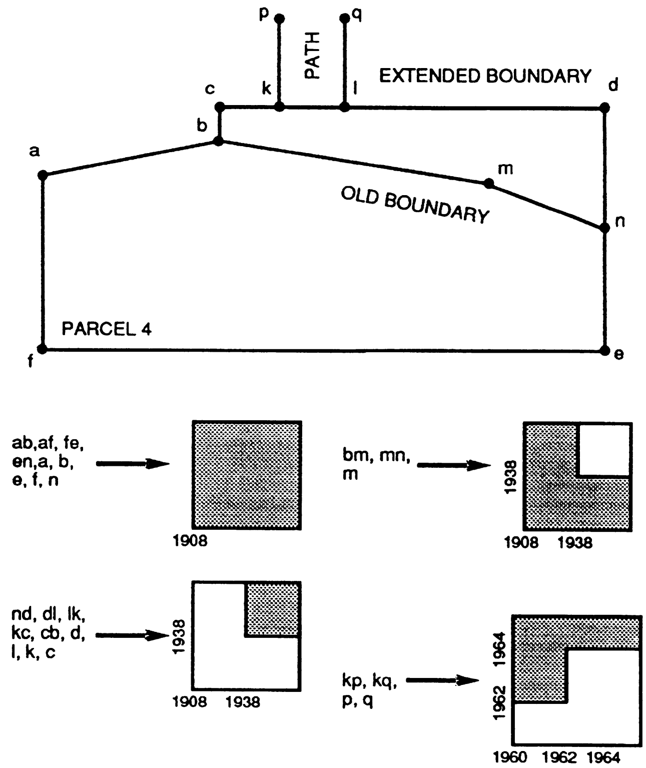

FIGURE 6. Parcel 4 boundary and path entry represented as ST-complex.

5. SPATIO-BITEMPORAL OPERATIONS

Table 2 classifies some of the possible operations on ST-complexes.

5.1. Equality and subset operations

Given two ST-complexes, C and C', the subset relationship between them is defined so that $C \sqsubset C'$ if, and only if, C is a subset of C', considering C and C' as embedded

TABLE 2. Classification of spatio-bitemporal relationships

Operator	*Operand*	*Operand*	*Resultant*
equals ($=$)	ST-complex	ST-complex	Boolean
subset ($\sqsubset$)	ST-complex	ST-complex	Boolean
boundary (∂)	ST-complex		ST-complex
S-project (π^s)	ST-complex		S-complex
T-project (π^t)	ST-complex		BTE
ST-β-product ($\times_\beta$)	ST-complex	ST-complex	ST-complex
ST-union ($\sqcup$)	ST-complex	ST-complex	ST-complex
ST-intersection ($\sqcap$)	ST-complex	ST-complex	ST-complex
ST-difference ($\backslash$)	ST-complex	ST-complex	ST-complex
S-select (σ^s_X)	ST-complex		ST-complex
T-select (σ^t_ϕ)	ST-complex		ST-complex

in four-dimensional space comprising two spatial and two temporal dimensions.

Definition 5 (ST-subset). For two ST-complexes, C and C', define $C \sqsubset C'$ iff for each $(x, y, z, w) \in \langle S, T \rangle \in C$, there is $\langle S', T' \rangle \in C'$ such that $(x, y, z, w) \in \langle S', T' \rangle$.

Definition 6 (ST-equals). Two ST-complexes, C and C', are defined to be equal if, and only if, $C \sqsubset C'$ and $C' \sqsubset C$.

5.2. Topological operations

We have included only one topological operator, namely boundary, although many others are possible.

Definition 7 (boundary). For ST-complex C, define

$$\partial C = \{\langle S, T \rangle \in C \mid S \in \partial \pi^s(C)\}$$

where we assume that the purely spatial boundary operation (also notated ∂) is already defined in the usual way.

5.3. Spatial and temporal projection

Purely spatial and bitemporal relationships may be calculated by firstly applying the relevant projection operators, π^s or π^t, and then continuing the analysis in the purely spatial or bitemporal domains. Intuitively, the spatial projection of an ST-complex is a complex representing the totality of its spatial extent, considered over all database and event time. The bitemporal projection of an ST-complex is a BTE representing the totality of its bitemporal extent (i.e. all database/event times at which parts of it have existed). The definitions, which extend the projection operators on simplexes given in Definition 3, are as below. Let complex $C = \{\langle S_1, T_1 \rangle, \ldots, \langle S_n, T_n \rangle\}$.

Definition 8 (spatial projection).

$$\pi^s(C) = \{S_1, \ldots, S_n\}$$

This projection is a spatial simplicial complex.

Definition 9 (temporal projection).

$$\pi^t(C) = \bigcup_{1 \leqslant i \leqslant n} T_i$$

The union is taken of the BTEs which are components of C. The result is a BTE.

5.4. Spatio-bitemporal β-product

The next operation allows the composition of two spatio-temporal complexes in a way which is parameterized by β-operations on BTEs. (Recall that a β-operation is a binary operation taking BTEs as arguments and returning a BTE.) Firstly, a preliminary construction is needed.

Definition 10 (common refinement). Let C_1 and C_2 be two purely spatial simplicial complexes. A *common refinement* of C_1 and C_2 is a simplicial complex which has the same planar embedding as the union of the embeddings of C_1 and C_2.

This construction is in general not unique (see, e.g. Figure 7).

Definition 11 (ST-β-product). Let C_1 and C_2 be two ST-complexes. Let β be a Boolean set operation on BTEs. Let simplicial complex R be a common refinement of $\pi^s(C_1)$ and $\pi^s(C_2)$. Then, define $C_1 \times_\beta C_2$ to be the smallest ST-complex (with respect to the ST-subset relation defined earlier) which contains the set of ST-simplexes

$$\{\langle S, T_1^S \beta T_2^S \rangle \mid S \in R\}$$

where T_1^S and T_2^S are the BTEs associated with the spatially smallest faces of $\pi^s(C_1)$ and $\pi^s(C_2)$, respectively, which contain S. (The result $C_1 \times_\beta C_2$ is dependent upon the choice of common refinement R. However, the results will all be ST-equal. Strictly, these operations act not on individual ST-complexes but upon equivalence classes of ST-equal ST-complexes. For notational simplicity, we allow operations to act upon single ST-complexes. All operations discussed in this paper are well-defined in this respect.)

Notice that the set of ST-simplexes $\{\langle S, T_1^S \beta T_2^S \rangle \mid S \in R\}$ does not necessarily form an ST-complex. However, there is always a unique minimal ST-complex which contains this set.

In order to provide some examples of the β-product, consider the two ST-complexes shown in Figure 8. Object C_1 has a triangular spatial projection and its bitemporal extent is represented by the BTEs attached to the simplicial components of C_1. The object represented by C_2 has a linear spatial projection with bitemporal extent as given by the associated BTEs. Figure 9 shows a common refinement of the spatial projections of C_1 and C_2.

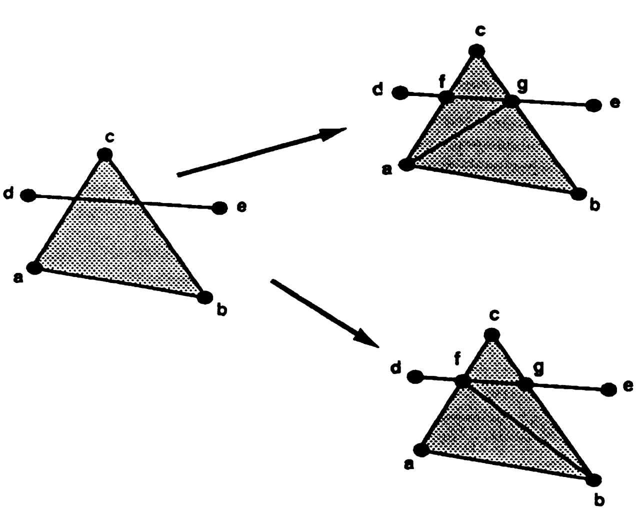

FIGURE 7. Two minimal common refinements for the complexes *abc* and *de*.

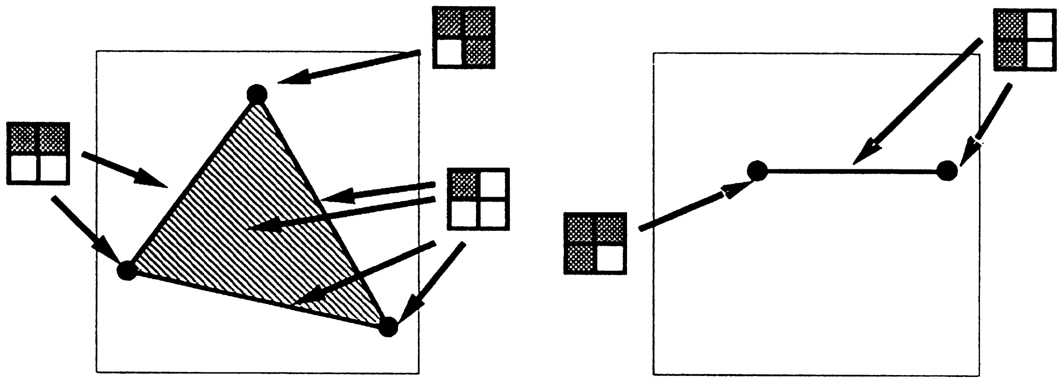

FIGURE 8. ST-complexes, C_1 and C_2.

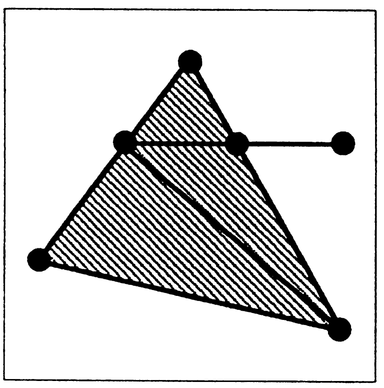

FIGURE 9. A common refinement of the spatial projections of C_1 and C_2.

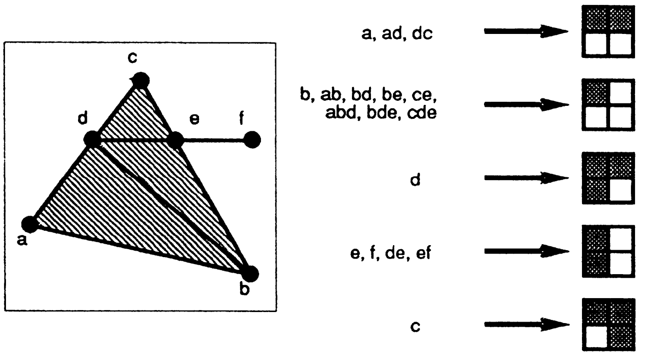

FIGURE 10. The union of C_1 and C_2.

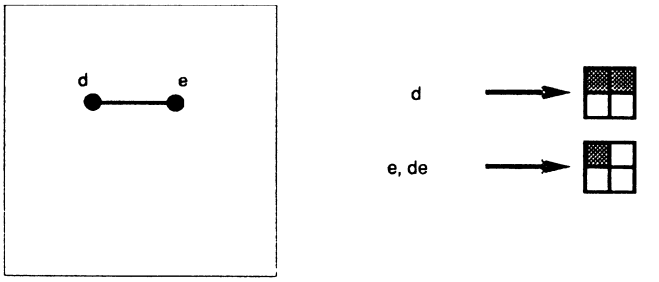

FIGURE 11. The intersection of C_1 and C_2.

5.4.1. Set-theoretical operations

The set-theoretic union, intersection and difference of two ST-complexes may be obtained by taking β to be the union, intersection or difference operator (respectively) between BTEs. Let C_1 and C_2 be two ST-complexes.

Definition 12 (ST-union).

$$C_1 \sqcup C_2 = C_1 \times_{\cup} C_2$$

Definition 13 (ST-intersection).

$$C_1 \sqcap C_2 = C_1 \times_{\cap} C_2$$

Definition 14 (ST-difference).

$$C_1 \backslash C_2 = C_1 \times_{\backslash} C_2$$

Figure 10 shows a ST-complex representing the union of C_1 and C_2. The idea here is that the union contains all the elements of the spatial projections of each of C_1 and C_2. Each spatial simplex has associated with it the union of the BTEs associated with that element in C_1 and C_2. Figure 11 shows an ST-complex representing the intersection of C_1 and C_2. Each spatial simplex has associated with it the intersection of the BTEs associated with that element in C_1 and C_2. If the empty BTE is calculated as being associated with a spatial simplex, then that simplex is omitted from the resulting complex.

In the case of ST-union and ST-intersection, the constituent simplexes of the result immediately form an ST-complex. In the case of ST-difference, it may be necessary to extend the set of simplexes in order that the conditions for an ST-complex are satisfied.

5.5. Spatial selection

Spatial selection σ_X^s may be expressed using the β-product operator. Suppose we wish to select from C_1 the part of the configuration which has spatial projection given by X, which is the spatial projection of C_2. Then, form the ST-complex, D, by associating the universal BTE, $T_D \times T_E$, with each simplex in X. Now take the ST-intersection (see above) of C_1 and D. The result, $\sigma_X^s(C_1)$, is again shown in Figure 11.

5.6. Temporal selection

Temporal selection, σ^t_ϕ, selects from an ST-complex the smallest ST-complex, each of whose simplicial components satisfies the temporal condition specified by the formula ϕ. More formally, let $\phi(t)$ be a first-order formula which may contain BTEs as constants, β-operations for functions and a single free variable t. Let C be an ST-complex. Then define $\sigma^t_\phi(C)$ to be the smallest (with respect to the ST-subset ordering) ST-complex containing the following set of ST-simplexes:

$$\{\langle S, T\rangle \in C \mid \phi(T)\}$$

For example, if BTE B is as shown in Figure 12 and C_1 as in Figure 8, then $\sigma^t_{t \supseteq B}(C_1)$ is shown in Figure 13.

6. CONCLUSIONS

This paper has discussed the modelling of a unified approach to spatiotemporal information, combining the modelling of two-dimensional space with two-dimensional time. We hope to have convinced the reader that a great deal of meaningful spatial information has an inseparable temporal component and that a model which handles only space is limited in scope. We hope also that the reader is convinced that more than one temporal dimension is necessary for many applications.

The model can be used as the basis of a query algebra, as the following examples show.

Land ownership queries

Query 1. Does the path currently pass through land that was ever part of Jane's house?

Let the ST-complexes representing the path and Jane's house be C_1 and C_2, respectively. The query can then be expressed algebraically as:

$$\pi^s(\sigma^t_{t \supseteq now_{DB}}(C_1)) \sqcap \pi^s(C_2) = \varnothing$$

where now_{DB} indicates a BTE representing all bi-times with database time *now* and the intersection operator is purely spatial intersection of spatial complexes.

Query 2. Has Jack's house ever shared a common boundary with the path?

Let the ST-complexes representing the path and Jack's house be C_1 and C_3, respectively. The query can then be expressed algebraically as:

$$\partial C_1 \sqcap \partial C_3 = \varnothing$$

Regarding the spatiotemporal operations, it is not claimed that the list of operations is complete in any sense, just as there is no complete list of purely spatial operators. The investigation of further operators and notions of completeness is the subject of current work.

Work is currently in progress to construct a pilot implementation of the model described here using administrative boundary areas as an application. This model is implemented using the object-based geographical information system, Smallworld, and UK Ordnance Survey spatiotemporal administrative boundary data.

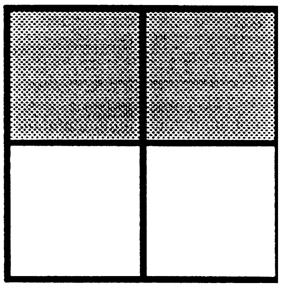

FIGURE 12. The BTE, B.

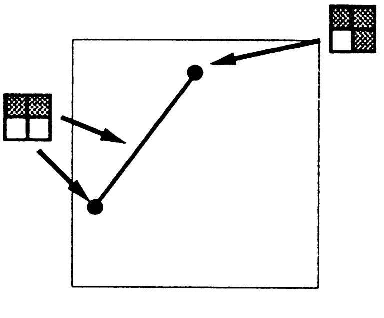

FIGURE 13. The temporal selection $\sigma^t_{t \supseteq B}(C_1)$.

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