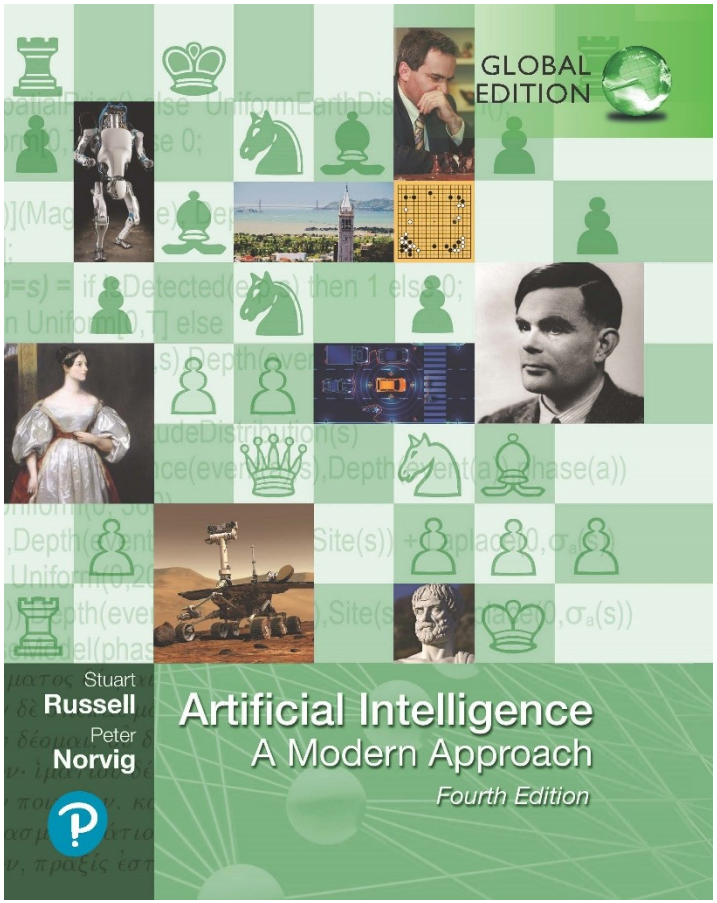
Artificial Intelligence: A Modern Approach

Fourth Edition, Global Edition



Chapter 1

Introduction

Lecture Presentations: Artificial Intelligence

Adapted from:
"Artificial Intelligence: A Modern Approach, Global Edition", 4th Edition by Stuart Russell and Peter Norvig © 2021 Pearson Education.

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Outline

- What is AI?
- A brief history
- The state of the art

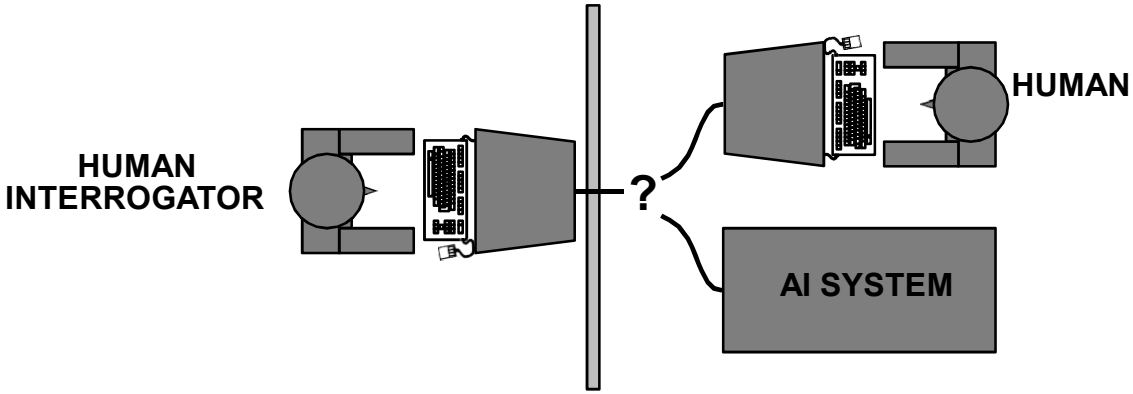
What is AI?

Systems that think like humans	Systems that think rationally
Systems that act like humans	Systems that act rationally

Acting humanly: The Turing test

Turing (1950) "Computing machinery and intelligence":

- "Can machines think?" −→ "Can machines behave intelligently?"
- Operational test for intelligent behavior: the Imitation Game



- Predicted that by 2000, a machine might have a 30% chance of fooling a lay person for 5 minutes
- Anticipated all major arguments against AI in following 50 years
- Suggested major components of AI: knowledge, reasoning, language understanding, learning

Problem: Turing test is not reproducible, constructive, or amenable to mathematical analysis

Thinking humanly: Cognitive Science

1960s "cognitive revolution": information-processing psychology replaced prevailing orthodoxy of behaviorism

Requires scientific theories of internal activities of the brain

- What level of abstraction? "Knowledge" or "circuits"?
- How to validate? Requires

  1)Predicting and testing behavior of human subjects (top-down)

  or 2) Direct identification from neurological data (bottom-up)

Both approaches (roughly, Cognitive Science and Cognitive Neuroscience) are now distinct from AI

Both share with AI the following characteristic:

the available theories do not explain (or engender) anything resembling human-level general intelligence

Hence, all three fields share one principal direction!

Thinking rationally: Laws of Thought

Normative (or prescriptive) rather than descriptive

Aristotle: what are correct arguments/thought processes?

Several Greek schools developed various forms of logic:

notation and rules of derivation for thoughts;

may or may not have proceeded to the idea of mechanization

Direct line through mathematics and philosophy to modern AI

Problems:

1) Not all intelligent behavior is mediated by logical deliberation
2) What is the purpose of thinking? What thoughts should I have out of all the thoughts (logical or otherwise) that I could have?

Acting rationally

Rational behavior: doing the right thing

The right thing: that which is expected to maximize goal achievement, given the available information

Doesn't necessarily involve thinking—e.g., blinking reflex—but thinking should be in the service of rational action

Aristotle (Nicomachean Ethics):

> Every art and every inquiry, and similarly every action and pursuit, is thought to aim at some good

Rational agents

An agent is an entity that perceives and acts

This course is about designing rational agents

Abstractly, an agent is a function from percept histories to actions:

$$f : P^* \rightarrow A$$

For any given class of environments and tasks, we seek the agent (or class of agents) with the best performance

Caveat: computational limitations make perfect rationality unachievable

→ design best program for given machine resources

AI prehistory

Philosophy	logic, methods of reasoning
	mind as physical system
	foundations of learning, language, rationality
Mathematics	formal representation and proof
	algorithms, computation, (un)decidability, (in)tractability
	probability
Psychology	adaptation
	phenomena of perception and motor control
	experimental techniques (psychophysics, etc.)
Economics	formal theory of rational decisions
Linguistics	knowledge representation
	grammar
Neuroscience	plastic physical substrate for mental activity
Control theory	homeostatic systems, stability
	simple optimal agent designs

Potted history of AI

1943	McCulloch & Pitts: Boolean circuit model of brain
1950	Turing's "Computing Machinery and Intelligence"
1952–69	Look, Ma, no hands!
1950s	Early AI programs, including Samuel's checkers program, Newell & Simon's Logic Theorist, Gelernter's Geometry Engine
1956	Dartmouth meeting: "Artificial Intelligence" adopted
1965	Robinson's complete algorithm for logical reasoning
1966–74	AI discovers computational complexity Neural network research almost disappears
1969–79	Early development of knowledge-based systems
1980–88	Expert systems industry booms
1988–93	Expert systems industry busts: "AI Winter"
1985–95	Neural networks return to popularity
1988–	Resurgence of probability; general increase in technical depth "Nouvelle AI": ALife, GAs, soft computing
1995–	Agents, agents, everywhere . . .
2003–	Human-level AI back on the agenda

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
- Drive safely along a curving mountain road

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
- Drive safely along a curving mountain road
- Drive safely along Telegraph Avenue

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
- Drive safely along a curving mountain road
- Drive safely along Telegraph Avenue
- Buy a week's worth of groceries on the web

State of the art

Which of the following can be done at present?

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- Buy a week's worth of groceries at Berkeley Bowl

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
- Drive safely along a curving mountain road
- Drive safely along Telegraph Avenue
- Buy a week's worth of groceries on the web
- Buy a week's worth of groceries at Berkeley Bowl
- Play a decent game of bridge

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
- Drive safely along a curving mountain road
- Drive safely along Telegraph Avenue
- Buy a week's worth of groceries on the web
- Buy a week's worth of groceries at Berkeley Bowl
- Play a decent game of bridge
- Discover and prove a new mathematical theorem

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
- Drive safely along a curving mountain road
- Drive safely along Telegraph Avenue
- Buy a week's worth of groceries on the web
- Buy a week's worth of groceries at Berkeley Bowl
- Play a decent game of bridge
- Discover and prove a new mathematical theorem
- Design and execute a research program in molecular biology

State of the art

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- Play a decent game of table tennis
- Drive safely along a curving mountain road
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- Discover and prove a new mathematical theorem
- Design and execute a research program in molecular biology
- Write an intentionally funny story

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- Discover and prove a new mathematical theorem
- Design and execute a research program in molecular biology
- Write an intentionally funny story
- Give competent legal advice in a specialized area of law

State of the art

Which of the following can be done at present?

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- Design and execute a research program in molecular biology
- Write an intentionally funny story
- Give competent legal advice in a specialized area of law
- Translate spoken English into spoken Swedish in real time

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
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- Discover and prove a new mathematical theorem
- Design and execute a research program in molecular biology
- Write an intentionally funny story
- Give competent legal advice in a specialized area of law
- Translate spoken English into spoken Swedish in real time
- Converse successfully with another person for an hour

State of the art

Which of the following can be done at present?

- Play a decent game of table tennis
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- Converse successfully with another person for an hour
- Perform a complex surgical operation

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- Give competent legal advice in a specialized area of law
- Translate spoken English into spoken Swedish in real time
- Converse successfully with another person for an hour
- Perform a complex surgical operation
- Unload any dishwasher and put everything away

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- Give competent legal advice in a specialized area of law
- Translate spoken English into spoken Swedish in real time
- Converse successfully with another person for an hour
- Perform a complex surgical operation
- Unload any dishwasher and put everything away

Risks and Benefits of AI

"First solve AI, then use AI to solve everything else." Demis Hassabis, CEO of Google DeepMind

Benefits:

- Decrease repetitive work
- Increase production of goods and services
- Accelerate scientific research (disease cures, climate change and resource shortages solutions)

Risks:

- Lethal autonomous weapons
- Surveillance and persuasion
- Biased decision making
- Impact on employment
- Safety-critical applications
- Cybersecurity threats

Risks and Benefits of AI

Development of an artificial superintelligence that surpasses human intelligence may pose a significant risk

Analogous to the "Gorilla problem"
Humans and gorillas evolved from the same species, but humans have more control than other primates.

Thus, we should design AI systems in such a way that they do not end up taking control in the way that Turing suggests they might.

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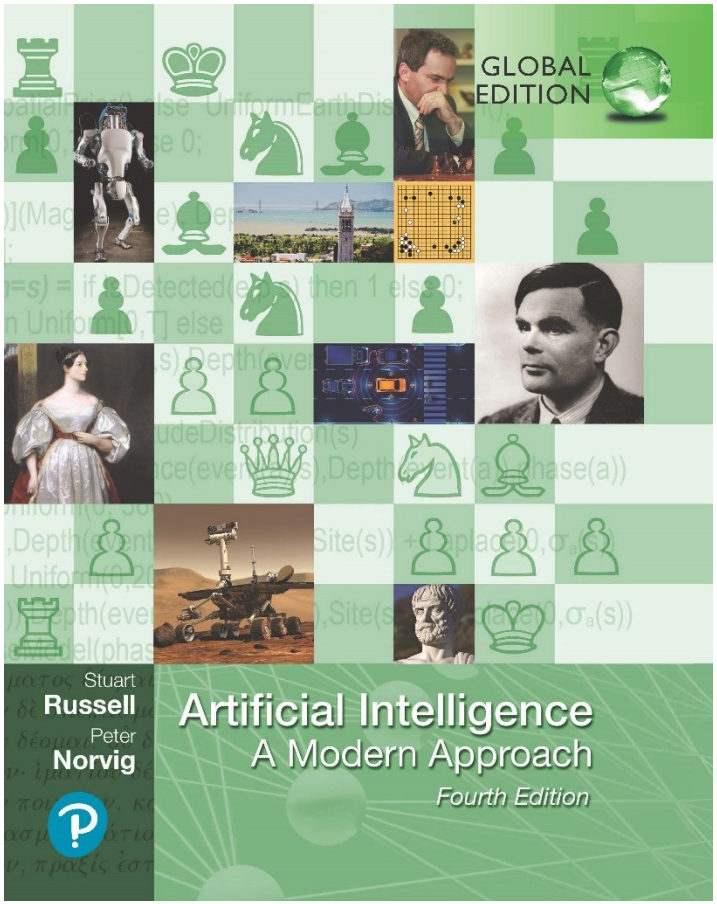

Chapter 2

Intelligent Agents

Lecture Presentations: Artificial Intelligence

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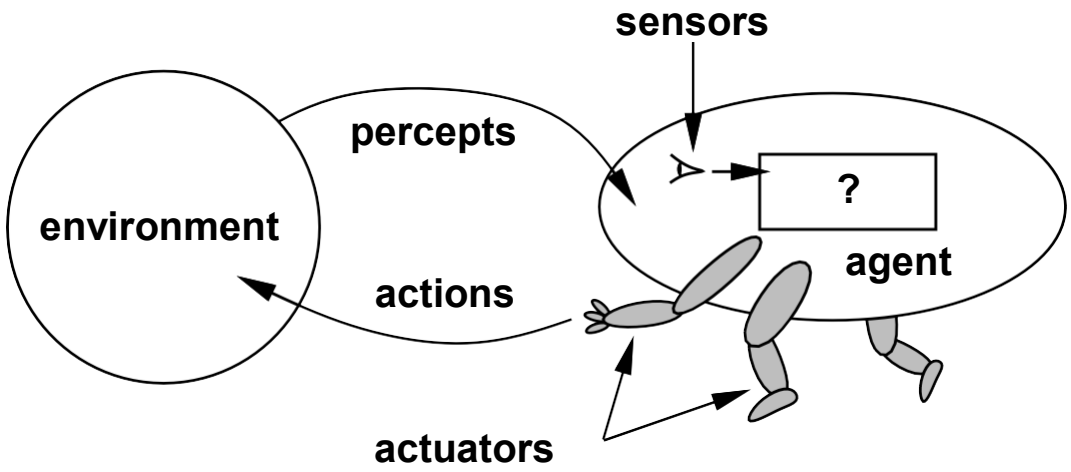
Outline

- Agents and environments
- Rationality
- PEAS (Performance measure, Environment, Actuators, Sensors)
- Environment types
- Agent types

Agents and environments



Agents include humans, robots, softbots, thermostats, etc.

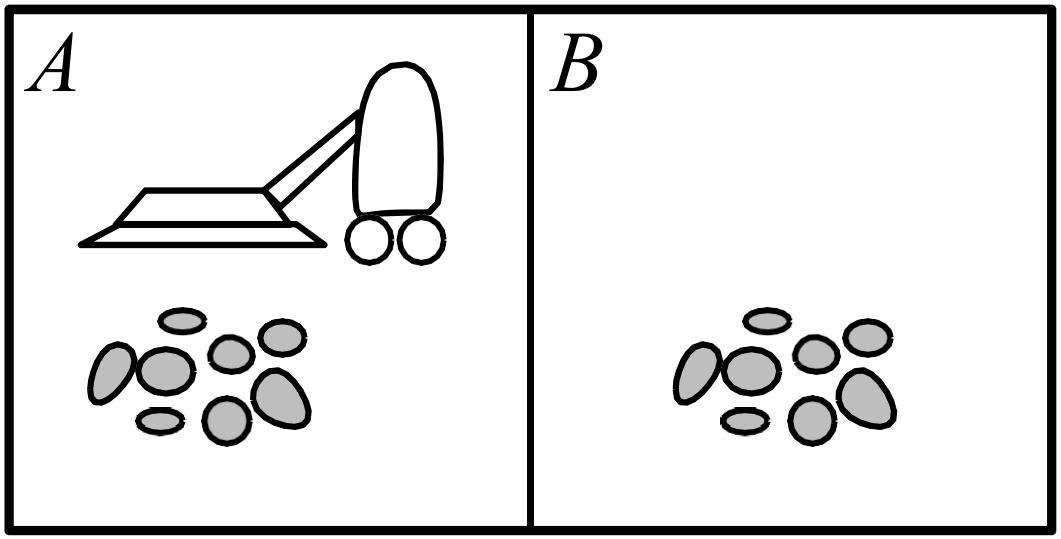
An agent can be anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators

The agent function maps from percept histories to actions:

$$f : \mathrm{P}^* \rightarrow \mathrm{A}$$

The agent program runs on the physical architecture to produce f

Vacuum-cleaner world



Percepts: location and contents, e.g., [*A*, *Dirty*]

Actions: *Left*, *Right*, *Suck*, *NoOp*

A vacuum-cleaner agent

Percept sequence	Action
[*A*, *Clean*]	*Right*
[*A*, *Dirty*]	*Suck*
[*B*, *Clean*]	*Left*
[*B*, *Dirty*]	*Suck*
[*A*, *Clean*], [*A*, *Clean*]	*Right*
[*A*, *Clean*], [*A*, *Dirty*]	*Suck*
.	.

```
function Reflex-Vacuum-Agent( [location,status]) returns an action
    if status = Dirty then return Suck
    else if location = A then return Right
    else if location = B then return Left
```

What is the right function?

Can it be implemented in a small agent program?

Rationality

Fixed performance measure evaluates the environment sequence

- one point per square cleaned up in time T?
- one point per clean square per time step, minus one per move?
- penalize for $> k$ dirty squares?

A rational agent chooses whichever action maximizes the expected value of the performance measure given the percept sequence to date

Rational /= omniscient

- percepts may not supply all relevant information

Rational /= clairvoyant

- action outcomes may not be as expected

Hence, rational /= successful

Rational $\Rightarrow$ exploration, learning, autonomy

PEAS

To design a rational agent, we must specify the task environment

Consider, e.g., the task of designing an automated taxi:

Performance measure??

Environment??

Actuators??

Sensors??

PEAS

To design a rational agent, we must specify the task environment

Consider, e.g., the task of designing an automated taxi:

Performance measure?? safety, destination, profits, legality, comfort, . . .

Environment?? US streets/freeways, traffic, pedestrians, weather, . . .

Actuators?? steering, accelerator, brake, horn, speaker/display, . . .

Sensors?? video, accelerometers, gauges, engine sensors, keyboard, GPS, . . .

Internet shopping agent

Performance measure??

Environment??

Actuators??

Sensors??

Internet shopping agent

Performance measure?? price, quality, appropriateness, efficiency

Environment?? current and future WWW sites, vendors, shippers

Actuators?? display to user, follow URL, fill in form

Sensors?? HTML pages (text, graphics, scripts)

Environment types

	Solitaire	Backgammon	Internet shopping	Taxi
Observable?? Deterministic?? Episodic?? Static?? Discrete?? Single-agent??				

Environment types

	Solitaire	Backgammon	Internet shopping	Taxi
Observable?? Deterministic?? Episodic?? Static?? Discrete?? Single-agent??	Yes	Yes	No	No

Environment types

	Solitaire	Backgammon	Internet shopping	Taxi
Observable??	Yes	Yes	No	No
Deterministic??	Yes	No	Partly	No
Episodic??				
Static??				
Discrete??				
Single-agent??				

Environment types

	Solitaire	Backgammon	Internet shopping	Taxi
Observable??	Yes	Yes	No	No
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Static??				
Discrete??				
Single-agent??				

Environment types

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Discrete??				
Single-agent??				

Environment types

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Discrete??	Yes	Yes	Yes	No
Single-agent??				

Environment types

	Solitaire	Backgammon	Internet shopping	Taxi
Observable??	Yes	Yes	No	No
Deterministic?	Yes	No	Partly	No
? Episodic??	No	No	No	No
Static??	Yes	Semi	Semi	No
Discrete??	Yes	Yes	Yes	
Single-agent??	No Yes	No	Yes (except auctions)	No

The environment type largely determines the agent design

The real world is (of course) partially observable, stochastic, sequential, dynamic, continuous, multi-agent

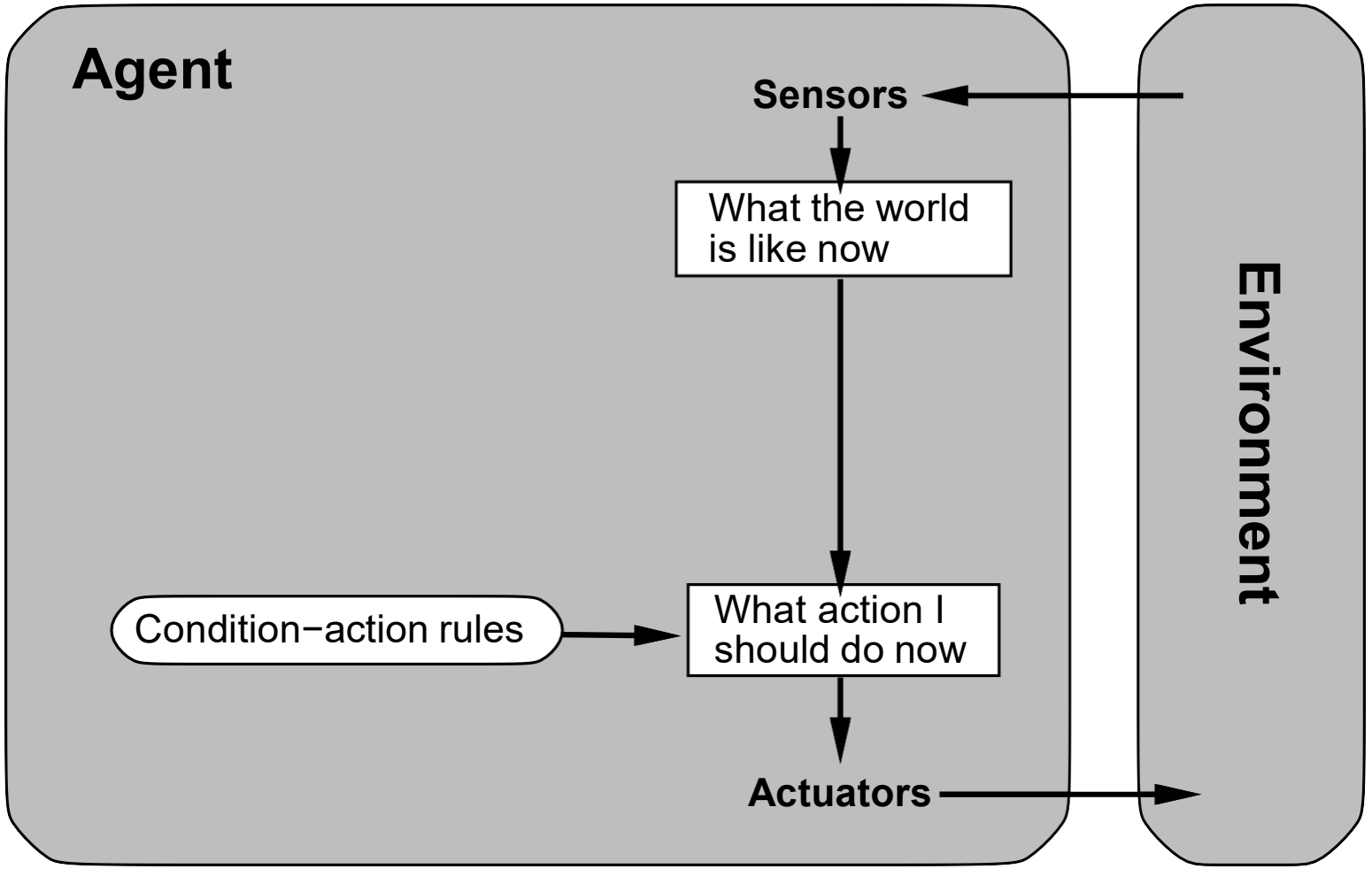
Agent types

Four basic types in order of increasing generality:

- simple reflex agents
- reflex agents with state
- goal-based agents
- utility-based agents

All these can be turned into learning agents

Simple reflex agents



Example

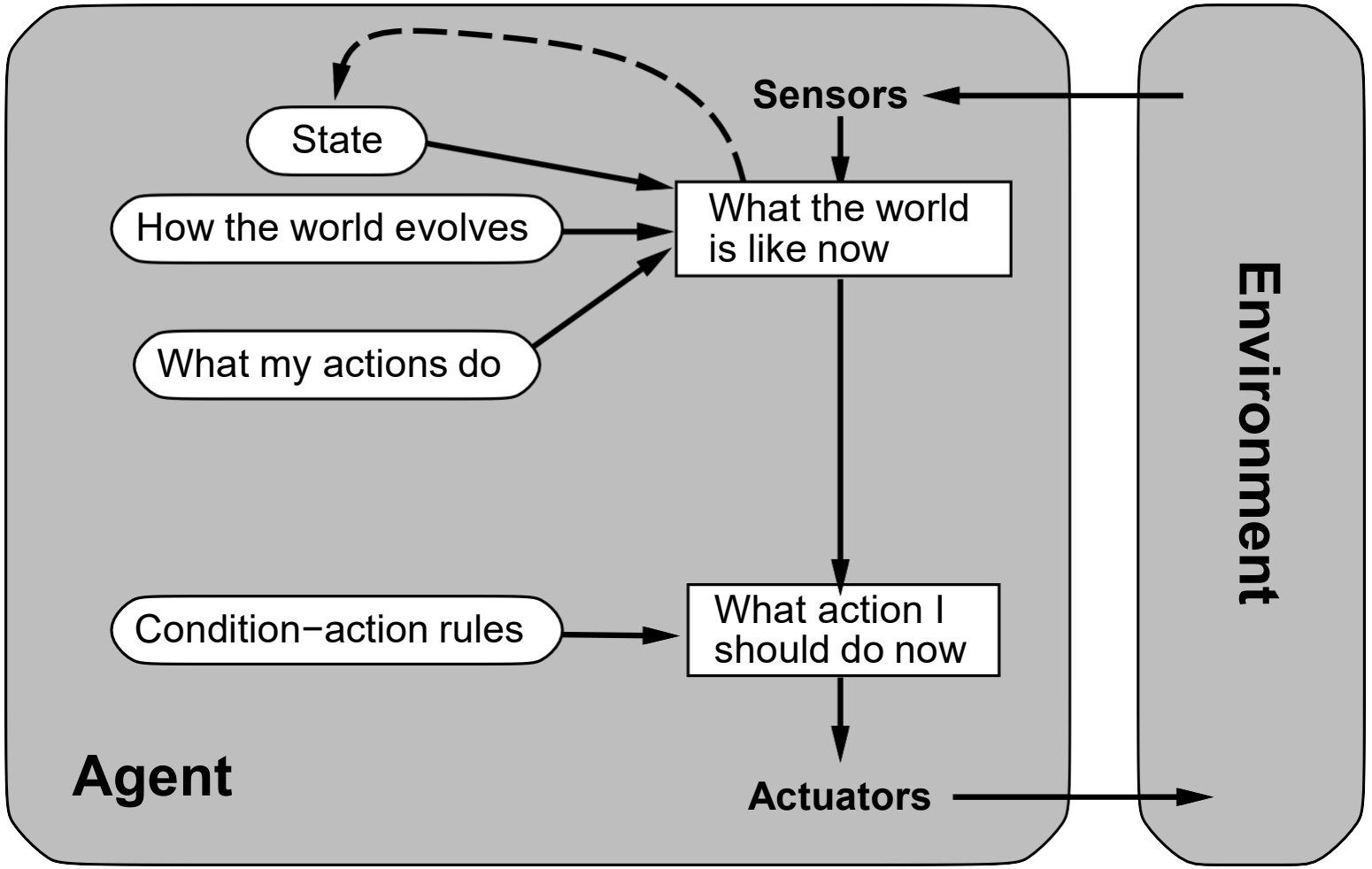
function Reflex-Vacuum-Agent([*location*,*status*]) returns an action

if *status* = *Dirty* then return *Suck*
else if *location* = *A* then return *Right*
else if *location* = *B* then return *Left*

```
(setq joe (make-agent :name 'joe :body (make-agent-body)
                      :program (make-reflex-vacuum-agent-program)))

(defun make-reflex-vacuum-agent-program ()
  #'(lambda (percept)
      (let ((location (first percept)) (status (second percept)))
        (cond ((eq status 'dirty) 'Suck)
              ((eq location 'A) 'Right)
              ((eq location 'B) 'Left)))))
```

Reflex agents with state



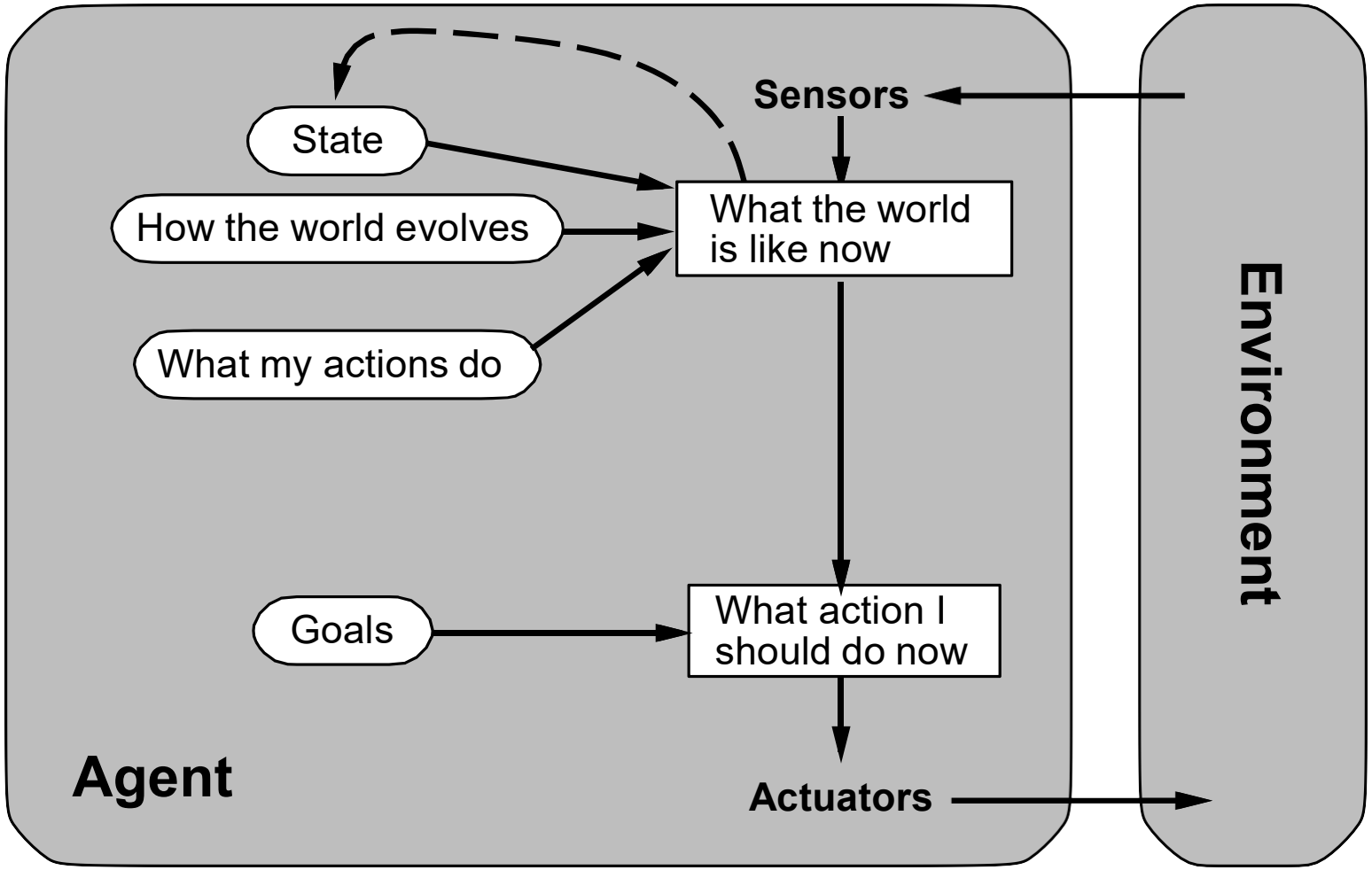
Example

function Reflex-Vacuum-Agent([*location*,*status*]) returns an action
static: *last_A*, *last_B*, numbers, initially ∞

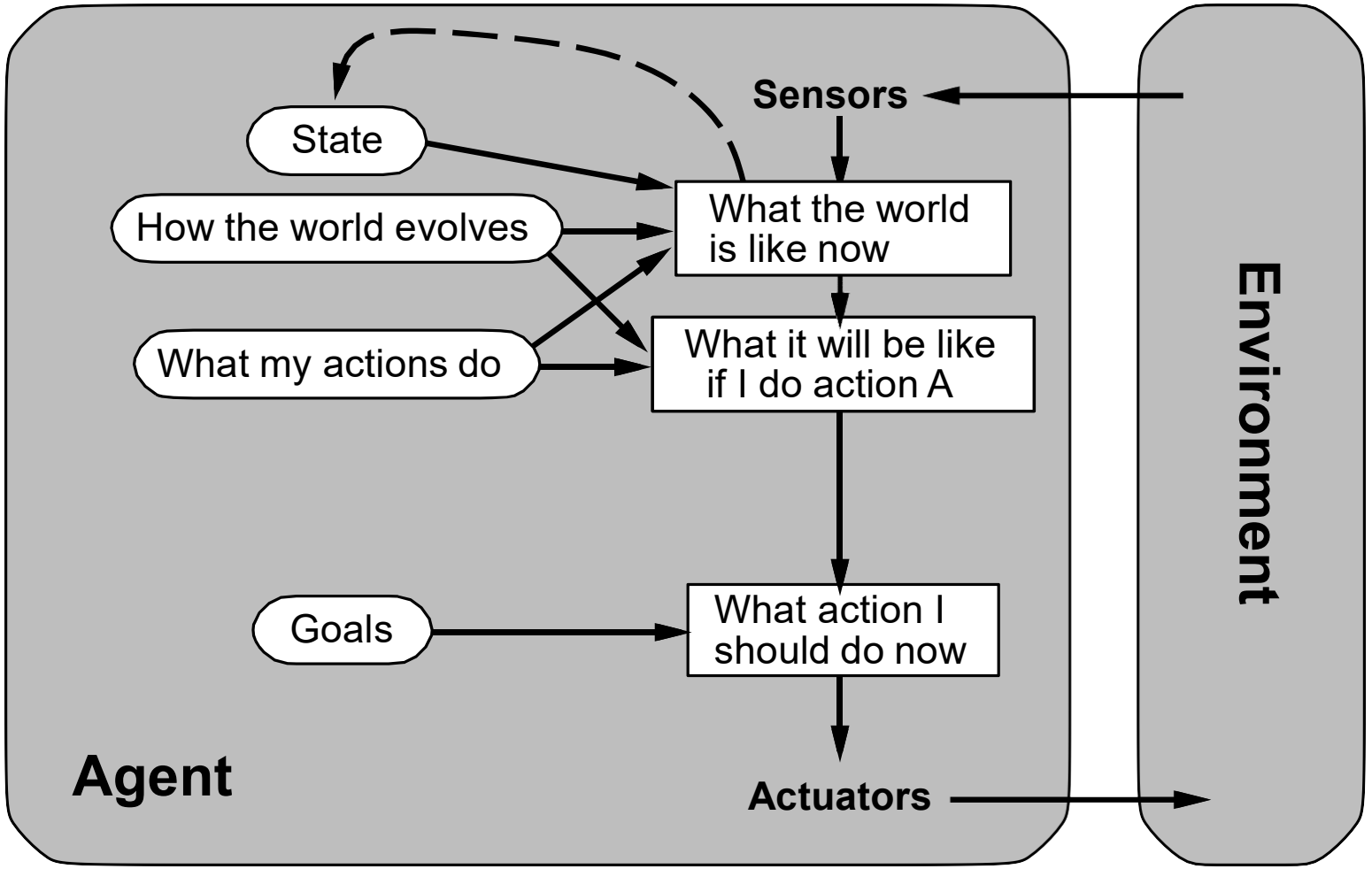
if *status* = *Dirty* then . . .

```
(defun make-reflex-vacuum-agent-with-state-program ()
  (let ((last-A infinity) (last-B infinity))
  #'(lambda (percept)
      (let ((location (first percept)) (status (second percept)))
        (incf last-A) (incf last-B)
        (cond
         ((eq status 'dirty)
          (if (eq location 'A) (setq last-A 0) (setq last-B 0))
          'Suck)
         ((eq location 'A) (if (> last-B 3) 'Right 'NoOp))
         ((eq location 'B) (if (> last-A 3) 'Left 'NoOp)))))))
```

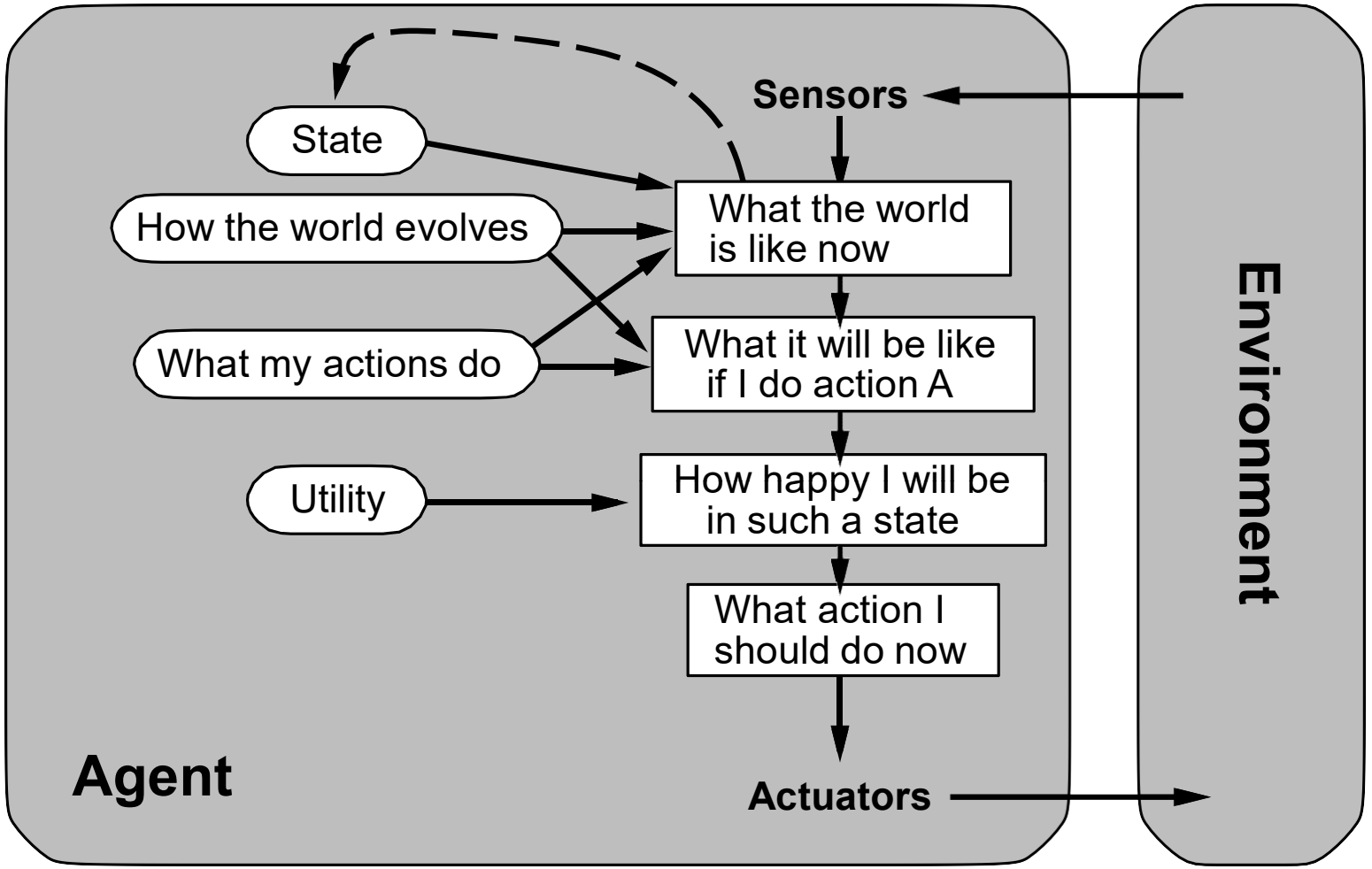
Model-based agents



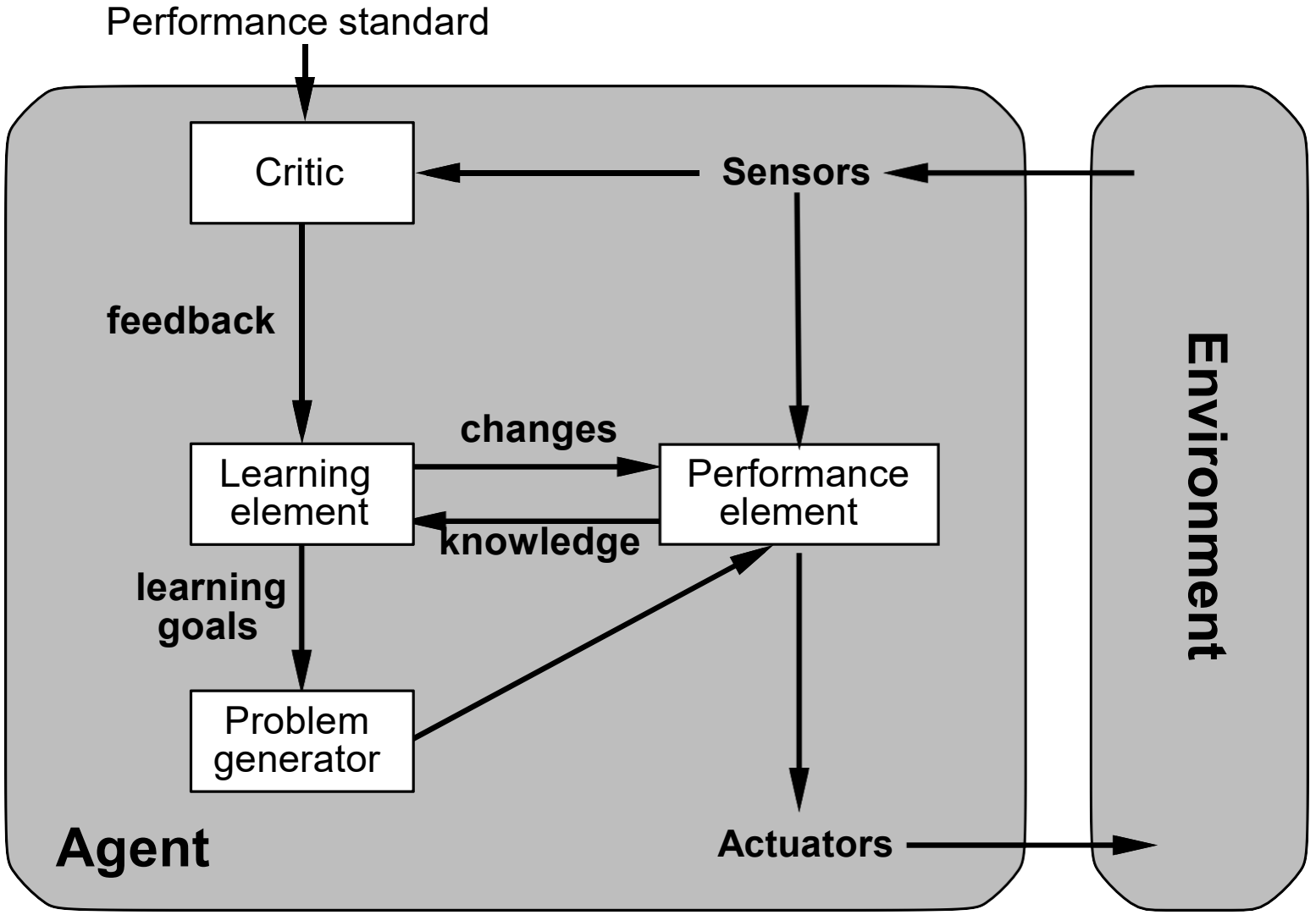
Goal-based agents



Utility-based agents



Learning agents



Summary

Agents interact with environments through actuators and sensors

The agent function describes what the agent does in all circumstances

The performance measure evaluates the environment sequence

A perfectly rational agent maximizes expected performance

Agent programs implement (some) agent functions

PEAS descriptions define task environments

Environments are categorized along several dimensions:

observable? deterministic? episodic? static? discrete? single-agent?

Several basic agent architectures exist:

reflex, reflex with state, goal-based, utility-based

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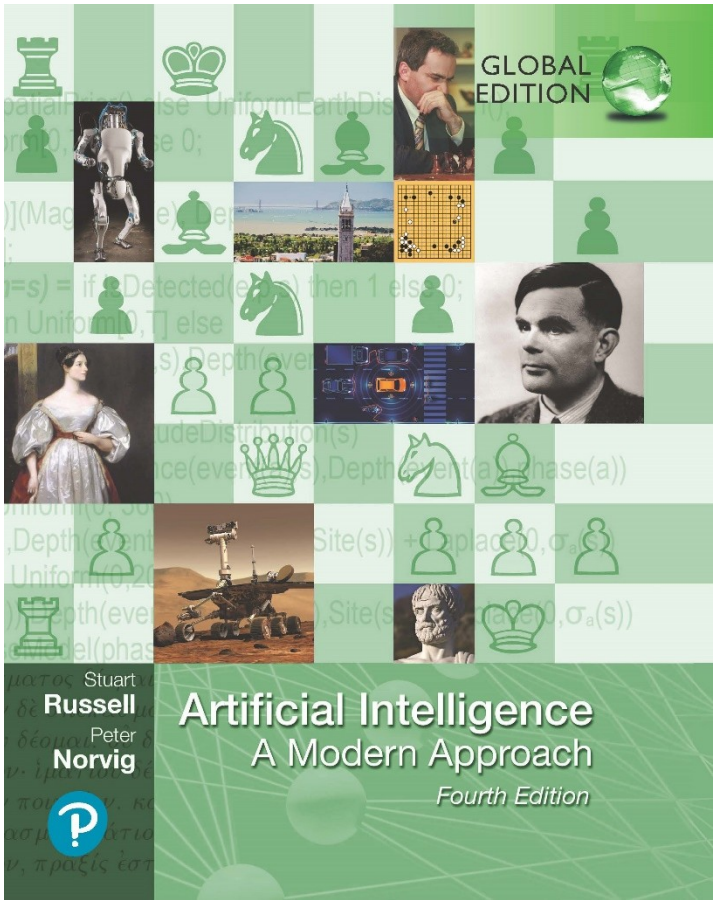

Chapter 3

Solving Problems By Searching

Lecture Presentations: Artificial Intelligence

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Outline

- Problem-solving agents
- Example Problems
- Problem formulation
- Search Algorithms
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies
- Heuristic Functions

Problem-solving agents

Restricted form of general agent:

```
function Simple-Problem-Solving-Agent(percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← Update-State(state, percept)
    if seq is empty then
        goal ← Formulate-Goal(state)
        problem ← Formulate-Problem(state, goal)
        seq ← Search(problem)
    action ← Recommendation(seq, state)
    seq ← Remainder(seq, state)
    return action
```

Note: this is offline problem solving; solution executed "eyes closed."
Online problem solving involves acting without complete knowledge.

Example: Romania

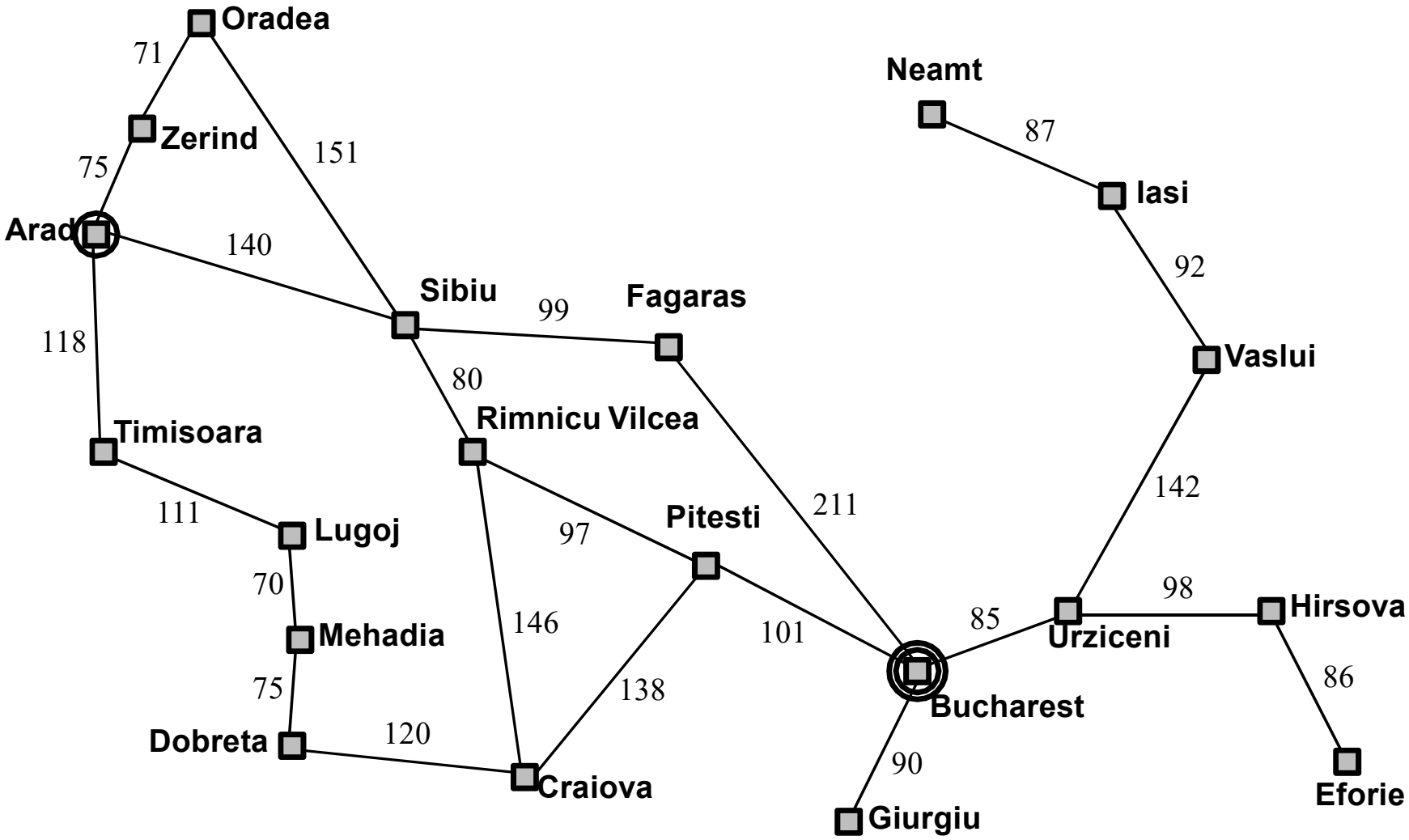
On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:
be in Bucharest

Formulate problem:
states: various cities
actions: drive between cities

Find solution:
sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



Problem types

Deterministic, fully observable ⟹ single-state problem

Agent knows exactly which state it will be in; solution is a sequence

Non-observable ⟹ conformant problem

Agent may have no idea where it is; solution (if any) is a sequence

Nondeterministic and/or partially observable ⟹ contingency problem

percepts provide new information about current state

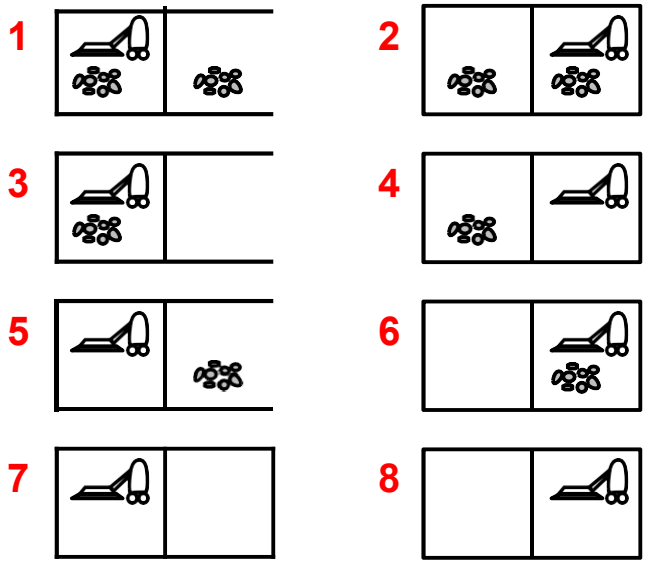
solution is a contingent plan or a policy

often interleave search, execution

Unknown state space ⟹ exploration problem ("online")

Example: vacuum world

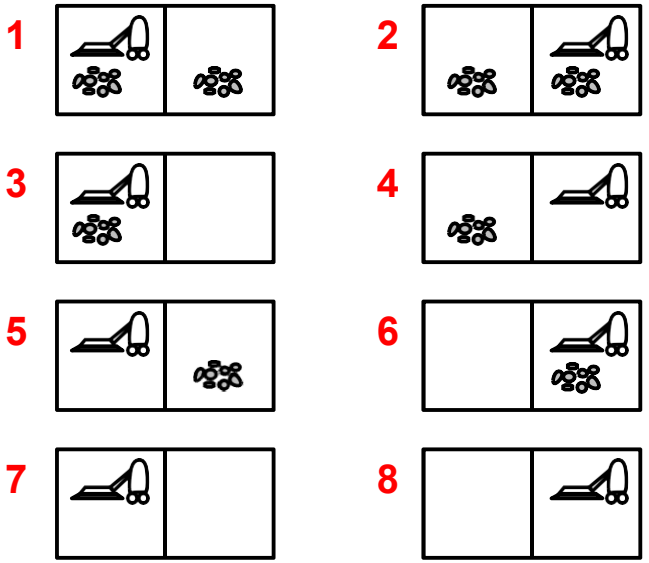
Single-state, start in #5. Solution??



Example: vacuum world

Single-state, start in #5. Solution??
[*Right, Suck*]

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., *Right* goes to $\{2, 4, 6, 8\}$. Solution??

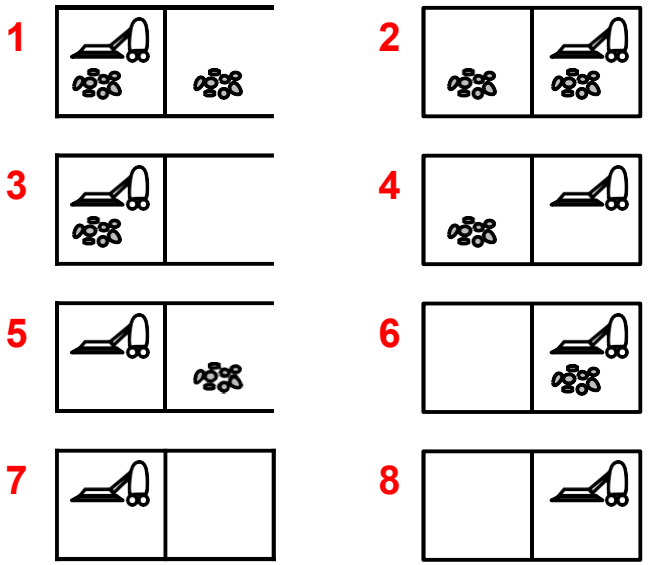


Example: vacuum world

Single-state, start in #5. Solution??
[*Right*, *Suck*]

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., *Right* goes to $\{2, 4, 6, 8\}$. Solution??
[*Right*, *Suck*, *Left*, *Suck*]

Contingency, start in #5
Murphy's Law: *Suck* can dirty a clean carpet
Local sensing: dirt, location only.
Solution??

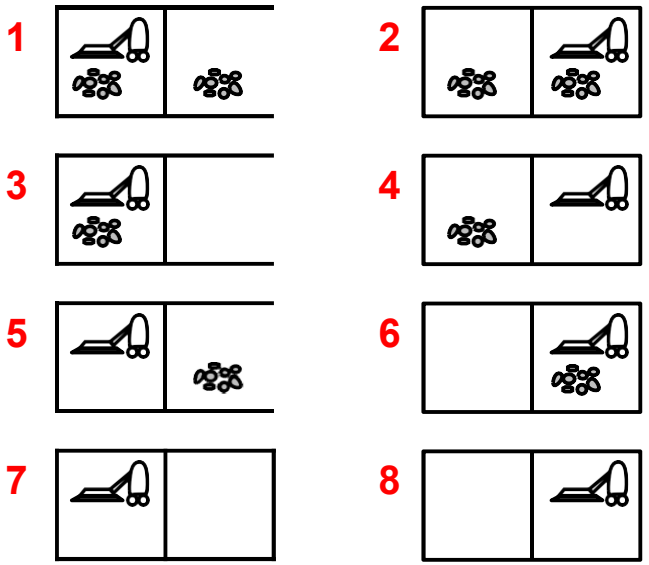


Example: vacuum world

Single-state, start in #5. Solution??
[*Right*, *Suck*]

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., *Right* goes to $\{2, 4, 6, 8\}$. Solution??
[*Right*, *Suck*, *Left*, *Suck*]

Contingency, start in #5
Murphy's Law: *Suck* can dirty a clean carpet
Local sensing: dirt, location only.
Solution??
[*Right*, if *dirt* then *Suck*]



Single-state problem formulation

A problem is defined by four items:

initial state e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs

e.g., $S(Arad) = \{(Arad \rightarrow Zerind, Zerind), \ldots\}$

goal test, can be

explicit, e.g., x = "at Bucharest"

implicit, e.g., $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of actions executed, etc.

$c(x, a, y)$ is the step cost, assumed to be ≥ 0

A solution is a sequence of actions
leading from the initial state to a goal state

Selecting a state space

Real world is absurdly complex

⇒ state space must be abstracted for problem solving

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions

e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.

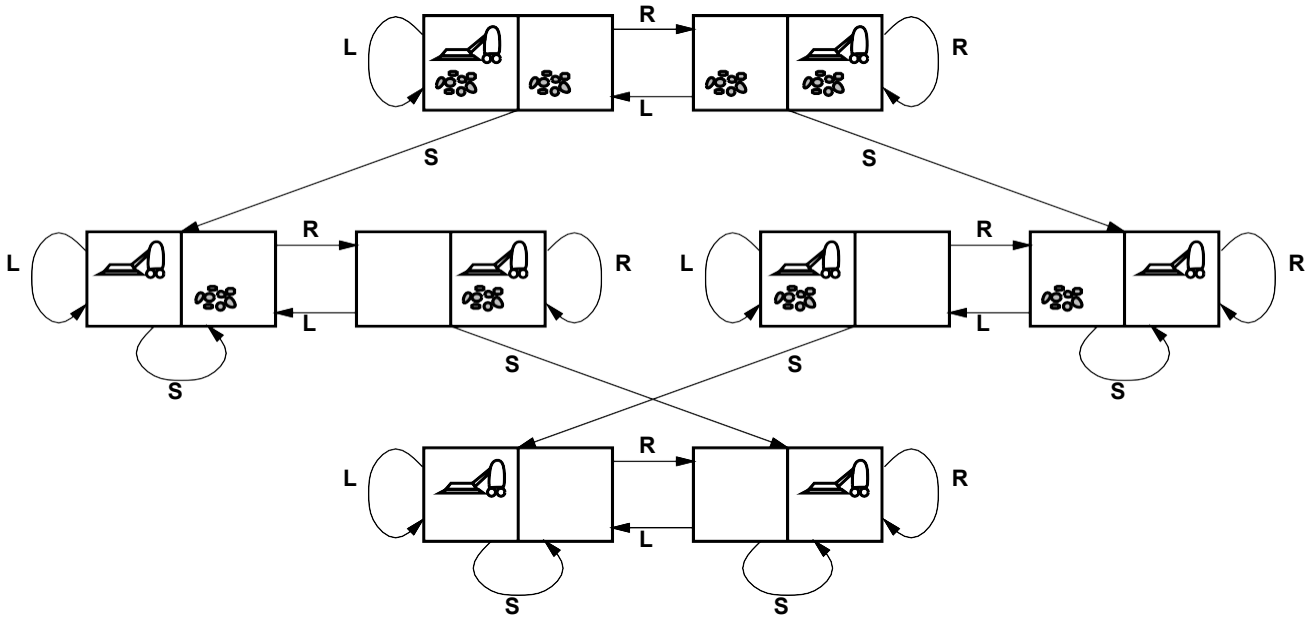
For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"

(Abstract) solution =

set of real paths that are solutions in the real world

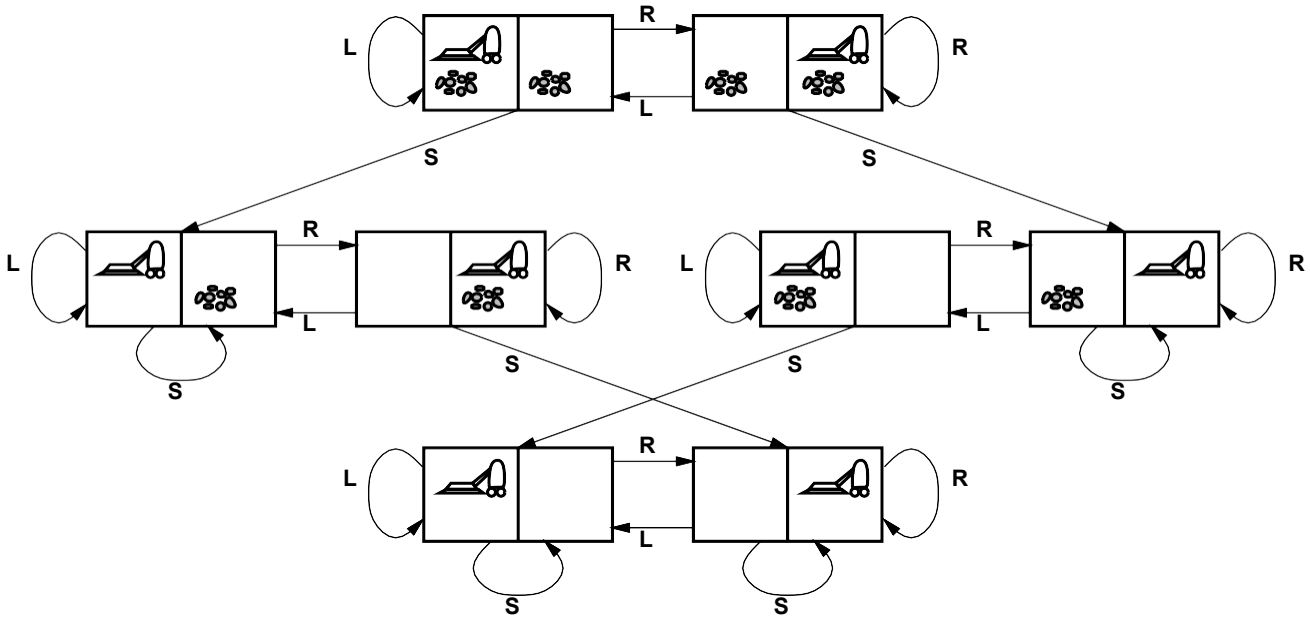
Each abstract action should be "easier" than the original problem!

Example: vacuum world state space graph



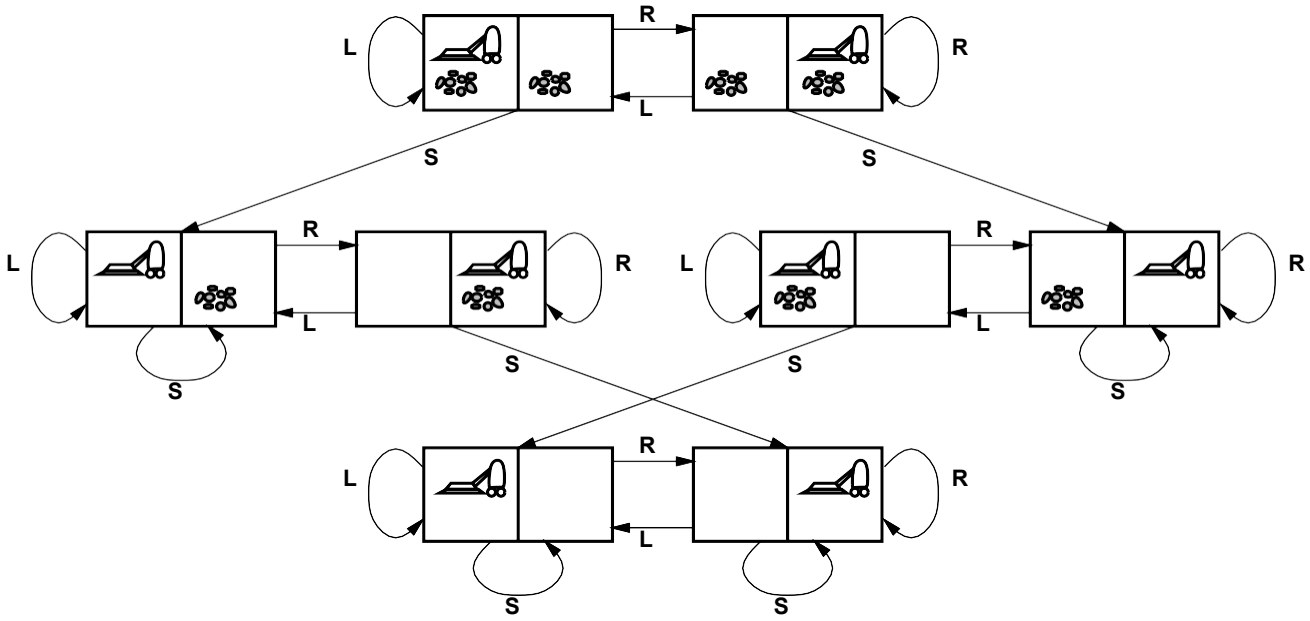
states??
actions??
goal test??
path cost??

Example: vacuum world state space graph



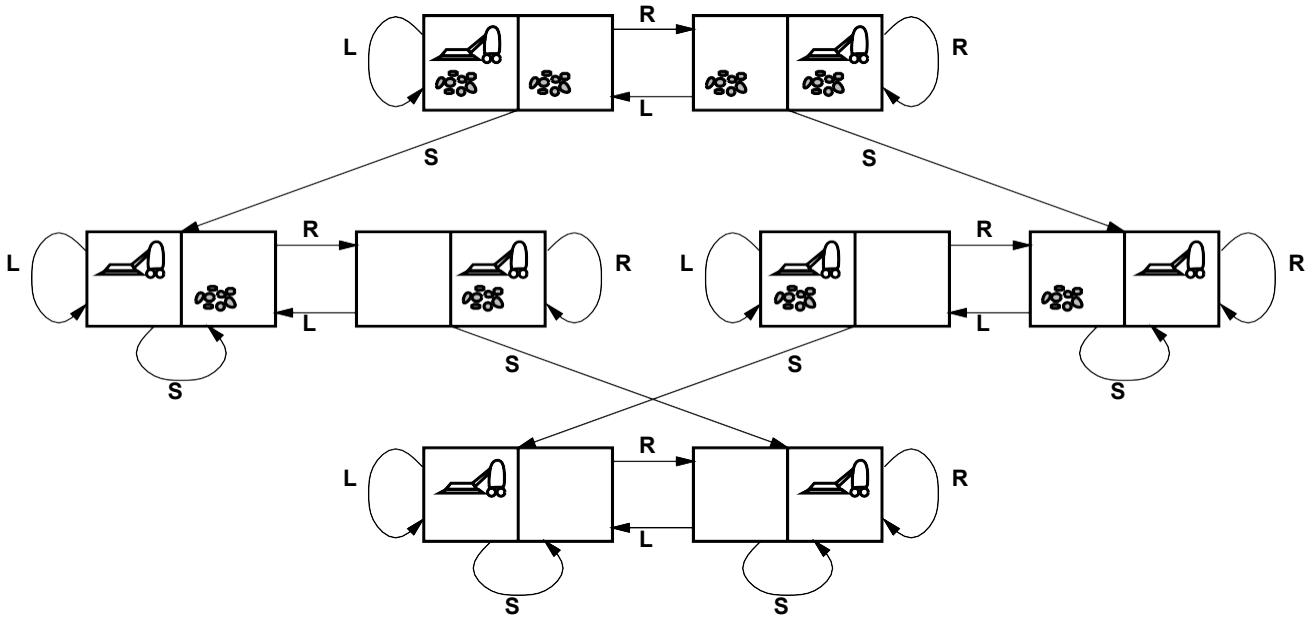
states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??
goal test??
path cost??

Example: vacuum world state space graph



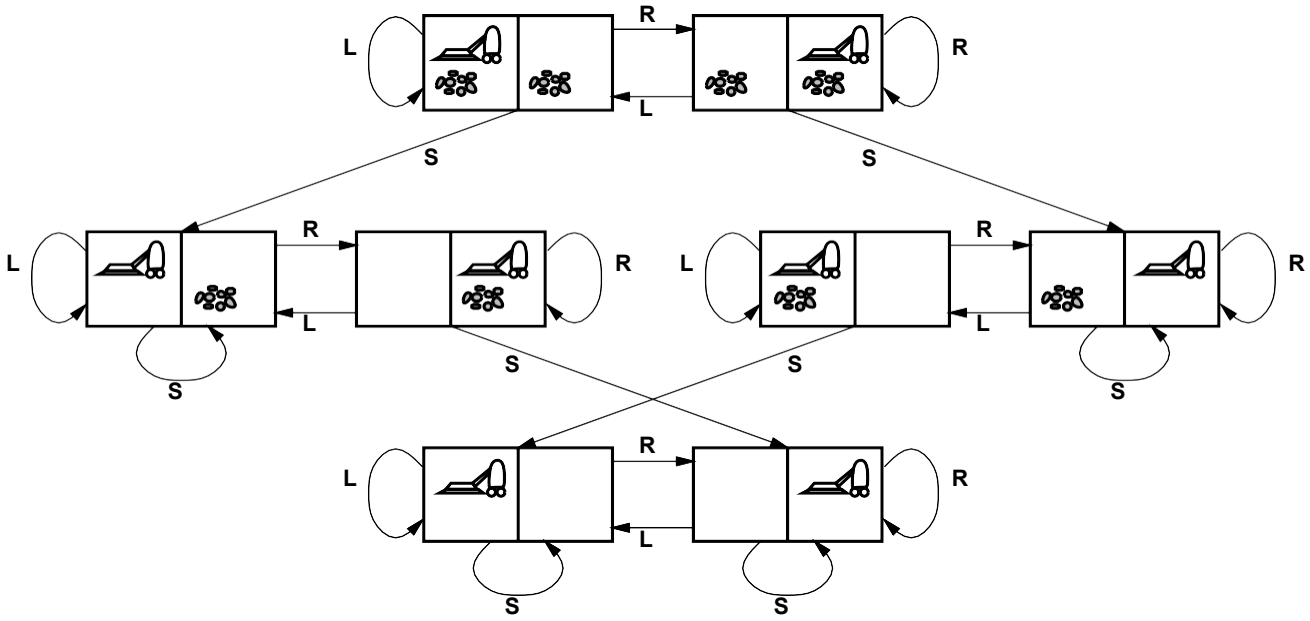
states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: *Left*, *Right*, *Suck*, *NoOp*
goal test??
path cost??

Example: vacuum world state space graph



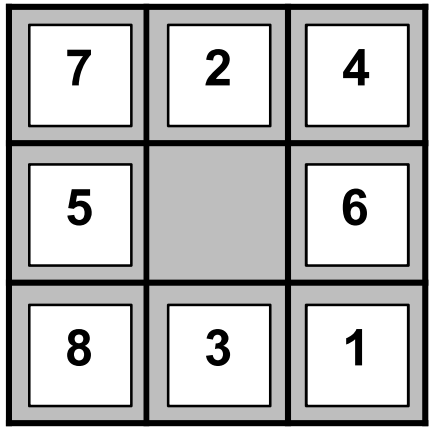
states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: *Left*, *Right*, *Suck*, *NoOp*
goal test??: no dirt
path cost??

Example: vacuum world state space graph

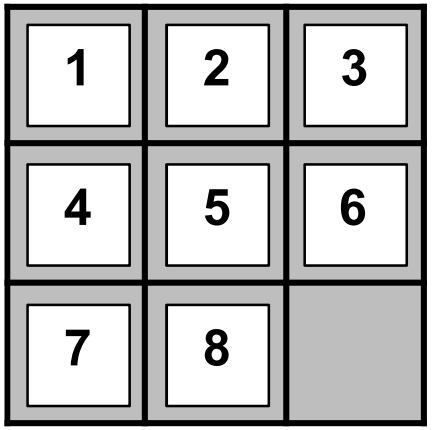


states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: *Left*, *Right*, *Suck*, *NoOp*
goal test??: no dirt
path cost??: 1 per action (0 for *NoOp*)

Example: The 8-puzzle



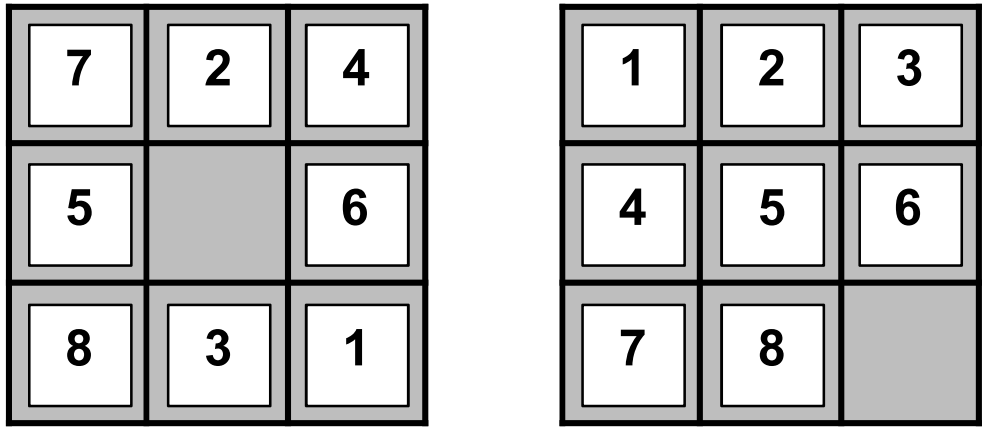
Start State



Goal State

states??
actions??
goal test??
path cost??

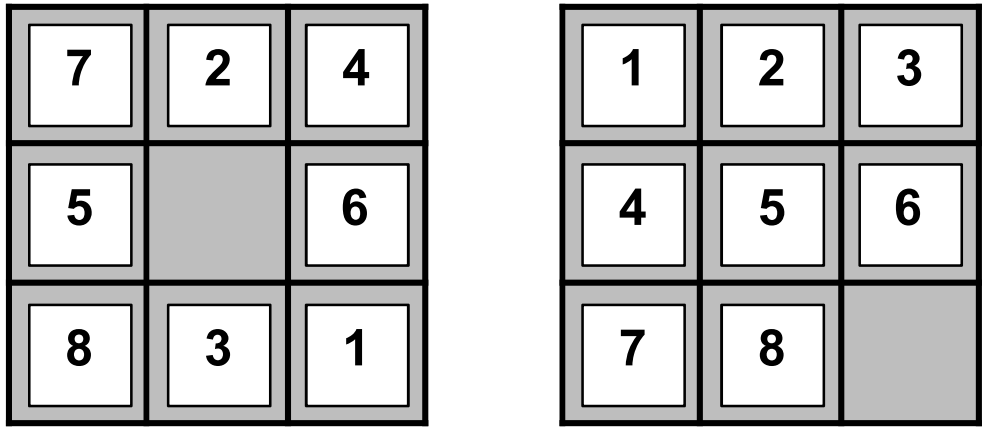
Example: The 8-puzzle



Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??
goal test??
path cost??

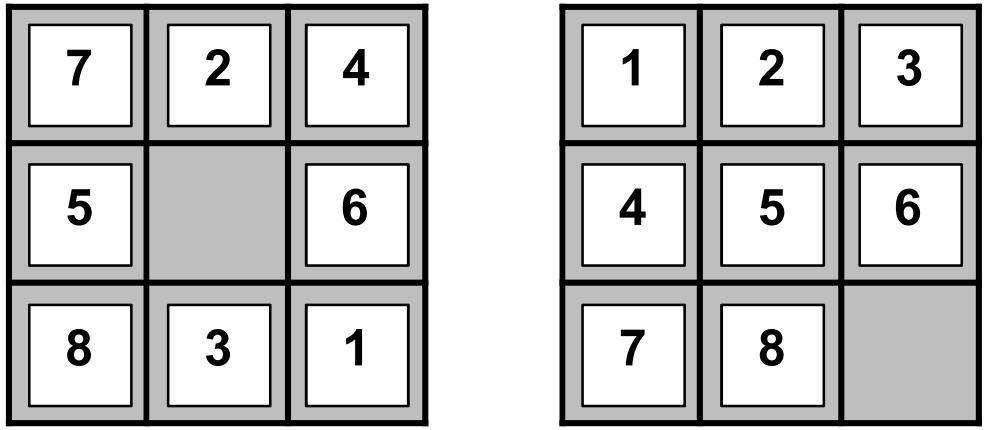
Example: The 8-puzzle



Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??
path cost??

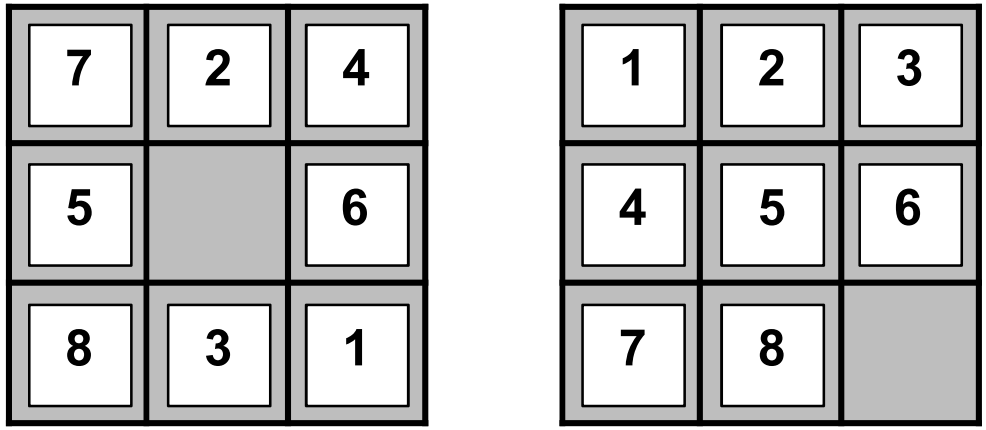
Example: The 8-puzzle



Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??

Example: The 8-puzzle

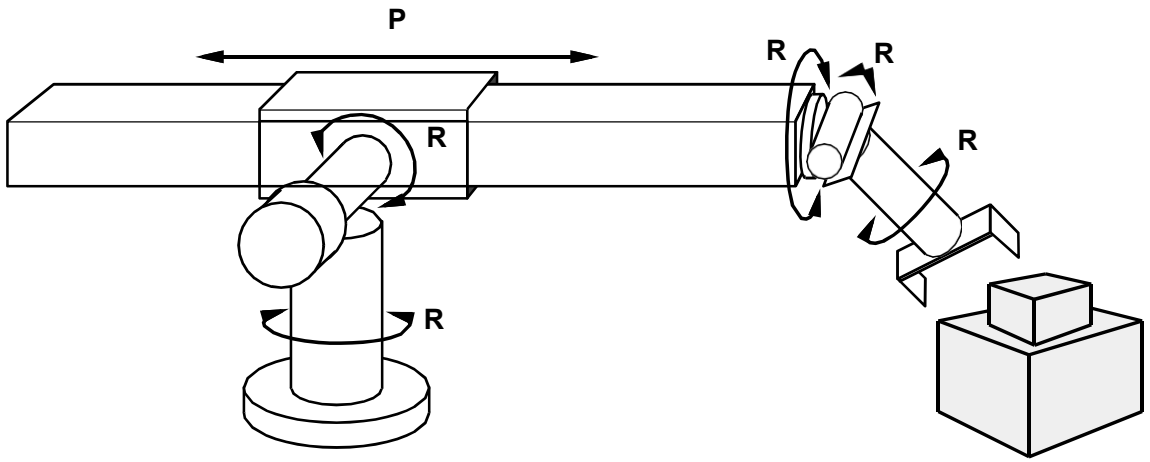


Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: robotic assembly



states??: real-valued coordinates of robot joint angles
parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly with no robot included!

path cost??: time to execute

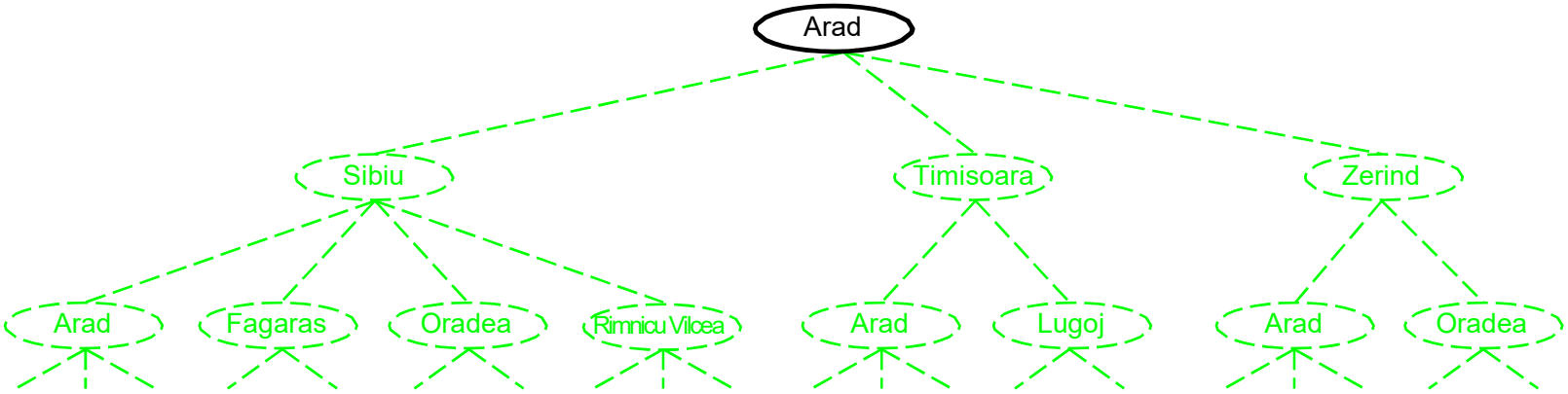
Tree search algorithms

Basic idea:

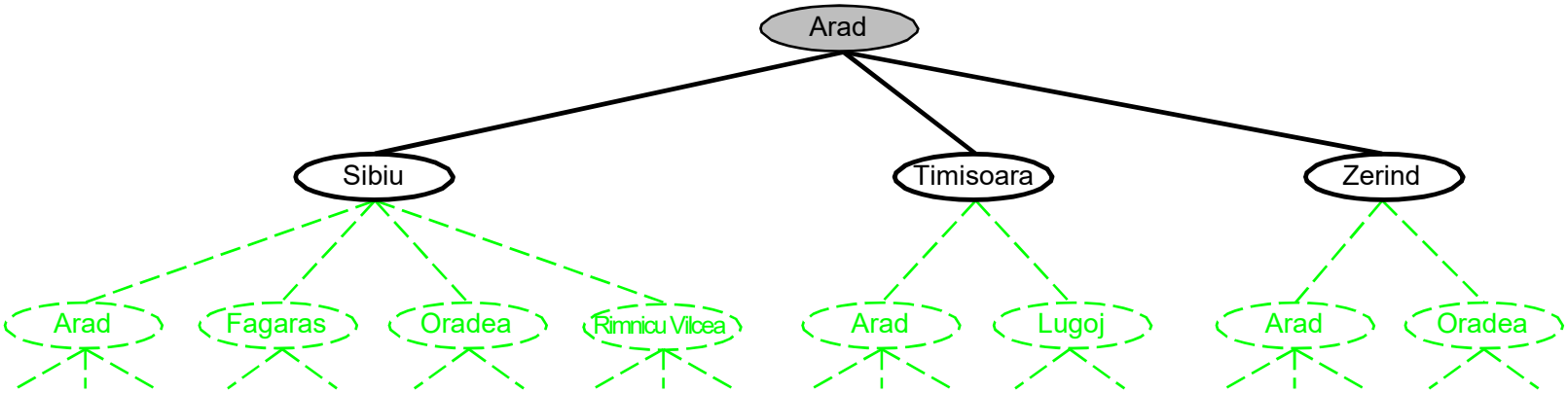
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

```
function Tree-Search( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

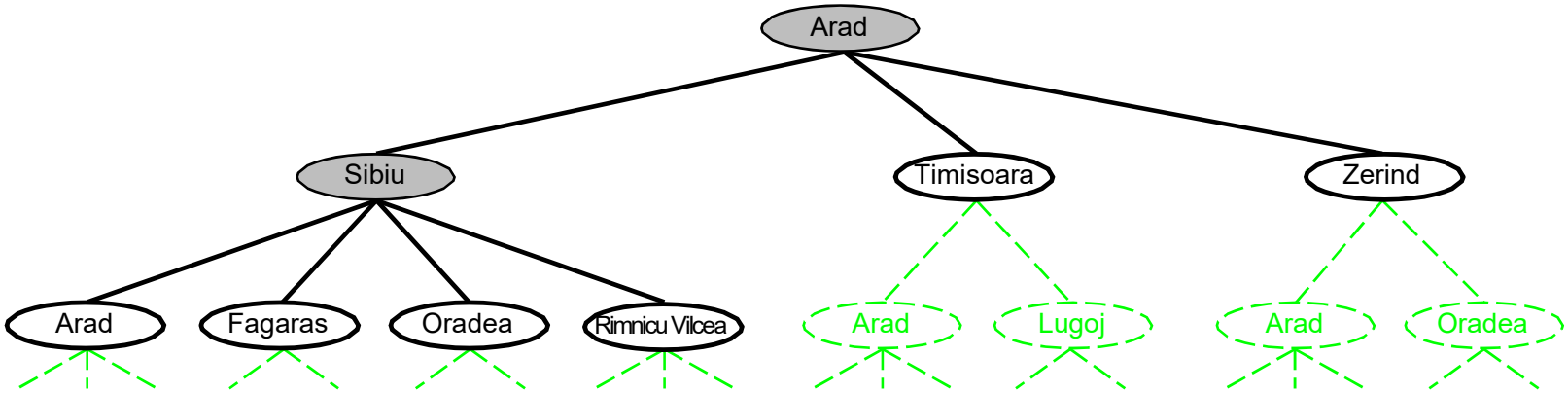
Tree search example



Tree search example

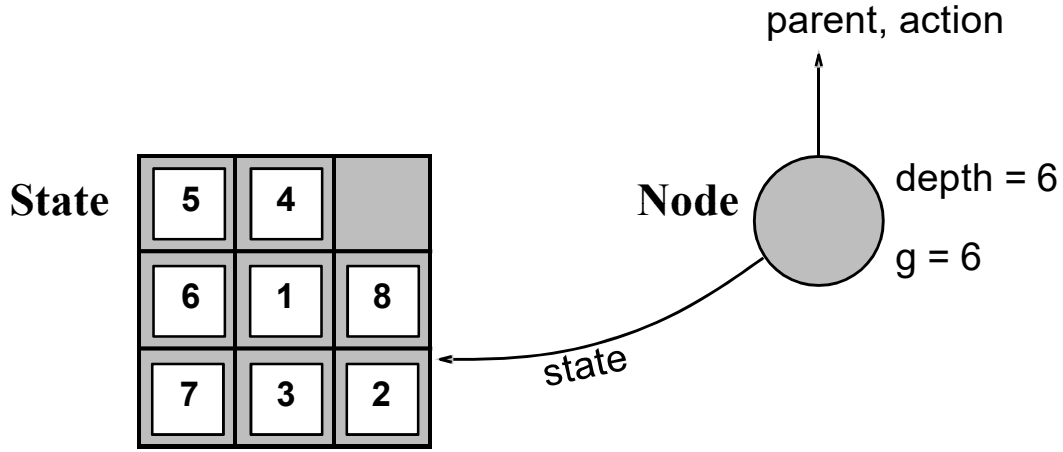


Tree search example



Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes parent, children, depth, path cost $g(x)$
States do not have parents, children, depth, or path cost!



The Expand function creates new nodes, filling in the various fields and using the SuccessorFn of the problem to create the corresponding states.

Implementation: general tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem, State(node)) then return node
        fringe ← InsertAll(Expand(node, problem), fringe)

---

function Expand(node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in Successor-Fn(problem, State[node]) do
        s ← a new Node
        Parent-Node[s] ← node; Action[s] ← action; State[s] ← result
        Path-Cost[s] ← Path-Cost[node] + Step-Cost(node, action, s)
        Depth[s] ← Depth[node] + 1
        add s to successors
    return successors
```

Search strategies

A strategy is defined by picking the order of node expansion

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

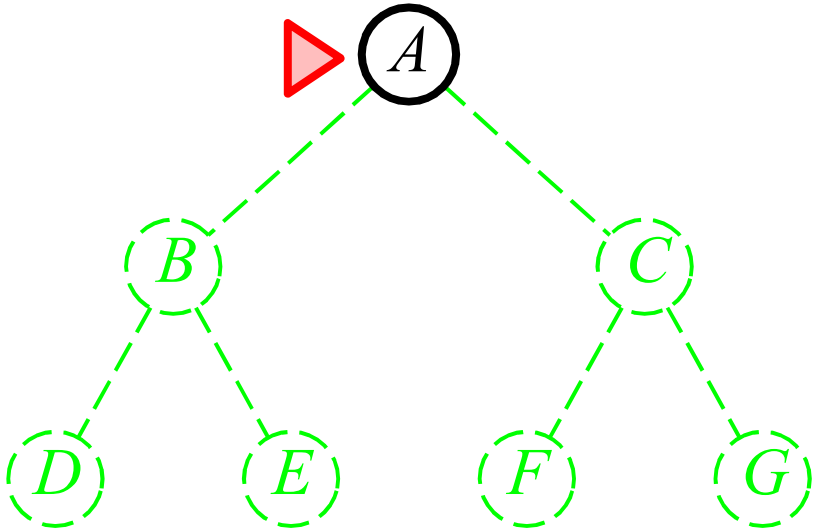
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end

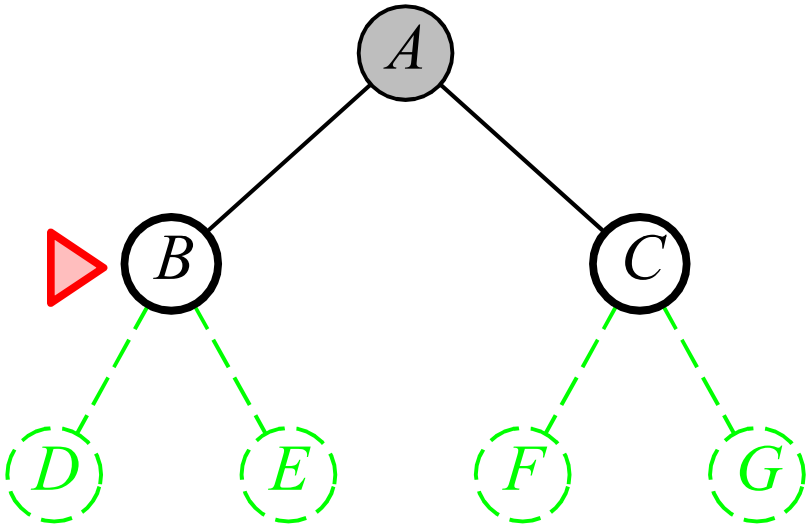


Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end



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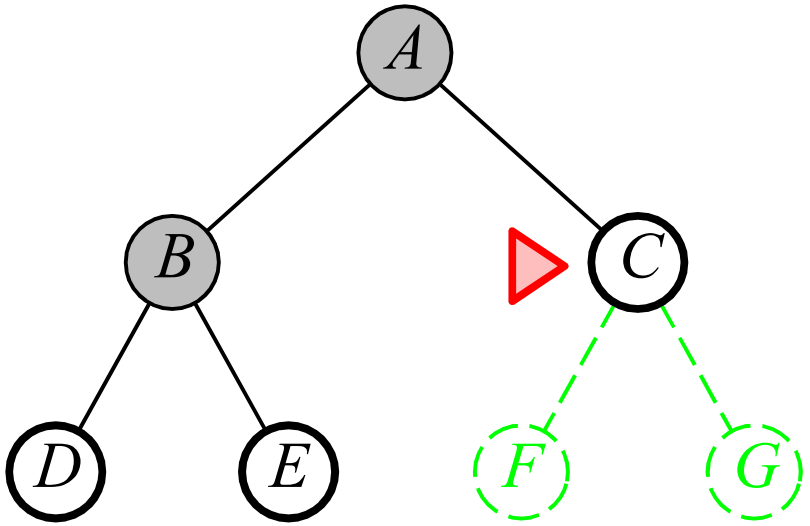

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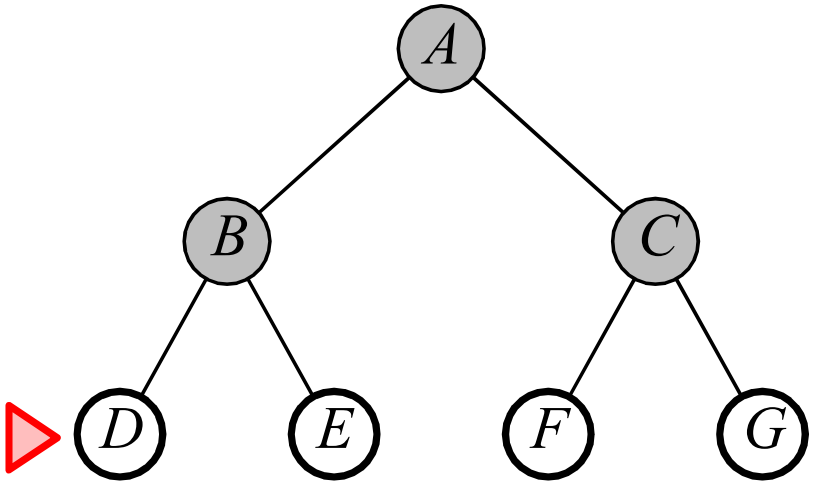


Properties of breadth-first search

Complete??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.

Uniform-cost search

Expand least-cost unexpanded node

Implementation:

fringe = queue ordered by path cost, lowest first

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

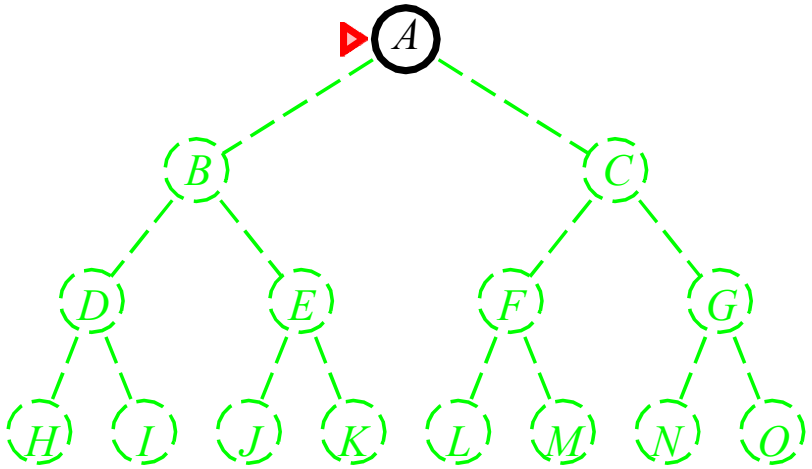
Optimal?? Yes—nodes expanded in increasing order of $g(n)$

Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

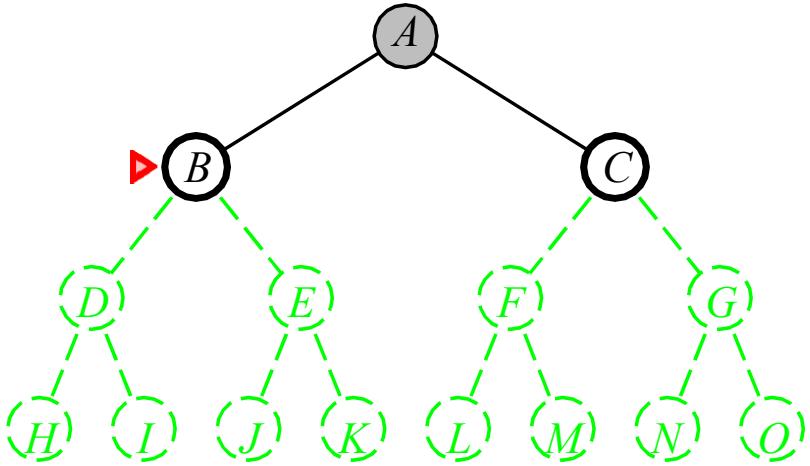


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front



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Expand deepest unexpanded node

Implementation:

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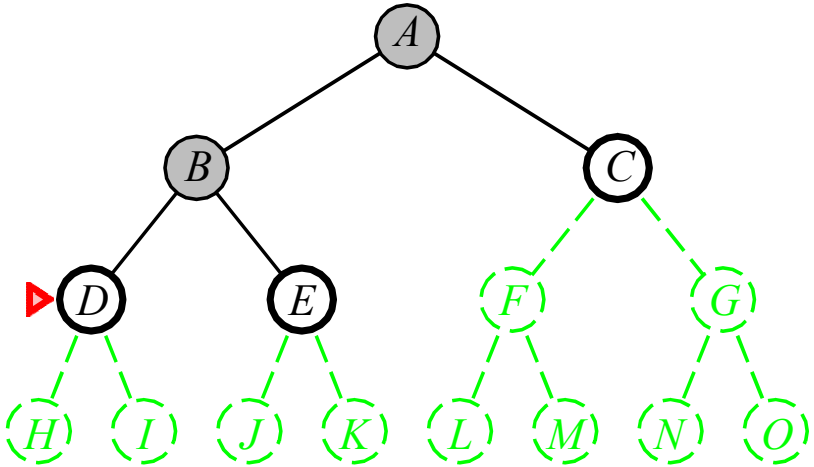

Depth-first search

Expand deepest unexpanded node

Implementation:

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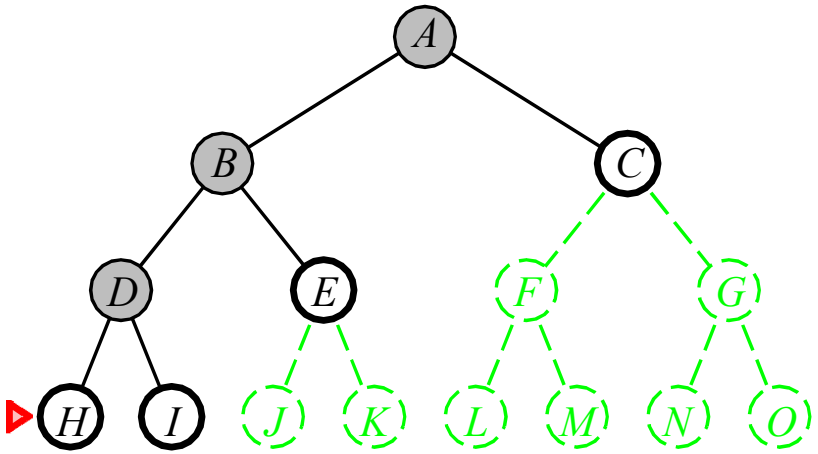
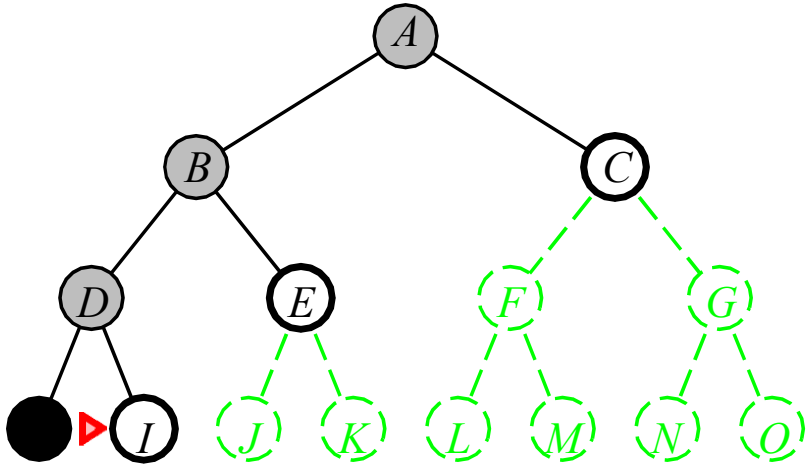


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

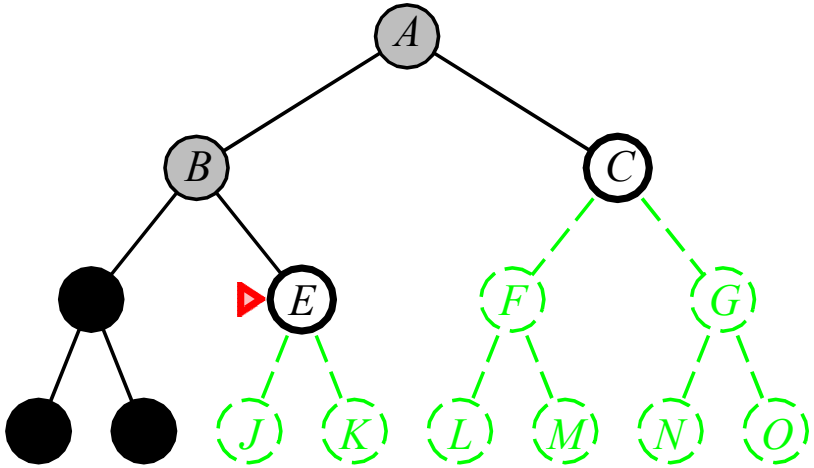


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

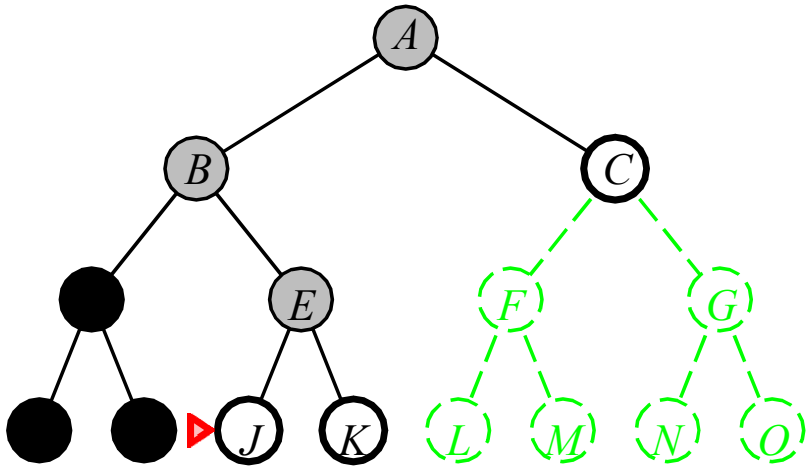


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

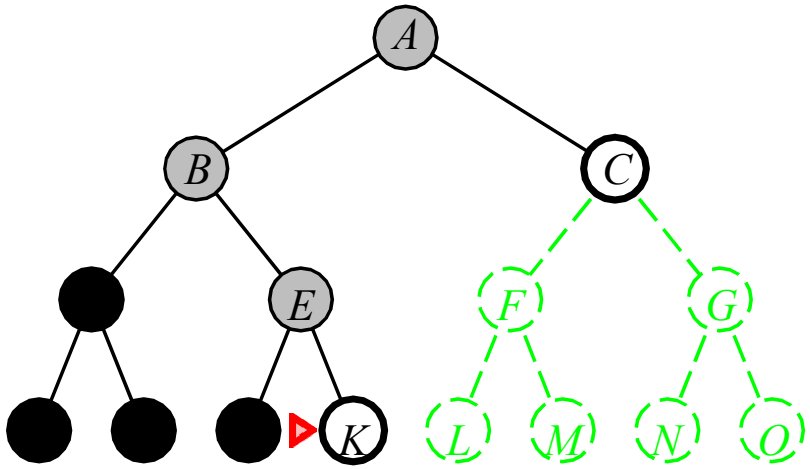


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

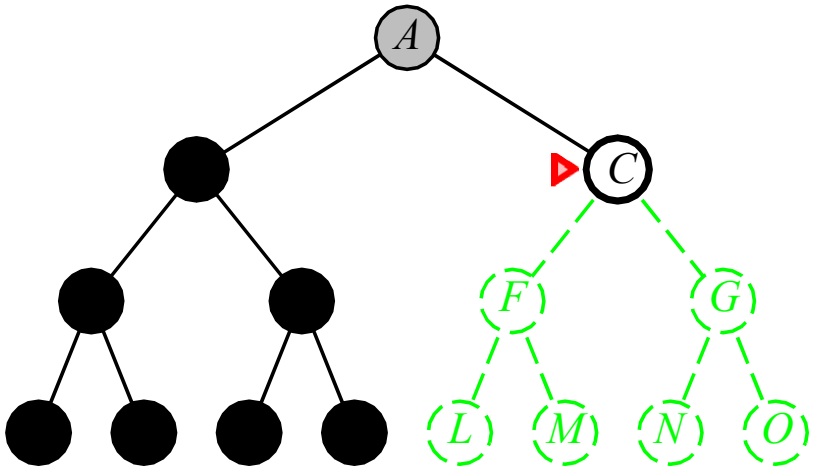


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

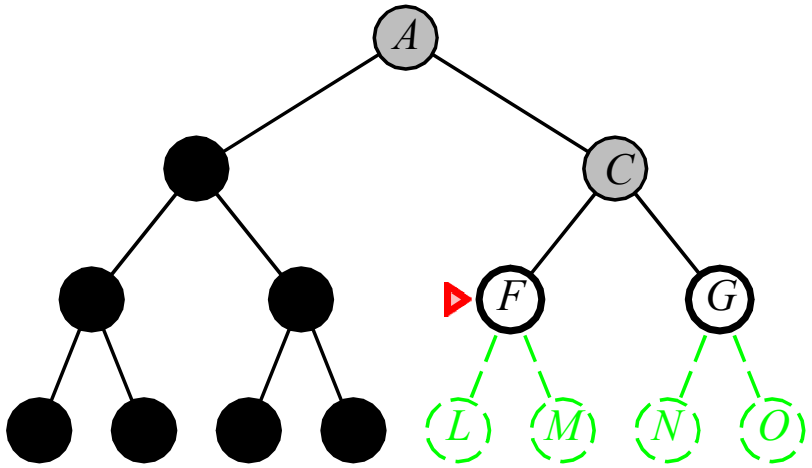


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

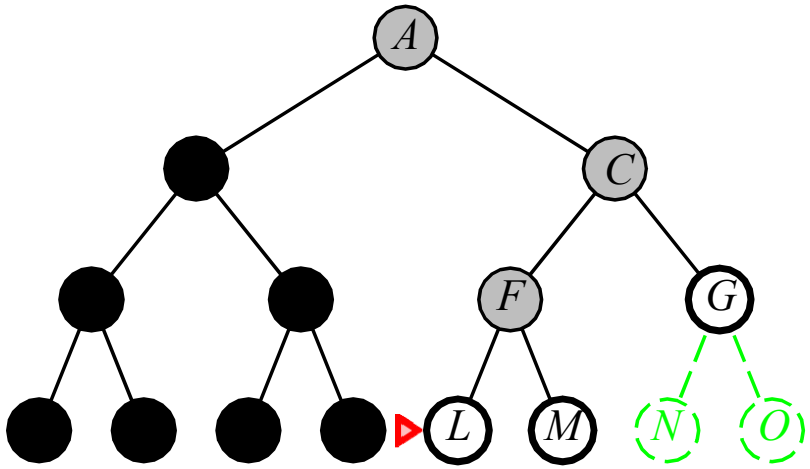


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front



Depth-first search

Expand deepest unexpanded node

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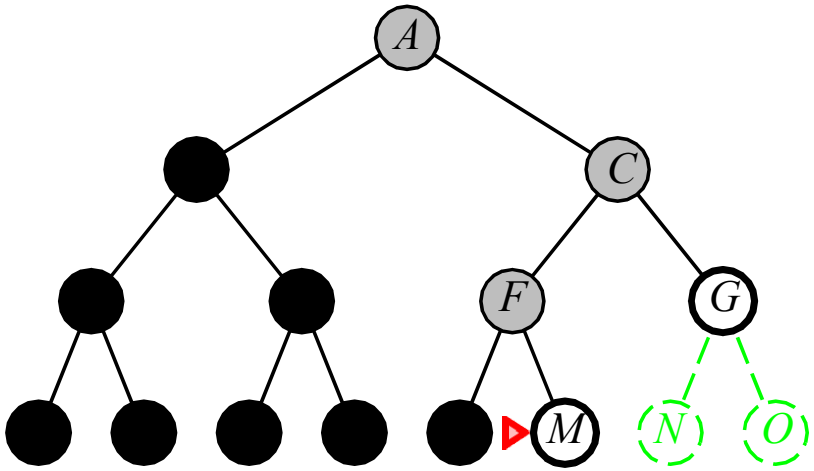


Properties of depth-first search

Complete??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

⇒ complete in finite spaces

Time??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No

Depth-limited search

= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors

Recursive implementation:

```
function Depth-Limited-Search(problem,limit) returns soln/fail/cutoff
    Recursive-DLS(Make-Node(Initial-State[problem]),problem,limit)

function Recursive-DLS(node,problem,limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if Goal-Test(problem,State[node]) then return node
    else if Depth[node] = limit then return cutoff
    else for each successor in Expand(node,problem) do
        result ← Recursive-DLS(successor,problem,limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result /= failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

```
function Iterative-Deepening-Search(problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← Depth-Limited-Search(problem, depth)
        if result /= cutoff then return result
    end
```

Iterative deepening search $l = 0$

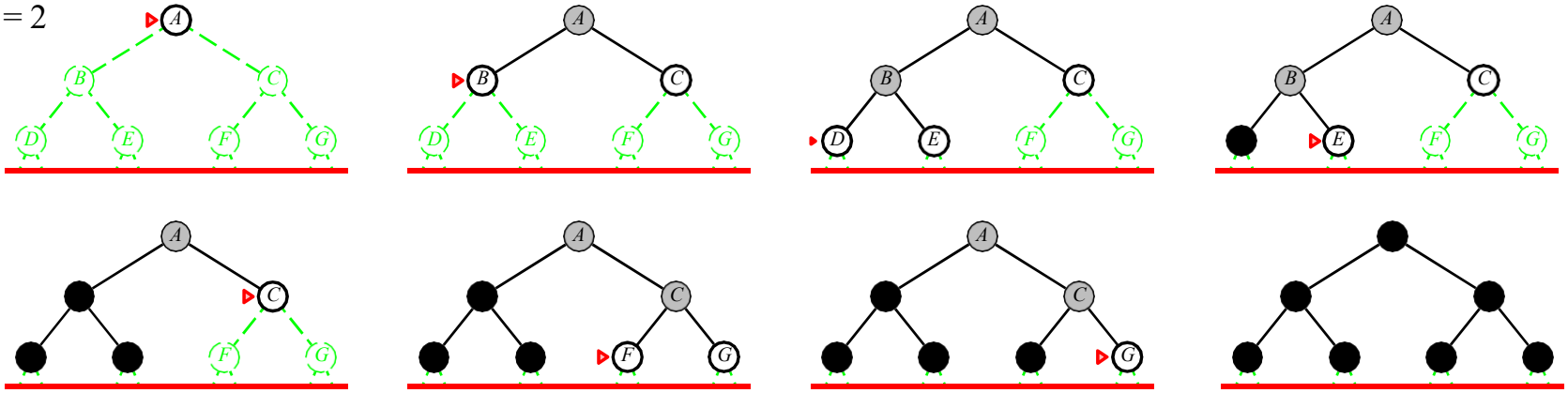
Limit = 0

Iterative deepening search $l = 1$

Limit = 1

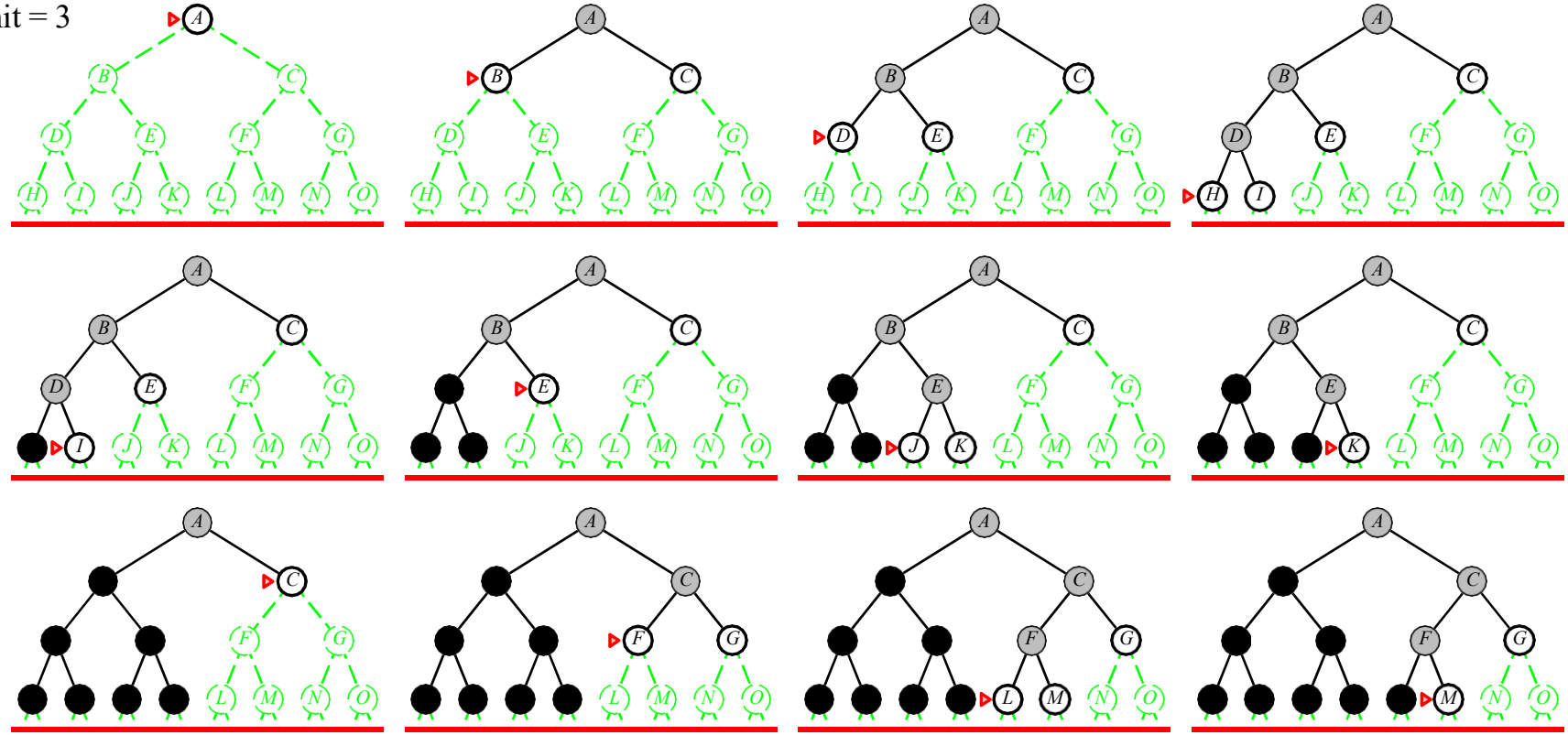
Iterative deepening search $l = 2$

Limit = 2



Iterative deepening search $l = 3$

Limit = 3



Properties of iterative deepening search

Complete??

Properties of iterative deepening search

Complete?? Yes

Time??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost = 1

Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

IDS does better because other nodes at depth d are not expanded

BFS can be modified to apply goal test when a node is generated

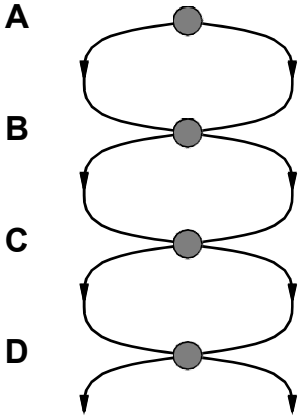
Summary of Uninformed Search algorithms

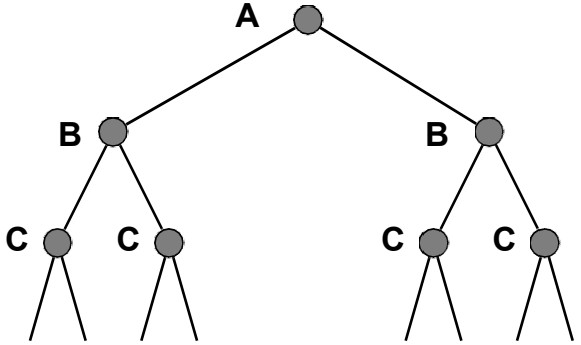
Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[1]	Yes[1,2]	No	No	Yes[1]	Yes[1,4]
Optimal cost?	Yes[3]	Yes	No	No	Yes[3]	Yes[3,4]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

Figure 3.15 Evaluation of search algorithms. b is the branching factor; m is the maximum depth of the search tree; d is the depth of the shallowest solution, or is m when there is no solution; ℓ is the depth limit. Superscript caveats are as follows: [1] complete if b is finite, and the state space either has a solution or is finite. [2] complete if all action costs are $\geq \epsilon > 0$; [3] cost-optimal if action costs are all identical; [4] if both directions are breadth-first or uniform-cost.

Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!





Graph search

```
function Graph-Search(problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem, State[node]) then return node
        if State[node] is not in closed then
            add State[node] to closed
            fringe ← InsertAll(Expand(node, problem), fringe)
    end
```

Review: Tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test[problem] applied to State(node) succeeds return node
        fringe ← InsertAll(Expand(node, problem), fringe)
```

A strategy is defined by picking the order of node expansion

Best-first search

Idea: use an evaluation function for each node
– estimate of "desirability"

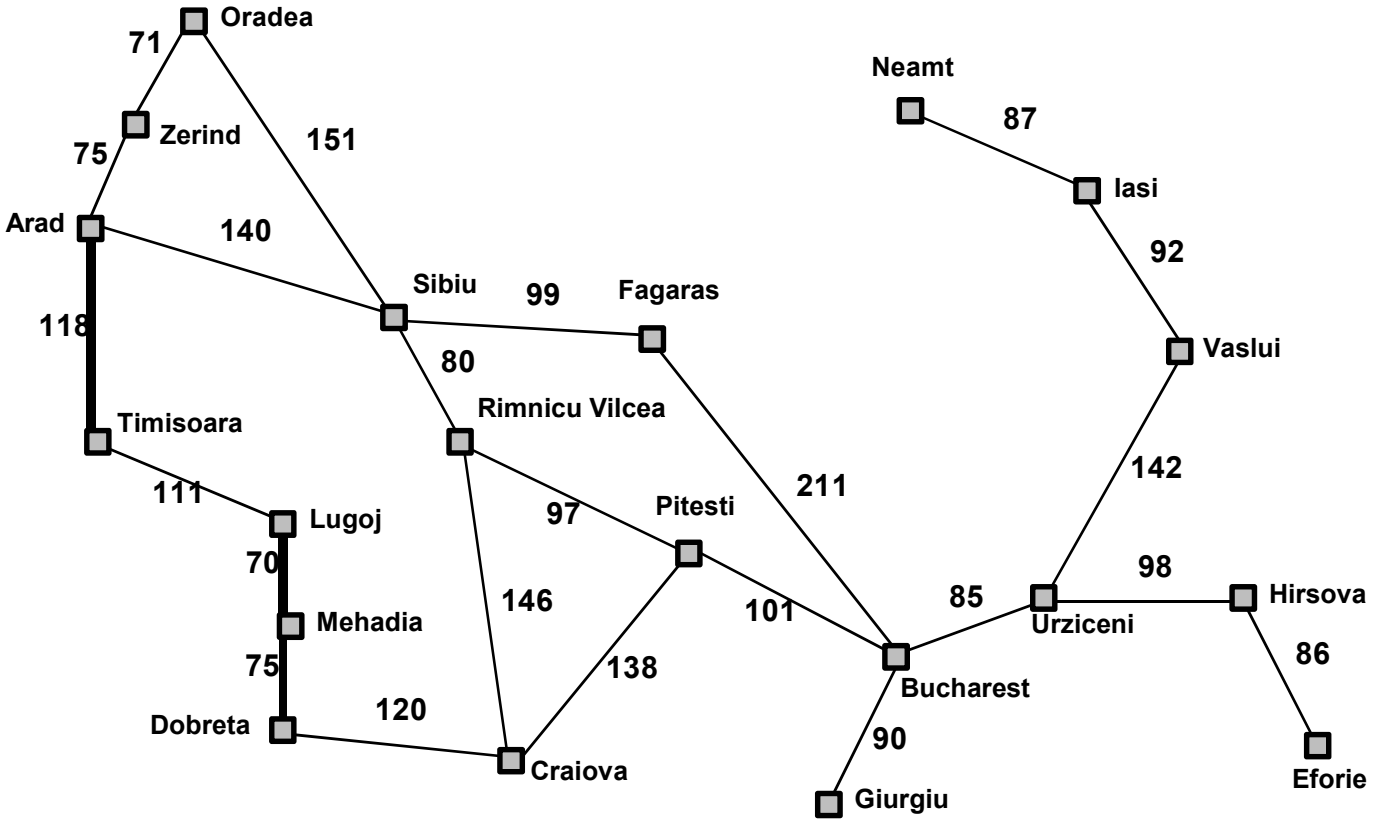
⇒ Expand most desirable unexpanded node

Implementation:

fringe is a queue sorted in decreasing order of desirability

Special cases:
greedy search
A* search

Romania with step costs in km



Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

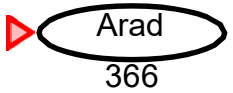
Greedy search

Evaluation function $h(n)$ (heuristic)
= estimate of cost from n to the closest goal

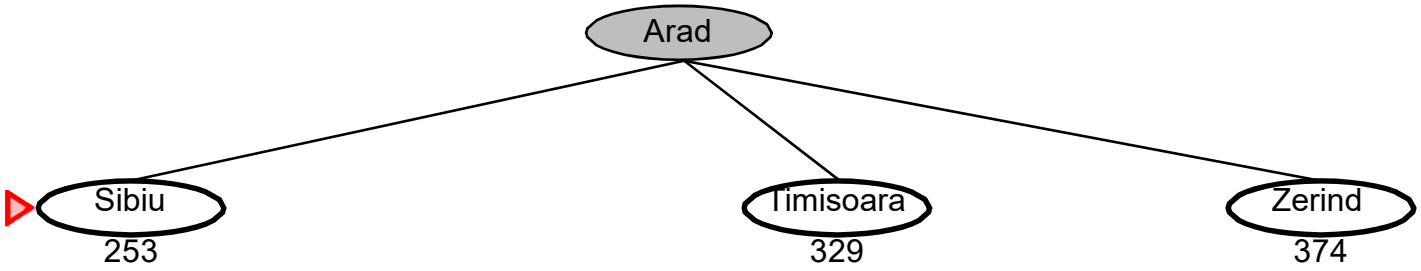
E.g., $h_{\mathrm{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that appears to be closest to goal

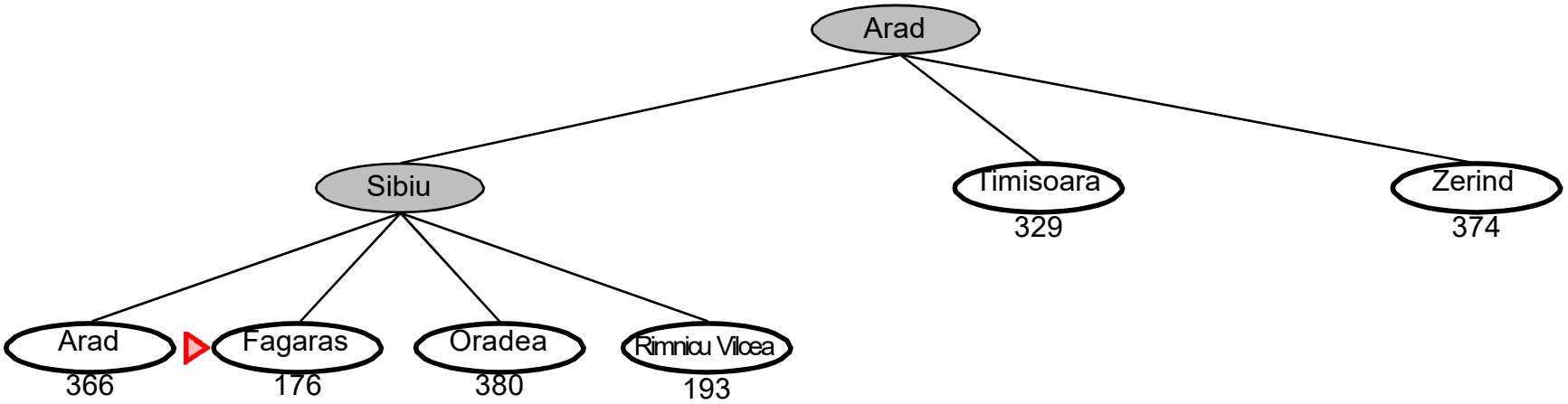
Greedy search example



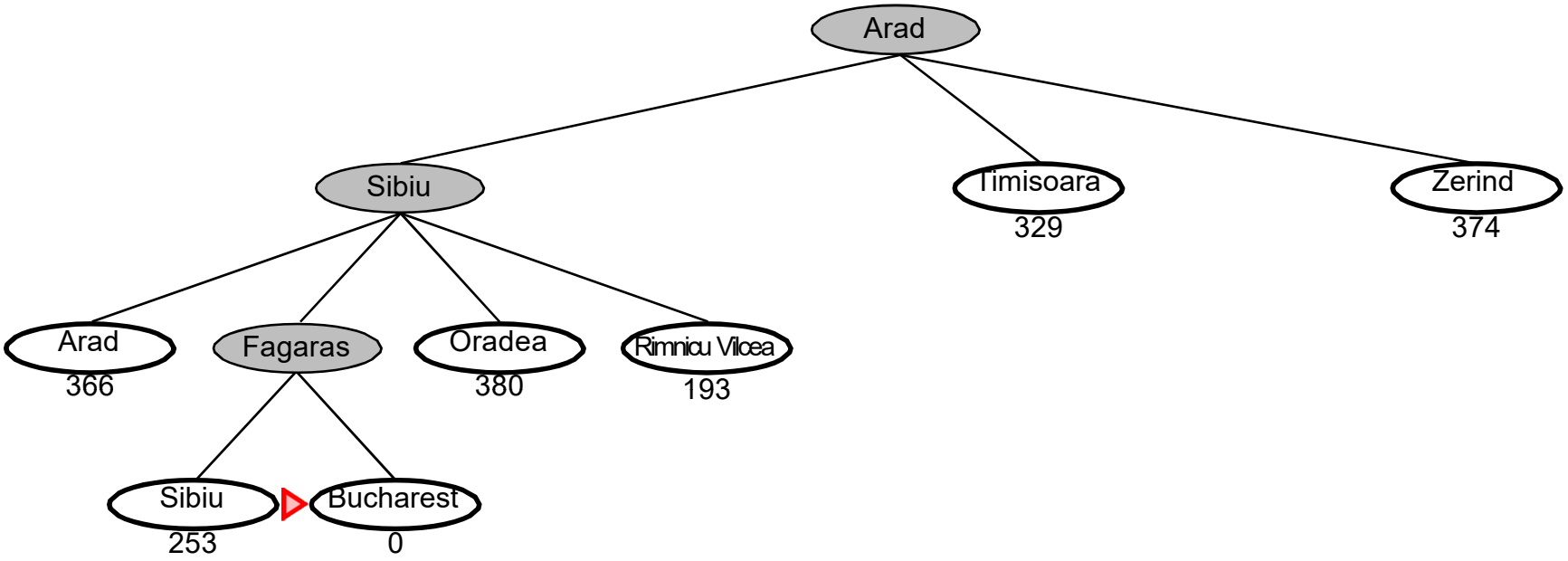
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g., with Oradea as goal,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

Properties of greedy search

Complete?? No–can get stuck in loops,
e.g., Iasi → Neamt → Iasi →
Neamt →
Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal??

Properties of greedy search

Complete?? No–can get stuck in loops,
e.g., Iasi → Neamt → Iasi →
Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n

$h(n)$ = estimated cost to goal from n

$f(n)$ = estimated total cost of path through n to goal

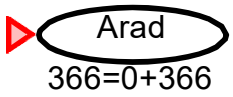
A* search uses an admissible heuristic

i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n. (Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

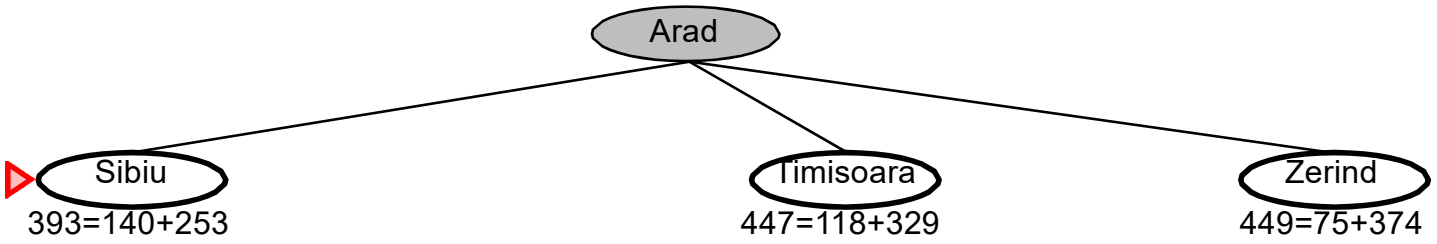
E.g., $h_{\text{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

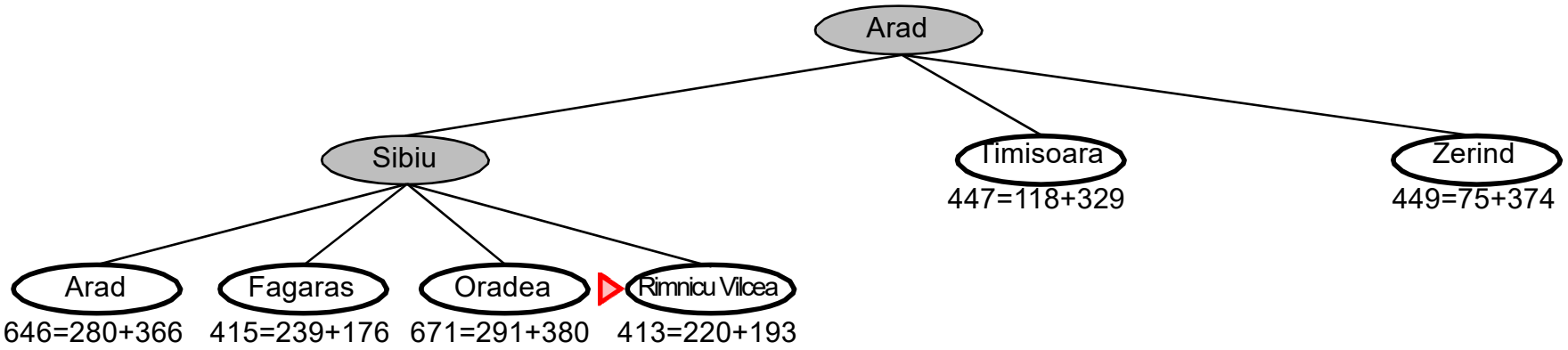
A* search example



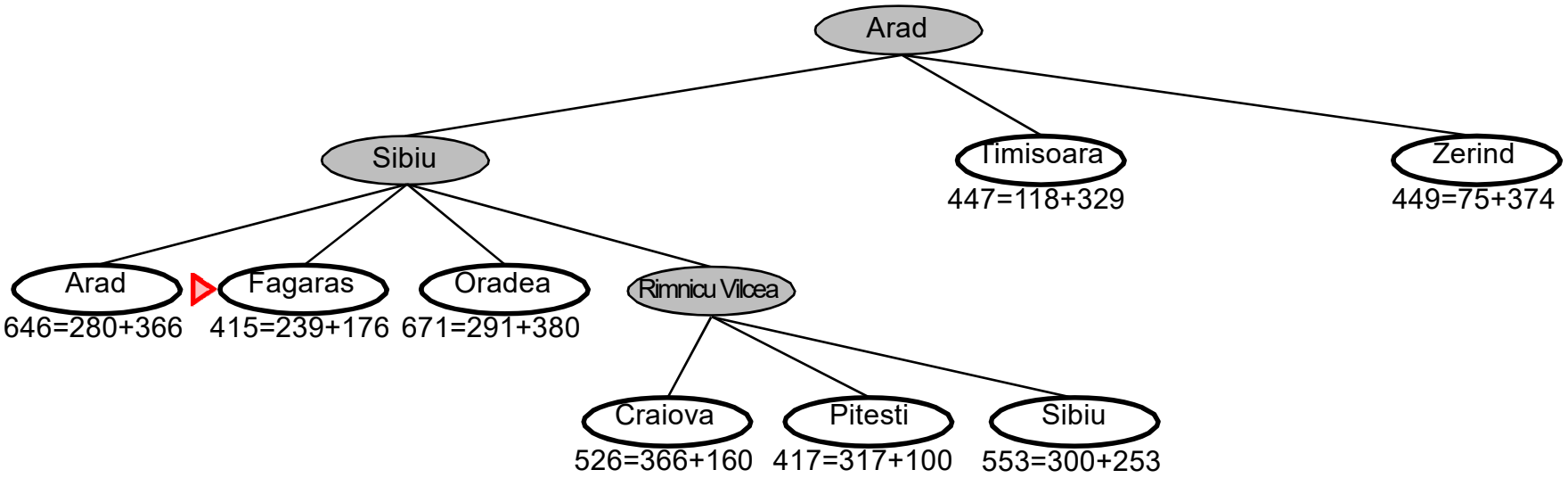
A* search example



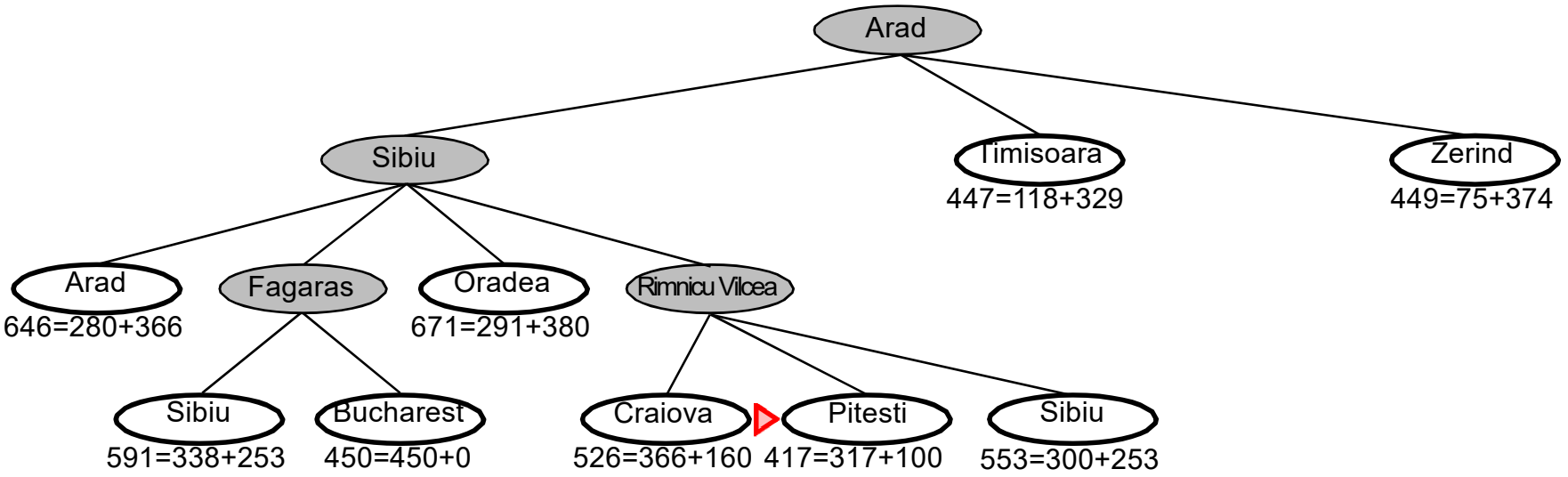
A* search example



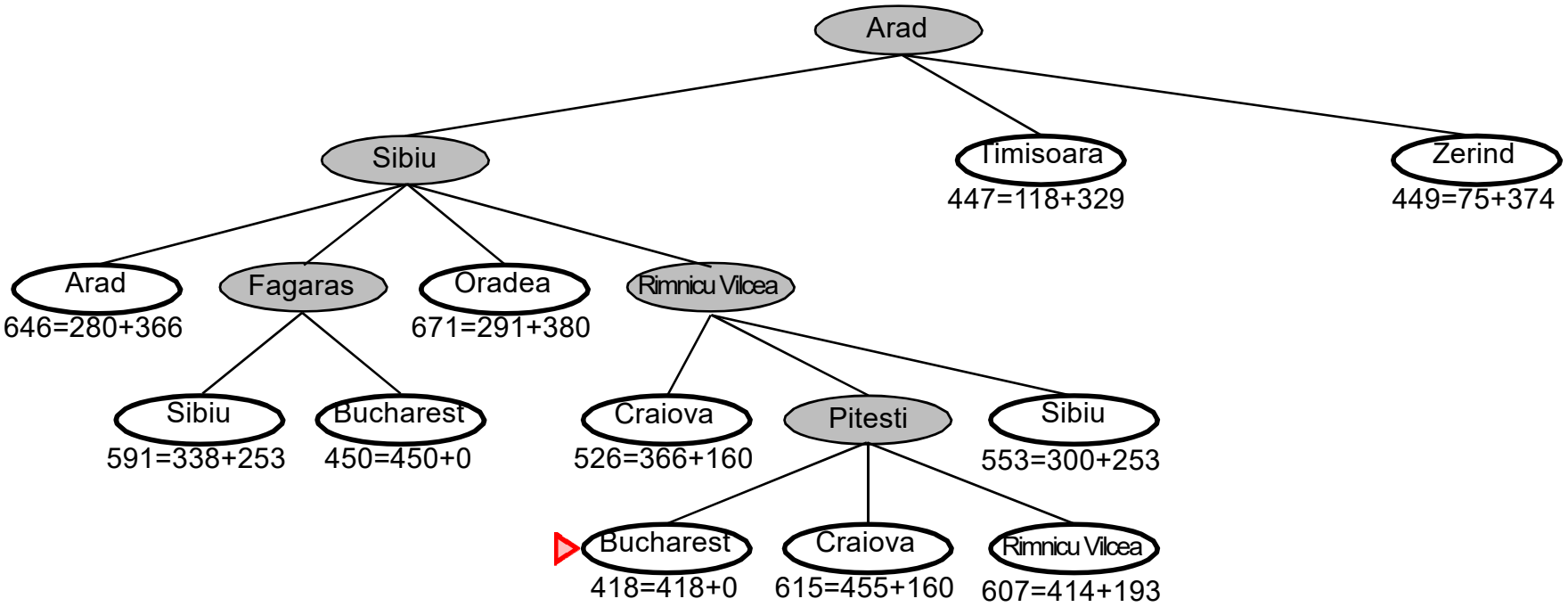
A* search example



A* search example

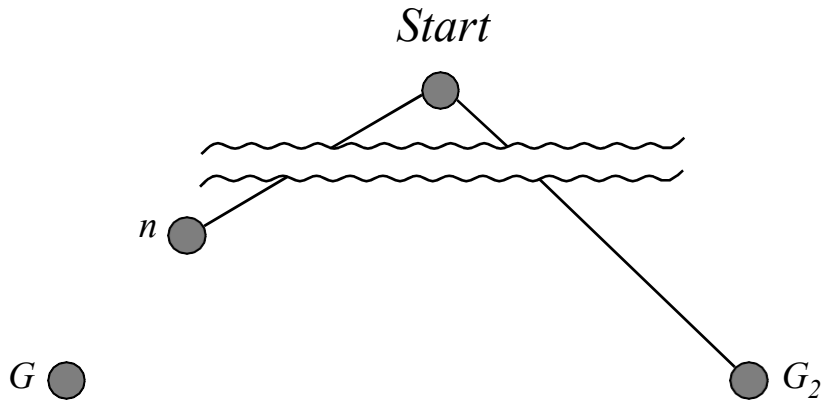


A* search example



Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



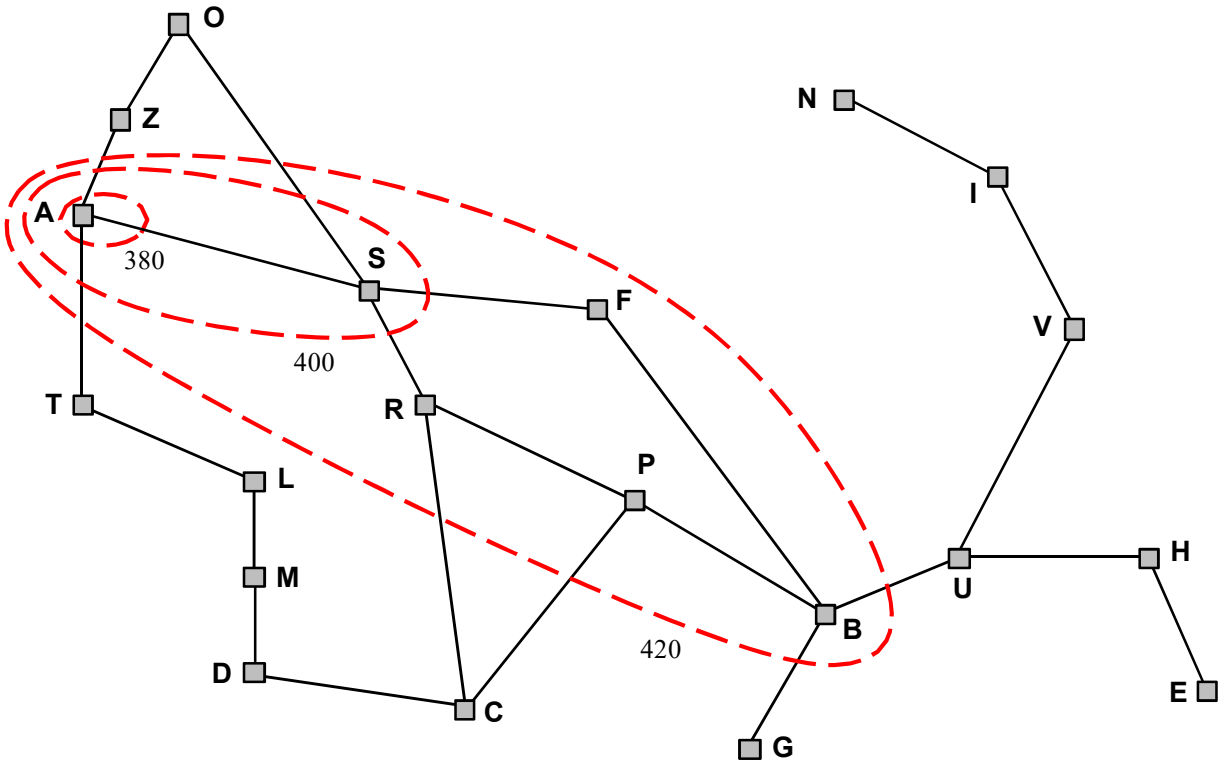
$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$

A*expands some nodes with $f(n) = C^*$

A*expands no nodes with $f(n) > C^*$

Proof of lemma: Consistency

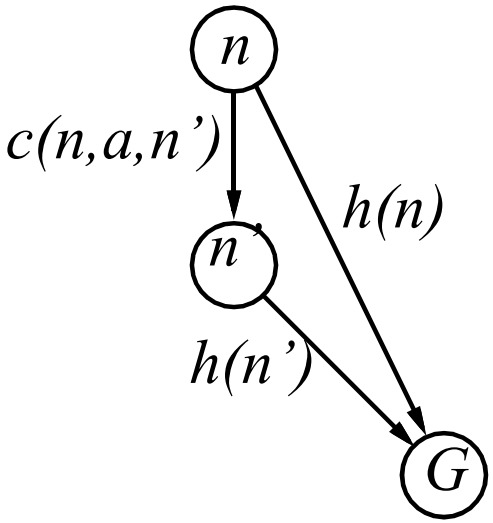
A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

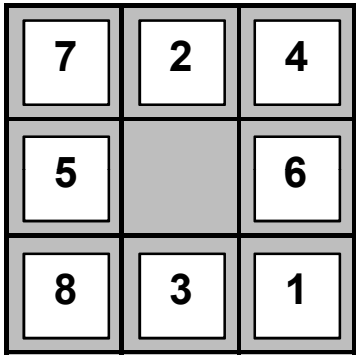
I.e., $f(n)$ is nondecreasing along any path.



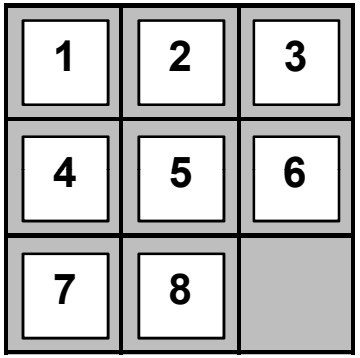
Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State



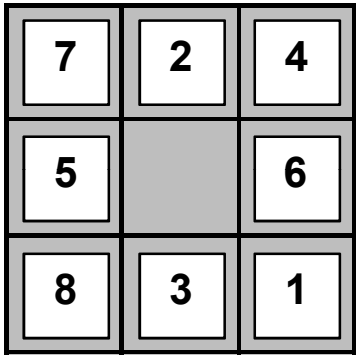
Goal State

$h_1(S)$ =??
$h_2(S)$ =??

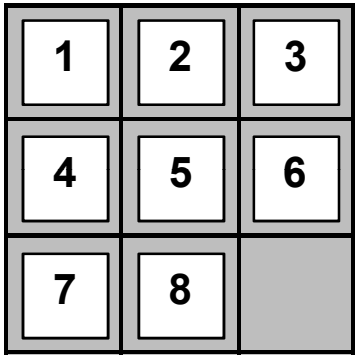
Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State



Goal State

$h_1(S)$ =?? 6
$h_2(S)$ =?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible), then h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A*(h_1) = 539 nodes
A*(h_2) = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A*(h_1) = 39,135 nodes
A*(h_2) = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem

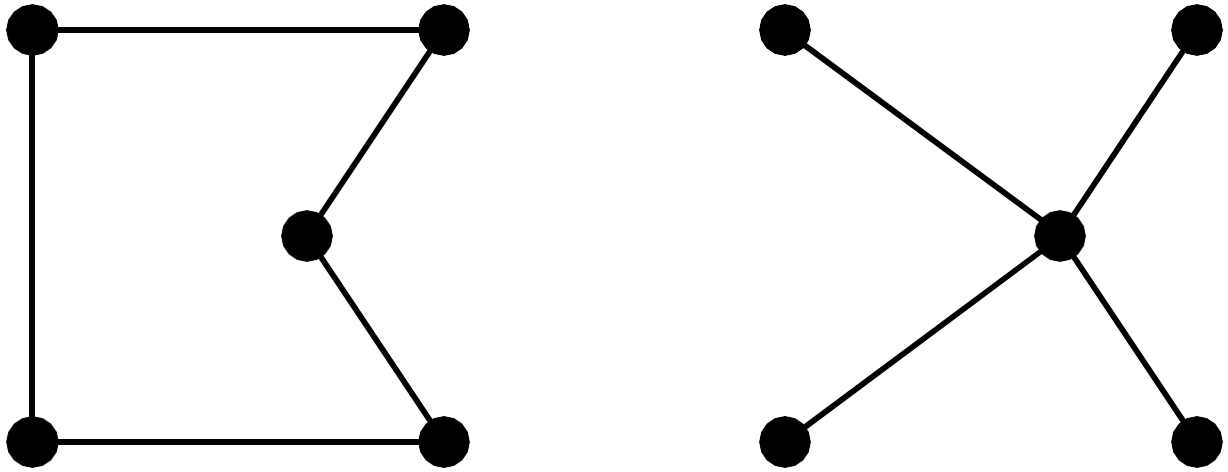
If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP) Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour

Summary

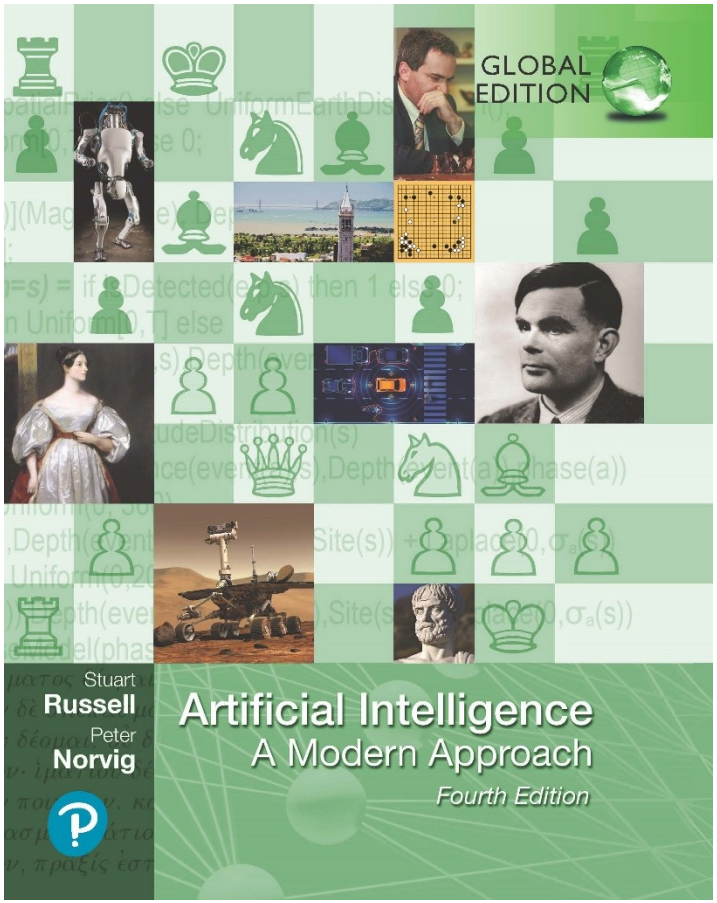
A problem consists of five parts: the **initial state**, a set of **actions**, a **transition model** describing the results of those actions, a set of **goal states**, and an **action cost function**.

Uninformed search methods have access only to the **problem definition**. Algorithms build a search tree in an attempt to find a solution.

Informed search methods have access to a **heuristic** function h(n) that estimates the cost of a solution from n.

Artificial Intelligence: A Modern Approach

Fourth Edition, Global Edition



Chapter 4

Search in Complex Environments

Lecture Presentations: Artificial Intelligence

Adapted from:
"Artificial Intelligence: A Modern Approach, Global Edition", 4th Edition by Stuart Russell and Peter Norvig © 2021 Pearson Education.

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Outline

- Local Search and Optimization Problems
  - Hill-climbing
  - Simulated annealing
  - Genetic algorithms
- Local search in continuous spaces
- Search with Nondeterministic Actions
- Search in Partially Observable Environments

Local Search and Optimization Problems

In many optimization problems, path is irrelevant; the goal state itself is the solution

Then state space = set of "complete" configurations; find optimal configuration, e.g., TSP
or, find configuration satisfying constraints, e.g., timetable

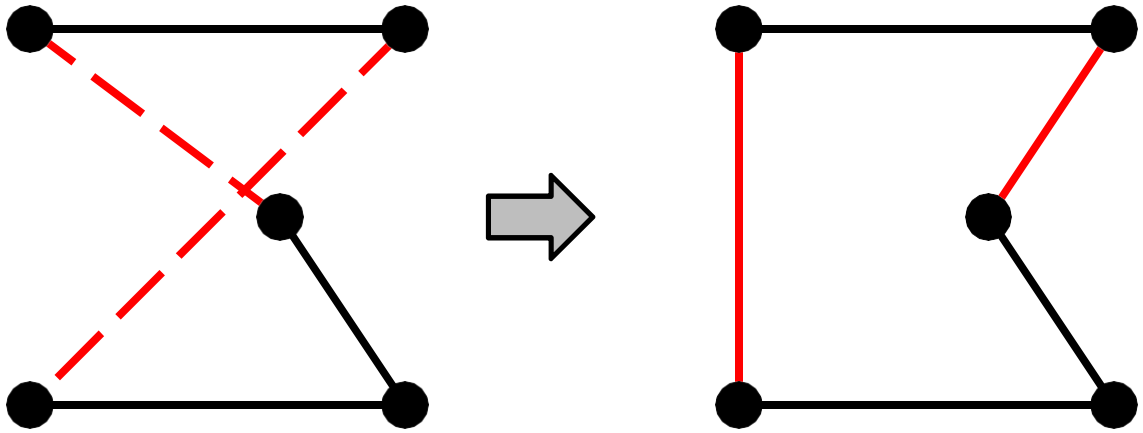
In such cases, one can use iterative improvement algorithms; keep a single "current" state, try to improve it

Local search algorithms operate by searching from a start state to neighboring states, without keeping track of the paths, nor the set of states that have been reached.

They are not systematic—they might never explore a portion of the search space where a solution actually resides.

Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

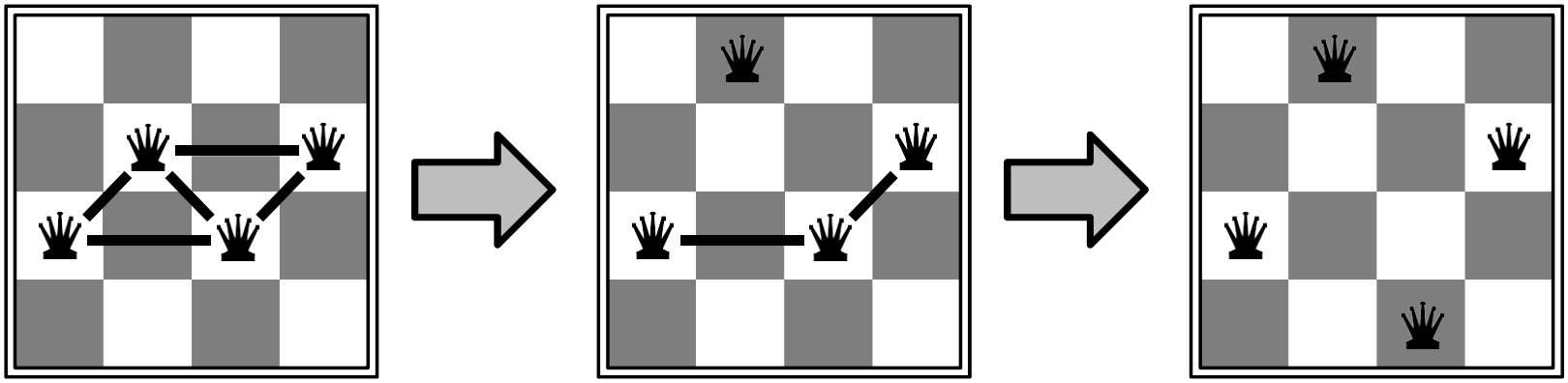


Variants of this approach get within 1% of optimal very quickly with thousands of cities

Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Move a queen to reduce number of conflicts



h = 5 h = 2 h = 0

Almost always solves n-queens problems almost instantaneously for very large n, e.g., $n = 1 million$

Hill-climbing (or gradient ascent/descent)

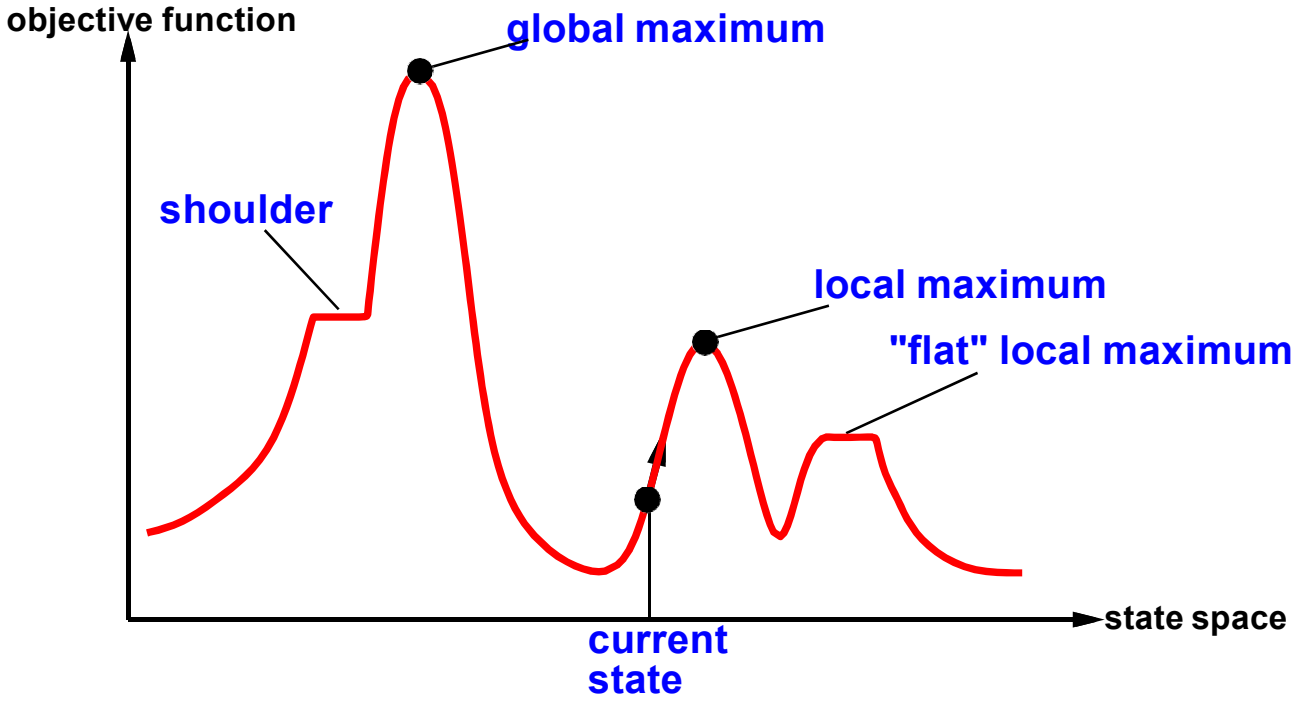
"Like climbing Everest in thick fog with amnesia"

```
function Hill-Climbing(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current ← Make-Node(Initial-State[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if Value[neighbor] ≤ Value[current] then return State[current]
        current ← neighbor
    end
```

Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves ☺ escape from shoulders ☹ loop on flat maxima

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
but gradually decrease their size and frequency

```
function Simulated-Annealing(problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← Make-Node(Initial-State[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← Value[next] – Value[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(ΔE/T)
```

Properties of simulated annealing

At fixed "temperature" T, state occupation probability reaches Boltzman distribution

$$p(x) = ae^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\Rightarrow$ always reach best state x^*
because $e^{\frac{E(x^*)}{kT}} / e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small T

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!
Searches that find good states recruit other searches to join them

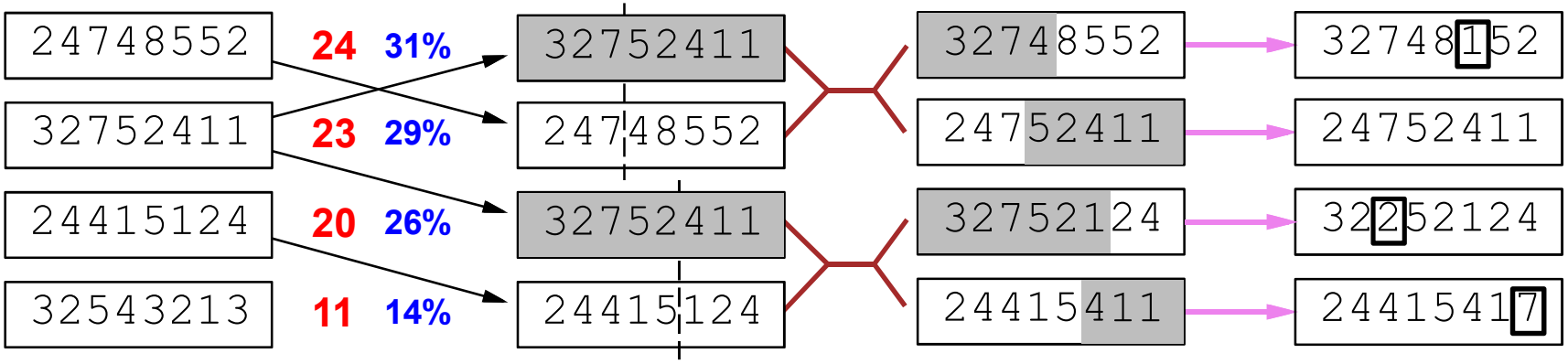
Problem: quite often, all k states end up on same local hill

Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic algorithms

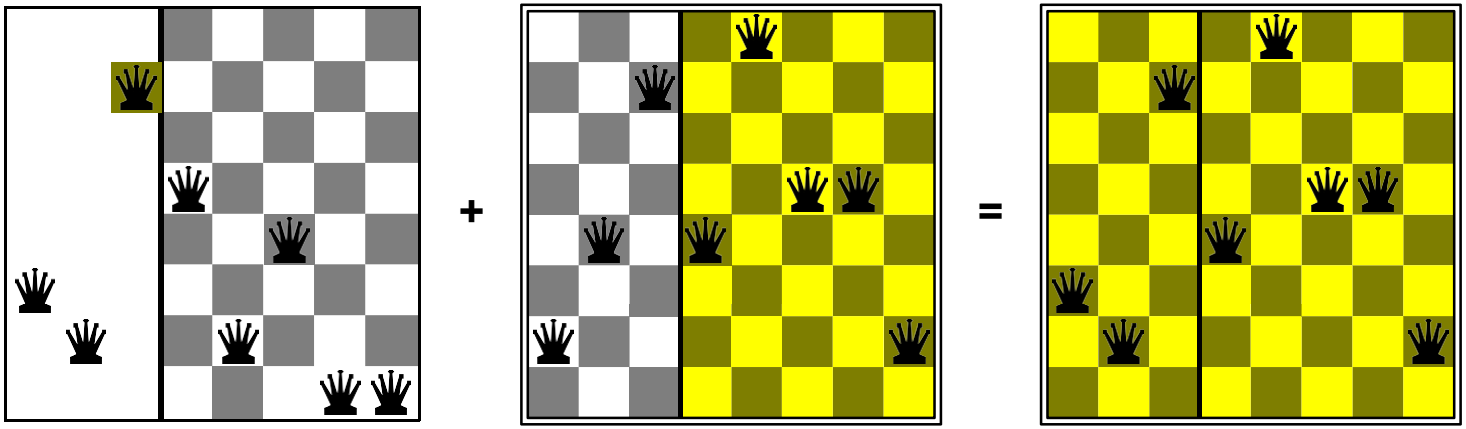
= stochastic local beam search + generate successors from pairs of states



Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)

Crossover helps iff substrings are meaningful components



GAs /= evolution: e.g., real genes encode replication machinery!

Continuous state spaces

Suppose we want to site three airports in Romania:

- 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3)$ = sum of squared distances from each city to nearest airport

Discretization methods turn continuous space into discrete space, e.g., empirical gradient considers $\pm\delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right]$$

to increase/reduce f, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + a\nabla f(\mathbf{x})$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x})\nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$

Search with Nondeterministic Actions

Agent doesn't know the state its transitioned to **after action**, the environment is **nondeterministic**.

Rather, it will know the possible states it will be in, which is called "**belief state**"

Examples:

- The **erratic vacuum world** (if-then-else) steps. If statement tests to know the current state.
- **AND-OR** search trees. Two possible actions (**OR nodes**). Branching that happens from a choice (**AND nodes)**.
- **Try, try again**. A cyclic plan where minimum condition (every leaf = goal state & reachable from other points in the plan)

Search in Partially Observable Environments

Problem of partial observability, where the agent's percepts are not enough to pin down the exact state**.**

Searching with no observation: Agent's percepts provide **no information at all**, sensorless problem (or a conformant problem).

- Solution: sequence of actions, not a conditional plan

Searching in partially observable environments requires a function that **monitors** or **estimates** the environment to maintain the belief state.

Summary

Local search methods keep only a **small number of states** in memory that are useful for optimization.

In **nondeterministic environments**, agents can apply **AND–OR search** to generate contingency plans that reach the goal regardless of which outcomes occur during execution.

Belief-state is the set of **possible states** that the agent is in for **partially observable environments**.

Standard search algorithms can be **applied directly to belief-state** space to solve **sensorless** problems.