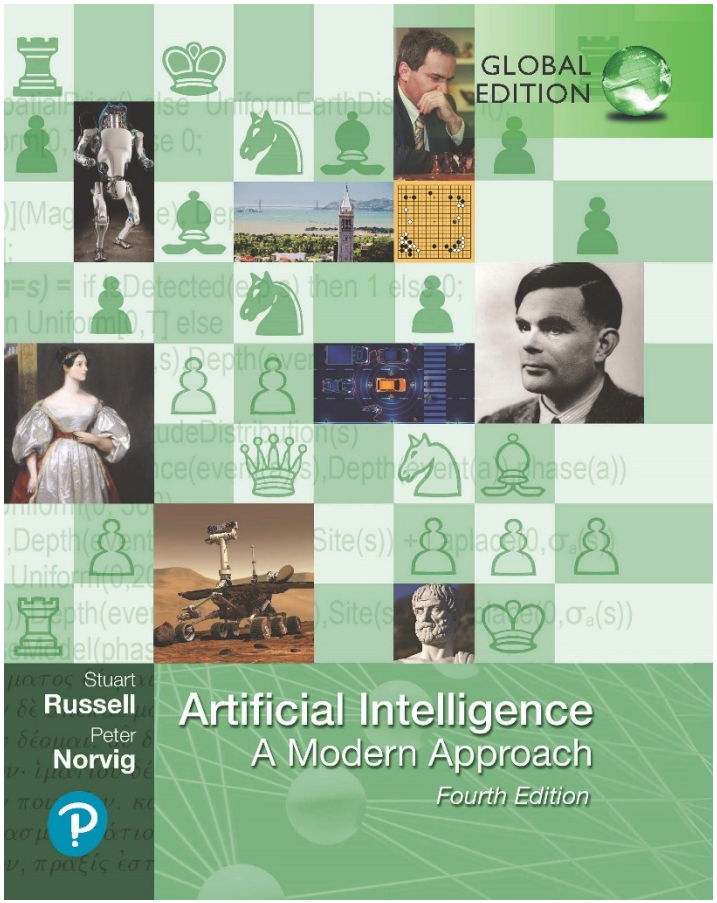
Artificial Intelligence: A Modern Approach

Fourth Edition, Global Edition



Chapter 6

Adversarial Search And Games

Lecture Presentations: Artificial Intelligence

Adapted from:
"Artificial Intelligence: A Modern Approach, Global Edition", 4th Edition by Stuart Russell and Peter Norvig © 2021 Pearson Education.

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Outline

- Game Theory
- Optimal Decisions in Games
  - minimax decisions
  - α–β pruning
  - Monte Carlo Tree Search (MCTS)
- Resource limits and approximate evaluation
- Games of chance
- Games of imperfect information
- Limitations of Game Search Algorithms

Games Theory

Two players
- Max-min
- Taking turns, fully observable

Moves: Action

Position: state

Zero sum:
- good for one player, bad for another
- No win-win outcome.

Games Theory

S_0: The initial state of the game

TO-MOVE(s): player to move in state s.

ACTIONS(s): The set of legal moves in state s.

RESULT(s, a): The transition model, resulting state

IS-TERMINAL(s): A terminal test to detect when the game is over

UTILITY(s; p): A utility function (objective/payoff)

Games vs. search problems

"Unpredictable" opponent ⇒ solution is a strategy specifying a move for every possible opponent reply

Time limits ⇒ unlikely to find goal, must approximate

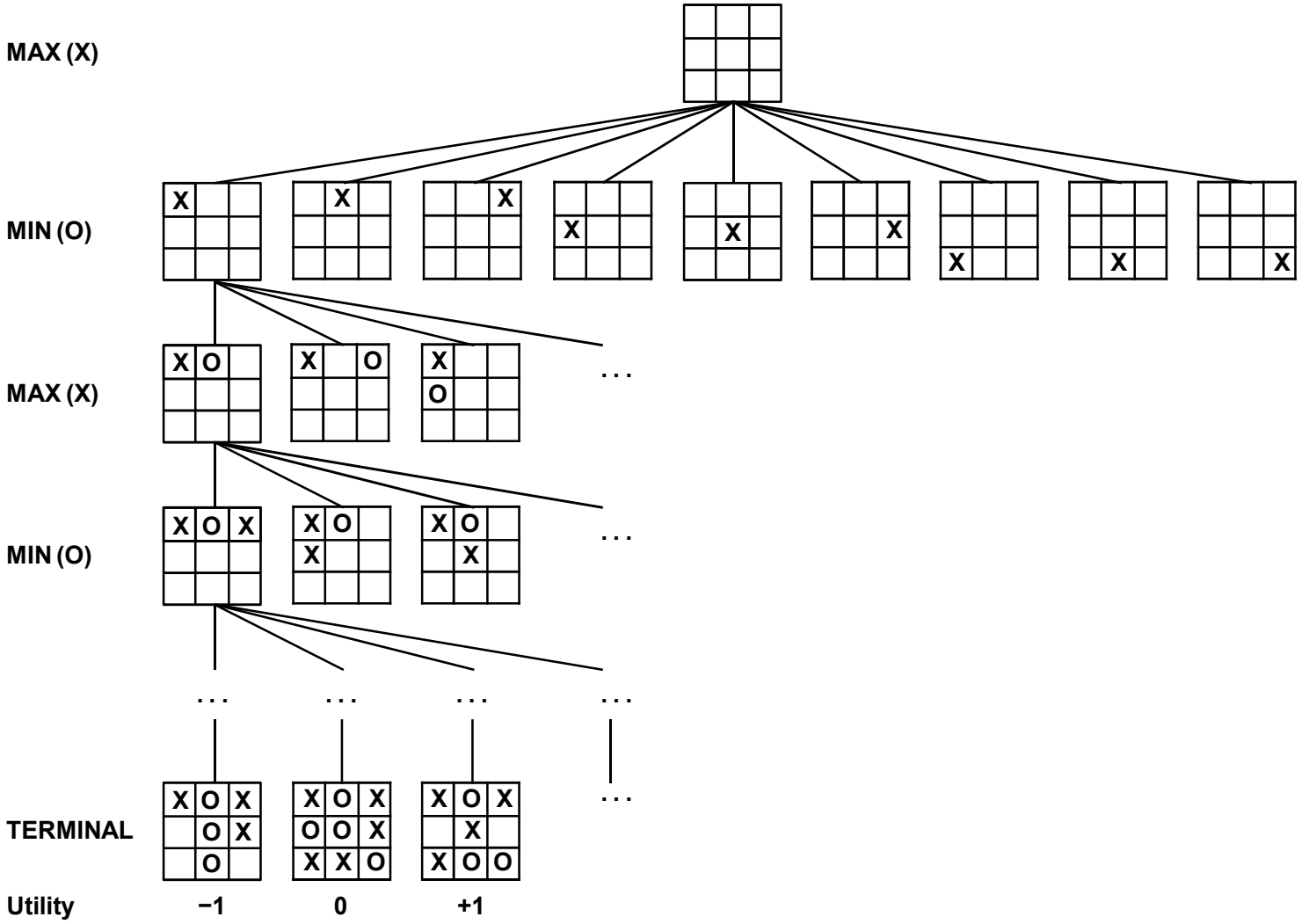
Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

Game tree (2-player, deterministic, turns)

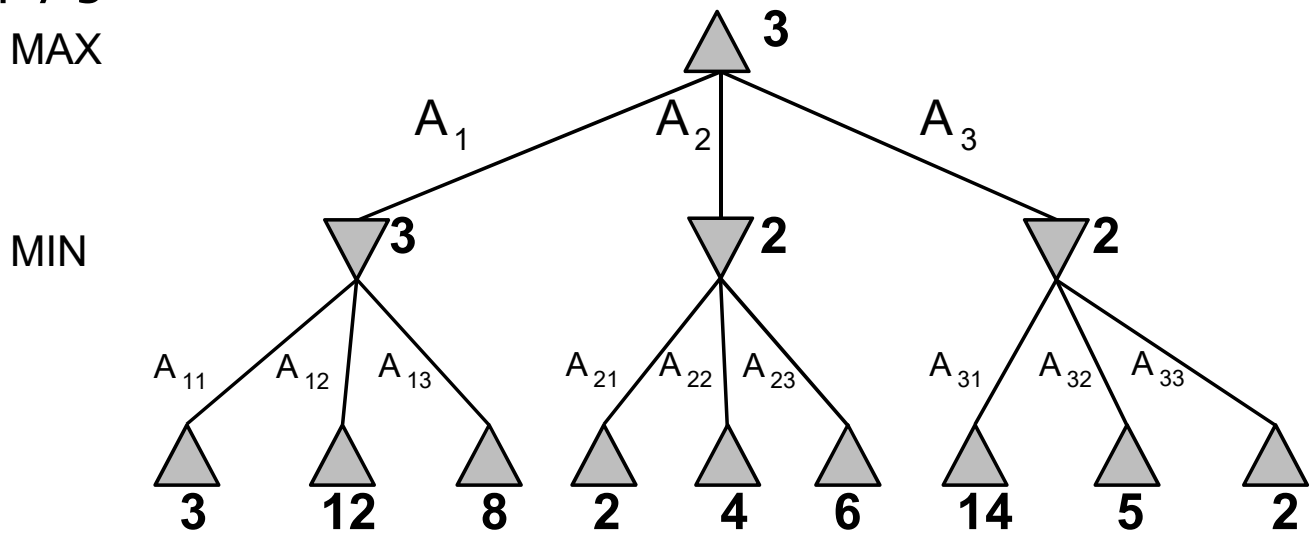


Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest minimax value
= best achievable payoff against best play

E.g., 2-ply game:



Minimax algorithm

```
function Minimax-Decision(state) returns an action
    inputs: state, current state in game

    return the a in Actions(state) maximizing Min-Value(Result(a, state))
```

```
function Max-Value(state) returns a utility value
    if Terminal-Test(state) then return Utility(state)
    v ← −∞
    for a, s in Successors(state) do v ← Max(v, Min-Value(s))
    return v
```

```
function Min-Value(state) returns a utility value
    if Terminal-Test(state) then return Utility(state)
    v ← ∞
    for a, s in Successors(state) do v ← Min(v, Max-Value(s))
    return v
```

Properties of minimax

Complete??

Properties of minimax

Complete?? Only if tree is finite (chess has specific rules for this).
NB a finite strategy can exist even in an infinite tree!

Optimal??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

Space complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

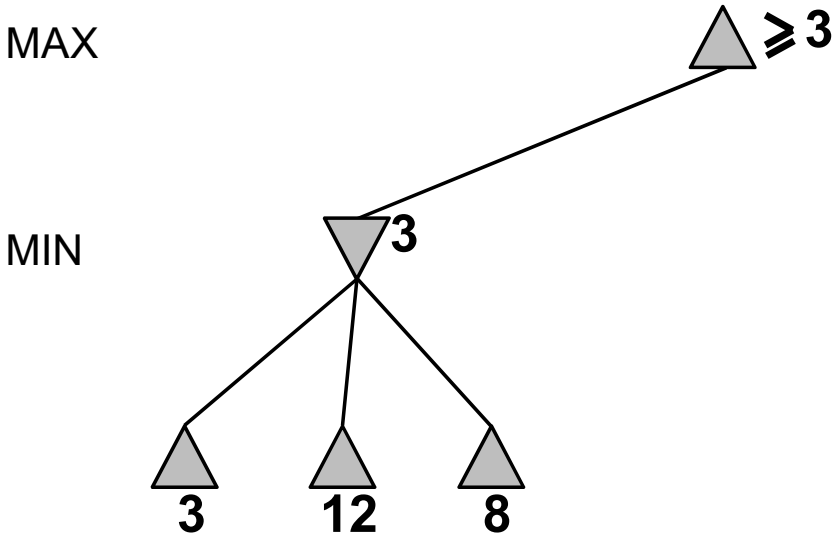
Space complexity?? $O(bm)$ (depth-first exploration)

For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games

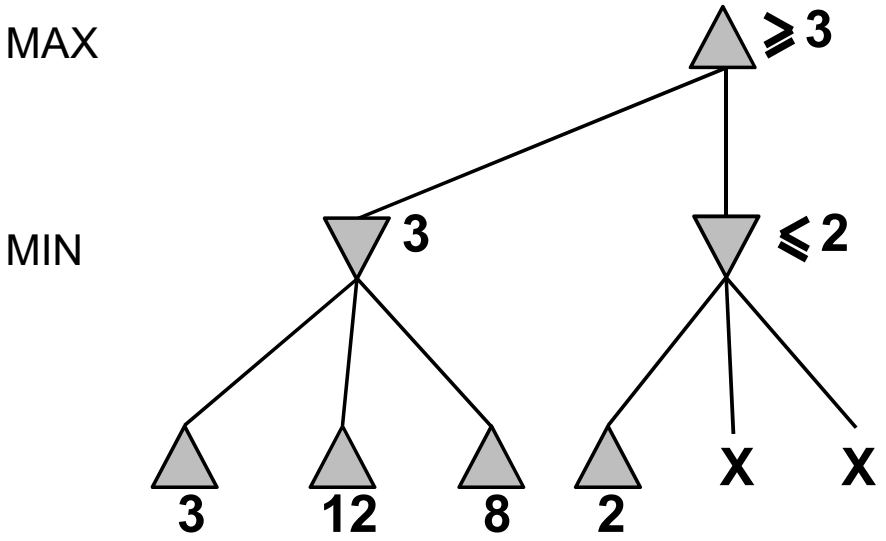
$\Rightarrow$ exact solution completely infeasible

But do we need to explore every path?

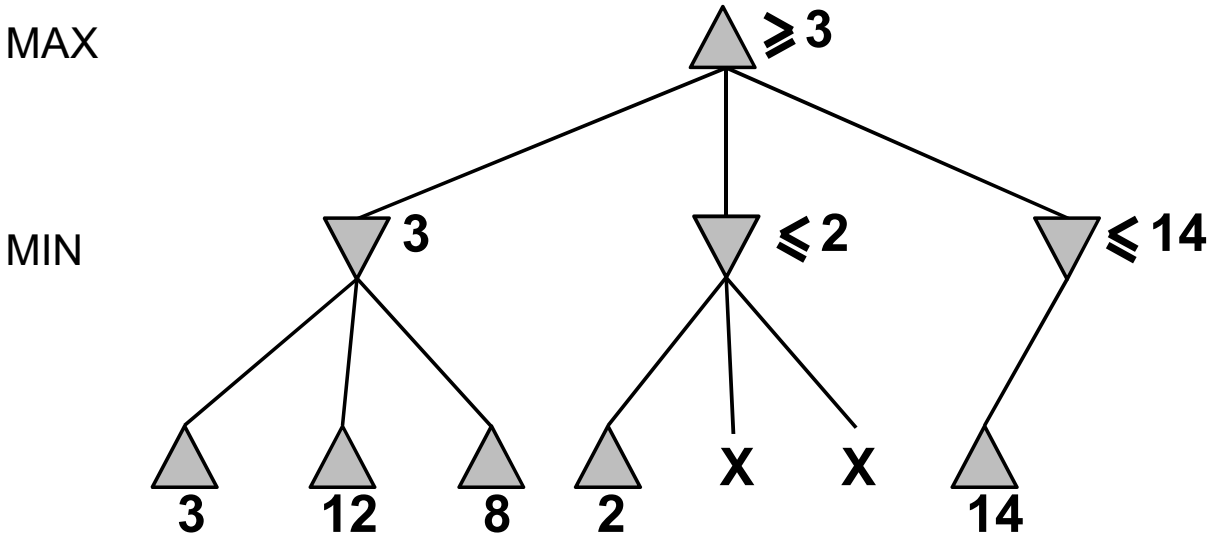
α–β pruning example



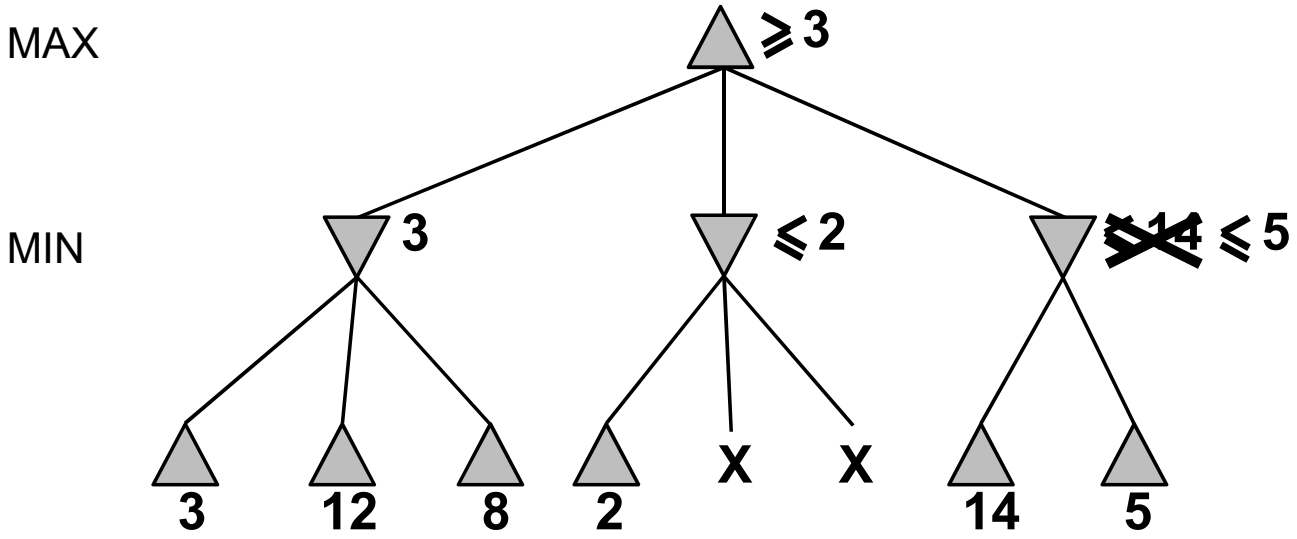
α–β pruning example



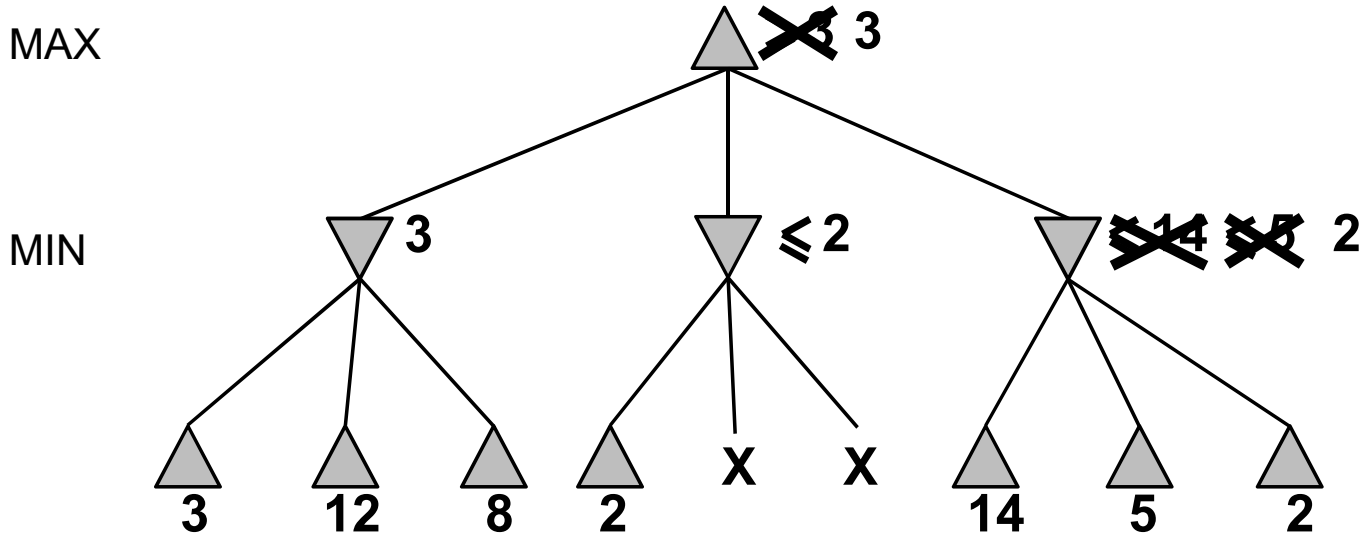
α–β pruning example



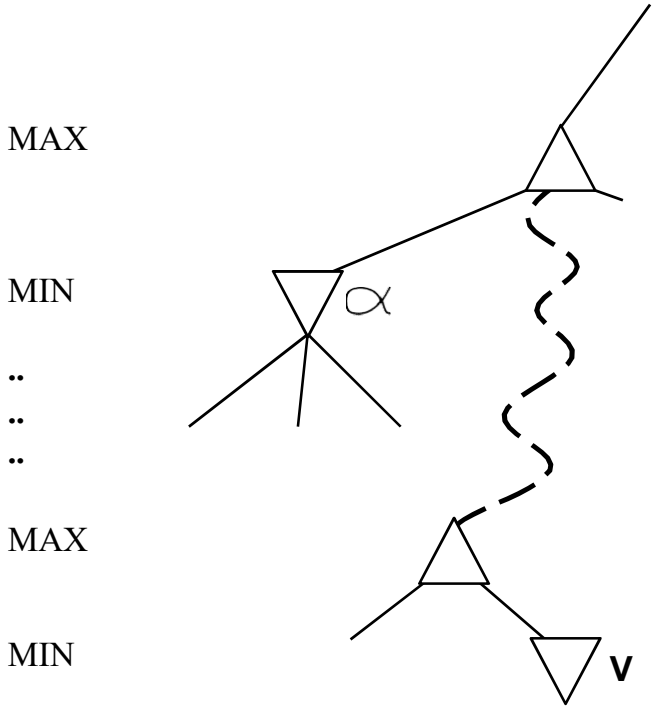
α–β pruning example



α–β pruning example



Why is it called α–β?



α is the best value (to max) found so far off the current path

If V is worse than α, max will avoid it $\Rightarrow$ prune that branch

Define β similarly for min

The α–β algorithm

```
function Alpha-Beta-Decision(state) returns an action
    return the a in Actions(state) maximizing Min-Value(Result(a, state))

function Max-Value(state, α, β) returns a utility value
    inputs: state, current state in game
            α, the value of the best alternative for max along the path to state
            β, the value of the best alternative for min along the path to state
    if Terminal-Test(state) then return Utility(state)
    v ← − ∞
    for a, s in Successors(state) do
        v ← Max(v, Min-Value(s, α, β))
        if v ≥ β then return v
        α ← Max(α, v)
    return v

function Min-Value(state, α, β) returns a utility value
    same as Max-Value but with roles of α, β reversed
```

Properties of α–β

Pruning does not affect final result

Good move ordering improves effectiveness of pruning

With "perfect ordering," time complexity = $O(b^{m/2})$

$\Rightarrow$ doubles solvable depth

A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Unfortunately, 35^{50} is still impossible!

Monte Carlo Tree Search

The basic Monte Carlo Tree Search (MCTS) strategy does not use a heuristic evaluation function. Value of a state is estimated as the average utility over number of simulations

- **Playout**: simulation that chooses moves until terminal position reached.
- **Selection**: Start of root, choose move (selection policy) repeated moving down tree
- **Expansion**: Search tree grows by generating a new child of selected node
- **Simulation**: playout from generated child node
- **Back-propagation**: use the result of the simulation to update all the search tree nodes going up to the root

Monte Carlo Tree Search

UCT: Effective selection policy is called “upper confidence bounds applied to trees”

UCT ranks each possible move based on an upper confidence bound formula UCT called UCB1

$$UCB1(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\text{PARENT}(n))}{N(n)}}$$

where U(n) is the total utility of all playouts that went through node n, N(n) is the number of playouts through node n, and PARENT(n) is the parent node of n in the tree.

Monte Carlo Tree Search

function MONTE-CARLO-TREE-SEARCH(state) **returns** an action
    tree ← NODE(state)
    while IS-TIME-REMAINING() **do**
        leaf ← SELECT(tree)
        child ← EXPAND(leaf)
        result ← SIMULATE(child)
        BACK-PROPAGATE(result, child)
    return the move in ACTIONS(state) whose node has highest number of playouts

Monte Carlo Tree Search

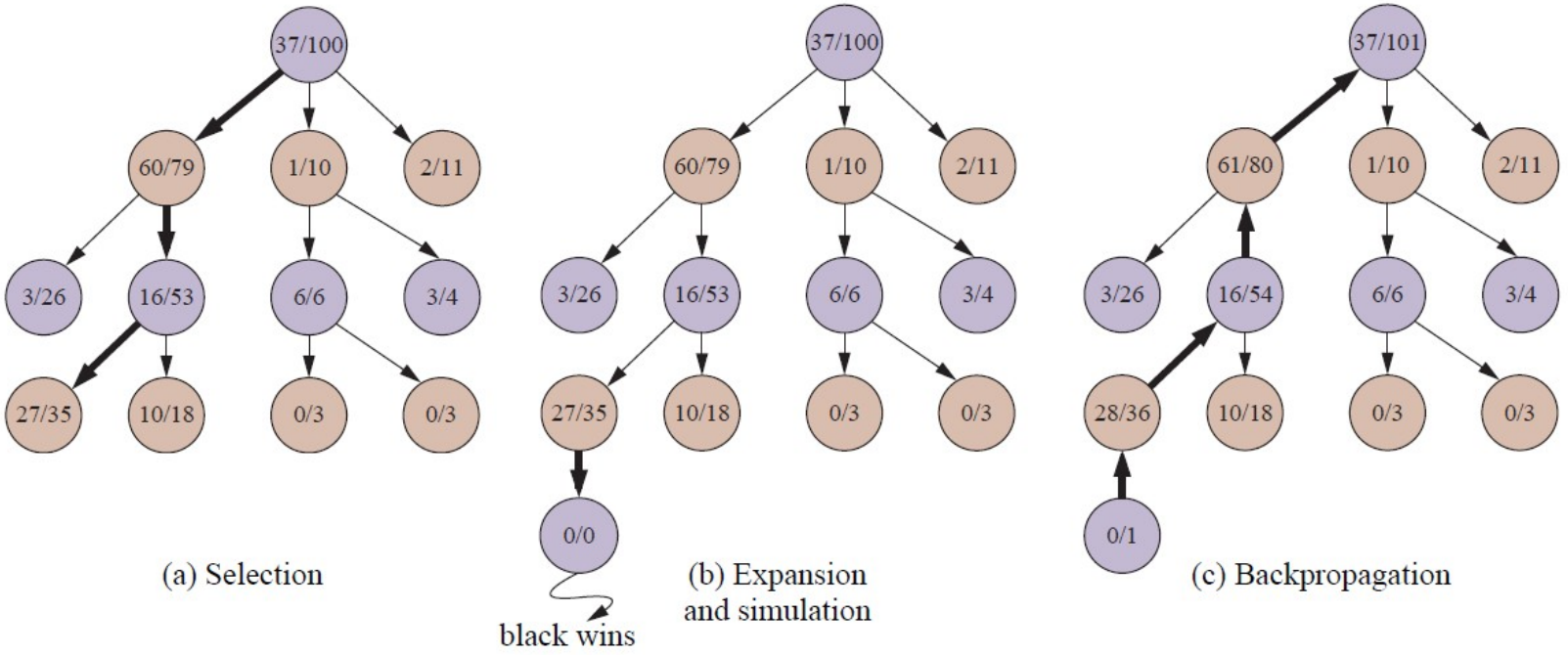


Figure 6.10 One iteration of the process of choosing a move with Monte Carlo tree search (MCTS) using the upper confidence bounds applied to trees (UCT) selection metric, shown after 100 iterations have already been done. In (a) we select moves, all the way down the tree, ending at the leaf node marked 27/35 (for 27 wins for black out of 35 playouts). In (b) we expand the selected node and do a simulation (playout), which ends in a win for black. In (c), the results of the simulation are back-propagated up the tree.

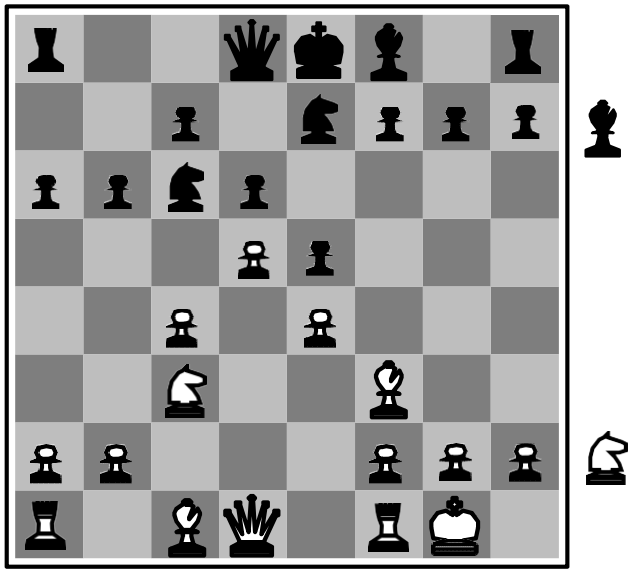
Resource limits

Standard approach:

- Use `Cutoff-Test` instead of `Terminal-Test`
  e.g., depth limit (perhaps add quiescence search)
- Use `Eval` instead of `Utility`
  i.e., evaluation function that estimates desirability of position

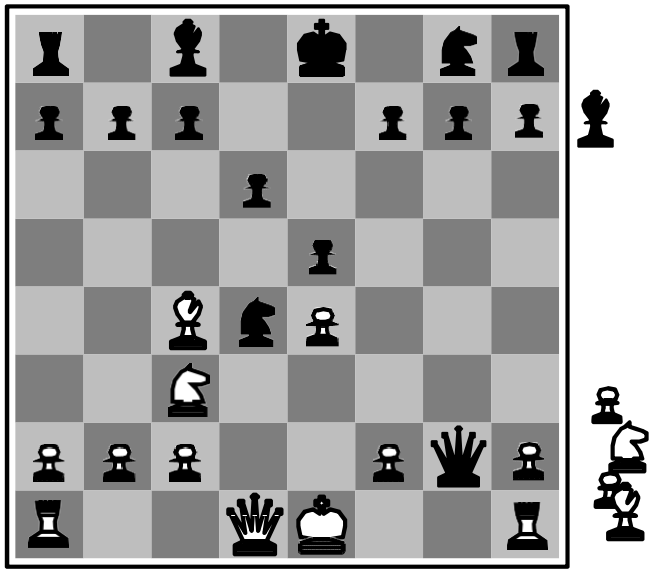
Suppose we have 100 seconds, explore 10^4 nodes/second
$\Rightarrow$ 10^6 nodes per move $\approx 35^{8/2}$
$\Rightarrow$ α–β reaches depth 8 $\Rightarrow$ pretty good chess program

Evaluation functions



Black to move

White slightly better



White to move

Black winning

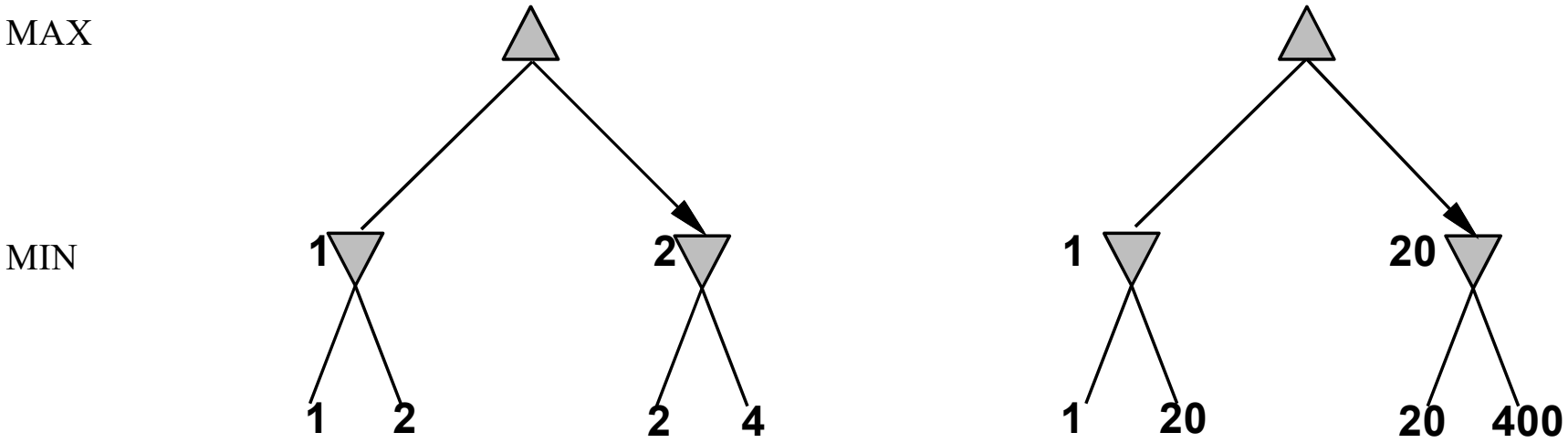
For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with

$f_1(s)$ = (number of white queens) – (number of black queens), etc.

Digression: Exact values don't matter



Behaviour is preserved under any monotonic transformation of Eval

Only the order matters:

payoff in deterministic games acts as an ordinal utility function

Deterministic games in practice

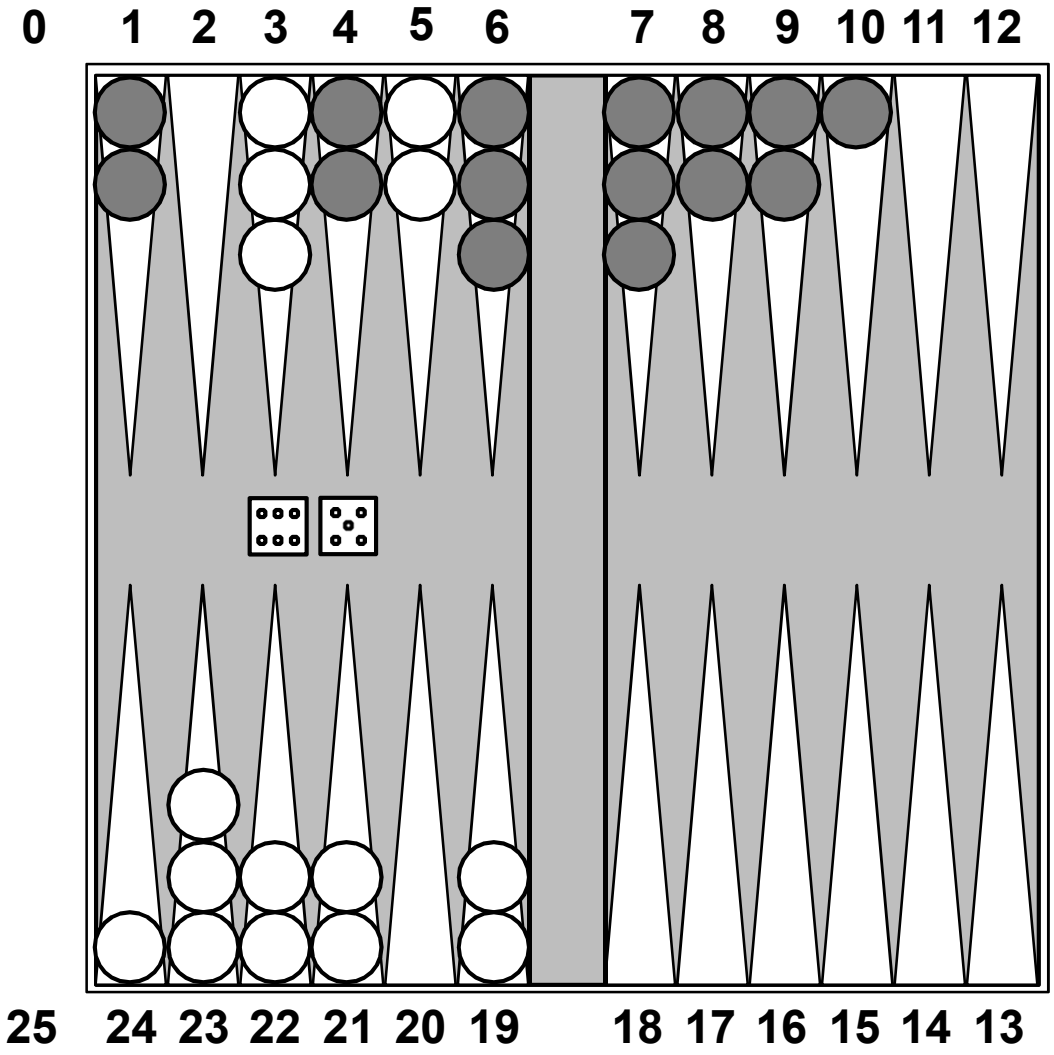
Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

Othello: human champions refuse to compete against computers, who are too good.

Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

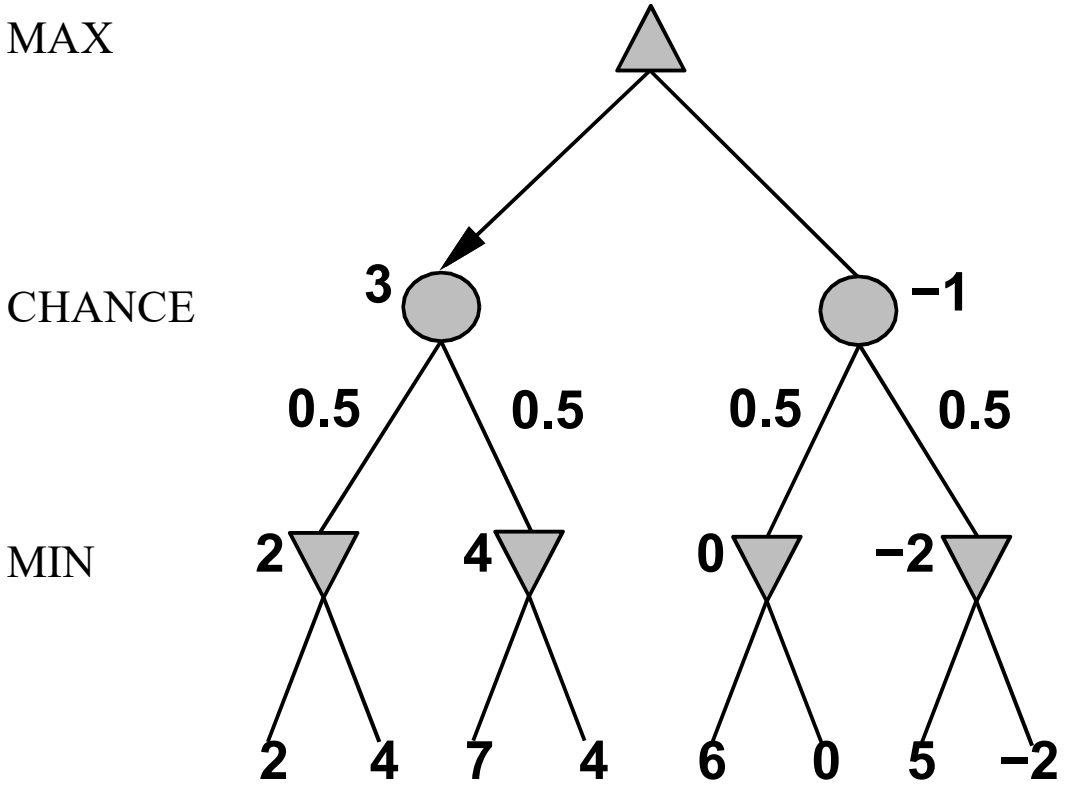
Nondeterministic games: backgammon



Nondeterministic games in general

In nondeterministic games, chance introduced by dice, card-shuffling

Simplified example with coin-flipping:



Algorithm for nondeterministic games

Expectiminimax gives perfect play

Just like Minimax, except we must also handle chance nodes:

```
...
if state is a Max node then
        return the highest ExpectiMinimax-Value of Successors(state)
if state is a Min node then
        return the lowest ExpectiMinimax-Value of Successors(state)
if state is a chance node then
        return average of ExpectiMinimax-Value of Successors(state)
...
```

Nondeterministic games in practice

Dice rolls increase b: 21 possible rolls with 2 dice
Backgammon $\approx$ 20 legal moves (can be 6,000 with 1-1 roll)

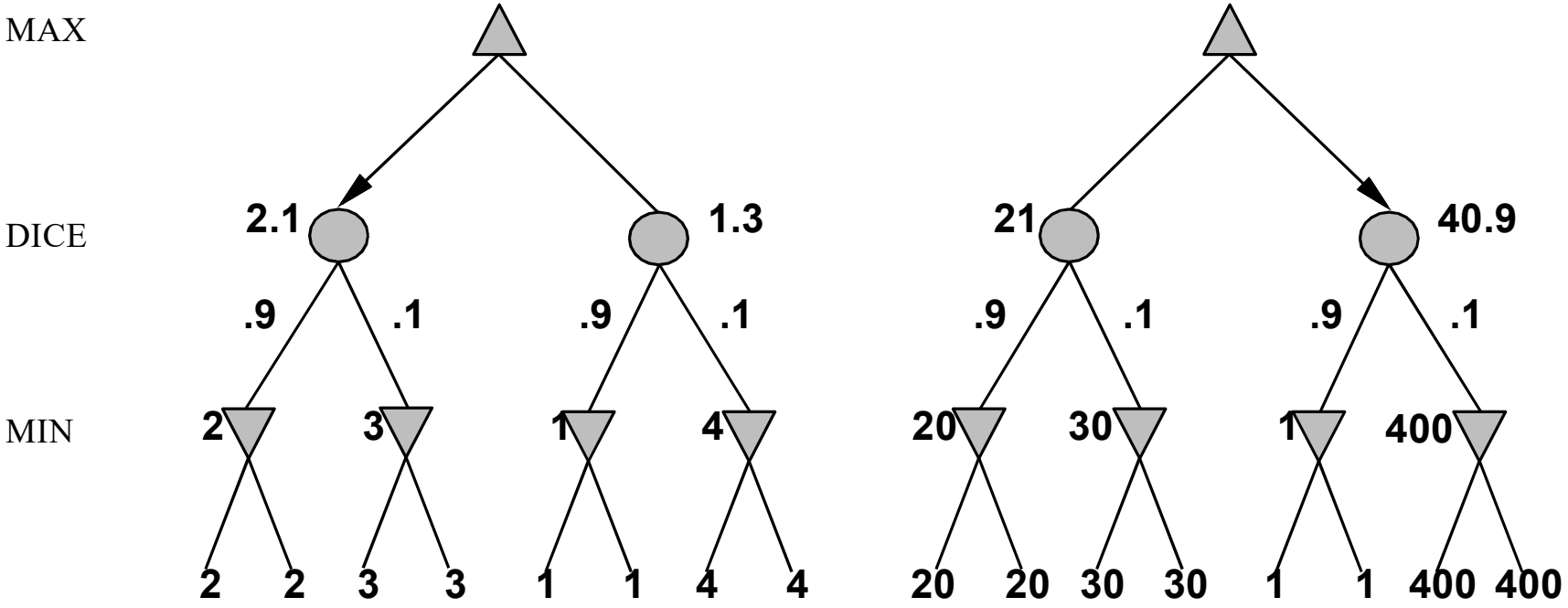
$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

As depth increases, probability of reaching a given node shrinks
$\Rightarrow$ value of lookahead is diminished

α–β pruning is much less effective

TDGammon uses depth-2 search + very good Eval
$\approx$ world-champion level

Digression: Exact values DO matter



Behaviour is preserved only by positive linear transformation of `Eval`

Hence `Eval` should be proportional to the expected payoff

Games of imperfect information

E.g., card games, where opponent's initial cards are unknown

Typically we can calculate a probability for each possible deal

Seems just like having one big dice roll at the beginning of the game*

Idea: compute the minimax value of each action in each deal,
then choose the action with highest expected value over all deals*

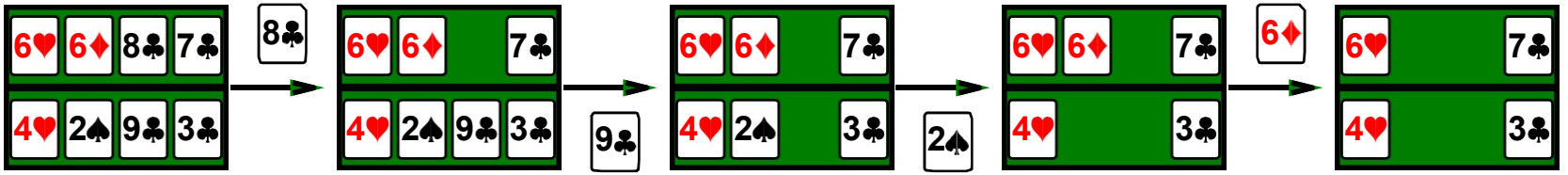
Special case: if an action is optimal for all deals, it's optimal.*

GIB, current best bridge program, approximates this idea by

1) generating 100 deals consistent with bidding information
2) picking the action that wins most tricks on average

Example

Four-card bridge/whist/hearts hand, Max to play first



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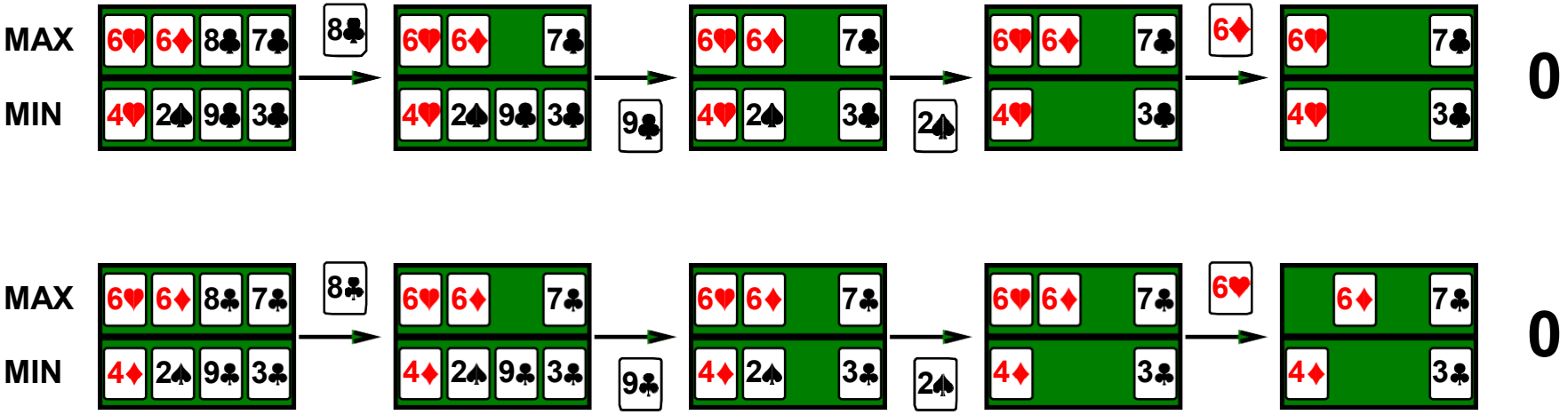
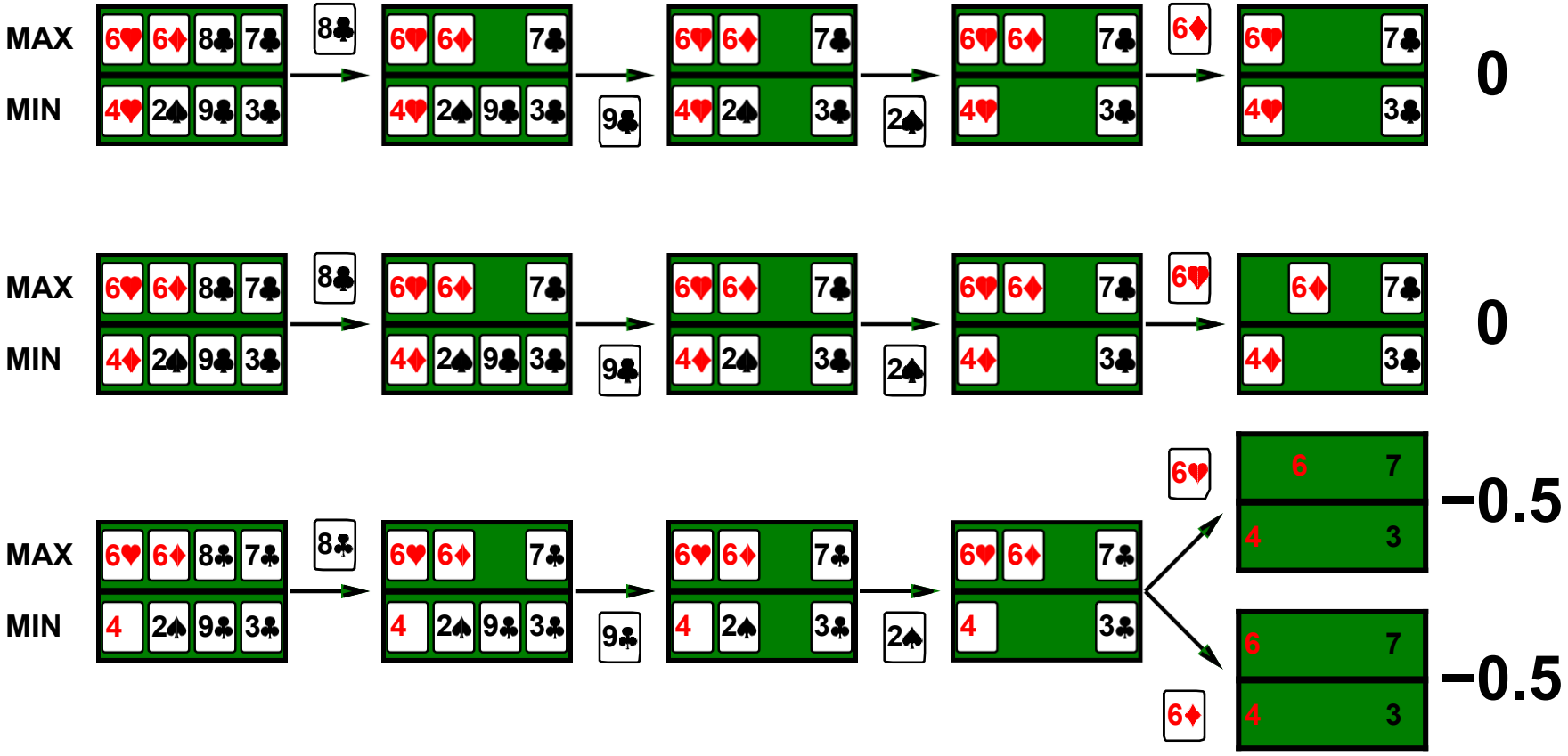
Example

Four-card bridge/whist/hearts hand, Max to play first



Example

Four-card bridge/whist/hearts hand, Max to play first



Common sense example

Road A leads to a small heap of gold pieces
Road B leads to a fork:
take the left fork and you'll find a mound of jewels;
take the right fork and you'll be run over by a bus.

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Common sense example

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Road A leads to a small heap of gold pieces
Road B leads to a fork:
take the left fork and you'll be run over by a bus;
take the right fork and you'll find a mound of jewels.

Road A leads to a small heap of gold pieces
Road B leads to a fork:
guess correctly and you'll find a mound of jewels;
guess incorrectly and you'll be run over by a bus.

Proper analysis

*Intuition that the value of an action is the average of its values in all actual states is WRONG

With partial observability, value of an action depends on the information state or belief state the agent is in

Can generate and search a tree of information states

Leads to rational behaviors such as

- Acting to obtain information
- Signalling to one's partner
- Acting randomly to minimize information disclosure

Limitations of Game Search Algorithms

- Alpha–beta search vulnerable to errors in the heuristic function.
- Waste of computational time for deciding best move where it is obvious (meta-reasoning).
- Reasoning done on individual moves. Humans reason on abstract levels.
- Possibility to incorporate Machine Learning into game search process.

Summary

Minimax algorithm: selects optimal moves by a depth-first enumeration of the game tree.

Alpha–beta algorithm: greater efficiency by eliminating subtrees

Evaluation function: a heuristic that estimates utility of state.

Monte Carlo tree search (MCTS): no heuristic, play game to the end with rules and repeated multiple times to determine optimal moves during playout.

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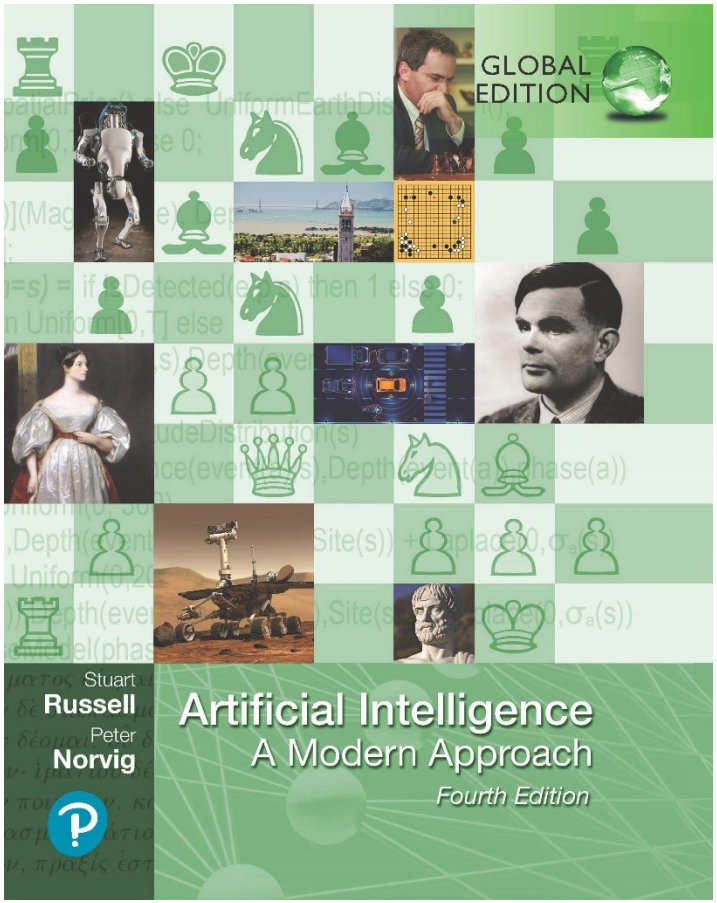

Chapter 5

Constraint Satisfaction Problems

Lecture Presentations: Artificial Intelligence

Adapted from:
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Outline

- Defining Constraint Satisfaction Problems (CSP)
- CSP examples
- Backtracking search for CSPs
- Local search for CSPs
- Problem structure and problem decomposition

Defining Constraint Satisfaction Problems

A constraint satisfaction problem (CSP) consists of three components, *X*, *D*, and *C:*

- *X* is a set of variables, $\{X_1 \ldots X_n\}$.
- *D* is a set of domains, $\{D_1, \ldots, D_n\}$, one for each variable
- *C* is a set of constraints that specify allowable combination of values

CSPs deal with assignments of values to variables.

- A complete assignment is one in which every variable is assigned a value, and a solution to a CSP is a consistent, complete assignment.
- A partial assignment is one that leaves some variables unassigned.
- Partial solution is a partial assignment that is consistent

Constraint satisfaction problems (CSPs)

Standard search problem:

state is a "black box"—any old data structure
that supports goal test, eval, successor

CSP:

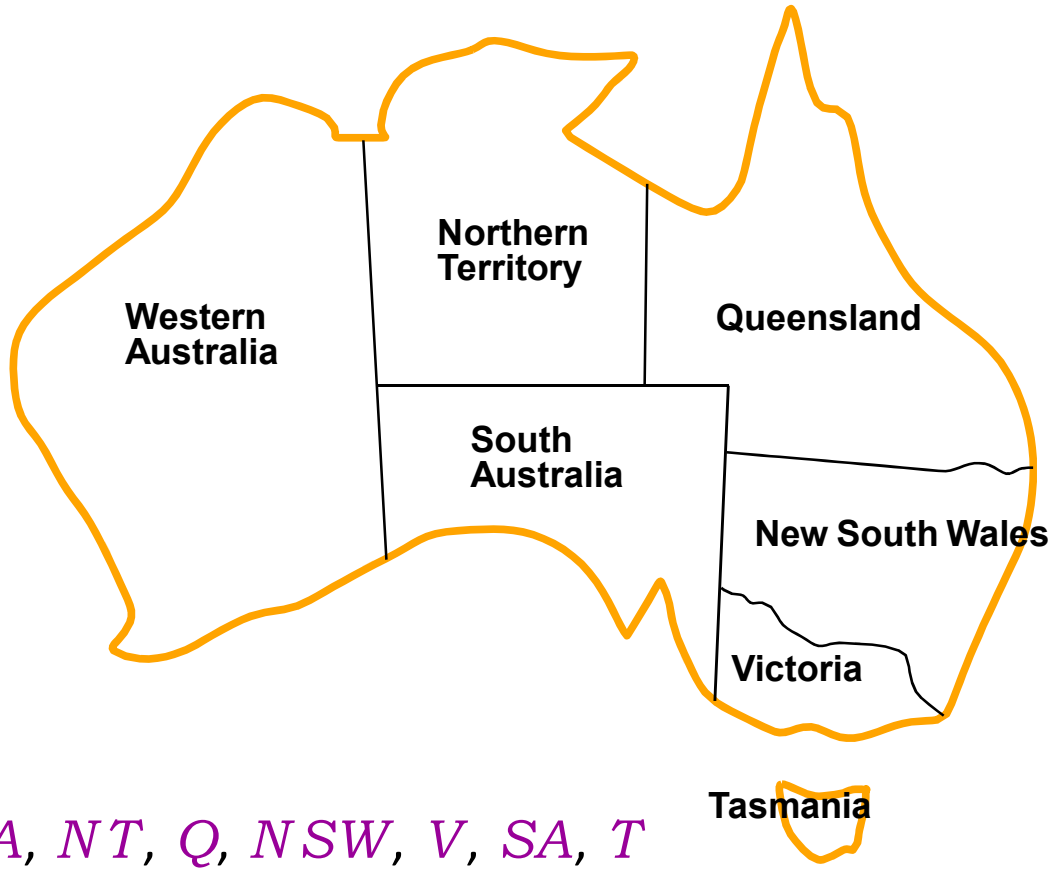
state is defined by variables X_i with values from domain D_i

goal test is a set of constraints specifying
allowable combinations of values for subsets of variables

Simple example of a formal representation language

Allows useful general-purpose algorithms with more power
than standard search algorithms

Example: Map-Coloring



Variables WA, NT, Q, NSW, V, SA, T

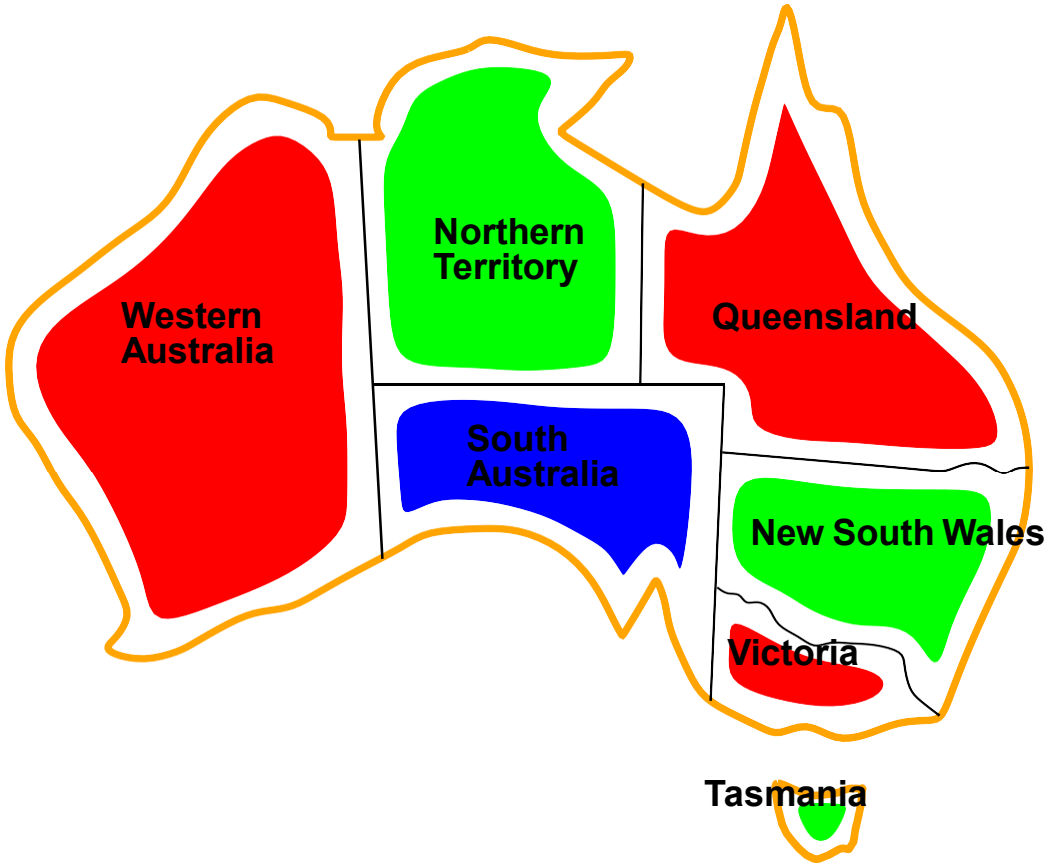
Domains $D_i = \{ red, green, blue \}$

Constraints: adjacent regions must have different colors

e.g., $WA \neq NT$ (if the language allows this), or

$(WA, NT) \in \{(red, green), (red, blue), (green, red), (green, blue), \ldots\}$

Example: Map-Coloring contd.



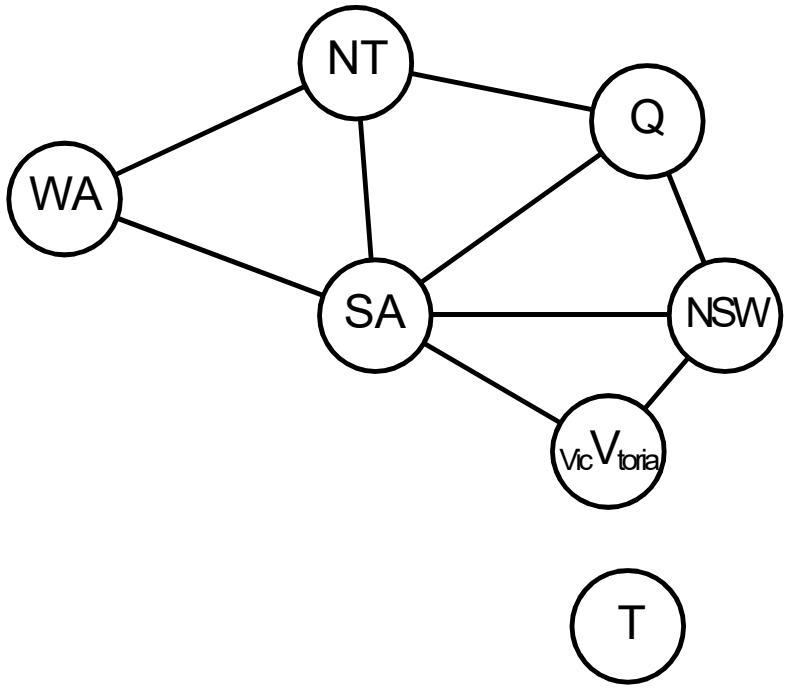
Solutions are assignments satisfying all constraints, e.g.,

$\{ WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green \}$

Constraint graph

Binary CSP: each constraint relates at most two variables

Constraint graph: nodes are variables, arcs show constraints



General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!

Varieties of CSPs

Discrete variables

finite domains; size d $\Rightarrow$ $O(d^n)$ complete assignments

- e.g., Boolean CSPs, incl. Boolean satisfiability (NP-complete)

infinite domains (integers, strings, etc.)

- e.g., job scheduling, variables are start/end days for each job
- need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$
- linear constraints solvable, nonlinear undecidable

Continuous variables

- e.g., start/end times for Hubble Telescope observations
- linear constraints solvable in poly time by LP methods

Varieties of constraints

Unary constraints involve a single variable,

e.g., $SA \neq green$

Binary constraints involve pairs of variables,

e.g., $SA \neq WA$

Higher-order constraints involve 3 or more variables,

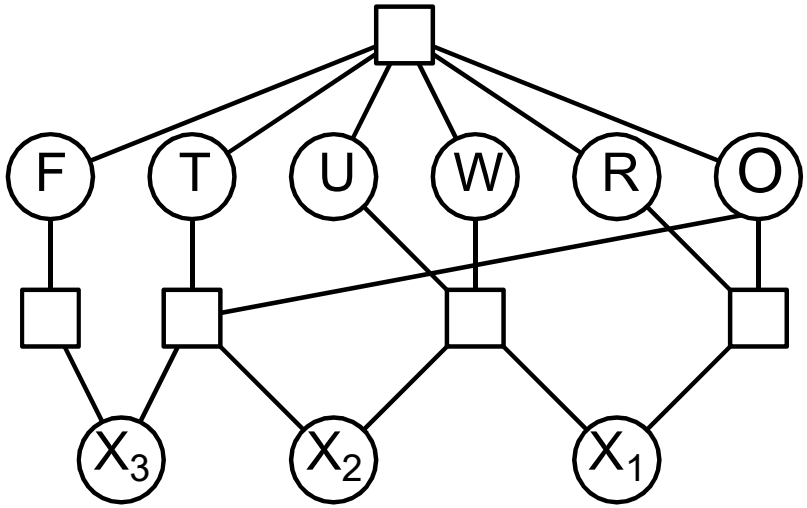
e.g., cryptarithmetic column constraints

Preferences (soft constraints), e.g., *red* is better than *green*
often representable by a cost for each variable assignment

→ constrained optimization problems

Example: Cryptarithmetic

```
   T W O
+  T W O
-------
F O U R
```



Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$

Domains: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Constraints

$alldiff(F, T, U, W, R, O)$

$O + O = R + 10 \cdot X_1$, etc.

Real-world CSPs

Assignment problems
    e.g., who teaches what class

Timetabling problems
    e.g., which class is offered when and where?

Hardware configuration

Spreadsheets

Transportation scheduling

Factory scheduling

Floorplanning

Notice that many real-world problems involve real-valued variables

Standard search formulation (incremental)

Let's start with the straightforward, dumb approach, then fix it

States are defined by the values assigned so far

- Initial state: the empty assignment, { }
- Successor function: assign a value to an unassigned variable that does not conflict with current assignment.
  ⇒ fail if no legal assignments (not fixable!)
- Goal test: the current assignment is complete

1) This is the same for all CSPs! 
2) Every solution appears at depth n with n variables
   ⇒ use depth-first search
3) Path is irrelevant, so can also use complete-state formulation
4) $b = (n - €)d$ at depth €, hence $n!d^n$ leaves!!!! 

Backtracking search

Variable assignments are commutative, i.e.,

[$WA = red$ then $NT = green$] same as [$NT = green$ then $WA = red$]

Only need to consider assignments to a single variable at each node

$\Rightarrow$ $b = d$ and there are d^n leaves

Depth-first search for CSPs with single-variable assignments is called backtracking search

Backtracking search is the basic uninformed algorithm for CSPs

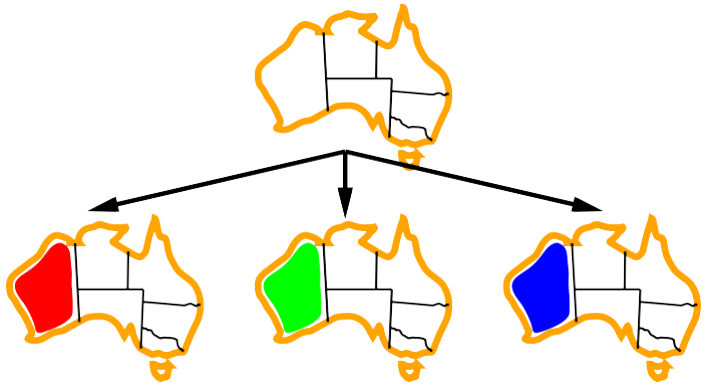
Can solve n-queens for $n \approx 25$

Backtracking search

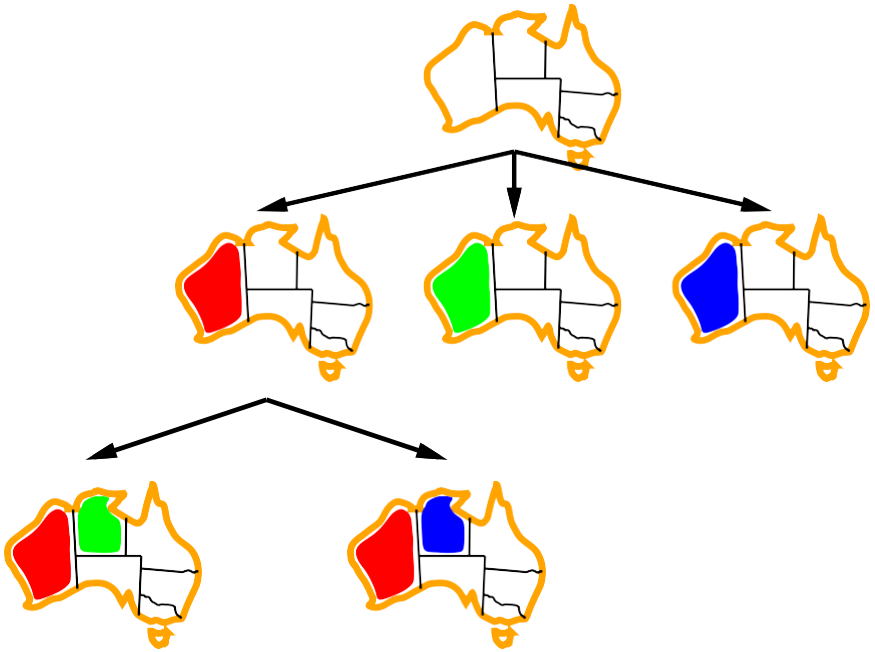
```
function Backtracking-Search(csp) returns solution/failure
    return Recursive-Backtracking({}, csp)

function Recursive-Backtracking(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← Select-Unassigned-Variable(Variables[csp], assignment, csp)
    for each value in Order-Domain-Values(var, assignment, csp) do
        if value is consistent with assignment given Constraints[csp] then
            add {var = value} to assignment
            result ← Recursive-Backtracking(assignment, csp)
            if result /= failure then return result
            remove {var = value} from assignment
    return failure
```

Backtracking example

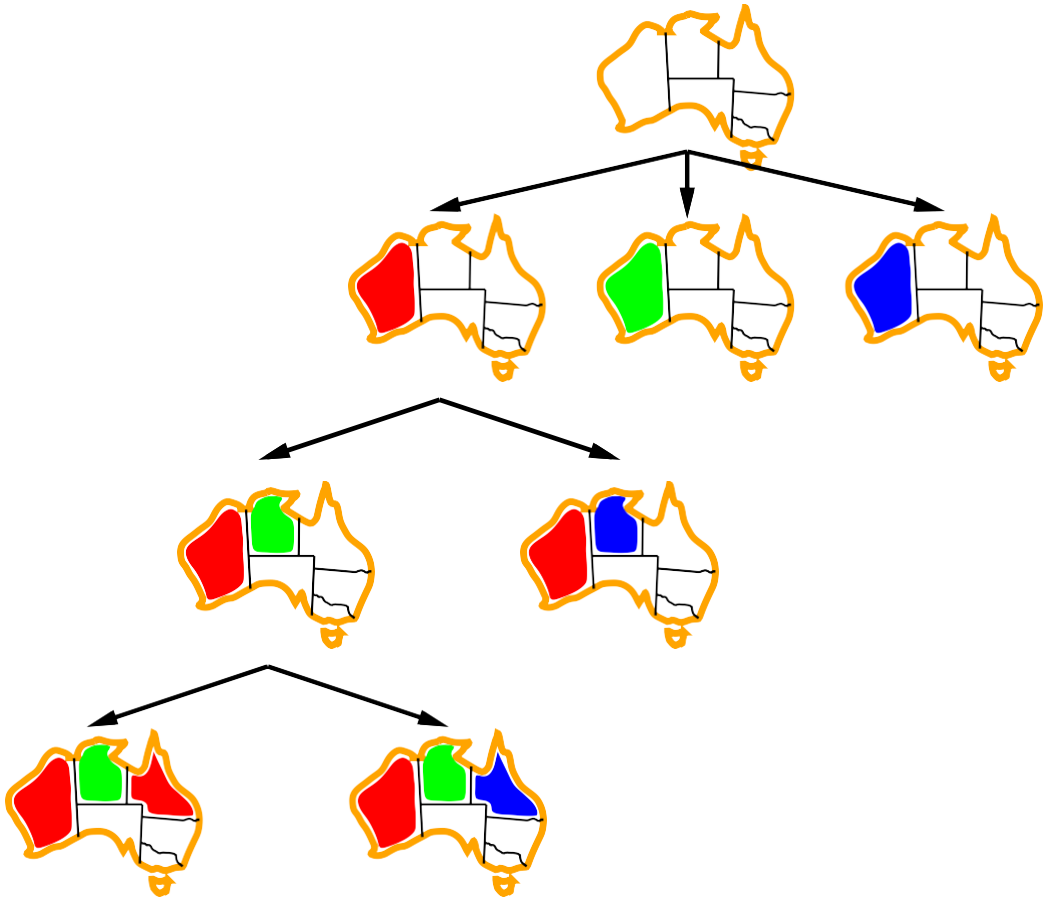
Backtracking example



Backtracking example



Backtracking example



Improving backtracking efficiency

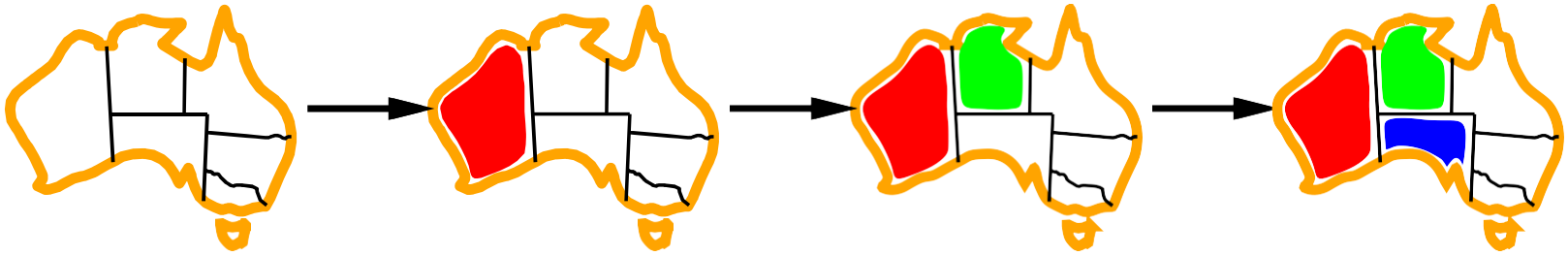
General-purpose methods can give huge gains in speed:

1. Which variable should be assigned next?
2. In what order should its values be tried?
3. Can we detect inevitable failure early?
4. Can we take advantage of problem structure?

Minimum remaining values

Minimum remaining values (MRV):

choose the variable with the fewest legal values

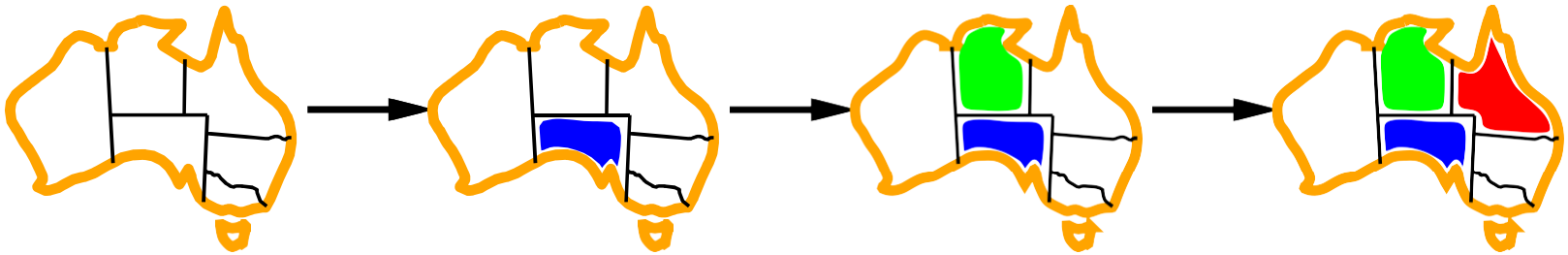


Degree heuristic

Tie-breaker among MRV variables

Degree heuristic:

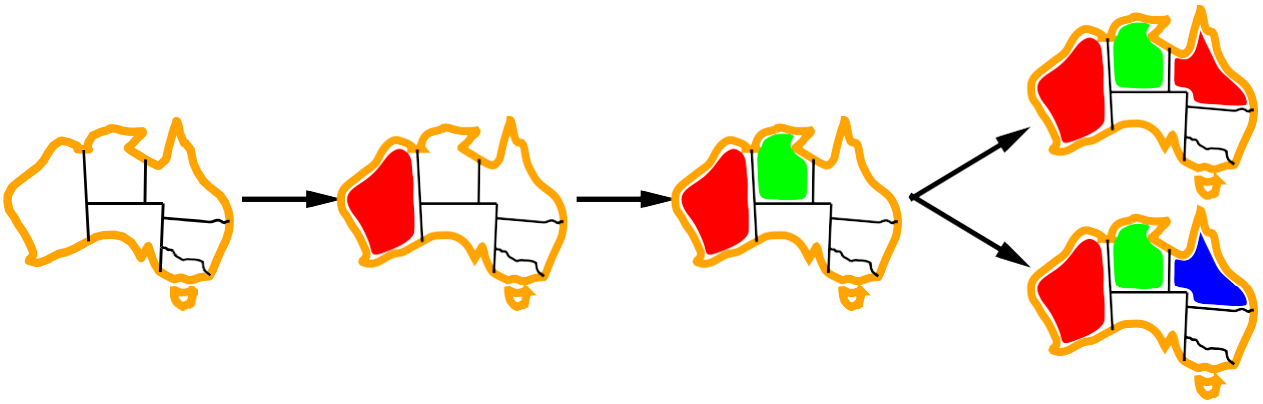
choose the variable with the most constraints on remaining variables



Least constraining value

Given a variable, choose the least constraining value:

the one that rules out the fewest values in the remaining variables

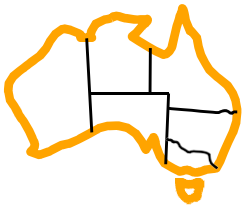


Combining these heuristics makes 1000 queens feasible

Forward checking

Idea: Keep track of remaining legal values for unassigned variables

Terminate search when any variable has no legal values



Forward checking

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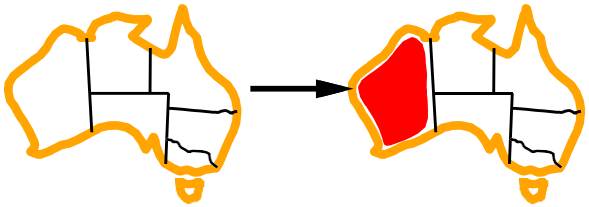

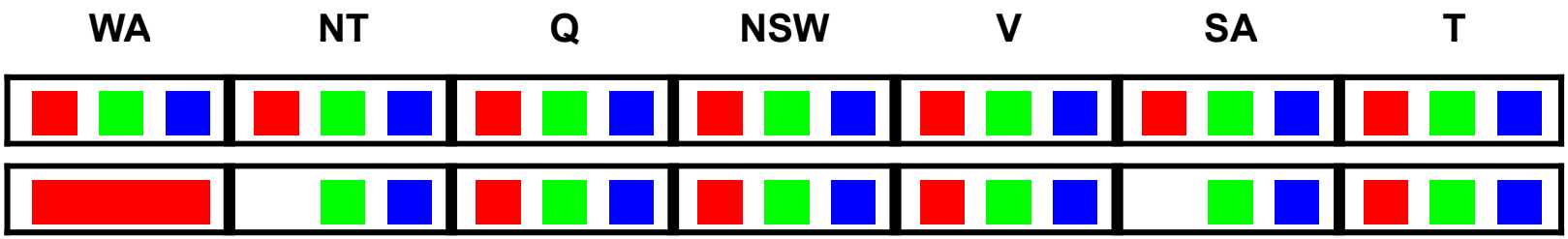

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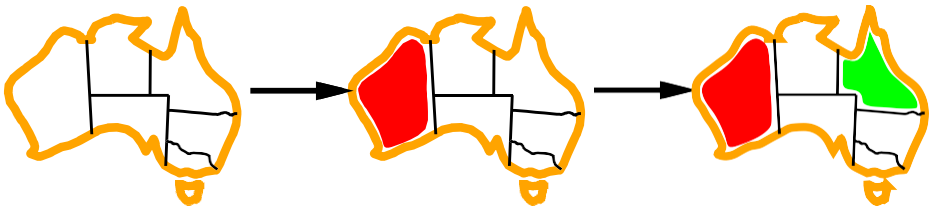

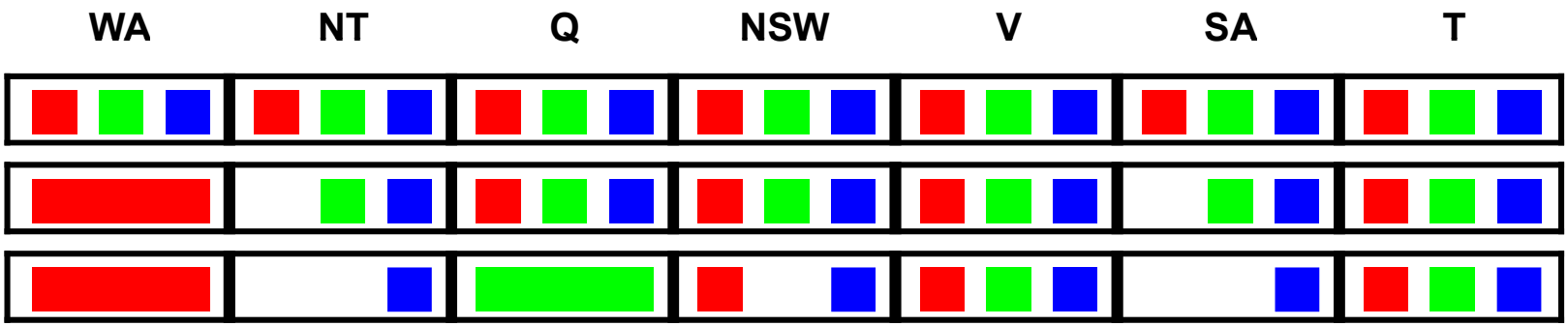

Forward checking

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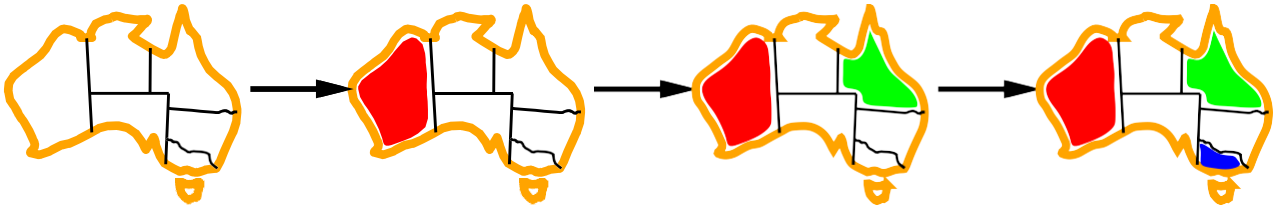
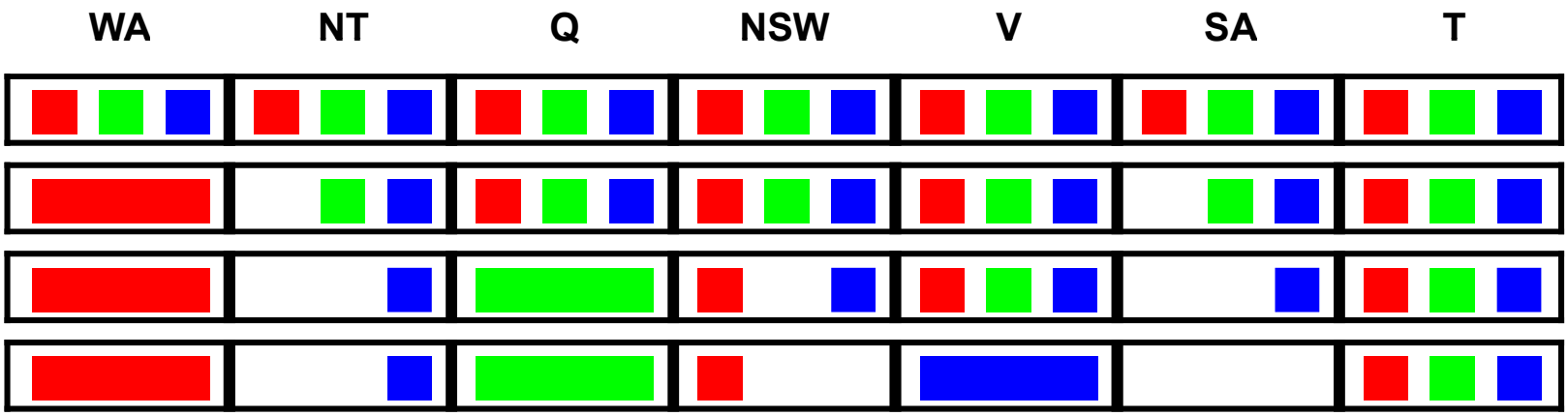




Constraint propagation

Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

NT and SA cannot both be blue!

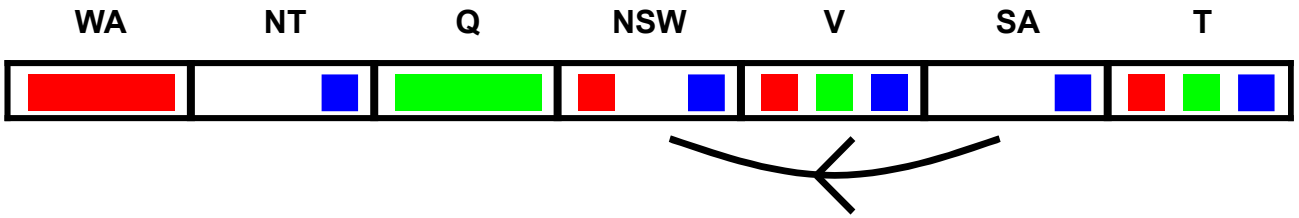
Constraint propagation repeatedly enforces constraints locally

Arc consistency

Simplest form of propagation makes each arc consistent

$X \rightarrow Y$ is consistent iff

for every value x of X there is some allowed y



Arc consistency

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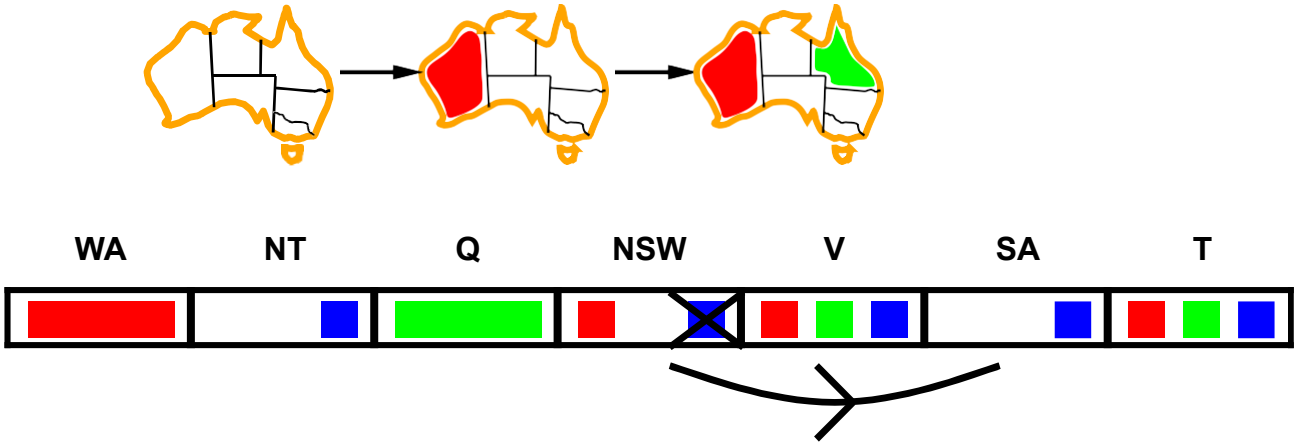

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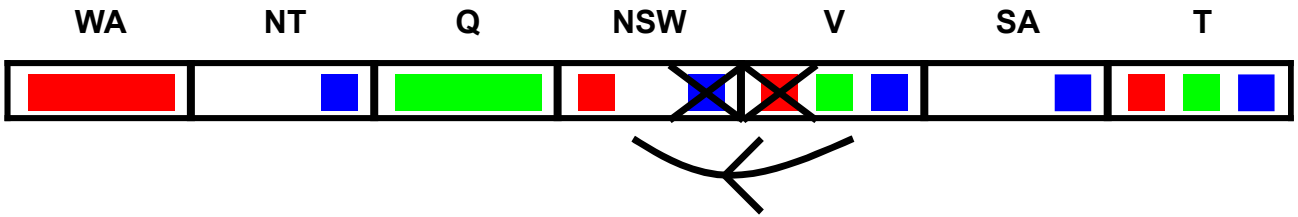

If X loses a value, neighbors of X need to be rechecked

Arc consistency

Simplest form of propagation makes each arc consistent

$X \rightarrow Y$ is consistent iff

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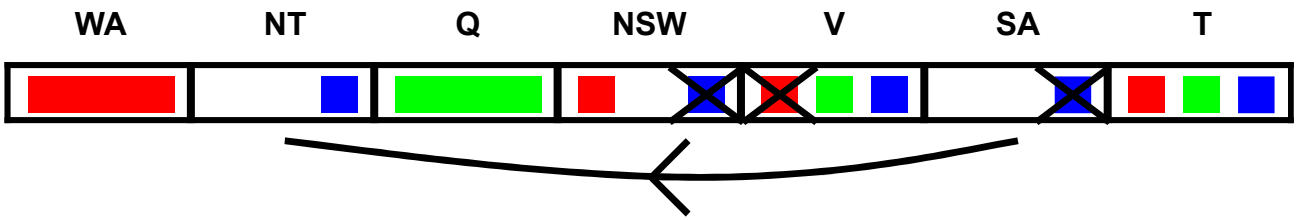


If X loses a value, neighbors of X need to be rechecked

Arc consistency detects failure earlier than forward checking

Can be run as a preprocessor or after each assignment

Arc consistency algorithm

```
function AC-3(csp) returns the CSP, possibly with reduced domains
  inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
  local variables: queue, a queue of arcs, initially all the arcs in csp

  while queue is not empty do
    (Xi, Xj) ← Remove-First(queue)
    if Remove-Inconsistent-Values(Xi, Xj) then
      for each Xk in Neighbors[Xi] do
        add (Xk, Xi) to queue
```

```
function Remove-Inconsistent-Values(Xi, Xj) returns true iff succeeds
  removed ← false
  for each x in Domain[Xi] do
    if no value y in Domain[Xj] allows (x,y) to satisfy the constraint Xi ↔ Xj
      then delete x from Domain[Xi]; removed ← true
  return removed
```

$O(n^2d^3)$, can be reduced to $O(n^2d^2)$ (but detecting all is NP-hard)

Local Search for CSPs

Local search algorithms can be very effective in solving many CSPs.

Local search algorithms use a complete-state formulation where each state assigns a value to every variable, and the search changes the value of one variable at a time.

Min-conflicts heuristic: value that results in the **minimum number** of **conflicts** with other variables that **brings us closer to a solution.**

- Usually has a series of **plateaus**

Plateau search: allowing sideways moves to another state with the same score.

- can help local search find its way off the plateau.

Constraint weighting aims to concentrate the search on the important constraints

- Each constraint is given a numeric weight, initially all 1.
- weights adjusted by incrementing when it is violated by the current assignment

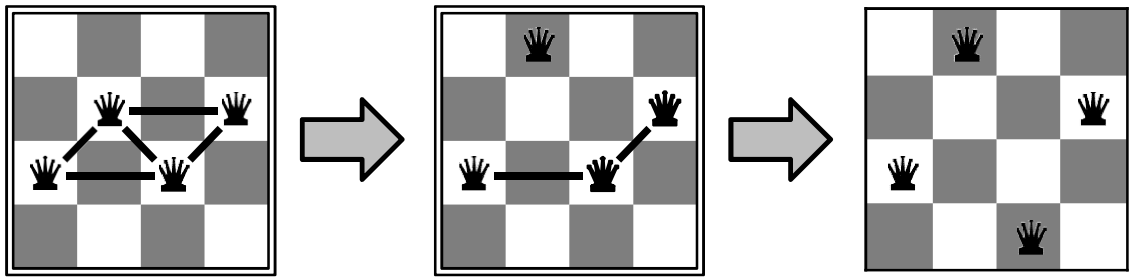
Example: 4-Queens

States: 4 queens in 4 columns ($4^4 = 256$ states)

Operators: move queen in column

Goal test: no attacks

Evaluation: $h(n)$ = number of attacks

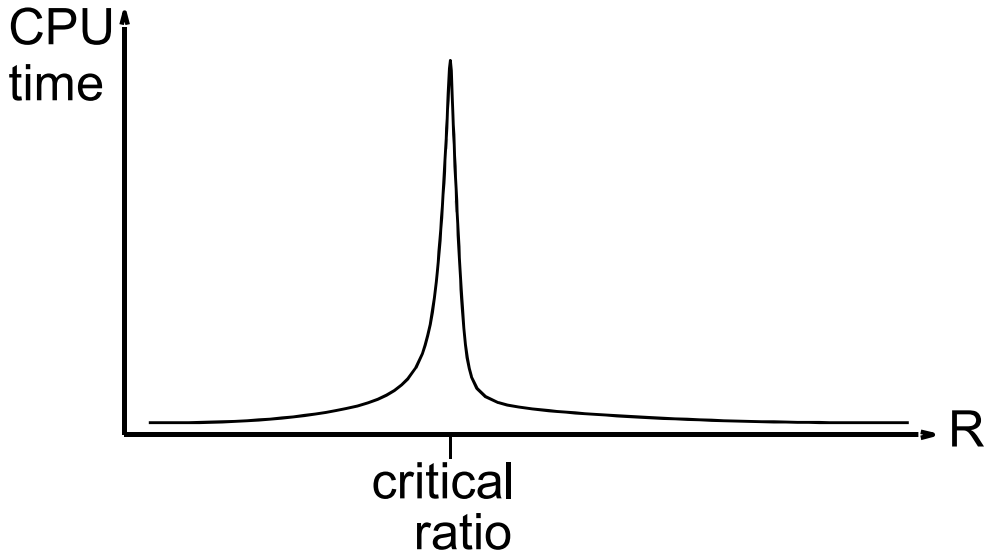


Performance of min-conflicts

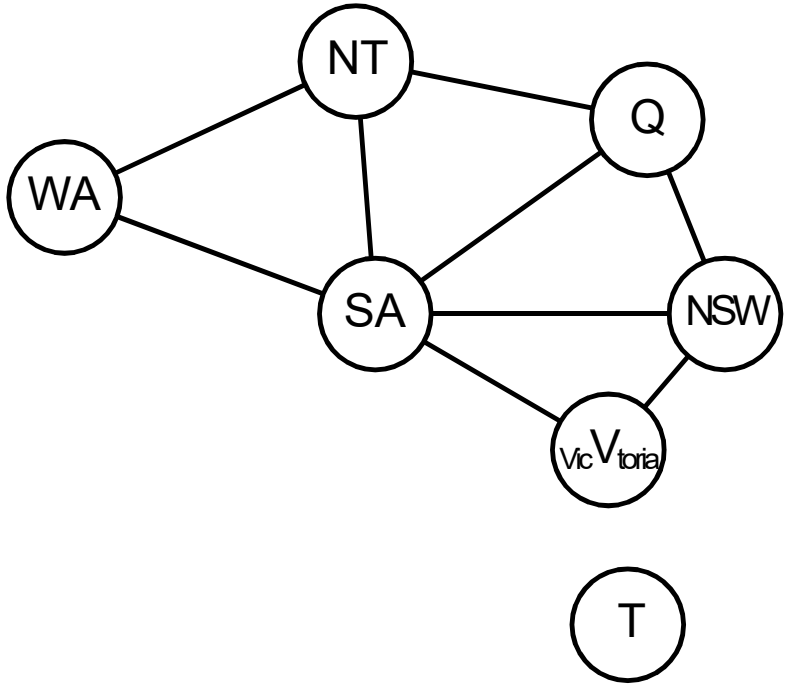
Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)

The same appears to be true for any randomly-generated CSP
except in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$



Problem structure



Tasmania and mainland are independent subproblems

Identifiable as connected components of constraint graph

Problem structure contd.

Suppose each subproblem has c variables out of n total

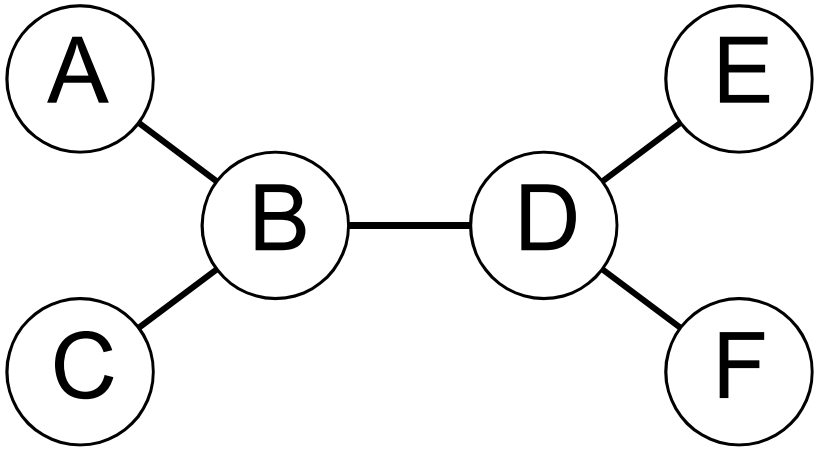
Worst-case solution cost is $n/c \cdot d^c$, linear in n

E.g., $n = 80$, $d = 2$, $c = 20$

2^{80} = 4 billion years at 10 million nodes/sec

$4 \cdot 2^{20}$ = 0.4 seconds at 10 million nodes/sec

Tree-structured CSPs



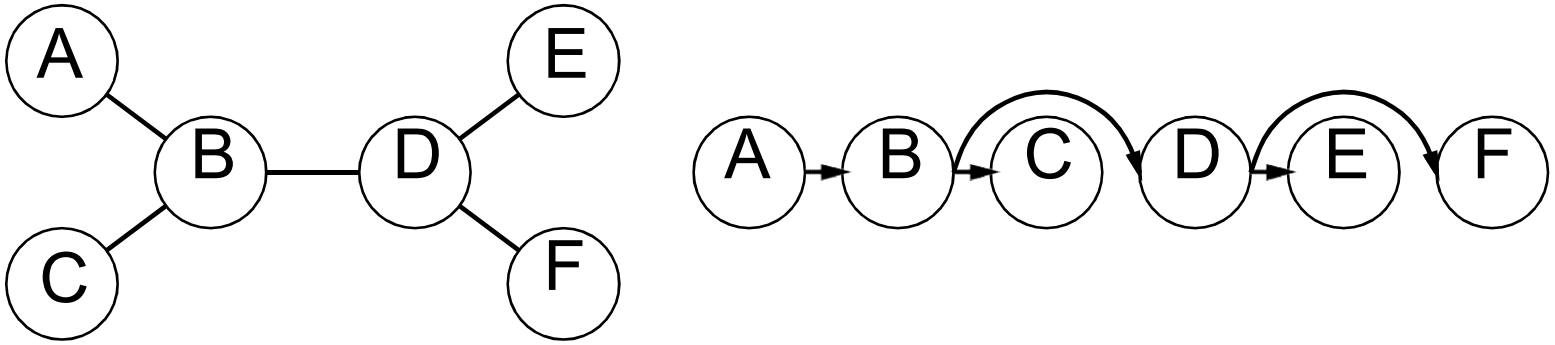
Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n d^2)$ time

Compare to general CSPs, where worst-case time is $O(d^n)$

This property also applies to logical and probabilistic reasoning:
an important example of the relation between syntactic restrictions
and the complexity of reasoning.

Algorithm for tree-structured CSPs

1. Choose a variable as root, order variables from root to leaves such that every node's parent precedes it in the ordering

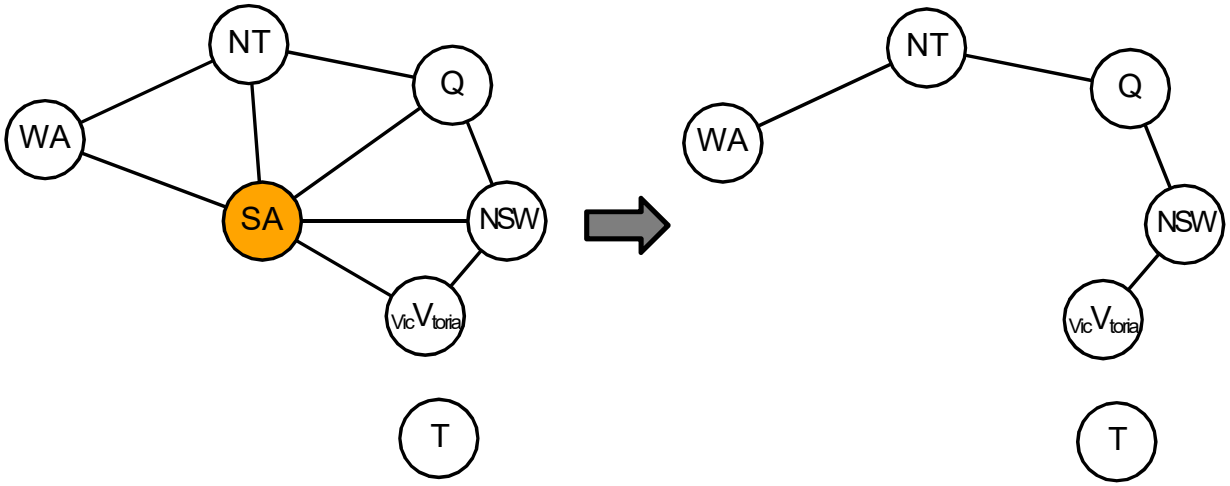


2. For j from n down to 2, apply $\text{RemoveInconsistent}(Parent(X_j), X_j)$

3. For j from 1 to n, assign X_j consistently with $Parent(X_j)$

Nearly tree-structured CSPs

Conditioning: instantiate a variable, prune its neighbors' domains



Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree

Cutset size c $\Rightarrow$ runtime $O(d^c \cdot (n-c)d^2)$, very fast for small c

Iterative algorithms for CSPs

Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned

To apply to CSPs:

allow states with unsatisfied constraints

operators reassign variable values

Variable selection: randomly select any conflicted variable

Value selection by min-conflicts heuristic:

choose value that violates the fewest constraints

i.e., hillclimb with $h(n)$ = total number of violated constraints

Summary

CSPs are a special kind of problem:
- states defined by values of a fixed set of variables
- goal test defined by constraints on variable values

Backtracking = depth-first search with one variable assigned per node

Variable ordering and value selection heuristics help significantly

Forward checking prevents assignments that guarantee later failure

Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies

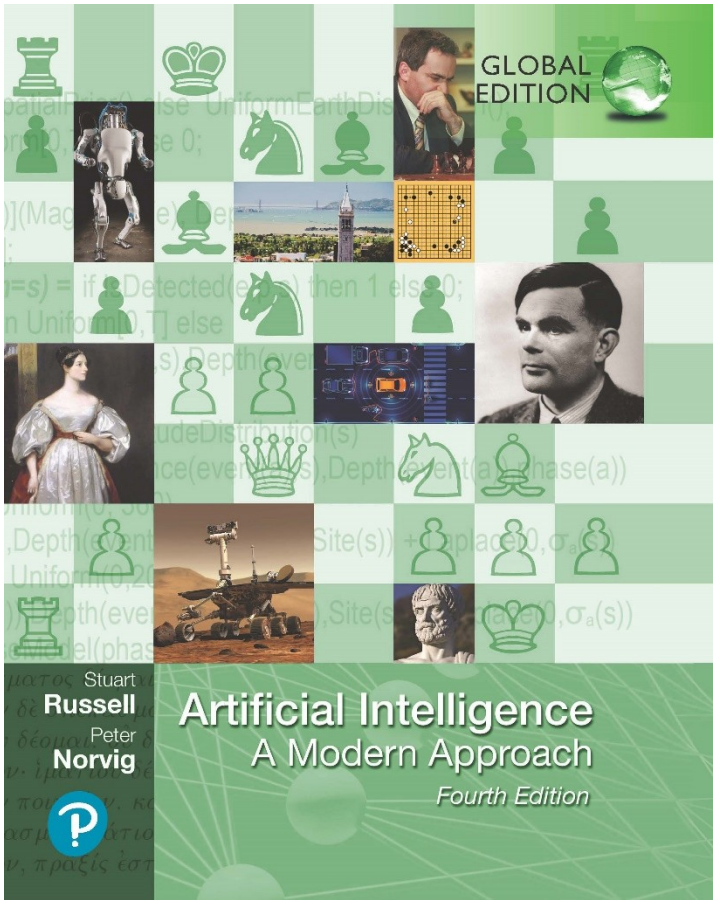
Local search using the min-conflicts heuristic has also been applied to constraint satisfaction problems with great success

The CSP representation allows analysis of problem structure

Tree-structured CSPs can be solved in linear time

Artificial Intelligence: A Modern Approach

Fourth Edition, Global Edition



Chapter 7

Logical agents

Lecture Presentations: Artificial Intelligence

Adapted from:
"Artificial Intelligence: A Modern Approach, Global Edition", 4th Edition by Stuart Russell and Peter Norvig © 2021 Pearson Education.

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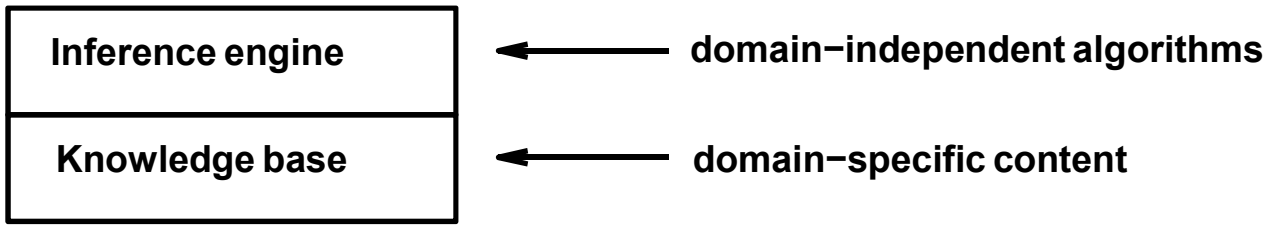
Outline

- Knowledge-based agents
- Wumpus world
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - resolution
  - forward chaining
  - backward chaining
- Effective Propositional Model Checking

Knowledge bases



Knowledge base = set of sentences in a formal language

Declarative approach to building an agent (or other system):

Tell it what it needs to know

Then it can Ask itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., what they know, regardless of how implemented

Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

```
function KB-Agent(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time
    Tell(KB, Make-Percept-Sentence(percept, t))
    action ← Ask(KB, Make-Action-Query(t))
    Tell(KB, Make-Action-Sentence(action, t))
    t ← t + 1
    return action
```

The agent must be able to:

- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

Wumpus World PEAS description

Performance measure

gold +1000, death -1000

-1 per step, -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

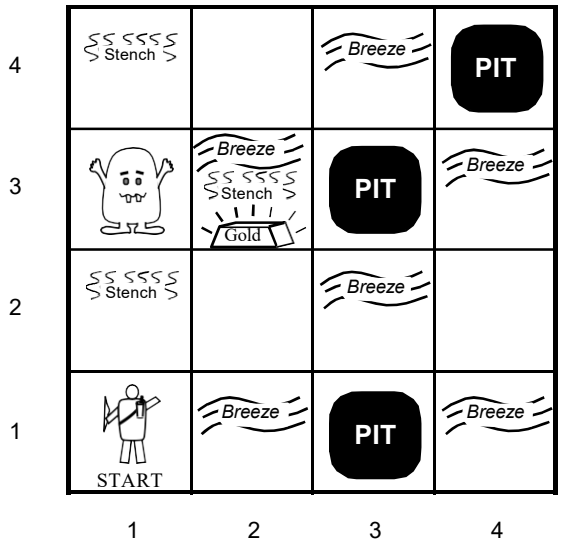
Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

Shooting uses up the only arrow

Grabbing picks up gold if in same square

Releasing drops the gold in same square



Actuators Left turn, Right turn,

Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell

Wumpus world characterization

Observable??

Wumpus world characterization

Observable?? No—only local perception

Deterministic??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

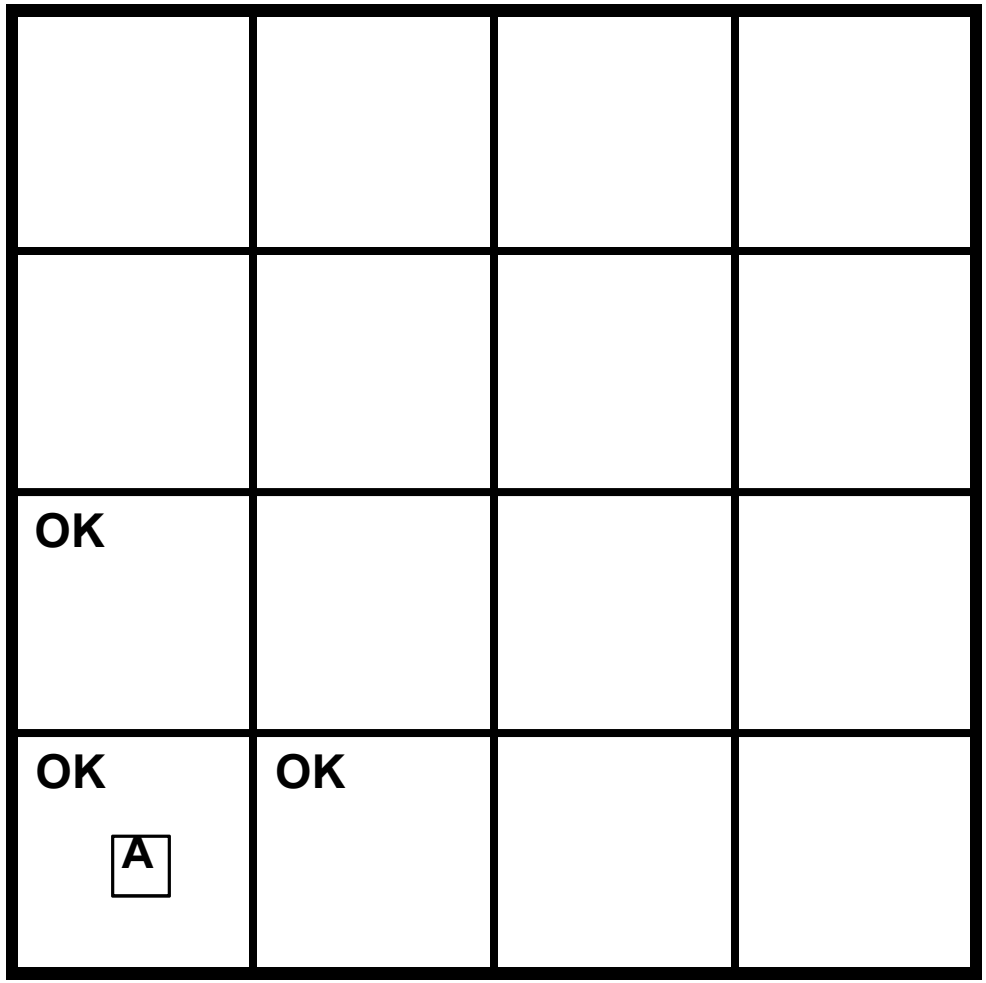
Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent?? Yes—Wumpus is essentially a natural feature

Exploring a wumpus world



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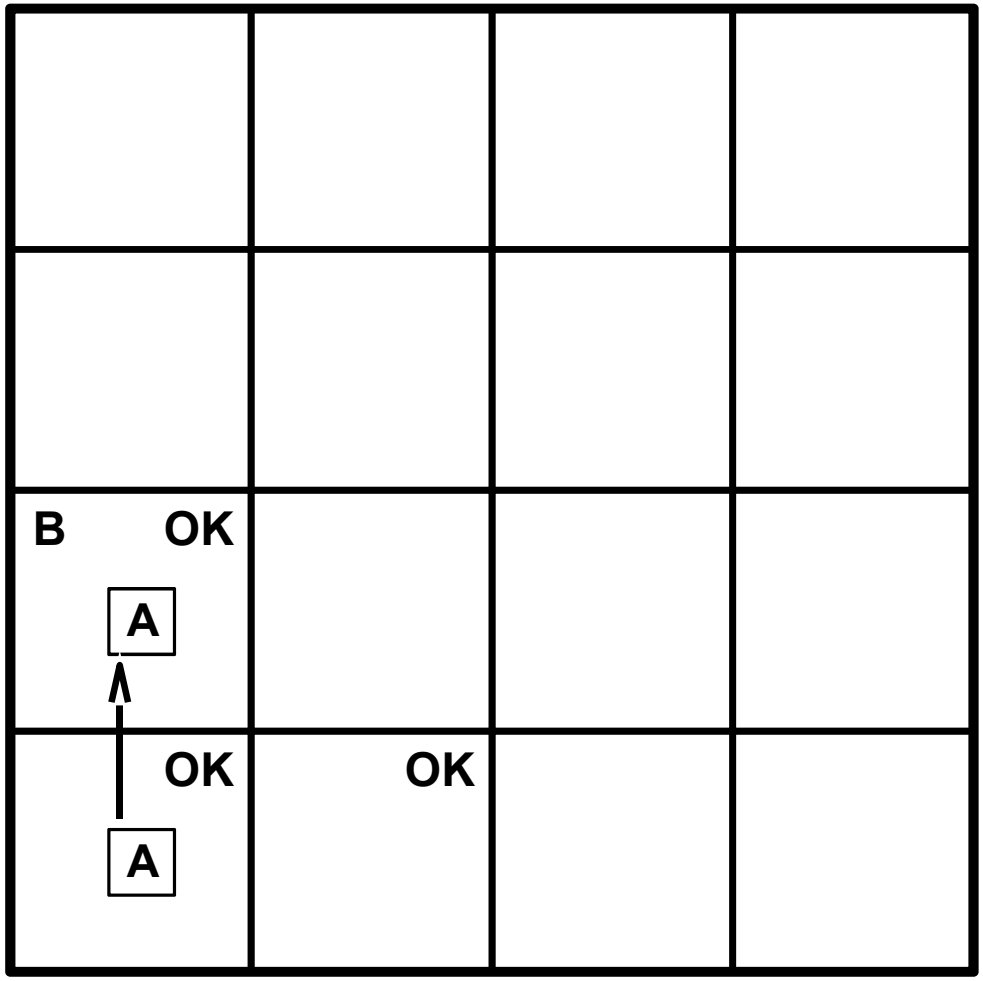

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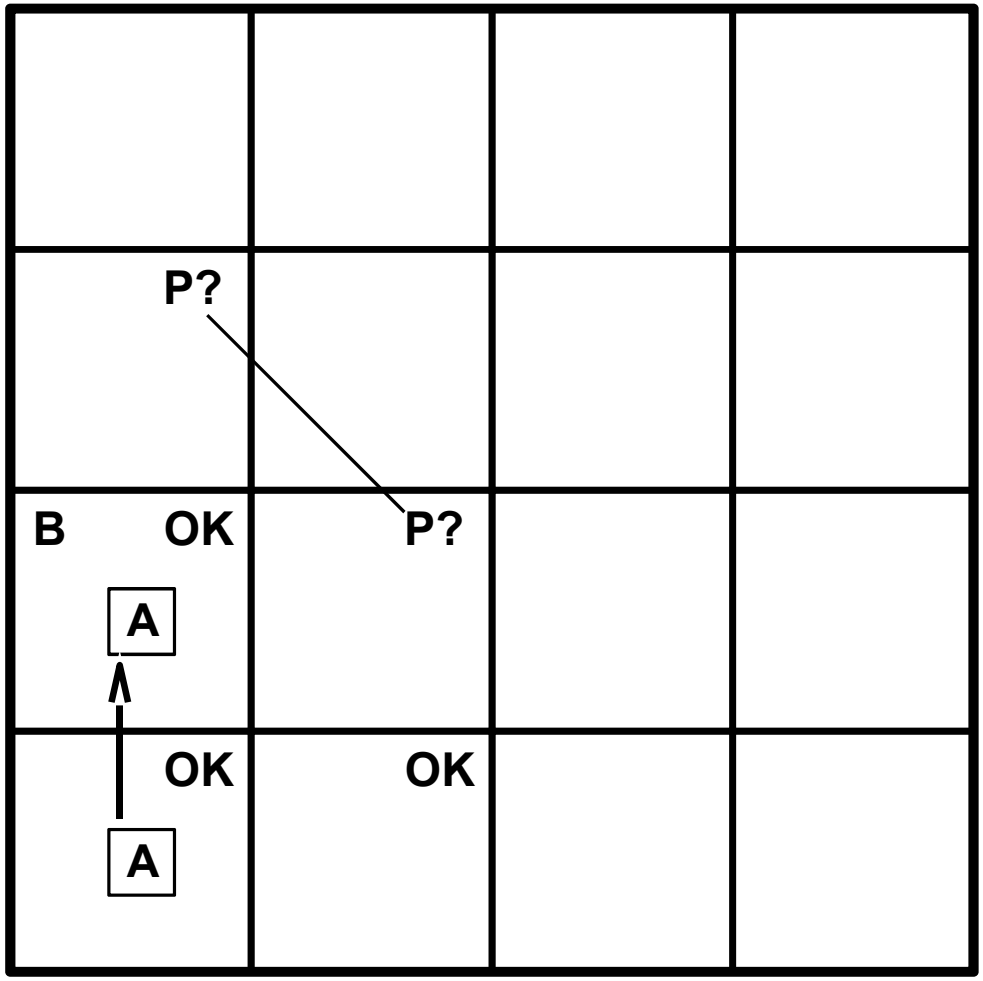

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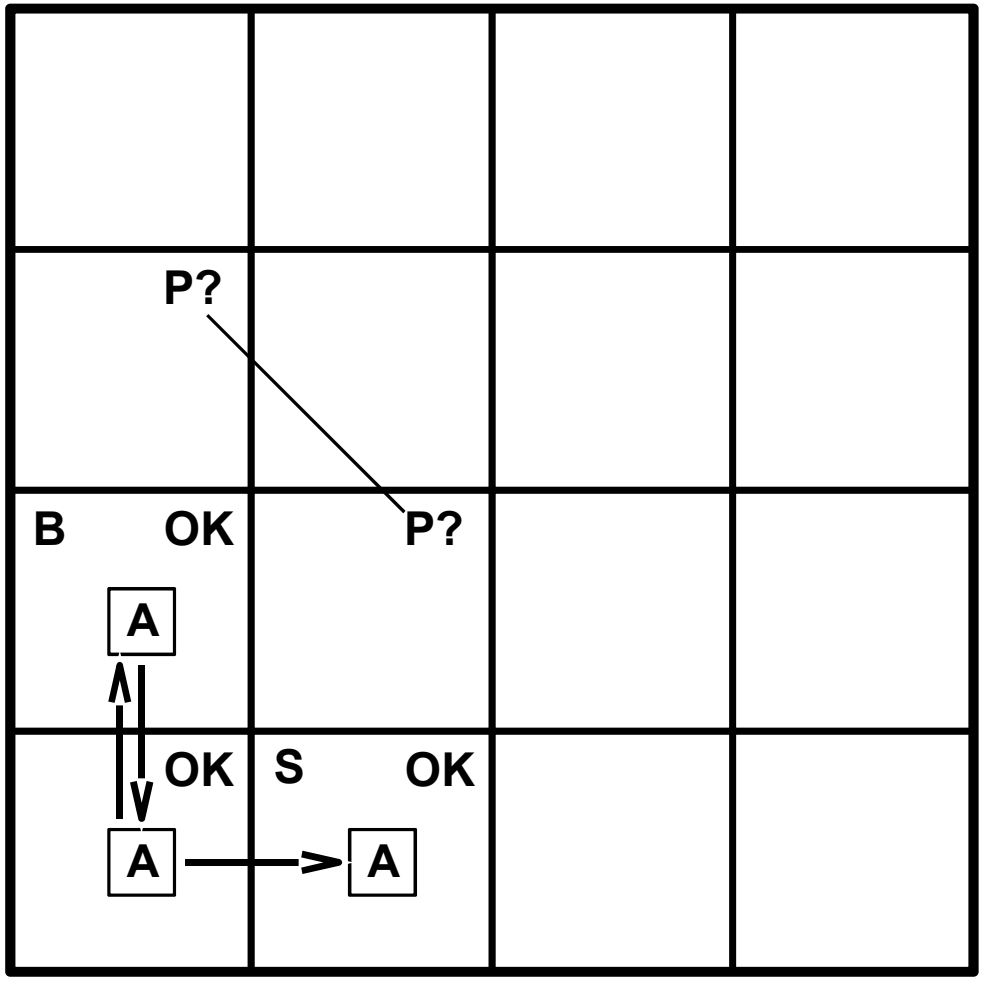
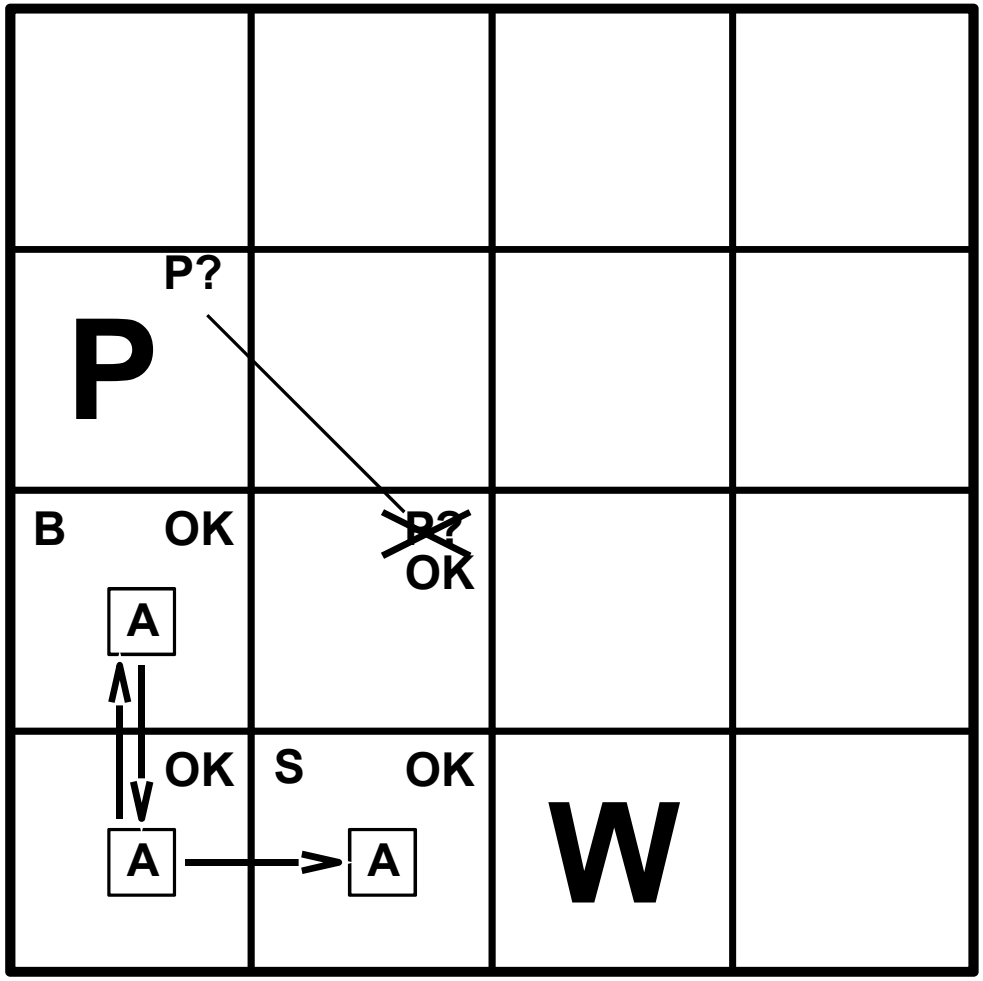
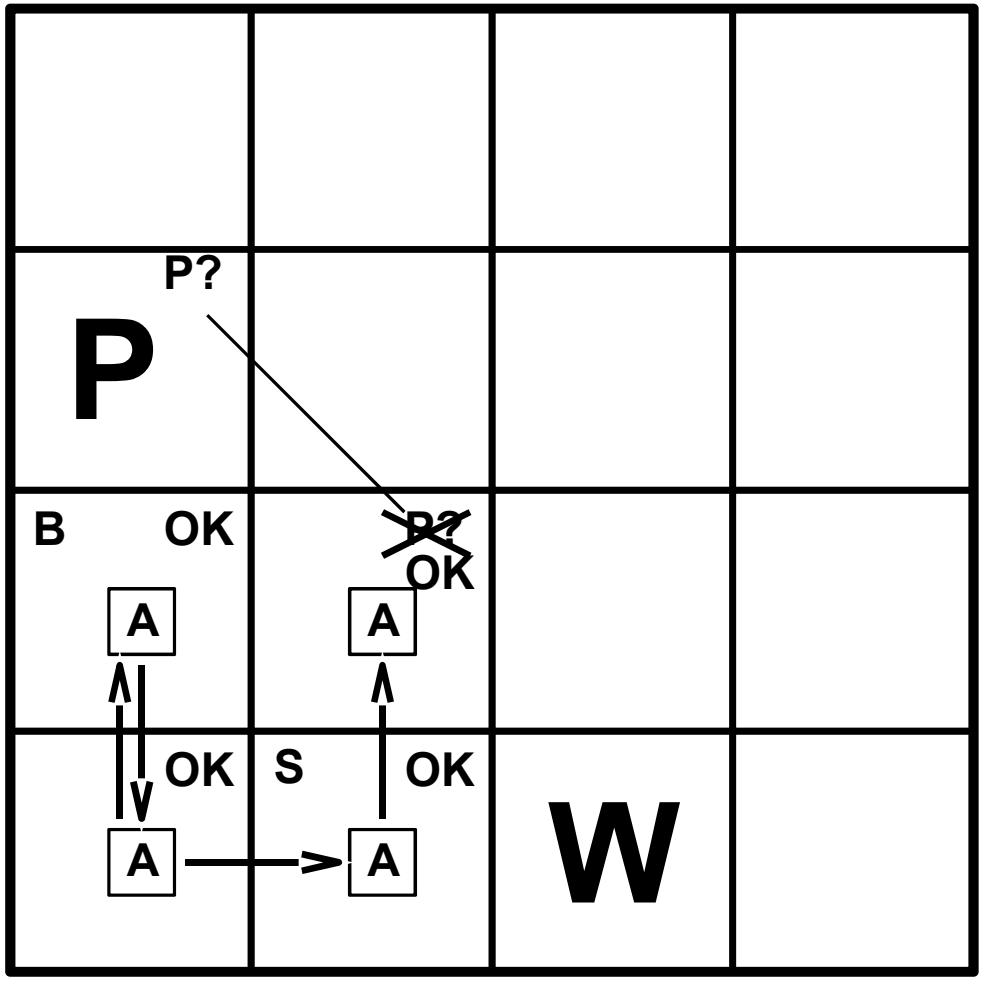


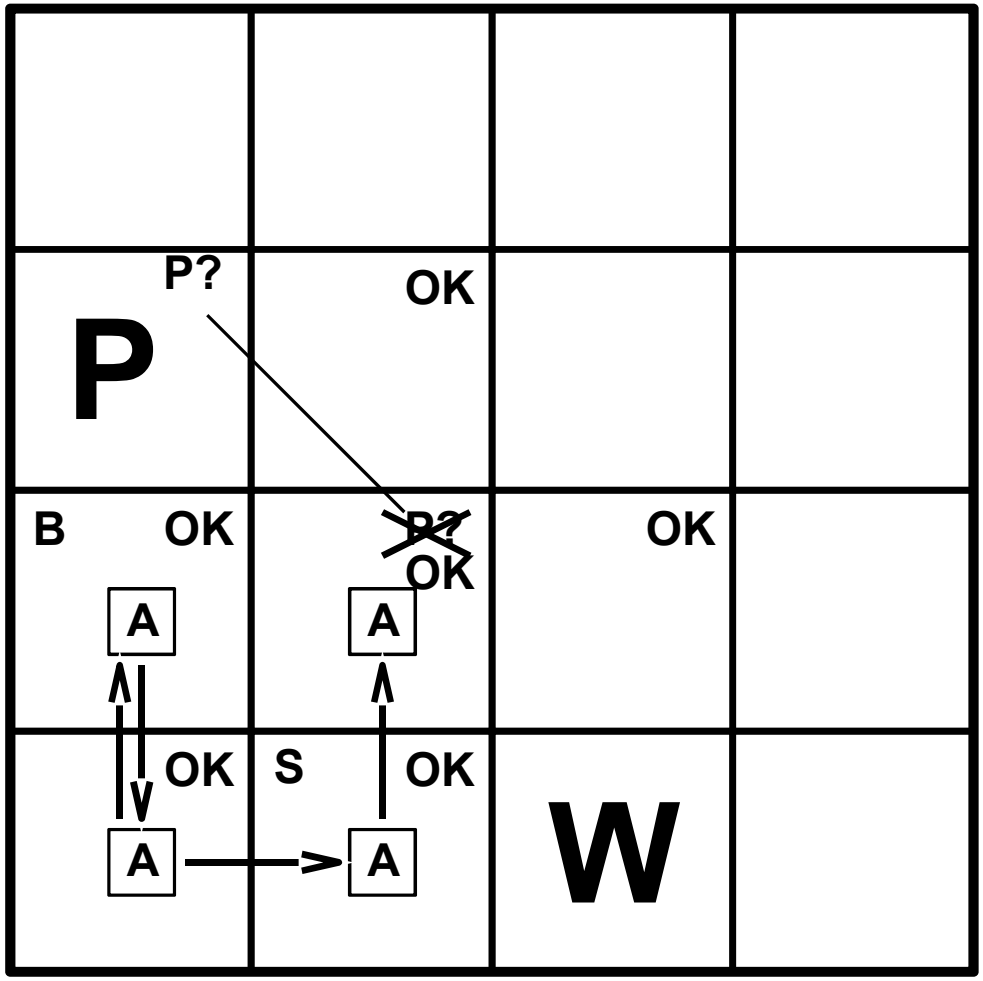
Exploring a wumpus world



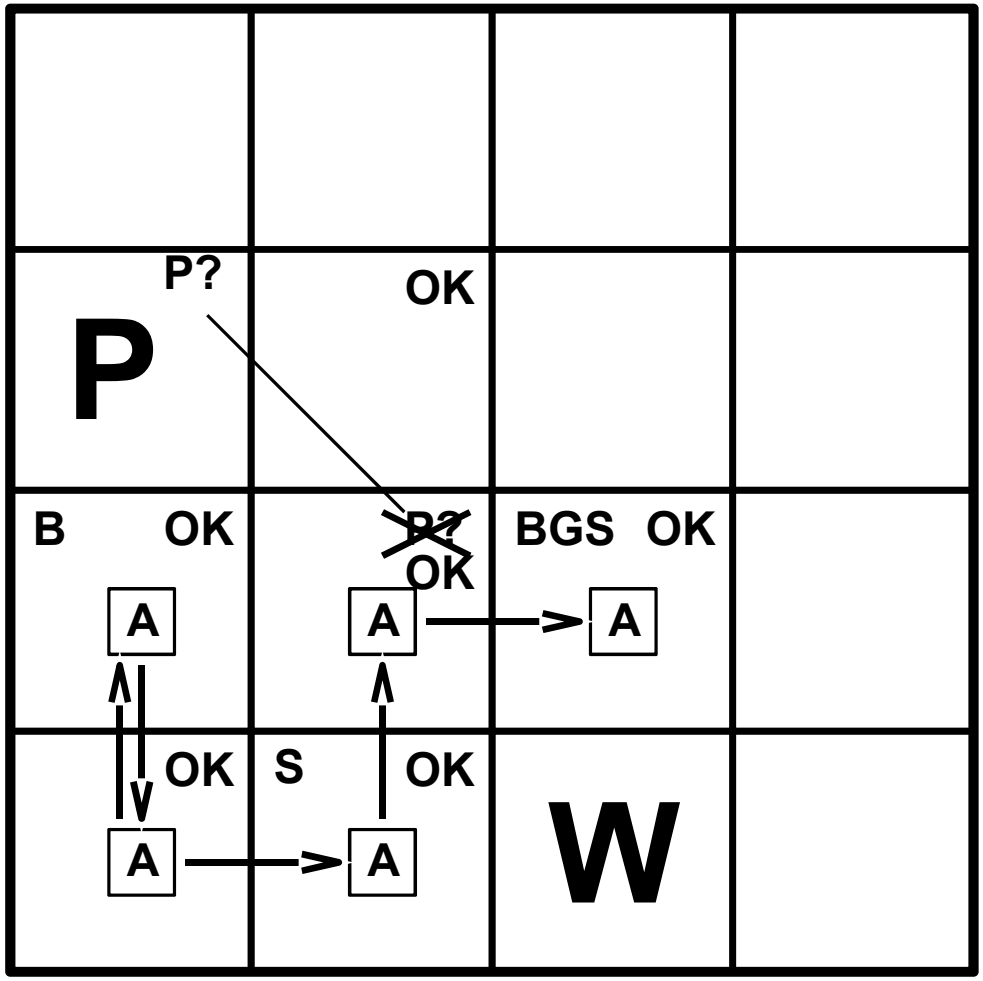
Exploring a wumpus world



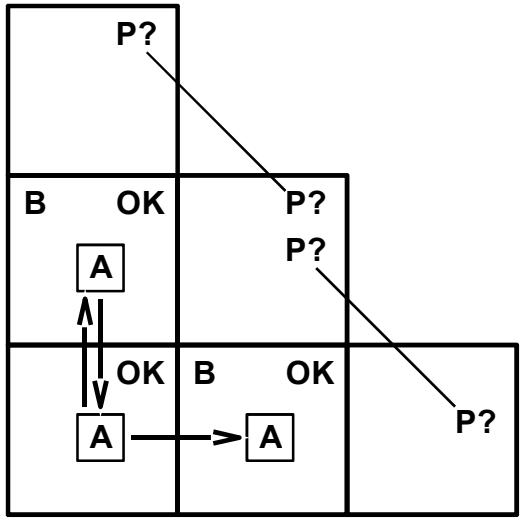
Exploring a wumpus world



Exploring a wumpus world

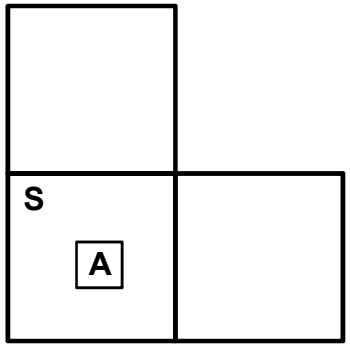


Other tight spots



Breeze in (1,2) and (2,1)
⇒ no safe actions

Assuming pits uniformly distributed,
(2,2) has pit w/ prob 0.86, vs. 0.31



Smell in (1,1)
⇒ cannot move
Can use a strategy of coercion:
shoot straight ahead
wumpus was there ⇒ dead ⇒ safe
wumpus wasn't there ⇒ safe

Logic in general

Logics are formal languages for representing information such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;

i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

$x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$

$x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

Entailment

Entailment means that one thing follows from another:

$$KB \models a$$

Knowledge base KB entails sentence a
if and only if
a is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

E.g., $x + y = 4$ entails $4 = x + y$

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Note: brains process syntax (of some sort)

Models

Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

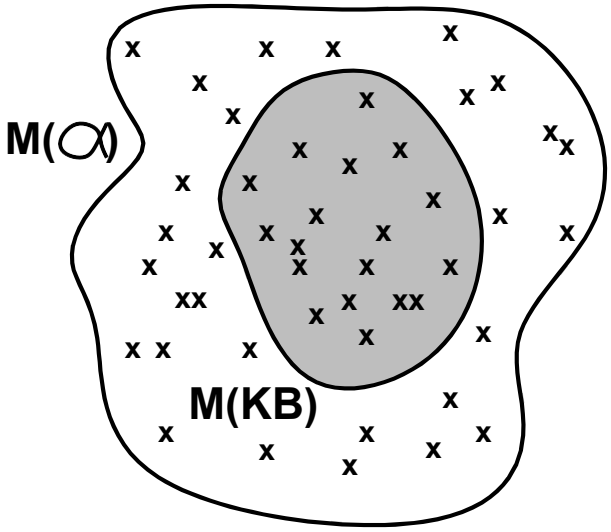
We say m is a model of a sentence a if a is true in m

$M(a)$ is the set of all models of a

Then $KB \models a$ if and only if $M(KB) \subseteq M(a)$

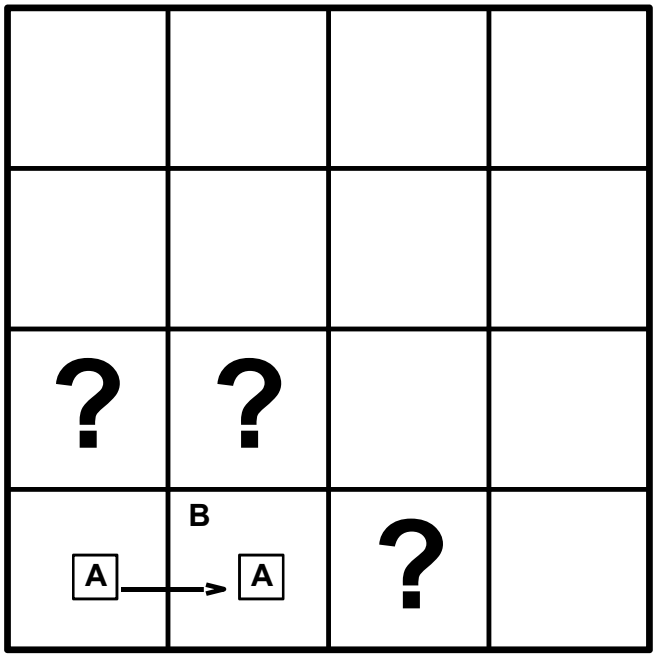
E.g. KB = Giants won and Reds won

a = Giants won



Entailment in the wumpus world

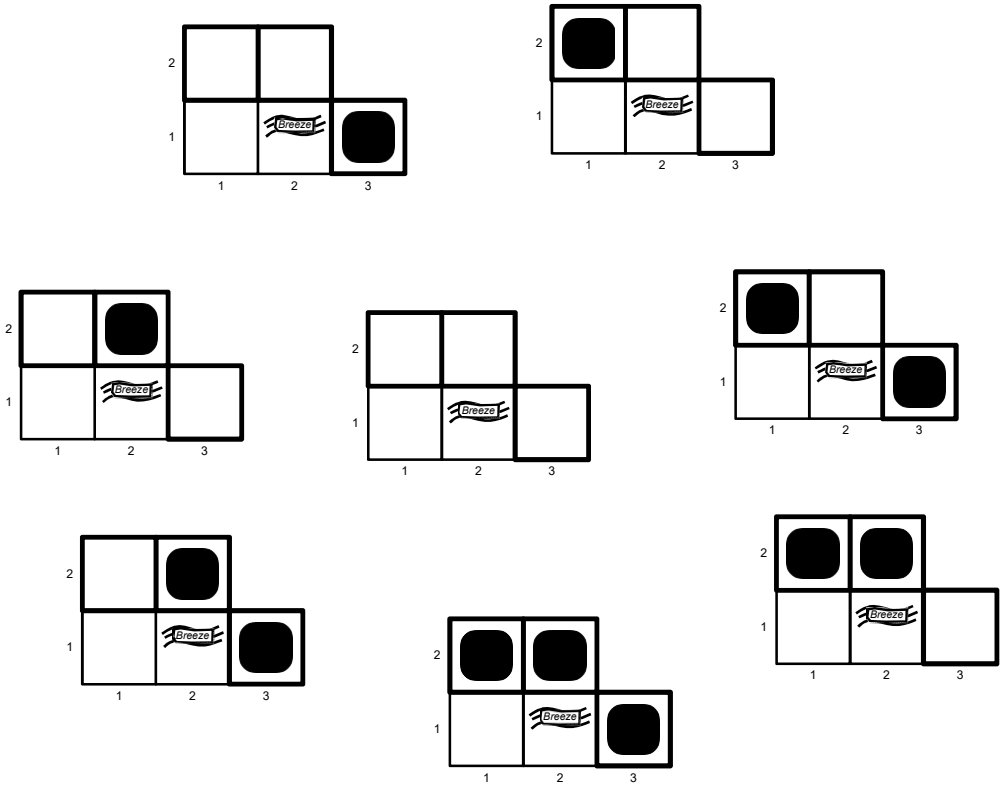
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]



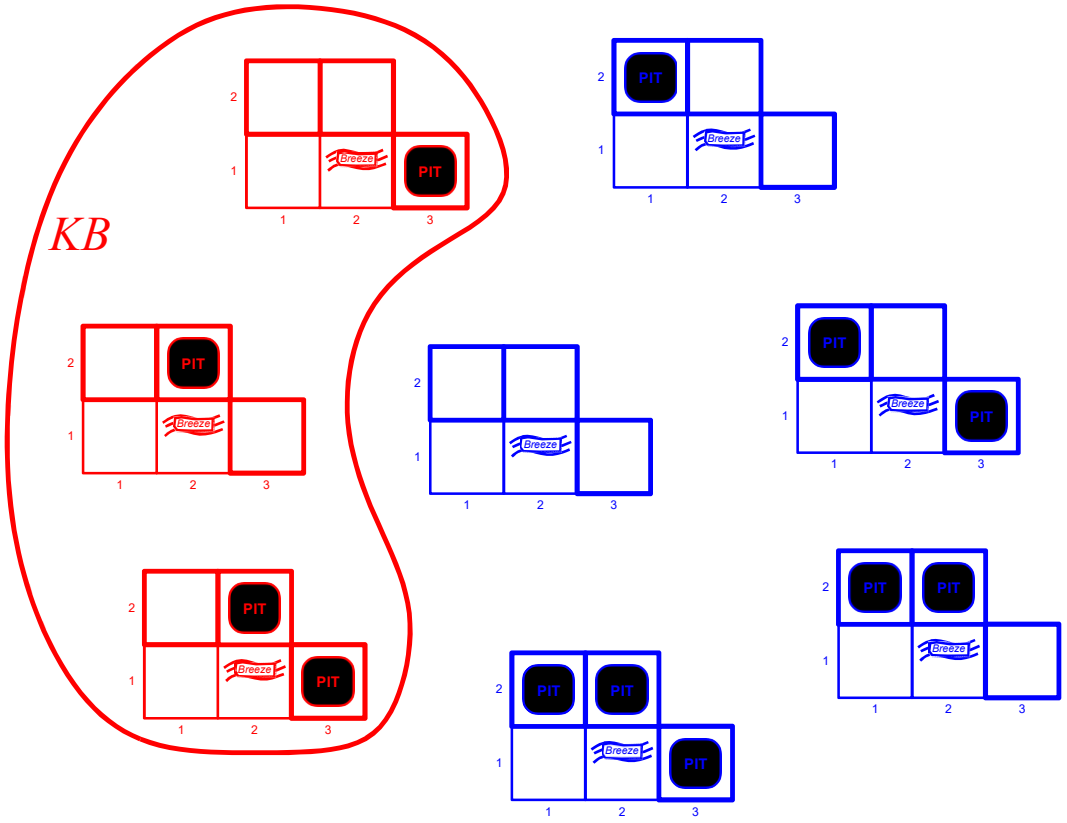
Consider possible models for ?s assuming only pits

3 Boolean choices ⇒ 8 possible models

Wumpus models

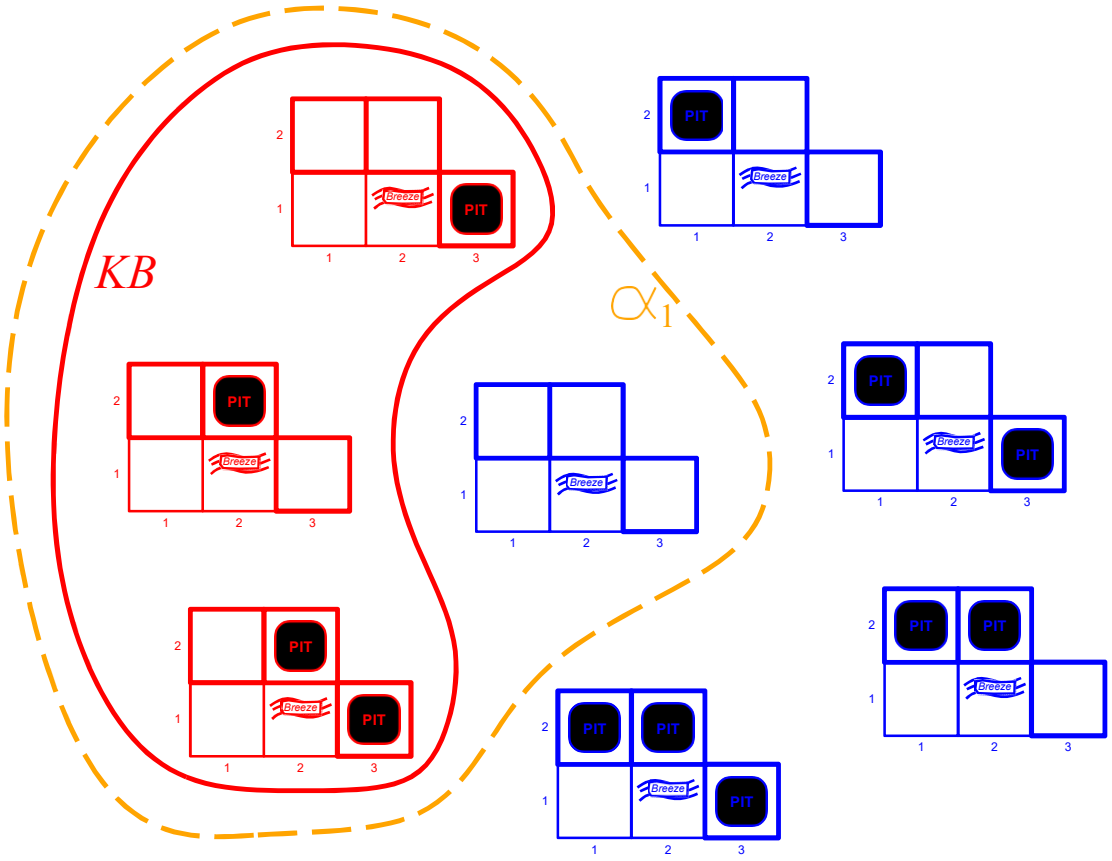


Wumpus models



KB = wumpus-world rules + observations

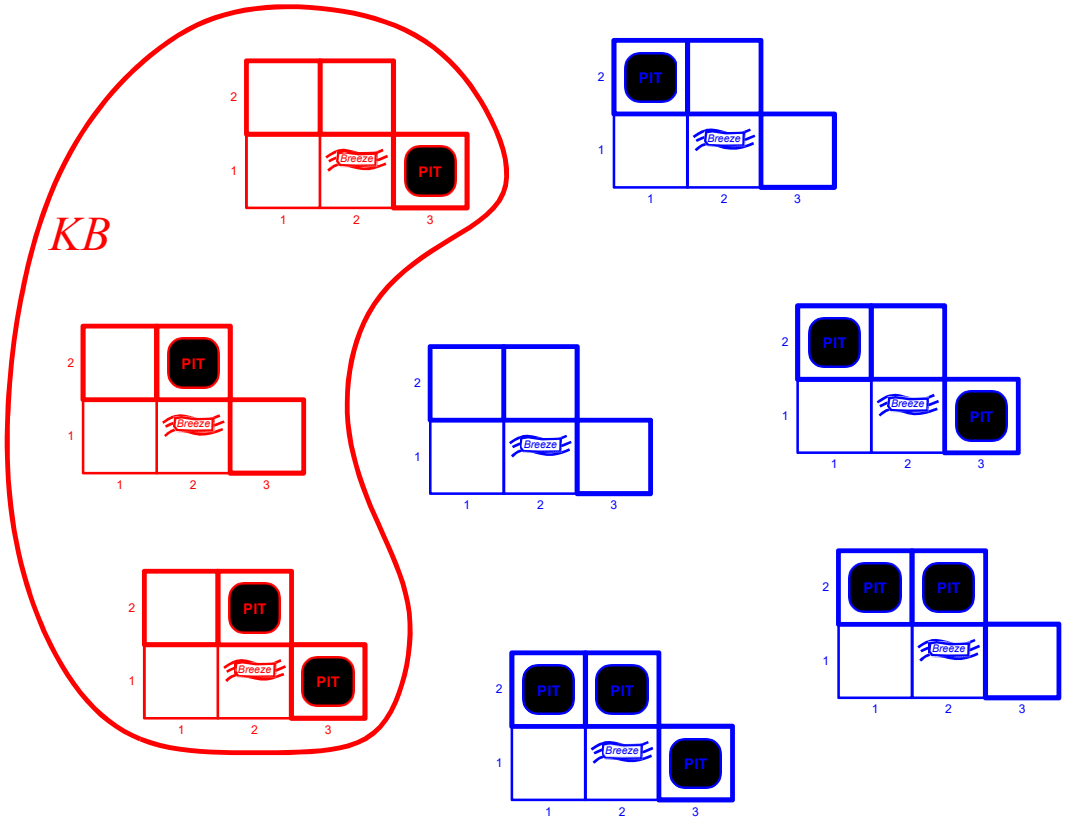
Wumpus models



KB = wumpus-world rules + observations

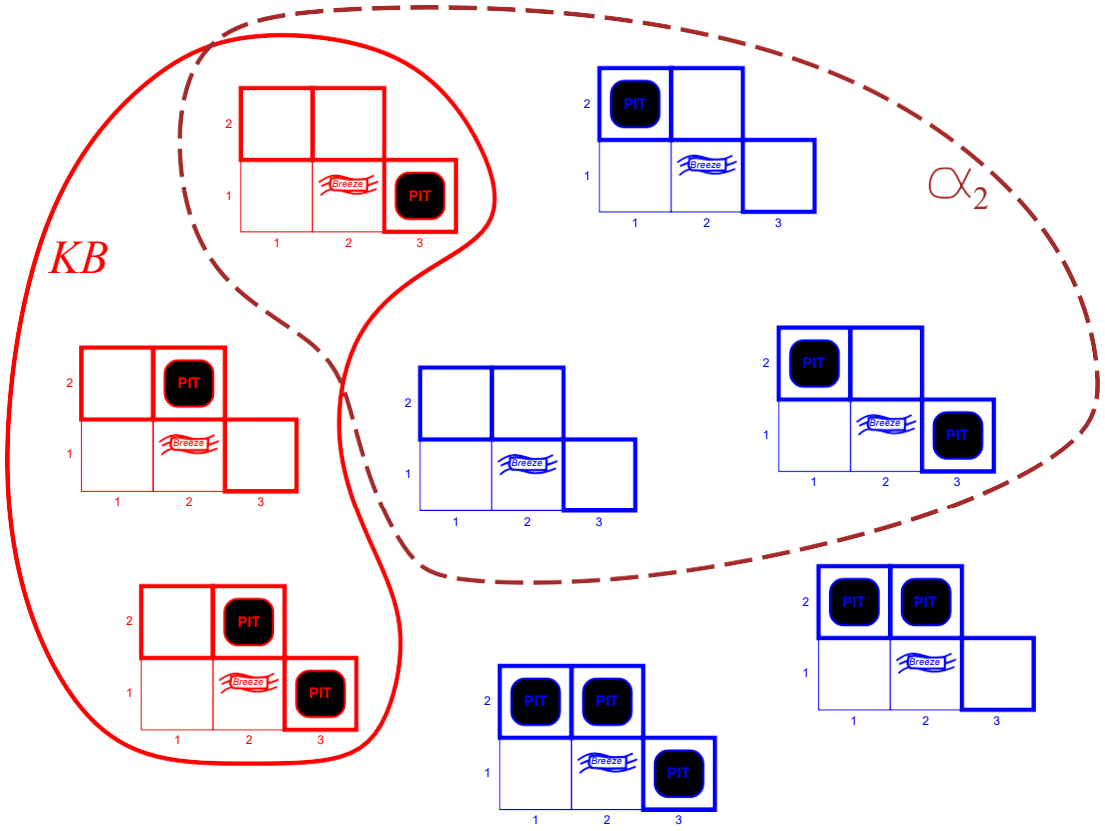
a_1 = "[1,2] is safe", $KB \models a_1$, proved by model checking

Wumpus models



KB = wumpus-world rules + observations

Wumpus models



KB = wumpus-world rules + observations

a_2 = "[2,2] is safe", $KB \models a_2$

Inference

$KB \vdash_i a$ = sentence a can be derived from KB by procedure i

Consequences of KB are a haystack; a is a needle.
Entailment = needle in haystack; inference = finding it

Soundness: i is sound if
whenever $KB \vdash_i a$, it is also true that $KB \models a$

Completeness: i is complete if
whenever $KB \models a$, it is also true that $KB \vdash_i a$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.

Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
true true false

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false					
$S_1 \wedge S_2$	is true iff	S_1	is true	and	S_2	is true		
$S_1 \vee S_2$	is true iff	S_1	is true	or	S_2	is true		
$S_1 \Rightarrow S_2$	is true iff	S_1	is false	or	S_2	is true		
i.e.,	is false iff	S_1	is true	and	S_2	is false		
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true	and	$S_2 \Rightarrow S_1$	is true		

Simple recursive process evaluates an arbitrary sentence, e.g.,
$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

"A square is breezy if and only if there is an adjacent pit"

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
.	.	.	.	.	.	.	.	.	.	.	.	.
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
fals	tru	fals	fals	fals	tru	fals	tru	tru	tru	tru	tru	tru
e	e	e	e	e	e	e	e	e	e	e	e	e
fals	tru	fals	fals	fals	tru	true	tru	tru	tru	tru	tru	tru
e	e	e	e	e	e		e	e	e	e	e	e
false	true	false	false	true	false	false	true	false	false	true	true	false
.	.	.	.	.	.	.	.	.	.	.	.	.
true	true	true	true	true	true	true	false	true	true	false	true	false

Enumerate rows (different assignments to symbols),
if KB is true in row, check that a is too

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-Entails?(KB, α) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            α, the query, a sentence in propositional logic

    symbols ← a list of the proposition symbols in KB and α
    return TT-Check-All(KB, α, symbols, [])

---

function TT-Check-All(KB, α, symbols, model) returns true or false
    if Empty?(symbols) then
        if PL-True?(KB, model) then return PL-True?(α, model)
        else return true
    else do
        P ← First(symbols); rest ← Rest(symbols)
        return TT-Check-All(KB, α, rest, Extend(P, true, model)) and
                    TT-Check-All(KB, α, rest, Extend(P, false, model))
```

$O(2^n)$ for n symbols; problem is co-NP-complete

Logical equivalence

Two sentences are logically equivalent iff true in same models:
$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

Validity and satisfiability

A sentence is valid if it is true in all models,

e.g., $True$, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models a$ if and only if $(KB \Rightarrow a)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models a$ if and only if $(KB \wedge \neg a)$ is unsatisfiable

i.e., prove a by *reductio ad absurdum*

Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
  Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Model checking

truth table enumeration (always exponential in n)

improved backtracking, e.g., Davis–Putnam–Logemann–Loveland

heuristic search in model space (sound but incomplete)
  e.g., min-conflicts-like hill-climbing algorithms

Resolution

Conjunctive Normal Form (CNF—universal)

conjunction of disjunctions of literals

clauses

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

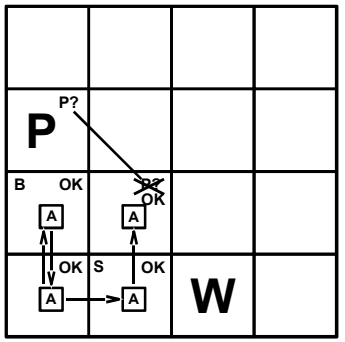
Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

Resolution is sound and complete for propositional logic



Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

Resolution algorithm

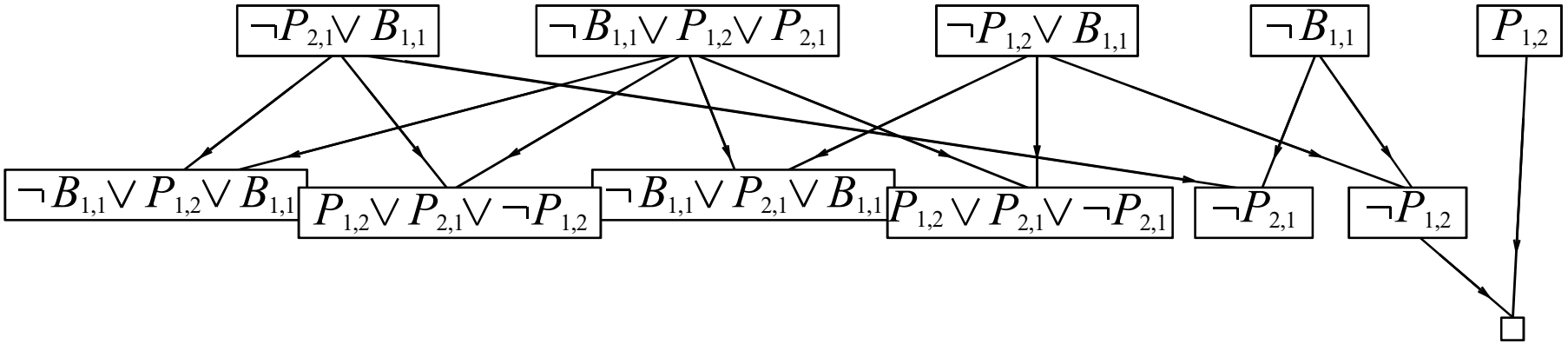
Proof by contradiction, i.e., show $KB \wedge \neg a$ unsatisfiable

```
function PL-Resolution(KB, a) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            a, the query, a sentence in propositional logic

    clauses ← the set of clauses in the CNF representation of KB ∧ ¬a
    new ← { }
    loop do
        for each Ci, Cj in clauses do
            resolvents ← PL-Resolve(Ci, Cj)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false
        clauses ← clauses ∪ new
```

Resolution example

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $a = \neg P_{1,2}$



Forward and backward chaining

Horn Form (restricted)

KB = conjunction of Horn clauses

Horn clause =

- proposition symbol; or
- (conjunction of symbols) $\Rightarrow$ symbol

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{a_1, \ldots, a_n, \quad a_1 \wedge \cdots \wedge a_n \Rightarrow \beta}{\beta}$$

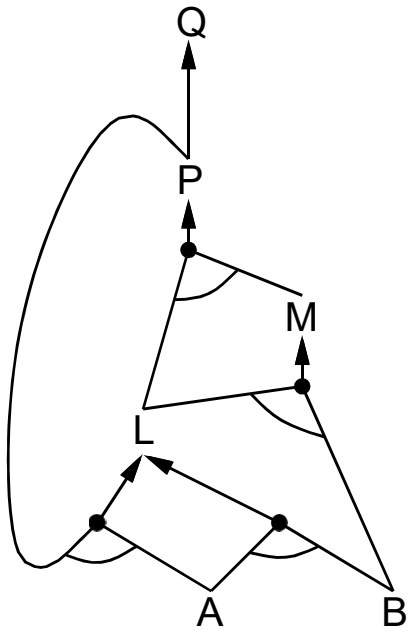
Can be used with forward chaining or backward chaining.

These algorithms are very natural and run in linear time

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB,
add its conclusion to the KB, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

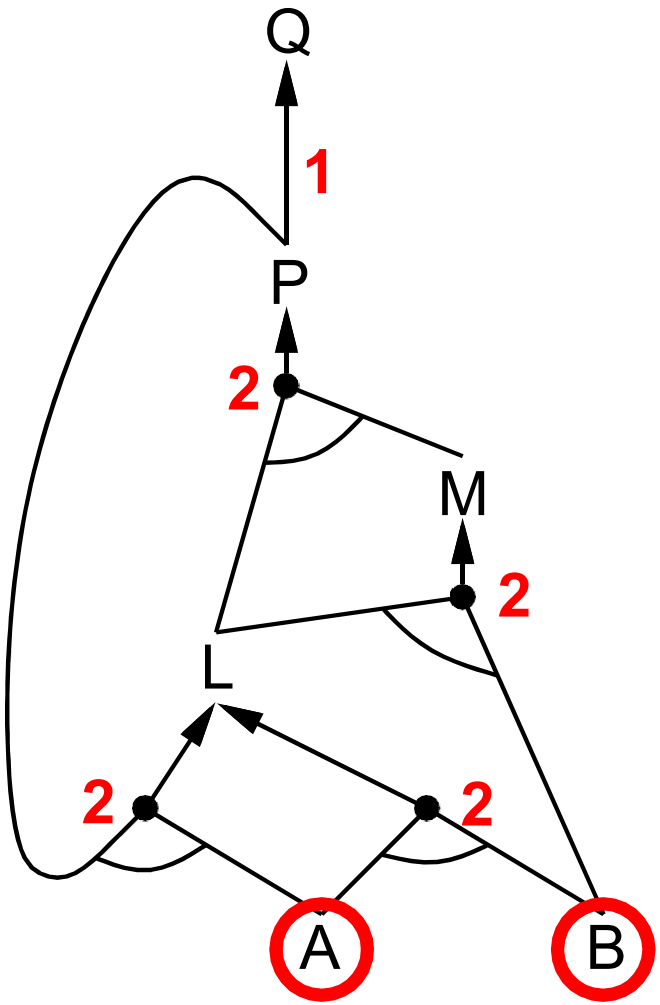


Forward chaining algorithm

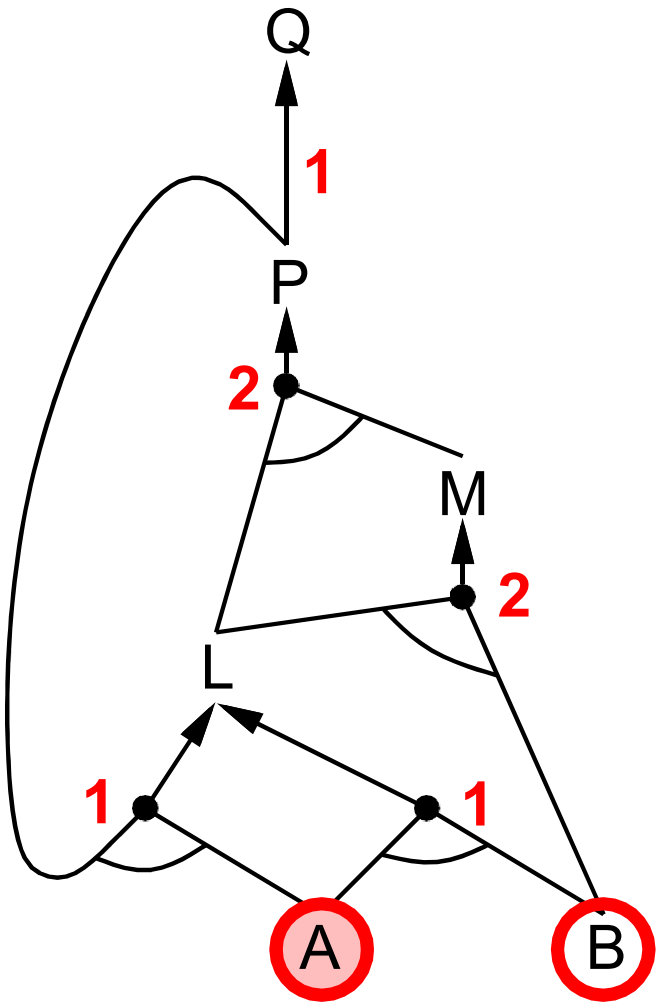
```
function PL-FC-Entails?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional Horn clauses
          q, the query, a proposition symbol
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known in KB

  while agenda is not empty do
      p ← Pop(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if Head[c] = q then return true
                  Push(Head[c], agenda)
  return false
```

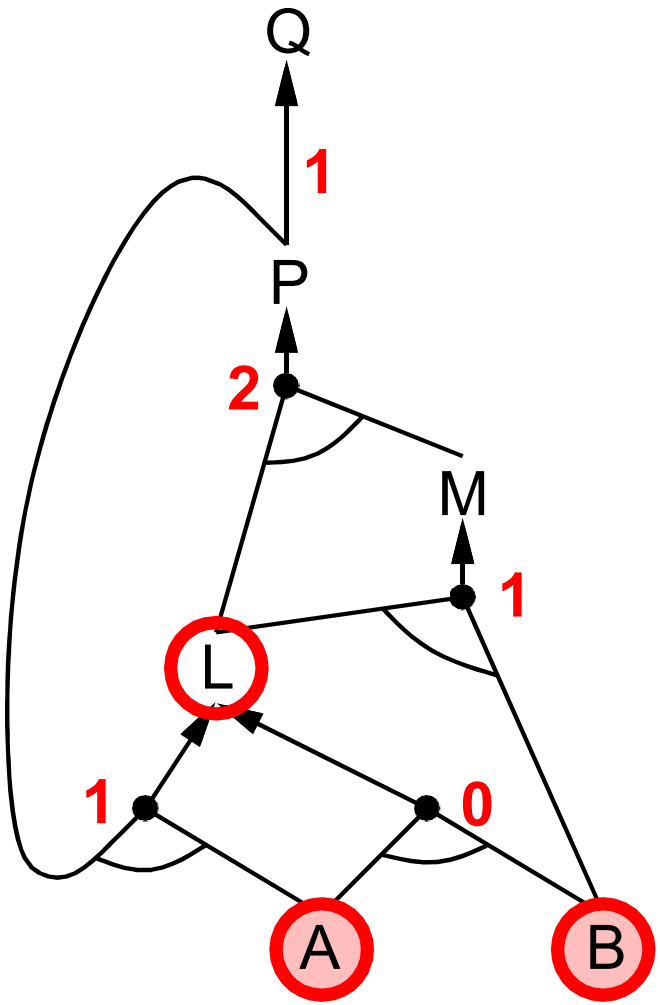
Forward chaining example



Forward chaining example



Forward chaining example



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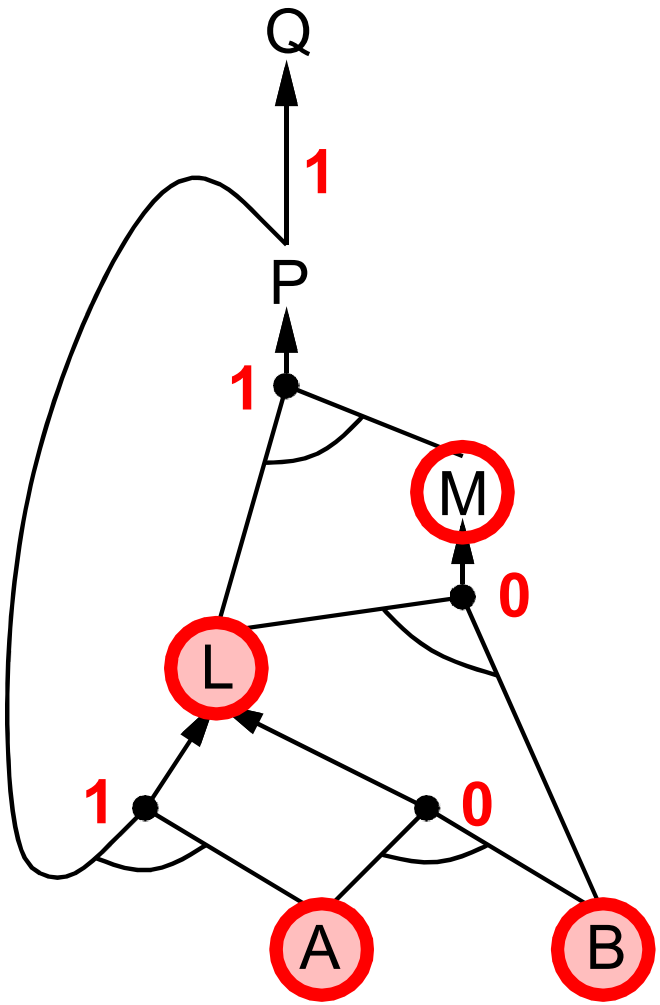
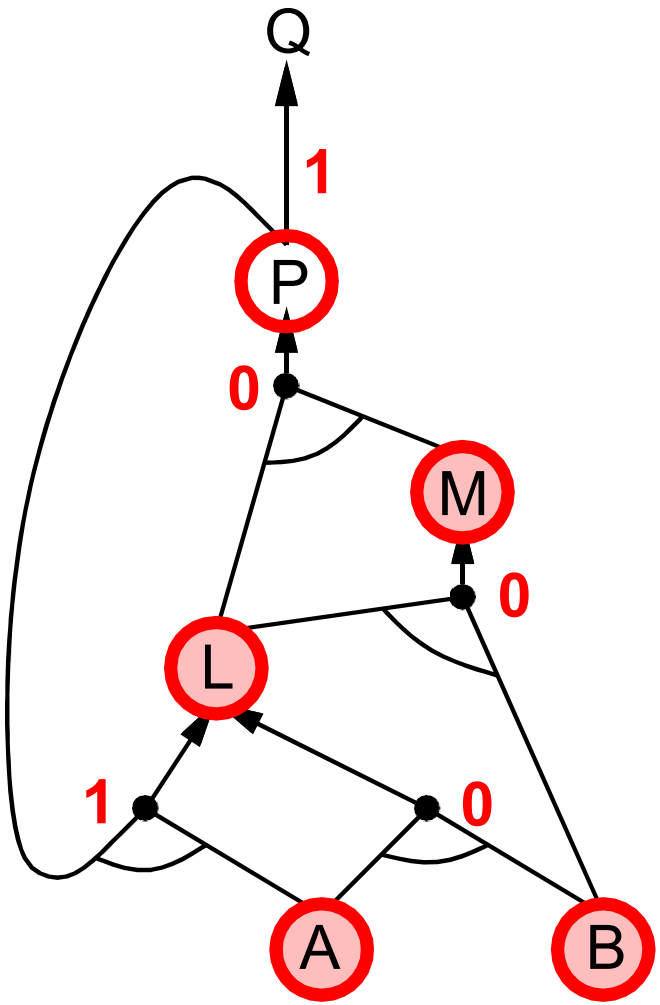
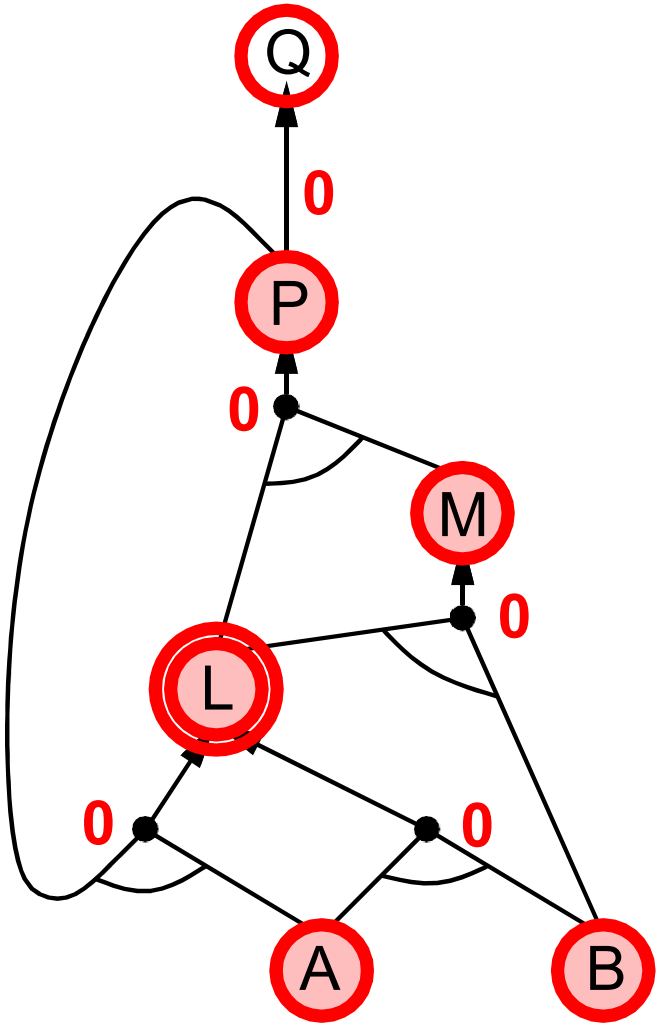


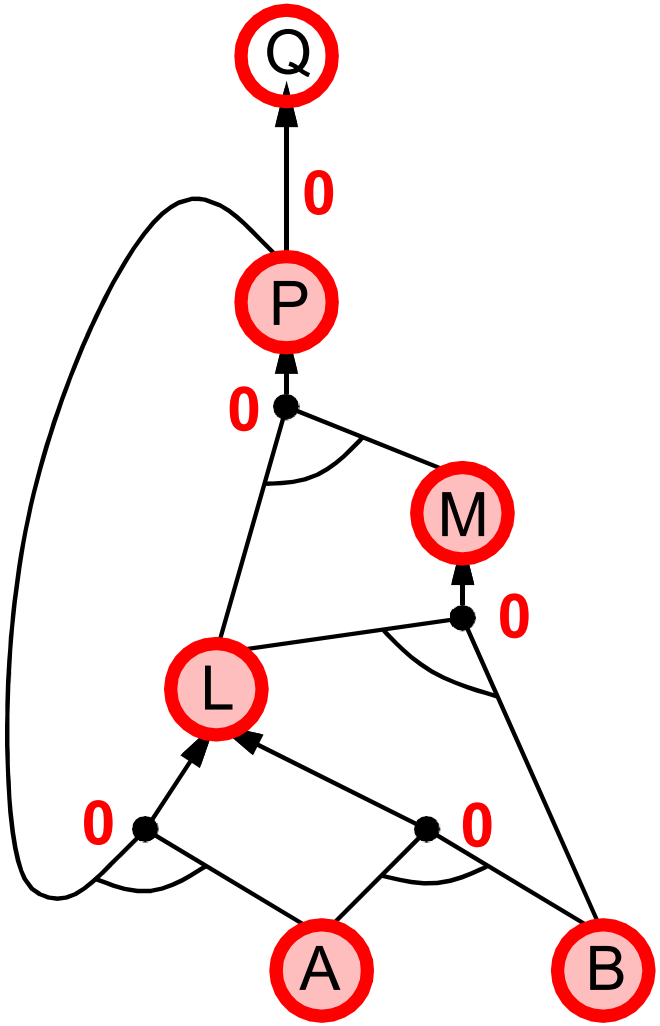
Forward chaining example



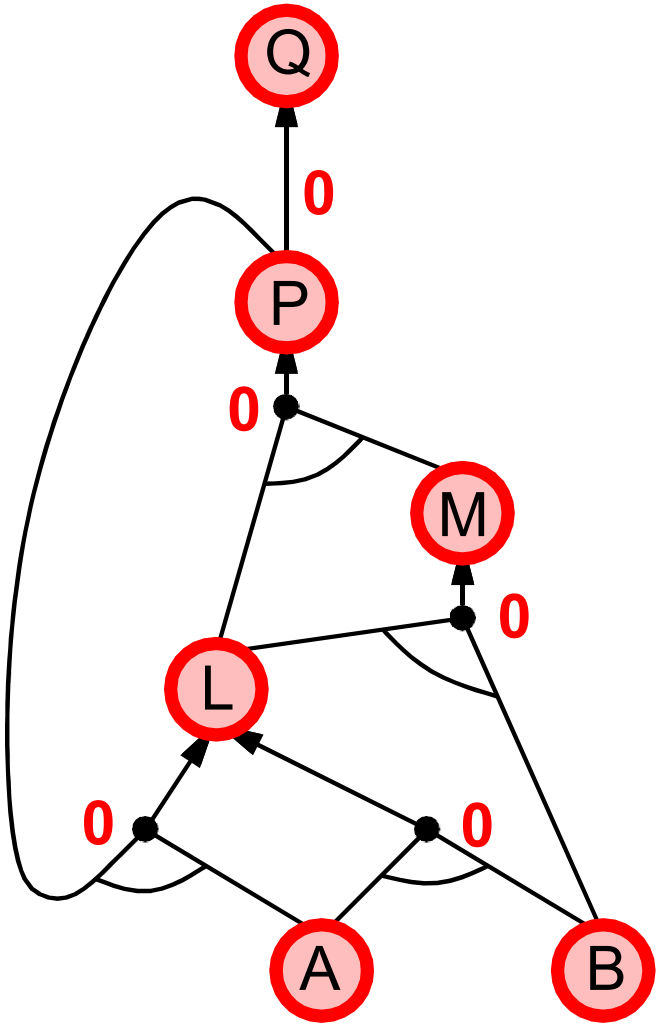
Forward chaining example



Forward chaining example



Forward chaining example



Proof of completeness

FC derives every atomic sentence that is entailed by KB

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model m, assigning true/false to symbols
3. Every clause in the original KB is true in m
   Proof: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in m
   Then $a_1 \wedge \ldots \wedge a_k$ is true in m and b is false in m
   Therefore the algorithm has not reached a fixed point!
4. Hence m is a model of KB
5. If $KB \models q$, q is true in every model of KB, including m

General idea: construct any model of KB by sound inference, check a

Backward chaining

Idea: work backwards from the query q:

to prove q by BC,

check if q is known already, or

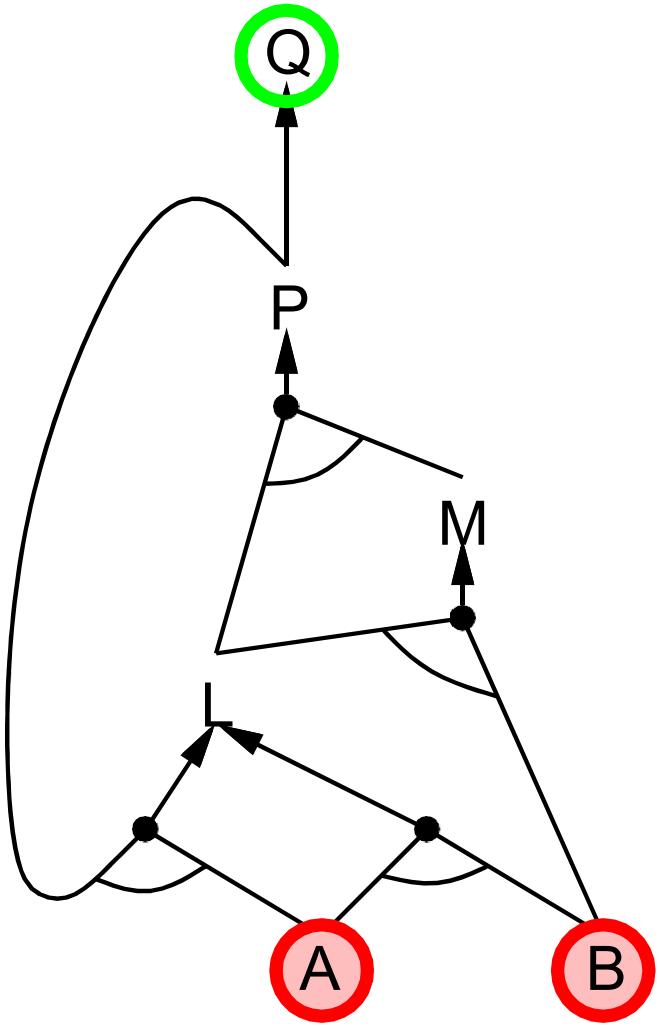
prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal

1) has already been proved true, or
2) has already failed

Backward chaining example



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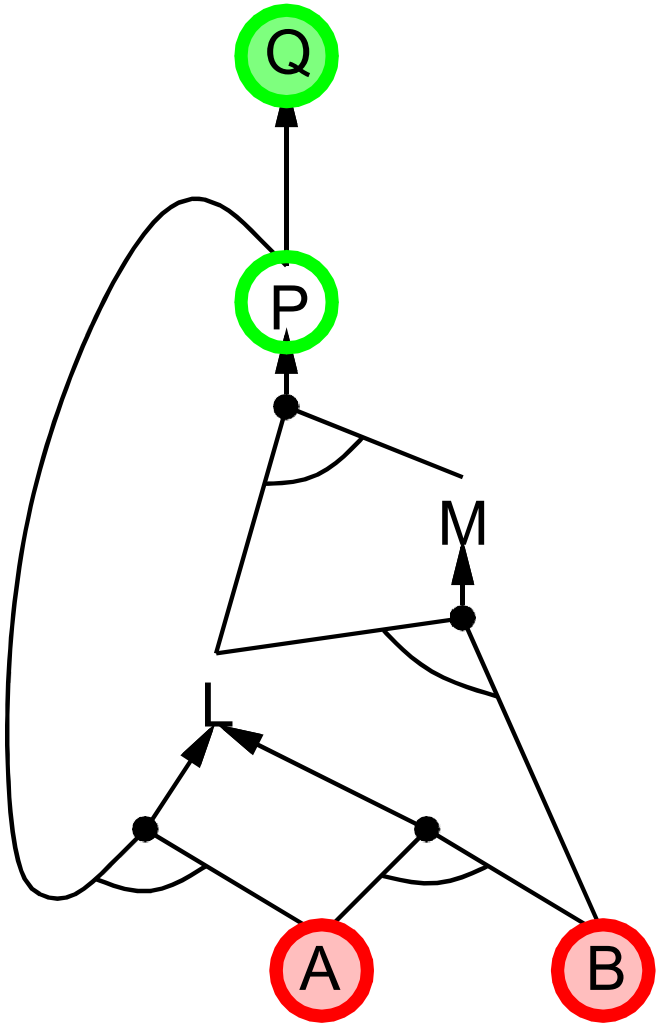

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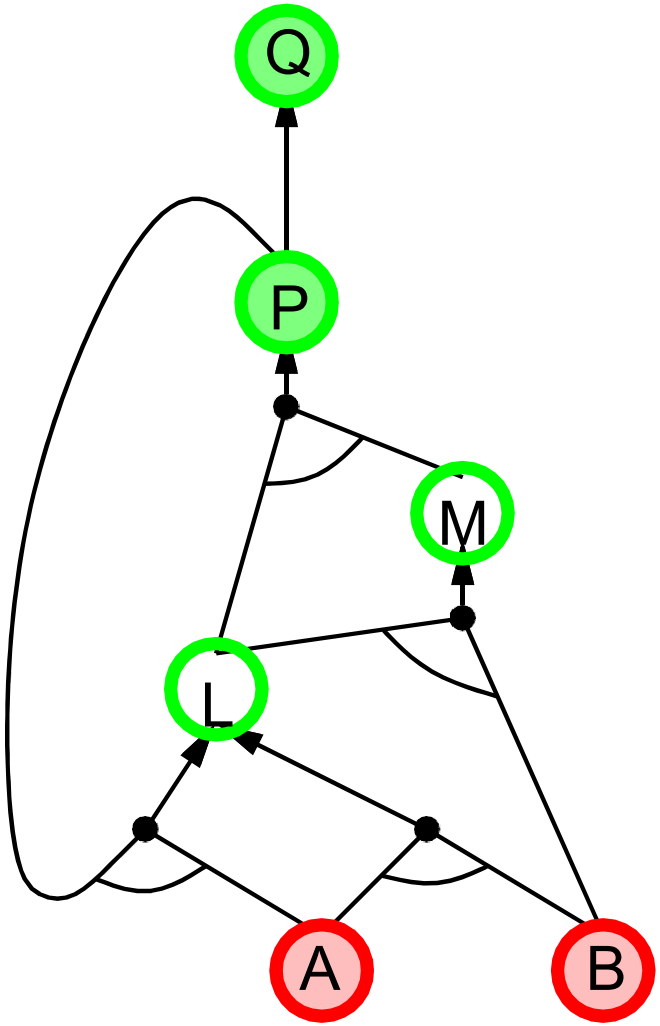

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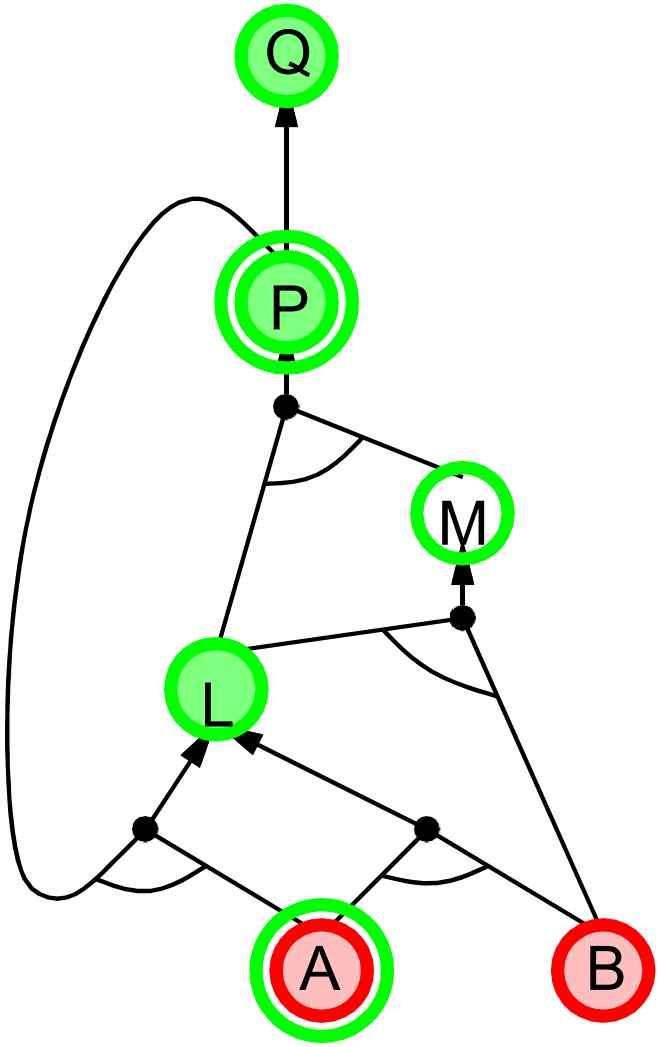

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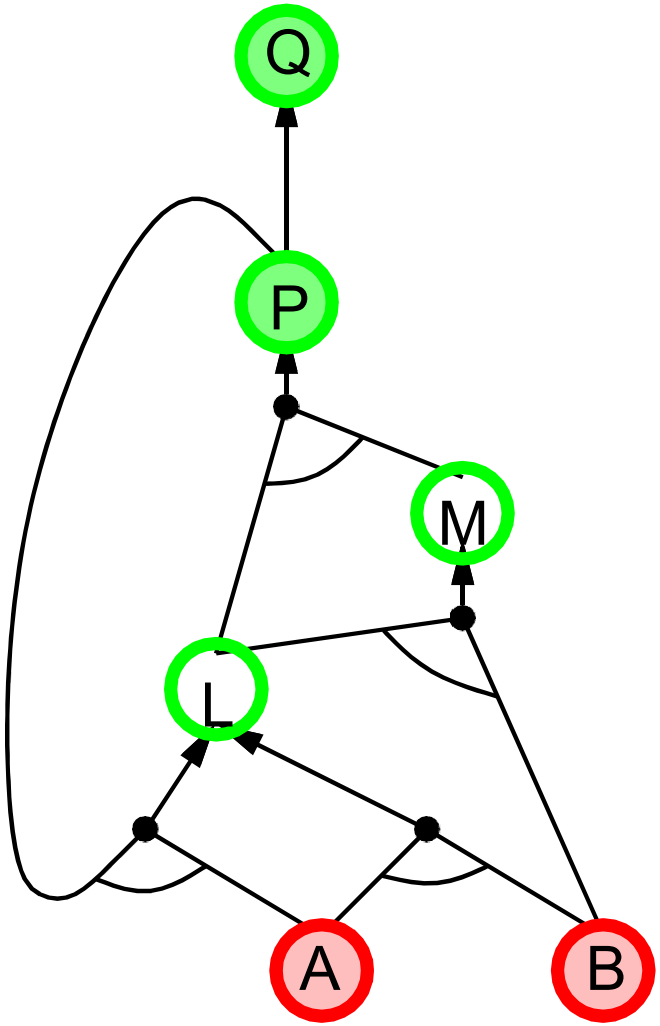

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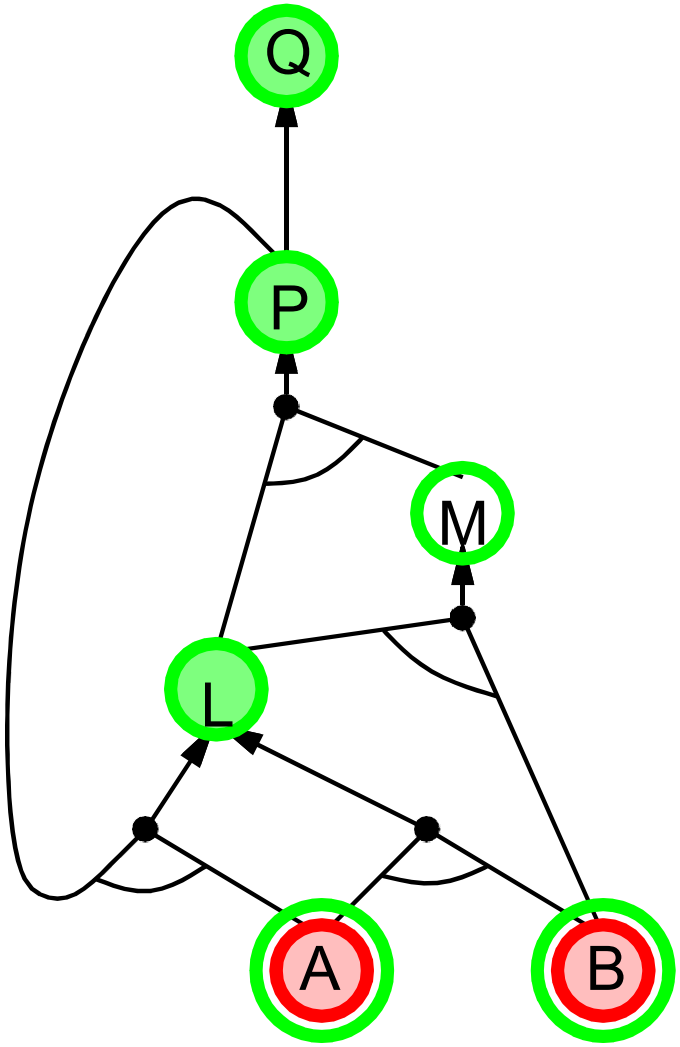

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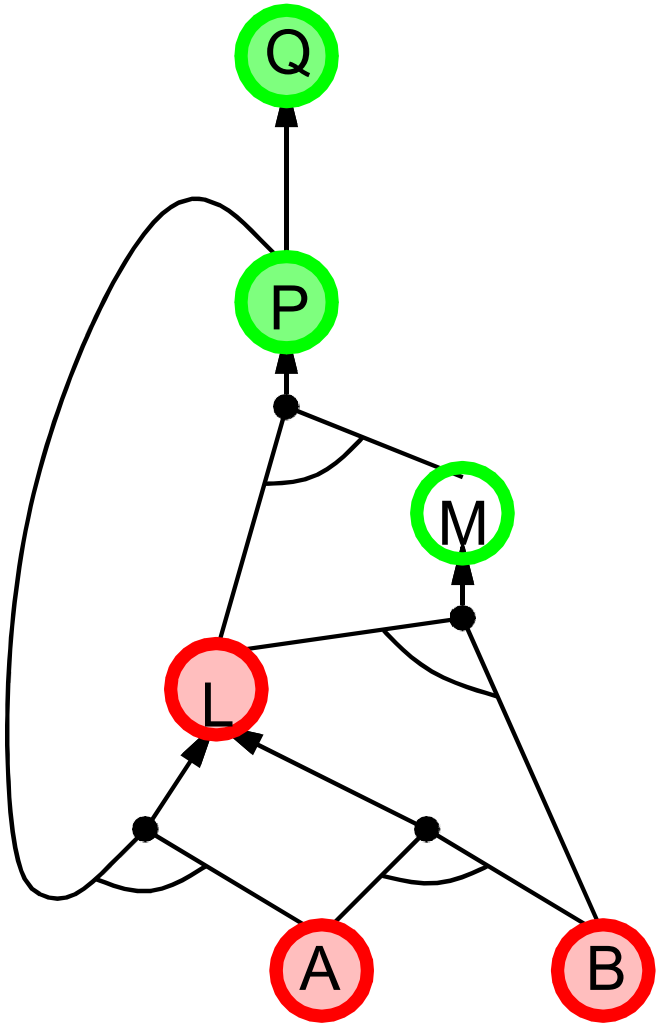

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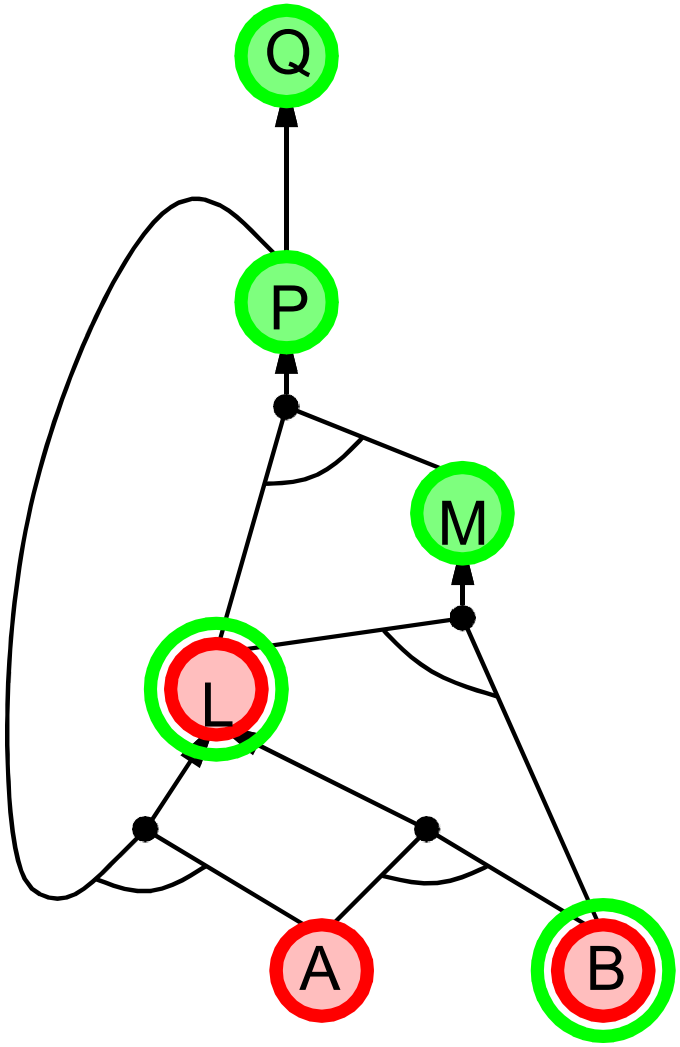

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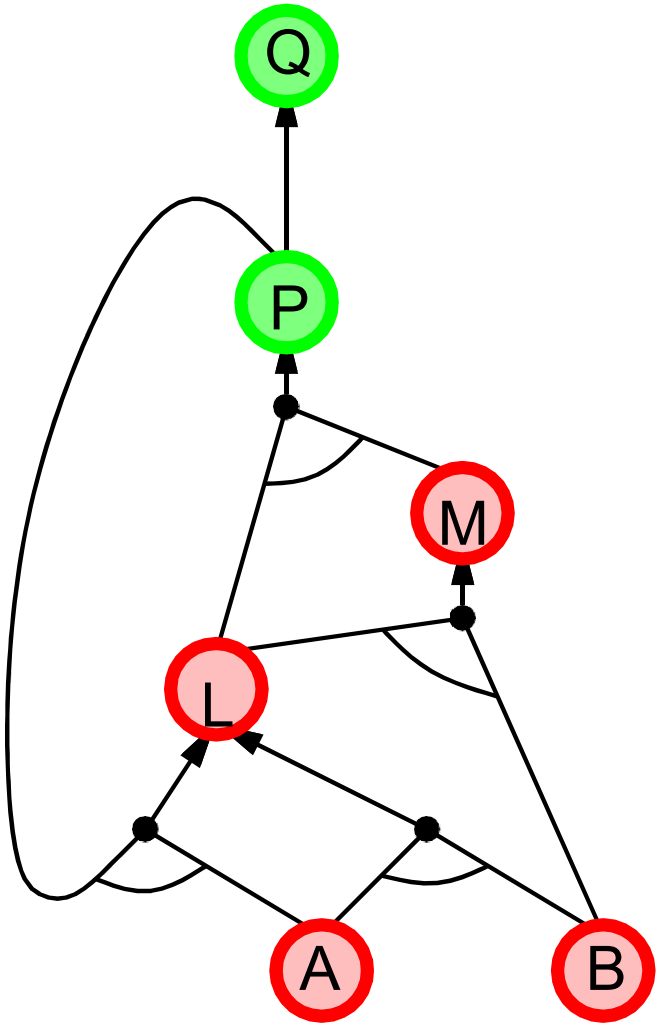

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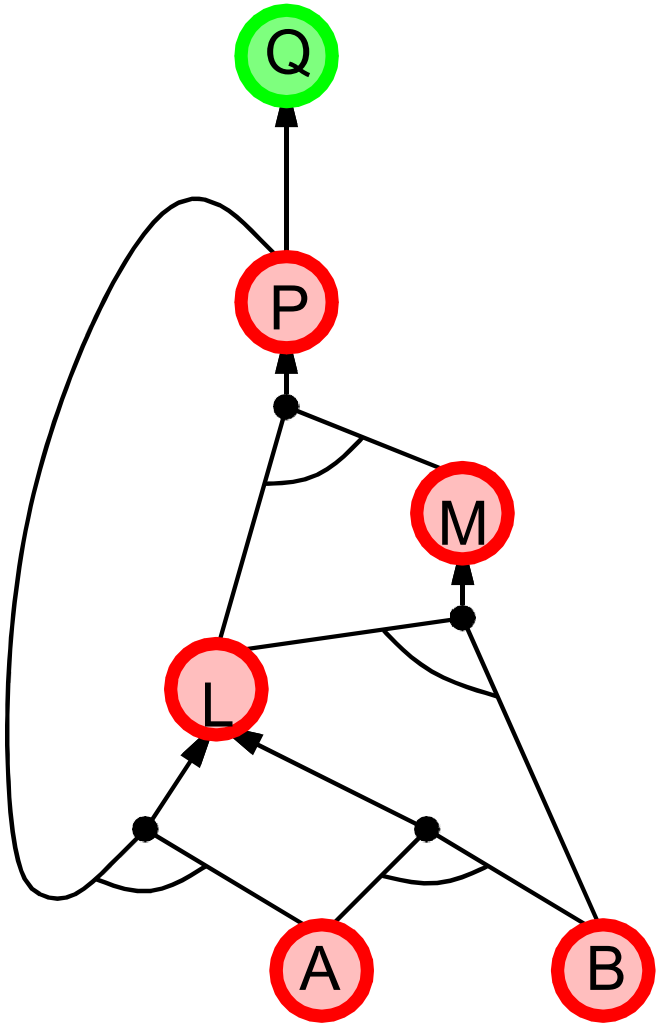

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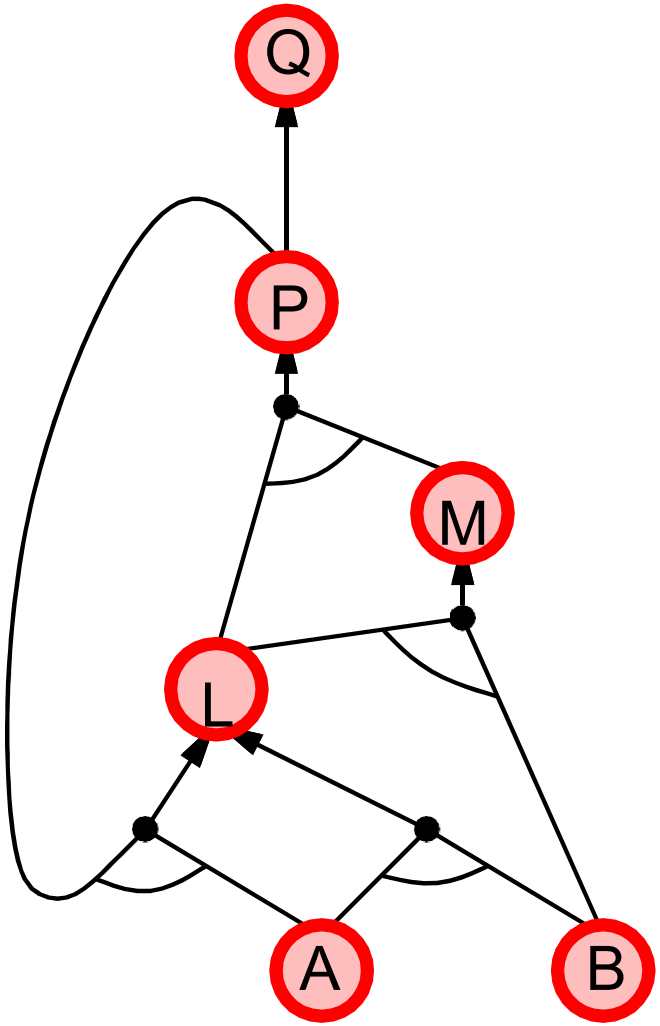

Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be much less than linear in size of KB

Effective Propositional Model Checking

Davis–Putnam algorithm with three improvements over TT-ENTAILS

- Early termination: detect T/F
- Pure symbol heuristic: same sign in all clauses
- Unit Clause heuristic: clause with one literal

Local search algorithms such as hill-climbing & simulated annealing.

In recent years, there has been a great deal of experimentation to find a good balance between greediness and randomness.

WALKSAT: On every iteration, the algorithm picks an unsatisfied clause and picks a symbol in the clause to flip.

Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:
- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses
Resolution is complete for propositional logic. Propositional logic lacks expressive power
Local search methods (WALKSAT) find solutions (sound but not complete).