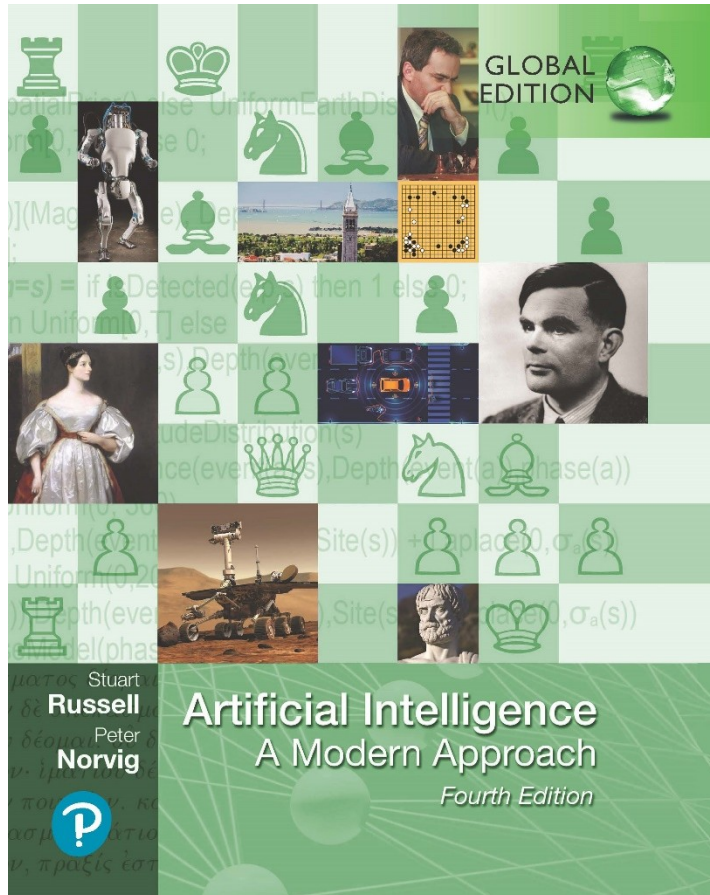


Artificial Intelligence: A Modern Approach

Fourth Edition, Global Edition



Chapter 3

Solving Problems By Searching

Lecture Presentations: Artificial Intelligence

Adapted from:

"Artificial Intelligence: A Modern Approach, Global Edition",
4th Edition by Stuart Russell and Peter Norvig © 2021
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Outline

- ◆ Problem-solving agents
- ◆ Example Problems
- ◆ Problem formulation
- ◆ Search Algorithms
- ◆ Uninformed Search Strategies
- ◆ Informed (Heuristic) Search Strategies
- ◆ Heuristic Functions

Problem-solving agents

Restricted form of general agent:

```
function Simple-Problem-Solving-Agent(percept) returns an action
  static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

  state ← Update-State(state, percept)
  if seq is empty then
    goal ← Formulate-Goal(state)
    problem ← Formulate-Problem(state, goal)
    seq ← Search(problem)
  action ← Recommendation(seq, state)
  seq ← Remainder(seq, state)
  return action
```

Note: this is **offline** problem solving; solution executed “eyes closed.”

Online problem solving involves acting without complete knowledge.

Example: Romania

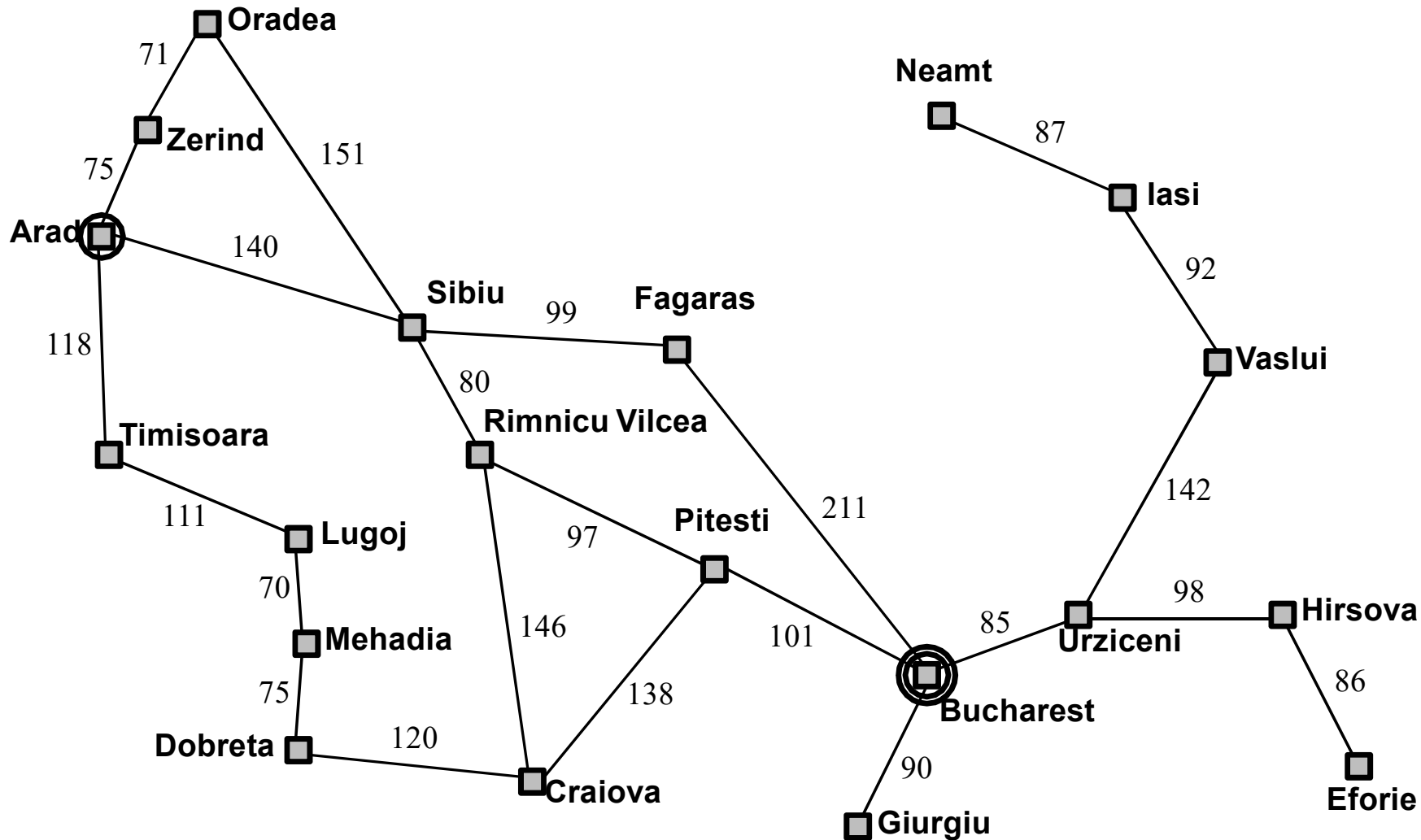
On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:
be in Bucharest

Formulate problem:
states: various cities
actions: drive between cities

Find solution:
sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



Problem types

Deterministic, fully observable \Rightarrow single-state problem

Agent knows exactly which state it will be in; solution is a sequence

Non-observable \Rightarrow conformant problem

Agent may have no idea where it is; solution (if any) is a sequence

Nondeterministic and/or partially observable \Rightarrow contingency problem

percepts provide new information about current state

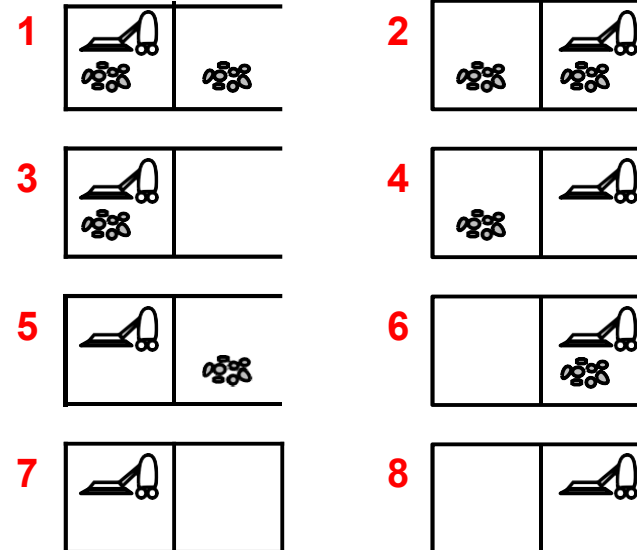
solution is a contingent plan or a policy

often interleave search, execution

Unknown state space \Rightarrow exploration problem ("online")

Example: vacuum world

Single-state, start in #5. Solution??



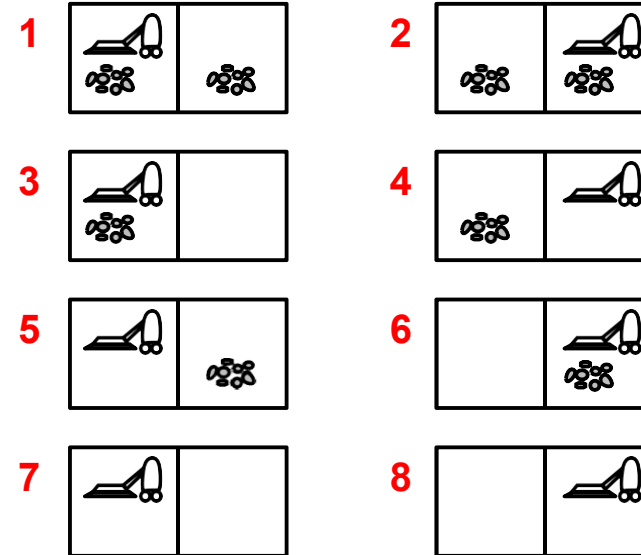
Example: vacuum world

Single-state, start in #5. Solution??

[*Right, Suck*]

Conformant, start in {1, 2, 3, 4, 5, 6, 7, 8}

e.g., *Right* goes to {2, 4, 6, 8}. Solution??



Example: vacuum world

Single-state, start in #5. Solution??

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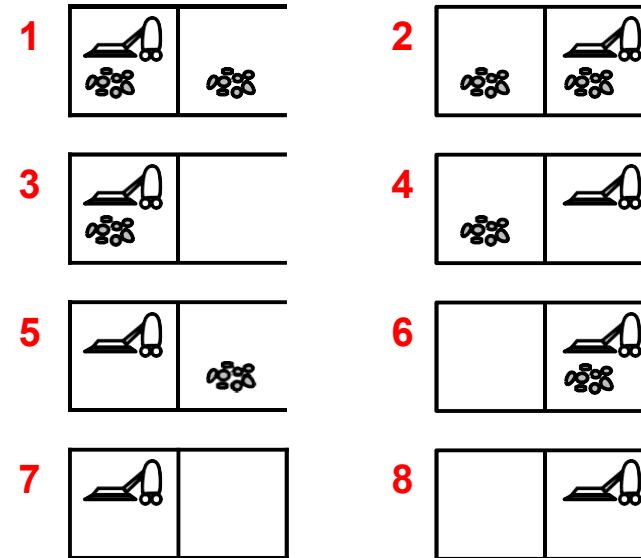
[*Right, Suck, Left, Suck*]

Contingency, start in #5

Murphy's Law: *Suck* can dirty a clean carpet

Local sensing: dirt, location only.

Solution??



Example: vacuum world

Single-state, start in #5. Solution??

[*Right, Suck*]

Conformant, start in {1, 2, 3, 4, 5, 6, 7, 8}

e.g., *Right* goes to {2, 4, 6, 8}. Solution??

[*Right, Suck, Left, Suck*]

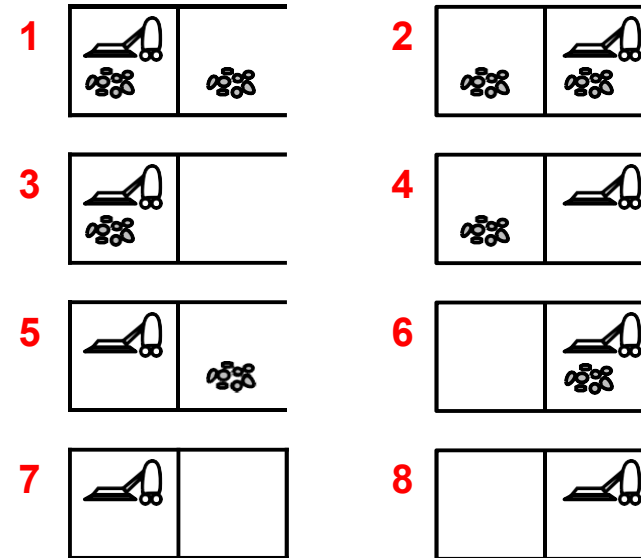
Contingency, start in #5

Murphy's Law: *Suck* can dirty a clean carpet

Local sensing: dirt, location only.

Solution??

[*Right, if dirt then Suck*]



Single-state problem formulation

A **problem** is defined by four items:

initial state e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs
e.g., $S(Arad) = \{(Arad \rightarrow Zerind, Zerind), \dots\}$

goal test, can be

explicit, e.g., $x = \text{"at Bucharest"}$

implicit, e.g., $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of actions executed, etc.

$c(x, a, y)$ is the **step cost**, assumed to be ≥ 0

A **solution** is a sequence of actions

leading from the initial state to a goal state

Selecting a state space

Real world is absurdly complex

⇒ state space must be **abstracted** for problem solving

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions

e.g., "Arad → Zerind" represents a complex set
of possible routes, detours, rest stops, etc.

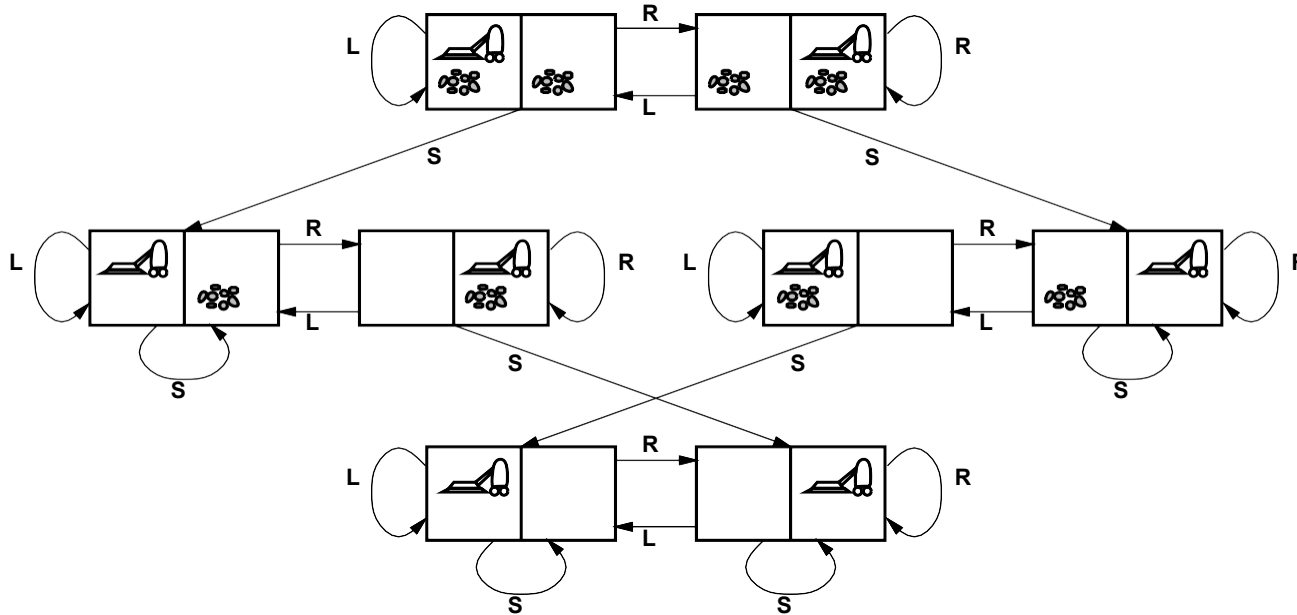
For guaranteed realizability, **any** real state "in Arad"
must get to **some** real state "in Zerind"

(Abstract) solution =

set of real paths that are solutions in the real world

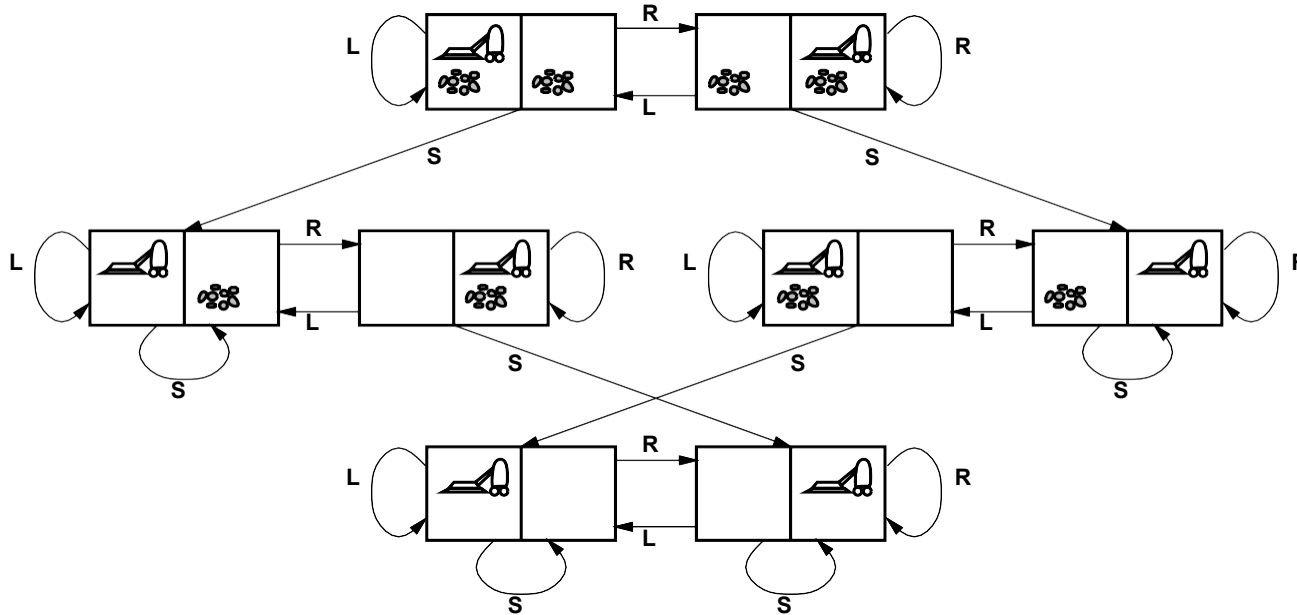
Each abstract action should be "easier" than the original problem!

Example: vacuum world state space graph



states??
actions??
goal test??
path cost??

Example: vacuum world state space graph



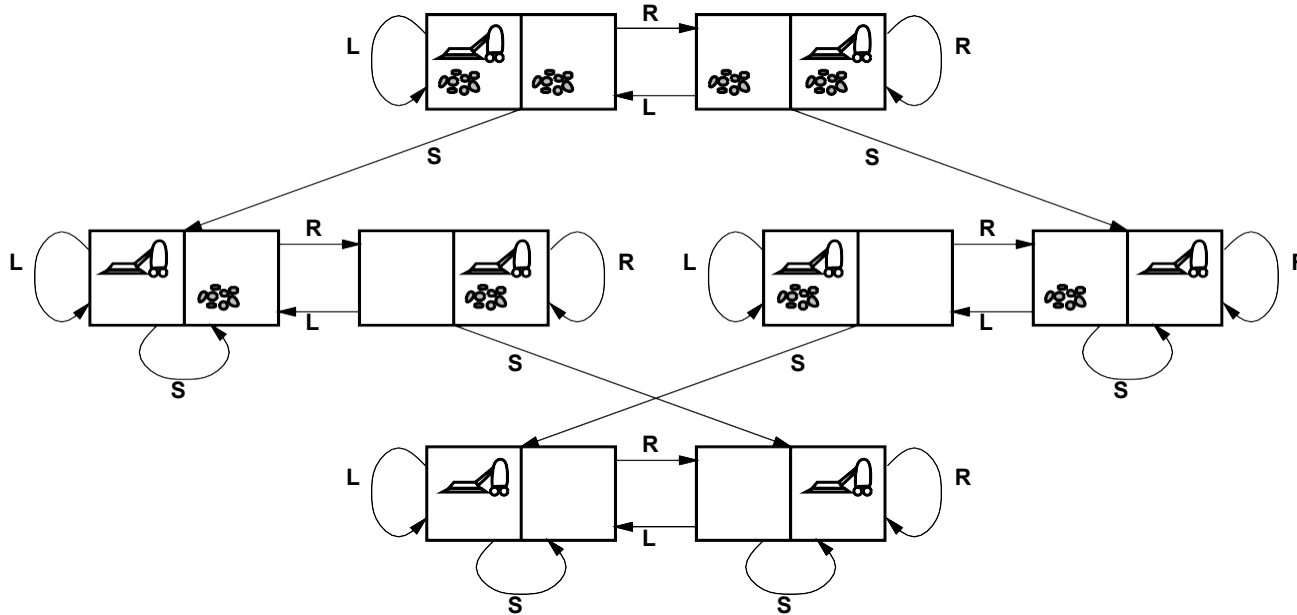
states??: integer dirt and robot locations (ignore dirt **amounts** etc.)

actions??

goal test??

path cost??

Example: vacuum world state space graph



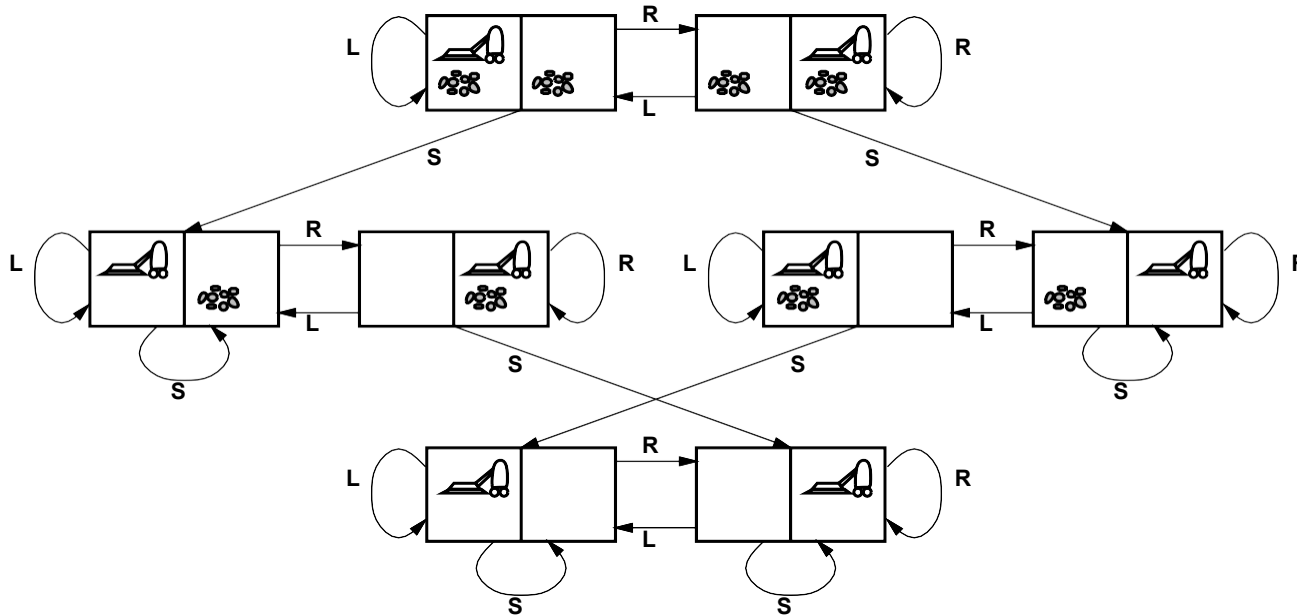
states??: integer dirt and robot locations (ignore dirt **amounts** etc.)

actions??: *Left, Right, Suck, NoOp*

goal test??

path cost??

Example: vacuum world state space graph



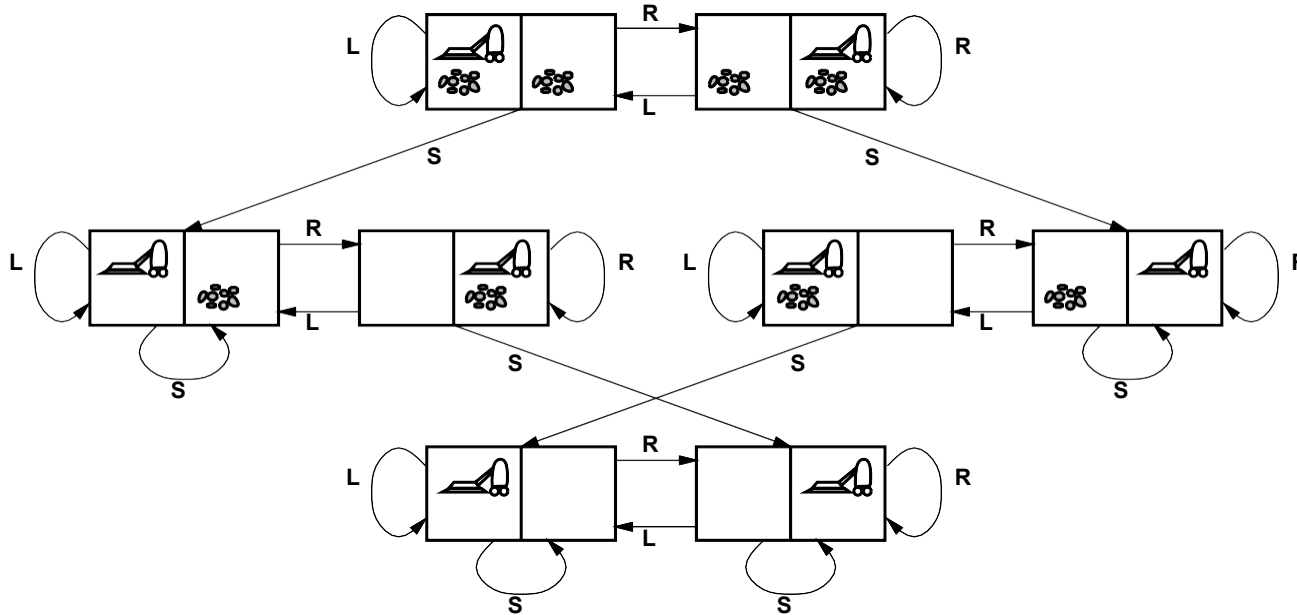
states??: integer dirt and robot locations (ignore dirt **amounts** etc.)

actions??: *Left, Right, Suck, NoOp*

goal test??: no dirt

path cost??

Example: vacuum world state space graph



states??: integer dirt and robot locations (ignore dirt **amounts** etc.)

actions??: *Left, Right, Suck, NoOp*

goal test??: no dirt

path cost??: 1 per action (0 for *NoOp*)

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states??
actions??
goal test??
path cost??

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
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Goal State

states??: integer locations of tiles (ignore intermediate positions)

actions??

goal test??

path cost??

Example: The 8-puzzle

7	2	4
5		6
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Start State

1	2	3
4	5	6
7	8	

Goal State

states??: integer locations of tiles (ignore intermediate positions)

actions??: move blank left, right, up, down (ignore unjamming etc.)

goal test??

path cost??

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states??: integer locations of tiles (ignore intermediate positions)

actions??: move blank left, right, up, down (ignore unjamming etc.)

goal test??: = goal state (given)

path cost??

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states??: integer locations of tiles (ignore intermediate positions)

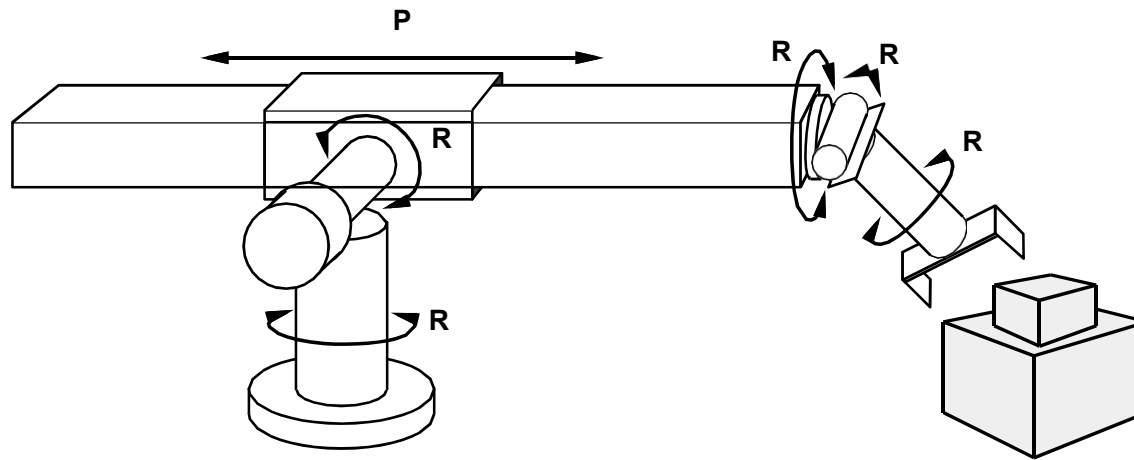
actions??: move blank left, right, up, down (ignore unjamming etc.)

goal test??: = goal state (given)

path cost??: 1 per move

[Note: optimal solution of n -Puzzle family is NP-hard]

Example: robotic assembly



states??: real-valued coordinates of robot joint angles
parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly **with no robot included!**

path cost??: time to execute

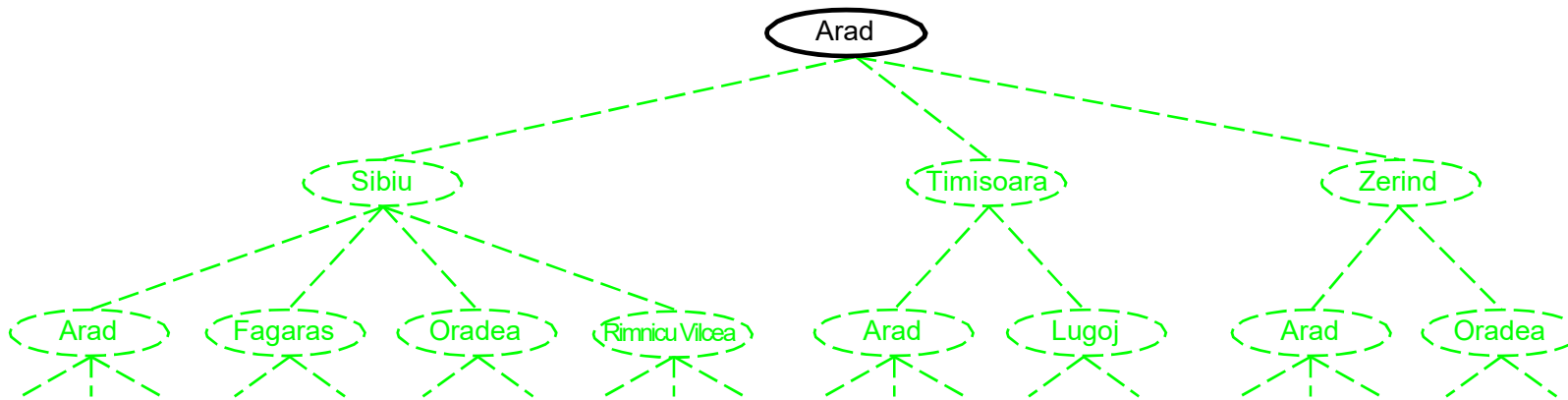
Tree search algorithms

Basic idea:

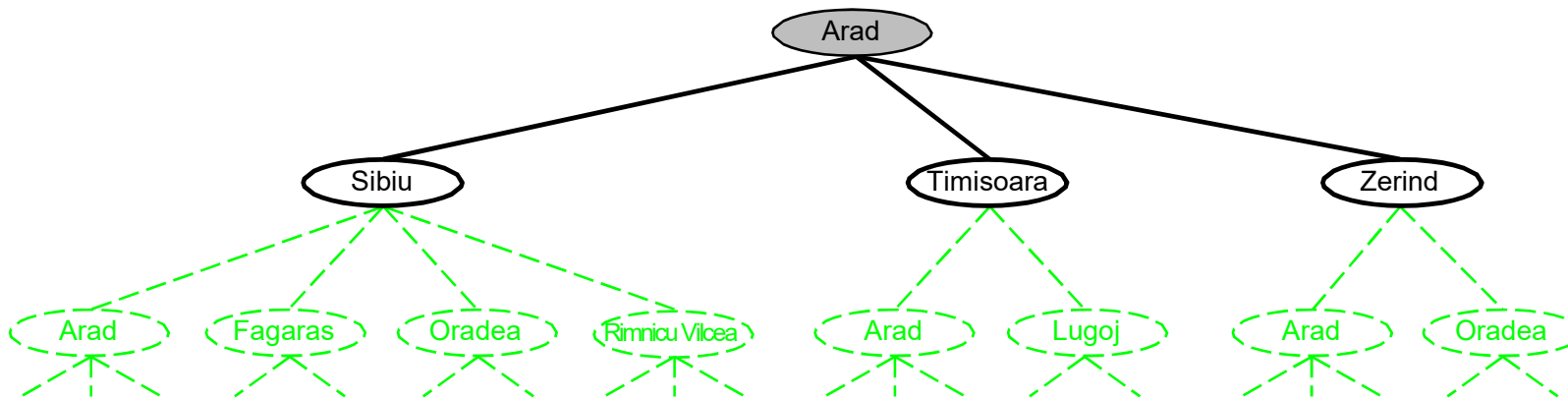
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. **expanding** states)

```
function Tree-Search( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

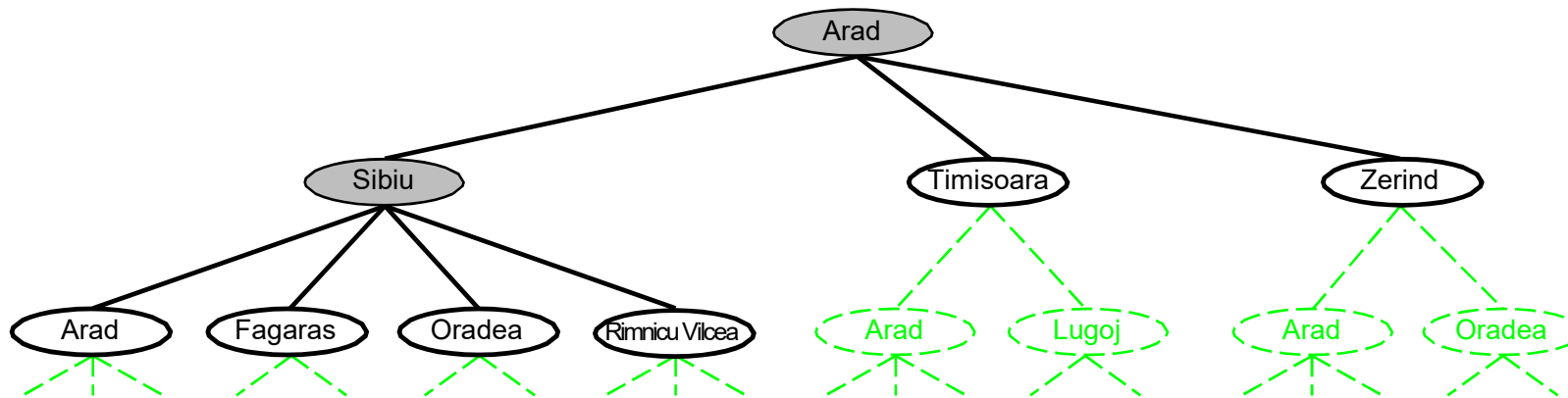
Tree search example



Tree search example



Tree search example



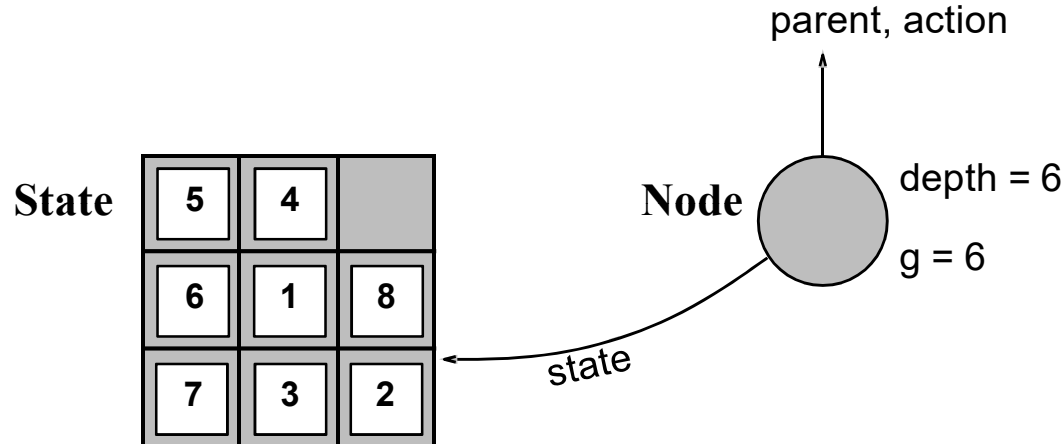
Implementation: states vs. nodes

A **state** is a (representation of) a physical configuration

A **node** is a data structure constituting part of a search tree

includes **parent**, **children**, **depth**, **path cost** $g(x)$

States do not have parents, children, depth, or path cost!



The Expand function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.

Implementation: general tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
  fringe ← Insert (Make-Node(Initial-State [problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← Remove-Front(fringe)
    if Goal-Test(problem, State(node)) then return node
    fringe ← Insert All(Expand(node, problem), fringe)
```

```
function Expand(node, problem) returns a set of nodes
  successors ← the empty set
  for each action, result in Successor-Fn(problem, State [node]) do
    s ← a new Node
    Parent-Node [s] ← node; Action [s] ← action; State [s] ← result
    Path-Cost [s] ← Path-Cost [node] + Step-Cost(node, action, s)
    Depth [s] ← Depth [node] + 1
    add s to successors
  return successors
```

Search strategies

A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:

completeness—does it always find a solution if one exists?

time complexity—number of nodes generated/expanded

space complexity—maximum number of nodes in memory

optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

b —maximum branching factor of the search tree

d —depth of the least-cost solution

m —maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

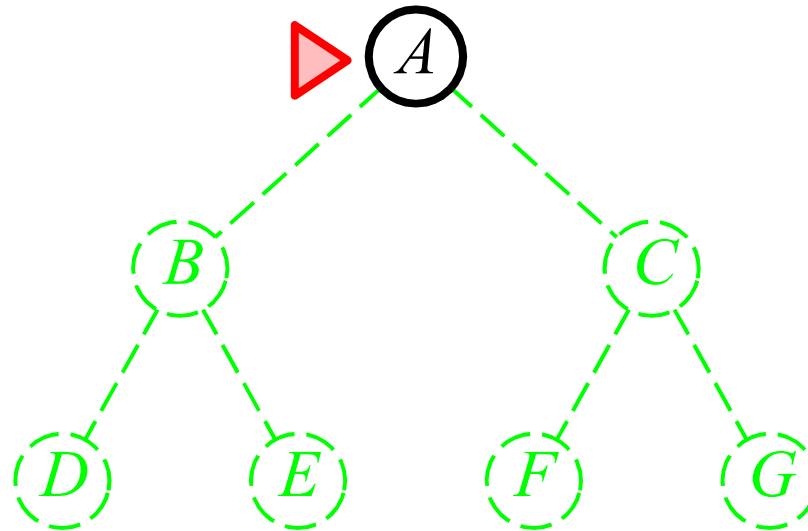
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end

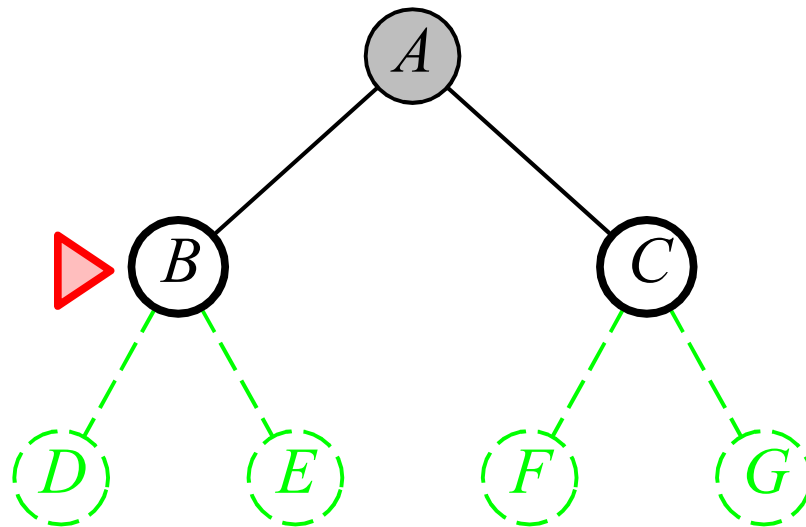


Breadth-first search

Expand shallowest unexpanded node

Implementation:

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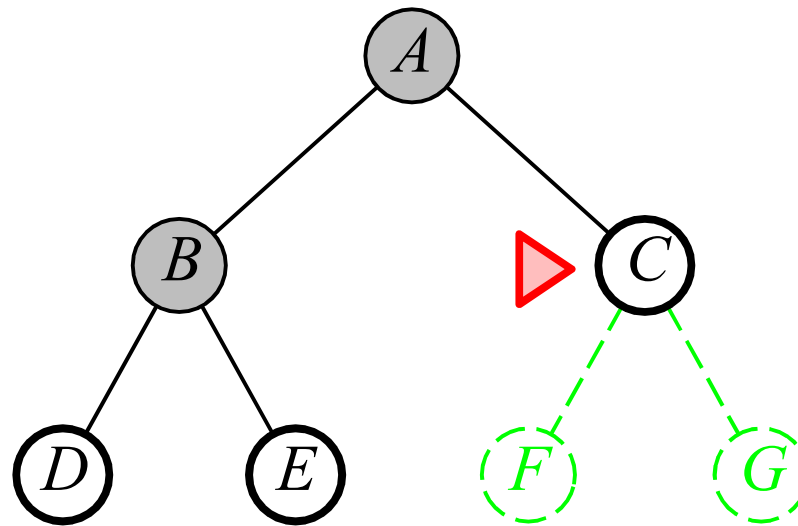


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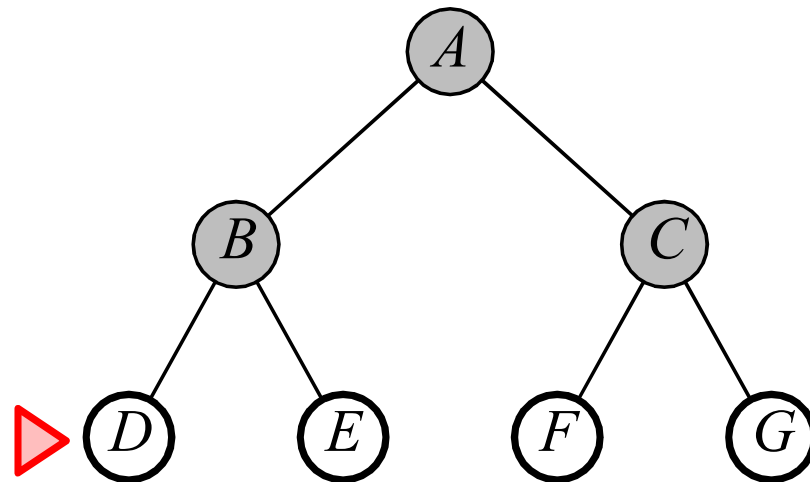


Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end



Properties of breadth-first search

Complete??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \dots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \dots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \dots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.

Uniform-cost search

Expand least-cost unexpanded node

Implementation:

fringe = queue ordered by path cost, lowest first

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq c$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{fC^*/d})$
where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{fC^*/d})$

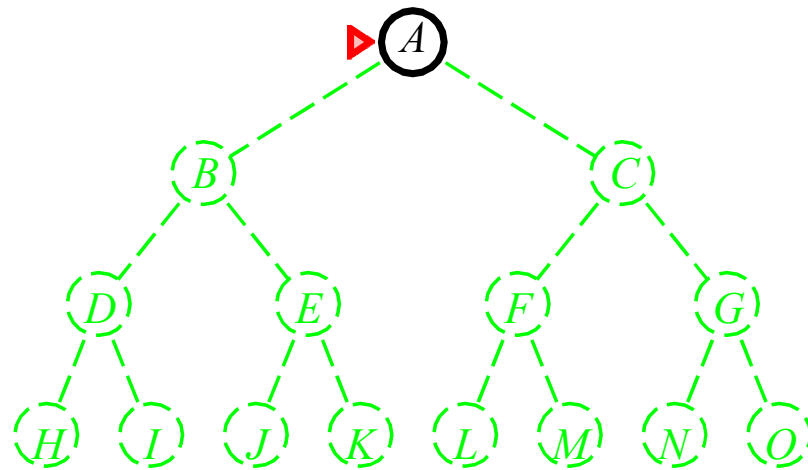
Optimal?? Yes—nodes expanded in increasing order of $g(n)$

Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

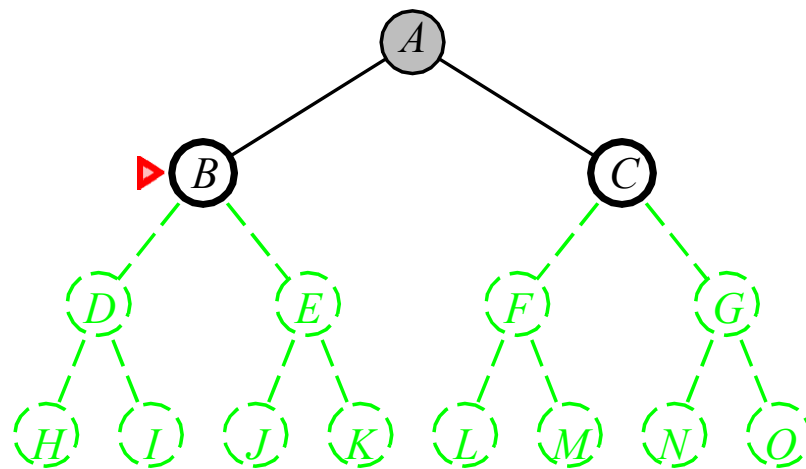


Depth-first search

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Implementation:

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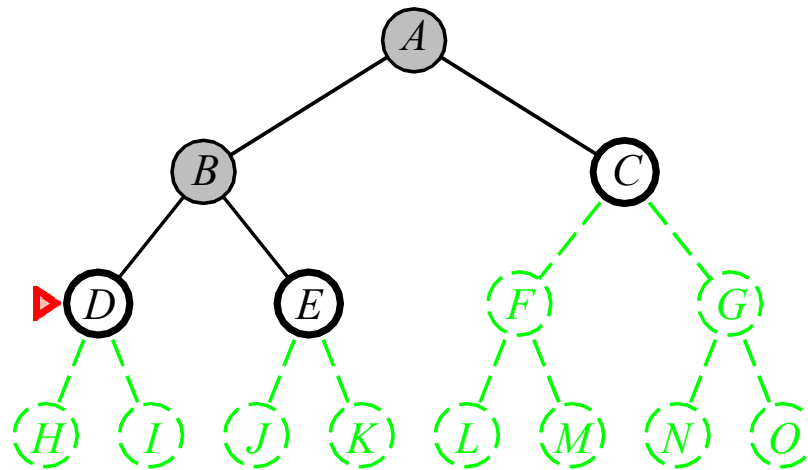


Depth-first search

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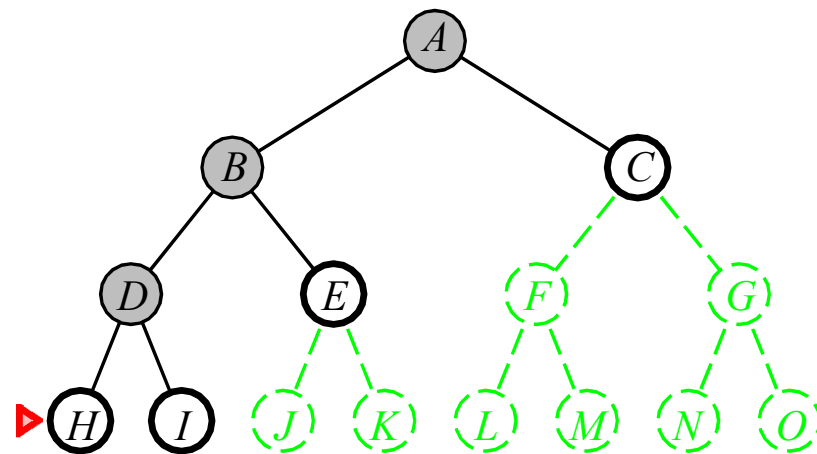


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Expand deepest unexpanded node

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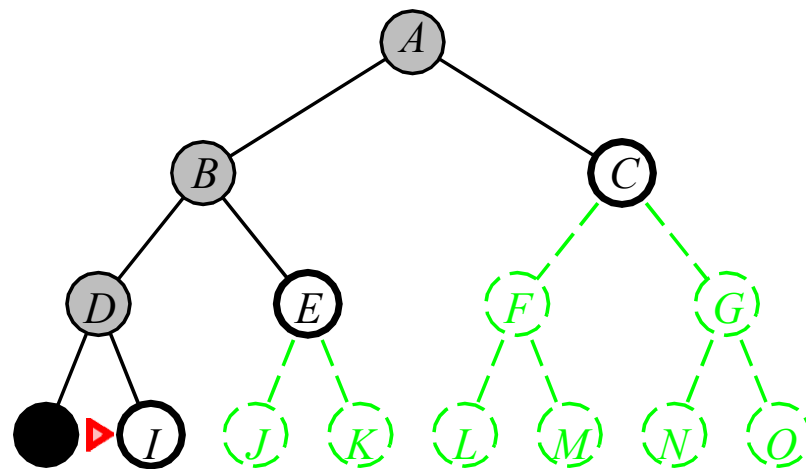


Depth-first search

Expand deepest unexpanded node

Implementation:

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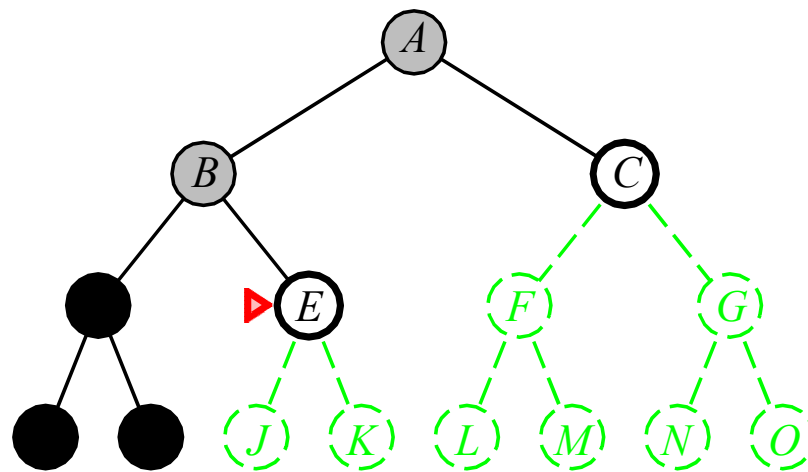


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

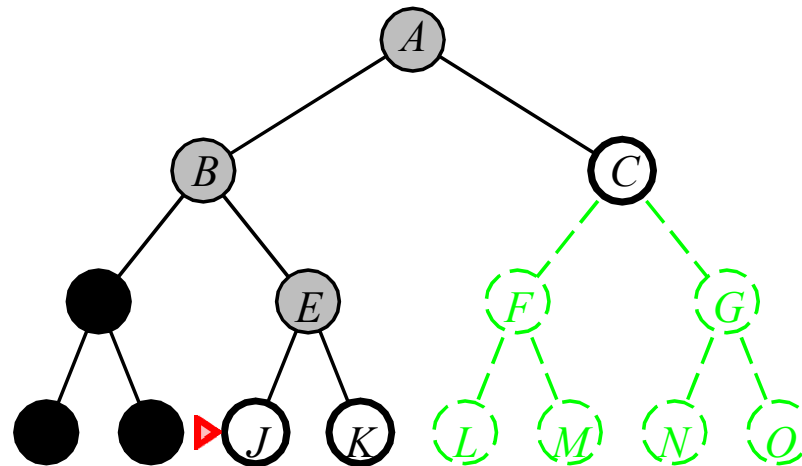


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

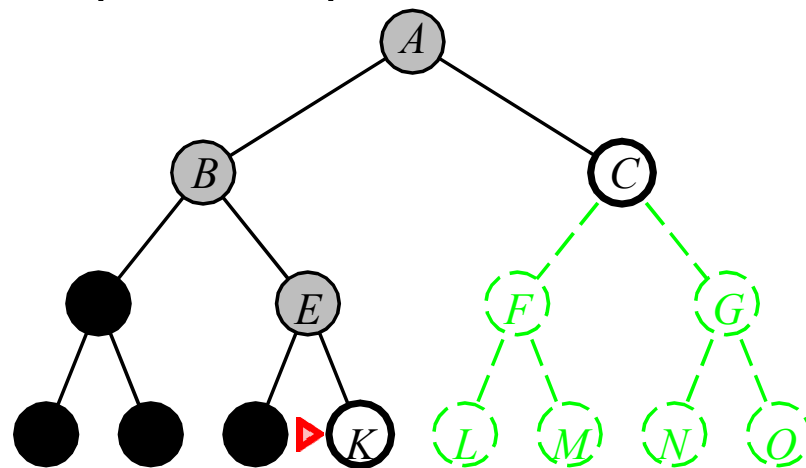


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

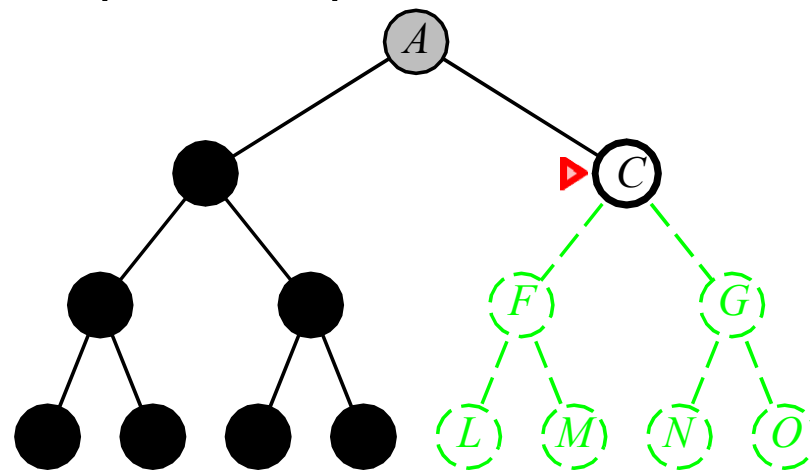


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

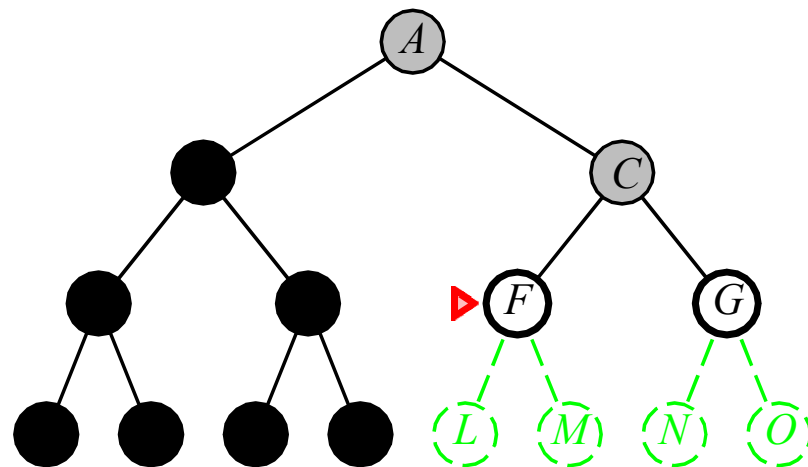


Depth-first search

Expand deepest unexpanded node

Implementation:

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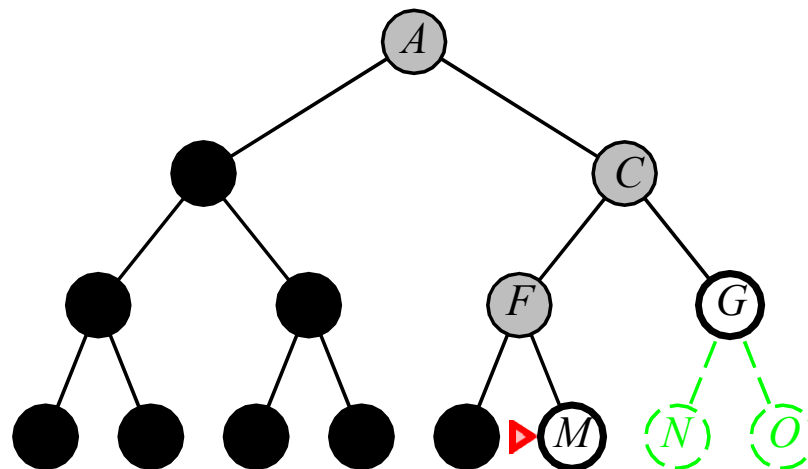
Implementation:

Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front



Properties of depth-first search

Complete??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

⇒ complete in finite spaces

Time??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

⇒ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

⇒ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

⇒ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No

Depth-limited search

= depth-first search with depth limit l ,
i.e., nodes at depth l have no successors

Recursive implementation:

```

function Depth-Limited-Search(problem,limit) returns soln/fail/cutoff
    Recursive-DLS(Make-Node(Initial-State [problem]),problem,limit)

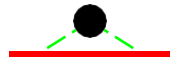
function Recursive-DLS(node,problem,limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if Goal-Test(problem,State[node]) then return node
    else if Depth[node] = limit then return cutoff
    else for each successor in Expand(node,problem) do
        result ← Recursive-DLS(successor,problem,limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
    
```

Iterative deepening search

```
function Iterative-Deepening-Search(problem) returns a solution
  inputs: problem, a problem
  for depth  $\leftarrow$  0 to  $\infty$  do
    result  $\leftarrow$  Depth-Limited-Search(problem, depth)
    if result  $\neq$  cutoff then return result
  end
```

Iterative deepening search $l = 0$

Limit = 0



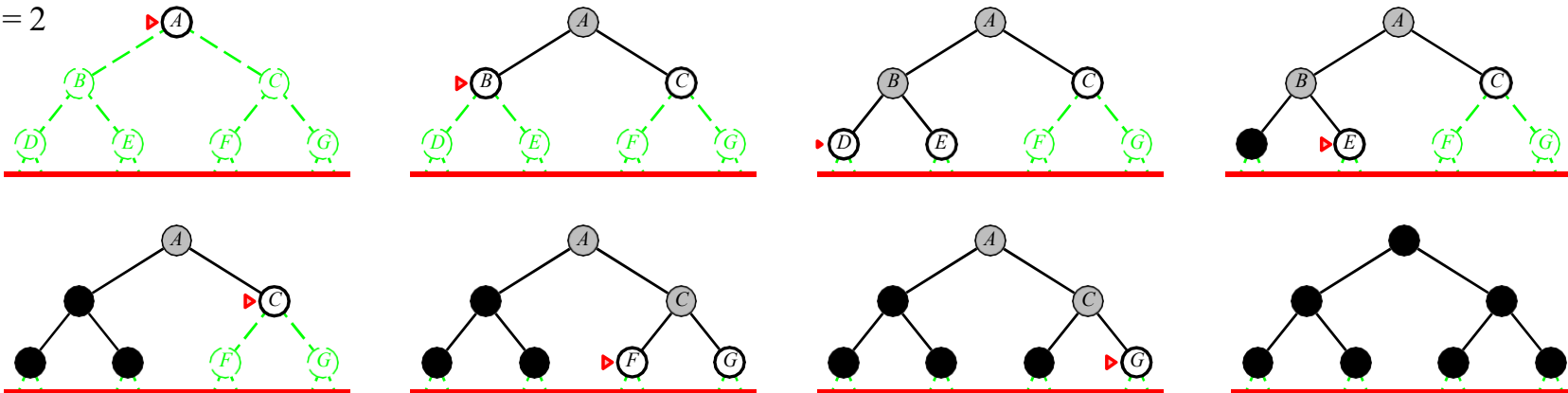
Iterative deepening search $l = 1$

Limit = 1



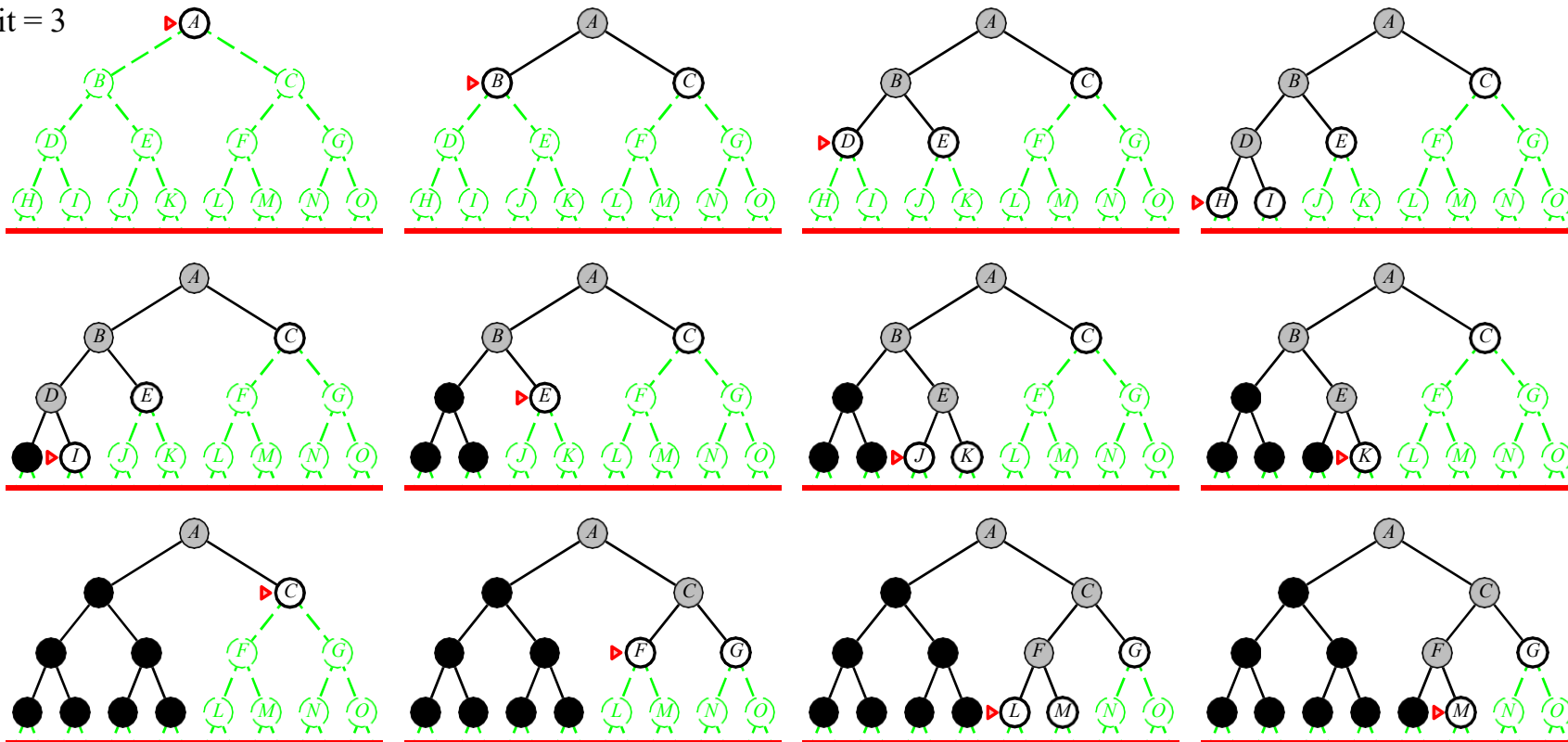
Iterative deepening search $l = 2$

Limit = 2



Iterative deepening search $l = 3$

Limit = 3



Properties of iterative deepening search

Complete??

Properties of iterative deepening search

Complete?? Yes

Time??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \dots + b^d = O(b^d)$

Space??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \dots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \dots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost = 1

Can be modified to explore uniform-cost tree

Numerical comparison for $b=10$ and $d=5$, solution at far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$

$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

IDS does better because other nodes at depth d are not expanded

BFS can be modified to apply goal test when a node is **generated**

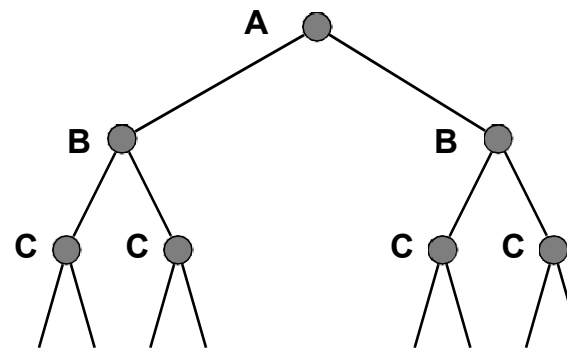
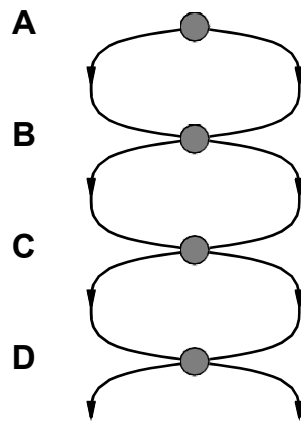
Summary of Uninformed Search algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes ¹	Yes ^{1,2}	No	No	Yes ¹	Yes ^{1,4}
Optimal cost?	Yes ³	Yes	No	No	Yes ³	Yes ^{3,4}
Time	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

Figure 3.15 Evaluation of search algorithms. b is the branching factor; m is the maximum depth of the search tree; d is the depth of the shallowest solution, or is m when there is no solution; ℓ is the depth limit. Superscript caveats are as follows: ¹ complete if b is finite, and the state space either has a solution or is finite. ² complete if all action costs are $\geq \epsilon > 0$; ³ cost-optimal if action costs are all identical; ⁴ if both directions are breadth-first or uniform-cost.

Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!



Graph search

```
function Graph-Search(problem,fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← Insert (Make-Node(Initial-State [problem]),fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem,State[node]) then return node
        if State[node] is not in closed then
            add State[node] to closed
            fringe ← Insert All (Expand(node,problem),fringe)
    end
```

Review: Tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
  fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← Remove-Front(fringe)
    if Goal-Test[problem] applied to State(node) succeeds return node
    fringe ← InsertAll(Expand(node, problem), fringe)
```

A strategy is defined by picking the **order of node expansion**

Best-first search

Idea: use an **evaluation function** for each node
– estimate of “desirability”

⇒ Expand most desirable unexpanded node

Implementation:

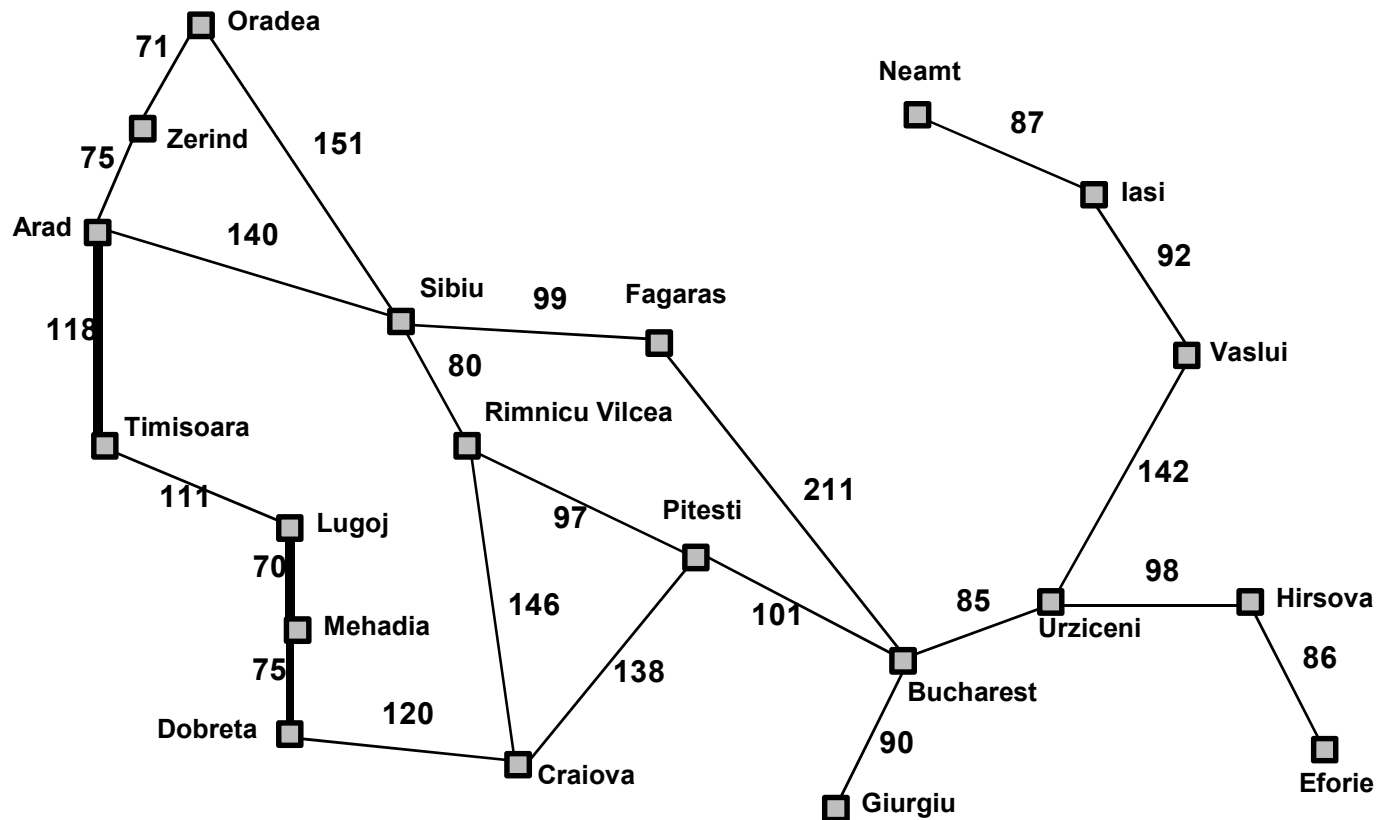
fringe is a queue sorted in decreasing order of desirability

Special cases:

greedy search

A* search

Romania with step costs in km



Straight-line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

Evaluation function $h(n)$ (heuristic)

= estimate of cost from n to the closest goal

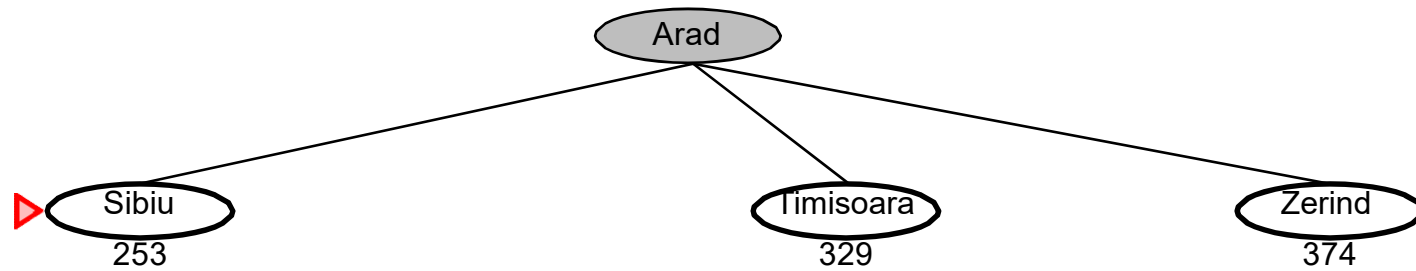
E.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that appears to be closest to goal

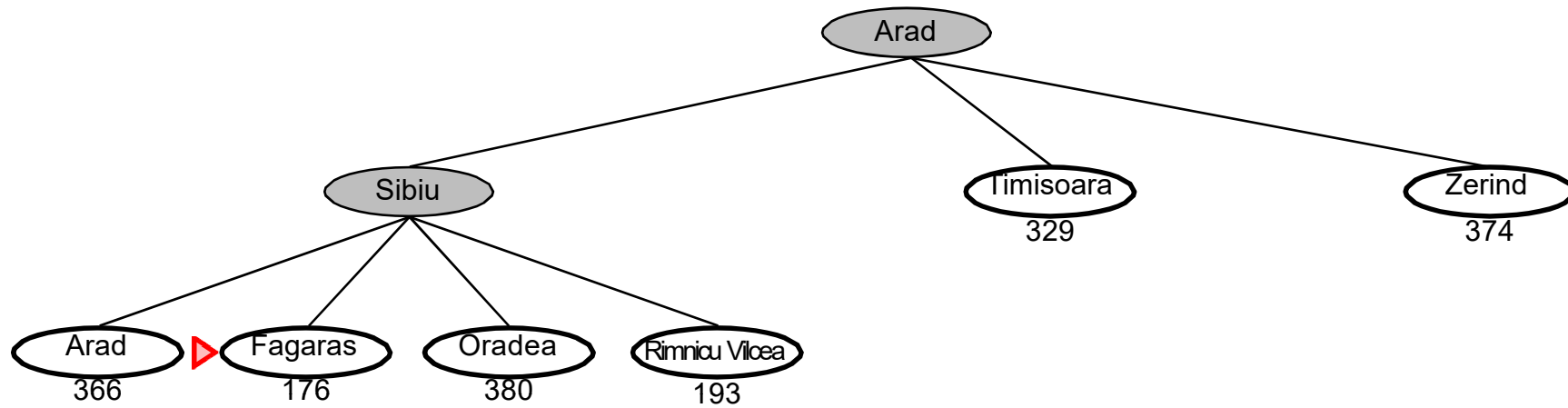
Greedy search example

▶ Arad
366

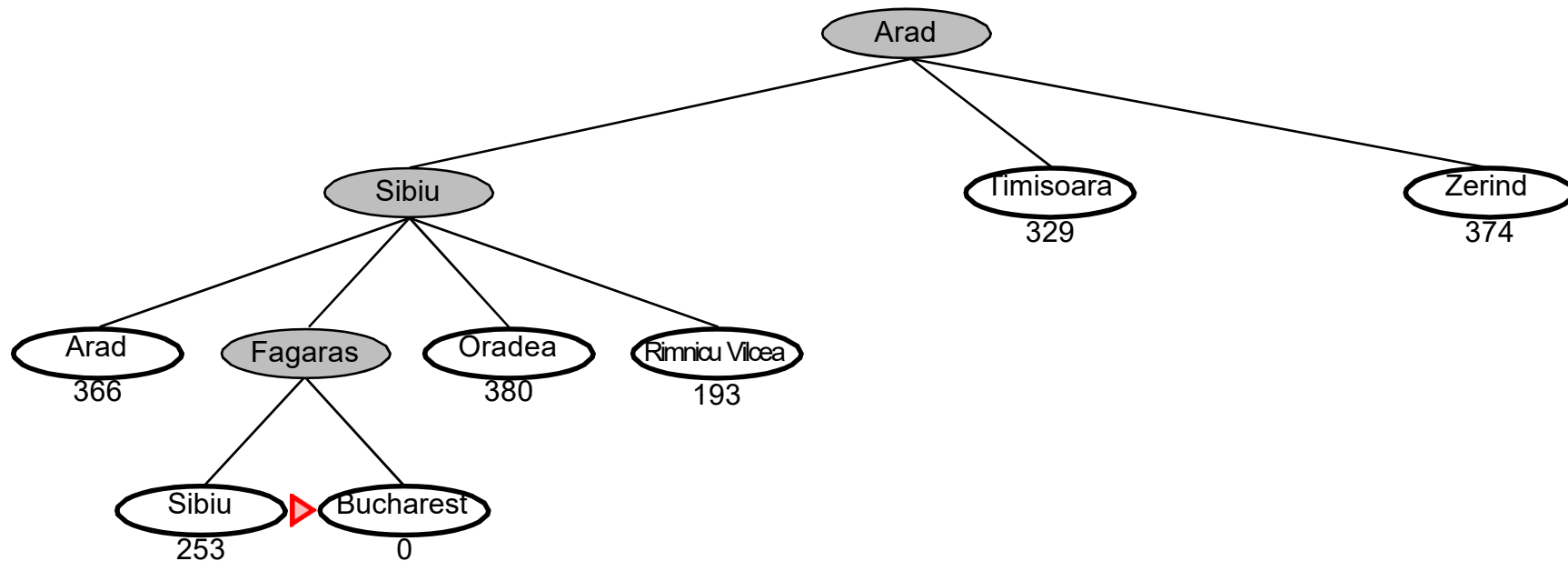
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Properties of greedy search

Complete?? No—can get stuck in loops, e.g., with Oradea as goal,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No—can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

Properties of greedy search

Complete?? No—can get stuck in loops,
e.g., Iasi → Neamt → Iasi →
Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic
improvement Space?? $O(b^m)$ —keeps all nodes in memory

Optimal??

Properties of greedy search

Complete?? No—can get stuck in loops,
e.g., Iasi → Neamt → Iasi →
Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic
improvement Space?? $O(b^m)$ —keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n

$h(n)$ = estimated cost to goal from n

$f(n)$ = estimated total cost of path through n to goal

A* search uses an **admissible** heuristic

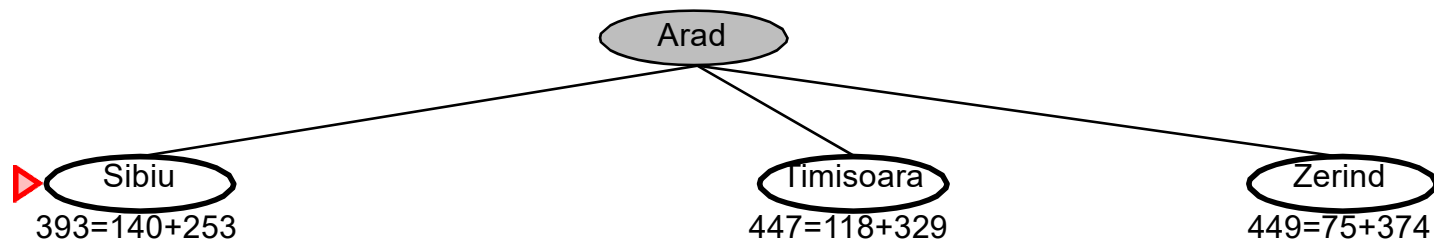
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n . (Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G .)

E.g., $h_{\text{SLD}}(n)$ never overestimates the actual road distance **Theorem:** A* search is optimal

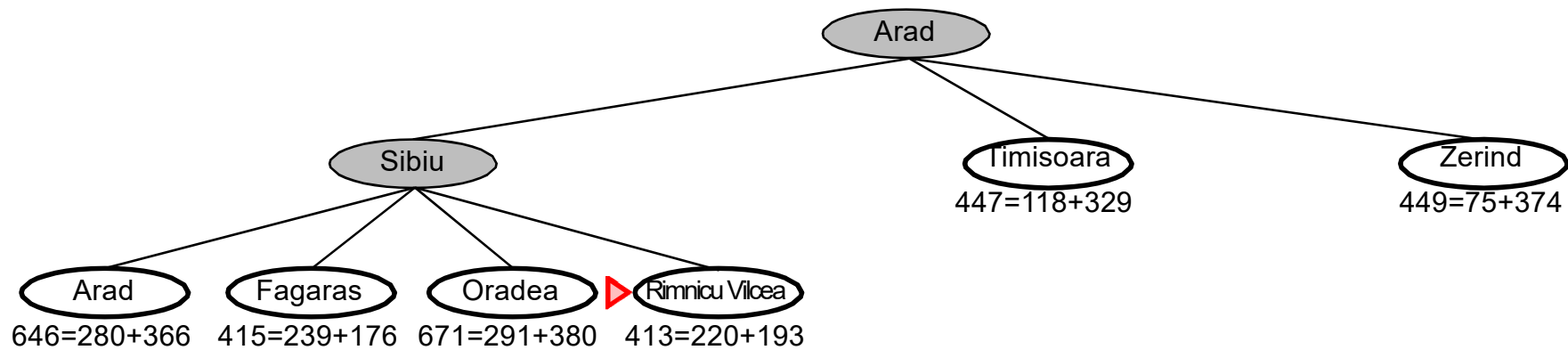
A* search example

▶ Arad
 $366 = 0 + 366$

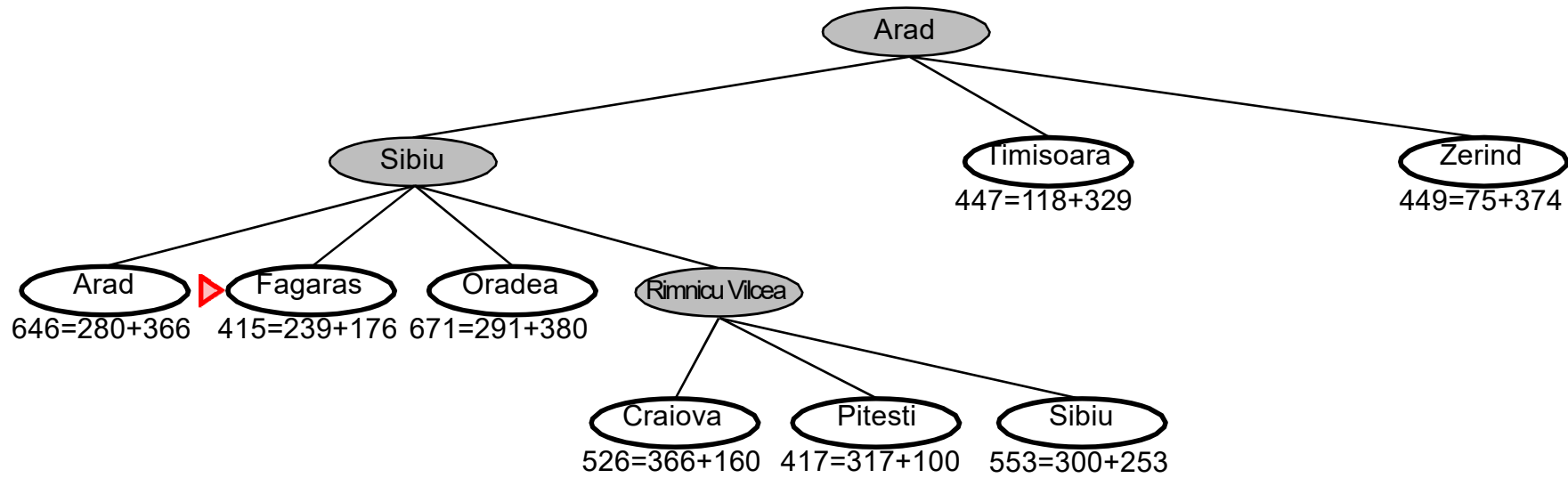
A* search example



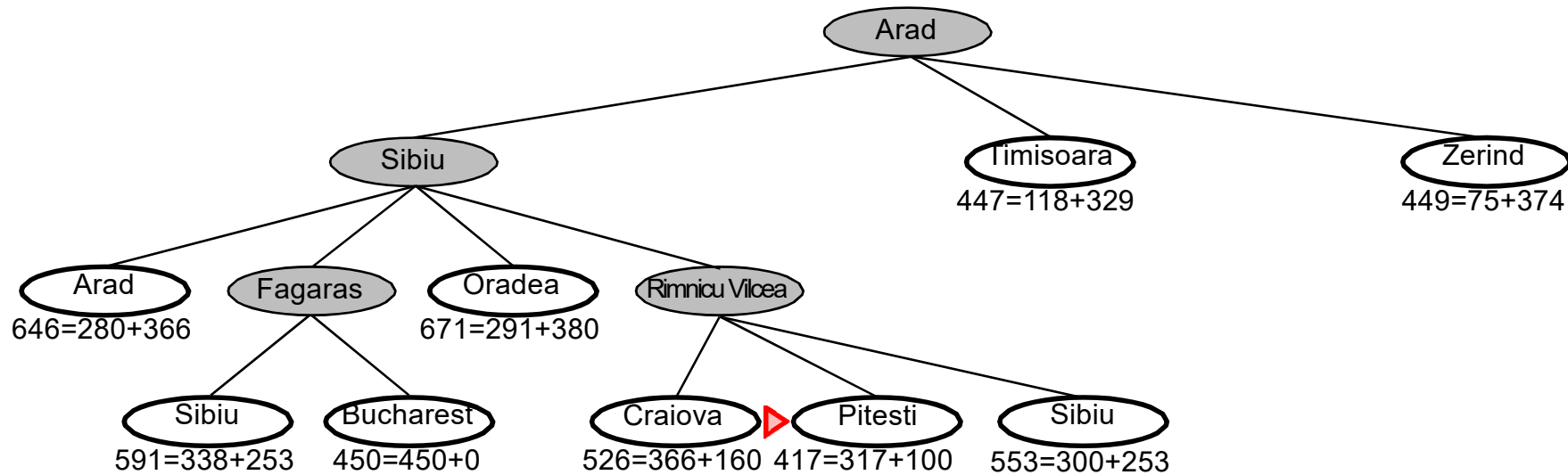
A* search example



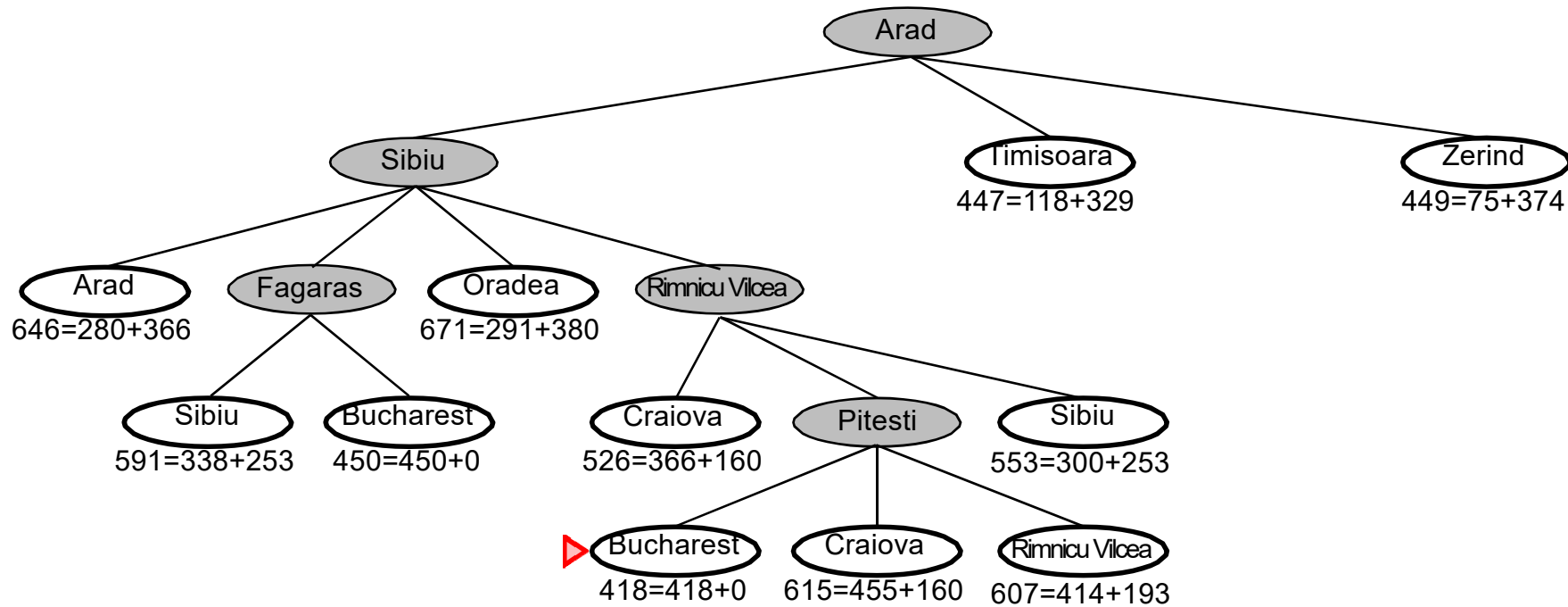
A* search example



A* search example

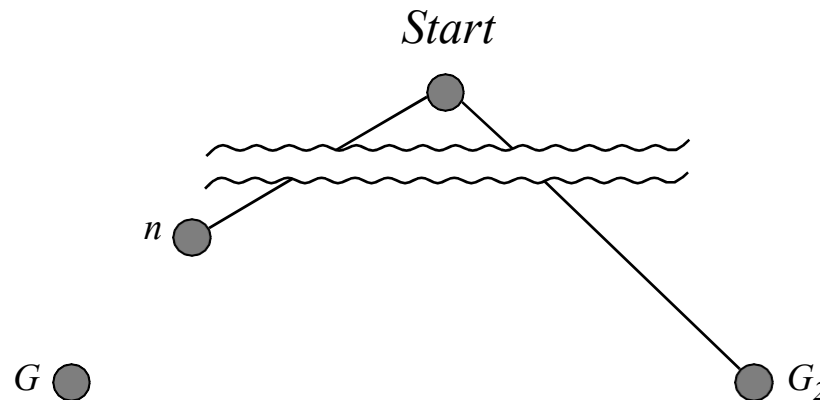


A* search example



Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1 .



$$\begin{aligned}
 f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
 &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
 &\geq f(n) && \text{since } h \text{ is admissible}
 \end{aligned}$$

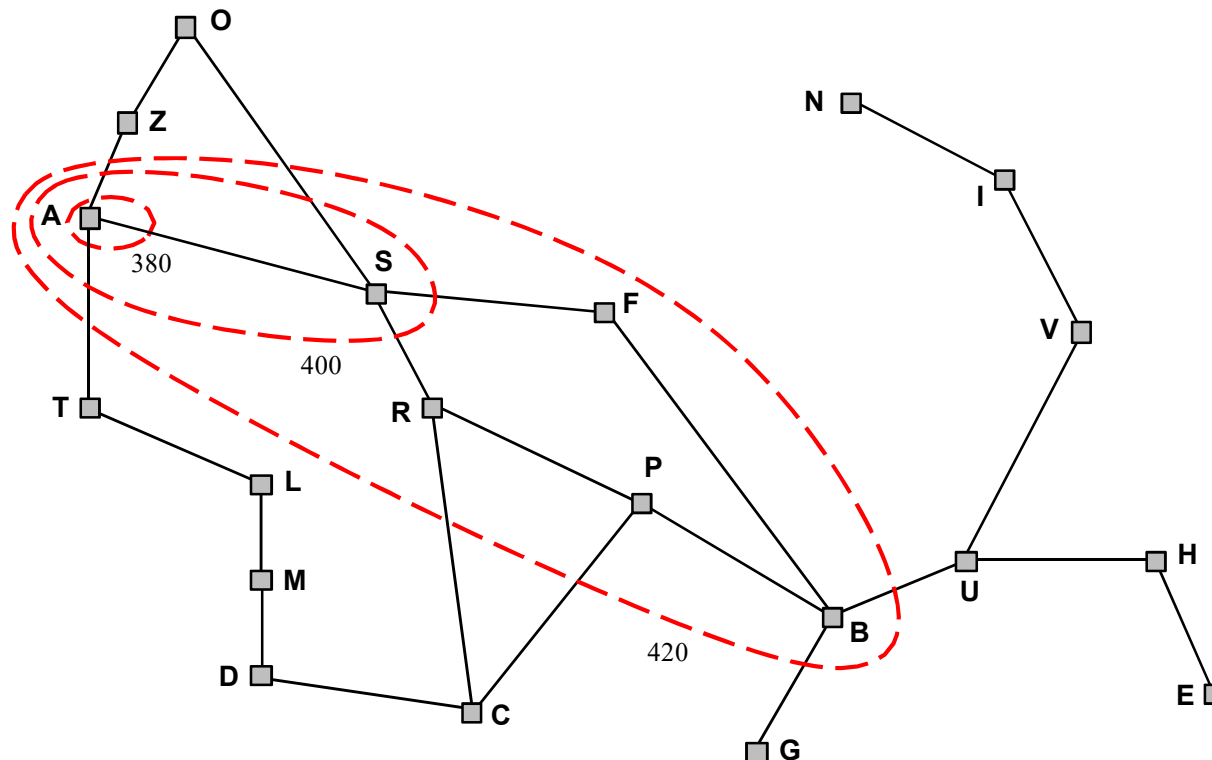
Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds " f -contours" of nodes (cf. breadth-first adds layers)

Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A^*

Complete??

Properties of A^*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time??

Properties of A^*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space??

Properties of A^*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$

A* expands some nodes with $f(n) = C^*$

A* expands no nodes with $f(n) > C^*$

Proof of lemma: Consistency

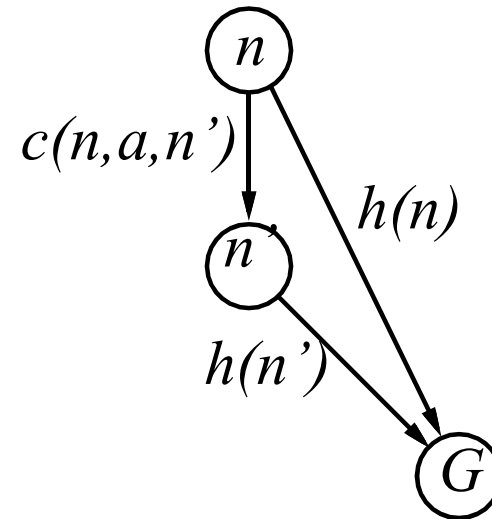
A heuristic is **consistent** if

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n) &= g(n) + h(n) \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

I.e., $f(n)$ is nondecreasing along any path.



Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total **Manhattan** distance

(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S) = ??$

$h_2(S) = ??$

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total **Manhattan** distance

(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S) = ??$ 6

$h_2(S) = ??$ $4+0+3+3+1+0+2+1 = 14$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible), then h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
 $A^*(h_1) = 539$ nodes
 $A^*(h_2) = 113$ nodes
 $d = 24$ IDS \approx 54,000,000,000 nodes
 $A^*(h_1) = 39,135$ nodes
 $A^*(h_2) = 1,641$ nodes

Given any admissible heuristics h_a, h_b ,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

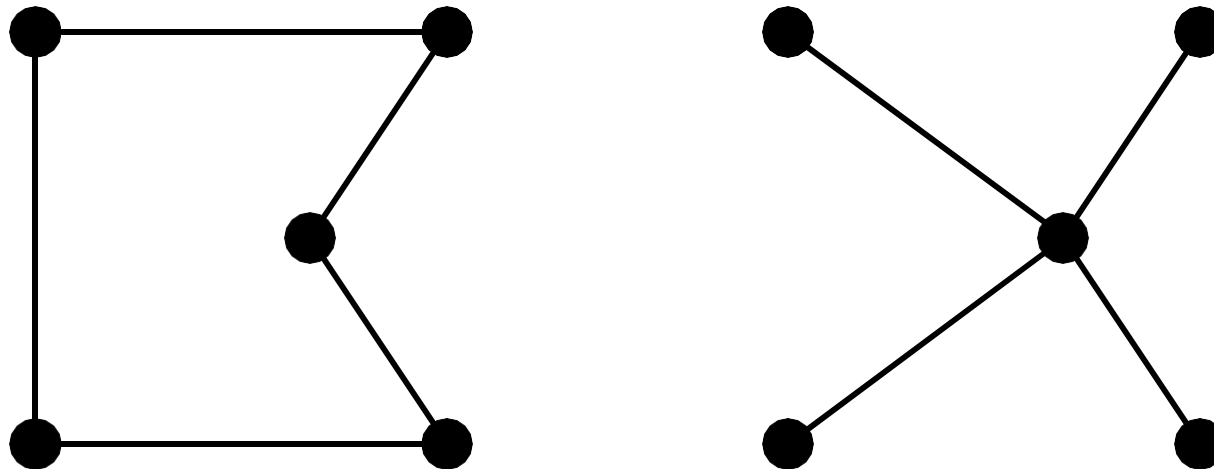
If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed problems contd.

Well-known example: **travelling salesperson problem** (TSP) Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour

Summary

A problem consists of five parts: the **initial state**, a set of **actions**, a **transition model** describing the results of those actions, a set of **goal states**, and an **action cost function**.

Uninformed search methods have access only to the **problem definition**. Algorithms build a search tree in an attempt to find a solution.

Informed search methods have access to a **heuristic** function $h(n)$ that estimates the cost of a solution from n .