Statistical report

Report 1:	Deficits, Discipline, and Design: A Statistical Comparison of UK and Luxembourg Fiscal Policy (2007–2022)
Report 2:	An investigation on any relationship between CO2 emissions and the growth in GDP.

Report 1:

Luxembourg and the UK differ significantly in size, structure, and fiscal behaviour. Comparing the government fiscal balances from 2007 to 2022 offers a stark contrast between a large, debt-prone economy and a small, fiscally disciplined one, revealing deeper insights into volatility, structural policy, and crisis response.

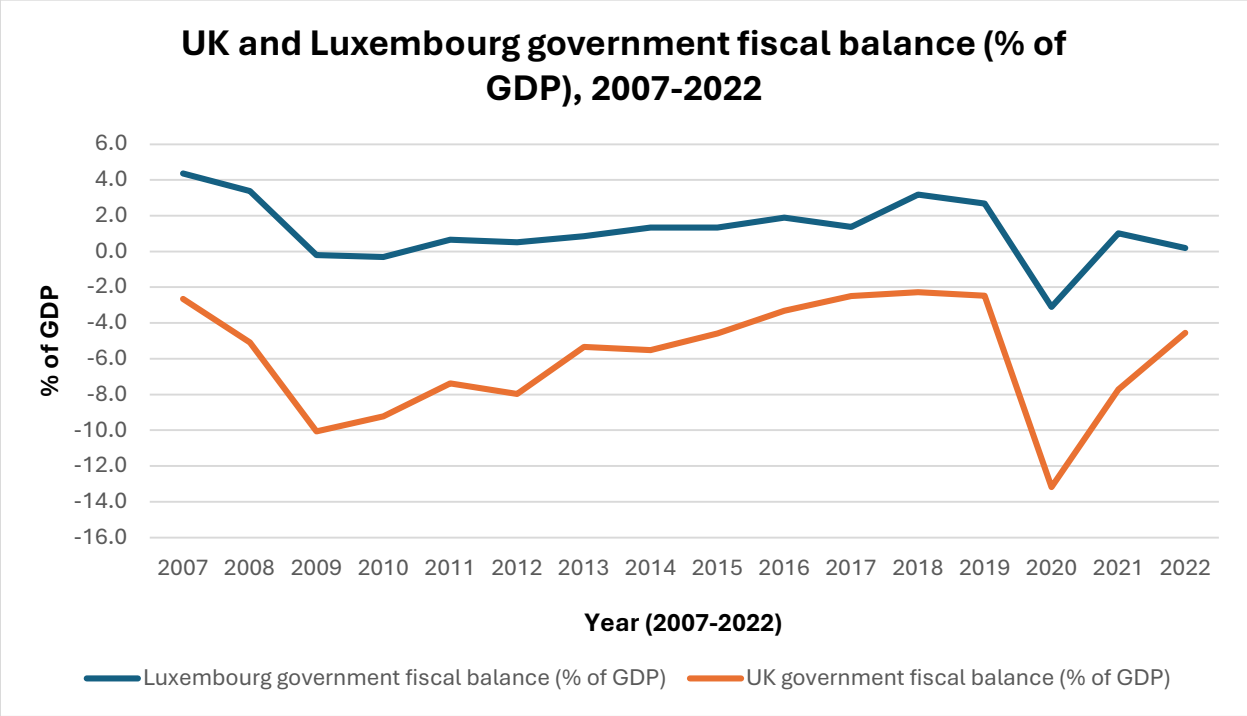


Figure 1: Source: OECD, See Appendix A for raw data.

Fiscal balance trends between the UK and Luxembourg reveal some outstanding differences. Both were impacted by the 2008-2009 financial crisis and the COVID-19 pandemic, but the severity and persistence of deficits were far more pronounced in the UK. During the financial crisis, the UK's fiscal balance fell to -10.06%, while Luxembourg dipped only to -0.21%. The lowest point in both series occurred during the pandemic: -13.17% for the UK and -3.09% for Luxembourg.

Numeric descriptive statistics for the government fiscal balance							
Countries	Number of Observations	Mean	Median	Max	Min	Sandard deviation	Skewness
Luxembourg	16	1.20	1.17	4.36	-3.09	1.75	-0.47
The UK	16	-5.86	-5.22	-2.27	-13.17	3.16	-0.83

Table 1, see Appendix C to find the formulas

Notably, the UK never entered surplus territory during the entire period, with its best fiscal balance being -2.27%. In contrast, Luxembourg not only avoided deep deficits but reached a surplus high of +4.36%.

This reflects fundamental differences in economic structure. The UK is a large, open, globalised economy with complex fiscal demands, greater welfare obligations, and a more active macroeconomic presence. Luxembourg, by contrast, is small, wealthy, and has fewer fiscal shocks to manage. Its fiscal policy is able to remain more surplus-oriented.

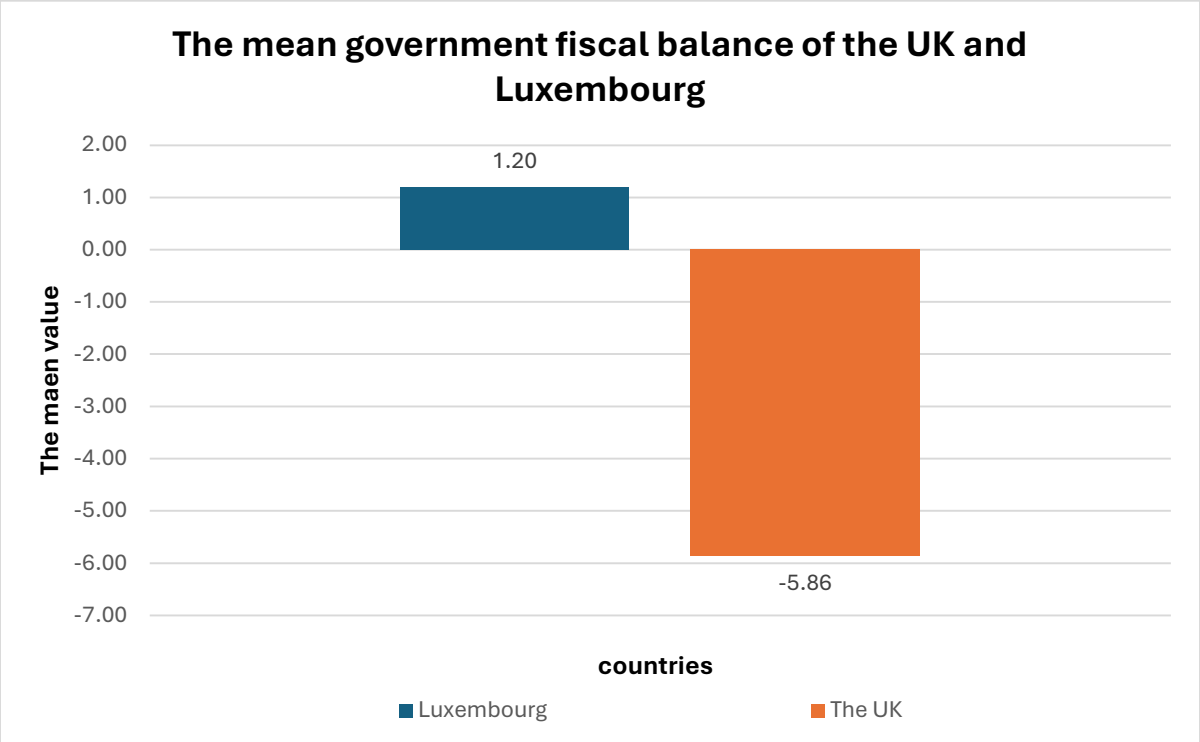


Figure 2, based on data from table 1

To simplify the comparison, the mean government fiscal balance across the 16-year period was calculated for both countries. The result is stark, the UK averaged a fiscal deficit of -5.86%, whereas Luxembourg averaged a surplus of +1.20%. This may suggest the UK runs unsustainable deficits, but means are blunt instruments, especially when outliers are involved.

Even if we assume individuals behave optimally, that does not imply, or even suggest, that the whole economy acts like a single optimizer under the simplest possible constraints (Solow, 2008). This reinforces the idea that fiscal deficits, particularly in a large and complex economy like the UK, can reflect rational macroeconomic responses rather than fiscal irresponsibility.

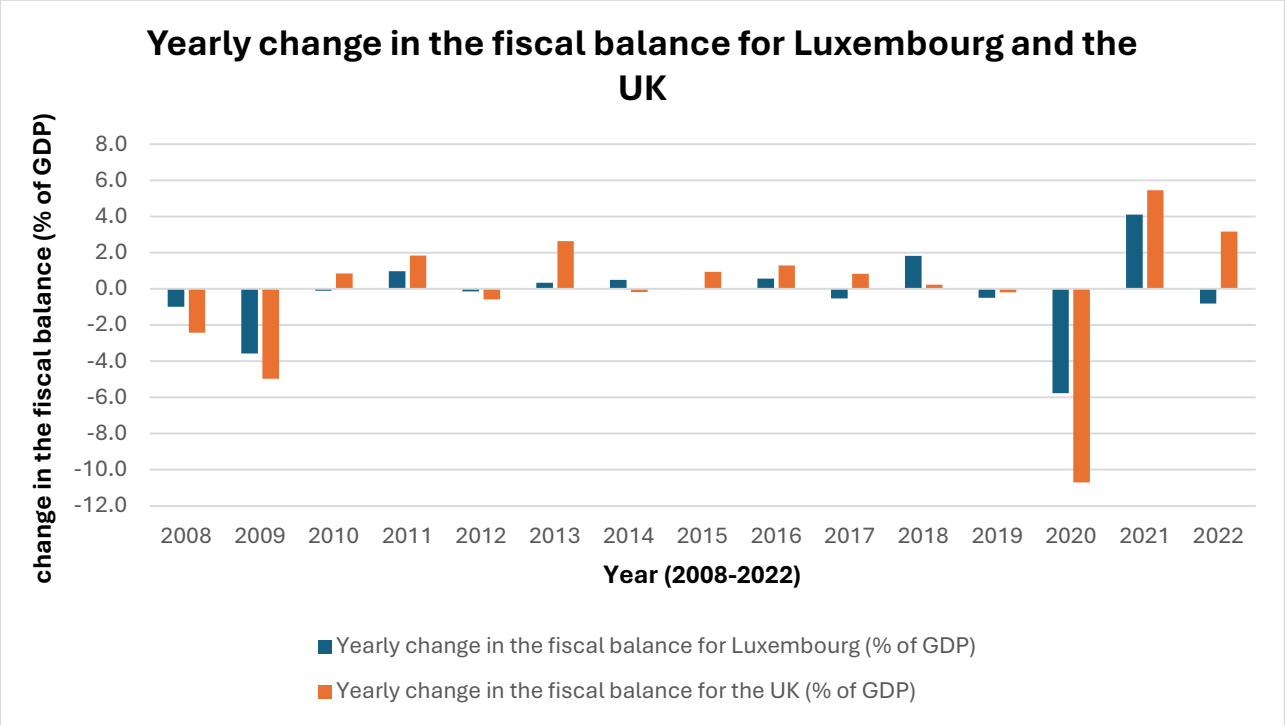


Figure 3, calculated based on raw data from Appendix 1, see Appendix C for the formula

Examining yearly changes reveals even more. The UK's fiscal shifts were sharp and often unpredictable, especially during crisis years. Luxembourg's changes, while responsive to global shocks, were not as drastic. This contrast is confirmed by the standard deviation of yearly changes, which is higher for the UK (3.16), highlighting its greater fiscal volatility.

Numeric descriptive statistics for the changes in the government fiscal balance							
Countries	Number of Observations	Mean	Median	Max	Min	Sandard deviation	Skewness
Luxembourg	15	-0.28	-0.11	4.11	-5.77	2.22	-0.80
The UK	15	-0.13	0.82	5.45	-10.70	3.77	-1.63

Table 2, see Appendix C to find the formula

The skewness of the yearly change further illustrates this point: -1.63 for the UK and -0.80 for Luxembourg. Negative skewness in this context means that large negative shifts in the fiscal balance occurred more frequently. The UK's more pronounced skew suggests a tendency toward severe fiscal response, such as increases in spending and borrowing, when faced with economic shocks.

Sample size:	16
Sample mean:	-5.86
Sample SD:	3.16
Hypothesized value:	-4.5
Standard Error:	0.79
t-test Statistics:	-1.73
degrees of freedom:	15
P-value (lower tail):	0.05
P-value (upper tail):	0.95
P-value (2 tail):	0.10

Table 3, one-sample hypothesis test

A one-sample hypothesis test was conducted to assess whether the UK's average fiscal balance was -4.5%.

Hypotheses: $H_{0:}\,\mu = -4.5;\ H_{1:}\,\mu \neq -4.5.$

With a P-value of 0.10 (above the 5% significance level), we fail to reject the null hypothesis, meaning there is insufficient evidence to challenge the claim.

	UK government fiscal balance (% of GDP)	*Luxembourg government fiscal balance (% of GDP)*
Mean	-5.86	1.19525
Variance	9.96	3.0687546
Observations	16	16
df	15	15
F	3.25	
P(F<=f) one-tail	0.01	
F Critical one-tail	2.86	

Table 4, F-Test Two-Sample for Variance

Next, a two-sample t-test was employed to compare the UK and Luxembourg's means with a 5% significance level.

The F-statistic (3.25) exceeds the critical value (2.86), so we assume unequal variances.

	UK government fiscal balance (% of GDP)	*Luxembourg government fiscal balance (% of GDP)*
Mean	-5.86	1.20
Variance	9.96	3.07
Observations	16	16
Hypothesized Mean Difference	0	
df	23	
t Stat	-7.82	
P(T<=t) one-tail	0.00	
t Critical one-tail	1.71	
P(T<=t) two-tail	0.00	
t Critical two-tail	2.07	

Table 5, t-Test: Two-Sample Assuming Unequal variances

The t-test result of -7.82 (beyond the critical value of -2.07) allows us to reject the null hypothesis, indicating that the means are different.

How we read macroeconomic data heavily depends on the model applied; there is no single shock dominating economic fluctuations (Blanchard, 2008). The influence of the models used is particularly relevant in the context of understanding fiscal balances within their broader economic environment, rather than treating them as the only thresholds of fiscal health.

This analysis challenges simplistic readings of fiscal data. The UK’s deficits are not random; they reflect a strategy of stabilisation and public investment in response to shocks. Luxembourg, by contrast, benefits from structural advantages that support consistent surpluses. In short, deficits are not always a failure, sometimes they are a feature.

Bibliography

Blanchard, Olivier J. *The State of Macro*. NBER Working Paper No. 14259. Cambridge, MA: National Bureau of Economic Research, August 2008. http://www.nber.org/papers/w14259.

Solow, Robert. “The State of Macroeconomics.” The Journal of Economic Perspectives, vol. 22, no. 1, 2008, pp. 243–46. JSTOR, http://www.jstor.org/stable/27648233. Accessed 1 May 2025.

Appendices

Time period	Luxembourg government fiscal balance (% of GDP)	UK government fiscal balance (% of GDP)
2007	4.36	-2.65
2008	3.37	-5.09
2009	-0.21	-10.06
2010	-0.32	-9.22
2011	0.66	-7.38
2012	0.51	-7.97
2013	0.84	-5.34
2014	1.34	-5.53
2015	1.33	-4.59
2016	1.89	-3.31
2017	1.37	-2.49
2018	3.18	-2.27
2019	2.68	-2.47
2020	-3.09	-13.17
2021	1.02	-7.72
2022	0.19	-4.55

Table 6, Appendix A, Raw data for figure 1

Time period	Yearly change in the fiscal balance for Luxembourg (% of GDP)	Yearly change in the fiscal balance for the UK (% of GDP)
2008	-0.99	-2.43
2009	-3.58	-4.97
2010	-0.11	0.84
2011	0.98	1.84
2012	-0.15	-0.59
2013	0.34	2.63
2014	0.50	-0.19
2015	0.00	0.94
2016	0.56	1.28
2017	-0.53	0.82
2018	1.81	0.22
2019	-0.50	-0.20
2020	-5.77	-10.70
2021	4.11	5.45
2022	-0.82	3.17

Table 7, Appendix B, Raw data for figure 2

Formula	Use
ΔFBt = FBt − FBt−1	calculate the yearly changes
=COUNT(range)	To calculate the number of observations
=AVERAGE(range)	To calculate the Mean
=MEDIAN(range)	To calculate the median
=MAX(range)	To calculate the maximum figure
=MIN(range)	To calculate the minimum figure
=STDEV.S(range)	To calculate the standard deviation
=SKEW(range)	To calculate the skewness
=sample SD/SQRT(sample size)	To calculate the standard error
=(sample mean-hypothesis value)/ standard error	To calculate the t-statistics
=sample size-1	To calculate the degrees of freedom
=T.DIST(E13,E14,TRUE)	To calculate the p-value (lower tail)
=1-E16	To calculate the p-value (upper tail)
=2*E16	To calculate the p-value
=VAR.S(B31:B46)	To calculate the variance

Table 8, list of formulas used in the first report

Report 2:

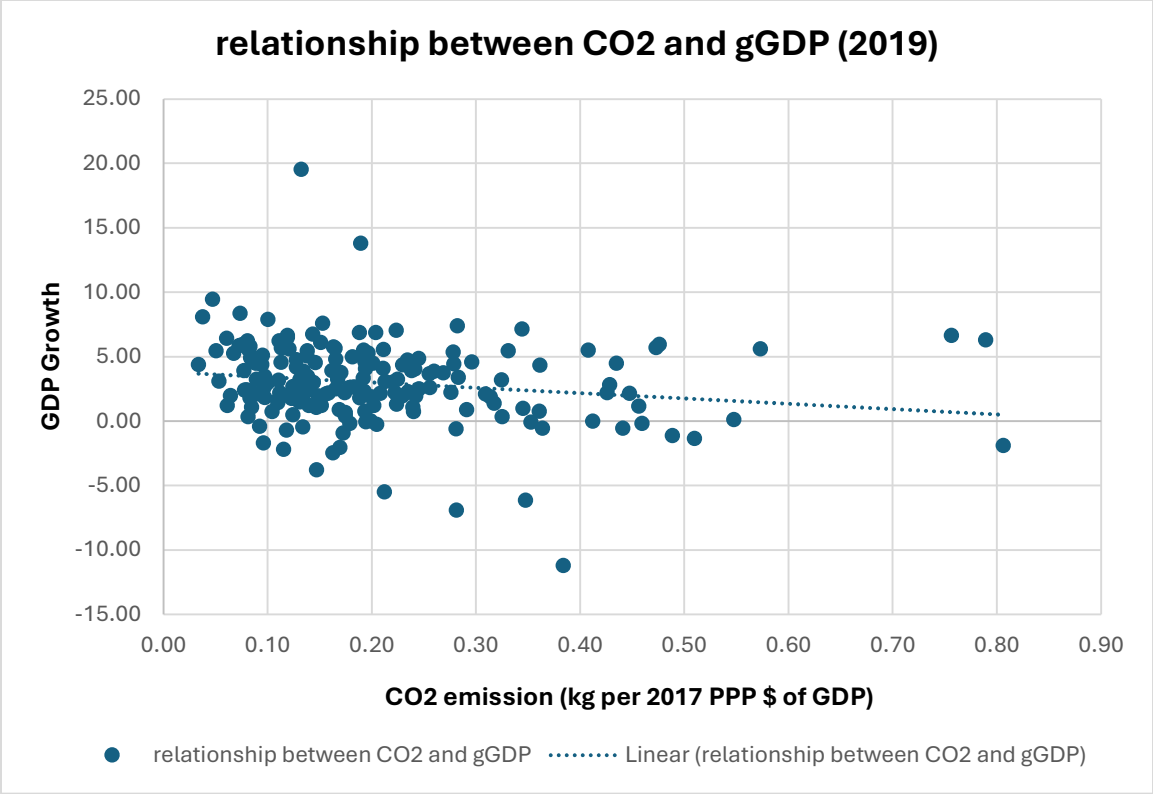
Climate change and economic development are two of the most pressing global challenges of our time. While economic growth often comes with increased resource use and emissions, the nature of this relationship is complex and varies across countries. This statistical brief investigates the relationship between CO2 emissions (kg per 2017 PPP $ of GDP) and growth in GDP (gGDP) across 182 countries in a single year. The findings contribute to a broader understanding of how sustainability and economic performance interact on a global scale.

Numeric descriptive statistics for or CO2 and gGDP variables.

variable	number of observations	mean	median	max	min	standard deviation	skewness
CO2	182	0.21	0.17	0.81	0.03	0.13	1.85
GDP	182	2.98	2.76	19.54	-11.20	3.24	0.12

Table 9, see Appendix, for the formulas used

The dataset covers 182 countries. CO2 emissions have a mean of 0.21 and are positively skewed (skewness = 1.85), suggesting most countries emit relatively little CO2 per GDP, with a few high outliers. gGDP has a mean of 2.98%, a wider range (from -11.20% to 19.54%), and is roughly symmetrical (skewness = 0.12), showing diverse but balanced growth patterns. Standard deviations indicate moderate variability for CO2 and higher variability for gGDP.



The scatter plot illustrates the relationship between CO2 emissions and gGDP across all countries. While the data points appear widely scattered with no obvious visual pattern, the line of best fit suggests a slight negative linear relationship, which is consistent with the calculated correlation coefficient of -0.17. This indicates that, on average, countries with higher CO2 intensity tend to have slightly lower GDP growth, although the relationship is weak.

A Chi-square test was conducted to examine the relationship between CO2Group and gGDPGroup using pivot table

H_0: $CO2Group\ and\ gGDPGroup\ are\ independent$, there is no relationship between a country's CO2 emission level and its GDP growth category.

H_1: $CO2Group\ and\ gGDPGroup\ are\ not\ independent$, there is a relationship between CO2 emission level and GDP growth category.

The test produced a p-value of 0.062, which is above the 5% significance level. Therefore, we fail to reject the null hypothesis and conclude that there is no statistically significant association between CO2 emissions level and GDP growth category. (See Appendix A and B for the pivot table and expected values.)

r	-0.17
n	182
dof	180
t-statistic	-2.33
p-value (lower tail)	0.01
p-value (upper tail)	0.99
p-value (two tail)	0.02

Table 10, using a t-test to analyse the statistical significance

A correlation coefficient between CO2 and GDP growth was conducted to examine the relationships of the two variables.

H_0: $\rho = 0$ assuming there is no relationship. H_1: $\rho \neq 0$ assumes there opposite.

To test this hypothesis, a t-test was conducted using a t-statistic of -2.33 with 180 degrees of freedom. The resulting two-tailed p-value was 0.02. Since this p-value is less than the 0.05 significance level, we reject the null hypothesis and conclude that there is a statistically significant linear relationship between CO2 emissions and GDP growth.

While the relationship is statistically significant, the correlation is weak (ρ = -0.17), indicating that although there is a negative relationship, the strength of the relationship is minimal. In other words, countries with higher CO2 emissions per unit of GDP tend to experience slightly lower GDP growth, but this pattern is not very strong. Other factors likely play a significant role in determining GDP growth across countries.

While traditional economic theory often assumes a positive relationship between economic growth and environmental degradation, recent findings suggest the dynamic is more nuanced. Beckerman (1992, p. 491.) provocatively argued that "although economic growth usually leads to environmental degradation in the early stages of the process, in the end the best—and probably the only—way to attain a decent environment in most countries is to become rich".

In conclusion, there is evidence of a weak but statistically significant negative linear relationship between CO2 emissions and GDP growth. Countries with higher CO2 emissions per unit of GDP tend to experience slightly lower GDP growth, although the relationship is minimal. No strong association was found when categorizing the data into

groups. Overall, CO2 emissions alone do not fully explain variations in economic growth across countries, suggesting that other factors are also at play.

Bibliography

Beckerman, W., 1992. Economic growth and the environment: Whose growth? Whose environment? World Development, 20(4), pp.481–496.

Stern, D.I., 2004. The rise and fall of the Environmental Kuznets Curve. World Development, 32(8), pp.1419–1439.

Appendices

Row Labels	Contraction	High	Moderate	Grand Total
High CO2	2.37	4.45	11.18	18
Low CO2	21.63	40.55	101.82	164
Grand Total	**24**	**45**	**113**	**182**

Table 11, Appendix A, expected values used to make the X^2 test

Row Labels	Contraction	High	Moderate	Grand Total
High CO2	5	6	7	18
Low CO2	19	39	106	164
Grand Total	**24**	**45**	**113**	**182**

Table 12, Appendix B, Pivot table used to conduct theX^2 test

Formula	Use
ΔFBt = FBt − FBt−1	calculate the yearly changes
=COUNT(range)	To calculate the number of observations
=AVERAGE(range)	To calculate the Mean
=MEDIAN(range)	To calculate the median
=MAX(range)	To calculate the maximum figure
=MIN(range)	To calculate the minimum figure
=STDEV.S(range)	To calculate the standard deviation
=SKEW(range)	To calculate the skewness
=CORREL(M3:M184,N3:N184)	To calculate the correlation coefficient
=(C4 * SQRT(C7 - 2)) / SQRT(1 - C4^2)	To calculate the t-statistics
=T.DIST(D18,D16,TRUE)	To calculate the p-value (lower tail)
=IF(B3>0.4,"High CO2","Low CO2")	To calculate the low and high figures for the CO2 in different countries
=IF(C2<0,"Contraction","")&IF(AND(C2>=0,C2<=5), "Moderate","")&IF(C2>5,"High","")	To categorise the GDP rate into construction, moderate, and high

Table 13, Appendix C, formulas used in report 2