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$\therefore T(n) = n$ for $n \le 5$

1 $T(n) = T(n-3) + 2$

Solution

$$T(n) = T(n-3) + 2$$

So, $T(n-3) = [T(n-3-3) + 2] + 2$

$T(n-3) = [T(n-6) + 2] + 2$

$T(n-3) = T(n-6) + 4$

Now Put $T(N-3)$ in eq.

$T(n) = [T(n-3-6) + \cancel{2}4] + \cancel{4}2$

$T(n) = [T(n-9) + 4] + 2$

$T(n) = T(n-9) + 6$

$T(n) = T(n-3K) + 2K$

As we can observe that ~~it~~ it is expanding by a rate of

$(n-K) + K$

Method # 1, 2

$T(n) = T(n-K) + K$

Now lets say $n-K=0$, $n=K$

$T(n) = T(n-n) + n$

$= T(0) + n$

As we know $T(n \le 5) = n$

$= n + n$

$= 2n$ (Ignore constants).

$n - 3K = 0$

$n = 3K$

$K = \frac{n}{3}$

$T(n) = T\left(n - \cancel{3}\left(\frac{n}{\cancel{3}}\right)\right) + 2\left(\frac{n}{3}\right)$

$T(n) = T(n-n) + 2\left(\frac{n}{3}\right)$

$T(n) = T(0) + 2\left(\frac{n}{3}\right)$

$T(n) = 0 + 2\left(\frac{n}{3}\right)$

$T(n) = O(n)$.

Tighest Bound.

$T(n) = n \Rightarrow T(n) = \Theta(n) \Rightarrow T(n) = O(n)$

(2) $T(n) = T(9n/10) + 1$

Sol:

$a=1, \quad b = 10/9 \quad f(n) = 1$

$n^{\log_b a} \qquad n^{\log_{10/9} 1} = n^0 = n^1$

$n^{\log_b a} \neq f(n)$ but $n^0 = 1$ so $1 = 1$.

Case #02 implies

$\Theta(n^{\log_b a} \log n) = \Theta(1 \log n)$

Choosing tighest bound

$O(\log n)$.

(3) $T(n) = T\left(\frac{n^{1/2}}{2}\right) + 1$

let $n = 2^m$

$T(n^m) = \frac{2^{m/2}}{2} + 1$

$T(2^m) = S(m)$

$S(m) = T(m/2 \cdot 2) + 1$

$= T(m/4) + 1$

$m^{\log_b a} \Rightarrow m^{\log_4 1} = m^0 = 1$

$\overset{n=m}{\left(n^{\log_b a} (\log_n)\right.} = m^{\log_b a} (\log_m) = 1 \log(m)$

but, $n = 2^m$ take log

$\log(n) = m \log_2 2 = m(1)$ Ans $= O(\log(\log(n)))$.

4 $T(n) = 3T(n-1)$

Sol:

$T(n-1) = 3T(n-2)$ $\therefore T(n-2) = 3T(n-3)$.

$T(n) = 3T(3T(n-2)$

$T(n-2) = 3T(n-3)$

$T(n) = 3T(3T(3T(n-3)$

expanding until K.

$T(n) = 3^K T(n-K)$

$n-K = 0$

$n = K$

$= 3^K T(n-n)$

$= 3^K T(0)$ $\therefore T(n \leq 5) = n$

$= 3^K \cdot n$

$T(n) = n3^K$

$T(n) = O(n3^K)$.

5 $T(n) = 2T(n/2) + 2$

Sol:

$n^{\log_b a} = n^{\log_2 2} = n^1 \Rightarrow n^{1-1} = n^0 = 1$

So it is Case # 1

$n^{\log_b a - \varepsilon} = f(n)$

$n^{1-1} = 1$

$1 = 1$

$T(n) = O(n)$. $\therefore n \Rightarrow n^{\log_b a}$ $\underline{n^1}$.

7 $T(n) = 5T(n/3) + n$

$n^{\log_b a} = n^{\log_3 5} = n^{1.5}$

$n^{\log_b a - \varepsilon} = f(n)$

$n^{1.5 - .5} = f(n)$

$n = n$

So

$= O(n^{\log_b a})$

$= O(n^{1.5})$

$= O(n^{3/2})$ Ans.

8 $T(n) = \cancel{8}\,8T(n/4) + n^2$

$n^{\log_b a} = n^{\log_4 8} = n^{1.5}$

$n^{\log_b a + \varepsilon} = f(n)$

$n^{1.5 + \varepsilon} = f(n)$

$af(n/b) \leq cf(n)$

$8(n/4)^2 \leq cn^2$

$\frac{\cancel{8}\,n^2}{\cancel{16}\,2} \leq cn^2$

$n^2/2 \leq n^2/2$ ✓

So

$T(n) = O(n^2)$

9 $T(n) = 27T(n/9) + n\log n$

$n^{\log_b a} = n^{\log_9 27} = n^{1.5}$

$n^{\log_b a - \varepsilon} = f(n)$

$n \approx n \log(n)$

$T(n) = O(n^{3/2})$ $\therefore n^{\log_b a}$

So, (10) $T(n) = 4T\left(\frac{n}{4}\right) + \frac{n}{\log}$ —(1)

Sol. $T(n) = \Theta(n\log(\log n))$

$T(n) \le c \cdot n\log(\log(n))$ —(2)

$n = \frac{n}{4}$

$T\left(\frac{n}{4}\right) \le c\frac{n}{4}\log\left(\log\left(\frac{n}{4}\right)\right)$

=> (1)

$= 4c\frac{n}{4}\log\left(\log\left(\frac{n}{4}\right)\right)$

$T(n) = cn\log\left(\log\frac{n}{4}\right) + \frac{n}{4}$

$= cn\log(\log n - \log 4) + \frac{n}{\log(n)}$

$\log(\log n - \log 4) \approx \log(\log 4)$.

$= cn\log(\log n) + \frac{n}{\log(n)}$

$= n\log(\log n)$ proved

$T(\sqrt{n})$

⑥ $T(n) = (T((n))^2$

Sol: Master therrem Not applieds

$$T(\sqrt{n}) = O(\log \sqrt{n}) = O(\tfrac{1}{2} \log n)$$

$$T(n) = (O(\tfrac{1}{2} \log n)^2) = O((\tfrac{1}{2} \log n)^2) = O(\log(n)^2)$$