

Thought for a few seconds ✓

Certainly! Let's analyze and solve the recurrence relation you've provided:

$$T(n) = (T(n))^2$$

You mentioned using the **Substitution Method** or the **Master Theorem** to solve this recurrence. However, it's important to note that the **Master Theorem** is specifically designed for divide-and-conquer recurrences of the form:

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

where:

- $a \geq 1$ is the number of subproblems,
- $b > 1$ is the factor by which the problem size is divided,
- $f(n)$ is the cost of the work done outside the recursive calls.

Given that your recurrence:

$$T(n) = (T(n))^2$$

does not fit this form (since it doesn't involve smaller subproblems or a division of the problem size), the **Master Theorem** is **not applicable** here. Instead, we'll approach this as a **functional equation** and use the **Substitution Method** (also known as the **Guess and Induction Method**) to find its solutions.

Understanding the Equation

The equation is:

$$T(n) = (T(n))^2$$

This implies that the value of $T(n)$ is equal to its own square. To find $T(n)$, we need to solve for $T(n)$ such that this equality holds.

Rewriting the Equation

Let's treat $T(n)$ as a variable and solve the equation algebraically.

1. Set Up the Equation:

$$T(n) = (T(n))^2$$

2. Rearrange the Equation:

Subtract $T(n)$ from both sides to bring all terms to one side:

$$(T(n))^2 - T(n) = 0$$

3. Factor the Equation:

Factor out $T(n)$:

$$T(n)(T(n) - 1) = 0$$

4. Find the Roots:

Set each factor equal to zero:

- $T(n) = 0$
- $T(n) - 1 = 0 \Rightarrow T(n) = 1$

Solutions

The equation $T(n) = (T(n))^2$ has two constant solutions:

1. $T(n) = 0$:

$$0 = (0)^2 \Rightarrow 0 = 0 \quad (\text{True})$$

2. $T(n) = 1$:

$$1 = (1)^2 \Rightarrow 1 = 1 \quad (\text{True})$$

Therefore, the general solutions are:

$$T(n) = 0 \quad \text{or} \quad T(n) = 1 \quad \text{for all } n$$

Interpretation

- **Constant Solutions:** The only solutions to this equation are constant functions. That is, $T(n)$ must be the same value for all n , either 0 or 1.

- **No Non-Constant Solutions:** There are **no non-constant solutions** to this equation. Any attempt to define $T(n)$ based on n (i.e., making $T(n)$ vary with n) would not satisfy the equation unless $T(n)$ remains at one of the constant values.

Why Substitution Method and Master Theorem Don't Apply Here

Substitution Method:

The **Substitution Method** involves making an educated guess about the form of the solution and then proving it rigorously using mathematical induction. However, in this case:

- Since the equation is purely functional and only admits constant solutions, there's no need for an inductive proof.
- The Substitution Method is typically used for more complex recurrences where the solution's form isn't immediately obvious.

Master Theorem:

As previously mentioned, the **Master Theorem** applies to divide-and-conquer recurrences of the form:

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

Your recurrence:

$$T(n) = (T(n))^2$$

does not involve smaller subproblems or a division of the problem size. Therefore, the Master Theorem is **not applicable** in this scenario.

Possible Misinterpretations or Typographical Errors

Given the simplicity of the equation $T(n) = (T(n))^2$, it's possible that there was a typographical error or misunderstanding in the problem statement. For instance, perhaps you intended to write a different recurrence relation, such as:

$$T(n) = 4T\left(\frac{n}{4}\right) + n^2$$

or

$$T(n) = T\left(\frac{n}{2}\right) + 1$$

If you intended a different form of the recurrence, please provide the corrected equation, and I'd be happy to assist you in solving it using the **Substitution Method**, **Master Theorem**, or any other appropriate method.

Conclusion

Given the equation as provided:

$$T(n) = (T(n))^2$$

the only valid solutions are the constant functions:

$$T(n) = 0 \quad \text{or} \quad T(n) = 1$$

Both solutions satisfy the equation for all values of n . Neither the **Substitution Method** nor the **Master Theorem** is necessary or applicable for solving this particular functional equation.

If this wasn't your intended recurrence relation, please provide the correct form, and I'll gladly help you solve it!