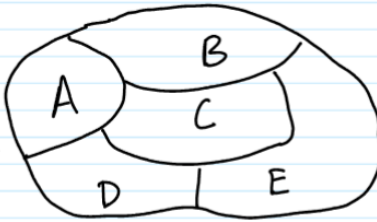


Discrete Lecture # 25

- Graph Coloring :
 - Graph coloring : No two adjacent vertices should have same colors
 - Chromatic Number : Least number of colors used such that no two vertices have the same color
 - Graph coloring is a concept of making sure minimum numbers of colors are used to color the graph such that adjacent vertices do not have the same color
- Chromatic number :
 - The number of colors used to color the graph
- Applications :

Graph Colouring :- P 606.

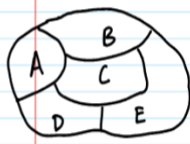


map.
 $A, B, \dots = \text{Regions.}$

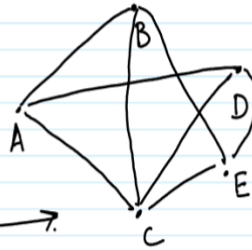
Problem:- No two Regions have the Same Colour. & we want to use minimum Number of Colours.

→ Graph:- Regions = Vertices.
Share Boundary = Edge.

- DUAL GRAPH CONCEPT :
 - Making graph from map

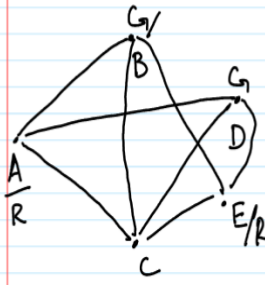


U



maps
Ex: Dual Graph.

Dual Graph.



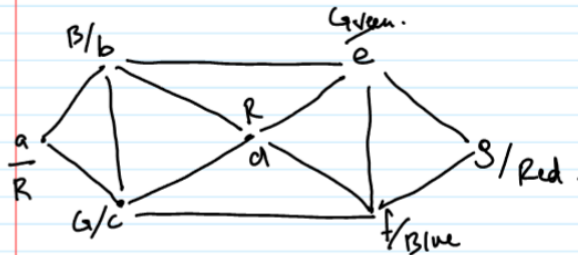
No two adjacent should have the same color.

R, G, B, ...

B

3-Colors.

Chromatic Number: least number of colours needed for colouring a Graph. = 3.



Chromatic Number = ? 3.

R, B, G, ...

-
- The above is the way to assign colors
- Finding the chromatic number

- Bi-Partile Graph :
 - All the bi-partile graph with KM,N formation forexample $K_{2,3}$ or $K_{3,3}$
 - Will always have a chromatic number of 2
- Cycle :
 -

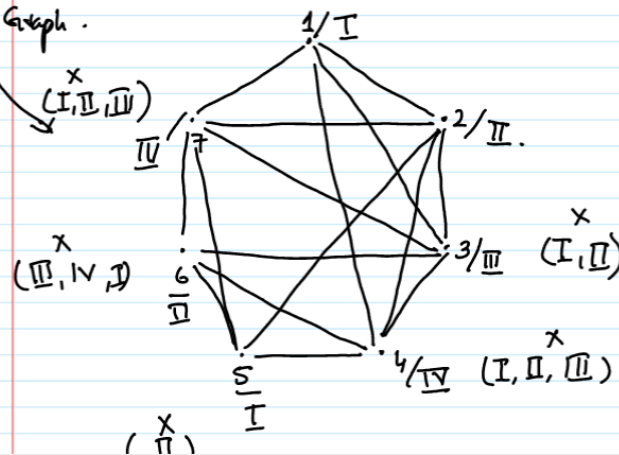
- All the C(EVEN) cycles will have a chromatic number of 2
- All the cycles with even number of vertices will have a chromatic number of 2
- All the C(ODD) cycles will have a chromatic number of 3
- All the cycles with odd number of vertices will have a chromatic number of 3
- Application of Graph Coloring :
 - There is a problem in a university that many students have the same classes and there is a clash of many students to not to have more than one class at the same time
 - The solution was concluded with graph coloring
 - Here colors are represented as SLOTS
 - S1 , S2 , S3 , S4.....
 - First make graph from
 - Take subjects as vertices
 - Take edges as common students in a class
 - Now make an edge where you find common students in a class
 - Color the graph with slot 1 2 3 or onwards
 - You can write the slots/graph color that cannot be assigned to vertex to make it easy to color/assign slot to the vertex given there are many edges

Application of Graph Colouring:-

Common Students in Courses:-

$1 \in 2, 2 \in 3, 3 \in 4, 4 \in 5$
 $1 \in 3, 2 \in 4, 3 \in 6, 4 \in 6$
 $1 \in 4, 2 \in 5, 3 \in 7$
 $1 \in 7, 2 \in 7$
 $5 \in 6, 6 \in 7$
 $5 \in 7$

Graph.



Courses 1

⋮

7.

Conduct Exam Using minimum Number of Slots

Vertices = Subjects.

Edges = if They have Common Students.

I, II, III, IV, ...

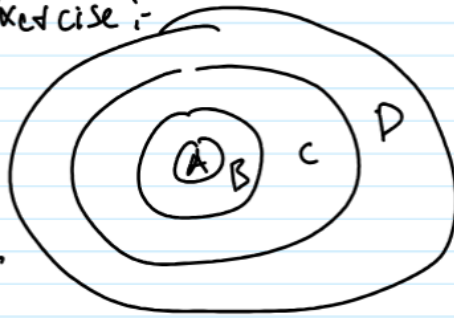
Slots	Courses
I	1, 5
II	2, 6

- Exercise for Making graph from map and then coloring it:

- As you can see

- The circles with letters :
 - Take letters as vertices
- The boundaries of those circles
 - Take boundary as an edge of two vertices
- Conduct a graph
- Color the graph as taught
- Write the chromatic number

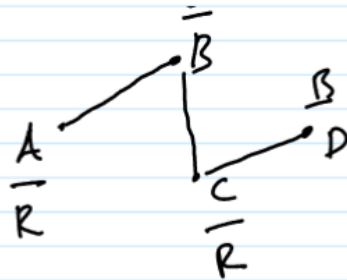
Exercise:-



⇓ minimum colours.

Step 2 Dual Graph.

= Graph Colouring.



Chromatic = 2.

Chromatic -

✓ Graph Colouring:- No two adjacent
should have same colour. ✓

✓ Chromatic Number:- least number of
colours that can be used such
that no two vertices have the colour. ✓

