lec # 18.

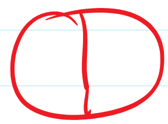
Let $R = \{(a,b) \mid a \leq b\}$. $A = \{a, b, c\}$.

Not Equivalence. $\{a, b\} \subseteq \{a, b, c\}$.

$R = \{(a,b). \ \mid |a - b| \in \mathbb{Z}\}$. $[4] = \{\mathbb{Z} \quad \}$.

(4,

one element in Partition.



Unique EC = 1.

$R = \{(a,b) \mid |a - b| = 0\}$. $A = \mathbb{Z}$.

$[4] = \{4\}$.

PARTIAL ORDER.

1) Reflexive
2) Anti Symmetric.
3). Transitive.

<u>Ex1</u>
P504 :

$R = \{(a,b) \mid a \text{ divides } b\}$. $A = \mathbb{Z}$.

$\geq$ ; $\leq$ ; $> \times$. $A = \mathbb{Z}$.

Reflexive $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{Z} \quad a$ divides $a$. ✓.

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in A$ if $a$ divides $b \wedge b$ divides $a \rightarrow a = b$.

$2 \div 2 \wedge 2 \div 2 \rightarrow 2 = 2$.

Transitive Check your self.

Partial Order Notation.

(PO. SET). $\rightarrow$ POSET.

$(a,b) \in R$.
$aRb$.

$(\mathbb{Z}, \geq)$ $\quad \preccurlyeq = \div$
$(\mathbb{Z}, \leq)$ $\quad \preccurlyeq = \geq$
$(\mathbb{Z}, \div)$ $\quad \preccurlyeq = \leq$
$(P(S), \subseteq)$. $\quad \preccurlyeq = \subseteq$.

$(S, \preccurlyeq)$.

$a \preccurlyeq_R b = (a,b) \in R$.

$(P(\{a,b,c\}), \subseteq)$. find PO.

$R = \{(a,b) \mid a \subseteq b\}$ $\quad P(A) = P(\{a,b,c\})$.

$R = \{(\phi,\phi), (\phi,\{a\}), (\phi,\{b\}), \ldots$ $\quad P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$.
$(\{a\},\{a\}), (\{a\},\{a,b\}), (\{a\},\{a,c\}),$
$(\{a\},\{a,b,c\}),$

$\vdots$
$\}$

Comparable:- Two element $a,b \in S$. are Comparable in $(S, \preccurlyeq)$ if.
$a \preccurlyeq b$ or $b \preccurlyeq a$.

Ex5 $(\mathbb{Z}^+, |)$ 5 and 7 are Comparable.

Ex5 $(Z^+, |)$ 5 and 7 are Comparable.
504
$5 \preceq 7$ or $7 \preceq 5$.
$5 | 7$ or $7 | 5$.
F $\vee$ F = F.

3 & 9 are Comparable.
$3 \preceq 9$ or $9 \preceq 3$.
$3 | 9$ or $9 | 3$.
F or T = T.

Total Order:-. If all element in S are Comparable.
$\forall a, b \in S$ $a \preceq b$ or $b \preceq a$.

Ex6:- $(Z, \leq)$. is a total order ?
505

$-1 \preceq 1$ or $1 \preceq -1$
$-1 \leq 1$ or $1 \leq -1$.
✓

$-\infty \leq -\infty + 1 \leq \cdots \leq -1 \leq 0 \leq +1 \leq +2 \leq \cdots \leq +\infty$.

Ex7 $(Z, |)$ is a total Order. X.
505

Lexicographic Order.

$A_1 \times A_1$ $(A_1, \preceq_1)$. $(A_2, \preceq_2)$. $A_2 \times A_2$.
$(a_1, a_2)$ $(b_1, b_2)$

$A_2 = \{b_1, b_2, \cdots b_n\}$.
$A_1 = \{a_1, a_2, \cdots a_n\}$.

$(A_1 \times A_2, \preceq_1)$

$(a_1, a_2) \preceq (b_1, b_2)$.
if $a_1 \prec_1 b_1$ or $(a_1 = b_1 \wedge a_2 \prec_2 b_2)$.

if $a_1 <_1 b_1$ or $(a_1 = b_1 \wedge a_2 <_2 b_2)$.

**Exa 9**
**508**

$(3,5) \preceq (4,8)$ $\quad (\mathbb{Z} \times \mathbb{Z}, \leq)$.

$a_1 = 3,\ a_2 = 5,\ b_1 = 4,\ b_2 = 8$.

$3 \leq 4 \vee (3 = 4 \wedge 5 \leq 8)$. ✓

$(3,8) \preceq (4,5)$. ✓

$(4,9) \preceq (4,10)$. ✓

$(5,10) \preceq (4,10)$.

$(a_1, a_2) \preceq (b_1, b_2)$.

if $a_1 <_1 b_1$ or $(a_1 = b_1 \wedge a_2 <_2 b_2)$.

$(\mathbb{Z} \times \mathbb{Z}, |)$.

$a_1 | b_1$ or $(a_1 = b_1 \wedge a_2 | b_2)$

General form.

$(A_1, \preceq_1), (A_2, \preceq_2), \ldots (A_n, \preceq_n)$.

$(a_1, a_2, a_3, \ldots, a_n) \preceq (b_1, b_2, b_3, \ldots, b_n)$.

$(a_1 <_1 b_1)$ or $(\exists i > 0 \quad a_1 = b_1 \wedge a_2 = b_2 \wedge \cdots \wedge a_i = b_i \wedge a_{i+1} \preceq b_{i+1})$

**Ex10:-**
**506**

$(1,2,3,5) < (1,2,4,3)$ $\quad (A_1 \times A_2 \times \cdots \times A_n, \leq)$.

$a_1, a_2, a_3, a_4 \quad b_1, b_2, b_3, b_4$.

$i = 2$.

$(a_1 < b_1)$ or $(a_1 = b_1 \wedge a_2 = b_2 \wedge a_3 < b_3)$

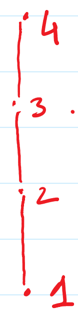
$(2<1)$ or $(1=1 \wedge 2=2 \wedge 3<4)$.

F or T ∧ T ∧ T ✓.

Dictionary. discreet < discrete. ✓

<

Hasse Diagram. (Visualization of Po).




$(\{1,2,3,4\}, \leq)$.

$R = \{\cancel{(1,1)}, (1,2), (1,3), (1,4),$
$\cancel{(2,2)}, (2,3), (2,4),$
$\cancel{(3,3)}, (3,4), \cancel{(4,4)}\}$.

**Ex 12**
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$(\{1,2,3,4,6,8,12\}, \mid)$.

$R = \{\cancel{(1,1)}, (1,2), (1,3), (1,4), (1,6), (1,8), (1,12),$
$\cancel{(2,2)}, (2,4), (2,6), (2,8), (2,12),$
$\cancel{(3,3)}, (3,6), (3,12),$
$\cancel{(4,4)}, (4,8), (4,12),$
$\cancel{(6,6)}\ (6,12)$
$\cancel{(8,8)},$
$\cancel{(12,12)}\}$.