Lec # 14:-

N-ary Relation.

$A = \{1, 2\}$.
$B = \{a, b\}$.
$C = \{x, y\}$.

$|A \times B \times C| = ?$ $8 = |A| \times |B| \times |C| = 2 \times 2 \times 2 = 8$.

possible Relations on $A \times B \times C$.

$Pow(A \times B \times C) = \{\phi, \{(1, a, x)\}, \{(1, a, y)\}, \ldots$

$|Pow(A \times B \times C)| = 2^{|A \times B \times C|}$

$= 2^{|A| \times |B| \times |C|}$

$= 2^{2 \times 2 \times 2}$

$= 2^8 = 256$.

Ex 1 P469

$R = \{(a, b, c) \mid a < b < c\}$. $\mathbb{N}$

$\mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

$(1, 2, 3) \in R$?

$(2, 4, 3) \notin R$?

Ex 2 469

$R = \{(a, b, c) \mid b = a + k,\ c = a + 2k\}$. $\mathbb{Z}$

$\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$.

$(1, 3, 5) \in R$? ✓ $k \in \mathbb{Z}$.

$(2, 5, 9) \in R$. ? ✗.

$3 = 1 + K \Rightarrow K = 2$

$5 = 1 + 2.2 \Rightarrow 5 = 5$

$5 = 2 + K \Rightarrow K = 3.$

$9 = 2 + 2.3 \Rightarrow 9 \neq 2 + 6$

$$2 \overline{)5}^{\,2}$$
$$\underline{-4}$$
$$1$$

$5 \bmod 2 = 1.$

$-4 \bmod +3 = 2.$

Ex3 : $R = \{(a,b,c) \mid a \equiv b \bmod c\}. \quad Z \times Z \times Z^+$
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$(8,2,3) \in R$? ✓

$8 \equiv 2 \bmod 3.$

$(-1, 9, 5) \in R$? ✓.

$-1 \equiv 9 \bmod 5.$

$(4, 0, 7) \in R$?

Relational Database.

Ex4 : $R \qquad A \times N \times S \times D \times T.$

→ (Turkish, TK-47, PEW, PXB, 22:00.) A = Set of Airlines.
N = " " flight #'s.
S = " " Starting Points.
D = " " Destinations.
T = " " Hours.

A N S D T.

Row. →

Representing Relations.

Matrices. 1) Square brackets.
2) Size, row X Col.

R is defined on $A \times B$. $A = \{a_1, a_2, \dots a_m\}$.
$B = \{b_1, b_2, \dots b_n\}$.

$M_R = [m_{ij}]$. rows = $|A|$.
Col = $|B|$.

M_R
$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R. \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

$(a_1, b_2) \in R$.

Ex 1
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$A = \{1, 2, 3\}$ $B = \{1, 2\}$.
$R = \{(2,1), (3,1), (3,2)\}$. R _ $A \times B$.

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Properties.

Reflexive.

$\forall x \quad (x, x) \in R.$

$\forall i \quad (a_i, a_i) \in R.$

$\forall i \quad m_{ij} = 1.$

$\underset{\text{row}}{i}, \; \underset{\text{col.}}{j}$

$[\;] \checkmark \qquad |A| = 0.$

$[1] \checkmark$

$[0] \times.$

$|A| = 2.$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \times \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \times \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \times.$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \times \quad \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \times \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \quad \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \times.$

$[\,0 \times 0\,] \quad [\,1 \times 1\,] \quad [\,1 \checkmark 0\,] \quad [\,0 \times 1\,] \quad [\,1 \checkmark 1\,]$

$\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ X $\quad \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ X $\quad \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ✓ $\quad \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ X $\quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ✓

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ✓.

Symmetric.

$\forall a, b \quad$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a_i, b_j \quad$ if $(a_i, b_j) \in R \rightarrow (b_j, a_i) \in R$.

$\forall i,j \quad$ if $m_{ij} = 1 \rightarrow m_{ji} = 1$.

$[\ ]$ ✓

$[0]$ ✓

$[1]$ ✓

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ✓ $\quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ✓ $\quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ X $\quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ X $\quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ ✓.

$$\begin{bmatrix}1 & \times & 1\\ 0 & & 0\end{bmatrix}\ \begin{bmatrix}1 & \times & 0\\ 1 & & 0\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 0\\ 0 & & 1\end{bmatrix}\ \begin{bmatrix}0 & \checkmark & 1\\ 1 & & 0\end{bmatrix}\ \begin{bmatrix}0 & \times & 1\\ 0 & & 1\end{bmatrix}.$$

$$\begin{bmatrix}0 & \times & 0\\ 1 & & 1\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 1\\ 1 & & 0\end{bmatrix}\ \begin{bmatrix}1 & \times & 0\\ 1 & & 1\end{bmatrix}\ \begin{bmatrix}0 & \checkmark & 1\\ 1 & & 1\end{bmatrix}\ \begin{bmatrix}1 & \times & 1\\ 0 & & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & \checkmark & 1\\ 1 & & 1\end{bmatrix}.$$

Anti Symmetric.

$\forall a_i, b_j \in A$ if $(a_i, b_j) \in R \wedge (b_j, a_i) \in R \Rightarrow \overset{i=j}{a = b}$.

$m_{11} = 1 \wedge m_{11} = 1 \rightarrow 1 = 1$.

$\forall i, j$ if $m_{ij} = 1 \wedge m_{ji} = 1 \rightarrow i = j$.

$$\begin{bmatrix}0 & \checkmark & 0\\ 0 & & 0\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 0\\ 0 & & 0\end{bmatrix}\ \begin{bmatrix}0 & \checkmark & 1\\ 0 & & 0\end{bmatrix}\ \begin{bmatrix}0 & \checkmark & 0\\ 1 & & 0\end{bmatrix}\ \begin{bmatrix}0 & \checkmark & 0\\ 0 & & 1\end{bmatrix}.$$

$$\begin{bmatrix}1 & \checkmark & 1\\ 0 & & 0\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 0\\ 1 & & 0\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 0\\ 0 & & 1\end{bmatrix}\ \begin{bmatrix}0 & \times & 1\\ 1 & & 0\end{bmatrix}\ \begin{bmatrix}0 & \checkmark & 1\\ 0 & & 1\end{bmatrix}.$$

$$\begin{bmatrix}0 & \checkmark & 0\\ 1 & & 1\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 0\\ 1 & & 0\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 0\\ 1 & & 1\end{bmatrix}\ \begin{bmatrix}0 & \times & 1\\ 1 & & 1\end{bmatrix}\ \begin{bmatrix}1 & \checkmark & 1\\ 0 & & 1\end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad X.$$