

Lec # 8.

Q9 (a). "Every body loves Jerry"
PS4.

for all x , x is a person. x loves Jerry.

$P(x) \equiv x \text{ loves Jerry.}$ $x \in \text{Set of Persons.}$

$\forall x P(x).$

Q9 (b) Every one loves Some one.

for all x , x is a person. there exist y , y is a person.
 x loves y .

$x, y \in \text{Set of Persons.}$

Let $\cdot P(x, y) \equiv x \text{ loves } y.$

$\forall x \exists y P(x, y).$

Rules of Inference.

1- Valid Argument.

(i) If you have the current password then you can log into the network.

You have the current password. p

therefore

You can log into the network. q .

$\rightarrow p \rightarrow q.$ } premises.

$\therefore q.$ } Conclusion.

P1 $p \rightarrow q$

P1

$$\frac{P_1 \quad P \rightarrow q \\ P_2 \quad P}{C \quad \therefore q}$$

$$\frac{P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_N}{\therefore C}$$

} Argument.

Valid Argument only when.

$$((P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_N) \rightarrow C) \rightarrow \text{tautology}.$$

$$((P \rightarrow a) \wedge P) \rightarrow q \quad \text{tautology}.$$

P	q	$P \rightarrow q$	$(P \rightarrow a) \wedge P$	$((P \rightarrow a) \wedge P) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T		T

$\frac{\text{Ex 2}}{P \cdot 60}$

if $\sqrt{2} > \frac{3}{2}$ then $(\sqrt{2})^2 > (\frac{3}{2})^2$

$\sqrt{2} > \frac{3}{2}$ P.

$\therefore (\sqrt{2})^2 = 2 > (\frac{3}{2})^2 = \frac{9}{4}$ q.

$$\frac{P \rightarrow q \\ P}{\therefore q}$$

Rules of Inference.

i) $P \rightarrow q$ Modus ponens. ii) $\frac{P}{P \vee q}$ Addition.

$$\frac{P}{\therefore q}$$

$$2) P \rightarrow q \quad \text{Modus tollens.} \quad 6) \frac{\neg q}{\therefore \neg p}$$

$$3) \frac{P \rightarrow q}{\begin{array}{l} \text{(Hypothetical)} \\ q \rightarrow r \\ \therefore P \rightarrow r \end{array}} \quad \text{Syllogism}$$

$$7) \frac{\begin{array}{l} P \\ q \end{array}}{\therefore P \wedge q} \quad \text{Conjunction.}$$

$$4) \frac{P \vee q}{\begin{array}{l} \text{Disjunctive} \\ \neg P \\ \therefore q \end{array}} \quad \text{Syllogism.}$$

$$8) \frac{\begin{array}{l} P \vee q \\ \neg q \vee r \end{array}}{\therefore P \vee r} \quad \text{Resolution.}$$

E.g. "It is not sunny this afternoon $\neg P$ \wedge it is colder than yesterday q ".
 "We will go swimming $\neg s$ \rightarrow P .
 (If) we do not go swimming $\neg s$ \rightarrow (then) we will take a canoe trip t .
 "If we take a canoe trip s then we will be home by sunset e ".

Leads to Conclusion

"We will be home by sunset".

$$\begin{array}{ll} P1: \neg P \wedge q & \Rightarrow P \rightarrow q \\ P2: \neg s \rightarrow p & \therefore P \\ P3: \neg s \rightarrow s & \therefore q \\ P4: s \rightarrow t & \therefore t \\ C: \therefore t & \end{array} \quad 2) P \rightarrow q \quad \text{Modus tollens.} \quad 6) \frac{\begin{array}{l} P \wedge q \\ \neg q \end{array}}{\therefore \neg P} \quad \text{Simplification.}$$

P5: From P1 $\neg P$ by Simplification. $\therefore \neg P$.

P6: From P2, P5 $\neg s$ by MT.

P7: From P3, P6 s by MP.

P8: From P4, P7 t by MP.

$$3) \frac{P \rightarrow q}{\begin{array}{l} \text{(Hypothetical)} \\ q \rightarrow r \\ \therefore P \rightarrow r \end{array}} \quad \text{Syllogism}$$

$$7) \frac{\begin{array}{l} P \\ q \end{array}}{\therefore P \wedge q} \quad \text{Conjunction.}$$

$$\begin{array}{c}
 \text{P8: From P4, P7 \& by MP.} \\
 \text{which is Conclusion -} \quad \text{(} \quad \text{-- P} \rightarrow \text{)} \quad \therefore \text{P} \wedge \text{q} \\
 \text{4) } \frac{\text{P} \vee \text{q}}{\neg \text{P}} \quad \text{Disjunctive} \quad \text{5) } \frac{\neg \text{P}}{\therefore \text{q}} \quad \text{Syllogism.} \\
 \text{6) } \frac{\text{P} \vee \text{q}}{\neg \text{q} \vee \text{P}} \quad \text{Resolution.} \\
 \text{7) } \frac{\neg \text{q} \vee \text{P}}{\therefore \text{P} \vee \text{q}}
 \end{array}$$

$$\begin{array}{c}
 \text{Bx7:} \\
 \text{X) } \frac{\text{P} \rightarrow \text{q}}{\text{P}} \quad \text{Modus ponens.} \quad \text{7) } \frac{\text{P}}{\therefore \text{P} \vee \text{q}} \quad \text{Addition.}
 \end{array}$$

$$\begin{array}{c}
 \text{P1 } \text{P} \rightarrow \text{q}, \checkmark \\
 \text{P2 } \neg \text{P} \rightarrow \text{r}, \checkmark \\
 \text{P3 } \text{r} \rightarrow \text{s} \\
 \hline
 \text{C: } \neg \text{q} \rightarrow \text{s}
 \end{array}
 \quad
 \begin{array}{c}
 \text{2) } \text{P} \rightarrow \text{q} \quad \text{Modus tollens.} \quad \text{6) } \frac{\text{P} \wedge \text{q}}{\therefore \text{P}} \quad \text{Simplification.} \\
 \text{X) } \frac{\neg \text{q}}{\therefore \neg \text{P}}
 \end{array}$$

$$\begin{array}{c}
 \text{P4: } \neg \text{q} \rightarrow \neg \text{p.} \quad \text{by CP.} \quad \text{3) } \frac{\text{P} \rightarrow \text{q}}{\frac{\text{q} \rightarrow \text{r}}{\therefore \text{P} \rightarrow \text{r}}} \quad \text{Hypothetical Syllogism} \quad \text{7) } \frac{\text{P}}{\text{q}} \quad \text{Conjunction.} \\
 \hline
 \text{C: } \neg \text{q} \rightarrow \text{s}
 \end{array}$$

$$\begin{array}{c}
 \text{P5: } \neg \text{q} \rightarrow \neg \text{r} \text{ by HS.} \quad \text{4) } \text{P} \vee \text{q} \quad \text{Disjunctive} \quad \text{8) } \frac{\text{P} \vee \text{q}}{\neg \text{q} \vee \text{r}} \quad \text{Resolution.} \\
 \text{P6: } \text{P5, P3 } \neg \text{q} \rightarrow \text{s} \text{ by HS.} \quad \frac{\neg \text{P}}{\therefore \text{q}} \quad \text{Syllogism.} \\
 \hline
 \text{C: } \neg \text{q} \rightarrow \text{s}
 \end{array}$$

Which is Conclusion.

Problem:

$$\begin{array}{c}
 \text{Bx8} \\
 \text{P1 } \frac{\text{P} \wedge \text{q}}{\text{P} \wedge \text{s}} \\
 \text{P2 } \text{T} \rightarrow (\text{M} \vee \text{E}) \checkmark \\
 \text{P3 } \text{S} \rightarrow \neg \text{E} \checkmark \\
 \hline
 \text{C: } \frac{\text{T} \wedge \text{S}}{\text{M.}}
 \end{array}
 \quad
 \begin{array}{c}
 \text{1) } \text{P} \rightarrow \text{q} \quad \text{Modus ponens.} \quad \text{7) } \frac{\text{P}}{\therefore \text{P} \vee \text{q}} \quad \text{Addition.} \\
 \text{2) } \text{P} \rightarrow \text{q} \quad \text{Modus tollens.} \quad \text{6) } \frac{\text{P} \wedge \text{q}}{\therefore \text{P}} \quad \text{Simplification.} \\
 \text{X) } \frac{\neg \text{q}}{\therefore \neg \text{P}}
 \end{array}$$

- C. I. M.
- P4 from P3 T by S ✓ $\frac{\neg q}{\therefore \neg P}$
- P5 from P3 S by S ✓ 3) $P \rightarrow q$ Hypothetical
- P6 from P1, P4 MVB by MP ✓ $\frac{q \rightarrow r}{\therefore P \rightarrow r}$ Syllogism
- P7 from P5, P6 $\neg E$ by II ✓ $\frac{\neg P}{\therefore P \wedge q}$ Conjunction.
- P8 from P6, P7 . M
- 4) $P \vee q$ Disjunctive Syllogism.
- $\frac{\neg P}{\therefore q}$
- 5) $P \vee q$ Resolution.
- $\frac{\neg q \vee r}{\therefore P \vee r}$
- which is Conclusion.

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