

lec # 14:

N-way Relation.

$$A = \{1, 2\}$$

$$B = \{a, b\}$$

$$C = \{x, y\}$$

$|A \times B \times C| = ?$   $8 = |A| \times |B| \times |C| = 2 \times 2 \times 2 = 8.$   
possible Relations. on  $A \times B \times C.$

$$\text{Pow}(A \times B \times C) = \{\emptyset, \{(1, a, x)\}, \{(1, a, y)\}\}.$$

$$\begin{aligned} |\text{Pow}(A \times B \times C)| &= 2^{|A \times B \times C|} \\ &= 2^{|A| \times |B| \times |C|} \\ &= 2^{2 \times 2 \times 2} \\ &= 2^8 = 256. \end{aligned}$$

$B \times A$   $R = \{(a, b, c) \mid a < b < c\}$ .  $N \times N \times N$ .

$$\begin{array}{ll} (1, 2, 3) \in R & ? \\ (2, 4, 3) \notin R & ? \end{array}$$

$B \times C$   $R = \{(a, b, c) \mid b = a + k, c = a + 2k\}$ .  $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ .

$$\begin{array}{ll} (1, 3, 5) \in R & ? \checkmark \\ (2, 5, 9) \in R & ? \times \end{array}$$

$$3 = 1 + K \Rightarrow K = 2$$

$$5 = 1 + 2 \cdot 2 \Rightarrow S = 5$$

$$5 = 2 + K \Rightarrow K = 3.$$

$$9 = 2 + 2 \cdot 3 \Rightarrow 9 \neq 2 + 6$$

$$\begin{array}{r} 2 \\ 2 \sqrt{5} \\ \underline{+4} \\ \hline 1 \end{array}$$

$$5 \bmod 2 = 1.$$

$$-4 \bmod +3 = 2.$$

$$\frac{\mathbb{B}r3}{469} \vdash R = \{ (a, b, c) \mid a \equiv b \bmod c \}, \quad \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^+$$

$$(8, 2, 3) \in R \quad ? \quad \checkmark$$

$$8 \equiv 2 \bmod 3.$$

$$(-1, 9, 5) \in R \quad ? \quad \checkmark$$

$$-1 \equiv 9 \bmod 5.$$

$$(4, 0, 7) \in R \quad ?$$

Relational Database.

$$\mathbb{B}xh \vdash R$$

$$A \times N \times S \times D \times I.$$

→ (Turkish, TK-47, PBN, PXB, 22:00.)  
 $A =$  Set of Airlines.  
 $N =$  " " " flight #'s  
 $S =$  " " " Starting Points.  
 $D =$  " " " Destinations.  
 $T =$  " " " Hours.

A            N            S            D            T

Row. →

Representing Relations.

Matrices.

- 1) Square brackets.
- 2) Size, rows X Col.

$R$  is defined on  $A \times B$ .  $A = \{a_1, a_2, \dots, a_m\}$ .  
 $B = \{b_1, b_2, \dots, b_n\}$ .

$$M_R = [m_{ij}]$$

rows =  $|A|$ .

cols =  $|B|$ .

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases} \quad (a_1, b_2) \in R.$$

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$$A = \{1, 2, 3\} \quad B = \{1, 2\}$$

$$R = \{(2, 1), (3, 1), (3, 2)\}$$

$R = A \times B$ .

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Properties.

Reflexive.

$$\forall x \quad (x, x) \in R.$$

$$\forall i \quad (a_i, a_i) \in R.$$

$$\textcircled{Hi} \quad m_{ii} = 1.$$

$\downarrow$   
Row      Col.

$$[ ] \checkmark \quad |A| = 0.$$

$$[1] \checkmark$$

$$[0] X.$$

$$|A| = 2.$$

$$\begin{bmatrix} 0 & x \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & x \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & x & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & x & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & \checkmark & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & x & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & x & 2 \\ 0 & 1 \end{bmatrix}.$$

$$\Gamma_0 \times \Gamma_1 \quad \Gamma_1 \times \Gamma_1 \quad \Gamma_1 \vee \Gamma_1 \quad \Gamma_0 \times \Gamma_1 \quad \Gamma_1 \vee \Gamma_1$$

$$\begin{bmatrix} 0 \times 0 \\ 1 \times 1 \end{bmatrix} \quad \begin{bmatrix} 1 \times 1 \\ 1 \times 0 \end{bmatrix} \quad \begin{bmatrix} 1 \vee 0 \\ 1 \vee 1 \end{bmatrix} \quad \begin{bmatrix} 0 \times 1 \\ 1 \times 1 \end{bmatrix} \quad \begin{bmatrix} 1 \vee 1 \\ 0 \vee 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \vee 1 \\ 1 \vee 1 \end{bmatrix}.$$

Symmetric.

$$H_{a,b} \quad \text{if } (a,b) \in R \rightarrow (b,a) \in R.$$

$$H_{a_i, b_j} \quad \text{if } (a_i, b_j) \in R \rightarrow (b_j, a_i) \in R.$$

$$H_{i,j} \quad \text{if } m_{ij} = 1 \rightarrow m_{ji} = 1.$$

$$[ ] \checkmark$$

$$[0] \checkmark$$

$$[1] \checkmark$$

$$\begin{bmatrix} 0 \vee 0 \\ 0 \vee 0 \end{bmatrix} \quad \begin{bmatrix} 1 \vee 0 \\ 0 \vee 0 \end{bmatrix} \quad \begin{bmatrix} 0 \times 1 \\ 0 \times 0 \end{bmatrix} \quad \begin{bmatrix} 0 \times 0 \\ 1 \times 0 \end{bmatrix} \quad \begin{bmatrix} 0 \vee 0 \\ 0 \vee 1 \end{bmatrix}$$

$$r_1 \times a_1 \quad r_1 \times 1 \quad r_1 \vee a_1 + n \vee a_1 \quad r_n \times a_1$$

$$\begin{bmatrix} 1 \times 1 \\ 0 \ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \times 0 \\ 2 \ 0 \end{bmatrix} \quad \begin{bmatrix} 2 \checkmark 0 \\ 0 \ 2 \end{bmatrix} \quad \begin{bmatrix} 0 \checkmark 1 \\ 1 \ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \times 2 \\ 0 \ \$ \end{bmatrix}, \\
 \begin{bmatrix} 0 \times 0 \\ 2 \ 2 \end{bmatrix} \quad \begin{bmatrix} 2 \checkmark 2 \\ 1 \ 0 \end{bmatrix} \quad \begin{bmatrix} 2 \times 0 \\ 2 \ 2 \end{bmatrix} \quad \begin{bmatrix} 0 \checkmark 1 \\ 2 \ 1 \end{bmatrix} \quad \begin{bmatrix} 2 \times 2 \\ 0 \ 1 \end{bmatrix} \\
 \begin{bmatrix} 2 \checkmark 1 \\ 1 \ 2 \end{bmatrix}.$$

Anti Symmetric.

$$\forall a_i, b_j \in A \quad \text{if } (a_i, b_j) \in R \wedge (b_j, a_i) \in R \Rightarrow a_i \stackrel{i \neq j}{\sim} b_j.$$

$$\begin{array}{c}
 \begin{array}{ccccc}
 m_{ii}=1 & \wedge & m_{jj}=1 & \rightarrow & i=j. \\
 m_{ij}=1 & \wedge & m_{ji}=1 & \rightarrow & i \neq j.
 \end{array} \\
 \begin{bmatrix} 0 \checkmark 0 \\ 0 \ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \checkmark 0 \\ 0 \ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \checkmark 1 \\ 0 \ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \checkmark 0 \\ 2 \ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \checkmark 0 \\ 0 \ 1 \end{bmatrix}, \\
 \begin{bmatrix} 1 \checkmark 1 \\ 0 \ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \checkmark 0 \\ 2 \ 0 \end{bmatrix} \quad \begin{bmatrix} 2 \checkmark 0 \\ 0 \ 2 \end{bmatrix} \quad \begin{bmatrix} 0 \times 1 \\ 1 \ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \checkmark 2 \\ 0 \ \$ \end{bmatrix}, \\
 \begin{bmatrix} 0 \checkmark 0 \\ 2 \ 2 \end{bmatrix} \quad \begin{bmatrix} 2 \checkmark 0 \\ 1 \ 0 \end{bmatrix} \quad \begin{bmatrix} 2 \checkmark 0 \\ 2 \ 2 \end{bmatrix} \quad \begin{bmatrix} 0 \times 1 \\ 2 \ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \checkmark 1 \\ 0 \ 1 \end{bmatrix}
 \end{array}$$

$$\left\{ \begin{bmatrix} 1 & - \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} - & 1 \\ 1 & \end{bmatrix}, \begin{bmatrix} - & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} - & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} - & 1 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & x \\ 1 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} x.$$