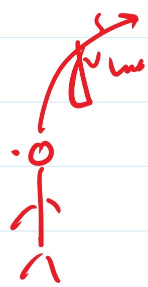
lec # 13.

Properties of |Relations|. ?

Reflexive, Symmetric.



Structures.
- Set
- Matrices
- Graph.
- Trees

→ CS.

Anti Symmetric :-

$\forall x, y$ if $\underbrace{\boxed{(x,y) \in R \wedge (y,x) \in R}}_{p} \rightarrow \overset{q}{\boxed{x = y}}$

$p \rightarrow q$.

F   T.

Ex7 :-
P461

$A = \{1,2,3,4\}$.

$R_1 = \{\}$ ✓

$R_2 = \{(1,1)\}$ ✓

$R_3 = \{(1,2)\}$. ✓

$R_4 = \{(1,2), (1,3), (1,4), (2,3)\}$.

Transitive Property.

$\forall x, y, z$ if $(x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$.

Ex7 :-
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$A = \{1,2,3,4\}$.

$R_1 = \{\}$. ✓

$R_2 = \{(1,1)\}$ ✓

$R_2 = \{(1,1)\}$ ✓

$R_3 = \{(1,2), (2,3), (1,3), (3,4), (1,4), (1,1)\}$. X.

$x \quad y \qquad\qquad y \quad z$.

1) Reflexive 2) Symmetric 3) Anti Symmetric
4) Transitive

Ex12: 463

$R = \{(a,b) \mid a \text{ divides } b\}$ $A = Z^+$.

$\geq$, $>$, $<$, $\leq$, $=$

Reflexive: $\forall x \in A \ (x,x) \in R$.
$\forall x \in Z^+$ x divides x. ✓

Symmetric: $\forall x,y \in A$ if $(x,y) \in R \rightarrow (y,x) \in R$.

$\forall x,y \in Z^+$ if x divides y $\rightarrow$ y divides x. X.
1 divides 2 $\rightarrow$ 2 divides 1.
T $\rightarrow$ F.
F.

Anti Symmetric $\forall x,y$ if $(x,y) \in R \wedge (y,x) \in R \rightarrow x = y$.
$\forall x,y \in Z^+$ if x divides y $\wedge$ y divides x $\rightarrow$ x = y. ✓

Transitive $\forall x,y,z$ if $(x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$.
$\forall x,y,z \in Z^+$ if x divides y $\wedge$ y divides z $\rightarrow$ x divides z. ✓

$\forall x,y,z \in \mathbb{Z}$ if x divides y $\wedge$ y divides z $\rightarrow$ x divides z. ✓

Inverse of a Relation R^{-1}

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$R = \{(1,1), (2,1)\}$. $\quad R^{-1} = \{(1,1), (1,2)\}$.

Complement of a Relation $\bar{R}$.

$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$.

$R = \{(1,1)\}$. $\quad A = \{1,2\}$.

$\bar{R} = \{(1,2), (2,1), (2,2)\}$. $\quad A \times A = \{\cancel{(1,1)}, (1,2), (2,1), (2,2)\}$.

$A \times A - R = \{(1,2), (2,1), (2,2)\}$.

Ex 467:

$x + y = 0$.

$R = \{(a,b) \mid x = \pm y\}$. $\quad A = \mathbb{R}$.

$R = \{(a,b) \mid x = y \text{ or } x = -y\}$.

Reflexive: $\forall x \in A \ (x,x) \in R$.

$\forall x \in \mathbb{R} \quad x = x \vee x = -x$ ✓

Symmetric: $\forall x,y \in A$ if $(x,y) \in R \rightarrow (y,x) \in R$.

$\forall x,y \in \mathbb{R}$

Anti Symmetric $\forall x,y$ if $(x,y) \in R \wedge (y,x) \in R \rightarrow x = y$.
$\forall x, y \in$

Transitive $\forall x,y,z$ if $(x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$.
$\forall x,y,z \in$

Ex 17 : $A = \{1,2,3\}$ $B = \{1,2,3,4\}$.
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$R_1 = \{(1,1),(2,2),(3,3)\}$.
$R_2 = \{(1,1),(1,2),(1,3),(1,4)\}$.

$R_1 \cap R_2 \{(1,1)\}$.
$R_1 \cup R_2 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(1,4)\}$.
$R_1 - R_2 = \{(2,2),(3,3)\}$.
$R_2 - R_1 = ?$

Ex 466- 468
1- 35.

Composite : R (a,b) $A \times B$.
S (b,c) $B \times C$.

$S \circ R$ (a,c).
$(a,c) \in S \circ R$ if $(a,b) \in R \wedge (b,c) \in S$.

Ex 20 R $A \times B$. $A = \{1,2,3\}$.
P 465 S $B \times C$. $B = \{1,2,3,4\}$.

[illegible] R $A \times B$. $A = \{1,2,3\}$.

P 465 S $B \times C$. $B = \{1,2,3,4\}$.

$C = \{0,1,2\}$.

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$.

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$.

$S \circ R = ?$

$S \circ R$ (a,c). $(1,1) \in R$ $\wedge$ $(1,0) \in S$.

$(a,c) \in S \circ R$ iff $\underline{(a,b) \in R}$ $\wedge$ $\underline{(b,c) \in S}$.

$(1,0) \in S \circ R$.

$S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$.

$R \circ S \neq S \circ R$.

$R^2 = R \circ R = ?$ $R \circ S = ?$

$R^3 = R^2 \circ R$.

$\vdots$

$R^n = R^{n-1} \circ R$.