lec #6

Negating Quantifiers.

De-Morgan's law

$\neg(p \wedge q) = \neg p \vee \neg q$.

$\neg(p_1 \wedge p_2 \wedge p_3 \wedge \cdots \wedge p_n) = \neg p_1 \vee \neg p_2 \vee \cdots \vee \neg p_n$

$\neg(p \vee q) = \neg p \wedge \neg q$

$\neg(p_1 \vee p_2 \vee p_3 \vee \cdots \vee p_n) = \neg p_1 \wedge \neg p_2 \wedge \cdots \wedge p_n$.

$\neg \forall x\, P(x) = \neg(P(1) \wedge P(2) \wedge \cdots \wedge P(N))$ $\quad x \in \{1, 2, 3, \cdots N\}$.

$= \underline{\neg P(1)} \vee \underline{\neg P(2)} \vee \cdots \vee \neg P(N)$

$=$

$= \exists_x \neg P(x)$.

① — $\neg \forall x P(x) = \exists x \neg P(x)$.

$\neg \exists x P(x) = \neg(P(1) \vee P(2) \vee P(3) \vee \cdots \vee P(N))$.

$= \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \cdots \wedge \neg P(N)$.

$= \forall x \neg P(x)$.

② — $\neg \exists x P(x) = \forall x \neg P(x)$.

Examples:- $\forall x \exists y \forall z\; P(x, y, z)$ Negate.

① — $\neg \forall x P(x) = \exists x \neg P(x)$.

② — $\neg \exists x P(x) = \forall x \neg P(x)$.

$\neg \forall x \boxed{\exists y \forall z\; P(x, y, z)}$ — $P(x)$

Assume

$P(x) = \exists y \forall z\, P(x, y, z)$ Apply ①.

$= \exists x \neg \exists y \boxed{\forall z\, P(x, y, z)}$ $P(x)$.

Assume $P(x) = \forall z P(x, y, z)$
Apply ②

$= \exists x\; \forall y \neg \forall z\; P(x, y, z)$.

$= \exists x\, \forall y\, \exists z\, \neg P(x, y, z)$.

$\forall x\, \exists y\, \neg \forall z\, \neg P(x, y, z)$ HW.

Ex 20 P39 "$\underbrace{\text{There is}}_{\exists}$ an $\underbrace{\text{honest}}_{\text{predicate}}$ $\underbrace{\text{politician}}_{\text{Domain}}$". find Negation.

$P(x) =$ ______ $x \in \{\ \}$.
Subject. Domain.

" There exist x, x is a politician, x is honest "

Let $P(x) =$ x is honest. x is a politician.
$x \in \{\text{politicain}\}$.

$\exists x P(x)$.

$\neg \exists x P(x) = \forall x \neg P(x)$.

For all x, x is a politician, x is not honest.

find Negation of "All American eat cheese burger".

"for all x, x is an American, x eat cheese burger"

Let $P(x) =$ x eat cheese burger $x \in \{\text{Americans}\}$.

$\forall x\, P(x)$.

$\neg \forall x\, P(x) = \exists x \neg P(x)$.

there exist x, x is an American, x does not eat cheese burger.

Ex21 p39 $\forall x\,(x^2 > x)$

let $P(x) = x^2 > x$.

$\forall x\, P(x)$ $\neg(>) = \leq$.

$\neg \forall x\, P(x) = \exists x\, \neg P(x)$.

$= \exists x\, \neg (x^2 > x)$.

$= \exists x\; x^2 \leq x$.

$\exists x\,(x^2 = 2)$.

Ex23 p40 HW.

Ex p43–45 1–40.

17: $P(x)$ $x \in \{0, 1, 2, 3, 4\}$.

i) $\exists x\, P(x)$. $\vee, \wedge, \neg$.

$P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$.

ii) $\exists x \neg P(x)$.

$\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$.

iii) $\neg \exists x\, P(x)$. $= \forall x \neg P(x)$.

$\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$.

Every one is studying DS.

For all x, x is ___ , x is study PS. True.
9.

$x \in \{human\}$ False.
$x \in \{$a student in This Class$\}$ True.

Some predicates are Propositions.

There exist x, x is a predicate, x is a Propositions.

Let $P(x) =$ x is a Proposition.

$\exists x\, P(x)$.

Q30 :- $\exists x\, P(x,3)$. $x, y \in \{1, 2, 3\}$.

$\neg, \wedge, \vee$.

$P(1,3) \vee P(2,3) \vee P(3,3)$.