Lecture 19:-

PARTIAL ORDER.

Ex 13 $(P(S), \subseteq)$ $S = \{a,b,c\}$.

P509 HASSE DIAGRAM.

$P(S) \times P(S)$. $= |P(S)| \times |P(S)| = 2^3 \times 2^3 = 2^6 = 64$.

$R = \{ \cancel{(\phi,\phi)}, (\phi,\{a\}), (\phi,\{b\}), \ldots,$

$\cancel{(\{a\},\{a\})}, (\{a\},\{a,b\}), (\{a\},\{a,c\}), (\{a\},\{a,b,c\}),$

$\cancel{(\{b\},\{b\})}, (\{b\},\{a,b\}), (\{b\},\{b,c\}), (\{b\},\{a,b,c\}),$

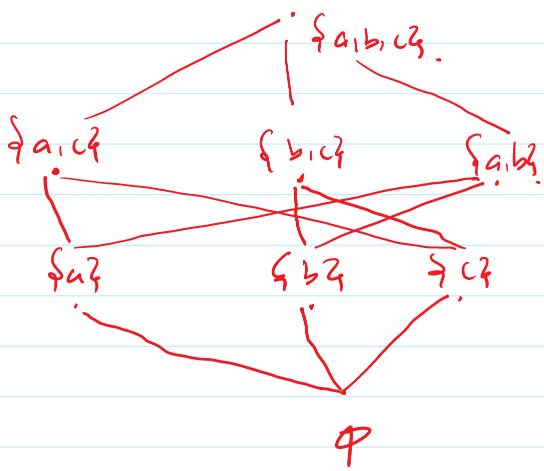
$\cancel{(\{c\},\{c\})}, (\{c\},\{a,c\}), (\{c\},\{b,c\}), (\{c\},\{a,b,c\}),$

$\cancel{(\{a,b\},\{a,b\})}, (\{a,b\},\{a,b,c\}),$

$\cancel{(\{a,c\},\{a,c\})}, (\{a,c\},\{a,b,c\}),$

$\cancel{(\{b,c\},\{b,c\})}, (\{b,c\},\{a,b,c\}),$

$\cancel{(\{a,b,c\},\{a,b,c\})}, \}$.

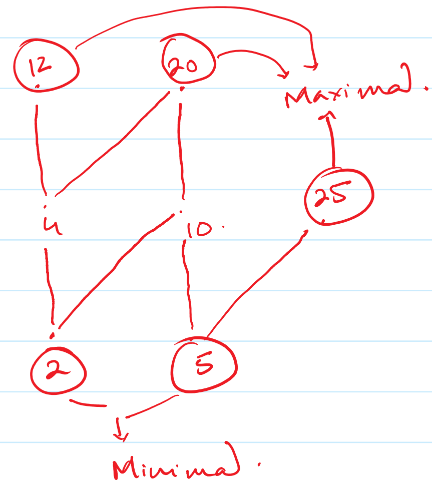


Maximal:- An element $a \in S$ is maximal in $(S, \leq)$ if $\neg\exists_{b \in S}$ such that $a < b$. $(a,b) \notin R$.

Minimal:- An element $a \in S$ is minimal in $(S, \leq)$ if $\neg\exists_{b \in S}$ such that $b < a$. $(b,a) \notin R$.

Ex14 :- $(\{2, 4, 5, 10, 12, 20, 25\}, \mid)$.
509

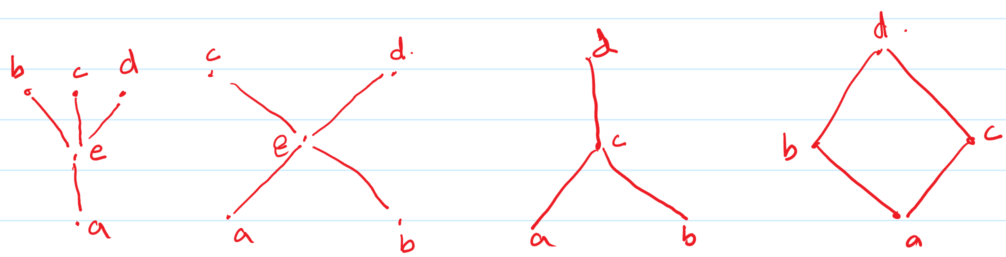
Minimal = ?
Maximal = ?



Greatest :- $a \in S$ is greatest in $(S, \preccurlyeq)$
if $\forall_{b \in S}$ $b \preccurlyeq a$.
$(b, a) \in R$.

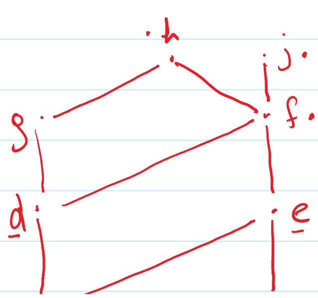
Least :- $a \in S$ is least in $(S, \preccurlyeq)$.
if $\forall_{b \in S}$ $a \preccurlyeq b$.

Ex

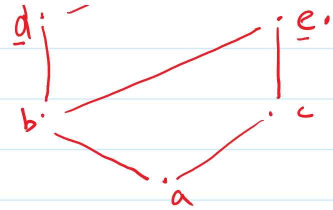


Max	b, c, d	c, d	d	d
Min	a	a, b	a, b	a
Greatest:	$\nexists$	$\nexists$	d	d
Least.	a	$\nexists$	$\nexists$	a.

Ex18



Lower Bound $\{a, b, c\}$.
$= \{a\}$.
Upper Bound $\{a, b, c\}$.
$= \{e, f, h, j\}$.



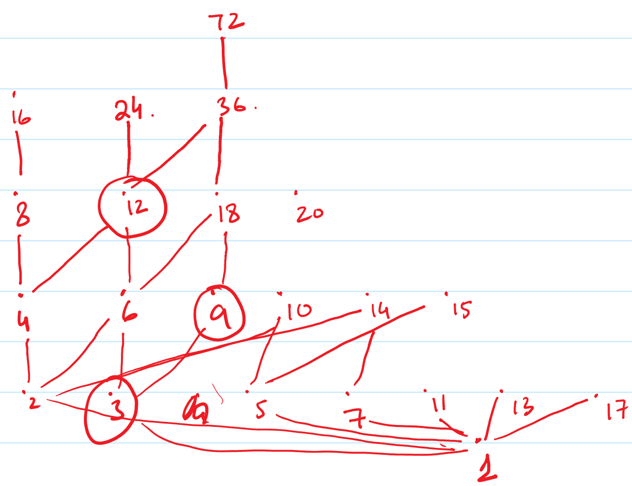
= {e, f, h, j}.

greatest lower Bound. {a, b, c}
= {a}.

least Upper Bound {a, b, c}.
= {e}.

least Upper Bound {d, e}. = {f, j, h}.
= f.

Ex 20 :- Greatest lower Bound & least Upper Bound
P511 of the Poset (Z^+ , |).
{3, 9, 12}.

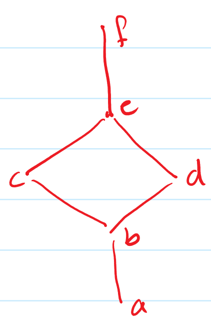


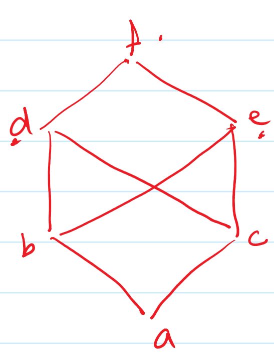
lower Bound {3, 9, 12}
= {1, 3}.
GLB = 3.
GCD = GLB.

Upper Bound {3, 9, 12}.
{36, 72, 108, 144
---}.

least UB = 36.
LCM = LUB.

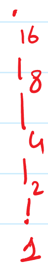
Lattice :- A partial Order in which each pair of elements has GLB & LUB.





UB{b, c} = {d, e, f}.
LUB{b, c} = ϕ.

Ex 23 $(\{1,2,4,8,16\}, |)$.



TO. z Lattice.

Quiz #4.

25-OCT-2023.