Lec # 8.

Q9 (a). PS4. "Every (body) loves Jerry"

for all x, x is a person. x loves jerry.

$P(x)$: x loves Jerry. $x \in$ Set of Persons.

$\forall x\, P(x)$.

Q9 (b) PS4 Every (one) loves Some (one).

for all x, x is a person. there exist y, y is a Person.
x loves y.

$x, y \in$ Set of Persons.

Let $P(x,y)$: x loves y.

$\forall x \exists y\, P(x,y)$.

Rules of Inference.

1- Valid Argument.

(if) you have the current password (p) (then) you can log into the network. (q)

you have the current password. (p)

therefore

you can log into the network. (q)

$p \rightarrow q$.
p
$\therefore q$.

} premises.
} Conclusion.

P1 $p \rightarrow q$ P1

p_1 $\quad p \to q$

p_2 $\quad p$

c $\quad \therefore q$

p_1

p_2

p_3

$\vdots$

p_N

$\therefore c$

} Argument.

Valid Argument only when.

$((p_1 \wedge p_2 \wedge p_3 \wedge \cdots \wedge p_n) \to c)$ → tautology.

$((p \to q) \wedge p) \to q$ tautology.

p	q	$p \to q$	$(p \to q) \wedge p$	$((p \to q) \wedge p) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Ex 2 p. 60

if $\sqrt{2} > \frac{3}{2}$ (1.4 < $\sqrt{2}$ < 1.5) then $(\sqrt{2})^2 > (\frac{3}{2})^2$ q.

$\sqrt{2} > \frac{3}{2}$ p.

$\therefore (\sqrt{2})^2 = 2 > (\frac{3}{2})^2 = \frac{9}{4}$ q.

$p \to q$

p

$\therefore q$.

Rules of Inference.

1) $p \to q$, p / $\therefore q$ Modus ponens.

2) p / $\therefore p \vee q$ Addition.

2) $p \to q$, $\neg q$, $\therefore \neg p$. Modus tollens.

3) $p \to q$, $q \to r$, $\therefore p \to r$. Hypothetical Syllogism

4) $p \vee q$, $\neg p$, $\therefore q$. Disjunctive Syllogism.

6) $p \wedge q$, $\therefore p$. Simplification.

7) p, q, $\therefore p \wedge q$. Conjunction.

8) $p \vee q$, $\neg q \vee r$, $\therefore p \vee r$. Resolution.

Ex 6 pg 2

"It is not Sunny (p) this afternoon and ($\wedge$) it is Colder (q) than yesterday"

"We will go Swimming (r) only if ($\to$) it is Sunny (p)"

"If ($\to$) we do not go swimming (r) then ($\to$) we will take a Canoe trip (s)".

"If we take a Canoe trip (s) then we will be home by Sunset (t)"

Leads to Conclusion

"We will be home by Sunset" (t).

P1: $\neg p \wedge q$ ✓
P2: $r \to p$ ✓
P3: $\neg r \to s$ ✓
P4: $s \to t$ ✓
C: $\therefore t$

P5: from P1 $\neg p$ by Simplification. ✓
P6: from P2, P5 $\neg r$ by MT ✓
P7: from P3, P6 s by MP. ✓
P8: from P4, P7 t by MP.

1) $p \to q$, p, $\therefore q$. Modus ponens.

2) $p \to q$, $\neg q$, $\therefore \neg p$. Modus tollens.

3) $p \to q$, $q \to r$, $\therefore p \to r$. Hypothetical Syllogism

4) $p \vee q$. Disjunctive

5) p, $\therefore p \vee q$. Addition.

6) $p \wedge q$, $\therefore p$. Simplification.

7) p, q, $\therefore p \wedge q$. Conjunction.

P8 :- from P4,P7 & by MP.

Which is Conclusion.

4) $\frac{p \vee q \quad \neg p}{\therefore q}$ Disjunctive Syllogism.

8) $\frac{p \vee q \quad \neg q \vee r}{\therefore p \vee r}$ Resolution.

Ex 7 :-

P1 $p \to q$ ✓

P2 $\neg p \to r$ ✓

P3 $r \to s$

C :. $\neg q \to s$

P4 :- $\neg q \to \neg p$. by CP ✓

P5 :- P4,P2 $\neg q \to r$ by HS.

P6 :- P5,P3 $\neg q \to s$ by HS.

Which is Conclusion.

1) $\frac{p \to q \quad p}{\therefore q}$ Modus ponens.

2) $\frac{p \to q \quad \neg q}{\therefore \neg p}$ Modus tollens.

3) $\frac{p \to q \quad q \to r}{\therefore p \to r}$ Hypothetical Syllogism

4) $\frac{p \vee q \quad \neg p}{\therefore q}$ Disjunctive Syllogism.

5) $\frac{p}{\therefore p \vee q}$ Addition.

6) $\frac{p \wedge q}{\therefore p}$ Simplification.

7) $\frac{p \quad q}{\therefore p \wedge q}$ Conjunction.

8) $\frac{p \vee q \quad \neg q \vee r}{\therefore p \vee r}$ Resolution.

Problem :-

Ex 8

P 65

P1 $T \to (M \vee E)$ ✓

P2 $S \to \neg E$ ✓

P3 $\frac{T \wedge S}{M.}$ ✓

C :.

C:. M1. $\neg q$ ✓ $\therefore \neg p$. $\therefore p$

P4 from P3 T by S ✓

P5 from P3 S by S ✓

P6 from P1, P4 MVE by MP ✓

P7 from P5, P2 $\neg E$ by II ✓

P8 from P6, P7. M

Which is Conclusion.

3) $p \to q$, $q \to r$ / $\therefore p \to r$ — Hypothetical Syllogism

4) $p \vee q$, $\neg p$ / $\therefore q$ — Disjunctive Syllogism.

7) p, q / $\therefore p \wedge q$ — Conjunction.

8) $p \vee q$, $\neg q \vee r$ / $\therefore p \vee r$ — Resolution.