lec #5 :- Propositional Equivalences.

1- Tautology :- Always True.

2- Contradiction :- " False.

3- Contingency :- Sometimes True & other times false.

Ex :-

p	$\neg p$	$p \vee \neg p$.	$p \wedge \neg p$.
T	F	T	F
F	T	T	F
		Tautology.	Contradiction.

p	q	$p \wedge q$.	$p \to q$	$p \vee q$	$\neg p \wedge q$	$\neg p \wedge \neg q$ - - - -
T	T	T				
T	F	F				
F	T	F				
F	F	F				
		Contingency.				

Equivalence :- Two Compound Propositions p & q. are logically equivalent if "$p \leftrightarrow q$". it becomes a tautology.

Ex2 :- $\neg(p \vee q)$ & $\neg p \wedge \neg q$ are equivalent.
P32 $\equiv$

p	q	$p \vee q$	$\neg(p \vee q)$.	$\neg p$	$\neg q$	$\neg p \wedge \neg q$.	$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$.
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T.	T

P32 (HW). P33 Ex3.

P32 (HW). P33 Ex3.
Ex2 HW

P34
Table.

p	T	$p \wedge T$
T	T	T
F	T	F

$p \wedge T \equiv p$ Identity laws.
$p \vee F \equiv p$

$p \vee T \equiv T$ Domination laws.
$p \wedge F \equiv F$

$p \vee p \equiv p$ Idempotent laws.
$p \wedge p \equiv p$

$\neg(\neg p) \equiv p$ Double Negation.

$p \vee q \equiv q \vee p$. Commutative laws. $p \to q \equiv q \to p$. $\to$ X.
$p \wedge q \equiv q \wedge p$. HW.
Truth table.

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
$p \vee (q \vee r) \equiv (p \vee q) \vee r$

$p \to (q \to r) \not\equiv (p \to q) \to r$

DeMorgan: $\neg(p \wedge q) \equiv \neg p \vee \neg q$.
$\neg(p \vee q) \equiv \neg p \wedge \neg q$.

Predicates:- Ex1 $x > 3$ $x \in \{1, 2, 3, 4\}$.
P31.

$\boxed{P(x) = x > 3}$

$P(1) = 1 > 3$. F
$P(2) = 2 > 3$ F
$P(3) = 3 > 3$ F

$P(2) = 2 > 3$ F
$P(3) = 3 > 3$ F
$P(4) = 4 > 3$. T.

General form $P(x) =$ ____ . Domain.

Subject. Condition = predicate.

Ex3 P31 $Q(x,y) = x = y + 3$ $Q(1,2)$ $Q(3,0)$.

$Q(1,2) = 1 = 2+3$. (F).

$Q(3,0) = 3 = 0+3$ (T).

Ex3 P31 let $P(x) =$ Computer x is under attack.

Suppose CS2 & Math1 are under attack. find truth value of.

$P(CS1) = ?$ F
$P(CS2) = ?$ T,

Ex 1, 4, 5 (P31) HW.

Quantifiers. Quality Quantity.
معيار مقدار

Universal. $\forall$ $P(x)$ $x \in \{1, 2, 3 \dots N\}$.
for all, for each, given any, for any.
$\forall x \, P(x) = P(1) \wedge P(2) \wedge P(3) \wedge \dots \wedge P(N)$.

Existential. $\exists$

there exist, for some. $\exists x\, P(x) = P(1) \vee P(2) \vee P(3) \vee \cdots \vee P(N).$

Ex8 Let $P(x) = x+1 > x$ $x \in \mathbb{R}.$

P33

$\forall x\, P(x) = T$

Ex9 $P(x) = x < 2$ $x \in \mathbb{R}.$

P34

$\forall x\, P(x) = F$

Ex10 $P(x) = x^2 > 9.$ $x \in \{1, 2, 3, 4\}.$

P34.

$\forall x\, P(x) = P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(N).$

$\forall x\, P(x) = P(1) \wedge P(2) \wedge P(3) \wedge P(4).$

$(1^2 > 9) \wedge (2^2 > 9) \wedge (3^2 > 9) \wedge (4^2 > 9).$

$F \wedge F \wedge F \wedge T = F.$

Ex. 14: $P(x) = x > 3$ $x \in \mathbb{R}.$

P35 $\exists x\, P(x) = x > 3.$

$100{,}000 > 3$ ✓.

Ex15 $P(x) = x = x+1.$ $x \in \mathbb{R}.$

P35 $\exists x\, P(x) = F$

Quiz #2. 05-Sep-2023.

if today is not friday and it is not raining Then if today is not friday and it is not raing then we are in Rom 5.

if the above statement is true then knights speak truth and knaves also speak truth otherwise both knaves & knights speak lies. Solve the case below.?

A	B
Knight	Knight

$p \rightarrow q$ = ?

$q \rightarrow p$. = ?

p = ? $\neg p$ = ?

q = ? $\neg q$. = ?