Lec # 7.

$x \in \{1, 2, \dots N\}$.

$\forall x\, P(x) = P(1) \wedge P(2) \wedge \dots \wedge P(N)$.

$\exists x\, P(x) = P(1) \vee P(2) \vee \dots \vee P(N)$.

Nested Quantifiers.

$\forall x \forall y\ P(x, y)$. $\qquad x, y \in \{1, 2, 3, \dots N\}$.

$= \forall x (P(x,1) \wedge P(x,2) \wedge P(x,3) \wedge \dots \wedge P(x,N))$.

$= \underline{\forall x\, P(x,1)} \wedge \forall x\, P(x,2) \wedge \forall x\, P(x,3) \dots \wedge \forall x\, P(x,N)$.

$= (P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \dots \wedge P(N,1)) \wedge$.

$P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \dots \wedge P(N,2) \wedge$.

$P(1,3) \wedge P(2,3) \wedge P(3,3) \wedge \dots \wedge P(N,3) \wedge$

$\vdots \qquad\qquad \vdots$

$P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \dots \wedge P(N,N)$.

$\forall x \exists y\ P(x,y) = \forall x (P(x,1) \vee P(x,2) \vee P(x,3) \vee \dots \vee P(x,N))$.

$= \forall x P(x,1) \vee \forall x\, P(x,2) \vee \forall x\, P(x,3) \vee \dots \vee \forall x\, P(x,N)$.

$= (P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \dots \wedge P(N,1)) \vee$.

$(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \dots \wedge P(N,2)) \vee$

$\vdots \qquad\qquad \vdots$

$(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \dots \wedge P(N,N))$.

HW. $\exists x \forall y\ P(x,y)$. ? $\qquad$ HW. $\exists x \forall y\ P(y,x)$?

HW $\exists x \exists y\ P(x,y)$?

Ex 1 P47 $\qquad \forall x \forall y\ (x+y = y+x)$: $\qquad x, y \in \mathbb{R}$

Let $P(x,y) = x+y = y+x$.

$\forall x \forall y\ P(x,y) = $ True.

$x = -4$

$y = +2$.

Ex 4 P48 : $\qquad Q(x,y) = x+y = 0$

$\exists y \forall x\ Q(x,y) = ?$ $\qquad x, y \in \mathbb{R}$.

P48

$\exists y \forall x\ Q(x,y) = ?$ $\qquad x, y \in \mathbb{R}$.

$\forall x \exists y\ Q(x,y) = ?$ $\qquad x + y = 0$.

برای   کدام

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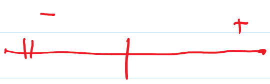

Ex5 : P49

$Q(x,y,z) = \quad x + y = z$. $\qquad x, y, z \in \mathbb{R}$.

$\forall x \forall y \exists z\ Q(x,y,z) = ?$

$\exists z \forall x \forall y\ Q(x,y,z) = ?$

Ex12 : P52

"Every one has exactly one best friend"

"for all x, x is a person, there exist y, y is a person, x and y are different, for all z,

if x is the best friend of y, then x can not best friend of z.

let $B(x,y) =$ x is the best friend of y.

$x, y, z \in$ persons.

$\forall x \exists y \forall z \left(B(x,y) \wedge x \neq y \right) \rightarrow \neg B(x,z)$.

Domain.

Ex15 (c) P56

$\boxed{\text{Every}}$ $\boxed{\text{Student in this class}}$ has taken $\boxed{\text{atleast one}}$ $\boxed{\text{CS Course}}$.

Domain.

for all x, x is a Student, there exist y, y is a CS Course, x has taken y.

let $P(x,y) =$ x has taken y.

$\forall x \exists y\ P(x,y)$.

$x \in$ Set of Student in This class.

$y \in$ Set of CS Courses.

Q15 (f) P56

"$\boxed{\text{there}}$ $\boxed{\text{is a Student in this class,}}$ who has been in every $\boxed{\text{room}}$ of atleast one $\boxed{\text{building on Campus}}$"

Q15 (f)
P56

"there is a Student in this class, who has been in every room of atleast one building on Campus"

$P(x, y, z)$. "x has been to y of z"

$x \in$ Set of Students in this class.
$y \in$ Set of rooms in a building.
$z \in$ Set of building on Campus.

"there exist x, X is a Student in the class. for all y, y is a room on Campus. there exist z, z is a building on Campus, x has been to y on z"

$\exists x \forall y \exists z \; P(x, y, z).$

$P(x, y, z) = \quad x + y \geq z$

$\forall x \exists y \forall z \; P(x, y, z) \qquad x, y, z \in \{1, 2\}.$

Expand this using $\wedge, \vee, \neg$.