lec # 17

Equivalence Relation.

1) Reflexive.
2) Symmetric
3) Transitive.

$\Big\}$ At a Single time.

Ex2 :-
494

$R = \{(a,b) \mid a-b \in \mathbb{Z}\}$. $A = \mathbb{Z}$

1) Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ $a-a \in \mathbb{Z}$.
$0 \in \mathbb{Z}$ ✓.

2) Symmetric. $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ $a-b \in \mathbb{Z} \rightarrow b-a \in \mathbb{Z}$. ✓

3) Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if $a-b \in \mathbb{Z} \wedge b-c \in \mathbb{Z} \rightarrow a-c \in \mathbb{Z}$.
✓.

E_R

Ex3:-

$R = \{(a,b) \mid a \equiv b \bmod (m)\}$. $m > 1$.
$A = \mathbb{Z}$.

1) Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ $a \equiv a \bmod m$. ✓

2) Symmetric. $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ $a \equiv b \bmod m \longrightarrow b \equiv a \bmod m$. ✓

3) Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}$ if $a \equiv b \bmod m \wedge b \equiv c \bmod m \to a \equiv c \bmod m$.

✓.

Ex 7 / 495 $\quad R = \{(a,b) \mid |a-b| < 1\}$. $\quad A = \mathbb{R}$.

1) Reflexive $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{R}$. $\quad |a-a| < 1$.

$|0| < 1$ ✓.

2) Symmetric. $\forall a,b \in A$ if $(a,b) \in R \to (b,a) \in R$.

$\forall a,b \in \mathbb{R}$ $\quad |a-b| < 1 \to |b-a| < 1$.

3) Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$.

$\forall a,b,c \in \mathbb{R}$.

$|a-b| < 1 \wedge |b-c| < 1 \to |a-c| < 1$.

$|1.5-0.6| < 1 \wedge |0.6-0.2| < 1 \to |1.5-0.1|$

$0.9 < 1 \qquad 0.5 < 1 \qquad 1.4 \not< 1$

$a = 1.5$

$b = 0.6$

$c = 0.2$.

Ex 6 / 495 (HW) Divides Relation.

Equivalence Classes.

$[a] = \{ s \mid (a,s) \in R \}$.

$(7,7), (7,-7),$

Ex 8 / 496 $\quad R = \{(a,b) \mid \overset{-7=-7}{a=b} \text{ or } \overset{-7=-(7)}{a=-b}\}$. $\quad A = \mathbb{Z}$.

$[7] = \{7, -7\}$

$[-7] = \{-7, 7\}$.

Ex9: 496.

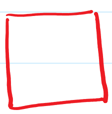
$R = \{(a,b) \mid a \equiv b \bmod m\}$. $A = \mathbb{Z}$.

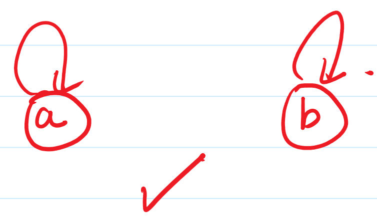
$0 \equiv b \bmod 4$.

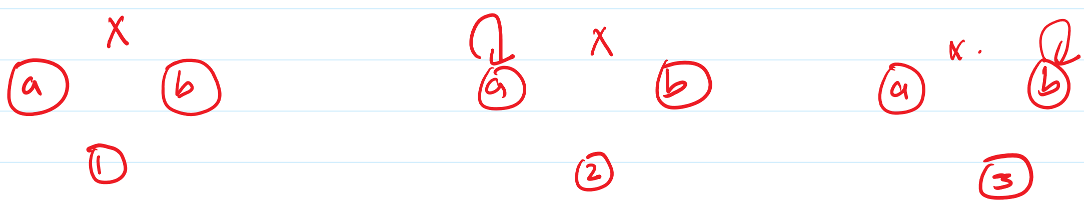
$[0] = \{0, \pm 4, \pm 8, \pm 12, \pm 16, \ldots\}$.

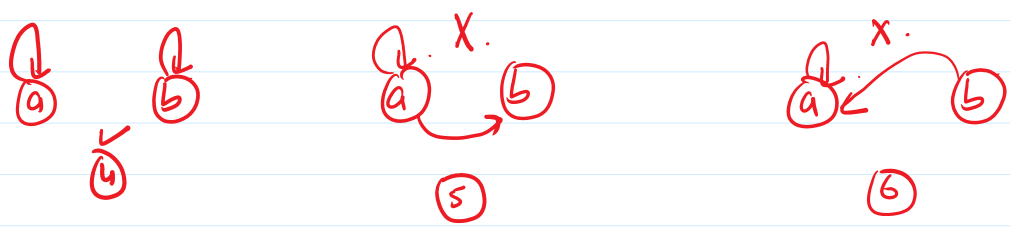
$[1] = \{1, 5, 9, 13, \ldots, -3, -7, -11, -15, \ldots\}$.

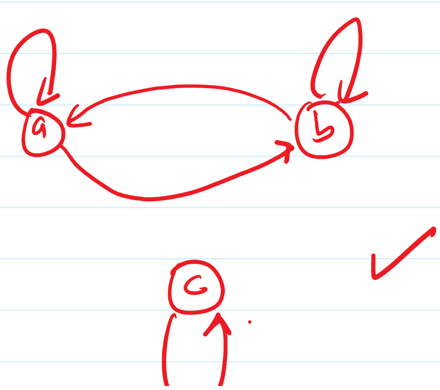
Graph.











EQ.

1) Reflexive

2) Symmetric

3). Transitive. ?

3). Transitive ?

$[a] = \{a, b\}$.
$[b] = \{a, b\}$.
$[c] = \{c\}$.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array} \quad X.$$

$[a] = \{a, b, c, d\}$.
$[b] =$
$[c] =$
$[d] =$

$[\ ]$

$$\begin{array}{c} \\ a \end{array} \begin{array}{c} a \\ \begin{bmatrix} 1 \end{bmatrix} \end{array}$$

$[a] = \{a\}$.

$$\begin{array}{c} a \\ b \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ \begin{array}{cc} a & b \end{array} \end{array}$$

$[a] = \{a, b\}$.
$[b] = \{a, b\}$

$$\begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$[a] = \{a, b\}. = [b]$.
$[c] = \{c\}$.

Partition. A family of Sets $\{P_1, P_2, P_3 \cdots, P_n\}$ is called a partition of Set S.

1) $\forall i \; P_i \neq \phi$.

2) $\forall i, j \; P_i \cap P_j = \phi$.

3). $\bigcup_{i=1}^{n} P_i = S$.

EX 13
P499.

$S = \{1, 2, 3, 4, 5, 6\}$.

$A_1 = \{1, 2, 3\}$.
$A_2 = \{4, 5\}$
$A_3 = \{6$

$A_1 \neq \phi \quad A_2 \neq \phi \quad A_3 \neq \phi$

$A_1 \neq \phi \wedge A_2 \neq \phi \wedge A_3 \neq \phi$.

$\{1,2,3\} \neq \phi \wedge \{4,5\} \neq \phi \wedge \{6\} \neq \phi$.

$T \wedge T \wedge T = T$.

$A_1 \cap A_2 = \phi \wedge A_1 \cap A_3 = \phi \wedge A_2 \cap A_3 = \phi$.

$T \wedge T \wedge T$

$= T$.

$A_1 \cup A_2 \cup A_3 = S$.

$\{1,2,3\} \cup \{4,5\} \cup \{6\} = \{1, \cdots 6\}$.

Equivalence classes creates a partition.