lec # 9 Rules of Inference.

Principle of Resolution.

- literal :- A Variable or negation of a Variable.
  p , ¬q

- clause :- Disjunction of literals.
  p ∨ q , ¬p ∨ q ∨ r
  , ¬p ∨ p

General form of an Argument.

p1
p2
p3
⋮
pN
∴ c

c1
c2
⋮
cM
¬c.

¬
∧ p ∧ q p
∨ q
→
↔.

p → q ≡ ¬p ∨ q.

p ↔ q ≡ p → q ∧ q → p.
¬p ∨ q ∧ ¬q ∨ p
¬p ∨ q
¬q ∨ p.

Ex:- 65 p1 :- p
p2 :- p → Q
c ∴ Q

c1 :- p ✓
c2 :- ¬p ∨ Q ✓
c3 :- ¬Q. ✓

c4 :- Q ✓ from c1, c2
c5 :- □ from c3, c4.

Resolution.
p ∨ q
¬q ∨ r
∴ p ∨ r.

Ex11 :- P65
p1 :- T → (M ∨ E)
p2 :- S → ¬E
p3 :- T ∧ S

P → Q ≡ ¬P ∨ Q.

P65 P2:- $S \rightarrow \neg E$

P3:- $T \wedge S$

C:- $\therefore M$.

C1:- $\neg T \vee M \vee E$ ✓

C2:- $\neg S \vee \neg E$. ✓

C3:- T ✓

C4:- S. ✓

C5:- $\neg M$. ✓

C6:- $\neg T \vee M \vee \neg S$ from C1, C2 ✓

C7:- $M \vee \neg S$ from C3, C6. ✓

C8:- M u C4, C7. ✓

C9:- $\square$ u C5, C8

Ex6 :- P62

P1:- $\neg p \wedge q$

P2:- $r \rightarrow p$

P3:- $\neg r \rightarrow s$

P4:- $s \rightarrow t$

C :- $\therefore t$

C1:- $\neg p$ ✓

C2:- q ?

C3:- $\neg r \vee p$ ✓

C4:- $r \vee s$ ✓

C5:- $\neg s \vee t$ ✓

C6:- $\neg t$ ✓

C7:- $\neg r$ C1, C3 ✓

C8:- s C4, C7. ✓

C9:- t C8, C5 ✓

C10:- $\square$ C6, C9

Ex 7 :- P62

P1 :- $p \to q$
P2 :- $\neg p \to r$
P3 :- $r \to s$
C :- $\neg q \to s$.

$\neg(\neg q \to s)$.
$\neg(q \vee s)$.
$= \neg q \wedge \neg s$.

C1 :- $\neg p \vee q$ ✓
C2 :- $p \vee r$ ✓
C3 :- $\neg r \vee s$. ✓
C4 :- $\neg q$ ✓
C5 :- $\neg s$. ✓

C6 :- $q \vee r$ from C1, C2. ✓
C7 :- $q \vee s$ from C3, C6. ✓
C8 :- s " C4, C7 ✓
C9 :- $\square$ " C8, C5

Ex 9 :- P62

P1 :- $L \to A$
P2 :- $E \to \neg I$
P3 :- $A \to E$
C :- $\therefore L \to \neg I$.

$\neg(L \to \neg I)$.
$\neg(\neg L \vee \neg I)$.
$= L \wedge I$.

$= L$

$= I$.

C1 :- $\neg L \vee A$ ✓
C2 :- $\neg E \vee \neg I$ ✓
C3 :- $\neg A \vee E$ ✓
C4 :- L ✓
C5 :- I. ✓

C6 :- $\neg L \vee E$ from C1, C3. ✓
C7 :- $\neg L \vee \neg I$ " C2, C6 ✓
C8 :- $\neg I$ " C7, C4 ✓
C9 :- $\square$ " C8, C5