

Lec # 4:-

Applications of Propositional Logic.

1- System Specification.

2- Logic Puzzles.

سقراط، لقراط، اور، اسطو
2200.

Deductive Inductive.

Smullyan 1919.



A Says " B is a Knight".
B Says " The two of us are opposite type".

Determine the type of

A is a Knight and B is a Knave. or -
B is a " " A " "

A	B
Knights	Knights
Knights	Knaves
Knaves	Knights
Knaves	Knaves

p = A is a Knight
 $\neg p$ = A is a Knave.
 q = B is a Knight
 $\neg q$ = B is a Knave.

① $\rightarrow q$
② $\rightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$.

	A	B
①	Knight	Knight.
②	$q = T$	
③	$(p \wedge \neg q) \vee (\neg p \wedge q) = T$	
	$T = T$	✓

$p = T$ $\neg p = F$
 $q = T$ $\neg q = F$.

$$(T \wedge F) \vee (F \wedge T) = T$$

$$P \vee P = T$$

$P \neq T$. Case does not hold.

A
Knight

B
Knave

$$\textcircled{1} \quad q = T$$

$$\textcircled{2} \quad (P \wedge \neg q) \vee (\neg P \wedge q) = F$$

$$P = T \quad \neg P = F$$

$$q = F \quad \neg q = T$$

$P \neq T$ Case not hold.

A
Knave

B
Knight.

$$\textcircled{1} \quad q = F$$

$$\textcircled{2} \quad (P \wedge \neg q) \vee (\neg P \wedge q) = T$$

$$P = F \quad \neg P = T$$

$$q = T \quad \neg q = F$$

$T \neq F$ Case does not hold.

A
Knave

B
Knave.

$$\textcircled{1} \quad q = F$$

$$\textcircled{2} \quad (P \wedge \neg q) \vee (\neg P \wedge q) = F$$

$$P = F \quad \neg P = T$$

$$q = F \quad \neg q = T$$

$$P = F \quad \checkmark$$

$$(F \wedge T) \vee (T \wedge F) = F$$

$$P \vee P = F$$

$$P = F \quad \checkmark$$

Case hold.

A is a Knave.

B " " " .

Ex 55-59.

P20.

A	B
Knights, Knights	✓
Knights, Knaves	✓
Knaves, Knight	
Knaves, Knaves	

A = Knight
B = ?

²² A	²¹ B	²⁰ C
Knight	Knight	Knight.
1	1	Knave.
1	Knave	Knight
1	1	1
1	1	1
1	1	1
1	1	1
Knave	Knave	Knave.

Values Variable.
 $2^3 = 8$.

A B.

Knight, Knave, Spy

$\neg p_1 \vee A$ is not a Knight $p_1 \vee A$ is a Knight.
 $\neg p_2 \vee A$ is not a Knave $p_2 \vee A$ is a Knave.
 $\neg p_3 \vee A$ is a Spy $p_3 \vee A$ is a Spy.

Q.W.
 $\neg q_1 \vee B$ is not a Knight $q_1 \vee B$ is a Knight.
 $\neg q_2 \vee B$ is not a Knave $q_2 \vee B$ is a Knave.
 $\neg q_3 \vee B$ is not a Spy $q_3 \vee B$ is a Spy.

A Says "I am not a Spy" $\neg p_3$

B Says "A is a Spy or I am a Knave" $p_3 \vee q_2$.

A
Knight

B.
Knight.

$$\neg P_3 = T$$

$$P_3 \vee Q_2 = T$$

$$T = T$$

$F \vee F \neq T$ Case does not hold.

$$P_1 = T$$

$$\neg P_1 = F$$

$$Q_1 = T \quad \neg Q_1 = F$$

$$P_2 = F$$

$$\neg P_2 = T$$

$$Q_2 = F \quad \neg Q_2 = T$$

$$P_3 = F$$

$$\neg P_3 = T$$

$$Q_3 = F \quad \neg Q_3 = T$$

Screen Cases (Hvt).

A
Spy

B.
Spy.

$$\neg P_3 = ?$$

$$P_3 \vee Q_2 = ?$$

$$P_1 = F$$

$$\neg P_1 = T$$

$$Q_1 = F \quad \neg Q_1 = T$$

$$P_2 = F$$

$$\neg P_2 = T$$

$$Q_2 = F \quad \neg Q_2 = T$$

$$P_3 = T$$

$$\neg P_3 = F$$

$$Q_3 = T \quad \neg Q_3 = F$$

Searching

→ Universities and CS.

Tools:-

Bag → Tools.