In [1]:
```
import numpy as np
import pandas as pd
from sklearn.preprocessing import PolynomialFeatures
from sklearn.model_selection import train_test_split, GridSearchCV, learning_cur
from sklearn.linear_model import LinearRegression
from sklearn.neighbors import KNeighborsRegressor
from sklearn.tree import DecisionTreeRegressor
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score
import matplotlib.pyplot as plt
import seaborn as sns
np.random.seed(42)
```

In [2]:
```
sns.set(style="whitegrid")
plt.rcParams['figure.figsize'] = (10, 6)
```

In [3]:
```
def generate_linear_data(n_samples=100, n_features=3):
    X = np.random.uniform(-10, 10, size=(n_samples, n_features))
    coefficients = np.array([2.5, -1.5, 3.0])
    noise = np.random.normal(0, 5, size=n_samples)
    y = X @ coefficients + noise
    return X, y
```

In [4]:
```
def generate_nonlinear_data(n_samples=100, n_features=3):
    X = np.random.uniform(-10, 10, size=(n_samples, n_features))
    noise = np.random.normal(0, 10, size=n_samples)
    y = (X[:, 0]**2) + np.sin(X[:, 1]) + np.log(np.abs(X[:, 2]) + 1) + noise
    return X, y
```

In [5]:
```
X_linear, y_linear = generate_linear_data()
X_nonlinear, y_nonlinear = generate_nonlinear_data()
```

In [6]:
```
feature_names = [f'Feature_{i+1}' for i in range(X_linear.shape[1])]
```

In [7]:
```
df_linear = pd.DataFrame(X_linear, columns=feature_names)
df_linear['Target'] = y_linear
```

In [8]:
```
df_nonlinear = pd.DataFrame(X_nonlinear, columns=feature_names)
df_nonlinear['Target'] = y_nonlinear
```

In [9]:
```
print("Linear Dataset Sample:")
display(df_linear.head())

print("\nNonlinear Dataset Sample:")
display(df_nonlinear.head())
```

```
Linear Dataset Sample:
```

	Feature_1	Feature_2	Feature_3	Target
0	-2.509198	9.014286	4.639879	-5.646928
1	1.973170	-6.879627	-6.880110	-8.645966
2	-8.838328	7.323523	2.022300	-16.294483
3	4.161452	-9.588310	9.398197	56.150280
4	6.648853	-5.753218	-6.363501	-3.964256

```
Nonlinear Dataset Sample:
```

	Feature_1	Feature_2	Feature_3	Target
0	-8.848825	0.990578	-1.169390	91.590310
1	7.754084	-2.981700	-7.658660	64.669369
2	-7.140166	5.230213	2.364361	54.702342
3	-7.977546	-8.317864	4.019383	60.241463
4	-8.544740	6.437201	4.124845	69.924025

In [10]:
```
def split_data(X, y, test_size=0.2):
    return train_test_split(X, y, test_size=test_size, random_state=42)
```

In [11]:
```
X_train_linear, X_test_linear, y_train_linear, y_test_linear = split_data(X_line
X_train_nonlinear, X_test_nonlinear, y_train_nonlinear, y_test_nonlinear = split
```

In [12]:
```
poly = PolynomialFeatures(degree=2, include_bias=False)
X_train_nonlinear_poly = poly.fit_transform(X_train_nonlinear)
X_test_nonlinear_poly = poly.transform(X_test_nonlinear)

poly_feature_names = poly.get_feature_names_out(feature_names)
df_train_nonlinear_poly = pd.DataFrame(X_train_nonlinear_poly, columns=poly_feat
df_test_nonlinear_poly = pd.DataFrame(X_test_nonlinear_poly, columns=poly_featur

print("Polynomial Features Sample (Nonlinear Dataset):")
display(df_train_nonlinear_poly.head())
```

```
Polynomial Features Sample (Nonlinear Dataset):
```

	Feature_1	Feature_2	Feature_3	Feature_1^2	Feature_1 Feature_2	Feature_1 Feature_3	Feature_2^2	Fe Fe
0	-6.961946	-7.223457	2.817495	48.468694	50.289315	-19.615248	52.178324	-2(
1	5.389859	-6.259125	-3.526415	29.050576	-33.735799	-19.006880	39.176646	22
2	-3.809448	6.275900	3.694623	14.511892	-23.907714	-14.074475	39.386926	23
3	0.403270	7.043630	1.038137	0.162627	2.840485	0.418649	49.612724	7
4	7.904137	0.233549	0.642270	62.475378	1.846002	5.076588	0.054545	(

In [13]:
```
def create_knn_model(X_train, y_train):
    knn = KNeighborsRegressor()
    param_grid = {'n_neighbors': list(range(1, 21))}
```

```
    grid_search = GridSearchCV(knn, param_grid, cv=5, scoring='neg_mean_squared_
    grid_search.fit(X_train, y_train)
    print(f"Optimal number of neighbors: {grid_search.best_params_['n_neighbors'
    return grid_search.best_estimator_
```

In [14]:
```
def create_dt_model(X_train, y_train):
    dt = DecisionTreeRegressor(random_state=42)
    param_grid = {'max_depth': list(range(1, 21))}
    grid_search = GridSearchCV(dt, param_grid, cv=5, scoring='neg_mean_squared_e
    grid_search.fit(X_train, y_train)
    print(f"Optimal max_depth: {grid_search.best_params_['max_depth']}")
    return grid_search.best_estimator_
```

In [15]:
```
def train_models(X_train, y_train, dataset_type='Linear'):
    print(f"\n=== Training Models on {dataset_type} Dataset ===")

    linear = LinearRegression()
    linear.fit(X_train, y_train)
    print("Linear Regression trained.")

    knn = create_knn_model(X_train, y_train)

    dt = create_dt_model(X_train, y_train)

    return linear, knn, dt
```

In [16]:
```
def evaluate_model(model, X_train, y_train, X_test, y_test, dataset_type='Linear
    metrics = {}
    y_pred_train = model.predict(X_train)
    y_pred_test = model.predict(X_test)

    metrics['Train_MAE'] = mean_absolute_error(y_train, y_pred_train)
    metrics['Train_MSE'] = mean_squared_error(y_train, y_pred_train)
    metrics['Train_R2'] = r2_score(y_train, y_pred_train)

    metrics['Test_MAE'] = mean_absolute_error(y_test, y_pred_test)
    metrics['Test_MSE'] = mean_squared_error(y_test, y_pred_test)
    metrics['Test_R2'] = r2_score(y_test, y_pred_test)

    print(f"\n--- Evaluation Metrics for {dataset_type} Dataset using {model_nam
    for key, value in metrics.items():
        print(f"{key}: {value:.4f}")

    return y_pred_test, metrics
```

In [17]:
```
performance_metrics = {
    'Dataset': [],
    'Model': [],
    'Train_MAE': [],
    'Train_MSE': [],
    'Train_R2': [],
    'Test_MAE': [],
    'Test_MSE': [],
    'Test_R2': []
}

linear_models_linear, knn_models_linear, dt_models_linear = train_models(X_train

y_pred_lin_lin, metrics_lin_lin = evaluate_model(linear_models_linear, X_train_l
```

```
performance_metrics['Dataset'].append('Linear')
performance_metrics['Model'].append('Linear Regression')
for key, value in metrics_lin_lin.items():
    performance_metrics[key].append(value)

y_pred_knn_lin, metrics_knn_lin = evaluate_model(knn_models_linear, X_train_line
performance_metrics['Dataset'].append('Linear')
performance_metrics['Model'].append('KNN Regressor')
for key, value in metrics_knn_lin.items():
    performance_metrics[key].append(value)

y_pred_dt_lin, metrics_dt_lin = evaluate_model(dt_models_linear, X_train_linear,
performance_metrics['Dataset'].append('Linear')
performance_metrics['Model'].append('Decision Tree Regressor')
for key, value in metrics_dt_lin.items():
    performance_metrics[key].append(value)

linear_models_nl, knn_models_nl, dt_models_nl = train_models(X_train_nonlinear_p

y_pred_lin_nl, metrics_lin_nl = evaluate_model(linear_models_nl, X_train_nonline
performance_metrics['Dataset'].append('Nonlinear')
performance_metrics['Model'].append('Linear Regression')
for key, value in metrics_lin_nl.items():
    performance_metrics[key].append(value)

y_pred_knn_nl, metrics_knn_nl = evaluate_model(knn_models_nl, X_train_nonlinear_
performance_metrics['Dataset'].append('Nonlinear')
performance_metrics['Model'].append('KNN Regressor')
for key, value in metrics_knn_nl.items():
    performance_metrics[key].append(value)

y_pred_dt_nl, metrics_dt_nl = evaluate_model(dt_models_nl, X_train_nonlinear_pol
performance_metrics['Dataset'].append('Nonlinear')
performance_metrics['Model'].append('Decision Tree Regressor')
for key, value in metrics_dt_nl.items():
    performance_metrics[key].append(value)
```

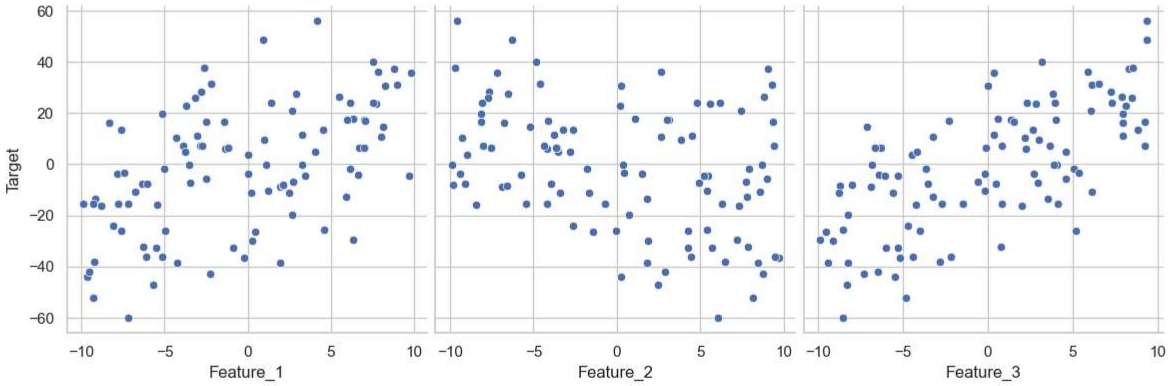
```
=== Training Models on Linear Dataset ===
Linear Regression trained.
Optimal number of neighbors: 3
Optimal max_depth: 5

--- Evaluation Metrics for Linear Dataset using Linear Regression ---
Train_MAE: 3.4698
Train_MSE: 17.5065
Train_R2: 0.9708
Test_MAE: 5.5219
Test_MSE: 50.4348
Test_R2: 0.9120

--- Evaluation Metrics for Linear Dataset using KNN Regressor ---
Train_MAE: 4.6206
Train_MSE: 30.1511
Train_R2: 0.9498
Test_MAE: 8.3129
Test_MSE: 98.7869
Test_R2: 0.8276

--- Evaluation Metrics for Linear Dataset using Decision Tree Regressor ---
Train_MAE: 3.1518
Train_MSE: 21.6989
Train_R2: 0.9639
Test_MAE: 10.8776
Test_MSE: 175.9244
Test_R2: 0.6930

=== Training Models on Nonlinear Dataset ===
Linear Regression trained.
Optimal number of neighbors: 4
Optimal max_depth: 3

--- Evaluation Metrics for Nonlinear Dataset using Linear Regression ---
Train_MAE: 7.5192
Train_MSE: 95.6348
Train_R2: 0.9212
Test_MAE: 8.7715
Test_MSE: 130.3549
Test_R2: 0.8865

--- Evaluation Metrics for Nonlinear Dataset using KNN Regressor ---
Train_MAE: 9.5773
Train_MSE: 158.5636
Train_R2: 0.8693
Test_MAE: 13.4882
Test_MSE: 355.1938
Test_R2: 0.6908

--- Evaluation Metrics for Nonlinear Dataset using Decision Tree Regressor ---
Train_MAE: 6.5628
Train_MSE: 68.0821
Train_R2: 0.9439
Test_MAE: 7.8045
Test_MSE: 107.1713
Test_R2: 0.9067
```

In [18]:

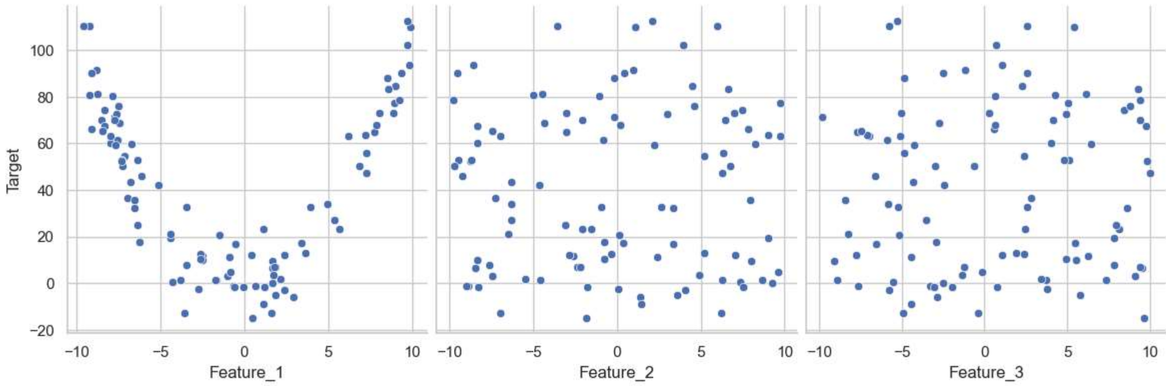
```
print("\n=== Pair Plot for Linear Dataset ===")
sns.pairplot(df_linear, x_vars=feature_names, y_vars='Target', height=4, aspect=
```

```
plt.show()

print("\n=== Pair Plot for Nonlinear Dataset ===")
sns.pairplot(df_nonlinear, x_vars=feature_names, y_vars='Target', height=4, aspe
plt.show()
```

```
=== Pair Plot for Linear Dataset ===
```

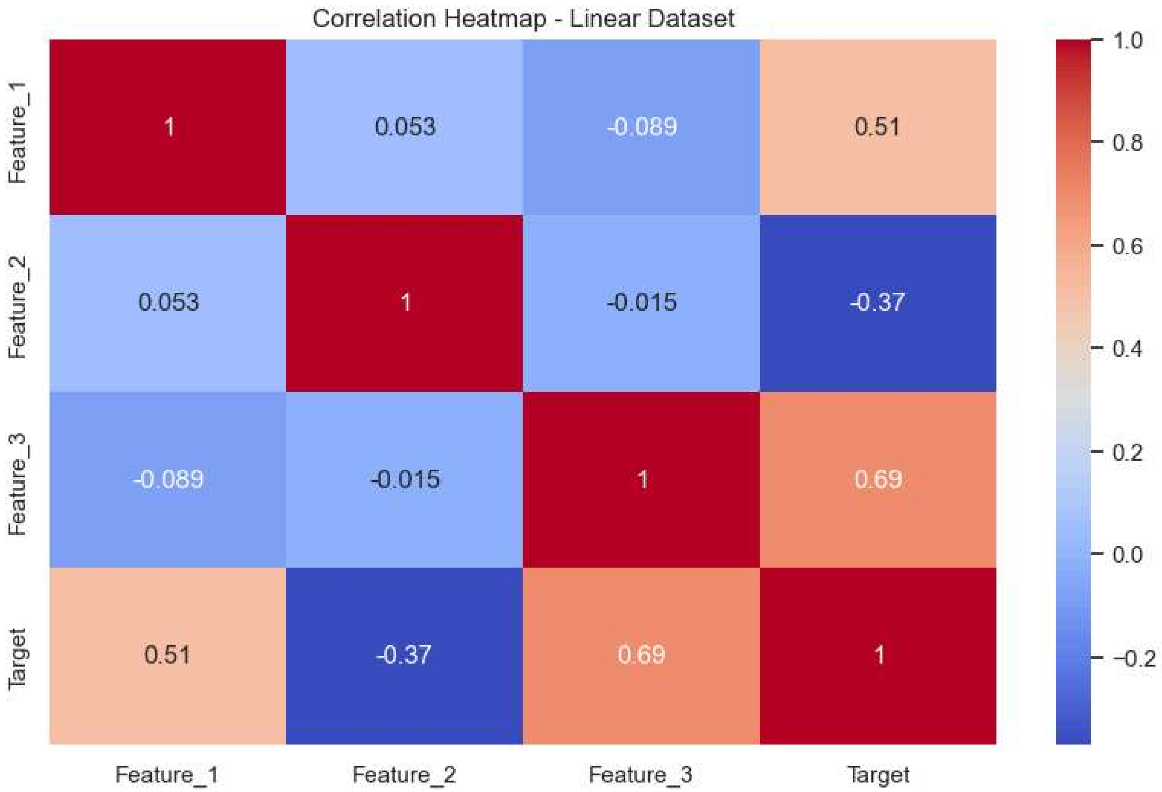


```
=== Pair Plot for Nonlinear Dataset ===
```

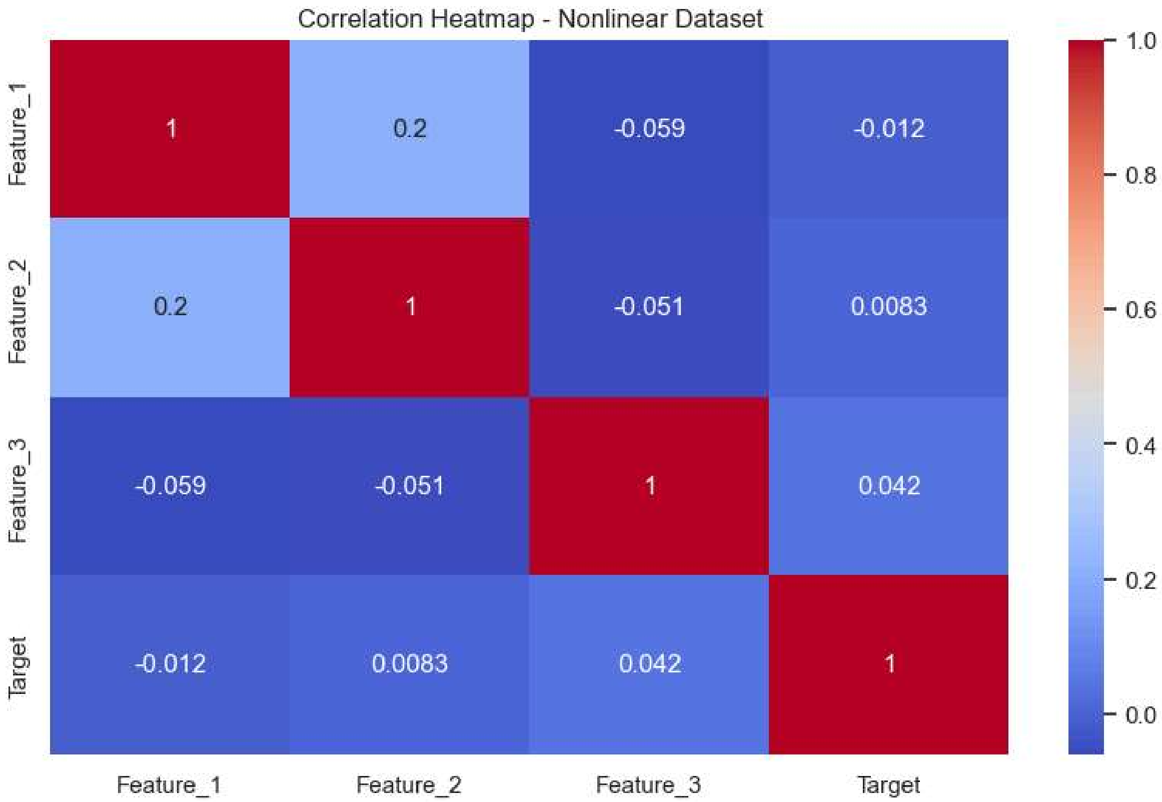


In [19]:
```
print("\n=== Correlation Heatmap for Linear Dataset ===")
corr_linear = df_linear.corr()
sns.heatmap(corr_linear, annot=True, cmap='coolwarm')
plt.title('Correlation Heatmap - Linear Dataset')
plt.show()

print("\n=== Correlation Heatmap for Nonlinear Dataset ===")
corr_nonlinear = df_nonlinear.corr()
sns.heatmap(corr_nonlinear, annot=True, cmap='coolwarm')
plt.title('Correlation Heatmap - Nonlinear Dataset')
plt.show()
```

```
=== Correlation Heatmap for Linear Dataset ===
```




```
=== Correlation Heatmap for Nonlinear Dataset ===
```

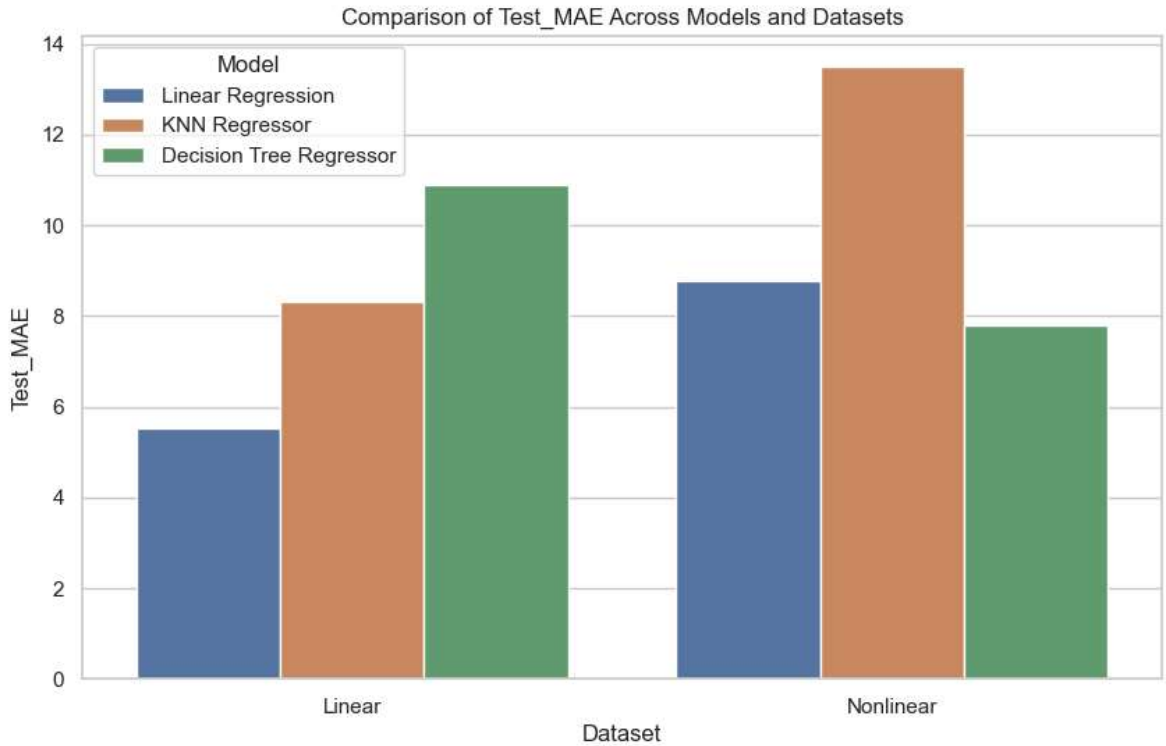


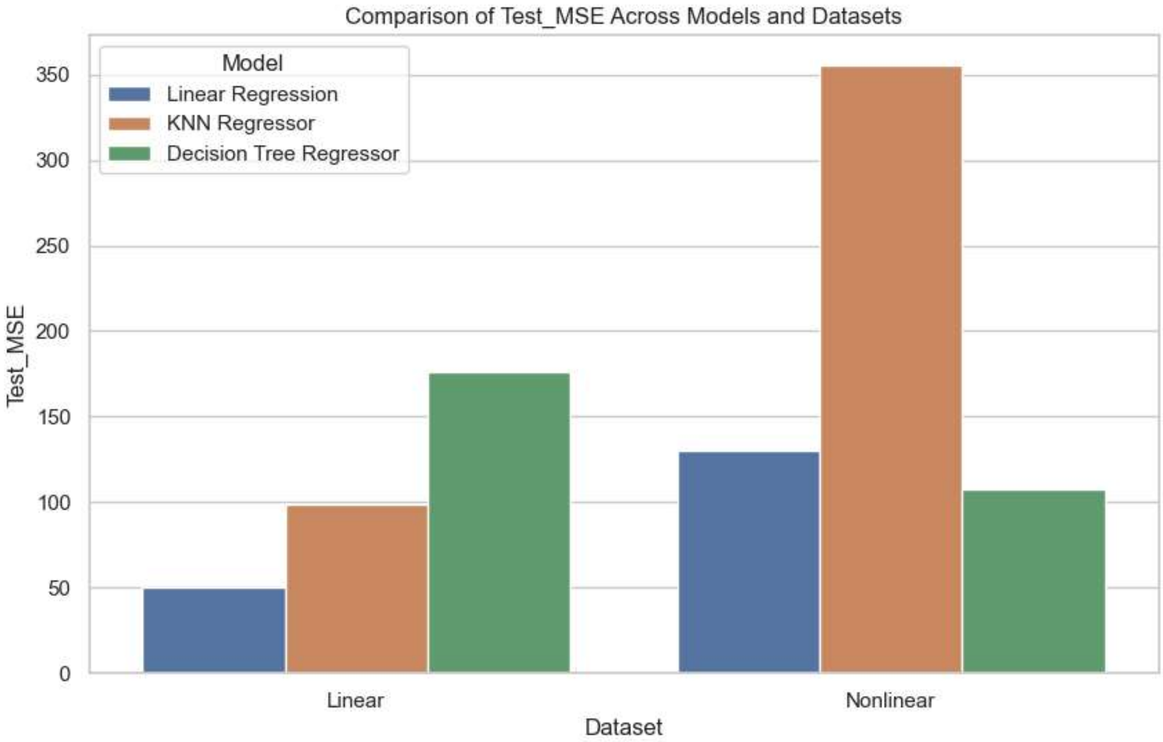
In [20]:
```
metrics_df = pd.DataFrame(performance_metrics)
print("\n=== Performance Metrics ===")
display(metrics_df)

metrics_to_plot = ['Test_MAE', 'Test_MSE', 'Test_R2']

for metric in metrics_to_plot:
    plt.figure(figsize=(10,6))
    sns.barplot(data=metrics_df, x='Dataset', y=metric, hue='Model')
    plt.title(f'Comparison of {metric} Across Models and Datasets')
```

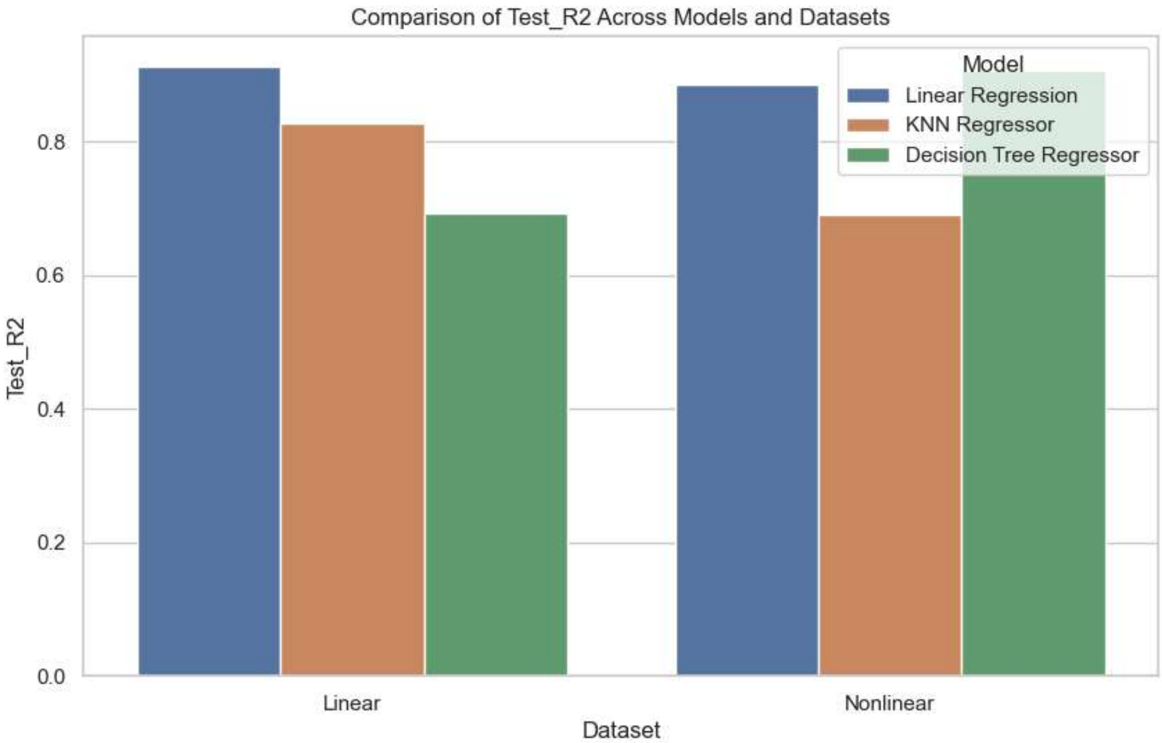
```
    plt.ylabel(metric)
    plt.xlabel('Dataset')
    plt.legend(title='Model')
    plt.show()
```

```
=== Performance Metrics ===
```

	Dataset	Model	Train_MAE	Train_MSE	Train_R2	Test_MAE	Test_MSE	Test_R
0	Linear	Linear Regression	3.469764	17.506518	0.970839	5.521900	50.434812	0.91199
1	Linear	KNN Regressor	4.620625	30.151124	0.949776	8.312862	98.786871	0.82761
2	Linear	Decision Tree Regressor	3.151797	21.698874	0.963855	10.877587	175.924428	0.69301
3	Nonlinear	Linear Regression	7.519228	95.634807	0.921193	8.771547	130.354859	0.88651
4	Nonlinear	KNN Regressor	9.577275	158.563551	0.869337	13.488216	355.193816	0.69075
5	Nonlinear	Decision Tree Regressor	6.562767	68.082062	0.943897	7.804486	107.171304	0.90669





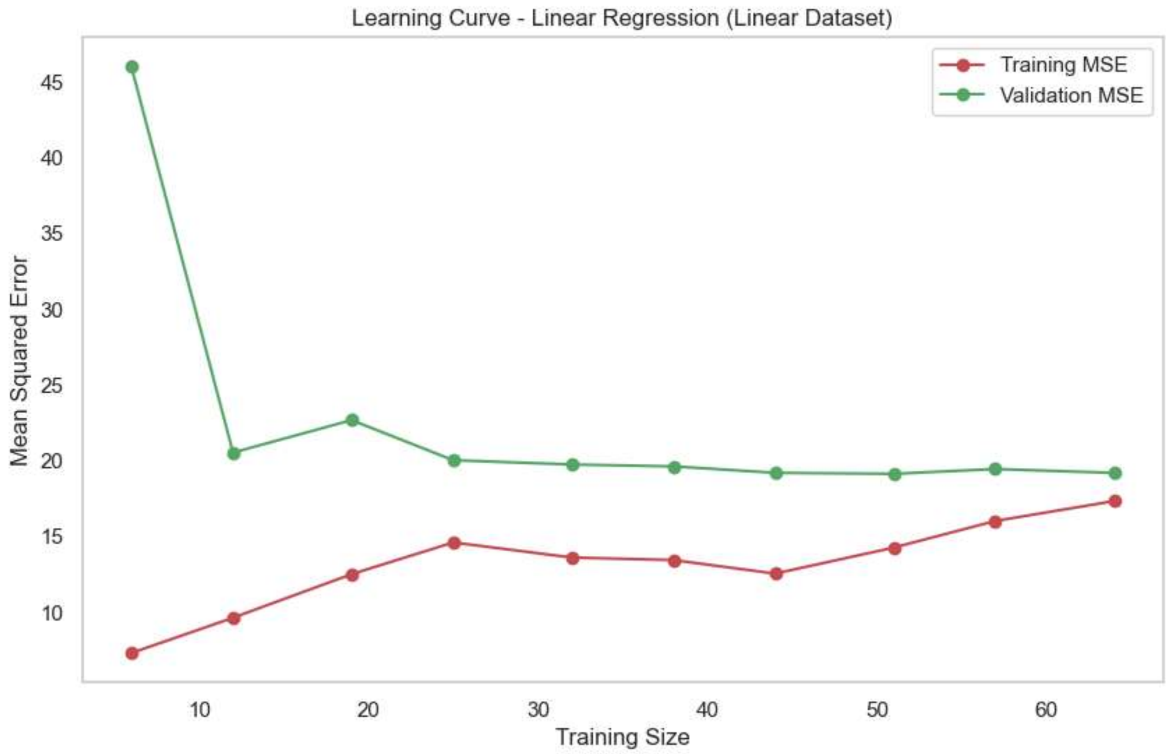


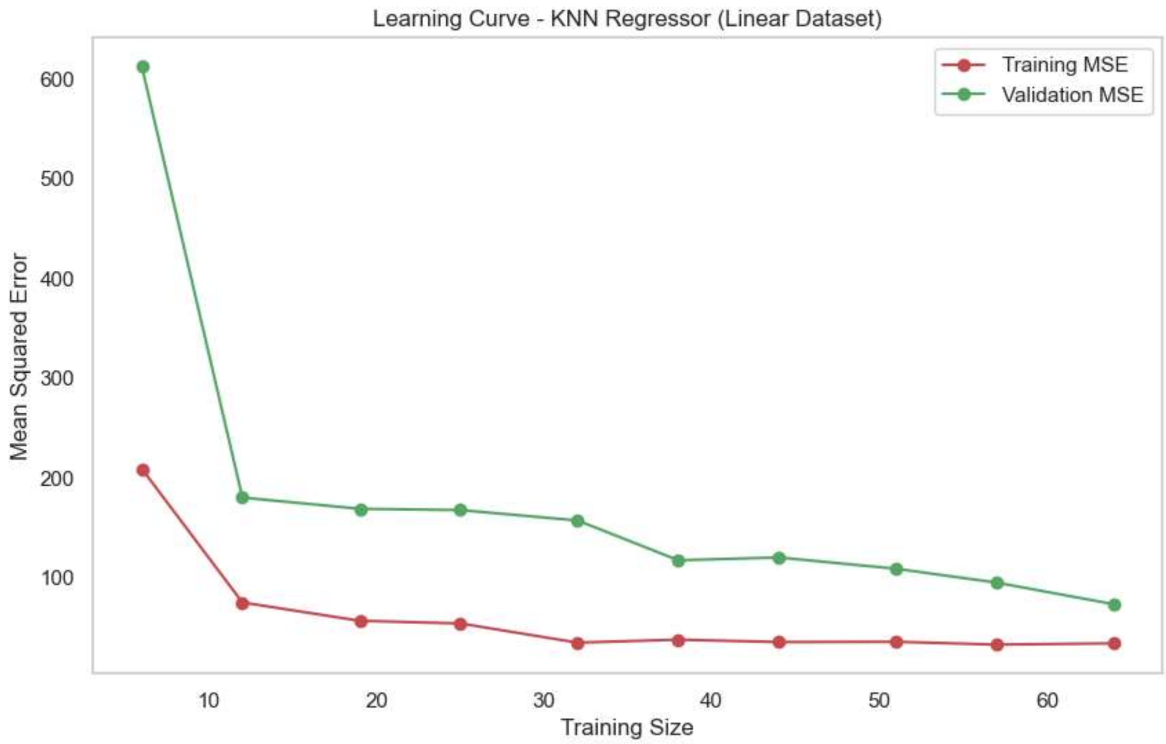
In [21]:

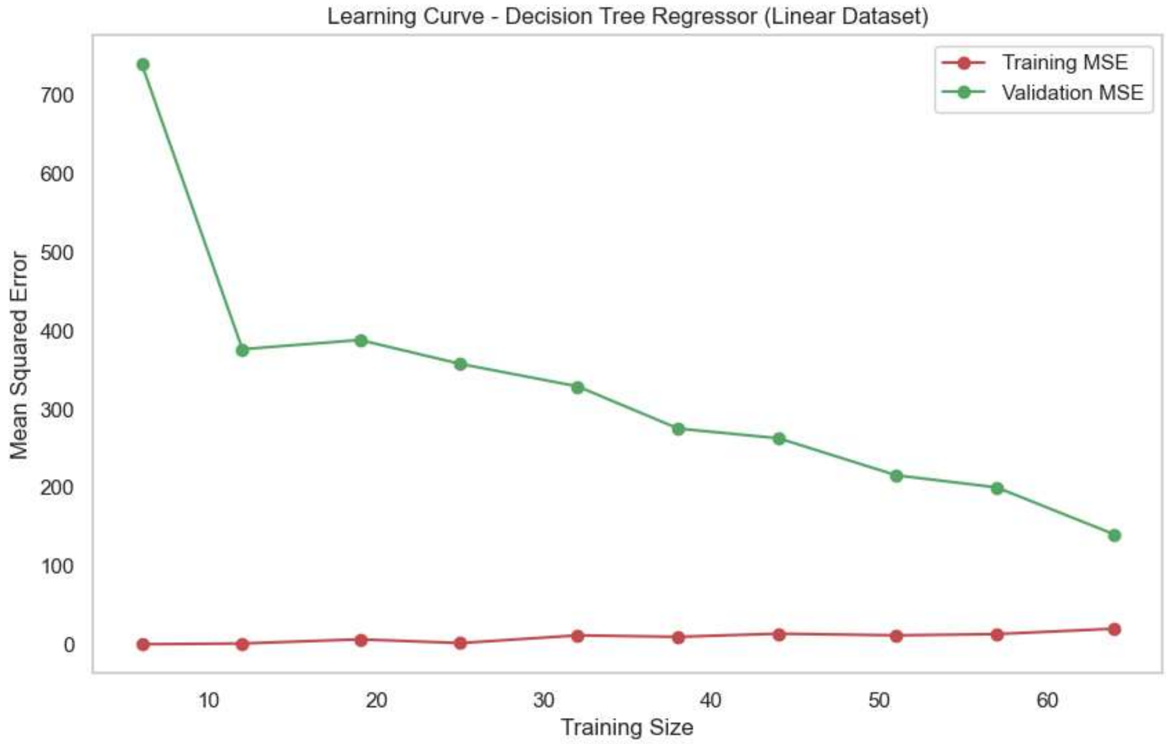
```
def plot_learning_curves(model, X, y, title):
    train_sizes, train_scores, test_scores = learning_curve(model, X, y, cv=5, s
                                                            train_sizes=np.linsp

    train_scores_mean = -train_scores.mean(axis=1)
    test_scores_mean = -test_scores.mean(axis=1)

    plt.figure(figsize=(10,6))
    plt.plot(train_sizes, train_scores_mean, 'o-', color='r', label='Training MS
    plt.plot(train_sizes, test_scores_mean, 'o-', color='g', label='Validation M
    plt.title(title)
    plt.xlabel('Training Size')
    plt.ylabel('Mean Squared Error')
    plt.legend(loc='best')
```

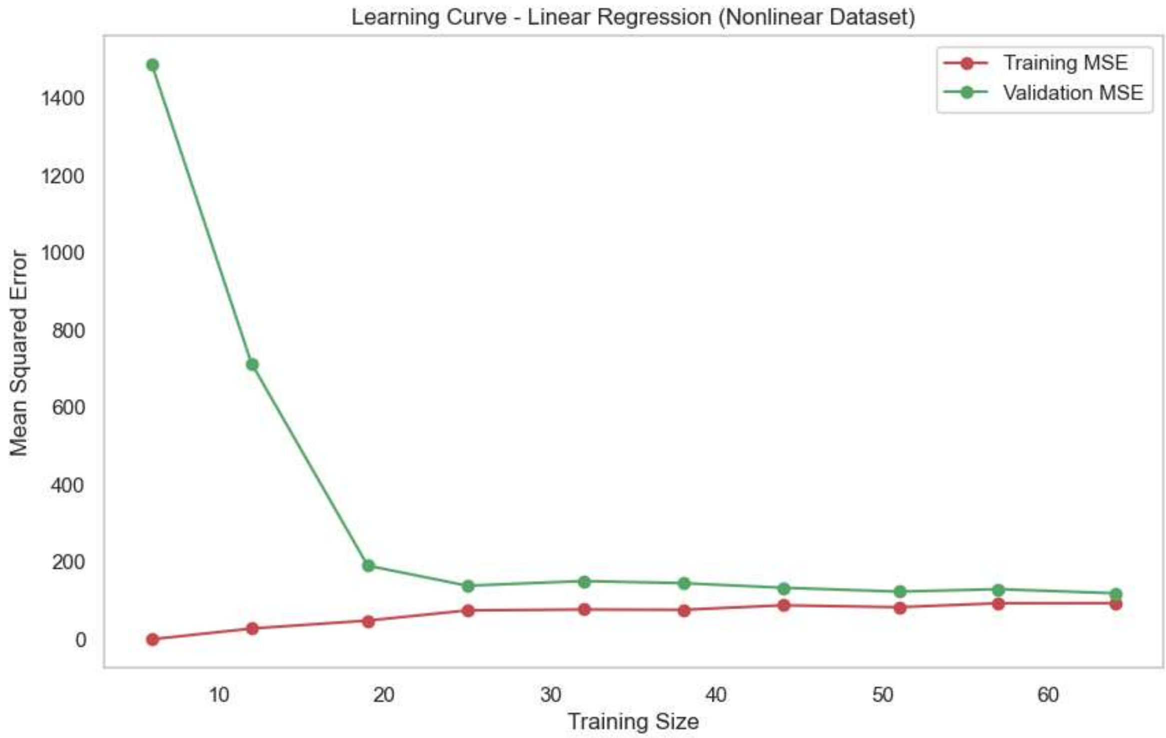
```
    plt.grid()
    plt.show()
```

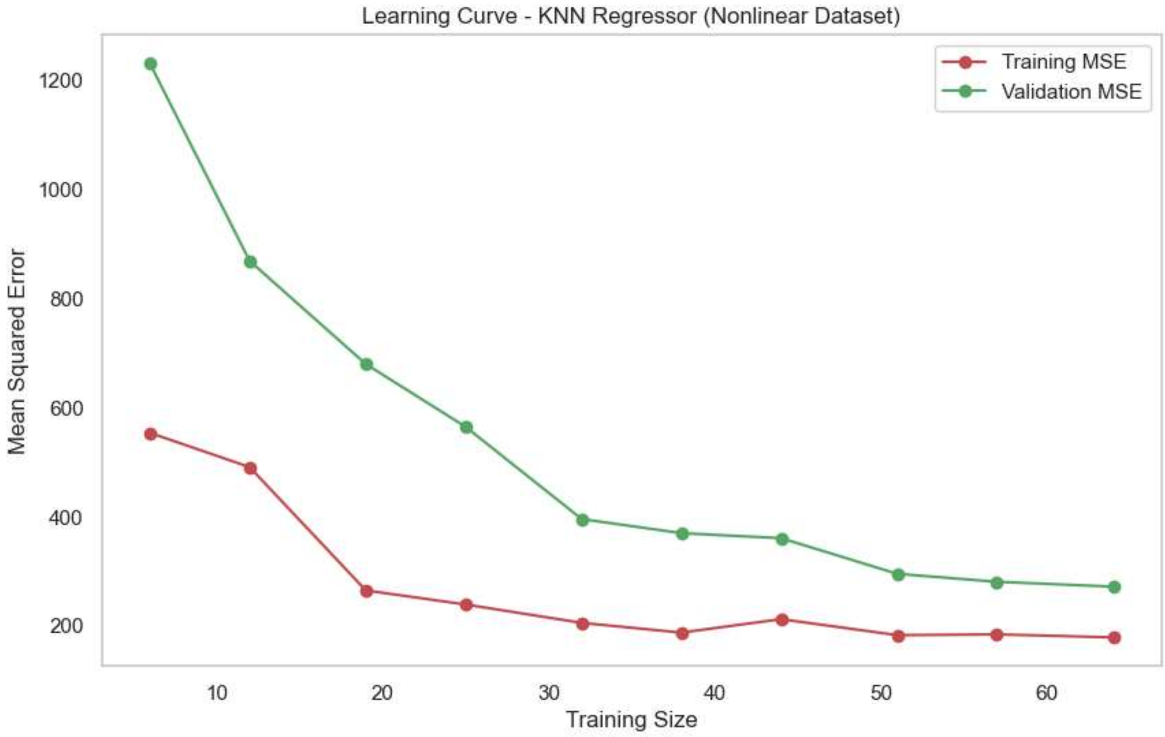
In [22]:
```
plot_learning_curves(LinearRegression(), X_train_linear, y_train_linear, 'Learni

plot_learning_curves(knn_models_linear, X_train_linear, y_train_linear, 'Learnin

plot_learning_curves(dt_models_linear, X_train_linear, y_train_linear, 'Learning

plot_learning_curves(LinearRegression(), X_train_nonlinear_poly, y_train_nonline

plot_learning_curves(knn_models_nl, X_train_nonlinear_poly, y_train_nonlinear, '

plot_learning_curves(dt_models_nl, X_train_nonlinear_poly, y_train_nonlinear, 'L
```

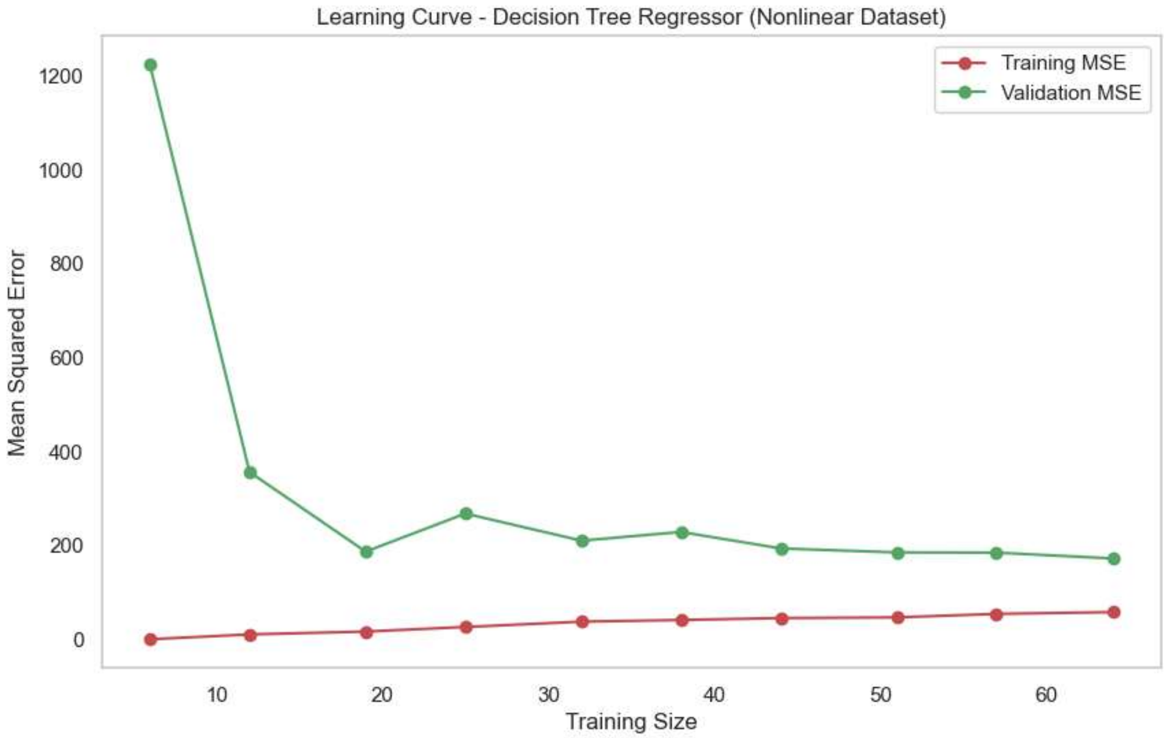





Learning Curve - Decision Tree Regressor (Linear Dataset)
Training MSE
Validation MSE
Mean Squared Error
0
100
200
300
400
500
600
700
10
20
30
40
50
60
Training Size


Learning Curve - Linear Regression (Nonlinear Dataset)
Training MSE
Validation MSE
Mean Squared Error
0
200
400
600
800
1000
1200
1400
10
20
30
40
50
60
Training Size


Learning Curve - KNN Regressor (Nonlinear Dataset)
Training MSE
Validation MSE
Mean Squared Error
200
400
600
800
1000
1200
10
20
30
40
50
60
Training Size


Learning Curve - Decision Tree Regressor (Nonlinear Dataset)
Training MSE
Validation MSE
Mean Squared Error
0
200
400
600
800
1000
1200
10
20
30
40
50
60
Training Size