Lecture 10

In this lecture we discuss rules of probability

Rules of Probability

On a table, there are a total of 30 distinct books: 9 math books, 10 physics books, and 11 chemistry books.

What is the probability of getting a book that is not a math book?

Probability of getting a physics or a math book?
First of all these are **mutually exclusive events → independent → no overlap**

$$P(\bar{m}) = P(H) + P(C) \qquad \text{no overlap!}$$
$$= \frac{10}{30} + \frac{11}{30} = 21/30$$

This is the **sum rule of probability (Handling OR)**

Generalize to include events that are <u>not</u> mutually exclusive:

A fair 20-sided dice is rolled.

What is the probability that the roll is an even number or prime number or both?

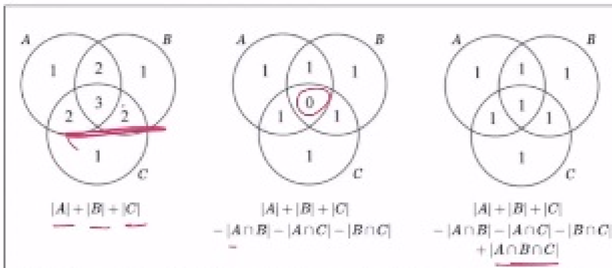
Now these are not mutually exclusive as 2 is both prime or even

$$P(E \cup R) = P(E) + P(R) - \overbrace{P(E \cap R)}^{\text{Double counted}}$$
$$= \frac{10}{20} + \frac{8}{20} - \frac{1}{20}$$
$$= \frac{17}{20}$$

2, 3, 5, 7, 11, 13, 17, 19

Now for the probability of **three** events

Inclusion Exclusion Principle :



Again, the number in each subset represents how many times that subset is counted. Here, the overlap of all three subsets must be added back in due to it being removed after the second step.

In the first we have to subtract the regions which **we counted twice like A∩B AND A∩C AND B∩C** but the region A∩B∩C is subtracted thrice so we have to add it once

Product rule : (Handling AND)

Two coin flips : Probability of both being heads :

$$P(HH) = P(H) * P(H)$$

Independent events : Knowing one has occured doesn't change the probability of the other !

Conversely if $P(A) * P(B) = P(A \cap B)$

then A and B are independent

Example : 20-sided dice is rolled. Probability that the number is even and prime.

$P(E) = \frac{10}{20}$ $\quad P(R) = \frac{8}{20}$ $\quad P(E \cap R) = \frac{1}{20}$

WE KNOW THAT EVEN NUMBERS ARE TOTAL 10 FROM 20 AND PRIME NUMBERS ARE 8 FROM 20 AND THERE IS ONLY ONE NUM WHICH IS BOTH EVEN AND PRIME.

20 20 20

$$P(E \cap R) \neq P(E) \cdot P(R)$$

$$\frac{1}{20} \neq \frac{10}{20} \cdot \frac{8}{20}$$

$$0.05 \neq 0.2$$

AS P(E∩R) IS NOT EQUAL TO P(E).P(R) SO THEY ARE NOT INDEPENDENT.

So, if we know one, the probability of the other changes.

$P(E) = \frac{1}{2} = 0.5$ given no other information.

If you are told that the number was a prime

$P(E) = \frac{1}{8} = 0.125$ (less likely now!)

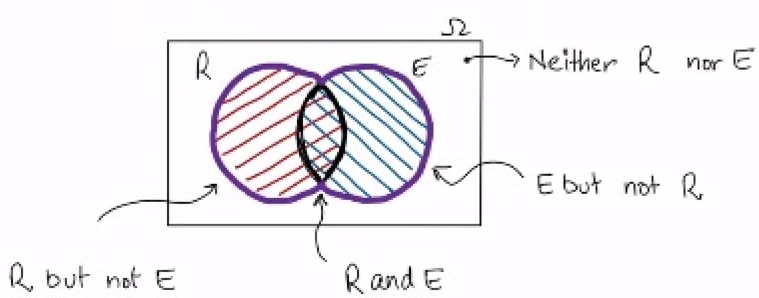
PROBABILITY OF BEING EVEN WAS ½
BUT IF I TOLD THAT THE NUMBER WAS ALSO A PRIME SO THE PROBABILITY OF PRIME CHANGES AS WE KNOW THERE IS ONLY ONE NO WHICH IS BOTH EVEN AND PRIME

SO PROBABILITY OF AN EVENT CHANGES BY THE OCCURRENCE OF ANOTHER EVENT

(*) Notation and Intuition:

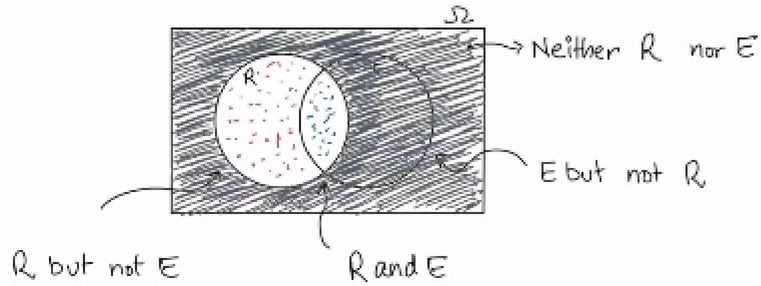
$P(E \mid R)$

probability of event given that this event is already known to have occurred!



R IS THE EVENT THAT HAS OCCURRED ALREADY U CANT SAY THAT NUMBER IS NOT PRIME IT IS ALREADY GIVEN THAT THE NO IS PRIME. PROBABILITY OF NON-PRIME IS ZERO.

R is already known to have occurred!



— R is our new "universe". There is no "not R".

— R is definite — P(R) has to be 1.

— **But** P(R) was 8/20 = 0.4

BUT P(R) HAS TO BE 1 TO SATISFY THE AXIOM
BECAUSE R IS OUR UNIVERSE NOW IT IS DEFINITE THAT IT HAS OCCUR
SO ITS PROBABILITY SHOULD BE 1.

PROBLEM : — our math does not work!

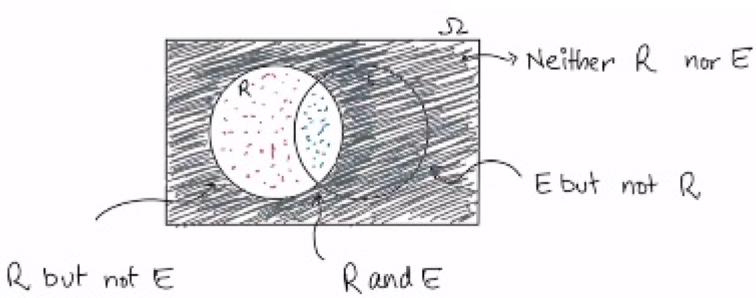
— Rescale everything so that P(R) becomes 1.

SO HOW TO

$P(R)$ becomes $\frac{P(R)}{P(R)} = \frac{0.4}{0.4} = 1$

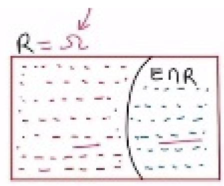
$P(R)$ becomes $\frac{P(R)}{P(R)}$ ***SO IT MAKES SENSE IF I SAY PROBABILITY OF R GIVEN THAT R HAS OCCURRED IS 1***

$P(R|R) = 1$

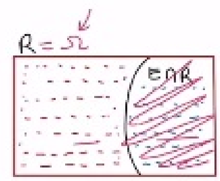


SO U HAVE TO TAKE THIS R PORTION AND STRETCH IT TO MAKE IT YOUR NEW UNIVERSE

(we are calling it universe bcz it has probability of 1 and universe probability is 1)



FOR STRETCHING THE R U TAKE THE P(R) AND DIVIDE BY THE P(R)
SIMILARLY FOR P(E∩R) TO STRETCH U DIVIDE IT BY P(R).



$$P(E \cap R) = \frac{1}{20} = 0.05$$

$$P(E|R) = \frac{\boxed{P(E \cap R)}}{P(R)} = \frac{0.05}{0.4} = 0.125$$

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

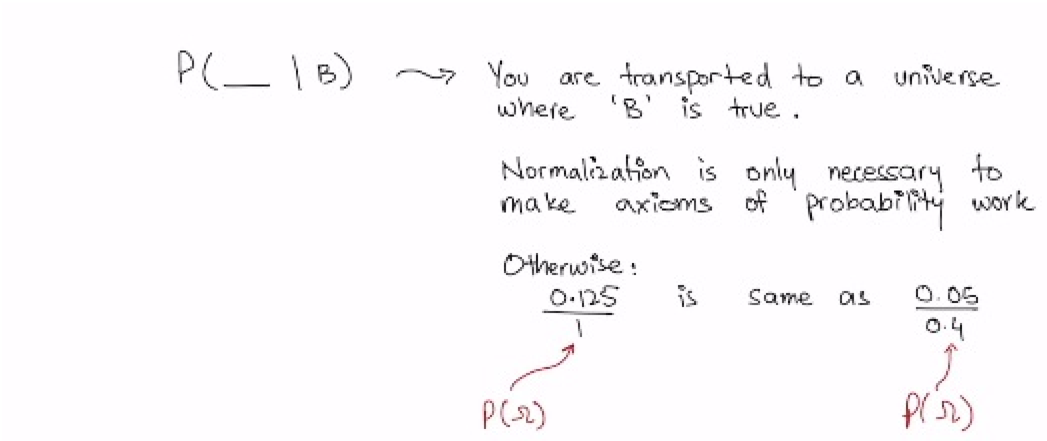
Conditional Probability

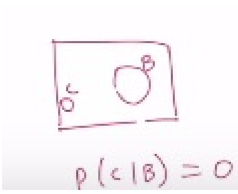
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Prob of A and B both happening

Prob of A given that B has occurred

normalized so that axioms of probability still hold





FOR INSTANCE IF WE SEE IN THIS DIAGRAM THERE IS NO OVERLAP SO P(C/B) IS 0 AS B WILL BE A NEW UNIVERSE FOR US WHERE IT IS TRUE SO THERE IS NO C .