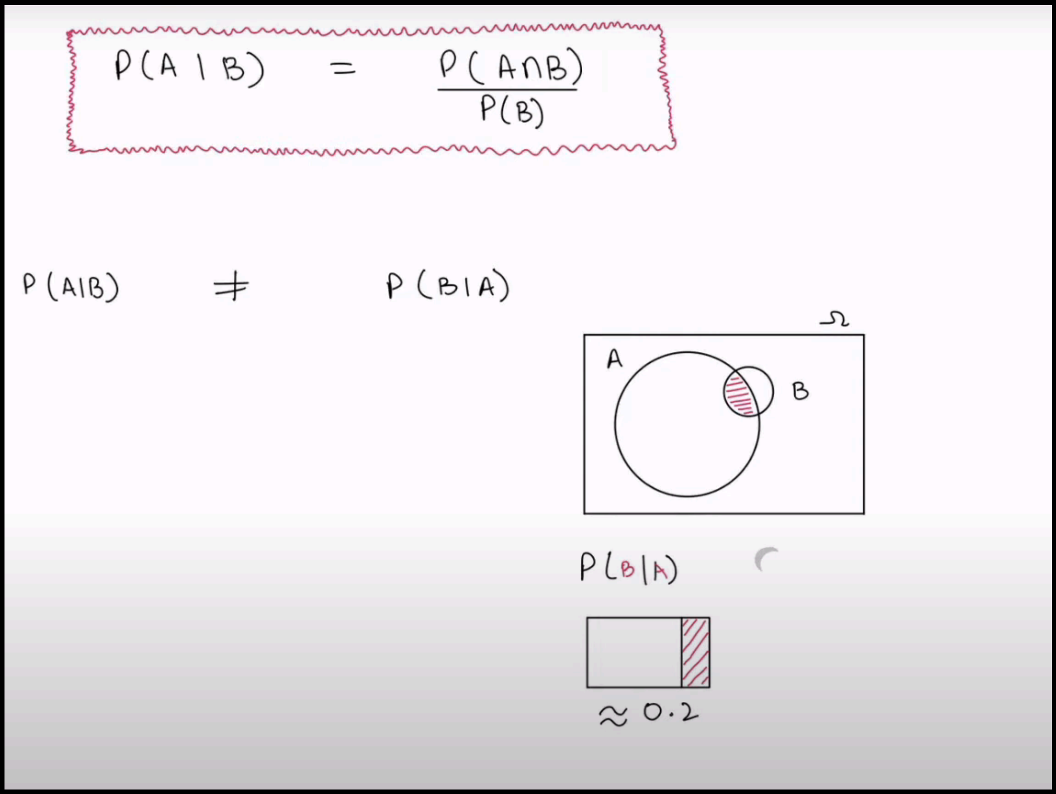
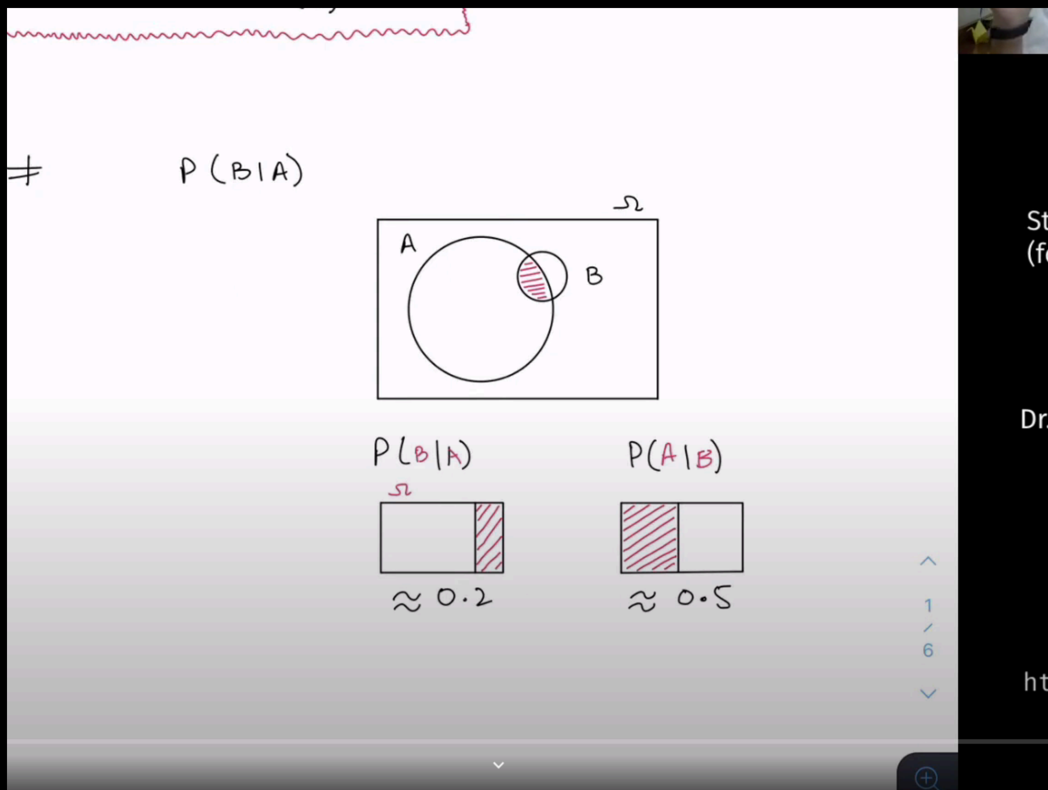
Probability And Statistics Lecture : 11

- Conditional Probability
    - The new rule says that probability of P(A | B) ≠ P(B | A)
    - Everyone universe has its own events
    - 

    - As you can see P (B | A) has a lesser probability of 0.2 because in the original universe A was a bigger event and B happening in it with a significantly lower probability , which is why the probability of A happening such that B has happened is relatively low . As B has a relatively lower span in the universe.



- Now if i talk about P(A | B) we can say that probability of A happening such that B has already happened is 0.5 because A has significantly greater probability of happening because it has a bigger event span in the universe as compared to B which is why A has greater probability of happening with respect to B

- Now after understanding we can clearly vividly see that both of the conditional probabilities are not equal to each other

- Interpretation

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

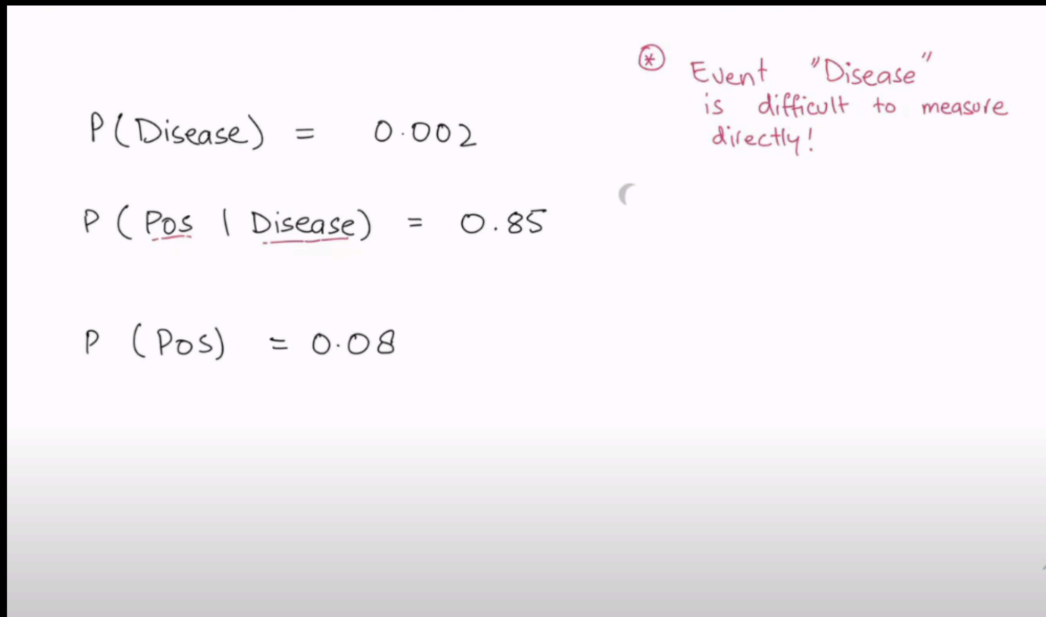
- We had discussed about it earlier but we were not able to solve it now we are gonna solve it properly as we have better understanding of the underlying concepts of probability

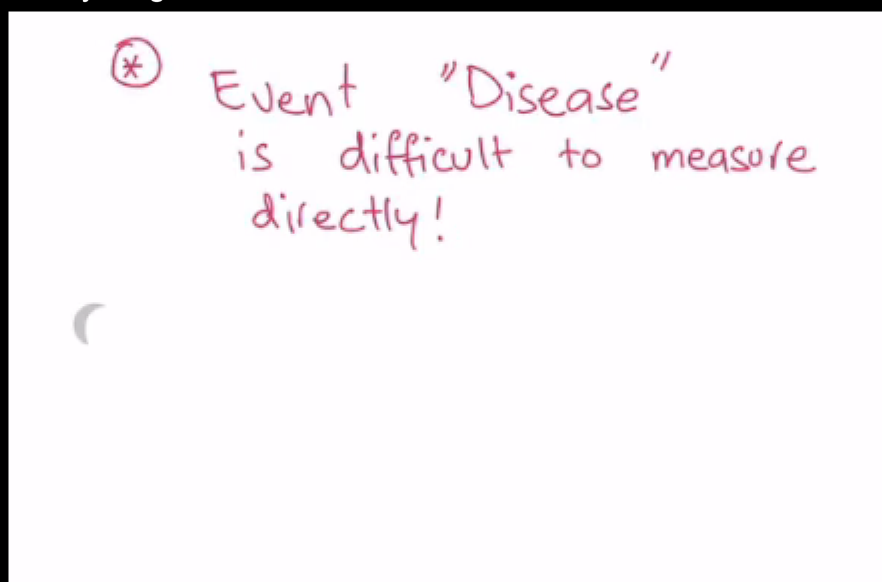
Problem :

- A disease is prevalent in 0.2% of a population.
- We have a test that, given to a sick person, gives a +ve result 85% of the time.
- Of all the people ever tested, 8% were positive.

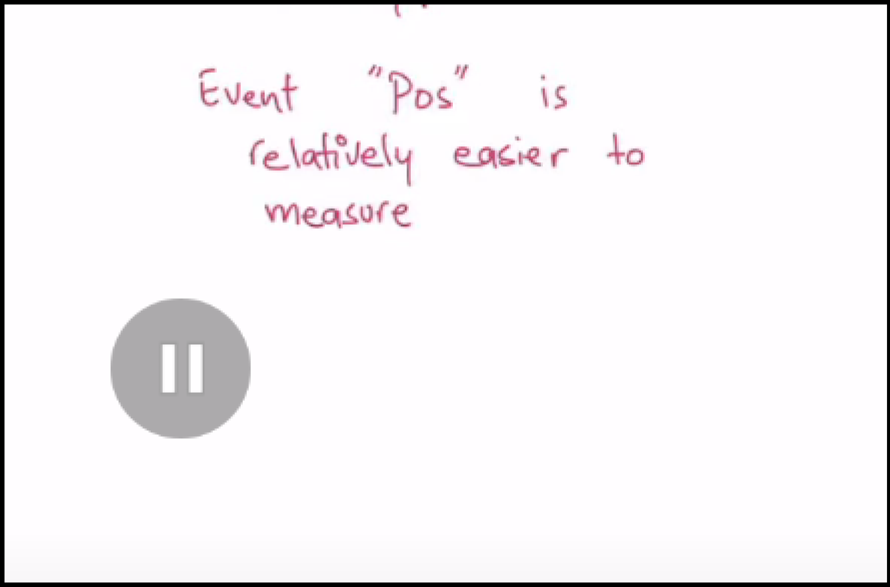
Q: If Nazo is tested and test comes back positive, what are the chances that she actually has the disease?

☐ 85% ☐ 77% ☐ 21% ☑ 2%

- 

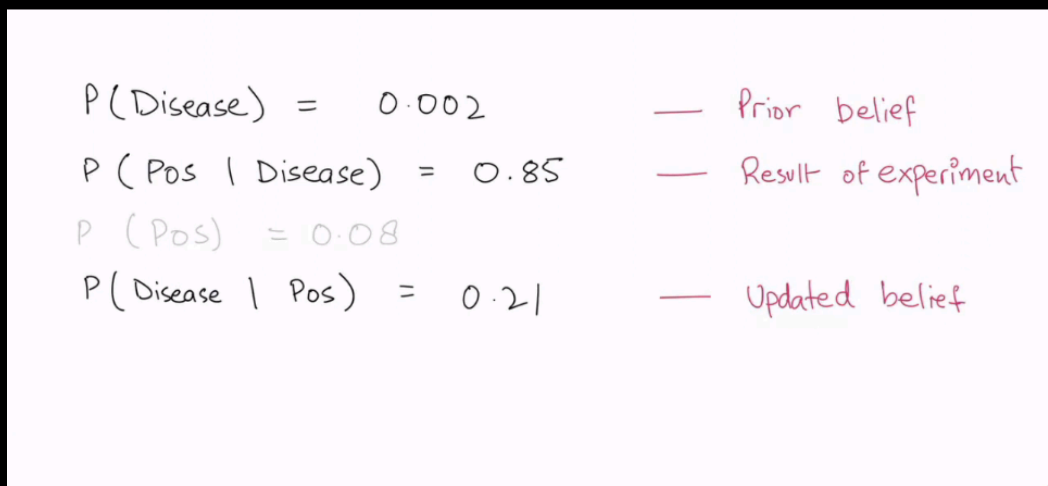
- After going through the question we can conclude this but before we conclude anything more than this first we have to talk about the following
    - 

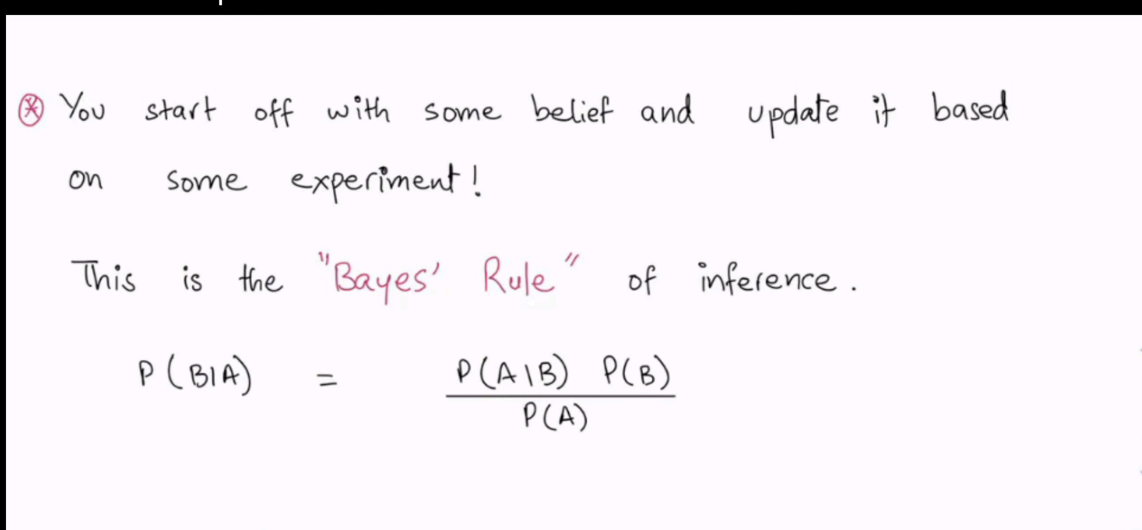
    - It is because the disease is hidden inside the body and we cannot get to it directly hence we conduct experiments and based on those experiments we find out what the results are and then we can conclude our theory

- 

    - It is easy because there are only two possibilities either positive or not
- Bayes Rule :

- $$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad P(A|B) = \frac{P(A \cap B)}{P(B)}$$
  $$P(A \cap B) = P(A|B)\,P(B)$$
  $$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$
- $$P(\overset{B}{\text{Disease}} \mid \overset{A}{\text{Pos}}) = \frac{P(\overset{A}{\text{Pos}} \mid \overset{B}{\text{Disease}})\,P(\overset{B}{\text{Disease}})}{P(\underset{A}{\text{Pos}})}$$
  $$= \frac{(0.85)(0.002)}{0.08}$$
  $$= 0.021 \qquad \underline{\underline{2.1\%}}$$
- With the help of mathematics we just rearrange the conditional formula and put it in the original one to form a bayes rule from it which is something like below
- $$\underbrace{P(\text{Disease} \mid \text{Pos})}_{\substack{\text{Posterior} \\ \text{(After)}}} = \frac{\overbrace{P(\text{Pos} \mid \text{Disease})}^{\text{Likelihood}}\ \overset{\text{Prior (Before)}}{P(\text{Disease})}}{\underset{\text{Normalizing factor}}{P(\text{Pos})}}$$
- Prior :

        - It is the probability that we conclude on information known before running any experiments or tests
    - Likelihood :
        - It is the actual result of the experiments done
    - Normalizing Factor :
        - It is just to maintain the probability axioms and to normalize our equation
    - Posterior :
        - It is the result we need the probability found after conducting tests and experimenting and then concluding onto some probability . This is what we are interested in
    - 

    - First we knew about the prob of the disease
    - Then we found the prob of positivity such that disease has occured
    - Then with the help of bayes rule we found out the prob of disease such that it is positive as well . which is 0.2%
- 

- Difference between Classical Statistics and Bayesian Rule

- The classical stats just concludes the result based on number of experiments which is not right
- In bayesian rule we keep prior knowledge before conducting any results
    - The higher the number of experiments you provide the more the prior effect will fade away eventually
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