

Lecture 8 :

Quantifying Chances

- Flip a coin:

Q: "What are the chances that it will land on its head?"

Not: "If I flip it 10 times, how many will be heads?"

Here are two questions first one is hard and the second one is easy

- first one is **hard** because u do one flip and before the event occur u have to guess either it will be head or tail
- second one is **easy** because u have to count how many heads will occur if u flip a coin 10 times(u just have to count)

Event: An event is an outcome or set of outcomes resulting from an experiment.

Mutually Exclusive: two or more events that cannot occur at the same time.

H, T

Events

Universe	
0.5	0.5
H	T

$\Omega = \{H, T\}$

This is an universe in which only two events occurs either **head** or **tail**

it is represented by Ω .

now it is given that the **both events are equally likely** → so it answers the question the chance of getting the head or tail is **50%**.

The chances of its lying on head is 0.5 and its lying on tail is 0.5.

either it will head or it will tail nothing else → looks like the binary number either 1 or 0

So flipping a single coin equals to 1 bit of information.

	H	T
Chance of it being heads	$P(H) = 0.5$	
" " " " tails	$P(T) = 0.5$	

So chance of being heads or probability of heads is 0.5. Similarly probability of tail is 0.5.

Axiom: Something u can't prove but u agree to it because it makes sense.

In probability there are **two** axioms((which we can't prove but everyone agrees to it because they are very rational)).

1. Probability of an event must lie between 0 and 1.

$0 \rightarrow$ it is definite that it will not occur.

$1 \rightarrow$ it is definite that it will occur.

(we are talking about chance so yeah what will be greater than if u know that something will definitely occur)

2. Sum of all events must be 1. (suppose take the example of coin so we already believe its in definition that there will be only two events head or tail so there both probability will be equals to 1 because what else is there we already say that there are only two events).

Now take the example of two coins

\rightarrow Flip it twice:	HH, HT, TH, TT	
		H H
		H T
The two flips are "independent".		T H
		T T

TRUTH TABLE

flip it twice now there are **four** possibilities.

Two events are independent means that the second event will not be effected by the first one .

- Now if we want to find an event that the **both flips are head.**
- As we now that sum of all will be equal to one and the no of possible outcomes is 4 so

→ Flip it twice:

$\{ \underline{HH}, \underline{HT}, \underline{TH}, \underline{TT} \}$

The two flips are "independent".

$P(HH)$

"Both flips are heads"

0.25	0.25
HH	HT
0.25	0.25
TH	TT

$$P(\text{favorable event}) = \frac{\# \text{ of ways event can occur}}{\# \text{ of all possible outcomes}}$$

$$= \frac{1}{4} = 0.25 = 25\%$$

probability will be $\frac{1}{4}$ as there is only one event in which both head occurs and all the four events are equally likely.

If we want to **find any of the flip is head**. So there are three events in which at least one head occurs so $\rightarrow \frac{3}{4}$

$$\begin{aligned}
 & \text{"Any of the flips is a head." } \} \text{ event} \\
 & A = \{ HH, HT, TH \} \\
 & P(A) = \frac{3}{4} = 0.75
 \end{aligned}$$

Alternate way :-

Alternate way / Intuition									
"Both flips are heads"	<table> <tr><td>HO</td><td>HO</td></tr> <tr><td>HO</td><td>TI</td></tr> <tr><td>TI</td><td>HO</td></tr> <tr><td>TI</td><td>TI</td></tr> </table>	HO	HO	HO	TI	TI	HO	TI	TI
HO	HO								
HO	TI								
TI	HO								
TI	TI								
First flip is H <u>and</u> second flip is H									

As we say that truth table is similar to it and also in the statement and is used so if we apply the and truth table to get to the answer

AND		
HO	HO	0
HO	TI	0
TI	HO	0
TI	TI	1

$$\begin{aligned}
 & \text{First flip is H and second flip is H} \\
 & P(H) * P(H) \\
 & = 0.5 * 0.5 = 0.25
 \end{aligned}$$

As **AND** is used and we **multiply** in **AND table** so if I multiply the probability of first event with the second and yeah we got the correct answer.

Two combine event probability will be less than the events independently.

Let's try to use this intuition for the second question

"Any of the flips is a head."

First flip is H or second flip is H

$$P(H) + P(H)$$
$$= 0.5 + 0.5$$
$$= 1$$

OR		
HO	HO	0
HO	TI	1
TI	HO	1
TI	TI	1

As OR is used so we add but we got the wrong answer ,Intuition here failed.

OR

HO	HO
HO	TI
TI	HO
TI	TI

We are counting the HH two times
for first flip is head → **HH**, HT
for second second flip is head → **HH** , TH

so remove the one HH

as we calculate above the probability of HH is 0.25

so

Remove the overlap

$$\begin{aligned}
 & P(H) + P(H) - P(HH) \\
 &= 0.5 + 0.5 - 0.25 \\
 &= 0.75
 \end{aligned}$$



0.75 the correct answer.

Now these two events are **no longer independent**.

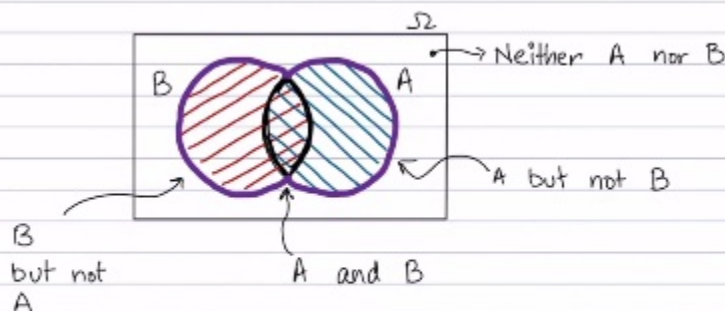
A and B are not independent!

Another view:

HH	HT
TH	TT

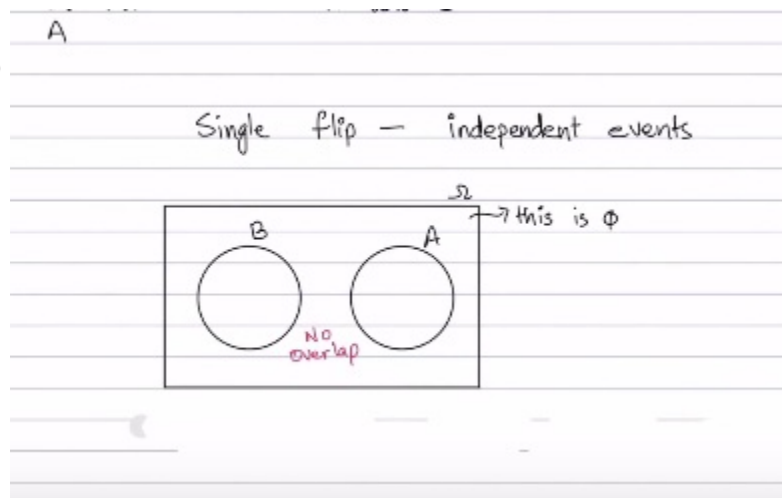
 first flip H A
 Second flip H B

Blue area is first flip is head
Red area second flip is head
HH area is overlapping



Represent above diagram in circles

independent
events → no overlap



so formula will be

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ subtracting the overlap area

Notation:	
A and B	$= A \cap B$
A or B	$= A \cup B$

Overview of the lecture:

- **defining a problem in which we see that quantifying the chance is necessary.**
- **Uncertainty.**
- **Example of flipping a single coin which looks like binary number.**
- **Chance is probability.**
- **Two axioms of probability.** 1 → probability lies btw 0 and 1 2. → sum of all events must be 1.
- **Two flips example.**
- **Compound Events .** Example : Both flips are head. Formula → # of ways an event can occur/ # of all possible outcomes → n/N
- **Alternate way using truth table.**
- **Overlap problem in OR case** → we are counting HH two times.
- **Venn diagram.**