CS 419: Production Computer Graphics Fall 2018
In-class Worksheet

Interpolation and Iterative Solvers

1. **Trilinear Interpolation**

Suppose you have the following sampled dat a

f(0,0,0)= 3	f(0,0,1)= 2
f(1,0,0)=19	f(1,0,1)= 10
f(0,1,0)= 4	f(0,1,1)= 24
f(1,1,0)= 8	f(1,1,1)= 0

Using trilinear interpolation, what is the value of f(1/4 ,1/2, 4/5)?

2. Evaluating Points on a Catmull-Rom Spline

Suppose we are evaluating a noise function using cubic Catmull-Rom splines with T=1/2. Derive an expression for the function evaluation at u=0, u=1/2, and u=1.

$$\mathbf{p}(s) = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\tau & 0 & \tau & 0 \\ 2\tau & \tau - 3 & 3 - 2\tau & -\tau \\ -\tau & 2 - \tau & \tau - 2 & \tau \end{bmatrix} \begin{bmatrix} \mathbf{p}_{i-2} \\ \mathbf{p}_{i-1} \\ \mathbf{p}_i \\ \mathbf{p}_{i+1} \end{bmatrix}$$

3. Jacobi Method

Suppose we have a linear system $Ax = b$ with initial approximate solution $x^{(0)}$

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 7 \end{bmatrix}, \; b = \begin{bmatrix} 11 \\ 13 \end{bmatrix} \quad \text{and} \quad x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Perform 2 iterations of the Jacobi method to find $x^{(2)}$. That is, using rational numbers (unless you want to use a calculator) find $\mathbf{x}^{(k+1)} = D^{-1}(\mathbf{b} - R\mathbf{x}^{(k)})$ where

$$A = D + R \qquad \text{where} \qquad D = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \text{ and } R = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ a_{21} & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 0 \end{bmatrix}$$