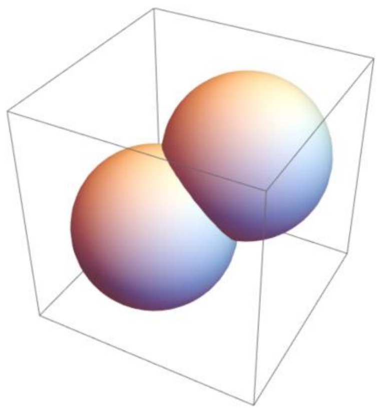
CS 419: Production Rendering Fall 2018
In-class Worksheet 6

Monte Carlo Methods

1. We want to calculate the volume of intersection between two spheres of radius 1. Sphere S1 is centered at (1,1,0) and S2 is at (1,2,0). Describe how you could use a Monte Carlo method to compute this volume.



Let V_b be the volume of a box from [0,2] x [0,3] x [-1,1]

Generate N samples (x,y,z) with
x = 2.0*rand()
y = 3.0*rand()
z = 2.0*rand() -1.0

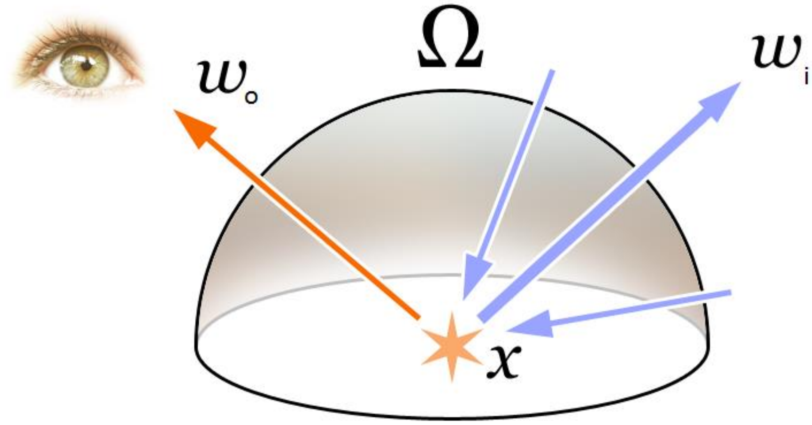
Let N_x be the number of samples that are within a distance of 1.0 from both sphere centers.

The volume of intersection V_x will then be V_x=12.0(N_x/N)

2. Suppose we generate an insufficiently converged approximation to an integral using a Monte Carlo method and N samples. If we wish to reduce the error by 1/3 with high probability, how many samples are needed?

9N

Rendering Equation



The hemisphere form of the rendering equation was developed by James Kajiya in1986. Describe, as best you can, what each of the components below represents:

$$L_{\text{o}}(\mathbf{x},\,\omega_{\text{o}},\,\lambda,\,t) \;=\; L_e(\mathbf{x},\,\omega_{\text{o}},\,\lambda,\,t) \;+\; \int_\Omega f_r(\mathbf{x},\,\omega_{\text{i}},\,\omega_{\text{o}},\,\lambda,\,t)\, L_{\text{i}}(\mathbf{x},\,\omega_{\text{i}},\,\lambda,\,t)\, (\omega_{\text{i}}\,\cdot\,\mathbf{n})\;\mathrm{d}\,\omega_{\text{i}}$$

- $L_{\text{o}}(\mathbf{x},\,\omega_{\text{o}},\,\lambda,\,t)$ is the total spectral radiance of wavelength λ directed outward along direction ω_{o} at time t, from a particular position $\mathbf{x}$
- $\mathbf{x}$ is the location in space
- ω_{o} is the direction of the outgoing light
- λ is a particular wavelength of light
- t is time
- $L_e(\mathbf{x},\,\omega_{\text{o}},\,\lambda,\,t)$ is emitted spectral radiance
- $\int_\Omega \dots\,\mathrm{d}\,\omega_{\text{i}}$ is an integral over Ω
- Ω is the unit hemisphere centered around $\mathbf{n}$ containing all possible values for ω_{i}
- $f_r(\mathbf{x},\,\omega_{\text{i}},\,\omega_{\text{o}},\,\lambda,\,t)$ is the bidirectional reflectance distribution function, the proportion of light reflected from ω_{i} to ω_{o} at position $\mathbf{x}$, time t, and at wavelength λ
- ω_{i} is the negative direction of the incoming light
- $L_{\text{i}}(\mathbf{x},\,\omega_{\text{i}},\,\lambda,\,t)$ is spectral radiance of wavelength λ coming inward toward $\mathbf{x}$ from direction ω_{i} at time t
- $\mathbf{n}$ is the surface normal at $\mathbf{x}$
- $\omega_{\text{i}}\cdot\mathbf{n}$ is the weakening factor of outward irradiance due to incident angle, as the light flux is smeared across a surface whose area is larger than the projected area perpendicular to the ray. This is often written as $\cos\theta_i$.