

## Geometric Modeling

1. Prove the average valence of a vertex in a manifold triangle mesh is 6. *Hint: Use the Euler characteristic  $V-E+F=2(1-g)$  to write  $E$  in terms of  $V$*

$$V+F = E+2 \text{ (let's just drop the 2)}$$

$$V+(2E/3) = E$$

$$V = (1/3)E$$

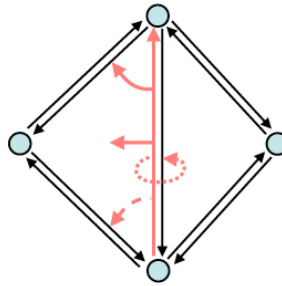
$$3V = E$$

Each edge has 2 endpoints so  $2E$  endpoints are distributed over  $V$  vertices, with each endpoint indicating a neighbor  
So, average valence is 6

- Write a pseudo-code function that uses the half-edge data structure below to find all neighboring vertices of a vertex  $v$ .

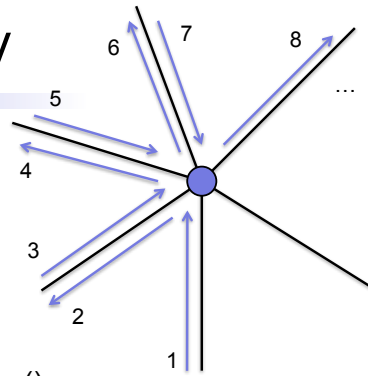
```
class HalfEdge {
    HalfEdge *opp;
    Vertex *end;
    Face *left;
    HalfEdge *next;
};

HalfEdge e;
```



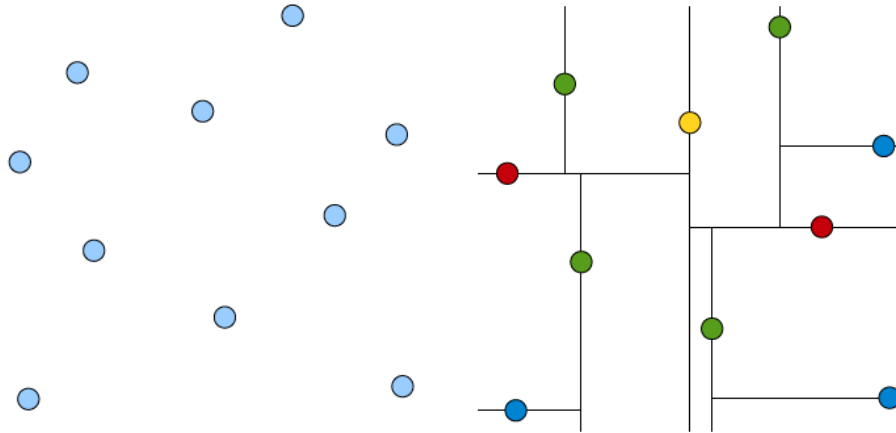
## Vertex Star Query

- $e$
- $e \rightarrow \text{next}()$
- $e \rightarrow \text{next}() \rightarrow \text{opp}()$
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- $e \rightarrow \text{next}() \rightarrow \text{opp}() \rightarrow \text{next}() \rightarrow \text{opp}() \rightarrow \text{next}() \rightarrow \text{opp}() \rightarrow \text{next}()$
- ... until  $e(-\rightarrow \text{next}() \rightarrow \text{opp}())^n == e$



## Spatial Partitions

3. Draw the 2D spatial partition that would be created by a kd-tree for the following point set. Assume you starting splitting the x-axis and split at the median point each time.



4. Suppose the overall bounding box for the tree is given by the corners  $(3, 5)$  and  $(8, 15)$  and the first split is at point  $(7, 11)$ . What is the bounding box of the right child of the root?  
  
 $(7, 5)$  to  $(8, 15)$
5. Compare the number of cells in a uniform grid containing  $n$  points and a kd-tree containing  $n$  points. Assume the uniform grid uses the allocation strategy discussed in lecture with a magic number  $m=2$ .

The kd-tree will generate  $n+1$  spatial cells while the grid will be around  $8n$  cells