

1  $-17 \bmod 23$

Soln: we know,

$$a \bmod m = (a+m) \bmod m \text{ if } a < 0$$

$$\therefore -17 \bmod 23 = (-17+23) \bmod 23$$

$$= 6 \bmod 23$$

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2 Multiplicative Inverse of  $-13 \bmod 23$

Soln: we want a number  $x$  such that:

$$(-13)x \equiv 1 \bmod 23$$

let's convert  $-13 \bmod 23$  into its positive equivalent:

$$-13 \bmod 23 = 10 \text{ (since } -13+23=10)$$

now,  $10 \cdot x \equiv 1 \bmod 23$

using Extended Euclidean Algorithm -

$$23 = 2 \times 10 + 3$$

$$10 = 3 \times 3 + 1$$

$$3 = 3 \times 1 + 0$$

Back-substitute:  $1 = 10 - 3 \times 3$

$$\Rightarrow 1 = 10 - 3(23 - 2 \times 10)$$

$$\Rightarrow 1 = 10 - 3 \times 23 + 6 \times 10$$

$$\Rightarrow 1 = 7 \times 10 - 3 \times 23$$

$$\Rightarrow 10^{-1} \bmod 23 = 7 \therefore (-13)^{-1} \bmod 23 = 7 \text{ Ans: } 7$$