Operations Research I: Models & Applications

Course Overview

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Road map

- **Motivation**.
- Business analytics.
- Mathematical programming.
- History.
- Preview for this course.

Resource allocation

- What is **management**?
- Someone said:
  - "Management is the attainment of organizational goals in an effective and efficient manner through planning, organizing, leading, and controlling organizational resources."[1] (Daft, 2014, p. 7)
- What are **resources** in an organization?

[1] Daft, R. L. (2014). *Management*.

Resource allocation

- Setting an objective requires strategic thinking, experiences, vision, etc.
- "Resource allocation" uses **Operations Research (OR)**.
- Names of similar subjects/ideas:
  - Management science.
  - Decision science.
  - Optimization method/algorithm.
  - Mathematical programming.
- Let's see some examples.

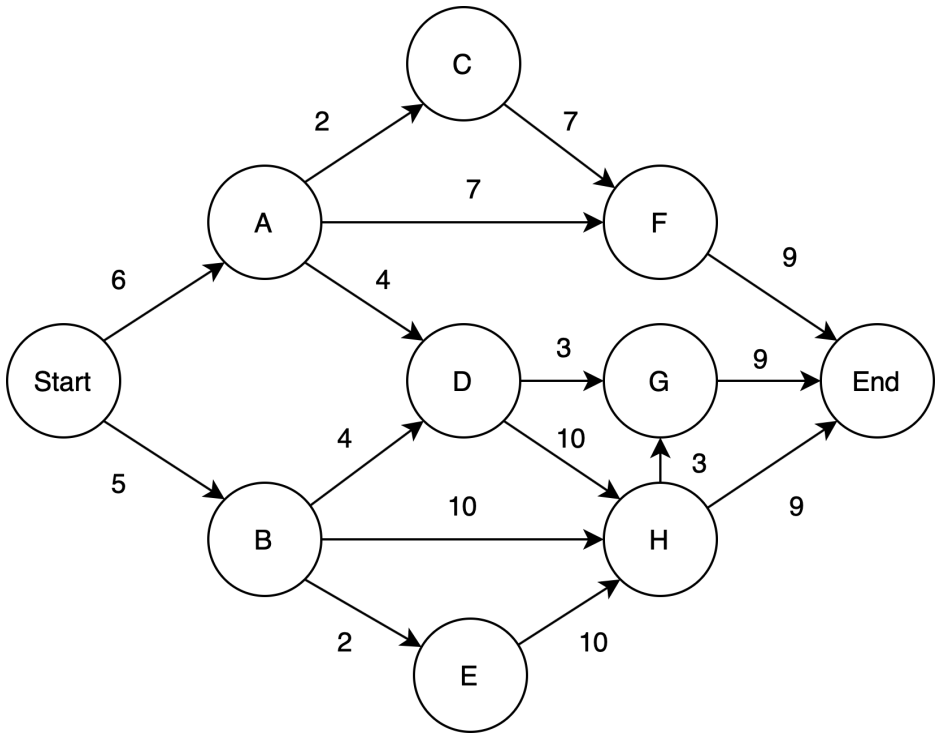
Example: job allocation

- Two people are going to hold an event, and they need to complete some tasks.
- One task must be assigned to exactly one person; one person can work on one task at a time.
- How to assign the tasks so that they can complete all tasks the fastest?
- What are the resources? What is the objective?

ID	task	processing time (min)
1	boiling boba	20
2	brewing milk tea	30
3	baking cookies	60
4	designing poster	15
5	renting handcart	25

Example: project management

- n workers are going to complete m jobs in a project.
  - Some jobs must be processed with precedence rules.
  - Some jobs cannot be done by certain workers.
  - Some jobs can be split and allocated to several workers.
  - Some jobs require different processing time if allocated to different workers.
- How many days does it take to complete this project?



Industry applications

- Key decisions:
  - How to deliver 6.5 millions items to more than 220 countries each day?
  - In each region, where to build distribution hubs?
  - In each distribution hub, how to classify and sort items?
  - In each city, how to choose routes?
- What do you need?
  - Well-designed information systems.
  - Operations Research!

Industry applications

- Key decisions:
  - How to determine the cities to connect?
  - How to schedule more than 2000 flights per day?
  - How to assign crews to flights?
  - How to reassign crews immediately when there is an emergency?
- What do you need?
  - Well-designed information systems.
  - Operations Research!

What is Operations Research?

- **Operations Research** (OR) is:
  - the methodology to "**allocate** the available **resources** to the various activities in a way that is most effective for the organization as a whole."
  - It is "applied to problems that concern how to conduct and coordinate the **operations** (i.e., activities) within an organization."[2]
- It aims to **support decision making**.
  - Typical tools: intuitions, business senses, and experiences.
  - And OR (and other quantitative tools)!
  - By doing OR studies, we generate some suggestions to **decision makers**.

[2] Both are quoted from the ninth edition of *Introduction to Operations Research* by Hillier and Lieberman.

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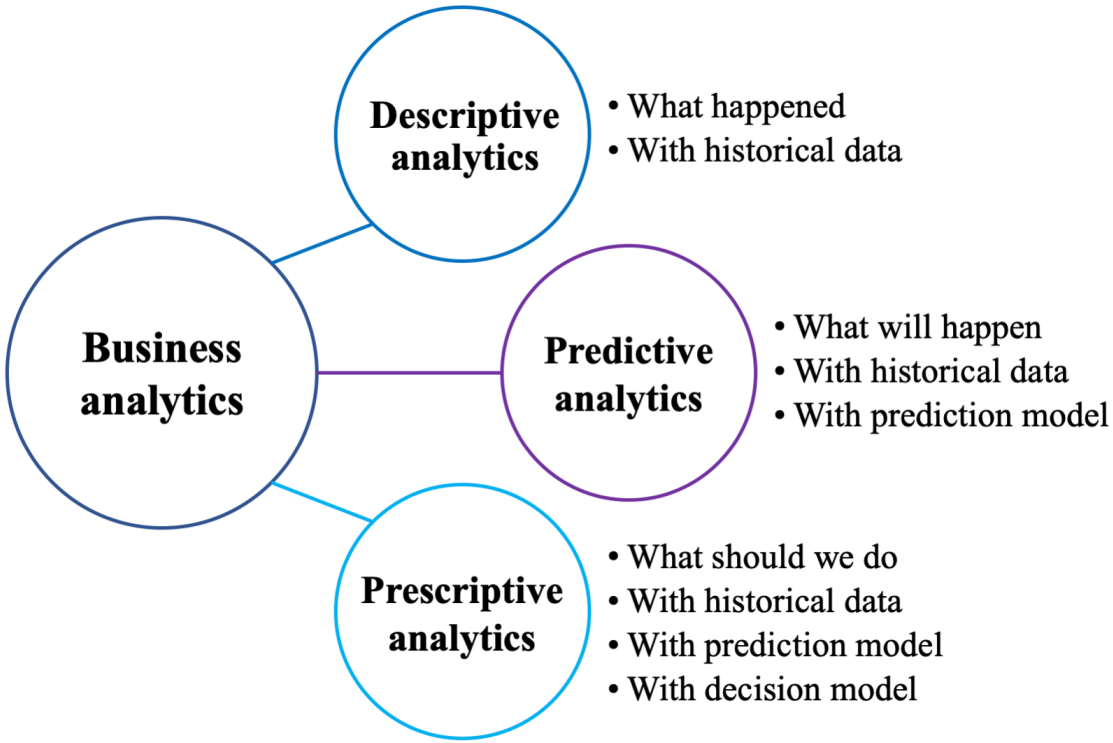
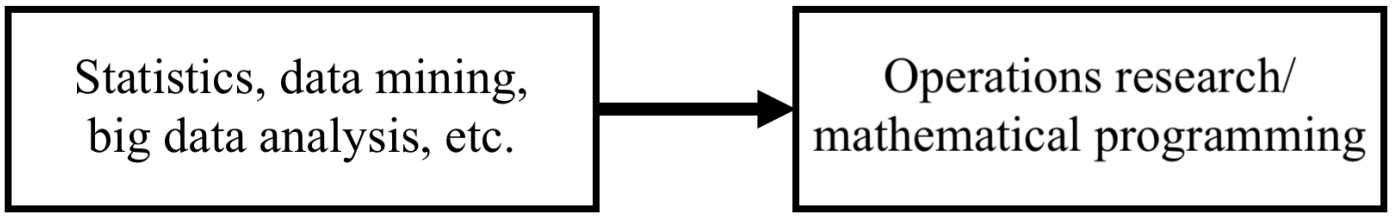
Business analytics

- Today, everybody talks about **business analytics**.
- Master of business administration (MBA) becomes master of business analytics (or master of science in business analytics).



Operations research/mathematical programming

- A typical process of decision making with business analytics:
  - Data analysis: Collect information and **understand** the problem.
  - Operations research: Allocate resources and **solve** the problem.



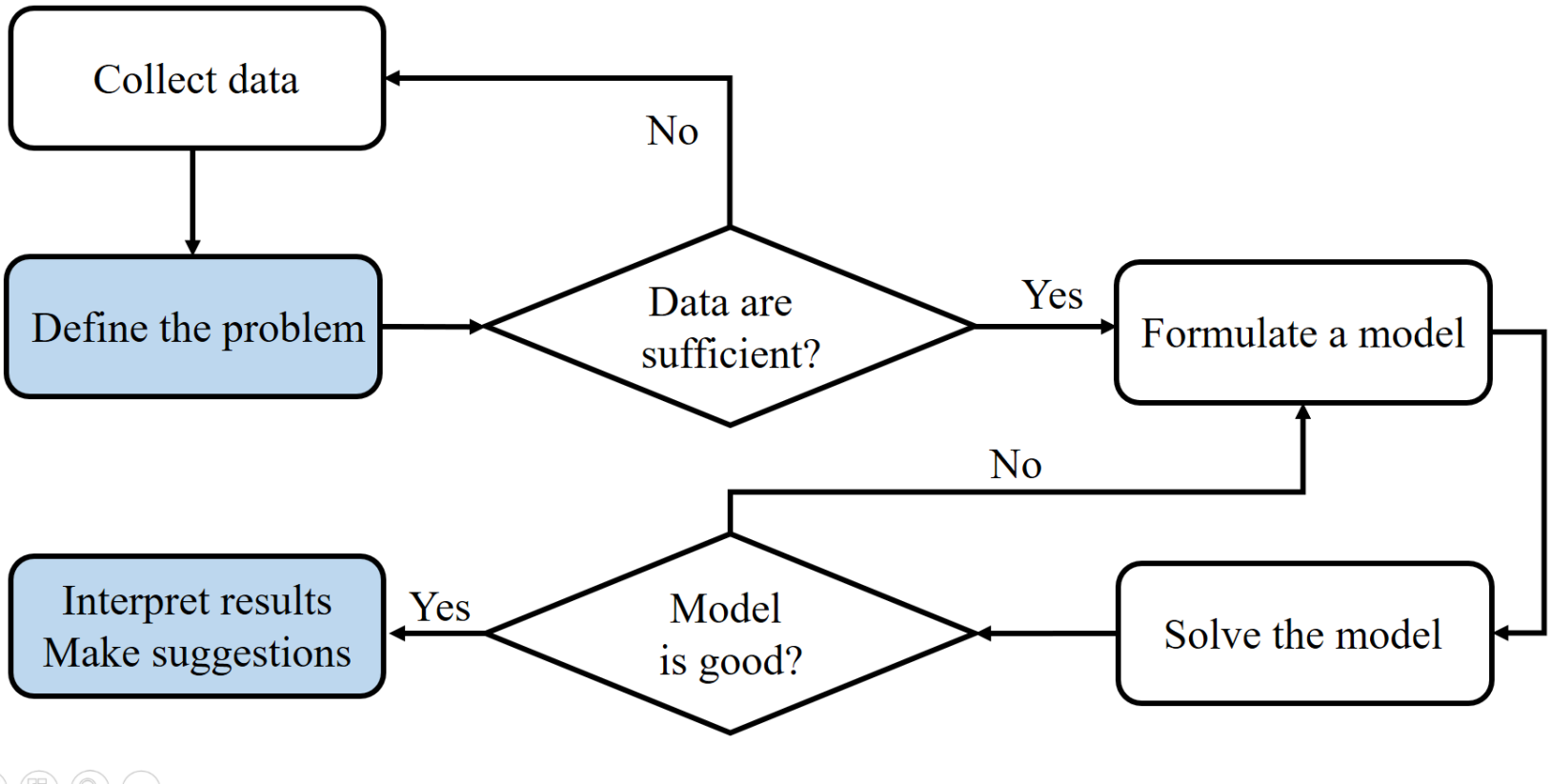
Example: multi-product inventory

- How to set the inventory levels of multiple products to maximize the total expected profit?
- Suppose that there is only one product.
  - Prevent understocking or overstocking.
  - Data analysis: **Estimate** the random amount of demand during one order cycle time.
  - Operations research: According to the random amount of demand, **find** the inventory level to maximize the expected profit.

Example: multi-product inventory

- When we have multiple products:
  - **Demand substitution**: "There is no more Coke. How about Pepsi?"
- Data analysis is difficult.
  - **Estimate** the probability of demand substitution between A and B, which is the probability for one to purchase B when A is sold out (or purchase A when B is sold out).
- Operations research is also difficult.
  - Given the substitution probabilities, **find** the best inventory levels of all products.

The process of conducting an OR study



Analytics is not everything

- Advantages:
  - It describes a problem in a precise and concise way (among people and between a person and a computer).
  - It facilitates the use of computers to solve a problem whose model has been formulated.
- Disadvantages:
  - Some of the problems are hard to be formulated into mathematical models.
  - Some critical information is missing.
- The aim of this course: Through various examples, we want to let people get an concrete idea about:
  - What **may** be solved by operations research.
  - What **cannot** be solved by operations research.

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Example: knapsack problem

- You preparing for hiking. There are some useful items, but your backpack (knapsack) can only carry 5 kilograms.
- An item cannot be split: Each item should be either chosen or discarded.
- Which items should you bring to maximize the total value?

ID	Item	Weight	Value
1	compass	0.5	6
2	hatchet	1.5	5
3	matches	0.4	4
4	tarpaulin	1	4
5	telescope	1.1	3
6	cylinder	1.6	4
7	Rilakkuma	0.8	1

Example: knapsack problem

- Choosing items according to their weight-value ratios results in items 1, 2, 3, 4, and 5. The total value is 22.
- The optimal solution is to bring items 1, 2, 3, 4, and 6. The total value is 23.
- How may we **model** or **formulate** this problem?

ID	Item	Weight	Value	Value / Weight
1	compass	0.5	6	12
2	hatchet	1.5	5	3.33
3	matches	0.4	4	10
4	tarpaulin	1	4	4
5	telescope	1.1	3	2.73
6	cylinder	1.6	4	2.5
7	Rilakkuma	0.8	1	1.25

A mathematical programming model

- **Decision variables**: What may we determine?

$$x_i = \begin{cases} 1 & \text{if item } i \text{ is chosen,} \\ 0 & \text{otherwise.} \end{cases}$$

- **Objective function**: What do we want?

$$\max \; 6x_1 + 5x_2 + 4x_3 + 4x_4 + 3x_5 + 4x_6 + x_7.$$

- **Constraints**: What are the limitations?

$$0.5x_1 + 1.5x_2 + 0.4x_3 + x_4 + 1.1x_5 + 1.6x_6 + 0.8x_7 \leq 5.$$

and

$$x_i \in \{0, 1\} \quad \forall i = 1, ..., 7.$$

A mathematical programming model

- Collectively, a complete **formulation** (**model**) is:

$$\begin{aligned} \max \quad & 6x_1 + 5x_2 + 4x_3 + 4x_4 + 3x_5 + 4x_6 + x_7 \\ \text{s.t.} \quad & 0.5x_1 + 1.5x_2 + 0.4x_3 + x_4 + 1.1x_5 + 1.6x_6 + 0.8x_7 \leq 5 \\ & x_i \in \{0, 1\} \quad \forall i = 1, ..., 7. \end{aligned}$$

A compact formulation (advanced)

- Let w_i and v_i be the weight and value of item i.
- Let n be the number of items and B be the maximum allowable weight.
- A **compact** (and more abstract) formulation is

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leq B \\ & x_i \in \{0,1\} \quad \forall i = 1, ..., n. \end{aligned}$$

Application: order selection

- Let w_i and v_i be the processing time and gross profit of order i.
- Let n be the number of orders and B be the factory capacity (the maximum total processing time that a factory has).
- The formulation is (still)

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leq B \\ & x_i \in \{0,1\} \quad \forall i = 1, ..., n. \end{aligned}$$

Application: portfolio optimization

- Let w_i and v_i be the maximum possible investment amount and expected return of stock i.
- Let n be the number of assets and B be the total budget.
- The formulation is

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leq B \\ & 0 \leq x_i \leq 1 \quad \forall i = 1, ..., n. \end{aligned}$$

- Note that investment decision is not **all-or-nothing**.
  - $x_i \in \{0, 1\}$ becomes $x_i \in [0, 1]$ or $0 \leq x_i \leq 1$.

Linear programming vs. integer programming

- This is a **linear program**:

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leq B \\ & 0 \leq x_i \leq 1 \quad \forall i = 1, ..., n. \end{aligned}$$

- This is an **integer program**:

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leq B \\ & x_i \in \{0, 1\} \quad \forall i = 1, ..., n. \end{aligned}$$

- Various types of mathematical programs will be introduced and compared in this course.

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Origin of linear programming

- George Dantzig (1914-2005).
  - Ph.D. in Statistics, University of California, Berkeley (1946).
  - He solved two well-known difficult problems which were mistakenly considered as homework assignment.
- During World War II, he served in the United States Air Force doing route planning for military aircraft.
  - Each plan is called a "program."
- He found:
  - Many problems are linear optimization problems.
  - No one is able to systematically solve large-scale optimization problems.
- He invented the simplex method.
  - The "first" effective solution for linear programming in the world.
  - Still an important part in commercial software nowadays.

- Numbers of personnel required at an airport vary a lot in a day.

0–6	6–8	8–10	10–12	12–14	14–16	16–18	18–20	22–24
6	10	15	20	16	24	28	20	10

- How many people will you hire?
  - Each person works for eight hours **continuously**.
  - They may start their shifts at different time.
- Linear programming is used by United Airlines to reduce the number of flight delays by 50% and save more than \$5 million per year in 1992.[3]

[3] Rakshit A., N. Krishnamurthy, and G. Yu. (1996) "System Operations Advisor: A Real-Time Decision Support System for Managing Airline Operations at United Airlines." *INFORMS Journal on Applied Analytics* **26**(2) 50-58.

- If you produce foods, what are important in getting an order from restaurants and retailers?
  - You have more than 8000 customers sharing your capacity and inventory.
  - Customers ask "When may I get them?" and "How much may I get?"
  - Once you **promise** a customer, you need to immediately **update** the availability information that are needed elsewhere.
  - Updating requires a lot of **planning** and calculations.
- Swift & Company increased its annual earning by more than 12 million U.S. dollars in 2006 with a linear programming solution.[4]

[4] Bixby A., B. Downs, and M. Self, (2006) "A Scheduling and Capable-to-Promise Application for Swift & Company." *INFORMS Journal of Applied Analytics* **36**(1) 69-86.

History of linear programming

- Linear programming has been widely used for government, business, and organizations.
- Two linear programming scholars, Leonid Kantorovich and Tjalling Koopmans, are awarded the 1975 Nobel Memorial Prize in Economic Sciences.
  - "For their contributions to the theory of **optimum allocation of resources**."
- How about Dantzig?
- At least they three took a photo together.[5]

[5] Gass, S. I. and Assad, A. A. (2005). *An Annotated Timeline of Operations Research: An Informal History*. Boston, MA: Springer.

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Course structure

Week	Topic	Computer
1	Course overview	
2	Linear programming	V
3	Integer programming	V
4	Nonlinear programming	V
5	Case studies	
6	Course summary and future directions	

Course contents

- In this course, we focus on **models** and **applications**.
  - We talk about the basic principles.
  - We formulate linear, integer, and nonlinear programs.
  - We experience solving mathematical programs using MS Excel solver.
  - We go though cases, applications, and business implications.
  - We (hopefully) let you know when to use (and when not to use) operations research.
- Prerequisites:
  - High-school mathematics (mainly algebra).
  - High-school English.

Let's learn operations research together!