Operations Research I: Models & Applications
Case Study: Call Center Personnel Scheduling

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Road map

- **Background, motivation, and research objective**.
- Problem description.
- Model formulation.
- Results.

CSR scheduling

- In a call center, many **customer service representatives** (CSR) take turns to serve customers.
- To meet the desired service level, historical in-coming calls have been analyzed to calculate the **ideal number of CSRs** in every time period.
- For example:

Hour	Mon.	Tue.	Wed.	Thu.	Fri.
9	46	36	36	37	35
10	46	36	38	36	35
11	47	37	38	35	11
12	29	23	24	23	22
13	36	31	31	28	29
14	20	35	37	36	35
15	38	35	35	31	35
16	35	32	32	32	30
17	28	23	24	23	7
18	18	18	18	15	15
19	6	5	16	16	5
20	20	4	15	15	4

CSR scheduling

- Even though we know the ideal number of CSRs in every time period, scheduling is still difficult:
  - One's work hours should be continuous in each day.
  - Some people are needed (not allowed) in some periods. E.g., there must be at least one manager in each night shift, pregnant women should not work at night, etc.
  - Fairness. E.g., one CSR should work at night for at most once in a week.
- Traditionally, CSR scheduling is done by hands.
  - Tedious and easy to make errors.
  - It takes one senior manager one whole day to create a (not so good) schedule: Sometimes there is shortage but sometimes there are extra CSRs.
  - Difficult to deal with emergent issues and accidents.

Scheduling with Operations Research

- This is an **allocation** problem.
  - The number of CSRs is limited.
  - Try to meet the demands under resource limitation: Minimizing the total shortage.
  - Allocation problem ⇒ **Linear programming**.
- This is a **selection** problem.
  - If one works in the morning and afternoon, she/he cannot work in the evening in that day.
  - If one works at night on Monday, she/he cannot do that from Tuesday to Friday in that week.
  - Selection problem ⇒ **Integer programming**.
- This is unlikely a **balancing** problem.
  - Nonlinear programming will not be used.

Research objective

- Let's formulate an **integer program**:
  - To tell each CSR which days to work (in which way) for the next month.
  - To minimizing the total shortage for the next month.
  - While following the scheduling rules set by the company.

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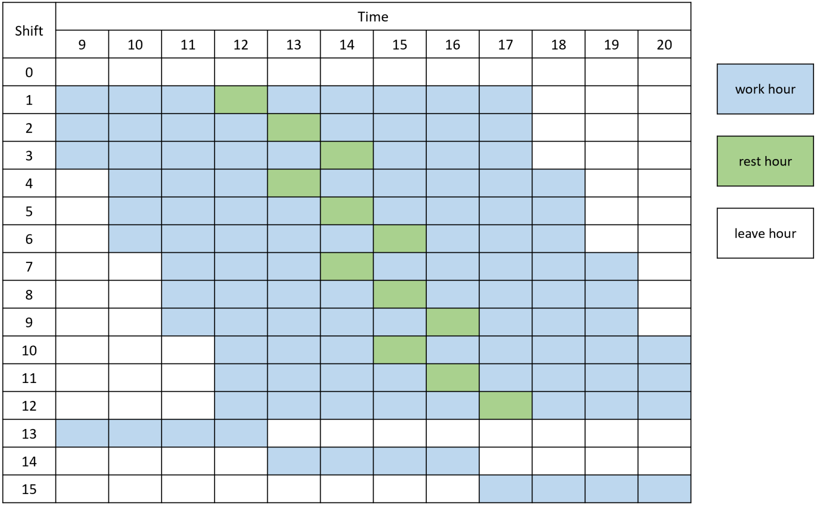
Decision making

- What exactly is the **decision** we need to make?
  - For each CSR, we should tell her/him which days to have days off.
  - For each work day of each CSR, we should tell her/him when to start work, leave the office, and take a break.
- In practice, managers list **shifts** to define work hours.
- As an example:
  - Shift 1: 8:00-17:00 with a lunch break at 12:00-13:00.
  - Shift 2: 8:30-17:30 with a lunch break at 12:30-13:30.
  - Shift 3: 8:30-17:30 with a lunch break at 12:00-13:00.
  - ...
  - Shift 0: a day off.
- The decision we need to make is to **assign one shift to each CSR for each day** (except holidays).

An illustration for shifts



Objective: minimizing the total shortage

- The number of CSRs needed (A) in a time period is known.
- After assigning shifts to CSRs, we may calculate the number of CSRs being on duty (B) in that period.
- The amount of **shortage** is thus $\max\{A - B, 0\}$ in each period.

CSR	Shift	Time											
		9	10	11	12	13	14	15	16	17	18	19	20
01	0												
02	1												
03	1												
04	4												
05	7												
06	7												
07	15												
Total CSRs		2	3	5	3	4	4	4	5	6	4	3	1
Demand		3	3	4	4	4	4	4	3	3	3	2	2
Shortage		1	0	0	1	0	0	0	0	0	0	0	1

Hard constraints (must have)

- Each CSR should be assigned to exactly one shift per day.
- A CSR may require a specific shift on a certain day.
  - E.g., a CSR chooses a day off in advance.
- Each CSR should be assigned to at most one night shift (a shift that asks a CSR to work after 18:00 for at least one period) per week.
- There are some requirements regarding some shifts in certain weekdays.
  - E.g., at most 3 CSRs may have a day off on each Monday.
  - E.g., a CSR can at most take 2 night shifts on Thursday in a month.
- There are some requirements regarding CSRs in some shifts.
  - E.g., at least one manager in every night shift.
  - E.g. CSRs with more than 2 years of experience in the call center should account for more than 50% of the total on-duty CSRs in a night period.

Soft constraints (nice to have)

- In many cases, managers also have some **soft constraints** in mind: They may be violated if needed, but hopefully they are satisfied.
- **Fairness among periods**: It is good to have similar extra on-duty CSRs for every period.
  - Rather than having 5 extra CSRs in period 1 and only 1 extra CSR in period 2, it is better to have 3 extra CSRs in each period.
- **Fairness among CSRs**: It is good to have similar numbers of night shifts for every CSR.
  - Rather than letting CSR A have 4 night shifts and CSR B have 2 night shifts, it is better to assign 3 night shifts to each of them.

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Sets, indices, and variables

- Let I, J, K, and T be the **sets** of CSRs, days in which the call center is open, shifts, and periods in a day.
- Let $i \in I$ denote the ith CSR, $j \in J$ denote the jth day, $k \in K$ denote the kth shift, and $t \in T$ denote the tth period. i, j, k, and t are the **indices**.
- The core **decision variable**:

$$x_{ijk} = \begin{cases} 1 & \text{if CSR } i \text{ is assigned to shift } k \text{ in day } j \\ 0 & \text{otherwise} \end{cases}, i \in I, j \in J, k \in K.$$

- One derived decision variable:

$$y_{jt} = \text{the number of CSR shortage in period } t \text{ of day } j,\ t \in T,\ j \in J.$$

Objective function

- The objective function should include the major **objective** (minimizing the total lack of CSRs) and two **soft constraints** (fairness among periods and fairness among CSRs).
  - Let w_1 be the maximum number of extra on-duty CSRs among all periods.
  - Let w_2 be the maximum number of night shifts among all CSRs.
- A **weighted average** form is often adopted.
- The objective function is

$$\min \ P_0 \sum_{j \in J} \sum_{t \in T} y_{jt} + P_1 w_1 + P_2 w_2,$$

where P_0, P_1, and P_2 are the weights to be determined by the manager.

Constraints

- While there are many constraints to be formulated, here we only take four sets of constraints as an example:
  - For each CSR and each day, she/he can only be assigned to one shift.
  - There should be some constraints to calculate the total shortage (i.e., to link x_{ijk} and y_{jt}).
  - There should be some constraints to calculate the maximum number of extra on-duty CSRs among periods (i.e., to link x_{ijk} and w_1).
  - There should be some constraints to calculate the maximum number of night shifts among CSRs (i.e., to link x_{ijk} and w_2).
- How will you formulate these constraints?
- We will formulate them and define **parameters** when necessary.

Constraints

- For each CSR and each day, one CSR can only be assigned to one shift:

$$\sum_{k \in K} x_{ijk} = 1 \quad \forall i \in I, j \in J.$$

- To calculate the total shortage:

$$\begin{aligned} & y_{jt} \geq D_{jt} - \sum_{k \in K} A_{kt} \sum_{i \in I} x_{ijk} \quad \forall j \in J, t \in T. \\ & y_{jt} \geq 0 \quad \forall j \in J, t \in T. \end{aligned}$$

  - D_{jt} is the number of CSRs needed in period t of day j.
  - A_{kt} is 1 if shift k covers period t or 0 otherwise.

Constraints

- To calculate the maximum number of extra on-duty CSRs:

$$w_1 \geq \sum_{k \in K} A_{kt} \sum_{i \in I} x_{ijk} - D_{jt} \quad \forall j \in J, t \in T.$$
$$w_1 \geq 0 \quad \forall j \in J, t \in T.$$

- To calculate the maximum number of night shifts:

$$w_2 \geq \sum_{j \in J} \sum_{k \in K^N} x_{ijk} \quad \forall i \in I.$$
$$w_2 \geq 0 \quad \forall j \in J, t \in T.$$

  - Let K^N be the set of night shifts.

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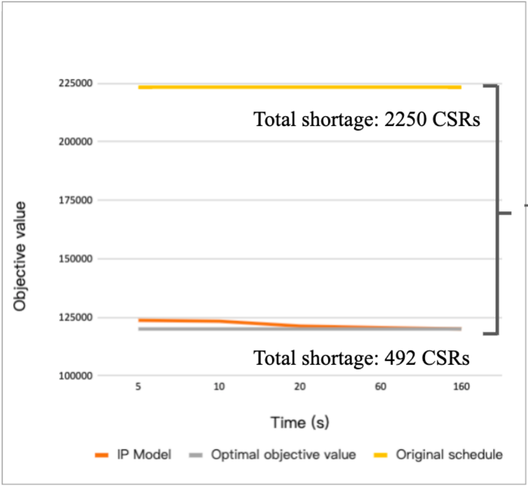
Performance evaluation (for one past month)



$$\frac{223400 - 119850}{223400} = 46\ \%$$

More benefits from the study

- With the shift scheduling system, one may better evaluate:
  - Whether to recruit more CSRs.
  - The impact of adjusting shift setting.
  - The impact of adjusting some policies.
- In general, being able to do better **operational** decisions helps us do better **strategic** decisions.