Operations Research I: Models & Applications
Using Excel to Solve Integer Programs

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Road map

- **Example 1: personnel scheduling**.
- Example 2: facility location.

Complete formulation

- According to the previous videos, let x_i be the number of people who start to work on day i for five consecutive days. x_i should be an integer.
- The formulation of this example is

$$
\begin{array}{lccccccccccccccc}
\min & x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & + & x_6 & + & x_7 & & \\
\text{s.t.} & x_1 & + & & & & & x_4 & + & x_5 & + & x_6 & + & x_7 & \geq & 110 \\
 & x_1 & + & x_2 & + & & & & & x_5 & + & x_6 & + & x_7 & \geq & 80 \\
 & x_1 & + & x_2 & + & x_3 & + & & & & & x_6 & + & x_7 & \geq & 150 \\
 & x_1 & + & x_2 & + & x_3 & + & x_4 & + & & & & & x_7 & \geq & 30 \\
 & x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & & & & & \geq & 70 \\
 & & & x_2 & + & x_3 & + & x_4 & + & x_5 & + & x_6 & & & \geq & 160 \\
 & & & & & x_3 & + & x_4 & + & x_5 & + & x_6 & + & x_7 & \geq & 120
\end{array}
$$

$$x_i \in \mathbb{Z}^+ \quad \forall i = 1, ..., 7.$$

- Let's use the Solver add-in to find an optimal solution!

Solve by the Solver add-in

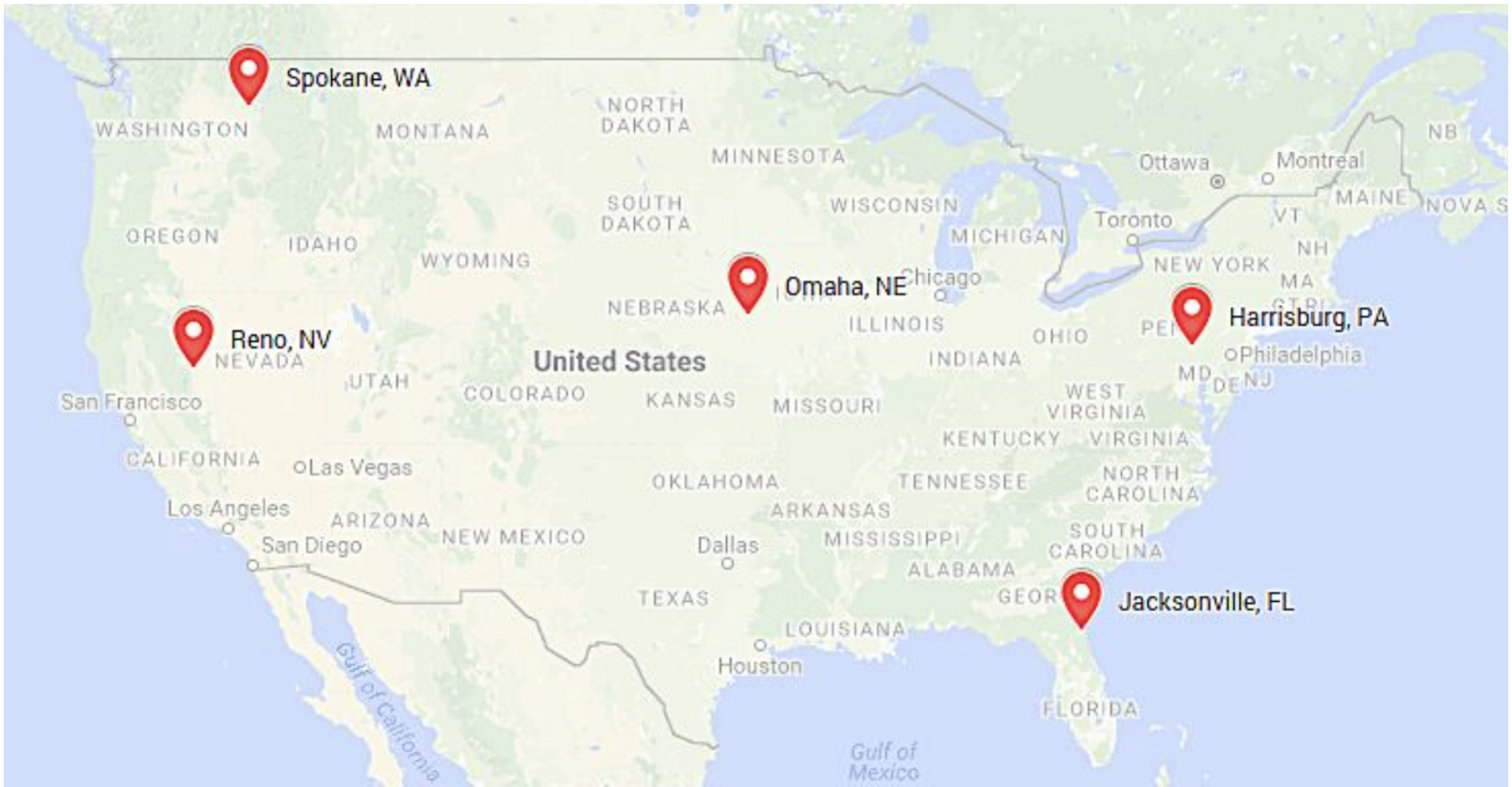
- An optimal solution of this IP is $(4, 0, 52, 0, 14, 94, 0)$. $z^* = 164$.

Constraints									
	x_1	x_2	x_3	x_4	x_5	x_6	x_7		
Mon	1			1	1	1	1	>=	110
Tue	1	1			1	1	1	>=	80
Wed	1	1	1			1	1	>=	150
Thu	1	1	1	1			1	>=	30
Fri	1	1	1	1	1			>=	70
Sat		1	1	1	1	1		>=	160
Sun			1	1	1	1	1	>=	120
Objective values									
min	1	1	1	1	1	1	1		
Integer Programing									
	x_1	x_2	x_3	x_4	x_5	x_6	x_7		
Decision variables	4	0	52	0	14	94	0		
Objective value	164								
Constraint 1	112								
Constraint 2	112								
Constraint 3	150								
Constraint 4	56								
Constraint 5	70								
Constraint 6	160								
Constraint 7	160								

Road map

- Example 1: personnel scheduling.
- **Example 2: facility location**.

Facility location



Complete formulation

- According to the previous videos, the parameters are

$$
\begin{aligned}
f_j &= \text{weekly operating cost of distribution center } j,\\
c_{ij} &= \text{shipping cost per book from distribution center } j \text{ to region } i,\\
K_j &= \text{capacity of distribution center } j,\\
D_i &= \text{book demand of region } i.
\end{aligned}
$$

- The decision variables are

$$
x_j = \begin{cases} 1 & \text{if a distribution center is built at location } j,\\ 0 & \text{otherwise.} \end{cases}
$$

$$
y_{ij} = \text{number of books shipped from distribution center } j \text{ to region } i.
$$

Complete formulation

- The formulation of this example is

$$\begin{aligned}
\min \quad & \sum_{j=1}^{5} f_j x_j + \sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij} y_{ij} & \\
\text{s.t.} \quad & \sum_{i=1}^{5} y_{ij} \leq K_j x_j & \forall j = 1, ..., 5 \\
& \sum_{j=1}^{5} y_{ij} \geq D_i & \forall i = 1, ..., 5 \\
& x_j \in \{0, 1\} & \forall j = 1, ..., 5 \\
& y_{ij} \geq 0 & \forall i = 1, ..., 5, j = 1, ..., 5.
\end{aligned}$$

- Let's use the Solver add-in to find an optimal solution!

Solve by the Solver add-in

- An optimal solution of this IP is obtained.

Integer Programing						
Objective value	268950					
Constraint 1	0	20000	0	25000	15000	
Decision variables	x_1	x_2	x_3	x_4	x_5	
	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	
	y_{21}	y_{22}	y_{23}	y_{24}	y_{25}	
	y_{31}	y_{32}	y_{33}	y_{34}	y_{35}	
	y_{41}	y_{42}	y_{43}	y_{44}	y_{45}	
	y_{51}	y_{52}	y_{53}	y_{54}	y_{55}	
	0	1	0	1	1	Constraint 2
	0	8000	0	0	0	8000
	0	12000	0	0	0	12000
	0	0	0	8000	1000	9000
	0	0	0	0	14000	14000
	0	0	0	17000	0	17000

An optimal solution