Operations Research III: Theory

Gurobi and Python for Network Flow

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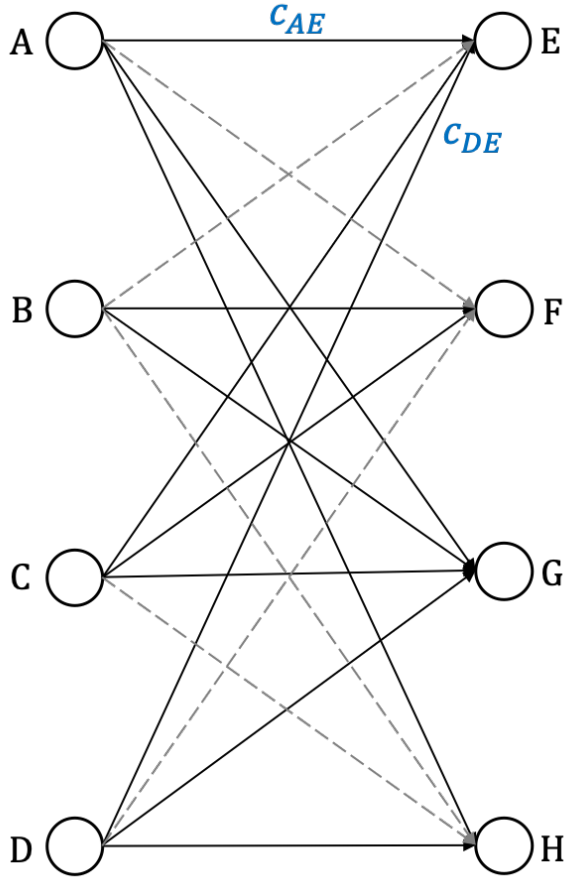
Road map

- **Assignment problems**.

Problem description

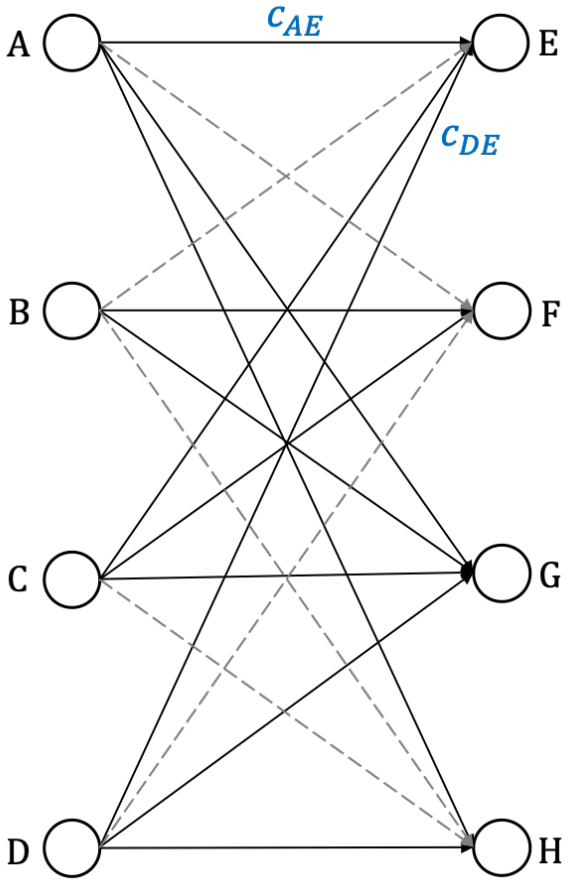
- In previous videos, we introduced the assignment problem as a special network flow model.
- In this instance, we need to assign four jobs to four workers.
- A job cannot be split, that is, the assignment must be one-to-one.
- The cost for worker j to complete job i is c_{ij}.
- The goal is to minimize the total costs.



Problem description

- The cost of each arc in this instance is below.

Arc	Cost	Arc	Cost
(A, E)	2	(C, E)	8
(A, F)	∞	(C, F)	2
(A, G)	10	(C, G)	5
(A, H)	7	(C, H)	∞
(B, E)	∞	(D, E)	7
(B, F)	4	(D, F)	∞
(B, G)	3	(D, G)	1
(B, H)	∞	(D, H)	6



IP formulation

- Let I and J be the sets of jobs and workers.
- For the assignment problem:

$$
\begin{aligned}
\min \quad & \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{m} x_{ij} = 1 \quad \forall i \in I \\
& \sum_{i=1}^{n} x_{ij} = 1 \quad \forall j \in J \\
& x_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J.
\end{aligned}
$$

Solve and interpret

- An optimal solution, which is illustrated as the set of red arcs in the figure, is obtained. The objective value is 13.
- When constructing the model, we may change the variable type from `BINARY` to `CONTINUOUS`.
- It means we relax the IP to an LP relaxation, and we still have an integer solution. Why?
- In previous videos, we introduced that the solution of the LP relaxation for MCNF must be an integer solution because of **total unimodularity**.