Operations Research III: Theory

Gurobi and Python for Shadow Prices

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Road map

- **Shadow prices**.
- Example 1: producing desks and tables.
- Example 2: personnel scheduling.

Shadow prices

- In previous videos, we introduced the idea of shadow prices.
- In this video, we use two instances to talk about how to get the shadow price for each constraint with `gurobipy`.
- We also compare the shadow price in the original setting and adjusted objective value if the RHS of that constraint is added by 1.

Road map

- Introduction.
- **Example 1: producing desks and tables**.
- Example 2: personnel scheduling.

Complete formulation

- Consider the problem we have introduced in *Operations Research I: Modeling and Applications*. Let

$$
\begin{aligned}
x_1 &= \text{number of desks produced in a day and} \\
x_2 &= \text{number of tables produced in a day.}
\end{aligned}
$$

- The formulation of this example is

$$
\begin{array}{rrcrcll}
\max & 700x_1 & + & 900x_2 & & & \\
\text{s.t.} & 3x_1 & + & 5x_2 & \leq & 3600 & \text{(wood)} \\
 & x_1 & + & 2x_2 & \leq & 1600 & \text{(labor)} \\
 & 50x_1 & + & 20x_2 & \leq & 48000 & \text{(machine)} \\
 & x_1 & & & \geq & 0 & \\
 & & & x_2 & \geq & 0. &
\end{array}
$$

Construct the model

- We construct the problem with `gurobipy` and get shadow prices from a constraint attribute in Gurobi optimizer, `Pi`.
- Let's construct the model and try it first.

Solve and interpret

- After building a new model with `gurobipy`, we have an optimal solution for this LP $(884.21, 189.47)$ and objective value is 789473.68.
- We also get shadow prices of the three constraints as below.

Constraint	Shadow price	Adjusted objective value
Wood	163.16	789636.84
Labor	0	789473.68
Machine	4.21	789477.89

- The shadow price of the second constraint (labor) is zero. By complementary slackness, the nonbinding constraint at the optimal solution implies that the shadow price equals zero.
- In this example, the shadow price of each constraint is equal to the amount of objective value increased when we add that constraint by 1.

Road map

- Introduction.
- Example 1: producing desks and tables.
- **Example 2: personnel scheduling**.

Complete formulation

- Consider the personnel scheduling problem we also have introduced in *Operations Research I: Modeling and Applications*.
- Let x_i be the number of people who work for five consecutive days starting from day i.
- The formulation of this example is

$$
\begin{array}{rccccccccccccccc}
\min & x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & + & x_6 & + & x_7 & & \\
\text{s.t.} & x_1 & + & & & & & x_4 & + & x_5 & + & x_6 & + & x_7 & \geq & 110 \\
& x_1 & + & x_2 & + & & & & & x_5 & + & x_6 & + & x_7 & \geq & 80 \\
& x_1 & + & x_2 & + & x_3 & + & & & & & x_6 & + & x_7 & \geq & 150 \\
& x_1 & + & x_2 & + & x_3 & + & x_4 & + & & & & & x_7 & \geq & 30 \\
& x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & & & & & \geq & 70 \\
& & & x_2 & + & x_3 & + & x_4 & + & x_5 & + & x_6 & & & \geq & 160 \\
& & & & & x_3 & + & x_4 & + & x_5 & + & x_6 & + & x_7 & \geq & 120
\end{array}
$$

$$x_i \geq 0 \quad \forall i = 1, ..., 7.$$

Solve and interpret

- After building a new model with `gurobipy`, we have an optimal solution to this LP $(3.33, 40, 13.33, 13.33, 0, 93.33, 0)$ and objective value is 163.33.
- This problem has **multiple optimal solutions**.

Solve and interpret

- We also get the shadow price of each constraint as below.

Day	RHS	Solutions	Shadow price	Adjusted Obj.
Monday	110	110	$\frac{1}{3}$	$163\frac{2}{3}$
Tuesday	80	$136\frac{2}{3}$	0	$163\frac{1}{3}$
Wednesday	150	150	$\frac{1}{3}$	$163\frac{2}{3}$
Thursday	30	70	0	$163\frac{1}{3}$
Friday	70	70	$\frac{1}{3}$	$163\frac{2}{3}$
Saturday	160	160	$\frac{1}{3}$	$163\frac{2}{3}$
Sunday	120	120	0	$163\frac{1}{3}$

Some Remarks

- The two instances are indeed the typical ones.
- From previous videos, we mentioned that the shadow price would be same as the amount of objective value increased when we add the RHS of that constraint by 1, **assuming** the current optimal basis remains optimal.
- However, shadow prices are still useful when we need to evaluate resource adjustments or answer “what-if” questions.