Operations Research III: Theory
Course Summary and Future Directions

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Road map

- **Summary and discussions**.
- Preview for the future.

Linear programming duality

- Topics:
  - Primal-dual pairs.
  - Duality theorems.
  - Shadow prices.
- Linear programming duality provides us a set of **optimality conditions** for linear programming.
  - For example, a solution is primal feasible if and only if (1) it is primal feasible, (2) there is a solution that is dual feasible, and (3) complementary slackness holds.
- It also helps us do sensitivity analysis.
- It also demonstrates how beautiful the theory is!

Sensitivity analysis and dual simplex

- Topics:
  - Evaluating a new variable.
  - Evaluating a new constraint: dual simplex method.
- In some cases, the instance to be solved is very similar to an instance that has been solved.
- The **existing** optimal solution for the old instance should be utilized.
- **Duality** is shown to help us do sensitivity analysis and speed up the branch-and-bound algorithm.
  - Utilizing existing solutions. Do not do everything from scratch!

Network flow models

- Topics:
  - Minimum-cost network flow (MCNF) problems.
  - An LP formulation for MCNF.
  - Special network flow models.
- Once the coefficient matrix of an integer program is **totally unimodular**, its LP relaxation guarantees to give an integer solution.
  - Network flow models all fall into this category.
  - This helps us solve the bipartite matching problem in polynomial time.
- Network flow models connect continuous optimization and discrete/combinatorial optimization.
  - Efficient mathematical models exist.
  - Efficient exact algorithms exists.

Convex analysis

- Topics:
  - Single-variate convex analysis.
  - Solving single-variate nonlinear programs.
  - Multi-variate convex analysis.
- Convex sets and convex functions form the key of **convex programming**.
  - For minimization problem with a convex objective function and no constraint, the **first-order necessary condition** becomes **sufficient**.
  - The convexity of a multi-variate function relies on the positive semi-definiteness of its Hessian matrix.
- Now we know when and why gradient descent and Newton's method fail.

Lagrangian duality and the KKT condition

- Topics:
  - Lagrangian relaxation.
  - The KKT condition.
  - Lagrangian duality vs. LP duality.
- To deal with constraints, we "move" them to the objective function and reward feasibility.
- **Lagrangian relaxation** helps us understand **the KKT condition**.
  - The constrained version of the first-order necessary condition.
- LP duality is actually a special case of Lagrangian duality.
  - How beautiful the theory is!

Case studies

- Many statistics and machine learning models are nonlinear programs.
  - The optimization for the model parameters is done by solving the nonlinear program.
  - Example 1: **linear regression**.
  - Example 2: **support vector machine**.
- Optimization is very important for **data science**.

Road map

- Summary and discussions.
- **Preview for the future**.

Advanced studies

- We have learned a lot!
- Of course, there are always more things to study.
- Each of the following subjects may form a one-semester course:
  - Linear programming.
  - Integer programming (and combinatorial optimization).
  - Graphs and network flow.
  - Convex programming.
  - Nonlinear programming.
- Many subjects (mostly having **uncertainty** involved) in operations research are not covered:
  - Stochastic processes.
  - Stochastic programming.
  - Robust programming.
  - Dynamic programming.

Advanced studies

- There are many related fields:
  - Scheduling.
  - Algorithms.
  - Theory of computation.
  - Machine learning.
- By adding **game theory** as an analytical tool, one may further investigate:
  - Supply chain management.
  - Competition.
  - Pricing.
  - And many more.
- All of these are so fascinating!

Foundations for advanced studies

- Do not forget to equip yourselves before starting your journey:
  - Calculus.
  - Discrete mathematics.
  - Linear algebra.
  - Probability.

It is a belief

- Operations research is a belief:
  - All decision problems are **optimization** problems.
  - Many real-world optimization problems may be **modeled**.
  - Most of these problems (after some simplification) may be **solved** (optimally or near-optimally).
  - **Decision making** may be supported by models and their solutions.
- Operation research is not perfect.
  - "All models are wrong, but some are **useful**" (George Box).
  - And it is full of **beauty**.

That's all. See you in future!