D'Alembert's principle: The reversed effective forces acting on each particle of the body and the external forces of the system are in equilibrium. This is D'Alemberts principle.

If F and R be respectively the external and internal forces, and if f be the acceleration of the system, then

$$P = mf$$

$$\Rightarrow F + R = mf$$

$$\Rightarrow F + R - mf = 0$$

i.e. F, R and mf are in equilibrium.

Again, summing up for all the particles of the body, we can write

$$\Sigma F + \Sigma R - \Sigma mf = 0$$

But the internal forces (by virtue of Newton's third law of motion)

$$\Sigma F - \Sigma mf = 0$$

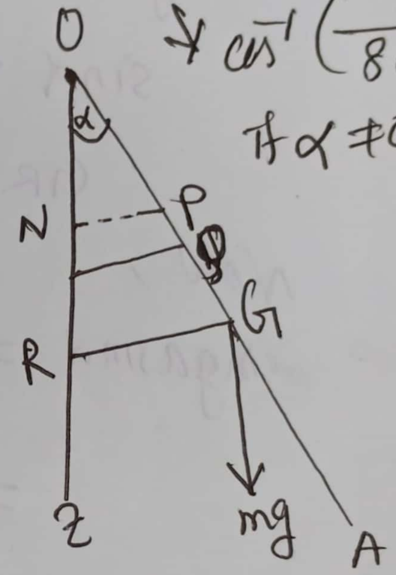
i.e. the external force together with the reversed force are in equilibrium.

Q. A uniform rod OA of length 2a free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ; find the value of α.

(4a)

Q. A uniform rod OA of length 2a and mass m, free to turn about its end O, revolved with uniform angular velocity ω about the vertical OZ through O and is inclined at a ~~distance~~ constance angle α to OZ. Show that $\alpha = \cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$. or zero.

Solution:



$\ne \cos^{-1}\left(\frac{3g}{8a\omega^2}\right)$

If $\alpha \neq 0$

Let, an element $PQ = \delta x$ whose distance from O is x. We draw PN perpendicular to OZ. By revolution of OA about the vertical axes OZ making angle α, PQ makes a circle of radius PN.

$$\sin\alpha = \frac{PN}{OP} \therefore PN = OP\sin\alpha = x\sin\alpha$$

Mass of the element $PQ = \frac{m}{2a}\delta x$.

As the rod OA revolves with ~~a~~ uniform angular velocity ω, so the acceleration of P is $\omega^2 PN = \omega^2 x\sin\alpha$.

So the effective force along PN is $\frac{m}{2a}\delta x \cdot \omega^2 x\sin\alpha$

Hence the reversed effective force is $\frac{m}{2a}\delta x \cdot \omega^2 x\sin\alpha$

By D'Alemberts principle, the reversed effec forces and the external forces are in equilibrium.

Hence taking the moment about O, we get

$$mg.\ GR = \sum \frac{m}{2a}\delta x \cdot \omega^2 x \sin\alpha \cdot ON$$

$$\sin\alpha = \frac{GR}{OG} = \frac{GR}{a}$$

$$\therefore GR = a\sin\alpha$$

$$\cos\alpha = \frac{ON}{OP}$$

$$\Rightarrow \cos\alpha = \frac{ON}{x}$$

$$\therefore ON = x\cos\alpha$$

Now,

$$mga\sin\alpha = \frac{m}{2a}\omega^2 \sin\alpha\cos\alpha \int_0^{2a} x^2 dx$$

$$= \frac{m}{2a}\omega^2\sin\alpha\cos\alpha \left[\frac{x^3}{3}\right]_0^{2a}$$

$$= \frac{m}{2a}\omega^2\sin\alpha\cos\alpha \frac{8a^3}{3}$$

$$\Rightarrow g\sin\alpha = \frac{4a}{3}\omega^2\sin\alpha\cos\alpha$$

$$\Rightarrow g\sin\alpha - \frac{4a}{3}\omega^2\sin\alpha\cos\alpha = 0$$

$$\Rightarrow g\sin\alpha\left(1 - \frac{4a}{3g}\omega^2\cos\alpha\right) = 0$$

Either $g\sin\alpha = 0$ or $1 - \frac{4a}{3g}\omega^2\cos\alpha = 0$

$$\Rightarrow \sin\alpha = 0$$

$$\Rightarrow \alpha = 0$$

$$\Rightarrow \frac{4a}{3g}\omega^2\cos\alpha = 1$$

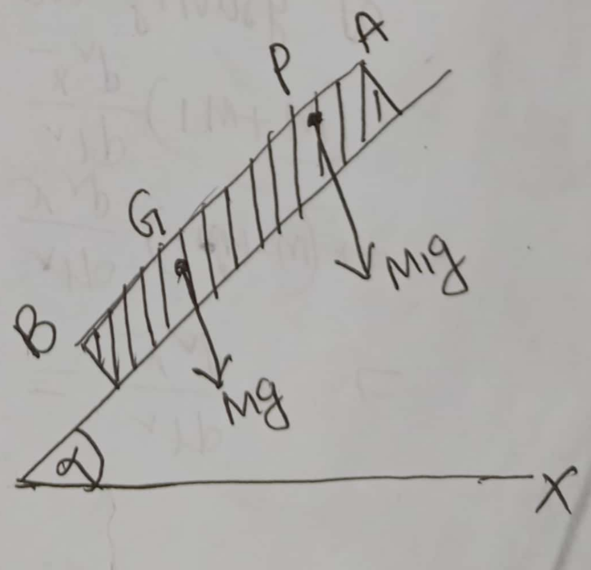
$$\Rightarrow \cos\alpha = \frac{3g}{4a\omega^2}$$

$$\therefore \alpha = \cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$$

$$\therefore \alpha = 0 \text{ or } \cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$$

Q. A plank of mass M is initially at rest along a line of the greatest slope of a smooth plane inclined at an angle α to the horizon, and a man of mass M_1, starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\dfrac{2M_1a}{(M+M_1)g\sin\alpha}}$ where a is the length of the plank.

[M ভরের একটি তক্তা আদিতে স্থির অবস্থায় একটি মসৃণ তলের উপর আনুভূমিকের সাথে সর্বোচ্চ α নতিতে অবস্থিত। M_1 ভরের একজন মানুষ তক্তার উপরের প্রান্ত হতে হেঁটে এরূপে নিচের প্রান্তে আসে যে তক্তাটি আদৌ নড়েনা। দেখাও যে, লোকটির তক্তার অপর প্রান্তে পৌঁছাতে $\sqrt{\dfrac{2M_1a}{(M+M_1)g\sin\alpha}}$ সময় লাগবে। যেখানে a হলো তক্তার দৈর্ঘ্য]



Solution:

Let centre of gravity of the plank AB is G whose length is 'a' and which is incl. at angle α with the horizon. And the man starts from A, he reaches at 'p' in time 't' where $AP = x$.

Let $\bar{x}$ be the combined centre of gravity of the plank and the man then we get

$$\bar{x} = \frac{M \cdot \frac{a}{2} + M_1 x}{M + M_1}$$

$$\Rightarrow \frac{d\bar{x}}{dt} = \frac{M_1}{M+M_1} \frac{dx}{dt}$$

$$\Rightarrow \frac{d^2\bar{x}}{dt^2} = \frac{M_1}{M+M_1} \frac{d^2x}{dt^2} \quad \text{--- (i)}$$

Again the equation of motion of the centre of gravity along the plane

$$(M+M_1)\frac{d^2\bar{x}}{dt^2} = Mg\sin\alpha + M_1 g\sin\alpha$$

$$\Rightarrow (M+M_1)\frac{d^2\bar{x}}{dt^2} = (M+M_1)g\sin\alpha$$

$$\Rightarrow \frac{d^2\bar{x}}{dt^2} = g\sin\alpha \quad \text{--- (ii)}$$

From the equation (i) and (ii) we get -

$$\frac{M_1}{M+M_1}\frac{d^2x}{dt^2} = g\sin\alpha$$

$$\Rightarrow \frac{d^2x}{dt^2} = \frac{M+M_1}{M_1} g\sin\alpha$$

$$\Rightarrow \frac{dx}{dt} = \frac{M+M_1}{M_1} gt\sin\alpha + C_1 \quad [\because \text{Integrating}]$$

Initially, as $t=0 \Rightarrow \frac{dx}{dt} = 0$

$$0 = \frac{M+M_1}{M_1} \cdot g \cdot 0 \cdot \sin\alpha + C_1$$

$$\Rightarrow C_1 = 0$$

$$\therefore \frac{dx}{dt} = \frac{M+M_1}{M_1} gt\sin\alpha$$

$$\Rightarrow \int dx = \frac{M+M_1}{M_1} g\sin\alpha \int t\,dt$$

$$\Rightarrow x = \frac{M+M_1}{M_1} g\sin\alpha \cdot \frac{t^2}{2} + C_2$$

As $t=0$, then $x=0$

$$\therefore 0 = 0 + C_2 \Rightarrow C_2 = 0$$

$$\therefore x = \frac{M+M_1}{2M_1} g t^2 \sin\alpha$$

P.T.O

$$\Rightarrow t = \sqrt{\frac{2m_1 x}{(M+m_1) g \sin\alpha}}$$

When the man reach at the other end of the plank then we get $x = a$

$$\therefore t = \sqrt{\frac{2m_1 a}{(M+m_1) g \sin\alpha}}$$ (Proved)