Mechanics -II

Simple Harmonic Motion

1. What is simple harmonic motion?
2. (v.v.i) Theorem-02 : When a particle moves in a straight line OA with an acceleration proportional to its distance from a fixed point O, in the straight line and is always directed away from O. If the particle starts from rest at x=a , find the motion.

3. (Theorem-03) : A particle moves in a straight line AO with an acceleration which is always directed towards O and varies inversely as the square of its distance from O . If initially the particle were at rest at A find its motion and the periodic time.

4. (Problem-05) : A particle whose mass is an m, acted upon by a force $m\mu(x + \frac{a^4}{x^3})$ towards the origin if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.
5. A particle moves with an acceleration which is always towards O and equal to μ divided by the distance from a fixed point O, If it starts from at a distance a from O ,show that it will arrive at O in time $a\sqrt{\frac{\pi}{2\mu}}$.
6. A particle falls from rest at a distance a from a centre of force, where the acceleration at distance x is $\mu x^{-\frac{5}{3}}$, When it reaches the centre. Show that its velocity is infinite and that the time it has taken is $\frac{2a^{\frac{4}{3}}}{\sqrt{3\mu}}$.

Motion of a Particle in a plane :

07. (Theorem-01): Find the velocity and acceleration of a particle along and perpendicular to the radius vector to it from a fixed origin O.

08. (Theorem -02): A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane .Find the differential equation of the path.

09. (v.v.i) : The velocity of a particle along and perpendicular to the radius from a fixed origin λr and $\mu\theta$ . . Find the path and show that the acceleration along and perpendicular to the radius vector are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta(\lambda + \frac{\mu}{r})$

10. (H.W) : Find the law of force towards the poll under which the following curves can be described

i) $r^n = a^n Cos(n\theta)$

ii) $r^n = ACos(n\theta) + BSin(n\theta)$

__Central Force__

11. Define Central Force with examples.

12. What is an Apse?

13. What is Apsidal Distance?

14. What is Apsidal Angle?

15. Write down the Kepler's laws of motion.

16.(Theorem) : A particle moves in a path so that the acceleration is always directed to a fixed point and is equal to $\frac{\mu}{distance^2}$ so that its path is a conic . Discuss the different cases.

17.(v.v.i) : A particle moves with central acceleration $\mu[3au^4 - 2(a^2 - b^2)u^5]$, a>b, and is projected from an apse at a distance (a+b) with velocity $\frac{\sqrt{\mu}}{a+b}$ show that its orbit is $r = a + bCos\theta$

18. (H.W) : A particle subject to a central force per unit if mass equal to $\mu[2(a^2 + b^2)u^5 - 3a^2b^2u^7]$ is projected at a distance a with a velocity $\frac{\sqrt{\mu}}{a}$ in a direction at right angles to the initial distance. Show that the path is the curve $r^2 = a^2Cos^2\theta + b^2Sin^2\theta$

19. (v.v.i) : A particle moves under a central repulsive force $\frac{m\mu}{distance^3}$,and is projected from an apse at a distance a with velocity V. Show that the equation

to the path is $rCos(p\theta) = a$, and that the angle described in time t is $\frac{1}{p} tan^{-1}(\frac{pV}{a} t)$, Where $p^2 = \frac{\mu + a^2V^2}{a^2V^2}$

20. (H.W) : A particle moves with a central acceleration $\mu(r^5 - c^4 r)$ being projected from an apse at a distance c with a velocity $\sqrt{\frac{2\mu}{3}}\ c^3$ .Show that it describes the path $x^4 + y^4 = c^4$

21. (v.v.i) : A particle describes an ellipse under a force $\frac{\mu}{distance^2}$ towards the focus. If it was projected with velocity v from the point distance r from the centre of force, Show that its periodic time is $\frac{2\pi}{\sqrt{\mu}} \left[\frac{2}{r} - \frac{v^2}{\mu}\right]^{-\frac{3}{2}}$

D'Alembarts Principal

22.(v.v.i) : State and prove D' Alembart's principle and hence deduce the general equation of motion.

23. (v.v.i) : A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M_1, starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M_1 a}{(M+M_1)gSin\alpha}}$,where a is the length of the plank.

24.(H.W) A uniform rod OA of length 2a and mass m, free to turn about its end O, revolved with uniform angular velocity ω about the vertical OZ through O and is inclined at constant angle α to OZ, show that $\alpha = Cos^{-1}(\frac{3g}{4a\ \omega^2})$ or zero. Find the value of α .

Or.

If length =4a then Prove that , $\alpha = Cos^{-1}(\frac{3g}{8a\ \omega^2})$

Motion about fixed axes :

25.Define Motion about the axis of rotation ?

26. (v.v.i) : A uniform rod of mass m and length 2a can turn freely about one end where h is fixed. If it starts with angular velocity ω from the position in which it hangs vertically, discuss the motion.

27. (v.v.i): A weightless straight rod ABC of length 2a is movable about the fixed end A and carries two particles at the same mass ,one fastened to the middle point B and other to the end C of the rod. If the road be held in the horizontal position and be then let go, show that its angular velocity when vertical is $\sqrt{\frac{6g}{5a}}$ and that is $\frac{5a}{3}$ the length of the simple equivalent pendulum.

28. (v.v.i) : A rigid body swing under gravity from a fixed horizontal axis, show that the time of complete small oscillation is $2\pi\sqrt{\frac{k^2}{hg}}$ where k is its radius of gyration about the fixed axis and h is the distance between the fixed axis and the centre of the inertia of the body.

Moment of Inertia and Product of Inertia :

29. (v.v.i) : What is moment of inertia?

30. (v.v.i) : What is product of inertia?

31. (v.v.i) : (Theorem-01) : If the moments and product of inertia about any line or lines, through the centre of inertia G of a body are known ,then obtain the corresponding quantities for any parallel line or lines.

32. (v.v.i) : Find the moment of inertia of a hollow sphere about a diameter, its external and internal radii being a and b.

33. (v.v.i) : Prove that at the vertex C of a triangle, ABC which is right angled at C, the principal axes are perpendicular to the plane and two other inclined to the sides at an angle $\frac{1}{2}tan^{-1}(\frac{ab}{a^2-b^2})$

34. Find the moment of inertia of the area bounded by $r^2 = a^2Cos2\theta$ about its axes.