Motion about fixed axes

* Motion about the axis of rotation

In any motion the moment of the effective forces about the axis is equal to the moment of the impressed forces. Hence we write

$$Mk^2 \frac{d^2\theta}{dt^2} = L,$$

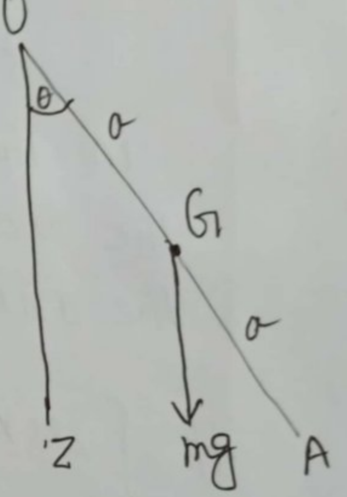
in the sense of θ increasing, and L is the moment of the impressed forces about the axis of rotation. external বাহ্যিক

$$\# \int \sec x \, dx = \ln|\sec x + \tan x| + c$$
$$= \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$$

Q-1: A uniform rod of mass m and length 2a, can turn freely about one end which is fixed. If it starts with angular velocity ω from the position in which it hangs vertically, discuss the motion.

Solution:

The only external force is the weight mg, whose moment L about the fixed axis is $mga\sin\theta$, when the rod has revolved through an angle θ, then



$$Mk^2 \frac{d^2\theta}{dt^2} = L$$

$$\Rightarrow m \cdot \frac{4a^2}{3} \cdot \frac{d^2\theta}{dt^2} = -mga\sin\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{3g}{4a}\sin\theta$$

Multiplying by $2\frac{d\theta}{dt}$,

Since the moment tends to lessen θ, the negative sign is taken. Also the moment of inertia of the rod is $m\frac{(2a)^2}{3} = mk^2$

$$\Rightarrow m\frac{4a^2}{3} = mk^2$$

$$\Rightarrow k^2 = \frac{4a^2}{3}$$

Let, $M = m$

P.T.O.

$$2\frac{d\theta}{dt}\frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right)^2 = -\frac{3g}{2a}\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{2a}\cos\theta + c \quad \text{[on integration]}$$

Initially $\theta = 0$, $\frac{d\theta}{dt} = \omega$

$$\Rightarrow \omega^2 = \frac{3g}{2a} + c$$

$$\Rightarrow c = \omega^2 - \frac{3g}{2a}$$, so we get

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{2a}\cos\theta + \omega^2 - \frac{3g}{2a}$$

$$\Rightarrow \omega_1^2 = \omega^2 - \frac{3g}{2a}(1-\cos\theta) \quad \cdots (i)$$

which gives the angular velocity at any instant. The angular velocity ω_1 gets less and less as θ gets bigger, and just vanishes when $\theta = \pi$, then

$$0 = \omega^2 - \frac{3g}{2a}(1-\cos\pi)$$

$$\Rightarrow 0 = \omega^2 - \frac{3g}{2a}(1+1)$$

$$\Rightarrow \omega^2 = \frac{3g}{2a} \times 2$$

$$\Rightarrow \omega^2 = \frac{3g}{a} \Rightarrow \omega = \sqrt{\frac{3g}{a}}$$

P.T.O.

With this value (i) becomes

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{a} - \frac{3g}{2a}(1-\cos\theta)$$

$$\Rightarrow \left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{a} - \frac{3g}{2a} + \frac{3g}{2a}\cos\theta$$

$$\Rightarrow \left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{2a} + \frac{3g}{2a}\cos\theta$$

$$\Rightarrow \left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{2a}(1+\cos\theta)$$

$$= \frac{3g}{2a} \times 2\cos^2\frac{\theta}{2}$$

$$\Rightarrow \frac{d\theta}{dt} = \sqrt{\frac{3g}{a}}\cos\frac{\theta}{2}$$

$$\Rightarrow \sec\frac{\theta}{2}\,d\theta = \sqrt{\frac{3g}{a}}\,dt$$

$$\Rightarrow \int_0^\theta \sec\frac{\theta}{2}\,d\theta = \sqrt{\frac{3g}{a}}\int_0^t dt \quad \text{[Integrating]}$$

$$\Rightarrow 2\ln\tan\left(\frac{\pi}{4}+\frac{\theta}{4}\right) = t\sqrt{\frac{3g}{a}}$$

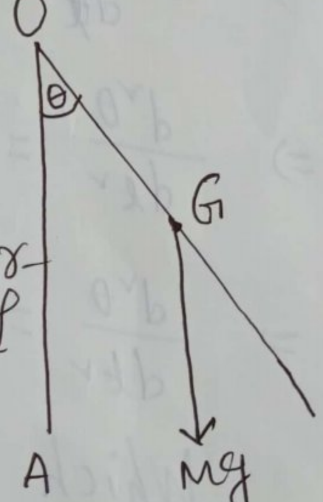
$$\Rightarrow t = 2\sqrt{\frac{a}{3g}}\ln\tan\left(\frac{\pi}{4}+\frac{\theta}{4}\right)$$

#

Q-2: If a rigid body swing under gravity, from a fixed horizontal axis, show that the time of complete small oscillation is $2\pi\sqrt{\frac{K^2}{hg}}$, where K is its radius of gyration about the fixed axis, and h is the distance between the fixed axis and the centre of the inertia of the body.

Solution:

Consider the plane of the paper be the plane through the centre of inertia G of the body perpendicular to the fixed axis and it meet the axis at O.



Let θ be the angle between the vertical OA and the line OG. That is θ be the angle between a plane fixed in the body and a plane fixed in the space.

P.T.O.

Let the weight of the body be Mg, Which acting at the centre of inertia G vertically downwards.

Let L be the moment of the body about the horizontal axes through O.

L = The moment of Mg acting at G

$= -Mgh\sin\theta$ [Negative sign taken as it has a tendency to decrease θ]

$$\Rightarrow MK^2 \frac{d^2\theta}{dt^2} = -Mgh\sin\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{gh}{k^2}\sin\theta = -\frac{gh\theta}{k^2}\frac{\sin\theta}{\theta}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{gh\theta}{k^2} \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

Which implies the equation of simple harmonic motion.

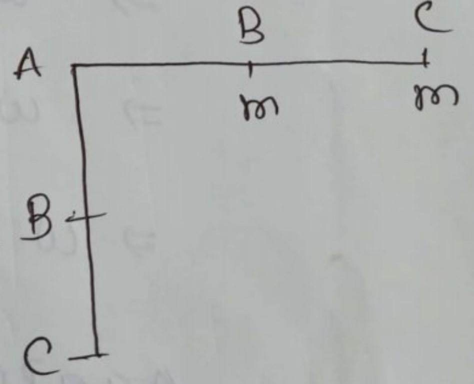
$\therefore$ Time of complete oscillation

$$T = \frac{2\pi}{\sqrt{\mu}} = \frac{2\pi}{\sqrt{gh/k^2}} \left[\because \frac{d^2x}{dt^2} = -\mu x\right]$$

$$= 2\pi\sqrt{\frac{k^2}{gh}}$$ (Proved)

Q3. A weightless straight rod ABC of length $2a$, is movable about the fixed end A and carries two particles at the same mass, one fastened to the middle point B and other to the end C of the rod. If the rod be held in the horizontal position and be then let go, show that its angular velocity when vertical is $\sqrt{\frac{6g}{5a}}$ and that $\frac{5a}{3}$ is the length of the simple equivalent pendulum.

Solution:



Let v_1, v_2 be the velocities of masses each equal to m at B and C respectively. If ω be the angular velocity when the rod is is vertical position, then $v_1 = a\omega$ and $v_2 = 2a\omega$

P.T.O.

Now by the principle of energy,
we write change in K.E. = work done

$$\Rightarrow \frac{1}{2}m(v_1^2+v_2^2) = mga + 2mga = m(ag+2ag)$$

$$\Rightarrow \frac{1}{2}(v_1^2+v_2^2) = 3ag$$

$$\Rightarrow v_1^2+v_2^2 = 6ag$$

$$\Rightarrow (a\omega)^2+(2a\omega)^2 = 6ag$$

$$\Rightarrow a^2\omega^2 + 4a^2\omega^2 = 6ag$$

$$\Rightarrow 5a^2\omega^2 = 6ag$$

$$\Rightarrow \omega^2 = \frac{6ag}{5a^2}$$

$$\Rightarrow \omega^2 = \frac{6g}{5a}$$

$$\Rightarrow \omega = \sqrt{\frac{6g}{5a}}$$, the angular velocity

Again for the radius of gyration,
we get

$$(m+m)K^2 = ma^2 + m(2a)^2$$ [$\because K$ is the radius of the gyration]

$$\Rightarrow 2mK^2 = ma^2 + 4a^2m$$

$$\Rightarrow 2K^2 = a^2 + 4a^2$$

$$\Rightarrow 2K^2 = 5a^2$$

p.T.o.

$\Rightarrow K^2 = \frac{5a^2}{2}$

If h be the distance of C.G from A,

$$h = \frac{m.a + m.2a}{m+m} = \frac{3a}{2}$$

∴ The length of the simple equivalent pendulum, $l = \frac{K^2}{h}$

$$= \frac{\frac{5a^2}{2}}{\frac{3a}{2}}$$

$$= \frac{5a^2}{2} \times \frac{2}{3a}$$

$$= \frac{5a}{3}$$

(Showed)

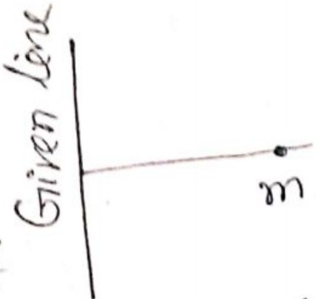
Q-3 Show that the length of a simple equivalent pendulum for a solid cone moving about the diameter of the base as axis is $\frac{2+3\tan^2\alpha}{5}h$, where α be the semi-vertical angle and h is the height of the cone.

[illegible]

Moments of inertia and product of inertia

Rigid body: A body is said to be rigid if the distance between any two particles of the body always remains the same.

Moment of Inertia:

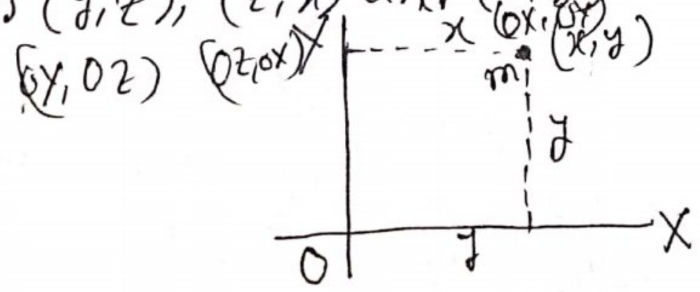


If r be the perpendicular distance from any given line of any element m of the mass of a body, then the quantity $\sum mr^2$ is called the moment of inertia of the body about the given line. If this sum be equal to MK^2, where M is the total mass of the body, then K is called the radius of gyration (ঘূর্ণন ব্যাসার্ধ) about the given line. Again, since the distance of the element from the axis of x is $\sqrt{y^2+z^2}$, the moment of inertia about the x-axis is $\sum m(y^2+z^2)$. Similarly for the other axes is $\sum m(z^2+x^2)_y$ and $\sum m(x^2+y^2)_z$.

Solu

(জড়তার গুণফল)

<u>Product of Inertia:</u> If three mutually perpendicular axes ox, oy, oz be taken; and if the coordinates of any element m of the system referred to these axes be x, y and z, then the quantities Σmyz, Σmzx and Σmxy are called the product of inertia with respect to the axes (y,z), (z,x) and (x,y) respectively.



<u>Theorem:</u> If the moments and products of inertia about any line or lines, through the centre of inertia G of a body are known then obtain the corresponding quantities for any parallel line or lines [যদি কোন বস্তুর জড়তা কেন্দ্র G গামী কোন রেখা বা রেখা সমূহের প্রেক্ষিতে জড়তা ভ্রামক ও গুনফলের মান জানা থাকে, তবে সমান্তরাল যে কোন রেখা বা রেখা সমূহের ক্ষেত্রে অনুরূপ মান সমূহ বের কর]

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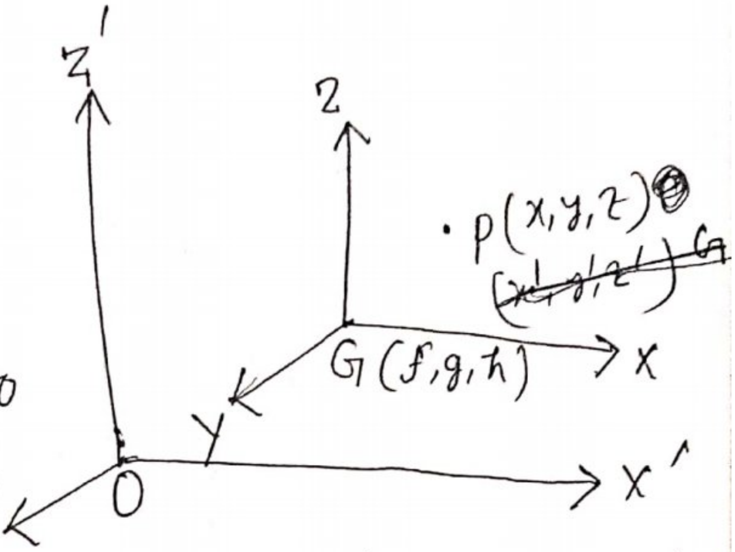
Solution: Let $p(x,y,z)$ be the position of any element m of the body referred to axes GX, GY, GZ



and $p(x',y',z')$ be that referred to parallel axes OX', OY', OZ' with O as the origin. Let the coordinates of the centre of gravity G be (f,g,h) referred to axes OX', OY', OZ'. The coordinates of the centre of inertia are

[We know statics] $\left(\dfrac{\Sigma mx}{\Sigma m}, \dfrac{\Sigma my}{\Sigma m}, \dfrac{\Sigma mz}{\Sigma m}\right)$ referred to G

as the origin $(0,0,0)$. Then we write

$$\Sigma mx = 0 = \Sigma my = \Sigma mz, \quad \text{--- (i)}$$

$$x' = x+f, \quad y' = y+g, \quad z' = z+h,$$

Now M. I. of the body about OX'

$$= \Sigma m(y'^2 + z'^2) = \Sigma m[(y+g)^2 + (z+h)^2]$$

$$= \Sigma m(y^2 + 2gy + g^2 + z^2 + 2zh + h^2).$$

P.T.O.

Some

$= \sum m(y^2+z^2) + \sum m(g^2+h^2) + 2g\sum my + 2h\sum mz$

$= \sum m(y^2+z^2) + M(g^2+h^2)$, [using (i)]

= (M.I. of the body about Gx) + M.I. of a mass M placed at G about Ox'

Also the product of inertia at Ox', Oy'

$= \sum m x' y' = \sum m(x+f)(y+g)$

$= \sum m(xy + xg + yf + fg)$

$= \sum mxy + f\sum my + g\sum mx + \sum mfg$

$= \sum mxy + Mfg$. [using (i)]

= (Product of inertia Gx, Gy) + P.I. of a mass M placed at G about Ox', Oy').

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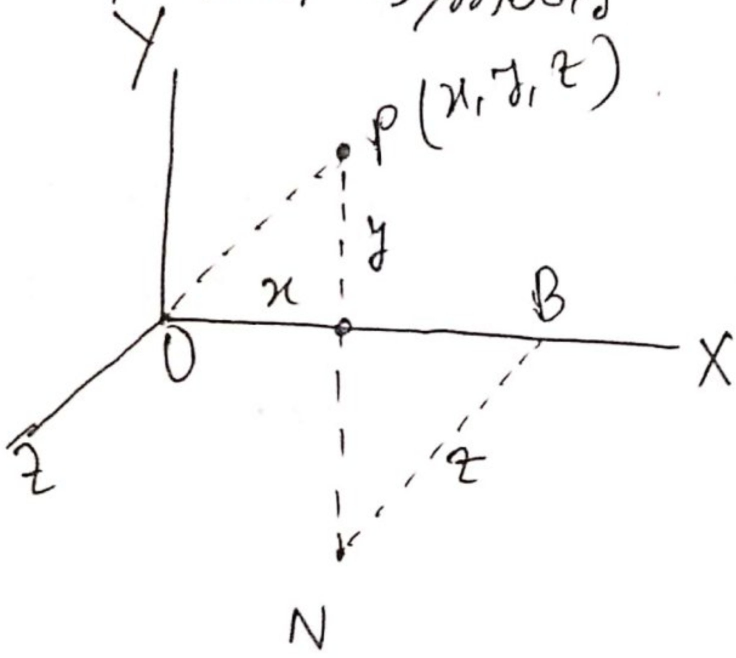
Some important symbols



Let a particle of mass m be placed at the point P (x, y, z) then the distance $OP = \sqrt{x^2+y^2+z^2}$

Distance of P from x-axis $= \sqrt{y^2+z^2}$

" " P " y- " $= \sqrt{z^2+x^2}$

" " P " z- " $= \sqrt{x^2+y^2}$

" " P " yz plane $= x$

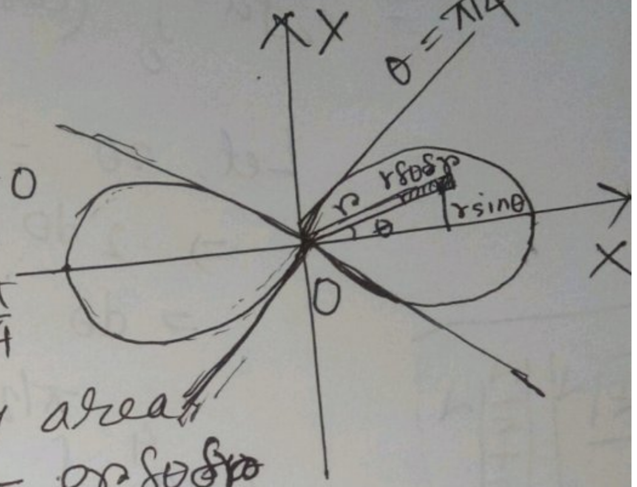
" " P " zx " $= y$

" " P. " xy " $= z$.

Q. Find the moment of inertia of the area bounded by $r^2 = a^2\cos 2\theta$ about its axes.

Solution:

The curve is as shown in the figure. The loop is formed between $\theta = \frac{\pi}{4}$ and $\theta = -\frac{\pi}{4}$.



$$r^2 = a^2\cos 2\theta = 0$$
$$\Rightarrow \cos 2\theta = 0$$
$$\Rightarrow \theta = \pm\frac{\pi}{4}$$

Consider the elementary area $r\delta\theta\delta r$. Its mass $= \rho r\delta\theta\delta r$

Its moment of inertia about OX

$= \rho r\delta\theta\delta r.(r\sin\theta)^2 = \rho r^3\sin^2\theta\delta\theta\delta r.$

Hence the moment of inertia of the whole area (both the loops) about OX

$$= 4\rho\int_0^{\pi/4}\int_0^{a\sqrt{\cos 2\theta}} r^3\sin^2\theta\, d\theta\, dr$$

$$= 4\rho\int_0^{\pi/4}\left[\frac{r^4}{4}\right]_0^{a\sqrt{\cos 2\theta}}\sin^2\theta\, d\theta$$

$$= \rho\int_0^{\pi/4} a^4\cos^2 2\theta\sin^2\theta\, d\theta$$

P.T.O

$$= \frac{1}{2}\rho a^4 \int_0^{\pi/4} \cos^2 2\theta \,(2\sin^2\theta)\, d\theta$$

$$= \frac{1}{2}\rho a^4 \int_0^{\pi/4} \cos^2 2\theta (1-\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\rho a^4 \int_0^{\pi/4} (\cos^2 2\theta - \cos^3 2\theta)\, d\theta$$

Let, $2\theta = z$

$\Rightarrow 2\, d\theta = dz$

$\Rightarrow d\theta = \frac{dz}{2}$

Limits :

When $\theta = 0$ then $z = 0$

" $\theta = \pi/4$ " $z = \frac{\pi}{2}$

$$\boxed{\int_0^{\pi/2} \sin^m\theta \cos^n\theta\, d\theta = \frac{\Gamma\frac{m+1}{2}\,\Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}}$$

$$= \frac{1}{4}\rho a^4 \int_0^{\pi/2} (\cos^2 z - \cos^3 z)\, dz$$

$$= \frac{1}{4}\rho a^4 \left[\frac{\Gamma\frac{1}{2}\,\Gamma\frac{3}{2}}{2\,\Gamma 2} - \frac{\Gamma\frac{1}{2}\,\Gamma 2}{2\,\Gamma\frac{5}{2}} \right]$$ $\because$ Using gamma beta formula

$$= \frac{1}{4}\rho a^4 \left[\frac{\sqrt{\pi}\cdot\frac{1}{2}\cdot\Gamma\frac{1}{2}}{2\cdot 1} - \frac{\sqrt{\pi}\cdot 1}{2\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2}} \right]$$

$\because \Gamma\frac{1}{2} = \sqrt{\pi}$

$\Gamma(n+1) = n!$

$\Gamma 2 = \Gamma(1+1) = 1!$
$= 1$

$$= \frac{1}{4}\rho a^4 \left[\frac{\sqrt{\pi}\cdot\sqrt{\pi}}{4} - \frac{\sqrt{\pi}}{\frac{3}{2}\sqrt{\pi}} \right]$$

$$= \frac{1}{4}\rho a^4 \left[\frac{\pi}{4} - \frac{2}{3} \right] \quad \text{......} \quad (i)$$

$$= \frac{1}{16}\rho a^4 \left[\pi - \frac{8}{3} \right]$$

P.T.O.

Now if M is the mass of the whole area (both the loops), then

$$M = 4\int_0^{\pi/4}\int_0^{a\sqrt{\cos 2\theta}} \rho r \, d\theta \, dr$$

$$= 4\rho \int_0^{\pi/4} \left[\frac{r^2}{2}\right]_0^{a\sqrt{\cos 2\theta}} d\theta$$

$$= 2\rho a^2 \int_0^{\pi/4} \cos 2\theta \, d\theta$$

$$= 2\rho a^2 \left[\frac{1}{2}\sin 2\theta\right]_0^{\pi/4}$$

$$= \rho a^2 \left(\sin\frac{\pi}{2} - \sin 0\right)$$

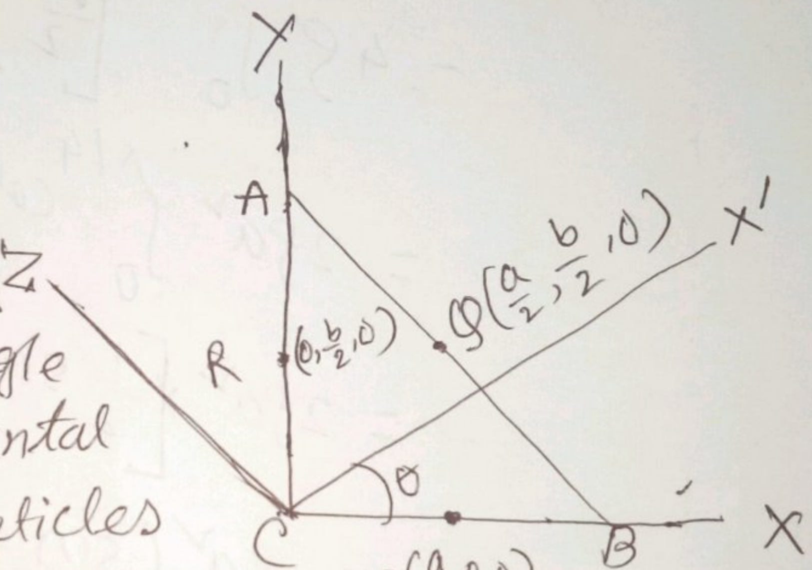
$$\therefore M = \rho a^2 (1-0) = \rho a^2.$$

Putting $M = \rho a^2$ in the equation (i), we get the required moment of inertia $= \frac{Ma^2}{16}\left(\pi - \frac{8}{3}\right)$.

(Ans)

Q. Prove that at the vertex C of a triangle ABC, which is right angled at C, the principal axes are a perpendicular to the plane and two others inclined to the sides at an angle $\frac{1}{2}\tan^{-1}\left(\frac{ab}{a^2-b^2}\right)$.

Proof:



Let M be the mass of the triangle ABC. Then the triangle is equi-momental to three particles each of mass $\frac{1}{3}M$ placed at the middle points P, Q, R of the sides of the triangle whose coordinates are clearly $\left(\frac{a}{2},0,0\right)$ $\left(\frac{a}{2},\frac{b}{2},0\right)$ & $\left(0,\frac{b}{2},0\right)$ respectively. Now we have to show that a line through C perpendicular to the z- plane is a principal axes at C. Clearly, z- coordinates of the centre of gravity of the triangle is zero. $\therefore D = E = 0$

Let CB and CA be the axis of x and y respectively and CB = a, CA = b.

$A = M\cdot I$ about $CX = 0 + \frac{1}{3}M\left(\frac{1}{2}b\right)^2 + \frac{1}{3}M\left(\frac{1}{2}b\right)^2$

$= \frac{1}{6}Mb^2$

$B = M\cdot I$ " $CY = \frac{1}{3}M\left(\frac{1}{2}a\right)^2 + \frac{1}{3}M\left(\frac{1}{2}a\right)^2 + 0$

$= \frac{1}{6}Ma^2$

F = Product of inertia about CX, CY

$= \frac{1}{3}M\left(\frac{1}{2}a \times 0\right) + \frac{1}{3}M\left(0 \times \frac{1}{2}b\right) + \frac{1}{3}M\left(\frac{1}{2}a \times \frac{1}{2}b\right)$

$= \frac{1}{12}Mab.$

If θ is the angle that a principal axis makes with CX, then by formula, we have

$$\tan 2\theta = \frac{2F}{B-A} = \frac{2 \times \frac{1}{12}Mab}{\frac{1}{6}Ma^2 - \frac{1}{6}Mb^2}$$

$$\Rightarrow 2\theta = \tan^{-1}\left(\frac{ab}{a^2-b^2}\right)$$

$$\Rightarrow \theta = \frac{1}{2}\tan^{-1}\left(\frac{ab}{a^2-b^2}\right)$$

(Proved)

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Mechanics - 2

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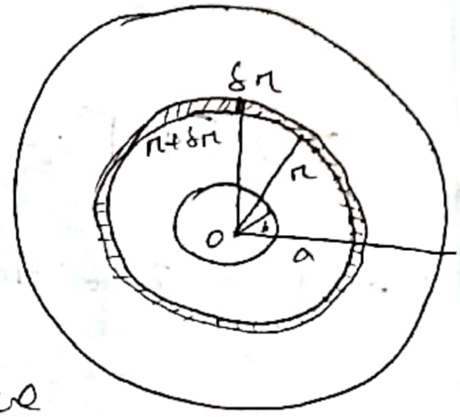
Find the M.I. of a hollow sphere about a diameters its external and ~~interal~~ internal radii being a and b



Soln: Let, mass of the hollow sphere is M whose external and internal radii being a and b.

$\therefore$ Volume of the hollow sphere is $\frac{4}{3}\pi(a^3-b^3)$

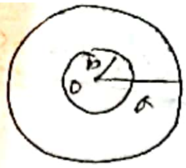
Consider, two concentric ~~space~~ sphere of ~~radii~~ radius r and $r+\delta r$ between the elementary shell to the hollow sphere.

~~$\therefore$ volume of the elementary shell included between two spheres~~

$\therefore$ Volume of the elementary shell included between two spheres $= \frac{4}{3}\pi\{(r+\delta r)^3 - r^3\}$

$= \frac{4}{3}\pi\{3r^2\delta r + 3r(\delta r)^2 + (\delta r)^3\}$

$= \frac{4}{3}\pi \cdot 3r^2\delta r$ [$\because \delta r$ is very small neglecting higher powers of δr]



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$= 4\pi r^2 dr$

$\therefore$ Mass per unit volume of the shell.

$$= \frac{M}{\frac{4}{3}\pi(a^3-b^3)} = \frac{3M}{4\pi(a^3-b^3)}$$

Therefore, mass of the elementary volume.

$$= \frac{3M}{4\pi(a^3-b^3)} \cdot 4\pi r^2 dr = \frac{3M}{a^3-b^3} r^2 dr$$

Hence the moment of inertia of the elementary volume about the diameter $= \frac{2}{3} \times \text{mass} \times (\text{radius})^2$

$$= \frac{2}{3}\left\{\frac{3M}{a^3-b^3} r^2 dr\right\} r^2$$

$$= \frac{2M}{a^3-b^3} r^4 dr$$

Thus the moment of inertia of the hollow sphere about the diameter.

$$= \frac{2M}{a^3-b^3}\int_a^b r^4 dr$$

$$= \frac{2M}{a^3-b^3}\left[\frac{r^5}{5}\right]_b^a = \frac{2M}{a^3-b^3}\left(\frac{a^5-b^5}{5}\right)$$

$$= \frac{2M}{5}\left(\frac{a^5-b^5}{a^3-b^3}\right)$$

(Ans:)

Mechanics -II

Simple Harmonic Motion

1. What is simple harmonic motion?
2. (v.v.i) Theorem-02 : When a particle moves in a straight line OA with an acceleration proportional to its distance from a fixed point O, in the straight line and is always directed away from O. If the particle starts from rest at x=a , find the motion.

3. (v.v.i)(Theorem-03) : A particle moves in a straight line AO with an acceleration which is always directed towards O and varies inversely as the square of its distance from O . If initially the particle were at rest at A find its motion and the periodic time.

4. (Imp.)(Problem-05) : A particle whose mass is an m, acted upon by a force $m\mu(x + \frac{a^4}{x^3})$ towards the origin if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.
5. A particle moves with an acceleration which is always towards O and equal to μ divided by the distance from a fixed point O, If it starts from at a distance a from O ,show that it will arrive at O in time $a\sqrt{\frac{\pi}{2\mu}}$.
6. A particle falls from rest at a distance a from a centre of force, where the acceleration at distance x is $\mu x^{-\frac{5}{3}}$, When it reaches the centre. Show that its velocity is infinite and that the time it has taken is $\frac{2a^{\frac{4}{3}}}{\sqrt{3\mu}}$.

Motion of a Particle in a plane :

07. (Theorem-01): Find the velocity and acceleration of a particle along and perpendicular to the radius vector to it from a fixed origin O.

08. (Imp)(Theorem -02): A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane .Find the differential equation of the path.

09. (v.v.i) : The velocity of a particle along and perpendicular to the radius from a fixed origin λr and $\mu\theta$ . . Find the path and show that the acceleration along and perpendicular to the radius vector are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta(\lambda + \frac{\mu}{r})$

10. (v.v.i)(H.W) : Find the law of force towards the poll under which the following curves can be described

i) $r^n = a^n Cos(n\theta)$

ii) $r^n = ACos(n\theta) + BSin(n\theta)$

Central Force

11. Define Central Force with examples.

12. What is an Apse?

13. What is Apsidal Distance?

14. What is Apsidal Angle?

15. Write down the Kepler's laws of motion.

16.(Theorem) : A particle moves in a path so that the acceleration is always directed to a fixed point and is equal to $\frac{\mu}{distance^2}$ so that its path is a conic . Discuss the different cases.

17.(v.v.i) : A particle moves with central acceleration $\mu[3au^4 - 2(a^2 - b^2)u^5]$, a>b, and is projected from an apse at a distance (a+b) with velocity $\frac{\sqrt{\mu}}{a+b}$ show that its orbit is $r = a + bCos\theta$

18. (H.W) : A particle subject to a central force per unit if mass equal to $\mu[2(a^2 + b^2)u^5 - 3a^2b^2u^7]$ is projected at a distance a with a velocity $\frac{\sqrt{\mu}}{a}$ in a direction at right angles to the initial distance. Show that the path is the curve $r^2 = a^2Cos^2\theta + b^2Sin^2\theta$

19. (v.v.i) : A particle moves under a central repulsive force $\frac{m\mu}{distance^3}$,and is projected from an apse at a distance a with velocity V. Show that the equation

to the path is $rCos(p\theta) = a$, and that the angle described in time t is $\frac{1}{p}tan^{-1}(\frac{pV}{a}t)$, Where $p^2 = \frac{\mu+a^2V^2}{a^2V^2}$

20. (H.W) : A particle moves with a central acceleration $\mu(r^5 - c^4r)$ being projected from an apse at a distance c with a velocity $\sqrt{\frac{2\mu}{3}}\ c^3$ .Show that it describes the path $x^4 + y^4 = c^4$

21. (v.v.i) : A particle describes an ellipse under a force $\frac{\mu}{distance^2}$ towards the focus. If it was projected with velocity v from the point distance r from the centre of force, Show that its periodic time is $\frac{2\pi}{\sqrt{\mu}}\left[\frac{2}{r} - \frac{v^2}{\mu}\right]^{-\frac{3}{2}}$

D'Alembarts Principal

22.(v.v.i) : State and prove D' Alembart's principle and hence deduce the general equation of motion.

23. (v.v.i) : A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M_1, starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M_1a}{(M+M_1)gSin\alpha}}$,where a is the length of the plank.

24.(H.W) A uniform rod OA of length 2a and mass m, free to turn about its end O, revolved with uniform angular velocity ω about the vertical OZ through O and is inclined at constant angle α to OZ, show that $\alpha = Cos^{-1}(\frac{3g}{4a\ \omega^2})$ or zero. Find the value of α .

Or.

If length =4a then Prove that , $\alpha = Cos^{-1}(\frac{3g}{8a\ \omega^2})$

Motion about fixed axes :

25.Define Motion about the axis of rotation ?

26. (v.v.i) : A uniform rod of mass m and length 2a can turn freely about one end where h is fixed. If it starts with angular velocity ω from the position in which it hangs vertically, discuss the motion.

27. (v.v.i): A weightless straight rod ABC of length 2a is movable about the fixed end A and carries two particles at the same mass ,one fastened to the middle point B and other to the end C of the rod. If the road be held in the horizontal position and be then let go, show that its angular velocity when vertical is $\sqrt{\frac{6g}{5a}}$ and that is $\frac{5a}{3}$ the length of the simple equivalent pendulum.

28. (v.v.i) : A rigid body swing under gravity from a fixed horizontal axis, show that the time of complete small oscillation is $2\pi\sqrt{\frac{k^2}{hg}}$ where k is its radius of gyration about the fixed axis and h is the distance between the fixed axis and the centre of the inertia of the body.

Moment of Inertia and Product of Inertia :

29. (v.v.i) : What is moment of inertia?

30. (v.v.i) : What is product of inertia?

31. (v.v.i) : (Theorem-01) : If the moments and product of inertia about any line or lines, through the centre of inertia G of a body are known ,then obtain the corresponding quantities for any parallel line or lines.

32. (v.v.i) : Find the moment of inertia of a hollow sphere about a diameter, its external and internal radii being a and b.

33. (v.v.i) : Prove that at the vertex C of a triangle, ABC which is right angled at C, the principal axes are perpendicular to the plane and two other inclined to the sides at an angle $\frac{1}{2}tan^{-1}(\frac{ab}{a^2-b^2})$

34. Find the moment of inertia of the area bounded by $r^2 = a^2Cos2\theta$ about its axes.