Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Improvement Examination-2017

Course No.: MTH 3201; Course Title: Abstract Algebra II

Full marks: 35 **Time: 2 Hours**

[N.B.: There are EIGHT questions. Answer any FIVE of the questions. The symbols and terms have their usual representations. Figures are in the right-hand margin indicates full marks.]

1. (a) Define ring with example. Is the set of natural number $\mathbb{N}$ form a ring? Explain. [3]

   (b) Let $*$ and o be two binary operations defined on Z by the following $a * b = a + b + 1$ and $aob = a + b + ab, \forall a, b \in Z$. Show that the system $(Z, *, o)$ is a commutative ring. [4]

2. (a) What do you mean by an ideal of a ring? Prove that every ideal is a subring, demonstrate by means of an example that not all subrings are ideals. [3]

   (b) If S is an ideal of a ring R and I is a subring of R, then show that S is an ideal of $S+T$. [4]

3. (a) Define subring. Show that the necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring of R are: $a - b \in S$ and $ab \in S$, $\forall a, b \in S$. [4]

   (b) Show that the intersection of two subrings of a ring R is also a subring of R. [3]

4. (a) Write down the definition of integral domain and field. Give an example of a ring which is not an integral domain. [3]

   (b) Prove that every field is an integral domain but the converse is not true always. [4]

5. (a) What is simple ring? Show that a field is a simple ring. [4]

   (b) Show that a communicative ring R with unity is a field if it has no proper ideals. [3]

6. (a) Define principal ideal, maximum ideal and prime ideals with examples. [3]

   (b) Let D be a communicative ring with unity and let I be a maximum ideal of D. Then show that D/I is a field [4]

7. (a) Define characteristics of a ring. Prove that characteristics of an integral domain is either zero or a prime number. [4]

   (b) Let R be a ring of characteristic 2. If $a, b \in R$ such that $ab = ba$ then show that $(a + b)^2 = a^2 + b^2 = (a - b)^2$. [3]

8. Define homomorphism and isomorphism on rings. State and prove the first isomorphism theorem on rings. [7]

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Department of Mathematics

3rd Year 2nd Semester Final Examination-2018

Course Code: MTH 3201 Course Title: Abstract Algebra-II

Full Marks: 70 **Time: 3 Hours**

*[N.B.: There are **EIGHT** questions. Answer any **FIVE** of the questions. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.]*

1.(a) Write down the name of the properties to form a ring. Is the set of all odd integers and the set of all even integers form ring? Explain your answer. **[04]**

(b) If $R = \{x : x = p + q\sqrt{2} + r\sqrt{3}, where\ p, q, r \in \mathbb{Z}\}$. Then determine whether R is ring or not with explanation. **[03]**

(c) Let $\oplus$ and $\odot$ be two binary operators on zdefined by $a \oplus b = a + b + 2$ and $a \odot b = a + ab + b, \forall a, b \in \mathbb{Z}$. Show that the system $(\mathbb{Z}, \odot, \otimes)$ is a commutative ring with unity. **[07]**

2.(a) Define ring with zero divisors and ring without zero divisors with example. Explain why the set of integer modulo 6 is ring with zero divisors? **[03]**

(b) When a ring is said to be commutative? Give an example of a ring which is not commutative with explanation. Is the relation $(a+b)^2 = a^2 + 2ab + b^2$true for a non-commutative ring? Justify your answer. **[05]**

(c) Define ring with unity. Give an example of ring without unity. What is Boolean ring? If R is a Boolean ring then show that **[06]**

(i) $a + a = 0, \forall a \in R$

(ii) $a + b = 0 \Rightarrow a = b$

3.(a) Define Subring and Ideal. Let $R = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{Z}\right\}$ be a ring. **[06]**

Suppose $S = \left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{Z}\right\}$ and $T = \left\{\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in \mathbb{Z}\right\}$ be two subsets of R.

Then determine whether S and T are subring or not?

(b) Prove that the intersection of two ideals in an ideal but the union of two ideals is an ideal if and only if one is contained in another. Also give an example in supporting of the above theorem. **[08]**

4.(a) Define Integral domain and Field. Give an example of a ring which is not an integral domain and also give an example of an integral domain which not a field with explanation. **[04]**

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(b) Let R be a commutative ring with unity whose only ideals are $\{0\}$ and R itself. Then show that R is a field. **[05]**

(c) Draw the composition tables for addition and multiplication of$(\mathbb{Z}_5, +_5, \times_5)$. From the multiplication table discuss the commutativity, existence of identity and existence of multiplicative inverse with respect to$\times_5$. Hence comment is it forms a field? **[05]**

5.(a) Define Proper and Improper Ideals. What is simple ring? Give example. Also give an example of an infinite integral domain and a finite integral domain. From the above example show that a finite integral domain is a field. **[07]**

(b) Let D be a commutative ring with unity and let I be an ideal of D. Then show that D/I is a field if and only if I is a maximal ideal of D. **[07]**

6.(a) What is quotient ring ? Suppose R and T are two rings and $f{:}R \to T$ is a ring homomorphism, then prove that $\dfrac{R}{\ker f} \cong \operatorname{Im} f$. **[07]**

(b) Let $f(x)$ be any polynomial and $g(x)$ be a non-zero polynomial domain $F(x)$ over the field F. Then show that there exist unique polynomial $q(x)$ and $r(x)$ in $F(x)$ such that $f(x) = q(x).g(x) + r(x)$ where either $r(x)=0$ *or* $deg(r(x))<deg(g(x))$. Also define monic polynomial and irreducible polynomial. **[07]**

7.(a) State Eisentein's irreducibility criterion. Let $F[x]$ be a polynomial domain over a field F and let $f(x), g(x)$ and $h(x)$ be any three non-zero polynomials in $F(x)$, such that $f(x) \mid g(x).h(x)$ and let $f(x)$ and $g(x)$ be relatively prime,then show that $f(x) \mid h(x)$. **[07]**

(b) State and prove unique factorization theorem for polynomials over a field. **[07]**

8.(a) Define ordered integral domain and ordered integral field. Show that $\{Z, +, \bullet\}$ is an ordered integral domain. **[07]**

(b) Writre short notes on principal ideal, prime ideal and maximal ideal of a ring with example. Prove that every ideal in the ring (Z,+,.) is a principal ideal. **[07]**

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Department of Mathematics

3rd Year 2nd Semester Final Examination-2019

Course No.: MTH 3201; Course Title: Abstract Algebra II

Full marks: 70 **Time: 3 Hours**

[*Answer any* ***05 (five)*** *questions. Figures are in the right-hand margin indicate full marks.*]

1. **(a)** Define a ring. Show that $\mathbb{Z}$, the set of integers, is a communicative ring under the binary operations $a \oplus b = a + b + 1$ and $a \otimes b = a + b + ab$. What is the multiplicative indemnity (unity) element, if any? [8]

   (b) Suppose R is a ring such that $x^2 = x$ holds $\forall x \in R$. Prove that $2x = 0$ holds $\forall x \in R$. Deduce that R is a commutative ring. [6]

2. **(a)** What do you mean by characteristic of a ring? Prove that the characteristic of an integral domain is either zero or a prime number. [5]

   (b) What is ring with zero divisors? Show that a ring $(R, +, .)$ is without zero divisors if and only if the cancellation laws hold on it. [5]

   (c) Show that $(\mathbb{Z}_5, +_5, \times_5)$ is a ring without zero divisors. [4]

3. **(a)** Define subring of a ring. If S is a non-empty subset of a ring R then prove that S is a subring of R if and only if [6]

   i. $a - b \in S$ for all $a, b \in S$
   ii. $ab \in S$ for all $a, b \in S$

   (b) What do you mean by an ideal of a ring? Demonstrate by means of an example that not all subrings are ideals. [4]

   (c) Prove that the set of all matrices of the form $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ with a, b, c are integers is a subring of the ring of 2×2 matrices having elements as integers. [4]

4. **(a)** Write down the definition of integral domain and field. [6]
   If $F = \{a + b\sqrt{2} \mid a,\ b \in \mathbb{Q}\}$, then show that $(F,\ +,\ .)$ is a field.

   (b) Prove that every field is an integral domain. Is the converse true in general? [4]

   (c) Prove that a field F has only two ideals $\{0\}$ and F itself. [4]

5. **(a)** What is principal ideal ring? Prove that the ring of integers $\mathbb{Z}$ is a principal ideal ring. [5]

   (b) Define prime ideal. Let D be an integral domain and I be an ideal of D, then show that D/I is an integral domain if and only if I is a prime ideal in D. [5]

   (c) What do you mean by kernel of a ring homomorphism? Let f be a homomorphism of a ring R into a ring R'. Then prove that $\ker f$ is an ideal of R. [4]

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6. (a) Let S be an ideal, prove that the set $\frac{R}{S} = \{S + a : a \in R\}$ is a ring for the operations in $\frac{R}{S}$ defined by the following ways: $(S + a) + (S + b) = S + (a + b)$, $(S + a)(S + b) = S + ab,\ \forall S + a,\ S + b \in \frac{R}{S}$. [6]

(b) If R is a communicative ring with unity and S is an ideal of R, then prove that $\frac{R}{S}$ is a field if and only if S is a maximal ideal of R. [8]

7. (a) Define ring isomorphism. Prove that every homomorphic image of an integral domain is an integral domain. [7]

(b) Prove that every homomorphic image of a ring R is isomorphic to some residue class ring (or quotient ring) of that ring. [7]

8. (a) Define Euclidean ring. Prove that the set $F[x]$ of all polynomials over a field F is a Euclidean domain. [8]

(b) Define prime element and irreducible element. Prove that every irreducible element of a principal ideal domain is prime. [6]

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Jagannath University, Dhaka
Department of Mathematics
3rd year 2nd Semester Examination-2020
Course Name: Abstract Algebra-II
Course Code: MTH-3201
Full Marks-70 Time: 3 Hours

There are eight questions. Answer any five of the questions. Figures in the right-hand margin indicate full marks.

1. a) What is ring with zero divisors? Let M be the set of all 2×2 matrices having their elements as integers, then show that $(M,+,.)$ is a non-commutative ring with unity and with zero divisors. 8

b) Define sub ring. If S is a non-empty subset of a ring R then prove that S is a sub ring of R if and only if 6

(i) $a-b \in S$ for all $a,b \in S$

(ii) $ab \in S$ for all $a,b \in S$

2. a) Let T be any non-empty subset of a ring R then prove that T is an ideal iff 6

(i) $x \in T, y \in T \Rightarrow x-y \in T$;

(ii) $x \in T, r \in R \Rightarrow xr \in T$ and $rx \in T$

b) Show that the set E of all even integers forms an ideal of the ring of integers. What about the set of all odd integers? 4

c) What is principal ideal of a ring? If R is a commutative ring with unity and $a \in R$, then prove that $Ra = \{ra : r \in R\}$ is a principal ideal of R generated by a. 4

3. a) Define integral domain and field. Prove that every finite integral domain is a field. 6

b) Prove that a field has no proper ideals. 4

c) Let a and b two elements of a commutative ring R of characteristic 2, show that $(a+b)^2 = a^2 + b^2 = (a-b)^2$. 4

4. a) Prove that the ring of integers Z is a principal ideal ring. 3

b) What do you mean by quotient ring? Let R be a ring and S be an ideal of R. Prove that quotient ring R/S is a ring. w.r.t. the operation 6

$$\left.\begin{aligned}(S+a)+(S+b) &= S+(a+b)\\ \text{and } (S+a)(S+b) &= S+ab\end{aligned}\right\} \forall\, S+a, S+b \in \frac{R}{S}$$

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c) Define prime ideal. Let D be an integral domain and I be an ideal of D Then show the D/I is an integral domain if and only if I is a prime ideal in D. 5

5 a) What is ring homomorphism? Prove that every homomorphic image of a commutative ring with unity is a commutative ring with unity. 6

b) If f is a homomorphism of a ring R into a ring R', then prove that 4
(i) $f(0) = 0'$, where 0 and $0'$ denote the zeros of R and R' respectively.
(ii) $f(-a) = -f(a), \forall a \in R$

c) What do you mean by an ideal of a ring? Demonstrate by means of an example that not all sub rings are ideal. 4

6 a) What do you mean by kernel of ring homomorphism? Let f is a homomorphism of a ring R into a ring R'. Then prove that $\ker f$ is an ideal of R. 5

b) Let $f: R \to R'$. f be a homomorphism from a ring R into a R' then show that $\frac{R}{\ker f} \cong R'$. 5

c) What do you mean by characteristic of ring? Prove that the characteristic of an integral domain is either zero or prime number. 4

7. a) Prove that the polynomial ring $Z[x]$ is not a principal ideal ring. 7

b) Let S be an ideal of a ring R and let T be an ideal of R containing S, then prove that $\frac{R}{T} \cong \frac{R/S}{T/S}$. 7

8. If R is anintegral domain and $R[x]$ is the set of all polynomials over R. Then prove that $R[x]$ is alsoan integral domain with respect to the operations of polynomial addition and polynomial multiplication. 14

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Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination 2021

Course Code: MTH 3201 Course Title: Abstract Algebra-II

Full Marks: 70 Time: 3 hours

*N.B.: There are **EIGHT** sets of questions. Answer any **FIVE**. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.*

1. (a) When a group is said to be a ring? Write the conditions which are not mandatory to form a ring but essential to form a field. Give separate examples of ring and field. [03]

(b) If $R=\{p+q\sqrt{2}\}$, where $p,q\in\mathbb{R}$, then show that R is a field. [05]

(c) Let $\oplus$ and $\odot$ be two binary operators on $\mathbb{Z}$ defined by $a\oplus b=a+b+2$ and $a\odot b=a+ab+b, \forall a,b\in\mathbb{Z}$. Show that the system $(\mathbb{Z},\oplus,\circledast)$ is a commutative ring with unity. [06]

2. (a) When is a ring said to be commutative? Give an example of a non-commutative ring with explanation. Is the relation $(a+b)^2=a^2+2ab+b^2$ true for a non-commutative ring? Justify your answer. [03]

(b) Define ring with zero divisors and ring without zero divisors. When is a ring said to be an integral domain? Is the set of all 2×2 matrices form a ring with zero divisors? Is it an integral domain also? Explain. [04]

(c) Prove that every field is an integral domain. Also show that the converse is not always true. What about when the integral domain is finite? [07]

3.(a) Define subring. If S is a nonempty subset of a ring R, then prove that S is a subring of R if and only if [06]

i) $a-b\in S$ for all $a,b\in S$

ii) $ab\in S$ for all $a,b\in S$

(b) What do you mean by an ideal of a ring? Demonstrate by means of an example that not all subrings are ideal. [04]

(c) Prove that the set of matrices $M=\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$, with a, b, c are integers, is a subring of the ring of 2×2 matrices having elements as integer. [04]

4. (a) What do you mean by quotient ring? Let R be be a ring and S be an ideal of R. Prove that the quotient ring R/S is a ring. [04]

(b) Let R be a commutative ring with unity whose only ideals are $\{0\}$ and R itself. Show that R is a field. [05]

(c) Show that in the ring $(\mathbb{Z}, +, \cdot)$ every ideal is a principal ideal. [05]

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5. (a) Define Homomorphism and Isomorphism of ring. Let $f:\mathbb{Z} \to M_2(\mathbb{Z})$ is a mapping defined by $f(x) = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}, \forall x \in \mathbb{Z}$ Prove that, f is a homomorphism. What about isomorphism? [06]

(b) State and prove the third law of isomorphism on rings. [08]

6. (a) Define irreducible polynomial and maximal ideal. Show that in a polynomial ring $F[x]$ overv a field F, an ideal $A = p(x)$ is a maximal ideal if and only if $p(x)$ is an irrducible polynomial over F. [07]

(b) What do you mean by unique factorization domain and primitive polynomial? Prove that the product of two primitive polynomials in $R[x], R$ is a unique factorization domain and primitive. [07]

7.(a) Define Euclidean ring. Prove that the set $F[x]$ of all polynomials over a field F is Euclidean ring. [06]

(b) Define prime element and irreducible element. Prove that every irreducible element of a principal ideal domain is prime. [08]

8. (a) Define Image and Kernel of ring homomorphis m. If $f: R \to R'$ is a ring homomorphism, then prove that, $(i)\ f(0) = 0', 0 \in R, 0' \in R'$ are additive identities. [07]

$(ii)\ f(-a) = -f(a), \forall a \in R$

$(iii)\ Imf$ is a subring of R'.

(b) State and prove the Division Algorithm on polynomial rings. [07]

*** *The End* ***

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Jagannath University, Dhaka
Department of Mathematics
3rd year 2nd Semester Final Examination-2017
Course No:: **MTH-3202**, Course Title: Real Analysis II
Full Marks: 70 Time: 3 Hours

There are **Eight** questions. Answer any **Five** of the questions. Figures in the right-hand margin indicate full marks.

1. a) Define continuity and uniform continuity of a function with examples. 4
   b) Discuss the uniform continuity of $f(x) = \frac{1}{x}$ on (0, 1] and [a, 1], $0 < a < 1$.. 4
   c). Discuss the continuity of the following function over $(-\infty, \infty)$ 2
   $$f(x) = \begin{cases} \sin x + x, & x \in \mathbb{Q} \\ x, & x \notin \mathbb{Q} \end{cases}$$
   d) Prove that a continuous function attains its bounds on a closed interval. 4

2. a) State and prove Rolle's theorem. 4
   b) Deduce Lagrange's Mean Value theorem from Cauchy's Mean value theorem. Using Mean Value theorem prove that 5
   $$x < \ln \frac{1}{1-x} < \frac{x}{1-x}, 0 < x < 1$$
   c) Write down the Leibnitz's rule to perform differentiation under the integral sign. Apply this rule to evaluate 5
   $$\int_0^\infty \frac{\sin ax}{x} dx \ (a > 0).$$

3. a) Define Riemann integral. 7
   If f is defined on $[0, a]$, $a > 0$ by $f(x) = x^3 \ \forall \ x \in [0, a]$, then prove that $f \in R[0, a]$ and $\int_0^a f(x)dx = \frac{a^4}{4}$.
   b) If $f:[a, b] \to R$ is monotonic on $[a, b]$, then prove that f is Riemann integrable on $[a, b]$. 7

4. a) Define Riemann-Stieltjes integral. Let f be Riemann integrable on $[a, b]$ and $\alpha(x)$ be monotonically non-decreasing function on $[a, b]$ such that it's derivative $\alpha'(x)$ is Riemann integrable on $[a, b]$, then show that f is Riemann-Stieltjes integrable with respect to α on $[a, b]$ and $\int_a^b f(x)d\alpha = \int_a^b f(x)\alpha'(x)dx$. 7

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b) Evaluate $\int_0^{\pi/2} x\,d(\sin x)$ by integration by parts. 5

c) Let $f(x)=x$ and $\alpha(x)=x^2$. Does $\int_0^1 f(x)\,d\alpha$ exist? If it exists, find its value by evaluating lower and upper Riemann-Stieltjes integrals. 7

5. a) Define point wise convergence and uniform convergence of functions. If a sequence of continuous functions $<f_n>$ is uniformly convergent to a function f on $[a, b]$ then prove that f is continuous on $[a, b]$. 7

b) If $f_n(x)=\frac{nx}{1+nx}$, where $0 \le x \le 1$ and $n = 1,2,3,\ldots$. Show that $<f_n(x)>$ converges point-wise but not uniformly on [0, 1].

6. a) State and prove Weierstrass's M-test to test the uniform convergence of series of functions. Show that 6

$$\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x^n}{n^2}\right) dx = \sum_{n=1}^{\infty} \frac{1}{n^2(n+1)}.$$

b) Show that the series $\sum f_n(x)$ for which $f_n(x) =$ [illegible] can be integrated term by term on [0, 1] although it is not uniformly convergent in this interval. 8

7. a) Define Euclidean n-space R^n. Show that R^n is a metric space with the metric $d(x, y) = \max_{1\le k\le n} |x_k - y_k|,\ x, y \in R^n$. 7

b) Show that Euclidean n-space R^n is a complete metric space.

8. a) State and prove the necessary and sufficient condition for a Jacobian to vanish. Show that the functions $u=x+y+z$, $v=xy+yz+zx$, $w=x^3+y^3+z^3-3xyz$, are not independent. Find a relation between them. 7

b) State the Implicit Function Theorem for $\mathbb{R}^3$ and explain its outcome. 3

c) State and prove Fubini's theorem for evaluating double integral. 4

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Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination-2018

Course Code: MTH 3202 Course Title: Real Analysis II

Full Marks: 70 **Time: 3 Hours**

*[N.B.: There are **EIGHT** questions. Answer any **FIVE** of the questions. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.]*

1. (a) Discuss the distinction between continuity and uniform continuity of functions. **3**

   (b) Prove that if a function f is one-one and continuous on $[a, b]$, then f is strictly monotonic on $[a, b]$. **5**

   (c) A function g is defined as **6**

$$g(x) = \begin{cases} 0, \text{when } x \text{ is zero or irrational} \\ \frac{1}{q}, \text{when } x \text{ is a nonzero rational number } \frac{p}{q} \text{ in its lowest form} \end{cases}$$

   Show that g is continuous at every irrational point and it is discontinuous at every rational point.

2. (a) State and prove Taylor's Theorem with Lagrange's form of remainder. **8**

   (b) Suppose f is defined in a neighborhood of x and suppose $f'(x)$ exists. Show that **4**

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

   (c) What is the specialty of the function $(x) = \sum a^n \cos(b^n \pi x)$? For this specialty what conditions a and b should satisfy? **2**

3. (a) Define Riemann integral. State and prove necessary and sufficient condition for which a function is said to be Riemann integrable. **7**

   (b) Prove that $\int_{-a}^{b} f(x) \le \int_{a}^{-b} f(x)dx$. If $f(x)$ is continuous on$[a, \ b]$, then show that $f(x)$ is also Riemann integrable. **7**

4. (a) Prove that, every monotonic function is Riemann-Stieltjesintegrable. **6**

   (b) Evaluate $\int_0^3 x \, d[x]$, where $[x]$ is the ceiling function of x. **4**

   (c) Show that $\int_0^\infty \frac{\tan^{-1} x}{x(1+x^2)} dx = \frac{\pi}{2} \ln 2$. **4**

5. (a) Define Limit Function of a sequence functions. Give an example of a sequence of continuous functions whose limit function is not continuous? **3**

   (b) State and prove M_n-test for sequence of functions. **6**

   (c) Show that the sequence $\langle f_n \rangle$ where $f_n(x) = \frac{x}{1+nx^2} \ \forall \ x \in \mathbb{R}$, converges uniformly on any closed interval $[a, b]$. **5**

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6. (a) State and prove Cauchy's criterion for uniform convergence. 7

(b) If $f_n(x)=\dfrac{nx}{1+nx}$, where $0<x<1$ and $n=1,2,3\ldots\ldots$ then show that $\langle f_n : n\in N\rangle$ converges uniformly but not pointwise. 7

7. (a) Define norm and induced metric on a vector space. Prove that for $x=(x_1,x_2)\in\mathbb{R}^2$, ρ is a norm in $\mathbb{R}^2$ where $\rho(x)=|x_1|+|x_2|$. 5

(b) Let $d:\mathbb{R}^2\to\mathbb{R}$ be defined by $d(x,y)=|\cos x-\cos y|$. Test whether d is a metric or pseudo-metric. 4

(c) Prove Minkowski's inequality in $\mathbb{R}^n$. 5

8. (a) State Fubini's Theorem for double integrals. 2

(b) State the Implicit Function Theorem for $\mathbb{R}^3$. Consider two three-variable functions $H(x,y,z)$ and $K(x,y,z)$ and the associated level surfaces 8

$$H(x,y,z)=a, K(x,y,z)=b.$$

Assuming that these surfaces intersect along a curve which contains the point (x_0,y_0,z_0), and that on some neighborhood of this point, the curve determines y as a function $y(x)$ of x. Derive a formula for y in terms of the partial derivatives of H and K.

(c) Define the Jacobian for n functions of n variables. If $f(0)=0$ and $f'(x)=\frac{1}{1+x^2}$, prove without using the method of integration, that 4

$$f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right).$$

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Jagannath University, Dhaka

Department of Mathematics

3^{rd} Year 2^{nd} Semester Final Examination 2019

Course No.: MTH-3202; Course Title: Real Analysis-II

Full marks: 70 **Time: 3 Hours**

[*N.B.: There are* ***EIGHT (8)*** sets *questions. Answer any* ***FIVE (5)*** sets *of the questions. The symbols and terms have their usual representations. Figures are in the right-hand margin indicate full marks.*]

1. (a) Define continuity at a point with its geometric meaning. If $f(x)$ is continuous in $[a,b]$, then show that it is bounded on that interval. **[05]**

   (b) State and prove that Bolzano's Intermediate Value Theorem. **[05]**

   (c) Prove that **[04]**

$$f(x)=\begin{cases} x\sin\frac{1}{x}; \text{when } x\neq 0 \\ 5\ ; \text{when } x=0 \end{cases}$$

   is not continuous at $x=0$. Can you redefined $f(0)$ so that $f(x)$ is continuous at $x=0$?

2. (a) Define continuity and uniform continuity and distinguish between them. **[04]**

   (b) If a function is continuous on a closed interval, then it attains its bounds. **[05]**

   (c) Show that $f(x)=\frac{1}{x}$ is continuous in an open interval but may fail to be uniformly continuous in the interval. **[05]**

3. (a) Define derivative of a function and give its graphical interpretation. **[03]**

   (b) State and prove Rolle's theorem with its geometric interpretation. **[06]**

   (c) Show that the function $f(x)$ defined by **[05]**

$$f(x)=\begin{cases} x^2\sin\frac{1}{x}; \text{when } x\neq 0 \\ 0\ ; \text{when } x=0 \end{cases}$$

   is differentiable everywhere.

4. (a) What is Riemann integral? State and prove the necessary and sufficient conditions for which a function is said to be Riemann integrable. **[07]**

   (b) If $f(x)$ be a real-valued and bounded function defined on $[a,b]$ and M,m be the supremum and infimum of $f(x)$ respectively on $[a,b]$, then prove that $m(b-a)\leq L(P,f)\leq U(P,f)\leq M(b-a);\ \forall P\in p[a,b]$. **[07]**

5. (a) If $f(x)$ is continuous on $[a, b]$, then show that it is also Riemann integrable. Also show that there exists $c\in[a, b]$ such that $\int_a^b f(x)\,dx=(b-a)f(c)$. **[07]**

   (b) If $f(x)=\begin{cases} 1; & \text{when } x \text{ is rational} \\ -1; & \text{when } x \text{ is irrational} \end{cases}$, then show that $f(x)\notin\Re$ on $[0, 1]$; where $\Re$ denotes Riemann integrability. Also show that $f(x)\notin\Re$ on $[0, 1]$; when $f(x)=\begin{cases} x; & \text{when } x \text{ is rational} \\ 1-x; & \text{when } x \text{ is irrational.} \end{cases}$ **[07]**

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6. (a) Define point wise convergence and uniform convergence of a sequence of functions. Also distinguish between them. **[04]**

(b) If $\{f_n(x): n \in \mathbb{N}\}$ is a sequence of function on S and M_n is a sequence of constant term and if $|f_n(x)| < M_n$ $(n = 1,2,3,\ldots)$ and $x \in S$, then show that $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly on S if the series of real numbers $\sum_{n=1}^{\infty} M_n$ converges. **[06]**

(c) If $f(x)$ is Riemann integrable on $[a,b]$, then the function F defined on $[a,b]$ by $F(x) = \int_a^x f(t)\,dt;\ \forall x \in [a,\ b]$ is continuous on $[a,b]$. **[04]**

7. (a) State and prove Cauchy's criterion for uniform convergence. **[06]**

(b) Show that the sequence of function $f_n(x) = \dfrac{nx}{1+n^2x^2}$ does not converge uniformly for all x. Also decide the condition for $\sum_{n=1}^{\infty} \dfrac{x}{1+n^2x^2}$. **[08]**

8. (a) Define Metric space and Normed space. Let (X,d) and (Y,d') be any two metric spaces. A mapping $f: X \to Y$ is continuous on X iff $f^{-1}(G)$ is open in X for every open set G in Y. **[06]**

(b) Show that the mapping $d(x,y): \mathbb{R}^n \to \mathbb{R}$ defined by **[04]**

$$d(x,y) = \left[\sum_{k=1}^{n} (x_k - y_k)^2\right]^{\frac{1}{2}};\forall x, y \in \mathbb{R}^n \text{ with } x_k, y_k \in \mathbb{R} \text{ is a metric.}$$

(c) Let (X,d) be a metric space. Show that (X,d_1) is a metric space where the mapping $d_1 : X \times X \to \mathbb{R}$ is defined by $d_1(x,y) = \dfrac{d(x,y)}{1+d(x,y)};\forall x, y \in X$. **[04]**

------*Best of Luck*------

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Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination-2020

Course Code: MTH 3202 Course Title: Real Analysis-II

Full Marks: 70 **Time: 3 Hours**

*[N.B.: There are **EIGHT** questions. Answer any **FIVE** of the questions. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.]*

1.(a) Discuss the graphical representation of $\varepsilon-\delta$ definition of limit. [05]

(b) If a function $f(x)$ is continuous on a closed and bounded interval $[a,b]$, then show that $f(x)$ is uniformly continuous on $[a,b]$. [05]

(c) Investigate the uniform continuity of $f(x)=x^2,\ 0<x<1$. [04]

2.(a) Define continuity, discontinuity of the 1st kind, 2nd kind, mixed discontinuity, removal discontinuity with example. [05]

(b) If a function is continuous on a closed interval, then show that it attains its bounds. [05]

(c) Show that the function $f(x)=\frac{1}{x^2}, \forall x\in(0,1)$ is continuous in an open interval may fail to be uniformly continuous in that interval. [04]

3.(a) Define derivative of a function and give its graphical interpretation. [03]

(b) State and prove Rolle's theorem with its graphical interpretation. [06]

(c) Show that the function $f(x)$ defined by $f(x)=\begin{cases} x^2 \sin\frac{1}{x}; & \text{when } x\neq 0 \\ 0 & \text{when } x=0 \end{cases}$ is differentiable everywhere. [05]

4.(a) What is Riemann Integral? Prove that every continuous function is Riemann integrable. [05]

(b) Show that every monotonic function is Riemann integrable. [05]

(c) If f is defined on $[a,b], b>0$ by $f(x)=x^3, \forall x\in[a,b]$, then prove that $f(x)$ Riemann integrable (R-I) and $\int_a^b f(x)dx=\frac{b^4}{4}$. [04]

5.(a) Prove that every monotonic function is Riemann-Stieljes integrable. [06]

(b) Evaluate $\int_0^3 x\,d[x]$, where $[x]$ is the ceiling function of x. [04]

(c) Show that $\int_0^\infty \frac{\tan^{-1}x}{x(1+x^2)}dx=\frac{\pi}{2}\ln 2$. [04]

6.(a) What is sequence of real valued function and point-wise convergence of a sequence of function? A series $\sum f_n(x)$ of function defined on an interval I_∞ converges uniformly on I if $\exists$ a convergent series $\sum_{n=1}^\infty M_n$ of positive numbers such that $|f_n(x)|\le M_n, \forall n\in\mathbb{N}, \forall x\in I$. [04]

(b) State and prove Sup-Span theorem. [05]

(c) Test for uniform convergence of the sequence $<f_n>$ defined by $f_n(x)=\frac{nx}{1+n^2x^2}, \forall n\in\mathbb{R}$. [05]

7.(a) State and prove Cauchy's Mean value theorem. [05]

(b) Let (X,d) and (Y,d') be any two metric spaces. Then show that the function $T:X\to Y$ is continuous at $x_0\in X$ iff $x_n\to x_0 \Rightarrow T_{x_n}\to T_{x_0}$. [05]

(c) Show that the mapping $d:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ i.e. $d:\mathbb{C}\times\mathbb{C}\to\mathbb{R}$ defined by $d(x,y)=\max\{|x_1-y_1|, |x_2-y_2|\}$ where $x=(x_1,x_2), y=(y_1,y_2)\in\mathbb{R}^2$ with $x_1,x_2,y_1,y_2\in\mathbb{R}$ is a metric on $\mathbb{R}^2$. [04]

8.(a) What is meant by compactness? Prove that continuous image of a compact space is compact. [05]

(b) What do you mean by $\mathbb{R}^n$? Show that $\mathbb{R}^n$ is a metric space with the metric $d(x,y)=\max_{1\le k\le n}(x_k-y_k), \forall x,y\in\mathbb{R}^n$. [04]

(c) Show that $\mathbb{R}^n$ is complete. [05]

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Jagannath University, Dhaka

Department of Mathematics

3^{rd} Year 2^{nd} Semester Final Examination-2021

Course No.: MTH-3202; Course Title: Real Analysis II

Full marks: 70 **Time: 03 Hours**

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*[N.B.: There are **EIGHT** questions. Answer any **FIVE** of the questions. The symbols and terms have their usual representations. Figures are in the right-hand margin indicates full marks.]*

1. (a) Give the sequential and $\delta-\varepsilon$ definitions of continuity. Prove the $\delta-\varepsilon$ definition of continuity. [7]
   (b) Let $f(x)=2x^2+1$ for $x \in \mathbb{R}$. Prove f is continuous on $\mathbb{R}$ by (i) Using the sequential definition, (ii) Using the $\varepsilon-\delta$ property. [7]

2. (a) Give the examples of local and global properties of a function. State and prove the global continuity theorem. [5]
   (b) Define uniform continuity. Prove that function f defined on $\mathbb{R}^+$ as $f(x)=\sin\left(\frac{1}{x}\right), \forall\, x>0$ is continuous but not uniformly continuous. [5]
   (c) State and prove Weierstrass *M*-test for uniform convergence of a sequence of real functions. [4]

3. (a) Make a comparative analysis on pointwise convergence and uniform convergence of a sequence of real valued functions. [3]
   (b) Let $f_n(x)=(x-1/n)^2$ for $x \in [0,1]$. [7]
   (i) Does the sequence (f_n) converge pointwise on the set $[0, 1]$? If so, give the limit function.
   (ii) Does (f_n) converge uniformly on $[0,1]$? Prove your assertion.
   (c) Let (f_n) be a sequence of continuous functions on $[a,b]$, and suppose $f_n \to f$ uniformly on $[a,b]$. Then prove that $\lim_{n\to\infty}\int_a^b f_n(x)dx=\int_a^b f(x)dx$. [4]

4. (a) State and prove Lagrange's Mean Value Theorem. [6]
   (b) Explain the geometrical significance of Lagrange's Mean Value Theorem. [4]
   (c) Discuss the applicability of Rolle's theorem to the function [4]
   $$f(x)=\begin{cases} x^2+1, & 0\le x\le 1\\ 3-x, & 1<x\le 2\end{cases}.$$

5. (a) What do you mean by the mesh of a partition? Show that a bounded function f on $[a,b]$ is integrable if and only if for each $\varepsilon>0$ there exists a $\delta>0$ such that mesh$(P)<\delta$ implies $U(f,P)-L(f,P)<\varepsilon$ for all partitions P of $[a,b]$. [6]
   (b) What is the concern of Fundamental theorem of calculus? State and prove the fundamental theorems of calculus. [8]

6. (a) What is Riemann-Stieltjes integral? If f is Riemann-Stieltjes integrable on $[a,b]$ then prove that $|f|$ is Riemann-Stieltjes integral on $[a,b]$ and $\left|\int_a^b f d\alpha\right| \le \int_a^b |f| d\alpha$. [7]

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(b) If f is a function defined on $[0,1]$ by $f(x) = 1$ if $x \neq \frac{1}{2}$ and $f(x) = 0$ if $x = \frac{1}{2}$ then show that f is Riemann integrable over $[0,1]$ and also evaluate $\int_0^1 f(x)dx$. [7]

7. (a) What do you mean by Metric and Metric space? Show that $\mathbb{R}^n$ is a metric space. [5]

(b) Define the terms Norm's and distance in R^n. State the relation between them. [3]

(c) Prove that every continuous mapping on a compact metric space X into R^n is closed and bounded. [6]

8. (a) State and prove Implicit Function Theorem. [7]

(b) State Fubini's theorem. Evaluate the Jacobin and using Fubini's theorem find the volume of the sphere of radius a over the region $S = \{(x,y): x^2 + y^2 \leq a^2\}$. [7]

Best of Luck

Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination 2017

Course No.: MTHT-3203; Course Title: Differential Geometry

Full marks:70 Time: 03 Hours

[N.B.: There areEIGHT questions Answer any FIVE of the questions. The symbols and terms have their usual representations. Figures are in the right-hand margin indicate full marks.]

1. (a) Define osculating plane at a point on a space curve. Derive the equation of the osculating plane of the space curve $\underline{x} = \underline{x}(u)$ 7

   (b) Find the equation of the tangent line, normal line and binormal line at $u=1$ to the curve $\underline{x} = (u, u^2, u^3)$. 7

2. (a) State and prove the Serret-Frenet formula. 8

   (b) Show that for space curve $\underline{x} = \underline{x}(s)$ 6

   (i) $[\underline{x}'\underline{x}''\underline{x}'''] = \kappa^2\tau[\underline{tnb}]$

   (ii) $\tau = \frac{1}{\kappa^2}[\underline{x}'\underline{x}''\underline{x}''']$

   (iii) $\tau^2 + \kappa^2 = \left(\frac{\underline{x}'''}{\kappa}\right)^2 - \left(\frac{\kappa'}{\kappa}\right)^2$

3. (a) Define a helix. Prove that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in a constant ratio. 7

   (b) Show that at any point s on a space curve $\underline{x} = \underline{x}(s)$ 7

   (i) $[\underline{t}'\underline{t}''\underline{t}'''] = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right)$ (ii) $[\underline{b}'\underline{b}''\underline{b}'''] = \tau^5 \frac{d}{ds}\left(\frac{\kappa}{\tau}\right)$

4. (a) What is meant by spherical indicatrix of the binormal of a curve? Show that the curvature and torsion of the spherical indicatrix of the binormal at s on the curve $\underline{x} = \underline{x}(s)$ are $\kappa_b = \dfrac{\sqrt{\kappa^2+\tau^2}}{\tau}$ and $\tau_b = \dfrac{\kappa'\tau - \kappa\tau'}{\tau(\kappa^2+\tau^2)}$. 6

   (b) Prove that if $\tau \neq 0$, $\alpha \neq 0$, $\alpha \neq \pi/2$, then on the Bertrand curve $\tau + (\kappa + 1/a)\tan\alpha = 0$, where a is the distance between corresponding points on two Bertrand curves. 4

   (c) Prove that, the necessary and sufficient condition for the curve to be a plane curve is $[\mathbf{r}', \mathbf{r}'', \mathbf{r}'''] = 0$. 4

5. (a) Define involute of a curve. Deduce the equation of involute. 4

   (b) Show that the evolutes of a plane curve are helices. 4

   (c) Define first fundamental metric of a surface. Show that the first fundamental form of a surface $x = x(u,v)$ is defined as $(ds)^2 = I = E(du)^2 + 2F du\, dv + G(dv)^2$. Also show that $G - F^2 > 0$. 6

6. (a) Find the first fundamental form, unit surface normal and element of surface area for the surface $z=f(x,y)$. 7

   (b) If the parametric curves are orthogonal then show that the differential equation of the curves, bisecting the angles of the parametric curves is, $E(du)^2 - G(dv)^2 = 0$ 7

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7. (a) Show that $A=\iint_S \sqrt{EG-F^2}\,du\,dv$ and find that the surface area of the sphere 5

$\underline{x}=(a\cos u\cos v, a\sin u\sin v, a\cos u)$, where $0\le u\le\pi, 0\le v\le 2\pi$.

(b) Define mean curvature. Find the mean curvature M and Gaussian curvature K for the surface $\underline{x}=(a\sin\theta\cos\phi, a\sin\theta\sin\phi, a\cos\theta)$. 5

(c) Show that the Rodrigue's formula is $\kappa d\mathbf{r}+d\mathbf{N}=0$. 4

8. (a) State and prove Euler's theorem on normal curvature. 8

(b) Define line of curvature. Show that the necessary and sufficient condition for the lines of curvature to be parametric curves is that $F=0, f=0$. 6

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Jagannath University, Dhaka
Department of Mathematics
3rd Year 2nd Semester Final Examination-2018
Course Code: MTH 3203 Course Title: Differential Geometry

Full Marks: 70 **Time: 3 Hours**

[N.B.: There are EIGHT questions. Answer any FIVE of the questions. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.]

1. (a) Derive the equations of the normal plane, rectifying plane and osculating plane of the space curve $\underline{x} = \underline{x}(u)$. 8

   (b) Find the equations of the normal plane, rectifying plane and osculating plane of the space curve $\underline{x} = (u - \cos u,\ 1 + \sin u,\ u)$ at $u = \frac{\pi}{2}$. 6

2. (a) Show that the necessary and sufficient condition for the curve x=x(u) to be a plane curve is $[\dot{x}\ddot{x}\dddot{x}]=0$ 7

   (b) Prove that, if the principal normals of a space curve are binormals of another then the relation $a(\kappa^2 + \tau^2) = b\kappa$ must hold, where a and b are constants. 7

3. (a) Show that at the point s on $\underline{x} = \underline{x}(s)$ 6

   i) $[\underline{t'}\underline{t''}\underline{t'''}] = [\underline{x''}\underline{x'''}\underline{x^{iv}}] = \kappa^3(\kappa\tau' - \kappa'\tau) = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right)$

   ii) $[\underline{b'}\underline{b''}\underline{b'''}] = \tau^3(\kappa'\tau - \kappa\tau') = \tau^5 \frac{d}{ds}\left(\frac{\kappa}{\tau}\right)$.

   (b) Find $\underline{t}$, $\underline{n}$, $\underline{b}$, κ and τ at a point on the space curve $\underline{x} = (a\cos u, a\sin u, bu)$. 8

4. (a) Find the evolute of the circular helix $x=(a\cos\theta, a\sin\theta, a\theta\tan\alpha)$. 7

   (b) What do you mean by involute and evolute of curve? Show that evolutes of a plane curve are helices. 7

5. (a) Define Bertrand curves. Show that – 10

   (i)The distance between corresponding points of the two Bertrand curves is constant.

   (ii)The angle between corresponding tangent lines to two Bertrand curves is constant.

   (iii)There exists a linear relationship between curvature and torsion of each of the Bertrand curve.

   (iv)The torsion of the two Bertrand curves have the same sign and their product is contant.

   (b) For the curve $x=(3u, 3u^2, 2u^3)$, prove that $R= -T=\frac{3}{2}(1+2u^2)^2$, where R, T are radius of curvature and radius of torsion respectively. 4

6. (a) Find the second fundamental form for the surface z=f(x,y) 5

   (b) Show that $eg - f^2 = [N N_u N_v]\sqrt{EG - F^2}$ 2

   (c) Calculate the fundamental coefficients for the right helicoid, x=(u cosv, u sinv, cv). Also find its 1st and 2nd fundamental forms. 7

7. (a) Define First curvature, Mean curvature and Gaussian curvature. Find the Mean curvature and Gaussian curvature for the surface x=(u+v,u-v,uv) at u=1 and v=1. 8

   (b) Deduce the Weingarten equations. 6

$$N_u = \frac{fF - eG}{EG - F^2}x_u + \frac{eF - fE}{EG - F^2}x_v$$

$$N_v = \frac{gF - fG}{EG - F^2}x_u + \frac{fF - gE}{EG - F^2}x_v.$$

8. (a) The necessary and sufficient condition for a curve on the surface to be a line of curvature is that the normals to the surface at any two consecutive points of the curve intersect. 7

   (b) Calculate the principal curvatures and principal directions for the surface $x=(u,v,4u^2+v^2)$ at u=0,v=0. 7

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Jagannath University, Dhaka

Department of Mathematics

3rd year 2nd Semester Final Examination-2019

Course No: MTH 3203; Course Title: Differential Geometry

Full marks: 70 **Time: 3 Hours**

[Note: There are ***Eight*** *questions. Answer any* ***Five****. Figures in the right margin indicate full marks of each question. The symbols and terms have their usual representations.]*

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1. (a) Define vector function of the single real variable with examples. Compute the vectors $\underline{x} = t^2\underline{e}_1 + (1-t)\underline{e}_2$ for t any integer between -4 and 4. Also sketch the curve traced by the terminal points of $\underline{x}$. **[5]**

   (b) Find the equation of the line tangent to the curve generated by $\underline{x} = t\underline{e}_1 + t^2\underline{e}_2 + t^3\underline{e}_3$ at $t=1$. **[4]**

   (c) Explain unit tangent vector on a curve C, tangent line and normal plane on curve C. **[5]**

2. (a) State and prove Serret-Frenet's formulae on the space curve $\vec{x} = \vec{x}(s)$ and hence show that $\kappa^2(\tau^2 + \kappa^2) = (x''')^2 - (\kappa')^2$. **[7]**

   (b) What do you mean by curvature and torsion of a curve? Compute $\vec{t}, \vec{n}, \vec{b}, \kappa,$ and τ at the point u on the space curve $\vec{x} = (e^u \cos u, e^u \sin u, u)$. **[7]**

3. (a) If a curve $\vec{x} = \vec{x}(s)$ lies on the sphere $(\vec{y} - \vec{c})\cdot(\vec{y} - \vec{c}) = a^2$, where $\vec{c}$ and a are the centre and radius of the sphere then prove that $\rho^2 + (\rho'\sigma)^2 = a^2$ and hence show that the tangent to the locus of $\vec{c}$ is parallel to the binormal to the curve $\vec{x} = \vec{x}(s)$ at the corresponding point. **[7]**

   (b) Define helix and circular helix. Show that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in a constant ratio. **[7]**

4. (a) What is meant by spherical indicatrix of the tangent of a curve? Show that the curvature and torsion of the spherical indicatrix of the tangent at s on the curve $\vec{x} = \vec{x}(s)$ are $\kappa_t = \dfrac{\sqrt{\kappa^2 + \tau^2}}{\kappa}$ and $\tau_t = \dfrac{\kappa\tau' - \kappa'\tau}{\kappa(\kappa^2 + \tau^2)}$. **[7]**

   (b) Define involute of a curve. Deduce the equation of involute in the form $\vec{y} = \vec{x} + (c - s)\vec{t}$ and hence find the equation of involute of the helix. **[7]**

5. (a) Describe first fundamental form geometrically and hence derive the first fundamental coefficients E, F and G. **[7]**

   (b) Show that the first fundamental form of the surface $\underline{x} = \underline{y}(s) + u\underline{b}(s)$ generated by the binormal $\underline{b}(s)$ of a curve $\underline{y} = \underline{y}(s)$ is $\underline{I} = (1 + u^2 r^2)ds^2 + du^2$. **[7]**

6. (a) Explain what is meant by normal curvature and geodesic curvature and hence establish the following identities (i) $R = R_n \cos\varphi$ and (ii) $d\vec{N} + \kappa_n d\vec{x} = 0$. **[7]**

   (b) Define curvature directions. Prove that the curvature directions are orthogonal and hence find the mean curvature and Gaussian curvature. **[7]**

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7. (a) What are elliptic, parabolic, and hyperbolic points on a surface? Find the first and second fundamental form of the surface $x = u, y = v, z = u^2 + v^3$ and discuss it at these three points. [7]

(b) Define lines of curvature. Show that the necessary and sufficient condition for the lines of curvature to be parametric curves is that $F = 0, f = 0$. [7]

8. (a) Derive Gauss-Weingarten equations for surfaces. [7]

(b) Determine the principal directions to $\underline{x} = u\underline{e}_1 + v\underline{e}_2 + (u^2 + v^2)\underline{e}_3$ at $\underline{u}=1$, $\underline{v}=1$ and verify Rodrigues' formula in each direction. [7]

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Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination-2020

Course Code: MTH 3203 Course Title: Differential Geometry

Full Marks: 70 **Time: 3 Hours**

[N.B.: There are ***EIGHT*** *questions. Answer any* ***FIVE*** *of the questions. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.]*

1.(a) Derive the equations of the normal plane, rectifying plane and osculating plane of the space curve $\underline{x} = \underline{x}(u)$. **[07]**

(b) Find the equations of the tangent line, normal line and binormal line at $u = 1$ to the curve $\underline{x} = (u, u^2, u^3)$. **[07]**

2.(a) Describe the equation of tangent line to a curve. Find the unit tangent vector to the circular helix $\boldsymbol{r} = (a\,cost, a\,sint, bt)$; $-\infty < t < \infty$, where $a > 0$ and write down the equation of the tangent at any point. Also find the length of one complete turn of helix. **[06]**

(b) If the principle normal of a space curve be binormal of another space curve, then show that the relation $c(\kappa^2 + \tau^2) = \kappa$ must hold, where c is any constant. **[04]**

(c) Prove that at any point s on a space curve $\underline{x} = \underline{x}(s)$ **[04]**

$$[\underline{x}''\ \underline{x}'''\ \underline{x}^{iv}] = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right).$$

3.(a) What is meant by curvature and torsion of a curve? For the curve $x = 4a\,cos^3u, y = 4a\,sin^3u, z = 3c\,cos2u$, prove that $\kappa = \frac{a}{6(a^2+c^2)sin2u}$. **[04]**

(b) State and prove Serret-Frenet formula and deduce it with Darboux vector $\boldsymbol{w} = \tau\boldsymbol{t} + \kappa\boldsymbol{b}$ **[06]**

(c) Prove that, the necessary and sufficient condition for the curve to be a plane curve is $[\boldsymbol{r}', \boldsymbol{r}'', \boldsymbol{r}'''] = 0$. **[04]**

4.(a) Derive the equation of the Evolutes in space curves. **[07]**

(b) What is called Bertrand curves? Prove that the torsions of the two associate Bertrand curves have the same sign and their product is a constant i. e. $\tau\tau_1 = \frac{1}{a^2}\sin^2\alpha$. **[07]**

5.(a) Define a helix. Prove that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in a constant ratio. **[07]**

(b) Show that curvature of the helix $\underline{x} = acos\theta, y = asin\theta, z = atan\alpha$ is $\frac{\cos^2\alpha}{a}$. Then deduce that the torsion is $\pm\frac{Sin\alpha Cos\alpha}{a}$. **[07]**

6.(a) Define spherical indicatrices of the tangent, principal normal and binormal. **[03]**

(b) Define involute of a curve. Deduce the equation of involute. **[05]**

(c) Find the curvature and torsion of the involute at son $\underline{x} = \underline{x}(s)$. **[06]**

7.(a) Define elliptic, parabolic and hyperbolic points on a surface. Obtain Dupinindicatrix and discus these points. **[05]**

(b) Define principle curvatures and lines of curvatures. Show that the parametric lines are lines of curvature if $F = 0,\ \ f = 0$. **[05]**

(c) Determine the first and second fundamental form of the surface $x = u,\ y = v,\ z = u^2 + v^3$ and discuss it at elliptic, parabolic and hyperbolic points. **[04]**

8.(a) What is called principal curvature? Write out, in determinant form, the quadratic equation in κ_n from which we may compute the mean curvature. **[07]**

(b) State and prove Euler's theorem on normal curvature. **[07]**

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Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination - 2021

Course No.: MTHT-3203; Course Title: Differential Geometry

Full marks: 70 **Time: 03 Hours**

[N.B.: There are EIGHT questions. Answer any FIVE of the questions. The symbols and terms have their usual representations. Figures are in the right-hand margin indicates full marks.]

1. (a) What is space curve? Find the arc length of a curve between the two points. [4]
   (b) Define tangent line to a curve. Find the unit tangent vector to a space curve. [4]
   (c) Find the equations of tangent line, normal line and binormal line at $u = 1$ of the space curve $\underline{x} = (u, u^2, u^3)$. [6]

2. (a) Derive the equations of the normal plane, rectifying plane and osculating plane of the space curve $\underline{x} = \underline{x}(u)$. [7]
   (b) Find the equations of the normal plane, rectifying plane and osculating plane of the space curve $\underline{x} = (u - sinu, 1 - cosu, u)$ at $u = \frac{\pi}{2}$. [7]

3. (a) Define curvature and torsion of the curve. Prove that the necessary and sufficient condition for a curve to be a straight line is that the curvature $\kappa = 0$ at all points of the curve. [7]
   (b) Find the radii of curvature and torsion at any point of the curve $x^2 + y^2 = a^2$, $x^2 - y^2 = az$. [7]

4. (a) State and prove the Serret-Frenet firmula. [7]
   (b) Show that at the point s on the space curve $\underline{x} = \underline{x}(s)$: [7]
   (i) $\underline{t} = \underline{x}'$ (ii)) $\underline{n} = \frac{\underline{x}''}{|\underline{x}''|}$ (iii) $\underline{b} = \frac{\underline{x}' \wedge \underline{x}''}{|\underline{x}''|}$ (iv) $\kappa = |\underline{x}''|$ and (v) $\tau = \frac{[\underline{x}' \ \underline{x}'' \ \underline{x}''']}{\underline{x}''.\underline{x}''}$.

5. (a) Define helix. Prove that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in a constant ratio. [4]
   (b) Show that $\frac{sin\theta}{sin\varphi}\frac{d\theta}{d\varphi} = -\frac{\kappa}{\tau}$, if the tangent and the binormal at a point of a curve make angles θ and φ respectively with a fixed direction. [3]
   (c) Show that $\underline{x} = (au, bu^2, cu^3)$is a helix if $3ac = \pm 2b^2$. [4]

6. (a) Define involute of a curve. Deduce the equation of involute. [4]
   (b) Show that the evolutes of a plane curve are helices. [4]
   (c) What is called Bertrand curves? Prove that the torsions of the two associate Bertrand curves have the same sign and their product is constant, i.e. $\tau\tau_1 = \frac{1}{a^2}sin^2\alpha$. [6]

7. (a) Define a surface. State and prove the properties of first fundamental form. [7]
   (b) Show that the family of curves given by $M\,du + N\,dv = 0$ are orthogonal to the family of curves given by $(EN - FM)du + (FN - GM)dv = 0$. Using this formula, find the orthogonal trajectories of the circles $r = a \cos\theta$. [7]

8. (a) Find the principal radii of curvature at origin of the hyperboloid $2z = 7x^2 + 6xy - y^2$. Hence show that the principal sections are $x = 3y, 3x = -y$. [7]
   (b) What is called mean curvature? Derive Rodrigues formula. [7]

Best of Luck

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JAGANNATH UNIVERSITY, DHAKA

Department of Mathematics

3rd Year 2nd Semester Final Examination 2017

Course Code: MTHT 3204 Course Title: Mechanics-II

Full Marks: 35 Time: 2 hours

- There are **FIVE** sets of questions in the following. Answer any **THREE** sets.
- Marks are equal for each set.

1. (a) Define central force. Find the velocities and accelerations of a particle along and perpendicular to the radius vector to it from a fixed origin O. 4

(b) A thin straight smooth tube made to revolve upwards with a constant velocity ω in a vertical plane about one extremity O. When it is in a horizontal position, a particle is at rest in it at a distance 'a' from the fixed end O. If ω be very small, show that it will reach O in a time $\left(\frac{6a}{g\omega}\right)^{\frac{1}{3}}$. 3

2. (a) A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane. Find the differential equation of its path. Also find the differential equation of the path in pedal form. 4

(b) Define apse. A particle moves with acceleration $\mu\left[3au^4 - 2(a^2 - b^2)u^5\right]$, $a > b$, and is projected from an apse at a distance $(a+b)$ with velocity $\frac{\sqrt{\mu}}{(a+b)}$. Show that its orbit is $r = a + b\cos\theta$. 3

3. (a) A particle moves on a smooth sphere under no forces except the pressure of the surface. Show that its path is given by the equation $\cot\theta = \cot\beta\cos\phi$, where θ and ϕ are its angular coordinates. 4

(b) A particle moves with a central acceleration $\frac{\mu}{(distance)^2}$. It is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is

$$\sin^{-1}\left[\frac{\mu}{VR\left(V - \frac{2\mu}{R}\right)^{\frac{1}{2}}}\right].$$ 3

4. (a) Define moment of inertia and product of inertia of a rigid body. Find the moment of inertia (M.I) of a right circular cylinder

(i) about its axis.

(ii) about a straight line through its centre of gravity perpendicular to its axis. 4

(b) Show that the momental ellipsoid at a point on the rim of a hemisphere is $2x^2 + 7(y^2 + z^2) - \frac{15}{4}xz = \text{Constant}$. 3

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5. State D' Alembert's principle. A, rod of length $2a$, revolves with a uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi vertical angle α. Show that $\omega^2 = \dfrac{3g}{4a\cos\alpha}$.

6. (a) Discuss Kepler's laws.

(b) A plank of mass M, is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man of mass M', starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\dfrac{2M'a}{(M+M')g\sin\alpha}}$

7. (a) If a rigid body with mass M rotates about a fixed axis, show that its equation of motion about the axis of rotation is $MK^2\dfrac{d^2\theta}{dt^2} = L$, where K is the radius of gyration about the axis, and L is the moment of external force about the axis.

(b) A solid homogenous cone, of length h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}\left(4+\tan^2\alpha\right)$.

8. (a) An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon. Determine the motion.

(b) A uniform sphere rolls down an inclined plane rough enough to prevent any sliding. Find the motion of the sphere.

*** The End ***

CS CamScanner

Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination-2018

Course Code: MTH-3204, Course Name: Mechanics-II

Full Marks: 35 **Time: 02 Hours**

*[N.B.: There are **EIGHT** questions. Answer any **FIVE** of the questions. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.]*

1. (a) A smooth straight thin tube resolves with angular velocity w in a vertical palne about an extremity which is fixed; if at zero time the tube be horizontal, and a particle inside it be at a distance a distance a from the fixed end, and be moving with velocity V along the tube, show that its distance at time t is $a\cosh(\omega t)+\left(\frac{V}{\omega}-\frac{g}{2\omega^2}\right)\sinh(\omega t)+\frac{g}{2\omega^2}\sin(\omega t)$. 4

   (b) Find the acceleration of a particle along and perpendicular to the radius vector to it from a fixed origin O. 3

2. (a) Define Apse. A particle moves with a central acceleration $\mu\left(r+\frac{a^4}{r^3}\right)$ being projected from an apse at a distance a with a velocity $2a\sqrt{\mu}$; show that it describes the curve $r^2\left[2+\cos\sqrt{3}\theta\right]=3a^2$. 3.5

   (b) A particle moves with a central acceleration $\mu(r^5-c^4r)$, being projected from an apse at c with a velocity $\sqrt{\frac{2\mu}{3}}c^3$; Show that its path is the curve $x^4+y^4=c^4$. 3.5

3. (a) A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane. Obtain the differential equation of its path. 3

   (b) A particle moving with a central acceleration $\frac{\mu}{(distance)^3}$; is projected from an apse at a distance a with velocity V; show that the path is $r\cosh\left[\frac{\sqrt{\mu-a^2v^2}}{av}\theta\right]=a$ or $r\cos\left[\frac{\sqrt{a^2v^2-\mu}}{av}\theta\right]=a$ according as V is ><the velocity from infinity. 4

4. (a) If the moments and products of inertia of a body about three perpendicular and concurrent axes known then find the moment of inertia about any other axis through their meeting point. 3

   (b) Find the moment of inertia of the area bounded by $r^2=a^2\cos 2\theta$ 4
   (i). About its axis
   (ii). About a line through the origin its plane and perpendicular to its axis.

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(iii). About a line through the origin and its plane.

5. (a) Define principle axes. Show that the principle axes at the node of a half loop of the lemniscate $r^2 = a^2 \cos 2\theta$ are inclined to the initial line at angle $\frac{1}{2}\tan^{-1}\frac{1}{2}$ and $\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{1}{2}$. 4

(b) Find the moment of inertia of the homogenous ellipsoid bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ about the normal at the point (x', y', z'). 3

6. (a) Deduce acceleration of a particle in terms of spherical polar coordinates. 3

(b) A particle moves on a smooth sphere under no forces except the pressure of the surface. Show that its path is given by the equation $\cot\theta = \cot\beta \cos\phi$, where θ and ϕ are its angular coordinates. 4

7. (a) A particle whose mass is m is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 3.5

(b) Find the equation of a motion of a rigid body rotating about a fixed axis. 3.5

8. (a) Define moment of inertia and product of inertia of a rigid body. Find the moment of inertia (M. I) of a truncated cone about its axis, the radii of its ends being a and b. 3

(b) At the vertex C of a triangle ABC, which is right angled at C, show that the principal axes are a perpendicular to the plane and two others inclined to the sides at an angle $\frac{1}{2}\tan^{-1}\frac{ab}{a^2-b^2}$. 4

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Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination 2019

Course No.: MTH-3204; Course Title: Mechanics II

Full marks: 35 **Time: 02 Hours**

[N.B.: There are EIGHT questions. Answer any FIVE of the questions. The symbols and terms have their usual representations. Figures are in the right-hand margin indicate full marks.]

1. (a) A particle moves in a straight line OA with an acceleration which is always directed towards O and varies inversely as the square of its distance from O; if initially the particle were at rest at A, find its motion and the periodic time. [3]

   (b) A particle falls from rest at a distance a from a centre of force, where the acceleration at distance x is $\mu x^{-\frac{5}{3}}$; when it reaches the centre then show that its velocity is infinite and that the time it has taken is $\frac{2a^{\frac{4}{3}}}{\sqrt{3\mu}}$. [4]

2. (a) A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane. Find the differential equation of its path. [3]

   (b) A particle moves with a central acceleration $\frac{\mu}{(\text{distance})^2}$; it is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is $\sin^{-1}\left[\frac{\mu}{VR\left(V^2-\frac{2\mu}{R}\right)^{1/2}}\right]$. [4]

3. (a) Define central force. Deduce the velocities and accelerations of a particle along and perpendicular to the radius vector to it from a fixed origin O. [3]

   (b) The velocities of a particle along and particular to the radius vector from a fixed origin are λr and $\mu\theta$. Find the path of the particle, and show that the acceleration along and perpendicular to the radius vector are $\lambda^2 r-\frac{\mu^2\theta^2}{r}$ and $\mu\theta\left[\lambda+\frac{\mu}{r}\right]$. [4]

4. (a) A particle moves with central acceleration $\mu\{3au^4-2(a^2-b^2)u^5\}$, $a>b$ and is projected from an apse at a distance $(a+b)$ with velocity $\frac{\sqrt{\mu}}{(a+b)}$. Show that its orbit is $r=a+b\cos\theta$. [5]

   (b) State Kepler's laws. [2]

5. (a) What do you mean by motions in two-dimension? A uniform sphere rolls down an inclined plane rough enough to prevent any sliding. Find the motion of the sphere. [3]

   (b) A solid homogeneous cone, of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4+\tan^2\alpha)$. [4]

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6. (a) Show that the momental ellipsoid at the centre of an ellipsoid is $(b^2+c^2)x^2+(c^2+a^2)y^2+(a^2+b^2)z^2 = \text{Constant}$. [3.5]

(b) Find law of force towards the pole under which the curve $r^n = a^n \cos n\theta$. [3.5]

7. (a) Find the moment of inertia of a hollow sphere about a diameter, its external and internal radii being a and b. [3]

(b) A wire is in the form of a semi-circle of a radius a; show that at the end of its diameter the principal axes in its plane are inclined to the diameter at angles $\frac{1}{2}\tan^{-1}\frac{4}{\pi}$ and $\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{4}{\pi}$. [4]

8. (a) State D' Alembert's principle. Deduce the general equation of a motion of a rigid body from D' Alembert's principle. [3]

(b) A uniform rod is OA of length 2α and mass m, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O and is inclined α at angle to OZ. Show that $\alpha = \cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$ or $\alpha = 0$. [4]

Best of Luck

CS CamScanner

Jagannath University, Dhaka

Department of Mathematics

3rd year 2nd Semester Final Examination-2020

Course No: MTH 3204; Course Title: Mechanics II

Full marks: 35 **Time: 2 Hours**

[Note: There are ***Eight*** *questions. Answer any* ***Five****. Figures in the right margin indicate full marks.]*

1. (a) Define central force. Find the velocities and accelerations of a particle along and perpendicular to the radius vector to it from a fixed origin O. [4]

   (b) Explain apses. A particle moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}, a > b$, and is projected from an apse at a distance $a+b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is $r = a + b\cos\theta$. [3]

2. (a) When a particle moves in a straight line OA with an acceleration proportion to its distance from a fixed point O in the straight line and is always directed away from O. If the particle starts from rest $x = a$ find the motion. [3]

   (b) A Particle, whose mass is m, is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin; if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. [4]

3. (a) What do you mean by Simple Harmonic Motion? [2]

   (b) A particle moves with an acceleration which is always towards O and equal to μ divide by the distance from a fixed point O. If it starts from at a distance a from O, show that it will arrive at O in time $a\sqrt{\frac{\pi}{2\mu}}$. [5]

4. (a) A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane. Obtain the differential equation of its path. [3.5]

   (b) Find the law of force towards the pole under where the following curve can be describe $r^n = A\cos n\theta + B\sin n\theta$. [3.5]

5. (a) What do you mean by moment of inertia and product of inertia? [3]

   (b) If the moments and products of inertia about any line or lines, through the center of inertia G of a body are known, then obtain the corresponding quantities for any parallel line or lines. [4]

6. (a) Show that the equation of the momental ellipsoid at the corner of a cube of side 2a referred to its principal axis is $2x^2 + 11(y^2 + z^2) = c$, where c is constant. [3]

   (b) Prove that at the vertex C of a triangle ABC, which is right angled at C, the principal axes are a perpendicular to the plane and two others inclined to the sides at an angle $\frac{1}{2}\tan^{-1}\left(\frac{ab}{a^2 - b^2}\right)$. [4]

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7. (a) Deduce the general equations of motion of a rigid body from D'Alemberts principle. [4]

(b) A rod of length 2a, revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremely of the rod so that it describes a cone of semi-vertical angle α, show that $\omega^2 = \frac{3g}{4a\cos\alpha}$. [3]

8. (a) A uniform rod, of mass m and length $2a$, can turn freely about one end which is fixed, it is started with angular velocity ω from the position in which it hangs vertically; find its angular velocity at any instant. [3]

(b) A weightless straight rod ABC of length 2a, is movable about the fixed end A and carries two particles at the same mass, one fastened to the middle point B and other to the end C of the rod. If the rod be held in the horizontal position and be then let go, show that its angular velocity when vertical is $\sqrt{\frac{6g}{5a}}$ and that $\frac{5a}{3}$ is the length of the simple equivalent pendulum. [4]

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Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination 2021

Course Code: MTH-3204 Course Title: Mechanics-II

Full Marks: 35 Time: 2 hours

- There are **Eight** sets of questions in the following. Answer any **Five** sets.
- Marks are equal for each set.

1\. (a) What is simple harmonic motion? [2]

(b) A particle falls from rest at a distance a from a centre of force, where the acceleration at distance x is $\mu x^{-\frac{5}{3}}$; when it reaches the centre then show that its velocity is infinite and that the time it has taken is $\frac{2a^{\frac{4}{3}}}{\sqrt{3\mu}}$. [5]

2\. (a) A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane. Find the differential equation of its path. [3]

(b) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$. Find the path of the particle, and show that the acceleration along and perpendicular to the radius vector are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left[\lambda + \frac{\mu}{r}\right]$. [4]

3\. (a) Deduce the velocities and accelerations of a particle along and perpendicular to the radius vector to it from a fixed origin O. [3]

(b) A particle moves with a central acceleration $\frac{\mu}{(dis\tan ce)^2}$. It is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is [4]

$$\sin^{-1}\left[\frac{\mu}{VR\left(V^2-\frac{2\mu}{R}\right)^{\frac{1}{2}}}\right].$$

4\.(a) Define moment of inertia and product of inertia of a rigid body. Find the moment of inertia (M.I) of a right circular cylinder [3]

(i) about its axis

(ii) a straight line through its centre of gravity perpendicular to its axis.

(b) Define an apse. A particle with a central acceleration $\mu(r^5 - c^4 r)$, being projected from an apse at c with a velocity $\sqrt{\frac{2\mu}{3}}c^3$; show that its path is the curve $x^4 + y^4 = c^4$. [4]

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5. (a) State D' Alembert's principle. Deduce the general equation of motion of a rigid body from D' Alembert's principle. [3]

(b) A uniform rod OA, of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that either $\alpha = 0$ or $\cos\alpha = \left(\frac{3g}{4a\omega^2}\right)$. [4]

6. (a) A uniform sphere rolls down an inclined plane rough enough to prevent any sliding. Find the motion of the sphere. Also find the equation of energy. [3]

(b) A wire is in the form of a semi-circle of radius a; show that at the end of its diameter the principal axes in its plane are inclined to the diameter at angles $\frac{1}{2}\tan^{-1}\frac{4}{\pi}$ and $\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{4}{\pi}$. [4]

7. (a) Discuss Kepler's laws. [2]

(b) Show that the momental ellipsoid at the centre of an ellipsoid is $(b^2+c^2)x^2+(c^2+a^2)y^2+(a^2+b^2)z^2=$Constant. [5]

8. A rod of length $2a$ is suspended by a string of length l attached to one end; if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that $\frac{3l}{a}=\frac{(4\tan\theta-3\tan\phi)\sin\phi}{(\tan\phi-\tan\theta)\sin\theta}$ [7]

*** The End ***

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Jagannath University, Dhaka

Department of Mathematics

3^{rd} Yr. 2^{nd} Semester Final Examination 2017

Course No.: MTHT-3205, Course title: Methods of Applied Mathematics-I

Full Marks: 70; Time: 3 hours

N.B: There are **EIGHT** questions. Answer any **FIVE**. Figures in the margin indicate full marks. Marks

1. (a) Define gamma and Beta function. [3]
   (b) Find the value of $\Gamma\left(\frac{1}{2}\right)$ and sketch the graph of Γn. [6]
   (c) Establish the relationship between Gamma function and Beta function. [5]

2. (a) Define error function and complementary error function with some real applications. [3]
   (b) Prove the following properties of error function:
   (i) $erf(x) + erfc(x) = 1$ [2]
   (ii) Error function is an odd function [2]
   (iii) $erf(x) = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3.1!} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + ...\right)$ [2]
   (c) Prove that $\int_0^{\pi/2} \sin^{2m-1}\theta d\theta = \frac{\sqrt{\pi}}{2}\cdot\frac{\Gamma m}{\Gamma(m+\frac{1}{2})}$ [5]

3. (a) Define Legendre function of 1^{st} kind and 2^{nd} kind. [2]
   (b) Show that $P_n(x)$ is the coefficient of t^n in the expansion of $(1-2xt+t^2)^{-\frac{1}{2}}$ in ascending power of t. [6]
   (c) Prove the following recurrence relations of the Legendre polynomials: [6]
   (i) $nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$. $n \geq 2$
   (ii) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$.

4. (a) What is Bessel's function of 1^{st} kind of order n? Prove that $J_{-n}(x) = (-1)^n J_n(x)$. [4]
   (b) Prove the following recurrence formula: [10]
   (i) $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$
   (ii) $J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right]$

5. (a) State Rodrigue's formula for Laguerre polynomials and hence find first two Laguerre polynomials. [3]
   (b) Write down Laguerre's differential equation and hence solve it. [6]
   (c) Prove that $\frac{e^{-\frac{xt}{1-t}}}{1-t} = \sum_{n=0}^{\infty} t^n L_n(x)$ [5]

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6.(a) Define Hermite polynomial. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$. [3]

(b) State and prove orthogonal property of Hermite polynomials. [5]

(c) Prove the following recurrence relations of the Hermite polynomials: [6]

(i) $H_n'(x) = 2nH_{n-1}(x), \quad n \geq 1$

(ii) $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x), \quad n \geq 1$

7. (a) Find the solution of the Legendre's differential equation [7]

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

(b) Prove the Rodrigue's formula for Legendre polynomial $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ [7]

8. (a) Define Hypergeometric function. If $|x| < 1$ and $\gamma > \beta > 0$, prove that [7]

$${}_2F_1(\alpha, \beta; \gamma; x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\, dt$$

(b) Prove that [7]

(i) $F(\alpha, \beta; \gamma; 1) = \frac{\Gamma(\gamma)\Gamma(\gamma-\beta-\alpha)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}$

(ii) $P_n(x) = {}_2F_1\left(-n,\ n+1;\ 1;\ \frac{1-x}{2}\right)$

*** *THE END* ***

Jagannath University, Dhaka

Department of Mathematics

3rd Year 2nd Semester Final Examination-2018

Course Code: MTH-3205, Course Name: Methods of Applied Mathematics - I

Full Marks: 70 **Time: 03 Hours**

[N.B.: There are EIGHT questions. Answer any FIVE of the questions. The symbols and terms have their usual representations. Figures in the right hand margin indicate full marks.]

1. (a) Define gamma function and hence find the value of $\Gamma\left(\frac{1}{2}\right)$. 4

   (b) Find the relation between gamma and beta functions: $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 4

   (c) State and prove Stirling's asymptotic formula for gamma function. 6

2. (a) Define error function and complimentary error function. 2

   (b) Prove the following properties of the error function: (i) $erf(-x) = -erf(x)$, (ii) $erf(0) = 0$ and (iii) $erf(\infty) = 1$. 6

   (c) Deduce the asymptotic expansion of the error function. 6

3. (a) Define Legendre function of 1st kind and 2nd kind. 2

   (b) Prove that $\int_{-1}^{1} P_m(x)P_n(x)\,dx = \begin{cases} 0, & if\ m \neq n \\ \frac{2}{2n+1}, & if\ m = n \end{cases}$ 6

   (c) Show that $P_n(x)$ is the coefficient of t^n in the expansion of $(1-2xt+t^2)^{-\frac{1}{2}}$ in ascending power of t. 6

4. (a) State and prove Rodrigue's formula for Legendre polynomials. 6

   (b) Find the values of $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$ using Rodrigue's formula. 2

   (c) Prove the following recurrence relations of the Legendre polynomials: 6

   (i) $nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x),\ n \geq 2$

   (ii) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$

5. (a) Define Bessel differential equation with some applications. 2

   (b) Prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\varphi - x\sin\varphi)d\varphi$ for all integral n. 6

   (c) Show that 6

   (i) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

   (ii) $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2 = \frac{2}{\pi x}$

6. (a) Write down Laguerre's differential equation and hence solve it. 8

   (b) Prove that $\int_0^{\infty} e^{-x}L_n(x)L_m(x)\,dx = \delta_{mn}$. 6

7. (a) What is meant by orthogonal function? Show that Hermite polynomials are orthogonal over $(-\infty, \infty)$ with respect to the weight function e^{-x^2} 8

   (b) Prove that $e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!}t^n$

8. (a) Define hypergeometric function in terms of Pochhammer symbol. Express the following functions as hypergeometric series: (i) $\tan^{-1}x$ (ii) $\ln\left(\frac{1+x}{1-x}\right)$. 7

   (b) If $|x| < 1$ and $\gamma > \beta > 0$, then prove that. 7

   $${}_2F_1(\alpha,\beta;\gamma;x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt.$$

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Jagannath University, Dhaka

Department of Mathematics

3rd year 2nd Semester Final Examination-2019

Course No: MTH 3205; Course Title: Methods of Applied Mathematics-I

Full marks: 70 **Time: 3 Hours**

[Note: There are ***Eight*** *questions. Answer any* ***Five****. Figures in the right margin indicate full marks.]*

1. (a) Write down the Euler's definition, Gauss's definition and Weiestrass's definition of gamma function. [3]
   (b) Deduce the Gauss's definition of gamma function from Euler's definition. [5]
   (c) State and prove the complement formula for gamma function. [6]

2. (a) Define error function and complementary error function. Prove that the following properties of error function [6]
   (i) $erf(\infty)=1$ (ii) $erf(x)+erf_c(x)=1$
   (b) Find the asymptotic expansion of error function for large $|z|$. [8]

3. (a) What is Legendre Polynomial? State and prove Rodrigue's formula for Legendre polynomials. Hence deduce first four Legendre polynomials using this formula. [1+6+2]
   (b) Show that Legendre polynomial is the coefficient of h^n in the expansion of $\left(1-2xh+h^2\right)^{-1/2}$. [5]

4. (a) Define Bessel differential equation. Write down the Bessel's functions of 1st and 2nd kind. Prove that $J_{-n}(x)=(-1)^n J_n(x)$ [7]
   (b) State and prove the orthogonality property of Bessel's function. [7]

5. (a) If $J_n(x)$ is the Bessel's functions of the first kind, then show that [4]
   (i) $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ (ii) $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$
   (b) Prove that $e^{x\left(t-\frac{1}{t}\right)/2}=\sum_{n=-\infty}^{\infty}J_n(x)t^n$, where n is a positive integer. [5]
   (c) Prove that $J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(n\varphi-x\sin\varphi)d\varphi$ for all integral n. [5]

6. (a) Write down the Hermite differential equation and hence solve it. [8]
   (b) Prove that $\int_{-\infty}^{\infty}e^{-x^2}H_n(x)H_m(x)dx=\begin{cases}0 & if\ m\neq n\\ 2^n\sqrt{\pi}n! & if\ m=n\end{cases}$ [6]

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7. (a) What is Laguerre's polynomial? [6]

Prove that $\int_0^\infty e^{-x} L_m(x) L_n(x) dx = \begin{cases} 0 & if\ m \neq n \\ 1 & if\ m = n \end{cases}$

(b) Prove that $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$ [4]

(c) Prove that (i) $L_n'(0) = -n$ (ii) $L_n''(0) = \frac{1}{2}n(n-1)$ [4]

8. (a) Define Pochhammer symbol and Hypergeometric function in terms of Pochhammer symbol. [4]

(b) Write down the integral formula for the hypergeometric function. State and prove the Gauss's theorem for Hypergeometric function. [10]

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Department of Mathematics

3rd year 2nd Semester Final Examination-2020

Course No: MTH 3205; Course Title: Methods of Applied Mathematics I

Full marks: 70 **Time: 3 Hours**

[Note: There are ***Eight*** *questions. Answer any* ***Five****. Figures in the right margin indicate full marks.]*

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1. (a) Define Gamma function. Hence find the value of $\Gamma(1/2)$ and sketch the graph of $\Gamma(n)$. [8]

   (b) Prove that, $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx = \frac{\Gamma(m)\Gamma(n)}{a^n(1+a)^m\Gamma(m+n)}$. [6]

2. (a) Write a short note on error function and its application. Prove that

   (i) $erf(\infty)=1$. (ii) $erf(x)=\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3.1!}+\frac{x^5}{5.2!}-\frac{x^7}{7.3!}+\dots\right]$. [3+4]

   (b) What do you mean by asymptotic expansion of a function? Find the asymptotic expansion of error function. [7]

3. (a) Define Legendre polynomial, $P_n(x)$ of order n. Hence find $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$. Also prove that, $\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$. [8]

   (b) Show that, $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$. [6]

4. (a) Define Bessel differential equation. Find the power series solution of the Bessel's differential equation and hence write down its general solution. [10]

   (b) Prove that $J_{-n}(x)=(-1)^n J_n(x)$ [4]

5. (a) Prove the followings:

   (i) $J_{3/2}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x}-\cos x\right)$. (ii) $J_n(x)=\frac{x}{2n}\left[J_{n-1}(x)+J_{n+1}(x)\right]$. [9]

   (iii) $J_n'(x)=\frac{1}{2}\left[J_{n-1}(x)-J_{n+1}(x)\right]$.

   (b) Find the generating function of Bessel's differential equation. [5]

6. (a) Prove that $\frac{1}{(1-t)}e^{\frac{-tx}{(1-t)}}=\sum_{n=0}^{\infty} t^n L_n(x)$. [4]

   (b) Prove the following recurrence relations:

   (i) $(n+1)L_{n+1}(x)=(2n+1-x)L_n(x)-nL_{n-1}(x)$. (ii) $xL_n'(x)=nL_n(x)-nL_{n-1}(x)$. [10]

7. (a) State and prove the Rodrigue's formula for Hermite Polynomial. [5]

   (b) Prove that $H_n(x)=2^n\exp\left(-\frac{1}{4}\frac{d^2}{dx^2}\right)x^n$. [3]

   (c) Prove that $\int_{-\infty}^{\infty} e^{-x^2}H_n(x)H_m(x)dx=\begin{cases}0 & if\ m\neq n\\ 2^n\sqrt{\pi}n! & if\ m=n\end{cases}$. [6]

8. (a) Define Hypergeometric function. If $|x|\le 1$ and $\gamma>\beta>0$, then show that,

   $${}_2F_1(\alpha,\beta;\gamma;x)=\frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}dt.$$ [8]

   (b) Prove that $P_n(\cos\theta)=\cos^n\theta\ {}_2F_1\left(-\frac{n}{2},\frac{n-1}{2},1,-\tan^2\theta\right)$. [6]

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Jagannath University, Dhaka

Department of Mathematics

B. Sc (Honors) Examination - 2021

Course Code: MTH- 3205

Cours Title: Methods of Applied Mathematics - I

Full Marks: 70 Time: 3 Hours

There are **eight** questions. Answer any **five** of the questions. Figures are in the right-hand margin indicates full marks.

1.	(a)	Write a short note on the origin of gamma function and also write down some applications of it.	[02]
	(b)	Write down the Euler's definition, Gauss's definition and Weiestrass's definition of gamma function. Show that these three definitions are equivalent.	[06]
	(c)	State and prove the duplication formula for gamma function.	[06]
2.	(a)	What is error function? Draw the graph of error function and hence write down importance of error function. Prove that $(i)\ erf\ (x) + erf_c\ (x) = 1.$ $(ii)\ erf\ (x) = \frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3.1!} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \dots\dots\dots\right]$	[4+4]
	(b)	What do you mean by asymptotic expansion of a function? Find the asymptotic expansion of error function.	[1+5]
3.	(a)	Define Legendre function of 1st kind and 2nd kind.	[02]
	(b)	Prove the following recurrence relations of the Legendre polynomials: $(i)\ nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x),\ \ n \geq 2$ $(ii)\ nP_n(x) = xP_n{}'(x) - P_{n-1}{}'(x)$	[06]
	(c)	Show that $P_n(x)$ is the coefficient of t^n in the expansion of $(1-2xt+t^2)^{-\frac{1}{2}}$ in ascending power of t.	[06]
4.	(a)	Define Bessel differential equation. Prove that $e^{\frac{x\left(t-\frac{1}{t}\right)}{2}} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$, where n is a positive integer.	[07]
	(b)	State and prove the orthogonality property of Bessel's function.	[07]
	(c)	Show that $(i)\ \frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ $(ii)\ J_{\frac{3}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\left(\frac{\sin x}{x} - \cos x\right)$	[05]
5.	(a)	What is Laguerre's polynomial? State and prove Rodrigue's formula for Laguerre polynomials. Hence deduce first four Laguerre's polynomials using this formula.	[06]

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	(b)	Prove that $\int_0^{\infty} e^{-x} L_m(x) L_n(x)\,dx = \begin{cases} 0 & if\ m \neq n \\ 1 & if\ m = n \end{cases}$	[05]
	(c)	Prove that (i) $L_n'(0) = -n$ (ii) $L_n''(0) = \frac{1}{2} n(n-1)$.	[03]
6.	(a)	Write down the Hermite differential equation and hence solve it.	[08]
	(b)	Write down the generating function of Hermite polynomial and hence prove it.	[06]
	(c)	Prove the following recurrence relations of the Hermite polynomials: (i) $H_n'(x) = 2n\, H_{n-1}(x)\,,\ n \geq 1$ (ii) $H_{n+1}(x) = 2x\, H_n(x) - 2n\, H_{n-1}(x),\ n \geq 1$	[05]
7.	(a)	Define Pochhammer symbol. Show that $\frac{d}{dx} F(\alpha,\beta;\gamma;x) = \frac{\alpha\beta}{\gamma} F(\alpha+1,\beta+1;\gamma+1;x)$ and deduce that (i) $\frac{d^n}{dx^n} F(\alpha,\beta;\gamma;x) = \frac{(\alpha)_n(\beta)_n}{(\gamma)_n} F(\alpha+n,\beta+n;\gamma+n;x)$ (ii) $\left[\frac{d^n}{dx^n} F(\alpha,\beta;\gamma;x)\right]_{x=0} = \frac{(\alpha)_n(\beta)_n}{(\gamma)_n}$	[08]
	(b)	Express the following functions as hypergeometric series: (i) e^x (ii) $\log(1-x)$ (iii) $\tan^{-1} x$	[06]
8.	(a)	What is hypergeometric function? If $\lvert x\rvert < 1$ and $\gamma > \beta > 0$, prove that ${}_2F_1(\alpha,\beta;\gamma;x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\,\Gamma(\gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt$	[07]
	(b)	Prove that (i) $F(\alpha,\beta;\gamma;1) = \frac{\Gamma(\gamma)\,\Gamma(\gamma-\beta-\alpha)}{\Gamma(\gamma-\alpha)\,\Gamma(\gamma-\beta)}$ (ii) $P_n(x) = {}_2F_1\left(-n, n+1; 1; \frac{1-x}{2}\right)$	[07]
		Good Luck	

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