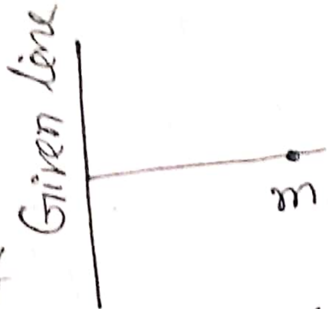
[illegible]

Moments of inertia and product of inertia

Rigid body: A body is said to be rigid if the distance between any two particles of the body always remains the same.

Moment of Inertia:

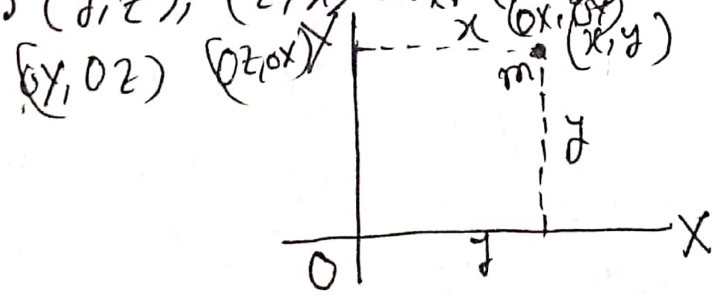


If r be the perpendicular distance from any given line of any element m of the mass of a body, then the quantity Σmr^2 is called the moment of inertia of the body about the given line. If this sum be equal to MK^2, where M is the total mass of the body, then K is called the radius of gyration [illegible] about the given line. Again, since the distance of the element from the axis of x is $\sqrt{y^2+z^2}$, the moment of inertia about the x-axis is $\Sigma m(y^2+z^2)$. Similarly for the other axes is $\Sigma m(z^2+x^2)_y$ and $\Sigma m(x^2+y^2)_z$.

Solu

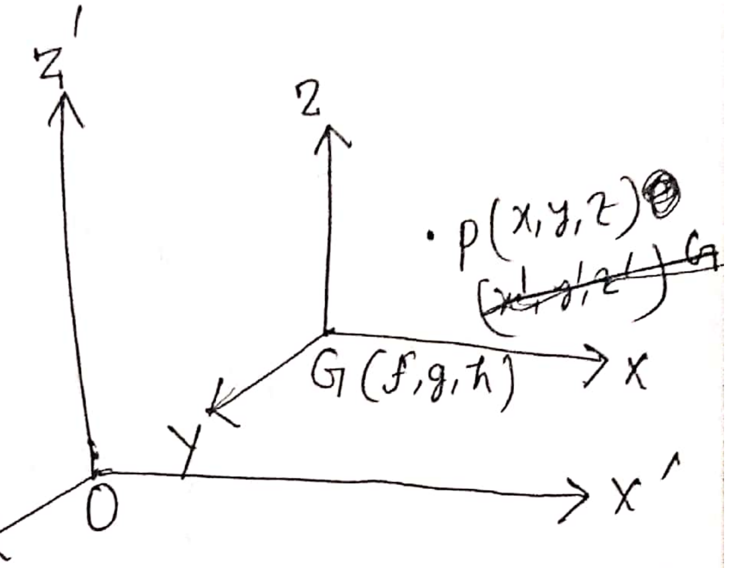
(জাড্য গুণফল)

Product of Inertia: If three mutually perpendicular axes OX, OY, OZ be taken; and if the coordinates of any element m of the system referred to these axes be x, y and z, then the quantities Σmyz, Σmzx and Σmxy are called the product of inertia with respect to the axes (y,z), (z,x) and (x,y) respectively.



Theorem 1: If the moments and products of inertia about any line or lines, through the centre of inertia G of a body are known, then obtain the corresponding quantities for any parallel line or lines [যদি কোন বস্তুর জড়তা কেন্দ্র G গামী কোন রেখা বা রেখা সমূহের প্রেক্ষিতে জড়তা ভ্রামক ও গুণফলের মান জানা থাকে, তবে সমান্তরাল যে কোন রেখা বা রেখা সমূহের ক্ষেত্রে অনুরূপ মান সমূহ বের কর]

Solution: Let



$p(x,y,z)$ be the position of any element m of the body referred to axes GX, GY, GZ and $p(x', y', z')$ be that referred to parallel axes OX', OY', OZ' with O as the origin. Let the coordinates of the centre of gravity G be (f, g, h) referred to axes OX', OY', OZ'. The coordinates of the centre of inertia are

[We know statics] $\left(\frac{\Sigma mx}{\Sigma m}, \frac{\Sigma my}{\Sigma m}, \frac{\Sigma mz}{\Sigma m}\right)$ referred to G

as the origin $(0,0,0)$. Then we write

$$\Sigma mx = 0 = \Sigma my = \Sigma mz, \quad \text{--- (i)}$$

$$x' = x+f, \quad y' = y+g, \quad z' = z+h,$$

Now M. I. of the body about OX'

$$= \Sigma m(y'^2 + z'^2) = \Sigma m[(y+g)^2 + (z+h)^2]$$

$$= \Sigma m(y^2 + 2gy + g^2 + z^2 + 2zh + h^2)$$

P.T.O.

$= \sum m(y^2+z^2) + \sum m(g^2+h^2) + 2g\sum my + 2h\sum mz$

$= \sum m(y^2+z^2) + M(g^2+h^2)$, [using (i)]

$=$ (M.I. of the body about Gx) + M.I. of a mass M placed at G about OX'

Also the product of inertia at OX', OY'

$= \sum mx'y' = \sum m(x+f)(y+g)$

$= \sum m(xy + xg + yf + fg)$

$= \sum mxy + f\sum my + g\sum mx + \sum mfg$

$= \sum mxy + Mfg$. [using (i)]

$=$ (Product of inertia Gx, Gy) + P.I. of a mass M placed at G about OX', OY').

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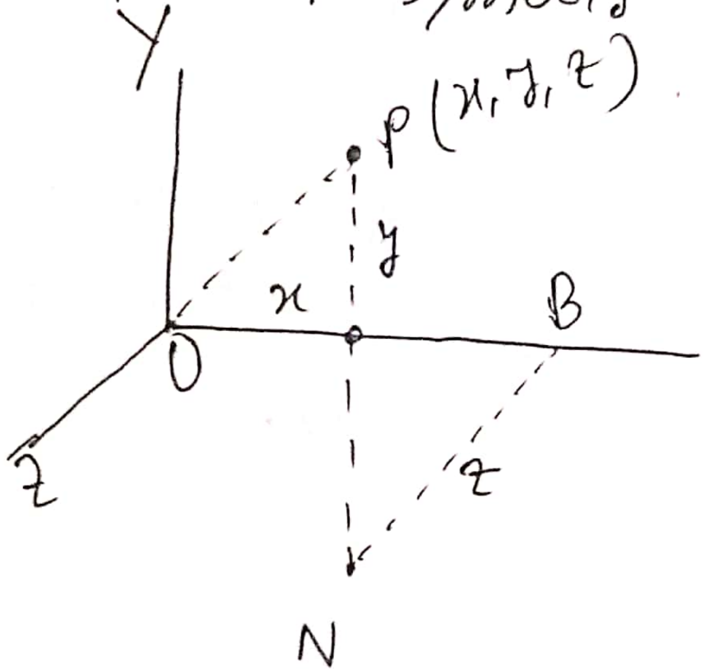
Some important symbols



Let a particle of mass m be placed at the point P (x, y, z) then the distance $OP = \sqrt{x^2+y^2+z^2}$

Distance of P from x-axis $= \sqrt{y^2+z^2}$

" " P " y- " $= \sqrt{z^2+x^2}$

" " P " z- " $= \sqrt{x^2+y^2}$

" " P " yz plane = x

" " P " zx " = y

" " P. " xy " = z.

"