Chapter 2: Kinematics of Linear Motion

Learning Outcomes (LO)

1. Define:
    a. instantaneous velocity, average velocity and uniform velocity; and
    b. instantaneous acceleration, average acceleration and uniform acceleration.
2. Derive and apply equations of motion with uniform acceleration
    $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$; $s = \frac{1}{2}(u + v)t$
3. Describe projectile motion launched at an angle, θ as well as special cases when θ=0°
4. Solve problems related to projectile motion.

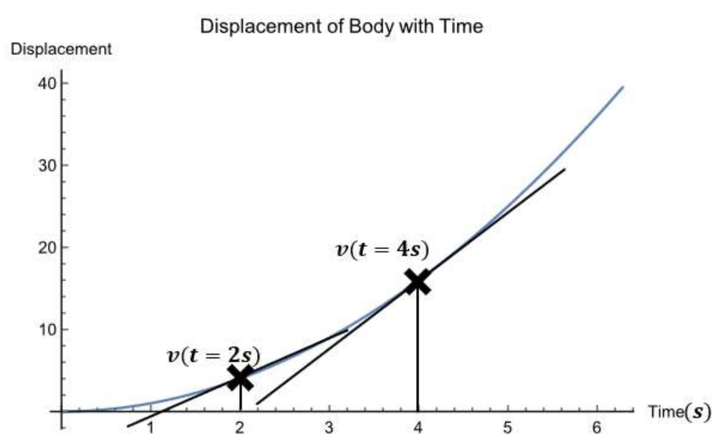
In this chapter, we talk about kinematics of linear motion. Dynamics is the study of motion of bodies under action of forces and their effects. One subbranch to the study of dynamics is kinematics. In the study of kinematics, we consider only the motion of the bodies without worrying too much about the forces that caused the bodies to move. We only worry about the geometry of the motion.

Instantaneous and Average Velocity (or acceleration)

Let us start with reminder of some ideas and terms that you have learnt in your SPM days. 3 mains terms – displacement (s), velocity (v) & acceleration (a). Displacement, denoted by x, simply refers to the change in position of a body. Velocity, v, refers to the rate of change of this change in position, i.e. $v = \frac{dx}{dt}$. Acceleration, a, is defined by the rate of change of velocity, which is the rate of change of the rate of change of position. That is $a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$.

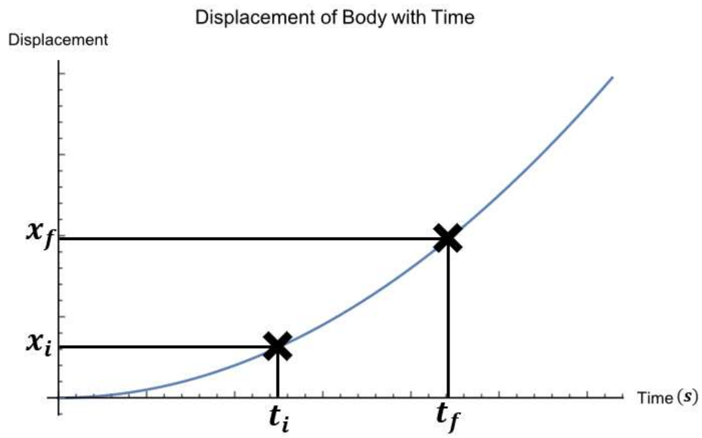
Once we have established that, we can further extend our ideas of velocity and acceleration by thinking about ***instantaneous*** velocity (or acceleration) and ***average*** velocity (or acceleration). By 'instantaneous', we mean 'at a particular instant in time'. When we combine it with velocity (or acceleration), what we mean is velocity (or acceleration) at a particular instant in time. On the other hand, when we say 'average', what we mean is 'over the course of a defined time span'. So, when we say 'average velocity', we usually would accompany it with 'between time t_a and t_b' or 'in 30 seconds', specifying a range of time.

Let us now have a graphical representation. Consider a body moving at constant velocity,



When we talk about instantaneous velocity, we are asking about a single point in time. From the displacement-time graph, the gradient represents the velocity of the body. As we can see from the graph, the instantaneous velocities when $t = 2s$ and $t = 4s$ are different. This is simply because the body is moving at a nonuniform velocity. If the instantaneous velocities are the same, then we call the motion is described as uniform velocity.

On the other, talking about ***average*** velocity, we simply define range of time, thus choosing two points in time rather than one. Then we take the difference in position and divide it by the difference in time to calculate the ***average*** velocity. That is to say, for the graph below,



We can calculate the average velocity as

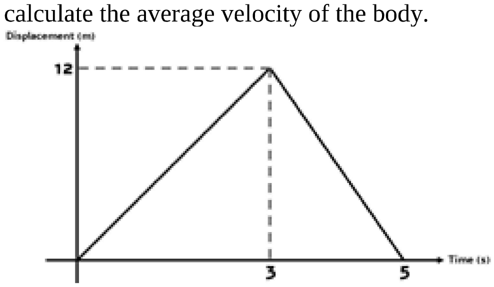
$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

We can take the same approach and understanding and apply it to acceleration, but with a velocity time graph rather than a displacement time graph.

<u>Sample Problem 2.1</u>
The motion of a body is described by the equation $v = 2t^2$, where v is in metres per second and t is in seconds. Calculate the instantaneous velocity of the body at $t = 3s$ and the average acceleration between $t = 2s$ and $t = 4s$.
Answer:

$$v_{instantaneous} = 2(3)^2 = 18ms^{-1}$$
$$v(t = 2s) = 2(2)^2 = 8ms^{-1}$$
$$v(t = 4s) = 2(4)^2 = 32ms^{-1}$$
$$a_{average} = \frac{\Delta v}{\Delta t} = \frac{v(t = 4s) - v(t = 2s)}{4 - 2} = \frac{32 - 8}{4 - 2}$$
$$a_{average} = 12ms^{-2}$$

<u>Sample Problem 2.2</u>
The motion of a body is shown in the graph shown in Figure 1. Calculate the displacement of the body and calculate the average velocity of the body.



Answer:
Displacement of the body can be calculated by recognising that the area under the velocity -time graph represents the displacement of the body. So all we need is to sum up all the areas under the graph.

$$s = \frac{1}{2}(12 \times 3) + \frac{1}{2}\left(12 \times (5 - 3)\right) = 30m$$

The average velocity can be calculated by simply dividing the displacement by the total time of motion.

$$v_{average} = \frac{30}{5} = 6ms^{-1}$$

Kinematic Equations

Now we want to look at kinematic equations, which are equations that relates variables that describes motion such as displacement, velocity and acceleration.

Derivation by calculus

We'd like to derive the equations from our understanding of linear motion and using calculus. We begin with the definition of acceleration

$$a = \frac{dv}{dt}$$

Assuming constant acceleration, we can rearrange then integrate both sides to yield

$$a\int_{t_{initial}}^{t_{final}} dt = \int_{v_{initial}}^{v_{final}} dv \Rightarrow a\left(t_{final} - t_{initial}\right) = v_{final} - v_{initial}$$

Adjusting such that $t_{initial} = 0$, $t_{final} = t$ and defining $v = v_{final}$, $v_{initial} = u$.

And then rearranging this equation yields

$$v = u + at$$

which is the same equation as the first equation found in LO2. Simply put, the final velocity of a body is initial velocity plus the product of acceleration and time difference.

We can take the same approach to find the third equation in LO2 using the first equation. We start with the definition of velocity and then rearranging it,

$$v = \frac{dx}{dt} \Rightarrow \int v\, dt = \int dx$$

Note that since velocity is not a constant, $v\, dt$ cannot be directly integrated. We therefore need an equation for velocity as a function of time (first equation).

$$\int u + a\, t\, dt = \int dx$$

Since u and a are constants, these integrals become

$$u\int_0^t dt + a\int_0^{\mathrm{t}} dt = \int_0^{\mathrm{x}} dx$$

Solving this integral gives

$$x = ut + \frac{1}{2}at^2.$$

For the second equation, we can start take advantage of calculus by starting with a time independent derivative,

$$\frac{dx}{dv} = \frac{dx}{dt}\frac{dt}{dv} = \frac{v}{a}$$

Rearranging this gives us the needed integral to solve

$$a\int_0^x dx = \int_u^v v\, dv \Rightarrow ax = \frac{1}{2}(v^2 - u^2)$$

Further rearrangement yields an equation

$$v^2 = u^2 + 2ax$$

matching with the third equation found in LO2.

Equation 4 of LO2 does not require any integration, rather we can obtain it using $s = ut + \frac{1}{2}at^2$ and $v = u + at$. This is left for the reader to do.

Geometric Derivation

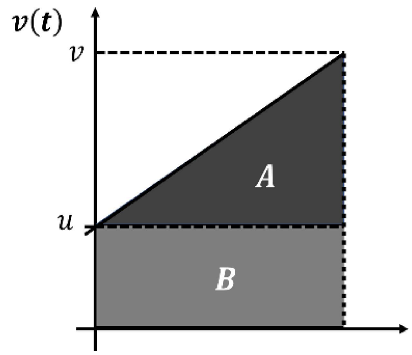
By definition,

$$a = \frac{v - u}{t}$$

Rearranging this gives

$$v = u + at$$

Consider an object that starts its motion with velocity u and maintains its constant acceleration a to a final velocity of v. We can describe its motion diagrammatically as below



Since the area under the graph represents displacement, all we need to do is to add up the area of A and B. If

$$\text{Area}_A = \frac{1}{2}(t)(v - u) = \frac{1}{2}(t)(at) = \frac{1}{2}at^2$$

$$\text{Area}_B = ut$$

then

$$s = ut + \frac{1}{2}at^2.$$

If, on the other hand, we consider

$$s = \frac{1}{2}(t)(v - u) + ut$$

Then we find that

$$s = \frac{1}{2}(v + u)t$$

For the equation of $v^2 = u^2 + 2as$, we can start the derivation by considering

$$v = u + at \Rightarrow t = \frac{v - u}{a}$$

And

$$s = \frac{1}{2}(u + v)t.$$

We can substitute time equation into the displacement to yield

$$s = \frac{1}{2}(u + v)\left(\frac{v - u}{a}\right) = \frac{v^2 - u^2}{2a} \Rightarrow v^2 = u^2 + 2as.$$

Sample Problem 2.3

A 2022 Honda Accord can travel down a $\frac{1}{4}$ mile track in $14.1s$ from rest. Calculate the acceleration (in SI units), assuming that its acceleration is constant.

Answer:

Values given ⇒ $s = \frac{1}{4}\, mile = 402.336km\,; t = 14.1s;\ u = 0ms^{-1}$

Choice of equation ⇒ $s = ut + \frac{1}{2}at^2$

$$402.336 = \frac{1}{2}a(14.1)^2$$

$$a = 4.04744ms^{-2}$$

Sample Problem 2.4

A car initially travels at $20ms^{-1}$. If the car undergoes constant acceleration of $1.2ms^{-2}$, determine the time the car need to reach double of its initial velocity.

Answer:

Values given ⇒ $\mathrm{u = 20ms^{-1};\ a = 1.2ms^{-2};\ v = 2u = 40ms^{-1}}$

Choice of equation $\Rightarrow v = u + at$

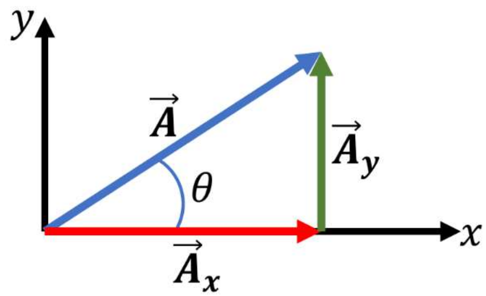
$$40 = 20 + 1.2t$$
$$\boldsymbol{t = \frac{50}{3} s}$$

<u>Projectile Motion (Motion in 2 Dimensions)</u>

When dealing with motion in two dimensions, the minimum that we need is the Pythagorean theorem as well as the definition of tangent. We consider a vector $\vec{A}$ defined by

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where $\vec{A}_x$ and $\vec{A}_y$ are component vectors of $\vec{A}$, each parallel to one of the axes in a rectangular coordinate system.



It then follows that the magnitude and direction of $\vec{A}$ can be related to its components by the Pythagorean theorem and the definition of tangent,

$$|\vec{A}| = \sqrt{|\vec{A}_x|^2 + |\vec{A}_y|^2}$$

$$tan\,\theta = \frac{|\vec{A}_y|}{|\vec{A}_x|}.$$

Conversely, we can work out the components of $\vec{A}$ from the magnitude of $\vec{A}$ and the angle θ,

$$|\vec{A}_x| = |\vec{A}|\,cos\,\theta$$
$$|\vec{A}_y| = |\vec{A}|\,sin\,\theta$$

One case study that we can do on two-dimensional motion is **projectile motion,** a motion that follows a parabolic path. The simplest case of projectile motion would be one where the air resistance and the rotation of the Earth is simply neglected, and that the motion is only affected by the Earth's gravity ($\vec{F}_{gravity} = m\vec{g}$). One important aspect of this case is that the horizontal (x-direction) and vertical (y-direction) motions are independent of each other. This means that the kinematics equation we have studied earlier can be dealt with separately for both x and y directions.

Keeping in mind that $u_x = u\,cos\,\theta$ and $u_y = u\,sin\,\theta$, we can then work out the 6 equations that describes the projectile motion:

x-direction (where $a_x = 0$)	y-direction (where $a_y = -g$)
$v_x = u_x$	$v_y = u_y - gt$
$s_x = u_x t$	$s_y = u_y t - \frac{1}{2}gt^2$
$v_x^2 = u_x^2$	$v_y^2 = u_y^2 - 2gs_y$

We can also work out the velocity of the projectile by keeping in mind that it merely follows from Pythagorean theorem

$$v^2 = v_x^2 + v_y^2.$$

If we substitute the equation for time-x-component, $t = \frac{s_x}{u_x}$, in the equation for displacement in y-direction, $s_y = u_y t - \frac{1}{2}gt^2$, what we get is the parabolic equation for the projectile motion path,

$$\left(\frac{u_y}{u_x}\right)s_x - \left(\frac{1}{2u_x^2}\right)s_x^2 - s_y = 0.$$

There are two more items that are of our interest:

1. If we were to look for the "**peak**" of the parabolic path, we can do so by applying $v_y = 0$ to the kinematics equations. This is simply because it is at this peak that $u_y = gt$ such that the velocity of the projectile is momentarily zero before the projectile falls back down towards the Earth.
2. Another item that would be of our interest is the **range** of the projectile motion. By range, what we are referring to is the point at which the projectile reaches back to ground or stop accelerating in the y-direction. This would differ from case to case, of course, and we shall demonstrate in the sample problems following this.

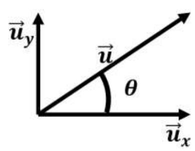
Sample Problem 2.5

An object is launched at a velocity of $21ms^{-1}$ in a direction making an angle of 30^o upward with the horizontal. Calculate

a. Initial velocity in x and y direction.
b. the location of the object at $t = 2s$.
c. the total horizontal range.
d. the velocity of the object just before it hits the ground.

Answer:

a.



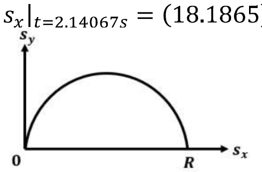
$$|\vec{u}_x| = |\vec{u}|\,cos\theta = 21\,cos\,30$$
$$|\vec{u}_x| = 18.1865ms^{-1}$$
$$|\vec{u}_y| = |\vec{u}|\,sin\theta\ = 21\,sin\,30$$
$$|\vec{u}_y| = 10.5ms^{-1}$$

b. $s_x = u_x t \Rightarrow s_x = (18.1865)(2) = 36.3730m$

$s_y = u_y t + \frac{1}{2}a_y t^2\ \Rightarrow s_y = (10.5)(2) + \frac{1}{2}(-9.81)(2^2) = 1.38m$

c. $s_y = u_y t + \frac{1}{2}a_y t^2\ \Rightarrow 0 = (10.5)(t) + \frac{1}{2}(-9.81)(t^2) \Rightarrow t\ =\ \{0{,}2.14067\}s$

$s_x|_{t=2.14067s} = (18.1865)(2.14067) = 38.9313m\ =\ R$



d. $v_x = u_x = 18.1865ms^{-1}$

$v_y = u_y + a_y t = 10.5 + (-9.81)(2.14067) = -10.5ms^{-1}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(18.1865)^2 + (-10.5)^2} = 21ms^{-1}$

$\theta = tan^{-1}\left(\frac{18.1865}{-10.5}\right) = -60^o$

Sample Problem 2.6

Compare the horizontal range of a ball thrown at velocity $35ms^{-1}$ if the angle of release is 15^o, 30^o, 45^o and 60^o.

Answer:

Condition for determining horizontal range $\Rightarrow s_y = 0$.

$s_y = u_y t + \frac{1}{2}a_y t^2 = (u\,cos\,\theta)t + \frac{1}{2}a_y t^2$

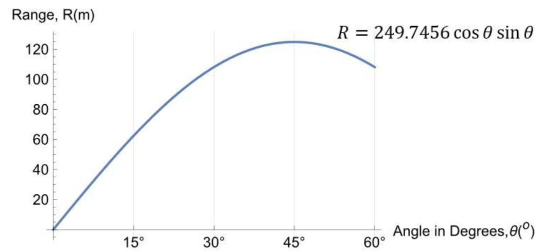
$0 = (35\,cos\,\theta)t - (4.905)t^2$

$t = \{0,\ 7.13558\,cos\,\theta\}s$

$R = s_x = u_x t = (u\,sin\,\theta)\mathrm{t}$

$R = (35\,sin\,\theta)(7.13558\cos\theta)$

$R = 249.7456\cos\theta\,sin\,\theta$



$\theta(^o)$	t (s)	$R(m)$
15^o	6.89244	62.4363
30^o	6.17959	108.143
45^o	5.04562	124.873
60^o	3.56779	108.143

The horizontal range peaks at 45^o.

Chapter 3: Dynamics of Linear Motion

Learning Outcomes

5. Define
   a. Momentum, $\vec{p} = m\vec{v}$
   b. Impulse, $J = F\Delta t$
6. Solve problem related to impulse and impulse-momentum theorem,
$$J = \Delta p = mv - mu$$
   **1D only*
7. Use *F-t* graph to determine impulse.
8. State:
   a. the principle of conservation of linear momentum.
   b. Newton's laws of motion.
9. Apply
   a. the principle of conservation of momentum in elastic and inelastic collisions in 2D collisions.
   b. Newton's laws of motion.
   **include static and dynamic equilibrium for Newton's first law motion*
10. Differentiate elastic and inelastic collisions. (remarks: similarities & differences)
11. Identify the forces acting on a body in different situations - Weight, *W;* Tension, *T;* Normal force, *N;* Friction, *f;* and External force (pull or push), *F*.
12. Sketch free body diagram.
13. Determine static and kinetic friction, $f_s \leq \mu_s N$, $f_k = \mu_k N$

In the previous chapters, we have looked at describing motion without the hassle of asking, "what force is causing the body to move?". In this chapter, we aim to expand our knowledge to a body's motion in that very aspect.

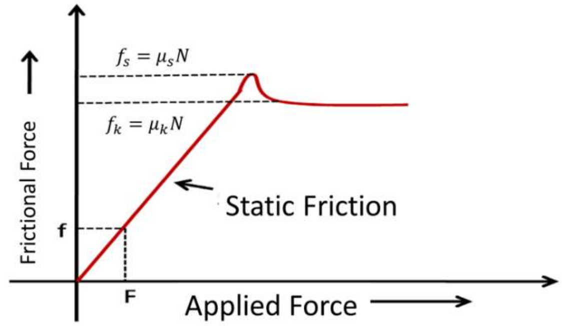
Types of Forces

$$\vec{F}_{net} = \vec{F}_1 - \vec{F}_2$$

We begin with asking the question, "what is force?", a simple answer would be to say force is a push and pull. Here, however, let us define force a bit further. Let us define force as **an agent for motion change**. Force is a vector quantity, that means **direction matters**. Two oppositely directed force acting on the same body work against each other. A body can experience multiple forces acting on it, however it is the net force, i.e., the resultant of all the forces acting on the body, that changes the motion of the body.

4 types of forces we'd consider in this chapter – gravitational (weight), tensional, normal and frictional. Their definitions and directions are as follows:

Forces	Definitions	Directions
Gravitational	Force exerted upon a body interacting with a gravitational field.	Towards the gravitational source.
Tensional	Force transmitted axially through a massless one-dimensional continuous element.	Along the one-dimensional continuous element.
Normal	Support force, perpendicular to the surface, exerted upon a body in contact with a stable object.	Perpendicular to the surface the body is in contact with.
Frictional	Force acting upon bodies that are in contact and moving relative to each other.	Against the direction of motion.

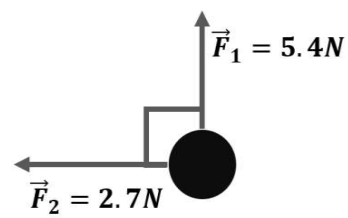


One particular type of force that maybe be of our interest is frictional force. This is because frictional force depends on the motion of the object. If the object is static, then it is subject to *static friction*. On the other hand, if the object is moving (relative to the surface it is in contact with) at some velocity, and therefore has some kinetic energy, then the object is subject to *kinetic friction*. Static friction is generally higher than kinetic friction because of the asperities (roughness) of the surfaces of the contacting bodies. This asperity enables the surfaces to interlock with each other, causing adhesion. This means that the force applied to the body must overcome this adhesion before the bodies can start moving relative to each other. This phenomenon can be observed by looking at the frictional force as a function of applied force graph:

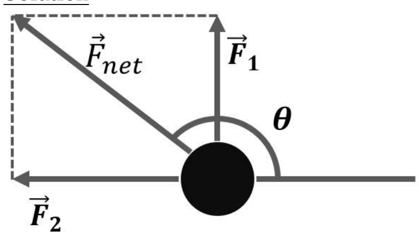
As the applied force is increased, so does the frictional force. This is true until a certain threshold is reached, after which the body will start to move. This threshold is exactly the unlocking of the asperities.

Sample Problem 3.1

Determine the resultant force exerted on the body shown in the diagram.

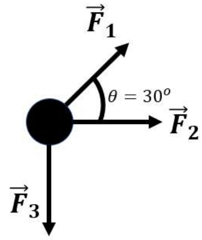


Solution



$$|\vec{F}_{net}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2}$$
$$|\vec{F}_{net}| = \sqrt{(5.4)^2 + (-2.7)^2}$$
$$|\vec{F}_{net}| = 6.03738N$$
$$\theta = 180^o - tan^{-1}\left(\frac{5.4}{2.7}\right)$$
$$\theta = 116.565^o$$

Sample Problem 3.2



Calculate the resultant force if $|\vec{F}_1| = 10N$, $|\vec{F}_2| = 12.5N$ and $|\vec{F}_3| = 17N$.

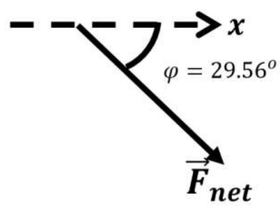
Solution

Force	x-components	y-components
F_1	$F_1 \, cos\, 30^o$ $= 10\, cos\, 30^o$ $\approx 8.66N$	$F_1 \, sin\, 30^o$ $= 10\, sin\, 30^o$ $\approx 5N$
F_2	$+12.5N$	0N
F_3	0N	$-17N$
Σ	21.16N	-12N

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{(21.16)^2 + (-12)^2}$$

$$F_{net} = 24.3258N$$

$$\varphi = tan^{-1}\left(\frac{12}{21.16}\right) = 29.56^o$$



Newton’s Law of Motion

There are laws of motion that a moving under force would generally follow. These laws were first introduced and came into its modern form via Newton’s *Principia*. In it, 3 laws of motions were found:

1. A body, when no external force is applied, will not undergo velocity change, i.e.
$$\vec{F}_{external} = 0 \Rightarrow \Delta\vec{v} = 0$$
2. When a force is acted upon it, a body will move in a manner such that rate of momentum change is equal to the said force, i.e.
$$\vec{F} = \frac{d\vec{p}}{dt}.$$
3. Forces exerted onto two interacting bodies will be equal in magnitude but opposite in direction. If $\vec{F}_{12}$ is force exerted onto body 1 by body 2, then
$$\vec{F}_{12} = -\vec{F}_{21}$$

These three laws form the foundation for what is known today as the *Newtonian Laws of Motion*.

Momentum

The first and third requires no further definitions of variable, however the second one, mentions an idea of ***momentum***. It seems useful to define this term at this point. What we mean by momentum at this point is the property of a moving body that rises from the product of the mass the body and its velocity, i.e.

$$\vec{p} = m\vec{v}.$$

If the velocity of the body changes (and therefore so does the momentum of the body in question), we quantify that change and call it ***impulse***:

$$\vec{J} = \Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t.$$

This also means that the area under the F-t graph represents impulse.

It is from Kinematics that we know a change in velocity means a non-zero acceleration. Knowing this as well as Newton's Second Law, we can say that force is present when acceleration is non-zero,

$$\vec{F} = m\vec{a}.$$

This statement, of course can be derived quite simply from Newton's Second Law of motion:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

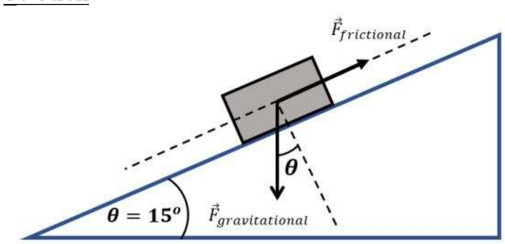
Here we can see that a change in mass may also produce force and that if mass change is zero, then what we have is the equation seen before,

$$\vec{F} = m\vec{a}.$$

Sample Problem 3.3

Suppose an object of 15 kg is placed on an incline plane of 15^o from the horizontal. Calculate the magnitude of frictional force that keeps the object from sliding down the incline plane.

Solution



$F_{net} = F_g \sin\theta - F_{frictional}$

Since the object stays static $\Rightarrow F_{net} = 0$

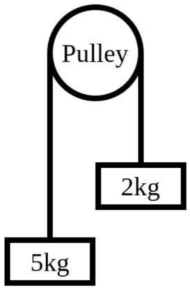
$F_{frictional} = F_g \sin\theta = mg \sin\theta$

$F_{frictional} = (15)(9.81)sin15^o$

$F_{frictional} = 38.0852N$

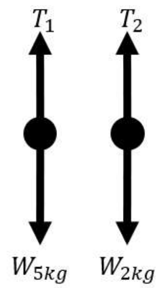
Sample Problem 3.4

Problem:
Based on the diagram, calculate the acceleration of both the objects.



Solution

FBD



Equation of motion for 5kg body:

$$F_{net} = W_{5kg} - T_1 = m_{5kg}a$$

Equation of motion for 5kg body:

$$F_{net} = T_2 - W_{2kg} = m_{2kg}a$$
$$T_1 = T_2 = T$$

Rearrange for a,

$$T = W_{5kg} - m_{5kg}a = W_{2kg} + m_{2kg}a$$
$$a = \frac{W_{5kg} - W_{2kg}}{m_{2kg} + m_{5kg}} = \frac{9.81(5-2)}{5+2}$$
$$a \approx 4.2ms^{-2}$$

The 2kg object will move **upwards** at $a \approx 4.2ms^{-2}$.
The 5kg object will move **downwards** at $a \approx 4.2ms^{-2}$.

Sample Problem 3.5

A 50g ball at $30ms^{-1}$ is travelling towards a wall. Upon striking the wall, the ball bounces back in the opposite direction at a speed of $10ms^{-1}$. Calculate the impulse.

Solution

$J = \Delta p = p_{final} - p_{initial}; p = mv$

$$J = m(v_{final} - v_{initial})$$
$$J = (0.05)((-10) - (+30))$$
$$J = -2kg\, ms^{-1}$$

Sample Problem 3.6

A footballer kicks a 300g ball from rest to a speed of $60ms^{-1}$ in a collision lasting $1.5ms$. Calculate the force generated by the footballer.

Solution

$$F\Delta t = \Delta p = p_{final} - p_{initial}$$

Since $p_{initial} = 0$, then

$$F(1.5 \times 10^{-3}) = (0.3)(60)$$
$$F = 12kN$$

One of the ways for bodies to interact is through collisions. When this happens, assuming this happens in an isolated system. The total momentum of the system doesn't change with the passage of time. The momenta of the participating bodies may change, but not the vector sum total momentum of the system. When we say that a quantity doesn't change, we say that the quantity is **conserved**. So, in this case, we say that the **total momentum is conserved**. Conservation of momentum is simply

$$\Delta(\Sigma p) = 0.$$

When we talk about collisions, we may consider two types of collision – elastic and inelastic collisions. Note that whilst momentum is conserved in **all** types of collisions, kinetic energy is not as it may be converted into other forms of energy (e.g., sound energy). It is this exact parameter from which we differentiate elastic from inelastic collisions. A perfectly elastic collision is defined by a collision in which both momentum and kinetic energy is conserved whilst a perfectly inelastic collision is a collision in which conservation of kinetic energy is not obeyed.

Sample Problem 3.7 (Conservation of momentum)

Ball A of mass 30g, travels at $3ms^{-1}$ collides head on with Ball B of 50g at rest. Calculate the velocity of Ball B after the collision if Ball A has the final velocity of $1.2ms^{-1}$.

Solution

$$\Delta\Sigma p = 0 \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$
$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$(0.03)(3) + 0 = (0.03)(1.2) + (0.05)v_B$$
$x = 1.08ms^{-1}$ in the same direction as Ball A.

Sample Problem 3.8 (Conservation of momentum in 2 Dimensions)

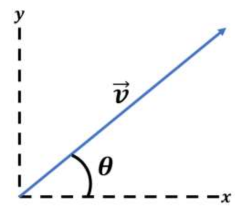
Ball A of mass 3kg, travels at $3ms^{-1}$ in the positive x direction collides with Ball B of 5kg travelling at $2ms^{-1}$ in the positive y direction. If the balls stick together after the collision, determine their velocity.

Solution

Need to apply conservation of momentum in x and y direction separately.

	X	Y
Momentum before	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_B = 0$ $\Sigma p = m_A u_A$	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_A = 0$ $\Sigma p = m_B u_B$
Momentum after	$\Sigma p = (m_A + m_B) v \cos\theta$	$\Sigma p = (m_A + m_B) v \sin\theta$
Final velocity	$m_A u_A = (m_A + m_B) v \cos\theta$ $v \cos\theta = \frac{m_A u_A}{m_A + m_B}$ ---(1)	$m_B u_B = (m_A + m_B) v \sin\theta$ $v \sin\theta = \frac{m_B u_B}{m_A + m_B}$ ---(2)

$$\frac{(2)}{(1)}: \tan\theta = \left(\frac{m_B u_B}{m_A + m_B}\right)\left(\frac{m_A + m_B}{m_A u_A}\right) = \frac{m_B u_B}{m_A u_A} = \frac{(5)(2)}{(3)(3)}$$
$$\theta = 48.0128^o$$
$$v \cos\theta = \frac{m_A u_A}{m_A + m_B} \Rightarrow v \cos(48.0128^o) = \frac{(3)(3)}{3(5)}$$
$$v = 0.897ms^{-1}$$



Sample Problem 3.9 (Perfectly Elastic Collision)

A ball, travelling at $3ms^{-1}$, collides head on with another ball of the same mass, travelling $2ms^{-1}$ in the opposite direction. Determine their velocities post-collision?

Solution

Let us assume that ball 1, travelling at $3ms^{-1}$, initially travels in the positive direction such that
$$u_1 = +3ms^{-1};\ u_2 = -2ms^{-1}$$
Since the balls are of the same mass,
$$m_1 = m_2 = m$$
Assuming the collision is elastic, the system would obey both the conservation of momentum and the conservation of kinetic energy.
$$\Delta(\Sigma \mathrm{p}) = 0 \Rightarrow \Sigma \mathrm{p}_{\mathrm{initial}} = \Sigma \mathrm{p}_{\mathrm{final}}$$
$$m(u_1 + u_2) = m(v_1 + v_2) \Rightarrow 3 + (-2) = v_1 + v_2$$
$$\Delta(\Sigma K) = 0 \Rightarrow \Sigma K_{initial} = \Sigma K_{final}$$
$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$
$$u_1^2 + u_2^2 = v_1^2 + v_2^2 \ \Rightarrow 3^2 + (-2^2) = v_1^2 + v_2^2$$
2 possible set of answers

Option 1: $v_1 = 3ms^{-1}$; $v_2 = -2ms^{-1}$, (which is just the initial case)

Option 2: $v_1 = -2ms^{-1}$; $v_2 = 3ms^{-1}$ (a more sensible answer)

Sample Problem 3.10 (Perfectly Inelastic collision)

A ball of mass 0.5kg, travelling in the +x direction at $2ms^{-1}$, collides with another ball of mass 0.2kg travelling in the opposite direction at $1.5ms^{-1}$. After the collision, the balls stick together and travels at the same speed. Determine the final velocity and its direction. Compare the kinetic energies before and after the collision.

Solution

Applying conservation of momentum,
$$\Delta(\Sigma p) \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$
$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v \Rightarrow (0.5)(2) + (0.2)(-1.5) = (0.5 + 0.2)v$$

$v = 1ms^{-1}$ in the positive x direction

To compare the kinetic energies, we can take their difference.

$$\Delta K = K_{final} - K_{initial}$$

$$\Delta K = \frac{v^2}{2}(m_1 + m_2) - \frac{1}{2}(m_1 u_1^2 + m_2 u_2^2)$$

$$\Delta K = \frac{1^2}{2}(0.5 + 0.2) - \frac{1}{2}\left((0.5)(2)^2 + (0.2)(-1.5)^2\right) = 0.35 - 1.225$$

$$\Delta K = -0.875J$$

This means 0.875J of kinetic energy has been converted into energies of other forms