Chapter 1: Physical Quantities & Measurements

Learning Outcomes

1. Define dimension, scalar and vector quantities.
2. Determine:
   (a) the dimensions of derived quantities.
   (b) resultant of vectors. (remarks: limit to three vectors only).
3. Verify the homogeneity of equations using dimensional analysis.
4. Resolve vector into two perpendicular components (*x* and *y* axes).

Dimensions

Dimensions refer to the physical nature of a quantity. Regardless of the unit used, the physical nature of a quantity remains the same. For example, a distance, measured in the unit of metres, or feet, is still a measurement of length. This measurement, therefore has the dimensions of length, most commonly represented by **L.** In this instance, the equation $[d] = L$ is simply states that "The dimension of d is length (**L**)". The following table shows some selected physical quantities and its dimensions:

Base Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Length	L	metre (m)	L
Mass	M	kilogram (kg)	M
Time	T	second (s)	T
Electric Current	I	ampere (A)	I
Temperature	T	Kelvin (K)	Θ
Amount of substance	n	mole (mol)	N
Luminosity	L	candela (cd)	J
Derived Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Velocity	$\vec{v}$	ms^{-1}	LT^{-1}
Acceleration	$\vec{a}$	ms^{-2}	LT^{-2}
Momentum	$\vec{p}$	Ns	MLT^{-1}
Angular acceleration	α	$rads^{-1}$	T^{-2}
Electric Charge	Q	Coulomb (A s)	TI
Energy	E	Joule ($J = kgm^2s^{-2}$)	ML^2T^{-2}

Once you understand what dimensions are and how to work with them, you can apply it to **verify the homogeneity of equations.** The word 'homogeneity' refers to 'of the same kind'. Let us consider the equation $s = ut + \frac{1}{2}at^2$ where *s* is displacement of a body, *t* is time taken for the displacement of the body, *u* is the initial velocity of the body and *a* is the acceleration of the body. To 'verify homogeneity', we can compare the dimensions the terms on the left-hand side and the right-hand side of the equation. That is to say, *s* must have the same dimensions as *ut* and $\frac{1}{2}at^2$. *s* has the dimension of L, so does *ut* as well as $\frac{1}{2}at^2$.

Sample Problem 1.1:
Identify the dimensions for power, P, defined by $P = \frac{E}{t}$ where E is energy and has dimensions of ML^2T^{-2} and t has dimension of time, T.

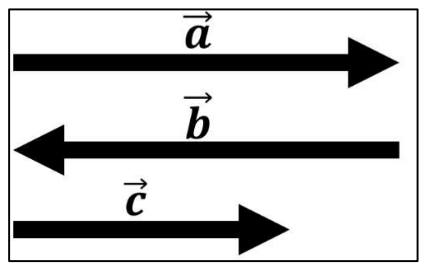
Solution:

$$[Power] = \left[\frac{E}{t}\right]$$
$$[Power] = \frac{ML^2T^{-2}}{T}$$
$$[Power] = ML^2T^{-3}$$

The dimensions for power are then ML^2T^{-3}.

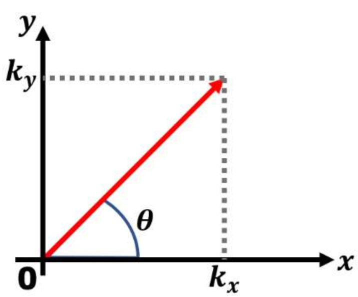
Scalars and Vectors

A scalar quantity is a quantity that is fully described by its magnitude. On the other hand, a vector quantity can only be fully described by both its magnitude **and** direction. When you talk about 5kg of rice, that statement is sufficient to describe the mass of the rice, this is where you can see that mass is a scalar quantity. If the rice is falling towards the Earth at a velocity of $2ms^{-1}$, 2 things matter here – how fast the rice is falling **and** the direction in which it is falling. Here you can see that velocity is a vector quantity.



A vector quantity is generally represented by a line segment with an arrowhead. The length of the line segment indicates its magnitude whereas its arrow head tells us the direction of the vector quantity. For example, say vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by the following arrows,

If we were to compare $\vec{b}$ and $\vec{c}$ to $\vec{a}$, we'd say that vector $\vec{b}$ has the same magnitude as $\vec{a}$ but is in the opposite direction, this would tell us that $\vec{b} = -\vec{a}$. Vector $\vec{c}$, on the other hand, is in the same direction as $\vec{a}$. But its magnitude is smaller that $\vec{a}$. The magnitude of vector $\vec{k}$ is denoted by $|\vec{k}|$. We can then relate vector $\vec{a}$ to $\vec{c}$ by the relation $|\vec{a}| > |\vec{c}|$.



Another method to represent vectors is to list the values of its elements in a sufficient number of difference directions, depending on the dimension of the vector. Consider a vector in a 2-dimensional Cartesian coordinate system, a vector $\vec{k}$ can then be represented by $\vec{k} = k_x\hat{\imath} + k_y\hat{\jmath}$ or $\vec{k} =< k_x, k_y >$, defining $\hat{\imath}$ and $\hat{\jmath}$ as unit vectors in the x and y directions respectively. From this notation, one can easily calculate the magnitude (length) of the 2-vector using Pythagoras' Theorem which gives

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2}.$$

Vector additions (or subtractions) can then be done by adding (or subtracting) corresponding components. That is to say, if we have vectors $\vec{a}$ and $\vec{b}$ defined by $\vec{a} =< a_x, a_y >$; $\vec{b} =< b_x, b_y >$, then the addition will yield

$$\vec{a} + \vec{b} =< a_x + b_x, a_y + b_y >.$$

The implication of this definition of vector addition are the following rules:

1. Commutativity of vectors $\Rightarrow \vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. Associativity of vectors $\Rightarrow (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + (-\vec{a}) = 0$

Resolution of vector $\vec{k}$ is then simply

$$k_x = |\vec{k}|cos(\theta);\ k_y = |\vec{k}|sin(\theta).$$

Multiplication of a vector

3 cases to consider when talking about multiplication of a vector:

1. The vector is multiplied by a scalar, then

$$k\vec{a} =< ka_x, ka_y >.$$

2. The **dot product** (also known as scalar or inner product) of two vectors, then

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = |\vec{a}||\vec{b}|\ cos(\theta_{ab}).$$

Note that in the dot product, the operation results in a scalar quantity.

3. The **cross product** (also known as the vector product) of two vectors, then

$$\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\hat{n} = |\vec{a}||\vec{b}|sin(\theta)\hat{n}$$

Note that in the cross product, the operation results in a vector quantity perpendicular to both the x and y axis.

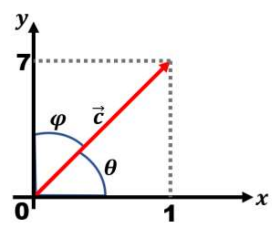
Sample Problem 1.2:
Calculate the magnitude and direction of vector $\vec{c}$ if it is defined by $\vec{c} = \vec{a} + \vec{b}$ where $\vec{a} = [2,3]$ and $\vec{b} = [-1,4]$.

Solution:
Magnitude:

$$\vec{c} =< 2 + (-1), 3 + 4 \geq< 1,7 >$$
$$|\vec{c}| = \sqrt{7^2 + 1^2} \approx 7.071$$

Direction:

$$\theta = tan^{-1}\left(\frac{7}{1}\right) \approx 81.87^o$$
$$\varphi = tan^{-1}\left(\frac{1}{7}\right) \approx 8.13^o$$



$\vec{c}$ has a magnitude (length) of 7.071 and is in the direction of 81.87^o from the x-axis or 8.13^o from the y-axis.

Unit Conversion

Unit conversions are so easy that we tend to overlook the importance of practicing it. Here's a simple reminder on how to do it. Say you are given that $1\ in\ =\ 2.54cm$ and you are asked to calculate 8cm in inches, here's how to do it:

$$1in\ =\ 2.54cm \leftarrow \text{divide both side by 2.54}$$
$$\frac{1}{2.54}in = \frac{2.54}{2.51}cm = 1cm \leftarrow \text{now multiply it by 8}$$
$$8cm = \frac{8}{2.54}in \approx 3.1496in$$

And that's how you do unit conversion.

In Physics, it is quite often that we are expected to work with values in form of scientific notation. For example, rather than writing down the speed of light as $300000000ms^{-1}$, we'd express this value as $3 \times 10^8 ms^{-1}$. One issue that may arise from working with scientific notation when using a calculator is the redundancy in typing out "10^x". To help with this, I would suggest that we take advantage of the **rules for exponents**. The example below demonstrates such application.

Sample Problem 1.3:

Evaluate c given that $c = \frac{a^2 b}{d}$ and $a = 3 \times 10^{-6}$, $b = 4 \times 10^3$ and $d = 12 \times 10^3$.

Solution

$$c = \frac{a^2 b}{d} = \frac{(3 \times 10^{-6})^2 (4 \times 10^3)}{(12 \times 10^3)}$$

Rather than evaluating this monstrosity as our input for the calculator, we can instead separate the coefficient from the base and its exponent to evaluate them separately.

$$c = \frac{a^2 b}{d} = \frac{(3)^2(4)}{(12)}\left[\frac{(10^{-6})^2(10^3)}{10^3}\right]$$

In this form, the evaluation can be done **easily** even without a calculator.

$$c = \frac{(3)^2(4)}{(12)}\left[\frac{(10^{-6})^2(10^3)}{10^3}\right] = \frac{36}{12}[10^{-6-6+3-3}]$$

$$c = 3 \times 10^{-12}$$

On Significant Figures

When we talk about the number of significant figures, we are talking about the number of digits whose values are known with certainty. This gives us information about the degree of accuracy of a reading in a measurement. In general, we should practice performing rounding off when the conditions call for it. This is to avoid false reporting. What we mean by false reporting is to give the illusion that our experiments are more sensitive than it actually it. For example, it would be very unlikely that our metre ruler to give reading in the micro scale.

Number	Number of significant figures		Number	Number of significant figures
2.32	3		2600	2
2.320	4		2602	4

When we do calculations, there are some rules (based on the operations) we should be aware of when stating the significant figures of the end value:

1. Multiplication / Divisions – number of significant figures in the result is the same as the least precise measurement in the least precise measurement used in the calculation.

   Example:

$$\frac{2.5(3.15)}{2.315} = 3.4$$

2. Addition / Subtraction – The result has the same number of decimal places as the least precise measurement used in the calculation.

   Example:

$$91.1 + 11.45 - 12.365 = 90.2$$

3. Logarithm / antilogarithm – Keep as many significant figures to the right of the decimal point as the are significant in the original number.

   Example:

$$ln(4.00) = 1.39;\ e^{0.0245} = 1.03$$