

Matriculation Physics (SP015)

*Study Notes
&
Exercises*

Shafiq R

Table of Contents

CHAPTER 1: PHYSICAL QUANTITIES & MEASUREMENTS	1
DIMENSIONS	1
SCALARS AND VECTORS	2
MULTIPLICATION OF A VECTOR	3
UNIT CONVERSION	3
ON SIGNIFICANT FIGURES	4
EXERCISES	5
CHAPTER 2: KINEMATICS OF LINEAR MOTION	6
INSTANTANEOUS AND AVERAGE VELOCITY (OR ACCELERATION)	6
KINEMATIC EQUATIONS	8
DERIVATION BY CALCULUS	8
GEOMETRIC DERIVATION	9
PROJECTILE MOTION (MOTION IN 2 DIMENSIONS)	10
EXERCISES	13
CHAPTER 3: DYNAMICS OF LINEAR MOTION	14
TYPES OF FORCES	14
NEWTON'S LAW OF MOTION	16
MOMENTUM	17
EXERCISES	21
CHAPTER 4: WORK, ENERGY AND POWER	23
WORK	23
WORK ENERGY THEOREM	25
ENERGIES	25
POWER	26
EXERCISES	27
CHAPTER 5: CIRCULAR MOTION	28
UNIFORM CIRCULAR MOTION	28

THE CASE FOR CONICAL PENDULUM	30
THE CASE FOR VERTICAL PENDULUM	31
EXERCISES	33
CHAPTER 6: ROTATION OF RIGID BODY	34
REVISIONS & DEFINITIONS	34
ANALOGY TO LINEAR KINEMATICS	35
ROTATIONAL DYNAMICS	35
EXERCISES	41
CHAPTER 7: OSCILLATIONS & WAVES	42
PART 1: SIMPLE HARMONIC MOTION	43
KINEMATICS OF SHM	43
ENERGY IN SHM	44
CASE STUDY	45
PART 1 EXERCISES (INCOMPLETE)	47
PART 2: WAVES	48
PROGRESSIVE WAVE	48
PRINCIPLE OF WAVE SUPERPOSITION	49
STANDING WAVE	49
TRAVELLING WAVE SOLUTION FOR STRING	49
SOUND WAVES	50
PART 2 EXERCISES (INCOMPLETE)	53
CHAPTER 8: PHYSICS OF MATTER	54
PART 1: MATERIAL CHANGES DUE TO FORCE	55
STRESS	55
STRAIN	55
YOUNG'S MODULUS	56
GRAPH	56
PART 1 EXERCISES (INCOMPLETE)	59
PART 2: MATERIAL CHANGES DUE TO HEAT	60
HEAT CONDUCTION	60
HEAT EXPANSION	61
PART 2 EXERCISES (INCOMPLETE)	63

MOLECULAR KINETIC THEORY	65
KINETIC THEORY OF GASES	65
KINETIC & INTERNAL ENERGY	67
THERMODYNAMICS	68
FIRST LAW OF THERMODYNAMICS	68
THERMODYNAMICAL PROCESSES	68

Chapter 1: Physical Quantities & Measurements

Learning Outcomes

1. Define dimension, scalar and vector quantities.
2. Determine:
 - (a) the dimensions of derived quantities.
 - (b) resultant of vectors. (remarks: limit to three vectors only).
3. Verify the homogeneity of equations using dimensional analysis.
4. Resolve vector into two perpendicular components (x and y axes).

Dimensions

Dimensions refer to the physical nature of a quantity. Regardless of the unit used, the physical nature of a quantity remains the same. For example, a distance, measured in the unit of metres, or feet, is still a measurement of length. This measurement, therefore has the dimensions of length, most commonly represented by **L**. In this instance, the equation $[d] = L$ simply states that “The dimension of d is length (**L**)”. The following table shows some selected physical quantities and its dimensions:

Base Quantities			
Quantity	Symbol	S.I. Base Unit	Dimensions
Length	L	metre (m)	L
Mass	M	kilogram (kg)	M
Time	T	second (s)	T
Electric Current	I	ampere (A)	I
Temperature	T	Kelvin (K)	Θ
Amount of substance	n	mole (mol)	N
Luminosity	L	candela (cd)	J
Derived Quantities			
Quantity	Symbol	S.I. Base Unit	Dimensions
Velocity	\vec{v}	ms^{-1}	LT^{-1}
Acceleration	\vec{a}	ms^{-2}	LT^{-2}
Momentum	\vec{p}	Ns	MLT^{-1}
Angular acceleration	α	$rad s^{-1}$	T^{-2}
Electric Charge	Q	Coulomb (A s)	TI
Energy	E	Joule ($J = kgm^2s^{-2}$)	ML^2T^{-2}

Once you understand what dimensions are and how to work with them, you can apply it to **verify the homogeneity of equations**. The word ‘homogeneity’ refers to ‘of the same kind’. Let us consider the equation $s = ut + \frac{1}{2}at^2$ where s is displacement of a body, t is time taken for the displacement of the body, u is the initial velocity of the body and a is the acceleration of the body. To ‘verify homogeneity’, we can compare the dimensions the terms on the left-hand side and the right-hand side of the equation. That is to say, s must have the same dimensions as ut and $\frac{1}{2}at^2$. s has the dimension of L, so does ut as well as $\frac{1}{2}at^2$.

Sample Problem 1.1:

Identify the dimensions for power, P , defined by $P = \frac{E}{t}$ where E is energy and has dimensions of ML^2T^{-2} and t has dimension of time, T .

Solution:

$$[Power] = \left[\frac{E}{t} \right]$$

$$[Power] = \frac{ML^2T^{-2}}{T}$$

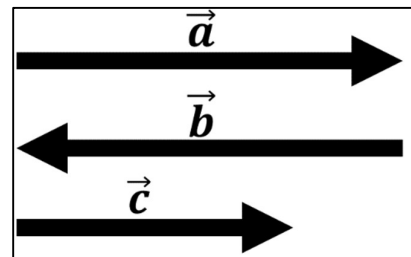
$$[Power] = ML^2T^{-3}$$

The dimensions for power are then ML^2T^{-3} .

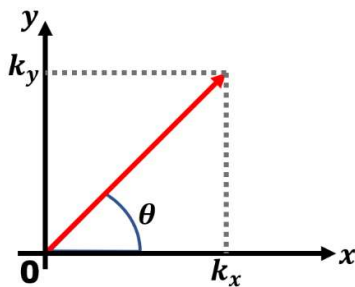
Scalars and Vectors

A scalar quantity is a quantity that is fully described by its magnitude. On the other hand, a vector quantity can only be fully described by both its magnitude **and** direction. When you talk about 5kg of rice, that statement is sufficient to describe the mass of the rice, this is where you can see that mass is a scalar quantity. If the rice is falling towards the Earth at a velocity of $2ms^{-1}$, 2 things matter here – how fast the rice is falling **and** the direction in which it is falling. Here you can see that velocity is a vector quantity.

A vector quantity is generally represented by a line segment with an arrowhead. The length of the line segment indicates its magnitude whereas its arrow head tells us the direction of the vector quantity. For example, say vectors \vec{a} , \vec{b} and \vec{c} are represented by the following arrows,



If we were to compare \vec{b} and \vec{c} to \vec{a} , we'd say that vector \vec{b} has the same magnitude as \vec{a} but is in the opposite direction, this would tell us that $\vec{b} = -\vec{a}$. Vector \vec{c} , on the other hand, is in the same direction as \vec{a} . But its magnitude is smaller than \vec{a} . The magnitude of vector \vec{k} is denoted by $|\vec{k}|$. We can then relate vector \vec{a} to \vec{c} by the relation $|\vec{a}| > |\vec{c}|$.



Another method to represent vectors is to list the values of its elements in a sufficient number of different directions, depending on the dimension of the vector. Consider a vector in a 2-dimensional Cartesian coordinate system, a vector \vec{k} can then be represented by $\vec{k} = k_x\hat{i} + k_y\hat{j}$ or $\vec{k} = \langle k_x, k_y \rangle$, defining \hat{i} and \hat{j} as unit vectors in the x and y directions respectively. From this notation, one can easily calculate the magnitude (length) of the 2-vector using Pythagoras' Theorem which gives

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2}.$$

Vector additions (or subtractions) can then be done by adding (or subtracting) corresponding components. That is to say, if we have vectors \vec{a} and \vec{b} defined by $\vec{a} = \langle a_x, a_y \rangle$; $\vec{b} = \langle b_x, b_y \rangle$, then the addition will yield

$$\vec{a} + \vec{b} = \langle a_x + b_x, a_y + b_y \rangle.$$

The implication of this definition of vector addition are the following rules:

1. Commutativity of vectors $\Rightarrow \vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. Associativity of vectors $\Rightarrow (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + (-\vec{a}) = 0$

Resolution of vector \vec{k} is then simply

$$k_x = |\vec{k}| \cos(\theta); k_y = |\vec{k}| \sin(\theta).$$

Multiplication of a vector

3 cases to consider when talking about multiplication of a vector:

1. The vector is multiplied by a scalar, then

$$k\vec{a} = \langle ka_x, ka_y \rangle.$$

2. The **dot product** (also known as scalar or inner product) of two vectors, then

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = |\vec{a}| |\vec{b}| \cos(\theta_{ab}).$$

Note that in the dot product, the operation results in a scalar quantity.

3. The **cross product** (also known as the vector product) of two vectors, then

$$\vec{a} \times \vec{b} = (a_x b_y - a_y b_x) \hat{n} = |\vec{a}| |\vec{b}| \sin(\theta) \hat{n}$$

Note that in the cross product, the operation results in a vector quantity perpendicular to both the x and y axis.

Sample Problem 1.2:

Calculate the magnitude and direction of vector \vec{c} if it is defined by $\vec{c} = \vec{a} + \vec{b}$ where $\vec{a} = [2, 3]$ and $\vec{b} = [-1, 4]$.

Solution:

Magnitude:

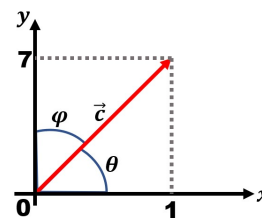
$$\vec{c} = \langle 2 + (-1), 3 + 4 \rangle = \langle 1, 7 \rangle$$

$$|\vec{c}| = \sqrt{1^2 + 7^2} \approx 7.071$$

Direction:

$$\theta = \tan^{-1}\left(\frac{7}{1}\right) \approx 81.87^\circ$$

$$\varphi = \tan^{-1}\left(\frac{1}{7}\right) \approx 8.13^\circ$$



\vec{c} has a magnitude (length) of 7.071 and is in the direction of 81.87° from the x-axis or 8.13° from the y-axis.

Unit Conversion

Unit conversions are so easy that we tend to overlook the importance of practicing it. Here's a simple reminder on how to do it. Say you are given that $1 \text{ in} = 2.54 \text{ cm}$ and you are asked to calculate 8cm in inches, here's how to do it:

$$1 \text{ in} = 2.54 \text{ cm} \leftarrow \text{divide both side by 2.54}$$

$$\frac{1}{2.54} \text{ in} = \frac{2.54}{2.54} \text{ cm} = 1 \text{ cm} \leftarrow \text{now multiply it by 8}$$

$$8 \text{ cm} = \frac{8}{2.54} \text{ in} \approx 3.1496 \text{ in}$$

And that's how you do unit conversion.

In Physics, it is quite often that we are expected to work with values in form of scientific notation. For example, rather than writing down the speed of light as $300000000 \text{ ms}^{-1}$, we'd express this value as $3 \times 10^8 \text{ ms}^{-1}$. One issue that may arise from working with scientific notation when using a calculator is the redundancy in typing out " 10^x ". To help with this, I would suggest that we take advantage of the **rules for exponents**. The example below demonstrates such application.

Sample Problem 1.3:

Evaluate c given that $c = \frac{a^2b}{d}$ and $a = 3 \times 10^{-6}$, $b = 4 \times 10^3$ and $d = 12 \times 10^3$.

Solution

$$c = \frac{a^2b}{d} = \frac{(3 \times 10^{-6})^2(4 \times 10^3)}{(12 \times 10^3)}$$

Rather than evaluating this monstrosity as our input for the calculator, we can instead separate the coefficient from the base and its exponent to evaluate them separately.

$$c = \frac{a^2b}{d} = \frac{(3)^2(4)}{(12)} \left[\frac{(10^{-6})^2(10^3)}{10^3} \right]$$

In this form, the evaluation can be done **easily** even without a calculator.

$$c = \frac{(3)^2(4)}{(12)} \left[\frac{(10^{-6})^2(10^3)}{10^3} \right] = \frac{36}{12} [10^{-6-6+3-3}]$$

$$c = 3 \times 10^{-12}$$

On Significant Figures

When we talk about the number of significant figures, we are talking about the number of digits whose values are known with certainty. This gives us information about the degree of accuracy of a reading in a measurement. In general, we should practice performing rounding off when the conditions call for it. This is to avoid false reporting. What we mean by false reporting is to give the illusion that our experiments are more sensitive than it actually is. For example, it would be very unlikely that our metre ruler to give reading in the micro scale.

Number	Number of significant figures	Number	Number of significant figures
2.32	3	2600	2
2.320	4	2602	4

When we do calculations, there are some rules (based on the operations) we should be aware of when stating the significant figures of the end value:

1. Multiplication / Divisions – number of significant figures in the result is the same as the least precise measurement in the least precise measurement used in the calculation.

Example:

$$\frac{2.5(3.15)}{2.315} = 3.4$$

2. Addition / Subtraction – The result has the same number of decimal places as the least precise measurement used in the calculation.

Example:

$$91.1 + 11.45 - 12.365 = 90.2$$

3. Logarithm / antilogarithm – Keep as many significant figures to the right of the decimal point as the are significant in the original number.

Example:

$$\ln(4.00) = 1.39; e^{0.0245} = 1.03$$

Exercises

- Two vectors lie on the x-y plane. Vector \vec{a} is 5 units long and points 25° above the x-axis. Vector \vec{b} is 8 units long and points upward 35° above the x-axis. What is the resultant of the two vectors?
- Show that the equation $v^2 = v_o^2 + 2ax$ is dimensionally correct. In this expression, a is acceleration, v and v_o are velocities and x is the distance.
- Derive the dimensions of k if $k = \frac{ab}{d}$ where $[a] = M^2T^2$, $[b] = M^{-1}T^{-1}$ and $[d] = MT^2$.
- A car is travelling at 35 miles per hour, what is its speed in metres per second if $1 \text{ mile} = 1.61 \text{ km}$.
- Express each of the following in grams
 - 3 lb if $1 \text{ g} = 2.2 \times 10^{-3} \text{ lb}$
 - 12.2 oz if $1 \text{ g} = 0.035 \text{ oz}$
- How many square centimetres are in a square inch if $1 \text{ cm} = 0.3937 \text{ inch}$.
- The length and the width of a rectangle is $(11 \pm 0.05) \text{ cm}$ and $(21 \pm 0.05) \text{ cm}$ respectively. Find the area (and its uncertainty) of the rectangle.