Matriculation Physics (SP025)

Short notes

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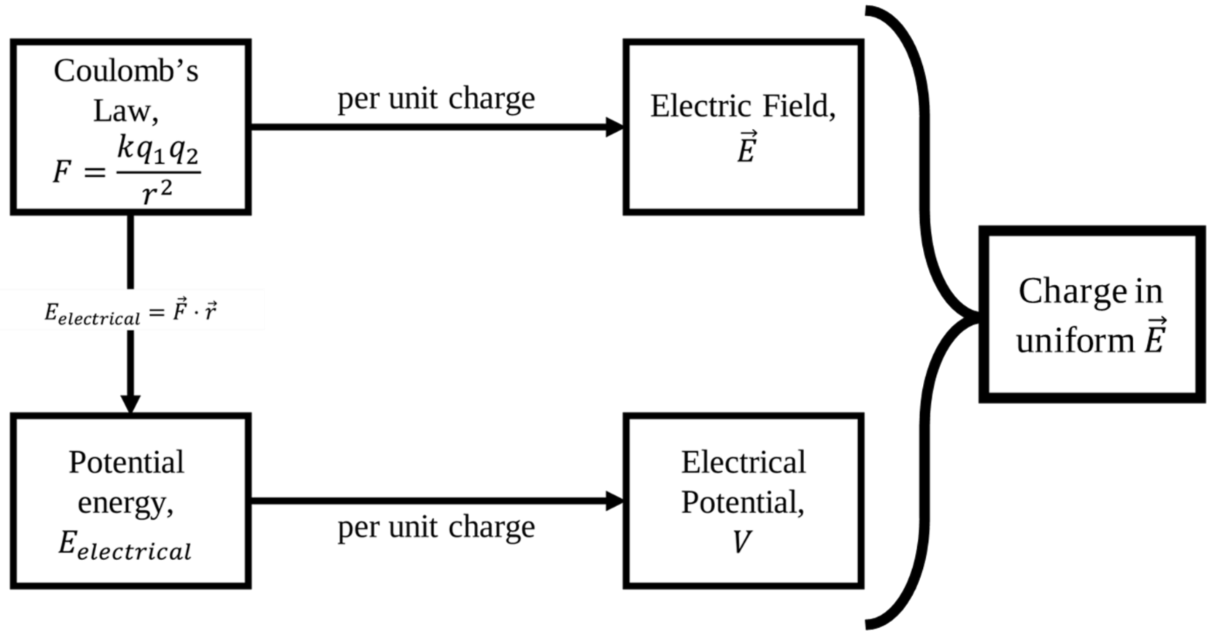
Chapter 1: Electrostatics



Coulomb's Law

Let it be known that Coulomb's Law allows us to measure forces between charged particles, this force is known as **Coulomb Force**. Mathematically, Coulomb's Law is

$$F_{Coulomb} = \frac{kq_1q_2}{r_{12}^2}$$

where q_i are the charges of interacting particles, r_{12} is the distance between the particles and k is the electrostatic constant. The electrostatic constant is

$$k = \frac{1}{4\pi\epsilon} = 8.98 \times 10^9\ kg\ m^3\ s^{-4}\ A^{-2}$$

On the note of direction of the Coulomb force,

Condition	Direction
$F_{Coulomb} < 0$	Towards each other
$F_{Coulomb} > 0$	Away from each other

For more than 2 particles, the Coulomb Force on particle j becomes

$$F_{Coulomb} = kq_j \sum_i \frac{q_i}{r_{ij}^2}$$

Electric Field

The electric field at a point in space $E(r)$, is defined as the electric force acting on a positive test charge placed at that point $F(r)$, divided by the test charge, q_{test}.

$$E(r) = \frac{F(r)}{q_{test}}$$

Rearranging this equation yields,

$$F(r) = q_{test}E(r)$$

which tells us that particle of charge q_{test} placed in a region of electric field $E(r)$ will experience a force of $F(r)$.
If the source of electric field has a charge of q_{source}, then the electric field at point r, $E(r)$ is

$$E(r) = \frac{kq_{source}}{r^2}$$

As in the case for Coulomb Force, for multiple, the electric field from multiple sources is simply additive,

$$E(r) = k\sum_i \frac{q_i}{r_i^2}$$

Electric Potential

Electric potential is the amount of word done to bring a test charge q_{test} from an infinite distance to a point at distance r from the source charged particle of charge q_{source}. This is found to be

$$V = \frac{W_{\infty\to r}}{q_{test}} = \frac{kq_{source}}{r}$$

Potential difference between positions $x = A$ and $x = B$ is then

$$V_{AB} = V_A - V_B = \frac{W_{\infty\to A}}{q_{test}} - \frac{W_{\infty\to B}}{q_{test}} = \frac{W_{\mathrm{A}\to B}}{q_{test}}$$

Electric potential energy is the energy a test charge would have positioned r distance away from a source,

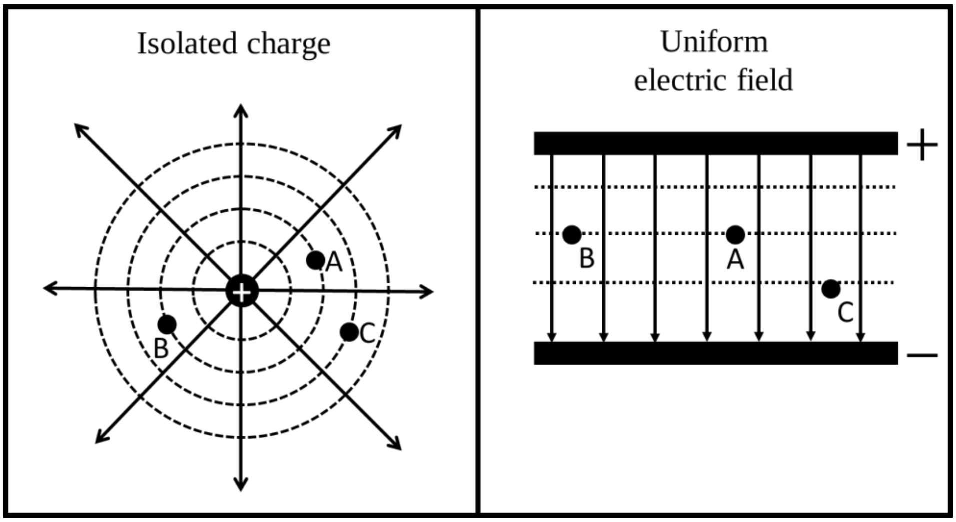
$$U = q_{test}V$$

For multiple sources, the potential and electrical potential energy at point r is simply,

$$V_{total} = k\sum_i \frac{q_i}{r_i};\ U_{total} = kq_j\sum_i \frac{q_i}{r_{ij}}$$

Equipotential lines and surfaces are graphical representation on which a particle on the line or surfaces is at the same potential.

- This means no work is done by the electric field when a charged particles are moved from on point of the equipotential line (or surface) to another point on the same line (or surface).
- Equipotential lines are always perpendicular to the electric field at all points.
- Examples:



In both examples,

$$V_A = V_B \neq V_C$$

Charge in Uniform Electric Field

For a uniform electric field produced by parallel plates of potential difference V, electric field strength is simply

$$E = \frac{V}{d}$$

where d is the distance between the parallel plates.

The following case studies involves a charged particle in a uniform electric field:

Case 1: Stationary charge

A stationary charged particle of charge q and mass m, placed in a uniform electric field E will experience force only from the electric field and therefore will move towards plate of its opposite charge (i.e. positive charged particle will move towards the negatively charged plate and vice versa).

Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

Case 2: Charge moving parallel to the field

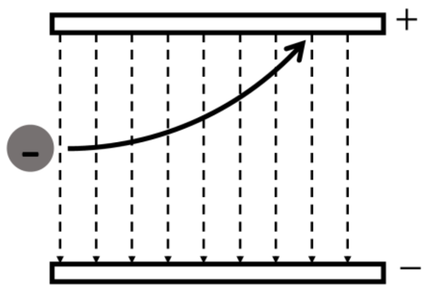
A charged particle of charge q and mass m, entering a uniform electric field E in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. It will either decelerate (if its velocity is in the opposite direction of its acceleration) or accelerate.

Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

Case 3: Charge moving perpendicularly to the field

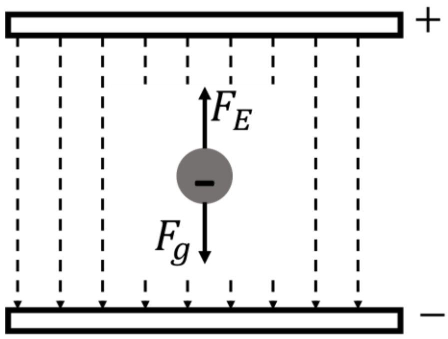
A charged particle of charge q and mass m, entering a uniform electric field E in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. Because of its initial velocity direction, it will follow a parabolic path, moving towards the plate of its opposite charge.



Case 4: Charge in dynamic equilibrium

In the case of dynamic equilibrium, the attractive Coulomb Force between the charged particle and the plate of opposite charge cancels out the weight of the charged particle,

$$F_{Coulomb} = W_{particle} \Rightarrow qE = mg$$



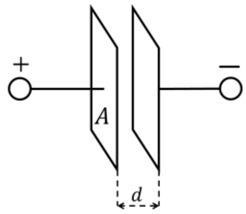
Chapter 2: Capacitors and Dielectrics

Parallel & Series

Capacitors are essentially batteries. Their ability to store charge is quantified by ***capacitance.*** Capacitance *C,* is the amount of charge *q* stored in one plate of a capacitor per unit potential difference between the plates, *V*,

$$C = \frac{Q_{single\ plate}}{V}$$

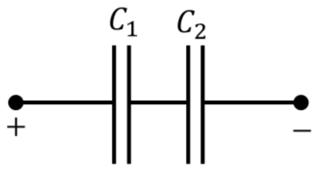
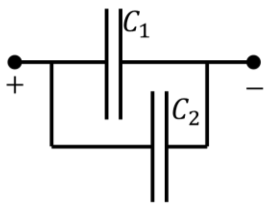
As a function of its geometry, capacitance of a parallel plate capacitor is



$$C = \frac{\varepsilon A}{d}$$

where ε is the permittivity of the space between the plates, A is the area of each plate and d is the distance between the parallel plates.

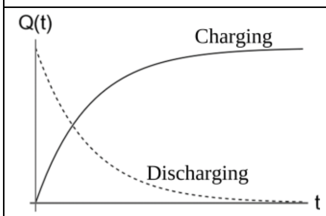
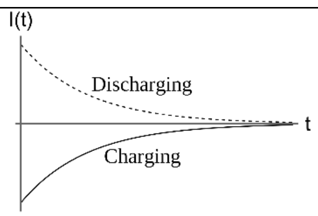
Multiple capacitors can be arranged either in parallel or series or combinations of them, and their effective capacitance can be calculated depending on their arrangement:

Arrangement	Effective Capacitance
Series 	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$ $C_{eff} = \left(\sum_{i=1}^{n} \frac{1}{C_i}\right)^{-1}$
Parallel 	$C_{eff} = C_1 + C_2 + \cdots + C_n$ $C_{eff} = \sum_{i=1}^{n} C_i$

The energy stored in a capacitor is then $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$.

Charging & Discharging Capacitors

Capacitors stores charges, now the question is how fast to charge it? Consider a simple circuit consisting of a power supply, a resistor of resistance *R* and a capacitor of capacitance *C*. Accumulation of charge with time for charging and discharging are as follows:

$Q - t$ graph	$I - t$ graph
 Discharging: $Q(t) = Q_o e^{-\frac{t}{RC}}$ Charging: $Q(t) = Q_o\left(1 - e^{-\frac{t}{RC}}\right)$	 Discharging: $I(t) = -\frac{dQ}{dt} = -\frac{Q_o}{RC}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$ Charging: $I(t) = \frac{Q_o}{RC}e^{-\frac{t}{RC}} = -I_o e^{-\frac{t}{RC}}$

Time constant, τ is defined as the time for the exponential term to drop to e^{-1} for discharging, or, for the charge to increase to $1 - e^{-1}$ for charging process, and is calculate by multiplying the R and C,

$$\tau = RC \text{ [seconds]}$$

Dielectrics

Dielectrics are electrically non-conductive materials placed in between the plates of capacitors to increase the capacitance of the capacitor.

We quantify the increase in capacitance as the **dielectric constant $\boldsymbol{\varepsilon_r}$**, define as the ratio of capacitance of capacitor with dielectric C, to the capacitance of capacitor with no dielectric (vacuum) C_o,

$$\varepsilon_r = \frac{C}{C_o} = \frac{\left(\frac{\varepsilon A}{d}\right)}{\left(\frac{\varepsilon_o A}{d}\right)} = \frac{\varepsilon}{\varepsilon_o}$$

Chapter 3: Current and DC Circuits

Electric Current

Current is the amount of charge ΔQ that passes through a surface area in time Δt,

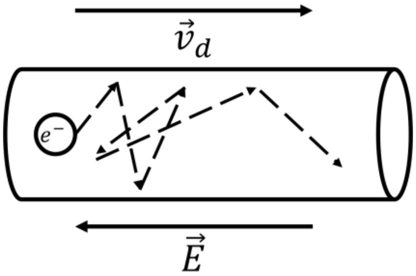
$$I = \frac{dQ}{dt}$$

Total charge Q is simply n multiples of electron charge e,

$$Q = ne$$

Without external electric field, the electron will drift through a conductor with kinetic energy equivalent to Fermi energy, which results in a net velocity of zero. With external electric field, the electron as a whole now gains a net velocity along the electric field. This 'net velocity' is what is known as 'drift velocity'.

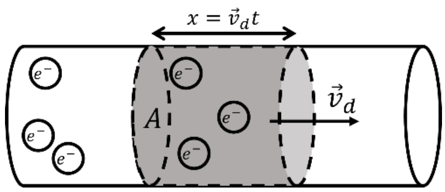
Consider an electron travelling through a conductor, on which an electric field of $\vec{E}$ is applied. The force on the electron is then $F = -qE \Rightarrow a = -\frac{eE}{m}$. Assuming the average time between collision is τ, we can show that



$$v_d = a\tau = \left(-\frac{eE}{m}\right)\tau$$

This means that applying a larger electric field, the larger the kinetic energy obtained by the electron due to a larger drift velocity. This also means that an increase in temperature, increases the collision frequency, decreases collision time and decreases drift velocity of the electrons.

Relating the idea of drift velocity to current can be done by considering a volume section of the conductor V, and the number of charges that flows through that section, n. We can work out that the amount of charge going through V is simply



$$\Delta Q = (ne)A\Delta x$$

where $\Delta x = v_d \Delta t$.

This means that current is

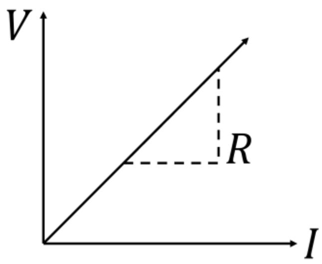
$$I = \frac{\Delta Q}{\Delta t} = neAv_d$$

Ohm's Law

Ohm's Law states that current I, is directly proportional to the potential difference V, is all conditions are constant.

$$V \propto I \ \Rightarrow V = IR$$

R, which is the proportionality constant to Ohm's Law, represents resistance which opposes current flow in a circuit.



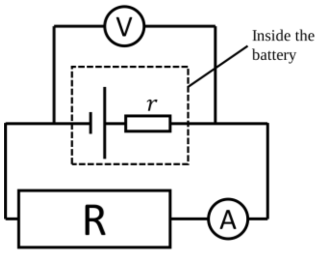
Resistance (Geometry & Temperature)

Resistance of a conductor in a circuit depends on 3 factors – geometry of the conductor, material of the conductor and the temperature of the conductor.

Factor	Equations
Material Geometry	For a resistor of resistivity ρ, length L and cross-sectional area A, the resistance is then $$R = \frac{\rho L}{A}$$
Temperature	When temperature of a conductor with coefficient of resistivity α (at $20^{o}C$), changes by ΔT, resistance changes by $$\Delta R = \alpha \Delta T$$

EMF, Internal Resistance and Potential Difference

Electromotive force (emf) is the electrical energy per unit charge generated by a power source generate current. Some of that electrical energy is used to overcome **internal resistance** within the power supply, the rest is then used for the rest of the circuit. That means the potential difference across the circuit is always less than the emf. This internal resistance may exist for a few reasons – distance between electrodes, temperature of the cell, effective area of the electrodes, irregularities found in the cell, etc.

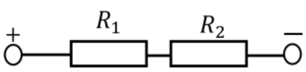
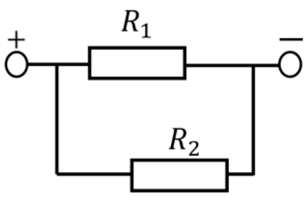


Consider a circuit consisting a voltmeter of reading V, an ammeter of reading I, a battery and a resistor of resistance R. The emf of the source is then

$$\varepsilon = IR + Ir = V + Ir$$

Parallel & Series

For systems of multiple resistors, they can be arranged in parallel, series or any combinations of the two. The effective resistance can then be calculated according to their arrangement.

Arrangement	Effective Capacitance
Series 	$$R_{eff} = R_1 + R_2$$ For n number of resistors in **series**, $$R_{eff} = R_1 + R_2 + \cdots + R_n = \sum_{i}^{n} R_i$$
Parallel 	$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$ For n number of resistors in **parallel**, $$R_{eff} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)^{-1} = \left(\sum_{i}^{n} \frac{1}{R_i}\right)^{-1}$$

Kirchhoff's Rules

Kirchhoff's Rules allows us to determine current flow around a circuit. The two rules are as follows:

Rules	Statement
First Rule – Junction Rule	Algebraic sum of currents in a network of conductors meeting at a junction is zero. $$\sum_i I_i = 0$$
Second Rule – Loop Rule	Algebraic sum of potential difference in any loop must equal to zero. $$\sum_i V_i = 0$$

Electrical Energy and Power

Electrical power P, can be calculated using the following equation

$$P = IV = I^2R = \frac{V^2}{R}.$$

Electrical energy E, is simply the product of electrical power and the time the electrical power was applied.

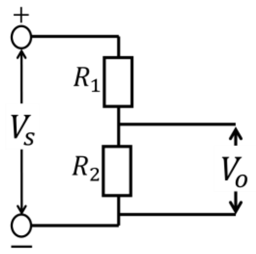
$$E = Pt$$

Potential Divider

A potential divider is used to produce a voltage of a fraction of the voltage provided by the power supply. This is achieved by using resistors of different resistances.

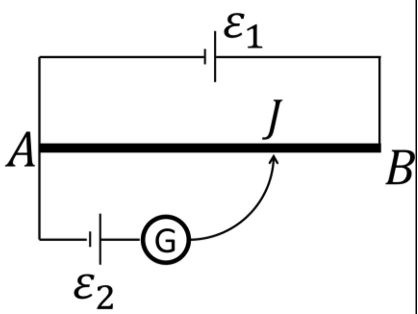
If the power supply provides potential difference of V_s, then the output voltage is simple

$$V_o = \frac{R_1}{R_1 + R_2} V_s$$



Potentiometer

A potentiometer can be used to measure potential differences by two or more cells.

How it works
Wire AB has a resistance of R. This means if the jockey is at point B, then $R_{AJ} = R_{AB}$, and thus $I = I_{maximum}$. As the jockey is slid to towards A, the galvanometer will show zero reading which indicates no current passes through the galvanometer and that the potentiometer is balanced. This means $V_{AJ} = \varepsilon_2$. This happens when $$\frac{V_{AJ}}{V_{AB}} = \frac{l_{AJ}}{l_{AB}}$$ 

We can also use a potentiometer to compare emfs between two cells.

This is done by the following setup.

compare emfs between cell 2 and 3

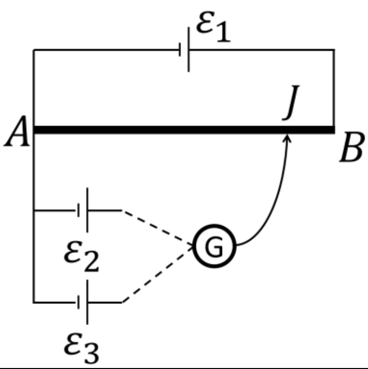
When the galvanometer is connected to ε_2 and balanced between A and J_1,

$$\varepsilon_2 = \frac{l_{AJ_1}}{l_{AB}} \varepsilon_1$$

When the galvanometer is connected to ε_2 and balanced between A and J_2,

$$\varepsilon_3 = \frac{l_{AJ_2}}{l_{AB}} \varepsilon_1$$

$$\Rightarrow \frac{\varepsilon_2}{\varepsilon_3} = \frac{l_{AJ_1}}{l_{AJ_2}}$$



Chapter 4: Magnetism

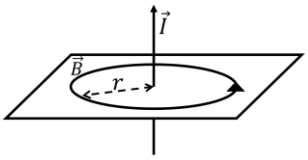
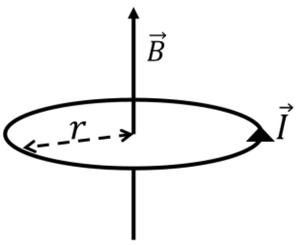
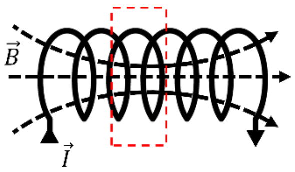
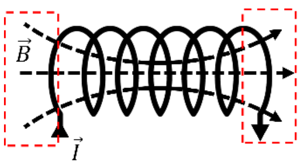
Magnetic Field

A magnetic field is a region of space in which a charged particle will experience magnetic force. They are generated by moving charged particles. Magnetic field lines are always drawn from its north pole to its south pole. When drawn on a 2D plane such as paper, we would generally represent a direction **into** the plane as $\otimes$, and direction **out** of the plane as $\odot$.

$\vec{B}$ from current-carrying conductor

Direction of magnetic field depends on the direction current flow – Right Hand Rule, where the thumb point to the current direction and curled fingers are the magnetic field lines.

4 cases to considering in calculating the magnitude of magnetic field

Situation	Equation
Long straight wire 	$B = \frac{\mu_o I}{2\pi r}$
Centre of circular coil 	$B = \frac{\mu_o I}{2r}$
Centre of solenoid 	$B = \mu_o I n$ Where n is the number of loops per unit length
End of solenoid 	$B = \frac{1}{2}\mu_o I n$ Where n is the number of loops per unit length

Magnetic Force

Force on a moving charged particle in uniform $\vec{B}$

Force on a particle with charge q moving at velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$, the magnetic force acting on it is

$$\vec{F}_{magnetic} = q(\vec{v} \times \vec{B}).$$

In the case of a large enough region, the magnetic force will cause the charged particle to travel in a circular motion. In such cases,

$$\vec{F}_{magnetic} = \vec{F}_{centripetal} \Rightarrow qvB = \frac{mv^2}{r^2}.$$

Force on a current carrying conductor in uniform $\vec{B}$

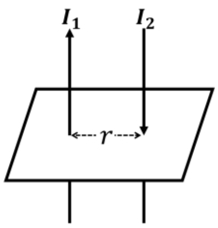
Consider a quantity of charge ΔQ travelling along a conductor of length *l* in a magnetic field $\vec{B}$ in time *t*. The magnetic force on the conductor is then

$$\vec{F}_{magnetic} = I(\vec{l} \times \vec{B}).$$

Force between two parallel current carrying conductors

Consider two current carrying conductors of length *l* in proximity such that their magnetic fields overlap, their resultant magnetic force on each other is then

$$\vec{F}_{magnetic} = \frac{\mu_o I_1 I_2}{2\pi r} l$$



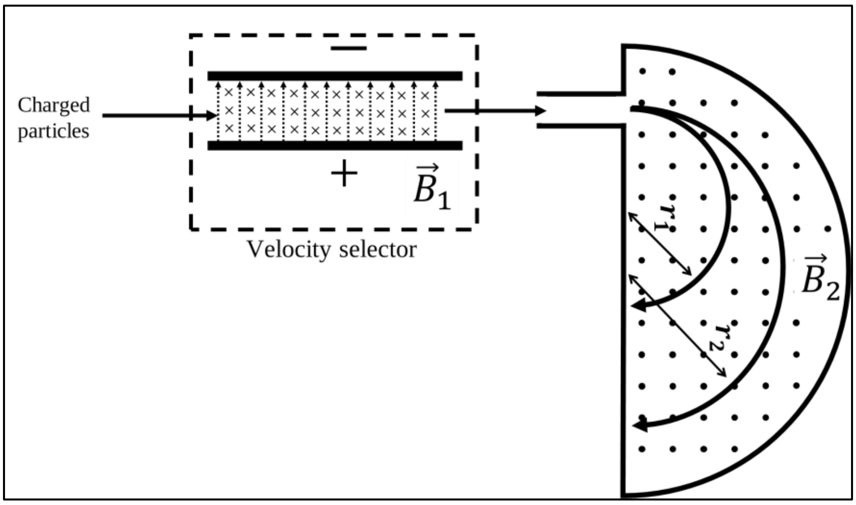
Bainbridge mass spectrometer

The Bainbridge mass spectrometer is used to accurately determine atomic mass.

The first part of the mass spectrometer is a velocity selector in which both electric field $\vec{E}$ and magnetic field $\vec{B}_1$. For charged particles to exit this velocity selector, their velocity must obey

$$v = \frac{E}{B_1}$$

This part of the mass spectrometer allows only charged particles with a certain velocity to enter the second region of only magnetic field $\vec{B}_2$.



The second part of the instrument takes advantage that charged particles of the same entry velocity but different mass will travel in circular path of different radius.

$$qvB_2 = \frac{mv^2}{r^2} \Rightarrow m = \frac{qB_2r^2}{v} = \frac{qB_1B_2r^2}{E}$$

Chapter 5: Electromagnetic Induction

Magnetic Flux

Magnetic flux is a measure of total magnetic field $\vec{B}$ passing through a given area $\vec{A}$, this is calculated with

$$\phi = \vec{B} \cdot \vec{A}.$$

In the case of N number of area of $\vec{A}$ of which $\vec{B}$ passes through, the total magnetic flux is called the **magnetic flux linkage $\boldsymbol{\Phi}$,** and is determine by

$$\boldsymbol{\Phi = N\phi = NBA\cos\theta}$$

Induced EMF

EMF is induced when magnetic flux changes with time. This is the core of Faraday's and Lenz's law of electromagnetic induction.

- Faraday's law tells us how much it emf is induced (magnitude) and Lenz's law tells us in what direction will the force act upon (direction of induced current).
- Faraday's law tells us that the magnitude of induced emf is equal to the rate of magnetic flux change and Lenz's law tells us that the induced current will be in the direction opposing the initial magnetic field.

Together, they are simply written as

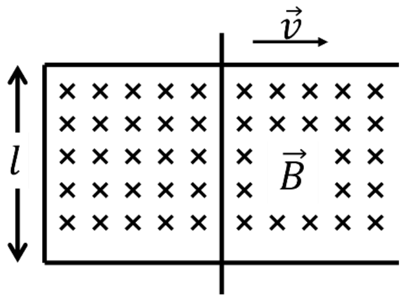
$$\varepsilon = -\frac{d\phi}{dt}$$

Induced emf in straight conductor

In a straight conductor, the area changes with time which causes the magnetic flux to change with time.

Consider a rectangular coil with one of its side movable and the opposite of the movable side has a length of l, in a region of magnetic field $\vec{B}$. If the movable side is moved at velocity $\vec{v}$, the area of the coil would change. The induced emf would then be

$$\varepsilon = -Bl\frac{dx}{dt} = Blv\sin\theta_{vB}$$



Induced emf in a coil

In a circular coil, the option for inducing emf comes from varying the magnetic field **and** the area of the coil, thus 2 equations can be found,

$$\varepsilon = -NB\frac{dA}{dt} \text{ or } \varepsilon = -NA\frac{dB}{dt}$$

Induced emf in a rotating coil

For a coil rotating at angular speed of ω, the emf induced is then

$$\varepsilon = NBA\omega\sin(\omega t)$$

Inductance

Self-induction

The idea of self-inductance is this – a magnetic field induces emf in a conductor, which in turns induces another magnetic field that opposes the initial induced emf. The conductor 'self induces' a magnetic field. The ability of a conductor to do this is quantified by ***self-inductance L,***

$$\boldsymbol{L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}}$$

Generally, this means that

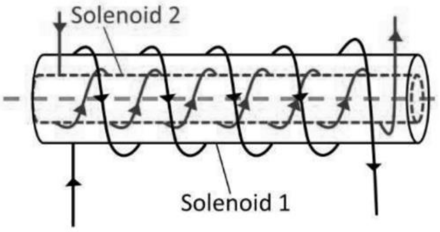
$$LI = N\phi$$

For more specific cases, 2 are considered:

1. For a coil of N turns with a cross sectional area of *A* and radius of *r*,
$$L = \frac{\mu_o N^2 A}{2r}$$
2. For a solenoid of N turns with a cross sectional area of *A* and length *l*,
$L = \frac{\mu_o N^2 A}{l}$

Mutual induction

Mutual inductance happens between 2 conductors, when the magnetic field induced by one conductor induces current in the other conductor. Consider two coaxial solenoids, a magnetic field is generated by solenoid 1 and thus solenoid 2 respond by an induced emf, if solenoid 2 has a cross sectional area of A_2, then the mutual inductance between solenoid 1 and 2 is



$$M_{21} = \frac{\mu_o N_1 N_2 A_2}{l},$$

where *l* is the length of the solenoid.

Energy Stored in Inductor

The energy stored in an inductor of inductance *L* and with current *I* running through it, is simply

$$U = \frac{1}{2}LI^2$$

Chapter 6: Alternating Current

Alternating Current

Alternating current (AC) is defined as an electric current that periodically reverses its direction with respect to time.

Root Mean Square Values

In AC circuits, the idea of voltage and current now are functions of time:

$$I \mapsto I(t) = I_{peak} \sin(\omega t)$$

$$V \mapsto V(t) = V_{peak} \sin(\omega t)$$

Resistance is then defined as

$$R = \frac{V_o}{I_o}$$

In calculation of power, where $P_{DC} = IV$, for AC circuits,

$$P_{AC} = I_{rms}V_{rms}$$

where,

$$I_{rms} = \frac{I_o}{\sqrt{2}} \text{ and } V_{rms} = \frac{V_o}{\sqrt{2}}$$

Impedance

In DC circuit, our main concern for opposition of current flow is only resistance R.

In AC circuits, we now have what is known as **impedance *Z***, which is defined by

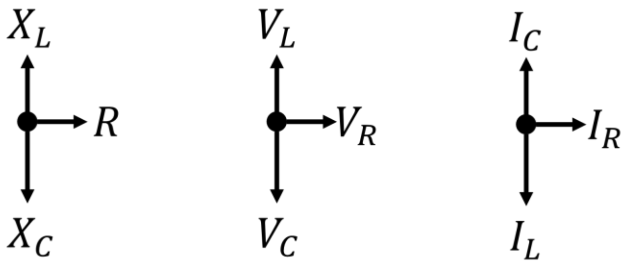
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where R is the resistance, X_L is the inductive reactance and X_C is the capacitive reactance found in the circuit.

The following table shows how to calculate these values

Reactance	Equation
Capacitance reactance for a capacitor of capacitance C	$X_C = \frac{1}{2\pi f C}$
Inductive reactance for an inductor of inductance L	$X_L = 2\pi f L$

The phasor diagram for an RLC circuit is as follows,



Phasor Diagram
for RLC Circuit

Which means that the phase angle between current and voltage is

$$\theta_{IV} = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Resonance occurs when $X_L = X_C \Rightarrow \omega = \frac{1}{\sqrt{LC}} = 2\pi f$.

Power & Power Factor

2 types of power that be calculate in the case of AC circuits,

1. Instantaneous power

$$P = I(t) \times V(t)$$

2. Average power

$$P_{ave} = I_{rms}V_{rms}\, cos\,(\theta_{IV})$$

The power factor is simply

$$cos\,\theta_{IV} = \frac{P_{real}}{P_{apparent}} = \frac{P_{ave}}{I_{rms}V_{rms}}$$

Chapter 7: Optics

Geometrical Optics: Reflection

Definitions:

1. Centre of curvature, *C* = a point on the principal (or optical) axis that is positioned at distance equal to the radius of curvature *R*, of the spherical mirror.
2. Focal point, *f* = a point on the principal axis at which light rays travelling parallel to the principal axis will converge onto or diverge from, after reflecting on the surface of the spherical mirror.

f and *R* are related by the following equation:

$$R = 2f$$

2 types of mirrors:

1. **Convex mirror**, of which its radius is located behind the mirror.
2. **Concave mirror**, of which it's radius of curvature is located in front of the mirror.

Conventions	
Focal length, *f*	+ for concave; - for convex
Curvature Radius, *R*	

Lateral magnification m, refers to the ratio between the height of the image to the height if the object. In equation form,

$$m = \frac{h_i}{h_o}$$

$$m > 0 \Rightarrow \text{ upright image; } m < 0 \Rightarrow \text{ inverted image}$$

Geometrical Optics: Refraction

An extension to Snell's law will be the refraction at a spherical surface. The following equation allows us to relate distances, refractive indices and radius of curvature of the spherical surface:

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

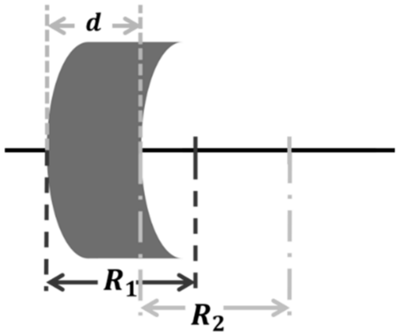
In the equation above n_i, refers to refractive indices, *u* and *v* refers to object and image distances respectively and *R* refers to the radius of curvature.

Conventions	
Curvature Radius, *R*	+ for convex, i.e. *C* opposite side as incoming light - for concave, i.e. *C* same side as incoming light

For the refractive indices, subscript 1 refers to the refractive index on the side of the incoming light rays and subscript 2 refers to the refractive index on the side of the outgoing rays.

Geometrical Optics: Thin lenses

The thin lens equation assumes that the thickness measured between two vertex of the spherical surface of a lens is much smaller than the product of the radii of the spherical lenses, that is $d \ll R_1 R_2$.



For thin lenses,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Conventions	
Focal length, f	+ for convex, i.e. same side as incoming light - for concave, i.e. opposite side as incoming light

On the other hand, using the lens maker's equation,

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

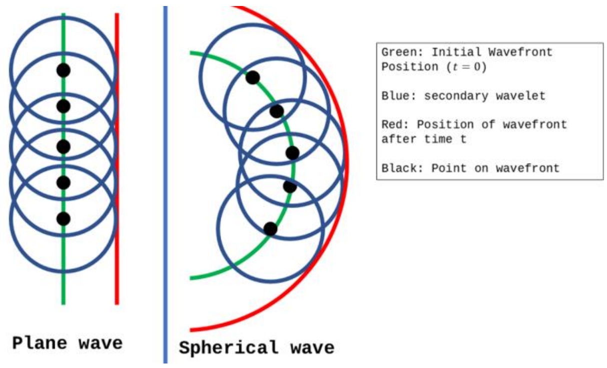
one can determine the focal length f, of the lens from

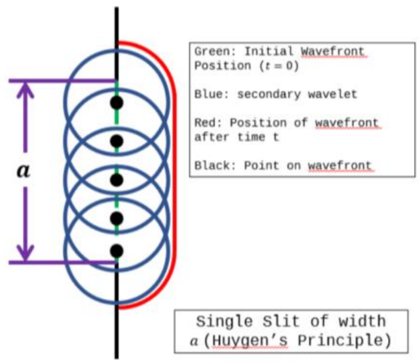
1. the radii of the lens surfaces, R_1 and R_2,
2. the ratio of the refractive index of the lens material to the refractive index of the surrounding, $\frac{n_{material}}{n_{medium}}$

Conventions	
Curvature Radius, R	- if curvature same side as incoming light + if curvature opposite side as incoming light

Physical Optics: Huygens's Principle

Huygen's Principle states that "each point on the wavefront acts as the source of secondary wavelets that spread out in all directions in spherical waves with a speed equal to the speed of wave propagation."



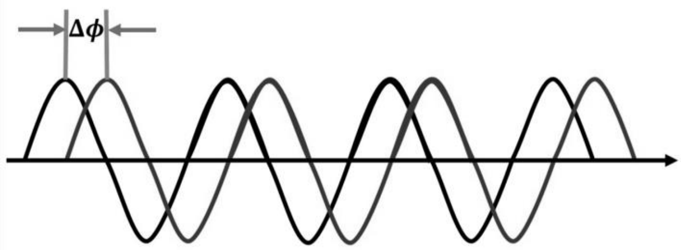


Physical Optics: Interferences

Coherence between 2 waves refers to the condition of constant phase difference between 2 waves with respect to time, that is to say $\frac{d\phi}{dt} = 0$. This property is the ideal property for stationary interference.

For a stable interference pattern, the following conditions are required:

1. Coherence, that is to say the two interacting light waves are of the same phase difference, $\frac{d\phi}{dt} = 0$.



2. Monochromatic, that is to say that the two interacting light waves are of the same wavelength, i.e. $\lambda_1 = \lambda_2$

For purely **constructive interference**, it is empirical that the phase difference between the interacting waves is either 0 or $n\lambda$. On the other hand, for purely **destructive interference**, it is required that the phase difference between the two interacting wave is $\frac{n\lambda}{2}$.(*n* is both cases refers to integer values.)

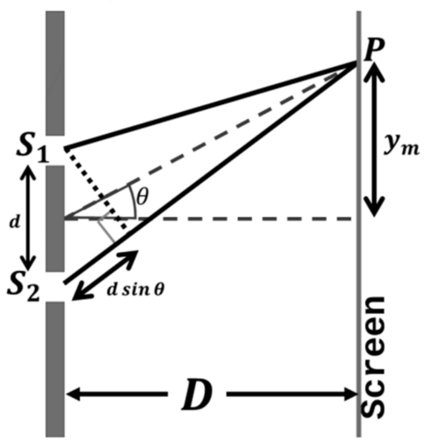
Physical Optics: Slits

Double Slit

We now consider the case for Young's double slit experiment.

Here we define the following variables:

D	distance from slit to screen
d	slit separation
y_m	distance from central maximum tu the mth fringe



We know that in order to determine what type of fringe forms at P, we need to look at the path difference and from the figure, we can say that the figure,

$$\Delta\phi = S_2P - S_1P = dsin\theta = d\left(\frac{y_m}{D}\right)$$

For **bright fringes**,

$$\Delta\phi = \frac{y_m d}{D} = m\lambda$$

Rearranging this allows us to find fringe distance as a function of *d* and *D* with *m* having any integer value indicating **mth bright fringe** from central maximum:

$$y_m = \frac{m\lambda D}{d}$$

Shifting one the waves by 0.5λ give us the equation for **dark fringes**,

$$y_m = \frac{(m + 0.5)\lambda D}{d}.$$

Lastly, we'd want to calculate the fringe separation. This can be done by considering $\Delta y = y_{m+1} - y_m$, which results in

$$\Delta y = \frac{\lambda D}{d}$$

Single Slit

Diffraction is defined as the spreading or bending of waves as they pass through an aperture of a barrier. The diffracted waves then interfere with each other to produce a diffraction pattern.

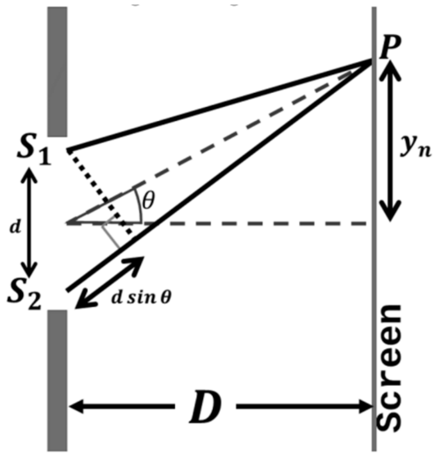
Light waves from one portion of the slit interacts with light waves from a different portion of the same slit to produce a diffraction pattern.

Here, we find that the dark fringes to forms when according to

$$d\,sin\,\theta = n\lambda.$$

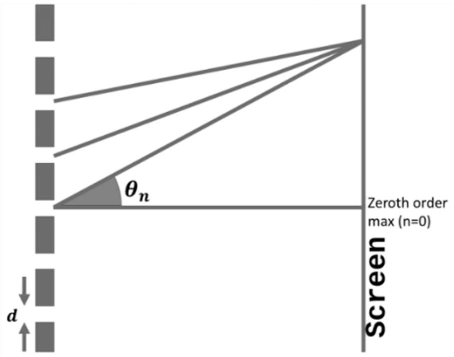
Geometrically, we also find that $tan\theta \approx sin\theta \approx \frac{n\lambda}{d} \approx \frac{y_n}{D}$.

As such, we can say that **dark fringes** forms at $y_n = \frac{n\lambda D}{d}$. This would also mean that **bright fringes** forms at $y_n = \frac{(n+0.5)\lambda D}{d}$.



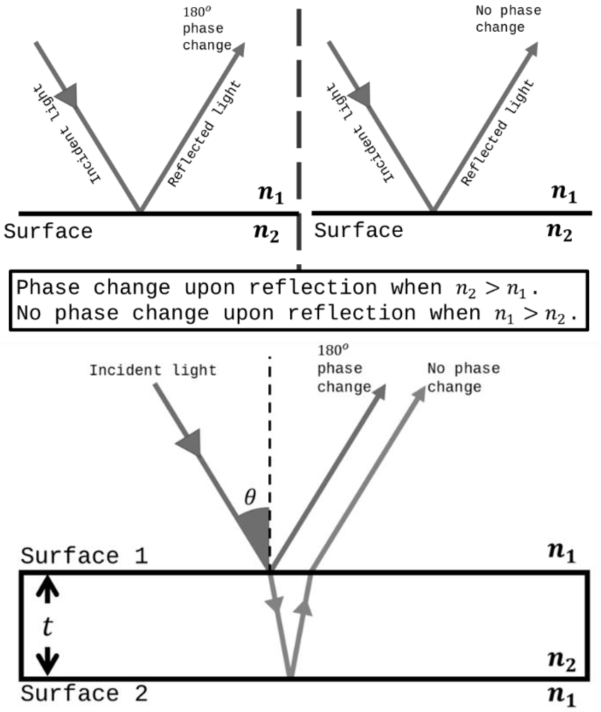
Diffraction Grating

In the case for diffraction grating, light waves from many slits and interfere at the screen to form fringes of equal width. The equation by which the pattern follows is $d\,sin\,\theta_n = n$ for **bright fringes** and shifted by 0.5λ for **dark fringes**. Note that the angle θ_n is measured from the normal line formed at the zeroth order maximum. Also note that, **maximum number of fringes** can be calculated by considering that $sin\,\theta_n < 1$.



Physical Optics: Thin Films

Referring to figure on thin films, we can see that the two reflected light waves has a phase difference of 0.5λ from reflections at surface 1 and 2. One must also take into consideration of the extra distance that the second (green) wave travelled, that is $2nt$. Therefore, the total phase difference between the reflected waves is then



$$\Delta\phi = 2nt - \frac{1}{2}\lambda.$$

For **constructive interference**, $\Delta\phi = 2nt - \frac{1}{2}\lambda = n$, which gives us the equation

$$2nt = \left(n + \frac{1}{2}\right)\lambda.$$

Dark fringes then appear between the bright fringes, i.e. they follow the equation

$$2nt = n\,\lambda.$$

Main application for this concept of thin film interference is in **optical coatings** as one can manipulate the thickness of the coating as to choose the level of constructive or destructive interference. These optical coatings can be applied onto both for reflective as well as refractive systems.

Chapter 8: Particle Waves

De Broglie Wavelength

Like light, matter also exist in dual form – as particles **and** waves.

Matter waves, known as "de Broglie wavelength", are calculated with

$$\lambda_{matter} = \frac{h}{p} = \frac{h}{mv}$$

For a particle with of mass m charge q accelerated by electric field of V volts,

$$\lambda_{matter} = \frac{h}{\sqrt{2qVm}}$$

Electron Diffraction

On de Broglie wavelength:

1. To show that particles may exhibit wave-like characteristics, Davisson and Germer designed an experiment in which they show that electrons diffracted.
2. They achieve this by directing a beam of electrons onto a nickel crystal.

On electron microscope:

1. Because of their short wavelength (1nm for electrons vs 400nm – 700nm for light microscope), electronic microscopes can offer physicists a higher resolution in probing specimens.
2. Optical microscope is made up of glass lenses, whereas components of an electron microscope are electromagnetic.

Chapter 9: Nuclear & Particle Physics

Binding Energy & Mass Defect

Mass defect, Δm = mass difference between the actual mass of an atomic nucleus and the sum of its components, i.e. protons and neutrons.

For an atomic nucleus of mass $m_{nucleus}$ with Z number of protons of mass m_{proton} and N number of neutrons of mass $m_{neutron}$, its mass defect is

$$\Delta m = \left(Zm_{proton} + Nm_{neutron}\right) - m_{nucleus}$$

Binding energy, $E_{binding}$ = energy found in the nucleus of an atom that binds its components together. This energy can be calculated from the mass defect,

$$E_{binding} = \Delta mc^2$$

As the masses of atomic nucleus is well, very small, and the speed of light is astronomical, it may be easier to perform calculations using atomic mass unit (amu) or Dalton (u) and MeV/c^2

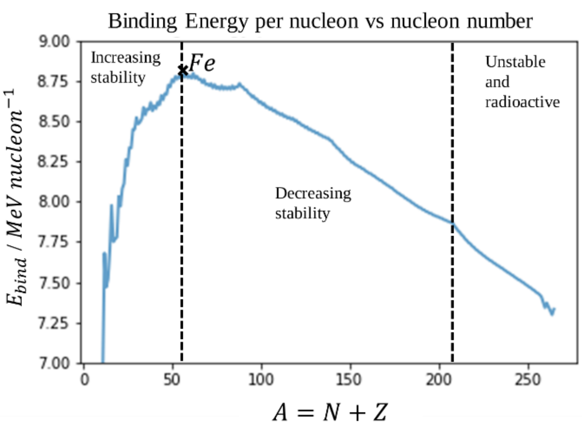
$$1kg = 6.022(10^{26})u = 5.60958(10^{29})\,MeV/c^2$$

$$1u = 1.66054(10^{-27})\,kg = 931.494\,MeV/c^2$$

Binding energy per nucleon, $\frac{E_{binding}}{A}$:

Nucleon number, A = Number of protons, Z + Number of neutrons, N

Binding energy per nucleon vs nucleon number graph:



- For nuclei lighter than that of iron (Fe), it is found that binding energy per nucleon increases with the nucleon number.
- After the iron limit, the binding energy per nucleon decreases.
- At $A \approx 209$ (nuclei of Bi), the binding energy per nucleon is too weak to keep the nuclei together and thus, are unstable and radioactive.

Radioactivity

The following table describes the types of decay of a radioactive substance

Type of decay	Process	Description
α	${}^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}He$ parent nucleus → daughter nucleus + α particle	In α decay, an α particle (Helium) is emitted when the parent nucleus decays into its daughter nucleus. Electrical charge is conserved throughout the process. Energy is released upon α decay.
β^-	$n \rightarrow p^+ + e^- + \bar{\nu}$ ${}^{A}_{Z}P \rightarrow {}_{Z+1}^{A}D + {}_{-1}^{0}e + \bar{\nu}$ parent nucleus → daughter nucleus + β^- particle + antineutrino	In β^- decay, an electron e^- and an antineutrino $\bar{\nu}$ is emitted when the parent nucleus decays into its daughter nucleus.
β^+	$p^+ \rightarrow n + e^+ + \nu$ ${}^{A}_{Z}P \rightarrow {}_{Z-1}^{A}D + {}_{+1}^{0}e + \nu$ parent nucleus → daughter nucleus + β^+ particle + neutrino	In β^+ decay, a positron e^+ and a neutrino ν is emitted when the parent nucleus decays into its daughter nucleus.
γ	${}^{A}_{Z}P * \rightarrow {}^{A}_{Z}P + \gamma$ nuclei of high energy state → nuclei of low energy state + γ ray	In γ decay, the emission is a photon (light ray). This happens because the nucleons lower its energy state.

In general, N number of radioactive particles will decay according to the **decay law**,

$$\frac{dN}{dt} = -\lambda N$$

where λ is the decay constant of the substance, which varies between isotopes.

The solution for the decay law is

$$N(t) = N_o e^{-\lambda t}$$

where $N_o = N(t = 0)$.

The rate of decay is known as **activity**

$$A = \left|\frac{dN}{dt}\right| = \left|\frac{dN_o}{dt}\right| e^{-\lambda t} = A_o e^{-\lambda t}$$

Half-life is simply the time it takes for the number of isotopes to decrease by half $T_{\frac{1}{2}}$,

$$N = \frac{1}{2}N_o = N_o e^{-\lambda T_{\frac{1}{2}}} \Rightarrow T_{\frac{1}{2}} = \frac{ln\ 2}{\lambda}$$

Particle Accelerator

Thermionic emission (Edison Effect) = emission of electrons on the surface of a metal by providing it sufficient thermal energy.

As mentioned before, a charged particle may be accelerated by the help of an electric and magnetic field. The acceleration would stem from Lorentz Force.

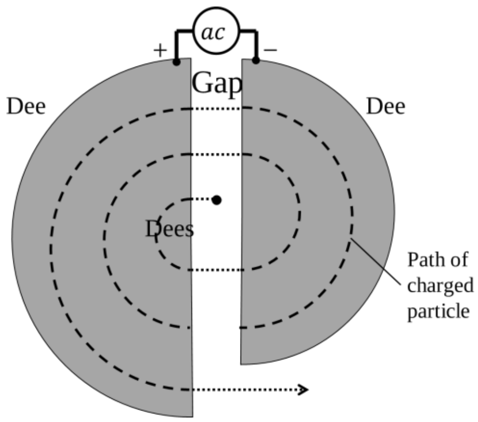
To probe subatomic particles, we need high energy because higher energy results in higher momentum which gives out smaller de Broglie wavelength. This means a higher resolution can be achieved.

2 types of particle accelerators:

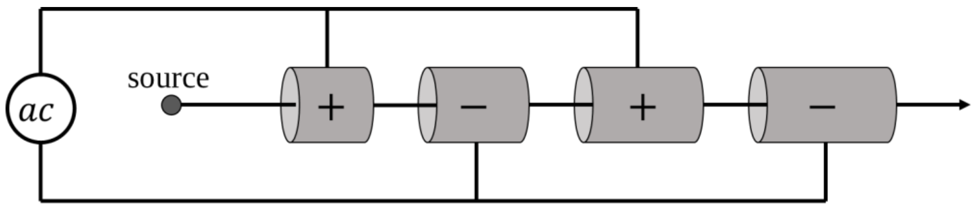
1. Cyclotron
   It uses magnetic field to maintain charged particles in nearly circular paths.

   A cyclotron is composed of 2 'dees', charged particles are accelerated in the region of space between the two 'dees', where an electric field is applied.

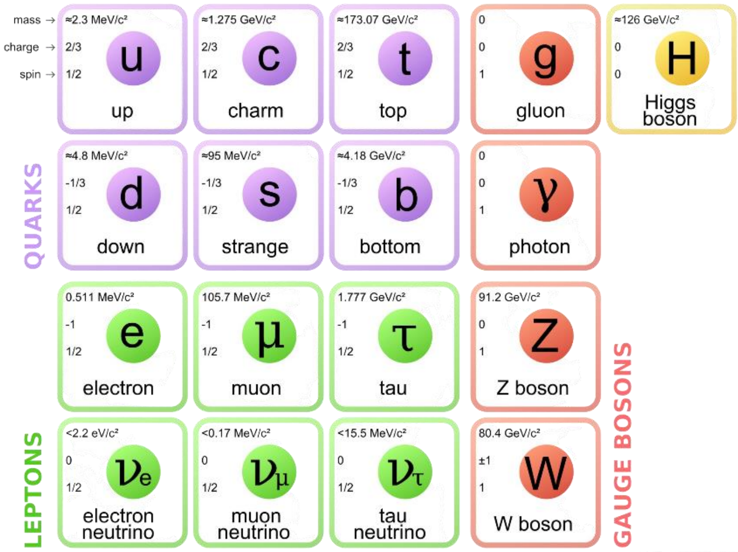
   Velocity of the charged particles when they are in the 'dees' is
   $$v = \frac{qBr}{m}$$
   Frequency of electric field is equal to the frequency of the circulating protons,
   $$f = \frac{qB}{2\pi m}$$
   which is known as the **cyclotron frequency**.

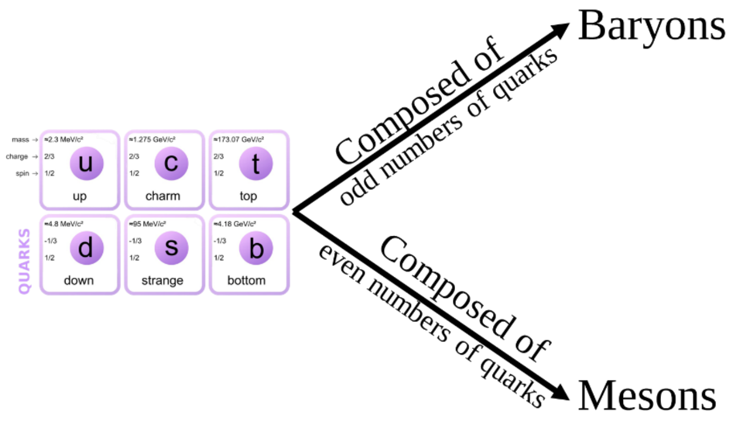


2. Linear Accelerator
   Charged particles are accelerated through a series of linear conductor tubes.
   Alternating voltage is applied to consecutive tubes so that when a charged particle reaches a gap, the tube they just left is now negatively charged and the tube they are heading into is positively charged.



Fundamental Particles





Particle-antiparticle pair:

They are pairs of particles that has opposite charge to each other. E.g., electron has a negative charge whereas its antiparticle, a positron has a positive charge. They interact by annihilating each other.

$$e^- + e^+ \rightarrow \gamma + \gamma$$