A note on error propagation

In your data analysis section of the lab report, you are expected to determine the uncertainty of the gradient. How do you then relate this to the physical variable you are studying? For example, in the free fall experiment, we study the relationship between height h and time of fall squared t^2, which we expect to follow

$$h = \frac{1}{2}gt^2$$

where g is the gravitational acceleration.

In our graph, we will have height h as our y-axis values and time squared t^2 as our y-axis values. This would mean that our gradient represented by

$$m = \frac{1}{2}g.$$

So now if we have some value for the uncertainty for the gradient, can we determine the uncertainty of the value of g. Yes, we can! Here I shall show you two ways of doing it, a simpler way and a more complicated method.

First method – simple

Say we have $m_{trendline} = 4.9ms^{-2}$ and $\Delta m = 0.2ms^{-2}$, we then determine that m would fall within the range of

$$m - \Delta \mathrm{m} \leq m \leq m + \Delta m \Rightarrow 4.7ms^{-2} \leq m \leq 5.1ms^{-1}$$

We can now evaluate values of g when $m = 4.9ms^{-2}$, $m = 4.7ms^{-2}$ and $m = 5.1ms^{-2}$, this would give

$$g = 2m \Rightarrow g|_{m=4.9ms^{-2}} = 9.8ms^{-2};\ g|_{m=4.7ms^{-2}} = 9.4ms^{-2}\ ;\ g|_{m=5.1ms^{-2}} = 10.2ms^{-2}$$

Allowing us to write the following for the range of g,

$$9.4ms^{-2} \leq g \leq 10.2ms^{-1}$$

We can then deduce that

$$g - \Delta g = 9.4 \Rightarrow \Delta g = 0.4ms^{-2}$$

Second method – derivative-based

The second method I shall present here is to use differentials to estimate errors.

What I mean by to use differentials to estimate errors is to take advantage of the following approximation for a function of y(x)

$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$$

If we have, say

$$m = \frac{1}{2}g$$

We can differentiate this equation with respect of g

$$\frac{dm}{dg} = \frac{1}{2}$$

And approximate it to

$$\frac{dm}{dg} = \frac{1}{2} = \frac{\Delta m}{\Delta g} \Rightarrow \Delta g \ = \ 2\Delta m$$

Substituting $\Delta m = 0.2ms^{-2}$ the yields

$$\Delta g = 2(0.2) = 0.4ms^{-2}.$$