Exercises

1. Two vectors lie on the x-y plane. Vector $\vec{a}$ is 5 units long and points 25^o above the x-axis. Vector $\vec{b}$ is 8 units long and points upward 35^o above the x-axis. What is the resultant of the two vectors?

2. Show that the equation $v^2 = v_o^2 + 2ax$ is dimensionally correct. In this expression, a is acceleration, v and v_o are
velocities and x is the distance.

3. Derive the dimensions of k if $k = \frac{ab}{d}$ where $[a] = M^2T^2$, $[b] = M^{-1}T^{-1}$ and $[d] = MT^2$.

4. A car is travelling at 35miles per hour, what is its speed in metres per second if $1mile = 1.61km$.

5. Express each of the following in grams
   a. $3lb$ if $1g = 2.2 \times 10^{-3}lb$
   b. $12.2oz$ if $1g = 0.035oz$
6. How many square centimetres are in a square inch if $1cm = 0.3937inch$.
7. The length and the width of a rectangle is $(11 \pm 0.05)cm$ and (21 ± 0.05) cm respectively. Find the area (and its uncertainty) of the rectangle.

Chapter 2: Kinematics of Linear Motion

Learning Outcomes (LO)

1. Define:
   a. instantaneous velocity, average velocity and uniform velocity; and
   b. instantaneous acceleration, average acceleration and uniform acceleration.
2. Derive and apply equations of motion with uniform acceleration
   $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$; $s = \frac{1}{2}(u + v)t$
3. Describe projectile motion launched at an angle, θ as well as special cases when θ=0°
4. Solve problems related to projectile motion.

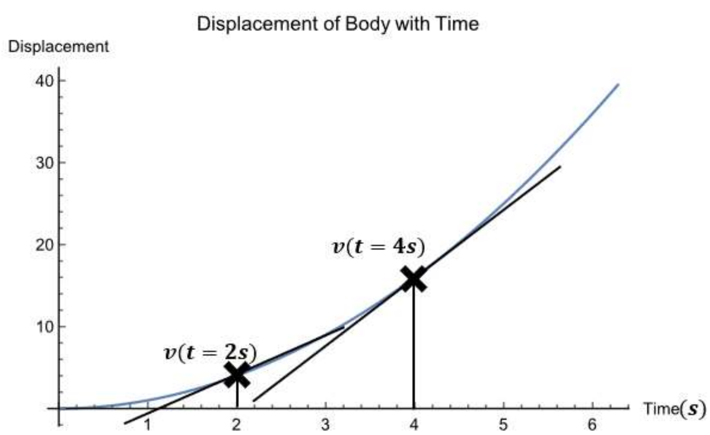
In this chapter, we talk about kinematics of linear motion. Dynamics is the study of motion of bodies under action of forces and their effects. One subbranch to the study of dynamics is kinematics. In the study of kinematics, we consider only the motion of the bodies without worrying too much about the forces that caused the bodies to move. We only worry about the geometry of the motion.

Instantaneous and Average Velocity (or acceleration)

Let us start with reminder of some ideas and terms that you have learnt in your SPM days. 3 mains terms – displacement (s), velocity (v) & acceleration (a). Displacement, denoted by x, simply refers to the change in position of a body. Velocity, v, refers to the rate of change of this change in position, i.e. $v = \frac{dx}{dt}$. Acceleration, a, is defined by the rate of change of velocity, which is the rate of change of the rate of change of position. That is $a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$.

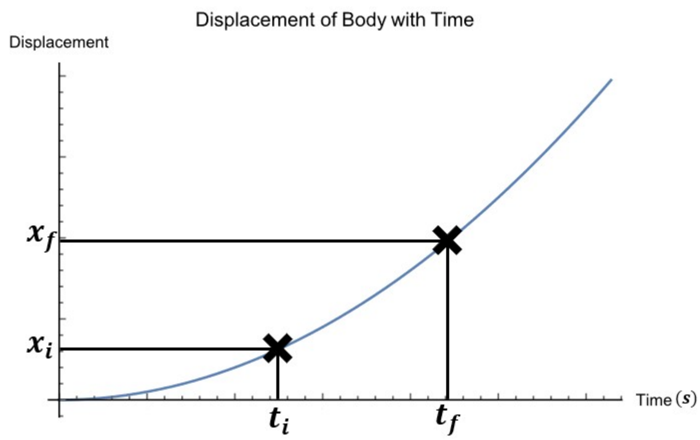
Once we have established that, we can further extend our ideas of velocity and acceleration by thinking about ***instantaneous*** velocity (or acceleration) and ***average*** velocity (or acceleration). By 'instantaneous', we mean 'at a particular instant in time'. When we combine it with velocity (or acceleration), what we mean is velocity (or acceleration) at a particular instant in time. On the other hand, when we say 'average', what we mean is 'over the course of a defined time span'. So, when we say 'average velocity', we usually would accompany it with 'between time t_a and t_b' or 'in 30 seconds', specifying a range of time.

Let us now have a graphical representation. Consider a body moving at constant velocity,



When we talk about instantaneous velocity, we are asking about a single point in time. From the displacement-time graph, the gradient represents the velocity of the body. As we can see from the graph, the instantaneous velocities when $t = 2s$ and $t = 4s$ are different. This is simply because the body is moving at a nonuniform velocity. If the instantaneous velocities are the same, then we call the motion is described as uniform velocity.

On the other, talking about ***average*** velocity, we simply define range of time, thus choosing two points in time rather than one. Then we take the difference in position and divide it by the difference in time to calculate the ***average*** velocity. That is to say, for the graph below,



We can calculate the average velocity as

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

We can take the same approach and understanding and apply it to acceleration, but with a velocity time graph rather than a displacement time graph.

Sample Problem 2.1

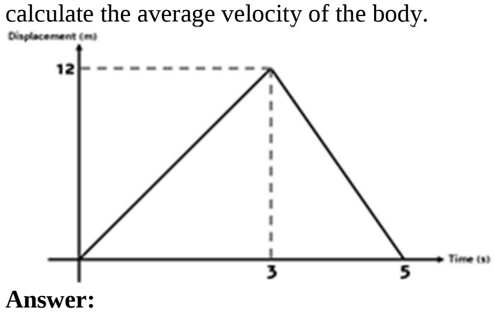
The motion of a body is described by the equation $v = 2t^2$, where v is in metres per second and t is in seconds. Calculate the instantaneous velocity of the body at $t = 3s$ and the average acceleration between $t = 2s$ and $t = 4s$.

Answer:

$$v_{instantaneous} = 2(3)^2 = 18ms^{-1}$$
$$v(t = 2s) = 2(2)^2 = 8ms^{-1}$$
$$v(t = 4s) = 2(4)^2 = 32ms^{-1}$$
$$a_{average} = \frac{\Delta v}{\Delta t} = \frac{v(t = 4s) - v(t = 2s)}{4 - 2} = \frac{32 - 8}{4 - 2}$$
$$a_{average} = 12ms^{-2}$$

Sample Problem 2.2

The motion of a body is shown in the graph shown in Figure 1. Calculate the displacement of the body and calculate the average velocity of the body.



Answer:

Displacement of the body can be calculated by recognising that the area under the velocity -time graph represents the displacement of the body. So all we need is to sum up all the areas under the graph.

$$s = \frac{1}{2}(12 \times 3) + \frac{1}{2}\big(12 \times (5 - 3)\big) = 30m$$

The average velocity can be calculated by simply dividing the displacement by the total time of motion.

$$v_{average} = \frac{30}{5} = 6ms^{-1}$$

Kinematic Equations

Now we want to look at kinematic equations, which are equations that relates variables that describes motion such as displacement, velocity and acceleration.

Derivation by calculus

We'd like to derive the equations from our understanding of linear motion and using calculus. We begin with the definition of acceleration

$$a = \frac{dv}{dt}$$

Assuming constant acceleration, we can rearrange then integrate both sides to yield

$$a\int_{t_{initial}}^{t_{final}} dt = \int_{v_{initial}}^{v_{final}} dv \Rightarrow a\left(t_{final} - t_{initial}\right) = v_{final} - v_{initial}$$

Adjusting such that $t_{initial} = 0$, $t_{final} = t$ and defining $v = v_{final}$, $v_{initial} = u$.

And then rearranging this equation yields

$$v = u + at$$

which is the same equation as the first equation found in LO2. Simply put, the final velocity of a body is initial velocity plus the product of acceleration and time difference.

We can take the same approach to find the third equation in LO2 using the first equation. We start with the definition of velocity and then rearranging it,

$$v = \frac{dx}{dt} \Rightarrow \int v\, dt = \int dx$$

Note that since velocity is not a constant, $v\ dt$ cannot be directly integrated. We therefore need an equation for velocity as a function of time (first equation).

$$\int u + a\, t\, dt = \int dx$$

Since u and a are constants, these integrals become

$$u\int_0^t dt + a\int_0^{\mathrm{t}} dt = \int_0^{\mathrm{x}} dx$$

Solving this integral gives

$$x = ut + \frac{1}{2}at^2.$$

For the second equation, we can start take advantage of calculus by starting with a time independent derivative,

$$\frac{dx}{dv} = \frac{dx}{dt}\frac{dt}{dv} = \frac{v}{a}$$

Rearranging this gives us the needed integral to solve

$$a\int_0^x dx = \int_u^v v\, dv \Rightarrow ax = \frac{1}{2}(v^2 - u^2)$$

Further rearrangement yields an equation

$$v^2 = u^2 + 2ax$$

matching with the third equation found in LO2.

Equation 4 of LO2 does not require any integration, rather we can obtain it using $s = ut + \frac{1}{2}at^2$ and $v = u + at$. This is left for the reader to do.

Geometric Derivation

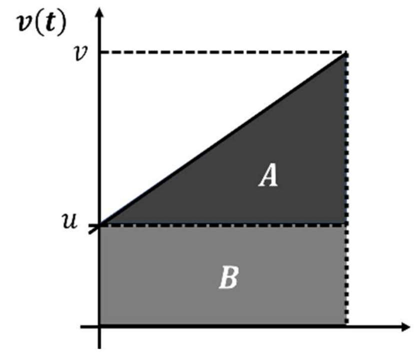
By definition,

$$a = \frac{v-u}{t}$$

Rearranging this gives

$$v = u + at$$

Consider an object that starts its motion with velocity u and maintains its constant acceleration a to a final velocity of v. We can describe its motion diagrammatically as below



Since the area under the graph represents displacement, all we need to do is to add up the area of A and B. If

$$\text{Area}_A = \frac{1}{2}(t)(v-u) = \frac{1}{2}(t)(at) = \frac{1}{2}at^2$$

$$\text{Area}_B = u\text{t}$$

then

$$\text{s} = u\text{t} + \frac{1}{2}at^2.$$

If, on the other hand, we consider

$$s = \frac{1}{2}(t)(v-u) + ut$$

Then we find that

$$s = \frac{1}{2}(v+u)t$$

For the equation of $v^2 = u^2 + 2as$, we can start the derivation by considering

$$v = u + at \Rightarrow t = \frac{v-u}{a}$$

And

$$s = \frac{1}{2}(u+v)t.$$

We can substitute time equation into the displacement to yield

$$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right) = \frac{v^2-u^2}{2a} \Rightarrow v^2 = u^2 + 2as.$$

<u>Sample Problem 2.3</u>

A 2022 Honda Accord can travel down a $\frac{1}{4}$ mile track in $14.1s$ from rest. Calculate the acceleration (in SI units), assuming that its acceleration is constant.

<u>Answer:</u>

Values given $\Rightarrow s = \frac{1}{4}\,mile = 402.336km\,; t = 14.1s;\ u = 0ms^{-1}$

Choice of equation $\Rightarrow s = ut + \frac{1}{2}at^2$

$$402.336 = \frac{1}{2}a(14.1)^2$$
$$a = 4.04744ms^{-2}$$

<u>Sample Problem 2.4</u>

A car initially travels at $20ms^{-1}$. If the car undergoes constant acceleration of $1.2ms^{-2}$, determine the time the car need to reach double of its initial velocity.

<u>Answer:</u>

Values given $\Rightarrow \text{u} = 20\text{ms}^{-1};\ \text{a} = 1.2\text{ms}^{-2};\ \text{v} = 2\text{u} = 40\text{ms}^{-1}$