Classical Dynamics and Thermodynamics

Shafiq R.*
Sarawak Matriculation College, Kuching, Sarawak
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Here, I present short notes to supplement lectures on the topic of Classical Dynamics and Thermodynamics found in the Malaysian Matriculation Programme Physics curriculum specification. The main reference text is the Cutnell's 2015 10^{th} edition textbook, *Introduction to Physics.*

Keywords: Newton's Laws, Energy, Oscillations, Gravity

CONTENTS

* Correspondence email address: shafiq@kmsw.matrik.edu.my

Part One: Dynamics and Solid Deformation

I. KINEMATICS OF LINEAR MOTION

Time allocation:
1.5h (Lecture) + 7h (Tutorial)
Learning Outcomes:

1. Define
   - Instantaneous Velocity/Acceleration
   - Average Velocity/Acceleration
   - Uniform Velocity/Acceleration
2. Discuss physical meaning of 3 graphs:
   - Displacement-Time
   - Velocity-Time
   - Acceleration-Time
3. Apply equations of motion with uniform acceleration:

$$v = u + at;\ s = ut + \frac{1}{2}at^2;\ v^2 = u^2 + 2as$$

4. Describe projectile motion launched at angle θ
5. Solve problems related to projectile motion

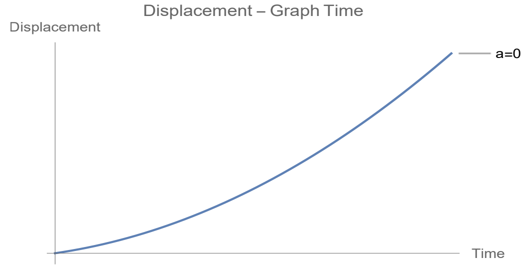
LO1:

1. Instantaneous (velocity/acceleration)
   = (Velocity/Acceleration) at a specific point in space and time.

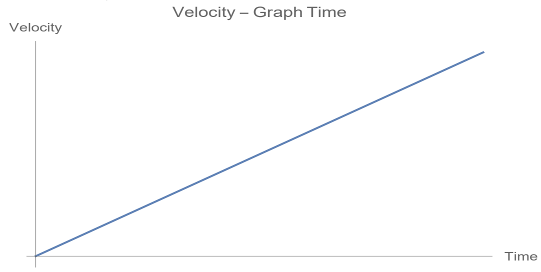
2. Average (velocity/acceleration)
   = (Velocity/Acceleration) averaged over time inteval

3. Uniform (velocity/acceleration)
   = constant rate of change of (displacement/velocity)

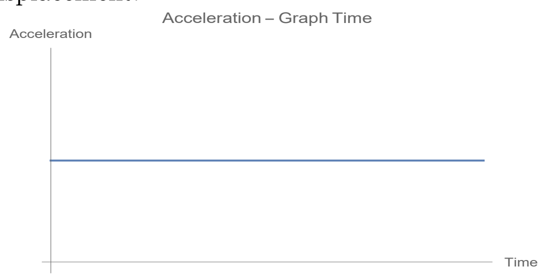
LO2:
Assumption: Non-zero, constant acceleration



Equation: $s = s_i + ut + \frac{1}{2}at^2$
Description: In the case of $a \neq 0$, the graph plotted follows a quadratic equation graph form with a y-intercept of s_i. The **gradient** of the plot at a certain point gives the velocity.



Equation: $v = u + at$
Description: The graph plotted is a linear plot when $a = 0$ with y-intercepts of the initial velocity, u. The **gradient** of the plot at a certain point gives the acceleration and the **area under the graph** represent the displacement.



Equation: $\frac{da}{dt} = 0$
Description: The **area under the graph** represent the velocity.
LO3 requires practice! But here we present a simple derivation that merely requires memorization of 1 equation:

$$\frac{dv}{dt} = a \rightarrow v = u + at$$
$$\frac{ds}{dt} = v = u + at \rightarrow s = s_i + ut + \frac{1}{2}at^2$$

Projectile Motion

Description: A projectile launched at an angle θ exhibits a parabolic path.
Problem Solving Strategy: Resolve ("Split") the v into its components, v_x and v_y, with $a_x = 0$ and $a_y = -g$, where g is the gravitational acceleration.

Components	x	y
u_0	$\frac{u_x}{cos(\theta)}$	$\frac{u_y}{sin(\theta)}$
Acceleration	0	-g
Velocity	$v_x = u_x$	$v_y = u_y - gt$
Displacement	$s_x = s_{0x} + u_x t$	$s_y = s_{0y} + u_y t - \frac{1}{2}gt^2$
Max height when $v_y = 0$		

II. MOMENTUM AND IMPULSE

Time allocation:
0.5h (Lecture) + 3h (Tutorial)
Learning Outcomes:

1. Define momentum and impulse, $\vec{J} = \vec{F}\Delta t$
2. Solve impulse/ impulse-momentum problems $\vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$
3. Determine impulse from $F - t$ graph
4. State the principle of linear momentum conservation, $\Delta(\sum\vec{p}) = 0 \rightarrow \sum\vec{p}_{final} = \sum\vec{p}_{initial}$
5. Apply $\frac{d}{dt}(\sum\vec{p}) = 0$ in 1D and 2D elastic and inelastic collisions.
6. Differentiate between elastic and inelastic collision

LO1 - Definitions:

Momentum	The vector product of mass of the body with it's velocity, $\vec{p} = m\vec{v}$.
Impulse	The change in momentum of a body with mass m when it changes velocity from $\vec{v}_{intial}$ to $\vec{v}_{final}$, $\vec{J} = \Delta\vec{p} = m(\vec{v}_{final} - \vec{v}_{initial}) = \vec{F}\Delta t$

LO3: Determine impulse from $F - t$ graph
Referring to $\vec{J} = \vec{F}\Delta t$, we'd deduce that the area under the $F - t$ graph give us the impulse.

LO4: Conservation of Linear Momentum
Statement: In a closed system, the total momentum is conserved regardless of the change in body mass of velocity. That is to say,

$$\Delta p = \vec{p}_{final} - \vec{p}_{initial} = 0 \rightarrow \vec{p}_{final} = \vec{p}_{initial}$$

LO5 and LO6: Application of momentum conservation law and categorizing collisions.
Begin with differentiating between elastic and inelastic collision:

Elastic	Obeys kinetic energy conservation, $\Delta(\sum E_{kinetic}) = 0$.
Inelastic	**Does not** obey kinetic energy conservation, $\Delta(\sum E_{kinetic}) \neq 0$

To solve problems, there are general key concepts to look for in the questions such as:

- A body comes to a stop $\rightarrow v_{final} = 0$.
- A body started at rest $\rightarrow v_{initial} = 0$
- sticks together $\rightarrow$ Inelastic equation, $\sum(\Delta E_{kinetic}) = 0$ not applicable here.

III. FORCES

Time allocation:
1h (Lecture) + 3h (Tutorial)
Learning Outcomes:

1. Identify forces (Weight, Tension, Normal, Friction, External) acting on a body in different situations.
2. Sketch Free Body Diagram (FBD)
3. Determine static and kinetic friction:

$$f_s \leq \mu_s N; \; f_k = \mu_k N$$

4. State Newton's Laws of Motion
5. Apply Newton's Laws of Motion

LO1, 4 types of forces and its definition:

1. Weight = Force exerted upon a body interacting with a gravitation field.
2. Tension force = Force transmitted axially through a massless one-dimensional continuous element.
3. Normal force = support force, perpendicular to the surface, exerted upon a body in contact with a stable object.
4. Frictional force = A force that stems from 2 rough surfaces that in is contact and moving relative to each other.

LO2, Sketching a free body diagram requires practice, however, herewith we present the directions of each type of forces:

1. Weight = towards the Earth/gravitational source.
2. Tension force = along the cable/string/cord and away from the body.
3. Normal force = Perpendicular to the surface the body is in contact with.
4. Frictional force = Against the direction of motion.

LO3, Determination of static and kinetic friction.
Static friction = Frictional force between two bodies that are in contact and not moving relative to each other.
Kinetic friction = Frictional force between two bodies that are in contact and moving relative to each other.
The equation for frictional force is

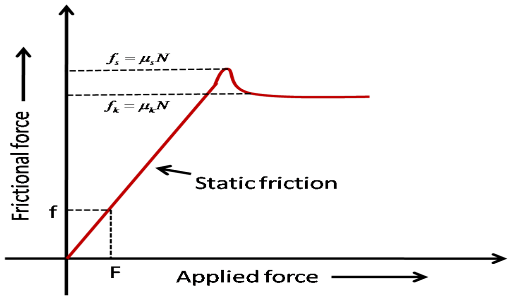
$$F_{friction} = \mu N$$

where the μ is friction coefficient. The value of this friction coefficinet depends on the material as well as whether or not the bodies are moving relative to each other.

Static friction are generally higher than kinetic friction (special cases exists! e.g. Rubber friction), and the reason is the asperities (roughness) interaction between 2 surfaces that are in contact. This interaction includes

- Asperities forms "cold welding" (intermolecular bonding).
- Asperities from both surfaces interlocks.

Both of which creates adhesion between the 2 surfaces. This adhesion must be overcome before the body can start moving relative to each other.



The diagram above shows how frictional force varies as applied force is increased for 2 surfaces in contact. The peak indicates the motion threshold which is the amount of force needed to start moving the body. This occurs at $\vec{F}_{friction} = \mu_{static}\vec{N}$. Once the body starts moving (relative to the surface it is in contact with), the frictional force decrease and becomes constant at some range of velocity/speed.

LO. 4 and 5, Newton's Laws of Motion:

1^{st} Law	An isolated body moves with constant velocity, $\vec{F} = 0 \Rightarrow \vec{v} = \textbf{constant}$
2^{nd} Law	The rate of change of linear momentum of a body is equal to the force acting on the body, $\vec{F} = \frac{d\vec{p}}{dt}$.
3^{rd} Law	Whenever two bodies interact, the force $\vec{F}_{21}$ which 1 exerts on 2 is equal and opposite to the force $\vec{F}_{21}$ which 2 exerts on 1, $\vec{F}_{12} = -\vec{F}_{21}$.

Application of these laws will be shown in tutorials.

IV. WORK, ENERGY AND POWER

Time allocation:
1.5h (Lecture) + 7h (Tutorial)
Learning Outcomes:

1. Define work done by a constant force, $W = \vec{F} \cdot \vec{s}$
2. Apply work done by a constant force and from a force-displacement graph
3. State the energy conservation principle
4. Apply energy conservation principle, involving mechanical energy and heat energy due to friction
5. State and apply the work-energy theorem, $W = \Delta K$
6. Define and use:
   - average power, $P_{av} = \frac{\Delta W}{\Delta t}$
   - Instantaneous Power, $P = \vec{F} \cdot \vec{v}$

LO1, LO3,LO6: Definitions and statements:

1. Work done, W, can be defined as the scalar product of force, F, and the displacement, $\vec{s}$, :
$$W = \vec{F} \cdot \vec{s}$$
2. Energy conservation statement - Total energy of an isolated system is conserved, that is $\Delta \sum E = 0$.
3. Average power, P_{av}, can be defined as the average amount of work done,ΔW, in a time interval, Δt, :
$$P_{av} = \frac{\Delta W}{\Delta t}$$
4. Work-energy theorem states that the change in kinetic energy is equal to the work done by the force, that is:
$$W_{i \to f} = \Delta E_k = \frac{1}{2} m(v_f^2 - v_i^2)$$
5. Instantaneous power, $P_{instant}$, is the scalar product of force, $\vec{F}$, and velocity, $\vec{v}$,:
$$P_{instant} = \vec{F} \cdot \vec{v}$$
6. Some useful equations:
   - $E_{kinetic} = \frac{1}{2} mv^2$
   - Generally, $F = -\frac{dE_{pot.}}{dx}$
   - $E_{grav.pot.} = mgh$
   - $E_{spring\ pot.} = \frac{1}{2} kx^2$

Note that for the definition of work done, W, and instantaneous power, $P_{instant}$ is a scalar produce of the two vector, this is a reminder that we are only interested in the components of force that is parallel to the displacement and that the angle between them must be taken into account when it is **not zero**.

LO2 - Force-displacement graph:
The area under the force-displacement graph represents the work done. The "proof" can be demonstrated if one were to consider the equation $dW = F \cdot dr$ and integrate both sides or sum up sections of the areas under the graph.

V. CIRCULAR MOTION

Time allocation:
1h (Lecture) + 2h (Tutorial)
Learning Outcomes:

1. Describe uniform circular motion
2. Convert units:

Degrees(^{o}) and radians $\rightleftharpoons$ revolution/rotation

3. Define centripetal acceleration
4. Solve centripetal force for uniform circular motion problems for the following cases:
   (a) horizontol circular motion
   (b) vertical circular motion
   (c) conical pendulum

LO2- Unit conversion:

$$1 \text{ revolution/rotation} = 360^o = 2\pi \text{ radian}$$

LO1- Uniform circular motion description:
Uniform circular motion is the motion of an object travelling at a constant (uniform) speed on a circular paths.An object travelling in a circular path constantly changes it's direction. Constant changes in its direction indicates a constant change in the velocity of the object. This constant change in object velocity is only allowed if there exist an acceleration, which we call *centripetal acceleration.*

LO2- Centripetal acceleration = a property in a body traversing in circular path, that is radially directed to the circle centre and has a magnitude, $|a_c| = \frac{v^2}{r}$.

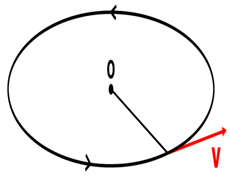
Using Newton's 2^{nd} Law of Motion, $\vec{F} = m\vec{a}$, we can then define centripetal force as follows:

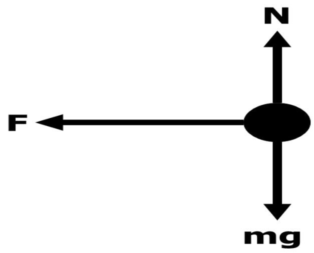
$$F_c = \frac{mv^2}{r}$$

with m = mass of body, v = linear velocity tangent to the circle and r = circle radius.

LO4- 3 cases to be considered:

1. Horizontal Circular Motion
   We can start by sketching out the path of the object as well as its Free Body Diagram:





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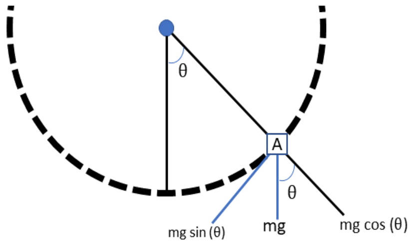
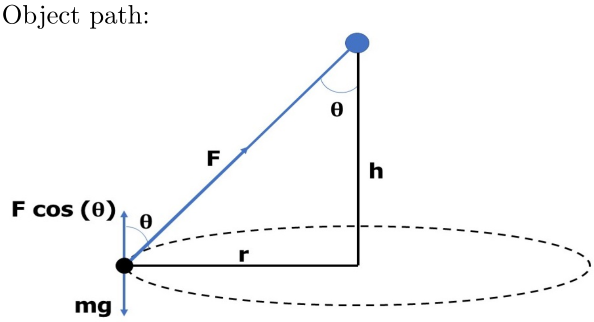
From these 2 diagrams, we can deduce that $F_c = \frac{mv^2}{r}$ and $N = mg$.

2. Vertical Circular Motion
   Again, we start with with sketching out the path of the object, but to ease calculations, only the bottom half of the path is presented here as it will be generalized to the rest of the pathway:



Here we see that $F_c = \frac{mv^2}{r} = T - mg\ cos(\theta)$ for a string and $F_c = \frac{mv^2}{r} = N - mg\ cos(\theta)$ for a body on a circular track, e.g. roller coaster ride.

3. Conical Pendulum
   Object path:



Here we see that $F\ cos(\theta) = mg$ and $F\ sin(\theta) = \frac{mv^2}{r}$. Three equations that may be obtained from this are:

(a) $tan\,(\theta) = \frac{v^2}{rg} = \frac{r}{h}$, from dividing the two equations,
(b) $F = mg\sqrt{1 + (\frac{r}{h})^2}$, from the fact that $sin^2(\theta) + cos^2(\theta) = 1$, and,
(c) Period, $T = 2\pi\sqrt{\frac{h}{g}}$, from $v = \omega r = \frac{2\pi}{T}r$ and $tan\,(\theta) = \frac{v^2}{rg}$

VI. NEWTONIAN GRAVITATION

Time allocation:
1h (Lecture) + 4h (Tutorial)
Learning Outcomes:

1. State and use Newton's Law of Gravitation, $F = G\frac{Mm}{r^2}$
2. State and use gravitational field strength, $a_g = G\frac{M}{r^2}$
3. Define and use gravitational potential energy, $U = -G\frac{Mm}{r}$
4. Derive and use escape velocity equation, $v_{esc}\sqrt{\frac{2GM}{R}} = \sqrt{2gR}$
5. Derive and use satellite motion equation:
   - Velocity, $v = \sqrt{\frac{GM}{r}}$
   - Period, $T = 2\pi\sqrt{\frac{r^3}{GM}}$

LO1, LO2- Newton's Law of Gravitation (F_g), gravitational field strength (a_g):

$$F_g = m(a_g) = m(G\frac{M}{r^2})$$

where m and M are masses of the interacting bodies, G is the Newtonian gravitational constant, and r is the displacement between the interacting bodies.

In words, gravitational field strength at a point is the gravitational force exerted experienced per unit mass at that point.

LO3-
Gravitational Potential Energy = energy of a body due to its position within the gravitational field. Derivation:

$$\begin{aligned} F_g &= -\frac{dU_g}{dr} \\ U_g &= -\int F_g\ dr \\ &= -GmM\int r^{-2}dr \\ &= -\frac{GMm}{r} \end{aligned}$$

LO4 - Derivation of escape velocity:
Idea: For a body overcome the gravitational attraction of a more massive body, the kinetic energy of that body must be equal to, or more than the gravitational potential energy, i.e. $E_k \geqslant U_g$.

The former describes a body escaping the gravitational attraction with the final velocity of zero.

$$\begin{aligned} E_k &= -U_g \\ \frac{1}{2}mv_{esc}^2 &= \frac{GmM}{R^2} \\ v_{esc} &= \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \end{aligned}$$

LO5- Satellite motion
Assumption: Satellite motion has a circular path, this simplifies equations.
Referring to the past chapter, we utilize centripetal force and compare it with the gravitational force equation to obtain the equation for satellite velocity:

$$\begin{aligned} F_{centripetal} &= F_{grav} \\ \frac{mv^2}{r} &= \frac{GMm}{r^2} \\ v &= \sqrt{\frac{GM}{r}} \end{aligned}$$

The period of the satellite motion can be derived by utilizing

$$v = \frac{2\pi r}{T}$$

to show that period of satellite is

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

VII. ROTATION OF RIGID BODIES

Time allocation:
1h (Lecture) + 4h (Tutorial)
Learning Outcomes:

1. Define and use:
   - Angular displacement, θ.
   - Average angular velocity, ω_{ave}.
   - Instantaneous angular velocity, ω.
   - Average angular acceleration, α_{ave}.
   - Instantaneous angular acceleration, α.
2. Relate rotational parameters with their corresponding linear parameters:
$$s = r\theta;\ v = r\omega;\ a_{trans.} = r\alpha,$$
$$a_{centri.} = r\omega^2 = \frac{v^2}{r}$$
3. Solve constant angular acceleration rotational motion problems:
$$\omega = \omega_o + \alpha t;$$
$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_o^2 + 2\alpha\theta$$
4. Define torque, $\vec{\tau} = \vec{r} \times \vec{F}$.
5. Solve uniform rigid body equilibrium problems.
6. Define and use the moment of inertia of a uniform rigid body (sphere, cylinder, ring, disc and rod).
7. State and use torque, $\tau = I\alpha$.
8. Define and use angular momentum, $L = I\omega$
9. State and use the principle of angular momentum conservation.

Note: For the most part of this chapter, the units are of radians, i.e. the units for angular displacement and angular velocity are radians (rad) and radians per second ($rads^{-1}$) respectively.

LO1, LO4, LO6, LO8 - Definitions:

1. LO1:
   - **Angular displacement,** θ = the angle through which a rigid object rotates about a fixed axis
   - Average angular (velocity, ω_{ave} / acceleration, α_{ave}) = angular (displacement / velocity) divided by the elapsed time during which the (displacement / velocity change) occurs:
$$\omega = \frac{\Delta\theta}{\Delta t};\ \alpha = \frac{\Delta\omega}{\Delta t}$$
   - Instantaneous angular (velocity, ω / acceleration, α) = the angular (velocity / acceleration) that exists at any given instant.
2. LO4 - Torque, τ, = a measure of force acting on a body rotating about an axis:
$$\vec{\tau} = \vec{r} \times \vec{F} = rFsin\theta$$
$$\tau = I\alpha = \frac{dL}{dt}$$
3. LO6- Moment of Inertia = a measure of a body's resistance to rotation, quantified by ratio of the net angular momentum of a system to its angular velocity along a principal axis, $\frac{L}{\omega}$.
$$I = \int_0^M r^2 dm$$
Common moments of Inertia:

Shape	Rotation Axis	Equation
Sphere (solid)	centre	$\frac{2}{5}MR^2$
Sphere (solid)	$\perp$ surface	$\frac{7}{5}MR^2$
Cylinder (solid) /disk	symmetry axis	$\frac{1}{2}MR^2$
Cylinder (solid)	Central diameter	$\frac{1}{4}MR^2 + \frac{1}{12}ML^2$
Ring	symmetry axis	MR^2
Rod	center	$\frac{1}{12}ML^2$
Rod	End	$\frac{1}{3}ML^2$

4. LO8 - Angular Momentum, L = the product of the body's moment of inertia, I, and its angular velocity, ω, with respect to that axis:
$$L = I\omega$$

LO2- Relate rotational parameters with their corresponding linear parameters.

Rotational parameters	Linear counterparts	Equation
Angular displacement, $\theta[rad]$	Linear displacement, $s[m]$	$s = r\theta$
Angular velocity, ω	linear velocity, v	$v = r\omega$

In this chapter, when speaking of acceleration, there are 2 types of acceleration - **translational acceleration**, $\alpha_{trans.}$, and **centripetal acceleration**, α_{centri}.

The latter is the same as the case for circular motion, $\alpha_{centri.} = \frac{v^2}{r} = r\omega^2$. The former, refers to the tangential component of acceleration. If $\alpha_{trans.} = 0$, the the motion is a **uniform circular motion**.

LO3- Constant angular accceleration rotational motion problems

The following equations describes rotational motion of which the angular acceleration is constant:

$$\omega = \omega_o + \alpha t;$$
$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

They resemble equations for constant acceleration, just with parameters redefined in the context of rotational motion.

LO5- Uniform Rigid Body Equilibrium:

The concept of the uniform rigid body Equilibrium is based on a few key ideas:

1. *Uniform* refers to a special case in which mass distribution is a constant.
2. *Rigid body* refers to the condition of the body to not shape its shape under external force, consequently neglecting strain/stress in the calculations.
3. *Equilibrium* refers to condition in which the sum of externally applied torques is 0, that is $\sum \tau = 0$ (and therefore $\sum F_x = 0$and $\sum F_y = 0$).

Guidelines to solving uniform rigid body equilibrium problems:

1. Select the object you'll be applying the equations to.
2. Draw free body diagram to show the external forces that acts upon the object chosen.
3. Make your choice of axes and resolve the external forces to its components on the chosen axes.
4. Apply $\sum F_x = 0$and $\sum F_y = 0$.
5. Select a rotation axis.
6. Apply $\sum \tau = 0$
7. Rinse and repeat until goal achieved.

LO9- Principle of angular momentum conservation:
Statement: When $\tau_{external} = 0$, $L_{final} = L_{initial}$.

VIII. SIMPLE HARMONIC MOTION

Time allocation:
1h (Lecture) + 8h (Tutorial)
Learning Outcomes:

1. Explain SHM
2. Solve problems related to SHM displacement equation, $x = A\ sin\ (\omega t)$
3. Derive equations :
   - Velocity, $v = \frac{dx}{dt} = \pm\omega\sqrt{A^2 - x^2}$.
   - Acceleration, $a = \frac{dv}{dt} = -\omega^2 x$.
   - Kinetic energy, $K = \frac{1}{2}m\omega^2(A^2 - x^2)$.
   - Potential energy, $U = \frac{1}{2}m\omega^2 x^2$
4. Emphasise the relationship between total SHM energy and the amplitude
5. Apply equations for $v, a, K$and U for SHM
6. Discuss the following graphs:
   - Displacement-time
   - Velocity-time
   - Acceleration-time
   - Energy-displacement
7. Use expression for SHM period for simple pendulum and single spring

LO 1 - Simple Harmonic Motion (SHM)
A SHM is a periodic motion in which the restoring force is directly proportional to its displacement and acts the opposite direction to the its displacement. It has the following general equation:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where the ω is the angular frequency.

Note that the negative sign in the general equation tells us that the force is acting on the opposite direction to the displacement. The solution to the SHM general equation is

$$x(t) = Asin(\omega t \pm \phi)$$

where A is the amplitude of oscillation and ϕ is the phase shift (which we will not be concerned of).

LO7- Single pendulum and single spring
One example of this type of motion is seen in the Hooke's Law describing the amount of reaction force of an extended/compressed spring, aptly giving the equation:

$$F = ma = m\frac{d^2x}{dt^2} = -kx$$

From this, we will obtain the equation

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

and obtain $\omega^2 = \frac{k}{m}$

Keeping in mind that the period-angular frequency equation, $T = \frac{2\pi}{\omega}$ gives us

$$T = 2\pi\sqrt{\frac{m}{k}}$$

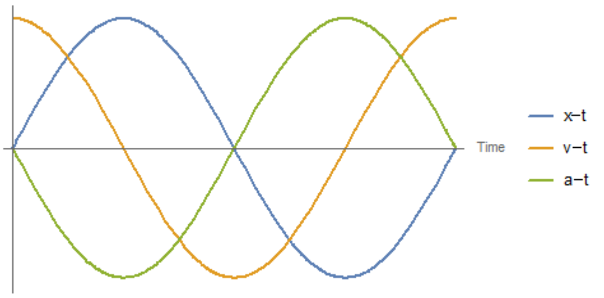
Another example noted here is the simple pendulum case, in which it has the equation $F = -mg(\frac{x}{L})$ describing the restoring force, where L is the length of the pendulum string. (The small angle approximation, $sin\ (\theta) \approx \theta = \frac{x}{L}$ is used here).
We then obtain the following equations:

$$\frac{d^2x}{dt^2} = -\frac{g}{L}x;$$
$$\omega^2 = \frac{g}{L}$$
$$T = 2\pi\sqrt{\frac{L}{g}}$$

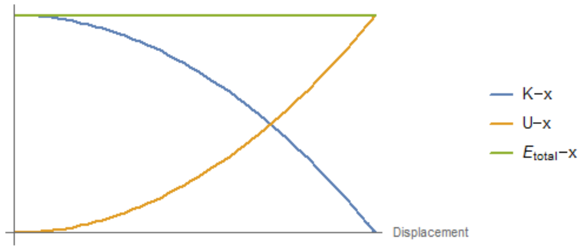
LO2- Deriving equations:
Equations for velocity, kinetic energy and potential energy can be done by knowing these equations:

- Velocity, $v = \frac{dx}{dt} \Rightarrow v = \pm\omega\sqrt{A^2 - x^2}$
- Kinetic Energy, $K = \frac{1}{2}mv^2 \Rightarrow K = \frac{1}{2}m\omega^2(A^2 - x^2)$
- Potential Energy, $U = \frac{1}{2}kx^2 \Rightarrow U = \frac{1}{2}m\omega^2x^2$

LO4- Conservation of energy states that $E_{[total]} = K + U$, utilizing this and applying the equations obtained above, gives $E_{total} = \frac{1}{2}kA^2$. We can see that the $E_{total} \propto A^2$ **LO6-** Graphs:



Displacement, velocity and acceleration against time.
$v \rightarrow 0$ twice, once at x_{max} and the second time when a_{max}.



Energy variation with displacement.
K decreases and U increases to keep E_{max} constant, as x increases.

IX. WAVES

Time allocation:
2.5h (Lecture) + 10h (Tutorial)
Learning Outcomes:

1. Define wavelength, λ, and wave number, k.
2. Solve problems related to the equation of progressive wave, $y(x,t) = Asin(\omega t \pm kx)$
3. Discuss and use the particle vibrational velocity and wave propagation velocity (group velocity)
4. Discuss the graphs of:
   - displacement-time, $y - t$
   - displacement-distance, $y - x$.
5. State the principle of superposition of waves for constructive and destructive interferences.
6. Use the standing wave equation, $y = Acos(kx)sin(\omega t)$
7. Discuss progressive and standing wave.
8. Define and use sound intensity
9. Discuss the dependence of intensity on amplitude and distance from a point source by using graphical illustrations.
10. Solve problems related to the fundamental and overtone frequencies for:
    - stretched string
    - air columns (open and closed)
11. Use wave speed in a stretched string, $v = \sqrt{\frac{T}{\mu}}$.
12. State Doppler effect for sound waves.
13. Apply Doppler effect equation, $f_{apparent} = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$, for relative motion between source and observer. Limit to stationary observer and moving source, and vice versa.

LO1, LO8- Definitions:

1. LO1: Wavelength, λ, is the horizontal length of one wave cycle - the distance over which wave's shape repeats.
2. LO1: Wave number, k, is the number of waves per unit distance, $k = \frac{2\pi}{\lambda}$.
3. LO8: Sound intensity, I, is defined to be the acoustic power that passes perpendicularly through a surface divided by the area of that surface, $I = \frac{P}{A}$

LO2, LO6, LO7- Waves:

1. Progressive wave:
   Sinusoidal equation: $y(x,t) = Asin(\omega t \pm kx)$
2. Standing wave:
   $y(x,t) = Acos(kx)sin(\omega t)$

where $y(x,t)$ is the displacement on an element at position x at time t, A is the amplitude, ω is the angular frequency, k is the wavenumber.

LO3- Velocities:
The aim here is to distinguish between particle vibrational velocities and wave propagation velocity. Picture, in your head, water waves. The water molecule, moves "up and down". This motion of "up and down" then causes the next water molecule to move "up and down". The velocity of moving "up and down" is called ***particle vibrational velocity*** whereas how fast the next water molecule reacts to the "up and down motion" of the water molecule before it is called ***wave propagation velocity***.
Essentialy,

- particle vibrational velocity = how fast the particle vibrate.
- wave propagation velocity = how fast the wave propagates.

LO4- Graphs:
In the displacement-time graph, the distance between peaks(troughs) is the **period** of the wave. In the displacement-distance graph, the distance between peaks (troughs) is the **wavelength** of the wave.

LO5- Principle of wave superposition:
The principle of (linear) wave superposition simply states that when multiple waves are present simultaneously at the same place (=interfere), the resultant disturbance is the sum of the disturbances from the individual waves.
Constructive (/Destructive) interference = when the interference results in a wave with amplitude higher(/lower) than the amplitude of the individual waves.

LO9- Sound Intensity Dependence:
Sound intensity is directly proportional to the square of the amplitude of the sound wave, $I \propto A^2$.
For a point source, one may assume the sound wave propagates uniformly in all directions and therefore may take the area of a surface of the sphere for the area, this give $I = \frac{P}{4\pi r^2}$. This shows the relation $I \propto \frac{1}{r^2}$.

LO10, LO11- Standing wave tones:

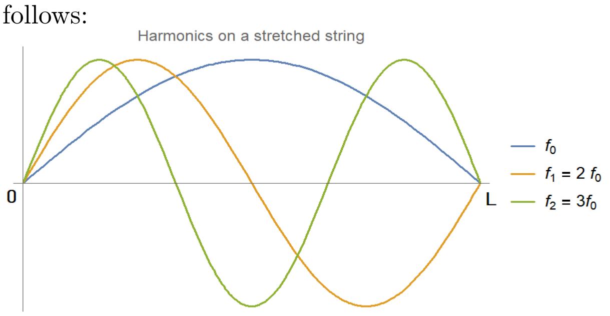
Nodes = points on a waveform where that does not vibrate at all.
Antinodes = points on a waveform where maximum vibration occurs.
Fundamental frequency, f_o = the lowest frequency of a periodic waveform.
Overtone = harmonic frequencies above the fundamental, $f = nf_o$ where $n \in \mathbb{Z}$.

Here, it is crucial one particular aspect, which is the naming of certain terms, that is worth mentioning,

Overtones	**Harmonics**
Fundamental	1^{st} Harmonic
1^{st} overtone	2^{nd} Harmonic

and so on and so forth. We can see that the nth overtone is the $(n+1)th$ harmonic.

In a **stretched string**, the boundary condition imposed is that the ends of the string must be a node, that is $y(0, L) = 0$. The fundamental and overtones are as follows:



As we can see, the string length is in the integer value of half a wavelength, that is $L = n\frac{\lambda}{2}$. We can then write down frequencies as a function of length using $f = \frac{v}{\lambda}$, where v is the wave speed to obtain:

$$f_n = \frac{nv}{2L} \text{ where } n \in \mathbb{Z}$$

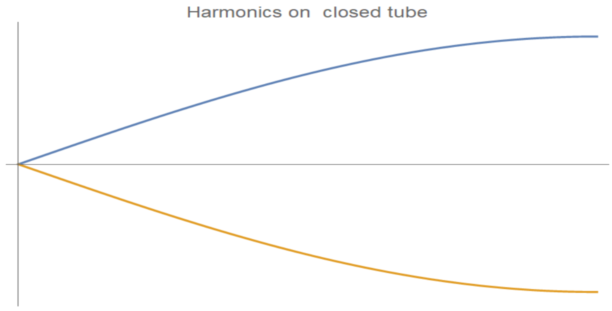
The wave speed on a string is given by:

$$v^2 = \frac{T}{\mu}$$

where T is the tension, $\mu (= \frac{m}{L})$ is the linear mass density.

For air columns, there are 2 situations to be considered:

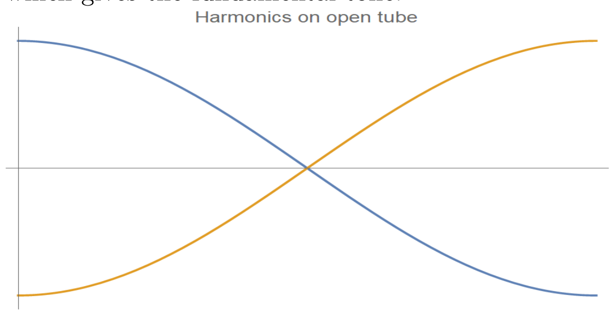
1. Tube with one open end and one closed end ("Closed Tube")
Boundary condition- a node on the closed end and an antinode on the open end, which gives the fundamental tone:



We can see that these boundary conditions requires $L = n\frac{\lambda}{4}$ which gives us:

$$f_n = (2n+1)\frac{v}{4L} \text{ where } n \in \mathbb{Z}$$

2. Tube with 2 open ends ("Open Tube")
Boundary condition- Antinodes on the open ends, which gives the fundamental tone:



We can see that these boundary conditions requires $L = n\frac{\lambda}{2}$ which gives us:

$$f_n = \frac{nv}{2L} \text{ where } n \in \mathbb{Z}$$

LO12, LO13- Doppler Effect:
Doppler Effect = change in frequency detected by an observer due to difference in velocity of the sound source and observer with respect to the medium of sound propagation.

The Doppler effect can be quantitatively described using the equation:

$$f_{apparent} = f_{source}\left(\frac{v \pm v_{observer}}{v \mp v_{source}}\right)$$

Where v is the sound wave speed ($\approx 343ms^{-1}$ in air of 20^oC).

In the numerator, the (plus/minus) sign is used when the observer moves (towards the / away from) source. In the denominator, the (minus/plus) sign is used if source moves (towards the / away from) the observer.

X. SOLID DEFORMATION

Time allocation:
2.5h (Lecture) + 2h (Tutorial)
Learning Outcomes:

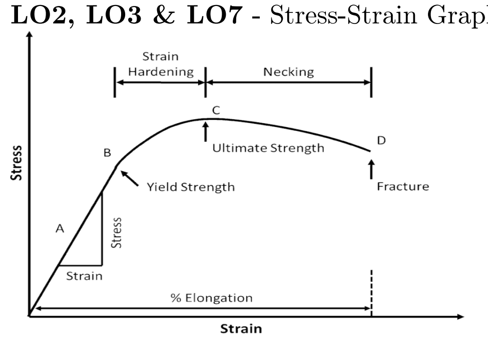
1. Distinguish between stress and strain for tensile and compression force.
2. Discuss the graph of stress-strain for a metal under tension.
3. Discuss elastic and plastic deformation.
4. Discuss graph of force-elongation for brittle and ductile materials.
5. Define Young's modulus.
6. Discuss strain energy from the force-elongation graph.
7. Discuss strain energy per unit volume from stress-strain graph.
8. Solve Young's modulus problems.

LO1, LO5- Definitions:

1. Stress, δ = the measure of internal forces acting on neighbouring particles in a continuous media, quantified by the ratio of the magnitude of the force to the area, $\delta = \frac{F}{A}$.
2. Strain, ϵ = the measure of the deformation of the material, quantified by the ratio of the change in a quantity to the quantity, $\epsilon = \frac{\Delta L}{L}$.
3. Young's modulus, Y = a measure of the stiffness of solid material quantified by the the ratio of stress to strain, $Y = \frac{\delta}{\epsilon}$.

LO2, LO3, LO4, LO6, LO7- Graphs:

1. **LO2, LO3 & LO7** - Stress-Strain Graph:



Discussion:
Young's modulus is applicable between the beginning of the graph to the **proportional limit** (yield strength). In this part of the graph, it shows a linear relation between stress and strain, by which one can calculate the Young's modulus from its gradient. It is in this region an object goes through **elastic deformation**.

Elastic deformation = deformation in which the shape change due to external force is **reversible**, temporary shape change

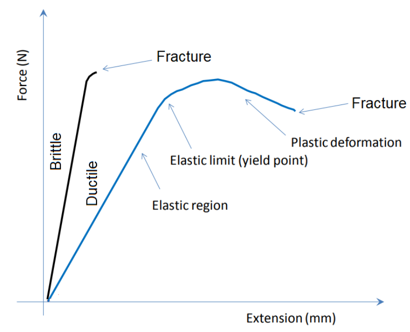
Non-linearity beyond the proportional limit represent the failure of Hooke's Law and the beginning of **plastic deformation**

Plastic deformation = deformation in which the the shape change due to the external force is **irreversible**, permanent shape change.

One can observe from the graph that the stress continues to increase with strain until it reaches a maximum, a point aptly named the **Ultimate Tensile Strength**(UTS). Between the yield strength and the UTS, the strain accumulates to contribute to **strain hardening**, which is caused by dislocation of crystal planes.

Beyond the UTS, necking occurs until a fracture takes place. Necking is simply when the cross-sectional area begins to become significantly lower than the average.

2. **LO4 & LO6** - Force-Elongation Graph:



Discussion:
The graph above shows that difference in the force-elongation curve for a brittle and ductile material. The brittle material fracture point lies close to the elastic limit where as for the ductile material, fracture point is quite an extent from the elastic limit. This means ductile goes through significant plastic deformation before fracturing. This is because ductile materials are able to contain much more strain energy than brittle materials. Necking also does not take place prior to fracturing in a brittle material.
From the difference in gradient, we can deduce that it takes a lot more force to extend brittle material than the ductile material, that is $Y_{ductile} > Y_{brittle}$. This allows us to say that under the same stress, the strain energy is lower in the brittle material,

$Y_{ductile} - Y_{brittle} > 0 \Rightarrow \delta\left(\frac{1}{\epsilon_{ductile}} - \frac{1}{\epsilon_{brittle}}\right) > 0$ which is obeyed iff $\epsilon_{brittle} < \epsilon_{ductile}$.

Part Two: Heat and Thermodynamics

XI. HEAT

Time allocation:
1h (Lecture) + 3h (Tutorial)
Learning Outcomes:

1. Define heat conduction
2. Solve problems related to rate of heat transfer, $\frac{dQ}{dt} = -kA\frac{dT}{dx}$, through a cross-sectional area. (Maximum 2 insulated objects in series)
3. Discuss temperature-distance, $T-x$ graph for for heat conduction through insulated and noninsulated rods. (Maximum two rods in series)
4. Define coefficient of linear, area and volume thermal expansion.
5. Solve problems related to thermal expansion of linear, area and volume (include expansion of liquid in a container):

$$\Delta L = \alpha L_o \Delta T;\ \beta = 2\alpha;\ \gamma = 3\alpha$$

LO1, LO4 & LO5- Definitions:

1. LO1 - Heat Conduction = Conduction is the process whereby heat is transferred directly through a material, with any bulk motion of the material playing no role in the transfer.
2. LO5 - Coefficient of (linear/area/volume) expansion = a measure of (linear/area/volumetric) fractional change per degree change in temperature at a constant temperature.

$$\Delta L = \alpha L_o \Delta T,$$
$$\Delta A = \beta A_o \Delta T,$$
$$\Delta V = \gamma V_o \Delta T,$$
$$\beta = 2\alpha;\ \gamma = 3\alpha$$

where L = length, A = Area, V = volume, T = temperature, $\{\alpha, \beta, \gamma\}$ = coefficient of {linear, area, volumetric} expansion.

LO2- Rate of heat transfer equation:
The equation for the rate of heat transfer is given by:

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

where $\frac{dQ}{dt}$ = heat change in a given body with respect to time, A = area of contact, $\frac{dT}{dx}$ = temperature gradient and k = heat transfer coefficient.
For a metal of length L and cross sectional area A, the heat conducted in time t is

$$Q = kAt\frac{\Delta T_{ends}}{L}$$

where ΔT_{ends} = temperature difference between ends of the metal.
This could be extended for 2 insulated objects in series (assuming steady state conduction) to give

$$Q = tA(\Delta T_{ends})\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)^{-1}$$

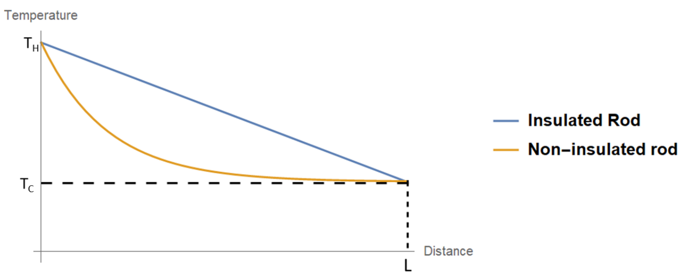
where the subscripts represents the objects involved.

This is possible because the assumption of steady state tells us that

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt}$$

$\frac{dQ}{dt}$ may be thought of as "heat currents" and functions analogously to an electrical system with resistances in series.So it can be said that the "heat current" going through both the material must be equal.

LO3- Temperature-distance graph:



For the insulated rod, the heat loss through the sides of the rod is negligible relative to the non-insulated rod. This means that the heat flows only through the cross sectional area from the hot end to the cooler end. What this means is that whilst the insulated rod temperature drops linearly with distance, the temperature drops exponentially for the non-insulated rod.

XII. KINETIC MODEL OF GAS

Time allocation:
1.5h (Lecture) + 4h (Tutorial)
Learning Outcomes:

1. Solve problems related to ideal gas equation, $pV = nRT$.
2. Discuss the following graphs of an ideal gas:
   - p-V graph at constant temperature
   - V-T graph at constant pressure
   - p-T graph at constant volume
3. State the assumptions of kinetic theory of gases.
4. Discuss root mean square (rms) speed of gas molecules
5. Solve problems related to root mean square (rms) speed of gas molecules.
6. Solve problems related to the equations:
   $$pV = \frac{1}{3}Nmv_{rms}^2;\ p = \frac{1}{3}\rho v_{rms}^2$$
7. Discuss translational kinetic energy of a molecule, $K_{tr} = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}k_BT$
8. Discuss degrees of freedom, f for monoatomic, diatomic and polyatomic gas molecules.
9. State the principle of equipartition of energy.
10. Discuss internal energy of gas.
11. Solve problems related to internal energy, $U = \frac{1}{2}fNk_BT$

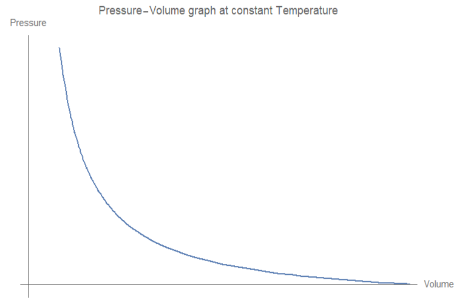
LO1 & LO2- Ideal Gas Equations:
The ideal gas equation is given by

$$pV = nRT$$

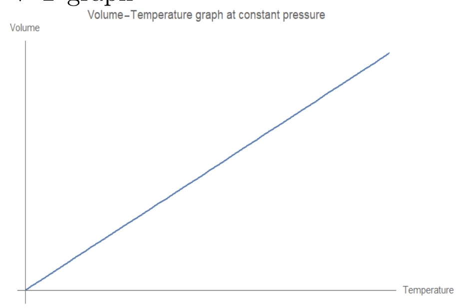
where p = absolute pressure of the gas , V = volume, T = temperature in Kelvin, R = universal gas constant, n = number of moles.
Standard temperature and pressure refers to $273K$ and $1atm$.

Graphs:

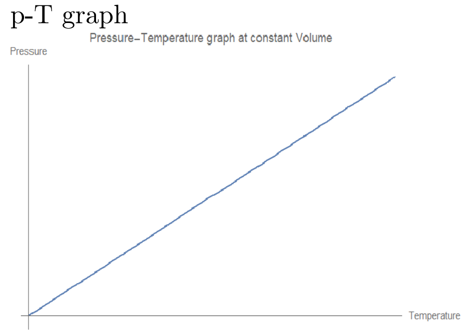
1. p-V graph



2. V-T graph



3. p-T graph



Comments: The relationships between variables shown by the graphs above are fairly obvious. That is the volume-temperature and pressure-temperature relations are linear in nature, assuming pressure is constant in the former and volume is constant for the latter. The pressure-volume graph, on the other hand, has a inversely proportional relation.

LO3- Assumptions of the kinetic theory of gases:
Gases are composed of **large number** of **non-null mass point-like** particles that **obeys Newton's laws** of motion and **elastically collides** with each other, of which their **average kinetic energy** solely depends on the **system's absolute temperature**.

It's called the **Kinetic** Theory of gases because the notion of kinetic energy is applied through Newton's laws of motion in collisional force between particles. The speed distribution curves follows that of **Maxwell-Boltzmann's distribution**.

LO4 & LO5- Root mean square (rms) speed of gas molecules:
The idea of rms speed of gas molecules stems from a notion of the same name in the study of statistics. It stems from the assumption for large number of particles, not all gas particles has the same speed, but its distribution follows that of what is called as the **Maxwell-Boltzmann distribution**. These gas particles all moves in random direction. Speaking of the particle velocities is then difficult due to directional component. It is easier then to consider the translational kinetic energy of the particles to equal to the average translational kinetic energy, such that $\frac{1}{2}mv^2 = \overline{\frac{1}{2}mv^2}$. This is achievable if one were to consider the root-mean-square speed of particles, v_{rms} which is equal to $\sqrt{\overline{v^2}}$. Note that $\overline{v^2} \neq \overline{v}^2$.

LO6:
Post-introduction of the rms speed, it might be useful (& necessary!) for one to cast the ideal gas equation with rms speed as one of its variable. To do so, one begins with $pV = nRT$, and then with consideration of notions of force (ipso facto utilization of Newton's Law of Motion) onto a wall by a cloud of gas molecules, one comes to the conclusion of the following equation:

$$pV = \frac{1}{3}Nmv_{rms}^2 \text{ and } p = \frac{1}{3}\rho v_{rms}^2$$

where the mass density, $\rho = \frac{Nm}{V}$ and number of particles, $N = n \times N_A$.

LO7 & LO9- Translational kinetic energy of a molecule and the equipartition theorem:
Moleculate move, and they move until they hit stuff (wall, another molecule etc.) but how much energy do they have when they move? Well, we know that kinetic energy,$K_{tr} = \frac{1}{2}mv^2$, so we know that $\overline{K_{tr}} = \frac{1}{2}mv_{rms}^2$. We can then have the following equation

$$pV = \frac{2}{3}N\overline{K_{tr}}$$

which when equated to the ideal gas equation gives

$$K_{tr} = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}k_BT$$

where $k_b = \frac{R}{N_A}$. This equation is important because it tells you that the kinetic energy of the molecules is solely dependent on the temperature (in Kelvin). In fact, this fact, $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$, is so crucial because it is the direct consequence of the ***classical equipartition theorem*** .

This theorem states that for every degree of freedom that a gas molecule has, it contributes to $\frac{1}{2}k_BT$ to the average energy. Since the velocity components are found in 3 directions, then the kinetic energy is $3 \times \frac{1}{2}k_BT$.

LO8, LO10 & LO11- Degrees of freedom and internal energy:
In the discussion of degrees of freedom, it is important for one to understand what it means to have N degrees of freedom. For gas molecules, degrees of freedom can be defined to be the number of physical variables that gives a particular system its characteristics. Mathematically, a degree of freedom is is any dynamical variable that contributes a squared term to the expression for the total energy of the molecule (e.g. $E_k = \frac{1}{2}mv^2$, $E_{rotation} = \frac{1}{2}I\omega^2$, $E_{hooke} = \frac{1}{2}kx^2$).

Classically, the calculation of internal energies is based on the number of degrees of freedom of the gas molecules, obeying

$$U = \frac{1}{2} f N_A k_B T$$

. To show its relationship with internal energy of a given system, let us consider 3 cases of the following:

1. Monatomic gas molecules:
For Monatomic gas molecules (e.g. Ne gas), the calculations are fairly straight forward. The assumption here is that there are **no molecular rotation** or **molecular vibration**. This means that the only contributor to the internal energy is the **translational kinetic energy**. The gas molecules here has 3 directions in which it can translates in - (x, y, z) direction. Then the **total internal energy for n moles of monoatomic gas molecules** is simply

$$U = K_{tr} = n \times \frac{3}{2} k_B T = \frac{3}{2} nRT$$

2. Diatomic gas molecules:
For the diatomic gas molecules (e.g. H_2 gas), there are 6 degrees of freedom - namely 3 translational motion and 2 corresponds to the rotational motion (about the x- and y-axis). Therefore, the **total internal energy for n moles of diatomic gas molecules** is given by

$$U = K_{tr} + E_{rotation} = n\frac{5}{2} RT$$

*This author would like to point out that this calculation is only limited to classical treatment of the internal energy of gases, which completely disregards the **quantization of energy** and leads to the consideration of vibrational energy. This consideration has been experimentally showed to take effect for Cl_2 gas at around 1500K, where the internal energy follows $\frac{7}{2}RT$. But this consideration is ignored at the current level. This represents the failure of classical physics.

3. Polyatomic gas molecules:
Essentially, one has to ask how many degrees of freedom (f) does the gas molecules have, break it down to its individual components (K_{tr}, $E_{rotation}$, etc...) and use the equipartition theorem. But classically, $U = \frac{1}{2} f N k_B T$ is sufficient.

XIII. THERMODYNAMICS

Time allocation:
1.5h (Lecture) + 3h (Tutorial)
Learning Outcomes:

1. State the first law of thermodynamics.
2. Solve problem related to first law of thermodynamics.
3. Define the following thermodynamics processes:
    - Isothermal
    - Isochoric
    - Isobaric
    - Adiabatic
4. Discuss p-V graph for all the thermodynamic processes.
5. Discuss work done in isothermal, isochoric and isobaric processes.
6. Solve problem related to work done in
    - isothermal process,
      $W = nRT\ ln\ \frac{V_2}{V_1} = nRT\ ln\ \frac{p_1}{p_2}$;
    - isobaric process,
      $W = \int p\, dV = p\,(V_2 - V_1)$;
    - isochoric process,
      $W = \int p\, dV = 0$.

LO1, LO2 & LO4- First law of thermodynamics:
The **first law of thermodynamics** states that:
Changes between states thermal equilibrium leads to internal energy change in the system that obeys

$$\Delta U = Q + W$$

in which Q = heat transferred ***to*** the system and W = total work done ***on*** the system.
Disclaimer:** Some reference texts may show $-W$ instead of $+W$ and this is due to defining W as the total work done *by* the system. But we choose this form as it clearly shows the *conservation of energy principle.* That is when heat is added to the system, the internal energy increase or when work is done on the system, the internal energy also increases.**

Solving problems related to the first law of thermodynamics requires certain strategies. Irregardless of the strategies involved, it might be useful to know that

1. for heat energies stemming from temperature change, $Q = mc\Delta T$
2. for heat energy stemming from phase change, $Q = ml$

3. Calculation of work done from volume changes, $W = \int pdV$

The third equation, $W = \int pdV$, gives us a piece of information on interpreting calculation of work done from the p-V graphs, that is the work done is the area under the graph, no matter how the graph looks like (i.e. what the thermodynamical process is involved).

LO3, LO4, LO5 & LO6-
In the matriculation syllabus, we consider 4 type of thermodynamical processes:

1. **Isothermal** ($\Delta T = \Delta U = 0 \Rightarrow Q = -W$)
   In this case, the **volume** of the gas container changes. So,

$$W = \int_{V_i}^{V_f} pdV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRTln\frac{V_2}{V_1}$$

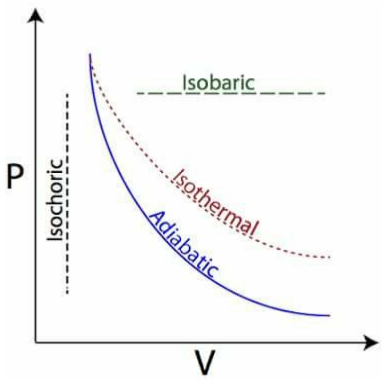
2. **Isochoric** ($\Delta V = 0 \Rightarrow \Delta U = Q$)
   This thermodynamical process is also known as **isovolumetric** process, which means the volume does not change and since work done that we consider depends on the change in volume, no work is done, $W = 0$.

3. **Isobaric** ($\Delta p = 0 \Rightarrow \Delta U = Q + W$)
   For the isobaric case, **pressure** is kept constant. work done is then

$$W = \int_{V_i}^{V_f} pdV = p(V_2 - V_1)$$

4. **Adiabatic** ($\Delta Q = 0 \Rightarrow \Delta U = W$)
   Adiabatic processes is a peculiar one, however the calculation of the work done is **not required** at the matriculation level. However, for those brave souls willing to give it a go, one can derive the equation of it. This can be done with the consideration that whilst $\Delta Q = 0$, $\Delta T \neq 0$ and $\Delta V \neq 0$, from the first law of thermodynamic, we can then write $dU = dW$. The ratio of specific heat capacities needs to be considered in this case to show that $\Delta(PV^\gamma) = 0$ where gamma is known as the **ratio of specific heat capacities**. This is the key to deriving the work done equation for the case of adiabatic processes.

The following shows p-V graph for all the processes:



End of notes