Mathematics

Rasulan, S. +60105520080 KMSw

1 Unit Conversion

Why is unit conversion?

Try Googling the Mars Climate Orbiter Crash & the Gimli Glider.

Questions:

- 1. Identify the dimensions the following values and calculate their values in another possible unit:
 - (a) Example: 2 metres

Answer: The unit "meter" has the dimensions of length. Alternative units for length may be inches and chain. Therefore,

 $2 m \approx 78.74 inches \approx 0.0994194 chain$

- (b) $20 \, {}^{o}C$
- (c) 15 kg
- (d) 54×10^3 seconds
- 2. Argue what unit would be suitable for each situation:
 - (a) Example: Between light years or metres, which one is more suitable for the length of an average person's arms span? Answer: To measure the length of an average person's arms span, the unit metres would be more appropriate, considering the average arm span is unlikely to reach the astronomical scale of that described using light years.
 - (b) Between Angstrom or centimetres, which one is more suitable for measuring the size of atoms?
 - (c) Between years or nanoseconds, which one is more suitable for estimating the average age of a human population?
 - (d) Between grams or ounces, which one is more suitable for explaining the weight of a cup of sugar to a Malaysian?

Fun exercise + extra knowledge: Name the few countries that still refuses to use S.I. unit nationwide.

2 Dimensional Analysis

- 1. What are the dimensions of the derived quantities below:
 - (a) Quantity c given that $c^2 = b^2 + a^2$, where a and b both has dimension of length.
 - (b) Quantity S given that $S = k_B \ln(W)$ where W is dimensionless and k_B has dimensions of $\left[\frac{Mass}{Temperature}\right]$.

(c) Quantity y given that $y = A \sin(\omega t)$, where A has dimension of length, t has the dimension of time and ω has the unit of $rad\ s^{-1}$.

3 Algebra, Geometry & Trigonometry

1. Algebra:

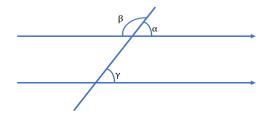
Writing down the quadratic formula used to solve for the zeroes of a second-degree polynomial function. Please include the general equation for a second-degree polynomial function in your answer.

2. Exponents:

Write down the exponent rules for multiplication, division, fractional exponent and negative exponent.

3. Geometry:

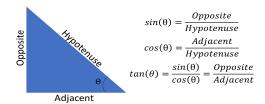
(a) Angles:



Based on the diagram above, write down equations that relates α to β , α to γ and γ to β .

- (b) Write down the equations for the surface area and volume of a cylinder, cone. sphere.
- (c) Write down the trigonometric identities, Law of cosines and Law of sines.

4. Trigonometry:



(a) Based on the diagram above, calculate θ is opposite is 18.4112cm and the hypotenuse is 32.0989cm.

- (b) Calculate the value of $tan(\theta)$ if the opposite is 21.98cm and the hypotenuse is 25.41cm.
- (c) sketch a graph describing $y = sin(\theta), y = cos(\theta)$ and $y = tan(\theta)$.

4 Scalar & Vectors

- 1. What is the difference between scalars and vectors? Give examples.
- 2. Given

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$
 and $\vec{b} = \langle b_1, b_2, b_3 \rangle$

Calculate \vec{c} where:

- (a) $\vec{c} = \vec{a} + \vec{b}$
- (b) $\vec{c} = \vec{a} \vec{b}$
- (c) $\vec{c} = \vec{a} \cdot \vec{b}$
- (d) $\vec{c} = \vec{a} \times \vec{b}$

Also, evaluate the magnitude $|\vec{c}|$ and the angle it makes, please specify from which axes you are referring to when working out the angle. Relate components of each vector in its unit vectors to the magnitude and angle.

3. Write down the properties of scalar product and vector product.

5 Calculus

- 1. Differentiation
 - (a) What are the differentiation rules for
 - i. A constant
 - ii. Power functions
 - iii. Product rule
 - iv. Chain Rules
 - (b) Evaluate the derivative with respect to x for functions:
 - f(x) = ln(x)
 - f(x) = sin(x)
 - f(x) = cos(x)
 - $f(x) = e^{kx}$ where k is a constant
- 2. Integration:

Evaluate the following integral:

- $\int x^n dx$ where $n \neq -1 \int x^{-1} dx$
- $\int e^x dx$
- $\int kx^n dx$ where k is a constant
- $\int sin(x) dx$
- $\int cos(x) dx$

Also, describe the method of integrating by parts.

6 Graphs

- 1. Given that (2,8) and (4,13) are coordinates of 2 points of a straight line on the x-y graph, determine the gradient and y-intercept of the graph.
- 2. Given the equation $k = \frac{df(x)}{dx}$ and $l = \int f(x)dx$, which one represents the area under the graph f(x) and which one determines the gradient at a given point of the graph f(x). Support your answer diagrammatically.



Mathematics

Solution

7 Unit Conversion

- 1. Conversions between unit are fairly straightforward and therefore requires only searches through the internet and textbooks. But the idea of this question was to evaluate if the student establishes the importance of making a choice for themselves.
- 2. Size of Atoms = Angstrom, diameter of atom are in the scale of Angstrom;

Average human age = years, nanoseconds are too short;

grams = keyword **Malaysian**. Because grams are more familiar for a Malaysian.

8 Dimensional Analysis

This part assess the student's ability to ensure homogeneity of dimensions. They are also presented equations unfamiliar to them to throw them off track and it is encouraged for them to look for solutions by themselves by *googling* keywords like "Addition in dimensional analysis", "Dimensional analysis involving logarithmic function" and "Dimensional analysis involving trigonometric function". This is to encourage self-learning.

Solution: Quantity c has dimensions of length, Quantity S has dimensions of $\left[\frac{Mass}{Temperature}\right]$, Quantity y has dimensions of length.

9 Algebra, Geometry & Trigonometry

1. This part assesses students' ability to perform when they are presented with a problem that is worded differently than the norm.

Second-degree polynomial function = Quadratic functions that has the general equation of

$$f(x) = ax^2 + bx + c$$

"Zeroes" indicates that the function equals to 0 and therefore gives $f(x) = ax^2 + bx + c = 0$ and

is solved by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 2. Obtainable from any maths textbook
- 3. (a) Obtainable from any maths textbook
 - (b) Obtainable from any maths textbook
 - (c) Obtainable from any maths textbook
- 4. 35° ; 0.86501

10 Scalar & Vectors

- 1. Textbook
- 2. This part was made to confuse students as they either have never seen or rarely see vectors presented in such a manner.

Actually, in Cartesian coordinates,

$$\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

and $\vec{b} = \langle b_1, b_2, b_3 \rangle = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$

It is in this form the students will have little to no difficulty solving this problem.

The angles requested here requires the students to clarify the origin of the angle, i.e. does it come from the x, y or z axes.

11 Calculus

This question on the description of the integrating by parts tests students understanding of the mathematical tools they have in hand, which allows them to do computation but is a reminder for them to understand the consequences of using such tools.

12 Graphs

- 1. m = 2.5; y-intercept = 3
- 2. k represents the gradient; l represents the area under the graph.