Chapter 8: Physics of Matter

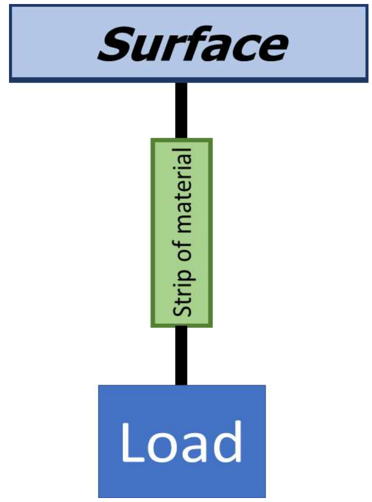
Learning Outcomes

Changes to material	No.	Learning Outcomes
Due to applied force	1.	Distinguish between stress, $\sigma = \frac{F}{A}$ and strain, $\epsilon = \frac{\Delta L}{L_o}$
	2.	Analyse the graph of: a. Stress-strain for metal under tension b. Force-elongation for brittle and ductile materials
	3.	Explain elastic and plastic deformations
	4.	Define and use Young's modulus, $Y = \frac{\sigma}{\epsilon}$
	5.	Apply: a. Strain energy from the force-elongation graph, $U = \frac{1}{2}F\Delta L$ b. Strain energy per unit volume from stress-strain graph, $\frac{U}{V} = \frac{1}{2}\sigma\epsilon$
Due to Heat	6.	Define: a. heat conduction b. coefficient of linear expansion, α c. coefficient of area expansion, β d. coefficient of volume expansion, γ
	7.	Solve problems: a. related to rate of heat transfer, $\frac{Q}{t} = -kA\left(\frac{\Delta T}{L}\right)$ through a cross-sectional area (remarks: maximum two insulated objects in series) b. related to thermal expansion of linear, area and volume $\Delta L = \alpha L_o \Delta T; \Delta A = \beta A_o \Delta T; \Delta V = \gamma V_o \Delta T; \alpha = \frac{\beta}{2} = \frac{\gamma}{3}$
	8.	Analyse graphs of temperature-distance (T-L) for heat conduction through insulated and non-insulated rods. *maximum two rods in series

Part 1: Material Changes due to Force

Stress

To begin, we start by talking about testing things to the point of deformation — we put them under some increasing force over some area of the thing and once it starts to deform, we stop and calculate the maximum amount of force that the thing starts to deform.



The most natural way to do this is merely to subject a strip/rod of the material (of length L and cross-sectional area A) to an axial load with the other end anchored to some surface. As the mass of the load is increased, the strip/rod deforms (becomes longer) and eventually breaks off (fracturing). So naturally we'd like to know, how much load can a strip of the given material, support?

Before answering this question, we can ask ourselves if there are any geometric variables that influences the ability of the strip of material to support load. We can then repeat the same experiment using strips/blocks of the same material (with varying cross-sectional area) and we would find that the axial strength increases with the increment of the cross-sectional area. This would make sense because as the cross-sectional area is increased, so does the number of bonding between each cross-sectional layer.

We now have a value for the amount of load a material can support relative to the cross-sectional area of that material sample.

It may be expressed mathematically as

$$F_{max} = \delta_{max} A_o$$

where F_{max} is the load at fracture, δ_{max} is the Ultimate Tensile stress and A_o is the initial cross-sectional area. This equation describes the maximum amount of stress that the strip/rod of material of cross-sectional A_o can handle and it is at this point that the material fails and fractures.

So, when the material is said to be put under stress, and that stress is less than δ_{max}, what we are referring to is

$$\delta = \frac{F}{A_o}$$

This new measure has the unit of Nm^{-2}.

Strain

In the last section, we quantified the amount of tensile stress a material may be be put under. In this section, let us quantify the "amount" of deformation that the material undergoes under some stress, that is we want to measure the stiffness.

Hooke's law gives us a great exposure to the deformation of a material with respect to the load that the material is put under. It is commonly written as

$$F = kx$$

Where k is the stiffness constant has units of Nm^{-1}.

We know from the previous section that this constant is not only affected by the type of material alone but also by the shape of the material. For now, we would like to normalize the measure for stiffness only by the deformation it undergoes independent of the shape. We can do this by considering the measure for the stretching of the material. This means we only consider the fractional change of the material when pun under stress, this is

$$\epsilon = \frac{\Delta L}{L_o}$$

where ΔL is the change in length and L_o is the initial length before the material was put under stress. This measure is what we call ***strain***.

Young's Modulus

We have now discussed the measure for stress the material undergoes as well as the material's deformation. We are now in the position to discuss how the stress that is put on the material affects the deformation observed in that material. That is to say, we want a calculable prediction on, *"If I put this amount of force, how big is the deformation I can expect? "*.

Experimental results show that for relatively small stress and strain, they are proportional to each other. This allows us to write

$$\delta \propto \epsilon.$$

This tells us that there is a proportionality constant between the two, let's call it Y. We can then define this proportionality constant to be

$$Y = \frac{\delta}{\epsilon}$$

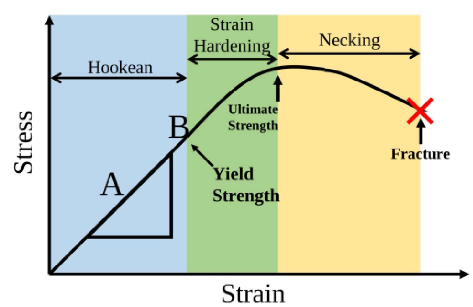
This proportionality constant is what we today call **Young's Modulus**.

Graph

2 graphs are studied in this section – stress-strain curve for a metal and the force-extension graph for brittle and ductile materials.

Stress-strain graph

Up to this point, we have considered stress and strain up within the Hookean limit. It is the best of our interest to also consider what happens further extension of the material after the Hookean limit (Yield limit) to the fracture point. The graph shows the stress-strain relation before and after the Hookean limit.



The first part (blue) shows the obeyance of the stress-strain curve to the Hookean law. Within this region, one can calculate the Young's modulus based on the gradient of the straight line. It is also in this region that the metal undergoes what is known as **elastic deformation**. Deforming elastically refers to the ability of the stretched metal to return to its initial length when the tensile stress is removed.

Beyond the Hookean limit, Hooke's law is no longer obeyed, and thus non-linearity is observed in the stress-strain curve. Beyond the Hookean limit, the metal undergoes **plastic deformation**, a type of deformation in which the stretched metal will not be able to return to its initial length even if the tensile stress is removed.

From the stress-strain curve, we observe that a non-linear increment of stress with the increment of strain until it reaches a peak, known as **Ultimate Tensile Strength**. The region (green) between the Hookean limit and the

UTS is where strain hardening takes place. This is the phenomenon where the metal is "strengthened" by the plastic deformation. "Strengthened" here refers to the dislocation of movements in the crystal structure of the material.

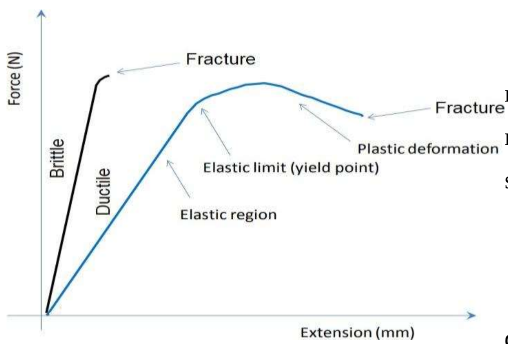
Beyond the peak, the strain still increases but the tensile stress decrease until the **fracture point**. Between the UTS and fracture point (pink), what is observed of the metal is that necking takes place. This is the phenomenon in which the local cross-sectional area becomes significantly smaller than average.

For some material, the plastic deformation occurring between the Hookean limit and the fracture point is so small, that it seems to the observer as if the plastic deformation region is non-existent. For such material, we call it **brittle**.

For other materials, the plastic deformation region is great and thus we call these materials, **ductile**.

Force-extension graph

As mentioned in the stress-strain graph section, the plastic deformation phase for brittle materials are very short whereas the same region for the ductile materials are relatively (and observably) significant.



One possible inference we can make from this is that ductile materials are able to contain much more strain energy than brittle materials. It is also from the force extension graph we can deduce the strain energy by the area under the force-extension graph, that is

$$U_{strain} = \frac{1}{2}F\Delta L$$

If were to find the strain energy per unit volume, we could easily do it by dividing both side by the volume of the material,

$$\frac{U_{strain}}{V} = \frac{1}{2}\frac{F\Delta L}{V}$$

and reminding ourselves that the volume is merely the product of cross-sectional area and length,

$$V = AL_o$$

We can see that the strain energy per unit volume is just the area under the stress strain curve,

$$\frac{U_{strain}}{V} = \frac{1}{2}\frac{F\Delta L}{V} = \frac{1}{2}\frac{F\Delta L}{AL_o}$$

$$\frac{U_{strain}}{V} = \frac{1}{2}\delta\epsilon$$

Sample Problem 8.1

A 50kg box is balanced on a pole of radius 25cm. Determine the stress that the pole is under.

Answer:

Stress is the amount of force onto a surface area. The weight of the box is $F = W = mg$ and the surface area, considering it is a circle, $A = \pi r^2$ and thus the stress on the pole is

$$\delta = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(50)(9.81)}{\pi(0.25)^2} = \frac{7848}{\pi} \approx 2498.1 Nm^{-2}$$

Sample Problem 8.2

Determine the strain on a piece of metal if is extended by 20% when some force is applied to it.

Answer:

Strain quantifies the fractional change to the geometry of the object.

$$\epsilon = \frac{\Delta L}{L_o} = \frac{0.2L_o}{L_o} = 0.2$$

Sample Problem 8.3

A piece of wire is hung with a mass of 2kg weight on its end. Because of the weight on its end, it stretches by 0.2cm from its original length of 5cm. If the cross-sectional area of the string is $6.25(10^{-8})\pi\, m^2$, calculate the Young's Modulus of the wire.

Answer:

Young's modulus is the ratio of stress to strain,

$$Y = \frac{\delta}{\epsilon} = \frac{\left(\frac{F}{A_o}\right)}{\left(\frac{\Delta L}{L_o}\right)} = \frac{FL_o}{A_o \Delta L} = \frac{(2)(9.81)(0.05)}{6.28(10^{-8})(\pi)(0.002)} = \frac{7.811 \times 10^9}{\pi}$$

$$Y \approx 2.486 \times 10^9\, Pa$$

Part 2: Material Changes due to Heat

Heat Conduction

Imagine a metal rod with one end heated. With time, the opposite end also gets hot even though it is not directly heated. The heat energy is transferred from one end to the other end. This happens through the 'jiggling' of the particles within the heated material. As the rod is heated, the particles begin to vibrate and collide with neighbouring particles. When they collide, they transfer some of the energy to the neighbouring particles and then the neighbouring particles starts to vibrate. This process is called **heat conduction.** It is the transfer of heat through agitations of the particles within the material without any motion of the material.

The variable that drives heat transfer is a temperature gradient across the material, that is to say there exist a difference in temperature between two parts of a conducting medium,

$$\Delta T > 0.$$

Referring to the rod heated on one end case, we can say that the heated end has temperature T_H and the non-heated end to have the temperature T_C. Fourier's law tells us that the local heat flux in a homogeneous body, q_h is in the direction of, and proportional to, the temperature gradient ∇T:

$$q_h \propto -\nabla T$$

In one-dimensional form,

$$q_h = -\kappa_x \frac{dT}{dx}$$

where κ_x is the thermal conductivity in the x-direction. Note that the minus sign is present due to the fact that heat flows from a higher temperature area to a lower one.

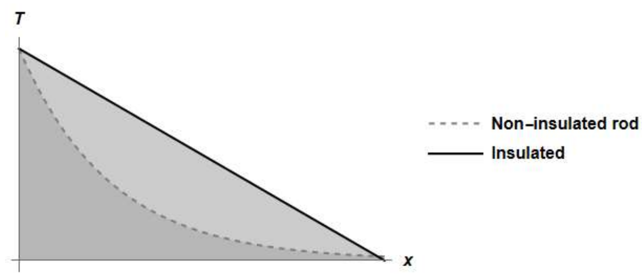
Local heat flux refers to rate of heat transfer per unit area,

$$q_h = \frac{\left(\frac{dQ}{dt}\right)}{A}$$

The equation for the rate of heat transfer is then

$$\frac{dQ}{dt} = -A\kappa_x \frac{dT}{dx}$$

Let us now try to understand the temperature-gradient graph for a rod heated on one end. For an uninsulated rod, the heat energy will be able to escape to the environment via the sides of the rod. This leads to temperature-distance gradient to be curved with a decreasing gradient, much like graphs exhibited by functions $f(x) = e^{-kx}$. On the other hand, for insulated rods, the heat loss to the environment is negligible. As a result, their temperature-distance graph is a linear graph with negative gradient.



Sample Problem 8.4

One end of a metal bar is kept at $250\,^{o}C$, while the other end is kept at a lower temperature. The cross-sectional area of the bar is $3 \times 10^{-4} m^2$. The heat loss through the sides of the bar is negligible due to the insulation around the metal bar. Heat flows through the bar at the rate of $2.5 Js^{-1}$. If the metal bar has a thermal conductivity value of 140 $Js^{-1}m^{-1\,o}C^{-1}$, calculate the temperature of the bar at a point 0.8 m from the hot end?

Answer:

$$\frac{dQ}{dt} = -A\kappa_x \frac{dT}{dx} \Rightarrow \frac{dQ}{dt} = A\kappa_x \frac{(T_{hot} - T_{cold})}{L}$$

$$2.5 = 3(10^{-4})(140)\frac{(250 - T_{cold})}{0.08}$$

$$T_{cold} \approx 245.24^{o}C$$

Heat Expansion

Things expand when they are heated, this phenomenon is known as **thermal expansion**. A great demonstration of this phenomenon is the Ring and Ball Experiment (https://www.youtube.com/watch?v=ne8oPFTM_AU). If the expansion happens in one dimension of space, we call it a **linear expansion**, where as expansion in two and three dimensions are known as **area** and **volume expansion.**

We can start by considering a small change in the object's length. By small, we mean relative to the object's initial dimensions. When that is the case, the change in the length is the proportional to the first power of the temperature change.

$$\Delta L \propto \Delta T$$

Say, that object has an initial length of L_o along some direction at temperature T_o. Then the change in length ΔL for a change in temperature ΔT is

$$\Delta L = \alpha L_o \Delta T$$

where α is known as **coefficient of linear expansion**.

So now how do we expand to area and volume thermal expansion? Well, note that $A = L^2$ and that $L = L_o + \Delta L$. As such,

$$A = L^2 = (L_o + \alpha L_o \Delta T)(L_o + \alpha L_o \Delta T)$$

$$A = L_o^2 + 2\alpha L_o^2 \Delta T + \alpha^2 L_o^2 \Delta T^2 = A_o + 2\alpha A_o \Delta T + \alpha^2 A_o^2 \Delta T^2$$

Since we are only considering only small changes, we can consider the last terms to be negligible such that

$$A = A_o + 2\alpha A_o \Delta T$$

which gives us the change in area

$$\Delta A = \beta A_o \Delta T$$

where $\beta = 2\,\alpha$ and is aptly named **coefficient of area expansion**. We can then imitate the same procedure to produce the equation for change in volume due to heat which will yield

$$\Delta V = \gamma V_o \Delta T$$

where $\gamma = 3\alpha$ and is named **coefficient of volume expansion.**

Sample Problem 8.6

A mug of filled with 200mL of tea at $90^{o}C$. If the tea has a coefficient of linear expansion of $69(10^{-6})K^{-1}$, calculate the volume of the tea when the tea has cooled down by $50^{o}C$.

Answer:

Change in tea temperature, $\Delta T = 50^{o}C = 50K$

Initial volume of water, $200mL = 0.0002m^{3}$

Coefficient of volumetric expansion, $\gamma = 3\alpha = 3(69)(10^{-6})K^{-1} = 2.07(10^{-4})K^{-1}$

$$\Delta V = \gamma V_{o}\Delta T \Rightarrow V_{final} - V_{initial} = \gamma V_{initial}\Delta T$$

$$V_{final} - (0.0002) = \left(2.07(10^{-4})\right)(0.0002)(10)$$

$$V_{final} = 2.0000207 \times 10^{-4}m^{3}$$