Chapter 4: Work, Energy and Power

Learning Outcomes

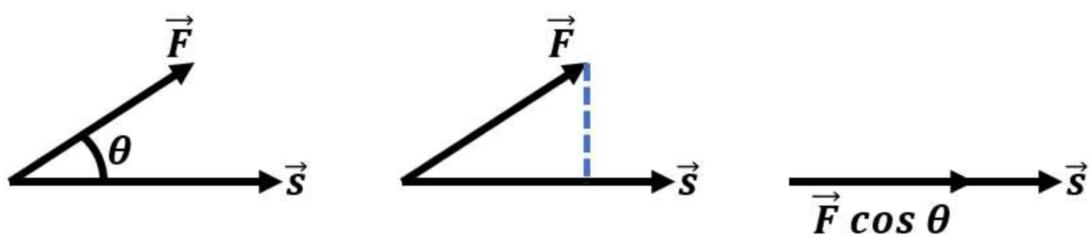
a) State:
   (a) the physical meaning of dot (scalar) product for work: $W = \vec{F} \bullet \vec{s} = Fscos\theta$
   (b) the principle of conservation of energy.
b) Define and apply
   (a) work done by a constant force.
   (b) Gravitational potential energy, $U = mgh$
   (c) Elastic potential energy for spring, $U_s = \frac{1}{2}kx^2 = \frac{1}{2}Fx$
   (d) Kinetic energy, $K = \frac{1}{2}mv^2$
   (e) work-energy theorem, $W = \Delta K$
   (f) average power, $P_{av} = \frac{\Delta W}{\Delta t}$ and instantaneous power, $P = \vec{F} \bullet \vec{v}$
c) Determine work done from a force-displacement graph.
b) Apply the principle of conservation of mechanical energy.

Work

Let us begin by defining work. The work on an object, W, is defined to be the product of magnitude of the displacement, s, and the force component parallel to the displacement of the object $F_{\parallel}$, i.e.

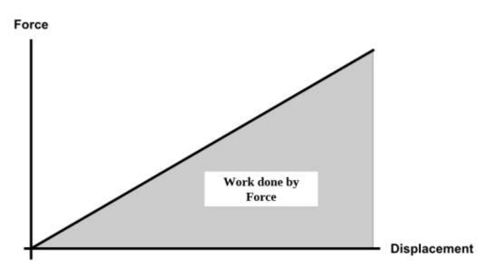
$$W = F_{\parallel}s = \vec{F} \cdot \vec{s}$$

Notice that it is not the product of force and displacement but the product of displacement and force component, the important characteristic of that particular force component is that it must be parallel to the displacement. The diagram below illustrates this point, we cannot simply multiply the magnitude of $\vec{F}$and $\vec{s}$. We must find the component of force that **is** parallel the displacement, and then we can find their product.



This of course means that in the force-displacement graph, work done by a force is equal to the area under the graph. That is to say, the work done to displace an object from $x_i$to x_f is simply

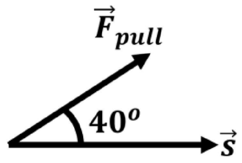
$$W = \int_{x_i}^{x_f} F_{\parallel}dx$$



Sample Problem 4.1

A block is pulled with a force of 50N (directed 40^o from the horizontal) on a smooth horizontal surface for 5m. Calculate the work done by the pulling force.

Answer:

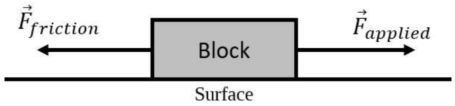


$$W = \vec{F} \cdot \vec{s} = (F\cos\theta)s$$
$$W_{50N} = (50\cos 40)5$$
$$W_{50N} = 191.511J$$

Sample Problem 4.2

A block is pushed with 5N in the positive x direction for $2m$ on a horizontal surface. If the block travels at constant speed, calculate the work done by frictional force, work done by the applied force and the total work done.

Answer



Since the block travels at constant speed,

$$F_{net} = 0 = F_{applied} - F_{friction}$$

That means the magnitude of frictional force is equal to the magnitude of applied force, but acts in the opposite direction.

$$F_{friction} = 5N$$
$$W_{friction} = \vec{F}_{friction} \cdot \vec{s} = F_{friction} s\cos\theta$$
$$W_{friction} = (5)(2)cos(180^o)$$
$$W_{friction} = -10Nm$$

Similarly,

$$W_{applied} = F_{applied} s\cos\theta = (5)(2)\cos 0 = 10Nm$$
$$W_{total} = W_{friction} + W_{applied} = -10 + 10 = 0Nm$$

Work Energy Theorem

When an object moves, we say it contains kinetic energy. Kinetic energy quantifies the amount of energy a moving object has. It depends on the velocity of the moving object,

$$E_k = K = \frac{1}{2}mv^2.$$

Now what we want to do is to show a relationship between the quantity related to moving object (kinetic energy) and another quantity related to the changes of object position (work).

We begin with the definition of work done on an object and Newton's Second Law of motion to show that

$$W = Fs;\ F = ma \Rightarrow W = mas$$

Assuming that the force is constant and therefore the acceleration is also constant, we can then apply equation of kinematics

$$v^2 = u^2 + 2as;\ W = m(as)$$

$$W = m\frac{v^2 - u^2}{2} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = K_{final} - K_{initial} = \Delta K$$

This show derivation brings about an important theorem, called the work-energy theorem. This theorem states that the work done onto a body is equal to the change in kinetic energy of the body.

Sample Problem 4.3

A 150g block begins travelling along a horizontal surface at $4.20ms^{-1}$. If the kinetic friction coefficient between the block and surface is 0.45, calculate the distance that the block moves before coming to a stop.

Answer:

$m_{block} = 0.15kg;\ u = 1.8ms^{-1};\ u_k = 0.45$

$$W = Fs = \Delta K = F_{final} - K_{initial} \Rightarrow (\mu_k m_{block} g)s = \frac{m_{block}}{2}(v^2 - u^2)$$

$$0.45(0.15)(9.81)s = \frac{0.15}{2}(0^2 - 4.2^2)$$

$$s = 1.99796m$$

Energies

Two more energies that are of interest to us. The first is the gravitational potential energy, which is the energy contained in an object due to its position, measure from a gravitational source. A more detailed analysis is found in the Newtonian Gravity part of this course. At this point, it is sufficient for us to know that if an object of mass *m* is position at height *h* from the surface of the Earth, then the gravitational potential energy found in that object is

$$E_{gp} = mgh$$

The second type of potential energy of interest is the elastic potential energy of a spring. By Hooke's Law, the force acting on a spring is direction proportional to its extension (or compression).

$$\text{Hooke's Law: } F = -kx$$

We can then utilize work energy theorem to find the elastic potential energy of a spring,

$$W = -\int F\,dx = \int kx\,dx \Rightarrow E_{ep} = \frac{1}{2}kx^2$$

Apart from the conservation of momentum, another important principle of conservation crucial to our study of moving bodies is the **principle of mechanical energy conservation.** The law simply states that the sum of all kinetic energy and all potential energy must remain constant at all times. That is to say

$$\Delta E_{total} = 0.$$

Sample Problem 4.4
An 2kg object was released from 20m height. Calculate its velocity just before striking the ground.

Answer:
Initially the object would have gravitational potential energy of

$$E_{gp} = mgh = 2(9.81)(20) = 392.4J$$

This energy is then converted fully into kinetic energy at $h = 0m$.
Therefore, the amount of kinetic energy possessed by the body will be 392.4J.

$$E_k = \frac{1}{2}mv^2 = 392.4J \Rightarrow \frac{1}{2}(2)v^2 = 392.4 \Rightarrow v = 19.8ms^{-1}$$

Power

Now that we have familiarize ourselves with work and energy, let us now talk about **power**, which is simply defined by the rate of work done. Average power refers to the work done within a time interval,

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{W_{final} - W_{initial}}{t_{final} - t_{initial}}.$$

On the other hand, instantaneous power refers to the mechanical power at one instant in time

$$P_{instantaneous} = \frac{dW}{dt} = \vec{F}\frac{d\vec{s}}{st} = \vec{F} \cdot \vec{v}$$

Sample Problem 4.5
Calculate the average power required to lift a 75kg man to a height of 10m in 2minutes.
Answer:
By work energy theorem,

$$W = mgh$$

$$P_{average} = \frac{W_f - 0}{t_f - 0} = \frac{(75)(9.81)(10)}{2(60)} = 61.3125W$$

Sample Problem 4.5
Calculate the instantaneous power required to lift a 75kg man at $0.09ms^{-1}$.
Answer:

$$P_{instant} = Fv = mgv = 75(9.81)(0.08) = 66.22W$$