

*Matriculation Physics (SP025)*

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*Short notes*

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*Shafiq R*

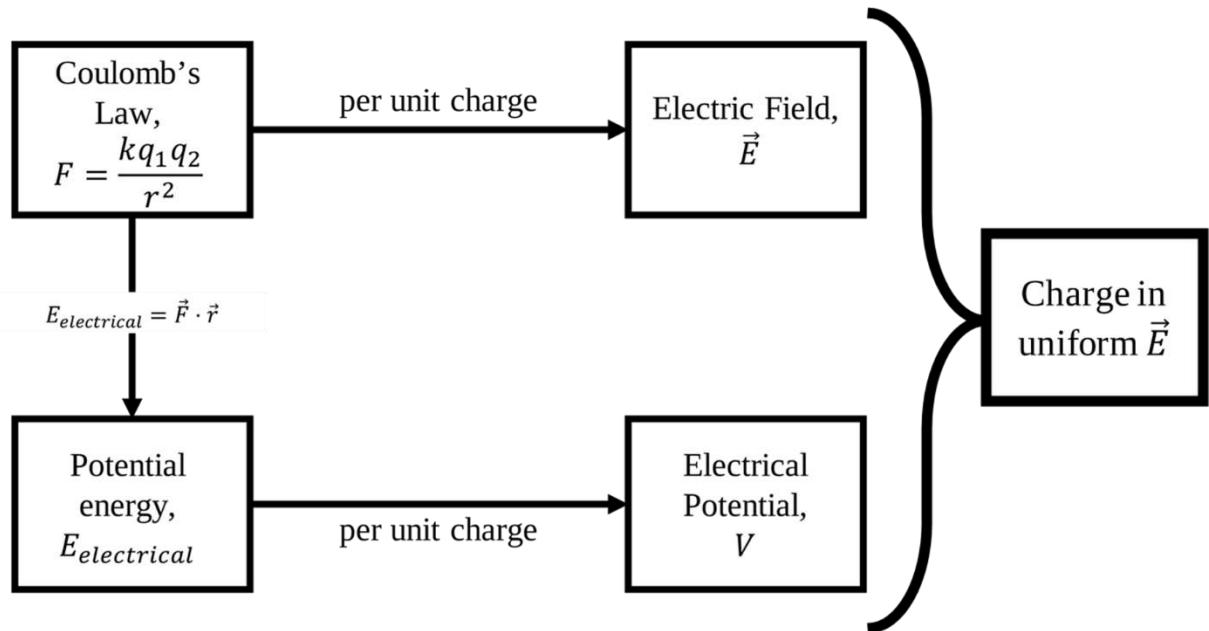
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## Chapter 1: Electrostatics



### Coulomb's Law

Let it be known that Coulomb's Law allows us to measure forces between charged particles, this force is known as **Coulomb Force**. Mathematically, Coulomb's Law is

$$F_{Coulomb} = \frac{kq_1q_2}{r_{12}^2}$$

where  $q_i$  are the charges of interacting particles,  $r_{12}$  is the distance between the particles and  $k$  is the electrostatic constant. The electrostatic constant is

$$k = \frac{1}{4\pi\epsilon_0} = 8.98 \times 10^9 \text{ kg m}^3 \text{ s}^{-4} \text{ A}^{-2}$$

On the note of direction of the Coulomb force,

Condition	Direction
$F_{Coulomb} < 0$	Towards each other
$F_{Coulomb} > 0$	Away from each other

For more than 2 particles, the Coulomb Force on particle  $j$  becomes

$$F_{Coulomb} = kq_j \sum_i \frac{q_i}{r_{ij}^2}$$

### Electric Field

The electric field at a point in space  $E(r)$ , is defined as the electric force acting on a positive test charge placed at that point  $F(r)$ , divided by the test charge,  $q_{test}$ .

$$E(r) = \frac{F(r)}{q_{test}}$$

Rearranging this equation yields,

$$F(r) = q_{test}E(r)$$

which tells us that particle of charge  $q_{test}$  placed in a region of electric field  $E(r)$  will experience a force of  $F(r)$ .

If the source of electric field has a charge of  $q_{source}$ , then the electric field at point  $r$ ,  $E(r)$  is

$$E(r) = \frac{kq_{source}}{r^2}$$

As in the case for Coulomb Force, for multiple, the electric field from multiple sources is simply additive,

$$E(r) = k \sum_i \frac{q_i}{r_i^2}$$

## Electric Potential

Electric potential is the amount of work done to bring a test charge  $q_{test}$  from an infinite distance to a point at distance  $r$  from the source charged particle of charge  $q_{source}$ . This is found to be

$$V = \frac{W_{\infty \rightarrow r}}{q_{test}} = \frac{kq_{source}}{r}$$

Potential difference between positions  $x = A$  and  $x = B$  is then

$$V_{AB} = V_A - V_B = \frac{W_{\infty \rightarrow A}}{q_{test}} - \frac{W_{\infty \rightarrow B}}{q_{test}} = \frac{W_{A \rightarrow B}}{q_{test}}$$

Electric potential energy is the energy a test charge would have positioned  $r$  distance away from a source,

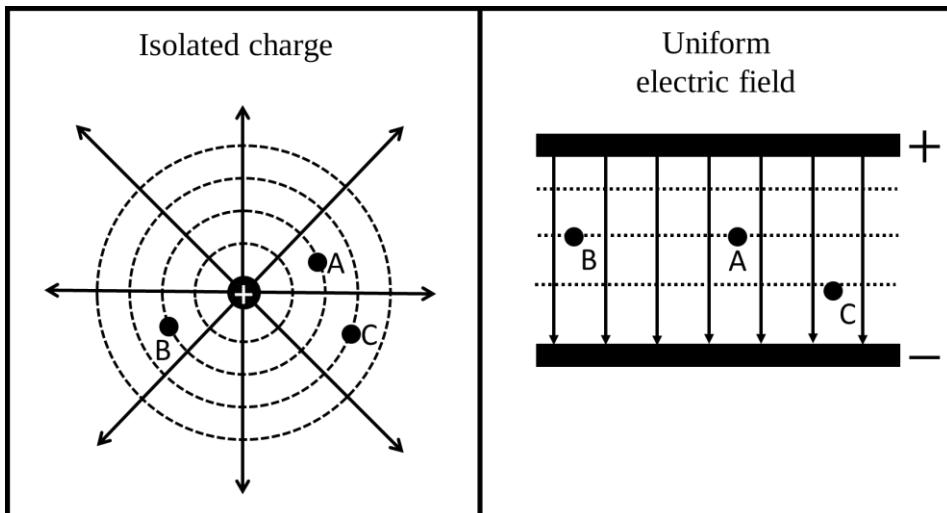
$$U = q_{test}V$$

For multiple sources, the potential and electrical potential energy at point  $r$  is simply,

$$V_{total} = k \sum_i \frac{q_i}{r_i}; U_{total} = kq_j \sum_i \frac{q_i}{r_{ij}}$$

Equipotential lines and surfaces are graphical representation on which a particle on the line or surfaces is at the same potential.

- This means no work is done by the electric field when a charged particles are moved from on point of the equipotential line (or surface) to another point on the same line (or surface).
- Equipotential lines are always perpendicular to the electric field at all points.
- Examples:



In both examples,

$$V_A = V_B \neq V_C$$

## Charge in Uniform Electric Field

For a uniform electric field produced by parallel plates of potential difference  $V$ , electric field strength is simply

$$E = \frac{V}{d}$$

where  $d$  is the distance between the parallel plates.

The following case studies involves a charged particle in a uniform electric field:

### Case 1: Stationary charge

A stationary charged particle of charge  $q$  and mass  $m$ , placed in a uniform electric field  $E$  will experience force only from the electric field and therefore will move towards plate of its opposite charge (i.e. positive charged particle will move towards the negatively charged plate and vice versa).

Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

### Case 2: Charge moving parallel to the field

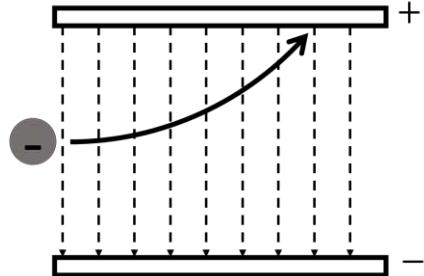
A charged particle of charge  $q$  and mass  $m$ , entering a uniform electric field  $E$  in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. It will either decelerate (if its velocity is in the opposite direction of its acceleration) or accelerate.

Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

### Case 3: Charge moving perpendicularly to the field

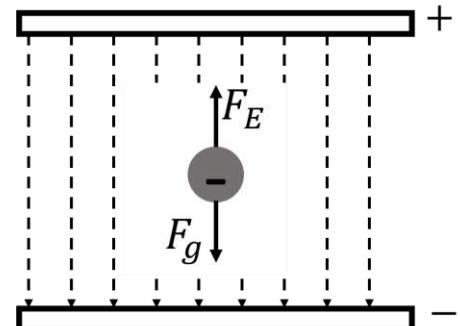
A charged particle of charge  $q$  and mass  $m$ , entering a uniform electric field  $E$  in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. Because of its initial velocity direction, it will follow a parabolic path, moving towards the plate of its opposite charge.



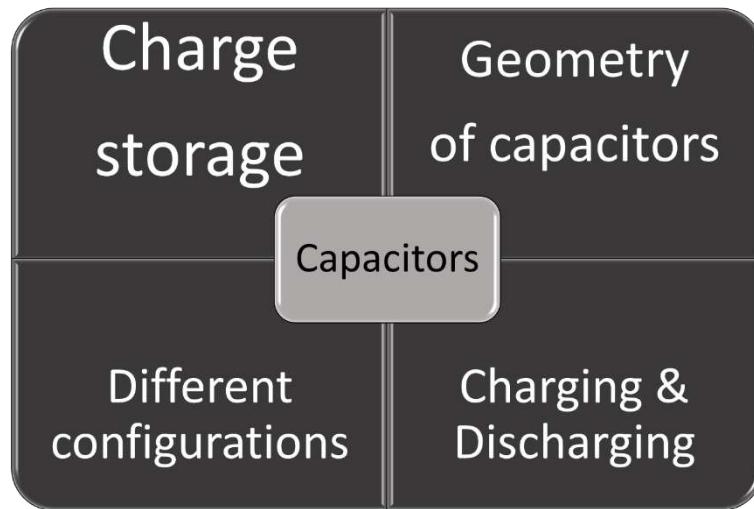
### Case 4: Charge in dynamic equilibrium

In the case of dynamic equilibrium, the attractive Coulomb Force between the charged particle and the plate of opposite charge cancels out the weight of the charged particle,

$$F_{Coulomb} = W_{particle} \Rightarrow qE = mg$$



## Chapter 2: Capacitors and Dielectrics



### Parallel & Series

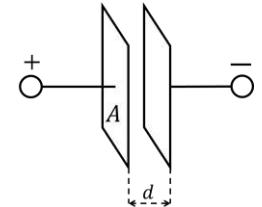
Capacitors are essentially batteries. Their ability to store charge is quantified by **capacitance**. Capacitance  $C$ , is the amount of charge  $q$  stored in one plate of a capacitor per unit potential difference between the plates,  $V$ ,

$$C = \frac{Q_{\text{single plate}}}{V}$$

As a function of its geometry, capacitance of a parallel plate capacitor is

$$C = \frac{\epsilon A}{d}$$

where  $\epsilon$  is the permittivity of the space between the plates,  $A$  is the area of each plate and  $d$  is the distance between the parallel plates.



Multiple capacitors can be arranged either in parallel or series or combinations of them, and their effective capacitance can be calculated depending on their arrangement:

Arrangement	Effective Capacitance
Series	$\frac{1}{C_{\text{eff}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$ $C_{\text{eff}} = \left( \sum_{i=1}^n \frac{1}{C_i} \right)^{-1}$
Parallel	$C_{\text{eff}} = C_1 + C_2 + \dots + C_n$ $C_{\text{eff}} = \sum_{i=1}^n C_i$

## Energy Stored in a Capacitor

Consider a pair of charged plate with charge

$$Q = CV$$

To deliver a small charge  $dQ$  at constant variable  $V$ , we require the amount of work

$$dW = V dQ = \left(\frac{Q}{C}\right) dQ$$

The total word done (i.e. energy stored in the capacitor) is then

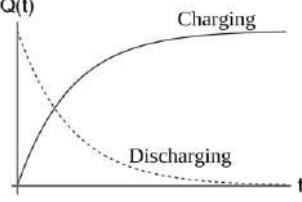
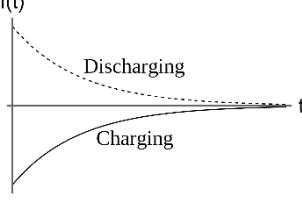
$$W = U = \int dW = \int_0^Q \left(\frac{Q}{C}\right) dQ = \frac{Q^2}{2C}$$

Considering  $Q = CV$ , energy stored in a capacitor can be expressed as

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

## Charging & Discharging Capacitors

Capacitors stores charges, now the question is how fast to charge it? Consider a simple circuit consisting of a power supply, a resistor of resistance  $R$  and a capacitor of capacitance  $C$ . Accumulation of charge with time for charging and discharging are as follows:

$Q - t$ graph	$I - t$ graph
 <p>Discharging: <math>Q(t) = Q_o e^{-\frac{t}{RC}}</math> Charging: <math>Q(t) = Q_o \left(1 - e^{-\frac{t}{RC}}\right)</math></p>	 <p>Discharging: <math>I(t) = -\frac{dQ}{dt} = -\frac{Q_o}{RC} e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}</math> Charging: <math>I(t) = \frac{Q_o}{RC} e^{-\frac{t}{RC}} = -I_o e^{-\frac{t}{RC}}</math></p>

Time constant,  $\tau$  is defined as the time for the exponential term to drop to  $e^{-1}$  for discharging, or, for the charge to increase to  $1 - e^{-1}$  for charging process, and is calculate by multiplying the  $R$  and  $C$ ,

$$\tau = RC \text{ [seconds]}$$

## Dielectrics

Dielectrics are electrically non-conductive materials placed in between the plates of capacitors to increase the capacitance of the capacitor.

We quantify the increase in capacitance as the **dielectric constant**  $\epsilon_r$ , define as the ratio of capacitance of capacitor with dielectric  $C$ , to the capacitance of capacitor with no dielectric (vacuum)  $C_o$ ,

$$\epsilon_r = \frac{C}{C_o} = \frac{\left(\frac{\epsilon A}{d}\right)}{\left(\frac{\epsilon_0 A}{d}\right)} = \frac{\epsilon}{\epsilon_0}$$

## Chapter 3: Current and DC Circuits

### Electric Current

Current is the amount of charge  $\Delta Q$  that passes through a surface area in time  $\Delta t$ ,

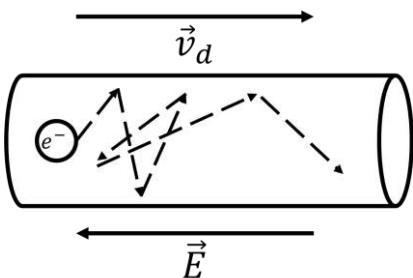
$$I = \frac{dQ}{dt}$$

Total charge  $Q$  is simply  $n$  multiples of electron charge  $e$ ,

$$Q = ne$$

Without external electric field, the electron will drift through a conductor with kinetic energy equivalent to Fermi energy, which results in a net velocity of zero. With external electric field, the electron as a whole now gains a net velocity along the electric field. This ‘net velocity’ is what is known as ‘drift velocity’.

Consider an electron travelling through a conductor, on which an electric field of  $\vec{E}$  is applied. The force on the



electron is then  $F = -qE \Rightarrow a = -\frac{eE}{m}$ . Assuming the average time between collision is  $\tau$ , we can show that

$$v_d = a\tau = \left(-\frac{eE}{m}\right)\tau$$

This means that applying a larger electric field, the larger the kinetic energy obtained by the electron due to a larger drift velocity. This also means that an increase in temperature, increases the collision frequency, decreases collision time and decreases drift velocity of the electrons.

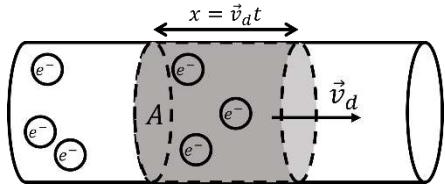
Relating the idea of drift velocity to current can be done by considering a volume section of the conductor  $V$ , and the number of charges that flows through that section,  $n$ . We can work out that the amount of charge going through  $V$  is simply

$$\Delta Q = (ne)A\Delta x$$

where  $\Delta x = v_d\Delta t$ .

This means that current is

$$I = \frac{\Delta Q}{\Delta t} = neAv_d$$

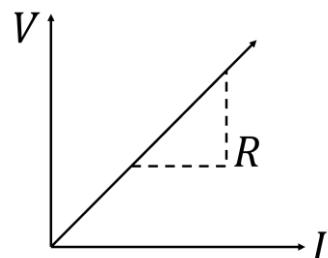


### Ohm's Law

Ohm's Law states that current  $I$ , is directly proportional to the potential difference  $V$ , if all conditions are constant.

$$V \propto I \Rightarrow V = IR$$

$R$ , which is the proportionality constant to Ohm's Law, represents resistance which opposes current flow in a circuit.



## Resistance (Geometry & Temperature)

Resistance of a conductor in a circuit depends on 3 factors – geometry of the conductor, material of the conductor and the temperature of the conductor.

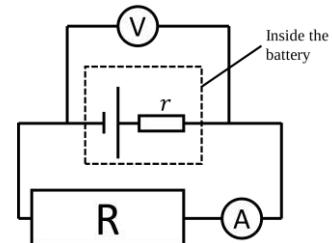
Factor	Equations
Material	For a resistor of resistivity $\rho$ , length $L$ and cross-sectional area $A$ , the resistance is then
Geometry	$R = \frac{\rho L}{A}$
Temperature	When temperature of a conductor with coefficient of resistivity $\alpha$ (at $20^{\circ}C$ ), changes by $\Delta T$ , resistance changes by $\Delta R = \alpha \Delta T$

## EMF, Internal Resistance and Potential Difference

Electromotive force (emf) is the electrical energy per unit charge generated by a power source generate current. Some of that electrical energy is used to overcome **internal resistance** within the power supply, the rest is then used for the rest of the circuit. That means the potential difference across the circuit is always less than the emf. This internal resistance may exist for a few reasons – distance between electrodes, temperature of the cell, effective area of the electrodes, irregularities found in the cell, etc.

Consider a circuit consisting a voltmeter of reading  $V$ , an ammeter of reading  $I$ , a battery and a resistor of resistance  $R$ . The emf of the source is then

$$\epsilon = IR + Ir = V + Ir$$



## Parallel & Series

For systems of multiple resistors, they can be arranged in parallel, series or any combinations of the two. The effective resistance can then be calculated according to their arrangement.

Arrangement	Effective Capacitance
Series	$R_{eff} = R_1 + R_2$ <p>For <math>n</math> number of resistors in <b>series</b>,</p> $R_{eff} = R_1 + R_2 + \dots + R_n = \sum_i^n R_i$
Parallel	$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$ <p>For <math>n</math> number of resistors in <b>parallel</b>,</p> $R_{eff} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right)^{-1} = \left( \sum_i^n \frac{1}{R_i} \right)^{-1}$

## Kirchhoff's Rules

Kirchhoff's Rules allows us to determine current flow around a circuit. The two rules are as follows:

Rules	Statement
First Rule – Junction Rule (Conservation of Charge)	Algebraic sum of currents in a network of conductors meeting at a junction is zero. $\sum_i I_i = 0$
Second Rule – Loop Rule (Conservation of Energy)	Algebraic sum of potential difference in any loop must equal to zero. $\sum_i V_i = 0$

## Electrical Energy and Power

Since work done (amount of energy) to deliver a small charge  $dq$  at constant variable  $V$  is  $W = E = VQ$ , and that current by definition is  $Q = It$ , we can see that energy will simply be

$$E = (VI)t$$

Since  $E = Pt$  and taking Ohm's Law ( $V = IR$ ) into consideration, electrical power is then

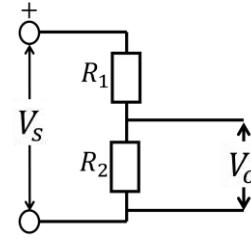
$$P = IV = I^2R = \frac{V^2}{R}.$$

## Potential Divider

A potential divider is used to produce a voltage of a fraction of the voltage provided by the power supply. This is achieved by using resistors of different resistances.

If the power supply provides potential difference of  $V_s$ , then the output voltage is simple

$$V_o = \frac{R_1}{R_1 + R_2} V_s$$

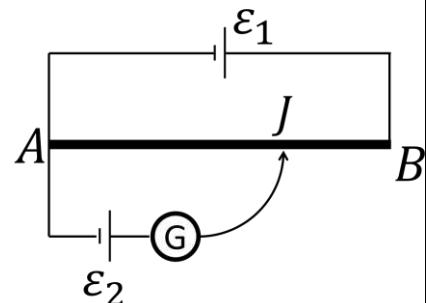


## Potentiometer

A potentiometer can be used to measure potential differences by two or more cells.

How it works
Wire AB has a resistance of $R$ . This means if the jockey is at point B, then $R_{AJ} = R_{AB}$ , and thus $I = I_{maximum}$ . As the jockey is slid to towards A, the galvanometer will show zero reading which indicates no current passes through the galvanometer and that the potentiometer is balanced. This means $V_{AJ} = \varepsilon_2$ . This happens when

$$\frac{V_{AJ}}{V_{AB}} = \frac{l_{AJ}}{l_{AB}}$$



We can also use a potentiometer to compare emfs between two cells.

This is done by the following setup.

compare emfs between cell 2 and 3
-----------------------------------

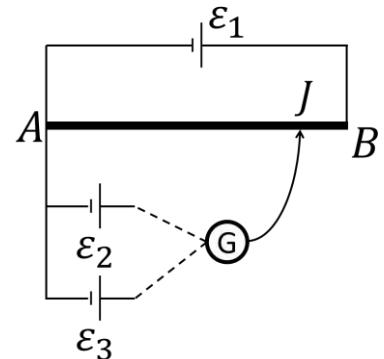
When the galvanometer is connected to  $\varepsilon_2$  and balanced between A and  $J_1$ ,

$$\varepsilon_2 = \frac{l_{AJ_1}}{l_{AB}} \varepsilon_1$$

When the galvanometer is connected to  $\varepsilon_2$  and balanced between A and  $J_2$ ,

$$\varepsilon_3 = \frac{l_{AJ_2}}{l_{AB}} \varepsilon_1$$

$$\Rightarrow \frac{\varepsilon_2}{\varepsilon_3} = \frac{l_{AJ_1}}{l_{AJ_2}}$$



## Chapter 4: Magnetism

### Magnetic Field

A magnetic field is a region of space in which a charged particle will experience magnetic force. They are generated by moving charged particles. Magnetic field lines are always drawn from its north pole to its south pole. When drawn on a 2D plane such as paper, we would generally represent a direction **into** the plane as , and direction **out** of the plane as .

### $\vec{B}$ from current-carrying conductor

Direction of magnetic field depends on the direction current flow – Right Hand Rule, where the thumb point to the current direction and curled fingers are the magnetic field lines.

4 cases to consider in calculating the magnitude of magnetic field

Situation	Equation
Long straight wire	$B = \frac{\mu_0 I}{2\pi r}$
Centre of circular coil	$B = \frac{\mu_0 I}{2r}$
Centre of solenoid	$B = \mu_0 In$ <p>Where <math>n</math> is the number of loops per unit length</p>
End of solenoid	$B = \frac{1}{2} \mu_0 In$ <p>Where <math>n</math> is the number of loops per unit length</p>

## Magnetic Force

### Force on a moving charged particle in uniform $\vec{B}$

Force on a particle with charge  $q$  moving at velocity  $\vec{v}$  in a uniform magnetic field  $\vec{B}$ , the magnetic force acting on it is

$$\vec{F}_{magnetic} = q(\vec{v} \times \vec{B}).$$

In the case of a large enough region, the magnetic force will cause the charged particle to travel in a circular motion. In such cases,

$$\vec{F}_{magnetic} = \vec{F}_{centripetal} \Rightarrow qvB = \frac{mv^2}{r}.$$

### Force on a current carrying conductor in uniform $\vec{B}$

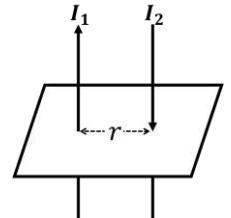
Consider a quantity of charge  $\Delta Q$  travelling along a conductor of length  $l$  in a magnetic field  $\vec{B}$  in time  $t$ . The magnetic force on the conductor is then

$$\vec{F}_{magnetic} = I(\vec{l} \times \vec{B}).$$

### Force between two parallel current carrying conductors

Consider two current carrying conductors of length  $l$  in proximity such that their magnetic fields overlap, their resultant magnetic force on each other is then

$$\vec{F}_{magnetic} = \frac{\mu_0 I_1 I_2}{2\pi r} l$$



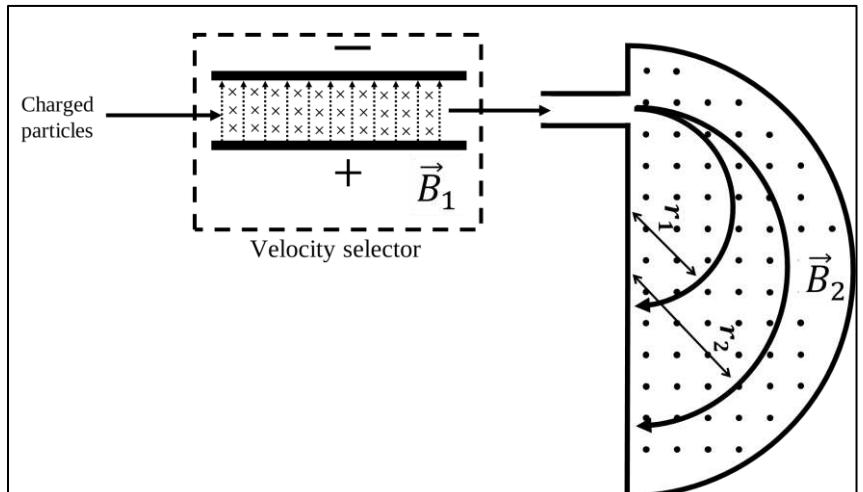
## Bainbridge mass spectrometer

The Bainbridge mass spectrometer is used to accurately determine atomic mass.

The first part of the mass spectrometer is a velocity selector in which both electric field  $\vec{E}$  and magnetic field  $\vec{B}_1$ . For charged particles to exit this velocity selector, their velocity must obey

$$v = \frac{E}{B_1}$$

This part of the mass spectrometer allows only charged particles with a certain velocity to enter the second region of only magnetic field  $\vec{B}_2$ .



The second part of the instrument takes advantage that charged particles of the same entry velocity but different mass will travel in circular path of different radius.

$$qvB_2 = \frac{mv^2}{r^2} \Rightarrow m = \frac{qB_2 r^2}{v} = \frac{qB_1 B_2 r^2}{E}$$

## Chapter 5: Electromagnetic Induction

### Magnetic Flux

**Magnetic flux** is a measure of total magnetic field  $\vec{B}$  passing through a given area  $\vec{A}$ , this is calculated with

$$\phi = \vec{B} \cdot \vec{A}.$$

In the case of  $N$  number of area of  $\vec{A}$  of which  $\vec{B}$  passes through, the total magnetic flux is called the **magnetic flux linkage**  $\Phi$ , and is determined by

$$\Phi = N\phi = NBA \cos \theta$$

### Induced EMF

EMF is induced when magnetic flux changes with time. This is the core of Faraday's and Lenz's law of electromagnetic induction.

- Faraday's law tells us how much emf is induced (magnitude) and Lenz's law tells us in what direction the force acts upon (direction of induced current).
- Faraday's law tells us that the magnitude of induced emf is equal to the rate of magnetic flux change and Lenz's law tells us that the induced current will be in the direction opposing the initial magnetic field.

Together, they are simply written as

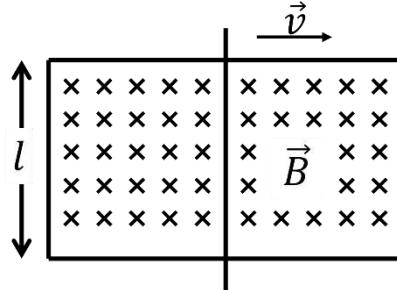
$$\varepsilon = -\frac{d\phi}{dt}$$

### **Induced emf in straight conductor**

In a straight conductor, the area changes with time which causes the magnetic flux to change with time.

Consider a rectangular coil with one of its sides movable and the opposite of the movable side has a length of  $l$ , in a region of magnetic field  $\vec{B}$ . If the movable side is moved at velocity  $\vec{v}$ , the area of the coil would change. The induced emf would then be

$$\varepsilon = -Bl \frac{dx}{dt} = Blv \sin \theta_{vB}$$



### **Induced emf in a coil**

In a circular coil, the option for inducing emf comes from varying the magnetic field **and** the area of the coil, thus 2 equations can be found,

$$\varepsilon = -NB \frac{dA}{dt} \text{ or } \varepsilon = -NA \frac{dB}{dt}$$

### **Induced emf in a rotating coil**

For a coil rotating at angular speed of  $\omega$ , the emf induced is then

$$\varepsilon = NBA\omega \sin(\omega t)$$

## Inductance

### Self-induction

The idea of self-inductance is this – a magnetic field induces emf in a conductor, which in turns induces another magnetic field that opposes the initial induced emf. The conductor ‘self induces’ a magnetic field. The ability of a conductor to do this is quantified by **self-inductance L**,

$$L = -\frac{\epsilon}{(\frac{dI}{dt})}$$

Generally, this means that

$$LI = N\phi$$

For more specific cases, 2 are considered:

1. For a coil of N turns with a cross sectional area of A and radius of r,

$$L = \frac{\mu_0 N^2 A}{2r}$$

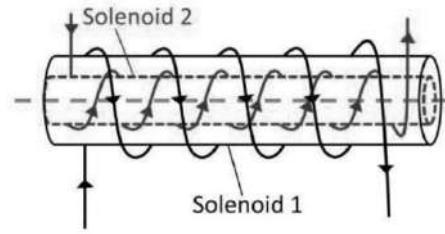
2. For a solenoid of N turns with a cross sectional area of A and length l,

$$L = \frac{\mu_0 N^2 A}{l}$$

### Mutual induction

Mutual inductance happens between 2 conductors, when the magnetic field induced by one conductor induces current in the other conductor.

Consider two coaxial solenoids, a magnetic field is generated by solenoid 1 and thus solenoid 2 respond by an induced emf, if solenoid 2 has a cross sectional area of  $A_2$ , then the mutual inductance between solenoid 1 and 2 is



$$M_{21} = \frac{\mu_0 N_1 N_2 A_2}{l},$$

where  $l$  is the length of the solenoid.

### Energy Stored in Inductor

The energy stored in an inductor of inductance  $L$  and with current  $I$  running through it, is simply

$$U = \frac{1}{2} LI^2$$

## Chapter 6: Alternating Current

### Alternating Current

Alternating current (AC) is defined as an electric current that periodically reverses its direction with respect to time.

### Root Mean Square Values

In AC circuits, rather than being of constant value (such found in DC Circuits), voltages and current now are functions of time:

$$I \mapsto I(t) = I_{peak} \sin(\omega t)$$

$$V \mapsto V(t) = V_{peak} \sin(\omega t)$$

Resistance is then defined as

$$R = \frac{V_o}{I_o}$$

In calculation of power, where  $P_{DC} = IV$ , for AC circuits,

$$P_{AC} = I_{rms} V_{rms}$$

where,

$$I_{rms} = \frac{I_o}{\sqrt{2}} \text{ and } V_{rms} = \frac{V_o}{\sqrt{2}}$$

### Impedance

In DC circuit, our main concern for opposition of current flow is only resistance  $R$ .

In AC circuits, we now have what is known as **impedance Z**, which is defined by

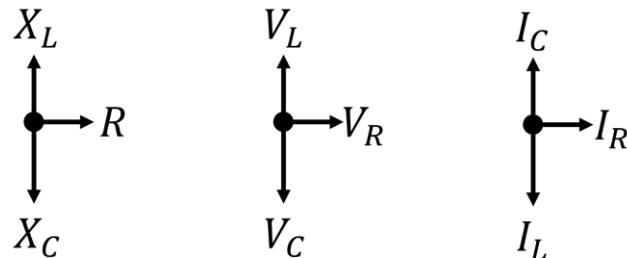
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where  $R$  is the resistance,  $X_L$  is the inductive reactance and  $X_C$  is the capacitive reactance found in the circuit.

The following table shows how to calculate these values

Reactance	Equation
Capacitance reactance for a capacitor of capacitance $C$	$X_C = \frac{1}{2\pi f C}$
Inductive reactance for an inductor of inductance $L$	$X_L = 2\pi f L$

The phasor diagram for an RLC circuit is as follows,



Phasor Diagram  
for RLC Circuit

Which means that the phase angle between current and voltage is

$$\theta_{IV} = \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$

Resonance occurs when  $X_L = X_C \Rightarrow \omega = \frac{1}{\sqrt{LC}} = 2\pi f$ .

## **Power & Power Factor**

2 types of power that be calculate in the case of AC circuits,

1. Instantaneous power

$$P = I(t) \times V(t)$$

2. Average power

$$P_{ave} = I_{rms}V_{rms} \cos(\theta_{IV})$$

The power factor is simply

$$\cos \theta_{IV} = \frac{P_{real}}{P_{apparent}} = \frac{P_{ave}}{I_{rms}V_{rms}}$$

## Chapter 7: Optics

### Geometrical Optics: Reflection

Definitions:

1. Centre of curvature,  $C$  = a point on the principal (or optical) axis that is positioned at distance equal to the radius of curvature  $R$ , of the spherical mirror.
2. Focal point,  $f$  = a point on the principal axis at which light rays travelling parallel to the principal axis will converge onto or diverge from, after reflecting on the surface of the spherical mirror.  
 $f$  and  $R$  are related by the following equation:

$$R = 2f$$

2 types of mirrors:

1. **Convex mirror**, of which its radius is located behind the mirror.
2. **Concave mirror**, of which it's radius of curvature is located in front of the mirror.

Conventions	
Focal length, $f$	+ for concave; - for convex
Curvature Radius, $R$	

Lateral magnification  $m$ , refers to the ratio between the height of the image to the height of the object. In equation form,

$$m = \frac{h_i}{h_o}$$

$m > 0 \Rightarrow$  upright image;  $m < 0 \Rightarrow$  inverted image

### Geometrical Optics: Refraction

An extension to Snell's law will be the refraction at a spherical surface. The following equation allows us to relate distances, refractive indices and radius of curvature of the spherical surface:

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

In the equation above  $n_i$ , refers to refractive indices,  $u$  and  $v$  refers to object and image distances respectively and  $R$  refers to the radius of curvature.

Conventions	
Curvature Radius, $R$	+ for convex, i.e. $C$ opposite side as incoming light - for concave, i.e. $C$ same side as incoming light

For the refractive indices, subscript 1 refers to the refractive index on the side of the incoming light rays and subscript 2 refers to the refractive index on the side of the outgoing rays.

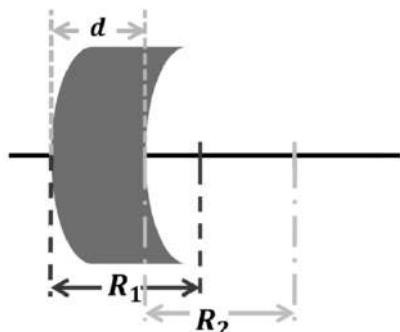
## Geometrical Optics: Thin lenses

The thin lens equation assumes that the thickness measured between two vertex of the spherical surface of a lens is much smaller than the product of the radii of the spherical lenses, that is  $d \ll R_1 R_2$ .

For thin lenses,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Conventions	
Focal length, $f$	+ for convex, i.e. same side as incoming light - for concave, i.e. opposite side as incoming light



On the other hand, using the lens maker's equation,

$$\frac{1}{f} = \left( \frac{n_{\text{material}}}{n_{\text{medium}}} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

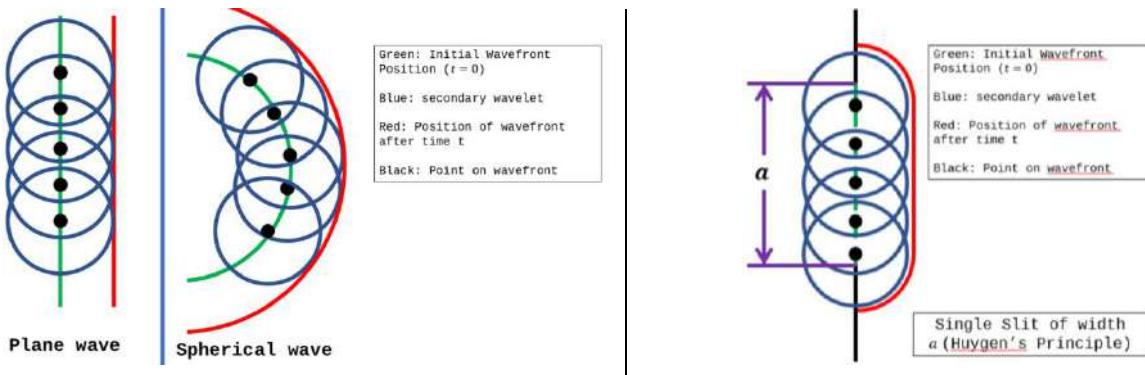
one can determine the focal length  $f$ , of the lens from

1. the radii of the lens surfaces,  $R_1$  and  $R_2$ ,
2. the ratio of the refractive index of the lens material to the refractive index of the surrounding,  $\frac{n_{\text{material}}}{n_{\text{medium}}}$

Conventions	
Curvature Radius, $R$	- if curvature same side as incoming light + if curvature opposite side as incoming light

## Physical Optics: Huygens's Principle

Huygen's Principle states that "each point on the wavefront acts as the source of secondary wavelets that spread out in all directions in spherical waves with a speed equal to the speed of wave propagation."

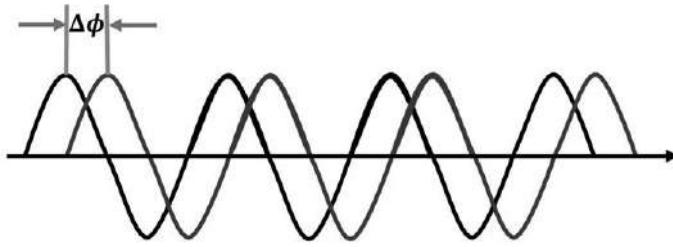


## Physical Optics: Interferences

Coherence between 2 waves refers to the condition of constant phase difference between 2 waves with respect to time, that is to say  $\frac{d\phi}{dt} = 0$ . This property is the ideal property for stationary interference.

For a stable interference pattern, the following conditions are required:

1. Coherence, that is to say the two interacting light waves are of the same phase difference,  $\frac{d\phi}{dt} = 0$ .



2. Monochromatic, that is to say that the two interacting light waves are of the same wavelength, i.e.  $\lambda_1 = \lambda_2$ .  
 For purely **constructive interference**, it is empirical that the phase difference between the interacting waves is either 0 or  $n\lambda$ . On the other hand, for purely **destructive interference**, it is required that the phase difference between the two interacting wave is  $\frac{n\lambda}{2}$ . ( $n$  is both cases refers to integer values.)

## Physical Optics: Slits

### Double Slit

We now consider the case for Young's double slit experiment.

Here we define the following variables:

$D$	distance from slit to screen
$d$	slit separation
$y_m$	distance from central maximum tu the mth fringe

We know that in order to determine what type of fringe forms at P, we need to look at the path difference and from the figure, we can say that the figure,

$$\Delta\phi = S_2P - S_1P = d\sin\theta = d\left(\frac{y_m}{D}\right)$$

For **bright fringes**,

$$\Delta\phi = \frac{y_m d}{D} = m\lambda$$

Rearranging this allows us to find fringe distance as a function of  $d$  and  $D$  with  $m$  having any integer value indicating **mth bright fringe** from central maximum:

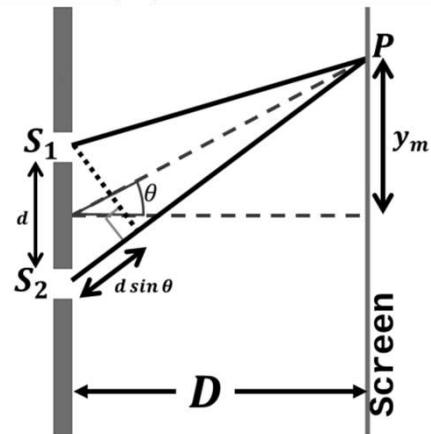
$$y_m = \frac{m\lambda D}{d}$$

Shifting one the waves by  $0.5\lambda$  give us the equation for **dark fringes**,

$$y_m = \frac{(m + 0.5)\lambda D}{d}.$$

Lastly, we'd want to calculate the fringe separation. This can be done by considering  $\Delta y = y_{m+1} - y_m$ , which results in

$$\Delta y = \frac{\lambda D}{d}$$



## Single Slit

Diffraction is defined as the spreading or bending of waves as they pass through an aperture of a barrier. The diffracted waves then interfere with each other to produce a diffraction pattern.

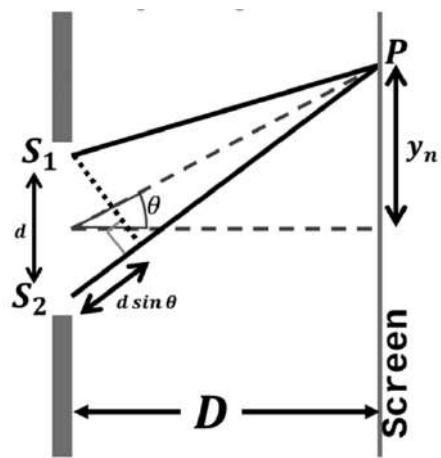
Light waves from one portion of the slit interact with light waves from a different portion of the same slit to produce a diffraction pattern.

Here, we find that the dark fringes forms when according to

$$d \sin \theta = n\lambda.$$

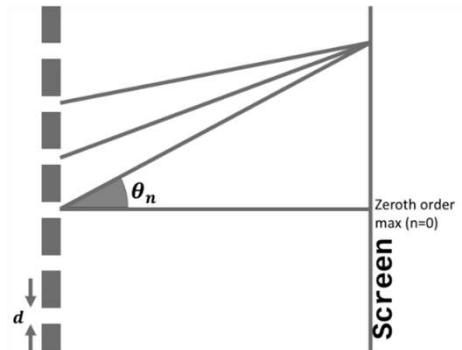
Geometrically, we also find that  $\tan \theta \approx \sin \theta \approx \frac{n\lambda}{d} \approx \frac{y_n}{D}$ .

As such, we can say that **dark fringes** forms at  $y_n = \frac{n\lambda D}{d}$ . This would also mean that **bright fringes** forms at  $y_n = \frac{(n+0.5)\lambda D}{d}$ .



## Diffraction Grating

In the case for diffraction grating, light waves from many slits and interfere at the screen to form fringes of equal width. The equation by which the pattern follows is  $d \sin \theta_n = n$  for **bright fringes** and shifted by  $0.5\lambda$  for **dark fringes**. Note that the angle  $\theta_n$  is measured from the normal line formed at the zeroth order maximum. Also note that, **maximum number of fringes** can be calculated by considering that  $\sin \theta_n < 1$ .



## Physical Optics: Thin Films

Referring to figure on thin films, we can see that the two reflected light waves has a phase difference of  $0.5\lambda$  from reflections at surface 1 and 2. One must also take into consideration of the extra distance that the second (green) wave travelled, that is  $2nt$ . Therefore, the total phase difference between the reflected waves is then

$$\Delta\phi = 2nt - \frac{1}{2}\lambda.$$

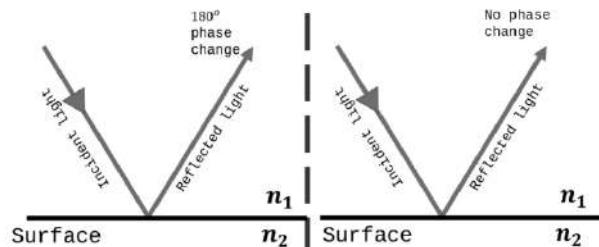
For **constructive interference**,  $\Delta\phi = 2nt - \frac{1}{2}\lambda = n$ , which gives us the equation

$$2nt = \left(n + \frac{1}{2}\right)\lambda.$$

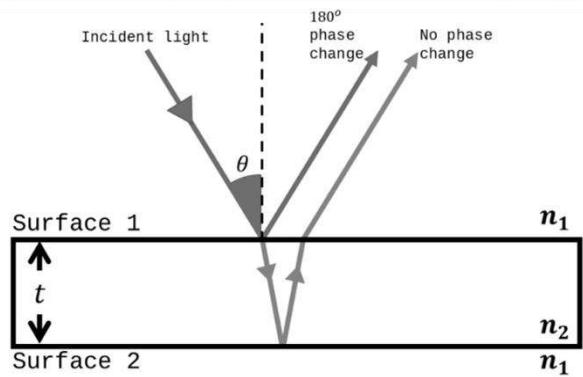
Dark fringes then appear between the bright fringes, i.e. they follow the equation

$$2nt = n\lambda.$$

Main application for this concept of thin film interference is in **optical coatings** as one can manipulate the thickness of the coating as to choose the level of constructive or destructive interference. These optical coatings can be applied onto both reflective as well as refractive systems.



Phase change upon reflection when  $n_2 > n_1$ .  
No phase change upon reflection when  $n_1 > n_2$ .



## Chapter 8: Particle Waves

### De Broglie Wavelength

Like light, matter also exist in dual form – as particles **and** waves.

Matter waves, known as “de Broglie wavelength”, are calculated with

$$\lambda_{matter} = \frac{h}{p} = \frac{h}{mv}$$

For a particle with mass  $m$  charge  $q$  accelerated by electric field of  $V$  volts,

$$\lambda_{matter} = \frac{h}{\sqrt{2qVm}}$$

### Electron Diffraction

On de Broglie wavelength:

1. To show that particles may exhibit wave-like characteristics, Davisson and Germer designed an experiment in which they show that electrons diffracted.
2. They achieve this by directing a beam of electrons onto a nickel crystal.

On electron microscope:

1. Because of their short wavelength (1nm for electrons vs 400nm – 700nm for light microscope), electronic microscopes can offer physicists a higher resolution in probing specimens.
2. Optical microscope is made up of glass lenses, whereas components of an electron microscope are electromagnetic.

## Chapter 9: Nuclear & Particle Physics

### Binding Energy & Mass Defect

**Mass defect**,  $\Delta m$  = mass difference between the actual mass of an atomic nucleus and the sum of its components, i.e. protons and neutrons.

For an atomic nucleus of mass  $m_{nucleus}$  with  $Z$  number of protons of mass  $m_{proton}$  and  $N$  number of neutrons of mass  $m_{neutron}$ , its mass defect is

$$\Delta m = (Zm_{proton} + Nm_{neutron}) - m_{nucleus}$$

**Binding energy**,  $E_{binding}$  = energy found in the nucleus of an atom that binds its components together. This energy can be calculated from the mass defect,

$$E_{binding} = \Delta mc^2$$

As the masses of atomic nucleus is well, very small, and the speed of light is astronomical, it may be easier to perform calculations using atomic mass unit (amu) or Dalton (u) and  $MeV/c^2$

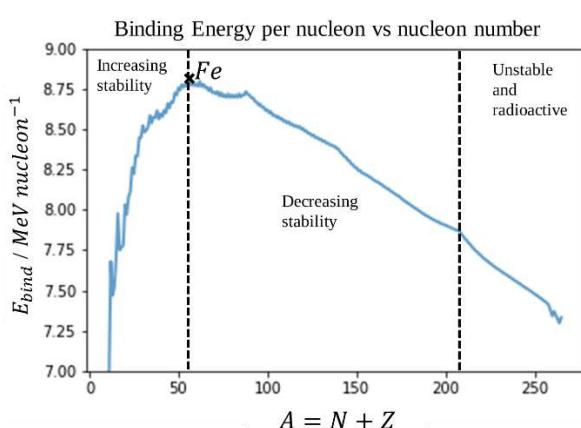
$$1kg = 6.022(10^{26})u = 5.60958(10^{29}) MeV/c^2$$

$$1u = 1.66054(10^{-27}) kg = 931.494 MeV/c^2$$

**Binding energy per nucleon**,  $\frac{E_{binding}}{A}$ :

Nucleon number,  $A$  = Number of protons,  $Z$  + Number of neutrons,  $N$

**Binding energy per nucleon vs nucleon number graph:**



- For nuclei lighter than that of iron (Fe), it is found that binding energy per nucleon increases with the nucleon number.
- After the iron limit, the binding energy per nucleon decreases.
- At  $A \approx 209$  (nuclei of Bi), the binding energy per nucleon is too weak to keep the nuclei together and thus, are unstable and radioactive.

## Radioactivity

The following table describes the types of decay of a radioactive substance

Type of decay	Process	Description
$\alpha$	$\begin{aligned} {}_Z^A P &\rightarrow {}_{Z-2}^{A-4} D + {}_2^4 He \\ \text{parent nucleus} &\rightarrow \text{daughter nucleus} \\ &+ \alpha \text{ particle} \end{aligned}$	In $\alpha$ decay, an $\alpha$ particle (Helium) is emitted when the parent nucleus decays into its daughter nucleus. Electrical charge is conserved throughout the process. Energy is released upon $\alpha$ decay.
$\beta^-$	$\begin{aligned} n &\rightarrow p^+ + e^- + \bar{\nu} \\ {}_Z^A P &\rightarrow {}_{Z+1}^{A-1} D + {}_{-1}^0 e + \bar{\nu} \\ \text{parent nucleus} &\rightarrow \text{daughter nucleus} \\ &+ \beta^- \text{ particle} \\ &+ \text{antineutrino} \end{aligned}$	In $\beta^-$ decay, an electron $e^-$ and an antineutrino $\bar{\nu}$ is emitted when the parent nucleus decays into its daughter nucleus.
$\beta^+$	$\begin{aligned} p^+ &\rightarrow n + e^+ + \nu \\ {}_Z^A P &\rightarrow {}_{Z-1}^{A-1} D + {}_{+1}^0 e + \nu \\ \text{parent nucleus} &\rightarrow \text{daughter nucleus} \\ &+ \beta^+ \text{ particle} \\ &+ \text{neutrino} \end{aligned}$	In $\beta^+$ decay, a positron $e^+$ and a neutrino $\nu$ is emitted when the parent nucleus decays into its daughter nucleus.
$\gamma$	$\begin{aligned} {}_Z^A P &\rightarrow {}_Z^A P + \gamma \\ \text{nuclei of high energy state} &\rightarrow \text{nuclei of low energy state} \\ &+ \gamma \text{ ray} \end{aligned}$	In $\gamma$ decay, the emission is a photon (light ray). This happens because the nucleons lower its energy state.

In general,  $N$  number of radioactive particles will decay according to the **decay law**,

$$\frac{dN}{dt} = -\lambda N$$

where  $\lambda$  is the decay constant of the substance, which varies between isotopes.

The solution for the decay law is

$$N(t) = N_o e^{-\lambda t}$$

where  $N_o = N(t = 0)$ .

The rate of decay is known as **activity**

$$A = \left| \frac{dN}{dt} \right| = \left| \frac{dN_o}{dt} \right| e^{-\lambda t} = A_o e^{-\lambda t}$$

**Half-life** is simply the time it takes for the number of isotopes to decrease by half  $T_{\frac{1}{2}}$ ,

$$N = \frac{1}{2} N_o = N_o e^{-\lambda T_{\frac{1}{2}}} \Rightarrow T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

## Particle Accelerator

Thermionic emission (Edison Effect) = emission of electrons on the surface of a metal by providing it sufficient thermal energy.

As mentioned before, a charged particle may be accelerated by the help of an electric and magnetic field. The acceleration would stem from Lorentz Force.

To probe subatomic particles, we need high energy because higher energy results in higher momentum which gives out smaller de Broglie wavelength. This means a higher resolution can be achieved.

2 types of particle accelerators:

1. Cyclotron

It uses magnetic field to maintain charged particles in nearly circular paths.

A cyclotron is composed of 2 'dees', charged particles are accelerated in the region of space between the two 'dees', where an electric field is applied.

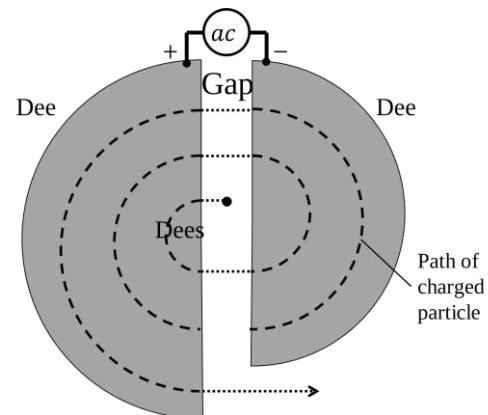
Velocity of the charged particles when they are in the 'dees' is

$$v = \frac{qBr}{m}$$

Frequency of electric field is equal to the frequency of the circulating protons,

$$f = \frac{qB}{2\pi m}$$

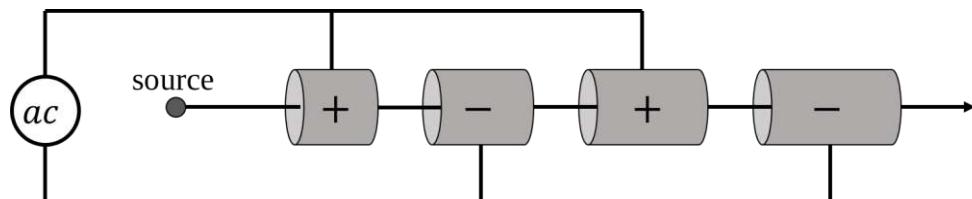
which is known as the **cyclotron frequency**.



2. Linear Accelerator

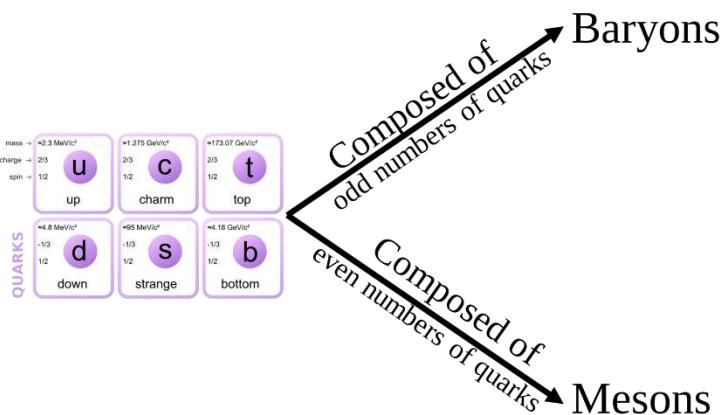
Charged particles are accelerated through a series of linear conductor tubes.

Alternating voltage is applied to consecutive tubes so that when a charged particle reaches a gap, the tube they just left is now negatively charged and the tube they are heading into is positively charged.



## Fundamental Particles

mass → $\approx 2.3 \text{ MeV}/c^2$	charge → 2/3	spin → 1/2	mass → $\approx 1.275 \text{ GeV}/c^2$	charge → 2/3	spin → 1/2	mass → $\approx 173.07 \text{ GeV}/c^2$	charge → 2/3	spin → 1/2	mass → 0	charge → 0	spin → 1	mass → $\approx 126 \text{ GeV}/c^2$	charge → 0	spin → 0
up	C	t	gluon	Higgs boson										
down	s	b	photon											
electron	$\mu$	$\tau$	Z boson											
electron neutrino	$\nu_e$	$\nu_\mu$	$\nu_\tau$											
strange														
bottom														



Particle-antiparticle pair:

They are pairs of particles that has opposite charge to each other. E.g., electron has a negative charge whereas its antiparticle, a positron has a positive charge. They interact by annihilating each other.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

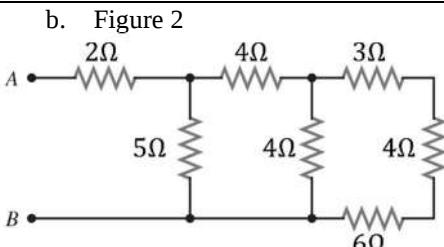
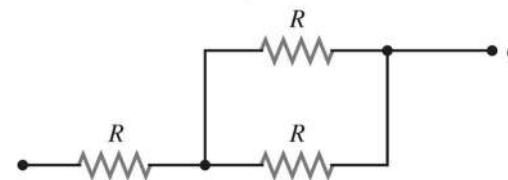
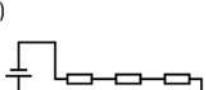
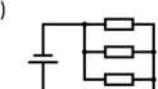
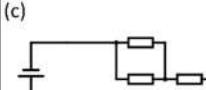
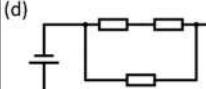
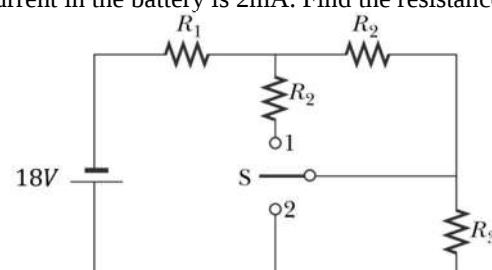
### Chapter 3 – Part 1: Electric Current, Ohm's Law & Resistance

No	Questions	Solutions
1	A defibrillator passes 22A of current through the torso of a person in 1.6ms. a. How much charge moves during this time? b. How many electrons pass through the wires connected to the patient?	$n = \frac{\Delta q}{e} = \frac{I\Delta t}{e} = \frac{22(1.8 \times 10^{-3})}{1.602 \times 10^{-19}}$ $n = 2.197 \times 10^{17} \text{ electrons}$
2	A coffee-maker contains a heating element that has a resistance of $12\Omega$ . This heating element is energized by a 240V outlet. What is the current in the heating element?	$V = IR$ $240 = I(12)$ $I = 20A$
3	The resistance across a cell wall of a cell is $5 \times 10^9\Omega$ . Determine the current and number of electrons flowing across the cell wall when the potential difference between the walls is 80mV in 2ms.	$V = IR$ $80(10^{-3}) = I(5(10^9))$ $I = 1.6 \times 10^{-11}A$ $n = \frac{I\Delta t}{e} = \frac{(1.6 \times 10^{-11})(2 \times 10^{-3})}{1.6 \times 10^{-19}}$ $n = 200000 \text{ electrons}$
4	A typical smartphone powerbank is rated 10000mAh. Determine the total charge in Coulombs that this battery can provide. Calculate the maximum current that the battery can provide for 30minutes.	$\Delta q = (10000 \times 10^{-3})(3600) = 36kC$ $I = \frac{\Delta q}{\Delta t} = \frac{36000}{30 \times 60} = 20A$
5	The drawing shows three situations in which the current takes different paths through a piece of material. Each of the rectangular pieces is made from a material whose resistivity is $1.8 \times 10^{-2}\Omega m$ , and the unit of length in the drawing is $L_0 = 2.8cm$ . Each piece of material is connected to a 6V battery. Find the resistance and the current in each case.	$R_a = \rho \frac{4L_0}{L_0 \times 2L_0} = \rho \frac{2}{L_0} = 1.8(10^{-2}) \left( \frac{2}{2.8 \times 10^{-2}} \right)$ $R_a = 1.286\Omega$  $R_b = \rho \frac{L_0}{2L_0 \times 4L_0} = \rho \frac{1}{8L_0}$ $R_b = 1.8(10^{-2}) \left( \frac{1}{8 \times 2.8 \times 10^{-2}} \right)$ $R_b = 0.0804\Omega$  $R_c = \rho \frac{2L_0}{L_0 \times 4L_0} = \rho \frac{1}{2L_0}$ $R_c = 1.8(10^{-2}) \left( \frac{1}{2 \times 2.8 \times 10^{-2}} \right)$ $R_c = 0.32143\Omega$
6	A cylindrical wire has a length of 2.8m and a radius of 0.8mm. It carries a current of 1.5A, when a voltage of 4.5V is applied across the ends of the wire. Determine the resistivity of the wire.	$\rho = \frac{RA}{l} = \frac{\left(\frac{V}{I}\right)(\pi r^2)}{l}$ $\rho = \frac{\left(\frac{4.5}{1.5}\right)(\pi)(0.8 \times 10^{-3})^2}{2.8}$ $\rho = 2.15 \times 10^{-6}\Omega m$
7	A piece of wire has a resistance of $30\Omega$ at $20^\circ C$ and $50\Omega$ at $60^\circ C$ . What is the temperature coefficient of resistivity?	$\frac{R}{R_o} = 1 + \alpha \Delta T$ $\frac{50}{30} = 1 + \alpha(60 - 20)$ $\alpha = 0.0167^\circ C^{-1}$
8	A wire spool of has 30m of insulated wire coiled in it. When connected to a DC power source, it is found that 2.2A of current is flowing through it. After some lengths of wire is used, it is found that the current flowing through the wires left reads 3.5A when it is connected to the same DC power source. Determine the length of wire left in the spool.	$\frac{L_f}{L_i} = \frac{VR_f A}{I_f \rho} \left( \frac{I_i \rho}{VR_i A} \right) = \frac{I_i}{I_f}$ $\frac{L_f}{30} = \frac{2.2}{3.5} = 18.86m$
9	What is the fractional change in the resistance ( $\frac{\Delta R}{R_o}$ ) of an iron filament ( $\alpha = 5(10^{-3})^\circ C^{-1}$ ) when its temperature changes from $25.0^\circ C$ to $50.0^\circ C$ ?	$R = R_o(1 + \alpha \Delta T)$ $\frac{R - R_o}{R_o} = \alpha \Delta T = (\alpha = 5(10^{-3})^\circ C^{-1})(50 - 25)$

		$\frac{R - R_o}{R_o} = 0.125$
10	The temperature coefficient of resistivity for the metal A is $0.0028^{\circ}\text{C}^{-1}$ , and for metal B it is $0.0048^{\circ}\text{C}^{-1}$ . The resistance of a wire of metal increases by 7.0% due to an increase in temperature. For the same increase in temperature, what is the percentage increase in the resistance of a wire of metal B?	<p>Percentages,</p> $p = \frac{R - R_o}{R_o} \times 100 = 100\alpha\Delta T$ $p_A = 7 = 100\alpha_A\Delta T$ $p_B = 100\alpha_B\Delta T$ $\frac{p_B}{p_A} = \frac{(100\alpha_B\Delta T)}{(100\alpha_A\Delta T)} = \frac{\alpha_B}{\alpha_A}$ $p_B = \frac{0.0048}{0.0028}(7) = 12\%$
11	A metal wire of resistance R is cut into three equal pieces that are then connected side by side to form a new wire the length of which is equal to one-third the original length. What is the resistance of this new wire?	$R_f = \frac{\rho \left(\frac{l}{3}\right)}{3A} = \frac{1}{9}R$
12	The temperature of a sample of tungsten ( $\rho = 5.6(10^{-8}) \Omega\text{m}$ , $\alpha = 4.5(10^{-3})^{\circ}\text{C}^{-1}$ ) is raised while a sample of copper ( $\rho = 1.7(10^{-8})\Omega\text{m}$ ) is maintained at $20.0^{\circ}\text{C}$ . At what temperature will the resistivity of the tungsten be 5 times that of the copper?	$\rho = \rho_o(1 + \alpha\Delta T)$ $T_f - T_i = \frac{1}{\alpha} \left( \frac{\rho}{\rho_o} - 1 \right)$ $T_f - 20^{\circ}\text{C} = \frac{1}{4.5(10^{-3})} \left( \frac{3(1.7)(10^{-8})}{5.6(10^{-8})} - 1 \right)$ $T_f = 135.08^{\circ}\text{C}$

### Chapter 3 – Part 2: EMFs & internal resistance

1	<p>Calculate the terminal voltage for a battery with an internal resistance of <math>1.2\Omega</math> and an emf of <math>12V</math> when the battery is connected in series with</p> <ol style="list-style-type: none"> <li>a <math>20\Omega</math> resistor,</li> <li>a <math>200\Omega</math> resistor.</li> </ol>	$\varepsilon = V + Ir = I(R + r) \Rightarrow I = \frac{\varepsilon}{R + r}$ $V = \varepsilon - Ir = \varepsilon - \left(\frac{\varepsilon}{R + r}\right)r$ $V(R = 20\Omega) = 12 - \left(\frac{12}{20 + 1.2}\right)(1.2) = 11.321V$ $V(R = 200\Omega) = 12 - \left(\frac{12}{200 + 1.2}\right)(1.2) = 11.928V$
2	<p>Three <math>1.5V</math> cells are connected in series to a <math>15\Omega</math> lightbulb. If the resulting current is <math>0.25 A</math>, what is the internal resistance of each cell, assuming they are identical and neglecting the resistance of the wires?</p>	<p>For each cell,</p> $V = \varepsilon - Ir$ <p>Total terminal voltage,</p> $V_{total} = 4(\varepsilon - Ir) = IR$ $3(1.5 - (0.25)r) = (0.25)(15)$ $r = 1A$
3	<p>What is the internal resistance of a <math>15V</math> battery whose terminal voltage drops to <math>9V</math> when the external resistors draw <math>30A</math>? What is the resistance of the starter?</p>	$V = \varepsilon - Ir \Rightarrow 9 = 15 - 30r \Rightarrow r = 0.2\Omega$ $V = IR \Rightarrow 9 = 30R \Rightarrow R = 0.3\Omega$
4	<p>A battery has an emf of <math>15V</math>. The terminal voltage of the battery is <math>12V</math> when it is delivering <math>27W</math> of power to an external load resistor <math>R</math>.</p> <ol style="list-style-type: none"> <li>What is the value of <math>R</math>?</li> <li>What is the internal resistance of the battery?</li> </ol>	$P = IV; V = IR$ $P = \frac{V^2}{R} \Rightarrow 27 = \frac{12^2}{R} \Rightarrow R = 5.3\Omega$ $V = \varepsilon - Ir = \varepsilon - \left(\frac{P}{V}\right)r$ $12 = 15 - \left(\frac{27}{12}\right)r$ $r = 1.33\Omega$
5	<p>What is the current in a <math>2.5\Omega</math> resistor connected to a battery that has a <math>0.2\Omega</math> internal resistance if the terminal voltage of the battery is <math>10V</math>? What is the emf of the battery?</p>	$V = IR \Rightarrow 10 = I(2.5) \Rightarrow I = 4A$ $V = \varepsilon - Ir \Rightarrow 10 = \varepsilon - (4)(0.2)$ $\varepsilon = 10.8V$
6	<p>Two <math>1.50-V</math> batteries — with their positive terminals in the same direction — are inserted in series into the barrel of a flashlight. One battery has an internal resistance of <math>0.255\Omega</math>, the other an internal resistance of <math>0.153\Omega</math>. When the switch is closed, a current of <math>750\text{ mA}</math> occurs in the lamp. What is the lamp's resistance?</p>	$R_{total} = \frac{V}{I} = R_{lamp} + r_{batteries}$ $\frac{2(1.5)}{0.75} = R_{lamp} + (0.254 + 0.382)$ $R_{lamp} = 3.364\Omega$
7	<p>Three resistors of resistance (<math>25\Omega</math>, <math>50\Omega</math> and <math>80\Omega</math>) are connected in series and a <math>81\text{mA}</math> current passes through them. Determine the equivalent resistance and the potential difference across each resistor.</p>	$V_1 = IR_1 = (0.081)(25) = 2.025V$ $V_2 = IR_2 = (0.081)(50) = 4.05V$ $V_3 = IR_3 = (0.081)(80) = 6.48V$ $V_T = V_1 + V_2 + V_3 = IR_T$ $\Rightarrow 2.025 + 4.05 + 6.48 = (0.081)R_T$ $R_T = 155\Omega$
8	<p>A <math>15\Omega</math> resistor and a <math>20\Omega</math> resistor are connected in series across a <math>120V</math> source of voltage. A <math>30\Omega</math> resistor is also connected across the <math>120V</math> source and is in parallel with the series combination. Sketch the circuit and find the total current supplied by the source of voltage.</p>	$V = IR_{eff} \Rightarrow 120 = I \left( \frac{1}{30} + \frac{1}{15+20} \right)^{-1}$ $I = 7.429A$
9	<p>Find the equivalent resistance between the points A and B in the figures shown.</p> <p>a. Figure 1</p> $R_{eff} = R_4 + \left( \frac{1}{R_3} + \frac{1}{R_1 + R_2} \right)^{-1}$ $R_{eff} = 20 + \left( \frac{1}{35} + \frac{1}{18 + 22} \right)^{-1} = 38.67\Omega$ <p>Figure 2:</p>	

	b. Figure 2 	$R_1 = 3 + 4 + 6 = 13\Omega$ $R_2 = 4 + \left(\frac{1}{4} + \frac{1}{13}\right)^{-1} = 7.06\Omega$ $R_{eff} = 2 + \left(\frac{1}{5} + \frac{1}{7.06}\right)^{-1} = 4.93\Omega$
10	The circuit in the drawing contains three identical resistors. Each resistor has a value of $10\Omega$ . Determine the equivalent resistance between the points a and b, b and c, and a and c. 	$R_{ab} = 10\Omega$ $R_{bc} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = 5\Omega$ $R_{ac} = R_{ab} + R_{bc} = 10 + 5 = 15\Omega$
11	Three $5\Omega$ resistors can be connected together in four different ways, making combinations of series and/or parallel circuits. What are these four ways, and what is the net resistance in each case?	(a)  (b)  (c)  (d)  $R_a = 5 + 5 + 5 = 15\Omega$ $R_b = 5 + \left(\frac{1}{5} + \frac{1}{5}\right)^{-1} = 7.5\Omega$ $R_c = \left(\frac{1}{5} + \frac{1}{5} + \frac{1}{5}\right)^{-1} = 1.67\Omega$ $R_d = \left(\frac{1}{5} + \frac{1}{5 + 5}\right)^{-1} = 3.33\Omega$
12	A $25\Omega$ and a $37\Omega$ resistor are connected in parallel; this combination is connected in series with a $14\Omega$ resistor. If circuit can only handle $13A$ , what is the maximum voltage that can be applied across the whole network?	$V = IR = (13)\left(14 + \left(\frac{1}{25} + \frac{1}{37}\right)^{-1}\right)$ $V = 375.95V$
13	Two resistors connected in series have an equivalent resistance of $690\Omega$ . When they are connected in parallel, their equivalent resistance is $150\Omega$ . Find the resistance of each resistor.	$690 = R_1 + R_2$ $150^{-1} = R_1^{-1} + R_2^{-1}$ $\{R_1, R_2\} = \{730.505, 19.495\}\Omega$ OR $\{R_1, R_2\} = \{250.251, 499.749\}\Omega$
14	A $18V$ battery supplies current to the circuit shown in the figure. When the double-throw switch S is open, as shown in the figure, the current in the battery is $1mA$ . When the switch is closed in position 1, the current in the battery is $1.20mA$ . When the switch is closed in position 2, the current in the battery is $2mA$ . Find the resistances. 	$V = IR$ $18 = (10^{-3})(R_1 + R_2 + R_3)$ $18 = (1.2)(10^{-3})\left(R_1 + \left(\frac{1}{R_2} + \frac{1}{R_2}\right)^{-1} + R_3\right)$ $18 = 2(10^{-3})(R_1 + R_2)$ $\{R_1, R_2, R_3\} = \{3, 6, 9\}k\Omega$

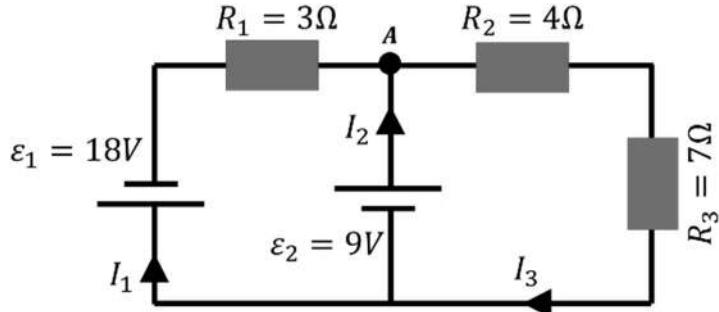
### Chapter 3 – Part 3: Electrical Energy & Power

No	Questions	Answers
1	A coffee cup heater and a lamp are connected in parallel to the same 240V outlet. Together, they use a total of 400W of power. The resistance of the heater is 500Ω. Find the resistance of the lamp.	$P_{total} = \frac{V^2}{R_{eff}}$ $P_{total} = V^2 \left( \frac{1}{R_{coffee}} + \frac{1}{R_{lamp}} \right)$ $400 = (240)^2 \left( \frac{1}{500} + \frac{1}{R_{lamp}} \right)$ $R_{lamp} = 202\Omega$
2	Two resistors, 15Ω and 65Ω, are connected in parallel. The current through the 65Ω resistor is 3A. Determine the current in the other resistor. What is the total power supplied to the two resistors?	$V_{15} = V_{65}$ $I_{65}R_{65} = I_{15}R_{15}$ $(3)(65) = I_{15}(15)$ $I_{15} = 13A$ $P_{total} = I_{65}^2 R_{65} + I_{15}^2 R_{15}$ $P_{total} = (3)^2(65) + (13)^2(15)$ $P_{total} = 3.12kW$
3	A blow-dryer and a vacuum cleaner each operate with a voltage of 240 V. The current rating of the blow-dryer is 6 A, and that of the vacuum cleaner is 4A. Determine the power consumed by the blow-dryer and the vacuum cleaner. Determine the ratio of the energy used by the blow dryer in 10minutes to the energy used by the vacuum cleaner in half an hour.	$P_{blowdryer} = 6(240) = 1440W$ $P_{vacuum} = 4(240) = 960W$ $\frac{E_{bd}}{E_{vc}} = \frac{(1440)(10)}{(960)(30)} = 0.5$
4	How many 75-W lightbulbs, connected in parallel to 240V source and each other, can be used without blowing a 13A fuse?	$I_{bulb} = \frac{P}{V}$ $I_{total} = N \frac{P}{V}$ $13 = N \left( \frac{75}{240} \right)$ $N = 41.6$ $N = 41 \text{ bulbs}$
5	What is the total amount of energy stored in a 5V, 1000mAh smartphone power bank when it is fully charged?	$E = QV = (1Ahr) \left( \frac{3600}{1hr} \right) (5)$ $E = 18kJ$
6	A 240V heater is rated at 1200W. Calculate the current through the heater when it is operating, and its resistance.	$P = IV \Rightarrow 1200 = I(240) \Rightarrow I = 5A$ $P = \frac{V^2}{R} \Rightarrow 1200 = \frac{240^2}{R} \Rightarrow R = 48\Omega$
7	In doing a load of clothes, a clothes dryer uses 16A of current at 240V for 30min. A personal computer, in contrast, uses 3A of current at 120V. With the energy used by the clothes dryer, how long (in hours) could you use this computer to “surf” the Internet?	$E = Pt = IVt$ $E = 16(240)(30(60)) = (3)(120)t$ $t = 19200s \approx 5.33\text{hrs}$
8	A piece of Nichrome wire (of resistivity $100 \times 10^{-8}\Omega m$ ) has a radius of 0.65mm. It is used in a laboratory to make a heater that uses 1.5kW of power when connected to a voltage source of 240V. Ignoring the effect of temperature on resistance, estimate the necessary length of wire.	$P = IV = \frac{V^2}{R}$ $R = \frac{V^2}{P} = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$ $\frac{240^2}{1500} = \frac{(100)(10^{-8})l}{\pi(0.65 \times 10^{-3})^2}$ $l = 50.97m$

## Notes on Kirchhoff's Rules

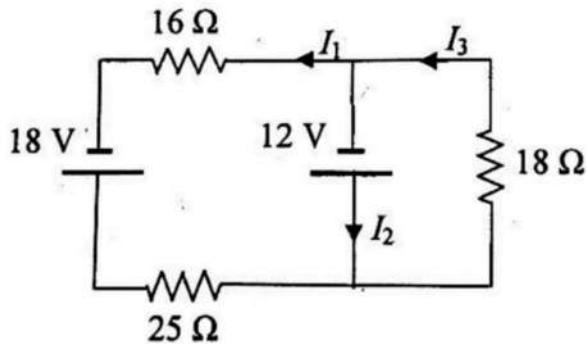
1. What is a junction/node?
2. What is a loop?
3. What does Kirchhoff's 1<sup>st</sup> Rule say?
4. What does Kirchhoff's 2<sup>nd</sup> Rule say?
5. [Sample Problem]

Calculate  $I_1$ ,  $I_2$  and  $I_3$ .

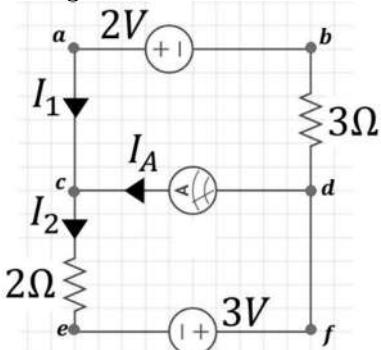


### 6. Practice

- a. [PSPM 14/15] Referring to the diagram below, determine  $I_1$ ,  $I_2$ ,  $I_3$  and the voltage across the  $18\Omega$  resistor.



- b. The figure shows a circuit consisting of two batteries, two resistors and an ammeter.



If the ammeter has internal resistance of  $5.0\Omega$ , what is the reading shown by the ammeter?

### Chapter 3 – Part 4: Kirchhoff's Rules

No	Questions	Answers
1	<p>Using Kirchhoff's current law, determine the unknown currents for the parallel network shown in the diagram.</p>	$12.6 = I_1 + 8.5$ $I_1 = 4.1mA$ $4 + I_2 = 8.5$ $I_2 = 4.5mA$
2	<p>Using Kirchoff's current law, find the unknown currents for the complex configurations shown in the diagram below.</p>	$9 - 12 + 2 + I_1 = 0$ $I_1 = 1A$ $I_2 - 1 + 1 - 3 = 0$ $I_2 = -3A$
3	<p>Using Kirchoff's current law, find the unknown currents for the complex configurations shown in the diagram below.</p>	$I_3 = 1.5 + 0.5 = 2\mu A$ $6 = I_2 + I_3 \Rightarrow I_2 = 4\mu A$ $I_2 + 1.5 = I_4 \Rightarrow I_4 = 5.5\mu A$ $I_1 = 6\mu A$
4	<p>The ammeter shown in the figure shown reads 1.8A. Find <math>I_1</math>, <math>I_2</math>, and <math>\epsilon</math>.</p>	$I_1 + I_2 = 1.8$ $15 = 7I_1 + 1.8(5)$ $I_1 = 0.86A$ $0.86 + I_2 = 1.8$ $I_2 = 0.94A$ $\epsilon = 0.94(2) + 1.8(5) = 10.8V$
5	<p>In the circuit shown below, find</p> <ol style="list-style-type: none"> <li>the current in the <math>3\Omega</math> resistor,</li> <li>the unknown emfs <math>\epsilon_1</math> and <math>\epsilon_2</math>,</li> <li>the resistance <math>R</math>. Note that three currents are given.</li> </ol>	$3 + 5 - I_3 = 0 \Rightarrow I_3 = 8A$ $2 + I_4 - 3 = 0 \Rightarrow I_4 = 1A$ $I_3 + I_4 + I_5 = 0 \Rightarrow I_5 = 7A$ $\epsilon_1 - 3(4) - I_3(3) = 0 \Rightarrow \epsilon_1 = 36V$ $\epsilon_2 - 5(6) - I_3(3) = 0 \Rightarrow \epsilon_2 = 54V$ $(-2)R - \epsilon_1 + \epsilon_2 = 0$ $R = 9\Omega$
6	<p>In the circuit shown, find</p>	$10 - (2 + 3)I_2 - (1 + 4)I_2 - 5 = 0$

	<p>a. the current in each branch b. the potential difference <math>V_{ab}</math> of point a relative to point b</p>	$I_1 + I_2 = 1A$ $5 + (1 + 4)I_2 - 10I_3 = 0$ $I_1 - 2I_3 = -1$ $I_1 = I_2 + I_3$ $I_i = \{0.8, 0.2, 0.6\}A$ $V_{ab} = -2(4) - (8)(3) = -3.2V$
7	Calculate the current through the $2\Omega$ resistor in the circuit shown.	$12 - 4I_1 - 2I_3 = 0$ $15 - 4I_2 + 2I_3 = 0$ $I_1 = I_2 + I_3$ $I_3 = I_{2\Omega} = -\frac{3}{8}A = -0.375A$
8	Referring to the figure below, determine the value of $\varepsilon$ at which the $200\Omega$ resistor dissipates no power.	$50 - 100I_1 - 200I_3 = 0$ $\varepsilon - 300I_2 - 200I_3 = 0$ Assume $I_3 = 0$ $I_1 = 0.5A$ $I_2 = -I_1$ $\varepsilon - 300(-0.5) - 200(0) = 0 \Rightarrow \varepsilon = -150V$
9	Referring to the figure shown, determine the current through each resistor.	$5 = I_{2\Omega}(4 + 2)$ $I_{2\Omega} = I_{4\Omega} = \frac{5}{6}A \approx 0.833A$ $8 - 5 = I_{5\Omega}(5)$ $I_{5\Omega} = \frac{3}{5}A = 0.6A$
10	Referring to the figure shown, determine the power used by each resistor.	

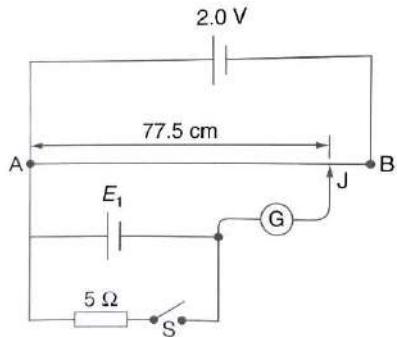
### Chapter 3 – Part 4: Potential Divider & Potentiometer

No	Questions	Answers
1	<p>The diagram shows a circuit consisting of 2 resistors, one 9V DC power source and 2 voltmeters.</p> <p>If <math>R_1 = 2.2\text{k}\Omega</math> and <math>R_2 = 6.8\text{k}\Omega</math>, determine the reading of voltmeter <math>V_1</math> and <math>V_2</math>.</p>	$V_i = \frac{R_i}{R_i + \dots + R_N} V_T$ $V_1 = \frac{2.2}{2.2 + 6.8} (9) = 2.2V$ $V_2 = \frac{6.8}{2.2 + 6.8} (9) = 6.8V$
2	<p>The diagram shows a potentiometer circuit. If <math>\epsilon = 12V</math> and <math>l_1</math> is one third of <math>l</math>, determine the reading on the voltmeter.</p>	$\frac{V}{\epsilon} = \frac{l_1}{l} = \frac{1}{3}$ $V = \frac{1}{3}(12) = 4V$
3	<p>The diagram shows a potentiometer circuit. The current through the <math>2\Omega</math> resistor is 750mA. If <math>\epsilon_1 = 12V</math> and length of AJ is one third of length of AB, determine the reading on the voltmeter.</p>	$\epsilon_1 = kl_{AB} + IR \Rightarrow \epsilon_1 - IR = kl_{AB}$ $V = kl_{AJ}$ $\frac{V}{\epsilon_1 - IR} = \frac{l_{AJ}}{l_{AB}}$ $\frac{V}{12 - (0.75)(2)} = \frac{1}{3}$ $V = 3.5V$
4	<p>The diagram shows a potentiometer circuit involving 2 DC power source. Determine the length <math>l_1</math> when the galvanometer shows zero reading if <math>\frac{\epsilon_1}{\epsilon_2} = 4</math> and <math>l = 28\text{cm}</math>.</p>	<p>The potentiometer is balanced when <math>\frac{\epsilon_1}{\epsilon_2} = \frac{l}{l_1}</math>.</p> $4 = \frac{0.28}{l_1}$ $l_1 = 0.07\text{m}$
5	<p>The diagram shows a potentiometer circuit involving 3 dry cells of different emf. When switch <math>S_1</math> is closed and switch <math>S_2</math> is opened, the length of AJ is 25cm. Conversely, when switch <math>S_1</math> is opened and switch <math>S_2</math> is closed, the length of AJ is 50cm. Determine the emf of <math>\epsilon_3</math> if <math>\epsilon_2</math> has an emf of 9V.</p>	$\epsilon_2 = kl_{AJ-1}; \epsilon_3 = kl_{AJ-2}$ $\frac{\epsilon_2}{\epsilon_3} = \frac{l_{AJ-1}}{l_{AJ-2}} \Rightarrow \frac{9}{\epsilon_3} = \frac{25}{50}$ $\epsilon_3 = 18V$

6

The emf  $E_1$  of a cell is measured using a potentiometer as shown in the figure. The driver cell has an emf of 2V and negligible resistance. When the switch S is open, the galvanometer G is balanced when the length AJ is 77.5cm. When the switch S is closed, the length AJ is 63.8 cm.

- Calculate the emf  $E_1$
- Calculate the internal resistance of the cell.



$$\frac{E_1}{2} = \frac{77.5}{100} \Rightarrow E_1 = 1.55V$$

$$\text{Opened Switch: } E_1 = kl_1$$

$$\text{Closed Switch: } V = kl_2 = E_1 - Ir$$

$$\text{By Ohm's Law, } E_1 = I(R + r) \Rightarrow I = \frac{E_1}{R+r}$$

$$V = E_1 - Ir = E_1 - \left(\frac{E_1}{R+r}\right)r$$

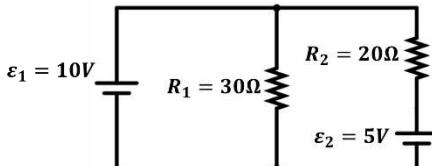
$$\frac{V}{E_1} = \frac{R}{R+r} \Rightarrow \frac{l_2}{l_1} = \frac{R}{R+r}$$

$$\frac{63.8}{77.5} = \frac{5}{5+r} \Rightarrow r = 1.07367\Omega$$

## Chapter 3 – Part 4: Kirchhoff's Rules

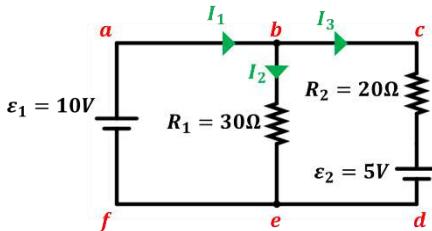
### Sample Problem:

Determine the current  $I_1$ ,  $I_2$  and  $I_3$  in the figure below.

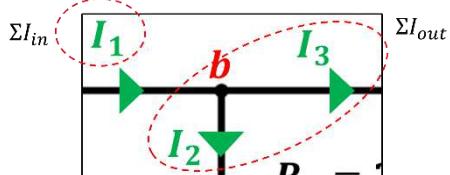


**Step-by-step solution with explanation:**

Label the diagram more for easier explanation.



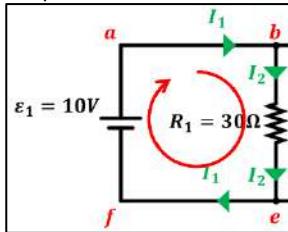
Apply KCL to junction at  $b$ ,



$$\Sigma I_{in} = \Sigma I_{out}$$

$$I_1 = I_2 + I_3$$

Apply KVL to loop  $abef$ ,



$$\Sigma V_{rise} = \Sigma V_{drop}$$

Since current flow is from  $\epsilon_1$  is the same as flow of  $I_1$  and  $I_2$ ,

$$\Sigma V_{rise} = \epsilon_1$$

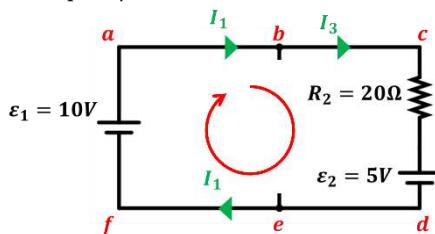
Voltage will always drop across resistor.

$$\Sigma V_{drop} = I_2 R_1$$

$$\Sigma V_{rise} = \Sigma V_{drop} \Rightarrow \epsilon_1 = I_2 R_1$$

$$10 = I_2(30) \Rightarrow I_2 = 0.333A$$

Apply KVL to loop  $acdf$ ,



$$\Sigma V_{rise} = \Sigma V_{drop}$$

Since current flow is from  $\epsilon_1$  is the same as flow of  $I_1$  and  $I_3$ ,

$$\Sigma V_{rise} = \epsilon_1$$

For  $V_{drop}$ ,

- $\epsilon_2$  pushes current against flow of  $I_1$  and  $I_3$
- Voltage will always drop across resistor.

$$\Sigma V_{drop} = \epsilon_2 + I_3 R_2$$

$$\Sigma V_{rise} = \Sigma V_{drop} \Rightarrow \epsilon_1 = \epsilon_2 + I_3 R_2$$

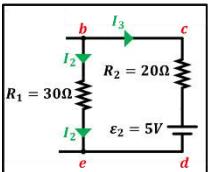
$$10 = 5 + I_3(20) \Rightarrow I_3 = 0.25A$$

### Optional

Consider applying KVL to loop  $bcde$ ,

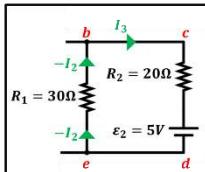
NOT A LOOP!

Because  $I_3$  goes clockwise but  $I_2$  goes counterclockwise.



Flip  $I_2$

This is a loop because both  $I_3$  and  $I_2$  goes clockwise.



No source to push current to flow clockwise,

$$\Sigma V_{rise} = 0$$

For  $V_{drop}$ ,

- $\epsilon_2$  pushes current against flow of  $I_1$  and  $I_3$
- Voltage will always drop across resistor.

$$\Sigma V_{drop} = \epsilon_2 + I_3 R_2 + (-I_2) R_1$$

$$\Sigma V_{rise} = \Sigma V_{drop} \Rightarrow 0 = \epsilon_2 + I_3 R_2 + (-I_2) R_1$$

$$0 = 5 + I_3(20) + (-0.333)(30) \Rightarrow I_3 = 0.25A$$

Revisiting KCL at junction  $b$ ,

$$I_1 = I_2 + I_3$$

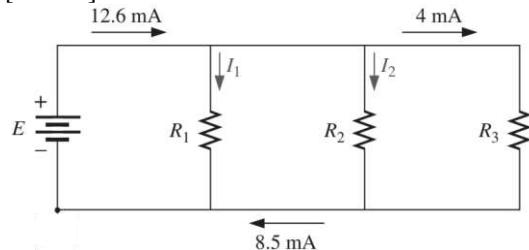
$$I_1 = 0.333 + 0.25$$

$$I_1 = 0.55A$$

### Kirchhoff's Rules Questions

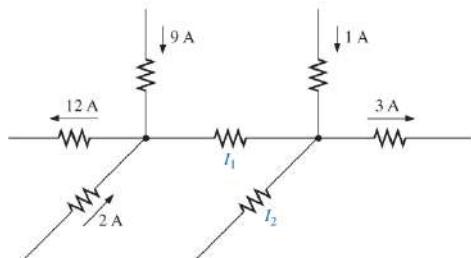
#### Problem 1:

Using Kirchhoff's current law, determine the unknown currents for the parallel network shown in the diagram. [4.5mA]



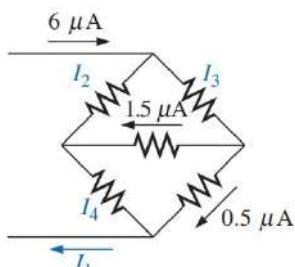
#### Problem 2:

Using Kirchoff's current law, find the unknown currents for the complex configurations shown in the diagram below. [1A; -3A ]



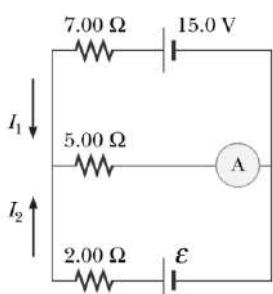
#### Problem 3:

Using Kirchoff's current law, find the unknown currents for the complex configurations shown in the diagram below. [{6, 4, 2, 5.5}μA]



#### Problem 4:

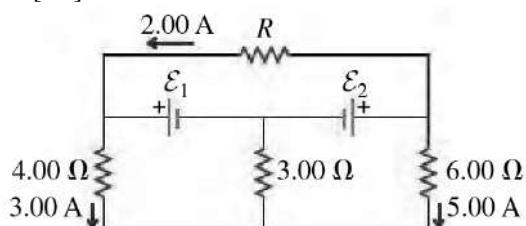
The ammeter shown in the figure shown reads 1.8A. Find  $I_1$ ,  $I_2$ , and  $\epsilon$ . [{0.86, 0.94}A; 10.8V]



#### Problem 5:

In the circuit shown below, find

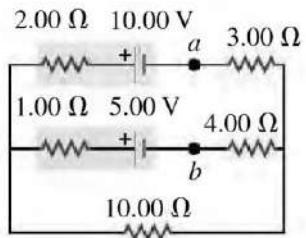
- the current in the  $3\Omega$  resistor, [8A]
- the unknown emfs  $\epsilon_1$  and  $\epsilon_2$ , [36V, 54V]
- the resistance  $R$ . Note that three currents are given. [9Ω]



#### Problem 6:

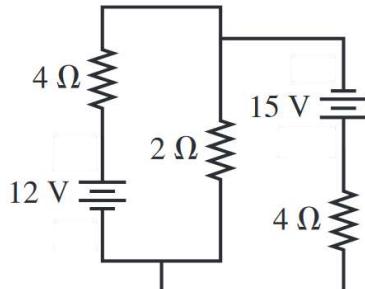
In the circuit shown, find

- the current in each branch [{0.8, 0.2, 0.6}A]
- the potential difference  $V_{ab}$  of point a relative to point b [-3.2V]



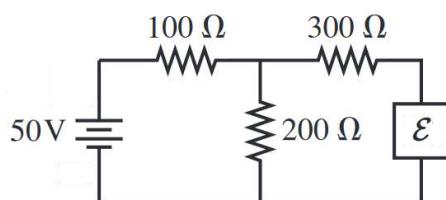
#### Problem 7:

Calculate the current through the  $2\Omega$  resistor in the circuit shown. [-0.375A]



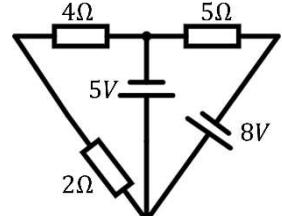
#### Problem 8:

Referring to the figure below, determine the value of  $\epsilon$  at which the  $200\Omega$  resistor dissipate no power. [-150V]



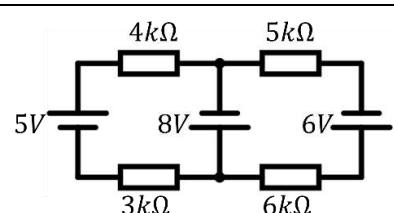
#### Problem 9:

Referring to the figure shown, determine the current through each resistor. [{0.833, 0.6}A]



#### Problem 10:

Referring to the figure shown, determine the power used by each resistor.



[{13.8, 0.17, 10.35, 0.198}mW]

### **KMSw: Numeracy Practice**

An electrical engineer is designing a DC electric motor.

The DC electric motor required a circular coil with the range of diameter between **9.0 cm to 11.0 cm**.

**TABLE 1** and **TABLE 2** show a set of data for two circular coils, coil A and coil B.

Based on the data given, use a suitable method to identify which coil is the most suitable for the DC electric motor.

**[20 marks]**

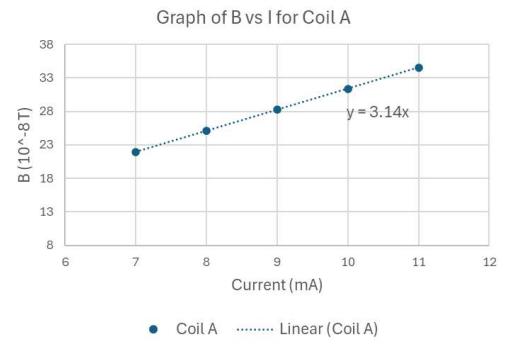
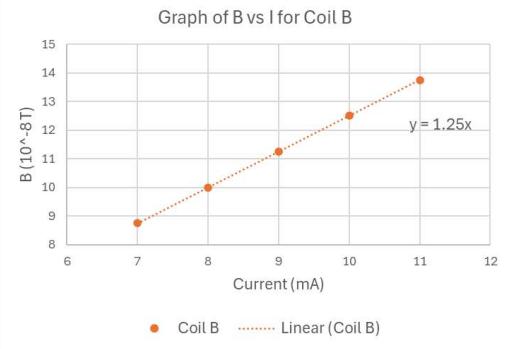
**TABLE 1:** Coil A

<b>B (<math>\times 10^{-8}</math> T)</b>	21.98	25.12	28.26	31.40	34.54
<b>I (mA)</b>	7	8	9	10	11

**TABLE 2:** Coil B

<b>B (<math>\times 10^{-8}</math> T)</b>	8.75	10.00	11.25	12.50	13.75
<b>I (mA)</b>	7	8	9	10	11

**Suggested Answer**

Suggested answer	Mark(s)	Suggested answer	Mark(s)
<b>Coil A</b>		<b>Coil B</b>	
Formulating the question, Magnetic field produced by circular coil $B = \frac{\mu_0 I}{2r}$	K1	Formulating the question, Magnetic field produced by circular coil $B = \frac{\mu_0 I}{2r}$	K1
Comparing with linear graph equation, $B = \frac{\mu_0 I}{2r} \Leftrightarrow y = mx + c$ $B \Leftrightarrow y$ $I \Leftrightarrow x$ $\frac{\mu_0}{2r} \Leftrightarrow m$	J1	Comparing with linear graph equation, $B = \frac{\mu_0 I}{2r} \Leftrightarrow y = mx + c$ $B \Leftrightarrow y$ $I \Leftrightarrow x$ $\frac{\mu_0}{2r} \Leftrightarrow m$	J1
	For each graph: x-axis and y-axis with correct unit – 1m 5 correct plotted points – 2 m 4 correct plotted points – 1 m 3 correct plotted points – 0 m		For each graph: x-axis and y-axis with correct unit – 1m 5 correct plotted points – 2 m 4 correct plotted points – 1 m 3 correct plotted points – 0 m
$m = \frac{\mu_0}{2r_A}$		$m = \frac{\mu_0}{2r_B}$	
$r_A = \frac{\mu_0}{2m}$	K1	$r_B = \frac{\mu_0}{2m}$	K1
$r_A = \frac{4\pi \times 10^{-7}}{2(3.14 \times 10^{-5})} = 0.02m$	JU1	$r_B = \frac{4\pi \times 10^{-7}}{2(1.25 \times 10^{-5})} = 0.05m$	JU1
$d_A = 2(r_A) = 0.04m; d_B = 2(r_B) = 2(0.05) = 0.10m$			JU1
Since $9.0cm < d_B < 11cm$ , coil B is the most suitable for the DC electric motor.			J1

# SPO25 Physics Tutorial Book



Student's name:

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Tutorial Class:

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Lecturer's Name:

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Contributors: Shafiq R, Mary Yusus, John Liew, Ahmad Syuhud



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## Chapter 1: Electrostatics (as)

- 1 Three point charges,  $q_1 = +3.0 \mu\text{C}$ ,  $q_2 = -4.0 \mu\text{C}$  and  $q_3 = -7.0 \mu\text{C}$  are placed 20 cm and 15 cm apart on a straight line in air as shown in **FIGURE 1.1**.
- Sketch the electric force diagram on charge  $q_1$  due to the other two charges.
  - What is the magnitude and direction of the net electrostatic force acting on charge  $q_1$ ? **(LO: 1.1)**
- 2 A  $-3 \mu\text{C}$  charge lies on the straight line between a  $2 \mu\text{C}$  charge and a  $4 \mu\text{C}$  charge. The separation between the  $2 \mu\text{C}$  and  $4 \mu\text{C}$  is  $0.06 \text{ m}$ . By sketching the electric force diagram, calculate the distance of the  $2 \mu\text{C}$  charge from  $-3 \mu\text{C}$  charge where net force acting on  $-3 \mu\text{C}$  charge is zero. **(LO: 1.2)**
- 3 Two equal positive point charges  $q_1 = q_2 = 2.0 \mu\text{C}$  are located at  $x = 0, y = 0.3 \text{ m}$  and  $x = 0, y = -0.3 \text{ m}$  respectively. What is the magnitude and direction of the total electric force that these charges exert on a third positive point charge  $Q = 4.0 \mu\text{C}$  at  $x = 0.4 \text{ m}, y = 0$ ? **(LO: 1.2)**
- 4 (a) When a test charge  $q = 2 \text{ nC}$  is placed at the origin, it experiences a force of  $8.0 \times 10^{-4} \text{ N}$ . Calculate the magnitude of electric field strength at the origin.
- (b) Two point charges of  $+2 \mu\text{C}$  and  $-5.0 \mu\text{C}$  are separated by a distance of  $6.0 \text{ cm}$ . Find the electric field strength at the midpoint between the charges. **(LO: 1.2)**
- 5 **FIGURE 1.2** shows four charges,  $q_1, q_2, q_3$ , and  $q_4$ , each of magnitude  $4 \mu\text{C}$  are placed at the respective corners of a square with sides  $20 \text{ cm}$ .
- Sketch the electric field strength diagram at the center of the square.
  - Find the magnitude of electric field strength at the center of the square.
  - Calculate the electric force acting on another charge of magnitude  $-4 \mu\text{C}$  placed at the center of the square **(LO: 1.2)**
- 6 Sketch the equipotential lines and surfaces of
- an isolated positive charge.
  - a uniform electric field.
- State the shape of the surfaces. **(LO: 1.2)**

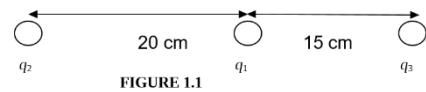


FIGURE 1.1

- 7 **FIGURE 1.3** shows two points A and B at a distance of  $0.4 \text{ m}$  and  $0.3 \text{ m}$  respectively from a point charge Q of  $5.0 \mu\text{C}$ . Determine the
- electric potential at point A and B
  - the potential difference between point B and A.
  - the work done to bring a proton from point A to B. **(LO: 1.3)**
- 8 Two point charges each of  $2 \mu\text{C}$  are located at coordinates  $(0.1, 0) \text{ m}$  and  $(-0.1, 0) \text{ m}$  respectively. Calculate the
- electric potential at coordinate  $(0, 0.5) \text{ m}$
  - change in electric potential energy of the system if a third charge of magnitude  $3 \mu\text{C}$  is brought from infinity to coordinate  $(0, 0.5) \text{ m}$  **(LO: 1.3)**
- 9 Three point charges of  $+q, +2q$  and  $-3q$  are arranged as shown in **FIGURE 1.4**. Calculate the electric potential energy of the system of three charges if  $q = 3 \mu\text{C}$  and  $a = 4 \text{ cm}$ . **(LO: 1.3)**
- 10 a) A small ball with mass  $25 \text{ g}$  has a total charge of  $+20 \mu\text{C}$  is placed between two parallel charged plates.
- In static equilibrium, determine the magnitude of the electric field in the plates.
  - Calculate the acceleration of the ball if it moves horizontally parallel with electric field.
- b) **FIGURE 1.5** shows a section of the deflection system of a cathode ray oscilloscope. An electron travelling at a certain speed enters the space between two parallel metal plates with distance  $20 \text{ mm}$ . The electric field between the plates is  $4.0 \times 10^3 \text{ V m}^{-1}$ .
- Copy the **FIGURE 1.5** and sketch the path of the electron in between the plates and after emerging from the space between the plates.
  - Calculate the acceleration of the electron.
  - Find the electric potential difference between the plates. **FIGURE 1.5**

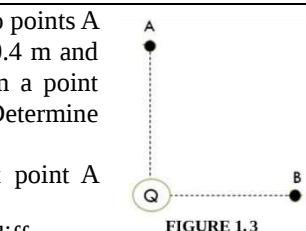


FIGURE 1.3

FIGURE 1.2

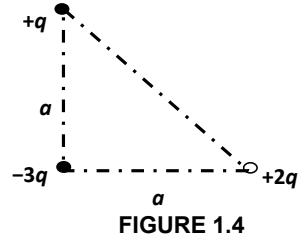
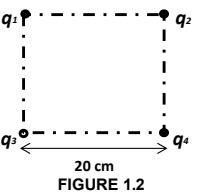
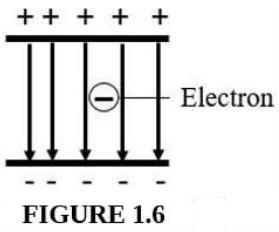


FIGURE 1.4

FIGURE 1.5

- c) FIGURE 1.6 shows a uniform electric field  $395 \text{ V m}^{-1}$  exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0 cm away. Calculate the
- acceleration of the electron.
  - potential difference between the plates.
  - work done by the electric field.



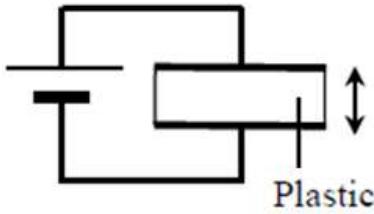
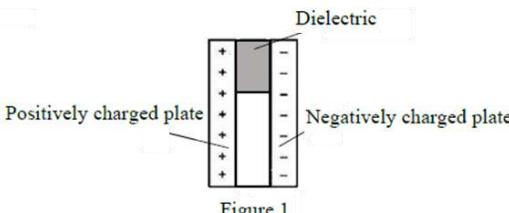
**(LO: 1.4)**

**Answer**

1	(a) Refer to lecturer (b) 5.7 N to the right
2	0.025 m
3	0.46 N to the right
4	(a) $4.0 \times 10^5 \text{ N C}^{-1}$ (b) $7.0 \times 10^7 \text{ N C}^{-1}$
5	(a) Refer to lecturer (b) $0 \text{ N C}^{-1}$ (c) 0 N
6	Refer to lecturer
7	(a) $1.125 \times 10^5 \text{ V}$ , $1.5 \times 10^5 \text{ V}$ (b) $3.75 \times 10^4 \text{ V}$ (c) $6 \times 10^{-15} \text{ J}$
8	(a) $7.06 \times 10^4 \text{ V}$ (b) 0.212 J
9	15.37 J
10	(a) (i) $1.23 \times 10^4 \text{ NC}^{-1}$ (ii) $9.8 \text{ ms}^{-2}$ (b) (i) You Think? (ii) $7.03 \times 10^{14} \text{ m s}^{-2}$ (iii) 80 V (c) (i) $6.94 \times 10^{13} \text{ ms}^{-2}$ (ii) 7.9V (iii) $1.264 \times 10^{-18} \text{ J}$

## Chapter 2: Capacitors (jl)

LO: 2.1abc		
1	<p>The figure shows an arrangement of three capacitors, <math>C_1</math>, <math>C_2</math>, and <math>C_3</math>. Calculate  a. total charge of these capacitors.  b. potential difference across capacitor <math>C_1</math>, <math>C_2</math>, and <math>C_3</math>.</p>	<p>iii. Determine the charges in each capacitor at maximum voltage.</p>
2	<p>Three isolated capacitors 100 <math>\mu\text{F}</math>, 200 <math>\mu\text{F}</math> and 300 <math>\mu\text{F}</math> have potential differences between plates 4.0 V, 6.0 V and 8.0 V respectively. Initially, the terminals of capacitors are connected as shown in the diagram below.</p> <p>(a) Determine the total capacitance across the capacitors.  (b) Calculate the total energy stored by the three capacitors after being connected.</p>	<p>FIGURE shows the variation of current in a circuit with the time during discharging process of a capacitor through a resistor of 2 <math>\text{M}\Omega</math>. Determine  i. the capacitance of the capacitor  ii. the time constant  iii. the time taken for the current to decrease to half of its maximum value.</p>
3	<p>Two capacitors <math>C_1</math> and <math>C_2</math> of capacitance 2.0 <math>\mu\text{F}</math> and 3.0 <math>\mu\text{F}</math> are connected in series to a 100 V source. What is the ratio of the energy stored in <math>C_1</math> to that in <math>C_2</math>?</p>	<p>6</p> <p>FIGURE 3 shows a circuit consisting of a switch S, 3 <math>\text{M}\Omega</math> resistor, 9V battery and a fully charged 6 <math>\mu\text{F}</math> capacitor. If the switch is opened, calculate the remaining charge in the capacitor after 6s.</p>
4	<p>Three capacitors, each of capacitance 48 <math>\mu\text{F}</math>, are connected as shown in figure 9. The maximum safe potential difference that can be applied across each capacitor is 6.0 V.</p> <p>i. Calculate the effective capacitance across AB.  ii. Calculate the maximum voltage across AB.</p>	<p>7</p> <p>FIGURE 3 shows an RC circuit with an emf source <math>\epsilon</math> and a switch S. If the capacitance and resistance is 150 <math>\mu\text{F}</math> and 25 <math>\text{k}\Omega</math> respectively, calculate the time taken for the charge to reach 60% of its maximum value.</p>
8	<p>A 150 <math>\mu\text{F}</math> capacitor is fully charged by a 12 V battery and is then discharged. What is the charge stored in the capacitor after two time constants of the circuit have passed since discharging began?</p>	
9	<p>A 2V battery is connected to a 2 <math>\mu\text{F}</math> capacitor and a 40 <math>\Omega</math> resistor. Calculate the charge in the capacitor at time <math>t = 90 \mu\text{s}</math>.</p>	

LO: 2.3acd	
10	 <p><b>FIGURE 2</b></p> <p>A parallel-plate capacitor filled with plastic is charged by 6.0 V battery as shown in FIGURE 2. The area of one plate is <math>150 \text{ cm}^2</math>. The thickness and dielectric constant of the plastic layer is 0.8 mm and 3.2 respectively. Calculate the energy stored in the capacitor.</p>
11	<p>A dielectric is inserted into an air-filled parallel plate capacitor of capacitance 1500 pF so that <math>\frac{1}{3}</math> of the space between the plates is filled as shown in Figure 1.</p>  <p>Figure 1</p>
12	<p>If the dielectric constant is 7.0, what is the new capacitance of the capacitor?</p> <p>A parallel plate air-filled capacitor whose capacitance, C is 400 pF is charged by a battery to a potential difference of 12 V between the two plates. Then the fully charged capacitor is disconnected and a porcelain slab with dielectric constant 3.2 is inserted between the plates. Calculate</p> <ol style="list-style-type: none"> <li>the capacitance of the capacitor after the slab is inserted.</li> <li>the energy stored in the capacitor after the slab is inserted.</li> <li>What happens to the potential difference if after the dielectric is inserted between the plates</li> </ol>
13	<p>An empty capacitor has an area of <math>6.00 \text{ cm}^2</math> plates that is separated by 4.00 mm. Dielectric material insert in between the parallel plate and the capacitance increase to 5.31 pF. What is the dielectric constant?</p>
14	<p>Calculate the capacitance of a capacitor consisting of two parallel plates separated by a layer of paraffin wax 1.5 cm thick, the area of each plate being <math>60 \text{ cm}^2</math>. The dielectric constant for the wax is 2.</p>

**Answer:**

1	(i) $57.6 \mu\text{C}$ (ii) $V_1=9.6 \text{ V}$ , $V_2=2.4 \text{ V}$ , $V_3=2.4 \text{ V}$
2	(a) $150 \mu\text{F}$ (b) $2.13 \times 10^{-3} \text{ J}$
3	3:2
4	(i) $32 \mu\text{F}$ (ii) $9 \text{ V}$ (iii) $Q_1=Q_2=140 \mu\text{C}$ , $Q_3=290 \mu\text{C}$
5	(i) $2.5 \mu\text{F}$ (ii) $5 \text{ s}$ (iii) $3.4657 \text{ s}$
6	$38.69 \mu\text{C}$
7	$3.436 \text{ s}$
8	$2.44 \times 10^{-4} \text{ C}$
9	$3.24 \times 10^{-5} \text{ C}$
10	$9.558 \times 10^{-9} \text{ J}$
11	$4500 \text{ pF}$
12	(i) $1280 \text{ pF}$ ii) $2.88 \times 10^{-8} \text{ J}$ (iii) decrease
13	4
14	$7.08 \times 10^{-12} \text{ F}$

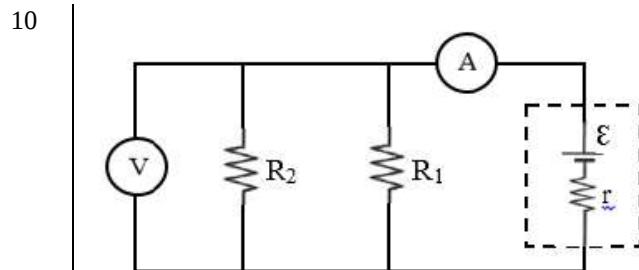
### Chapter 3: Electric Current and Direct Current Circuits (il)

LO: 3.1b, 3.1c	LO: 3.2a, 3.2b
1 A bulb drawn 0.5 A current from a 3 V battery in 7.5 minutes. Calculate the charge that flows in the circuit.	3 A potential difference of 3.0 V is applied across a tungsten wire of length 1.5 m with cross sectional area of $0.05 \text{ mm}^2$ . Calculate the current flow in the wire. <i>(Given the resistivity of tungsten is <math>5.7 \times 10^{-8} \Omega \text{ m}</math>)</i>
2 In 4500 ms, a charge of $6000 \mu\text{C}$ passes each point of a wire. Calculate i the current. ii the number of electrons flow through the point.	4 Two wires made of the same material have the same length but different diameters. The first wire has twice the diameter of the second wire. Compare the resistances of the two wires.  5 A copper wire has a length of 5.0 m and a cross-sectional area of $1.0 \times 10^{-6} \text{ m}^2$ . The resistivity of copper is $1.68 \times 10^{-8} \Omega \text{ m}$ (a) Calculate the resistance of the wire. (b) If the wire is connected to a 12 V battery, determine the current flowing through it.

### **LO: 3.3b**

6 The resistance of the tungsten filament of a bulb at $30^\circ\text{C}$ is $2.5 \Omega$ . Given the temperature coefficient of resistance of tungsten is $4.6 \times 10^{-3} \text{ K}^{-1}$ , what is the operating temperature of the bulb if the current in the filament is 1.6 A and the voltage is 6V?	8 A 12 V emf battery with a internal resistance of $2.0 \Omega$ is connected to a $8 \Omega$ resistor. Calculate the voltage across the battery terminal.
7 A wire of cross-sectional area $0.4 \text{ mm}^2$ is connected to a variable voltage, $V$ and the current, $I$ in the wire is measured. At $27^\circ\text{C}$ , the resistance of the wire is $0.25 \Omega$ and the resistivity is $4.5 \times 10^{-8} \Omega \text{ m}$ . Calculate the (i) length of the wire. (ii) change in the resistance of the wire when it is heated to $80^\circ\text{C}$ . The temperature coefficient of resistivity of the wire is $3.9 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ .	9 A battery of internal resistance $0.5 \Omega$ has a terminal voltage of 9.0 V when no current flows. A $15.0 \Omega$ resistor is connected across the battery terminals. Calculate the i) emf of the battery ii) current through the $10.0 \Omega$ resistor iii) new terminal voltage

### **LO: 3.4d**



**FIGURE 2**

The circuit in **FIGURE 2** shows a battery  $\text{emf}_\text{b} = 10 \text{ V}$  with unknown internal resistance,  $r$  that is connected to a circuit includes resistor  $R_1 = 10 \Omega$  and  $R_2 = 15 \Omega$ . The reading on the ideal voltmeter is 9 V.  
i What is the terminal voltage?  
ii What is the reading of ammeter?  
iii Determine the internal resistance,  $r$ .

**LO: 3.5a**

<p><b>11 Series Combination</b> Three resistors of <math>5\ \Omega</math>, <math>10\ \Omega</math>, and <math>15\ \Omega</math> are connected in series.</p> <p>a) Calculate the total resistance of the combination. b) If a <math>12\text{ V}</math> battery is connected across this series combination, calculate the total current.</p>	<p><b>14 Circuit with a Switch</b> Three resistors of <math>2\ \Omega</math>, <math>4\ \Omega</math>, and <math>6\ \Omega</math> are connected in a circuit. The <math>2\ \Omega</math> and <math>4\ \Omega</math> resistors are in parallel, and their combination is in series with the <math>6\ \Omega</math> resistor.</p> <p>a) Calculate the total resistance of the circuit. b) If a switch is added to bypass the <math>6\ \Omega</math> resistor, what will be the new total resistance when the switch is closed?</p>
<p><b>12 Parallel Combination</b> Two resistors, <math>6\ \Omega</math> and <math>12\ \Omega</math>, are connected in parallel.</p> <p>a) Determine the effective resistance of the combination. b) If the total current from the power supply is <math>4\text{ A}</math>, find the current through each resistor.</p>	<p><b>15</b></p>
<p><b>13 Combination of Series and Parallel</b> A <math>12\ \Omega</math> resistor is connected in series with a combination of two parallel resistors, <math>8\ \Omega</math> and <math>16\ \Omega</math>.</p> <p>a) Calculate the equivalent resistance of the entire circuit. b) If the circuit is connected to a <math>24\text{ V}</math> battery, find the total current supplied by the battery.</p>	<p><b>FIGURE 3.1</b></p> <p><b>FIGURE 3.1</b> shows a circuit with a <math>24\text{ V}</math> battery connected to four resistors of resistance, <math>R_1 = 8\ \Omega</math>, <math>R_2 = 10\ \Omega</math>, <math>R_3 = 3\ \Omega</math> and <math>R_4 = 7\ \Omega</math>. Calculate the (i) effective resistance. (ii) potential difference across the resistor <math>R_1</math>. (iii) power dissipated in the resistor <math>R_1</math>.</p>

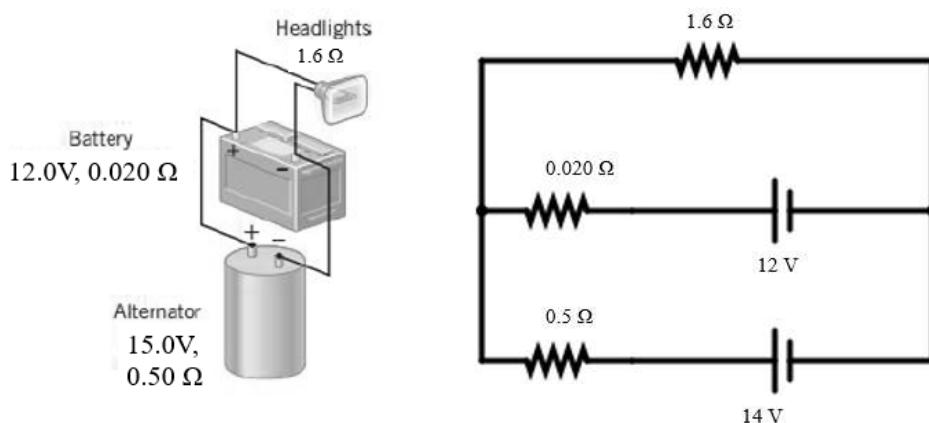
**LO: 3.6a**

<p><b>16</b></p> <p>The circuit diagram shows a circuit containing batteries and resistors. You may assume that the batteries have negligible internal resistance. Calculate the current <math>I_1</math>, <math>I_2</math>, and <math>I_3</math></p>	<p><b>17</b></p> <p><b>FIGURE</b> shows an electrical circuit of three resistors and two batteries. Calculate (i) Current, <math>I</math> (ii) emf, <math>\epsilon</math> (iii) resistance, <math>R</math></p>
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

**LO: 3.6a**

18

In a car, the headlights are connected to the battery and would discharge the battery if it were not for the alternator, which is run by the engine. Diagram below indicates how the car battery, headlights and alternator are connected.



The circuit includes an internal resistance of  $0.020\ \Omega$  for the car battery and a resistance of  $1.60\ \Omega$  for the headlights. For the sake of simplicity, the alternator is approximated as an additional  $14.00\ \text{V}$  battery with an internal resistance of  $0.500\ \Omega$ . Determine the currents through the car battery,  $I_B$ , the headlights,  $I_H$ , and the alternator,  $I_A$ .

**LO: 3.7ab**

No.

Question

19

A kettle with a power rating of  $1500\ \text{W}$  is used for 15 minutes to boil water. How much electrical energy is consumed by the kettle?

20

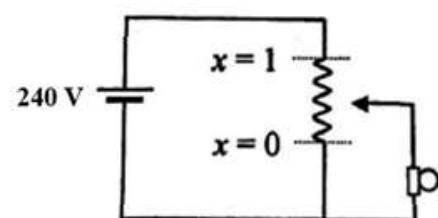
An electric motor uses a current of  $10\ \text{A}$  when connected to a  $240\ \text{V}$  supply. What is the power used by the motor?

21

A toaster uses  $1000\ \text{W}$  of power. How much time will it take to consume  $1.8\ \text{kWh}$  of electrical energy?

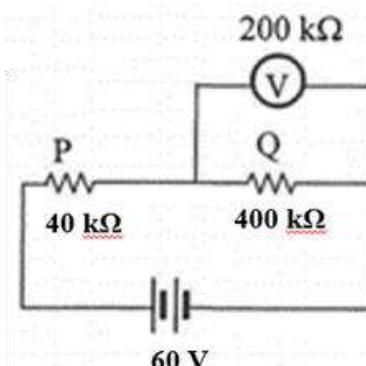
**LO: 3.8b**

22


**FIGURE 5**

**FIGURE 5** shows a light bulb dimmer consisting of a  $150\ \Omega$  variable resistor,  $240\ \text{V}$  voltage source and a light bulb. The slider moves between  $x = 0$  to  $x = 1.2$ . If it is at  $x = 0.4$ , calculate the voltage of the bulb.

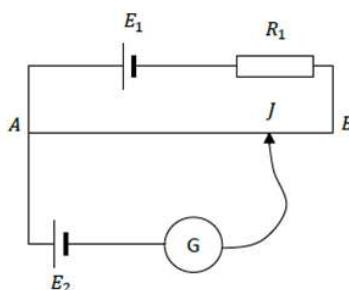
23



**FIGURE** shows a voltmeter with resistance  $200\ \text{k}\Omega$  connected across a resistor  $Q$ . Determine the voltmeter reading.

**LO: 3.9b**

24

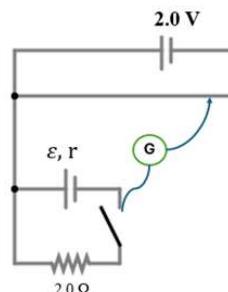


A battery with unknown e.m.f  $E_2$  is connected to a potentiometer as shown in the diagram below.

The driver cell  $E_1$  has an e.m.f of 6.0 V and negligible internal resistance. The length of sliding wire  $AB$  is 100.0 cm with a resistance of 10.0  $\Omega$ . The resistor  $R_1$  has a resistance of 20.0  $\Omega$ . The reading of galvanometer is zero when the length of  $AJ$  is 68.0 cm. Determine the e.m.f  $E_2$ .

25

Diagram below shows a 2.0 V potentiometer with a slide wire  $AB$  of length 100 cm is used for the determination of e.m.f and internal resistance of a cell. When the switch  $S$  is opened, the balance length of the cell is 60.0 cm. When the switch is closed, the balance length shifts to 40.0 cm of the potentiometer. Determine the e.m.f and internal resistance of the cell.



26

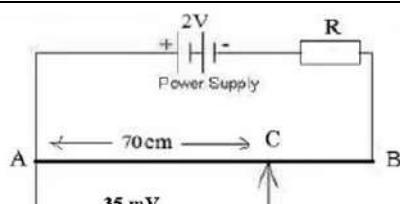


FIGURE 3

FIGURE 3 shows a potentiometer used to determine the emf of a cell. The sliding wire is 100 cm long and has resistance of 10  $\Omega$ . The internal resistance for power supply and the cell is negligible. AC is 70 cm long at equilibrium. Calculate

- the voltage in sliding wire AB.
- the value of current across resistor R.
- the value of R

27

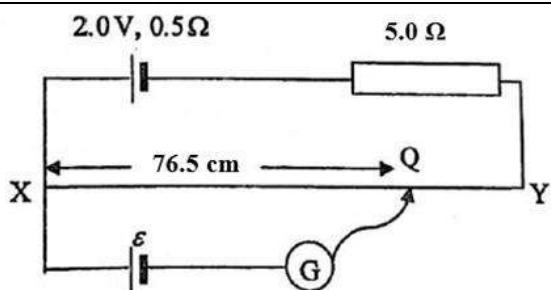


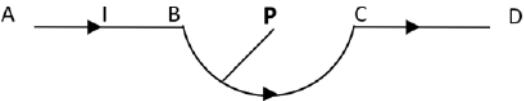
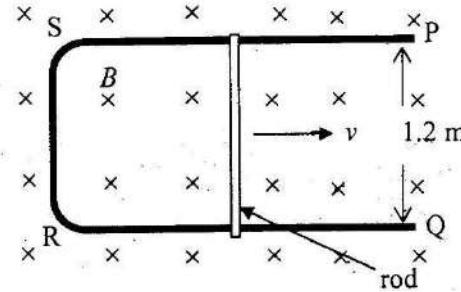
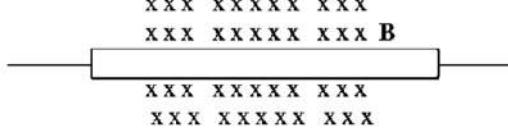
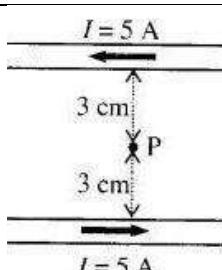
FIGURE shows a potential divider consisting of a wire XY of length 1.0 m and resistance 4.5  $\Omega$ . A cell of emf 2.0 V with internal resistance 0.5  $\Omega$  is connected in series with a 5.0  $\Omega$  resistor. When another cell with emf  $\epsilon$  is connected to potential divider the balance length XQ is 76.2 cm. Calculate  $\epsilon$ .

[ 2marks]

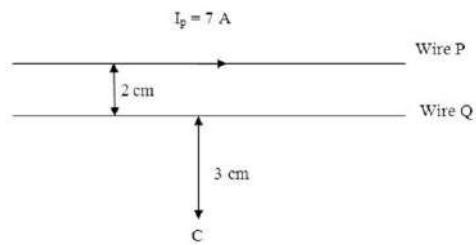
**Answer:**

1	225 C	15	(i) 13 $\Omega$ (ii) 14.77 V (iii) 27.27 W
2	(i) $1.33 \times 10^{-3}$ A (ii) $3.75 \times 10^{16}$ electrons	16	$I_1 = 0.750$ A, $I_2 = 1.25$ A, and $I_3 = 0.500$ A
3	1.754 A	17	(i) 1.875 A (ii) 3 V (iii) 2 $\Omega$
4	$R_2 = 4R_1$	18	$I_B = 3.325$ A $I_H = 7.458$ A and $I_A = 4.133$ A
5	(a) $0.084\Omega$ (b) $I \approx 142.9$ A	19	1.35MJ
6	138.7 °C	20	2.4kW
7	(i) 2.22 m (ii) 0.051675 $\Omega$	21	1.8hours
8	9.6 V	22	80 V
9	(i) 9.0 V (ii) 0.45 A (iii) 6.75 V	23	46.16 V
10	(i) 9 V (ii) 1.5 A (iii) 0.667 $\Omega$	24	1.36 V
11	(a) $30\Omega$ (b) 0.4A	25	1.2 V, 1 $\Omega$
12	(a) $4\Omega$ (b) 2.67A	26	(i) 50 mV (ii) 5 mA (iii) 390 $\Omega$
13	(a) $17.33\Omega$ (b) 1.38A	27	0.6885 V
14	(a) $7.33\Omega$ (b) 1.33Ω		

## Chapter 4: Magnetic Field (as)

No	Question	
1	 <p>Determine the magnitude and direction of the magnetic field at point P due to current, 1.0A flowing in a semi-circle wire with radius, R = 2 cm as shown in <b>FIGURE</b> above.</p> <p style="text-align: right;"><b>(LO:4.2)</b></p>	
2	<p>(a) Explain why a charged particle is moving in a circular path in a uniform magnetic field.</p> <p>(b) A particle of charge <math>3.2 \times 10^{-19}</math> C and velocity of <math>2 \times 10^5</math> ms<math>^{-1}</math> enters a uniform magnetic field of magnetic field strength, 0.2 T. If the particle moves in a circular path of radius 4.0 cm, calculate the mass of the charged particle.</p> <p style="text-align: right;"><b>(LO:4.3)</b></p>	
3	<p>a) Write the expression for the force acting on an electron carrying a charge <math>e</math> moving with velocity <math>v</math> when it enters perpendicularly into</p> <ol style="list-style-type: none"> <li>an electric field of intensity <math>E</math></li> <li>a magnetic field of intensity <math>B</math>.</li> </ol> <p>b) Sketch two separate diagrams of the direction of field, velocity and path of the electron and the force acting on it for the motion in part (a) (i) and (ii).</p> <p>c) With reference to your diagrams, state two differences between the electric field and magnetic field.</p> <p>d) A helium ion of charge <math>+2e</math> and mass <math>6.6 \times 10^{-27}</math> kg is accelerated by a voltage of 2400V. The accelerated ion then moves into a perpendicular uniform magnetic field of magnitude 0.24 T. Determine</p> <ol style="list-style-type: none"> <li>the radius of the circular path of the helium ion.</li> <li>the period of revolution of the helium ion.</li> </ol> <p>[Given charge of electron = <math>1.60 \times 10^{-19}</math> C]</p> <p style="text-align: right;"><b>(LO:4.3)</b></p>	
4	 <p>A 1.2 m conducting rod rests on metal rails SP and RQ as shown in <b>FIGURE</b> 8. The rod is placed in a uniform magnetic field <math>B=2.5</math> T perpendicular to the plane of the paper. The rod is pulled to the right at a uniform velocity <math>v=2</math> m s<math>^{-1}</math>. If the resistance of the circuit PQRS is 6 <math>\Omega</math>, determine</p> <ol style="list-style-type: none"> <li>the induced current in the rod.</li> <li>the magnitude and direction of the force required to keep the rod moving to the right at constant velocity 2 m s<math>^{-1}</math>.</li> </ol> <p style="text-align: right;"><b>(LO: 4.4)</b></p>	
5	 <p>A copper rod of mass 0.08 kg and length 0.20 m is attached to two thin current carrying wires, as shown in <b>the figure</b>. The rod is perpendicular to a uniform magnetic field of 0.60T. Determine the magnitude and direction of the electric current to keep the copper rod fixed and stay horizontal.</p> <p style="text-align: right;"><b>(LO: 4.4)</b></p>	
6	 <p>The figure shows two long parallel wires, each carrying a 5 A current in opposite direction. Determine the magnetic field, at point P, 3 cm from both wires.</p> <p style="text-align: right;"><b>(LO: 4.5)</b></p>	

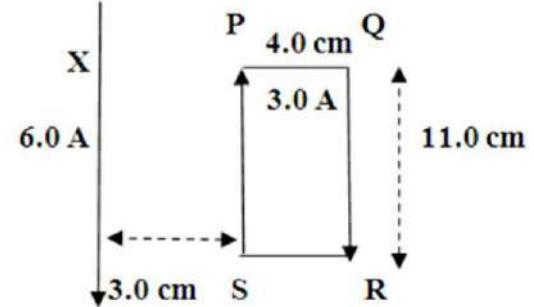
7



**FIGURE** above shows two parallel wires 2 cm apart. Wire P carries a current 7 A to the right. Point C is 3 cm from wire Q and in the same plane as wire P and Q. Determine the magnitude and direction of current in wire Q so that the magnetic field at C is zero.

(LO: 4.5)

- (a) Write an expression for the force between two parallel conductors. Explain the physical quantities that you have used.  
 (b) State two differences between the force due to an electric field and the force due to a magnetic field on a charged particle.



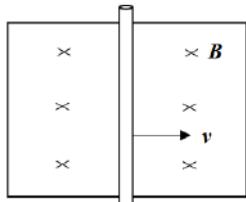
- (c) The **FIGURE** above shows a long straight conductor, X, carrying a current of 6.0 A. A rectangular metal frame PQRS is suspended with PS 3.0 cm from X. The sides of PQ and QR are 4.0 cm and 11.0 cm respectively, and a current of 3.0 A flows in the rectangular coil as indicated. The conductor X and the metal frame PQRS are in the same plane. Calculate  
 (i) the force on the wire PS,  
 (ii) the force on the wire QR.  
 Hence, calculate the resultant force on PQRS and determine its direction.

(LO: 4.5)

Answer:

No	Answer:
1	<b>1. <math>5 \times 10^{-5}</math> T, out of the plane paper</b>
2	(b) $1.28 \times 10^{-26}$ kg
3	(d) $4.14 \times 10^{-2}$ m, $5.39 \times 10^{-7}$ s
4	(a) $I = 1\text{A}$ , (b) $F = 3\text{ N}$ (to the right)
5	<b>6.54 A to the right</b>
6	$6.66 \times 10^{-5}\text{ T}$ (out of page)
7	<b>4.2 A to the left</b>
8	(c) $1.32 \times 10^{-5}\text{ N}$ , $0.5657 \times 10^{-5}\text{ N}$ , $7.54 \times 10^{-6}\text{ N}$ to the right

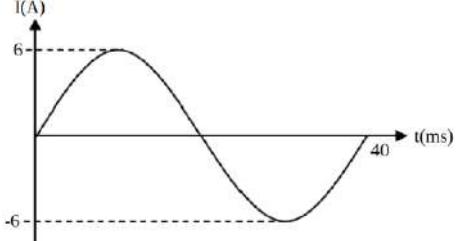
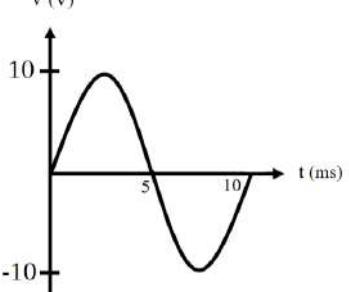
## Chapter 5: Electromagnetic Induction (sr)

1	<p>A circular coil of 80 turns with a plane area of <math>8 \times 10^{-3} m^2</math> is placed in a region of 0.2T magnetic field. Calculate the magnetic flux linkage through the coil when the plane of the coil makes an angle of <math>30^\circ</math> to the magnetic field.</p> <p>[LO 5.1]</p>	5	<p>A coil of 200 turns is placed perpendicularly in a magnetic field region of 5T. The cross-sectional area of the coil is <math>0.4 cm^2</math>. The magnetic field is reduced to 3.5T after 3s. Determine the induced emf in the coil and the induced current if resistance is <math>20\Omega</math>.</p> <p>[LO 5.2d]</p>
2	<p>Find the maximum induced emf if a 50-turn circular coil of plane <math>8 \times 10^{-3} m^2</math> is rotated about an axis of rotation perpendicular to the uniform magnetic field at <math>10 revs^{-1}</math>.</p> <p>[LO 5.2d]</p>	6	 <p>A wire of 0.4m length moves to the right with a uniform velocity of 0.25m/s, as shown in the figure. If the resistance of the wire is <math>5\Omega</math> and the magnetic field strength is 4T, calculate the induced emf and current in the wire.</p> <p>[LO 5.2d]</p>
3	<p>A and B are two adjacent coaxial coils and has 1000 and 5000 turns respectively. If each turn in coil B has a flux of 0.035mWb due to a 2A current in coil A, determine the mutual inductance of the coils.</p> <p>[LO 5.5b]</p>	7	<p>A solenoid has a length of 8cm and cross sectional area of <math>6 \times 10^{-2} m^2</math>. The solenoid has a turn density of 1300 per meter. Determine the self inductance of the solenoid.</p> <p>[LO 5.3b]</p>
4	<p>Each turn of a coil produces a magnetic flux of 0.045mWb when a current of 3A flows through it. Determine the self-inductance of this coil and the energy stored in it.</p> <p>[5.3b]</p>	8	<p>An inductor coil of 4 turns and 15mm in diameter, carries a 6A current. Calculate the inductance of the coil, the energy stored in the inductor and the induced emf if current drops to 3A in <math>15\mu s</math>.</p> <p>[LO 5.9d, 5.3b, 5.4a]</p>

Answers:

1	0.064Wb	5	4mV; 0.2mA
2	5.03V	6	0.4V; 0.08A upwards
3	0.087H	7	0.496H
4	0.015H; 0.0675J	8	$2.63 \times 10^{-8} H$ , $4.67 \times 10^{-6} J$ , 5.26mV

## Chapter 6: AC Circuits (sr)

- 1 An AC circuit contains  $12\Omega$  resistor, a  $3mF$  capacitor, and a  $0.33H$  inductor connected in series to an AC power supply with  $75V$  and frequency  $2.2\text{Hz}$ . Calculate the rms current, rms voltage across the capacitor and power factor with phase angle.  
Sketch a labelled phasor diagram for the supply voltage and current.  
[6.2b; 6.3a, 6.4aiii)]
- 2 An AC generator with a frequency of  $40\text{Hz}$  and a voltage of  $120\text{V}$  is connected to an RLC series circuit with a  $20 \Omega$  resistor, a  $0.45\text{H}$  inductor, and a  $1.4 \text{ mF}$  capacitor. Calculate the impedance in the circuit, average power dissipated and value of current when the circuit is at resonance.  
[6.3c, 6.3d, 6.4ai]
- 3 An unknown resistor and an inductor are connected in series to an AC generator with rms voltage of  $12\text{V}$ . The inductive reactance and impedance in the circuit are  $36 \Omega$  and  $95 \Omega$ , respectively. Determine the resistance of the unknown resistor. Calculate the phase angle.  
[6.3c]
- 4 
- The figure shows a graph of current against time of an AC generator. Calculate the rms current value and frequency of the source.  
[25Hz]  
[6.3b]
- 5 A series RLC circuit is connected to an AC supply of  $220\text{V}$ ,  $50\text{ Hz}$ . If given  $R = 360\Omega$ ,  $L = 1.8\text{H}$  and  $C = 3.2\mu\text{F}$ , calculate the inductive and capacitive reactance, impedance, phase angle and the power factor.  
[6.3c]
- 6 

The figure shows a graph of voltage against time of an AC circuit. Determine the frequency and the root mean square voltage.

[6.2b]

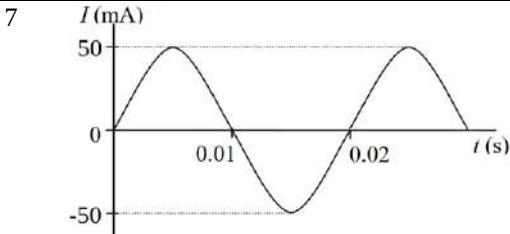
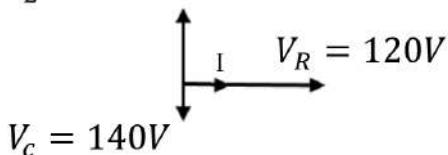


FIGURE 5 shows a graph of current against time of an AC generator. Determine the peak current, the frequency and the alternating current equation.

[6.2b]

- 8 An AC source of peak voltage  $120\text{ V}$  and frequency  $60.0\text{ Hz}$  is connected in series to an  $880\Omega$  resistor,  $2.36\text{H}$  inductor and  $15\mu\text{F}$  capacitor. Calculate the impedance of the circuit, the phase angle of the circuit, the power factor of the circuit [0.777]  
[6c; 6d]

9  $V_L = 310\text{V}$



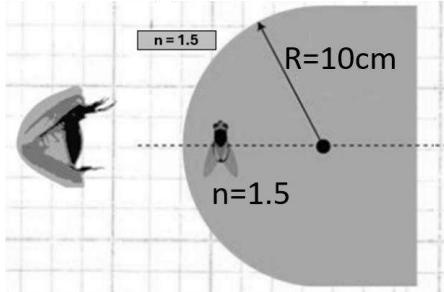
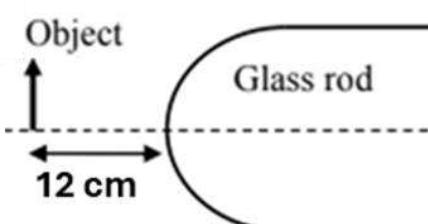
$V_c = 140\text{V}$

The figure shows the values of root means square voltage across the resistor R, inductor L and capacitor, C. Calculate the source voltage and phase angle of the source voltage with respect to the current. Does the source voltage lag or lead the current? Determine power factor of the circuit.

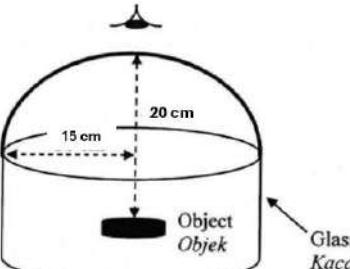
[6c; 6d]

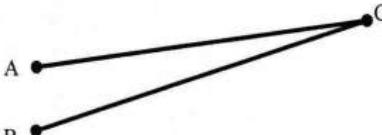
- 10 A sinusoidal voltage  $V(t) = 20 \sin(55\pi t)$  is applied to a series RLC circuit with  $R = 670\Omega$ ,  $L = 245\text{ mH}$  and  $C = 50\mu\text{F}$ . Calculate the impedance of the circuit and phase angle of the circuit [-6.2°]  
[6c; 6d]

## Chapter 7: Optics (mgy)

1	The radius of curvature of a convex mirror is 20 cm. A virtual image formed 6 cm away from the mirror. <ol style="list-style-type: none"> <li>Calculate the focal length of the mirror.</li> <li>Calculate the object distance.</li> <li>Calculate the height of the image if the height of the object is 10 cm.</li> <li>State three characteristics of the image formed.</li> </ol> <p><b>(LO: 7.1)</b></p>
2	An object is placed at -18 cm in front of a mirror, an upright image is twice the size of the object is formed. Calculate the radius of curvature of the mirror. <b>(LO: 7.1)</b>
3	A spherical mirror has a focal length of 6.0 cm. Determine the distance at which you hold the mirror from your face in order to see your upright image with a magnification of 5.
4	A convex mirror of focal length 9.0 cm produces a virtual image, 4.0 cm from the mirror. <ol style="list-style-type: none"> <li>Sketch a labelled ray diagram to show the formation of the image.</li> <li>Calculate the object distance</li> </ol> <p><b>(LO: 7.1)</b></p>
5	
	Refer to <b>FIGURE 1</b> , a fly is in a ball of amber with a radius of curvature of 10 cm. The amber has an index of refraction of $n = 1.5$ . To an observer outside the amber, the fly appears to be located 3 cm from the surface. Where is fly actually located? <b>(LO: 7.2)</b>
6	An object is placed 5 cm from a glass rod with hemispherical tip as shown in <b>FIGURE 2</b> . If the radius of curvature and refractive index of glass rod is 12 cm and 1.49 respectively, calculate the image distance.  <b>(LO: 7.2)</b>
7	A point object is placed at the centre of a glass sphere of radius 2 cm and refractive index of 1.5. Calculate the image distance of the point object.

## (LO: 7.2)

8	<b>FIGURE 3</b> shows an object embedded in a solid glass with a hemispherical end of radius 15 cm and refractive index 1.49. The object is 20 cm inside the glass. Calculate the image distance. Refractive index of air is 1.0.  <b>(LO: 7.2)</b>
9	A toy is placed 25.0 cm in front of a converging (convex) lens with a focal length of 16.0 cm. <ol style="list-style-type: none"> <li>Sketch a ray diagram to show the formation of an image</li> <li>Determine the image distance</li> <li>Determine the magnification of the image</li> <li>State two characteristics of the image</li> </ol> <p><b>(LO: 7.2)</b></p>
10	A biconvex lens made from glass with refractive index 1.47 has a focal length of 9 cm in air. Given the refractive index of air is 1.00. Calculate the focal length of the lens when it is submerged in a liquid with refractive index 1.35. <b>(LO: 7.3)</b>
11	An object is placed at 11.0 cm from a thin biconcave lens with radius of curvature 35.0 cm. The refractive index of the lens is 1.89. Determine the focal length of the lens. <b>(LO: 7.3)</b>
12	The convex meniscus lens has a 12.5 cm radius for convex surface and 20.0 cm for the concave surface. The lens is made of glass with a refractive index, $n=1.42$ in air. Refractive index of air is 1.0. Determine the focal length of the lens. <b>(LO: 7.3)</b>
13	An apple is placed in front of a thin lens. The distance between the apple and the lens is 15 cm. The index of refraction of the lens, $n = 1.40$ , radius of the nearer lens surface is -20 cm, and the radius of the farther lens surface is +16 cm. Determine <ol style="list-style-type: none"> <li>the focal length of the lens.</li> <li>the image distance.</li> <li>the magnification of the apple.</li> </ol> <p><b>(LO: 7.3)</b></p>

14		<p><b>FIGURE 4</b> shows two paths of in phase coherent lights from points A and B that produce an interference pattern at point C. Determine whether it is a constructive or destructive interference if AC and BC are <math>4.2\lambda</math> and <math>6.7\lambda</math>.</p> <p><b>(LO: 7.3)</b></p>
15	<p>In a Young's Double slit experiment, the slit separation is 0.05 cm and the distance of the double slit from the screen is 200 cm. When a green light is used, the distance of the first bright fringe from the central maximum is 0.4 cm.</p> <ul style="list-style-type: none"><li>i) Calculate the wavelength of the green light.</li><li>ii) Calculate the distance of the third dark fringes from the central maximum.</li><li>iii) Explain the changes to the interference pattern if the screen is moved further from the slits.</li></ul> <p><b>(LO: 7.6)</b></p>	
16	<p>In a Young's Double Slit experiment, the distance measured between the central bright maximum and the fifth dark fringe is 5.0 cm. The slits are separated by a distance of 0.175 mm. The distance between the slits and the screen is 3.5 m. Calculate</p> <ul style="list-style-type: none"><li>i) the wavelength of light.</li><li>ii) the fringe separation.</li></ul> <p><b>(LO: 7.6)</b></p>	
17	<p>A 550 nm monochromatic light is incident on a thin film (<math>n=1.40</math>). Calculate the minimum thickness of the film which produce destructive interference of the reflected light in the following condition:</p>	

18	<p>An gasoline film (<math>n=1.40</math>) floats on the water puddle (<math>n=1.33</math>) You notice that green light with wavelength 623 nm is absent in the reflection. What is the minimum thickness of the gasoline film.</p> <p><b>(LO: 7.7)</b></p>	<p>i) the film in air. ii) the film is a coating on a glass of reflective index 1.60.</p>
19	<p>A monochromatic light of wavelength 520 nm is incident on a slit 0.3 mm wide. A diffraction pattern is produced on a screen placed at a distance of 3.5 m from the slit. What is the distance between consecutive dark fringes on both sides of the central bright fringe?</p> <p><b>(LO: 7.8)</b></p>	
20	<p>A single-slit diffraction pattern is obtained on a screen placed at a distance of 15 cm from the slit of width 7 <math>\mu\text{m}</math>. The wavelength of the monochromatic light used is <math>6.6 \times 10^{-7} \text{ m}</math></p> <ul style="list-style-type: none"><li>i) Calculate the angular separation between the first and second minima.</li><li>ii) What is the width of the central bright fringe?</li><li>iii) How many dark fringes are found on the screen?</li></ul> <p><b>(LO: 7.8)</b></p>	
21	<p>A monochromatic green light is diffracted by a slit of width 0.65 mm. The diffraction pattern forms on a screen 2.5 m away from the slit. The width of central bright fringe is 3.7 mm. Calculate the distance between the central bright fringe and the 3rd dark fringe.</p> <p><b>(LO: 7.8)</b></p>	
22	<p>A diffraction grating has <math>6000 \text{ lines cm}^{-1}</math>. A monochromatic light with wavelength of 769 nm is incident on the grating. A screen is placed 2.00 m away from the grating. Determine the number of bright fringes can be observed on a screen.</p> <p><b>(LO: 7.9)</b></p>	

### Answer

1	10m, 15 cm, 4cm, Virtual, Upright, Diminished	12	65.19 cm
2	-72 cm	13	22.22 m, -8.95 cm, 0.597
3	4.8 cm	14	Destructive interference
4	7.2 cm	15	$1 \times 10^{-6} \text{ m}$ ; 0.01 m Fringe separation $\Delta y$ increases and the maximum intensity of the bright fringes decreases.
5	5.29 cm	16	$5.56 \times 10^{-7} \text{ m}$ ; $1.11 \times 10^{-2} \text{ m}$
6	-9.36 cm	17	196 nm; 98.21 nm
7	-2 cm	18	$2.23 \times 10^{-7} \text{ m}$
8	-23.9 cm	19	$1.21 \times 10^{-2} \text{ m}$
9	44.44 m; -1.78	20	$5.47^\circ$ ; $2.83 \times 10^{-2} \text{ m}$ ; 20
10	47.59 cm	21	$5.55 \times 10^{-3} \text{ m}$
11	-19.66 cm	22	5

### Chapter 8: Matter Waves (sr)

1	A beam of electrons is accelerated through a potential difference of 6500 V in an electron microscope. Calculate the speed of the electrons and their de Broglie wavelength.	6	What is the wavelength of a photon that has the same momentum as an electron moving with a speed of $1500\text{ms}^{-1}$ ?
2	The speed of an electron is $2.8(10^6)\text{ms}^{-1}$ . Calculate the momentum and the de Broglie wavelength of the electron.	7	An electron has energy of $400\text{eV}$ . Calculate the de Broglie wavelength associated with the electron.
3	Given that the de Broglie wavelength of a photon is 900nm, determine the momentum of the photon.	8	An electron and a proton are accelerated through the same voltage. If the de Broglie wavelength of the electron is 0.15nm, what is the de Broglie wavelength of the proton?
4	Calculate the de Broglie wavelength for a 7.5g bullet moving at a speed of $400\text{ms}^{-1}$ .	9	The de Broglie wavelength of an electron is 1nm, determine its kinetic energy.
5	An electron is accelerated in vacuum through a potential difference of 1kV. If the potential difference is tripled, calculate the ratio of the electron's new speed to its original speed and the new wavelength of the electron.		

### Chapter 9: Mass defect, Binding Energy & Radioactivity (sr)

Binding Energy		Radioactivity	
1	Determine the mass defect and the binding energy of Bromine nucleus, $^{81}_{35}\text{Br}$ in Joule ( $J$ ), given that the mass of bromine nucleus is 80.916291u.	1	A 2.5g sample of radioactive element has a half-life of 82hours and has a molar mass of $65\text{gmol}^{-1}$ . Calculate the decay constant and the number of atoms remained in the sample after 30hours.
2	The binding energy of an aluminium nucleus ( $^{27}_{13}\text{Al}$ ) is 224.93MeV. Calculate the mass of its nucleus in atomic mass unit.	2	The half-life of an isotope is 90minutes, determine how long it will take for 75% of the nuclei of the isotope to decay from its initial activity.
3	The mass of $^{56}_{26}\text{Fe}$ nucleus is 55.9349u. Calculate the binding energy per nucleon in MeV per nucleon.	3	Suppose a $^{51}_{24}\text{Cr}$ sample has an activity of $6.74 \times 10^9$ decay per days. If the sample has a half-life of 28 days, determine the number of nuclei the sample initially contains and the sample activity in Bq after 1 year.
4	Given that the mass of $B - 11$ nucleus is 11.009305u and has 5 protons, calculate the binding energy per nucleon in MeV per nucleon.	4	$^{199}_{78}\text{Pt}$ has a half-life of 30.8 min and an activity of $9 \times 10^{15}\text{Bq}$ . Calculate the mass of the sample.
5	The nuclear mass of Radon ( $^{226}_{88}\text{Rn}$ ) is 226.025403u. Determine the mass defect in kilograms for Radon.	5	A sample of a radioactive source has $3 \times 10^{23}$ atoms. If the half-life of the sample nuclei is $1.5 \times 10^{19}\text{s}$ , determine the initial activity of the sample in Bq and the number of the sample atoms remaining after $3.2 \times 10^{17}\text{s}$ .
		6	A $10^{-6}\text{g}$ radioactive sample has a nucleon number of 239 and an activity of 3000 particles per second, determine the decay constant and the time required for the activity of the source to decay to 20decay per second.

### Answer:

Chapter 8		Chapter 9: Binding Energy		Chapter 9 - Radioactivity	
1	$4.78 \times 10^7\text{ms}^{-1}$ , $1.523 \times 10^{-11}\text{m}$	1	$0.736994\text{u}$ , $1.1 \times 10^{10}\text{J}$	1	$8.453 \times 10^{-3}\text{hours}^{-1}$ , $1.79676 \times 10^{22}$ nuclei
2	$2.55(10^{-24})\text{ms}^{-1}$ ; $2.6 \times 10^{-10}\text{m}$	2	26.97u	2	180 minutes
3	$7.37 \times 10^{-28}\text{Ns}$	3	8.55MeV per nucleon	3	$2.7 \times 10^{11}\text{atoms}$ , 8.49Bq
4	$2.21 \times 10^{-34}\text{m}$	4	6.696MeV per nucleon	4	7.9g
5	$1.73$ ; $2.23 \times 10^{-11}\text{m}$	5	$3 \times 10^{-27}\text{kg}$	5	$13862\text{Bq}$ ; $2.95596 \times 10^{23}$ atoms
6	485nm			6	$1.49 \times 10^{-12}\text{s}^{-1}$ ; $4.2 \times 10^{12}\text{s}$
7	0.0619nm				
8	$3.5 \times 10^{-12}\text{m}$				
9	$1.314 \times 10^{-22}\text{J}$				

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## Geometrical Optics Module

### Reflection upon Curved Mirror

What is the relationship between the radius of curvature of a curved mirror and its focal length?

$$R = \boxed{\phantom{0}} f$$

**Label the number of lines! (Line 1 has been labelled for you)**

**Line 1:**

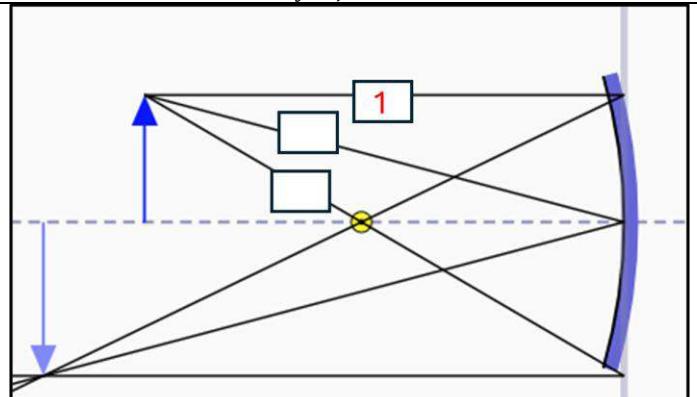
The principal ray line that is parallel to the paraxial line and crosses the focal point after reflecting on the curved mirror.

**Line 2:**

The principal ray line that goes through the focal point and is reflected parallel to the paraxial line.

**Line 3:**

The principal ray line that has the same reflection angle and incoming angle relative to the paraxial line.



**Complete the following equations!**

Focal Length, object distance and image distance

$$\frac{1}{f} = \frac{1}{\boxed{u}} + \frac{1}{\boxed{v}}$$

Magnification

$$M = \frac{h_i}{\boxed{u}} = -\frac{\boxed{v}}{h_o}$$

**Complete the following table of sign conventions!**

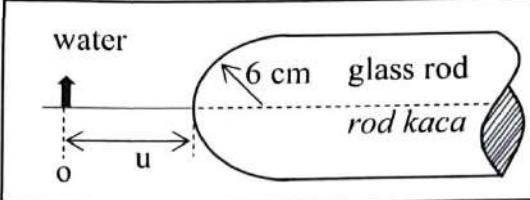
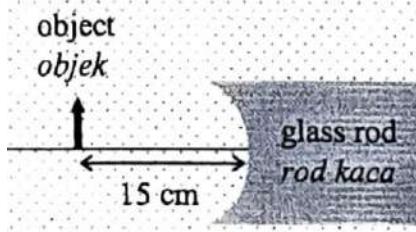
	<b>Positive when</b>	<b>Negative when</b>
<b>Focal length, <i>f</i></b>	Focal length is at same side as incoming light (concave)	
<b>Object Distance, <i>u</i></b>		Object is placed on the opposite side as incoming light ('behind', 'virtual object')
<b>Image Distance, <i>v</i></b>	Image is placed same side as incoming light ('in front', 'real image')	
<b>Magnification</b>		

PSPM 23/24	PSPM 22/23	PSPM 21/22
A spherical mirror is placed in front of a table lamp and reflecting an inverted image on the wall. The radius of the mirror is 180cm and the size of the image is half the size of the table lamp. Sketch the ray diagram and calculate the object distance.	A student who is standing 1.52m in front of a spherical mirror produces an inverted image 18cm from the mirror. Determine his new position from the mirror to get an upright image that is twice his actual size.	An object is placed 5cm from a curved mirror. An image which is twice the size of the object is formed behind the mirror. Explain if the mirror is convex or concave and determine the radius of the curvature of the mirror.

## Refraction at a spherical surface

Complete the following equation!	$\frac{n_1}{\boxed{\phantom{0}}} + \frac{\boxed{\phantom{0}}}{v} = \frac{\boxed{\phantom{0}}}{\boxed{\phantom{0}}} - \frac{\boxed{\phantom{0}}}{\boxed{\phantom{0}}}$
----------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------

Complete the following table of sign conventions!		
	<b>Positive when</b>	<b>Negative when</b>
<b>Radius, <math>R</math></b>	Radius at opposite side as incoming light ('convex')	
<b>Object Distance, <math>u</math></b>		Object is placed on the opposite side as incoming light ('behind', 'virtual object')
<b>Image Distance, <math>v</math></b>	Image is placed on the opposite side as incoming light ('behind', 'real image')	

PSPM 21/22	PSPM 19/20
 <p>The figure shows a long rod with a convex surface of radius of curvature of 6cm at one end and is made from glass with refractive index of 1.6. The glass rod is placed in water with refractive index of 1.33. An object placed along the rod's axis is to be imaged 53cm inside the rod. Calculate the object distance.</p>	 <p>The figure shows an object and a glass rod immersed in a liquid. The rod has a refractive index of 1.7 and radius of curvature of 8cm. If the object distance is 15cm and the virtual distance is 13cm, determine the refractive index of the liquid.</p>

## Thin Lenses

Label the number of lines! (Line 1 has been labelled)

**Line 1:**

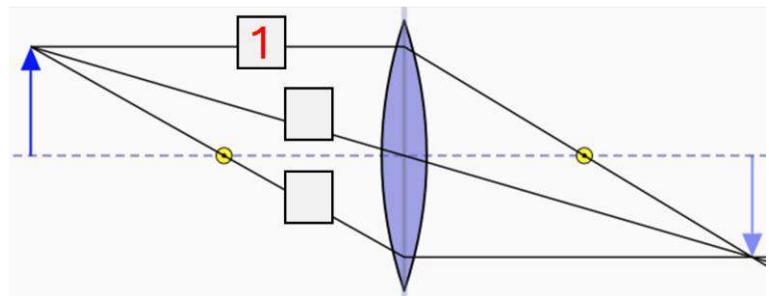
Ray 1 leaves one point on object going parallel to the axis, then refracts through focal point behind the lens.

**Line 2:**

Ray 2 passes through the focal point in front of the lens; therefore it is parallel to the axis behind the lens.

**Line 3:**

Ray 3 passes straight through the center of the lens (assumed very thin).



Complete the following equations!

$f(u, v)$

$$\frac{1}{f} = \frac{1}{\boxed{u}} + \frac{1}{\boxed{v}}$$

Magnification

$$M = \frac{h_i}{\boxed{u}} = -\frac{\boxed{h_o}}{\boxed{v}}$$

Lensmaker's Equation:  $f(R_1, R_2)$

$$\frac{1}{f} = \left( \frac{\boxed{n_{med}}}{\boxed{R_1}} \right) \left( \frac{1}{\boxed{u}} - \frac{1}{\boxed{v}} \right)$$

Complete the following table of sign conventions!

	Positive when	Negative when
<b>Focal length, <math>f</math></b>		Diverging lens
<b>Object Distance, <math>u</math></b>	Object is placed same side as incoming light ('in front', 'real object')	
<b>Image Distance, <math>v</math></b>		Image is placed same side as incoming light ('in front', 'opposite image')
<b>Radii of surfaces, <math>R_1</math> &amp; <math>R_2</math></b>		Radii is on the opposite side of the incoming light ('behind')

PSPM 23/24	PSPM 22/23	PSPM 21/22
A biconvex lens of radii 35cm is immersed in chloroform ( $n = 1.44$ ). Its focal length is doubled when it is placed in an unknown liquid. If the refractive index of the glass is 1.57, determine the focal length of the lens in the liquid.	A biconvex lens has surfaces with radii of curvature 18cm and 20cm. When an object is placed in front of the lens, a real image is formed 32cm from the lens. Determine the focal length and refractive index.	A converging meniscus lens is made from a glass of refractive index 1.52 having a radius 7cm and 4cm. An object is placed 24cm in front of the lens. Calculate the position of the image from the lens and justify if the image is magnified or diminished in size.

## Worksheet

### Part 1: Reflection

No	Questions
1	<p>A concave mirror has a focal length of 30cm. The distance between an object and its image is 45cm. Find the object and image distances, assuming that the object lies</p> <ol style="list-style-type: none"> <li>beyond the center of curvature [<b>{+90, +30}cm</b>]</li> <li>between the focal point and the mirror. [<b>{+15, -30}cm</b>]</li> </ol> <p><math display="block">d_o - d_i = 45.0 \text{ cm}, \quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}, \quad \frac{1}{d_o} + \frac{1}{d_o - 45.0 \text{ cm}} = \frac{1}{30.0 \text{ cm}}</math></p> $d_o^2 - 105 d_o + 1350 = 0, \quad d_o = (105 \pm 75)/2.$ <p>a. When the object lies beyond the center of curvature we have</p> $d_{o+} = (1.80 \times 10^2 \text{ cm})/2 = \boxed{+9.0 \times 10^1 \text{ cm}} \quad \text{and} \quad d_{i+} = \boxed{+45 \text{ cm}}$ <p>b. When the object lies within the focal point</p> $d_{o-} = (3.0 \times 10^1 \text{ cm})/2 = \boxed{+15 \text{ cm}}, \quad \text{and} \quad d_{i-} = \boxed{-3.0 \times 10^1 \text{ cm}}$
2	<p>A spherical mirror is polished on both sides. When the concave side is used as a mirror, the magnification is +2.0. What is the magnification when the convex side is used as a mirror, the object remaining the same distance from the mirror? [<b>+0.67</b>]</p> <p><math display="block">f_{\text{convex}} = -\frac{1}{2}R \quad f_{\text{concave}} = \frac{1}{2}R</math></p> $f_{\text{convex}} = -\frac{1}{2}R = -f_{\text{concave}} \quad \text{or} \quad \frac{1}{f_{\text{convex}}} = -\frac{1}{f_{\text{concave}}}$ $m = -\frac{d_i}{d_o} \quad \text{or} \quad d_i = -m d_o$ $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{(-m d_o)} = \frac{1}{d_o} \left(1 - \frac{1}{m}\right)$ $\underbrace{\frac{1}{d_o} \left(1 - \frac{1}{m_{\text{convex}}}\right)}_{1/f_{\text{convex}}} = \underbrace{-\frac{1}{d_o} \left(1 - \frac{1}{m_{\text{concave}}}\right)}_{-1/f_{\text{concave}}} \quad \text{or} \quad m_{\text{convex}} = \frac{m_{\text{concave}}}{2m_{\text{concave}} - 1} = \frac{+2.0}{2(+2.0) - 1} = \boxed{+0.67}$
3	<p>A tall tree is growing across a river from you. You would like to know the distance between yourself and the tree, as well as its height, but are unable to make the measurements directly. However, by using a mirror to form an image of the tree and then measuring the image distance and the image height, you can calculate the distance to the tree as well as its height. Suppose that this mirror produces an image of the sun, and the image is located 0.90m from the mirror. The same mirror is then used to produce an image of the tree. The image of the tree is 0.91m from the mirror.</p> <ol style="list-style-type: none"> <li>How far away is the tree? [<b>82m</b>]</li> <li>The image height of the tree has a magnitude of 0.12 m. How tall is the tree? [<b>11m</b>]</li> </ol> <p>a. The distance to the tree is given by the mirror equation as</p> $\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{0.9000 \text{ m}} - \frac{1}{0.9100 \text{ m}} \quad \text{so} \quad d_o = \boxed{82 \text{ m}}$ <p>b. Since <math>h_o = h_i/m</math> and <math>m = -d_i/d_o</math>, we have that</p> $h_o = \frac{h_i}{m} = \frac{h_i}{\left(-\frac{d_i}{d_o}\right)} = h_i \left(-\frac{d_o}{d_i}\right)$ <p>Now <math>h_i = -0.12 \text{ m}</math>, where the minus sign has been used since the image is inverted relative to the tree (see Figure 25.18b). Thus, the height of the tree is</p> $h_o = h_i \left(-\frac{d_o}{d_i}\right) = (-0.12 \text{ m}) \left(-\frac{82 \text{ m}}{0.9100 \text{ m}}\right) = \boxed{11 \text{ m}}$
4	<p>A dentist's mirror is placed 2cm from a tooth. The enlarged image is located 5.6cm behind the mirror.</p> <ol style="list-style-type: none"> <li>What kind of mirror (plane, concave, or convex) is being used? Explain.</li> <li>Determine the focal length of the mirror. [<b>+3.1cm</b>]</li> <li>What is the magnification? [<b>+2.8</b>]</li> </ol>

	<p>d) How is the image oriented relative to the object? Explain. [upright]</p> <p>a. Since the image of the tooth is enlarged, it cannot be a plane mirror, for which the object and the image would have the same size. Convex mirrors produce smaller images in all cases. Therefore, the enlarged image means that the mirror must be <b>concave</b>.</p> <p>b. Using the mirror equation (Equation 25.3), we find for the focal length that</p> $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{2.0 \text{ cm}} + \frac{1}{-5.6 \text{ cm}} = 0.32 \text{ cm}^{-1} \quad \text{or} \quad f = \frac{1}{0.32 \text{ cm}^{-1}} = \boxed{+3.1 \text{ cm}}$ <p>c. Using the magnification equation (Equation 25.4), we find for the magnification that</p> $m = -\frac{d_i}{d_o} = -\frac{-5.6 \text{ cm}}{2.0 \text{ cm}} = \boxed{+2.8}$ <p>d. Since <math>m</math> is positive, the image is <b>upright</b> relative to the object.</p>
5	<p>An object is located 14cm in front of a convex mirror, the image being 7cm behind the mirror. A second object, twice as tall as the first one, is placed in front of the mirror, but at a different location. The image of this second object has the same height as the other image. How far in front of the mirror is the second object located? <b>[+42cm]</b></p> $\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{14.0 \text{ cm}} + \frac{1}{-7.00 \text{ cm}} = \frac{1}{f} \quad d_{i2} = d_{o2} \left( \frac{d_{i1}}{d_{o1}} \right) \left( \frac{h_{o1}}{h_{o2}} \right) \quad h_{o2} = 2h_{o1}$ $f = -14.0 \text{ cm} \quad d_{i2} = d_{o2} \left( \frac{d_{i1}}{d_{o1}} \right) \left( \frac{h_{o1}}{h_{o2}} \right) = d_{o2} \left( \frac{-7.00 \text{ cm}}{14.0 \text{ cm}} \right) \left( \frac{h_{o1}}{2h_{o1}} \right) = -0.250 d_{o2}$ $h_{i2} = h_{i1} \quad \frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f}$ $\left( \frac{-d_{i2}}{d_{o2}} \right) h_{o2} = \left( \frac{-d_{i1}}{d_{o1}} \right) h_{o1} \quad \frac{1}{d_{o2}} + \frac{1}{-0.250 d_{o2}} = \frac{1}{-14.0 \text{ cm}}$ $\boxed{d_{o2} = +42.0 \text{ cm}}$
6	<p>A convex mirror has a focal length of 27cm. Find the magnification produced by the mirror when the object distance is 9cm and 18cm. <b>[0.75, 0.6]</b></p> $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{fd_o} \quad \text{or} \quad d_i = \frac{fd_o}{d_o - f}$ $m = -\frac{d_i}{d_o} = -\frac{fd_o / (d_o - f)}{d_o} = \frac{f}{f - d_o}$ <p><b>Smaller object distance</b> <math>m = \frac{f}{f - d_o} = \frac{-27.0 \text{ cm}}{(-27.0 \text{ cm}) - (9.0 \text{ cm})} = \boxed{0.750}</math></p> <p><b>Greater object distance</b> <math>m = \frac{f}{f - d_o} = \frac{-27.0 \text{ cm}}{(-27.0 \text{ cm}) - (18.0 \text{ cm})} = \boxed{0.600}</math></p>

## Part 2: Refraction

No	Questions
1	<p>One end of a long glass rod (<math>n = 1.50</math>) is formed into a convex surface with a radius of curvature of 6.00 cm. An object is located in air along the axis of the rod. Find the image positions corresponding to object distances of</p> <ul style="list-style-type: none"> <li>a) 20.0 cm,</li> <li>b) 10.0 cm,</li> <li>c) 3.00 cm from the end of the rod.</li> </ul> <p><math>\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}</math> becomes <math>\frac{1.00}{p} + \frac{1.50}{q} = \frac{1.50 - 1.00}{6.00 \text{ cm}} = \frac{1}{12.0 \text{ cm}}</math></p> <p>(a) <math>\frac{1.00}{20.0 \text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0 \text{ cm}}</math> or <math>q = \frac{1.50}{[(1.00/12.0 \text{ cm}) - (1.00/20.0 \text{ cm})]} = \boxed{45.0 \text{ cm}}</math></p> <p>(b) <math>\frac{1.00}{10.0 \text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0 \text{ cm}}</math> or <math>q = \frac{1.50}{[(1.00/12.0 \text{ cm}) - (1.00/10.0 \text{ cm})]} = \boxed{-90.0 \text{ cm}}</math></p> <p>(c) <math>\frac{1.00}{3.0 \text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0 \text{ cm}}</math> or <math>q = \frac{1.50}{[(1.00/12.0 \text{ cm}) - (1.00/3.0 \text{ cm})]} = \boxed{-6.00 \text{ cm}}</math></p>
2	<p>A glass sphere (<math>n = 1.50</math>) with a radius of 15cm has a tiny air bubble 5cm above its center. The sphere is viewed looking down along the extended radius containing the bubble. What is the apparent depth of the bubble below the surface of the sphere?</p> <p><math>\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}</math>      <math>p = 10.0 \text{ cm}</math>.</p> <p><math>q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}</math>      <math>q = \frac{(1.00)(-15.0 \text{ cm})(10.0 \text{ cm})}{(10.0 \text{ cm})(1.00 - 1.50) - (1.50)(-15.0 \text{ cm})} = -8.57 \text{ cm}</math>.</p> <p><math>n_1 = 1.50, n_2 = 1.00, R = -15.0 \text{ cm}</math>      <span style="border: 1px solid black; padding: 2px;">apparent depth is 8.57 cm</span>.</p>
3	<p>A simple model of the human eye ignores its lens entirely. Most of what the eye does to light happens at the outer surface of the transparent cornea. Assume that this surface has a radius of curvature of 6mm, and assume that the eyeball contains just one fluid with a refractive index of 1.40. Prove that a very distant object will be imaged on the retina, 21mm behind the cornea. Describe the image.</p> <p><math>\frac{1.00}{\infty} + \frac{1.40}{21.0 \text{ mm}} = \frac{1.40 - 1.00}{6.00 \text{ mm}}</math></p> <p><math>0.0667 = 0.0667</math>.</p> <p style="border: 1px solid black; padding: 2px;">The image is inverted, real and diminished.</p>
4	<p>A goldfish is swimming at 2cm/s toward the front wall of a rectangular aquarium. What is the apparent speed of the fish measured by an observer looking in from outside the front wall of the tank? The index of refraction of water is 1.33.</p> <p>For a plane surface, <math>\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}</math> becomes <math>q = -\frac{n_2 p}{n_1}</math>.</p> <p>Thus, the magnitudes of the rate of change in the image and object positions are related by</p> $\left  \frac{dq}{dt} \right  = \frac{n_2}{n_1} \left  \frac{dp}{dt} \right .$ <p>If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by</p> $v_{\text{image}} = \left  \frac{dq}{dt} \right  = \frac{1.00}{1.33} (2.00 \text{ cm/s}) = \boxed{1.50 \text{ cm/s}}.$

Part 3: Thin lenses

No	Questions
1	<p>A camera is supplied with two interchangeable lenses, whose focal lengths are 35mm and 150mm. A woman whose height is 1.6m stands 9m in front of the camera.</p> <p>a) Show that the height of the image is</p> $h_i = \left( \frac{f}{f - u} \right) h_o$ <p>where <math>h_o</math> is the height of the object, <math>f</math> is the focal length of the lens and <math>u</math> is the object distance from the camera.</p> <p>b) What is the height of her image on the image sensor, as produced by the 35.0-mm lens and the 150.0-mm lens?</p> <p><math display="block">\frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \text{or} \quad h_i = h_o \left( -\frac{d_i}{d_o} \right)</math></p> $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{fd_o} \quad \text{or} \quad d_i = \frac{fd_o}{d_o - f}$ $h_i = h_o \left( -\frac{d_i}{d_o} \right) = h_o \left( -\frac{1}{d_o} \right) \left( \frac{fd_o}{d_o - f} \right) = h_o \left( \frac{f}{f - d_o} \right)$ $h_i = h_o \left( \frac{f}{f - d_o} \right) = (1.60 \text{ m}) \left[ \frac{35.0 \times 10^{-3} \text{ m}}{(35.0 \times 10^{-3} \text{ m}) - 9.00 \text{ m}} \right] = [-0.00625 \text{ m}]$ $h_i = h_o \left( \frac{f}{f - d_o} \right) = (1.60 \text{ m}) \left[ \frac{150.0 \times 10^{-3} \text{ m}}{(150.0 \times 10^{-3} \text{ m}) - 9.00 \text{ m}} \right] = [-0.0271 \text{ m}]$ <p>Both heights are negative because the images are inverted with respect to the object.</p>
2	<p>An object is placed in front of a converging lens in such a position that the lens (<math>f = 12\text{cm}</math>) creates a real image located 21cm from the lens. Then, with the object remaining in place, the lens is replaced with another converging lens (<math>f = 16\text{cm}</math>). A new, real image is formed. What is the image distance of this new image?</p> <p>The image distance <math>d_{i2}</math>, produced by the 2nd lens is related to the object distance <math>d_{o2}</math> and the focal length <math>f_2</math> by the thin-lens equation:</p> $\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}}$ <p>Since <math>d_{o2} = d_{o1}</math> (the image distance for the 1st lens), Equation 26.6 can be written as</p> $\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o1}}$ $\frac{1}{d_{o1}} = \frac{1}{f_1} - \frac{1}{d_{i1}}$ $\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o1}} = \frac{1}{f_2} - \left( \frac{1}{f_1} - \frac{1}{d_{i1}} \right) = \frac{1}{16.0 \text{ cm}} - \left( \frac{1}{12.0 \text{ cm}} - \frac{1}{21.0 \text{ cm}} \right)$ <p>Solving for <math>d_{i2}</math> gives <math>d_{i2} = [37.3 \text{ cm}]</math>.</p>
3	<p>The distance between an object and its image formed by a diverging lens is 49cm. The focal length of the lens is <math>-233\text{cm}</math>. Find the image distance and the object distance.</p> <p><math display="block">\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{49.0 - d_i} + \frac{1}{d_i} = \frac{1}{-233.0}</math></p> $\frac{d_i + 49.0 - d_i}{d_i(49.0 - d_i)} = \frac{49.0}{d_i(49.0 - d_i)} = \frac{1}{-233.0}$ $d_i(49.0 - d_i) = -11417$ $d_i = \frac{-(-49.0) \pm \sqrt{(-49.0)^2 - 4(1.00)(-11417)}}{2(1.00)} = [-85.1 \text{ cm}]$ $d_o = 49.0 \text{ cm} - d_i = (49.0 \text{ cm}) - (-85.1 \text{ cm}) = [134.1 \text{ cm}]$
4	<p>An object is placed 96.5 cm from a glass lens with one concave surface of radius 22cm and one convex surface of radius 18.5cm. Where is the final image? What is the magnification?</p>

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow$$

$$f = \frac{1}{(n-1) \left( \frac{R_1 R_2}{R_1 + R_2} \right)} = \frac{1}{(1.52-1) \left( \frac{(-22.0 \text{ cm})(+18.5 \text{ cm})}{(-22.0 \text{ cm}) + (+18.5 \text{ cm})} \right)} = 223.6 \text{ cm}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{(96.5 \text{ cm})(223.6 \text{ m})}{96.5 \text{ cm} - 223.6 \text{ cm}} = -169.77 \text{ cm} = \boxed{-170 \text{ cm}}$$

$$m = -\frac{d_i}{d_o} = -\frac{-169.77 \text{ cm}}{96.5 \text{ cm}} = 1.759 = \boxed{+1.8}$$

The image is virtual, in front of the lens, and upright.

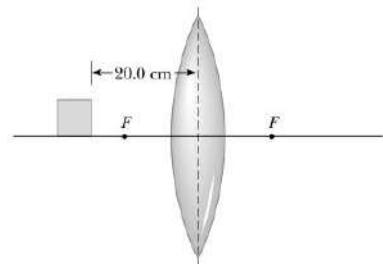
- 5 A planoconvex lens with  $n = 1.55$  is to have a focal length of 16.3 cm. What is the radius of curvature of the convex surface?

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{f} = (n-1) \left( \frac{1}{R_1} + \frac{1}{\infty} \right) = \frac{(n-1)}{R_1} \rightarrow$$

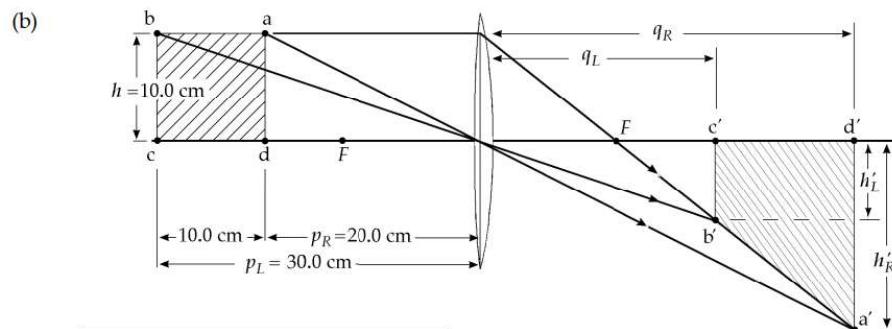
$$R_1 = (n-1)f = (1.55-1)(16.3 \text{ cm}) = 8.965 \text{ cm} \approx \boxed{9.0 \text{ cm}}$$

- 6 The figure shows a thin glass ( $n=1.5$ ) converging lens for which the radii of curvature are  $R_1 = 15 \text{ cm}$  and  $R_2 = -12 \text{ cm}$ . To the left of the lens is a cube having a face area of  $100 \text{ cm}^2$ . The base of the cube is on the axis of the lens, and the right face is 20cm to the left of the lens.

- Determine the focal length of the lens.
- Draw the image of the square face formed by the lens. What type of geometric figure is this?
- Determine the area of the image.



(a)  $\frac{1}{f} = (n-1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right] = (1.50-1) \left[ \frac{1}{15.0 \text{ cm}} - \frac{1}{(-12.0 \text{ cm})} \right] \rightarrow f = 13.3 \text{ cm}$



The square is imaged as a trapezoid.

- (c) To find the area, first find  $q_R$  and  $q_L$ , along with the heights  $h'_R$  and  $h'_L$ , using the thin lens equation.

$$\frac{1}{p_R} + \frac{1}{q_R} = \frac{1}{f} \quad \text{becomes} \quad \frac{1}{20.0 \text{ cm}} + \frac{1}{q_R} = \frac{1}{13.3 \text{ cm}} \quad \text{or} \quad q_R = 40.0 \text{ cm}$$

$$h'_R = h M_R = h \left( \frac{-q_R}{p_R} \right) = (10.0 \text{ cm})(-2.00) = -20.0 \text{ cm}$$

$$\frac{1}{p_L} + \frac{1}{q_L} = \frac{1}{f} \quad \text{or} \quad q_L = 24.0 \text{ cm}$$

$$h'_L = h M_L = (10.0 \text{ cm})(-0.800) = -8.00 \text{ cm}$$

Thus, the area of the image is: Area =  $|q_R - q_L| |h'_L| + \frac{1}{2} |q_R - q_L| |h'_R - h'_L| = \boxed{224 \text{ cm}^2}$ .

## Physical Optics Module

Huygen's Principle Statement:

Every **1** on a wave front can be considered as a source of tiny **2** that spread out in the **3** direction at the speed of the **4** itself. The new **5** is the **6** of all the wavelets—that is, the **7** to all of them.

Match the correct words to their numbers!

1	Wavelets
2	Wavefront
3	Envelope
4	Tangent
5	Point
6	Forward
7	Wave

### Fill in the blank!

#### **Conditions for Interference:**

Term	Meaning
Coherence	
	Two interacting light waves are of the same wavelength

#### **Types of Interference:**

Constructive Interference	
	Phase difference between the two monochromatic interacting wave is $\frac{n\lambda}{2}$ , where $n \in N^+$

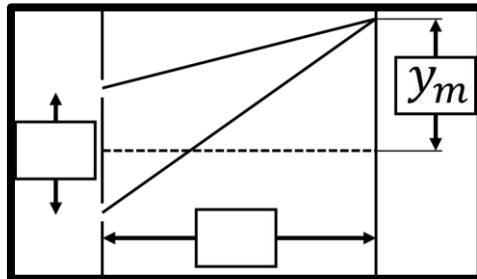
#### Diffraction?

## Double Slit Interference

Complete the table below!

Equations		
Bright Fringes	Dark Fringes	Fringe Separation
$y_m = \frac{m\lambda D}{d}$	$y_m =$	$\Delta y =$

Write the symbol from your equations into the diagram below.



## Thin Film Interference

Phase change upon reflection

Incoming ray	Outgoing ray	If $n_2 > n_1$ ,	If $n_2 < n_1$ ,
		$\pi$ rad phase change occurs upon reflection.	

Situations on Thin Films

Diagrams		
Conditions	Case 1: $n_3 > n_2 > n_1$	Case 2:
Phase difference	$\Delta\phi =$	$\Delta\phi = 180^\circ = \pi \text{ rad} = \frac{\lambda}{2}$

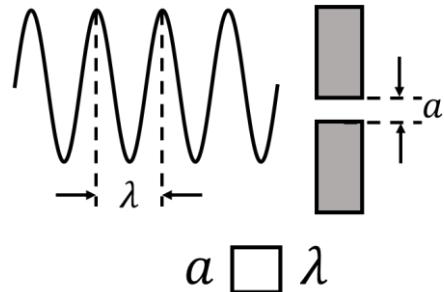
Constructive and Destructive Interferences for thin films

	Types	Equation
Case 1: Non-reflective coating	Bright Fringes	$2nt = m\lambda$
	Dark Fringes	
Case 2: Reflective coating	Bright Fringes	
	Dark Fringes	

## Single Slit Diffraction

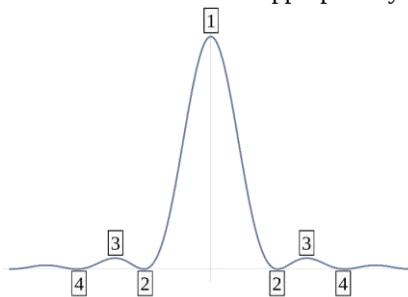
Understanding Diffraction

Definition	The bending of waves behind obstacles into the “shadow region” is known as diffraction.
Condition	Diffraction is most prominent when the size of the opening is on the order of the wavelength of the wave.



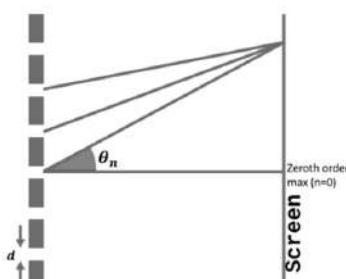
Complete the following table		
Type of Fringe	Equations	Values of $m$
Bright Fringes	$y_m = \frac{(m + \boxed{\quad}) \lambda D}{\boxed{\quad}}$	
Dark Fringes	$y_m = \frac{\boxed{\quad} \lambda \boxed{\quad}}{a}$	

Label the values of  $m$  appropriately



Number in Diagram	Value of $m$
1	
2	
3	
4	

## Diffraction Grating



Complete the following table		
Type of Fringe	Equations	Condition for maximum number of fringes on one side
Bright Fringes	$d \sin \boxed{\quad} = m \boxed{\quad}$ $\boxed{\quad} = \frac{1}{N}$	$\sin \theta \boxed{\quad} \leq 1$

## Past Year Questions on Interferences & Diffractions

### Double Slit & Thin Films

23/24	<b>Double Slit</b>	Two narrow slits 0.8mm apart are illuminated by a monochromatic light. The image of evenly distributed dark and bright fringes are displayed on the screen 50cm away. If the distance between two consecutive dark fringes is 0.304mm, determine the wavelength of the light and the distance between the third dark fringe from the central bright fringe.
	<b>Thin Films</b>	A glass of refractive index 1.6 is coated with a thin film to reflect blue light. If the wavelength of blue light is 460nm and the refractive index of film is 1.58, what is the minimum thickness of the film?
22/23	<b>Double Slit</b>	A 632nm wavelength light passes through a double slit at normal incident. Interference pattern is formed on the screen at 1.4m away. The distance between second order bright fringes is 23mm. Calculate the slits separation.
	<b>Thin Films</b>	A thin film of soap floating in air has a refractive index 1.33. Calculate the minimum thickness of the film that will reflect yellow light of wavelength 590nm.
21/22	<b>Double Slit</b>	A 475nm light passes through two narrow slits. The interference pattern is observed on a screen at a distance 85cm from the slits. The second order bright fringe is seen at $\pm 2.01\text{cm}$ from the central bright fringe. Calculate the slit separation and the width of the second order dark fringe.
	<b>Thin Films</b>	A flat glass with index of refraction 1.5 is coated with a transparent material of refractive index 1.25, in order to eliminate reflection of light of wavelength 680nm. Determine the minimum thickness of the coating.
20/21	<b>Double Slit</b>	Two narrow slits separated by 2.4mm are illuminated by a light with $\lambda = 512\text{nm}$ . The screen is placed 6.5m from the slits. Determine the <ol style="list-style-type: none"> <li>Distance between adjacent bright fringes on a screen.</li> <li>Distance of the fifth dark fringe from the central bright fringe.</li> </ol>
19/20	<b>Double Slit</b>	In a double slit experiment, the incident wavelength is 660nm, the slit separation is 0.25mm and the screen is placed 90cm away from the slits. Calculate the distance from the second to the third destructive interference fringe.
	<b>Thin Films</b>	A soap film with refractive index 1.3 and minimum thickness $0.177\mu\text{m}$ appears reddish under white light. Calculate the wavelength of light that is missing from the reflection.

### Single Slit & Diffraction Grating

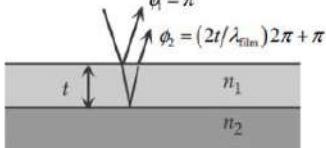
23/24	<b>Diffraction Grating</b>	A series of bright fringes is observed on a screen 1m away from a diffraction grating. A monochromatic light of wavelength $5.8 \times 10^{-7}\text{m}$ is incident normally on the grating with 200 lines per mm. Calculate the diffraction angle of the second order bright fringe and the maximum order of bright fringe.
22/23	<b>Single Slit</b>	For demonstration, a student uses a razor blade to create a slit of width $9.35 \times 10^{-5}\text{m}$ on an aluminium foil. When he shines a laser pointer with wavelength 680nm on the slit, a diffraction pattern observed on a screen. The width of the central bright fringe is 8cm. Calculate the distance of the screen from the slit.
21/22	<b>Diffraction Grating</b>	A monochromatic light 600nm is incident on a diffraction grating with 400 lines $\text{mm}^{-1}$ . Calculate the angle for the first bright order of diffraction and the maximum number of diffraction pattern that be formed.
20/21	<b>Single Slit</b>	A monochromatic light of wavelength 620nm is incident on a single slit and forms a diffraction pattern on a screen 1.2m away. The distance of seventh dark fringe from the central maximum is 18mm. Determine the size of the single slit and the distance of the second bright fringe from the central maximum.
19/20	<b>Diffraction Grating</b>	In a double slit experiment, the incident wavelength is 660nm, the slit separation is 0.25mm and the screen is placed 90cm away from the slits. The double slit is now replaced with a diffraction grating, if the maximum number of bright fringes is 15, calculate the slit separation of the grating.

## Worksheet on Interferences & Diffraction

### Young's Double Slit

<p>1 Two narrow, parallel slits separated by 0.250mm are illuminated by green light (<math>\lambda = 546.1\text{nm}</math>). The interference pattern is observed on a screen 1.20m away from the plane of the slits. Calculate the distance</p> <ol style="list-style-type: none"> <li>from the central maximum to the first bright region on either side of the central maximum</li> <li>between the first and second dark bands.</li> </ol>	<p>For the bright fringe,</p> $y_{\text{bright}} = \frac{m\lambda L}{d} \text{ where } m=1$ $y = \frac{(546.1 \times 10^{-9} \text{ m})(1.20 \text{ m})}{0.250 \times 10^{-3} \text{ m}} = 2.62 \times 10^{-3} \text{ m} = \boxed{2.62 \text{ mm}}$	<p>For the dark bands, <math>y_{\text{dark}} = \frac{\lambda L}{d} \left( m + \frac{1}{2} \right)</math>; <math>m = 0, 1, 2, 3, \dots</math></p> $y_2 - y_1 = \frac{\lambda L}{d} \left[ \left( 1 + \frac{1}{2} \right) - \left( 0 + \frac{1}{2} \right) \right] = \frac{\lambda L}{d} (1)$ $= \frac{(546.1 \times 10^{-9} \text{ m})(1.20 \text{ m})}{0.250 \times 10^{-3} \text{ m}}$ $\Delta y = \boxed{2.62 \text{ mm}}.$
<p>2 Two slits are separated by 0.32mm. A beam of 500nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range <math>-30^\circ &lt; \theta &lt; 30.0^\circ</math>.</p>	<p>At <math>30.0^\circ</math>, <math>d \sin \theta = m\lambda</math></p> $(3.20 \times 10^{-4} \text{ m}) \sin 30.0^\circ = m(500 \times 10^{-9} \text{ m}) \quad \text{so} \quad m = 320$ <p>There are 320 maxima to the right, 320 to the left, and one for <math>m=0</math> straight ahead.</p> <p>There are <math>\boxed{641 \text{ maxima}}</math>.</p>	
<p>3 Light with wavelength 442nm passes through a double-slit system that has a slit separation <math>d = 0.4\text{mm}</math>. Determine how far away a screen must be placed in order that a dark fringe appear directly opposite both slits, with just one bright fringe between them.</p>	$m = 0, y = 0.200 \text{ mm}$ $L \approx \frac{2dy}{\lambda} = \frac{2(0.400 \times 10^{-3} \text{ m})(0.200 \times 10^{-3} \text{ m})}{442 \times 10^{-9} \text{ m}} = 0.362 \text{ m}$ $L \approx \boxed{36.2 \text{ cm}}$	
<p>4 In Young's experiment a mixture of orange light (611 nm) and blue light (471 nm) shines on the double slit. The centers of the first-order bright blue fringes lie at the outer edges of a screen that is located 0.50m away from the slits. However, the first-order bright orange fringes fall off the screen. By how much and in which direction (toward or away from the slits) should the screen be moved so that the centers of the first-order bright orange fringes will just appear on the screen?</p>	<p>The first-order orange fringes occur farther out from the center than do the first-order blue fringes. Therefore, the screen must be moved toward the slits so that the orange fringes will appear on the screen. The distance between the screen and the slits is <math>L</math>, and the amount by which the screen must be moved toward the slits is <math>L_{\text{blue}} - L_{\text{orange}}</math>. We know that <math>L_{\text{blue}} = 0.500 \text{ m}</math>, and must, therefore, determine <math>L_{\text{orange}}</math>.</p>	$\frac{\lambda}{d} \approx \frac{y}{L} \quad \text{or} \quad L = \frac{yd}{\lambda}$ $\frac{L_{\text{blue}}}{L_{\text{orange}}} = \frac{yd / \lambda_{\text{blue}}}{yd / \lambda_{\text{orange}}} = \frac{\lambda_{\text{orange}}}{\lambda_{\text{blue}}}$ $L_{\text{blue}} - L_{\text{orange}} = L_{\text{blue}} - \left( \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}} \right) L_{\text{blue}} = L_{\text{blue}} \left( 1 - \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}} \right)$ $= (0.500 \text{ m}) \left[ 1 - \frac{471 \text{ nm}}{611 \text{ nm}} \right] = \boxed{0.115 \text{ m}}$

## Thin Film Interference

1	A nonreflective coating of magnesium fluoride ( $n=1.38$ ) covers the glass ( $n=1.52$ ) of a camera lens. Assuming that the coating prevents reflection of yellow-green light (wavelength in vacuum= 565 nm), determine the minimum nonzero thickness that the coating can have.
	Thus, in this case, the minimum condition for destructive interference is $2t = \frac{1}{2} \lambda_{\text{film}}$ $\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n} = \frac{565 \text{ nm}}{1.38} = 409 \text{ nm}$ $t = \frac{1}{4} \lambda_{\text{film}} = \frac{1}{4} (409 \text{ nm}) = 102 \text{ nm}$
2	A film of oil lies on wet pavement. The refractive index of the oil exceeds that of the water. The film has the minimum nonzero thickness such that it appears dark due to destructive interference when viewed in red light (wavelength=640nm in vacuum). Assuming that the visible spectrum extends from 380 to 750 nm, for which visible wavelength(s) in vacuum will the film appear bright due to constructive interference?
	the condition for destructive interference becomes $2t = \lambda_{\text{film}} = \frac{640.0 \text{ nm}}{n_{\text{film}}}$ The condition for constructive interference is $\underbrace{2t}_{\substack{\text{Extra distance} \\ \text{traveled by wave} \\ \text{in the film}}} + \underbrace{\frac{1}{2} \lambda'_{\text{film}}}_{\substack{\text{Half-wavelength} \\ \text{net phase change} \\ \text{due to reflection}}} = \underbrace{m \lambda'_{\text{film}}}_{\substack{\text{Condition for} \\ \text{constructive} \\ \text{interference}}} \quad m = 1, 2, 3, \dots$ where $\lambda'_{\text{film}}$ is the wavelength that produces constructive interference in the film. $2t = \left(m - \frac{1}{2}\right) \lambda'_{\text{film}} = \left(m - \frac{1}{2}\right) \frac{\lambda'_{\text{vacuum}}}{n_{\text{film}}}$ $\frac{640.0 \text{ nm}}{n_{\text{film}}} = \left(m - \frac{1}{2}\right) \frac{\lambda'_{\text{vacuum}}}{n_{\text{film}}} \quad \text{or} \quad \lambda'_{\text{vacuum}} = \frac{640.0 \text{ nm}}{m - \frac{1}{2}}$ For $m = 1$ , $\lambda'_{\text{vacuum}} = 1280 \text{ nm}$ ; for $m = 2$ , $\lambda'_{\text{vacuum}} = 427 \text{ nm}$ ; for $m = 3$ , $\lambda'_{\text{vacuum}} = 256 \text{ nm}$ . Values of $m$ greater than 3 lead to values of $\lambda'_{\text{vacuum}}$ that are smaller than 256 nm. Thus, the only wavelength in the visible spectrum (380 to 750 nm) that will give constructive interference is 427 nm.
3	A uniform thin film of alcohol ( $n = 1.36$ ) lies on a flat glass plate ( $n = 1.56$ ). When monochromatic light, whose wavelength can be changed, is incident normally, the reflected light is a minimum for $\lambda = 525\text{nm}$ and a maximum for $\lambda = 655\text{nm}$ . What is the minimum thickness of the film?
	For constructive interference, the net phase change must be an even nonzero integer multiple of $\pi$ . $\phi_{\text{net}} = \phi_2 - \phi_1 = \left[ \left( \frac{2t}{\lambda_{1\text{film}}} \right) 2\pi + \pi \right] - \pi = m_1 2\pi \rightarrow t = \frac{1}{2} \lambda_{1\text{film}} m_1 = \frac{1}{2} \frac{\lambda_1}{n_{\text{film}}} m_1, \quad m_1 = 1, 2, 3, \dots$ For destructive interference, the net phase change must be an odd-integer multiple of $\pi$ . $\phi_{\text{net}} = \phi_2 - \phi_1 = \left[ \left( \frac{2t}{\lambda_{2\text{film}}} \right) 2\pi + \pi \right] - \pi = (2m_2 + 1)\pi \rightarrow t = \frac{1}{4} \frac{\lambda_2}{n_{\text{film}}} (2m_2 + 1), \quad m_2 = 0, 1, 2, \dots$ $\frac{1}{2} \frac{\lambda_1}{n_{\text{film}}} m_1 = \frac{1}{4} \frac{\lambda_2}{n_{\text{film}}} (2m_2 + 1) \rightarrow \frac{2m_2 + 1}{2m_1} = \frac{\lambda_2}{\lambda_1} = \frac{(655 \text{ nm})}{(525 \text{ nm})} = 1.2476 \approx 1.25 = \frac{5}{4}$  <p>Thus we see that <math>m_1 = m_2 = 2</math>, and the thickness of the film is  <math display="block">t = \frac{1}{2} \frac{\lambda_1}{n_{\text{film}}} m_1 = \frac{1}{2} \left( \frac{655 \text{ nm}}{1.36} \right) (2) = 481.6 \text{ nm} \quad \text{or} \quad t = \frac{1}{4} \frac{\lambda_2}{n_{\text{film}}} (2m_2 + 1) = \frac{1}{4} \left( \frac{525 \text{ nm}}{1.36} \right) (5) = 482.5 \text{ nm}</math></p> <p>The average thickness, with 3 significant figures, is 482 nm.</p>
4	A thin oil slick ( $n_o = 1.5$ ) floats on water ( $n_w = 1.33$ ). When a beam of white light strikes this film at normal incidence from air, the only enhanced reflected colors are red (650 nm) and violet (390 nm). From this information, deduce the (minimum) thickness of the oil slick.
	When illuminated from above, the light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$ . A ray reflected at the oil-water interface undergoes no phase shift due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left( \frac{2t}{\lambda_{\text{oil}}} \right) 2\pi$ . For constructive interference to occur, the net phase change must be a multiple of $2\pi$ . $\phi_{\text{net}} = \phi_2 - \phi_1 = \left[ \left( \frac{2t}{\lambda_{\text{oil}}} \right) 2\pi \right] - \pi = m(2\pi) \rightarrow t = \frac{1}{2} \left( m + \frac{1}{2} \right) \lambda_{\text{oil}} = \frac{1}{2} \left( m + \frac{1}{2} \right) \frac{\lambda}{n_o}$ For $\lambda = 650 \text{ nm}$ , the possible thicknesses are as follows: $t_{650} = \frac{1}{2} \left( m + \frac{1}{2} \right) \frac{650 \text{ nm}}{1.50} = 108 \text{ nm}, 325 \text{ nm}, 542 \text{ nm}, \dots$ For $\lambda = 390 \text{ nm}$ , the possible thicknesses are as follows: $t_{390} = \frac{1}{2} \left( m + \frac{1}{2} \right) \frac{390 \text{ nm}}{1.50} = 65 \text{ nm}, 195 \text{ nm}, 325 \text{ nm}, 455 \text{ nm}, \dots$ The minimum thickness of the oil slick must be 325 nm.

### Single slit & Diffraction grating

1	In a single-slit diffraction pattern, the central fringe is 450 times as wide as the slit. The screen is 18 000 times farther from the slit than the slit is wide. What is the ratio $\frac{\lambda}{W}$ , where $\lambda$ is the wavelength of the light shining through the slit and $W$ is the width of the slit?
	It is given that $2y = 450W$ and $L = 18\ 000W$ . We know $\lambda/W = \sin \theta$ . Now $\sin \theta \approx \tan \theta = y/L$ , so $\frac{\lambda}{W} = \frac{y}{L} = \frac{225\ W}{18\ 000\ W} = \boxed{0.013}$
2	How many dark fringes will be produced on either side of the central maximum if light ( $\lambda = 651\text{ nm}$ ) is incident on a single slit that is $5.47\mu\text{m}$ wide?
	The angle $\theta$ that specifies the location of the $m^{\text{th}}$ dark fringe is given by $\sin \theta = m\lambda/W$ , where $\lambda$ is the wavelength of the light and $W$ is the width of the slit. When $\theta$ has its maximum value of $90.0^\circ$ , the number of dark fringes that can be produced is a maximum. Solving Equation 27.4 for $m$ , and setting $\theta = 90.0^\circ$ , we have $m = \frac{W \sin 90.0^\circ}{\lambda} = \frac{(5.47 \times 10^{-6}\text{ m}) \sin 90.0^\circ}{651 \times 10^{-9}\text{ m}} = 8.40$ Therefore, the number of dark fringes is $\boxed{8}$ .
3	Light waves with two different wavelengths, $632\text{ nm}$ and $474\text{ nm}$ , pass simultaneously through a single slit whose width is $71.5\mu\text{m}$ and strike a screen $1.20\text{ m}$ from the slit. Two diffraction patterns are formed on the screen. What is the distance (in cm) between the common center of the diffraction patterns and the first occurrence of a dark fringe from one pattern falling on top of a dark fringe from the other pattern?
	$\begin{aligned} \sin \theta &= m_1 \frac{\lambda_1}{W} & \text{and} & \sin \theta = m_2 \frac{\lambda_2}{W} \\ m_1 \frac{\lambda_1}{W} &= m_2 \frac{\lambda_2}{W} & \text{or} & \frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{474\text{ nm}}{632\text{ nm}} = 0.75 \end{aligned}$ The first dark fringes of the two diffraction patterns do not coincide, because setting $m_1 = m_2 = 1$ yields a ratio of $m_1/m_2 = 1/1 = 1$ , which does not satisfy Equation (3). But we can see that other dark fringes do coincide, because Equation (3) is satisfied when $m_1 = 3$ and $m_2 = 4$ ( $m_1/m_2 = 3/4 = 0.75$ ), or when $m_1 = 6$ and $m_2 = 8$ ( $m_1/m_2 = 6/8 = 0.75$ ), and so forth. The first time the dark fringes overlap occurs when $m_1 = 3$ and $m_2 = 4$ . Solving the first of Equations (2) for $\theta$ , and taking $m_1 = 3$ yields $\theta = \sin^{-1} \left( m_1 \frac{\lambda_1}{W} \right) = \sin^{-1} \left[ 3 \left( \frac{632 \times 10^{-9}\text{ m}}{7.15 \times 10^{-5}\text{ m}} \right) \right] = 1.52^\circ$ $y = L \tan \theta = (1.20\text{ m}) \tan 1.52^\circ = 3.18 \times 10^{-2}\text{ m} = \boxed{3.18\text{ cm}}$
4	For a wavelength of $420\text{ nm}$ , a diffraction grating produces a bright fringe at an angle of $26^\circ$ degrees. For an unknown wavelength, the same grating produces a bright fringe at an angle of $41^\circ$ degrees. In both cases the bright fringes are of the same order $m$ . What is the unknown wavelength?
	$\begin{aligned} \sin \theta_1 &= \frac{m\lambda_1}{d} & \text{and} & \sin \theta_2 = \frac{m\lambda_2}{d} \\ \frac{\sin \theta_2}{\sin \theta_1} &= \frac{m\lambda_2/d}{m\lambda_1/d} = \frac{\lambda_2}{\lambda_1} \\ \lambda_2 &= \lambda_1 \frac{\sin \theta_2}{\sin \theta_1} = (420\text{ nm}) \frac{\sin 41^\circ}{\sin 26^\circ} = \boxed{630\text{ nm}} \end{aligned}$
5	The light shining on a diffraction grating has a wavelength of $495\text{ nm}$ (in vacuum). The grating produces a second-order bright fringe whose position is defined by an angle of $9.34^\circ$ degrees. How many lines per centimeter does the grating have?

	$\sin \theta = m \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \dots$ $d = \frac{m\lambda}{\sin \theta} = \frac{2(495 \times 10^{-9} \text{ m})}{\sin 9.34^\circ} = 6.10 \times 10^{-6} \text{ m} \quad \text{or} \quad 6.10 \times 10^{-4} \text{ cm}$ $N = \frac{1}{d} = \frac{1}{6.10 \times 10^{-4} \text{ cm}} = \boxed{1640 \text{ lines/cm}}$
6	<p>Two gratings A and B have slit separations <math>d_A</math> and <math>d_B</math>, respectively. They are used with the same light and the same observation screen. When grating A is replaced with grating B, it is observed that the first-order maximum of A is exactly replaced by the second-order maximum of B.</p> <ol style="list-style-type: none"> <li>Determine the ratio <math>\frac{d_B}{d_A}</math> of the spacings between the slits of the gratings.</li> <li>Find the next two principal maxima of grating A and the principal maxima of B that exactly replace them when the gratings are switched. Identify these maxima by their order numbers.</li> </ol> <p>a. The angular positions of the specified orders are equal, so <math>\lambda/d_A = 2\lambda/d_B</math>, or</p> $\frac{d_B}{d_A} = \boxed{2}$ <p>b. Similarly, we have for the <math>m_A</math> order of grating A and the <math>m_B</math> order of grating B that <math>m_A\lambda/d_A = m_B\lambda/d_B</math>, so <math>m_A = m_B/2</math>.</p> <p>The next highest orders which overlap are</p> $m_B = \boxed{4}, \quad m_A = \boxed{2} \quad \text{and} \quad m_B = \boxed{6}, \quad m_A = \boxed{3}$

====End of Module====

# **Physical Optics**

## **Interference & Diffraction**

## **Huygen's Principle:**

"Every **point** on a wave front can be considered as a source of tiny **wavelets** that spread out in the **forward** direction at the speed of the **wave** itself. The new **wavefront** is the **envelope** of all the wavelets—that is, the **tangent** to all of them."

## **Conditions for interference:**

### **1. Coherent:**

The light sources must be coherent, meaning they maintain a constant phase difference. This ensures stable and consistent interference patterns.

### **2. Monochromatic:**

The sources should emit light of the same wavelength (or frequency) to produce clear and stable interference fringes.

## **Types of Interferences:**

### **1. Constructive:**

Phase difference of  $n\lambda$

### **2. Destructive:**

Phase difference of  $(n + \frac{1}{2})\lambda$  or  $\frac{n\lambda}{2}$

# Contents

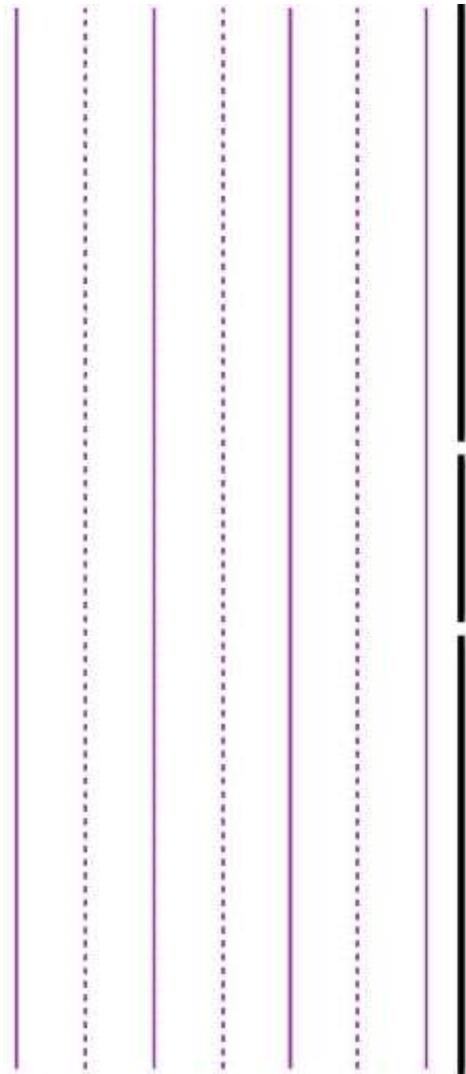
Young's  
Double Slit  
Interference

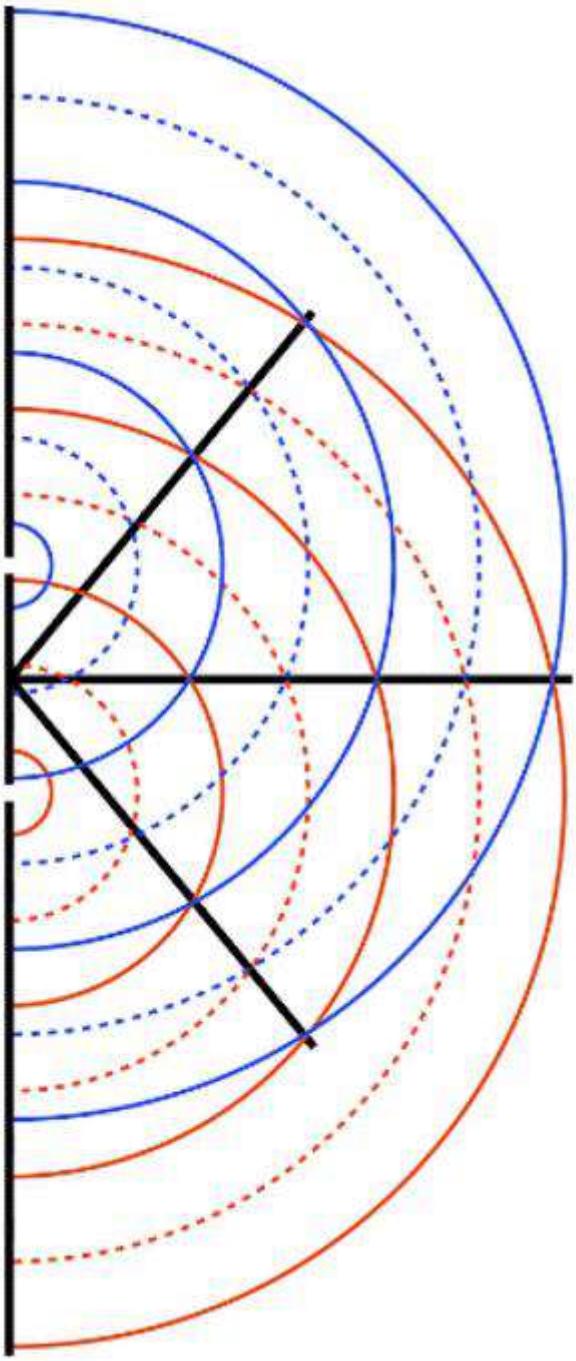
Single Slit  
Diffraction

Diffraction  
Gratings

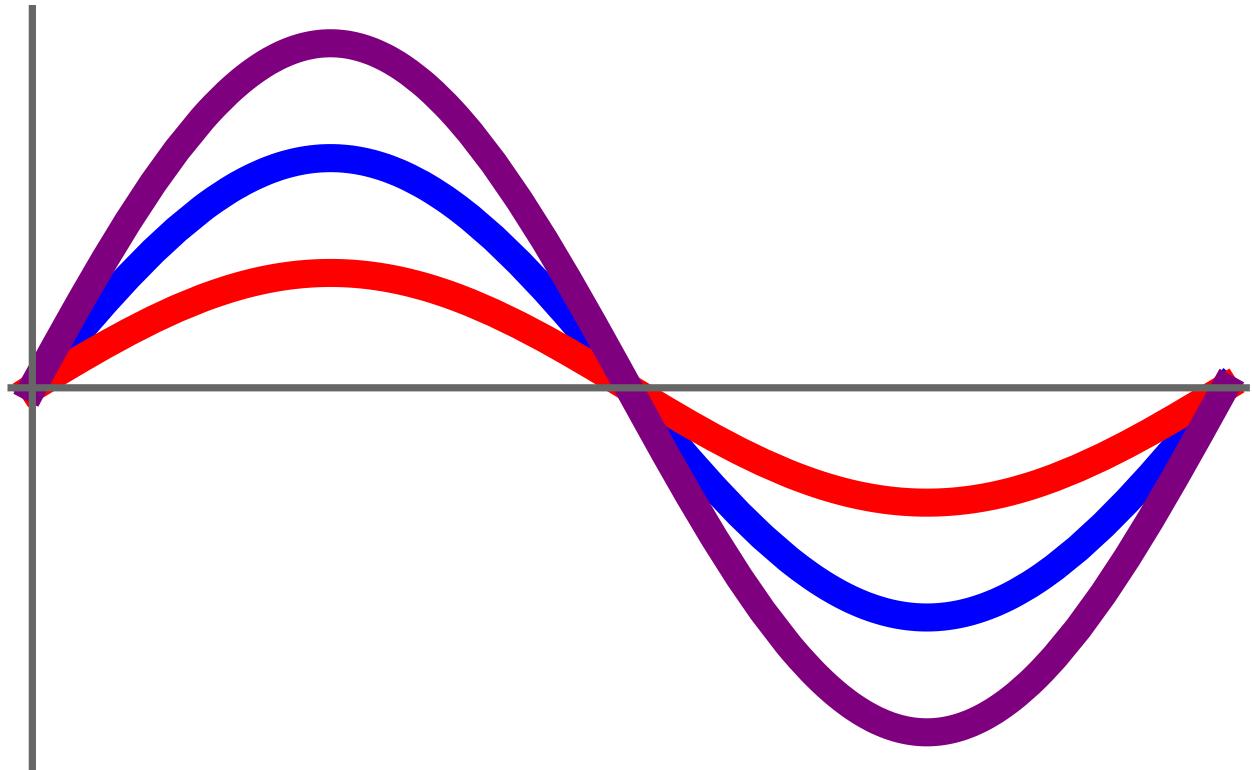
Thin Films

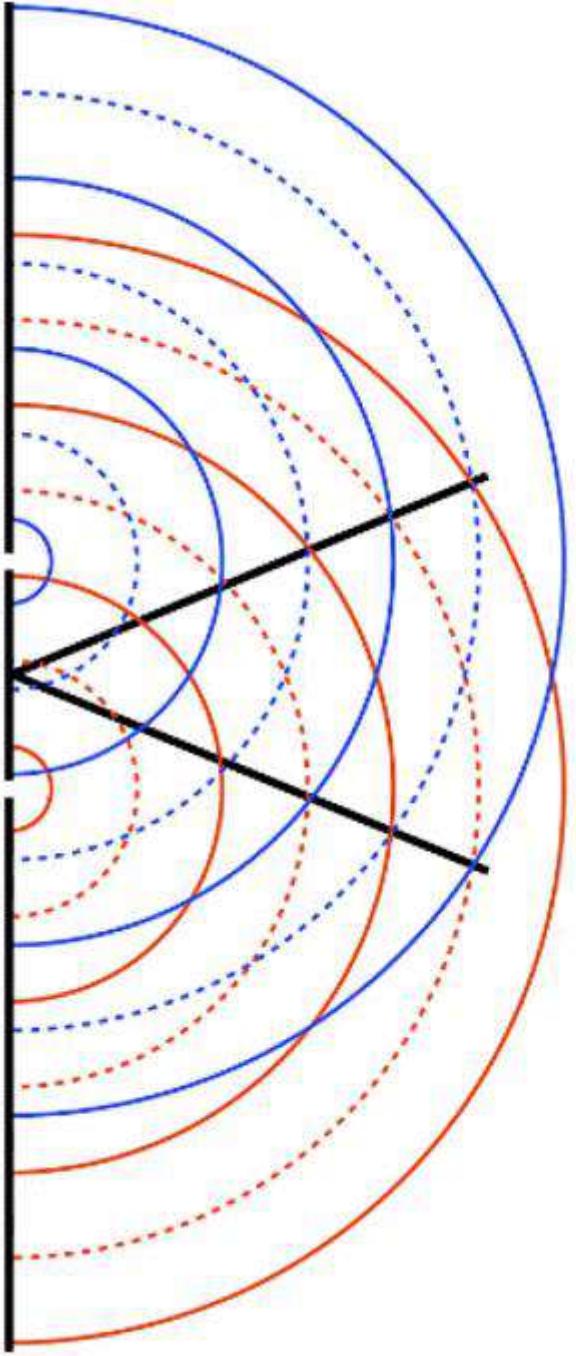
# Young's Double Slit Interference



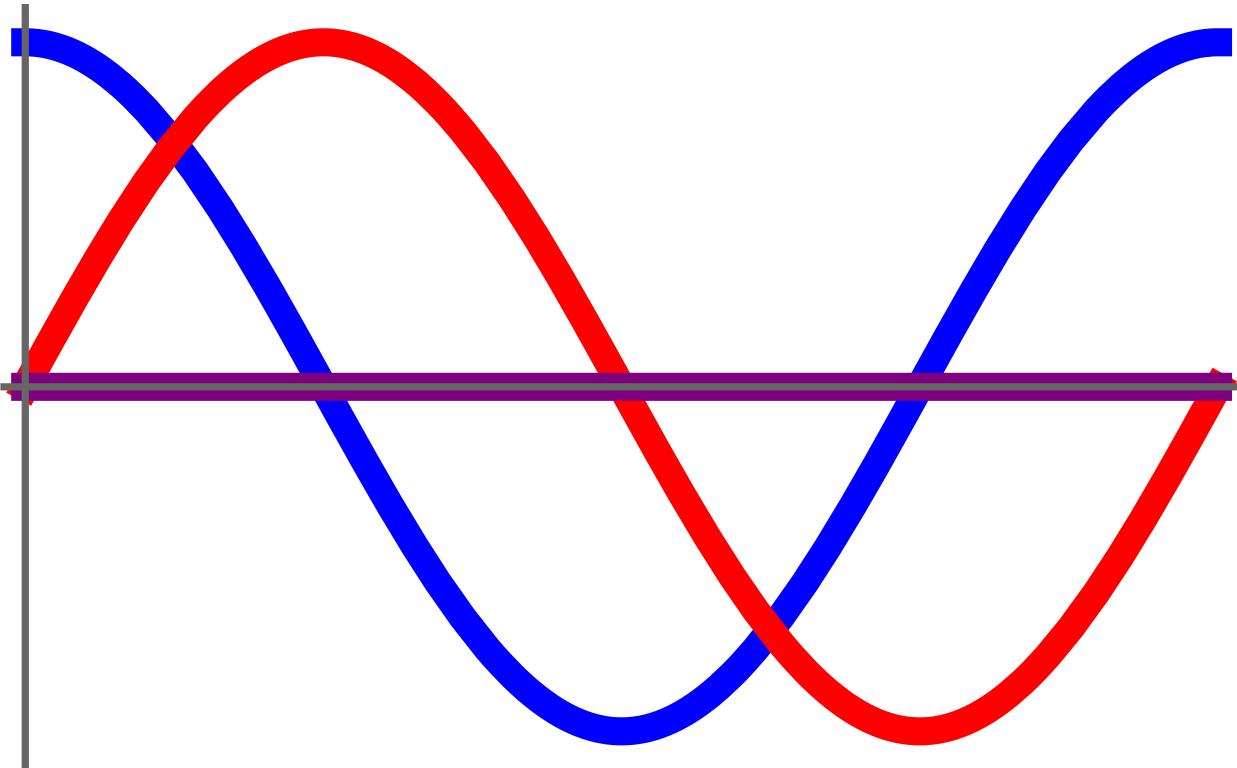


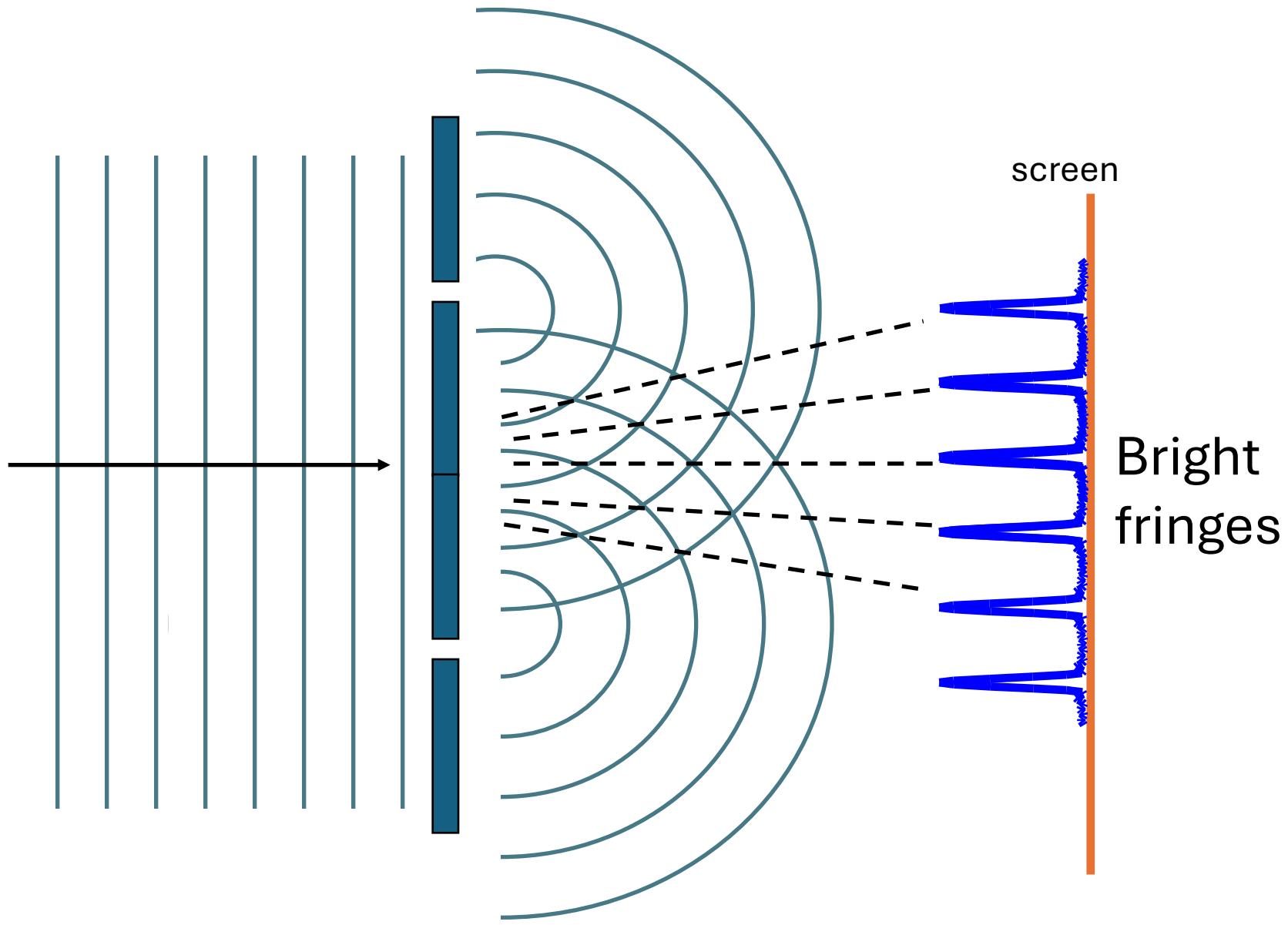
**constructive  
interference**

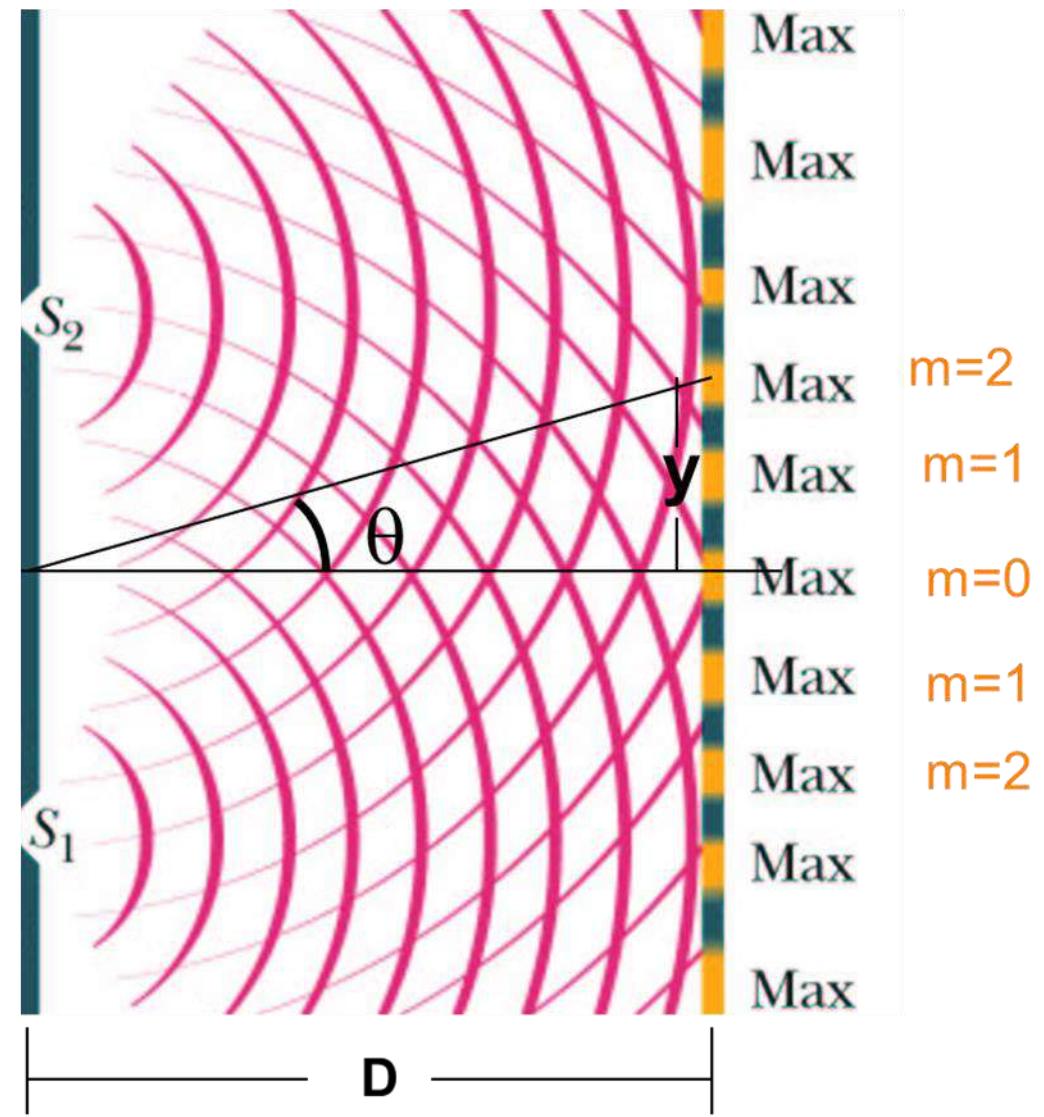
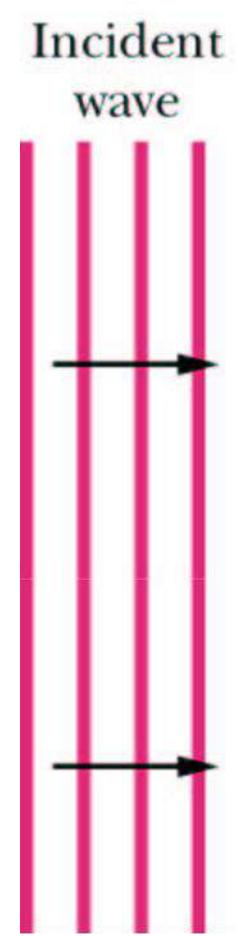




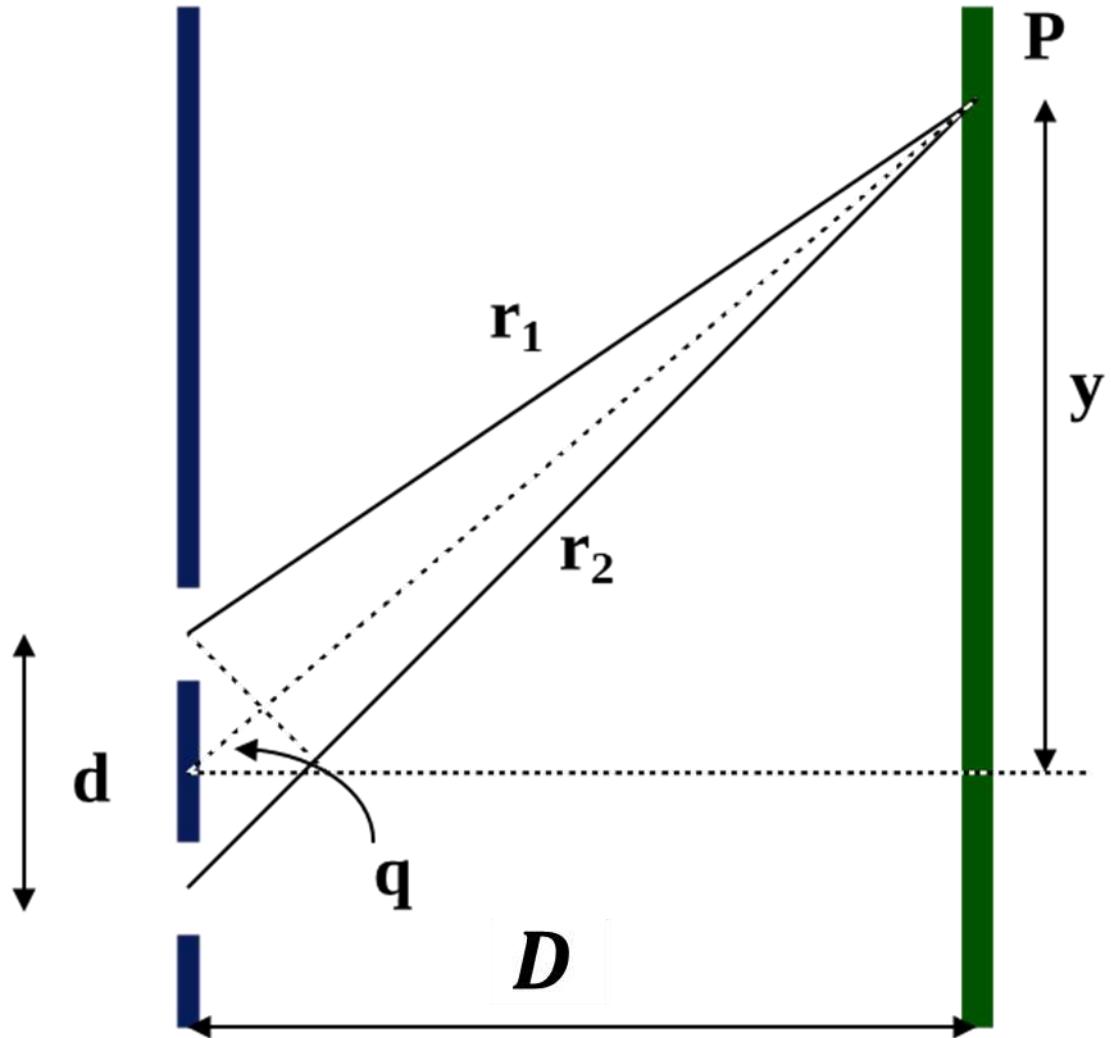
destructive  
interference







$$\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 \approx d \sin \theta$$



When the path difference is a multiple of the wavelength these add constructively, and when it's a half-multiple they cancel.

$$d \sin \theta = m\lambda \text{ (bright fringes)}$$

$$d \sin \theta = (m + \frac{1}{2})\lambda \text{ (dark fringes)}$$

$$y = L \tan \theta$$

for small  $y$ ,

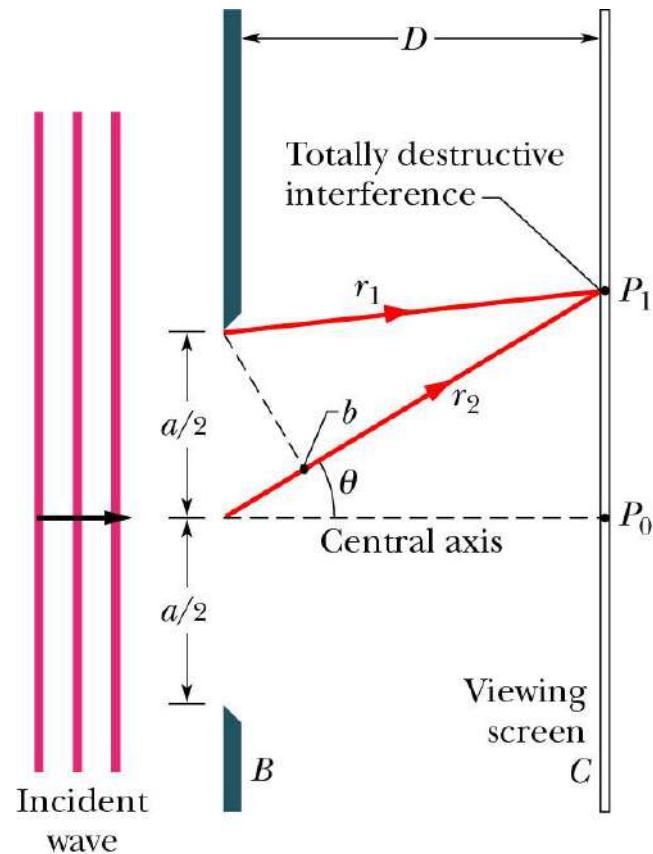
$$\sin \theta \approx \tan \theta = \frac{y}{D}$$

$$y = \frac{m\lambda D}{d} \text{ (bright fringes)}$$

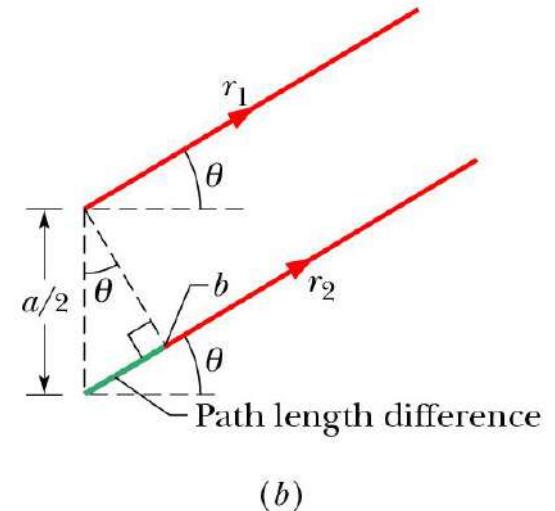
$$y = \frac{(m + \frac{1}{2})\lambda D}{d} \text{ (dark fringes)}$$

**Contents**

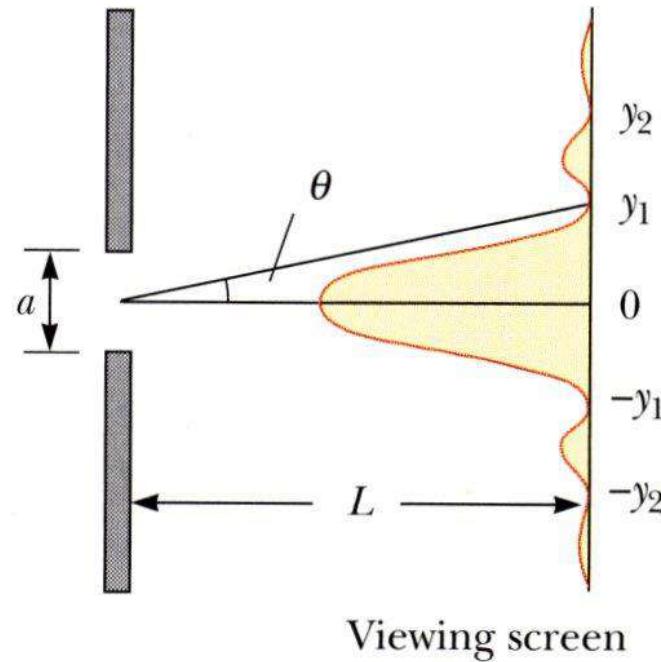
# Single Slit Diffraction



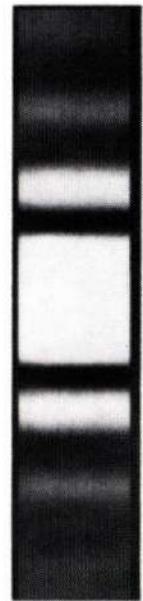
$$\Delta r = r_2 - r_1 = \frac{a}{2} \sin \theta = \frac{n\lambda}{2} \text{ (minima)}$$
$$\sin \theta = \frac{n\lambda}{a} = \frac{y_n}{D}$$
$$y_n = \frac{n\lambda D}{a}$$



(b)

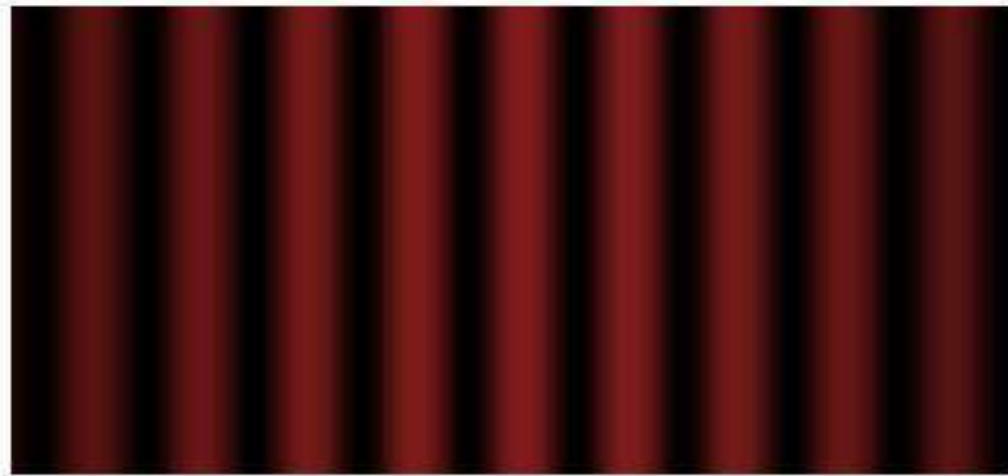


$\sin \theta = 2\lambda/a$
$\sin \theta = \lambda/a$
$\sin \theta = 0$
$\sin \theta = -\lambda/a$
$\sin \theta = -2\lambda/a$

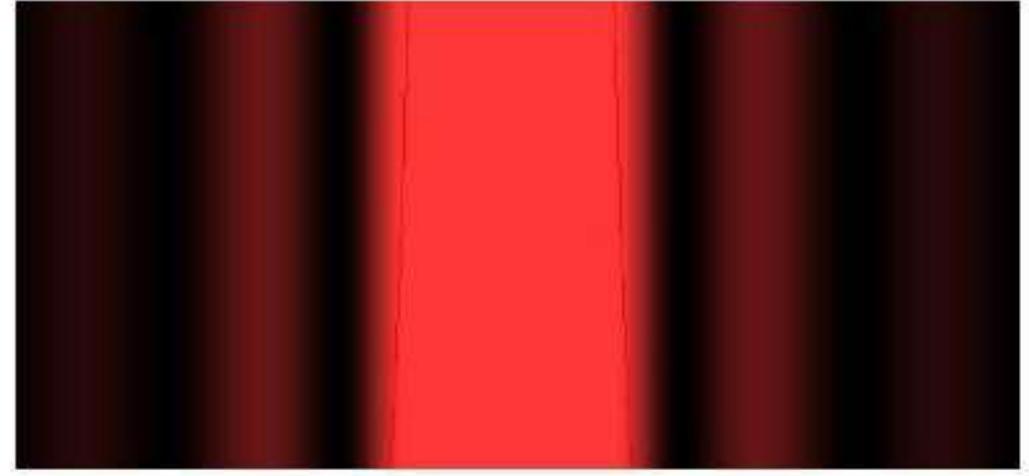


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# Single Slit Diffraction vs Double Slit Interference



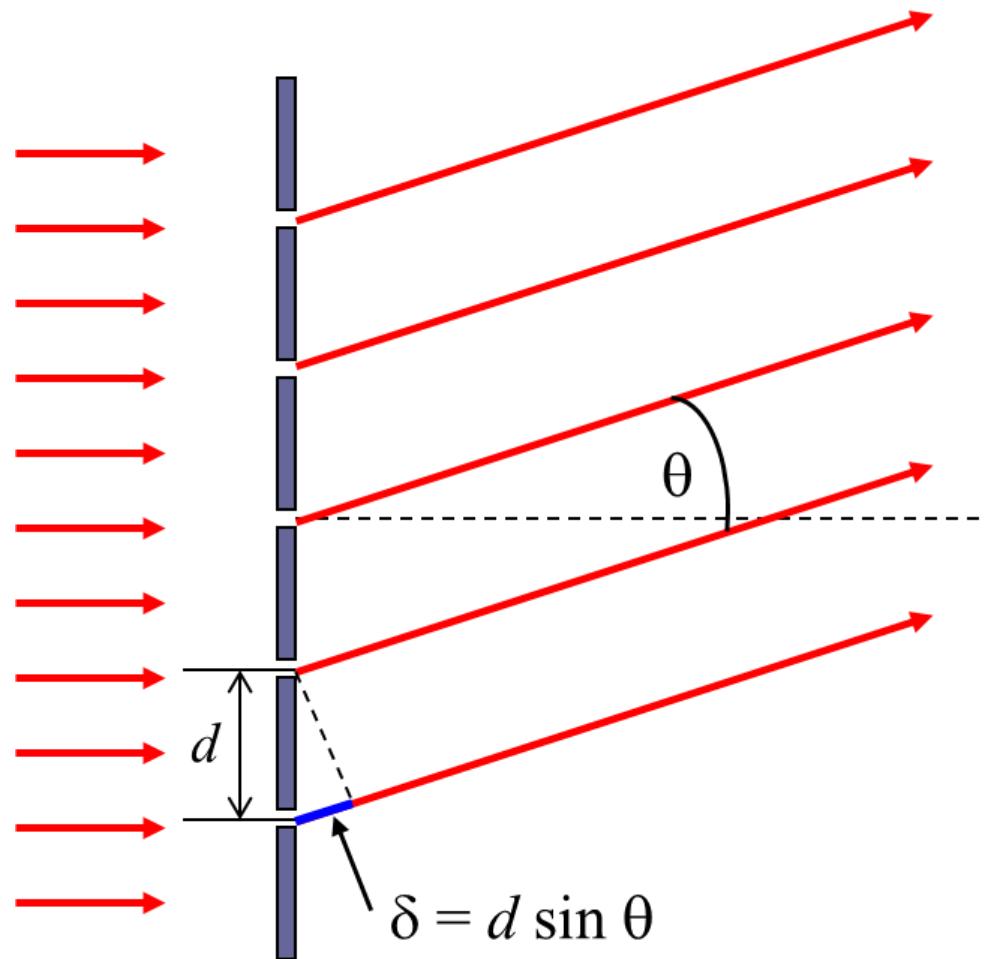
Double Slit  
Interference



Single Slit Diffraction

# Diffraction Grating

A diffraction grating consists of a large number of equally spaced parallel slits.



The path difference between rays from any two adjacent slits is

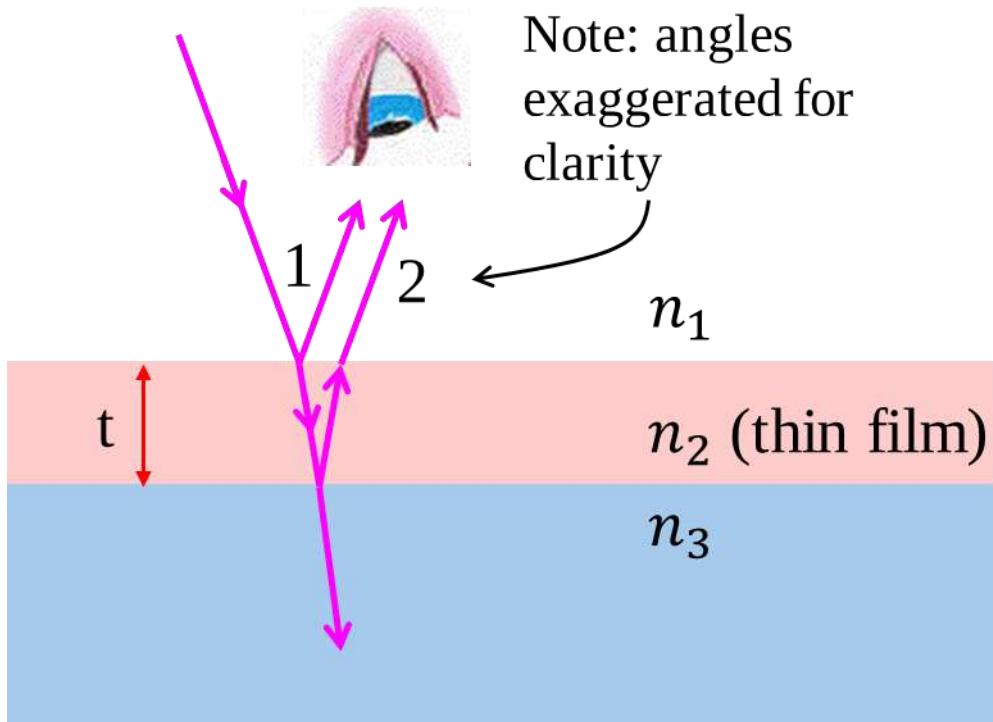
$$\delta = \Delta r = d \sin \theta$$

If  $\delta$  is equal to some integer multiple of the wavelength then waves from all slits will arrive in phase at a point on a distant screen.

Interference maxima occur for  
 $d \sin \theta = m\lambda$

# Thin Film Interference

Light is incident close to normal to a thin film

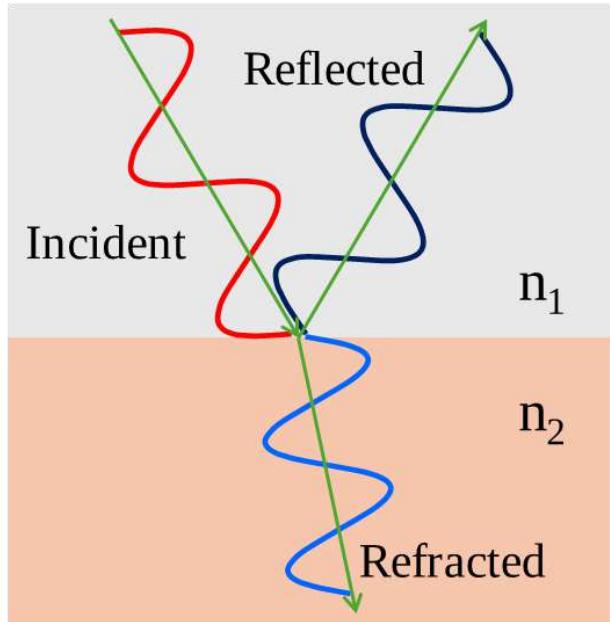


Get two waves by reflection off two different interfaces: interference!

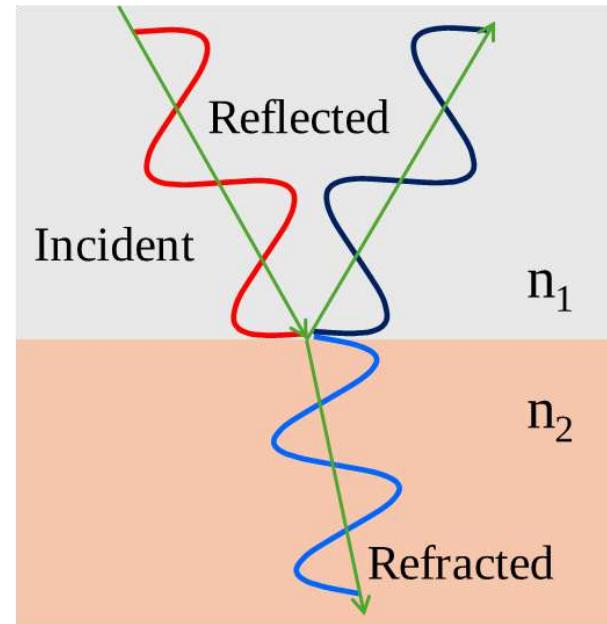
Ray 2 travels approximately  $2t$  further than ray 1.

# Thin Film Interference

Upon reflection from a boundary between two transparent materials, the phase of the reflected light may change.



If  $n_1 > n_2$  – no phase change upon reflection



If  $n_1 < n_2$  –  $180^\circ$  phase change upon reflection (shift by  $\frac{\lambda}{2}$ )

# Thin Film Interference

Reflection      Distance

$$\text{Ray 1: } r_1 = \left(0 + \frac{1}{2}\right) + 0$$
$$\text{Ray 2: } r_2 = \left(0 + \frac{1}{2}\right) + \frac{2t}{\lambda_{film}}$$

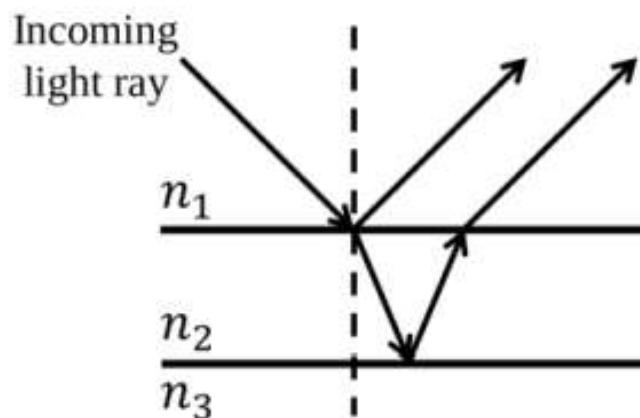
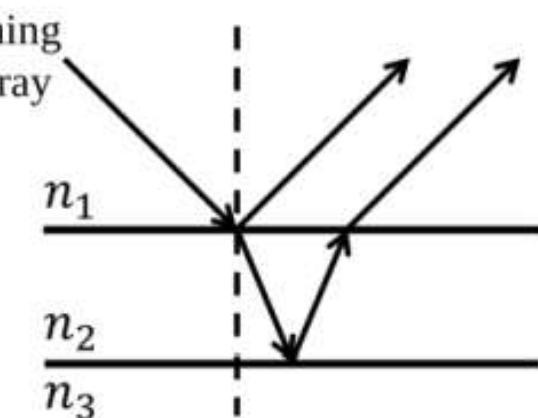
Note: this is wavelength in film!  
 $(\lambda_{film} = \frac{\lambda_o}{n_2})$

If  $\Delta r = |r_2 - r_1| = m\lambda$  (constructive)

If  $\Delta r = |r_2 - r_1| = \frac{m\lambda}{2}$  (destructive)

# Thin Film Interference

## Situations on Thin Films

Diagrams		
Conditions	Case 1: $n_3 > n_2 > n_1$	Case 2: $n_2 > n_1 \wedge n_2 > n_3$
Phase difference	$\Delta\phi = 0^\circ$	$\Delta\phi = 180^\circ = \pi \text{ rad} = \frac{\lambda}{2}$

Case	Refractive Indices	Type of Fringes	Equations
Non-Reflective Coating	$n_3 > n_2 > n_1$	Bright Fringes	$2n_2 t = m\lambda$
		Dark Fringes	$2n_2 t = \left(m + \frac{1}{2}\right)\lambda$
Reflective Coating	$n_2 > n_1 \wedge n_2 > n_3$	Bright Fringes	$2n_2 t = \left(m + \frac{1}{2}\right)\lambda$
		Dark Fringes	$2n_2 t = m\lambda$

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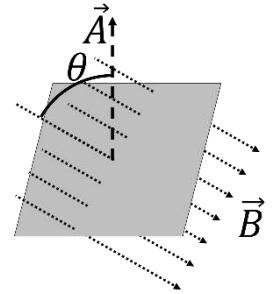
# Interference vs Diffraction

Aspect	Interference	Diffraction
Definition	Superposition of waves from two or more coherent sources.	Bending of waves around obstacles or through slits, producing a wave pattern.
Required Sources	Requires at least two coherent sources.	Can occur with a single wavefront (e.g., through a single slit).
Pattern	Alternating bright and dark fringes of equal width.	Central bright fringe is wider, and intensity decreases for higher orders.
Cause	Overlapping of waves from different sources.	Interaction of waves with an obstacle or aperture.
Fringe Width	Generally uniform.	Varies, with a central maximum often wider than the others.
Dependency	Depends on the path difference between the waves.	Depends on the size of the aperture and wavelength of the wave.
Example	Young's double-slit experiment.	Single-slit diffraction, diffraction grating.
Applications	Thin film interference, anti-reflective coatings.	Spectroscopy, optical instruments.

# Chapter 5 Notes & Exercises

**Magnetic flux** (unit:  $Tm^2$  or Wb) is a measure of total magnetic field  $\vec{B}$  (unit: T) passing through a given area  $\vec{A}$  (unit:  $m^2$ ), this is calculated with

$$\phi =$$



In the case of  $N$  number of area of  $\vec{A}$  of which  $\vec{B}$  passes through, the total magnetic flux is called the **magnetic flux linkage  $\Phi$** , and is determined by

$$\Phi =$$

## Induced EMF

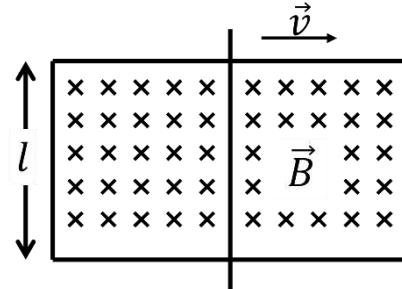
Faraday's Law		
Lenz's Law		$\varepsilon =$

### Induced emf in straight conductor

In a straight conductor, the area changes with time which causes the magnetic flux to change with time.

Consider a rectangular coil with one of its sides movable and the opposite of the movable side has a length of  $l$ , in a region of magnetic field  $\vec{B}$ . If the movable side is moved at velocity  $\vec{v}$ , the area of the coil would change. The induced emf would then be

$$\varepsilon =$$



### Induced emf in a coil

In a circular coil, the option for inducing emf comes from varying the magnetic field **and** the area of the coil, thus 2 equations can be found,

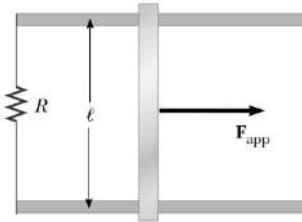
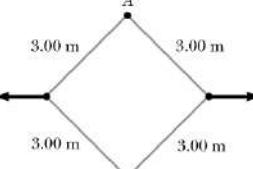
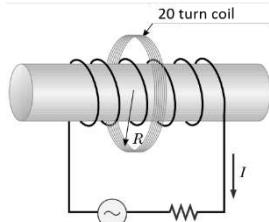
$$\varepsilon =$$

### Induced emf in a rotating coil

For a coil rotating at angular speed of  $\omega$ , the emf induced is then

$$\varepsilon =$$

## Part 1: Magnetic Flux & Induced EMFs

<p><b>Problem 1:</b></p> <p>A 50-turn rectangular coil of dimensions <math>2.5\text{cm} \times 10\text{cm}</math> is allowed to fall from a position where <math>B=0\text{T}</math> to a new position where <math>B=2.5\text{T}</math> and the magnetic field is directed perpendicular to the plane of the coil. Calculate the magnitude of the average emf that is induced in the coil if the displacement occurs in <math>0.5\text{s}</math>. [0.625V]</p>	<p><b>Problem 6:</b></p> <p>A 700-turn solenoid, <math>20\text{cm}</math> long, has a diameter of <math>2.5\text{cm}</math>. A 14turn coil is wound tightly around the centre of the solenoid. If the current in the solenoid increases uniformly from 0 to <math>25\text{A}</math> in <math>8\text{ms}</math>, what will be the induced current in the short coil during this time if the short coil has a resistance of <math>0.3\Omega</math>? [315mA]</p>
<p><b>Problem 2:</b></p> <p>A <math>1.2\text{m}</math> aluminum bar is held with its length parallel to the east–west direction and dropped from a bridge. Just before the bar hits the river below, its speed is <math>25\text{ms}^{-1}</math>, and the emf induced across its length is <math>1.11\text{mV}</math>. Assuming the horizontal component of the earth’s magnetic field at the location of the bar points directly north,</p> <ul style="list-style-type: none"> <li>a) determine the magnitude of the horizontal component of the earth’s magnetic field, [<math>37\mu\text{T}</math>]</li> <li>b) state whether the east end or the west end of the bar is positive</li> </ul>	<p><b>Problem 7:</b></p> <p>Consider the arrangement shown in the figure. Assume that <math>R = 5\Omega</math>, <math>l = 1.5\text{m}</math>, and a uniform <math>4\text{T}</math> magnetic field is directed into the page.</p>  <ul style="list-style-type: none"> <li>a) At what speed should the bar be moved to produce a current of <math>0.5\text{A}</math> in the resistor? [<math>0.417\text{ms}^{-1}</math>]</li> <li>b) Calculate the applied force required to move the bar to the right at a constant speed of <math>3\text{m/s}</math>. [21.6N]</li> </ul>
<p><b>Problem 3:</b></p> <p>A circular loop in the plane of the paper lies in a <math>0.70\text{-T}</math> magnetic field pointing into the paper. The loop’s diameter changes from <math>20\text{cm}</math> to <math>4\text{cm}</math> in <math>0.15\text{ s}</math>. What is</p> <ul style="list-style-type: none"> <li>a) the direction of the induced current,</li> <li>b) the magnitude of the average induced emf, [141mV]</li> <li>c) the average induced current if the coil resistance is <math>2.5\Omega</math>? [56.4mA]</li> </ul>	<p><b>Problem 8:</b></p> <p>The square loop in the figure shown is made of wires with total series resistance <math>10\Omega</math>. It is placed in a uniform <math>1.2\text{T}</math> magnetic field directed perpendicularly into the plane of the paper. The loop, which is hinged at each corner, is pulled as shown until the separation between points A and B is <math>3\text{m}</math>. If this process takes <math>90\text{ms}</math>, what is the average current generated in the loop? What is the direction of the current? [1.6A]</p> 
<p><b>Problem 4:</b></p> <p>A 416-loop circular armature coil with an <math>8\text{cm}</math> diameter rotates at <math>120\text{ rev s}^{-1}</math> in a uniform magnetic field of strength <math>0.64\text{T}</math>. Determine the peak induced emf. [1.009kV]</p>	
<p><b>Problem 5:</b></p> <p>A coil of 20 turns and radius <math>10\text{cm}</math> surrounds a long solenoid of radius <math>2\text{cm}</math> and <math>1000</math> turns/meter, as shown in the figure. The current in the solenoid changes from <math>120\text{A}</math> to <math>0\text{A}</math> in <math>3\text{ms}</math>. Find the induced emf in the 20-turn coil. [-1.00531V]</p> 	

# Inductance

## Self-induction

The idea of self-inductance is this – a magnetic field induces emf in a conductor, which in turns induces another magnetic field that opposes the initial induced emf. The conductor ‘self induces’ a magnetic field. The ability of a conductor to do this is quantified by **self-inductance L** (unit: Henry, H),

$$L =$$

Generally, this means that

$$LI =$$

For a coil of N turns with a cross-sectional area of A and radius of r,

$$L =$$

For a solenoid of N turns with a cross-sectional area of A and length l,

$$L =$$

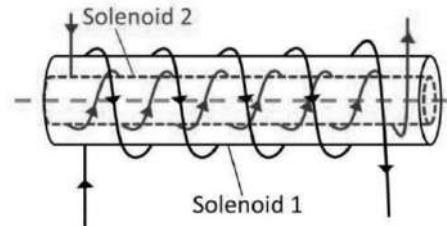
## Mutual induction

Mutual inductance happens between 2 conductors, when the magnetic field induced by one conductor induces current in the other conductor.

Consider two coaxial solenoids, a magnetic field is generated by solenoid 1 and thus solenoid 2 respond by an induced emf, if solenoid 2 has a cross sectional area of  $A_2$ , then the mutual inductance between solenoid 1 and 2 is

$$M_{21} =$$

where  $l$  is the length of the solenoid.

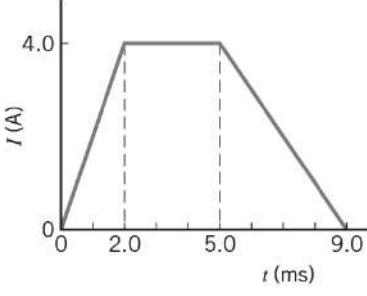


## Energy Stored in Inductor

The energy stored (unit: Joule, J) in an inductor of inductance  $L$  and with current  $I$  running through it, is simply

$$U =$$

## Part 2: Inductance & Energy Stored in Inductor

<p>Problem 1:</p> <ol style="list-style-type: none"> <li>An air-filled cylindrical inductor has 2600 turns, and it is 2.5cm in diameter and 28.2cm long. What is its inductance? [15mH]</li> <li>An air-filled cylindrical inductor has 4000 turns, and it is 30cm in diameter and 2cm long. What is its inductance? [1.51H]</li> <li>A coil has 200-mH inductance. If the current is 3.00 A and is increasing at a rate of <math>3.8A\text{s}^{-1}</math>, what is the potential difference across the coil at this moment? [0.6V]</li> </ol>	<p>Problem 5:</p> <p>Two coils of wire are placed close together. Initially, a current of 2.5A exists in one of the coils, but there is no current in the other. The current is then switched off in a time of <math>3.7 \times 10^{-2}\text{s}</math>. During this time, the average emf induced in the other coil is 1.7 V. What is the mutual inductance of the two-coil system? [25mH]</p>										
<p>Problem 2:</p> <p>A long thin solenoid of length 20cm and cross-sectional area <math>20\text{cm}^{-2}</math> contains 200 closely packed turns of wire. Wrapped tightly around it is an insulated coil 650 of turns. Assume all the flux from coil 1 (the solenoid) passes through coil 2, and calculate the mutual inductance. [1.6336mH]</p>	<p>Problem 6:</p> <p>A constant current 15A of exists in a solenoid whose inductance is 3.1H. The current is then reduced to zero in a certain amount of time.</p> <ol style="list-style-type: none"> <li>If the current goes from 15 to 0 A in a time of 75ms, what is the emf induced in the solenoid? [620V]</li> <li>How much electrical energy is stored in the solenoid? [350J]</li> <li>At what rate must the electrical energy be removed from the solenoid when the current is reduced to 0A in a time of 75ms? [4700W]</li> </ol>										
<p>Problem 3:</p> <p>The current through a 3.2-mH inductor varies with time according to the graph shown in the drawing.</p> <p>What is the average induced emf during the time intervals</p> <ol style="list-style-type: none"> <li>0-2ms [-6.4V]</li> <li>2-5ms [0V]</li> <li>5-9ms [+3.2V]</li> </ol>  <table border="1"> <caption>Data points for Problem 3 graph</caption> <thead> <tr> <th>t (ms)</th> <th>I (A)</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>2.0</td><td>4.0</td></tr> <tr><td>5.0</td><td>4.0</td></tr> <tr><td>9.0</td><td>0</td></tr> </tbody> </table>	t (ms)	I (A)	0	0	2.0	4.0	5.0	4.0	9.0	0	<p>Problem 7:</p> <p>Suppose you wish to make a solenoid whose self-inductance is 1.4mH. The inductor is to have a cross-sectional area of <math>1.2 \times 10^{-3}\text{m}^2</math> and a length of 5.2cm. How many turns of wire are needed? [220 turns]</p> <p>Problem 8:</p> <p>A <math>54\mu\text{H}</math> solenoid is constructed by wrapping 65 turns of wire around a cylinder with a cross-sectional area of <math>9\text{cm}^2</math>. When the solenoid is shortened by squeezing the turns closer together, the inductance increases to <math>86\mu\text{H}</math>. Determine the change in the length of the solenoid. [33mm]</p>
t (ms)	I (A)										
0	0										
2.0	4.0										
5.0	4.0										
9.0	0										
<p>Problem 4:</p> <p>Two coils of wire are placed close together. Initially, a current of 2.5A exists in one of the coils, but there is no current in the other. The current is then switched off in a time of <math>3.7 \times 10^{-2}\text{s}</math>. During this time, the average emf induced in the other coil is 1.7 V. What is the mutual inductance of the two-coil system? [25mH]</p>											

Name : .....

Practicum: .....

Matric Number: .....

## **EXPERIMENT 1: MEASUREMENT AND UNCERTAINTY**

**Course Learning Outcome:**

Solve problems related to Physics of motion, force and energy, waves, matter and thermodynamics  
**(C4, PLO 4, CTPS 3, MQF LOD 6)**

**Learning Outcomes:**

At the end of this lesson, students will able to describe technique of measurement and determine uncertainty of length of various objects.

**Student Learning Time:**

Face-to-face	Non face-to-face
1 hour	1 hour

**Direction:** Read over the lab manual and then answer the following question.

Introduction1. Complete **Table 1**

Basic Quantity	Symbol	SI Unit (with symbol)	Measuring Instrument
Length	<i>l</i>		
Mass	<i>m</i>		
Time	<i>t</i>		
Electric Current	<i>I</i>		
Temperature	<i>T</i>		

**Table 1**

2. .... is used to measure the diameter of a coin.
3. Micrometer screw gauge is usually used to measure the ..... of a thin wire or the ..... of paper.
4. Complete **Table 2**

Measuring Apparatus	Sensitivity	Uncertainty
Meter rule	0.1 cm	$\pm 0.1\text{cm}$
Vernier calipers	0.01 cm	
Micrometer screw gauge		$\pm 0.01\text{mm}$
Travelling microscope		$\pm 0.01\text{mm}$
Thermometer	0.1°C	
Voltmeter	0.1 V	
Ammeter		$\pm 0.1\text{A}$

Measuring Apparatus	Sensitivity	Uncertainty
Electronic Balance	0.01 g	

**Table 2**

5. State **TWO** types of reading;

- i. ....
- ii. ....

6. The repeated reading for a measurement is given as  $a, b, c, d, e$ , and  $f$ . Write the equation of Average Value and Uncertainty.

	EQUATION
Average Value, $\bar{x}$	
Uncertainty, $\Delta\bar{x}$	

### Experiment

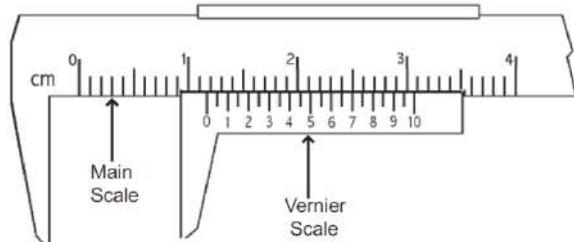
7. Complete **Table 3**

Measurement	Measuring Instrument	Uncertainty/Smallest scale	Type of reading (single point/two point/Vernier scale)
Length of a metal rod			Two points
Length and width of a laboratory book			Two points
Mass of a ball bearing			Single Point
Diameter of a ball bearing			Vernier scale
Diameter of a coin			Vernier scale
External diameter of a glass rod			Vernier scale

**Table 3**

8. Determine the reading for the following measurements:

i.

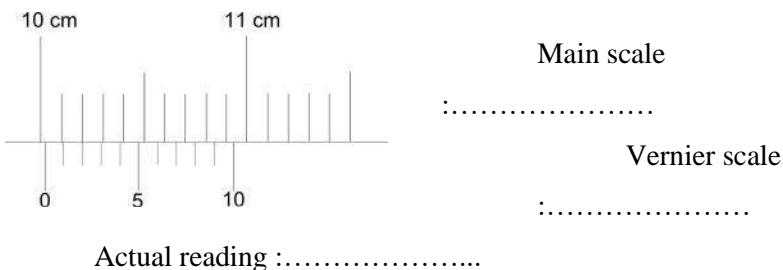


Main scale : .....

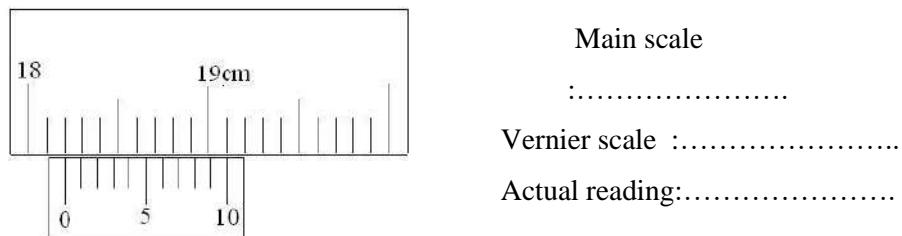
Vernier scale : .....

Actual reading : .....

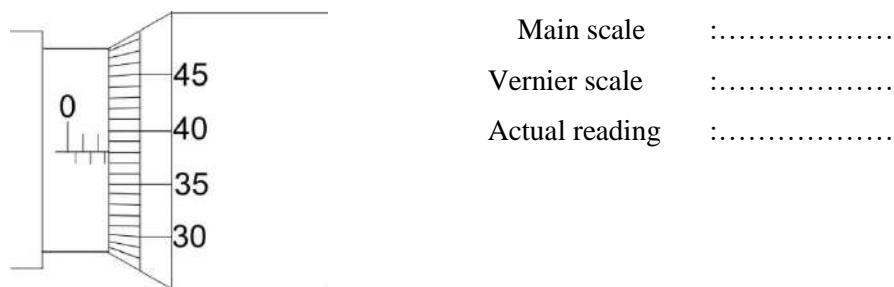
ii.



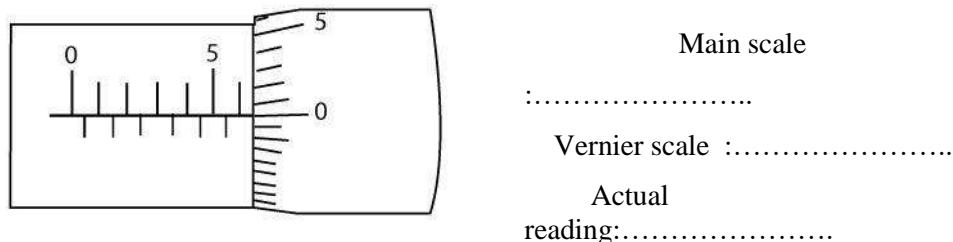
iii.



iv.



v.



9. The repeated readings of the diameter,  $d$  of a ball bearing are 2.50 mm, 2.52 mm, 2.51 mm and 2.50 mm.

- i. Calculate the Average Value and Uncertainty. Write the result as  $(\bar{d} \pm \Delta d)$

ii. What instrument/apparatus is used for this measurement?

.....

iii. From 10.1, calculate the volume,  $V$  of the ball bearing.

iv. Write the result as  $(\bar{V} \pm \Delta \bar{V})$

.....

### Data Analysis

10. Complete **Table 4**.

No	Length of Scientific Calculator (Model Casio fx-570ES PLUS), $L$ (cm)	$ \bar{L} - L_i $ (cm)
1	15.42	
2	15.55	
3	15.30	
4	15.48	
5	15.49	
6	15.45	
7	15.55	
	Average, $\bar{L}=$	$\Delta \bar{L}=$

**Table 4**

11. Express your answer as  $(\bar{L} \pm \Delta \bar{L})$

12. Calculate the percentage of uncertainty,

13. State THREE precautions of this experiment:

- i. .....
- ii. .....
- iii. .....

Name : .....

Practicum: .....

Matric Number: .....

**EXPERIMENT 2 : FREE FALL AND PROJECTILE MOTION****Course Learning Outcome:**

Solve problems related to **Physics of motion**, force and energy, waves, matter and thermodynamics  
**(C4, CLO 2, PLO 4, CTPS 3, MQF LOD 6)**

**Learning Outcomes:**

At the end of this lesson, students will able to describe experiment to determine acceleration due to gravity using free fall and projectile motion.

**Student Learning Time:**

Face-to-face	Non face-to-face
1 hour	1 hour

**Direction:** Read over the lab manual and then answer the following question.

**Introduction**

1. What is meant by free fall motion?

.....  
.....

2. Under free fall motion the acceleration of an object is also known as gravitational acceleration or acceleration due to gravity. What is the symbol and SI unit of this type of acceleration?

.....

3. What is the value of acceleration due to gravity at the surface of Earth?

.....

4. Projectile motion of an object is the motion of an object which is projected or thrown. Under a gravitational field **when the air resistance is not present**, projectile motion can be considered as a free fall motion. State **TWO** differences between free fall motion and projectile motion?

.....  
.....  
.....  
.....  
.....

5. State the law applied in these experiment

.....

**Experiment**

6. How do we release the steel ball to form

- (a) free fall motion

.....  
.....

- (b) Projectile motion

.....  
.....  
.....

7. State the measurement *apparatus* involved. (e.g. type / name of equipment) for both experiment.

.....  
.....  
.....

8. State the related variables that need to be recorded in this experiment?

	Free fall motion	Projectile motion
Manipulated variable (change on purpose)		
Responding variable (what is measured)		

9. Construct the table to record the related values for free fall and projectile motion experiment.

- (a) Free Fall Motion

- (b) Projectile Motion

10. How do you obtained the value of  $t$  for projectile motion from the graph of free fall motion experiment?
- .....  
.....

### **Data Analysis**

11. a) Write the equations related to both experiments in order to determine the acceleration due to gravity,  $g$ .
- b) Sketch a suitable graph for  
 i) Free fall motion                                  ii) Projectile motion
- c) How the acceleration due to gravity,  $g$  can be determine from the graphs.
12. List down the precautions of the experiments.
- a) .....  
 b) .....  
 c) .....

Name : .....

Practicum: .....

Matric Number: .....

**EXPERIMENT 3: ENERGY****Course Learning Outcome:**

Solve problems related to Physics of motion, **force and energy**, waves, matter and thermodynamics  
**(C4, PLO 4, CTPS 3, MQF LOD 6)**

**Learning Outcomes:**

At the end of this lesson, students will able to explain the experiment to determine the acceleration due to gravity,  $g$  from the experiment.

**Student Learning Time:**

Face-to-face	Non face-to-face
1 hour	1 hour

**Direction:** Read over the lab manual and then answer the following question.

**Introduction**

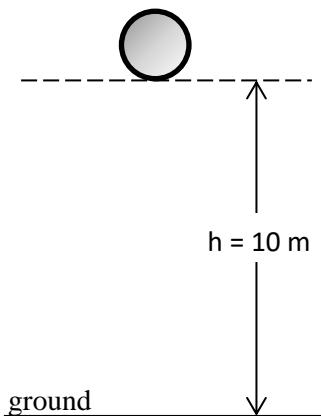
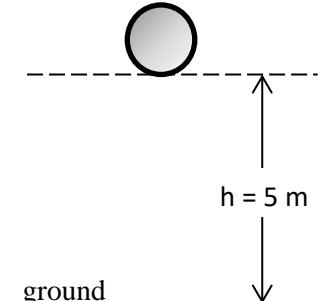
1. State the law of conservation of energy.
- .....

2. State the gravitational potential energy and kinetic energy.
- .....
- .....

3. What is the symbol and SI unit of gravitational potential energy and kinetic energy?

Energy	gravitational potential energy	kinetic energy
Symbol		
Unit		

4. Based on the situations below, answer the questions:

**SITUATION A****SITUATION B**

- a) Using the conservation of energy, determine the velocity of the ball just before it reaches the ground.
  
- b) From the answers calculated in question (a), what can we deduce about the relation between the released height and the velocity of the ball before hitting the ground?  
.....

### **Experiment**

5. What is the energy owned by the ball bearing when it is attached to the free fall adapter?  
.....

6. What is the usage of the photo gate?  
.....

7. State the change in mechanical energy in this experiment.  
.....

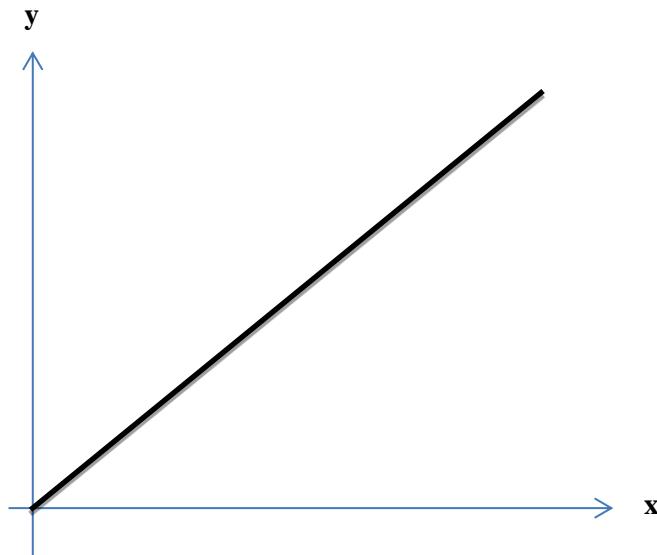
8. State the related variables that need to be recorded in this experiment?  
a) Manipulated variable  
.....

b) Responding variable  
.....

9. How the final velocity of ball bearing is determined?

**Data Analysis**

10. An equation for a straight line graph is  $y = mx + c$ , where  $y$  is the quantity on the vertical axis and  $x$  is the quantity on the horizontal axis as shown in **FIGURE 1**.

**FIGURE 1**

The velocity of ball bearing,  $v$  is related to the height of released ( $h$ ) by the following equation:

$$v^2 = 2gh \quad (1)$$

where  $g$  is the acceleration due to the gravity.

- a) Based on the equation (1) and the graph, determine the variables for  $x$  axis and  $y$  axis

- b) From the graph what does the gradient,  $m$  represents?
- .....

- c) From the gradient of the graph, how can we determine the value of  $g$ .

11. List **THREE** precautions of the experiment:

- i. ....
- ii. ....
- iii. ....

12. State two types of errors during experiment and give an example for each error.
- .....
- .....

13. Based on the situation below identify either random or systematic error.

Situation	Random Error/Systematic Error
Wind keeps blowing in the surrounding using the experiment. This shall affect the velocity measured in this experiment. The best way to solve this is by conducting this experiment in the closed area or vacuum space.	
Some of the numbers on the timer's display was broken and missing. Thus the reading can be taken only to the nearest decimal point.	
Instead of using the hand to release the ball bearing, it is suggested that the ball can be released using the automatic control or trigger.	
Sometimes the time measured is hardly detected by the photo gates. This is due to the position of the gates where the ball bearing failed to hit the motion sensor. Therefore, the free fall adapter and photo gates must be realigned properly.	

Name : .....

Practicum: .....

Matric Number: .....

### **EXPERIMENT 4: ROTATIONAL MOTION OF A RIGID BODY**

**Course Learning Outcome:**

Solve problems related to Physics of motion, **force and energy**, waves, matter and thermodynamics  
**(C4, PLO 4, CTPS 3, MQF LOD 6)**

**Learning Outcomes:**

At the end of this lesson, students will able to explain the experiment to determine the moment of inertia of a fly-wheel from experiment.

**Student Learning Time:**

Face-to-face	Non face-to-face
1 hour	1 hour

**Direction:** Read over the lab manual and then answer the following question.

**Introduction**

1. What is a rigid body?

.....

2. What is meant by moment of inertia?

.....  
.....

3. What is the symbol and SI unit for moment of inertia?

.....

4. Moment of inertia depends on ..... and .....

5. Complete **TABLE 4** with correct analogues between linear motion and rotational motion.

Linear Motion	Rotational Motion
Mass, m	
Acceleration, a	
Net force, F	

6. A motor capable of producing a constant torque of 100 Nm is connected to a flywheel which rotates with an angular acceleration of  $1000 \text{ rad s}^{-2}$ . Calculate moment of inertia of the flywheel.

**Experiment**

7. Sketch a free body diagram for fly-wheel and falling slotted mass.  
 a) Free body diagram of fly-wheel      b) Free body diagram of falling slotted mass
8. By referring to the free body diagram in 7(a) and 7(b), deduce equation by using Newton's 2<sup>nd</sup> Law of motion.
9. For this experiment, identify
- a) the manipulated variable  
 .....  
 b) the responding variable  
 .....
10. Complete the observation table with the suitable equation.

Acceleration	Angular acceleration	Tension in the string

**Data Analysis**

11. Write the equation of the graph of  $\alpha$  against  $T$
12. Base on the linear graph equation  $y = mx + c$ , fill in the suitable quantity by referring the equation in question 11 :
- a)  $y$ -axis : .....  
 b)  $x$ -axis : .....  
 c) gradient,  $m$  : .....  
 d)  $y$ -interception : .....

13. How do we determine the value of inertia of a fly-wheel from this graph?
14. List **THREE** precautions of this experiment
- i. ....
  - ii. ....
  - iii. ....

Name : .....

Practicum: .....

Matric Number: .....

### **EXPERIMENT 5: SIMPLE HARMONIC MOTION (SHM)**

**Course Learning Outcome:**

Solve problems related to Physics of motion, force and energy, **waves**, matter and thermodynamics

**(C4, PLO 4, CTPS 3, MQF LOD 6)**

**Learning Outcomes:**

At the end of this lesson, students will able to:

1. explain the experiment to determine the acceleration due to gravity,  $g$  using a simple pendulum.
2. describe the effect of large amplitude oscillation to the accuracy of  $g$  obtained from the experiment

**Student Learning Time:**

Face-to-face	Non face-to-face
1 hour	1 hour

**Direction:** Read over the lab manual and then answer the following question.

**Introduction**

1. What is a simple pendulum?

.....

2. Motion of an object that returns to its initial position after a fixed time interval is called

.....

3. In SHM, state two quantities that proportional to the object's displacement

i. ....

ii. ....

4. The condition for the simple pendulum to perform SHM are

a) The mass of the spherical bob is .....

b) The ..... of the string is negligible

c) Amplitude of oscillation is .....

5. Does the period of oscillation of simple pendulum depend on mass?

(Yes / No)

**Experiment**

6. How to determine the period of a simple pendulum for a given number, n of oscillation?

.....

7. If we vary the length of a pendulum, the period will change. Construct an appropriate table to record the data of length,  $l$ , time taken,  $t$  and corresponding  $T$  and  $T^2$ .

8. What is the title of the graph that needs to be plotted in this experiment?

.....

9. Which procedure that investigates the effect of large amplitude of oscillation and state the related angle used.

.....

**Data Analysis**

10. How to determine the value of  $g$  from the gradient of the graph.

11. How to calculate the percentage of error between the value  $g_{\text{experiment}}$  and  $g_{\text{standard}}$ ? Take  $g_{\text{standard}} = 9.81 \text{ m s}^{-2}$ .

12. Predict what would happen to the value of  $g$  if **large amplitude** is used.

.....

13. List **THREE** precautions of this experiment

- i. .....
- ii. .....
- iii. .....

Name : .....

Practicum: .....

Matric Number: .....

## **EXPERIMENT 6: STANDING WAVES**

**Course Learning Outcome:**

Solve problems related to Physics of motion, force and energy, **waves**, matter and thermodynamics  
**(C4, PLO 4, CTPS 3, MQF LOD 6)**

**Learning Outcomes:**

At the end of this lesson, students will able to explain the experiment to investigate standing waves formed in stretched string.

**Student Learning Time:**

Face-to-face	Non face-to-face
1 hour	1 hour

**Direction:** Read over the lab manual and then answer the following question.

**Introduction**

1. What is the meaning of standing waves?

.....

2. Sketch standing wave formed in a stretch string and label the node (N) and antinode (A).

.....  
.....  
.....

3. How standing wave is formed?

.....

**Experiment**

4. What is the symbol and SI unit for mass per unit length?

.....

.....

.....

.....

6. Construct the table for the value of  $m$  and  $l$ .
  
  
  
  
  
  
7. Sketch free body diagram to show that  $T = W$ .
  
  
  
  
  
  
8. Suggest a way to determine the actual value for mass per unit length of the string/wire used in this experiment.

.....  
.....

9. Suggest how to identify the position of two consecutive nodes formed in the string / wire.

.....

### **Data analysis**

10. Write the equation that relates period,  $T$  and frequency,  $f$ .

11. Sketch the graph to show the relationship between  $T$  and  $\ell^2$ .

12. Construct the observation table.

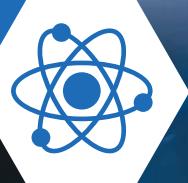
13. How do you determine the mass per unit length from this graph?

.....

14. Throughout the experiment the terminals are connected to AC power supply. In your opinion why does this essential?



MINISTRY OF EDUCATION  
MATRICULATION DIVISION



# PHYSICS

LABORATORY MANUAL

**SP015 &  
SP025**

13th EDITION



**MATRICULATION DIVISION  
MINISTRY OF EDUCATION MALAYSIA**

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**PHYSICS  
LABORATORY MANUAL  
SEMESTER I & II  
SP015 & SP025**

**MINISTRY OF EDUCATION MALAYSIA  
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## **NATIONAL EDUCATION PHILOSOPHY**

Education in Malaysia is an on-going effort towards further developing the potential of individuals in a holistic and integrated manner, so as to produce individuals who are intellectually, spiritually and physically balanced and harmonious based on a firm belief in and devotion to God. Such an effort is designed to produce Malaysian citizens who are knowledgeable and competent, who possess high moral standards and who are responsible and capable of achieving a high level of personal well-being as well as being able to contribute to the betterment of the family, society and the nation at large.

## **NATIONAL SCIENCE EDUCATION PHILOSOPHY**

In consonance with the National Education Philosophy, science education in Malaysia nurtures a science and technology culture by focusing on the development of individuals who are competitive, dynamic, robust and resilient and able to master scientific knowledge and technological competency.

## **FOREWORD**

I am delighted to write the foreword for the Laboratory Manual, which aimed to equip students with knowledge, skills, and the ability to be competitive undergraduates.

This Laboratory Manual is written in such a way to emphasise students' practical skills and their ability to read and understand instructions, making assumptions, apply learnt skills and react effectively in a safe environment. Science process skills such as making accurate observations, taking measurement in correct manner, using appropriate measuring apparatus, inferring, hypothesizing, predicting, interpreting data, and controlling variables are further developed during practical session. The processes are incorporated to help students to enhance their Higher Order Thinking Skills such as analytical, critical and creative thinking skills. These 21<sup>st</sup> century skills are crucial to prepare students to succeed in Industrial Revolution (I.R.) 4.0.

The manipulative skills such as handling the instruments, setting up the apparatus correctly and drawing the diagrams can be advanced through practical session. The laboratory experiments are designed to encourage students to have enquiry mind. It requires students to participate actively in the science process skills before, during and after the experiment by preparing the pre-report, making observations, analysing the results and in the science process skills before, during, after the experiment by preparing the pre-report, making observations, analysing the results and drawing conclusions.

It is my hope and expectation that this manual will provide an effective learning experience and referenced resource for all students to equip themselves with the skills needed to fulfil the prerequisite requirements in the first-year undergraduate studies.



DR HAJAH ROSNARIZAH BINTI ABDUL HALIM  
Director  
Matriculation Division

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### Semester I

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## **1.0 Student Learning Time (SLT)**

Students will be performing the experiment within the time allocated for each practical work.

<b>Face-to-face</b>	<b>Non face-to-face</b>
2 hours	0

## **2.0 Learning Outcomes**

### **2.1 Matriculation Science Programme Educational Objectives**

Upon a year of graduation from the programme, graduates are:

- i. Knowledgeable and technically competent in science disciplines study in-line with higher educational institution requirement.
- ii. Able to apply information and use data to solve problems in science disciplines.
- iii. Able to communicate competently and collaborate effectively in group work to compete in higher education environment.
- iv. Able to use basic information technologies and engage in life-long learning to continue the acquisition of new knowledge and skills.
- v. Able to demonstrate leadership skills and practice good values and ethics in managing organisations.

### **2.2 Matriculation Science Programme Learning Outcomes**

At the end of the programme, students should be able to:

- i. Acquire knowledge of science and mathematics as a fundamental of higher level education.  
(MQF LOC i – Knowledge and understanding)
- ii. Apply logical, analytical and critical thinking in scientific studies and problem solving.  
(MQF LOC ii – Cognitive skills)
- iii. Demonstrate manipulative skills in laboratory works.  
(MQF LOC iii a – Practical skills)

- iv. Collaborate in group work with skills required for higher education.  
(MQF LOC iii b – Interpersonal skills)
- v. Deliver ideas, information, problems and solution in verbal and written communication.  
(MQF LOC iii c – Communication skills)
- vi. Use basic digital technology to seek and analyse data for management of information.  
(MQF LOC iii d – Digital skills)
- vii. Interpret familiar and uncomplicated numerical data to solve problems.  
(MQF LOC iii e – Numeracy skills)
- viii. Demonstrate leadership, autonomy and responsibility in managing organization.  
(MQF LOC iii f – Leadership, autonomy and responsibility)
- ix. Initiate self-improvement through independent learning.  
(MQF LOC iv – Personal and entrepreneurial skills)
- x. Practice good values attitude, ethics and accountability in STEM and professionalism.  
(MQF LOC v – Ethics and professionalism)

### **2.3 Physics 1 Course Learning Outcome**

At the end of the course, student should be able to:

- 1. Describe basic concepts of mechanics, waves, matters, heat and thermodynamics.  
(C2, PLO 1, MQF LOC i)
- 2. Solve problems related to mechanics, waves, matters, heat and thermodynamics.  
(C4, PLO 2, MQF LOC ii)
- 3. Apply the appropriate scientific laboratory skills in physics experiments.  
(P3, PLO 3, MQF LOC iii a)

## **2.4 Physics 2 Course Learning Outcome**

At the end of the course, student should be able to:

1. Explain basic concepts of electricity, magnetism, optics and modern physics.  
(C2, PLO 1, MQF LOC i)
2. Solve problems of electricity, magnetism, optics and modern physics.  
(C4, PLO 2, MQF LOC ii)
4. Apply the appropriate scientific laboratory skills in physics experiments.  
(P3, PLO 3, MQF LOC iii a)
3. Interpret and use familiar and uncomplicated numerical and graphical data to solve problems in basic physics.  
(C4, PLO 7, MQF LOC iii e)

## **2.5 Physics Practical Learning Outcomes**

Physics experiment is to give the students a better understanding of the concepts of physics through experiments. The **aims** of the experiments in this course are to be able to:

1. introduce students to laboratory work and to equip them with the practical skills needed to carry out experiment in the laboratory.
2. determine the best range of readings using appropriate measuring devices.
3. recognise the importance of single and repeated readings in measurement.
4. analyse and interpret experimental data in order to deduce conclusions for the experiments.
5. make conclusions in line with the objective(s) of the experiment which rightfully represents the experimental results.
6. verifying the correct relationships between the physical quantities in the experiments.
7. identify the limitations and accuracy of observations and measurements.

8. familiarise student with standard experimental techniques.
9. choose suitable apparatus and to use it correctly and carefully.
10. gain scientific trainings in observing, measuring, recording and analysing data as well as to determine the uncertainties (errors) of various physical quantities observed in the experiments.
11. handle apparatus, measuring instruments and materials safely and efficiently.
12. present a good scientific report for the experiment.
13. follow instructions and procedures given in the laboratory manual.
14. gain confidence in performing experiments.

### **3.0      Guidance for Students**

#### **3.1      Ethics in the laboratory**

- a. Follow the laboratory rules.
- b. Students must be punctual for the practical session. Students are not allowed to leave the laboratory before the practical session ends without permission.
- c. Co-operation between members of the group must be encouraged so that each member can gain experience in handling the apparatus and take part in the discussions about the results of the experiments.
- d. Record the data based on the observations and not based on any assumptions. If the results obtained are different from the theoretical value, state the possible reasons.
- e. Get help from the lecturer or the laboratory assistant should any problems arise during the practical session.

## **3.2 Preparation for experiment**

### **3.2.1 Planning for the practical**

#### **a. Before entering the laboratory**

- i) Read and understand the objectives and the theory of the experiment.
- ii) Think and plan the working procedures properly for the whole experiment. Make sure you have appropriate table for the data.
- iii) Prepare a jotter book for the data and observations of the experiments during pre-lab discussion.

#### **b. Inside the laboratory**

- i) Check the apparatus provided and note down the important information about the apparatus.
- ii) Arrange the apparatus accordingly.
- iii) Conduct the experiment carefully.
- iv) Record all measurements and observations made during the experiment.

### **3.3 Report writing**

The report must be written properly and clearly in English and explain what has been carried out in the experiment. Each report must contain **name, matriculation number, number of experiment, title, date and practicum group.**

**The report must also contain the followings:**

- i) Objective
  - state clearly
- ii) Theory
  - write concisely in your own words
  - draw and label diagram if necessary
- iii) Apparatus
  - name, range, and sensitivity, e.g  
    Voltmeter: 0.0 – 10.0 V
  - Sensitivity:  $\pm 0.1$  V
- iv) Procedure
  - write in passive sentences about all the steps taken during the experiment
- v) Observation
  - data tabulation with units and uncertainties
  - data processing (plotting graph, calculation to obtain the results of the experiments and its uncertainties).
  - Calculation of uncertainties using LSM method can refer attachment A
- vi) Discussion
  - give comments about the experimental results by comparing it with the standard value
  - state the source of mistake(s) or error(s) if any as well as any precaution(s) taken to overcome them
  - answer all the questions given
- vii) Conclusion
  - state briefly the results with reference to the objectives of the experiment

**Reminder:** NO PLAGIARISM IS ALLOWED.

## 4.0 Significant Figures

The significant figures of a number are those digits carry meaning contributing to its precision. Therefore, the most basic way to indicate the precision of a quantity is to write it with the correct number of significant figures.

The significant figures are all the digits that are known accurately plus the one estimated digit. For example, we say the distance between two towns is 200 km, that does not mean we know the distance to be exactly 200 km. Rather, the distance is 200 km *to the nearest kilometres*. If instead we say that the distance is 200.0 km that would indicate that we know the distance to the nearest *tenth* of a kilometre.

More significant figures mean greater precision.

### Rules for identifying significant figures:

1. Nonzero digits are always significant.
2. Final or ending zeros written to the right of the decimal point are significant.
3. Zeros written on either side of the decimal point for the purpose of spacing the decimal point are not significant.
4. Zeros written between significant figures are significant.

**Example:**

Value	Number of significant figures	Remarks
0.5	1	Implies value between 0.45 and 0.55
0.500	3	Implies value between 0.4995 and 0.5005
0.050	2	Implies value between 0.0495 and 0.0505
5.0	2	Implies value between 4.95 and 5.05
1.52	3	Implies value between 1.515 and 1.525
$1.52 \times 10^4$	3	Implies value between 15150 and 15250
150	2 or 3 (ambiguous)	The zero may or may not be significant. If the zero is significant, the value implied is between 149.5 and 150.5. If the zero is not significant, the value implied is between 145 and 155.

**5.0 Uncertainty in Measurements**

No matter how careful or how accurate are the instruments, the results of any measurements made at best are only close enough to their true values (actual values). Obviously, this is because the instruments have certain smallest scale by which measurement can be made. Chances are, the true values lie within the smallest scale. Hence, we have uncertainties in our measurements.

The uncertainty of a measurement depends on its type and how it is done. For a quantity  $x$  with uncertainty  $\Delta x$ , the measurement should be recorded as  $x \pm \Delta x$  with appropriate unit.

The relative uncertainty of the measurement is defined as  $\frac{\Delta x}{x}$ .

and therefore its percentage of uncertainty, is given by  $\frac{\Delta x}{x} \times 100\%$ .

## 5.1 Single Reading

- (a) If the reading is taken from a single point or at the end of the scale we use:

$$\Delta x = \frac{1}{2} \times (\text{smallest division of the scale})$$

- (b) If the readings are taken from two points on the scale:

$$\Delta x = 2 \times \left[ \frac{1}{2} \times (\text{smallest division from the scale}) \right]$$

- (c) If the apparatus has a vernier scale:

$$\Delta x = 1 \times (\text{smallest unit of the vernier scale})$$

## 5.2 Repeated Readings

For a set of  $n$  repeated measurements, the best value is the average value, that is

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

where:  $n$  is the number of measurements taken  
 $x_i$  is the  $i^{\text{th}}$  measurement value

The uncertainty is given by

$$\Delta x = \frac{\sum_{i=1}^n |\bar{x} - x_i|}{n}$$

The result should be written in the form of

$$x = \bar{x} \pm \Delta x$$

### 5.3 Combination of uncertainties

We adopt maximum uncertainty.

- (a) Addition or subtraction

$$x = a + b - c \Rightarrow \Delta x = \Delta a + \Delta b + \Delta c$$

- (b) Multiplication with constant  $k$

$$x = ka \Rightarrow \Delta x = k\Delta a$$

- (c) Multiplication or division

$$x = \frac{ab}{c} \Rightarrow \frac{\Delta x}{x} = \left( \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c} \right)$$

- (d) Powers

$$x = a^n \Rightarrow \frac{\Delta x}{x} = n \left( \frac{\Delta a}{a} \right)$$

### 5.4 Uncertainty gradient and y-intercept using Least Square Method (LSM)

#### 5.4.1 Formula uncertainty for gradient and y-intercept

Straight line graphs are very useful in data analysis for many physics experiments.

From straight line equation, that is,  $y = mx + c$  we can easily determine the gradient  $m$  of the graph and its intercept  $c$  on the vertical axis.

When plotting a straight line graph, the line does not necessarily pass through all the points. Therefore, it is important to determine the uncertainties  $\Delta m$  and  $\Delta c$  for the gradient of the graph and the  $y$ -intercept respectively.

Consider the data obtained is as follows:

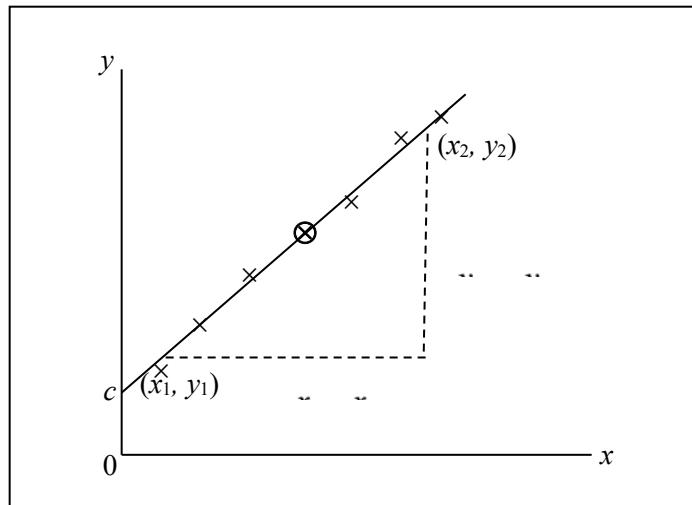
$x$	$x_1$	$x_2$	$x_3 \dots \dots \dots x_n$
$y$	$y_1$	$y_2$	$y_3 \dots \dots \dots y_n$

- (a) Find the centroid  $(\bar{x}, \bar{y})$ , where

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad \text{and} \quad \bar{y} = \frac{\sum_{i=1}^n y_i}{n}$$

- (b) Draw the best straight line passing through the centroid and balance.
- (c) Determine the gradient of the line by drawing a triangle using dotted lines. The gradient is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



**Figure A**

- (d) The uncertainty of the slope,  $\Delta m$  can be calculated using the following equation

$$\Delta m = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{(n-2) \sum_{i=1}^n (x_i - \bar{x})^2}}$$

where  $n$  is the number of readings and  $\bar{x}$  is the average value of  $x$  given by

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

and the estimated value of  $y$ ,  $\hat{y}_i$  is given by,

$$\hat{y}_i = \hat{m}x_i + \hat{c}$$

- (e) The uncertainty of the y-intercept,  $\Delta c$  can be calculated using the following equation

$$\Delta c = \sqrt{\frac{1}{n-2} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \left( \frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \right)}$$

#### 5.4.2 Procedure to draw a straight line graph and to determine its gradient with its uncertainty

- (a) Choose appropriate scales to use at least 80% of the sectional paper. Draw, label, mark the two axes, and give the units. Avoid using scales of 3, 7, 9, and the likes or any multiple of them. Doing so will cause difficulty in plotting the points later on.
- (b) Plot all points clearly with  $\times$ . At this stage you can see the pattern of the distribution of the graph points. If there is a point which is clearly too far-off from the rest, it is necessary to repeat the measurement or omit it.
- (c) Calculate the centroid and plot it on the graph.

**Example:**

Suppose a set of data is obtained as below. Graph of  $T^2$  against  $\ell$  is to be plotted.

$\ell$ ( $\pm 0.1$ cm)	10.0	20.0	30.0	40.0	50.0	60.0
$T^2$ ( $\pm 0.01$ s $^2$ )	0.33	0.80	1.31	1.61	2.01	2.26

From the data:

$$\bar{\ell} = \frac{10.0 + 20.0 + 30.0 + 40.0 + 50.0 + 60.0}{6} = 35.0 \text{ cm}$$

$$\bar{T^2} = \frac{0.33 + 0.80 + 1.31 + 1.61 + 2.01 + 2.26}{6} = 1.39 \text{ s}^2$$

Therefore, the centroid is (35.0 cm, 1.39 s $^2$ ).

- (d) Draw a best straight line through the centroid and balance. Points above the line are roughly in equal number and positions to those below the line.
- (e) Determine the gradient of the line. Draw a fairly large right-angle triangle with part of the line as the hypotenuse.

From the graph in **Figure B**, the gradient of the line is as follows:

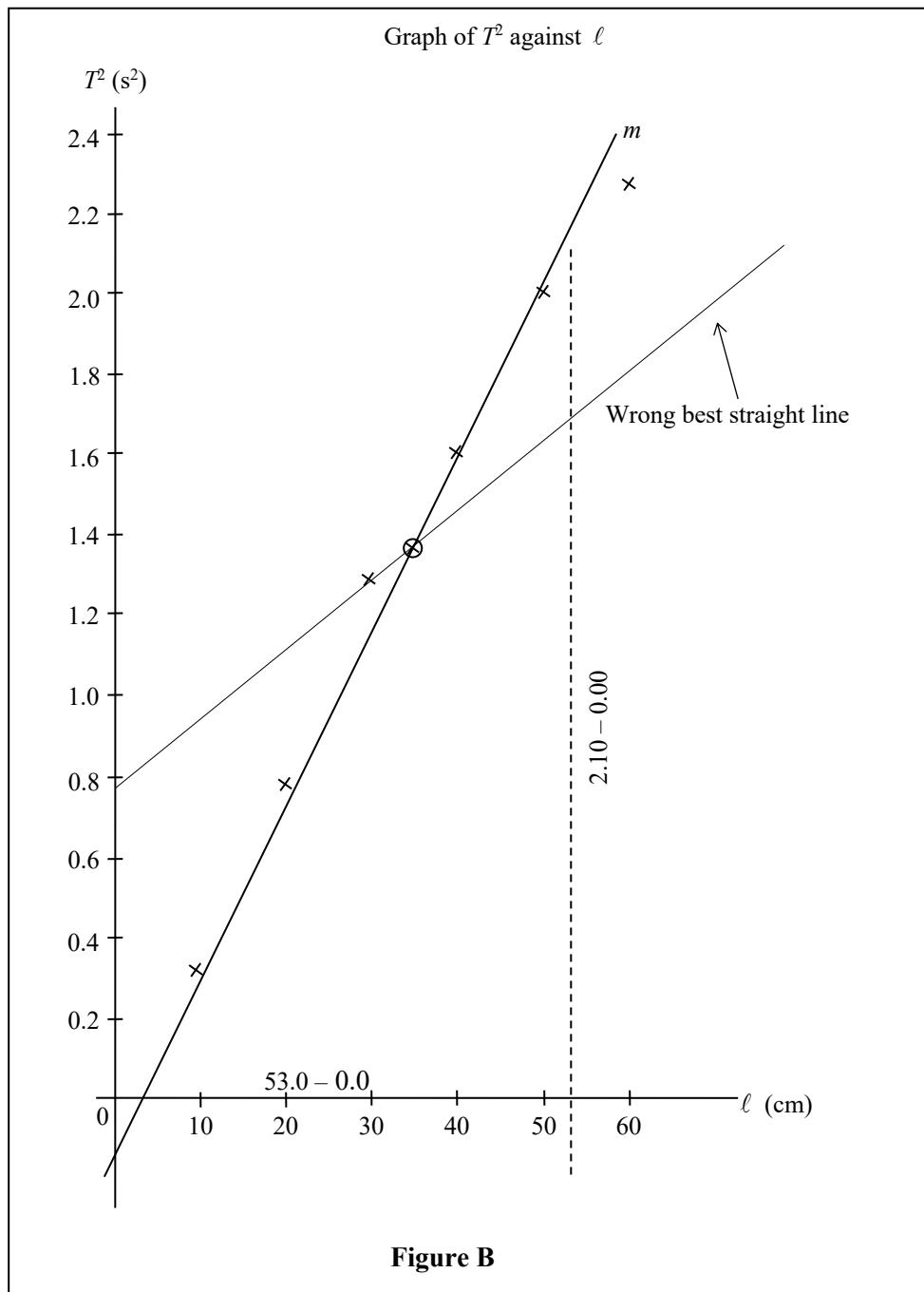
For the best line:

$$m = \frac{(2.10 - 0.00) \text{ s}^2}{(53.0 - 0.0) \text{ cm}} \\ = 0.040 \text{ s}^2 \text{ cm}^{-1}$$

The gradient of the graph and its uncertainty should be written as follows:

$$m = (0.040 \pm \underline{\hspace{1cm}}) \text{ s}^2 \text{ cm}^{-1}$$

Take extra precaution so that the number of significant figures for the gradient and its uncertainty are in consistency.



### (f) Calculation of uncertainties

Rewrite the data in the form of

$\ell$	$\ell - \bar{\ell}$	$(\ell - \bar{\ell})^2$	$T^2$	$\widehat{T^2}$	$T^2 - \widehat{T^2}$	$(T^2 - \widehat{T^2})^2$
10.0	-25.0	625.0	0.33	0.4	-0.070	0.0049
20.0	-15.0	225.0	0.80	0.8	0.000	0.0000
30.0	-5.0	25.0	1.31	1.2	0.110	0.0121
40.0	5.0	25.0	1.61	1.6	0.010	0.0001
50.0	15.0	225.0	2.01	2.0	0.010	0.0001
60.0	25.0	625.0	2.26	2.4	-0.140	0.0196
<b><math>\Sigma=210.0</math></b>		<b><math>\Sigma=1750.0</math></b>				<b><math>\Sigma=0.0368</math></b>

Where,  $\bar{\ell}$  is the average of  $\ell$ ,

$$\bar{\ell} = \frac{210}{6} = 35.0 \text{ cm}$$

Where,  $\widehat{T^2}$  is the expected value of  $T^2$

$$\widehat{T^2} = 0.04\ell$$

Calculate the uncertainty of slope,  $\Delta m$

$$\Delta m = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{(n-2) \sum_{i=1}^n (x_i - \bar{x})^2}}$$

$$= \sqrt{\frac{0.0368}{(6-2)(1750)}}$$

$$= \pm 0.002$$

Then, calculate the uncertainty of y-intercept,  $\Delta c$

$$\Delta c = \sqrt{\left(\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}\right) \left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^n (x_i - \bar{x})^2}\right)}$$

$$\Delta c = \sqrt{\left(\frac{0.0368}{6-2}\right) \left(\frac{1}{6} + \frac{35^2}{1750}\right)}$$

$$\Delta c = \pm 0.09$$

The data given in section 5.4.2(e) was obtained from an experiment to verify the relation between  $T^2$  and  $\ell$ . Theoretically, the quantities obey the following relation,

$$T^2 = \left( \frac{k}{p} \right) \ell$$

where  $k$  is a natural number equals 39.48 and  $p$  is a physical constant. Calculate  $p$  and its uncertainty.

**Solution:**

From the equation, we know that

$$\begin{aligned} \frac{k}{p} &= \text{gradient } m \\ p &= \frac{k}{m} \\ &= \frac{39.48}{0.040} \\ &= 987 \text{ cm s}^{-2} \end{aligned}$$

Since  $k$  is a natural number which has no uncertainties, that is  $\Delta k = 0$ .

$$\begin{aligned} \Delta p &= \left( \frac{\Delta k}{k} + \frac{\Delta m}{m} \right) p \\ &= \left( 0 + \frac{0.002}{0.040} \right) 987 \\ &= 49.35 \end{aligned}$$

so we write,

$$p = (987 \pm 49.35) \text{ cm s}^{-2} \quad \text{or} \quad p = (1000 \pm 50) \text{ cm s}^{-2}$$

### **5.5 Percentage of difference:**

When comparing an experimental result to a value determined by theory or to an accepted known value, the difference between the experimental value and the theoretical value can be determined by:

$$\text{Percentage of difference} = \left| \frac{X_{\text{Theory}} - X_{\text{Experiment}}}{X_{\text{Theory}}} \right| \times 100\%$$

**PHYSICS 1**

**SP015**

---

## EXPERIMENT 1: MEASUREMENT AND UNCERTAINTY

---

**Objective:**

To measure and determine the uncertainty of physical quantities.

**Theory:**

Measuring some physical quantities is part and parcel of any physics experiment. It is important to realise that not all measured values are the same as the actual values. This could be due to errors that we made during the measurement, or perhaps the apparatus that we used may not be accurate or sensitive enough. Therefore, as a rule, the uncertainty of a measurement must be taken, and it has to be recorded together with the measured value.

The uncertainty of a measurement depends on the type of measurement and how it is done. For a quantity  $x$  with the uncertainty  $\Delta x$ , its measurement is recorded as below:

$$x \pm \Delta x$$

The relative uncertainty of the measurement is defined as:

$$\frac{\Delta x}{x}$$

and therefore, its percentage of uncertainty is  $\frac{\Delta x}{x} \times 100\%$ .

### 1.1 Single Reading

- (a) If the reading is taken from a single point or at the end of the scale,

$$\Delta x = \frac{1}{2} \times (\text{smallest division from the scale})$$

- (b) If the readings are taken from two points on the scale,

$$\Delta x = 2 \times \left[ \frac{1}{2} \times (\text{smallest division from the scale}) \right]$$

- (c) If the apparatus uses a vernier scale,

$$\Delta x = 1 \times (\text{smallest unit from the vernier scale})$$

## 1.2 Repeated Readings

For a set of  $n$  repeated measurements of  $x$ , the best value is the average value given by

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad 1.1$$

where  $n$  = the number of measurements taken

$x_i$  = the  $i^{th}$  measurement

The uncertainty is given by

$$\Delta x = \frac{\sum_{i=1}^n |\bar{x} - x_i|}{n} \quad 1.2$$

The result should be written as

$$x = \bar{x} \pm \Delta x \quad 1.3$$

### Apparatus:

- A metre rule
- A vernier callipers
- A micrometer screw gauge
- A travelling microscope
- A coin
- A glass rod
- A ball bearing
- A capillary tube (1 cm long)

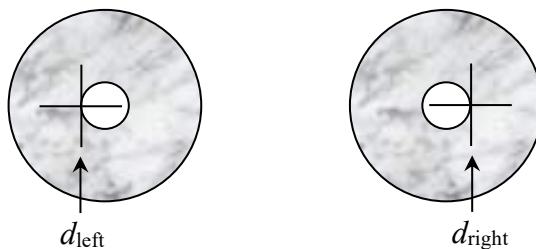
**Procedure:**

1. Choose the appropriate instrument for measurement of
  - (i) length of a laboratory manual.
  - (ii) diameter of a coin.
  - (iii) diameter of a glass rod.
  - (iv) diameter of a ball bearing.
2. For task (i) to (iv), perform the measurement and record the data in a suitable table for at least 5 readings. (Refer to **Table 1.1** as an example)

**Table 1.1**

No.	Length of the laboratory manual, $l$ ( $\pm \dots\dots\dots$ )	$ \bar{l} - l_i $ ( $\dots\dots\dots$ )
1		
2		
3		
4		
5		
Average	$\bar{l} = \frac{\sum_{i=1}^n l_i}{n} = \dots\dots\dots$	$\Delta l = \frac{\sum_{i=1}^n  \bar{l} - l_i }{n} = \dots\dots\dots$

3. Determine the percentage of uncertainty for each set of readings.
4. Use travelling microscope to measure the internal diameter of the capillary tube. Adjust the microscope so that the cross-hairs coincide with the left and right edge of the internal diameter of the tube as shown in **Figure 1.1**. Record  $d_{\text{left}}$  and  $d_{\text{right}}$ .



The internal diameter,  

$$d = |d_{\text{right}} - d_{\text{left}}|$$

**Figure 1.1**

Determine the uncertainty,  $\Delta d$  and the percentage of uncertainty of the internal diameter of the capillary tube.

**EXPERIMENT 2: FREE FALL AND PROJECTILE MOTIONS****Objective:**

To determine the acceleration due to gravity,  $g$  using free fall and projectile motions.

**Theory:****A. Free fall motion**

When a body of mass  $m$  falls freely from a certain height  $h$  above the ground, it experiences a linear motion. The body will obey the equation of motion,

$$s = ut + \frac{1}{2}at^2 \quad 2.1$$

By substituting the following into equation 2.1,

$s = -h$  (downward displacement of the body from the falling point to the ground)

$u = 0$  (the initial velocity of the body)

$a = -g$  (the downward acceleration due to gravity)

we obtain,

$$h = \frac{1}{2}gt^2 \quad 2.2$$

Evidently, a graph of  $h$  against  $t^2$  is a straight line of gradient equals  $\frac{1}{2}g$ .

## B. Projectile motion

According to **Figure 2.2**, from the law of conservation of energy, the potential energy of a steel ball of mass  $m$  equals its kinetic energy,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 \quad 2.3$$

where     $h$  is the height of the release point above the track  
 $v$  is the velocity of the steel ball at the end of the track

**Note:** The rotational kinetic energy for solid sphere is  $\frac{1}{5}mv^2$ .

The range,  $R$  of the steel ball is given by

$$R = vt \quad 2.4$$

Solving equations 2.3 and 2.4, we obtain

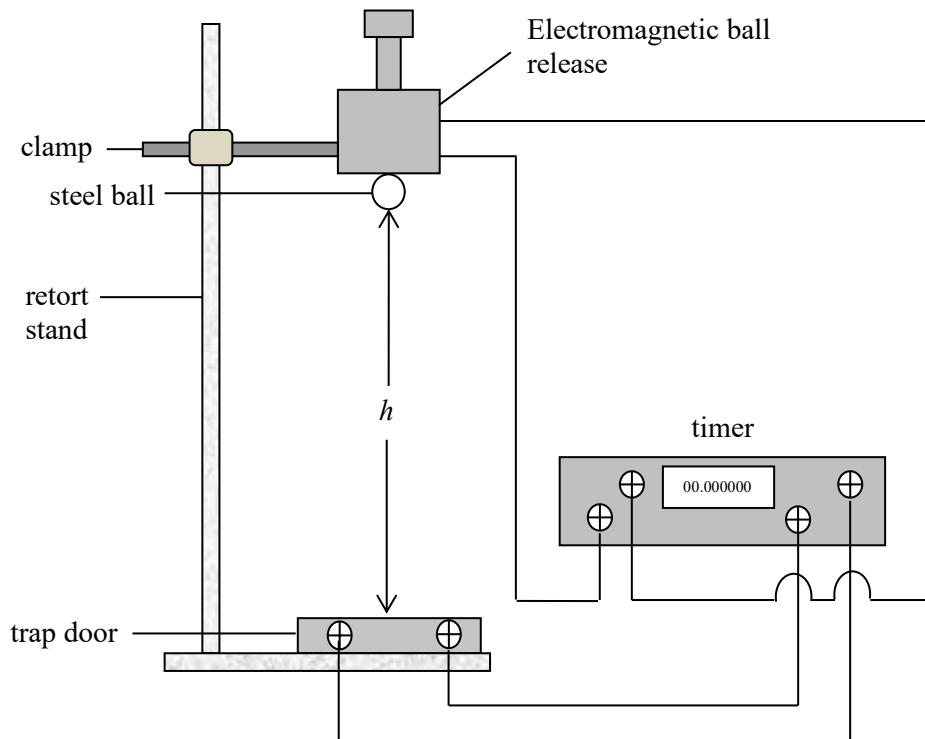
$$h = \frac{7}{10} \frac{R^2}{gt^2} \quad 2.5$$

where  $t$  is the time taken for the steel ball from the end of the curved track to reach the ground.

Evidently, a graph of  $h$  against  $R^2$  is a straight line of gradient equals  $\frac{7}{10gt^2}$ .

**Apparatus:**

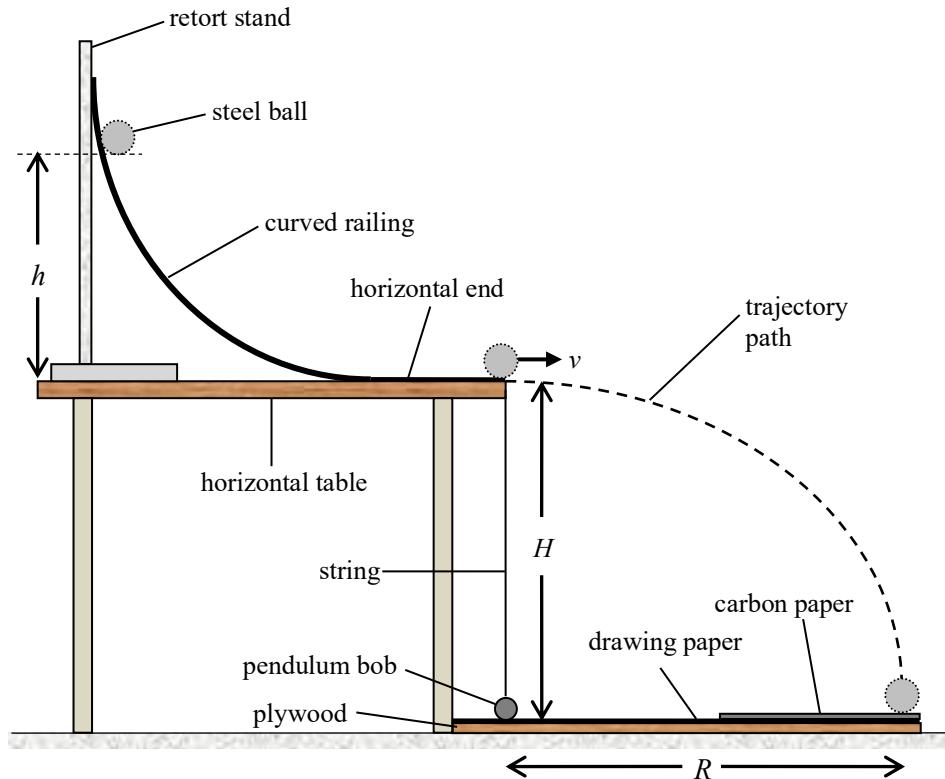
- A retort stand with a clamp
- A timer
- A metre rule
- A free fall adaptor (electromagnetic ball release and trap door)
- A horizontal table
- A steel ball
- A curved railing (**Note:** *The lower end of the track must be horizontal*)
- A piece of carbon paper
- A piece of drawing paper
- Cellophane tape
- Plasticine
- A pair of scissors or a cutter
- A piece of string
- A pendulum bob
- A plywood

**Procedure:****A. Free fall motion****Figure 2.1**

**Note:** Refer to Figure 2.3 for free fall apparatus with separate power supply for the electromagnet.

1. Set up the apparatus as in **Figure 2.1**.
2. Switch on the circuit and attach the steel ball onto the upper contact.
3. Adjust the height,  $h$  of the electromagnet above the point of impact.
4. Switch off the circuit and let the ball fall. Record the height,  $h$  and time,  $t$ .
5. Repeat step (3) and (4) for at least six different height,  $h$ .
6. Tabulate the data.
7. Plot a graph of  $h$  against  $t^2$ .
8. Determine the acceleration due to gravity,  $g$  from the gradient of the graph.
  
9. Determine the uncertainty of acceleration due to gravity,  $\Delta g$ .

## B. Projectile Motion



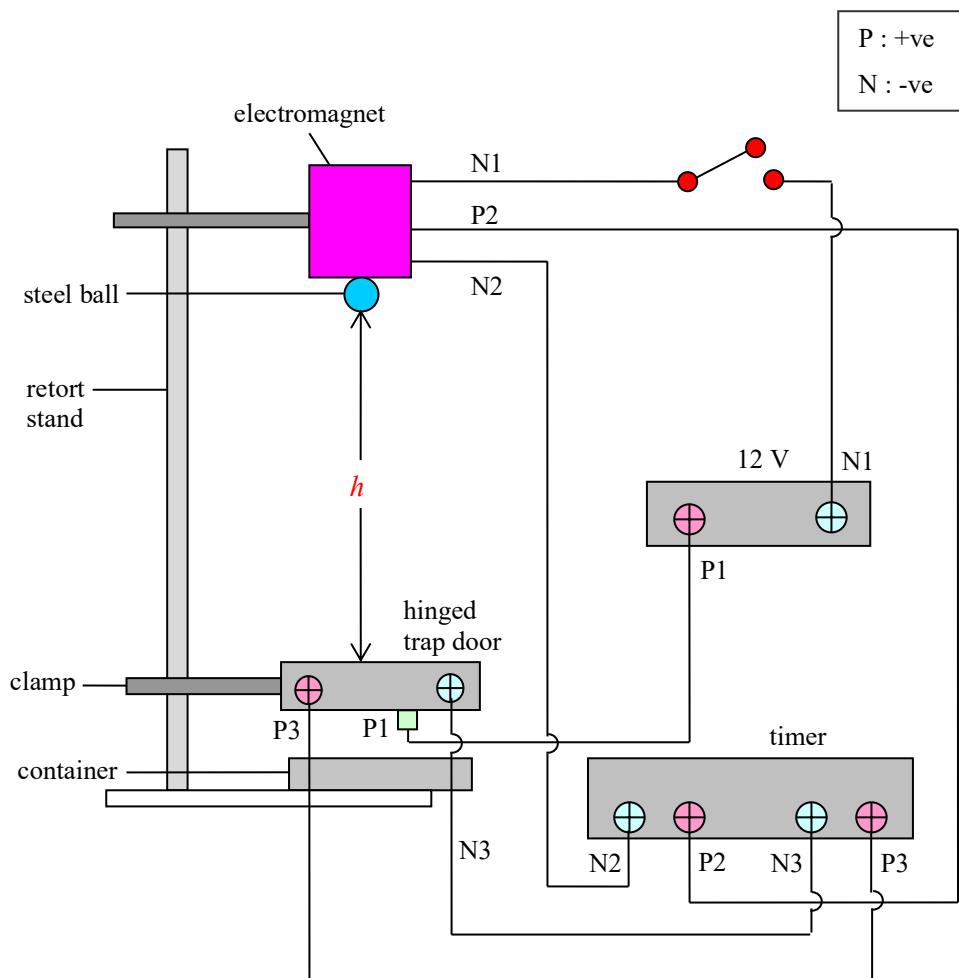
**Figure 2.2**

1. Set up the apparatus as in **Figure 2.2**.
2. Release the steel ball on the curvature railing at least six different heights,  $h$  and record the range,  $R$ .
3. Tabulate the data.
4. Plot a graph of  $h$  against  $R^2$ .
5. Measure the height of the table,  $H$  (the edge of the railing to the landing surface).
6. By referring to the graph of  $h$  against  $t^2$  from experiment A, obtain the value of  $t^2$  for  $H$  using extrapolation.
7. Determine the acceleration due to gravity,  $g$  from the gradient of the graph.

8. Determine the uncertainty of acceleration due to gravity,  $\Delta g$ .
9. Compare the acceleration due to gravity,  $g$  obtained from both experiments with the standard value. Write the comments.

**Alternative set-up:**

Set-up for free fall apparatus with separate power supply to electromagnet.



**Figure 2.3**

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## EXPERIMENT 3: ENERGY

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**Objective:**

To verify the law of conservation of energy by using free fall motion.

**Theory:**

Consider a steel ball of mass,  $m$  initially at rest at height,  $h$  vertically above a velocity detector. By taking the position of the velocity detector as the reference point, the potential energy is  $mgh$  and the kinetic energy of the ball is zero. Thus, the total initial energy,  $E_1$  of the steel ball is given by

$$E_1 = mgh \quad 3.1$$

When the steel ball is released, it falls freely with acceleration due to gravity,  $g$ . At the instance it reaches the velocity detector, the gravitational potential energy is zero and its kinetic energy is  $\frac{1}{2}mv^2$ . Hence, the total final energy,  $E_2$  of the steel ball is given by

$$E_2 = \frac{1}{2}mv^2 \quad 3.2$$

According to the law of conservation of energy, in the absence of external force the total energy of a system remains constant. In this case, the law is verified if we demonstrate experimentally that  $E_1$  equals  $E_2$ , that is,

$$\frac{1}{2}mv^2 = mgh$$

And we obtain

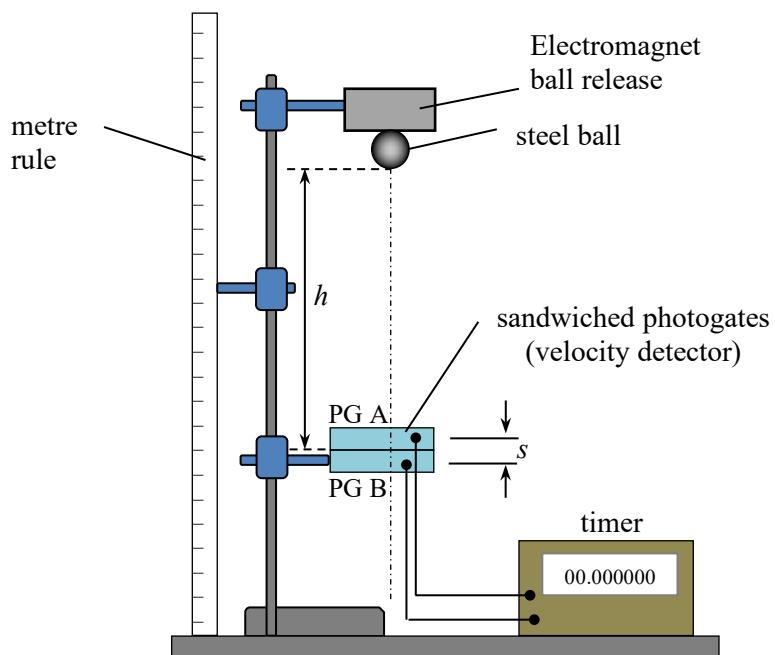
$$v^2 = 2gh \quad 3.3$$

Consequently, if a graph of  $v^2$  against  $h$  is plotted, we should obtain a straight line passing through the origin with gradient equals  $2g$ .

**Apparatus:**

- A steel ball
- A metre rule
- A free fall adaptor (electromagnetic ball release)
- Two photogates PG A and PG B (Velocity detector)
- A timer
- A retort stand

**Procedure:**

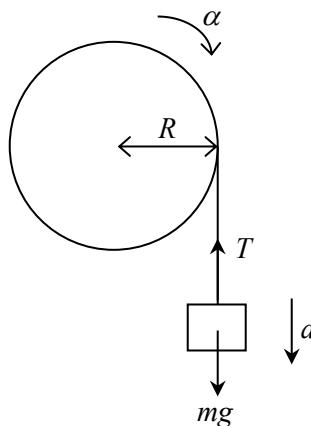


**Figure 3.1**

1. Construct a velocity detector by sandwiching photogates (PG) A and B using binding tape. Measure the distance,  $s$  between the photogates.
2. Set up the apparatus as shown in **Figure 3.1**.
3. Switch on the timer and reset to zero. Set the falling distance,  $h$  at 15 cm. Release the steel ball and record the time,  $t$ . Repeat the process to obtain the average time.
4. Repeat step (3) for falling distance,  $h = 20, 25, 30, 35, 40$ , and  $45$  cm.
5. For each falling distance,  $h$ , calculate the velocity,  $v$  using  $v = \frac{s}{t}$
6. Tabulate the data.
7. Plot a graph of  $v^2$  against  $h$ .
8. Determine the acceleration due to gravity,  $g$  from the gradient of the graph.
9. Determine the uncertainty for acceleration due to gravity,  $\Delta g$  obtained in (8).
10. Verify the law of conservation of energy by comparing the acceleration due gravity,  $g$  obtained from the experiment with standard value. Write the comments.

**EXPERIMENT 4: ROTATIONAL MOTION OF A RIGID BODY****Objective:**

To determine the moment of inertia of a fly-wheel,  $I$ .

**Theory:**

**Figure 4.1**

By referring to **Figure 4.1**, apply Newton's second law for linear motion,

$$mg - T = ma$$

$$T = m(g - a) \quad 4.1$$

and applying Newton's second law for rotational motion,

$$TR - \tau = I\alpha \quad 4.2$$

where  $a$  is the downward linear acceleration

$\tau$  is the frictional torque (unknown)

$\alpha$  is the angular acceleration

$T$  is the tension in the string

$R$  is the radius of the axle

$I$  is the moment of inertia of the fly-wheel

Therefore,

$$\alpha = \left( \frac{R}{I} \right) T - \left( \frac{\tau}{I} \right) \quad 4.3$$

The graph  $\alpha$  against  $T$  is a straight line graph with gradient  $\frac{R}{I}$ .

Moment of inertia of the fly-wheel,

$$I = \frac{R}{\text{gradient}} \quad 4.4$$

From kinematics,  $s = ut + \frac{1}{2}(-a)t^2$  (negative sign means the acceleration is downward)

By substituting,  $s = -h$  and  $u = 0$  into the equation above, we obtain

$$h = \frac{1}{2}at^2$$

Hence the linear acceleration,

$$a = \frac{2h}{t^2} \quad 4.5$$

where  $h$  is the height of mass

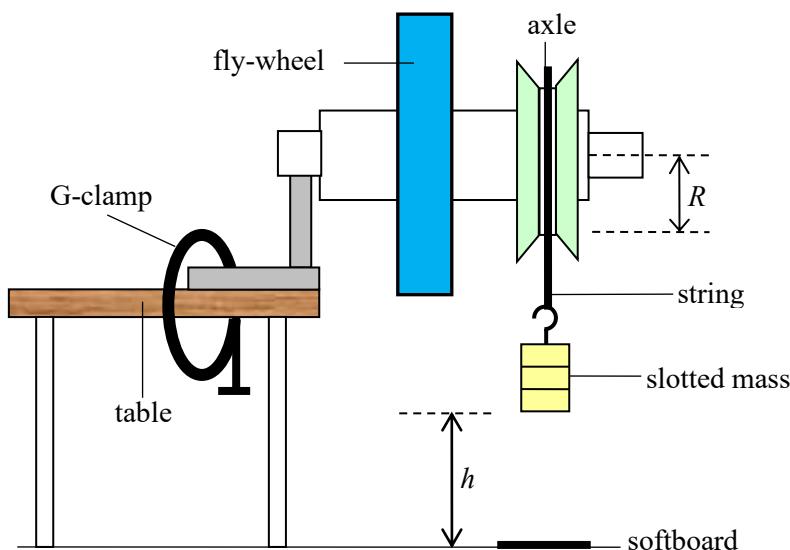
$t$  is the time taken for the mass to fall to the floor

Angular acceleration,

$$\alpha = \frac{a}{R} \quad 4.6$$

**Apparatus:**

- A fly-wheel
- A stopwatch
- A set of slotted mass with hook (**Note:** Use suitable masses for the fly-wheel to rotate at a suitable rate)
- A metre rule
- A G-clamp
- A piece of inelastic string to hang the mass to the fly-wheel
- A piece of softboard or plywood
- A vernier callipers

**Procedure:****Figure 4.2**

1. Set up the apparatus as in **Figure 4.2**.
2. Measure the diameter,  $d$  of the axle and calculate its radius,  $R$ .
3. Record the falling slotted mass,  $m$ .
4. Choose a fixed point at a height,  $h$  above the floor. Record height,  $h$ .
5. Release the slotted mass,  $m$  from the fixed height,  $h$  after the string has been wound around the axle.

6. Record the time,  $t$  for the slotted mass,  $m$  to reach the floor.
7. Calculate the linear acceleration,  $a$ , tension,  $T$  and angular acceleration,  $\alpha$  using equations 4.5, 4.1 and 4.6 respectively.
8. Repeat steps (3) to (7) for at least six different slotted mass,  $m$ .
9. Tabulate the data.
10. Plot a graph of  $\alpha$  against  $T$ .
11. Determine the moment of inertia of the fly-wheel,  $I$  from the gradient of the graph.
12. Determine the uncertainty of moment of inertia of the fly-wheel,  $\Delta I$ .
13. Compare the moment of inertia of the fly-wheel,  $I$  to the standard value.  
Write the comments.

**EXPERIMENT 5: SIMPLE HARMONIC MOTION (SHM)****Objectives:**

- (i) To determine the acceleration,  $g$  due to gravity using simple pendulum.
- (ii) To investigate the effect of large amplitude oscillation to the accuracy of acceleration due to gravity,  $g$  obtained from the experiment.

**Theory:**

An oscillation of a simple pendulum is an example of a simple harmonic motion (SHM) if

- (i) the mass of the spherical bob is a point mass
- (ii) the mass of the string is negligible
- (iii) amplitude of the oscillation is small ( $< 10^\circ$ )

According to the theory of SHM, the period of oscillation of a simple pendulum,  $T$  is given by

$$T = 2\pi \sqrt{\frac{l}{g}} \quad 5.1$$

where  $l$  is the length of pendulum  
 $g$  is the acceleration due to gravity

Rearrange equation 5.1, we obtain

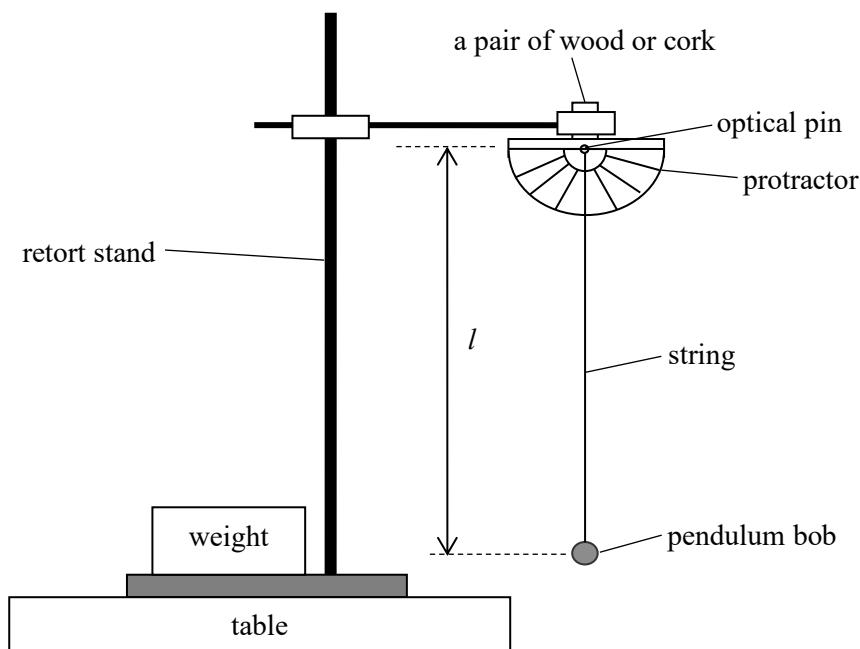
$$T^2 = \frac{4\pi^2}{g} \quad 5.2$$

Evidently, a graph of  $T^2$  against  $l$  is a straight line of gradient equals  $\frac{4\pi^2}{g}$ .

Hence, from the gradient of the graph, the acceleration due to gravity,  $g$  can be calculated.

**Apparatus:**

A piece of string ( $\approx 105$  cm)  
 A small pendulum bob  
 A pair of small flat pieces of wood or cork  
 A retort stand with a clamp  
 A stopwatch  
 A metre rule  
 A protractor with a hole at the centre of the semicircle  
 An optical pin  
 A pair of scissors or a cutter  
 A stabilizing weight or a G-clamp

**Procedure:****Figure 5.1**

1. Set up a simple pendulum as in **Figure 5.1**.
2. Measure the length,  $l$  of the pendulum at 40 cm.
3. Release the pendulum at less than  $10^\circ$  from the vertical in one plane and measure the time,  $t$  for 10 complete oscillations.

**Note:** Start the stopwatch after several complete oscillations.

4. Calculate the average time,  $t$ .
5. Calculate the period of oscillation,  $T$  of the pendulum.
6. Repeat step (3) to (5) for length,  $l$  of 50 cm, 60 cm, 70 cm, 80 cm and 90 cm.
7. Tabulate the data.
8. Plot a graph of  $T^2$  against  $l$ .
9. Determine the acceleration due to gravity,  $g$  from the gradient of the graph.
10. Determine the uncertainty of acceleration due to gravity,  $\Delta g$ .
11. Fix the length,  $l$  of pendulum at 100 cm.
12. Release the pendulum through a large arc of about  $70^\circ$  from the vertical and measure the time,  $t$  for 5 complete oscillations. Repeat the step for three times.
13. Calculate the average time,  $t$  and the period of oscillation,  $T$  of the pendulum.
14. Calculate the acceleration due to gravity,  $g$  using equation 5.1 by using the length,  $l$  and period,  $T$  from step (11) to (13).
15. Compare the acceleration due to gravity,  $g$  obtained from step (9) and (14) with the standard value. Write the comments.

**EXPERIMENT 6: STANDING WAVES**

---

**Objectives:**

- (i) To investigate standing waves formed in a stretched string.
- (ii) To determine the mass per unit length,  $\mu$  of the string.

**Theory:**

When a stretched string is vibrated at a frequency,  $f$  the standing waves formed have both ends as nodes. The frequency in the string obeys the following relation

$$f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$$

Hence, the tension of the string,

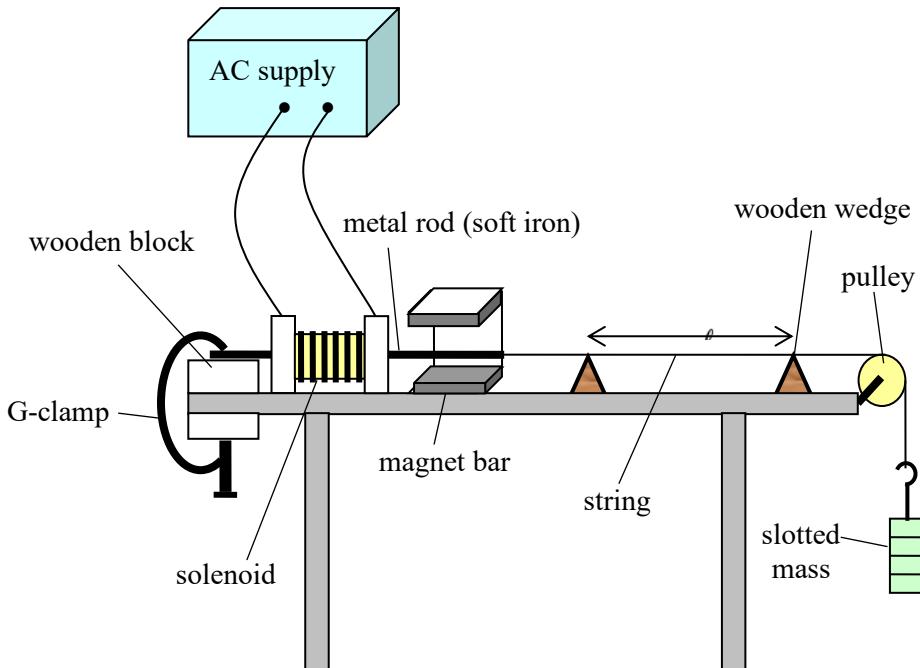
$$T = 4\mu f^2 l^2 \quad 6.1$$

where  $f$  is the frequency  
 $l$  is the length between two nodes  
 $T$  is the tension in the string  
 $\mu$  is the mass per unit length

Evidently, a graph of  $T$  against  $l^2$  is a straight line of gradient equals  $4\mu f^2$ .  
Hence, the mass per unit length,  $\mu$  can be calculated.

**Apparatus:**

A G-clamp  
A solenoid (about 100 turns) or ticker timer  
An AC supply (2 – 4 V)  
A metal rod (soft iron)  
Two bar magnets  
A magnet holder  
A piece of string approximately 2 m long  
A pulley with clamp  
A wooden wedge  
Set of slotted mass 2 g, 5 g, 10 g and 20 g  
A metre rule  
Connecting wires

**Procedure:****Figure 6.1**

1. Set up the apparatus as in **Figure 6.1**.
  2. Connect the terminals of the solenoid to the AC power supply (2 V, 50 Hz).
- Caution:** *Do not exceed 4 V to avoid damage to the solenoid.*
3. Place the metal rod between the two bar magnets.
  4. Tie one end of the string to the rod and the other to the hook of the slotted mass. Make sure that the length of the string from the end of the rod to the pulley is **not less than 1.5 m**.
  5. Clamp the metal rod properly. Switch on the power supply. Adjust the position of the metal rod to get maximum vibration.
  6. Place the wooden wedges below the string **as close as possible to the pulley**.

7. Adjust the position of the wooden wedges until a clear single loop standing wave (fundamental mode) is observed.
8. Record the distance,  $l$  between the wedges and total mass,  $m$  (mass of the hook and the slotted mass). Calculate weight,  $W$  where  $W = mg$ .

**Note:**  $Weight, W = Tension, T$

9. Add a small mass, preferably 10 g to the hook and repeat step (7) and (8) for at least six different readings.
10. Tabulate the data.
11. Plot a graph of  $T$  against  $l^2$ .
12. Determine the mass per unit length,  $\mu$  from the gradient of the graph and its uncertainty,  $\Delta\mu$  if the frequency of the vibration is 50 Hz.
13. Weigh the mass of the string and measure the total length of the string. Calculate the mass per unit length,  $\mu$  of the string.
14. Compare the mass per unit length,  $\mu$  in step (12) with the result obtained in step (13). Write the comments.

# **PHYSICS 2**

# **SP025**

**EXPERIMENT 1: CAPACITOR****Objectives:**

- (i) To determine the time constant,  $\tau$  of an RC circuit.
- (ii) To determine the capacitance,  $C$  of a capacitor using an RC circuit.

**Theory:**

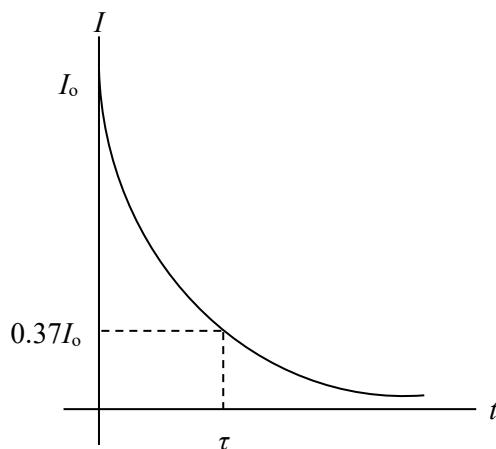
Time constant is defined as the time taken of a discharge current decreases to 37% of its maximum current. The time constant can be calculated by using

$$\tau = RC \quad 1.1$$

Where  $\tau$  is time constant

$R$  is the resistance of a resistor

$C$  is the capacitance of a capacitor



**Figure 1.1**

During discharging, the magnitude of the current,  $I$  varies with time as shown in **Figure 1.1**.

From Figure 1.1, the magnitude of the discharge current is

$$I = I_o e^{-\frac{t}{\tau}} \quad 1.2$$

Rearrange equation 1.2 we obtain

$$\ln\left(\frac{I_o}{I}\right) = \frac{t}{\tau} \quad 1.3$$

Where  $I_o$  is the maximum current in the circuit

$I$  is current in the circuit at time  $t$

By using equation 1.3, the time constant can be determined from the gradient of the straight-line graph.

### Apparatus:

A DC power supply (4 – 6 V)

A switch

A DC microammeter

A digital stopwatch

A 100 kΩ resistor

Connecting wires

Two capacitors labelled  $C_1$  and  $C_2$  (470 – 1000 μF)

### Procedure:

**Note:** Before starting or repeating this experiment, make sure that the capacitors are fully discharged. This can be attained by short circuiting the capacitors.

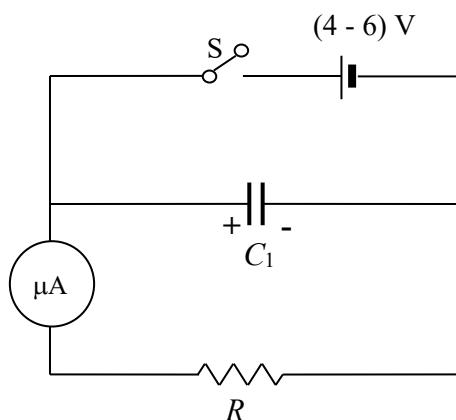
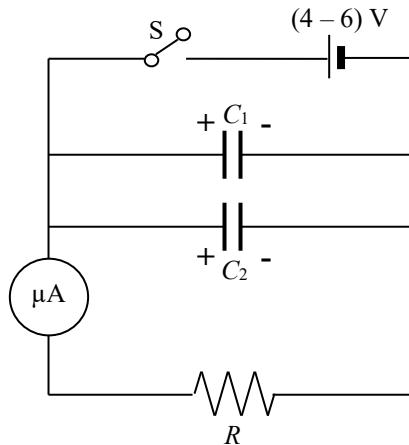


Figure 1.2

1. Set up the circuit as shown in **Figure 1.2**.
2. Close switch S to fully charge the capacitor  $C_1$ . Record the reading of the microammeter for maximum current,  $I_o$ .
3. Open switch S and start the stopwatch simultaneously.
4. Use the ‘lap’ function on the stopwatch when the current reaches at least six different values. Record the time for each value of current.
5. Repeat steps (1) to (4) and calculate the average time,  $t$ .
6. Tabulate the data.
7. Plot a graph of  $\ln\left(\frac{I_o}{I}\right)$  against  $t$ .
8. Determine the time constant,  $\tau$  from the gradient of the graph.
9. Calculate the capacitance of the capacitor  $C_1$  by using equation 1.1.
10. Connect capacitor  $C_2$  to the circuit as shown in **Figure 1.3**.

**Figure 1.3**

11. Repeat steps (1) to (6) to obtain the readings of the microammeter  $I'$  and the stopwatch  $t'$ . Record the readings.
12. Tabulate the data.

13. Plot a graph of  $\ln\left(\frac{I_o}{I}\right)'$  against  $t'$ .
14. Determine the time constant,  $\tau'$  from the gradient of the graph.
15. Calculate the effective capacitance,  $C_{\text{eff}}$  of capacitors by using equation 1.1.
16. Compare all the results with their respective standard values. Write a comment.

**EXPERIMENT 2: OHM'S LAW**

---

**Objectives:**

- (i) To sketch V-I graph.
- (ii) To verify Ohm's law.
- (iii) To determine the effective resistance,  $R_{\text{eff}}$  of the resistors in series and parallel by graphing method.

**Theory:**

At constant temperature, the potential difference  $V$  across a conductor is directly proportional to the current  $I$  that flows through it. The constant of proportionality is known as the resistance of the conductor denoted by  $R$ .

Mathematically,  $V \propto I$

$$V = IR \quad 2.1$$

For resistors in series, the effective resistance is

$$R_{\text{eff}} = R_1 + R_2 + R_3 + \dots + R_n \quad 2.2$$

For resistors in parallel, the effective resistance is

$$\frac{1}{R_{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n} \quad 2.3$$

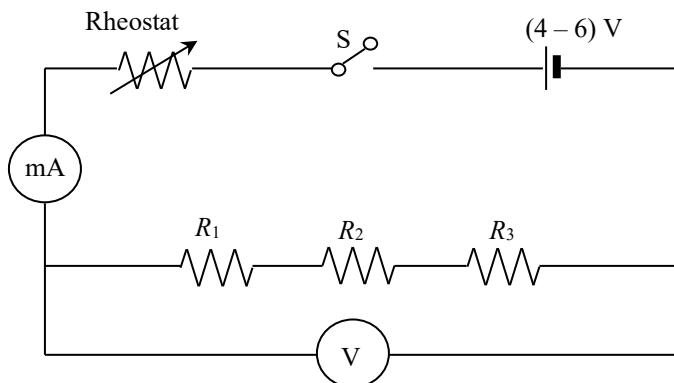
**Apparatus:**

- A DC power supply (4 – 6 V)
- Three resistors of the same resistance (27 – 100  $\Omega$ )
- A DC milliammeter
- A DC ammeter (1 A)
- A DC voltmeter
- A rheostat
- A switch
- Connecting wires

**Procedure:**

1. Determine the resistance,  $R$  of each resistor from their colour bands.
2. Set up the circuit as in **Figure 2.1**. Connect the three resistors in series.

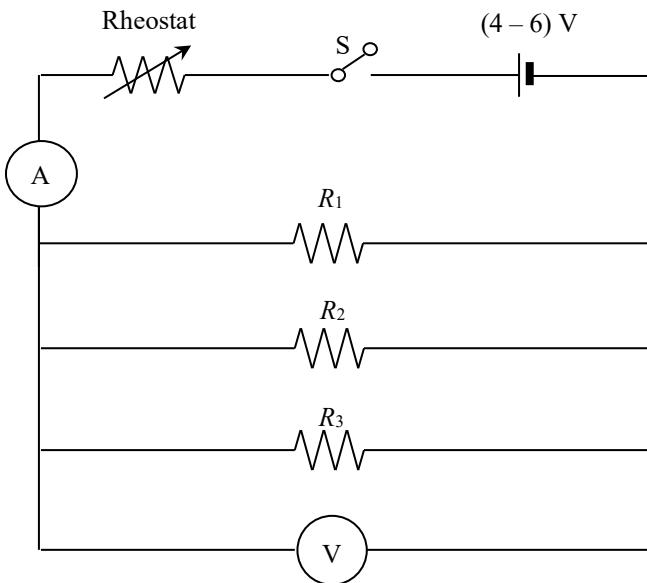
**Note:** Ask your lecturer to check the circuit before switching ON the power.



**Figure 2.1**

3. Adjust sliding contact on the rheostat to change the resistance values to obtain a minimum reading of the milliammeter. Record the reading of the voltmeter,  $V$  and the milliammeter,  $I$ .
4. Adjust sliding contact on the rheostat to change the resistance values to obtain at least six different values of  $V$  and  $I$ .
5. Tabulate the data.
6. Plot a graph of  $V$  against  $I$ .
7. Determine the effective resistance,  $R_{\text{eff}}$  of the three resistors connected in series from the gradient of graph.

8. Set up the circuit as in **Figure 2.2**.

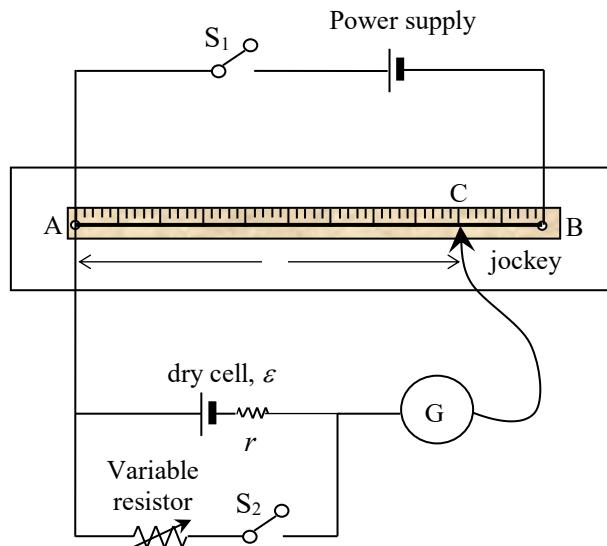


**Figure 2.2**

9. Repeat steps (3) to (6).
10. Determine the effective resistance,  $R_{\text{eff}}$  of the three resistors connected in parallel from the gradient of the graph.
11. Compare all the results with their respective standard values.
12. Verify Ohm's law from the plotted graphs. Write a comment.

**EXPERIMENT 3: POTENTIOMETER****Objective:**

To determine the internal resistance  $r$  of a dry cell by using a potentiometer.

**Theory:****Figure 3.1**

Let  $\varepsilon$  be the electromotive force (emf) and  $r$  the internal resistance of the dry cell. The emf of the dry cell is balanced by the potential difference across wire AB provided by the power supply when the jockey is tapped at balance point, C with  $S_1$  close and  $S_2$  open. The balance condition is indicated when there is no deflection in the galvanometer. If  $l_o$  is the length of the wire from A to C,

$$\varepsilon \propto l_o$$

Hence,

$$\varepsilon = kl_o \quad 3.1$$

where  $k$  is a constant.

With both  $S_1$  and  $S_2$  closed, the new length of wire at the balance point is equal to  $l$ . Hence,

$$V \propto l$$

$$V = kl \quad 3.2$$

$$\varepsilon = V + Ir$$

3.3

Rearrange equation 3.1, 3.2 and 3.3, we obtain

$$\frac{l_o}{l} = r \left( \frac{1}{R} \right) + 1 \quad 3.4$$

The graph  $\frac{l_o}{l}$  against  $l$  is a straight-line graph and its gradient is  $r$ .

### **Apparatus:**

A potentiometer

A variable resistor ( $0 - 1 \Omega$ ) (A breadboard, six  $1 \Omega$  resistor and jumpers)

Two switches

A jockey

A regulated power supply

A  $1.5 \text{ V}$  dry cell

A galvanometer

Connecting wires

### **Procedure:**

1. Set up the apparatus as shown in **Figure 3.1**.

#### **Note:**

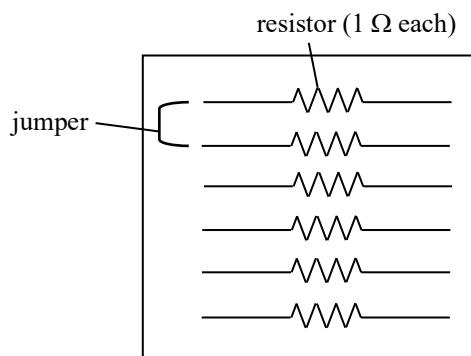
- i. *Make sure the polarity of the batteries is connected in right configuration. Ask your lecturer to check the circuit before switch ON the power.*
  - ii. *Make sure that the galvanometer deflected at both sides when the jockey is tapped at two different extreme points.*
2. With  $S_1$  closed and  $S_2$  opened, tap the jockey along the wire until the galvanometer reading is zero (balanced) and determine  $l_o$ .
  3. With both  $S_1$  and  $S_2$  closed, repeat step (2) and determine  $l$  for at least six different values of  $R$ .
  4. Tabulate the data.

#### **Note:**

*Use 1  $\Omega$  resistor in parallel combination to obtain resistance*

$1\ \Omega$ ,  $\frac{1}{2}\ \Omega$ ,  $\frac{1}{3}\ \Omega$ ,  $\frac{1}{4}\ \Omega$ ,  $\frac{1}{5}\ \Omega$  and  $\frac{1}{6}\ \Omega$  as shown in **Figure 3.2**.

5. Plot a graph of  $\frac{l_o}{l}$  against  $\left(\frac{1}{R}\right)$ .
6. Determine the internal resistance of the dry cell,  $r$  from the gradient of the graph. Write a comment.



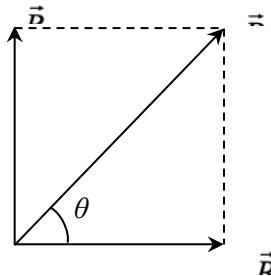
**Figure 3.2**

**EXPERIMENT 4 (a): MAGNETIC FIELD****Objective:**

To determine the value of the horizontal component of the earth magnetic field  $\vec{B}_E$ .

**Theory:**

The magnetic field strength  $\vec{B}$  is a vector quantity so the addition of two magnetic fields obeys the parallelogram law. For example, if  $\vec{B}_E$  is the horizontal component of earth magnetic field and  $\vec{B}_C$  is the magnetic field of a coil which is perpendicular to  $\vec{B}_E$  then the resultant of the two fields  $\vec{B}$  is as shown in **Figure 4.1**. A compass needle is situated at the place where the two fields meet will be aligned to the direction of the resultant field  $\vec{B}$ .



**Figure 4.1**

From Biot-Savart's Law, the magnetic field strength of the coil at the centre as shown in **Figure 4.2** is given by

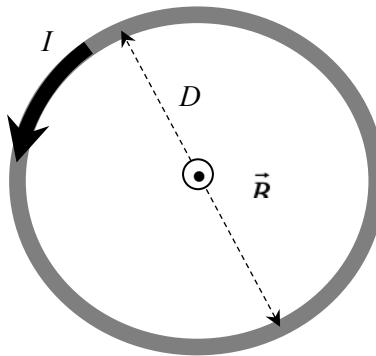
$$B_c = \frac{\mu_0 NI}{D} \quad 4.1$$

where  $\mu_0 = 4\pi \times 10^{-7}$  H m<sup>-1</sup> (permeability of free space)

$I$  is the current in ampere

$N$  is the number of turns in the coil

$D$  is the diameter of the coil

**Figure 4.2**

From **Figure 4.1**,

$$\begin{aligned} \tan \theta &= \frac{B_c}{B_E} \\ \tan \theta &= \frac{\mu_0 N}{D(B_E)} I \end{aligned} \quad 4.2$$

The gradient of graph  $\tan \theta$  against  $I$

$$m = \frac{\mu_0 N}{D(B_E)} \quad 4.3$$

Therefore,

$$B_E = \frac{\mu_0 N}{Dm} \quad 4.4$$

### **Apparatus:**

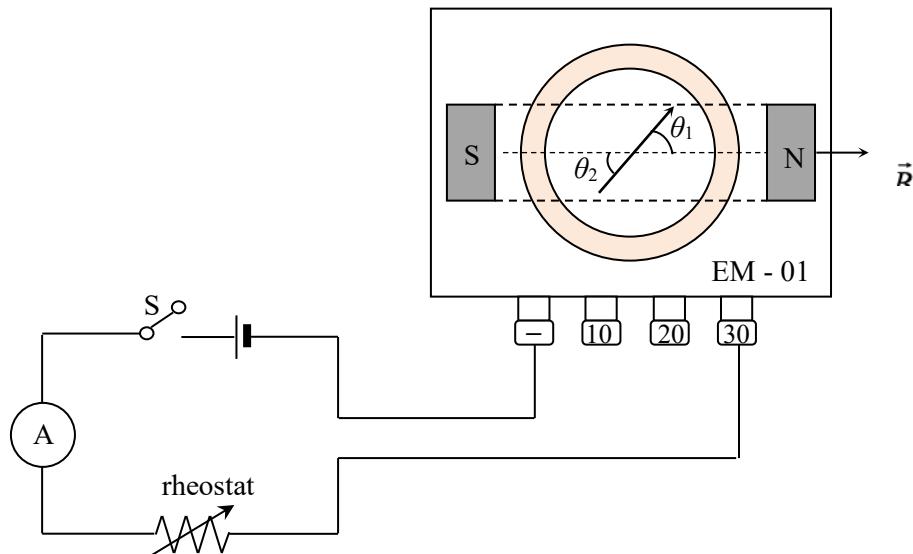
- Earth magnetic field measurement kit (EM-01)
- Connecting wires of about 50 cm long with crocodile clips
- A DC ammeter (0 – 1 A)
- A rheostat
- A DC power supply (2 - 4 V)

**Procedure:**

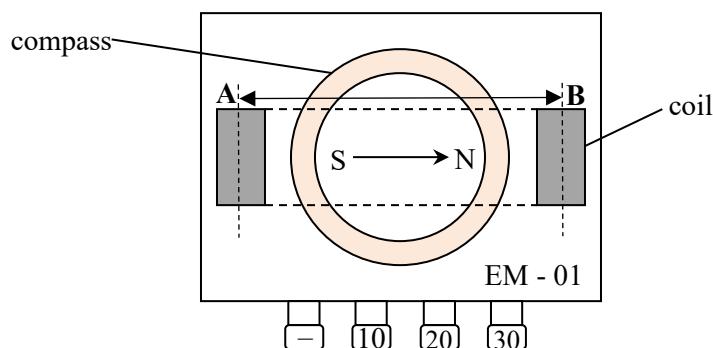
- Set up apparatus as shown in **Figure 4.3**.

**Note:**

*Make sure the EM-01 kit is located far away from other electrical devices to avoid magnetic disturbance.*

**Figure 4.3**

- Align the compass needle of EM-01 until it is pointed North as in **Figure 4.4**.

**Figure 4.4**

3. Set the rheostat to its maximum value and switch on the circuit. Reduce the resistance of rheostat to increase the current  $I$  and hence the corresponding value of  $\theta_1$  for at least six sets of readings. Record the readings of the ammeter  $I$  and the angle of deflection  $\theta_1$  in **Table 4.1**.

**Note:** The deflection angle should not be more than  $80^\circ$ .

4. Repeat step (3) by changing the polarity of the power supply. Record the angle  $\theta_2$ , pointed by the compass needle in **Table 4.1**.
5. Measure the diameter of the coil.

**Note:** Make sure the diameter is measured from A to B as in **Figure 4.4**

Diameter of coil  $D = (\dots\dots\dots \pm \dots\dots\dots)$  cm

Number of turns  $N = 30$

No.	current, $I$ ( $\pm \dots\dots\dots$ )	$\theta_1$ ( $\pm \dots\dots\dots$ )	$\theta_2$ ( $\pm \dots\dots\dots$ )	average $\theta_A$ ( $\dots\dots\dots$ )	$\tan \theta_A$
1					
2					
3					
4					
5					
6					
7					
8					
9					

**Table 4.1**

6. Plot a graph of  $\tan \theta$  against  $I$ .
7. Determine  $B_E$  from the gradient of the graph.
8. Compare the result with the standard value given by the lecturer. Write a comment.

**EXPERIMENT 4(b): MAGNETIC FIELD****Objective:**

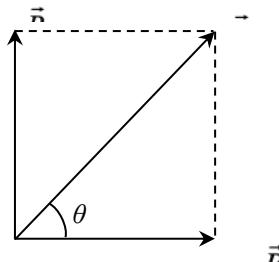
To determine the value of the horizontal component of the earth magnetic field,  $\vec{B}_E$ .

**Student Learning Time (SLT):**

Face-to-face	Non face-to-face
2 hours	0

**Theory:**

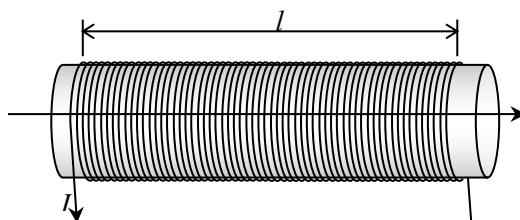
The magnetic field strength  $\vec{B}$  is a vector quantity so the addition of two magnetic fields obeys the parallelogram law. For example, if  $\vec{B}_E$  is the horizontal component of earth magnetic field and  $\vec{B}_s$  is the magnetic field of a solenoid which is perpendicular to  $\vec{B}_E$  then the resultant of the two fields  $\vec{B}$  is as shown in **Figure 4.1**. A compass needle is situated at the place where the two fields meet will be aligned to the direction of the resultant field  $\vec{B}$ .



**Figure 4.1**

The magnetic field strength at the end of an  $N$ -turn solenoid of length  $l$  and carries current  $I$  as shown in **Figure 4.2** is given by

$$B_s = \frac{1}{2} \left( \frac{\mu_0 N I}{l} \right) \quad 4.1$$



**Figure 4.2**

From **Figure 4.1**,

$$\tan \theta = \frac{B_s}{B_E}$$

$$\tan \theta = \frac{\frac{1}{2} \mu_0 \left( \frac{N}{l} \right) I}{B_E} \quad 4.2$$

The gradient of graph  $\tan \theta$  against  $I$  is

$$m = \frac{\frac{1}{2} \mu_0 \left( \frac{N}{l} \right)}{B_E} \quad 4.3$$

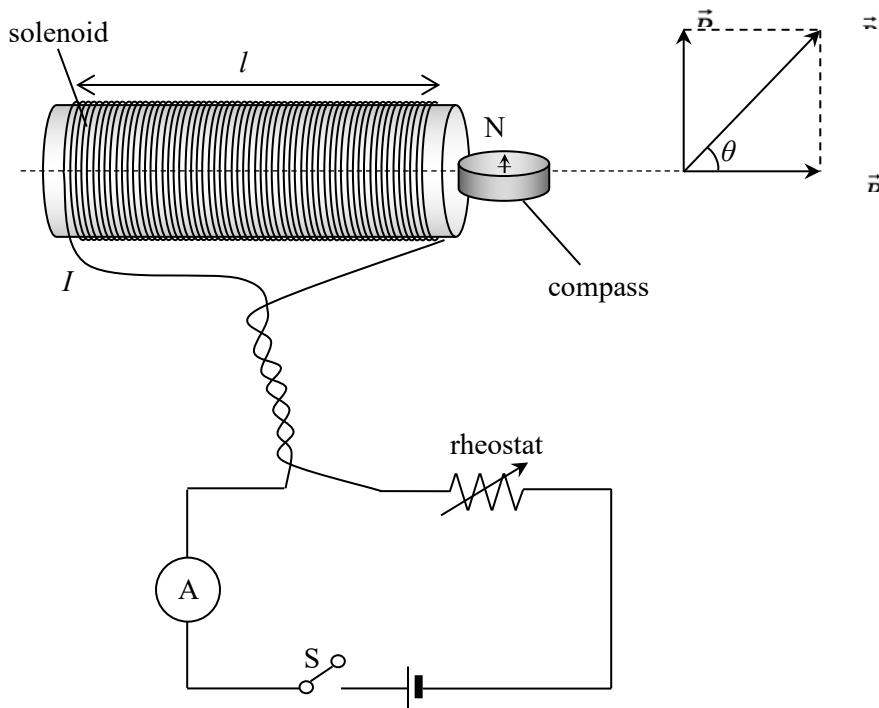
Therefore,

$$B_E = \frac{\frac{1}{2} \mu_0 \left( \frac{N}{l} \right)}{lm} \quad 4.4$$

where  $\mu_0 = 4\pi \times 10^{-7}$  H m<sup>-1</sup> (permeability of free space).

### **Apparatus:**

- A 50 turns or 100 turns solenoid
- A DC power supply (2 – 4 V)
- A DC ammeter (0 – 1 A)
- A switch
- Connecting wires of about 50 cm long with crocodile clips
- A rheostat
- A compass

**Procedure:****Figure 4.3**

1. Place a compass at one end of the solenoid. Let the compass stay still in N–S direction where the magnet pointer is perpendicular to the axis of the solenoid. The north direction of the compass must be pointed to the north.

**Note:** Choose a position to place your compass away from any iron structure to avoid any influence on the alignment of the compass needle.

2. Connect the solenoid in series with the rheostat, the ammeter, the power supply and the switch. The ammeter must be at least 50 cm away from the compass. A complete set up is as in **Figure 4.3**.
3. Set the rheostat to its maximum value and switch on the circuit. Reduce the resistance of rheostat to increase the current  $I$  and hence the corresponding value of  $\theta_1$  for at least six sets of readings. Record the readings of the ammeter  $I$  and the angle of deflection  $\theta_1$  in **Table 4.1**.

**Note:** The deflection angle should not be more than  $80^\circ$ .

4. Repeat step (3) by changing the polarity of the power supply. Record the angle  $\theta_2$ , pointed by the compass needle in **Table 4.1**.

**Table 4.1**

No.	Current, $I$ ( $\pm$ .....)	$\theta_1$ ( $\pm$ .....)	$\theta_2$ ( $\pm$ .....)	Average $\theta_A$ (....)	$\tan \theta_A$
1					
2					
3					
4					
5					
6					
7					
8					
9					

5. Remove the solenoid from the clamp and measure the length,  $l$  of the solenoid.
6. Plot a graph of  $\tan \theta$  against  $I$ .
7. Determine  $B_E$  from the gradient of the graph.
8. Compare the result with the standard value given by the lecturer. Write a comment.

**EXPERIMENT 5: GEOMETRICAL OPTICS**

---

**Objective:**

To determine the focal length,  $f$  of a convex lens.

**Theory:**

From the thin lens equation,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad 5.1$$

where  $f$  is the focal length

$u$  is the object distance

$v$  is the image distance

Multiply equation 5.1 with  $v$ ,

$$\frac{v}{f} = \frac{v}{u} + 1$$

$$M = -\frac{v}{f} + 1 \quad 5.2$$

Where  $M = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_o} = -\frac{v}{u}$  is the linear magnification.

For this experiment the image formed is always real, then negative sign for magnification indicates that the image is inverted.

Hence the graph  $M$  against  $v$  is a straight-line graph.

The equation also shows that  $M$  is proportional to  $v$ .

When  $v = 2f$ ,  $M = -1$ .

**Apparatus:**

A convex lens

A piece of card with narrow triangle shaped slit or any suitable objects

A screen

A light source

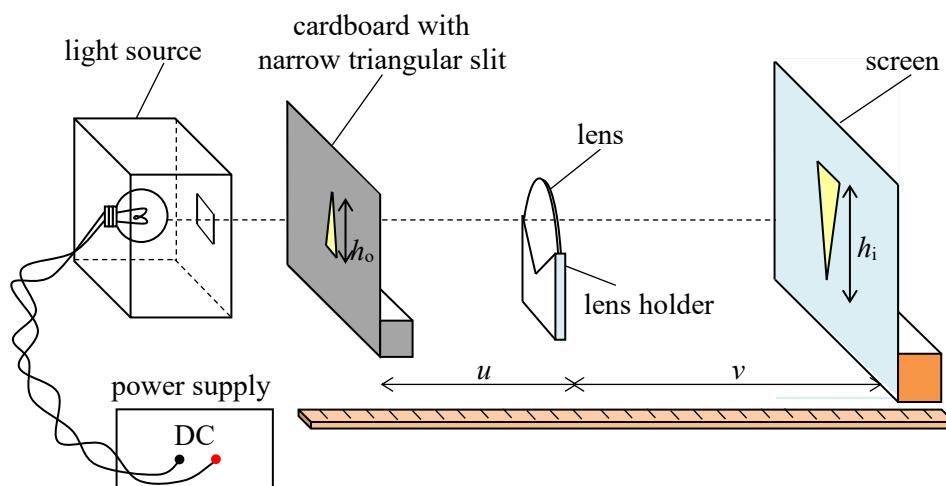
A metre rule

A lens holder

Plasticine

### Procedure:

1. Use the convex lens to focus a distant object such as a tree outside the laboratory on a screen. The distance between the screen and the lens is the estimated focal length,  $f_0$  of the lens. Record the estimated focal length.



**Figure 5.1**

2. Set up the apparatus as in **Figure 5.1**.
3. Place the object in front of the lens at a suitable distance ( $f_0 < u < 2f_0$ ) and adjust the position of the screen so that a sharp real, inverted image is projected on the screen.
4. Record the measurement for the object distance  $u$  and the image distance  $v$ .
5. Calculate the magnification of the image,  $M = -\frac{v}{u}$ .
6. Change the location of the object. Repeat steps (4) and (5) until six sets of  $u$ ,  $v$  and  $M$  are obtained.
7. Tabulate the data.

8. Plot a graph of  $M$  against  $v$ .
9. Determine the focal length of the lens,  $f_1$  from the gradient of the graph.
10. Determine the image distance  $v$  from the graph by using extrapolation when  $M = -1$  and calculate the focal length,  $f_2$  by using equation 5.2.
11. Compare the results  $f_1$  and  $f_2$  with  $f_0$ . Write a comment.

**EXPERIMENT 6: DIFFRACTION GRATING****Objectives:**

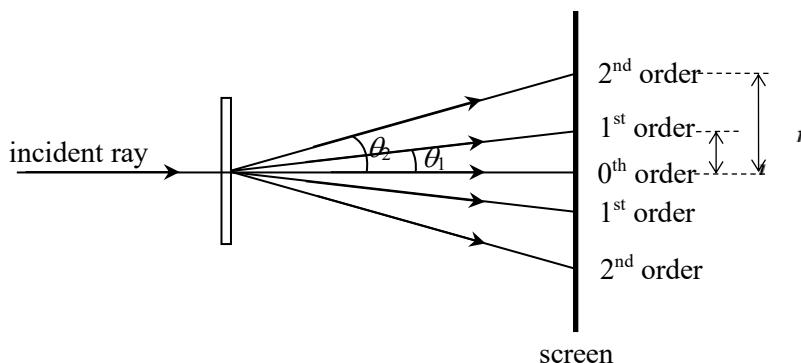
- (i) To determine the wavelength,  $\lambda$  of laser beam using a diffraction grating.
- (ii) To the number of diffraction grating lines per unit length,  $N$ .

**Student Learning Time (SLT):**

Face-to-face	Non face-to-face
2 hour	0

**Theory:**

When a laser beam is incident on a diffraction grating, a diffraction pattern in the form of a series of bright dots can be seen on the screen as shown in **Figure 6.1**

**Figure 6.1**

The relationship between the angle  $\theta_n$  of the  $n^{\text{th}}$  order and the wavelength of laser  $\lambda$  is

$$\sin \theta_n = \frac{n\lambda}{d} \quad 6.1$$

where  $d$  is the distance between two consecutive lines of the diffraction grating, known as grating spacing.

Usually, the grating spacing is specified in number of lines per meter, such as  $N$  lines per meter. Hence,

$$N = \frac{1}{d} \quad 6.2$$

Then

$$\sin \theta_n = N n \lambda \quad 6.3$$

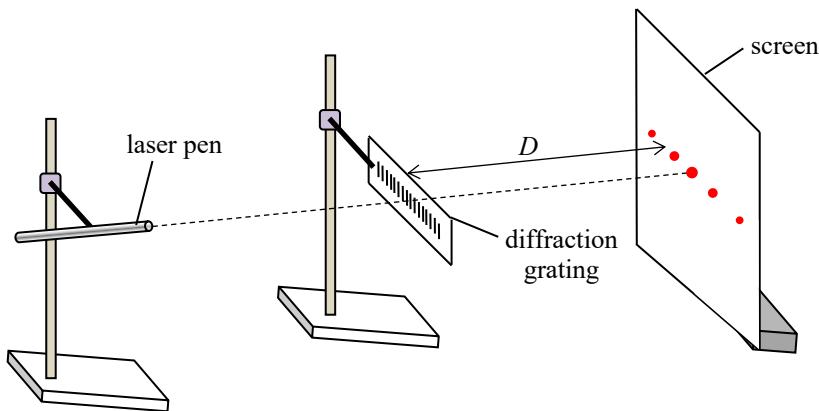
By measuring the angle  $\theta_n$  for each order of diffraction  $n$ ,  $\lambda$  can be determined.

### Apparatus:

A laser pen  
 Two retort stands with clamps  
 A metre rule  
 A screen  
 Two diffraction gratings (A and B)

**Note:** Suggestion A is 100 lines/mm and B is 300 lines/mm.

### Procedure:



**Figure 6.2**

1. Set up the apparatus as shown in **Figure 6.2**. Ensure that the laser ray is pointed perpendicularly to the diffraction grating A.

**Note:** Make sure that

- i) the incident ray is normal to the diffraction grating.
- ii) the screen is parallel to the diffraction grating.

2. The distance  $D$  from the diffraction grating to screen must be adjusted so that the spacing between the spots on the screen is as far as possible from one another. Measure and record the value of  $D$ .

**Caution:** A laser pen is NOT a toy. It is dangerous to look directly at the laser beam because it may cause permanent damage to your eyesight.

3. Measure the distance  $l_1, l_2, l_3, \dots, l_n$  that correspond to the diffraction order of  $n = 1, 2, 3, \dots$  where  $l_n$  is the distance between spots of order,  $n$  to the centre spot.
4. Determine values of  $\sin \theta_n$  for order of  $n = 1, 2, 3, \dots$  using equation

$$\sin \theta_n = \frac{l_n}{\sqrt{(l_n)^2 + D^2}}$$

5. Tabulate the data.
6. Plot a graph of  $\sin \theta_n$  against  $n$ .
7. Determine the wavelength,  $\lambda$  of the laser beam from the gradient of the graph.

**Note:** Take the value of  $N$  printed on the grating A.

8. Repeat steps (1) to (6) using grating B.
9. Determine the number of lines per mm,  $N$  of grating B from the gradient of the graph by using the value of  $\lambda$  in step (7).
10. Compare the value of  $\lambda$  and  $N$  for grating B with their standard values. Write a comment.

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- Serway, R. A. & Jewett, J. A. (2019). *Physics for Scientists and Engineers* (10<sup>th</sup> ed.). International Student Edition. USA: Brooks/Cole Cengage Learning.
- Giancoli, D. C. (2016). *Physics – Principles with Application* (7<sup>th</sup> ed.). Prentice Hall.

## **ACKNOWLEDGEMENTS**

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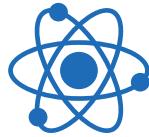
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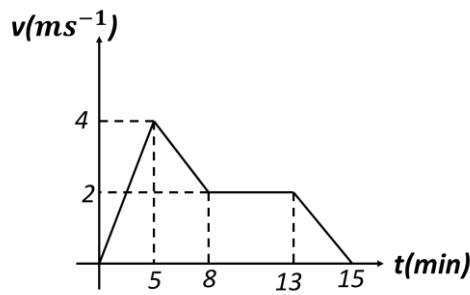
**SP015**

**PAST YEAR QUESTIONS**

WORKSHEET 1: MATHEMATICS .....	ERROR! BOOKMARK NOT DEFINED.
WORKSHEET 2: 1D KINEMATICS .....	ERROR! BOOKMARK NOT DEFINED.
WORKSHEET 3: 2D KINEMATICS & PROJECTILE MOTION.....	ERROR! BOOKMARK NOT DEFINED.
WORKSHEET 4: PYQ KINEMATICS.....	3
WORKSHEET 5: MOMENTUM & ITS CONSERVATION .....	ERROR! BOOKMARK NOT DEFINED.
WORKSHEET 6: FREE BODY DIAGRAM .....	ERROR! BOOKMARK NOT DEFINED.
WORKSHEET 7: WORK, ENERGY, $\Sigma W = \Delta K$ & $\Delta \Sigma E = 0$ .....	ERROR! BOOKMARK NOT DEFINED.
WORKSHEET 8: MECHANICAL POWER.....	ERROR! BOOKMARK NOT DEFINED.
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WORKSHEET 13: ROTATIONAL DYNAMICS & $\Delta \Sigma L = 0$ .....	ERROR! BOOKMARK NOT DEFINED.
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WORKSHEET 15: WAVES – BASICS.....	ERROR! BOOKMARK NOT DEFINED.
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## Worksheet 4: PYQ Kinematics

21/22	<p>a. A bus is moving with an initial speed <math>u</math> begins to slow down at a uniform rate of <math>3ms^{-2}</math>. Calculate <math>u</math> if it takes 6.67s to travel at a distance of 67m.</p> <p>b. A ball is thrown horizontally from the top of a building with a speed of <math>20ms^{-1}</math>. After 7s, the ball hits the ground. What is the height of the building?</p> <p>c. The figure below shows the path taken by an object projected with an initial speed, <math>u</math> at an angle <math>30^\circ</math> to the horizontal.</p>  <p>What is the speed of the object at point A in terms of <math>u</math>?</p> <p>d. A motorcycle accelerates from rest to <math>5ms^{-1}</math> in 4.5s and then continues at this speed for another 4.5s. Calculate the total distance travelled by the motorcycle.</p>
	<p>a. Method 1</p> $v^2 = u^2 + 2as \Rightarrow v^2 = u^2 + 2(-3)(67)$ $v = u + at \Rightarrow v = u + (-3)(6.67)$ $v = 0.04ms^{-1}; u = 20.05ms^{-1}$ <p>Method 2</p> $s = ut + \frac{1}{2}at^2$ $\Rightarrow 67 = 6.67(u) + \frac{1}{2}(-3)(6.67)^2$ $u = 20.05ms^{-1}$ <p>b.</p> $s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow s_y = 0 + \frac{1}{2}(-9.81)7^2 \Rightarrow$ $s_y = -240.345m$ <p>c.</p> $v_y = u_y + a_y t = u \sin \theta - gt; v_x = u_x = u \cos \theta \Rightarrow v = \sqrt{v_x^2 + v_y^2}$ $v = \sqrt{u^2 - 2gt \sin \theta + g^2 t^2}$ <p>d.</p> $s_1 = \frac{1}{2}(u + v)t = \frac{1}{2}(0 + 5)4.5 = 11.25m$ $s_2 = vt = 5(4.5) = 22.5m$ $s_{total} = s_1 + s_2 = 11.25 + 22.5$ $s_{total} = 33.75m$
20/21	<p>A student took 15minutes to cycle from his house to school. He starts from rest and reaches a maximum speed of <math>4ms^{-1}</math> in 5 minutes at constant acceleration. After reaching the maximum speed, he decelerates uniformly to <math>2.0ms^{-1}</math> in 3 minutes and continues cycling with this speed for 5 minutes. He then took 2 minutes to decelerate uniformly to stop.</p> <p>A. Sketch a labelled graph of speed versus time for the whole journey.</p> <p>B. Calculate the acceleration of the bicycle for the time segments of 0-5 minutes and 13-15 minutes.</p> <p>C. Determine the total distance from his house to school.</p>
	A.



B.

$$a_{0-5\text{min}} = \frac{v-u}{t} = \frac{4-0}{5 \times 60} = 0.013\text{ms}^{-2}$$

$$a_{13-15\text{min}} = \frac{v-u}{t} = \frac{0-2}{2 \times 60} = -0.017\text{ms}^{-2}$$

C.

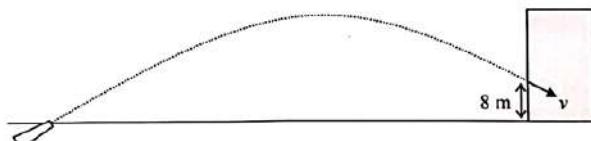
$$s_{total} = s_{0-5\text{mins}} + s_{5-8\text{mins}} + s_{8-13\text{mins}} + s_{13-15\text{mins}}$$

$$s_{total} = \left(\frac{1}{2}(v+u)t\right) + \left(\frac{1}{2}(v+u)t\right) + vt + \frac{1}{2}(v+u)t$$

$$s_{total} = \left(\frac{1}{2}(4+0)(5 \times 60)\right) + \left(\frac{1}{2}(2+4)(3 \times 60)\right) + 2(5 \times 60) + \frac{1}{2}(0+2)(2 \times 60)$$

$$s_{total} = 1860\text{m}$$

- 19/20 A. A boat with an initial speed of  $30\text{ms}^{-1}$ , decelerates at  $3.5\text{ms}^{-2}$  for 4.5s before reaching a buoy. Calculate the speed of the boat at the buoy.  
B. The figure below shows a stream of water hitting a wall at a height of 8m with a velocity of  $40\text{ms}^{-1}$  at an angle of  $35^\circ$  below the horizontal.



Determine the initial velocity of the water as it leaves the nozzle.

A.  $v = u + at \Rightarrow v = 30 + (-3.5)(4.5) = 14.25\text{ms}^{-1}$

B.  $v_y^2 = u_y^2 - 2gs_y$

$$\Rightarrow (40 \sin(-35^\circ))^2 = (u \sin \theta_i)^2 - 2(9.81)(8)$$

$$\Rightarrow u \sin \theta_i = 26.141\text{ms}^{-1} \quad [1]$$

$$u_x = v_x = v \cos \theta_f = 40 \cos(-35^\circ)$$

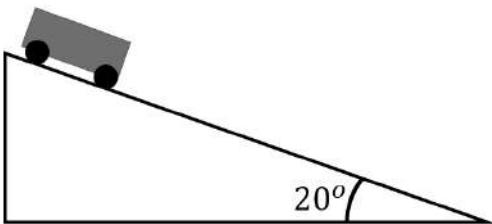
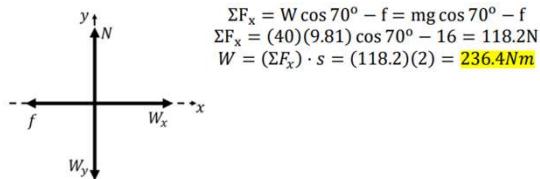
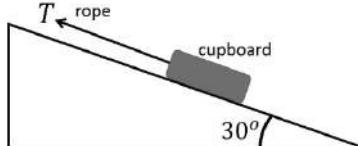
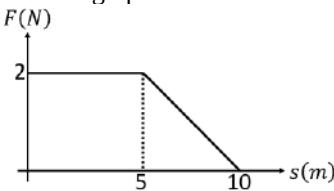
$$\Rightarrow u \cos \theta_i = 32.766\text{ms}^{-1} \quad [2]$$

Solving equations 1 and 2 simultaneously yields,

$$u = 41.9161\text{ms}^{-1}; \theta_i = 38.5832^\circ$$

### Worksheet 9: PYQ Momentum, Newton's Law, and Energy

- 21/22 A. A 500g soccer ball is kicked horizontally at the speed of  $12\text{ms}^{-1}$  towards a wall. It rebounds off the wall at the speed of  $2\text{ms}^{-1}$ . Calculate the magnitude of the impulse on the ball.  
B. Two rugby players with mass 75kg and 100kg run directly towards each other with velocities of  $6\text{ms}^{-1}$  to the right and  $8\text{ms}^{-1}$  to the left respectively. If they grab each other as they collide, calculate the combined velocity of the two players just after the collision.  
C. A man of mass 75kg and a woman of mass 55kg stand facing each other on a smooth horizontal surface, both wearing roller blades. The woman pushes the man to the right with a horizontal force of 85N. Determine the acceleration of the woman.  
D. How large of a net force required to accelerate a 600N at a rate  $0.7\text{ms}^{-2}$  on a smooth horizontal surface?

	<p>E. A shopping trolley with a total mass of 40kg is released from rest and rolls down a 2m long surface which is inclined at <math>20^\circ</math> as shown in the figure below.</p>  <p>Calculate the work done to stop the trolley at the bottom of the surface if it experiences a constant frictional force of 16N.</p> <p>F. A man is lifting three boxes each weighing 80N to a 1.2m high shelf in 2s. Calculate the power required by the man to lift the boxes.</p> <p>G. Calculate the release height of a falling of a 2kg sphere if its kinetic energy is 300J just before striking the ground. The air resistance can be ignored.</p>
	<p>A. <math>\vec{J} = m(\vec{v} - \vec{u}) = 0.5((-2) - (12)) \Rightarrow J = -7Ns</math></p> <p>B. <math>m_1u_1 + m_2u_2 = (m_1 + m_2)v \Rightarrow (75)(6) + (100)(-8) = (100 + 75)v \Rightarrow v = -2ms^{-1}</math></p> <p>C. <math>F_{12} = -F_{21} \Rightarrow m_{woman}a_{woman} = -85 = (55)a_{woman} \Rightarrow a_{woman} \approx 1.54ms^{-2}</math></p> <p>D. <math>mg = m(9.81) = 600N \Rightarrow m = 61.2kg</math></p> <p>E.</p>  $\Sigma F_x = W \cos 70^\circ - f = mg \cos 70^\circ - f$ $\Sigma F_x = (40)(9.81) \cos 70^\circ - 16 = 118.2N$ $W = (\Sigma F_x) \cdot s = (118.2)(2) = 236.4Nm$ <p>F. Total Weight, <math>W_{total} = 3(80) = 240N</math>  Work Done, <math>W = \Delta K = (mg)h = (240)(1.2)</math>  Power required, <math>P = \frac{W}{t} = \frac{240(1.2)}{2}</math>  <math>P = 144W</math></p> <p>G. <math>E_k = E_{gp} \Rightarrow 300 = mgh = (2)(9.81)(h) \Rightarrow h = 15.3m</math></p>
20/21	<p>A. A 20g bullet is fired and travels with speed of <math>800ms^{-1}</math>. It hits 5kg wooden block initially at rest and is stuck inside, causing the block to move. Determine the final velocity of the bullet and the block immediately after the block is hit and show that the collision is inelastic.</p> <p>B. The figure shows a 40kg cupboard being pulled along a rough inclined plane <math>30^\circ</math> to the horizontal by a light rope.</p>  <p>The cupboard moves at a constant velocity. The coefficient of kinetic friction between the cupboard and inclined plane is 0.5.</p> <ol style="list-style-type: none"> <li>Draw a free body diagram of the cupboard.</li> <li>Determine the magnitude of the frictional force and tension acting on the cupboard.</li> </ol> <p>C. A 2kg object moving with an initial velocity of <math>5ms^{-1}</math> is acted on by a force of 2N. The force-displacement of the motion is shown in the graph below.</p>  <p>Determine the velocity of the object at 10m displacement.</p> <p>D. A car with mass 1500kg is moving with a constant force, <math>F</math> acting on it along its direction of motion. Upon achieving a speed of <math>20ms^{-1}</math>, it delivers a maximum power of <math>100kW</math>. Later the car enters a 50m rough road and decelerates to a speed of <math>10ms^{-1}</math>. Determine the constant force <math>F</math> and the work done to overcome the frictional force on the rough road.</p>

	<p>A. <math>m_1u_1 + m_2u_2 = (m_1 + m_2)v \Rightarrow (0.02)(800) + (5)(0) = (5 + 0.02)v \Rightarrow v = 3.19ms^{-1}</math></p> $\Sigma K_{initial} = \frac{1}{2}m_1u_1^2 = \frac{1}{2}(0.02)(800)^2 = 6.4kJ \quad [1]$ $\Sigma K_{final} = \frac{1}{2}m_2v^2 = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(5 + 0.02)(3.19)^2 = 25.542J \quad [2]$ <p>Since <math>\Sigma K_{initial} \neq \Sigma K_{final} \Rightarrow</math> inelastic collision</p> <p>B.</p> <p><math>\Sigma F_x = \Sigma F_y = 0</math></p> $\Sigma F_x = 0 \Rightarrow T = W_x + f = W \sin 30^\circ + f$ $\Sigma F_y = 0 \Rightarrow N = W_y = W \cos 30^\circ$ $f = \mu N = \mu W \cos 30^\circ = 0.5(40)(9.81) \cos 30^\circ$ $f = 169.9N$ $T = mg \sin 30^\circ + f$ $T = (40)(9.81) \sin 30^\circ + 169.9$ $T = 562.3N$ <p>C. Area under graph, <math>A = 5(2) + \frac{1}{2}(2)(5) = 15J</math></p> $W = \Delta K = \frac{1}{2}m(v^2 - u^2) \Rightarrow 15 = \frac{1}{2}(2)(v^2 - (5)^2) \Rightarrow v = 2\sqrt{10}ms^{-1}$ <p>D. <math>P = 100(10^3) = Fv = F(20) \Rightarrow F_{engine} = 5kN</math></p> $F_{net} = ma = m \frac{v^2 - u^2}{2s} = (1500) \frac{10^2 - 20^2}{2(50)} = -4.5kN$ $F_{engine} = F_{friction} + F_{net} \Rightarrow W_{engine} = W_{friction} + W_{net}$ $5kN(50) = W_{friction} + (4.5kN)(50) \Rightarrow W_{friction} = 25kN$
19/20	<p>A. The figure below shows a <math>0.52kg</math> ball P moving at <math>0.69ms^{-1}</math> collides with a stationary ball Q.</p> <p>Before collision Sebelum pelanggaran</p> <p>After collision Selepas pelanggaran</p> <p>After the collision, the velocity of the balls P and Q are <math>0.3ms^{-1}</math> and <math>0.45ms^{-1}</math> respectively. Determine the mass of ball Q.</p> <p>B. A man drags a <math>23kg</math> suitcase with a <math>45N</math> force at constant speed as shown in the figure below.</p> <p>The frictional force on the suitcase is <math>18N</math>. With the help of a free body diagram, calculate the coefficient of kinetic friction between the suitcase and the floor.</p> <p>C. The figure below shows a <math>15kg</math> blocks being pulled by a <math>100N</math> force at an initial speed of <math>2ms^{-1}</math> up an inclined plane.</p> <p>The block travels a distance of <math>6.2m</math> parallel to the inclined plan. The coefficient of kinetic friction is <math>0.14</math>. By using the work-energy theorem, calculate the change in the kinetic energy of the block.</p> <p>D. A <math>120kg</math> motorcycle accelerates uniformly from rest to <math>25ms^{-1}</math> in <math>5s</math>. Calculate the instantaneous power of the motorcycle at time <math>t = 3s</math>.</p>

	<p>A. 2 possible solutions:</p> <p>x: <math>m_p u_p = m_p v_p \cos 20^\circ + m_Q v_Q \cos 30^\circ</math>  <math>0.52(0.69) = (0.52)(0.3) \cos 20^\circ + m_Q(0.45) \cos 30^\circ</math>  <math>m_Q = 0.545\text{kg}</math></p> <p>y: <math>0 = m_p v_p \sin(20^\circ) - m_Q v_Q \sin(30^\circ)</math>  <math>(0.52)(0.3) \sin(20^\circ) = m_Q(0.45) \sin(30^\circ)</math>  <math>m_Q = 0.237\text{kg}</math></p> <p>B.</p> <p>constant speed: <math>\Sigma F_x = \Sigma F_y = 0</math>  <math>\Sigma F_x = 0</math>  <math>\Rightarrow f = 18 = T_x = T \cos \theta = 45 \cos \theta</math>  <math>\theta = 66.42^\circ</math>  <math>\Sigma F_y = 0</math>  <math>\Rightarrow N = W - T_y = mg - T \sin \theta</math>  <math>f = 18 = \mu N = \mu(mg - T \sin \theta)</math>  <math>18 = \mu((23)(9.81) - (45) \sin(66.42))</math>  <math>\mu = 0.098</math></p> <p>C.</p> <p><math>\Sigma F_y = 0</math>  <math>\Rightarrow N = W_y = W \cos 65^\circ = mg \cos 65^\circ = (15)(9.81) \cos 65^\circ</math>  <math>N = 133.4\text{N}</math>  <math>\Sigma F_x = F - f - W_x = F - \mu N - mg \sin 65^\circ</math>  <math>\Sigma F_x = 100 - (0.14)(133.4) - (15)(9.81) \sin 65^\circ = 19.1\text{N}</math>  <math>\Delta K = W = \Sigma F_x \cdot s = (19.1)6.2</math>  <math>\Delta K = 118.42\text{J}</math></p> <p>D. <math>v = u + at \rightarrow 25 = 0 + a(5) \rightarrow a = 5\text{ms}^{-2}</math>  <math>v(t = 3\text{s}) = u + at = 0 + 5(3) = 15\text{ms}^{-1}</math>  <math>P = Fv = ma(v) = (120)(5)(15)</math>  <math>P = 9\text{kW}</math></p>
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## Worksheet 11: PYQ Circular Motion

21/22	In a bike race, a racer and his bike of mass 230kg moves around a curve on a level track with a velocity of $80\text{kmh}^{-1}$ . If the radius of the curve is 90m, what is the frictional force acting on the bike at the curve?
	$F_c = F_f = \frac{mv^2}{r} \Rightarrow F_f = \frac{(230) \left(\frac{80 \times 1000}{60 \times 60}\right)^2}{90}$ $F_f = 126.2\text{N}$
20/21	A 20g stone tied at the end of an inelastic string rotates in a horizontal circle. The length of the string is 1.0m and the stone rotates with a constant angular velocity of 2 revolutions per second. <ul style="list-style-type: none"> <li>a. Draw a free body diagram of the stone.</li> <li>b. Calculate the centripetal acceleration of the stone.</li> </ul>
	<ul style="list-style-type: none"> <li>a. Refer to lecturer</li> <li>b.</li> </ul> $a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 = (1)(2 \times 2\pi)^2$ $a = 16\pi^2 \approx 157.9\text{ rads}^{-2}$
19/20	A 16g ball is swung vertically using a 0.5m string. Calculate the <ul style="list-style-type: none"> <li>a. Minimum tension in the string if the speed of the ball is <math>1.5\text{ms}^{-1}</math></li> <li>b. Speed of the ball when the string breaks.</li> </ul>
	a.

Minimum tension  $\rightarrow$  ball position at top  $\Rightarrow F_c = \frac{mv^2}{r} = T + W$

$$\frac{0.016(1.5^2)}{0.5} = T + (0.016)(9.81) \Rightarrow T = -0.085N$$

b.

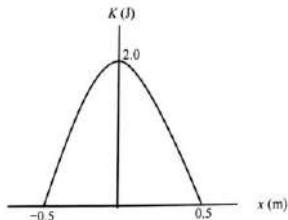
$$a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 = (1)(2 \times 2\pi)^2$$

$$a = 16\pi^2 \approx 157.9 \text{ rads}^{-2}$$

## Worksheet 17: PYQ Waves

21/22

- A. A mass of 200g is attached to a spring. When the mass displaced a certain distance from equilibrium and released, it oscillates at a period of 0.85s. What is the constant of the spring?
- B. The figure shows a particle of mass 4.0kg moves in simple harmonic motion and its kinetic energy,  $K$  varies with position,  $x$ .



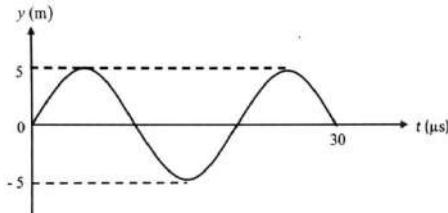
Determine the period.

- C. What is the speed of a transverse wave in a rope of length 5.00m and mass 55.00g under a tension of 600.00N?
- D. A transverse wave is represented by the following equation:

$$y = 7 \sin(5t - 3x)$$

where  $y$  and  $x$  are measured in centimetres and  $t$  in seconds. What is the maximum vibrational velocity of a particle in the wave?

- E. The figure shows how the displacement,  $y$  of a particle varies with time,  $t$  when a wave passes through the particle at speed  $6\text{km s}^{-1}$ . The wave is reflected and superimposed with an incident wave.



What is the equation of the standing wave formed?

- F. The security alarm in a parking area produces a siren with frequency of 980Hz. As a car drives away, the driver observes the frequency changes to 850Hz. The speed of sound is in  $345\text{ms}^{-1}$ . What is the speed of the car?

A.  $T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow 0.85 = 2\pi \sqrt{\frac{0.2}{k}} \Rightarrow k = 10.93 \text{ Nm}^{-1}$

- B. When  $x = A$ ,

$$K = E_{total} = \frac{1}{2}m\omega^2 A^2$$

$$K = \frac{1}{2}m \left(\frac{2}{T}\right)^2 A^2 \Rightarrow 2 = \frac{1}{2}(4) \left(\frac{2}{T}\right)^2 (0.5)^2 \Rightarrow T = 3.1416\text{s}$$

- C.

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T l}{m}} = \sqrt{\frac{600(5)}{0.055}} \Rightarrow v = 233.55\text{ms}^{-1}$$

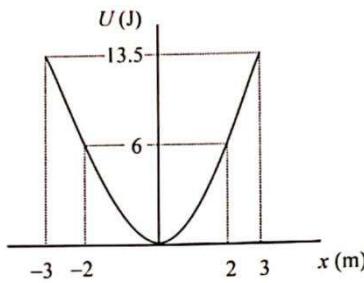
- D.

$$v_{vibration} = \frac{dy}{dt} = \frac{d}{dt}(7 \sin(5t - 3x))$$

$$v_{vibration} = 35 \cos(5t - 3x)$$

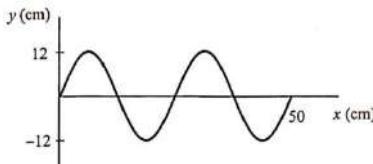
$$v_{max-vibration} = 35\text{cm s}^{-1}$$

	<p>E. From diagram <math>\Rightarrow T = 20 \mu s</math>; <math>A = 5m</math>; <math>v = 6km s^{-1} = 6000ms^{-1}</math>  General equation for standing wave: <math>y(x, t) = 2A \cos(kx) \sin(\omega t)</math></p> $k = \frac{2\pi}{\lambda} = \frac{2\pi}{vT} = \frac{2\pi}{(6000)(20(10^{-6}))} = \frac{50}{3}\pi m^{-1}$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{20(10^{-6})} = \pi(10^{-5}) \text{ rad s}^{-1}$ $y(x, t) = 10 \cos\left(\frac{50}{3}\pi x\right) \sin(\pi(10^{-5})t)$ <p>Where x is in metres and t is in seconds.</p> <p>F.</p> $\frac{f_{app}}{f_{source}} = \frac{v \pm v_{observer}}{v \mp v_{source}}$ $\frac{850}{980} = \frac{345 - v_{observer}}{345 + 0} \Rightarrow v_{observer} = 45.77ms^{-1}$
20/21	<p>A. The figure shows the displacement-time graph of a body performing simple harmonic motion.</p> <p>a. Determine the amplitude, angular frequency and maximum speed of the motion.  b. Deduce the expression for the motion.  c. Sketch the velocity-time graph for the motion.</p> <p>B. A stretched wire of length 1.0m is fixed at both ends. The speed of the transverse wave in the wire is <math>10ms^{-1}</math>. If the mode of vibration is third overtone, calculate the  a. Wavelength  b. Frequency of the third overtone  c. Lowest resonant frequency of the wire.</p> <p>C. A train with a velocity of <math>40ms^{-1}</math> is approaching an observer standing on a platform. The frequency of the siren from the train is 1600Hz. Assuming the speed of sound in air is <math>330ms^{-1}</math>, determine the  a. Frequency of the sound heard by the observer.  b. Frequency of the sound heard by the observer when the train is leaving the platform.</p>
	<p>A.</p> $A = 2m; T = 5s$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{5} \Rightarrow \omega = 0.4\pi \text{ rads}^{-1}$ $v = \frac{dy}{dt} = \frac{d}{dt}(2 \sin 0.4\pi t)$ $v_{max} = 0.8\pi \approx 2.51ms^{-1}$ $y(t) = 2 \sin 0.4\pi t$ <p>B.</p> $2\lambda = L \Rightarrow \lambda = 0.5L = 0.5(1) \Rightarrow \lambda = 0.5m$ $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{10}{0.5} \Rightarrow f = 20Hz$ <p>Lowest resonant <math>\Rightarrow</math> fundamental frequency</p> $\lambda = 2L = 2m \Rightarrow f = \frac{v}{\lambda} = \frac{10}{2} \Rightarrow f = 5Hz$ <p>C.</p> $\frac{f_{observed}}{f_{source}} = \frac{v \pm v_{observer}}{v \mp v_{source}}$ $\frac{f_{observed}}{f_{source}} = \frac{330 \pm 0}{330 - 40}$ $f_{observed} = 1820.69Hz$ $\frac{f_{observed}}{f_{source}} = \frac{v \pm v_{observer}}{v \mp v_{source}}$ $\frac{f_{observed}}{f_{source}} = \frac{330 \pm 0}{330 + 40}$ $f_{observed} = 1427.03Hz$
19/20	A. The figure shows the potential energy of 0.5kg object that undergoes a simple harmonic motion.



Determine the

- Velocity when time  $t = 2s$
  - Kinetic energy of the object when displacement  $x = 1.5m$
- B. An oscillating pendulum has length 0.3m and 240g bob. If the total energy is 0.06J, calculate the amplitude of the oscillation.
- C.



The figure shows a graph of displacement  $y$  against distance  $x$  for a progressive wave propagating to the right in a string with mass 920g, length 3m and tension 15N. Determine the wave equation.

- D. A 1.53m closed pipe makes a humming sound at frequency 282Hz when the wind blows across the open end. The speed of sound is  $343ms^{-1}$ . With the help of a diagram, determine the number of nodes in the standing wave.
- E. The frequency of whistle by a moving train and the frequency heard by a stationary observer are 520Hz and 460Hz respectively. If the speed of sound in the air is  $343ms^{-1}$ , calculate the speed of the train.

A.

$$A = 3$$

$$U(x) = \frac{1}{2}m\omega^2x^2 \Rightarrow 13.5 = \frac{1}{2}(0.5)\omega^2(3^2)$$

$$\omega = 2.45 \text{ rads}^{-1}$$

$$\text{General equation } \Rightarrow y(t) = A \sin(\omega t) = 3 \sin(2.45t)$$

$$v = \frac{dy}{dt} = \frac{d}{dt}(3 \sin(2.45t)) = 7.35 \cos(2.45t)$$

$$v(t = 2s) = 7.35 \cos(2.45(2)) \Rightarrow v(t = 2s) = 1.37ms^{-1}$$

$$K = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$K(x = 1.5m) = \frac{1}{2}(0.24)(2.45)^2(3^2 - 1.5^2)$$

$$K(x = 1.5m) = 10.129J$$

B.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\sqrt{\frac{l}{g}}} \Rightarrow \omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{0.3}} = 5.718 \text{ rads}^{-1}$$

$$E_{total} = \frac{1}{2}m\omega^2A^2 \Rightarrow 0.06 = \frac{1}{2}(0.24)(5.718)^2A^2 \Rightarrow A = 0.124m$$

C. General Equation  $\Rightarrow y(x, t) = A \sin(\omega t \pm kx)$

$$A = 12 \text{ cm}; k = \frac{2\pi}{\lambda} = \frac{2\pi}{25} \text{ cm}^{-1}$$

$$\omega = 2\pi f = 2\pi \left( \frac{v}{\lambda} \right) = 2\pi \left( \frac{\left( \frac{F_g}{\mu} \right)}{\lambda} \right) = 2\pi \left( \frac{\left( \frac{15}{0.92} \right)}{0.25} \right) = 55.95\pi \text{ rads}^{-1}$$

$$y(x, t) = 12 \sin \left( 55.95\pi t - \frac{2}{25}\pi x \right), \text{ where } x \text{ and } y \text{ are in centimetres and } t \text{ is in seconds.}$$

D.  $L = 1.53m; f_n = 282 \text{ Hz}; v_{sound} = 343ms^{-1}$   
For fundamental frequency  $\Rightarrow L = \frac{\lambda}{4} \Rightarrow \lambda = 4L \Rightarrow v = f_0\lambda_0 \Rightarrow f_0 = \frac{v}{\lambda_0} = \frac{v}{4L} = \frac{343}{4(1.53)} = 56 \text{ Hz}$

$$n = \frac{f_n}{f_0} = \frac{282}{56} \approx 5 \Rightarrow 5^{\text{th}} \text{ Harmonic} \Rightarrow 3 \text{ nodes}$$

## Worksheet 20: PYQ Physics of Matter

21/22	<p>A. A 5.0m long wire a cross sectional area of <math>4.0 \times 10^{-4} m^2</math>. The wire extended by 0.5cm. Calculate the Young's modulus of the wire when a 200kg is suspended at its one end.</p> <p>B. An aluminium rod of radius 0.5cm and length 20cm is welded end-to-end with a steel rod of the same dimensions. The free end of the aluminium rod is held at <math>100^\circ C</math> while the steel free end is placed in a ice bath. When the system is at steady state, calculate the temperature at the aluminium-steel interface. [Given the thermal conductivity of aluminium is <math>250 W m^{-1} K^{-1}</math> and the thermal conductivity of steel is <math>14 W m^{-1} K^{-1}</math>]</p> <p>C. An aluminium tube of external diameter 3.00cm at <math>25^\circ C</math> is heated to <math>80^\circ C</math>. Calculate the external area of the tube at <math>80^\circ C</math> if the coefficient of linear expansion for aluminium is <math>2.4 \times 10^{-5} K^{-1}</math>.</p>
	<p>A. <math display="block">Y = \frac{Fl}{A\Delta l} = \frac{(200)(9.81)(5)}{(4 \times 10^{-4})0.5(10^{-2})} = 4.905 \times 10^9 N m^{-2}</math></p> <p>B.</p> $\frac{dQ_{Al}}{dt} = \frac{dQ_{steel}}{dt} \Rightarrow k_{Al}A_{Al} \frac{(T_1 - T)}{l_{Al}} = k_{steel}A_{steel} \frac{(T - T_2)}{l_{steel}}$ $k_{Al}(T_1 - T) = k_{steel}(T - T_2) \Rightarrow 250((100 + 273.15) - T) = 14(T - (0 + 273.15))$ $T = 367.85K$ <p>C.</p> $A_o = \pi \left(\frac{0.03}{2}\right)^2 = 0.000225\pi; \frac{\Delta A}{A_o} = \frac{A_f - A_o}{A_o} = 2\alpha\Delta T$ $\frac{A_f - 0.000225\pi}{0.000225\pi} = 2(2.4 \times 10^{-5})(80 - 25) \Rightarrow A_f = 0.00708724 m^2$
20/21	<p>A. A 1.5m steel wire is stretched 2.0mm by force <math>F</math>. The diameter of the steel wire is 4.0mm. The Young's modulus of steel is <math>2.0 \times 10^{11} N m^{-2}</math>. Determine the force <math>F</math> applied on the wire.</p> <p>B. A perfectly insulated aluminium rod has length 50cm and cross-sectional area <math>3.0 cm^2</math>. At the steady state, the temperatures at 0cm and 50cm ends are <math>150^\circ C</math> and <math>50^\circ C</math>. (Thermal conductivity of aluminium is <math>210 W m^{-1} K^{-1}</math>)</p> <ol style="list-style-type: none"> <li>Sketch a labelled graph of temperature against distance.</li> <li>Calculate the temperature gradient along the rod.</li> <li>Calculate the rate of heat flow in the rod.</li> </ol>
	<p>A. <math display="block">Y = \frac{Fl}{A\Delta l} = \frac{Fl}{\left(\frac{\pi d^2}{4}\right)\pi\Delta l} \Rightarrow 2(10^{11}) = \frac{F(1.5)}{\left(\frac{(4 \times 10^{-3})^2}{4}\right)\pi(0.002)} \Rightarrow F = 3351N</math></p> <p>B.</p> <p>a.</p> $\frac{dT}{dx} = \frac{323.15 - 423.15}{0.5 - 0} = -200 K m^{-1}$ <p>b.</p> $\frac{dQ}{dt} = -k_{Al}A_{Al} \frac{dT}{dx} = -(210)(3(10^{-2})^2)(-200)$ $\Rightarrow \frac{dQ}{dt} = 63.5 J s^{-1} \text{ (towards the cold end)}$
19/20	<p>A. The diameter of a circular shoe heel is 13mm. If both heels support 70% of the weight of a 54kg woman, calculate the stress on both heels.</p> <p>B. A gold rod is in contact with a silver rod. The gold end and the silver end of the compound is at <math>90^\circ C</math> and <math>30^\circ C</math> respectively. The silver rod has thermal conductivity <math>427 W m^{-1} K^{-1}</math>, length 2.5cm and cross-sectional area <math>7.85 \times 10^{-5} m^2</math>. If 341.3J heat flows through the gold rod in 10s, calculate the temperature at the contact surface.</p> <p>C. The area of a metal plate changes from <math>120 m^2</math> to <math>120.059 m^2</math> when the temperature increases by <math>30^\circ C</math>. Calculate the coefficient of linear expansion of the metal.</p>
	<p>A.</p> $\delta = \frac{F}{A} = \frac{0.7(54)(9.81)}{2\left(\frac{13(10^{-3})}{2}\right)^2} = 1.397 \times 10^6 N m^{-2}$ <p>B.</p>

	$\frac{dQ_{gold}}{dt} = \frac{dQ_{silver}}{dt} = \frac{341.3J}{10s} = 34.13Js^{-1} = -k_{silver}A_{silver} \frac{dT}{dx} = -(427)(7.85 \times 10^{-5}) \frac{30-T_2}{0.025}$ $\Rightarrow T_2 = 55.46^{\circ}\text{C}$
C.	$\frac{\Delta A}{A_0} = \frac{A_f - A_0}{A_0} = 2\alpha\Delta T \Rightarrow \frac{120.059 - 120}{120} = 2\alpha(30) \Rightarrow \alpha = 8.19444(10^{-6})\text{K}^{-1}$

### Worksheet 23: PYQ Molecular Energies & Thermodynamics

21/22	<p>A. What is the pressure of one mole ideal gas in the container of volume <math>4 \times 10^{-4}\text{m}^3</math> at temperature <math>363.15\text{K}</math>?</p> <p>B. Given the molar mass of oxygen is <math>32\text{ g mol}^{-1}</math>. What is the root mean square speed of the oxygen molecules at a temperature of <math>333\text{K}</math>.</p> <p>C. A balloon contains helium gas at <math>30^{\circ}\text{C}</math> and <math>2 \times 10^{-5}\text{ Pa}</math>. Calculate the number of helium gas molecules per unit volume.</p> <p>D. The figure shows a graph of pressure, <math>p</math> against volume, <math>V</math> of an ideal gas.</p> <p>When the gas changes from state X to state Y, the amount of heat transfer into the gas is <math>1.0\text{ kJ}</math>. What is the internal energy of the gas?</p> <p>E. The figure shows a graph of pressure versus volume of an ideal gas undergoing a cyclic thermodynamic process ABCDA.</p> <p>Calculate the work done by the gas.</p>
	<p>A. <math>pV = nRT \Rightarrow p = \frac{nRT}{V} = \frac{(1)(8.31)(363.15)}{4(10^{-4})}</math>  <math>\Rightarrow p = 75.4(10^5)\text{ Pa} \approx 75.4\text{ atm}</math></p> <p>B. <math>1\text{ mol} = 32\text{ g} = 6.02 \times 10^{23}\text{ oxygen gas molecules}</math>  <math>1\text{ molecule} = \frac{32}{6.02(10^{23})}(10^{-3})\text{ kg}</math>  <math>\frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_B T</math>  <math>\Rightarrow \frac{1}{2}\left(\frac{32}{6.02(10^{23})}(10^{-3})\right)v_{rms}^2 = \frac{3}{2}(1.38 \times 10^{-23})(333)</math>  <math>v_{rms} \approx 509.27\text{ ms}^{-1}</math></p> <p>C. <math>pV = Nk_B T \Rightarrow \frac{N}{V} = \frac{p}{k_B T} = \frac{2(10^{-5})}{(1.38 \times 10^{-23})(30+273.15)}</math>  <math>\frac{N}{V} = 4.78(10^{15})\text{ molecules m}^{-3}</math></p> <p>D. <math>\Delta U = \Delta Q + \Delta W = \Delta Q - \int_{V_i}^{V_f} p dV</math>  <math>\Delta U = \Delta Q - p(V_f - V_i)</math>  <math>\Delta U = (+10^3) - (20)(10^3)(75 - 50)(10^{-3})</math>  <math>\Delta U = 500\text{ J}</math></p> <p>E. <math>W_{total} = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow D} + W_{D \rightarrow A}</math>  <math>W_{total} = 0 + 2P(3V - V) + 0 + P(V - 3V)</math>  <math>W_{total} = 2PV</math></p>
20/21	<p>1. A container contains <math>3.0\text{ mol}</math> of nitrogen gas at <math>30^{\circ}\text{C}</math>. If nitrogen gas behaves like an ideal gas, calculate the</p>

	<p>a. Total translational kinetic energy of the gas molecules.      b. Internal energy of the gas.      c. Root mean square speed of the nitrogen molecules if the mass is 28 g per mole.</p> <p>2. The pressure of a tyre rises from 200kPa to 400kPa at constant temperature <math>30^{\circ}C</math>. Assuming the air in the tyre acts as an ideal gas, calculate the      a. Work done per mole of the air.      b. Heat transferred in this process.</p>
	<p>1.</p> <p>a. <math>\Sigma K = \frac{3}{2} nRT = 3 \left(\frac{3}{2}\right) (8.31)(30 + 273.15)</math>  <math>\Rightarrow \Sigma K = 11.336 kJ</math></p> <p>b. <math>U = \frac{5}{2} nRT = \frac{5}{2} (8.31)(30 + 273.15)</math>  <math>\Rightarrow U = 18.893 kJ</math></p> <p>c. <math>\frac{1}{2} \left(\frac{28 \times 10^{-3}}{6.02 \times 10^{23}}\right) v_{rms}^2 = \frac{3}{2} (1.38 \times 10^{-23})(30 + 273.15)</math>  <math>\Rightarrow v_{rms} \approx 519.455 ms^{-1}</math></p> <p>2.</p> <p>a. Work done by the gas,  <math>W = \int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \ln\left[\frac{V_f}{V_i}\right]</math>      Constant temperature <math>\Rightarrow p_i V_i = p_f V_f</math>  <math>W = nRT \ln\left[\frac{p_i}{p_f}\right] = (1)(8.31)(30 + 273.15) \ln\left[\frac{200000}{400000}\right]</math>  <math>\Rightarrow</math> Work done by the gas, <math>W = -1746 J</math></p> <p>b. Constant temperature <math>\Rightarrow \Delta U = 0</math>  <math>\Delta U = \Delta Q - \Delta W \Rightarrow \Delta Q = \Delta W = -1746 J</math></p>
19/20	<p>A sealed cylinder contains <math>1.2 \times 10^{24}</math> helium atoms at initial pressure <math>1.04 \times 10^5 Pa</math>. The cylinder is heated until the final temperature and the change in the internal energy of the helium gas are <math>315K</math> and <math>1.6 \times 10^3 J</math>. The molar mass of helium is <math>4 g mol^{-1}</math>. Calculate the</p> <ol style="list-style-type: none"> <li>Density of the helium gas.</li> <li>Final pressure of the helium gas.</li> </ol> <p>B. A <math>0.8m^3</math> container at <math>60^{\circ}C</math> is filled with 0.6 mol ideal gas. The gas is isothermally compressed to a volume of <math>0.2m^3</math>. Then the gas expands isobarically to its initial volume. Calculate the total work done in the processes.</p> <p>A.</p> <p>i. <math>N = 1.2(10^{24})</math> atoms; <math>p_i = 1.05(10^5) Pa</math>;  <math>\Delta V = 0</math>; <math>T_f = 315K</math>; <math>\Delta U = 1.6(10^3) J</math></p> $\rho = \frac{m_{gas}}{V}; n = \frac{N}{N_A} = \frac{12 \times 10^{23}}{6.02 \times 10^{23}} = 1.993 mol;$ $n = \frac{m_{gas}}{m_{molar}} = \frac{m_{molar}}{\rho V}$ $pV = nRT = \left(\frac{\rho V}{m_{molar}}\right) RT \Rightarrow p_i = \left(\frac{\rho}{m_{molar}}\right) RT_i$ $\Delta U = \frac{3}{2} nR(T_f - T_i) \Rightarrow T_i = T_f - \frac{2\Delta U}{3nR}$ $p_i = \left(\frac{\rho}{m_{molar}}\right) R \left(T_f - \frac{2\Delta U}{3nR}\right)$ $\Rightarrow 1.05(10^5) = \left(\frac{\rho}{0.004}\right) (8.31) \left(315 - \frac{2(1.6)(10^3)}{3(1.993)(8.31)}\right)$ $\rho = 0.202 kg m^{-3}$ <p>iii. <math>\Delta V = 0 \Rightarrow \frac{p}{T} = \frac{nR}{V} = \text{constant}</math></p> $\frac{p_i}{T_i} = \frac{p_f}{T_f} \Rightarrow \frac{1.05(10^5)}{\left(315 - \frac{2(1.6)(10^3)}{3(1.993)(8.31)}\right)} = \frac{p_f}{315}$ $p_f = 131986 Pa$ <p>B.</p>

$$\begin{aligned}
W_{total} &= W_1 + W_2 \\
W_{total} &= \left( \int_{0.8}^{0.2} p \, dV \right) + \left( \int_{0.2}^{0.8} p \, dV \right) \\
W_{total} &= \left( \int_{0.8}^{0.2} \frac{nRT}{V} \, dV \right) + \left( p_2 \int_{0.2}^{0.8} \, dV \right) \frac{p_1}{p_2} = \frac{\left( \frac{nRT_1}{0.8} \right)}{p_2} = \frac{0.8}{0.2} = 4 \\
W_{total} &= \left( nRT \ln \left( \frac{0.2}{0.8} \right) \right) + \left( \frac{nRT_1}{4(0.8)} (0.8 - 0.2) \right) \\
W_{total} &= \left( (0.6)(8.31)(60 + 273.15) \ln \left( \frac{0.2}{0.8} \right) \right) + \\
&\quad \frac{(0.6)(8.31)(60+273.15)}{4(0.8)} (p(0.8 - 0.2)) \\
\text{Work done by the gas, } W_{total} &= -1991.3J
\end{aligned}$$

