Study Notes
&
Exercises
On Linear Motion

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Chapter 1: Physical Quantities & Measurements

Learning Outcomes

1. Define dimension, scalar and vector quantities.
2. Determine:
    (a) the dimensions of derived quantities.
    (b) resultant of vectors. (remarks: limit to three vectors only).
3. Verify the homogeneity of equations using dimensional analysis.
4. Resolve vector into two perpendicular components (*x* and *y* axes).

Dimensions

Dimensions refer to the physical nature of a quantity. Regardless of the unit used, the physical nature of a quantity remains the same. For example, a distance, measured in the unit of metres, or feet, is still a measurement of length. This measurement, therefore has the dimensions of length, most commonly represented by **L.** In this instance, the equation $[d] = L$ is simply states that "The dimension of d is length (**L**)". The following table shows some selected physical quantities and its dimensions:

Base Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Length	L	metre (m)	L
Mass	M	kilogram (kg)	M
Time	T	second (s)	T
Electric Current	I	ampere (A)	I
Temperature	T	Kelvin (K)	Θ
Amount of substance	n	mole (mol)	N
Luminosity	L	candela (cd)	J
Derived Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Velocity	$\vec{v}$	ms^{-1}	LT^{-1}
Acceleration	$\vec{a}$	ms^{-2}	LT^{-2}
Momentum	$\vec{p}$	Ns	MLT^{-1}
Angular acceleration	α	$rads^{-1}$	T^{-2}
Electric Charge	Q	Coulomb (A s)	TI
Energy	E	Joule ($J = kgm^2s^{-2}$)	ML^2T^{-2}

Once you understand what dimensions are and how to work with them, you can apply it to **verify the homogeneity of equations.** The word 'homogeneity' refers to 'of the same kind'. Let us consider the equation $s = ut + \frac{1}{2}at^2$ where s is displacement of a body, t is time taken for the displacement of the body, u is the initial velocity of the body and a is the acceleration of the body. To 'verify homogeneity', we can compare the dimensions the terms on the left hand side and the right hand side of the equation. That is to say, s must have the same dimensions as ut and $\frac{1}{2}at^2$. s has the dimension of L, so does ut as well as $\frac{1}{2}at^2$.

Sample Problem 1.1:

Identify the dimensions for power, *P*, defined by $P = \frac{E}{t}$ where E is energy and has dimensions of ML^2T^{-2} and *t* has dimension of time, *T*.

Solution:

$$[Power] = \left[\frac{E}{t}\right]$$

$$[Power] = \frac{ML^2T^{-2}}{T}$$

$$[Power] = ML^2T^{-3}$$

The dimensions for power are then ML^2T^{-3}.

Scalars and Vectors

A scalar quantity is a quantity that is fully described by its magnitude. On the other hand, a vector quantity can only be fully described by both its magnitude **and** direction. When you talk about 5kg of rice, that statement is sufficient to describe the mass of the rice, this is where you can see that mass is a scalar quantity. If the rice is falling towards the Earth at a velocity of $2ms^{-1}$, 2 things matter here – how fast the rice is falling **and** the direction in which it is falling. Here you can see that velocity is a vector quantity.

A vector quantity is generally represented by a line segment with an arrowhead. The length of the line segment indicates its magnitude whereas its arrow head tells us the direction of the vector quantity. For example, say vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by the following arrows,
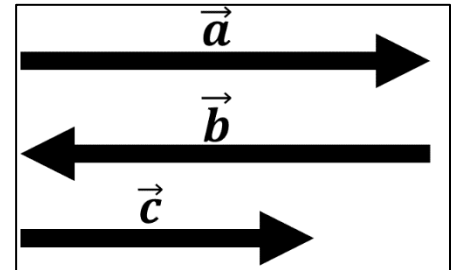
If we were to compare $\vec{b}$ and $\vec{c}$ to $\vec{a}$, we'd say that vector $\vec{b}$ has the same magnitude as $\vec{a}$ but is in the opposite direction, this would tell us that $\vec{b} = -\vec{a}$. Vector $\vec{c}$, on the other hand, is in the same direction as $\vec{a}$. But its magnitude is smaller that $\vec{a}$. The magnitude of vector $\vec{k}$ is denoted by $|\vec{k}|$. We can then relate vector $\vec{a}$ to $\vec{c}$ by the relation $|\vec{a}| > |\vec{c}|$.
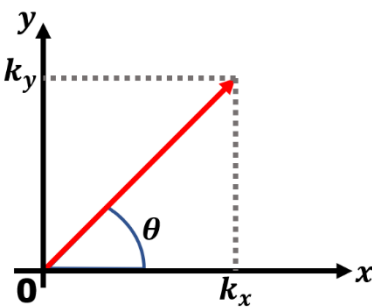
Another method to represent vectors is to list the values of its elements in a sufficient number of difference directions, depending on the dimension of the vector. Consider a vector in a 2-dimensional Cartesian coordinate system, a vector $\vec{k}$ can then be represented by $\vec{k} = k_x\hat{\imath} + k_y\hat{\jmath}$ or $\vec{k} = <k_x, k_y>$, defining $\hat{\imath}$ and $\hat{\jmath}$ as unit vectors in the x and y directions respectively. From this notation, one can easily calculate the magnitude (length) of the 2-vector using Pythagoras' Theorem which gives

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2}.$$

Vector additions (or subtractions) can then be done by adding (or subtracting) corresponding components. That is to say, if we have vectors $\vec{a}$ and $\vec{b}$ defined by $\vec{a} = <a_x, a_y>$; $\vec{b} = <b_x, b_y>$, then the addition will yield

$$\vec{a} + \vec{b} = <a_x + b_x, a_y + b_y>.$$

The implication of this definition of vector addition are the following rules:

1. Commutativity of vectors $\Rightarrow \vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. Associativity of vectors $\Rightarrow \left(\vec{a} + \vec{b}\right) + \vec{c} = \vec{a} + \left(\vec{b} + \vec{c}\right)$
3. $\vec{a} + (-\vec{a}) = 0$

Resolution of vector $\vec{k}$ is then simply

$$k_x = |\vec{k}|cos(\theta); \; k_y = |\vec{k}|sin(\theta).$$

Multiplication of a vector

3 cases to consider when talking about multiplication of a vector:

1. The vector is multiplied by a scalar, then
$$k\vec{a} = < ka_x, ka_y >.$$
2. The **dot product** (also known as scalar or inner product) of two vectors, then
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = |\vec{a}||\vec{b}| \, cos(\theta_{ab}).$$
Note that in the dot product, the operation results in a scalar quantity.
3. The **cross product** (also known as the vector product) of two vectors, then
$$\vec{a} \times \vec{b} = \left(a_x b_y - a_y b_x\right)\hat{n} = |\vec{a}||\vec{b}|sin(\theta)\hat{n}$$
Note that in the cross product, the operation results in a vector quantity perpendicular to both the x and y axis.

Sample Problem 1.2:

Calculate the magnitude and direction of vector $\vec{c}$ if it is defined by $\vec{c} = \vec{a} + \vec{b}$ where $\vec{a} = [2,3]$ and $\vec{b} = [-1,4]$.

Solution:

Magnitude:
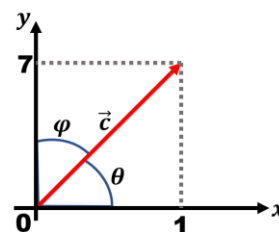$$\vec{c} = < 2 + (-1), 3 + 4 \geq < 1,7 >$$
$$|\vec{c}| = \sqrt{7^2 + 1^2} \approx 7.071$$

Direction:
$$\theta = tan^{-1}\left(\frac{7}{1}\right) \approx 81.87^o$$

$$\varphi = tan^{-1}\left(\frac{1}{7}\right) \approx 8.13^o$$

Additional notes:

Unit conversions are so easy that we tend to overlook the importance of practicing it. Here's a simple reminder on how to do it. Say you are given that $1 \, in = 2.54cm$ and you are asked to calculate 8cm in inches, here's how to do it:

$$1in = 2.54cm \leftarrow \text{divide both side by 2.54}$$

$$\frac{1}{2.54} in = \frac{2.54}{2.51} cm = 1cm \leftarrow \text{now multiply it by 8}$$

$$8cm = \frac{8}{2.54} in \approx 3.1496in$$

And that's how you do unit conversion.

In Physics, it is quite often that we are expected to work with values in form of scientific notation. For example, rather than writing down the speed of light as $300000000ms^{-1}$, we'd express this value as $3 \times 10^8 ms^{-1}$. One issue that may arise from working with scientific notation when using a calculator is the redundancy in typing out "10^x". To help with this, I would suggest that we take advantage of the **rules for exponents**. The example below demonstrates such application.

Sample Problem 1.3:

Evaluate c given that $c = \frac{a^2 b}{d}$ and $a = 3 \times 10^{-6}$, $b = 4 \times 10^3$ and $d = 12 \times 10^3$.

Solution

$$c = \frac{a^2 b}{d} = \frac{(3 \times 10^{-6})^2 (4 \times 10^3)}{(12 \times 10^3)}$$

Rather than evaluating this monstrosity as our input for the calculator, we can instead separate the coefficient from the base and its exponent to evaluate them separately.

$$c = \frac{a^2 b}{d} = \frac{(3)^2 (4)}{(12)} \left[\frac{(10^{-6})^2 (10^3)}{10^3} \right]$$

In this form, the evaluation can be done **easily** even without a calculator.

$$c = \frac{(3)^2 (4)}{(12)} \left[\frac{(10^{-6})^2 (10^3)}{10^3} \right] = \frac{36}{12} [10^{-6-6+3-3}]$$

On Significant Figures

When we talk about the number of significant figures, we are talking about the number of digits whose values are known with certainty. This gives us information about the degree of accuracy of a reading in a measurement. In general, we should practice performing rounding off when the conditions call for it. This is to avoid false reporting. What we mean by false reporting is to give the illusion that our experiments are more sensitive than it actually it. For example, it would be very unlikely that our metre ruler to give reading in the micro scale.

Number	Number of significant figures		Number	Number of significant figures
2.32	3		2600	2
2.320	4		2602	4

When we do calculations, there are some rules (based on the operations) we should be aware of when stating the significant figures of the end value:

1. Multiplication / Divisions – number of significant figures in the result is the same as the least precise measurement in the least precise measurement used in the calculation.
    Example:

    $$\frac{2.5(3.15)}{2.315} = 3.4$$

2. Addition / Subtraction – The result has the same number of decimal places as the least precise measurement used in the calculation.
    Example:
    $$91.1 + 11.45 - 12.365 = 90.2$$

3. Logarithm / antilogarithm – Keep as many significant figures to the right of the decimal point as the are significant in the original number.
    Example:
    $$ln(4.00) = 1.39; \; e^{0.0245} = 1.03$$

Exercises

1. Two vectors lie on the x-y plane. Vector $\vec{a}$ is 5 units long and points 25° above the x-axis. Vector $\vec{b}$ is 8 units long and points upward 35° above the x-axis. What is the resultant of the two vectors?

2. Show that the equation $v^2 = v_o^2 + 2ax$ is dimensionally correct. In this expression, a is acceleration, v and v_o are velocities and x is the distance.

3. Derive the dimensions of k if $k = \frac{ab}{d}$ where $[a] = M^2T^2$, $[b] = M^{-1}T^{-1}$ and $[d] = MT^2$.

4. A car is travelling at 35miles per hour, what is its speed in metres per second if $1mile = 1.61km$.

5. Express each of the following in grams
   a. $3lb$ if $1g = 2.2 \times 10^{-3}lb$
   b. $12.2oz$ if $1g = 0.035oz$
6. How many square centimetres are in a square inch if $1cm = 0.3937inch$.
7. The length and the width of a rectangle is $(11 \pm 0.05)cm$ and (21 ± 0.05) cm respectively. Find the area (and its uncertainty) of the rectangle.

Chapter 2: Kinematics of Linear Motion

Learning Outcomes (LO)

1. Define:
   a. instantaneous velocity, average velocity and uniform velocity; and
   b. instantaneous acceleration, average acceleration and uniform acceleration.
2. Derive and apply equations of motion with uniform acceleration

   $$v = u + at \; ; \; v^2 = u^2 + 2as \; ; \; s = ut + \frac{1}{2}at^2 \; ; \; s = \frac{1}{2}(u + v)t$$

3. Describe projectile motion launched at an angle, θ as well as special cases when θ=0°
4. Solve problems related to projectile motion.

I

n this chapter, we talk about kinematics of linear motion. Dynamics is the study of motion of bodies under action of forces and their effects. One subbranch to the study of dynamics is kinematics. In the study of kinematics, we consider only the motion of the bodies without worrying too much about the forces that caused the bodies to move. We only worry about the geometry of the motion.

Instantaneous and Average Velocity (or acceleration)

Let us start with reminder of some ideas and terms that you have learnt in your SPM days. 3 mains terms – displacement (), velocity () & acceleration (). Displacement, denoted by x, simply refers to the change in position of a body. Velocity, v, refers to the rate of change of this change in position, i.e. $v = \frac{dx}{dt}$. Acceleration, a, is defined by the rate of change of velocity, which is the rate of change of the rate of change of position. That is $a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$.

Once we have established that, we can further extend our ideas of velocity and acceleration by thinking about ***instantaneous*** velocity (or acceleration) and ***average*** velocity (or acceleration). By 'instantaneous', we mean 'at a particular instant in time'. When we combine it with velocity (or acceleration), what we mean is velocity (or acceleration) at a particular instant in time. On the other hand, when we say 'average', what we mean is 'over the course of a defined time span'. So, when we say 'average velocity', we usually would accompany it with 'between time t_a and t_b' or 'in 30 seconds', specifying a range of time.

Let us now have a graphical representation. Consider a body moving at constant velocity,

Displacement of Body with Time
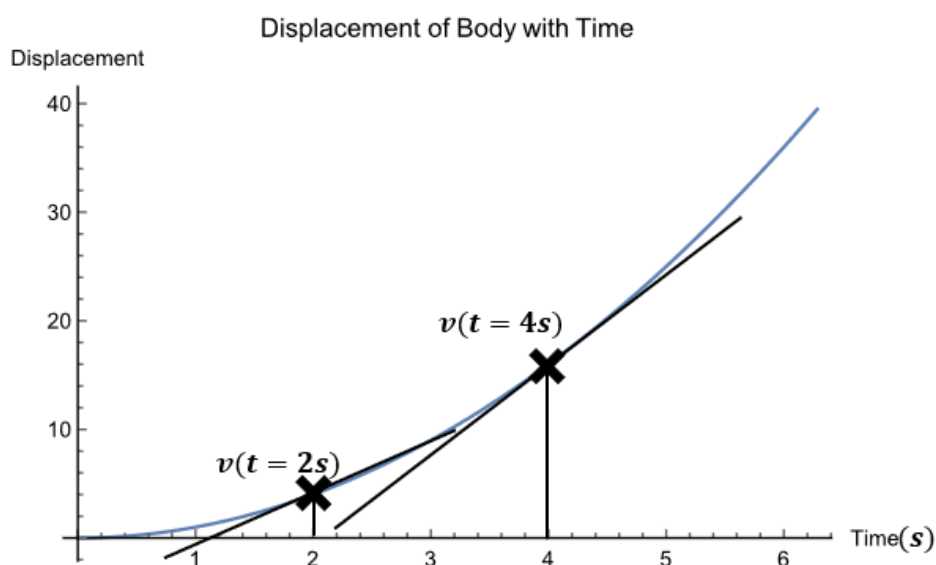
When we talk about instantaneous velocity, we are asking about a single point in time. From the displacement-time graph, the gradient represents the velocity of the body. As we can see from the graph, the instantaneous velocities when $t = 2s$ and $t = 4s$ are different. This is simply because the body is moving at a non-uniform velocity. If the instantaneous velocities are the same, then we call the motion is described as uniform velocity.

On the other, talking about ***average*** velocity, we simply define range of time, thus choosing two points in time rather than one. Then we take the difference in position and divide it by the difference in time to calculate the ***average*** velocity. That is to say, for the graph below,

Displacement of Body with Time
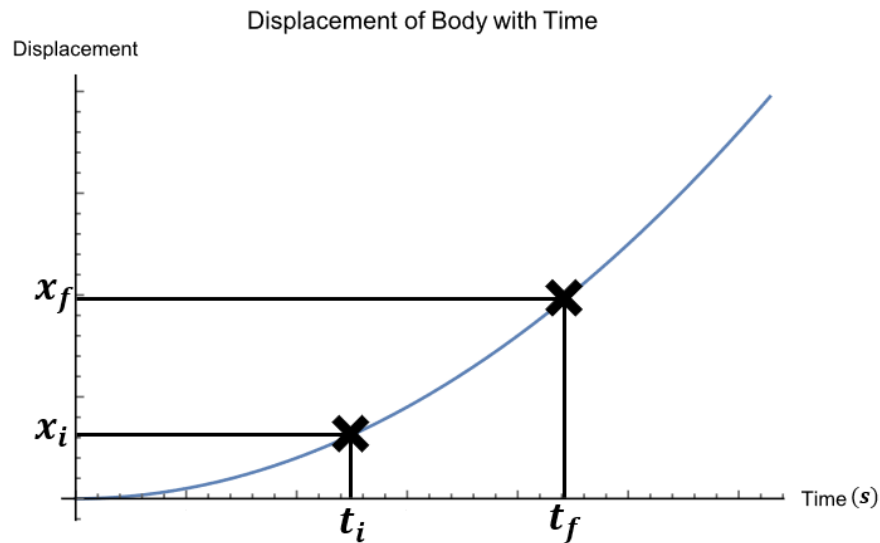
We can calculate the average velocity as

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

We can take the same approach and understanding and apply it to acceleration, but with a velocity time graph rather than a displacement time graph.

Sample Problem 2.1

The motion of a body is described by the equation $v = 2t^2$, where v is in metres per second and t is in seconds. Calculate the instantaneous velocity of the body at $t = 3s$ and the average acceleration between $t = 2s$ and $t = 4s$.

Answer:

$$v_{instantaneous} = 2(3)^2 = 18ms^{-1}$$
$$v(t = 2s) = 2(2)^2 = 8ms^{-1}$$
$$v(t = 4s) = 2(4)^2 = 32ms^{-1}$$
$$a_{average} = \frac{\Delta v}{\Delta t} = \frac{v(t = 4s) - v(t = 2s)}{4 - 2} = \frac{32 - 8}{4 - 2}$$
$$a_{average} = 12ms^{-2}$$

Sample Problem 2.2

The motion of a body is shown in the graph shown in Figure 1. Calculate the displacement of the body and calculate the average velocity of the body.
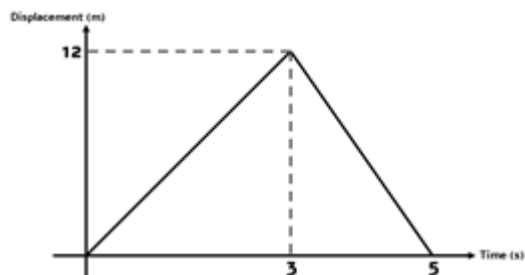
Answer:

Displacement of the body can be calculated by recognising that the area under the velocity -time graph represents the displacement of the body. So all we need is to sum up all the areas under the graph.

$$s = \frac{1}{2}(12 \times 3) + \frac{1}{2}\big(12 \times (5 - 3)\big) = 30m$$

The average velocity can be calculated by simply dividing the displacement by the total time of motion.

$$v_{average} = \frac{30}{5} = 6ms^{-1}$$

Kinematic Equations

Now we want to look at kinematic equations, which are equations that relates variables that describes motion such as displacement, velocity and acceleration.

Derivation by calculus

We'd like to derive the equations from our understanding of linear motion and using calculus. We begin with the definition of acceleration

$$a = \frac{dv}{dt}$$

Assuming constant acceleration, we can rearrange then integrate both sides to yield

$$a \int_{t_{initial}}^{t_{final}} dt = \int_{v_{initial}}^{v_{final}} dv \Rightarrow a\big(t_{final} - t_{initial}\big) = v_{final} - v_{initial}$$

Adjusting such that $t_{initial} = 0$, $t_{final} = t$ and defining $v = v_{final}$, $v_{initial} = u$.

And then rearranging this equation yields

$$v = u + at$$

which is the same equation as the first equation found in LO2. Simply put, the final velocity of a body is initial velocity plus the product of acceleration and time difference.

We can take the same approach to find the third equation in LO2 using the first equation. We start with the definition of velocity and then rearranging it,

$$v = \frac{dx}{dt} \Rightarrow \int v \, dt = \int dx$$

Note that since velocity is not a constant, $v \, dt$ cannot be directly integrated. We therefore need an equation for velocity as a function of time (first equation).

$$\int u + a \, t \, dt = \int dx$$

Since u and a are constants, these integrals become

$$u \int_0^t dt + a \int_0^t dt = \int_0^x dx$$

Solving this integral gives

$$x = ut + \frac{1}{2}at^2.$$

For the second equation, we can start take advantage of calculus by starting with a time independent derivative,

$$\frac{dx}{dv} = \frac{dx}{dt}\frac{dt}{dv} = \frac{v}{a}$$

Rearranging this gives us the needed integral to solve

$$a\int_0^x dx = \int_u^v v\,dv \Rightarrow ax = \frac{1}{2}(v^2 - u^2)$$

Further rearrangement yields an equation

$$v^2 = u^2 + 2ax$$

matching with the third equation found in LO2.

Equation 4 of LO2 does not require any integration, rather we can obtain it using $s = ut + \frac{1}{2}at^2$ and $v = u + at$. This is left for the reader to do.
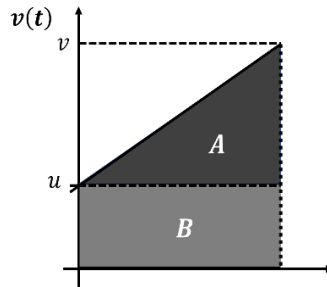
Geometric Derivation

By definition,

$$a = \frac{v - u}{t}$$

Rearranging this gives

$$v = u + at$$

Consider an object that starts its motion with velocity u and maintains its constant acceleration a to a final velocity of v. We can describe its motion diagrammatically as below

Since the area under the graph represents displacement, all we need to do is to add up the area of A and B. If

$$\text{Area}_A = \frac{1}{2}(t)(v - u) = \frac{1}{2}(t)(at) = \frac{1}{2}at^2$$

$$\text{Area}_B = ut$$

then

$$s = ut + \frac{1}{2}at^2.$$

If, on the other hand, we consider

$$s = \frac{1}{2}(t)(v - u) + ut$$

Then we find that

$$s = \frac{1}{2}(v + u)t$$

For the equation of $v^2 = u^2 + 2as$, we can start the derivation by considering

$$v = u + at \Rightarrow t = \frac{v - u}{a}$$

And

$$s = \frac{1}{2}(u + v)t.$$

We can substitute time equation into the displacement to yield

$$s = \frac{1}{2}(u + v)\left(\frac{v - u}{a}\right) = \frac{v^2 - u^2}{2a} \Rightarrow v^2 = u^2 + 2as.$$

Sample Problem 2.3

A 2022 Honda Accord can travel down a $\frac{1}{4}$ mile track in $14.1s$ from rest. Calculate the acceleration (in SI units), assuming that its acceleration is constant.

Answer:

Values given $\Rightarrow s = \frac{1}{4}\ mile = 402.336km\,; t = 14.1s;\ u = 0ms^{-1}$

Choice of equation $\Rightarrow s = ut + \frac{1}{2}at^2$

$$402.336 = \frac{1}{2}a(14.1)^2$$
$$a = 4.04744ms^{-2}$$

Sample Problem 2.4

A car initially travels at $20ms^{-1}$. If the car undergoes constant acceleration of $1.2ms^{-2}$, determine the time the car need to reach double of its initial velocity.

Answer:

Values given $\Rightarrow$ u = 20ms^{-1}; a = 1.2ms^{-2}; v = 2u = 40ms^{-1}

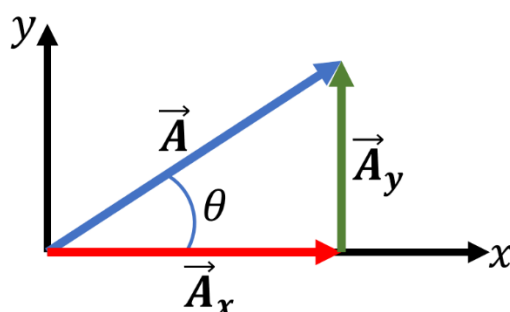Choice of equation $\Rightarrow v = u + at$

$$40 = 20 + 1.2t$$
$$\boldsymbol{t = \frac{50}{3}s}$$

Projectile Motion (Motion in 2 Dimensions)

When dealing with motion in two dimensions, the minimum that we need is the Pythagorean theorem as well as the definition of tangent. We consider a vector $\vec{A}$ defined by

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where $\vec{A}_x$ and $\vec{A}_y$ are component vectors of $\vec{A}$, each parallel to one of the axes in a rectangular coordinate system.

It then follows that the magnitude and direction of $\vec{A}$ can be related to its components by the Pythagorean theorem and the definition of tangent,

$$|\vec{A}| = \sqrt{|\vec{A}_x|^2 + |\vec{A}_y|^2}$$

$$\tan\theta = \frac{|\vec{A}_y|}{|\vec{A}_x|}.$$

Conversely, we can work out the components of $\vec{A}$ from the magnitude of $\vec{A}$ and the angle θ,

$$|\vec{A}_x| = |\vec{A}|\cos\theta$$

$$|\vec{A}_y| = |\vec{A}|\sin\theta$$

One case study that we can do on two-dimensional motion is **projectile motion,** a motion that follows a parabolic path. The simplest case of projectile motion would be one where the air resistance and the rotation of the Earth is simply neglected, and that the motion is only affected by the Earth's gravity ($\vec{F}_{gravity} = m\vec{g}$). One important aspect of this case is that the horizontal (x-direction) and vertical (y-direction) motions are independent of each other. This means that the kinematics equation we have studied earlier can be dealt with separately for both x and y directions.

Keeping in mind that $u_x = u\cos\theta$ and $u_y = u\sin\theta$, we can then work out the 6 equations that describes the projectile motion:

x-direction (where $a_x = 0$)	y-direction (where $a_y = -g$)
$v_x = u_x$	$v_y = u_y - gt$
$s_x = u_x t$	$s_y = u_y t - \frac{1}{2}gt^2$
$v_x^2 = u_x^2$	$v_y^2 = u_y^2 - 2gs_y$

We can also work out the velocity of the projectile by keeping in mind that it merely follows from Pythagorean theorem

$$v^2 = v_x^2 + v_y^2.$$

If we substitute the equation for time-x-component, $t = \frac{s_x}{u_x}$, in the equation for displacement in y-direction, $s_y = u_y t - \frac{1}{2}gt^2$, what we get is the parabolic equation for the projectile motion path,

$$\left(\frac{u_y}{u_x}\right)s_x - \left(\frac{1}{2u_x^2}\right)s_x^2 - s_y = 0.$$

There are two more items that are of our interest:

1. If we were to look for the "**peak**" of the parabolic path, we can do so by applying $v_y = 0$ to the kinematics equations. This is simply because it is at this peak that $u_y = gt$ such that the velocity of the projectile is momentarily zero before the projectile falls back down towards the Earth.
2. Another item that would be of our interest is the **range** of the projectile motion. By range, what we are referring to is the point at which the projectile reaches back to ground or stop accelerating in the y-direction. This would differ from case to case, of course, and we shall demonstrate in the sample problems following this.
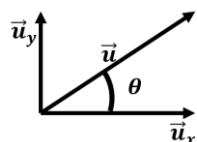
Sample Problem 2.5

An object is launched at a velocity of $21ms^{-1}$ in a direction making an angle of 30° upward with the horizontal. Calculate

a. Initial velocity in x and y direction.
b. the location of the object at $t = 2s$.
c. the total horizontal range.
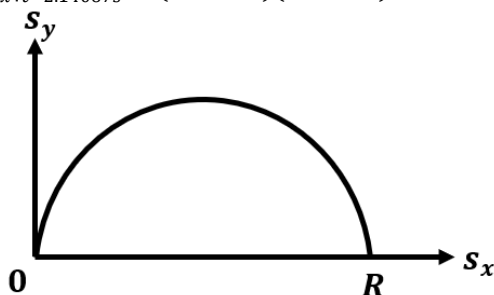d. the velocity of the object just before it hits the ground.

Answer:

a.

$$|\vec{u}_x| = |\vec{u}|\cos\theta = 21\cos 30$$
$$|\vec{u}_x| = 18.1865ms^{-1}$$
$$|\vec{u}_y| = |\vec{u}|\sin\theta = 21\sin 30$$
$$|\vec{u}_y| = 10.5ms^{-1}$$

b. $s_x = u_x t \Rightarrow s_x = (18.1865)(2) = 36.3730m$

$s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow s_y = (10.5)(2) + \frac{1}{2}(-9.81)(2^2) = 1.38m$

c. $s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow 0 = (10.5)(t) + \frac{1}{2}(-9.81)(t^2) \Rightarrow t = \{0, 2.14067\}s$

$s_x|_{t=2.14067s} = (18.1865)(2.14067) = 38.9313m = R$

d. $v_x = u_x = 18.1865ms^{-1}$

$v_y = u_y + a_y t = 10.5 + (-9.81)(2.14067) = -10.5ms^{-1}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(18.1865)^2 + (-10.5)^2} = 21ms^{-1}$

$\theta = \tan^{-1}\left(\frac{18.1865}{-10.5}\right) = -60^o$

Sample Problem 2.6

Compare the horizontal range of a ball thrown at velocity $35ms^{-1}$ if the angle of release is 15°, 30°, 45° and 60°.

Answer:

Condition for determining horizontal range $\Rightarrow s_y = 0$.

$s_y = u_y t + \frac{1}{2}a_y t^2 = (u\cos\theta)t + \frac{1}{2}a_y t^2$
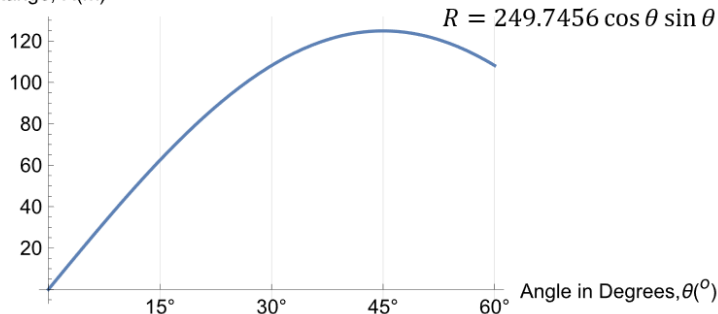
$0 = (35\cos\theta)t - (4.905)t^2$

$t = \{0, 7.13558\cos\theta\}s$

$R = s_x = u_x t = (u\sin\theta)t$

$R = (35\sin\theta)(7.13558\cos\theta)$

$R = 249.7456\cos\theta\sin\theta$

$\theta(^o)$	t (s)	$R(m)$
15°	6.89244	62.4363
30°	6.17959	108.143
45°	5.04562	124.873
60°	3.56779	108.143

The horizontal range peaks at 45°.

Exercises

1. A car accelerates uniformly from rest for 5.88s for a distance of 150m. Calculate the acceleration of the car.
2. An object is accelerated uniformly from $2ms^{-1}$ to $8ms^{-1}$ over a distance of 38m. Calculate the acceleration of the object.
3. Car A, moving at a constant velocity of $10ms^{-1}$, passes by Car B at rest. 2 seconds after Car A passes, Car B starts accelerating at $5ms^{-2}$ in the same direction as Car A. When does Car B surpasses Car A?
4. A car, initially at rest at position A, accelerates at $2ms^{-2}$ for 5 seconds and then maintains its velocity for 10 seconds. After which the car decelerates back to $0ms^{-1}$ in 3s. Sketch the velocity-time graph of the car and calculate the total displacement of the car from position A.
5. The Tailgating Problem [1]:
    a. Determine the stopping distance of a BMW M3 if the car can decelerate at a rate of $9.2ms^{-2}$ from $97kmh^{-1}$.
    b. Consider the reaction time of a driver is 0.55s, calculate the stopping distance of the BMW M3 mentioned in the previous question.
    c. "When pigs fly" – A driver driving a car travelling at $28ms^{-1}$ suddenly notices a pig (oink oink!) $28m$ on the road ahead. With what velocity would the car hit the pig if the car decelerated at $8ms^{-2}$ and the driver's reaction time was $0.8s$?
6. A projectile is launched at a velocity of $20ms^{-1}$ at angle of 30° to the horizontal. Calculate the
    a. Time of flight of the projectile.
    b. Horizontal range of the projectile.
    c. The maximum height of the projectile.
    d. The speed of projectile at a third of the maximum height.
7. A diver, on a pier 10m from the water, jumps up at an angle 27° from the horizontal with an initial velocity of $4.2ms^{-1}$. Calculate the
    a. Time at which the diver hits the water.
    b. The speed of the diver just before he hits the water.
8. A projectile is fired at velocity v and it has a horizontal range of 10m. If the projectile is again fired at the twice its velocity before, determine the new horizontal range.

Chapter 3: Dynamics of Linear Motion

Learning Outcomes

5. Define
   a. Momentum, $\vec{p} = m\vec{v}$
   b. Impulse, $J = F\Delta t$
6. Solve problem related to impulse and impulse-momentum theorem,
$$J = \Delta p = mv - mu$$

   1D only

7. Use *F-t* graph to determine impulse.
8. State:
   a. the principle of conservation of linear momentum.
   b. Newton's laws of motion.
9. Apply
   a. the principle of conservation of momentum in elastic and inelastic collisions in 2D collisions.
   b. Newton's laws of motion.
      include static and dynamic equilibrium for Newton's first law motion

10. Differentiate elastic and inelastic collisions. (remarks: similarities & differences)
11. Identify the forces acting on a body in different situations - Weight, *W;* Tension, *T;* Normal force, *N;* Friction, *f;* and External force (pull or push), *F*.
12. Sketch free body diagram.
13. Determine static and kinetic friction, $f_s \le \mu_s N, f_k = \mu_k N$

In the previous chapters, we have looked at describing motion without the hassle of asking, "what force is causing the body to move?". In this chapter, we aim to expand our knowledge to a body's motion in that very aspect.

Types of Forces

We begin with asking the question, "what is force?", a simple answer would be to say force is a push and pull. Here, however, let us define force a bit further. Let us define force as **an agent for motion change**. Force is a vector quantity, that means **direction matters**. Two oppositely directed force acting on the same body work against each other. A body can experience multiple forces acting on it, however it is the net force, i.e., the resultant of all the forces acting on the body, that changes the motion of the body.

$$\vec{F}_{net} = \vec{F}_1 - \vec{F}_2$$
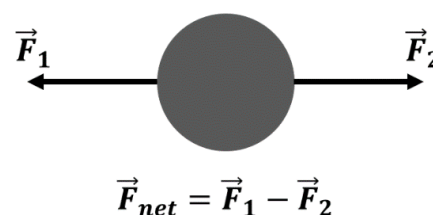
4 types of forces we'd consider in this chapter – gravitational (weight), tensional, normal and frictional. Their definitions and directions are as follows:

Forces	Definitions	Directions
Gravitational	Force exerted upon a body interacting with a gravitational field.	Towards the gravitational source.
Tensional	Force transmitted axially through a massless one-dimensional continuous element.	Along the one-dimensional continuous element.
Normal	Support force, perpendicular to the surface, exerted upon a body in contact with a stable object.	Perpendicular to the surface the body is in contact with.
Frictional	Force acting upon bodies that are in contact and moving relative to each other.	Against the direction of motion.

One particular type of force that maybe be of our interest is frictional force. This is because frictional force depends on the motion of the object. If the object is static, then it is subject to *static friction*. On the other hand, if the

object is moving (relative to the surface it is in contact with) at some velocity, and therefore has some kinetic energy, then the object is subject to *kinetic friction*. Static friction is generally higher than kinetic friction because of the asperities (roughness) of the surfaces of the contacting bodies. This asperity enables the surfaces to interlock with each other, causing adhesion. This means that the force applied to the body must overcome this adhesion before the bodies can start moving relative to each other. This phenomenon can be observed by looking at the frictional force as a function of applied force graph:
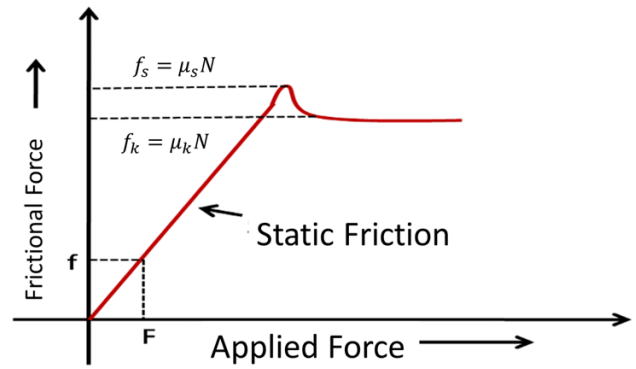
As the applied force is increased, so does the frictional force. This is true until a certain threshold is reached, after which the body will start to move. This threshold is exactly the unlocking of the asperities.

Sample Problem 3.1

Determine the resultant force exerted on the body shown in the diagram.

$\vec{F}_1 = 5.4N$

$\vec{F}_2 = 2.7N$

Solution

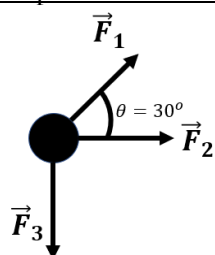$\vec{F}_{net}$ $\vec{F}_1$

θ

$\vec{F}_2$
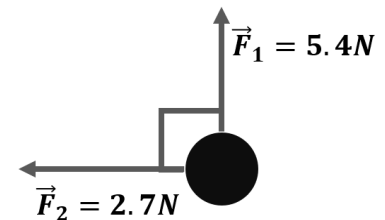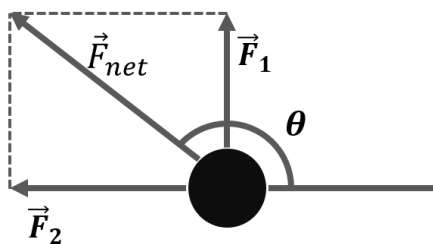
$$|\vec{F}_{net}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2}$$
$$|\vec{F}_{net}| = \sqrt{(5.4)^2 + (-2.7)^2}$$
$$|\vec{F}_{net}| = 6.03738N$$
$$\theta = 180^o - tan^{-1}\left(\frac{5.4}{2.7}\right)$$
$$\theta = 116.565^o$$

Sample Problem 3.2

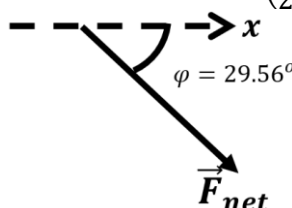Calculate the resultant force if $|\vec{F_1}| = 10N$, $|\vec{F_2}| = 12.5N$ and $|\vec{F_3}| = 17N$.

Solution

Force	x-components	y-components
F_1	$F_1 \cos 30^o$ $= 10 \cos 30^o$ $\approx 8.66N$	$F_1 \sin 30^o$ $= 10 \sin 30^o$ $\approx 5N$
F_2	$+12.5N$	$0N$
F_3	$0N$	$-17N$
Σ	21.16N	-12N

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{(21.16)^2 + (-12)^2}$$
$$F_{net} = 24.3258N$$
$$\varphi = tan^{-1}\left(\frac{12}{21.16}\right) = 29.56^o$$

$\varphi = 29.56^o$

$\vec{F}_{net}$

Newton's Law of Motion

There are laws of motion that a moving under force would generally follow. These laws were first introduced and came into its modern form via Newton's *Principia*. In it, 3 laws of motions were found:

1. A body, when no external force is applied, will not undergo velocity change, i.e.
$$\vec{F}_{external} = 0 \Rightarrow \Delta\vec{v} = 0$$
2. When a force is acted upon it, a body will move in a manner such that rate of momentum change is equal to the said force, i.e.
$$\vec{F} = \frac{d\vec{p}}{dt}.$$
3. Forces exerted onto two interacting bodies will be equal in magnitude but opposite in direction.
    If $\vec{F}_{12}$ is force exerted onto body 1 by body 2, then
$$\vec{F}_{12} = -\vec{F}_{21}$$

These three laws form the foundation for what is known today as the *Newtonian Laws of Motion*.

Momentum

The first and third requires no further definitions of variable, however the second one, mentions an idea of **momentum**. It seems useful to define this term at this point. What we mean by momentum at this point is the property of a moving body that rises from the product of the mass the body and its velocity, i.e.

$$\vec{p} = m\vec{v}.$$

If the velocity of the body changes (and therefore so does the momentum of the body in question), we quantify that change and call it **impulse**:

$$\vec{J} = \Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t.$$

This also means that the area under the F-t graph represents impulse.

It is from Kinematics that we know a change in velocity means a non-zero acceleration. Knowing this as well as Newton's Second Law, we can say that force is present when acceleration is non-zero,

$$\vec{F} = m\vec{a}.$$

This statement, of course can be derived quite simply from Newton's Second Law of motion:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

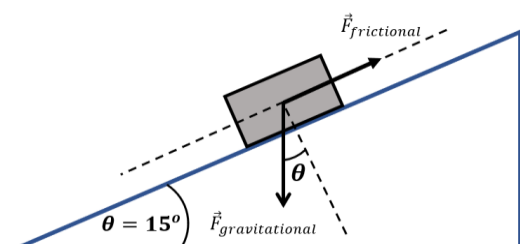$$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

Here we can see that a change in mass may also produce force and that if mass change is zero, then what we have is the equation seen before,

$$\vec{F} = m\vec{a}.$$

Sample Problem 3.3

Suppose an object of 15 kg is placed on an incline plane of 15° from the horizontal. Calculate the magnitude of frictional force that keeps the object from sliding down the incline plane.

Solution

$F_{net} = F_g \sin\theta - F_{frictional}$
Since the object stays static $\Rightarrow F_{net} = 0$
$F_{frictional} = F_g \sin\theta = mg \sin\theta$
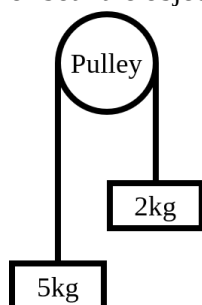$F_{frictional} = (15)(9.81)sin15^o$
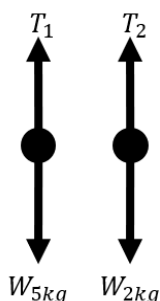$F_{frictional} = 38.0852N$

Sample Problem 3.4

Problem:
Based on the diagram, calculate the acceleration of both the objects.

Solution

FBD

Equation of motion for 5kg body:
$$F_{net} = W_{5kg} - T_1 = m_{5kg}a$$
Equation of motion for 5kg body:
$$F_{net} = T_2 - W_{2kg} = m_{2kg}a$$
$$T_1 = T_2 = T$$
Rearrange for a,
$$T = W_{5kg} - m_{5kg}a = W_{2kg} + m_{2kg}a$$
$$a = \frac{W_{5kg} - W_{2kg}}{m_{2kg} + m_{5kg}} = \frac{9.81(5-2)}{5+2}$$
$$a \approx 4.2ms^{-2}$$
The 2kg object will move **upwards** at $a \approx 4.2ms^{-2}$.
The 5kg object will move **downwards** at $a \approx 4.2ms^{-2}$.

Sample Problem 3.5

A 50g ball at $30ms^{-1}$ is travelling towards a wall. Upon striking the wall, the ball bounces back in the opposite direction at a speed of $10ms^{-1}$. Calculate the impulse.

Solution
$J = \Delta p = p_{final} - p_{initial}; p = mv$

$$J = m(v_{final} - v_{initial})$$
$$J = (0.05)((-10) - (+30))$$
$$J = -2kg\,ms^{-1}$$

Sample Problem 3.6
A footballer kicks a 300g ball from rest to a speed of $60ms^{-1}$ in a collision lasting $1.5ms$. Calculate the force generated by the footballer.

Solution

$$F\Delta t = \Delta p = p_{final} - p_{initial}$$

Since $p_{initial} = 0$, then

$$F(1.5 \times 10^{-3}) = (0.3)(60)$$
$$F = 12kN$$

One of the ways for bodies to interact is through collisions. When this happens, assuming this happens in an isolated system. The total momentum of the system doesn't change with the passage of time. The momenta of the participating bodies may change, but not the vector sum total momentum of the system. When we say that a quantity doesn't change, we say that the quantity is **conserved**. So, in this case, we say that the **total momentum is conserved**. Conservation of momentum is simply

$$\Delta(\Sigma p) = 0.$$

When we talk about collisions, we may consider two types of collision – elastic and inelastic collisions. Note that whilst momentum is conserved in **all** types of collisions, kinetic energy is not as it may be converted into other forms of energy (e.g., sound energy). It is this exact parameter from which we differentiate elastic from inelastic collisions. A perfectly elastic collision is defined by a collision in which both momentum and kinetic energy is conserved whilst a perfectly inelastic collision is a collision in which conservation of kinetic energy is not obeyed.

Sample Problem 3.7 (Conservation of momentum)
Ball A of mass 30g, travels at $3ms^{-1}$ collides head on with Ball B of 50g at rest. Calculate the velocity of Ball B after the collision if Ball A has the final velocity of $1.2ms^{-1}$.
Solution

$$\Delta\Sigma p = 0 \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$
$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$(0.03)(3) + 0 = (0.03)(1.2) + (0.05)v_B$$
$$x = 1.08ms^{-1} \text{ in the same direction as Ball A.}$$

Sample Problem 3.8 (Conservation of momentum in 2 Dimensions)
Ball A of mass 3kg, travels at $3ms^{-1}$ in the positive x direction collides with Ball B of 5kg travelling at $2ms^{-1}$ in the positive y direction. If the balls stick together after the collision, determine their velocity.
Solution
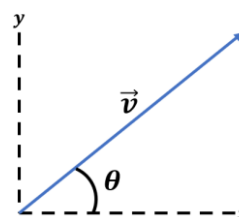Need to apply conservation of momentum in x and y direction separately.

	X	Y
Momentum before	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_B = 0$ $\Sigma p = m_A u_A$	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_A = 0$ $\Sigma p = m_B u_B$
Momentum after	$\Sigma p = (m_A + m_B)v \cos\theta$	$\Sigma p = (m_A + m_B)v \sin\theta$
Final velocity	$m_A u_A = (m_A + m_B)v \cos\theta$ $v \cos\theta = \frac{m_A u_A}{m_A + m_B}$ ---(1)	$m_B u_B = (m_A + m_B)v \sin\theta$ $v \sin\theta = \frac{m_B u_B}{m_A + m_B}$ ---(2)

$$\frac{(2)}{(1)}: \tan\theta = \left(\frac{m_B u_B}{m_A + m_B}\right)\left(\frac{m_A + m_B}{m_A u_A}\right) = \frac{m_B u_B}{m_A u_A} = \frac{(5)(2)}{(3)(3)}$$
$$\theta = 48.0128^o$$
$$v \cos\theta = \frac{m_A u_A}{m_A + m_B} \Rightarrow v \cos(48.0128^o) = \frac{(3)(3)}{3(5)}$$
$$v = 0.897ms^{-1}$$

Sample Problem 3.9 (Perfectly Elastic Collision)

A ball, travelling at $3ms^{-1}$, collides head on with another ball of the same mass, travelling $2ms^{-1}$ in the opposite direction. Determine their velocities post-collision?

Solution

Let us assume that ball 1, travelling at $3ms^{-1}$, initially travels in the positive direction such that
$$u_1 = +3ms^{-1}; u_2 = -2ms^{-1}$$

Since the balls are of the same mass,
$$m_1 = m_2 = m$$

Assuming the collision is elastic, the system would obey both the conservation of momentum and the conservation of kinetic energy.

$$\Delta(\Sigma p) = 0 \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$
$$m(u_1 + u_2) = m(v_1 + v_2) \Rightarrow 3 + (-2) = v_1 + v_2$$
$$\Delta(\Sigma K) = 0 \Rightarrow \Sigma K_{initial} = \Sigma K_{final}$$
$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$
$$u_1^2 + u_2^2 = v_1^2 + v_2^2 \Rightarrow 3^2 + (-2^2) = v_1^2 + v_2^2$$

2 possible set of answers

Option 1: $v_1 = 3ms^{-1}; v_2 = -2ms^{-1}$, (which is just the initial case)

Option 2: $v_1 = -2ms^{-1}; v_2 = 3ms^{-1}$ (a more sensible answer)

Sample Problem 3.10 (Perfectly Inelastic collision)

A ball of mass 0.5kg, travelling in the +x direction at $2ms^{-1}$, collides with another ball of mass 0.2kg travelling in the opposite direction at $1.5ms^{-1}$. After the collision, the balls stick together and travels at the same speed. Determine the final velocity and its direction. Compare the kinetic energies before and after the collision.

Solution

Applying conservation of momentum,
$$\Delta(\Sigma p) \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$
$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v \Rightarrow (0.5)(2) + (0.2)(-1.5) = (0.5 + 0.2)v$$
$$v = 1ms^{-1} \text{ in the positive x direction}$$

To compare the kinetic energies, we can take their difference.
$$\Delta K = K_{final} - K_{initial}$$
$$\Delta K = \frac{v^2}{2}(m_1 + m_2) - \frac{1}{2}(m_1 u_1^2 + m_2 u_2^2)$$
$$\Delta K = \frac{1^2}{2}(0.5 + 0.2) - \frac{1}{2}\left((0.5)(2)^2 + (0.2)(-1.5)^2\right) = 0.35 - 1.225$$
$$\Delta K = -0.875J$$

This means 0.875J of kinetic energy has been converted into energies of other forms

Exercises

1. A car of mass 850kg on a slope inclined at 25°. Determine the acceleration of the car if the slope is frictionless. If the car was initially at rest and the slope is $20m$ in length, calculate the velocity of the car at the bottom of the slope.

2. Block m_1 with mass 2kg is connected to another block m_2 of mass x kg via a light string that passes over a frictionless pulley as shown in the figure.
   Find the mass of m_2 if the objects accelerate at $7ms^{-2}$ and the coefficient of kinetic friction between the table surface and block m_1 is 0.275.
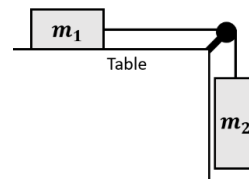
3. A block of mass 2kg rides on top of a second block of mass 5kg and the coefficient of friction between the two blocks is 0.33. A string is attached to the bottom block and pulls the string horizontally across the frictionless table surface. Determine the maximum force that can be applied such that the top block does not slip.
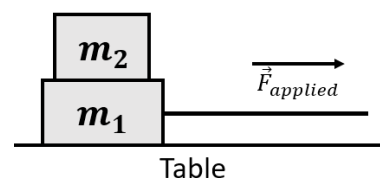
4. The diagram shows a man on a platform lifting himself upwards. The man pulls the rope with 800N force. If the mass of the man and the platform is 80kg and 60kg respectively, calculate

   a. the acceleration of the cradle
   b. force the man exerts on the floor
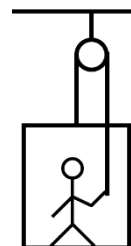
5. A smaller ball of 50g is held just above a bigger ball of mass 250g with their centres vertically aligned. The balls are then released from rest to fall through 2m as shown in the figure. Upon colliding elastically with the ground, the bigger ball rebounds and collides with the smaller ball that is currently still moving downwards. calculate the height of the rebound of the smaller ball.
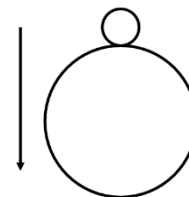
6. In cardiology research, the mass of the blood per pump stroke can be determined through a *ballistocardiograph*. The instrument works by having the patient lie on a horizontal platform floating on a film of air such that the air friction is negligible. The patient along with the horizontal platform are initially static. When the heart expels blood into the aorta in one direction, the patient's body along with the platform moves respond by moving in the opposite direction. If the speed in which the heart expels the blood is $59\,cms^{-1}$, determine the mass of the blood that leaves the heart given that the mass of the body and platform is 75kg and that response is that the body with the platform moves $60\mu m$ in 0.180s.

7. A large car traveling at $15ms^{-1}$ collides head on perfectly inelastically with a smaller car travelling at the same speed in the opposite direction. The collision time was 0.15s, compare the magnitude of the forces that the seat belts exert onto the drivers of each car if the total mass of the large car is three times the total mass of the smaller car.

Chapter 4: Work, Energy and Power

Work

Let us begin by defining work. The work on an object, *W*, is defined to be the product of magnitude of the displacement, *s*, and the force component parallel to the displacement of the object $F_{\parallel}$, i.e.
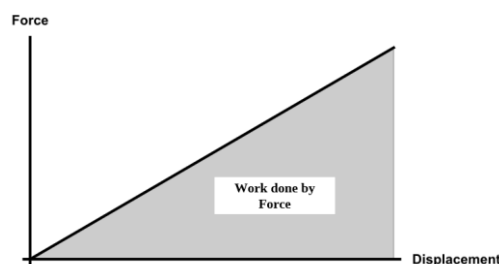
$$W = F_{\parallel}s = \vec{F} \cdot \vec{s}$$

Notice that it is not the product of force and displacement but the product of displacement and force component, the important characteristic of that particular force component is that it must be parallel to the displacement. The diagram below illustrates this point, we cannot simply multiply the magnitude of $\vec{F}$ and $\vec{s}$. We must find the component of force that **is** parallel the displacement, and then we can find their product.
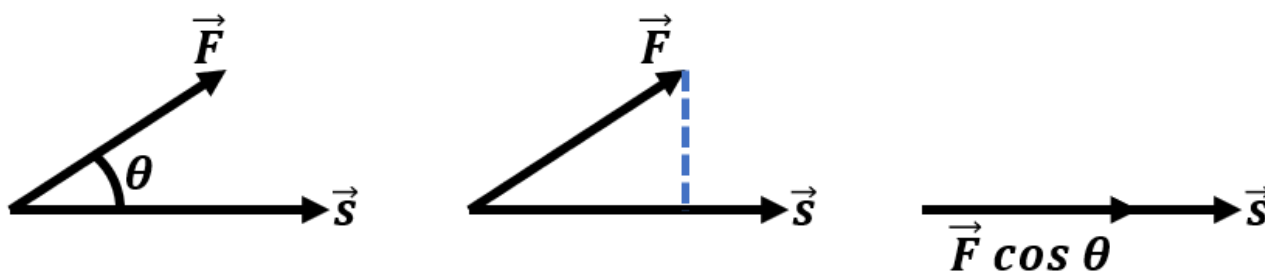
This of course means that in the force-displacement graph, work done by a force is equal to the area under the graph. That is to say, the work done to displace an object from x_i to x_f is simply
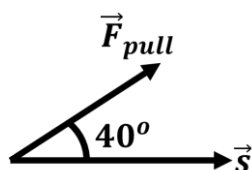
$$W = \int_{x_i}^{x_f} F_{\parallel}dx$$

Sample Problem 4.1

A block is pulled with a force of 50N (directed 40° from the horizontal) on a smooth horizontal surface for 5m. Calculate the work done by the pulling force.

Answer:

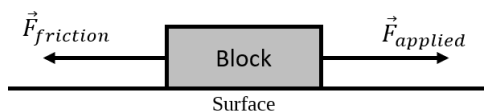$$W = \vec{F} \cdot \vec{s} = (F \cos \theta)s$$
$$W_{50N} = (50 \cos 40)5$$
$$W_{50N} = 191.511J$$

Sample Problem 4.2

A block is pushed with 5N in the positive x direction for $2m$ on a horizontal surface. If the block travels at constant speed, calculate the work done by frictional force, work done by the applied force and the total work done.

Answer

Since the block travels at constant speed,
$$F_{net} = 0 = F_{applied} - F_{friction}$$
That means the magnitude of frictional force is equal to the magnitude of applied force, but acts in the opposite direction.
$$F_{friction} = 5N$$
$$W_{friction} = \vec{F}_{friction} \cdot \vec{s} = F_{friction}s \cos \theta$$
$$W_{friction} = (5)(2)cos(180^o)$$
$$W_{friction} = -10Nm$$

Similarly,
$$W_{applied} = F_{applied}s \cos \theta = (5)(2) \cos 0 = 10Nm$$
$$W_{total} = W_{friction} + W_{applied} = -10 + 10 = 0Nm$$

Work Energy Theorem

When an object moves, we say it contains kinetic energy. Kinetic energy quantifies the amount of energy a moving object has. It depends on the velocity of the moving object,

$$E_k = K = \frac{1}{2}mv^2.$$

Now what we want to do is to show a relationship between the quantity related to moving object (kinetic energy) and another quantity related to the changes of object position (work).

We begin with the definition of work done on an object and Newton's Second Law of motion to show that

$$W = Fs; \; F = ma \Rightarrow W = mas$$

Assuming that the force is constant and therefore the acceleration is also constant, we can then apply equation of kinematics

$$v^2 = u^2 + 2as; W = m(as)$$

$$W = m\frac{v^2 - u^2}{2} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = K_{final} - K_{initial} = \Delta K$$

This show derivation brings about an important theorem, called the work-energy theorem. This theorem states that the work done onto a body is equal to the change in kinetic energy of the body.

Sample Problem 4.3
A 150g block begins travelling along a horizontal surface at $4.20ms^{-1}$. If the kinetic friction coefficient between the block and surface is 0.45, calculate the distance that the block moves before coming to a stop.

Answer:
$m_{block} = 0.15kg; u = 1.8ms^{-1}; u_k = 0.45$
$$W = Fs = \Delta K = F_{final} - K_{initial} \Rightarrow (\mu_k m_{block}g)s = \frac{m_{block}}{2}(v^2 - u^2)$$
$$0.45(0.15)(9.81)s = \frac{0.15}{2}(0^2 - 4.2^2)$$
$$s = 1.99796m$$

Energies
Two more energies that are of interest to us. The first is the gravitational potential energy, which is the energy contained in an object due to its position, measure from a gravitational source. A more detailed analysis is found in the Newtonian Gravity part of this course. At this point, it is sufficient for us to know that if an object of mass *m* is position at height *h* from the surface of the Earth, then the gravitational potential energy found in that object is

$$E_{gp} = mgh$$

The second type of potential energy of interest is the elastic potential energy of a spring. By Hooke's Law, the force acting on a spring is direction proportional to its extension (or compression).

$$\text{Hooke's Law: } F = -kx$$

We can then utilize work energy theorem to find the elastic potential energy of a spring,

$$W = -\int F\,dx = \int kx\,dx \Rightarrow E_{ep} = \frac{1}{2}kx^2$$

Apart from the conservation of momentum, another important principle of conservation crucial to our study of moving bodies is the **principle of mechanical energy conservation.** The law simply states that the sum of all kinetic energy and all potential energy must remain constant at all times. That is to say

$$\Delta E_{total} = 0.$$

Sample Problem 4.4
An 2kg object was released from 20m height. Calculate its velocity just before striking the ground.

Answer:
Initially the object would have gravitational potential energy of
$$E_{gp} = mgh = 2(9.81)(20) = 392.4J$$

This energy is then converted fully into kinetic energy at $h = 0m$.
Therefore, the amount of kinetic energy possessed by the body will be 392.4J.
$$E_k = \frac{1}{2}mv^2 = 392.4J \Rightarrow \frac{1}{2}(2)v^2 = 392.4 \Rightarrow v = 19.8ms^{-1}$$

Power

Now that we have familiarize ourselves with work and energy, let us now talk about **power**, which is simply defined by the rate of work done. Average power refers to the work done within a time interval,

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{W_{final} - W_{initial}}{t_{final} - t_{initial}}.$$

On the other hand, instantaneous power refers to the mechanical power at one instant in time

$$P_{instantaneous} = \frac{dW}{dt} = \vec{F}\frac{d\vec{s}}{st} = \vec{F} \cdot \vec{v}$$

Sample Problem 4.5
Calculate the average power required to lift a 75kg man to a height of 10m in 2minutes.
Answer:
By work energy theorem,

$$W = mgh$$
$$P_{average} = \frac{W_f - 0}{t_f - 0} = \frac{(75)(9.81)(10)}{2(60)} = 61.3125W$$

Sample Problem 4.5
Calculate the instantaneous power required to lift a 75kg man at $0.09ms^{-1}$.
Answer:
$$P_{instant} = Fv = mgv = 75(9.81)(0.08) = 66.22W$$

Exercises

1. The figure shows a bullet shot at a wooden block. After the bullet makes contact, the bullet and block travels 20cm in the initial direction of the bullet before coming to a stop. Determine the velocity $\vec{v}$ at which the bullet was shot if the coefficient of kinetic friction is 0.2. The mass of the bullet and block is 5g and 2kg respectively.

2. A ball is kicked off of a pier horizontally 10m above water with an initial speed $15ms^{-1}$. Using the principle of energy conservation, calculate the speed at which the ball hits the surface of the water. Compare this solution to a different approach, that is by analysing it as a projectile.

3. Ali (105kg) jumps off a ledge of 0.75m high and land with his knees unbent. If the compressing joint material is compressed 1.25cm, calculate the force experienced by each knee. If he were to bend his knees, he could extend his stopping distance to 0.25m, calculate the force on his knee if he bent his knees. Compare the force experienced by his knee in both cases.

4. A man is able to throw a 10kg ball by accelerating it from rest to $9ms^{-1}$ in 1.5s while raising the ball 0.5m in height. Calculate the power need to do this.

5. An Axia with mass 850kg climbs a 5° slope at a constant $25ms^{-1}$. If the frictional force is 350N, calculate the power output.

6. A 10kg weight is released 1m from the tip of a vertically positioned spring, determine the compression if the spring has a spring constant of $82.4\ kNm^{-1}$