SP015 ENTRANCE QUIZ

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1. What are the correct units for force?
A. joules
B. newtons
C. watts
D. m/s

2. Which of these is the correct formula linking force and acceleration?
A. force = mass x acceleration
B. acceleration = mass x force
C. force = weight x acceleration
D. acceleration = weight x force

3. What force is needed to make a 2kg firework rocket accelerate at 4 ms^{-2}?
A. 0.2 N
B. 2 N
C. 4 N
D. 8 N

4. A 100g toy car uses an electric motor that produces a force of 3 N. What is the car's acceleration?
A. 30 ms^{-2}
B. 300 ms^{-2}
C. 3 ms^{-2}
D. 0.3 ms^{-2}

5. When the astronaut Buzz Aldrin first stood on the Moon, he could feel a difference in the pull of gravity. Which statement is correct?
A. He was weightless.
B. His mass was now zero.
C. His mass was the same but his weight was less.
D. His weight was the same but his mass was less.

6. Which of these statements give the correct description and unit for mass and weight?

	Mass (unit)	Weight (unit)
A	Pull of gravity (N)	amount of matter (kg)
B	Pull of gravity (kg)	amount of matter (N)
C	amount of matter (N)	Pull of gravity (kg)
D	amount of matter (kg)	Pull of gravity (N)

7. Which of these is the correct formula linking mass and weight?
A. weight = mass x gravitational field strength (W = mg)
B. weight = mass ÷ gravitational field strength (W= m/g)
C. mass = gravitational field strength x weight (m = gW)
D. mass = gravitational field strength ÷ weight (m = g/W)

8-10. Different planets have different strengths of gravity. Rocks on the surface may be easier or harder to lift up. What are the missing values in the table below?

	Planet	Gravitational Field Strength (N/kg)	Mass of rock (kg)	weight (N)
8.	Mars	4	8	?
9.	Earth	10	?	1000
10.	Mercury	?	240	888

11. What is the unit used to measure energy and work done?
A. Watts (W)
B. Newtons (N)
C. Pascals (Pa)
D. Joules (J)

12. Which of these is the correct formula for gravitational potential energy (G.P.E.)?
(g is the 'Gravitational Field Strength' of the Earth)
A. G.P.E.= mass x g x height change
B. G.P.E.= mass x height change
C. G.P.E.= mass x height change ÷ g
D. G.P.E.= g x height change ÷ mass

13. If I lift a school bag of mass 5kg onto a shelf 2m up, the increase in G.P.E. will be....
A. 10 J
B. 20 J
C. 50 J
D. 100 J

14. A 100 g apple falls from a tree.
If it loses 2 J of G.P.E. What distance has it fallen?
A. 0.002 m
B. 2 m
C. 5 m
D. 500 cm

15. To calculate kinetic energy (K.E.) we need the velocity (v) and the mass (m).
Using this notation, the formula for K.E. is:
A. K.E. = $\frac{1}{2}mv$
B. K.E. = mv
C. K.E. = $\frac{1}{2}mv^2$
D. K.E. =m^2v

Q16-18 This table shows the kinetic energy of different sports balls in a competition. What are the missing values?

	Type of Ball	Mass	Velocity	Energy
16	Bowling ball	2 kg	3 m/s	?
17	Tennis ball	100 g	10 m/s	?
18	Netball	?	2 m/s	1 J

Q19 & 20. These questions are about a basketball drop.
19. A basketball of mass 0.6 kg is lifted to a height of 5 m. What is the G.P.E. gain at this height?
A. 0.5 J
B. 3 J
C. 5 J
D. 30 J

20. The ball is then dropped and all of this energy is eventually transferred (shifted) to kinetic energy. Ignoring air friction, what would be the maximum velocity just before it hits the floor?
A. 7.07 m/s
B. 10 m/s
C. 3.16 m/s
D. 100 m/s

Answer Sheet

Name :	
Tutorial Class :	

Results:

Out of 20

Instruction:
Tick (x) in the boxes that are your answers. For questions that needs numerical answers, just fill in your answer in the boxes.

	A	**B**	**C**	**D**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

	A	**B**	**C**	**D**
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

SP015 NOTES

Matriculation Physics (SP015)

Study Notes

Shafiq R.

$\vec{A} \cdot \vec{B} = AB \cos\theta$; $\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$; $v = u + at$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$; $s = \frac{1}{2}(u + v)t$; $p = mv$; $J = F\Delta t$; $J = \Delta p = mv - mu$; $f = \mu N$; $W = Fs\cos\theta$; $K = \frac{1}{2}mv^2$; $U = mgh$; $U_x = \frac{1}{2}kx^2 = \frac{1}{2}Fx$; $P_{ave} = \frac{\Delta W}{\Delta t}$; $P = Fv$; $F_c = \frac{mv^2}{r} = mv\omega = mr\omega^2$; $s = r\theta$; $v = r\omega$; $a_t = r\alpha$; $\omega = \omega_o + \alpha t$; $\theta = \omega_o t + \frac{1}{2}\alpha t^2$; $\omega^2 = \omega_o^2 + 2\alpha\theta$; $\tau = rF\sin\theta$; $I = \Sigma mr^2$; $I_{\text{solid sphere}} = \frac{2}{5}MR^2$; $I_{\text{solid cylinder/disk}} = \frac{1}{2}MR^2$; $I_{\text{ring}} = MR^2$; $I_{\text{rod}} = \frac{1}{12}ML^2$; $\tau = I\alpha$; $L = I\omega$; $x = A\sin\omega t$; $v = \frac{dx}{dt} = \pm\sqrt{A^2 - x^2}$; $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 x$; $K = \frac{1}{2}m\omega^2(A^2 - x^2)$; $U = \frac{1}{2}m\omega^2x^2$; $E = \frac{1}{2}m\omega^2A^2$; $\omega = \frac{2\pi}{T} = 2\pi f$; $T = 2\pi\sqrt{\frac{l}{g}}$; $T = 2\pi\sqrt{\frac{m}{k}}$; $k = \frac{2\pi}{\lambda}$; $v = f\lambda$; $y(x,t) = A\sin(\omega t \pm kx)$; $y = A\cos kx \sin\omega t$; $I = \frac{P}{A}$; $f = \frac{nv}{2l}$; $f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$; $f = \frac{nv}{4l}$; $v = \sqrt{\frac{T}{\mu}}$; $\mu = \frac{m}{l}$; $f_n = \left(\frac{v \pm v_o}{v \pm v_s}\right)f$; $\sigma = \frac{F}{A}$; $\varepsilon = \frac{e}{l_o}$; $Y = \frac{\sigma}{\varepsilon}$; $U = \frac{1}{2}Fe$; $\Delta L = \alpha L_o \Delta T$; $\Delta A = \beta A_o \Delta T$; $\Delta V = \gamma V_o \Delta T$; $\beta = 2\alpha$; $\gamma = 3\alpha$; $pV = nRT$; $n = \frac{m}{M} = \frac{N}{N_A}$; $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$; $pV = \frac{1}{3}Nmv_{rms}^2$; $p = \frac{1}{2}\rho v_{rms}^2$; $K_{tr} = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT$; $U = \frac{1}{2}fNkT = \frac{1}{2}fnRT$; $Q = \Delta U + W$; $W = nRT\ln\frac{V_2}{V_1} = nRT\ln\frac{p_1}{p_2}$; $W = \int p\, dV = p(V_2 - V_1)$

Table of Contents

CHAPTER 1: PHYSICAL QUANTITIES & MEASUREMENTS...1
- DIMENSIONS...1
- SCALARS AND VECTORS...3
- UNIT CONVERSION...5
- ON SIGNIFICANT FIGURES...6

CHAPTER 2: KINEMATICS OF LINEAR MOTION...8
- INSTANTANEOUS AND AVERAGE VELOCITY (OR ACCELERATION)...8
- KINEMATIC EQUATIONS...10
- PROJECTILE MOTION (MOTION IN 2 DIMENSIONS)...14

CHAPTER 3: DYNAMICS OF LINEAR MOTION...17
- TYPES OF FORCES...18
- NEWTON'S LAW OF MOTION...20
- MOMENTUM...21

CHAPTER 4: WORK, ENERGY AND POWER...23
- WORK...23
- WORK ENERGY THEOREM...24
- ENERGIES...25
- POWER...26

CHAPTER 5: CIRCULAR MOTION...27
- UNIFORM CIRCULAR MOTION...27
- THE CASE FOR CONICAL PENDULUM...30
- THE CASE FOR VERTICAL PENDULUM...32

CHAPTER 6: ROTATION OF RIGID BODY...35
- REVISIONS & DEFINITIONS...36
- ANALOGY TO LINEAR KINEMATICS...37
- ROTATIONAL DYNAMICS...37

CHAPTER 7: OSCILLATIONS & WAVES...41
- PART 1: SIMPLE HARMONIC MOTION...43
- PART 2: WAVES...49

CHAPTER 8: PHYSICS OF MATTER...61
- PART 1: MATERIAL CHANGES DUE TO FORCE...63
- PART 2: MATERIAL CHANGES DUE TO HEAT...69

CHAPTER 9: KINETIC THEORY OF GASES & THERMODYNAMICS...73
- MOLECULAR KINETIC THEORY...75
- THERMODYNAMICS...81

Chapter 1: Physical Quantities & Measurements

Dimensions

Dimensions refer to the physical nature of a quantity. Regardless of the unit used, the physical nature of a quantity remains the same. For example, a distance, measured in the unit of metres, or feet, is still a measurement of length. This measurement, therefore has the dimensions of length, most commonly represented by **L.** In this instance, the equation $[d] = L$ is simply states that "The dimension of d is length (**L**)". The following table shows some selected physical quantities and its dimensions:

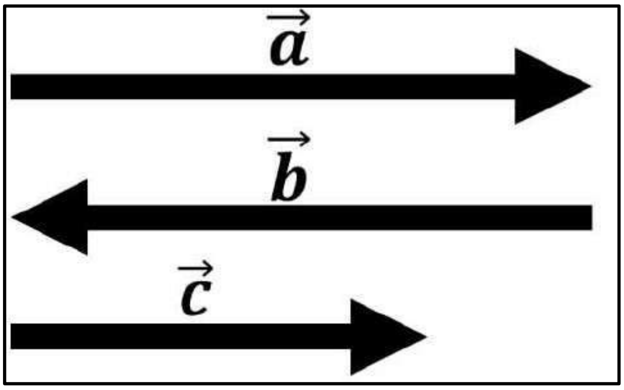
Base Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Length	L	metre (m)	L
Mass	M	kilogram (kg)	M
Time	T	second (s)	T
Electric Current	I	ampere (A)	I
Temperature	T	Kelvin (K)	Θ
Amount of substance	n	mole (mol)	N
Luminosity	L	candela (cd)	J
Derived Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Velocity	$\vec{v}$	ms^{-1}	LT^{-1}
Acceleration	$\vec{a}$	ms^{-2}	LT^{-2}
Momentum	$\vec{p}$	Ns	MLT^{-1}

Angular acceleration	α	$rads^{-1}$	T^{-2}
Electric Charge	Q	Coulomb (A s)	TI
Energy	E	Joule ($J = kgm^2s^{-2}$)	ML^2T^{-2}

Once you understand what dimensions are and how to work with them, you can apply it to **verify the homogeneity of equations.** The word ‘homogeneity’ refers to ‘of the same kind’. Let us consider the equation $s = ut + \frac{1}{2}at^2$ where *s* is displacement of a body, *t* is time taken for the displacement of the body, *u* is the initial velocity of the body and *a* is the acceleration of the body. To ‘verify homogeneity’, we can compare the dimensions the terms on the left-hand side and the right-hand side of the equation. That is to say, *s* must have the same dimensions as *ut* and $\frac{1}{2}at^2$. *s* has the dimension of L, so does *ut* as well as $\frac{1}{2}at^2$.

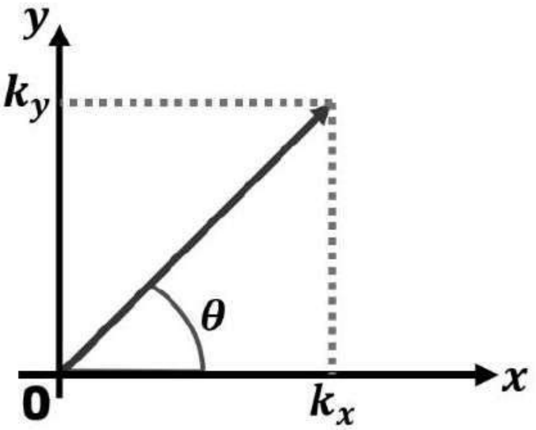
<u>**Scalars and Vectors**</u>

A scalar quantity is a quantity that is fully described by its magnitude. On the other hand, a vector quantity can only be fully described by both its magnitude **and** direction. When you talk about 5kg of rice, that statement is sufficient to describe the mass of the rice, this is where you can see that mass is a scalar quantity. If the rice is falling towards the Earth at a velocity of $2ms^{-1}$, 2 things matter here – how fast the rice is falling <u>**and**</u> the direction in which it is falling. Here you can see that velocity is a vector quantity.



A vector quantity is generally represented by a line segment with an arrowhead. The length of the line segment indicates its magnitude whereas its arrow head tells us the direction of the vector quantity. For example, say vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by the following arrows,

If we were to compare $\vec{b}$ and $\vec{c}$ to $\vec{a}$, we'd say that vector $\vec{b}$ has the same magnitude as $\vec{a}$ but is in the opposite direction, this would tell us that $\vec{b} = -\vec{a}$. Vector $\vec{c}$, on the other hand, is in the same direction as $\vec{a}$. But its magnitude is smaller that $\vec{a}$. The magnitude of vector $\vec{k}$ is denoted by $|\vec{k}|$. We can then relate vector $\vec{a}$ to $\vec{c}$ by the relation $|\vec{a}| > |\vec{c}|$.



Another method to represent vectors is to list the values of its elements in a sufficient number of difference directions, depending on the dimension of the vector. Consider a vector in a 2-dimensional Cartesian coordinate system, a vector $\vec{k}$ can then be represented by $\vec{k} = k_x\hat{\imath} + k_y\hat{\jmath}$ or $\vec{k} =< k_x, k_y >$, defining $\hat{\imath}$ and $\hat{\jmath}$ as unit vectors in the x and y directions respectively. From this notation, one can easily calculate the magnitude (length) of the 2-vector using Pythagoras' Theorem which gives

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2}.$$

Vector additions (or subtractions) can then be done by adding (or subtracting) corresponding components. That is to say, if we have vectors $\vec{a}$ and $\vec{b}$ defined by $\vec{a} =< a_x, a_y >$; $\vec{b} =< b_x, b_y >$, then the addition will yield

$$\vec{a} + \vec{b} =< a_x + b_x, a_y + b_y >.$$

The implication of this definition of vector addition are the following rules:

1. Commutativity of vectors $\Rightarrow \vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. Associativity of vectors $\Rightarrow (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + (-\vec{a}) = 0$

Resolution of vector $\vec{k}$ is then simply

$$k_x = |\vec{k}|cos(\theta);\ k_y = |\vec{k}|sin(\theta).$$

Multiplication of a vector

3 cases to consider when talking about multiplication of a vector:

1. The vector is multiplied by a scalar, then
$$k\vec{a} =< ka_x, ka_y >.$$
2. The **dot product** (also known as scalar or inner product) of two vectors, then
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = |\vec{a}||\vec{b}| \, cos(\theta_{ab}).$$
Note that in the dot product, the operation results in a scalar quantity.
3. The **cross product** (also known as the vector product) of two vectors, then
$$\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\hat{n} = |\vec{a}||\vec{b}| sin(\theta)\hat{n}$$
Note that in the cross product, the operation results in a vector quantity perpendicular to both the x and y axis.

Unit Conversion

Unit conversions are so easy that we tend to overlook the importance of practicing it. Here's a simple reminder on how to do it. Say you are given that $1\ in = 2.54cm$ and you are asked to calculate 8cm in inches, here's how to do it:

$$1in = 2.54cm \leftarrow \text{divide both side by 2.54}$$
$$\frac{1}{2.54} in = \frac{2.54}{2.51} cm = 1cm \leftarrow \text{now multiply it by 8}$$
$$8cm = \frac{8}{2.54} in \approx 3.1496in$$

And that's how you do unit conversion.

In Physics, it is quite often that we are expected to work with values in form of scientific notation. For example, rather than writing down the speed of light as $300000000ms^{-1}$, we'd express this value as

$3 \times 10^8 ms^{-1}$. One issue that may arise from working with scientific notation when using a calculator is the redundancy in typing out "10^x". To help with this, I would suggest that we take advantage of the **rules for exponents**. The example below demonstrates such application.

On Significant Figures

When we talk about the number of significant figures, we are talking about the number of digits whose values are known with certainty. This gives us information about the degree of accuracy of a reading in a measurement. In general, we should practice performing rounding off when the conditions call for it. This is to avoid false reporting. What we mean by false reporting is to give the illusion that our experiments are more sensitive than it actually it. For example, it would be very unlikely that our metre ruler to give reading in the micro scale.

Number	Number of significant figures		Number	Number of significant figures
2.32	3		2600	2
2.320	4		2602	4

When we do calculations, there are some rules (based on the operations) we should be aware of when stating the significant figures of the end value:

1. Multiplication / Divisions – number of significant figures in the result is the same as the least precise measurement in the least precise measurement used in the calculation.

Example:

$$\frac{2.5(3.15)}{2.315} = 3.4$$

2. Addition / Subtraction – The result has the same number of decimal places as the least precise measurement used in the calculation.

   Example:

$$91.1 + 11.45 - 12.365 = 90.2$$

3. Logarithm / antilogarithm – Keep as many significant figures to the right of the decimal point as the are significant in the original number.

   Example:

$$ln(4.00) = 1.39;\ e^{0.0245} = 1.03$$

Chapter 2: Kinematics of Linear Motion

Learning Outcomes (LO)

1. Define:
   a. instantaneous velocity, average velocity and uniform velocity; and
   b. instantaneous acceleration, average acceleration and uniform acceleration.
2. Derive and apply equations of motion with uniform acceleration
   $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$; $s = \frac{1}{2}(u + v)t$
3. Describe projectile motion launched at an angle, θ as well as special cases when θ=0°

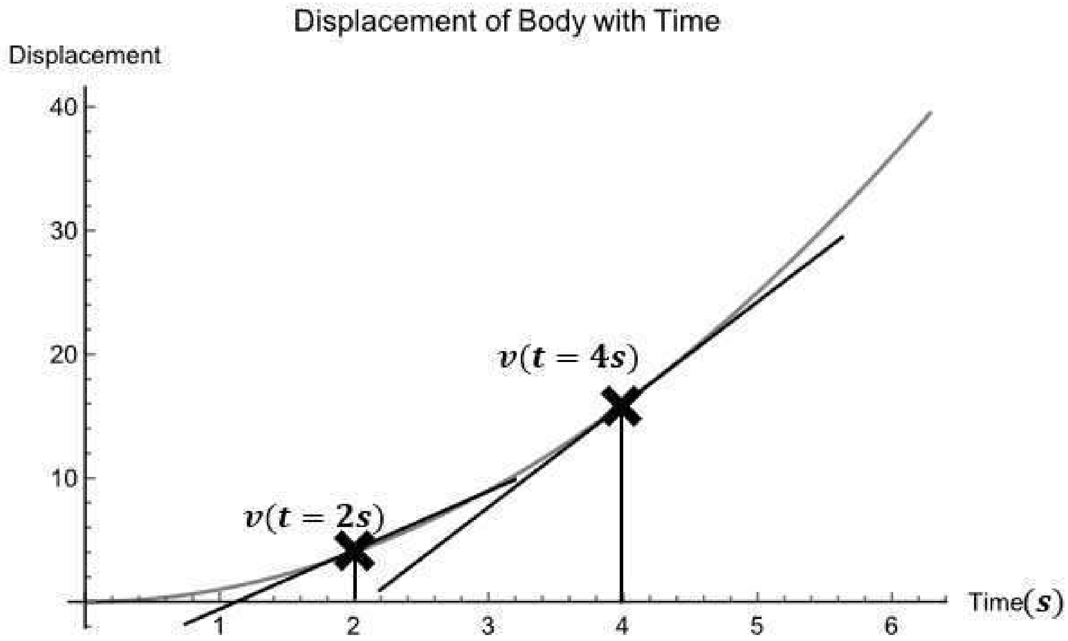
In this chapter, we talk about kinematics of linear motion. Dynamics is the study of motion of bodies under action of forces and their effects. One subbranch to the study of dynamics is kinematics. In the study of kinematics, we consider only the motion of the bodies without worrying too much about the forces that caused the bodies to move. We only worry about the geometry of the motion.

Instantaneous and Average Velocity (or acceleration)

Let us start with reminder of some ideas and terms that you have learnt in your SPM days. 3 mains terms – displacement (s), velocity (v) & acceleration (a). Displacement, denoted by x, simply refers to the change in position of a body. Velocity, v, refers to the rate of change of this change in position, i.e. $v = \frac{dx}{dt}$. Acceleration, a, is defined by the rate of change of velocity, which is the rate of change of the rate of change of position. That is $a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$.

Once we have established that, we can further extend our ideas of velocity and acceleration by thinking about ***instantaneous*** velocity (or acceleration) and ***average*** velocity (or acceleration). By 'instantaneous', we mean 'at a particular instant in time'. When we combine it with velocity (or acceleration), what we mean is velocity (or acceleration) at a particular instant in time. On the other hand, when we say 'average', what we mean is 'over the course of a defined time span'. So, when we say 'average velocity', we usually would accompany it with 'between time t_a and t_b' or 'in 30 seconds', specifying a range of time.

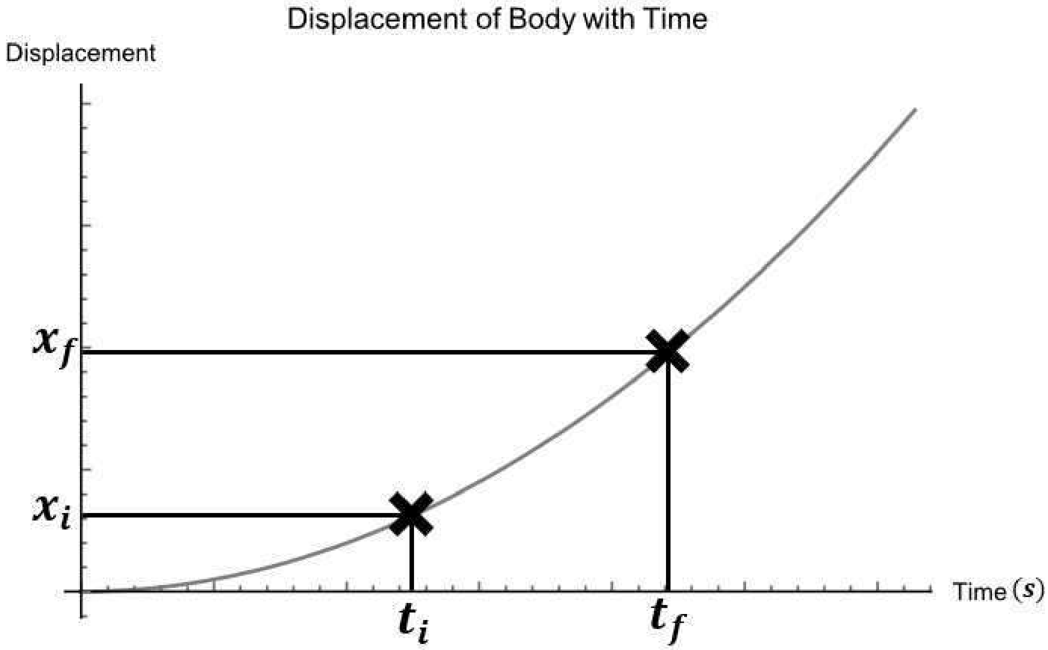
Let us now have a graphical representation. Consider a body



moving at constant velocity,

When we talk about instantaneous velocity, we are asking about a single point in time. From the displacement-time graph, the gradient represents the velocity of the body. As we can see from the graph, the instantaneous velocities when $t = 2s$ and $t = 4s$ are different. This is simply because the body is moving at a non-uniform velocity. If the instantaneous velocities are the same, then we call the motion is described as uniform velocity.

On the other, talking about ***average*** velocity, we simply define range of time, thus choosing two points in time rather than one. Then we take the difference in position and divide it by the difference in time to



calculate the ***average*** velocity. That is to say, for the graph below,
We can calculate the average velocity as

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

We can take the same approach and understanding and apply it to acceleration, but with a velocity time graph rather than a displacement time graph.

Kinematic Equations

Now we want to look at kinematic equations, which are equations that relates variables that describes motion such as displacement, velocity and acceleration.

Derivation by calculus

We'd like to derive the equations from our understanding of linear motion and using calculus. We begin with the definition of acceleration

$$a = \frac{dv}{dt}$$

Assuming constant acceleration, we can rearrange then integrate both sides to yield

$$a\int_{t_{initial}}^{t_{final}} dt = \int_{v_{initial}}^{v_{final}} dv \Rightarrow a\left(t_{final} - t_{initial}\right) = v_{final} - v_{initial}$$

Adjusting such that $t_{initial} = 0$, $t_{final} = t$ and defining $v = v_{final}$, $v_{initial} = u$.

And then rearranging this equation yields

$$v = u + at$$

which is the same equation as the first equation found in LO2. Simply put, the final velocity of a body is initial velocity plus the product of acceleration and time difference.

We can take the same approach to find the third equation in LO2 using the first equation. We start with the definition of velocity and then rearranging it,

$$v = \frac{dx}{dt} \Rightarrow \int v\,dt = \int dx$$

Note that since velocity is not a constant, $v\ dt$ cannot be directly integrated. We therefore need an equation for velocity as a function of time (first equation).

$$\int u + a\,t\,dt = \int dx$$

Since u and a are constants, these integrals become

$$u\int_0^t dt + a\int_0^{\mathrm{t}} dt = \int_0^{\mathrm{x}} dx$$

Solving this integral gives

$$x = ut + \frac{1}{2}at^2.$$

For the second equation, we can start take advantage of calculus by starting with a time independent derivative,

$$\frac{dx}{dv} = \frac{dx}{dt}\frac{dt}{dv} = \frac{v}{a}$$

Rearranging this gives us the needed integral to solve

$$a\int_0^x dx = \int_u^v v\,dv \Rightarrow ax = \frac{1}{2}(v^2 - u^2)$$

Further rearrangement yields an equation

$$v^2 = u^2 + 2ax$$

matching with the third equation found in LO2.

Equation 4 of LO2 does not require any integration, rather we can obtain it using $s = ut + \frac{1}{2}at^2$ and $v = u + at$. This is left for the reader to do.

Geometric Derivation

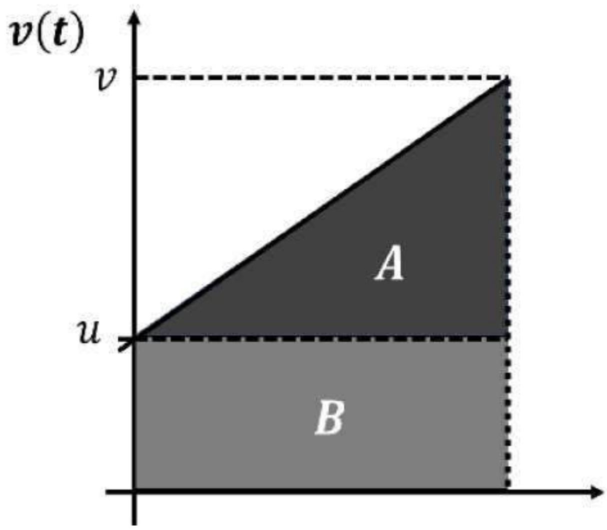
By definition,

$$a = \frac{v-u}{t}$$

Rearranging this gives

$$v = u + at$$

Consider an object that starts its motion with velocity u and maintains its constant acceleration a to a final velocity of v. We can describe its motion diagrammatically as below



Since the area under the graph represents displacement, all we need to do is to add up the area of A and B. If

$$\text{Area}_A = \frac{1}{2}(t)(v-u) = \frac{1}{2}(t)(at) = \frac{1}{2}at^2$$

$$\text{Area}_B = u\text{t}$$

then

$$\text{s} = u\text{t} + \frac{1}{2}at^2.$$

If, on the other hand, we consider

$$s = \frac{1}{2}(t)(v-u) + ut$$

Then we find that

$$s = \frac{1}{2}(v+u)t$$

For the equation of $v^2 = u^2 + 2as$, we can start the derivation by considering

$$v = u + at \Rightarrow t = \frac{v-u}{a}$$

And

$$s = \frac{1}{2}(u+v)t.$$

We can substitute time equation into the displacement to yield

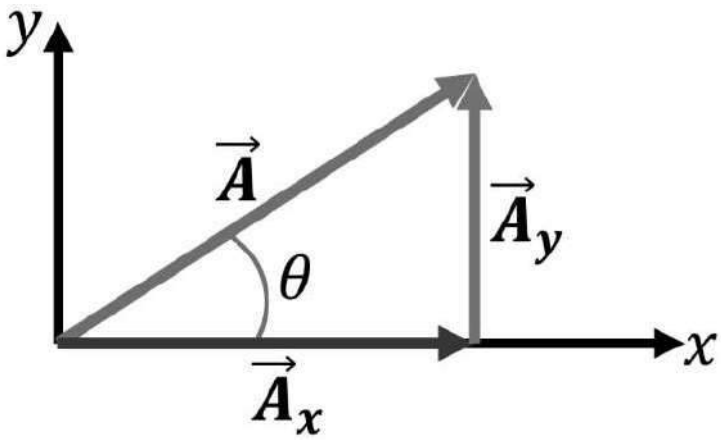
$$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right) = \frac{v^2-u^2}{2a} \Rightarrow v^2 = u^2 + 2as.$$

Projectile Motion (Motion in 2 Dimensions)

When dealing with motion in two dimensions, the minimum that we need is the Pythagorean theorem as well as the definition of tangent. We consider a vector $\vec{A}$ defined by

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where $\vec{A}_x$ and $\vec{A}_y$ are component vectors of $\vec{A}$, each parallel to one of the axes in a rectangular coordinate system.



It then follows that the magnitude and direction of $\vec{A}$ can be related to its components by the Pythagorean theorem and the definition of tangent,

$$|\vec{A}| = \sqrt{|\vec{A}_x|^2 + |\vec{A}_y|^2}$$

$$tan\,\theta = \frac{|\vec{A}_y|}{|\vec{A}_x|}.$$

Conversely, we can work out the components of $\vec{A}$ from the magnitude of $\vec{A}$ and the angle θ,

$$|\vec{A}_x| = |\vec{A}|\, cos\,\theta$$

$$|\vec{A}_y| = |\vec{A}|\, sin\,\theta$$

One case study that we can do on two-dimensional motion is **projectile motion,** a motion that follows a parabolic path. The simplest case of projectile motion would be one where the air resistance and the rotation of the Earth is simply neglected, and that the motion is only affected by the Earth's gravity ($\vec{F}_{gravity} = m\vec{g}$). One important aspect of this case is that the horizontal (x-direction) and vertical (y-direction) motions are independent of each other. This means that the kinematics equation we have studied earlier can be dealt with separately for both x and y directions.

Keeping in mind that $u_x = u\,cos\,\theta$ and $u_y = u\,sin\,\theta$, we can then work out the 6 equations that describes the projectile motion:

x-direction (where $a_x = 0$)	y-direction (where $a_y = -g$)
$v_x = u_x$	$v_y = u_y - gt$
$s_x = u_x t$	$s_y = u_y t - \frac{1}{2}gt^2$
$v_x^2 = u_x^2$	$v_y^2 = u_y^2 - 2gs_y$

We can also work out the velocity of the projectile by keeping in mind that it merely follows from Pythagorean theorem

$$v^2 = v_x^2 + v_y^2.$$

If we substitute the equation for time-x-component, $t = \frac{s_x}{u_x}$, in the equation for displacement in y-direction, $s_y = u_y t - \frac{1}{2} g t^2$, what we get is the parabolic equation for the projectile motion path,

$$\left(\frac{u_y}{u_x}\right) s_x - \left(\frac{1}{2u_x^2}\right) s_x^2 - s_y = 0.$$

There are two more items that are of our interest:

1. If we were to look for the "**peak**" of the parabolic path, we can do so by applying $v_y = 0$ to the kinematics equations. This is simply because it is at this peak that $u_y = gt$ such that the velocity of the projectile is momentarily zero before the projectile falls back down towards the Earth.
2. Another item that would be of our interest is the **range** of the projectile motion. By range, what we are referring to is the point at which the projectile reaches back to ground or stop accelerating in the y-direction. This would differ from case to case, of course, and we shall demonstrate in the sample problems following this.

Chapter 3: Dynamics of Linear Motion

Learning Outcomes

5. Define
   a. Momentum, $\vec{p} = m\vec{v}$
   b. Impulse, $J = F\Delta t$
6. Solve problem related to impulse and impulse-momentum theorem,

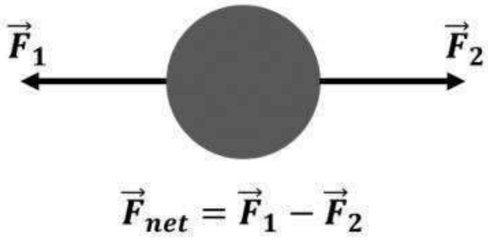
$$J = \Delta p = mv - mu$$

   **1D only*

7. Use F-t graph to determine impulse.
8. State:
   a. the principle of conservation of linear momentum.
   b. Newton's laws of motion.
9. Apply
   a. the principle of conservation of momentum in elastic and inelastic collisions in 2D collisions.
   b. Newton's laws of motion.
   **include static and dynamic equilibrium for Newton's first law motion*
10. Differentiate elastic and inelastic collisions. (remarks: similarities & differences)
11. Identify the forces acting on a body in different situations - Weight, *W;* Tension, *T;* Normal force, *N;* Friction, *f;* and External force (pull or push), *F*.
12. Sketch free body diagram.
13. Determine static and kinetic friction, $f_s \leq \mu_s N$, $f_k = \mu_k N$

In the previous chapters, we have looked at describing motion without the hassle of asking, "what force is causing the body to move?". In this chapter, we aim to expand our knowledge to a body's motion in that very aspect.

Types of Forces

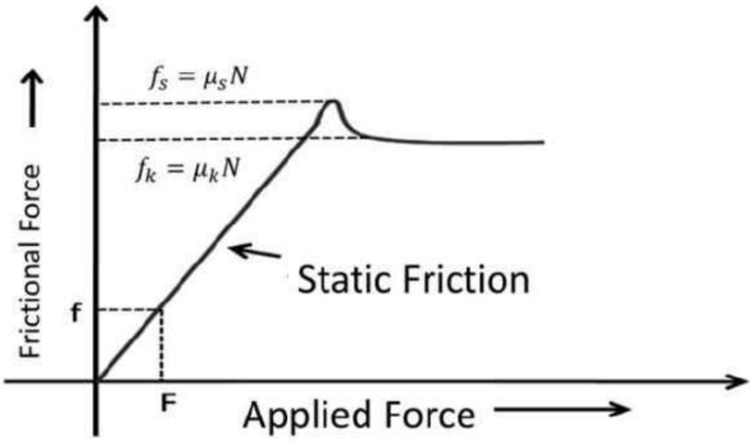


We begin with asking the question, "what is force?", a simple answer would be to say force is a push and pull. Here, however, let us define force a bit further. Let us define force as **an agent for motion change**. Force is a vector quantity, that means **direction matters**. Two oppositely directed force acting on the same body work against each other. A body can experience multiple forces acting on it, however it is the net force, i.e., the resultant of all the forces acting on the body, that changes the motion of the body.

4 types of forces we'd consider in this chapter – gravitational (weight), tensional, normal and frictional. Their definitions and directions are as follows:

Forces	Definitions	Directions
Gravitational	Force exerted upon a body interacting with a gravitational field.	Towards the gravitational source.
Tensional	Force transmitted axially through a massless one-dimensional continuous element.	Along the one-dimensional continuous element.
Normal	Support force, perpendicular to the surface, exerted upon a body in contact with a stable object.	Perpendicular to the surface the body is in contact with.

Frictional	Force acting upon bodies that are in contact and moving relative to each other.	Against the direction of motion.



One particular type of force that maybe be of our interest is frictional force. This is because frictional force depends on the motion of the object. If the object is static, then it is subject to *static friction*. On the other hand, if the object is moving (relative to the surface it is in contact with) at some velocity, and therefore has some kinetic energy, then the object is subject to *kinetic friction*. Static friction is generally higher than kinetic friction because of the asperities (roughness) of the surfaces of the contacting bodies. This asperity enables the surfaces to interlock with each other, causing adhesion. This means that the force applied to the body must overcome this adhesion before the bodies can start moving relative to each other. This phenomenon can be observed by looking at the frictional force as a function of applied force graph:

As the applied force is increased, so does the frictional force. This is true until a certain threshold is reached, after which the body will start to move. This threshold is exactly the unlocking of the asperities.

Newton's Law of Motion

There are laws of motion that a moving under force would generally follow. These laws were first introduced and came into its modern form via Newton's *Principia*. In it, 3 laws of motions were found:

1. A body, when no external force is applied, will not undergo velocity change, i.e.

$$\vec{F}_{external} = 0 \Rightarrow \Delta\vec{v} = 0$$

2. When a force is acted upon it, a body will move in a manner such that rate of momentum change is equal to the said force, i.e.

$$\vec{F} = \frac{d\vec{p}}{dt}.$$

3. Forces exerted onto two interacting bodies will be equal in magnitude but opposite in direction.

   If $\vec{F}_{12}$ is force exerted onto body 1 by body 2, then

$$\vec{F}_{12} = -\vec{F}_{21}$$

These three laws form the foundation for what is known today as the *Newtonian Laws of Motion*.

Momentum

The first and third requires no further definitions of variable, however the second one, mentions an idea of ***momentum***. It seems useful to define this term at this point. What we mean by momentum at this point is the property of a moving body that rises from the product of the mass the body and its velocity, i.e.

$$\vec{p} = m\vec{v}.$$

If the velocity of the body changes (and therefore so does the momentum of the body in question), we quantify that change and call it ***impulse***:

$$\vec{J} = \Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t.$$

This also means that the area under the F-t graph represents impulse.

It is from Kinematics that we know a change in velocity means a non-zero acceleration. Knowing this as well as Newton's Second Law, we can say that force is present when acceleration is non-zero,

$$\vec{F} = m\vec{a}.$$

This statement, of course can be derived quite simply from Newton's Second Law of motion:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

Here we can see that a change in mass may also produce force and that if mass change is zero, then what we have is the equation seen before,

$$\vec{F} = m\vec{a}.$$

One of the ways for bodies to interact is through collisions. When this happens, assuming this happens in an isolated system. The total momentum of the system doesn't change with the passage of time. The momenta of the participating bodies may change, but not the vector sum total momentum of the system. When we say that a quantity doesn't change, we say that the quantity is **conserved**. So, in this case, we say that the **total momentum is conserved**. Conservation of momentum is simply

$$\Delta(\Sigma p) = 0.$$

When we talk about collisions, we may consider two types of collision – elastic and inelastic collisions. Note that whilst momentum is conserved in **all** types of collisions, kinetic energy is not as it may be converted into other forms of energy (e.g., sound energy). It is this exact parameter from which we differentiate elastic from inelastic collisions. A perfectly elastic collision is defined by a collision in which both momentum and kinetic energy is conserved whilst a perfectly inelastic collision is a collision in which conservation of kinetic energy is not obeyed.

Chapter 4: Work, Energy and Power

Learning Outcomes

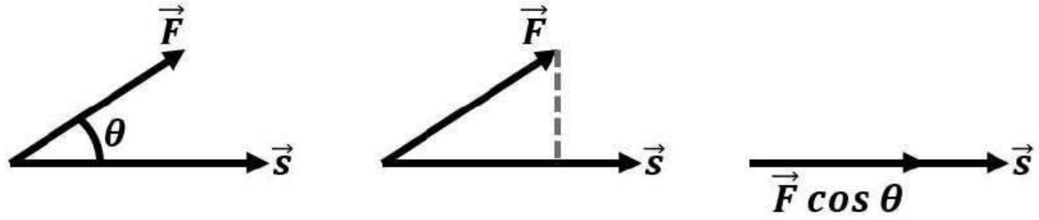
a) State:
   (a) the physical meaning of dot (scalar) product for work: $W = \vec{F} \bullet \vec{s} = Fscos\theta$
   (b) the principle of conservation of energy.
b) Define and apply
   (a) work done by a constant force.
   (b) Gravitational potential energy, $U = mgh$
   (c) Elastic potential energy for spring, $U_s = \frac{1}{2}kx^2 = \frac{1}{2}Fx$
   (d) Kinetic energy, $K = \frac{1}{2}mv^2$
   (e) work-energy theorem, $W = \Delta K$
   (f) average power, $P_{av} = \frac{\Delta W}{\Delta t}$ and instantaneous power, $P = \vec{F} \bullet \vec{v}$
c) Determine work done from a force-displacement graph.
b) Apply the principle of conservation of mechanical energy.

Work

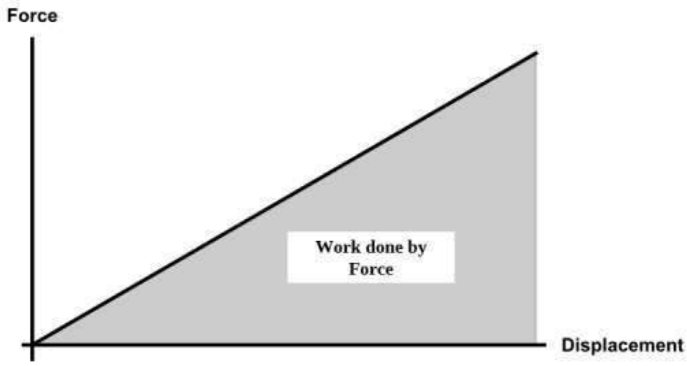
Let us begin by defining work. The work on an object, *W*, is defined to be the product of magnitude of the displacement, *s*, and the force component parallel to the displacement of the object $F_{\parallel}$, i.e.

$$W = F_{\parallel}s = \vec{F} \cdot \vec{s}$$

Notice that it is not the product of force and displacement but the product of displacement and force component, the important characteristic of that particular force component is that it must be parallel to the displacement. The diagram below illustrates this point, we cannot simply multiply the magnitude of $\vec{F}$and $\vec{s}$. We must find the component of force that **is** parallel the displacement, and then we can find their product.



This of course means that in the force-displacement graph, work done by a force is equal to the area under the graph. That is to say, the work done to displace an object from $x_i$to x_f is simply



$$W = \int_{x_i}^{x_f} F_{\parallel} dx$$

Work Energy Theorem

When an object moves, we say it contains kinetic energy. Kinetic energy quantifies the amount of energy a moving object has. It depends on the velocity of the moving object,

$$E_k = K = \frac{1}{2}mv^2.$$

Now what we want to do is to show a relationship between the quantity related to moving object (kinetic energy) and another quantity related to the changes of object position (work).

We begin with the definition of work done on an object and Newton's Second Law of motion to show that

$$W = Fs\,; \; F = ma \Rightarrow W = mas$$

Assuming that the force is constant and therefore the acceleration is also constant, we can then apply equation of kinematics

$$v^2 = u^2 + 2as;\ W = m(as)$$

$$W = m\frac{v^2 - u^2}{2} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = K_{final} - K_{initial} = \Delta K$$

This show derivation brings about an important theorem, called the work-energy theorem. This theorem states that the work done onto a body is equal to the change in kinetic energy of the body.

Energies

Two more energies that are of interest to us. The first is the gravitational potential energy, which is the energy contained in an object due to its position, measure from a gravitational source. A more detailed analysis is found in the Newtonian Gravity part of this course. At this point, it is sufficient for us to know that if an object of mass m is position at height h from the surface of the Earth, then the gravitational potential energy found in that object is

$$E_{gp} = mgh$$

The second type of potential energy of interest is the elastic potential energy of a spring. By Hooke's Law, the force acting on a spring is direction proportional to its extension (or compression).

$$\text{Hooke's Law: } F = -kx$$

We can then utilize work energy theorem to find the elastic potential energy of a spring,

$$W = -\int F\,dx = \int kx\,dx \Rightarrow E_{ep} = \frac{1}{2}kx^2$$

Apart from the conservation of momentum, another important principle of conservation crucial to our study of moving bodies is the **principle of mechanical energy conservation.** The law simply states that the sum of all kinetic energy and all potential energy must remain constant at all times. That is to say

$$\Delta E_{total} = 0.$$

Power

Now that we have familiarize ourselves with work and energy, let us now talk about **power**, which is simply defined by the rate of work done. Average power refers to the work done within a time interval,

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{W_{final} - W_{initial}}{t_{final} - t_{initial}}.$$

On the other hand, instantaneous power refers to the mechanical power at one instant in time

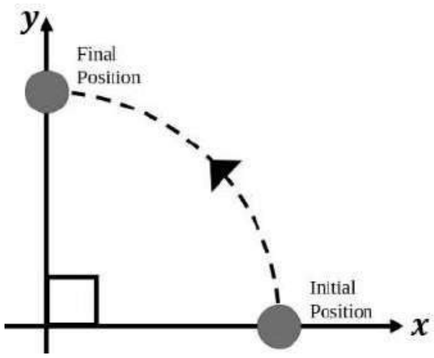
$$P_{instantaneous} = \frac{dW}{dt} = \vec{F}\frac{d\vec{s}}{st} = \vec{F} \cdot \vec{v}$$

Chapter 5: Circular Motion

Learning Outcomes

14. Define and use:
    a. angular displacement, θ
    b. period, *T*
    c. frequency, *f*
    d. angular velocity, ω
15. Describe uniform circular motion.
16. Convert units between degrees, radian, and revolution or rotation.
17. Explain centripetal acceleration and centripetal force, $a_c = \frac{v^2}{r} = r\omega^2 = v\omega$
    and $F_c = \frac{mv^2}{r} = mr\omega^2 = mv\omega$
18. Solve problems related to centripetal force for uniform circular motion cases: horizontal circular motion, vertical circular motion and conical pendulum.
    **exclude banked curve*

Uniform Circular Motion



Consider moving a body from coordinates (0,r) to (r,0) whilst keeping the same distance *r*, from the origin (0,0). The motion that has taken place is what is known as a **rotation** and the path the body has taken is what we consider to be **circular**. We call it **circular** simply because throughout the motion, a fixed distance *r* was kept between the origin and the object. This transformation may be easy enough to see and describe as it is simply a 90^o rotation. However, dealing with rotations using Cartesian coordinates can get really complicated. So let us propose new method of describing such motion.

In linear dynamics, we started with displacement x and took derivatives of it twice over to obtain acceleration. In dealing with rotational motion, let us instead begin with angular displacement, θ. The rate of change of this angular displacement, we can then call **angular velocity** ω , and the rate of change of angular velocity is what is known as **angular acceleration**.

$$\omega = \frac{d\theta}{dt};\ \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

If we define θ in radian, then we can work out the arc length of the object’s path using

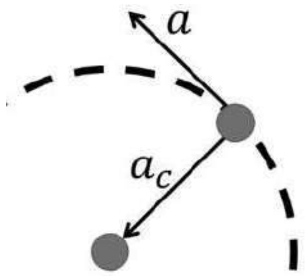
$$s = r\theta.$$

It is at this point, it is useful for use to know the conversion between angles in radians and angles in degrees, which is $2\pi\text{rad} = 360^o$. If we differentiate the arc length with respect to time, what we get is quite simple the tangential velocity v, which is the speed at which the body covers said length.

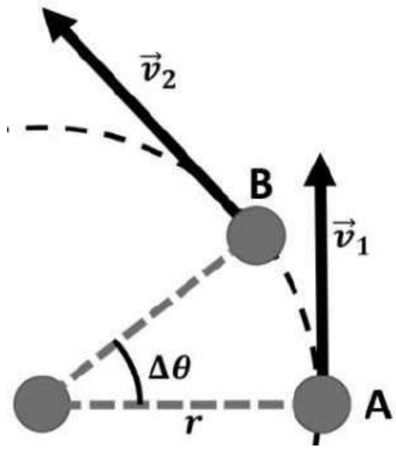
$$v = \frac{d}{dt}s = r\frac{d\theta}{dt} \Rightarrow v = r\omega$$

We may apply the same logic to find the tangential acceleration a and relate it to angular acceleration α,

$$a = \frac{d}{dt}v = r\frac{d\omega}{dt} \Rightarrow a\ =\ r\alpha.$$



Now we ask what direction are these quantities, tangential velocity and tangential acceleration? As the name suggests, they have the direction tangent to the circular path. Now, tangential acceleration alone will not be enough to ensure the body in motion to follow a circular path, we now need an acceleration towards the centre of the circle, called a **radial (centripetal) acceleration,** generally denoted by a_c. Together with the tangential acceleration, they combined and ensures the body follows a circular path.

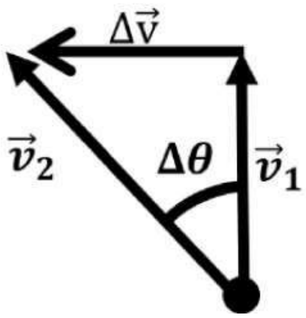


We can now work out the equation for this centripetal acceleration. We can begin by reminding ourselves that the effect of centripetal acceleration is the change in direction. Mind that the speed does not change, but the direction changes. We can see that if v is relatively small,

$$a_c = \frac{dv}{dt} = \frac{\Delta v}{\Delta t}$$

$$\Delta v \approx v\Delta\theta \text{ (by geometry)}$$

We also know that the change in arc length is related to the change in the angle,



$$\Delta\theta = \frac{\Delta s}{r} \approx \frac{v\Delta t}{r} \Rightarrow \Delta t \approx \frac{r\Delta\theta}{v}$$

This means that

$$a_c = \frac{v^2}{r}.$$

The force associated with this centripetal acceleration is known as the **centripetal acceleration** and follows the equation

$$F = ma_c = \frac{mv^2}{r}.$$

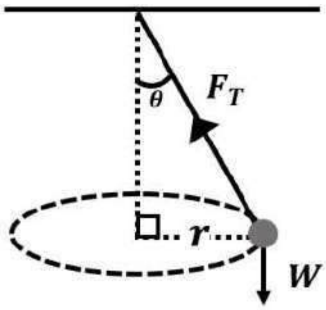
Centripetal force is **not** a type of force per se. Rather it is a way to say a force is acting as a centripetal force. For example, the gravitational force causes the moon to curve and travel in a circular path around the earth. In this instance, the gravitational force acts as a centripetal force. The way we can think about this is when we talk about retarding force, that retarding force in linear motion could be friction force or any other applied force acting opposing to the direction of motion. In the case of centripetal force, any force could act as centripetal force if it is the force that causes the body to follow a circular path.

When we work with bodies following a circular path, we know that after $\Delta\theta = 2\pi$, the object has returned to its initial position. We say that it has undergo one full revolution. The time the body take to travel one revolution is what we call **period** *T*, and the number of revolutions per unit time is what we call **frequency** ***f*.** Frequency and period is merely the inverse of each other.

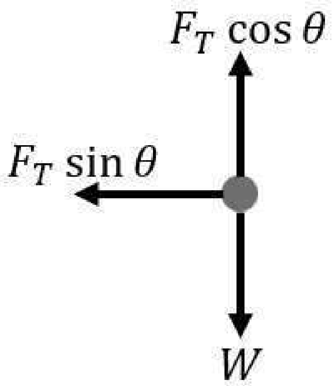
$$T = \frac{1}{f}$$

The case for conical pendulum

The conical pendulum is a system of pendulum in which rather than having the pendulum bob swing back and forth in a single place, the path of the pendulum bob is circular about a center, whereby the string along with the pendulum bob traces a cone.



Consider a conical pendulum consisting of a bob of mass *m* revolving without friction in a circular path at constant speed *v* on a string of length *l* at an angle θ from the vertical. We can see that two forces acting on the pendulum bob, tension along the string and weight of the pendulum bob. The tensional force can be resolved into its horizontal component $T\ sin\ \theta$, and its vertical component $T\ cos\ \theta$.



Applying Newton's second law, we find that

$$F_T cos\,\theta = mg;\ F_T\,sin\,\theta = \frac{mv^2}{r} = \frac{mv^2}{l\,sin\,\theta}$$

To find the angle θ from the vertical,

$$\frac{F_T\ sin\ \theta}{F_T\ cos\ \theta} = tan\ \theta = \left(\frac{v^2}{gr}\right)$$

$$\theta = tan^{-1}\left(\frac{v^2}{gr}\right)$$

To find the period for the pendulum,

$$F_T = \frac{mg}{cos\,\theta} = \frac{mv^2}{r\,sin\,\theta};\ v = r\omega = \frac{2\pi r}{T}$$

$$\frac{g}{cos\,\theta} = \frac{1}{r\,sin\,\theta}\left(\frac{4\pi^2 r^2}{T^2}\right) \Rightarrow T(r,\theta) = 2\pi\sqrt{\frac{r}{g\ tan\,\theta}}$$

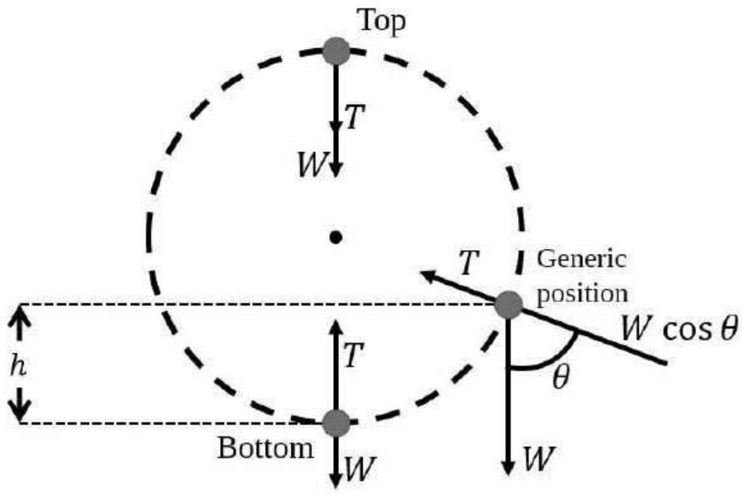
Noting that $r\ =\ l\,sin\,\theta$, the period of the oscillation is therefore

$$T(l,\theta) = 2\pi\sqrt{\frac{l\,cos\theta}{g}}$$

We can see that in the case for the conical pendulum, the period is independent of the mass used, rather it depends on the length of the string used.

The case for vertical pendulum

Consider swinging a ball of mass *m* vertically via a string of negligible mass, such that it follows a circular path with radius *r*. The path could be illustrated as shown in a diagram.



When the ball is at the top of the path, we can see that the tensional force is directed in the same direction as the ball's weight. On the other hand, when the ball is at the bottom of the path, the tensional force is directed in the opposite direction of the weight of the ball.

We can apply compare the velocities of the ball at any generic position on the path using conservation of energy.

$$\frac{1}{2}mv_{bottom}^2 = mgh + \frac{1}{2}mv_{generic}^2$$

$$v_{generic} = \sqrt{v_{bottom}^2 - 2gh}$$

We can see that the velocity of the ball is not constant as it follows the vertical circular. In this case, we do recognize that it is the ball undergoes circular motion but not **uniform** circular motion. This is of course if the tensional force along the string is constant. If different tension is applied along the circular path, then it is possible to ensure **uniform circular motion**.

Let us compare then the tension needed at the top and the bottom of the circular path. At the bottom, the tension is pointing upwards and the weight is pointing downwards. We then have $F_c = T - mg$. At the top, the tensional force and the weight is pointing in the same direction (downwards) and therefore we have $F_c = T + mg$.

Now if we consider the forces acting on the ball at the generic position,

$$F_{net} = F_c = \frac{m\, v_{generic}^2}{r} = T - mg\, cos\, \theta$$

From the figure, we find $cos\, \theta$ to be

$$cos\, \theta = \frac{r - h}{r} = 1 - \frac{h}{r}$$

As such, we may express the tensional force along the string to be

$$T = m\left(\frac{v_{generic}^2}{r} + g - \frac{gh}{r}\right)$$

Expressing this in terms of the speed at the bottom gives,

$$T = m\left(\frac{v_{bottom}^2}{r} + g - \frac{3gh}{r}\right)$$

$$T = \frac{mg}{r}\left(\frac{v_{bottom}^2}{g} + r - 3h\right)$$

What can we do with this information? Well one of the things we can do is to talk about the **minimum speed** at the bottom of the motion to ensure the ball completes one loop.

At the top of the loop, we want the tensional force to be positive, such that

$$T_h = T_{highest} \geq 0$$

At this point,

$$h = 2r \Rightarrow T_h = \frac{mg}{r}\left(\frac{v_{bottom}^2}{g} + r - 6r\right) = \frac{mg}{r}\left(\frac{v_{bottom}^2}{g} - 5r\right)$$

Since we know $\frac{mg}{r} > 0$, then for $T_h \geq 0$, we need

$$\frac{v_{bottom}^2}{g} - 5r \geq 0$$

So the **minimum speed** required at the bottom of the motion to ensure the ball completes one loop must follow the condition of

$$v \geq \sqrt{5gr}$$

We shall deal with rotational kinematics in the following chapter.

Chapter 6: Rotation of Rigid Body

Learning Outcomes

1. Define and use:
   a. angular displacement, θ;
   b. average angular velocity, ω_{av};
   c. instantaneous angular velocity, ω;
   d. average angular acceleration, α_{av}; and
   e. instantaneous angular acceleration, α
   f. torque
   g. moment of inertia, $I = \sum mr^2$
   h. net torque, $\sum \tau = I\alpha$
   i. angular momentum, $L = I\omega$
2. Analyse parameters in rotational motion with their corresponding quantities in linear motion:

   $s = r\theta, v = r\omega, a_t = r\alpha, a_c = r\omega^2 = \frac{v^2}{r}$
3. Solve problem related to rotational motion with constant angular acceleration:

   $\omega = \omega_o + \alpha t, \theta = \omega_o + \frac{1}{2}\alpha t^2, \omega^2 = {\omega_o}^2 + 2\alpha\theta, \theta = \frac{1}{2}(\omega_o + \omega)t$
4. State and apply:
   a. the physical meaning of cross (vector) product for torque, $|\vec{\tau}| = rFsin\theta$
   b. the conditions for equilibrium of rigid body, $\sum F = 0, \sum \tau = 0$
   c. the principle of conservation of angular momentum.
5. Solve problems related to equilibrium of a uniform rigid body.

   **Limit to 5 forces*
6. Use the moment of inertia of a uniform rigid body.

   (Sphere, cylinder, ring, disc, and rod).

Revisions & Definitions

In the previous chapters, we have familiarised ourselves with the idea of instantaneous quantities, average quantities, angular displacement, angular velocity as well as angular acceleration. We recap those ideas in this section.

When we say angular velocity, what we mean is the rate of change of angular displacement θ,

$$\omega = \frac{d\theta}{dt}.$$

We may find the **average** angular velocity if we are only concerned about the final state and the initial state of θ, i.e.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_{final} - \theta_{initial}}{t_{final} - t_{final}}.$$

Such distinctions can also be done for angular acceleration, i.e.

$$\alpha_{instant.} = \frac{d\omega}{dt};\ \alpha_{ave} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_{final} - \omega_{initial}}{t_{final} - t_{initial}}$$

If we consider the relationship between angular displacement, θ and the arc length of a motion, s, we can quite quickly workout the relationship between the linear speed v and the angular speed ω.

$$s = r\theta \overset{\frac{d}{dt}}{\Rightarrow} v = r\omega$$

Similar operations can be done to fine the relationship between tangential acceleration, a_t and angular acceleration, α.

$$v = r\omega \overset{\frac{d}{dt}}{\Rightarrow} a_t = r\alpha$$

Analogy to linear kinematics

We now have the ingredients we need to work out the **equations for rotational motion with constant angular acceleration**. Because ω and α may be defined analogously to their linear counterparts, v and a_t, equations for linear kinematics may be applied when we make substitutions θ for s, ω for v, ω_o for u and α for a. Below we present the results of the substitutions

$$v = u + at \Rightarrow \omega = \omega_o + \alpha t$$

$$s = ut + \frac{1}{2}at^2 \Rightarrow \theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$v^2 = u^2 + 2as \Rightarrow \omega^2 = \omega_o^2 + 2\alpha$$

Rotational Dynamics

Considering we have analogous cases between linear kinematics and rotational kinematics, i.e., θ for x, ω for v and α for a, surely, we must have analogous quantities for describing a body's motion.

If we recall Newton's 2nd Law of Motion, whereby we say a force accelerates a body, we can now ask what quantity brings about changes to the angular acceleration? We'd be right in this line of thinking and what we will eventually find is a quantity called **torque, τ**. Much like rotational kinematics, we can relate torque to its linear counterpart,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Where $\vec{r}$ is the distance between the force applied and the rotation axis and $\vec{F}$ is the force vector applied. The direction of the torque will follow mathematical convention of cross products.

Now we ask, what is the rotational analogue to the Newton's 2nd law of motion? Considering we know $F = ma$, we know we can substitute τ for F and α for a. But what do we substitute m with? We substitute it with the **moment of inertia, $\boldsymbol{I}$**. In that case, we shall have

$$F = ma \Rightarrow \tau = I\,\alpha.$$

Just as in Newton's law of motion, equilibrium dictates $\Sigma F = 0$, equilibrium in the rotation of rigid body dictates

$$\Sigma\tau = \tau_{clockwise} - \tau_{anticlockwise} = 0.$$

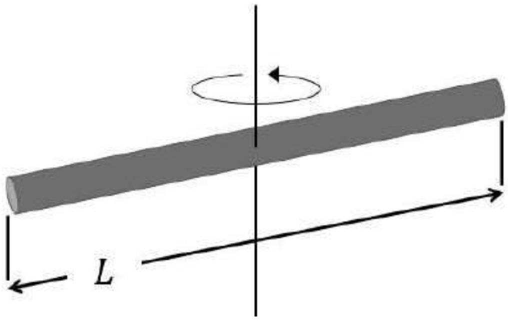
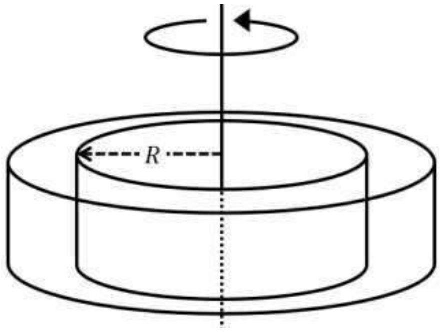
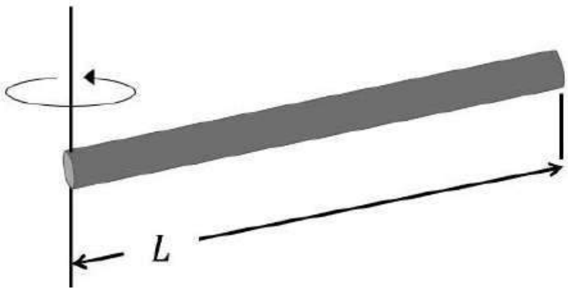
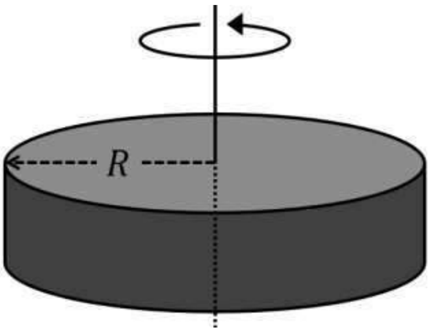
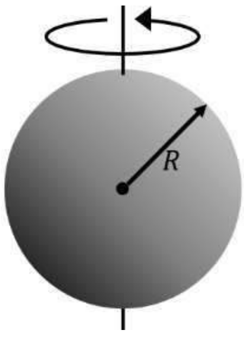
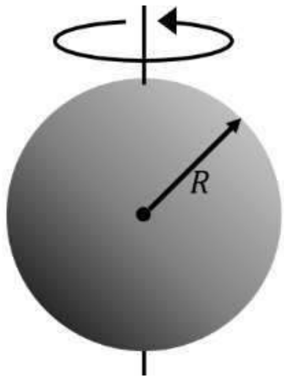
But what is this moment of inertia? If mass can be defined to be property of the body that resists linear acceleration, then moment of inertia can be defined to be as the property of the body (or system) to resist angular acceleration. If the system consists of discrete individual mass points, then

$$I = \Sigma m_i r_i^2$$

If the system consists of a continuous distribution of matter, then

$$I = \int r^2 dm$$

Considering that this is an algebra-based physics course, you are not expected to be able to derive equations for moment of inertia for a system of continuous distribution of matter (though I highly recommend you trying as you should know integration from your maths course!). As such, moment of inertia equations commonly used in this course is provided in the table below:

<table>
<tr><th>Description and Diagram</th><th></th></tr>
<tr><td>Thin rod of mass M about its centre

$I = \frac{1}{12}ML^2$</td><td>Thin ring about its centre axis

$I = MR^2$</td></tr>
<tr><td>Thin rod of mass M about its end

$I = \frac{1}{3}ML^2$</td><td>Disk/ solid cylinder about its axis

$I = \frac{1}{2}MR^2$</td></tr>
<tr><td>Solid Sphere

$I = \frac{2}{5}MR^2$</td><td>Hollow spherical shell

$I = \frac{2}{3}MR^2$</td></tr>
</table>

Note: For those of you who are keen on learning more about moment of inertia and how their equations are derived, feel free explore

integrations related to rotational inertia, **the parallel axis theorem**, and **the perpendicular axis theorem**.

Since we have talked about rotational analog to kinematics as well as forces, it is natural to proceed to asking if there exist a rotational analog to linear momentum. There is! It is called **angular momentum, *L*,** and is defined as

$$L = I\omega = \mathrm{rp}\,\sin\theta.$$

Conservation law also exist for this quantity,

$$\Delta L \ = 0.$$

Chapter 7: Oscillations & Waves

Learning Outcomes

SHM	1.	Explain SHM.
	2.	Apply SHM displacement equation, $x(t) = A\ sin\ \omega t$
	3.	Derive, use and apply equations: a. velocity, $v = \omega Acos\omega t = \pm\omega\sqrt{A^2 - x^2}$ b. acceleration, $a = -\omega^2 A\ sin\ sin\ \omega t\ = -\omega^2 x$ (remarks: No calculus. Derive use algebra and trigonometry method, refer reference book Cutnell) c. kinetic energy, $K = \frac{1}{2}m\omega^2(A^2 - x^2)$ d. potential energy, $U = \frac{1}{2}m\omega^2 x^2$ e. period of SHM, T for simple pendulum, $T\ = 2\pi\sqrt{\frac{l}{g}}$ f. period of SHM, T for mass-spring system : $T\ = 2\pi\sqrt{\frac{m}{k}}$
	4.	Emphasise the relationship between total SHM energy and amplitude.
	5.	Analyse the following graphs: a. displacement-time; b. velocity-time; c. acceleration-time; and d. energy-displacement.

Learning Outcomes

Waves	1.	Define/state: a. wavelength b. wavenumber c. the principle of wave propagation for constructive and destructive interference d. Doppler Effect for sounds waves
	2.	Solve problems: a. related to progressive wave equation, $y(x,t) = Asin(\omega t \pm kx)$ b. related to the fundamental and overtone frequencies for stretched string ($f_n = \frac{nv}{2L}$) and open ($f_n = \frac{nv}{2L}$) and closed ($f_n = \frac{nv}{4L}$) ended air columns.
	3.	Distinguish/compare: a. between particle vibrational velocity and wave propagation velocity b. progressive and standing waves
	4.	Use: a. wavenumber, $k = \frac{2\pi}{\lambda}$ b. particle vibrational velocity, $v_y = A\omega cos(\omega t \pm kx)$ c. propagation velocity, $v = f\,\lambda$ d. the standing wave equation, $y = 2Acos(kx)sin(\omega t)$ e. wave speed in a stretched string, $v = \sqrt{\frac{T}{\mu}}$ f. Doppler Effect equation, $f_{apparent} = \left(\frac{v \pm v_{observer}}{v \mp v_{source}}\right) f$, for relative motion between source and observer. Limited to stationary observer and moving source and vice versa.
	5.	Analyse graphs of a. displacement-time, $y - t$ b. displacement-distance, x-t

Part 1: Simple Harmonic Motion

When we observe a motion in which the restoring force acting upon a system is directly proportional to the magnitude of its displacement and acts towards the initial position, then the situation at hand we say to have **simple harmonic motion (SHM).** Mathematically, systems undergoing SHM will obey

$$F_{restoring} \propto -x$$

The importance of understanding SHM is that it is foundational for the understand and analysis of more complex periodic motion, which is typically analysed using Fourier Analysis. Applying Newton's 2nd law of motion to the equation above gives us the differential equation

$$m\frac{d^2x}{dt^2} = -kx$$

where k is just the constant of proportionality for the relations above.

The solution for this differential equation is then

$$x(t) = A\,cos\,(\omega t) \text{ where } \omega = \sqrt{\frac{k}{m}}$$

Since we know the limits to the cosine function is $-1 \leq\ cos\,\omega t \leq 1$, a full cycle requires ωt to go from 0 to 2π and that a period is defined to be the time for one full cycle, we can make the conclusion that

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

And of course, frequency f is simply the inverse of period,

$$f = \frac{\omega}{2\pi} = 2\pi\sqrt{\frac{k}{m}}$$

Kinematics of SHM

Once we have defined an equation for displacement, we can quite easily proceed to equations for velocity and acceleration. This can be achieved by 2 methods – by calculus and by algebra.

Let us first work with **calculus**.

For velocity, it is simply the first derivative of displacement with respect to time and therefore has the form

$$v = \frac{dx}{dt} = \omega A\, cos\, (\omega t)$$

and acceleration is simply the second derivative of displacement,

$$a = \frac{d^2x}{dt^2} = -\omega^a A\, sin\, (\omega t)$$

As for functions of displacement,

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} \Rightarrow v\frac{dv}{dx} = -\omega^2 x$$

Solving this differential equation yields

$$\int v\ dv = -\omega^2 \int x\ dx$$

This leads to

$$\frac{v^2}{2} = -\omega^2 \frac{x^2}{2} + C$$

We can set such that at $x = A,\ v = 0$.

$$\frac{0^2}{2} = -\omega^2 \frac{A^2}{2} + C \Rightarrow C = \omega^2 \frac{A^2}{2}$$

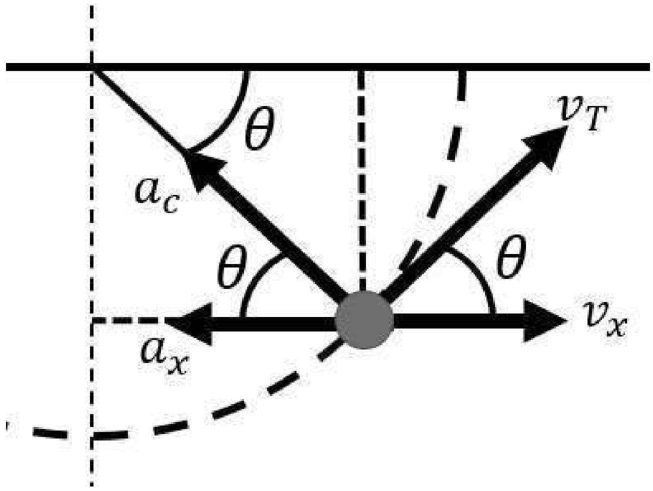
We then have

$$\frac{v^2}{2} = -\omega^2 \frac{x^2}{2} + \omega^2 \frac{A^2}{2}$$

Rearranging for v give

$$v = \pm\omega\sqrt{A^2 - x^2}$$

Now, let us derive the equations for velocity and acceleration for an object under SHM **without calculus**, only algebraically and trigonometrically. Let us consider the motion of a simple pendulum shown in the diagram.



We can now use this vector diagram to relate several physical quantities together. Note that $\theta = \omega t$. First, note that the displacement of the pendulum is expressed as

$$\cos\omega t = \frac{x}{-A} \Rightarrow x(t) = -A\cos\omega t$$

For velocity, we see that

$$\sin\omega t = \frac{v_x}{v_T} = \frac{v_x}{\omega A}$$

And since $v_T = \omega r$ where $r = A$,

$$\sin\omega t = \frac{v_x}{\omega A} \Rightarrow v_x(t) = \omega A\sin\omega t$$

Lastly, we can work out the linear acceleration in the x direction as a function of time by noting that

$$\cos\omega t = \frac{-a_x}{-a_c}$$

And since $a_c = r\omega^2$ where $r = A$,

$$cos\,\omega t = \frac{-a_x}{-A\omega^2} \Rightarrow \mathrm{a_x} = \mathrm{A}\omega^2\cos\omega t$$

For the equation describing velocity as a function of displacement, we start with the

$$v(t) = \omega A\, sin\, \omega t$$

and then we would utilise a trigonometric identity

$$sin^2 \omega t + cos^2 \omega t = 1$$

Rearrange it for $sin\, \omega t$,

$$sin\, \omega t = \pm\sqrt{1 - cos^2 \omega t}$$

and substitute it back into the $v(t)$ equation

$$v(x) = \pm\omega\sqrt{A^2 - (-A\, cos\, \omega t)^2} = \pm\omega\sqrt{A^2 - x^2}$$

Energy in SHM

If we consider the equation for kinetic energy,

$$E_k = \frac{1}{2}mv^2$$

It is quite easy to see how one would be able to get the variation of kinetic energy as the object undergoes SHM. This can be achieved simply by substituting v with $v(t)$ or $v(x)$.

For $x(t) = A\, sin\, \omega t$,

$$E_k = \frac{1}{2}m(\omega A\, cos\, \omega t)^2 \Rightarrow E_k = \frac{1}{2}m\omega^2 A^2\, cos^2 \omega t$$

$$E_k = \frac{1}{2}m\left(\pm\omega\sqrt{A^2 - x^2}\right)^2 \Rightarrow E_k = \frac{1}{2}m\omega^2(A^2 - x^2)$$

On the other hand, if the restoring force acting upon the system is described by $F_{restoring} = -kx$, then the potential energy that provides the system with such restoring force must have obey the equation

$$U = \frac{1}{2}kx^2$$

since $F = -\frac{dU}{dx}$. As such we can substitute k with $m\omega^2$ and x with $x(t) = A\, sin\, \omega t$, which yields,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(m\omega^2)x^2 = \frac{1}{2}m\omega^2 A^2\, sin^2 \omega t$$

By law of energy conservation, we can find the equation for total mechanical energy

$$E_{total} = E_k + U = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2 \Rightarrow E_{total} = \frac{1}{2}m\omega^2 A^2.$$

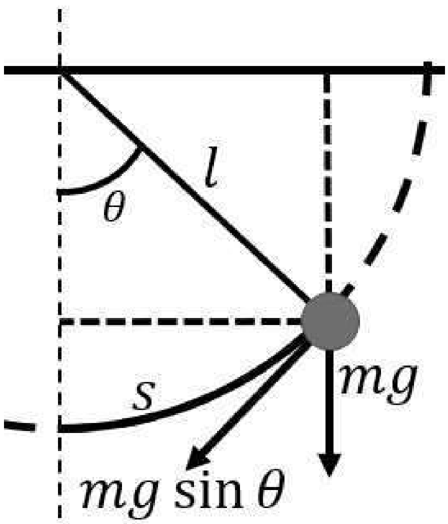
It is from this equation that whilst the kinetic energy and potential energy of the system depends on the displacement, the total energy only depends on the amplitude, angular velocity, and mass of the system.

Case study

For this section of the topic, we are interested in 2 cases – simple pendulum and spring mass system. For both cases, we aim to derive their equations for the period of their oscillation.

Case 1: Simple Pendulum

Consider the motion of a simple pendulum based on the diagram given.



In this diagram, the restoring force is a component of the weight of the pendulum bob,

$$F_{restoring} = -mg\ sin\,\theta = ma$$

If we apply **small angle approximation, i.e.**

$$\boldsymbol{sin\,\theta \approx \theta}$$

we find that

$$a = -g\theta \approx -g\left(\frac{s}{l}\right) \Rightarrow a \approx -\frac{g}{l}s$$

We can now compare with the SHM equation, $a = -\omega^2 x$, and find that

$$\omega^2 = \frac{g}{l}$$

And since $\omega = \frac{2\pi}{T}$, the expression for period of oscillation for a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

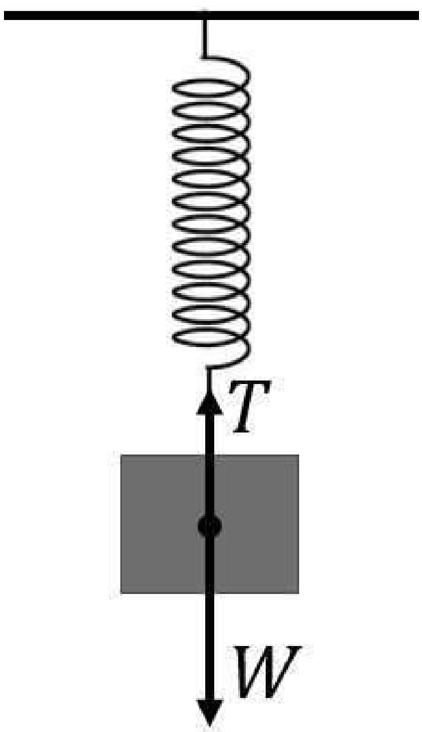
Case 2: Spring-mass system

Consider the motion of a mass attached to a spring. At equilibrium,

$$T = mg = ke.$$

Let us now introduce an extension x to the system. Newton's 2nd law of motion, when applied will result in

$$mg - ke - kx = ma \Rightarrow -kx = ma \Rightarrow a = -\frac{k}{m}x$$



Comparing this to the SHM equation, $a = -\omega^2 x$, we find that

$$\omega = \sqrt{\frac{k}{m}}$$

And since $\omega = \frac{2\pi}{T}$, the expression for period of oscillation for the spring mass system is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Part 2: Waves

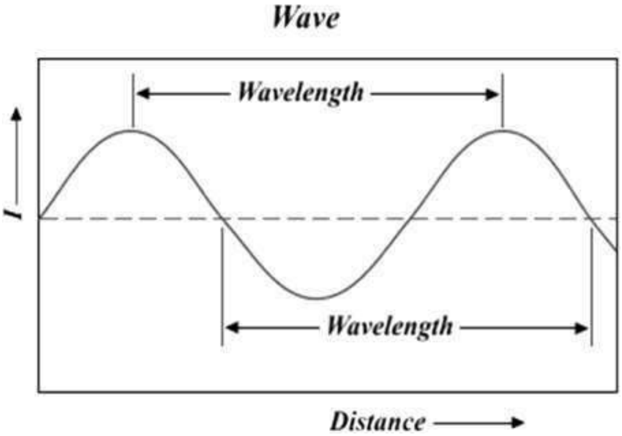
As discussed before, the solution of the SHM equation can be written as

$$y(t) = A\,sin\,(\omega t).$$

Here, the ω represents the rate of change of the sinusoidal waveform. It answers the question of "How big is the phase change in 1 second?". ω is related to the period T, and frequency f by the following equation:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Mechanical waves work by energy transfer from one point to another point some distance away. Take water waves for example. the disturbance causes the water particles to oscillate up and down, but it doesn’t traverse or spread out. This oscillation is merely the transfer of kinetic energy from particles at one point to another in space. The velocity of the particle’s up and down motion is known as the particle vibrational velocity. A snapshot of this oscillation may produce a graph as shown below



The distance between corresponding points in successive wave-form is known as the wavelength. If the oscillation occurs at frequency f, in one second the wave has move forward by fλ. This means the velocity of the wave (also known as the wave propagation velocity) is related to the frequency and wavelength according to

$$v = f\lambda.$$

Wave number is the number of wave per unit distance. This is similar to the case of angular frequency, what angular frequency is to period, is what wave number is to wavelength. Difference is in the dimension, where wave number and wavelength are in the dimensions of space and angular frequency and period are in the dimension of time.

Progressive Wave

A progressive wave is a wave where the wave profile moves along with the speed of the wave. Its equation takes the following form

$$y(x,t) = A\,sin\,(\omega t \pm kx)$$

Different from the SHM where the variation of y is only dependent on x, here the equation for progressive wave is a function of both x and t. This is y varies with t. Because of this variation, at any point in x from the origin, the particle is displaced by a phase of kx.

To determine whether the wave is moving towards positive x direction or negative x direction, we may revisit the equation for velocity:

$$v = f\lambda = \frac{\omega}{k}$$

If $\frac{\omega}{k} > 0$ then v > 0. This is achievable if ωt − kx = 0. This means if the wave is moving in the positive x direction then the general equation takes the form

y(x, t) = A sin (ωt − kx).

$$y(x,t) = A\,sin\,(\omega t - kx)$$

Principle of Wave Superposition

The Principle of Wave Superposition states that the resultant displacement at any point is the sum of the individual wave displacements. That is to say

$$y_{resultant} = \Sigma_i A_i\,sin\,(\omega_i t \pm k_i x)$$

Standing Wave

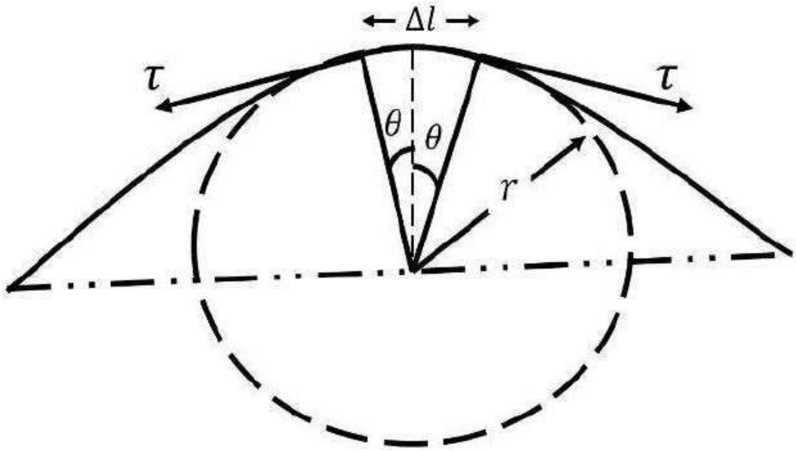
Whilst the progressive wave is a wave whose wave profile moves along the speed of the wave,, the standing wave is a case where the wave profile does not move in space. The peak amplitude of the wave oscillation at any point in space is independent of time.

The locations of minimum amplitudes are known as nodes and the locations of maximum amplitudes are called antinodes. A standing wave can be produce by having 2 progressive waves (of the same amplitude, frequency and wave number but travelling at opposite direction) superpose. This gives a resultant wave of the following equation:

$$y_{standing\,wave} = Asin(\omega t - kx) + Asin(\omega t + kx)$$
$$= 2Asin(\omega t)cos(kx)$$

Travelling Wave Solution for String

Consider a single symmetrical pulse on a stretched string that moves in the ±x direction.



Take a small string element Δl within the pulse. This string element has the mass of Δm, where with μ = string mass density, is given by

$$\Delta m \ = \mu \Delta l$$

The end points of the element forms an arc of a circle of radius r, subtending 2θ at the centre of that circle. Assuming constant velocity, the horizontal components of τ cancels and its vertical component is the restoring force,

$$F = 2\tau \ sin\,\theta$$

Making small angle approximation and consider the equation ofan arc length, $s = r\theta$, gives

$$F \approx (2\theta)\tau \approx \left(\frac{\Delta l}{R}\right)$$

Since the element Δl is moving in an arc of the circle, its centripetal acceleration is given by

$$a_{centri.} = \frac{v^2}{r}$$

Thus

$$F = ma = (\mu\Delta l)\left(\frac{v^2}{R}\right) = \frac{\tau\Delta l}{R} \Rightarrow v = \sqrt{\frac{\tau}{\mu}}$$

Sound Waves

One example of a longitudinal wave is sound wave. Sound wave here refers to the transmission of energy through the adiabatic compression and decompression of a medium. We can charac- terise sound in 3 aspects - loudness (amplitude), quality (over- lapping of overtones) and pitch (frequency). In this section, we shall discuss these 3 characters of sound waves to some extent.

Loudness

Sensed as loudness, acoustic intensity, I, is defined as the power, P , propagated by the sound wave per unit area, A, in the direction normal to that area. In equation form,

$$I = \frac{P}{A}$$

We can expand this to show the relationship between intensity of sound wave to its amplitude, y_{max}, by considering the mass and velocity of an air layer as it reaches a point some distance away from the source. That layer of air vibrates at with simple harmonic motion. If the mass of the air layer is m and the velocity of the air layer is $v = \omega y_{max}$, then the intensity of the sound wave is

$$I = \frac{P}{A} = \frac{E_{kinetic}}{tA} = \frac{m\omega^2}{2tA} y_{max}^2$$

This tells us for any wave

$$I \propto y_{max}^2.$$

Shifting our attention to area, we can assume that the sound wave is spherical, then the intensity of sound was at a distance r from the source is

$$I(r) = \frac{P}{4\pi r^2}$$

because the area of a sphere of radius r is $4\pi r^2$. Here, we see it follows the inverse square law that states

$$I \propto \frac{1}{r^2}$$

Quality

The quality of the sound, or timbre, describes the characteristic of a sound which allows us to distinguish sounds that has the same pitch and loudness. We shall study the primary contributor to the timbre of a sound, which is the its harmonic content. When a musical instrument is heard, what our ears pick up is not the wave of single frequency but the the superposition product of sound waves of multiple frequencies. This is what is known as the harmonic content. Reversing the process, the harmonic content of a sound can broken down into its individual pure tones by Fourier transform.

Our interest, however, is to consider pure tones. What we mean by pure tone here is that instead of considering a combina- tion of sound waves of various frequencies, we consider a sound which is made up of a sound of a single frequency. We shall consider the boundary conditions of 3 systems and deduce the allowable frequencies of the sound produced. These 3 systems are chosen because most a large percentage of the all the musical instruments fundamentally works based on these 3 systems.

Before specifying the systems, let us recall some general ideas of waves:

1. Frequency ($f = \frac{v}{\lambda}$) ⇒ number of oscillation per second, unit: Hz
2. Wavelength ($\lambda = \frac{v}{f}$) distance between corresponding points in successive wave form.
3. Nodes (N) = location on a standing wave at which minimum amplitudes are observed.
4. Anti nodes (AN)= location on a standing wave at which maximum amplitudes are observed.

Additional terms that will be used in the analysis of the systems are

1. Fundamental frequency, f_o = the lowest frequency
2. Harmonics = whole number multiples of the f_o
3. Overtone = any frequency produced by the system which is greater than f_o
4. End correction, e = a short distance added to the actual length of a pipe due to resonant vibration at any open end of a pipe.

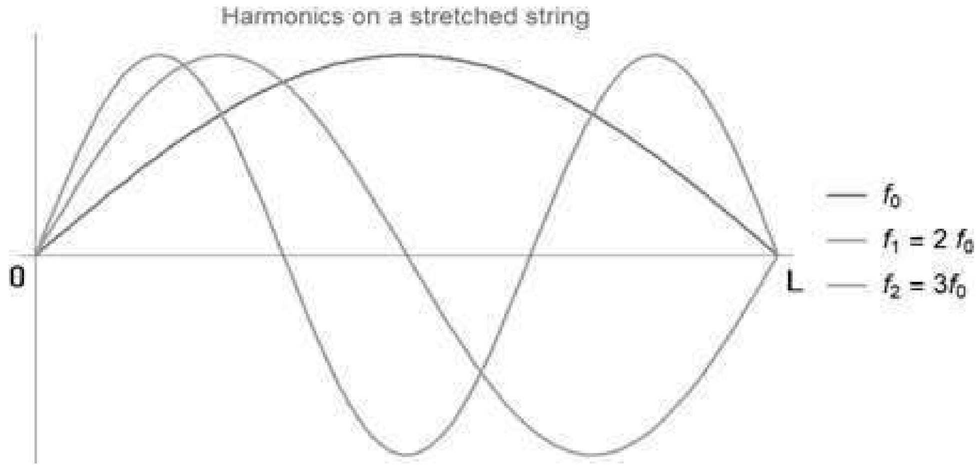
Generally speaking, the nth overtone of any system is the (n + 1)th harmonic of that system.

System 1: Stretched String

The first of the system we're considering is the stretched string. Examples of musical instruments that works on this system includes the piano and the guitar.

The boundary condition imposed here is that the ends must be composed of nodes, that is

$$y(0,0) = y(0, L) = 0.$$



We find that the string length must be integer of half wavelength, this means that

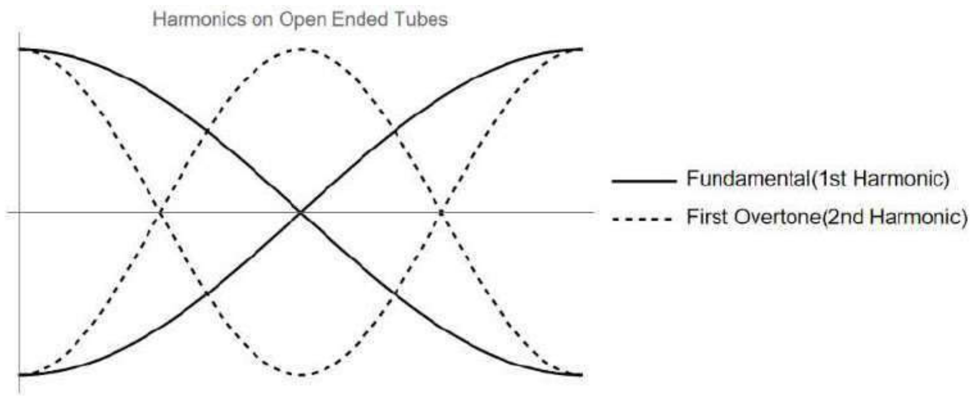
$$L = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

This allows us to write down the allowed frequencies for this system, and it is

$$f_n = n\frac{v}{2L} = \frac{\mathrm{n}}{2\mathrm{L}}\sqrt{\frac{\mathrm{T}}{\mu}} \Leftrightarrow n = \{0,1,2, \dots\}$$

System 2: Open Pipe

The second of the system we're considering is the open-ended pipe. This system can be found applied in the construction of a flute. The boundary condition imposed here is that the ends must be composed of anti-nodes.



We find that the pipe length must be integer of half wavelength,this means that

$$L = n\frac{\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

This allows us to write down the allowed frequencies for this system, and it is

$$f_n = n\frac{v}{2L} \Leftrightarrow n = \{0,1,2,\dots\}$$

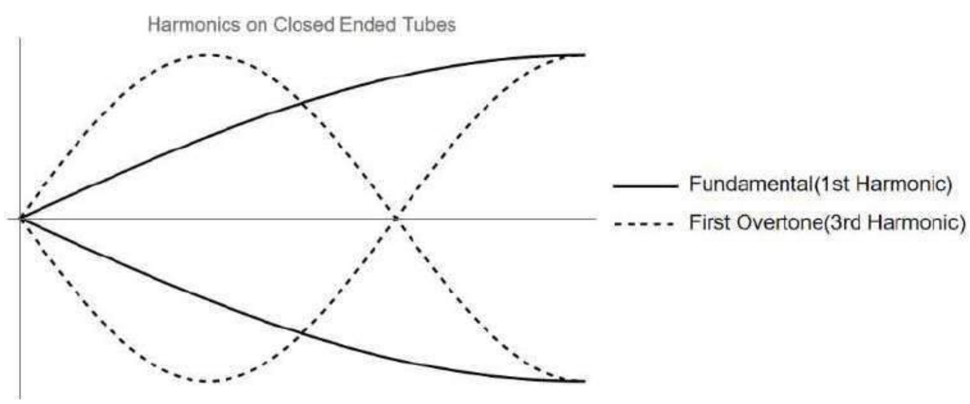
With end correction,

$$L = \frac{n}{2}\lambda - 2e \Rightarrow \lambda = \frac{2(L+2e)}{n}$$

$$f_n = n\frac{v}{2(L+2e)}$$

System 3: Closed Pipe

The system found in some organs/clarinet is the closed pipe system. This is essentially a pipe with one of the pipe ends sealed off. The boundary condition imposed here is that an antinode must be found at the open and a node must be found at the open end. This gives the following diagram



We find that the pipe length must be integer of a quarter wave- length, this means that

$$L = n\frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{n}$$

However, looking at the following harmonics we find the n does not take the values of all integers, only the odd integers, i.e. $n = \{1,3,5, \ldots\}$. This also means that the second overtone is the 3rd harmonic and the third overtone is the 5th harmonic and so on. So we will have to make adjustments to our equation to only consider odd integers. That adjustment is

$$n \to 2n + 1$$

This yields only odd integer as we consider $n = \{0,1,2, \ldots\}$, resulting in

$$L = \frac{2n+1}{4}\lambda \Rightarrow \lambda = \frac{4L}{2n+1} \Leftrightarrow n = \{0,1,2, \ldots\}$$

With this modification, we can write down the allowed frequencies for this system, and it is

$$f_n = (2n+1)\frac{v}{4L} \Leftrightarrow n = \{0,1,2, \ldots\}$$

With end correction,

$$L = \frac{2n+1}{4} - e\lambda \Rightarrow \lambda = \frac{4(L+e)}{2n+1}$$

$$f_n = \frac{(2n+1)v}{4(L+e)}$$

Pitch

Frequency of waves are observed to correspond to pitch of sound. Sound waves with high frequency are observed as high pitched, and vice versa. One phenomenon of this character of sound wave is the **doppler effect**.

Imagine an ambulance, emitting a sound of frequency, moves towards you. As the ambulance approaches you, you hear that the pitch gets higher. Once the ambulance passes by you and moves away from you, the pitch becomes lower. This change in pitch is known as the *Doppler effect*.

The calculation for the change of pitch will be dealt soon, but for now, let us look at a graphical representation of what actually happen. The following figure shows how the frequency that you observe (= apparent frequency, f_{app}) depends on your position relative to the motion of the sound source.

As the source moves at a steady speed from position 1 to 4, four circular (coloured) waves are produced, which has point {1,2,3,4} as their centres. If you, as the observer positions yourself at point µ, and therefore the sound wave source is moving towards you, you will then observe sound wave in frame A. Conversely, positioning yourself at point γ means that the sound wave is observed according to frame B.

Comparing the wavelengths between wave fronts in frame A and B tells us that $\lambda_A < \lambda_B$. Because the frequency is inversely proportional to the wavelength, we then know that for apparent frequencies, $f_A > f_B$.

The most general case for the Doppler effect is when both the observer and the source is moving, therefore the approach taken here to quantify the apparent frequency to the observer will be to consider such as case.

The apparent frequency, f_{app}, can be found using the equation

$$f_{app} = \frac{v'}{\lambda_{app}}$$

where v′ is the sound wave velocity relative to the observer and λ_{app} is the wavelength that reaches the observer.

Assuming source is moving at velocity ±vs and the observer moves at a velocity $\pm v_o$, then

$$v' = v \mp u_o$$

$$\lambda = \frac{v \mp u_s}{f}$$

$$f_{app} = \frac{v \mp u_o}{v \mp u_s} f$$

In the numerator, u_o is added to v when the observer is moving towards the source, and vice versa. In the denominator us is added to v when the source is moving away from the observer.

We might also find determining the plus-minus signs in the Doppler effect equation easier if we take into consideration of the f_{app} to f ratio. This takes the form

$$\frac{f_{app}}{f} = \frac{v \mp u_o}{v \mp u_s}$$

which is bigger than 1 if a higher frequency is expected to observed and less than 1 if a lower frequency is expected to observed.

Chapter 8: Physics of Matter

Learning Outcomes

Changes to material	No.	Learning Outcomes
Due to applied force	1.	Distinguish between stress, $\sigma = \frac{F}{A}$ and strain, $\epsilon = \frac{\Delta L}{L_o}$
	2.	Analyse the graph of: a. Stress-strain for metal under tension b. Force-elongation for brittle and ductile materials
	3.	Explain elastic and plastic deformations
	4.	Define and use Young's modulus, $Y = \frac{\sigma}{\epsilon}$
	5.	Apply: a. Strain energy from the force-elongation graph, $U = \frac{1}{2}F\Delta L$ b. Strain energy per unit volume from stress-strain graph, $\frac{U}{V} = \frac{1}{2}\sigma\epsilon$

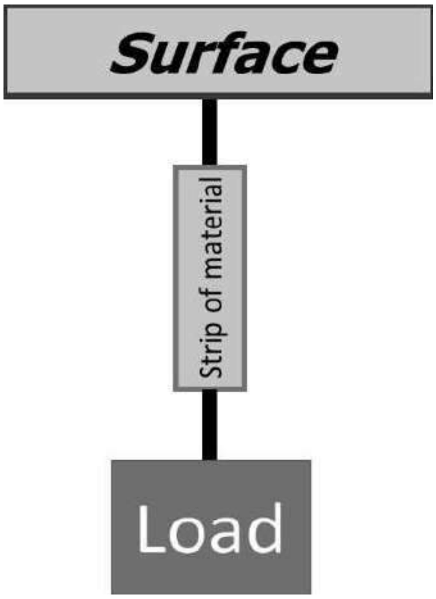
Learning Outcomes

Changes to material	No.	Learning Outcomes
Due to Heat	6.	Define: a. heat conduction b. coefficient of linear expansion, α c. coefficient of area expansion, β d. coefficient of volume expansion, γ
	7.	Solve problems: a. related to rate of heat transfer, $\frac{Q}{t} = -kA\left(\frac{\Delta T}{L}\right)$ through a cross-sectional area (remarks: maximum two insulated objects in series) b. related to thermal expansion of linear, area and volume $\Delta L = \alpha L_o \Delta T;\quad \Delta A = \beta A_o \Delta T;\quad \Delta V = \gamma V_o \Delta T;$ $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$
	8.	Analyse graphs of temperature-distance (T-L) for heat conduction through insulated and non-insulated rods. *maximum two rods in series

Part 1: Material Changes due to Force

Stress

To begin, we start by talking about testing things to the point of deformation — we put them under some increasing force over some area of the thing and once it starts to deform, we stop and calculate the maximum amount of force that the thing starts to deform.



The most natural way to do this is merely to subject a strip/rod of the material (of length L and cross-sectional area A) to an axial load with the other end anchored to some surface. As the mass of the load is increased, the strip/rod deforms (becomes longer) and eventually breaks off (fracturing). So naturally we'd like to know, how much load can a strip of the given material, support?

Before answering this question, we can ask ourselves if there are any geometric variables that influences the ability of the strip of material to support load. We can then repeat the same experiment using strips/blocks of the same material (with varying cross-sectional area) and we would find that the axial strength increases with the increment of the cross-sectional area. This would make sense because as the cross-sectional area is increased, so does the number of bonding between each cross-sectional layer.

We now have a value for the amount of load a material can support relative to the cross-sectional area of that material sample.

It may be expressed mathematically as

$$F_{max} = \delta_{max} A_o$$

where F_{max} is the load at fracture, δ_{max} is the Ultimate Tensile stress and A_o is the initial cross-sectional area. This equation describes the maximum amount of stress that the strip/rod of material of cross-sectional A_o can handle and it is at this point that the material fails and fractures.
So, when the material is said to be put under stress, and that stress is less than δ_{max}, what we are referring to is

$$\delta = \frac{F}{A_o}$$

This new measure has the unit of Nm^{-2}.

Strain

In the last section, we quantified the amount of tensile stress a material may be be put under. In this section, let us quantify the "amount" of deformation that the material undergoes under some stress, that is we want to measure the stiffness.

Hooke's law gives us a great exposure to the deformation of a material with respect to the load that the material is put under. It is commonly written as

$$F = kx$$

Where k is the stiffness constant has units of Nm^{-1}.

We know from the previous section that this constant is not only affected by the type of material alone but also by the shape of the material. For now, we would like to normalize the measure for stiffness only by the deformation it undergoes independent of the shape. We can do this by considering the measure for the stretching of the material. This means we only consider the fractional change of the material when pun under stress, this is

$$\epsilon = \frac{\Delta L}{L_o}$$

where ΔL is the change in length and L_o is the initial length before the material was put under stress. This measure is what we call ***strain***.

Young's Modulus

We have now discussed the measure for stress the material undergoes as well as the material's deformation. We are now in the position to discuss how the stress that is put on the material affects the deformation observed in that material. That is to say, we want a calculable prediction on, *"If I put this amount of force, how big is the deformation I can expect? "*.

Experimental results show that for relatively small stress and strain, they are proportional to each other. This allows us to write

$$\delta \propto \epsilon.$$

This tells us that there is a proportionality constant between the two, let's call it *Y*. We can then define this proportionality constant to be

$$Y = \frac{\delta}{\epsilon}$$

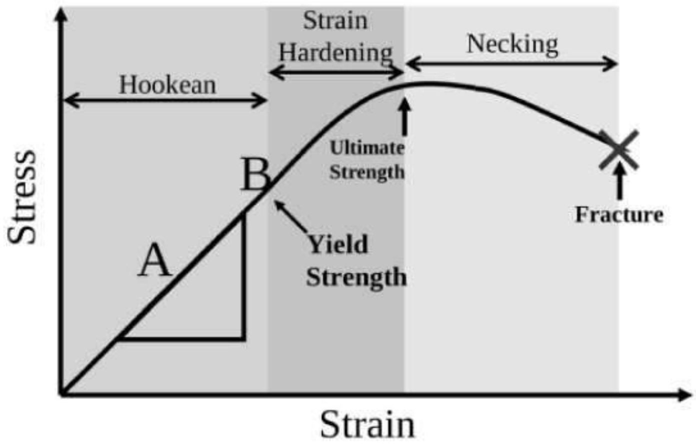
This proportionality constant is what we today call **Young's Modulus**.

Graph

2 graphs are studied in this section – stress-strain curve for a metal and the force-extension graph for brittle and ductile materials.

Stress-strain graph

Up to this point, we have considered stress and strain up within the Hookean limit. It is the best of our interest to also consider what happens further extension of the material after the Hookean limit (Yield limit) to the fracture point. The graph shows the stress-strain relation before and after the Hookean limit.



The first part (blue) shows the obeyance of the stress-strain curve to the Hookean law. Within this region, one can calculate the Young's modulus based on the gradient of the straight line. It is also in this region that the metal undergoes what is known as **elastic deformation**. Deforming elastically refers to the ability of the stretched metal to return to its initial length when the tensile stress is removed.

Beyond the Hookean limit, Hooke's law is no longer obeyed, and thus non-linearity is observed in the stress-strain curve. Beyond the Hookean limit, the metal undergoes **plastic deformation**, a type of deformation in which the stretched metal will not be able to return to its initial length even if the tensile stress is removed.

From the stress-strain curve, we observe that a non-linear increment of stress with the increment of strain until it reaches a peak, known as **Ultimate Tensile Strength**. The region (green) between the Hookean limit and the UTS is where strain hardening takes place. This is the phenomenon where the metal is "strengthened" by the plastic deformation. "Strengthened" here refers to the dislocation of movements in the crystal structure of the material.

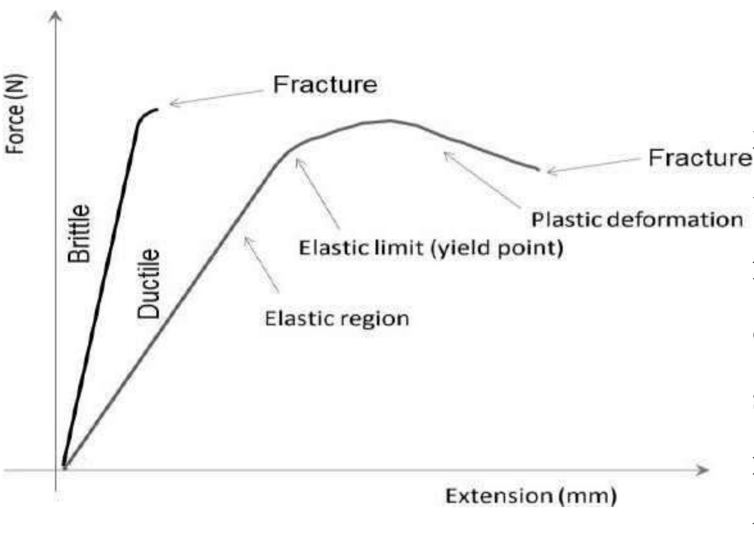
Beyond the peak, the strain still increases but the tensile stress decrease until the **fracture point**. Between the UTS and fracture point (pink), what is observed of the metal is that necking takes place. This is the phenomenon in which the local cross-sectional area becomes significantly smaller than average.

For some material, the plastic deformation occurring between the Hookean limit and the fracture point is so small, that it seems to the observer as if the plastic deformation region is non-existent. For such material, we call it **brittle**.

For other materials, the plastic deformation region is great and thus we call these materials, **ductile**.

Force-extension graph

As mentioned in the stress-strain graph section, the plastic deformation phase for brittle materials are very short whereas the same region for the ductile materials are relatively (and observably) significant.



One possible inference we can make from this is that ductile materials are able to contain much more strain energy than brittle materials. It is also from the force extension graph we can deduce the strain energy by the area under the force-extension graph, that is

$$U_{strain} = \frac{1}{2} F\Delta L$$

If were to find the strain energy per unit volume, we could easily do it by dividing both side by the volume of the material,

$$\frac{U_{strain}}{V} = \frac{1}{2}\frac{F\Delta L}{V}$$

and reminding ourselves that the volume is merely the product of cross-sectional area and length,

$$V = AL_o$$

We can see that the strain energy per unit volume is just the area under the stress strain curve,

$$\frac{U_{strain}}{V} = \frac{1}{2}\frac{F\Delta L}{V} = \frac{1}{2}\frac{F\Delta L}{AL_o}$$

$$\frac{U_{strain}}{V} = \frac{1}{2}\delta\epsilon$$

Part 2: Material Changes due to Heat

Heat Conduction

Imagine a metal rod with one end heated. With time, the opposite end also gets hot even though it is not directly heated. The heat energy is transferred from one end to the other end. This happens through the 'jiggling' of the particles within the heated material. As the rod is heated, the particles begin to vibrate and collide with neighbouring particles. When they collide, they transfer some of the energy to the neighbouring particles and then the neighbouring particles starts to vibrate. This process is called **heat conduction.** It is the transfer of heat through agitations of the particles within the material without any motion of the material.

The variable that drives heat transfer is a temperature gradient across the material, that is to say there exist a difference in temperature between two parts of a conducting medium,

$$\Delta T > 0.$$

Referring to the rod heated on one end case, we can say that the heated end has temperature T_H and the non-heated end to have the temperature T_C. Fourier's law tells us that the local heat flux in a homogeneous body, q_h is in the direction of, and proportional to, the temperature gradient ∇T:

$$q_h \propto -\nabla T$$

In one-dimensional form,

$$q_h = -\kappa_x \frac{dT}{dx}$$

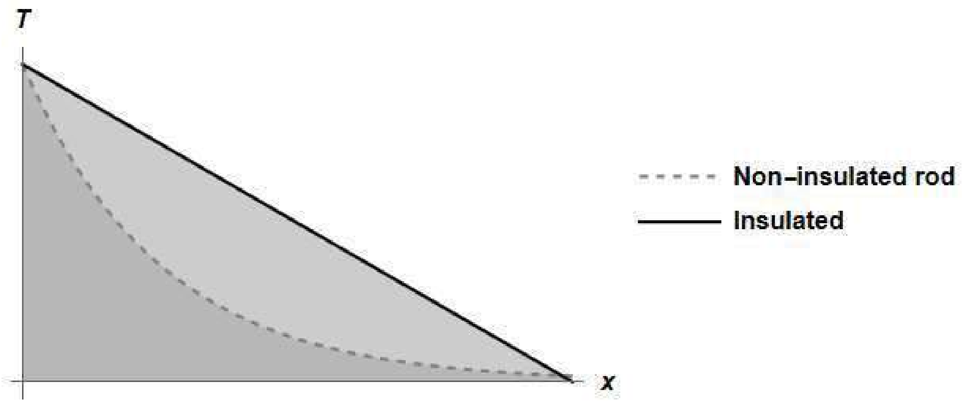
where κ_x is the thermal conductivity in the x-direction. Note that the minus sign is present due to the fact that heat flows from a higher temperature area to a lower one.

Local heat flux refers to rate of heat transfer per unit area,

$$q_h = \frac{\left(\frac{dQ}{dt}\right)}{A}$$

The equation for the rate of heat transfer is then

$$\frac{dQ}{dt} = -A\kappa_x \frac{dT}{dx}$$



Let us now try to understand the temperature-gradient graph for a rod heated on one end. For an uninsulated rod, the heat energy will be able to escape to the environment via the sides of the rod. This leads to temperature-distance gradient to be curved with a decreasing gradient, much like graphs exhibited by functions $f(x) = e^{-kx}$. On the other hand, for insulated rods, the heat loss to the environment is negligible. As a result, their temperature-distance graph is a linear graph with negative gradient.

Heat Expansion

Things expand when they are heated, this phenomenon is known as **thermal expansion**. A great demonstration of this phenomenon is the Ring and Ball Experiment (https://www.youtube.com/watch?v=ne8oPFTM_AU). If the expansion happens in one dimension of space, we call it a **linear expansion**, where as expansion in two and three dimensions are known as **area** and **volume expansion.**

We can start by considering a small change in the object's length. By small, we mean relative to the object's initial dimensions. When that is the case, the change in the length is the proportional to the first power of the temperature change.

$$\Delta L \propto \Delta T$$

Say, that object has an initial length of L_o along some direction at temperature T_o. Then the change in length ΔL for a change in temperature ΔT is

$$\Delta L = \alpha L_o \Delta T$$

where α is known as **coefficient of linear expansion**.

So now how do we expand to area and volume thermal expansion? Well, note that $A = L^2$ and that $L = L_o + \Delta L$. As such,

$$A = L^2 = (L_o + \alpha L_o \Delta T)(L_o + \alpha L_o \Delta T)$$

$$A = L_o^2 + 2\alpha L_o^2 \Delta T + \alpha^2 L_o^2 \Delta T^2 = A_o + 2\alpha A_o \Delta T + \alpha^2 A_o^2 \Delta T^2$$

Since we are only considering only small changes, we can consider the last terms to be negligible such that

$$A = A_o + 2\alpha A_o \Delta T$$

which gives us the change in area

$$\Delta A = \beta A_o \Delta T$$

where $\beta = 2\ \alpha$ and is aptly named **coefficient of area expansion**. We can then imitate the same procedure to produce the equation for change in volume due to heat which will yield

$$\Delta V = \gamma V_o \Delta T$$

where $\gamma = 3\alpha$ and is named **coefficient of volume expansion.**

Chapter 9: Kinetic Theory of Gases & Thermodynamics

Learning Outcomes

Molecular kinetic theory	1.	Define/State a. The assumptions of kinetic theory of gases. b. The principle of equipartition of energy c. Degrees of freedom
	2.	Describe/Explain: a. Root mean square (rms) speed of gas molecules, $v_{rms} = \sqrt{< v^2 >}$ b. Translational kinetic energy of a molecule, $E_K = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT$ c. Internal energy of gas
	3.	Solve problems related to: a. rms speed of gas molecules, $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$ b. the equations, $PV = \frac{1}{3}Nmv_{rms}^2$; $P = \frac{1}{3}\rho v_{rms}^2$ c. Translational kinetic energy of a molecule, $E_K = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT$ d. Internal energy, $U = \frac{1}{2}fNkT$
	4.	Identify number of degrees of freedom for monoatomic, diatomic and polyatomic gas molecules.

Learning Outcomes

Thermodynamics	1.	Define/State: a. First Law of Thermodynamic, $\Delta U = Q - W$ b. Isothermal process c. Isochoric process d. Isobaric process d. Adiabatic process
	2.	Solve problems related to: a. First Law of Thermodynamics b. Isothermal process, $W = nRTln\left(\frac{V_f}{V_i}\right) = nRTln\left(\frac{\mathrm{p_i}}{\mathrm{p_f}}\right)$ c. Isobaric process, $W = \int PdV = P\left(V_f - V_i\right)$ d. Isochoric process, $W = \int PdV = 0$
	3.	Analyse $P - V$ graph for all the thermodynamic processes
	4.	Derive equation of work done in isothermal, isochoric and isobaric processes from $P - V$ graph.

Molecular Kinetic Theory

Kinetic Theory of Gases

Because atoms are very light, it is often useful to use the **atomic mass unit** for the masses of the atomic scale. The atomic mass unit is defined as $\frac{1}{12}$ of the mass of a carbon-12 atom This atomic mass unit (a.m.u.) is related to the SI kilogram by

$$1u = 1.660539 \times 10^{-27} kg$$

Apart from that, in our daily lives, quite often we deal with a large number of atoms/molecules/particles. So rather than describing is numerically by the number of particles, we often describe the number of atoms relative to the **Avogadro's Constant,**

$$N_A = 6.022 \times 10^{23} \text{ particle per mol.}$$

For example, instead of saying there are $5(10^{23})$ gas particles in a container, it is easier to say 0.83mol of gas particles in the container. These two new ways of quantifying the light-mass but large number particle systems leads to a very interesting result, that is **the mass per mole of any substance and the atomic (or molecular) mass unit has the same numerical value.** For example, the oxygen atom has a mass of 16u and therefore has a mass of $16g\,mol^{-1}$.

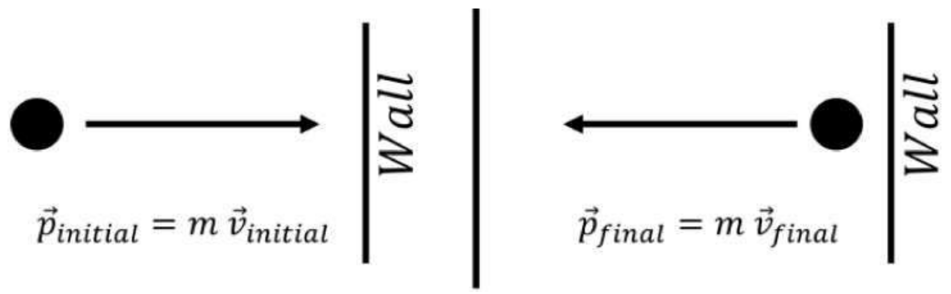
Macroscopically, the simplest model for gases is the **ideal gas law,** which states

$$pV = nRT = Nk_BT$$

where p is the pressure of the gas (in Pa), V is the volume of the gas (in m^3), n is the number of mole of the gas, T is the temperature in Kelvin of the gas, R is the gas constant ($J\, mol^{-1}\, K^{-1}$) and k_B is the Boltzmann constant (defined as $k_B = \frac{R}{N_A}$).

Microscopically, though we want to start considering kinetic energies of the gas particles. Let us first lay down assumptions for our kinetic theory of gas. In one sentence, let us consider **gas to be composed of large numbers of non-null mass point-like particles that obeys Newton's laws of motion and interact elastically with each other where the average kinetic energy of the gas particles depends solely on the absolute temperature of the gas particle system.**

Consider gas particles in a cube container of side lengths L. We can think of the interaction between the gas particle and the container wall to be like that of a ball hitting a wall. We can determine the force exerted by the particle onto the container wall and then divide it by the area to determine the pressure.



We can see that the change in momentum is $\Delta\vec{p} = m\big(-v - (+v)\big) = -2mv$

. Considering the speed of the particle is v and the distance between walls of the container are L, the time between the collisions will simply be $t = \frac{2L}{v}$. The force exerted on the wall by the particle will then be

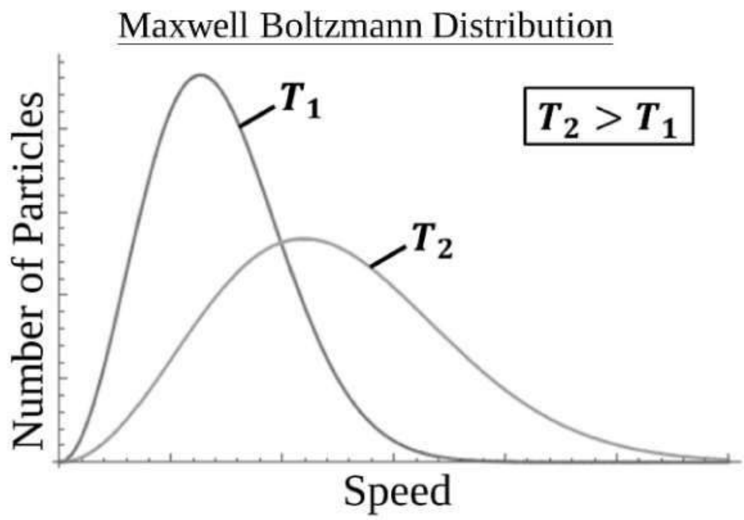
$$F = \frac{\Delta\vec{p}}{t} = \frac{-2mv}{\left(\frac{2L}{v}\right)} = \frac{-mv^2}{L}$$

If there are N particles in the container, then the total force exerted on the container walls is

$$F = \frac{-Nm\overline{v^2}}{3L}$$

2 major changes happened - $\overline{v^2}$ has replaced v^2 and a factor of $\frac{1}{3}$ seemed to popped up. Here are the reasons:

a. $\overline{v^2}$ has replaced v^2 because in a system of many particles, not all the particles will have the same speed. Their speed will follow the **Maxwell-Boltzmann distribution**.



So, to take that into account, we will use the **rms speed (v_{rms})** of the particle rather than the peak or average speed. This rms speed is defined by

$$v_{rms} = \sqrt{\overline{v^2}}$$

b. On the other hand, the factor of $\frac{1}{3}$ popped us because the particles can move in 3 dimensions. This means the velocities of the particles can happen inn x, y or z axis. Considering the speeds are random, this would mean $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$, and since $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$, this would mean that

$$\overline{v^2} = 3\overline{v_x^2} \Rightarrow \overline{v_x^2} = \frac{\overline{v^2}}{3}$$

And thus the force can be written as

$$F = \frac{Nmv_{rms}^2}{3L}$$

We can now divide this force by the area

$$P = \frac{F}{L^2} = \frac{\left(\frac{Nmv_{rms}^2}{3L}\right)}{L^2} = \frac{Nmv_{rms}^2}{3L^3} = \frac{Nmv_{rms}^2}{3V}$$

Defining mass density as $\rho = \frac{Nm}{V}$,

$$P = \frac{Nmv_{rms}^2}{3V} \Rightarrow P = \frac{1}{3}\rho v_{rms}^2.$$

Rearranging the pressure – rms speed equation yields

$$PV = \left(\frac{2}{3}N\right)\left(\frac{1}{2}mv_{rms}^2\right) = \left(\frac{2}{3}N\right)(\bar{E}_{kinetic})$$

Comparing this to $pV = Nk_BT$ gives us an expression of the average kinetic energy as a function of the temperature,

$$Nk_BT = \left(\frac{2}{3}N\right)(\bar{E}_{kinetic}) \Rightarrow \bar{E}_{kinetic} = \frac{3}{2}k_BT \text{ (for a single particle)}$$

$$\text{Total Translational Kinetic Energy of N gas molecules: } \Sigma E_{kinetic}$$
$$= \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

This result is significant because now we really do have a **kinetic** theory of gas, where the temperature of the gas is expressed in terms of the motion of the gas particles.

Kinetic & Internal Energy

Before talking about the classical principle equipartition of energy, we need to address what we mean when we say **degrees of freedom**. In the context of gas motion, degrees of freedom are the dynamical variables that contributes a squared term to the expression for the total particle energy. Classically, there two that we'd consider are

a. translational kinetic energy,

$$K_{translational} = \frac{1}{2}mv^2$$

b. Molecular rotational energy,

$$K_{rotational} = \frac{1}{2}I\omega^2$$

Though, in the quantum regime, we need to consider vibrational energy, in which the bonds between molecules may be treated as "springs" and that would add 2 more degrees of freedom.

The following table shows the cases and the number of degrees of freedom

Cases	Number of Degrees of Freedom
Monoatomic	3 (Only translational) $$\frac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right)$$
Diatomic*	5 (3 translational + 2 rotational) $$\frac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right) + \frac{1}{2}\omega\left(I_x^2 + I_y^2\right)$$

*in the classical regime. Taking into consideration quantization of energy leads to consideration for vibrational energy.

Based on the idea of degrees of freedom, we can now extend our discussion to the classical theorem of energy equipartition which states **at**

equilibrium, each degree of freedom contributes $\frac{1}{2}\boldsymbol{k_B T}$ of energy per molecule.

That is to say, for N number of gas molecules of f degrees of freedom, the total internal energy is

$$U = fN\frac{1}{2}k_B T = \frac{f}{2}nRT$$

Here what we mean by internal energy is simply the sum of all the kinetic energy of all the molecules accounted for in the systems. If we were to talk about translational kinetic energy, we mean kinetic energy associated with translation of the particle ($f = 3$). If we were to talk about the total kinetic energy, we are referring to the internal energy of the system, where the degree of freedom depends on the type of particle.

Thermodynamics

Thermodynamics is the study of heat and energy transformation. In this section, we discuss 3 things. Firstly, the 0^{th} law of thermodynamics, which essentially provides us with the idea of **thermal equilibrium**. Secondly, we shall look at how the idea of adding (or taking our) energy to a system relates to the internal energy of the system, i.e. the 1^{st} law of thermodynamics. Lastly, we shall consider the thermodynamical processes and lay foundations to understanding heat engines.

Zeroth law of Thermodynamics

The 0^{th} law of thermodynamics focuses on the idea of a thermal equilibrium. If the temperature of system A is equal to system B, and that system B has a temperature equal to system C, then system C and A is said to be at **thermal equilibrium**.

$$T_A = T_B \;\&\; T_B = T_C \Rightarrow T_A = T_C$$

In other words, two systems are are to be in thermal equilibrium with each other if they have the same temperature. When two systems are in thermal equilibrium with each other, the net heat flow between them is essentially 0.

First Law of Thermodynamics

At this point, what we want to do is to relate the internal energy of the system with the external factors. By External factors, we mean whether we apply heat to the system, or take heat away from it or changing the geometry of the system (whether increasing it or decreasing it). Much like the conservation of energy, the sum total of the energy of an isolated system must be conserved. Therefore, when some heat ΔQ is added to the system and some work ΔW is added to the system, the change in internal energy ΔU can be calculate by

$$\Delta U = \Delta Q + \Delta W$$

Though some books may have a minus sign instead of a positive sign for the ΔW and the reason for that is that they have defined ΔW to be work done by the system.

Equation	Terms definition
$\Delta U = \Delta Q + \Delta W$	ΔW = work done **onto** the system
$\Delta U = \Delta Q - \Delta W$	ΔW = work done **by** the system

Thermodynamical Processes

Looking back at the first law of thermodynamics, we want to be able to do some calculations related to it. And thus, we shall quantitatively defined *U, Q* and *W*. By internal energy, we take the definition defined previously, that is internal energy of the gas depends on the temperature of the system,

$$\Delta U(\Delta T) = \frac{1}{2} f N k_B \Delta T = \frac{f}{2} nR\Delta T$$

By heat added, what we refer to is the heat transfer into or out of the system. This heat is defined by

$$\Delta Q = mc\, \Delta\theta$$

where m is the mass of the system, c is the heat capacity of the system and θ is the temperature of the system. By work done onto (or by) the system, we define it to be related to the change in volume of the system,

$$\Delta W = p\, \Delta V$$

In general, we want to consider 4 case studies on thermodynamical processes:

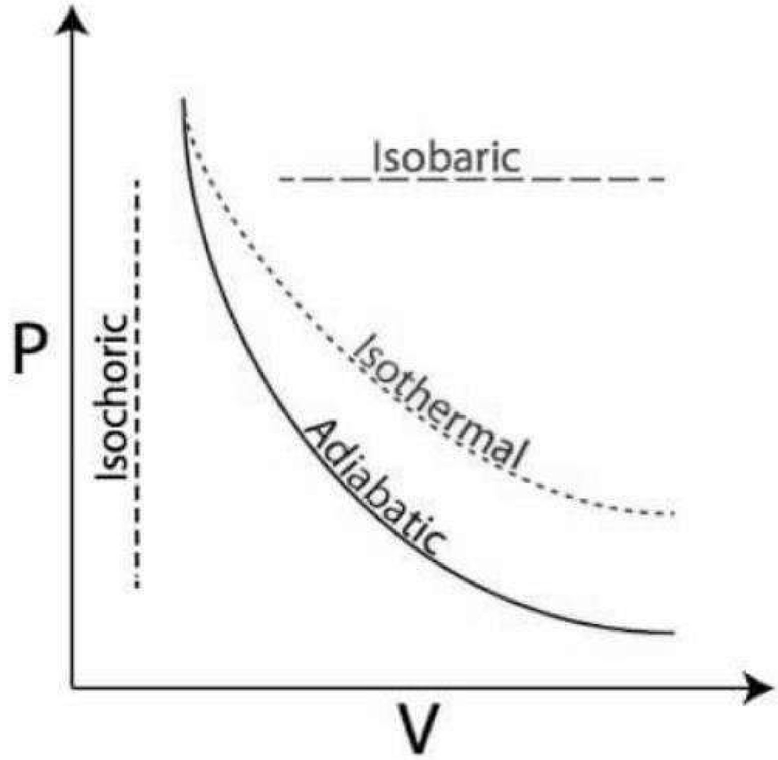
1. Isothermal
   In isothermal expansion/compression, temperature of the system is kept constant, $\Delta T = 0$. Since the change in internal energy depends solely on change in temperature, this means that $\Delta U = 0$ and thus
   $$\Delta U \ = \ 0 \ = \ \Delta Q + \Delta W$$
   $$\Rightarrow \Delta Q \ = -\, \Delta W$$
   In the isothermal case, pressure is not a constant, we can define pressure as a function of volume via the ideal gas law
   $$p = \frac{nRT}{V}$$
   And thus the work done is
   $$W = \int_{V_i}^{V_f} p(V)\, dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \int_{V_i}^{V_f} \frac{1}{V} dV$$
   $$= nRT\, ln\left(\frac{V_f}{V_i}\right)$$

2. Isochoric / Isovolumetric
   In the isochoric case, the volume of the gas is kept constant, $\Delta V = 0$ and thus
   $$\Delta U = \Delta Q$$
3. Isobaric
   In the isobaric expansion/compression cases, the pressure of the gas is kept constant, $\Delta p \ = \ 0$. This means that for the calculation of work done onto (or by) the system is simply
   $$W = \int_{V_f}^{V_i} p\, dV = p \int_{V_f}^{V_i} dV = p\big(V_f - V_i\big)$$
4. Adiabatic
   For the adiabatic process, what is kept constant is the heat transfer into and out of the system, $\Delta Q = 0$. This means that $\Delta U = \Delta W$. Since

$$\Delta U = \Delta W = \frac{f}{2}\text{nR}\Delta T$$

$$\Rightarrow W = \int_{T_i}^{T_f} \frac{f}{2} nR\, dT = \frac{f}{2} nR\left(T_f - T_i\right)$$

The p-V graph of each thermodynamical processes is shown in the diagram below.



===End of Short Notes===

SP015 TUTORIALS

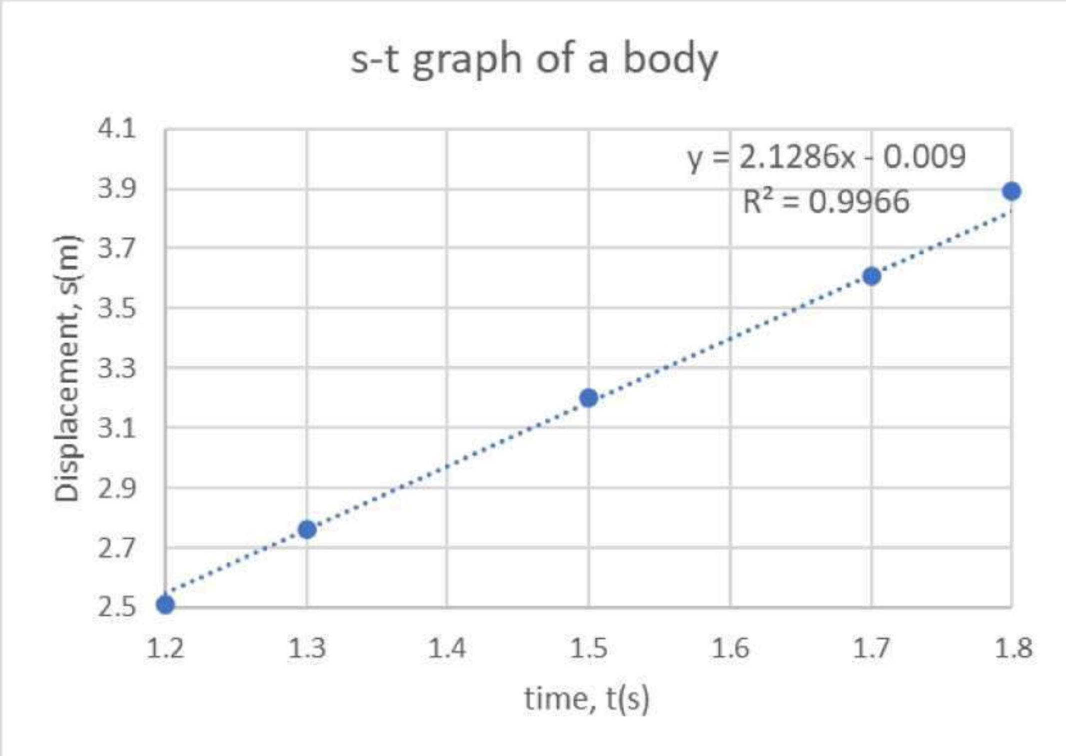
WORKSHEET 1: MATHEMATICS 2
WORKSHEET 2: 1D KINEMATICS 6
WORKSHEET 3: 2D KINEMATICS & PROJECTILE MOTION 10
WORKSHEET 4: PYQ KINEMATICS 13
WORKSHEET 5: MOMENTUM & ITS CONSERVATION 16
WORKSHEET 6: FREE BODY DIAGRAM 20
WORKSHEET 7: WORK, ENERGY, $\Sigma W = \Delta K$ & $\Delta \Sigma E = 0$ 26
WORKSHEET 8: MECHANICAL POWER 30
WORKSHEET 9: PYQ MOMENTUM, NEWTON'S LAW, AND ENERGY 33
WORKSHEET 10: CIRCULAR MOTION 38
WORKSHEET 11: CIRCULAR MOTION 42
WORKSHEET 12: ROTATIONAL KINEMATICS 42
WORKSHEET 13: ROTATIONAL DYNAMICS & $\Delta \Sigma L = 0$ 45
WORKSHEET 14: SHM: GRAPHS & CASES 49
WORKSHEET 15: WAVES – BASICS 53
WORKSHEET 16: WAVES – CASES & DOPPLER EFFECT 58
WORKSHEET 17: SHM 62
WORKSHEET 19: STRESS, STRAIN & YOUNG'S MODULUS 66
WORKSHEET 19: HEAT CONDUCTION & THERMAL EXPANSION 68
WORKSHEET 20: PYQ PHYSICS OF MATTER 71
WORKSHEET 21: MOLECULAR ENERGIES 73
WORKSHEET 22: THERMODYNAMICS 75
WORKSHEET 23: PYQ MOLECULAR ENERGIES & THERMODYNAMICS 79

Worksheet 1: Mathematics

1	Suppose a man's scalp hair grows at a rate of 0.35 mm per day. What is this growth rate in feet per decade?
	Solution: $\text{Growth rate} = \left(0.35\,\frac{\cancel{\text{mm}}}{\cancel{\text{d}}}\right)\left(\frac{1\,\cancel{\text{m}}}{10^3\,\cancel{\text{mm}}}\right)\left(\frac{1\text{ ft}}{0.3048\,\cancel{\text{m}}}\right)\left(\frac{365.24\,\cancel{\text{d}}}{1\,\cancel{\text{y}}}\right)\left(\frac{100\,\cancel{\text{y}}}{\text{century}}\right) = \boxed{42\text{ ft/century}}$
2	You are driving into Kuala Lumpur, and in the distance, you see the famous Merdeka 118 building. This building rises to a height of 670m. You estimate your line of sight with the top of the arch to be 15^o above the horizontal. Approximately how far (in kilometres) are you from the base of the building?
	Solution: $$r = \frac{670}{tan\ 15^o} = 2.5km$$
3	Estimate the number of litres of water a human drink in a lifetime.
	Solution: A person should drink eight 8-oz. glasses of water each day. That is about 2 quarts, or 2 liters of water per day. Approximate the lifetime as 70 years. $(70\text{ yr})(365\text{ d}/1\text{ yr})(2\text{ L}/1\text{ d}) \approx \boxed{5\times10^4\text{ L}}$
4	An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kgm^{-3}. What is the volume of the room in cubic feet and the weight of air in the room in pounds?
	Solution: $V = (40.0\text{ m})(20.0\text{ m})(12.0\text{ m}) = 9.60\times10^3\text{ m}^3$ $V = 9.60\times10^3\text{ m}^3(3.28\text{ ft}/1\text{ m})^3 = \boxed{3.39\times10^5\text{ ft}^3}$ $m = \rho_{\text{air}}V = (1.20\text{ kg/m}^3)(9.60\times10^3\text{ m}^3) = 1.15\times10^4\text{ kg}$. $F_g = mg = (1.15\times10^4\text{ kg})(9.80\text{ m/s}^2) = 1.13\times10^5\text{ N}$.
5	The speed of an object is given by the equation $v = At^3 - Bt$, where t refers to time. What are the dimensions of A and B? What are the SI units for the constants A and B?
	Solution:

	For the equation $v = At^3 - Bt$, the units of At^3 must be the same as the units of v. So the units of A must be the same as the units of v/t^3, which would be $\boxed{L/T^4}$. Also, the units of Bt must be the same as the units of v. So the units of B must be the same as the units of v/t, which would be $\boxed{L/T^2}$. For A, the SI units would be $\boxed{\text{m/s}^4}$, and for B, the SI units would be $\boxed{\text{m/s}^2}$.
6	Vector $\vec{A}$ has a magnitude of 63 units and points due west, while vector $\vec{B}$ has the same magnitude and points due south. Find the magnitude and direction of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$. Specify the directions relative to due west.
	Solution: $\text{Magnitude of } \mathbf{A}+\mathbf{B} = \sqrt{(63\text{ units})^2 + (63\text{ units})^2} = \boxed{89\text{ units}}$ $\theta = \tan^{-1}\left(\frac{63\text{ units}}{63\text{ units}}\right) = \boxed{45^\circ\text{ south of west}}$ $\text{Magnitude of } \mathbf{A}-\mathbf{B} = \sqrt{(63\text{ units})^2 + (63\text{ units})^2} = \boxed{89\text{ units}}$ $\theta = \tan^{-1}\left(\frac{63\text{ units}}{63\text{ units}}\right) = \boxed{45^\circ\text{ north of west}}$
7	A particle undergoes the following consecutive displacements: 3.50m south, 8.20m northeast, and 15.0m west. What is the resultant displacement?
	Solution:

	$d_1 = \left(-3.50\hat{\mathbf{j}}\right)$ m $d_2 = 8.20\cos 45.0°\,\hat{\mathbf{i}} + 8.20\sin 45.0°\,\hat{\mathbf{j}} = \left(5.80\hat{\mathbf{i}} + 5.80\hat{\mathbf{j}}\right)$ m $d_3 = \left(-15.0\hat{\mathbf{i}}\right)$ m $\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\hat{\mathbf{i}} + (5.80 - 3.50)\hat{\mathbf{j}} = \boxed{\left(-9.20\hat{\mathbf{i}} + 2.30\hat{\mathbf{j}}\right) \text{ m}}$ (or 9.20 m west and 2.30 m north) The magnitude of the resultant displacement is $$\lvert\mathbf{R}\rvert = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48 \text{ m}}.$$ The direction is $\theta = \arctan\left(\frac{2.30}{-9.20}\right) = \boxed{166°}$.
8	The following dataset describes the displacement of a body of constant velocity with time. <table><tr><td>$t(s)$</td><td>$s(m)$</td></tr><tr><td>1.2</td><td>2.51</td></tr><tr><td>1.3</td><td>2.76</td></tr><tr><td>1.5</td><td>3.2</td></tr><tr><td>1.7</td><td>3.61</td></tr><tr><td>1.8</td><td>3.89</td></tr><tr><td>2.1</td><td>4.41</td></tr></table>Draw a graph, include a line of best fit and its equation. Then determine the uncertainty of the velocity from the graph.
	Solution: Equation of graph: $s(t) = vt + s_o$ Comparing with $y = mx + c$, $y = s(t)$, $m = v$, $x = t$, $c = s_o$



Uncertainty of velocity = uncertainty of gradient

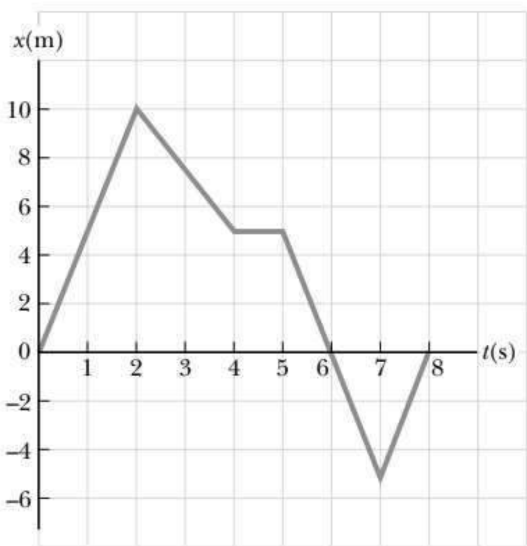
t	$t - t_{average}$	$(t - t_{average})^2$	S	$\hat{s}$	$s - \hat{s}$	$(s - \hat{s})^2$
1.2	**-0.4**	**0.2**	**2.51**	**2.55**	**-0.04**	**0.0012**
1.3	**-0.3**	**0.1**	**2.76**	**2.76**	**0.00**	**0.0000**
1.5	**-0.1**	**0.0**	**3.2**	**3.18**	**0.02**	**0.0003**
1.7	**0.1**	**0.0**	**3.61**	**3.61**	**0.00**	**0.0000**
1.8	**0.2**	**0.0**	**3.89**	**3.82**	**0.07**	**0.0046**
2.1	**0.5**	**0.3**	**4.41**	**4.46**	**-0.05**	**0.0026**
$\Sigma = 9.6$		**$\Sigma = 0.56$**				**$\Sigma = 0.0087$**

$$\Delta m = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

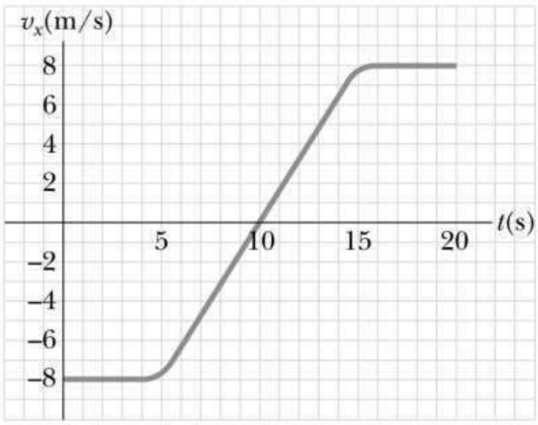
$$\Delta m = \sqrt{\frac{0.087}{(6-2)(0.56)}} \approx 0.2ms^{-1}$$

$$\Delta v = \Delta m = 0.2ms^{-1}$$

Worksheet 2: 1D Kinematics

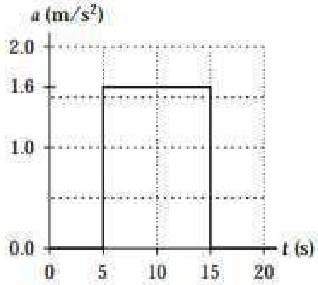
1	A bird can fly 25km/h. How long does it take to fly 3.5 km?
	Solution: $\bar{v} = \Delta x/\Delta t \quad \rightarrow \quad \Delta t = \Delta x/\bar{v} = (3.5\text{ km})/(25\text{ km/h}) = \boxed{0.14\text{ h}} = 8.4\text{ min}$
2	You are driving home from school steadily at 95km/h for 180 km. It then begins to rain and you slow to 65km/h. You arrive home after driving 4.5 h. How far is your hometown from school? What was your average speed?
	Solution: $\bar{v}_1 = \frac{d_1}{\Delta t_1} \quad \rightarrow \quad \Delta t_1 = \frac{d_1}{\bar{v}_1} = \frac{180\text{ km}}{95\text{ km/h}} = 1.895\text{ h} = 113.7\text{ min}$ $\Delta t_2 = \Delta t_{\text{total}} - \Delta t_1 = 4.5\text{ h} - 1.895\text{ h} = 2.605\text{ h} = 156.3\text{ min}$ $\bar{v}_2 = \frac{d_2}{\Delta t_2} \quad \rightarrow \quad d_2 = \bar{v}_2 \Delta t_2 = (65\text{ km/h})(2.605\text{ h}) = 169.3\text{ km} \approx 170\text{ km}$ The total distance is then $d_{\text{total}} = d_1 + d_2 = 180\text{ km} + 169.3\text{ km} = 349.3\text{ km} \approx \boxed{350\text{ km}}$. The average speed is NOT the average of the two speeds. Use the definition of average speed, Eq. 2–1. $\bar{v} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}} = \frac{349.3\text{ km}}{4.5\text{ h}} = 77.62\text{ km/h} \approx \boxed{78\text{ km/h}}$
3	The position versus time for a certain particle moving along the x axis is shown in the figure. Find the average velocity in the time intervals (a) 0 to 2s, (b) 0 to 4s, (c) 2s to 4s, (d) 4s to 7s, (e) 0 to 8s. 
	Solution:

	(a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10\text{ m}}{2\text{ s}} = \boxed{5\text{ m/s}}$ (b) $\bar{v} = \frac{5\text{ m}}{4\text{ s}} = \boxed{1.2\text{ m/s}}$ (c) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$ (d) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5\text{ m} - 5\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$ (e) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0\text{ m/s}}$
4	In a car race, a Myvi and a Alza are moving side by side down a straight away at $71.5ms^{-1}$. The driver of the Myvi realizes he must make a pit stop, and he smoothly slows to a stop over 250m. He spends 5.00s in the pit and then accelerates out, reaching his previous speed of $71.5ms^{-1}$ after 350 m. At this point, how far has the Myvi fallen behind the Alza, which has continued at a constant speed?
	Solution: For the Myvi, $x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$ $t = \frac{2\Delta x}{v_{xi} + v_{xf}} = \frac{2(250\text{ m})}{71.5\text{ m/s} + 0} = 6.99\text{ s}.$ $t = \frac{2(350\text{ m})}{0 + 71.5\text{ m/s}} = 9.79\text{ s}.$ $250 + 350\text{ m} = 600\text{ m}$ For the Alza, $6.99\text{ s} + 5.00\text{ s} + 9.79\text{ s} = 21.8\text{ s}$ $x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(71.5 + 71.5)(\text{m/s})(21.8\text{ s})$ $= 1\,558\text{ m}$ $1\,558\text{ m} - 600\text{ m} = \boxed{958\text{ m}}.$
5	A student throws a set of keys vertically upward to her roommate, who is in a window 4m above. The keys are caught 1.50s later by the roommate's outstretched hand. Determine the initial velocity of the keys thrown? What was the velocity of the keys just before they were caught?
	Solution:

	$y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0 \text{ m/s upward}}$. $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$ $v_f = \boxed{4.68 \text{ m/s downward}}$
6	A bowling ball traveling with constant speed hits the pins at the end of a bowling lane 16.5m long. The bowler hears the sound of the ball hitting the pins 2.80s after the ball is released from his hands. What is the speed of the ball, assuming the speed of sound is $340ms^{-1}$?
	$v_{\text{sound}} = \Delta x/\Delta t$ $\Delta t_{\text{sound}} = \frac{\Delta x}{v_{\text{sound}}} = \frac{16.5 \text{ m}}{340 \text{ m/s}} = 4.85 \times 10^{-2} \text{ s}$ $\Delta t_{\text{ball}} = \Delta t_{\text{total}} - \Delta t_{\text{sound}} = 2.80 \text{ s} - 4.85 \times 10^{-2} \text{ s} = 2.7515 \text{ s}.$ $v_{\text{ball}} = \frac{\Delta x}{\Delta t_{\text{ball}}} = \frac{16.5 \text{ m}}{2.7515 \text{ s}} = 5.9967 \text{ m/s} \approx \boxed{6.00 \text{ m/s}}$
7	A velocity–time graph for an object moving along the x axis is shown in the figure. a) Plot a graph of the acceleration versus time. b) Determine the average acceleration of the object in the time intervals t = 5.00 s to t = 15.0 s and t = 0 to t = 20.0 s. 

<table>
<tr><td></td><td>

(a) Acceleration is the slope of the graph of v vs t.
For $0 < t < 5.00\ \mathrm{s}$, $a = 0$.
For $15.0\ \mathrm{s} < t < 20.0\ \mathrm{s}$, $a = 0$.
For $5.0\ \mathrm{s} < t < 15.0\ \mathrm{s}$, $a = \dfrac{v_f - v_i}{t_f - t_i}$.
$a = \dfrac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60\ \mathrm{m/s^2}$



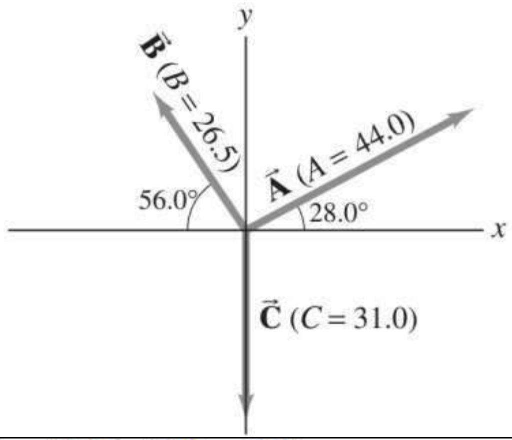
(b) $a = \dfrac{v_f - v_i}{t_f - t_i}$
(i) For $5.00\ \mathrm{s} < t < 15.0\ \mathrm{s}$, $t_i = 5.00\ \mathrm{s}$, $v_i = -8.00\ \mathrm{m/s}$,
$t_f = 15.0\ \mathrm{s}$
$v_f = 8.00\ \mathrm{m/s}$
$a = \dfrac{v_f - v_i}{t_f - t_i} = \dfrac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60\ \mathrm{m/s^2}}$.
(ii) $t_i = 0$, $v_i = -8.00\ \mathrm{m/s}$, $t_f = 20.0\ \mathrm{s}$, $v_f = 8.00\ \mathrm{m/s}$
$a = \dfrac{v_f - v_i}{t_f - t_i} = \dfrac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800\ \mathrm{m/s^2}}$

</td></tr>
<tr><td>8</td><td>A ball is seen to pass upward by a window with a vertical speed of $14ms^{-1}$. If the ball was thrown by a person $18m$ below on the street, what was its initial speed, what altitude does it reach, when was it thrown, and when does it reach the street again?</td></tr>
<tr><td></td><td>

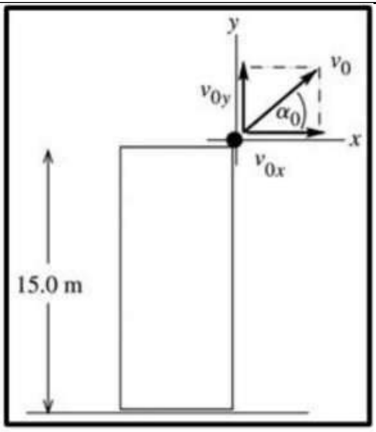
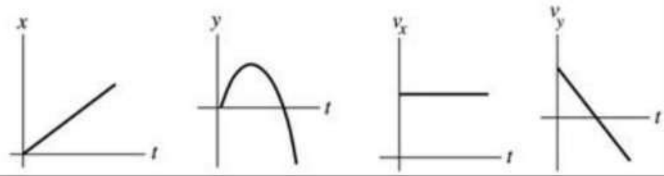
$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$
$v_0 = \pm\sqrt{v^2 - 2a(y - y_0)} = \pm\sqrt{(14\ \mathrm{m/s})^2 - 2(-9.80\ \mathrm{m/s^2})(18\ \mathrm{m})} = 23.43\ \mathrm{m/s} \approx \boxed{23\ \mathrm{m/s}}$
$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \dfrac{v^2 - v_0^2}{2a} = 0 + \dfrac{0 - (23.43\ \mathrm{m/s})^2}{2(-9.80\ \mathrm{m/s^2})} = \boxed{28\ \mathrm{m}}$
$v = v_0 + at \rightarrow t = \dfrac{v - v_0}{a} = \dfrac{14\ \mathrm{m/s} - 23.43\ \mathrm{m/s}}{-9.80\ \mathrm{m/s^2}} = 0.9622\ \mathrm{s} \approx \boxed{0.96\ \mathrm{s}}$
$v = v_0 + at \rightarrow t = \dfrac{v - v_0}{a} = \dfrac{-23.43\ \mathrm{m/s} - 14\ \mathrm{m/s}}{-9.80\ \mathrm{m/s^2}} = 3.819\ \mathrm{s} \approx \boxed{3.8\ \mathrm{s}}$

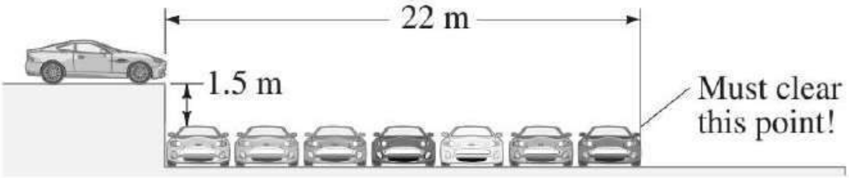
</td></tr>
</table>

Worksheet 3: 2D Kinematics & Projectile Motion

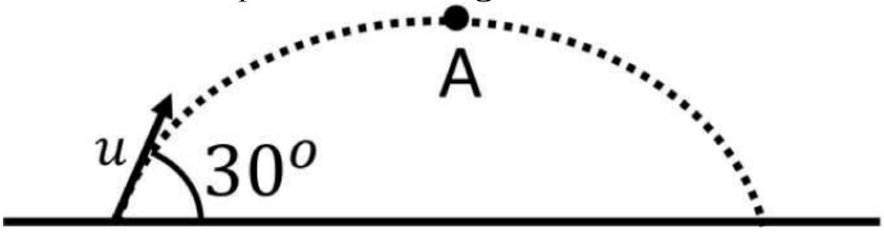
1	A spaceship is traveling east through the atmosphere at $18.3 kms^{-1}$ while descending at a rate of $11.5\ kms^{-1}$. What is its speed, in kms^{-1}?
	Let east be the $+x$ direction, and up be the $+y$ direction. $v_x = +18.3$ km/s and $v_y = -11.5$ km/s. Pythagorean theorem $v^2 = v_x^2 + v_y^2$. $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(+18.3\text{ km/s})^2 + (-11.5\text{ km/s})^2} = \boxed{21.6\text{ km/s}}$
2	Three vectors are shown in the figure. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of a) components b) magnitude and angle with the ±x axis 
	$A_x = 44.0 \cos 28.0° = 38.85$ $A_y = 44.0 \sin 28.0° = 20.66$ $B_x = -26.5 \cos 56.0° = -14.82$ $B_y = 26.5 \sin 56.0° = 21.97$ $C_x = 31.0 \cos 270° = 0.0$ $C_y = 31.0 \sin 270° = -31.0$ $(\vec{A}+\vec{B}+\vec{C})_x = 38.85 + (-14.82) + 0.0 = 24.03 = \boxed{24.0}$ $(\vec{A}+\vec{B}+\vec{C})_y = 20.66 + 21.97 + (-31.0) = 11.63 = \boxed{11.6}$ $\lvert\vec{A}+\vec{B}+\vec{C}\rvert = \sqrt{(24.03)^2 + (11.63)^2} = \boxed{26.7}$ $\theta = \tan^{-1}\frac{11.63}{24.03} = \boxed{25.8°}$
3	A volleyball is spiked so that it has an initial velocity of 15 m/s directed downward at an angle of 55^o below the horizontal. What is the horizontal component of the ball's velocity when the opposing player fields the ball?
	$v_x = v_0 \cos\theta = (15\text{ m/s})\cos 55° = \boxed{8.6\text{ m/s}}$
4	A diver running $2.5 ms^{-1}$ dives out horizontally from the edge of a vertical cliff and 3s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?

	$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \frac{1}{2}(9.80 \text{ m/s}^2)(3.0 \text{ s})^2 = \boxed{44 \text{ m}}$ $\Delta x = v_x t = (2.5 \text{ m/s})(3.0 \text{ s}) = \boxed{7.5 \text{ m}}$
5	A diver leaves the end of a 4m-high diving board and strikes the water 1.3s later, 3m beyond the end of the board. Determine: a) his initial velocity, b) the maximum height reached, c) the velocity with which he enters the water.
	$\Delta x = v_x t \quad \rightarrow \quad v_x = \frac{\Delta x}{t} = \frac{3.0 \text{ m}}{1.3 \text{ s}} = 2.308 \text{ m/s}$ $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = 4.0 \text{ m} + v_{y0}(1.3 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.3 \text{ s})^2 \quad \rightarrow$ $v_{y0} = 3.293 \text{ m/s}$ $v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(2.308 \text{ m/s})^2 + (3.293 \text{ m/s})^2} = \boxed{4.0 \text{ m/s}}$ $\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{3.293 \text{ m/s}}{2.308 \text{ m/s}} = \boxed{55° \text{ above the horizontal}}$ $v_y^2 = v_{y0}^2 + 2a\Delta y \quad \rightarrow \quad 0 = (3.293 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(y_{max} - 4.0 \text{ m}) \quad \rightarrow$ $y_{max} = \boxed{4.6 \text{ m}}$ $v_y = v_{y0} + at = 3.293 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.3 \text{ s}) = -9.447 \text{ m/s}$
6	A dive bomber has a velocity of $280ms^{-1}$ at an angle θ below the horizontal. When the altitude of the aircraft is 2.15km, it releases a bomb, which subsequently hits a target on the ground. The magnitude of the displacement from the point of release of the bomb to the target is 3.25 km. Find the angle θ.
6	$x_f = \sqrt{(3.25 \text{ km})^2 - (2.15 \text{ km})^2} = 2.437 \text{ km}$ $y_f = x_f \tan\theta - \frac{g x_f^2}{2v_i^2 \cos^2\theta_i}$ $-2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - \frac{(9.8 \text{ m/s}^2)(2\,437 \text{ m})^2}{2(280 \text{ m/s})^2 \cos^2\theta_i}$ $\therefore -2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - (371.19 \text{ m})(1 + \tan^2\theta_i)$ $\therefore \tan^2\theta - 6.565\tan\theta_i - 4.792 = 0$ $\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945.$ Select the negative solution, since θ_i is below the horizontal. $\therefore \tan\theta_i = -0.662\,,\ \boxed{\theta_i = -33.5°}$

7	A man stands on the roof of a 15m-tall building and throws a rock with a speed of $30ms^{-1}$ at an angle of 33° above the horizontal. Ignore air resistance. Calculate a) the maximum height above the roof that the rock reaches b) the speed of the rock just before it strikes the ground, c) the horizontal range from the base of the building to the point where the rock strikes the ground. d) Draw $x - t$, $y - t$, $v_x - t$, and $v_y - t$ graphs for the motion.
	$a_x = 0$ and $a_y = -g$. $v_{0x} = v_0 \cos\alpha_0 = 25.2$ m/s $v_{0y} = v_0 \sin\alpha_0 = 16.3$ m/s At the maximum height $v_y = 0$. $a_y = -9.80 \text{ m/s}^2$, $v_y = 0$, $v_{0y} = +16.3$ m/s, $y - y_0 = ?$ $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (16.3 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +13.6$ m  $\boxed{v_x = v_{0x} = 25.2 \text{ m/s}}$ (since $a_x = 0$) $v_y = ?$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -15.0$ m, $v_{0y} = 16.3$ m/s $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ $v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ $v_y = -\sqrt{(16.3 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-15.0 \text{ m})} = -23.7$ m/s $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2 \text{ m/s})^2 + (-23.7 \text{ m/s})^2} = \boxed{34.6 \text{ m/s.}}$ $t = ?$, $v_y = -23.7$ m/s (from part (b)), $a_y = -9.80 \text{ m/s}^2$, $v_{0y} = +16.3$ m/s $t = \frac{v_y - v_{0y}}{a_y} = \frac{-23.7 \text{ m/s} - 16.3 \text{ m/s}}{-9.80 \text{ m/s}^2} = +4.08$ s $t = 4.08$ s, $v_{0x} = 25.2$ m/s, $a_x = 0$, $x - x_0 = ?$ $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2 \text{ m/s})(4.08 \text{ s}) + 0 = \boxed{103 \text{ m}}$ Graphs of x versus t, y versus t, v_x versus t and v_y versus t: 
8	A stunt driver wants to make his car jump over 7 cars parked side by side below a horizontal ramp. With what minimum speed must he drive off the horizontal ramp? The vertical height of the ramp is 1.5m above the cars and the horizontal distance he must clear is 22m. If the ramp

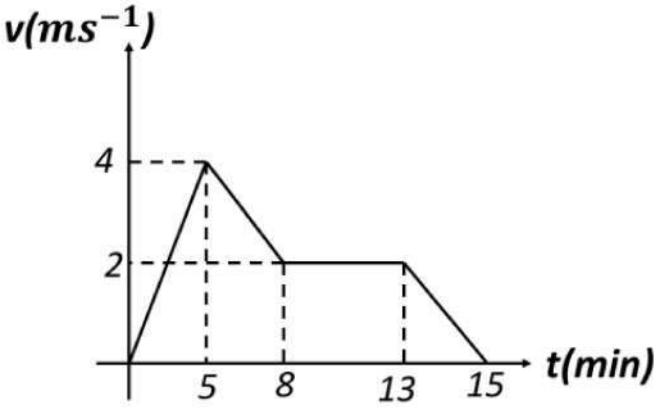
	is now tilted upward, so that "takeoff angle" is 7.0° above the horizontal, what is the new minimum speed? 
	$y=-1.5\text{ m}.$ $v_{y0}=0,\ a_y=-g,\ v_x=v_0,$ and $\Delta x=22\text{ m}.$ $y=y_0+v_{y0}t+\frac{1}{2}a_yt^2 \ \rightarrow\ y=0+0+\frac{1}{2}(-g)(\Delta x/v_0)^2 \ \rightarrow$ $v_0=\sqrt{\frac{-g(\Delta x)^2}{2(y)}}=\sqrt{\frac{-(9.80\text{ m/s}^2)(22\text{ m})^2}{2(-1.5\text{ m})}}=39.76\text{ m/s}\approx\boxed{4.0\times10^1\text{ m/s}}$ $y=-1.5\text{ m}.$ $v_{y0}=v_0\sin\theta_0,\ a_y=-g,\ v_x=v_0\cos\theta_0,$ and $\Delta x=22\text{ m}.$ $\theta_0=7.0°.$ $\Delta x=v_xt \ \rightarrow\ t=\frac{\Delta x}{v_0\cos\theta_0}$ $y=y_0+v_{y0}t+\frac{1}{2}a_yt^2 \ \rightarrow\ y=v_0\sin\theta_0\frac{\Delta x}{v_0\cos\theta_0}+\frac{1}{2}(-g)\left(\frac{\Delta x}{v_0\cos\theta_0}\right)^2 \ \rightarrow$ $v_0=\sqrt{\frac{g(\Delta x)^2}{2(\Delta x\tan\theta_0-y)\cos^2\theta_0}}=\sqrt{\frac{(9.80\text{ m/s}^2)(22\text{ m})^2}{2((22\text{ m})\tan 7.0°+1.5\text{ m})\cos^2 7.0°}}=\boxed{24\text{ m/s}}$

Worksheet 4: PYQ Kinematics

21/22	a.	A bus is moving with an initial speed u begins to slow down at a uniform rate of $3ms^{-2}$. Calculate u if it takes 6.67s to travel at a distance of 67m.
	b.	A ball is thrown horizontally from the top of a building with a speed of $20ms^{-1}$. After 7s, the ball hits the ground. What is the height of the building?
	c.	The figure below shows the path taken by an object projected with an initial speed, u at an angle 30^o to the horizontal.  What is the speed of the object at point A in terms of u?

	d. A motorcycle accelerates from rest to $5ms^{-1}$ in 4.5s and then continues at this speed for another 4.5s. Calculate the total distance travelled by the motorcycle.
	a. Method 1 $$v^2 = u^2 + 2as \Rightarrow v^2 = u^2 + 2(-3)(67)$$ $$v = u + at \Rightarrow v = u + (-3)(6.67)$$ $$v = 0.04ms^{-1};\ u = 20.05ms^{-1}$$ Method 2 $$s = ut + \frac{1}{2}at^2$$ $$\Rightarrow 67 = 6.67(u) + \frac{1}{2}(-3)(6.67)^2$$ $$u = 20.05ms^{-1}$$ **b.** $$s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow s_y = 0 + \frac{1}{2}(-9.81)7^2 \Rightarrow$$ $$s_y = -240.345m$$ **c.** $$v_y = u_y + a_y t = u\sin\theta - gt;\ v_x = u_x = u\cos\theta \Rightarrow v = \sqrt{v_x^2 + v_y^2}$$ $$v = \sqrt{u^2 - 2gt\sin\theta + g^2t^2}$$ **d.** $$s_1 = \frac{1}{2}(u+v)t = \frac{1}{2}(0+5)4.5 = 11.25m$$ $$s_2 = vt = 5(4.5) = 22.5m$$ $$s_{total} = s_1 + s_2 = 11.25 + 22.5$$ $$s_{total} = 33.75m$$
20/21	A student took 15minutes to cycle from his house to school. He starts from rest and reaches a maximum speed of $4ms^{-1}$ in 5 minutes at constant acceleration. After reaching the maximum speed, he decelerates uniformly to $2.0ms^{-1}$ in 3 minutes and continues cycling with this speed for 5 minutes. He then took 2 minutes to decelerate uniformly to stop. A. Sketch a labelled graph of speed versus time for the whole journey.

<table>
<tr><td></td><td>B. Calculate the acceleration of the bicycle for the time segments of 0-5 minutes and 13-15 minutes.
C. Determine the total distance from his house to school.</td></tr>
<tr><td></td><td>A.

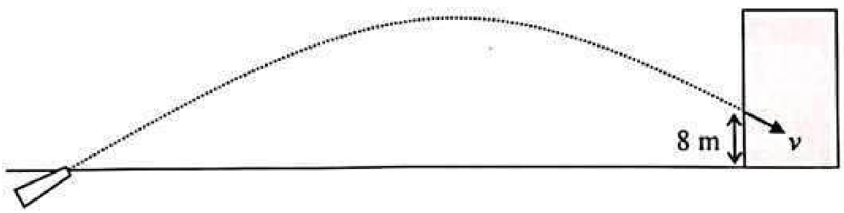


B.

$$a_{0-5min} = \frac{v-u}{t} = \frac{4-0}{5\times 60} = 0.013ms^{-2}$$
$$a_{13-15min} = \frac{v-u}{t} = \frac{0-2}{2\times 60} = -0.017ms^{-2}$$

C.

$$s_{total} = s_{0-5mins} + s_{5-8mins} + s_{8-13mins} + s_{13-15mins}$$
$$s_{total} = \left(\frac{1}{2}(v+u)t\right) + \left(\frac{1}{2}(v+u)t\right) + vt + \frac{1}{2}(v+u)t$$
$$s_{total} = \left(\frac{1}{2}(4+0)(5\times 60)\right) + \left(\frac{1}{2}(2+4)(3\times 60)\right) + 2(5\times 60) + \frac{1}{2}(0+2)(2\times 60)$$
$$s_{total} = 1860m$$</td></tr>
<tr><td>19/20</td><td>A. A boat with an initial speed of $30ms^{-1}$, decelerates at $3.5ms^{-2}$ for 4.5s before reaching a buoy. Calculate the speed of the boat at the buoy.
B. The figure below shows a stream of water hitting a wall at a height of 8m with a velocity of $40ms^{-1}$ at an angle of 35^o below the horizontal.</td></tr>
</table>

	 Determine the initial velocity of the water as it leaves the nozzle.
	A. $v = u + at \Rightarrow v = 30 + (-3.5)(4.5) = 14.25ms^{-1}$ B. $v_y^2 = u_y^2 - 2gs_y$ $\Rightarrow \left(40\,sin\,(-35^o)\right)^2 = (u\,sin\,\theta_i)^2 - 2(9.81)(8)$ $\Rightarrow u\,sin\,\theta_i = 26.141ms^{-1} - - - [1]$ $u_x = v_x = v\,cos\,\theta_f = 40\,cos(-35^o)$ $\Rightarrow u\,cos\,\theta_i = 32.766ms^{-1} - - - [2]$ Solving equations 1 and 2 simultaneously yields, $u = 41.9161ms^{-1};\ \theta_i = 38.5832^o$

Worksheet 5: Momentum & its Conservation

1	a. b.	A 0.1kg ball is thrown straight up into the air with an initial speed of 15.0m/s. Find the momentum of the ball at its maximum height and halfway up to its maximum height. A basketball (m = 0.60 kg) is dropped from rest. Just before striking the floor, the ball has a momentum whose magnitude is 3.1kg m/s. At what height was the basketball dropped?
		a. At maximum height $\mathbf{v} = 0$, so $\mathbf{p} = \boxed{0}$. Its original kinetic energy is its constant total energy, $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.100)\text{kg}(15.0\ \text{m/s})^2 = 11.2\ \text{J}$. At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic: $K = 5.62\ \text{J} = \frac{1}{2}(0.100\ \text{kg})v^2$ $v = \sqrt{\frac{2 \times 5.62\ \text{J}}{0.100\ \text{kg}}} = 10.6\ \text{m/s}$ $\mathbf{p} = m\mathbf{v} = (0.100\ \text{kg})(10.6\ \text{m/s})\hat{\mathbf{j}}$ $\mathbf{p} = \boxed{1.06\ \text{kg}\cdot\text{m/s}\,\hat{\mathbf{j}}}$.

	b. $$v_y^2 = v_{0y}^2 + 2a_y y$$ $$v_{0y} = 0.$$ $$v_y = p/m.$$ $$y = \frac{v_y^2}{2a_y} = \frac{(p/m)^2}{2a_y} = \frac{\left[(3.1\ \text{kg}\cdot\text{m/s})\ /\ (0.60\ \text{kg})\right]^2}{2\ (9.80\ \text{m/s}^2)} = \boxed{1.4\ \text{m}}$$
2	A model rocket is constructed with a motor that can provide a total impulse of 29.0Ns. The mass of the rocket is 0.175 kg. What is the speed that this rocket achieves when launched from rest? Neglect the effects of gravity and air resistance.
	$$\left(\Sigma\bar{\mathbf{F}}\right)\Delta t = m\mathbf{v_f} - m\mathbf{v_0}$$ $$\mathbf{v_0}\ =\ 0\ \ \text{m/s}$$ $$\left(\Sigma\bar{\mathbf{F}}\right)\Delta t = m\mathbf{v_f}$$ $$v_f = \frac{\left(\Sigma\bar{F}\right)\Delta t}{m} = \frac{29.0\ \text{N}\cdot\text{s}}{0.175\ \text{kg}} = \boxed{166\ \text{m/s}}$$
3	A dump truck is being filled with sand. The sand falls straight downward from rest from a height of 2.00 m above the truck bed, and the mass of sand that hits the truck per second is 55.0 kg/s. The truck is parked on the platform of a weight scale. By how much does the scale reading exceed the weight of the truck and sand?
	The excess weight of the truck is due to the force exerted on the truck by the sand. $$\bar{\mathbf{F}}\Delta t = m(\mathbf{v_f} - \mathbf{v_0})$$ $$v_0 = \sqrt{2gh} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m})}\ =\ 6.26\ \text{m/s}$$ $$\mathbf{v_f}\ = 0\ \text{m/s}.$$ $$\bar{\mathbf{F}} = \left(\frac{m}{\Delta t}\right)(\mathbf{v_f} - \mathbf{v_0}) = (55.0\ \text{kg/s})[(0\ \text{m/s}) - (-6.26\ \text{m/s})] = \boxed{+344\ \text{N}}$$
4	A 40.0-kg boy, riding a 2.50-kg skateboard at a velocity of 5.30 m/s across a level sidewalk, jumps forward to leap over a wall. Just after leaving contact with the board, the boy's velocity relative to the sidewalk is 6.00 m/s, 9.5^o above the horizontal. Ignore any friction between the skateboard and the sidewalk. What is the skateboard's velocity relative to the sidewalk at this instant?

$$m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{01x} + m_2 v_{02x}$$

		Horizontal velocity component	
	Mass	Initial	Final
Boy	$m_1 = 40.0$ kg	$v_{01x} = v_0 = +5.30$ m/s	$v_{f1x} = +(6.00 \text{ m/s})(\cos 9.50°) = +5.92$ m/s
Skateboard	$m_2 = 2.50$ kg	$v_{02x} = v_0 = +5.30$ m/s	$v_{f2x} = ?$

$$m_2 v_{f2x} = m_1 v_0 + m_2 v_0 - m_1 v_{f1x} \qquad \text{or} \qquad v_{f2x} = \frac{m_1(v_0 - v_{f1x}) + m_2 v_0}{m_2}$$

$$v_{f2x} = \frac{m_1(v_0 - v_{f1x})}{m_2} + v_0 = \frac{(40.0 \text{ kg})(+5.30 \text{ m/s} - 5.92 \text{ m/s})}{2.50 \text{ kg}} + 5.30 \text{ m/s} = \boxed{-4.6 \text{ m/s}}$$

5 A 144g baseball moving at $32ms^{-1}$ strikes a stationary 5.25kg brick resting on small rollers so it moves without significant friction. After hitting the brick, the baseball bounces straight back, and the brick moves forward at $1.1ms^{-1}$. What is the baseball's speed after the collision? Find the total kinetic energy before and after the collision.

$$v_A = \frac{(0.144)(32) - (5.25)(1.1)}{0.144} = -8.1m/s$$

$$K_i = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.144)(32) = 73.73J$$

$$K_f = \frac{1}{2}(m_A v_A^2 + m_B v_B^2) = \frac{1}{2}\left((0.144)(8.1)^2 + (5.25)(1.1)^2\right) = 7.9J$$

6 A 50kg skater is traveling due east at a speed of 3m/s. A 70kg skater is moving due south at a speed of 7m/s. They collide and hold on to each other after the collision, managing to move off at an angle south of east, with a speed of v_f. Find the angle and the speed v_f, assuming that friction can be ignored.

$$P_{fx} = P_{0x}$$

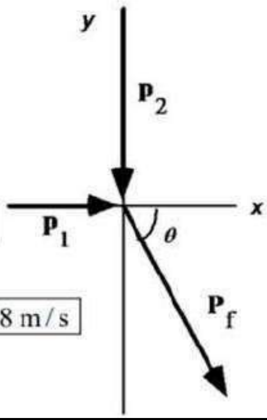
$$(m_1 + m_2)v_f \cos\theta = m_1 v_{01}$$

$$P_{fy} = P_{0y}$$

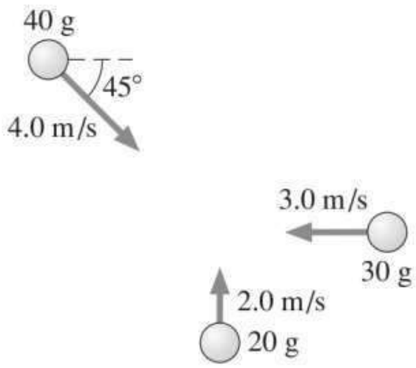
$$(m_1 + m_2)v_f \sin\theta = m_2 v_{02}$$

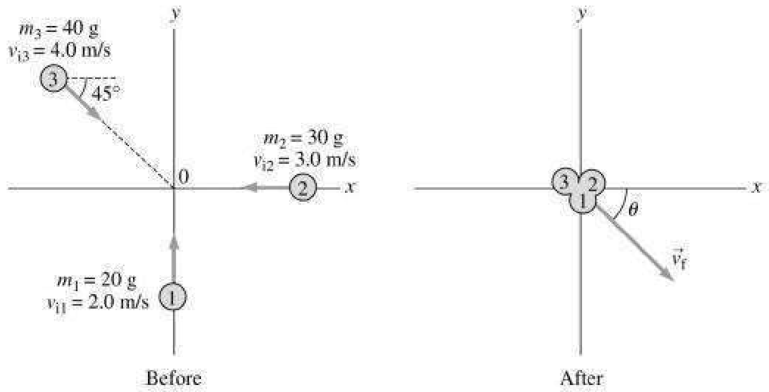
$$\theta = \tan^{-1}\left(\frac{m_2 v_{02}}{m_1 v_{01}}\right) = \tan^{-1}\left[\frac{(70.0 \text{ kg})(7.00 \text{ m/s})}{(50.0 \text{ kg})(3.00 \text{ m/s})}\right] = \boxed{73.0°}$$

$$v_f = \frac{m_1 v_{01}}{(m_1 + m_2)\cos\theta} = \frac{(50.0 \text{ kg})(3.00 \text{ m/s})}{(50.0 \text{ kg} + 70.0 \text{ kg})(\cos 73.0°)} = \boxed{4.28 \text{ m/s}}$$



<table>
<tr>
<td>7</td>
<td>The figure shows a collision between three balls of clay. The three hit simultaneously and stick together. What are the speed and direction of the resulting blob of clay?


</td>
</tr>
<tr>
<td></td>
<td>Pictorial representation



$$\vec{p}_{i1} = m_1\vec{v}_{i1} = (0.020\text{ kg})(2.0\text{ m/s})\hat{j} = 0.040\hat{j}\text{ kg m/s}$$

$$\vec{p}_{i2} = m_2\vec{v}_{i2} = (0.030\text{ kg})(-3.0\text{ m/s }\hat{i}) = -0.090\hat{i}\text{ kg m/s}$$

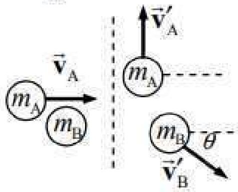
$$\vec{p}_{i3} = m_3\vec{v}_{i3} = (0.040\text{ kg})[(4.0\text{ m/s})\cos 45°\hat{i} - (4.0\text{ m/s})\sin 45°\hat{j}] = (0.113\hat{i} - 0.113\hat{j})\text{ kg m/s}$$

Since $\vec{p}_f = \vec{p}_i = \vec{p}_{i1} + \vec{p}_{i2} + \vec{p}_{i3}$, we have

$$(m_1 + m_2 + m_3)\vec{v}_f = (0.023\hat{i} - 0.073\hat{j})\text{ kg m/s} \quad\Rightarrow\quad \vec{v}_f = (0.256\hat{i} - 0.811\hat{j})\text{ m/s}$$

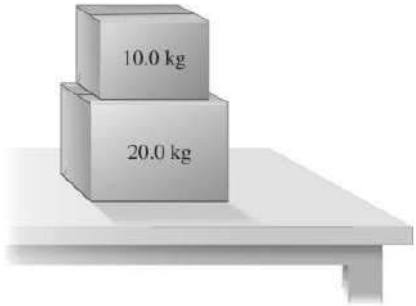
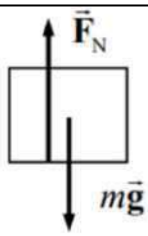
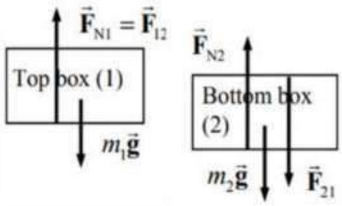
$$v_f = \sqrt{(0.256\text{ m/s})^2 + (-0.811\text{ m/s})^2} = 0.85\text{ m/s}$$

$$\theta = \tan^{-1}\frac{|v_{fy}|}{v_{fx}} = \tan^{-1}\frac{0.811}{0.256} = 72°\text{ below the } x\text{-axis.}$$</td>
</tr>
<tr>
<td>8</td>
<td>An atomic nucleus of mass m traveling with speed v collides elastically with a target particle of mass 2m (initially at rest) and is scattered at 90°. At what angle does the target particle move after the collision? What are the final speeds of the two particles? What fraction of the initial kinetic energy is transferred to the target particle?</td>
</tr>
</table>

	p_x: $m_A v_A = m_B v'_B \cos\theta \;\rightarrow\; v = 2v'_B \cos\theta$ p_y: $0 = m_A v'_A - m_B v'_B \sin\theta \;\rightarrow\; v'_A = 2v'_B \sin\theta$ KE: $\frac{1}{2}m_A v_A^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B \;\rightarrow\; v^2 = v'^2_A + 2v'^2_B \;\rightarrow\; v^2 - v'^2_A = 2v'^2_B$ $v = 2v'_B \cos\theta;\; v'_A = 2v'_B \sin\theta \;\rightarrow\; v^2 = 4v'^2_B \cos^2\theta;$ $v'^2_A = 4v'^2_B \sin^2\theta \;\rightarrow\; v^2 + v'^2_A = 4v'^2_B$ $v^2 - v'^2_A = 2v'^2_B;\; v^2 + v'^2_A = 4v'^2_B \;\rightarrow\; 2v^2 = 6v'^2_B \;\rightarrow\; v'_B = \frac{v}{\sqrt{3}}$  $\cos\theta = \frac{v}{2v'_B} = \frac{v}{2\frac{v}{\sqrt{3}}} = \frac{\sqrt{3}}{2} \;\rightarrow\; \boxed{\theta = 30^\circ}$ $v'_A = 2v'_B \sin\theta = 2\frac{v}{\sqrt{3}}\sin 30^\circ = \boxed{v'_A = \frac{v}{\sqrt{3}}}$ $\frac{\text{KE}_{\text{target}}}{\text{KE}_{\text{original}}} = \frac{\frac{1}{2}m_B v'^2_B}{\frac{1}{2}m_A v_A^2} = \frac{\frac{1}{2}(2m_A)(v^2/3)}{\frac{1}{2}m_A v^2} = \boxed{\frac{2}{3}}$

Worksheet 6: Free Body Diagram

1	a. According to a simplified model of a mammalian heart, at each pulse approximately 20g of blood is accelerated from $0.25ms^{-1}$ to $0.35ms^{-1}$ during a period of 0.1s. What is the magnitude of the force exerted by the heart muscle? b. What average force is required to stop a 950kg car in 8s if the car is traveling at $95kmh^{-1}$.
	a. $a = \frac{v - v_0}{t} = \frac{0.35\text{ m/s} - 0.25\text{ m/s}}{0.10\text{ s}} = 1.0\text{ m/s}^2.$ $F = ma = (20 \times 10^{-3}\text{ kg})(1.0\text{ m/s}^2) = \boxed{0.02\text{ N}}$ b. $v = 0$ $v_0 = (95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 26.39\text{ m/s}$ $a_{\text{avg}} = \frac{v - v_0}{t} = \frac{0 - 26.39\text{ m/s}}{8.0\text{ s}} = -3.299\text{ m/s}^2$ $F_{\text{avg}} = ma_{\text{avg}} = (950\text{ kg})(-3.299\text{ m/s}^2) = -3134\text{ N} \approx \boxed{-3100\text{ N}}$
2	A 20kg box rests on a table.

	a. What is the weight of the box and the normal force acting on it? b. A 10kg box is placed on top of the 20kg box, as shown in the figure shown. Determine the normal force that the table exerts on the 20kg box and the normal force that the 20kg box exerts on the 10kg box. 
	(a) $mg = (20.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{196\ \text{N}}$.  (b)  $\Sigma F_1 = F_{N1} - m_1 g = 0$ $F_{N1} = m_1 g = (10.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{98.0\ \text{N}} = F_{12} = F_{21}$ $\Sigma F_2 = F_{N2} - F_{21} - m_2 g = 0$ $F_{N2} = F_{21} + m_2 g = 98.0\ \text{N} + (20.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) = \boxed{294\ \text{N}}$
3	Arlene is to walk across a “high wire” strung horizontally between two buildings 10m apart. The sag in the rope when she is at the midpoint is 10°, as shown in the figure shown.  If her mass is 50kg, what is the tension in the rope at this point?

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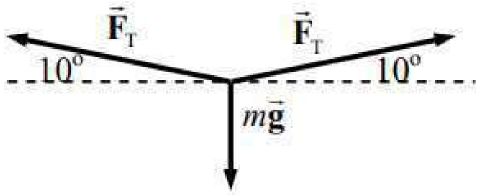

$$\sum F = F_T \sin 10.0^\circ + F_T \sin 10.0^\circ - mg = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{2 \sin 10.0^\circ} = \frac{(50.0\ \text{kg})(9.80\ \text{m/s}^2)}{2 \sin 10.0^\circ} = \boxed{1410\,\text{N}}$$
</td>
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<td>4</td>
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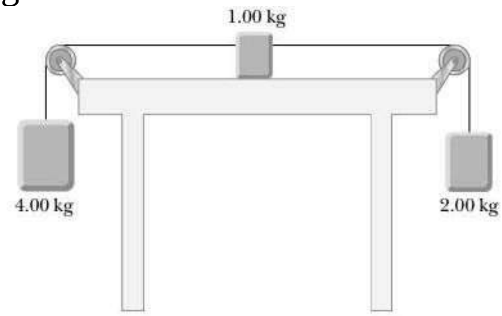
Three objects are connected on the table as shown in the figure. The table is rough and has a coefficient of kinetic friction of 0.35.



The objects have masses of 4kg, 1kg, and 2kg, as shown, and the pulleys are frictionless. Draw free-body diagrams of each of the objects.

a. Determine the acceleration of each object and their directions.

b. Determine the tensions in the two cords.
</td>
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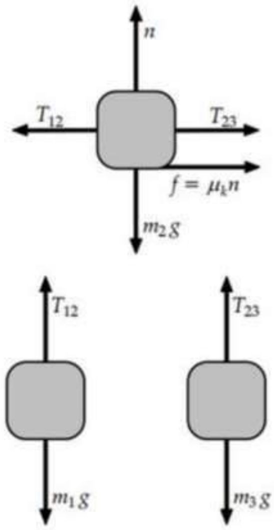
For m_1, $\sum F_y = ma_y$ $\quad +T_{12} - m_1 g = -m_1 a$
For m_2, $\sum F_x = ma_x$ $\quad -T_{12} + \mu_k n + T_{23} = -m_2 a$
and $\sum F_y = ma_y$ $\quad n - m_2 g = 0$
for m_3, $\sum F_y = ma_y$ $\quad T_{23} - m_3 g = +m_3 a$
$-T_{12} + 39.2\text{ N} = (4.00\text{ kg})a$
$+T_{12} - 0.350(9.80\text{ N}) - T_{23} = (1.00\text{ kg})a$
$+T_{23} - 19.6\text{ N} = (2.00\text{ kg})a$.
(a) $+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$
$a = \boxed{2.31\text{ m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}$.
(b) Now $-T_{12} + 39.2\text{ N} = (4.00\text{ kg})(2.31\text{ m/s}^2)$
$\boxed{T_{12} = 30.0\text{ N}}$
and $T_{23} - 19.6\text{ N} = (2.00\text{ kg})(2.31\text{ m/s}^2)$
$\boxed{T_{23} = 24.2\text{ N}}$.


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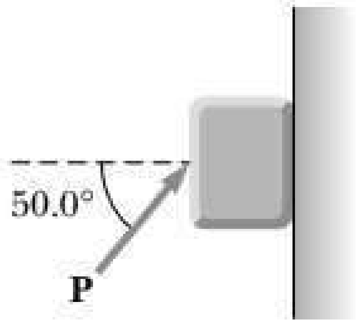
<tr><td>5</td><td>
A block of mass 3kg is pushed up against a wall by a force P that makes a 50° angle with the horizontal as shown in the figure shown. The coefficient of static friction between the block and the wall is 0.25. Determine the possible values for the magnitude of P that allow the block to remain stationary.


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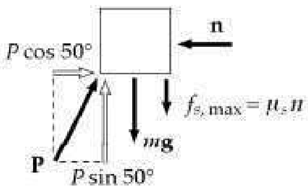
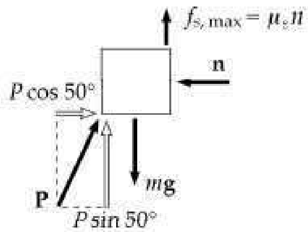
(Case 1, impending upward motion)
$\sum F_x = 0$: $\quad P\cos 50.0° - n = 0$
$f_{s,\max} = \mu_s n$: $\quad f_{s,\max} = \mu_s P\cos 50.0°$
$= 0.250(0.643)P = 0.161P$
$\sum F_y = 0$: $\quad P\sin 50.0° - 0.161P - 3.00(9.80) = 0$
$P_{\max} = \boxed{48.6\text{ N}}$


(Case 2, impending downward motion)
$f_{s,\max} = 0.161P$
$\sum F_y = 0$: $\quad P\sin 50.0° + 0.161P - 3.00(9.80) = 0$
$P_{\min} = \boxed{31.7\text{ N}}$


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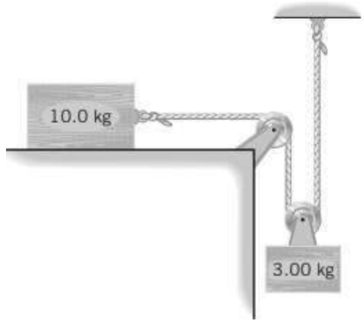
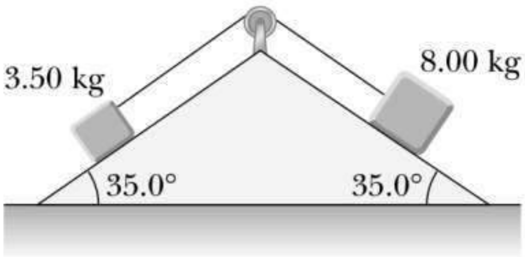
6	In the drawing, the rope and the pulleys are massless, and there is no friction. Find the tension in the rope and the acceleration of the 10kg block. 
	Newton's second law for block 1 (10.0 kg) is $T = m_1 a$ Block 2 (3.00 kg) has two ropes attached each carrying a tension T. Also, block 2 only travels half the distance that block 1 travels in the same amount of time so its acceleration is only half of block 1's acceleration. Newton's second law for block 2 is then $2T - m_2 g = -\frac{1}{2} m_2 a$ $T = \frac{\frac{1}{2} m_2 g}{1+\frac{1}{4}(m_2/m_1)} = \boxed{13.7\text{ N}}$ $a = \frac{T}{m_1} = \frac{13.7\text{ N}}{10.0\text{ kg}} = \boxed{1.37\text{ m/s}^2}$
7	a. A child slides down a slide with a 34° incline, and at the bottom her speed is precisely half what it would have been if the slide had been frictionless. Calculate the coefficient of kinetic friction between the slide and the child. b. Two blocks of mass 3.5kg and 8.0kg are connected by a massless string that passes over a frictionless pulley.  The inclines are frictionless. Find the magnitude of the acceleration of each block and the tension in the string.
	a.

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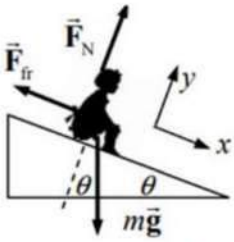


$$\sum F_x = mg \sin\theta = ma \quad \rightarrow \quad a = g\sin\theta$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$v_{(\text{No friction})} = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{2g\sin\theta(x - x_0)}$$

$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = ma = mg\sin\theta - F_{fr} = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k\cos\theta)$$

$$v_{(\text{friction})} = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{2g(\sin\theta - \mu_k\cos\theta)(x - x_0)}$$

$$v_{(\text{friction})} = \frac{1}{2}v_{(\text{No friction})}$$

$$\rightarrow \quad \sqrt{2g(\sin\theta - \mu_k\cos\theta)(x - x_0)} = \frac{1}{2}\sqrt{2g(\sin\theta)(x - x_0)}$$

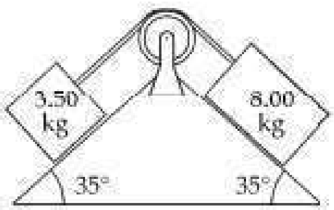
$$2g(\sin\theta - \mu_k\cos\theta)(x - x_0) = \frac{1}{4}2g(\sin\theta)(x - x_0)$$

$$\mu_k = \frac{3}{4}\tan\theta = \frac{3}{4}\tan 34^\circ = \boxed{0.51}$$

b.

$$\sum F_1 = m_1 a_1: \quad -m_1 g\sin 35.0^\circ + T = m_1 a$$

$$\sum F_2 = m_2 a_2: \quad m_2 g\sin 35.0^\circ - T = m_2 a$$



$$-(3.50)(9.80)\sin 35.0^\circ + T = 3.50a$$

$$(8.00)(9.80)\sin 35.0^\circ - T = 8.00a.$$

$$+45.0\ \text{N} - 19.7\ \text{N} = (11.5\ \text{kg})a.$$

$$\boxed{a = 2.20\ \text{m/s}^2}.$$

$$-19.7\ \text{N} + T = (3.50\ \text{kg})(2.20\ \text{m/s}^2) = 7.70\ \text{N}.$$

$$\boxed{T = 27.4\ \text{N}}.$$

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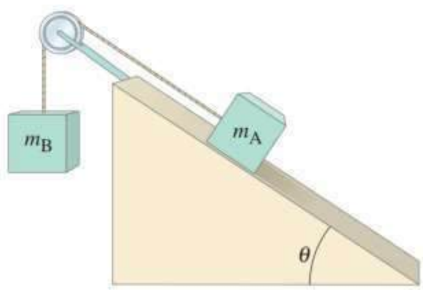
Suppose the coefficient of kinetic friction between m_A and the plane in the figure shown is $\mu_k = 0.15$ and that $m_A = m_B = 2.7kg$. As m_B moves down, determine the magnitude of the acceleration of m_A and m_B, given $\theta = 34^o$. What smallest value of μ will keep the system from accelerating?



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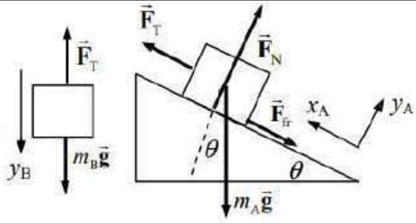
	$a_{yB} = a_{xA} = a$ $\Sigma F_{yB} = m_B g - F_T = m_B a \rightarrow F_T = m_B g - m_B a$ $\Sigma F_{xA} = F_T - m_A g \sin\theta - F_{fr} = m_A a$ $\Sigma F_{yA} = F_N - m_A g\cos\theta = 0 \rightarrow F_N = m_A g\cos\theta$  $m_B g - m_B a - m_A g\sin\theta - \mu_k m_A g\cos\theta = m_A a \rightarrow$ $a = \frac{m_B g - m_A g\sin\theta - m_A g\mu_k\cos\theta}{(m_A + m_B)} = \frac{1}{2}g(1-\sin\theta - \mu_k\cos\theta)$ $a = \frac{1}{2}(9.80\ m/s^2)(1-\sin 34° - 0.15\cos 34°) = \boxed{1.6\ m/s^2}$ $a = \frac{1}{2}g(1-\sin\theta-\mu_k\cos\theta) = 0 \rightarrow 1-\sin\theta-\mu_k\cos\theta = 0 \rightarrow$ $\mu_k = \frac{1-\sin\theta}{\cos\theta} = \frac{1-\sin 34°}{\cos 34°} = \boxed{0.53}$

Worksheet 7: Work, Energy, $\mathbf{\Sigma W = \Delta K}$ & $\mathbf{\Delta(\Sigma E) = 0}$

1	a. During a tug-of-war, team A pulls on team B by applying a force of 1100N to the rope between them. The rope remains parallel to the ground. How much work does team A do if they pull team B toward them a distance of 2.0m? b. The brakes of a truck cause it to slow down by applying a retarding force of $3.0 \times 10^3 N$ to the truck over a distance of 850 m. What is the work done by this force on the truck? Is the work positive or negative? Why?
	a. $W = (F\cos\theta)s = (1100\,N)(\cos 0°)(2.0\ m) = \boxed{2.2\times 10^3\ J}$ b. $W = (F\cos\theta)s$ $\theta = 180°$ $W = (F\cos\theta)s = (3.0\times 10^3\ N)(\cos 180°)(850\ m) = \boxed{-2.6\times 10^6\ J}$ $\boxed{\text{negative}}$, because the retarding force is directed opposite to the direction of the displacement of the truck.
2	a. What is the minimum work needed to push a 950kg car 710 m up along a 9.0° incline? Ignore friction. b. A 1.00$\times 10^2$kg crate is being pushed across a horizontal floor by a force $\vec{P}$ that makes an angle of 30^o below the horizontal. The coefficient of kinetic friction is 0.2. What should be the magnitude of $\vec{P}$, so that the net work done by it and the kinetic frictional force is zero?
	a.

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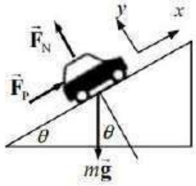
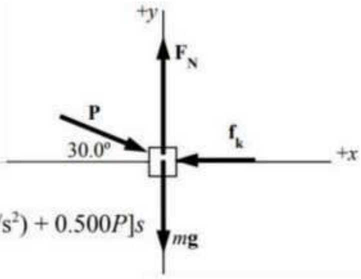
$\sum F_x = F_P - mg\sin\theta = 0 \rightarrow F_P = mg\sin\theta$
$W_P = F_P d\cos 0° = mgd\sin\theta = (950\ \text{kg})(9.80\ \text{m/s}^2)(710\ \text{m})\sin 9.0° = \boxed{1.0\times10^6\ \text{J}}$


b.
$W_P = (P\cos 30.0°)\,s = 0.866\,P\,s$
$W_f = (f_k\cos 180°)\,s = -f_k\,s$
$F_N - mg - P\sin 30.0° = 0$
$F_N = mg + P\sin 30.0°$
$f_k = \mu_k F_N = \mu_k\,(mg + P\sin 30.0°)$
$W_f = -f_k\,s = -(0.200)[(1.00\times10^2\ \text{kg})(9.80\ \text{m/s}^2) + 0.500P]s$
$W_f = -(0.100P + 196)s$
$W_P + W_f = 0.866\,Ps - (0.100P + 196)s = 0$
$\boxed{P = 256\ \text{N}}$.


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a. Two bullets are fired at the same time with the same kinetic energy. If one bullet has twice the mass of the other, which has the greater speed and by what factor? Which can do the most work?
b. The hammer throw is a track-and-field event in which a 7.3-kg ball (the “hammer”), starting from rest, is whirled around in a circle several times and released. In one throw, the hammer is given a speed of 29 m/s. Determine the work done to launch the motion of the hammer.
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a.
Bullet 1 is the heavier bullet.
$m_1 = 2m_2 \quad \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1^2 \rightarrow m_2v_2^2 = 2m_2v_1^2 \rightarrow v_2^2 = 2v_1^2 \rightarrow v_2 = v_1\sqrt{2}$
The lighter bullet has the higher speed, by a factor of the square root of 2. Both bullets can do the same amount of work.
b.
$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m\left(v_f^2 - v_0^2\right) = \tfrac{1}{2}(7.3\ \text{kg})\left[(29\ \text{m/s})^2 - (0\ \text{m/s})^2\right]$
$W = \boxed{3.1\times10^3\ \text{J}}$
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<td>4</td>
<td>
A 1.60-m-tall person lifts a 1.65-kg book off the ground so it is 2.20 m above the ground. What is the potential energy of the book relative to
a. the ground, and
b. the top of the person’s head?
c. How is the work done by the person related to the answers in parts (a) and (b)?
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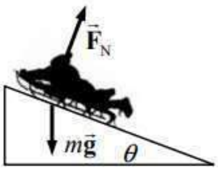
<table>
<tr><td></td><td>(a) $PE_G = mg(y_{book} - y_{ground}) = (1.65\text{ kg})(9.80\text{ m/s}^2)(2.20\text{ m}) = \boxed{35.6\text{ J}}$
(b) $PE_G = mg(y_{book} - y_{head}) = (1.65\text{ kg})(9.80\text{ m/s}^2)(2.20\text{ m} - 1.60\text{ m}) = \boxed{9.7\text{ J}}$
(c) The work done by the person in lifting the book from the ground to the final height is the same as the answer to part (a), $\boxed{35.6\text{ J}}$. In part (a), the potential energy is calculated relative to the starting location of the application of the force on the book. The work done by the person is not related to the answer to part (b), because the potential energy is not calculated relative to the starting location of the application of the force on the book.</td></tr>
<tr><td>5</td><td>A 35-kg girl is bouncing on a trampoline. During a certain interval after she leaves the surface of the trampoline, her kinetic energy decreases to 210J from 440J. How high does she rise during this interval? Neglect air resistance.</td></tr>
<tr><td></td><td>$\underbrace{KE_f + PE_f}_{E_f} = \underbrace{KE_0 + PE_0}_{E_0}$ or $PE_f - PE_0 = KE_0 - KE_f$ or $mg(h_f - h_0) = KE_0 - KE_f$
$h_f - h_0 = \dfrac{KE_0 - KE_f}{mg} = \dfrac{440\text{ J} - 210\text{ J}}{(35\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.67\text{ m}}$</td></tr>
<tr><td>6</td><td>a. A sled is initially given a shove up a frictionless 23° incline. It reaches a maximum vertical height 1.22m higher than where it started at the bottom. What was its initial speed?
b. A 0.48-kg ball is thrown with a speed of at an upward angle of 36°. Using conservation of energy, determine its speed at its highest point, and how high does it go?</td></tr>
<tr><td></td><td>a.
$y_1 = 0,\ \upsilon_2 = 0,$ and $y_2 = 1.12$ m.
$\frac{1}{2}m\upsilon_1^2 + mgy_1 = \frac{1}{2}m\upsilon_2^2 + mgy_2 \ \rightarrow\ \frac{1}{2}m\upsilon_1^2 + 0 = 0 + mgy_2$
$\upsilon_1 = \sqrt{2gy_2} = \sqrt{2(9.80\text{ m/s}^2)(1.22\text{ m})} = \boxed{4.89\text{ m/s}}$


b.
$\upsilon_{top} = \upsilon_0 \cos\theta = (8.8\text{ m/s})\cos 36° = \boxed{7.1\text{ m/s}}$
$\upsilon_1 = 8.8\text{ m/s},\ y_1 = 0,$ and $\upsilon_2 = \upsilon_1\cos\theta.$
$E_1 = E_2 \ \rightarrow\ \frac{1}{2}m\upsilon_1^2 + mgy_1 = \frac{1}{2}m\upsilon_2^2 + mgy_2$
$\frac{1}{2}m\upsilon_1^2 + 0 = \frac{1}{2}m\upsilon_1^2\cos^2\theta + mgy_2$
$y_2 = \dfrac{\upsilon_1^2(1-\cos^2\theta)}{2g} = \dfrac{(8.8\text{ m/s})^2(1-\cos^2 36°)}{2(9.80\text{ m/s}^2)} = 1.365\text{ m} \approx \boxed{1.4\text{ m}}$
This is the height above its throwing level.</td></tr>
<tr><td>7</td><td>a. A pitcher throws a 0.140-kg baseball, and it approaches the bat at a speed of 40.0 m/s. The bat does W_{nc} =70.0 J of work on the ball in hitting it. Ignoring air resistance, determine the speed of the ball after the ball leaves the bat and is 25.0 m above the point of impact.</td></tr>
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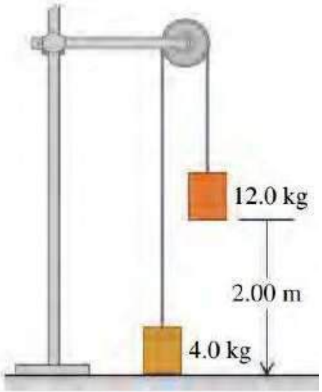
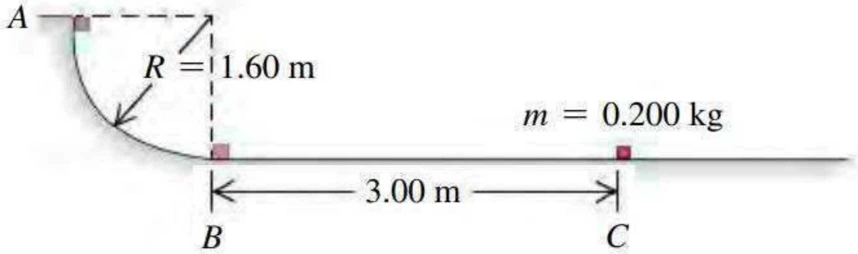
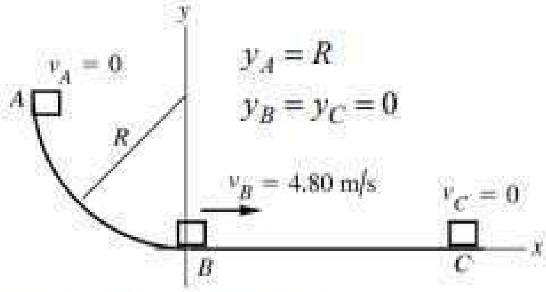
<table>
<tr><td></td><td>b. A system of two paint buckets connected by a lightweight rope is released from rest with the 12kg bucket 2m above the floor. Use the principle of conservation of energy to find the speed with which this bucket strikes the floor. Ignore friction and the mass of the pulley.

</td></tr>
<tr><td></td><td>a.
$h_0 = 0$ m at the level of the bat,
$v_0 = 40.0$ m/s just before the bat strikes the ball
v_f to be the speed of the ball at $h_f = 25.0$ m
$$W_{nc} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 + mg(h_f - h_0)$$
$$v_f = \sqrt{\frac{2W_{nc}}{m} + v_0^2 - 2gh_f}$$
$$= \sqrt{\frac{2(70.0\ \text{J})}{0.140\ \text{kg}} + (40.0\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(25.0\ \text{m})} = \boxed{45.9\ \text{m/s}}$$
b.
The tension force does positive work on the 4.0 kg bucket and an equal amount of negative work on the 12.0 kg bucket, so the net work done by the tension is zero.
$W_{other} = 0$ and $U = U_{grav}$.
$K_1 = 0$, $K_2 = \frac{1}{2}m_A v_{A,2}^2 + \frac{1}{2}m_B v_{B,2}^2$
$v_{A,2} = v_{B,2} = v_2$.
$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = (8.00\ \text{kg})v_2^2$.
$U_1 = m_A g y_{A,1} = (12.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) = 235.2$ J.
$U_2 = m_B g y_{B,2} = (4.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) = 78.4$ J.
Putting all this into $K_1 + U_1 + W_{other} = K_2 + U_2$ gives $U_1 = K_2 + U_2$.
$235.2\ \text{J} = (8.00\ \text{kg})v_2^2 + 78.4\ \text{J}$. $v_2 = \sqrt{\frac{235.2\ \text{J} - 78.4\ \text{J}}{8.00\ \text{kg}}} = 4.4$ m/s</td></tr>
<tr><td>8</td><td>In a truck-loading station at a post office, a small 0.200-kg package is released from rest at point A on a track that is one quarter of a circle with radius 1.60 m. The size of the package is much less than 1.60 m, so the package can be treated as a particle. It slides down the track and reaches point B with a speed of $4.80 ms^{-1}$. From point B, it slides on a level surface a distance of 3.00 m to point C, where it comes to rest.</td></tr>
</table>

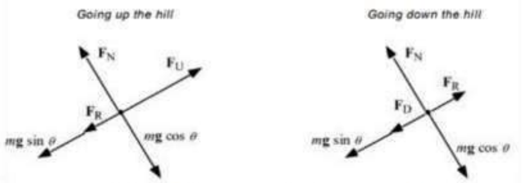
	 What is the coefficient of kinetic friction on the horizontal surface? How much work is done on the package by friction as it slides down the circular arc from A to B?
	 $K_B + U_B + W_{\text{other}} = K_C + U_C.$ $W_{\text{other}} = W_f = f_k(\cos\phi)s = \mu_k mg(\cos 180°)s = -\mu_k mgs$ $K_B = \frac{1}{2}mv_B^2, \ K_C = 0$ $U_B = 0, \ U_C = 0$ Thus $K_B + W_f = 0$ $\frac{1}{2}mv_B^2 - \mu_k mgs = 0$ $\mu_k = \frac{v_B^2}{2gs} = \frac{(4.80\text{ m/s})^2}{2(9.80\text{ m/s}^2)(3.00\text{ m})} = 0.392.$ $K_A + U_A + W_{\text{other}} = K_B + U_B$ $K_A = 0, \ K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(4.80\text{ m/s})^2 = 2.304\text{ J}$ $U_A = mgy_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(1.60\text{ m}) = 3.136\text{ J}, \ U_B = 0$ Thus $U_A + W_f = K_B$ $W_f = K_B - U_A = 2.304\text{ J} - 3.136\text{ J} = -0.83\text{ J}$

Worksheet 8: Mechanical Power

1	You are working out on a rowing machine. Each time you pull the rowing bar (which simulates the oars) toward you, it moves a distance of 1.2m in a time of 1.5s. The readout on the display

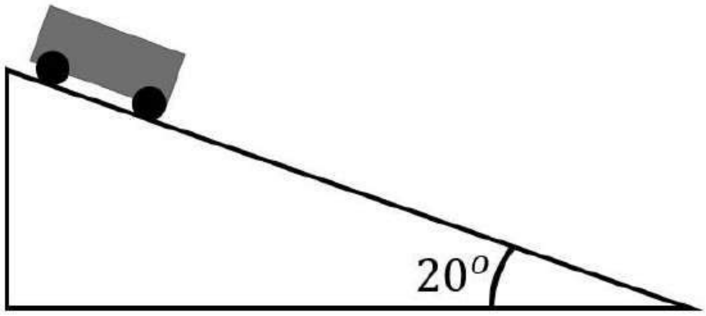
	indicates that the average power you are producing is 82W. What is the magnitude of the force that you exert on the handle?
	$$F=\frac{W}{(\cos\theta)s}$$ $$W=\bar{P}t$$ $$F=\frac{W}{(\cos\theta)s}=\frac{\bar{P}t}{(\cos\theta)s}$$ $$F=\frac{\bar{P}t}{(\cos\theta)s}=\frac{(82\text{ W})(1.5\text{ s})}{(\cos 0°)(1.2\text{ m})}=\boxed{1.0\times10^{2}\text{ N}}$$
2	If a car generates 18 hp when traveling at a steady $95kmh^{-1}$, what must be the average force exerted on the car due to friction and air resistance?
	$F=P/\upsilon.$ $$F=\frac{P}{\upsilon}=\frac{(18\text{ hp})(746\text{ W/1 hp})}{(95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)}=\boxed{510\text{ N}}$$
3	A driver notices that her 1080kg car, when in neutral, slows down from 95km/h to 65km/h in about 7s on a flat horizontal road. Approximately what power (watts and hp) is needed to keep the car traveling at a constant 80 km/h?
	The energy transfer from the engine must replace the lost kinetic energy. $$\upsilon_1=95\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=26.39\text{ m/s}\quad \upsilon_2=65\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=18.06\text{ m/s}$$ $$\Delta\text{KE}=\tfrac{1}{2}m\upsilon_2^2-\tfrac{1}{2}m\upsilon_1^2=\tfrac{1}{2}(1080\text{ kg})[(18.06\text{ m/s})^2-(26.39\text{ m/s})^2]=-1.999\times10^5\text{ J}$$ $$P=\frac{W}{t}=\frac{1.999\times10^5\text{ J}}{7.0\text{ s}}=2.856\times10^4\text{ W, or }(2.856\times10^4\text{ W})\frac{1\text{ hp}}{746\text{ W}}=38.29\text{ hp}$$ $\boxed{2.9\times10^4\text{ W}}$, or $\boxed{38\text{ hp}}$
4	A 650kg elevator starts from rest. It moves upward for 3s with constant acceleration until it reaches its cruising speed of 1.75 m/s. What is the average power of the elevator motor during this period? How does this power compare with the motor power when the elevator moves at its cruising speed?

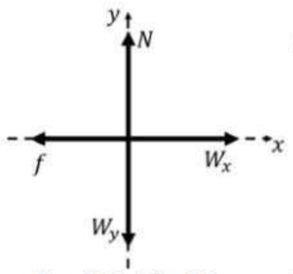
	$$\Delta y = \bar{v}t = \left[\frac{0+1.75\ \text{m/s}}{2}\right](3.00\ \text{s}) = 2.63\ \text{m}.$$ $$\frac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$ $$W_{\text{motor}} = \frac{1}{2}(650\ \text{kg})(1.75\ \text{m/s})^2 - 0 + (650\ \text{kg})g(2.63\ \text{m}) = 1.77\times10^4\ \text{J}$$ $$W = \bar{\mathcal{P}}t$$ $$\bar{\mathcal{P}} = \frac{W}{t} = \frac{1.77\times10^4\ \text{J}}{3.00\ \text{s}} = \boxed{5.91\times10^3\ \text{W}} = 7.92\ \text{hp}.$$ at constant speed $(v = 1.75\ \text{m/s})$ the applied force equals the weight $= (650\ \text{kg})(9.80\ \text{m/s}^2) = 6.37\times10^3\ \text{N}$ $$\mathcal{P} = Fv = (6.37\times10^3\ \text{N})(1.75\ \text{m/s}) = \boxed{1.11\times10^4\ \text{W}} = 14.9\ \text{hp}$$
5	An energy-efficient lightbulb, taking in 28W of power, can produce the same level of brightness as a conventional bulb operating at power 100 W. The lifetime of the energy efficient bulb is 10000h and its purchase price is \$17, whereas the conventional bulb has lifetime 750h and costs \$0.42per bulb. Determine the total savings obtained by using one energy-efficient bulb over its lifetime, as opposed to using conventional bulbs over the same time period. Assume an energy cost of \$0.080 per kilowatt-hour.
	For the 28.0 W bulb: $$\text{Energy used} = (28.0\ \text{W})(1.00\times10^4\ \text{h}) = 280\ \text{kilowatt}\cdot\text{hrs}$$ $$\text{total cost} = \$17.00 + (280\ \text{kWh})(\$0.080/\text{kWh}) = \$39.40$$ For the 100 W bulb: $$\text{Energy used} = (100\ \text{W})(1.00\times10^4\ \text{h}) = 1.00\times10^3\ \text{kilowatt}\cdot\text{hrs}$$ $$\#\ \text{bulb used} = \frac{1.00\times10^4\ \text{h}}{750\ \text{h/bulb}} = 13.3$$ $$\text{total cost} = 13.3(\$0.420) + (1.00\times10^3\ \text{kWh})(\$0.080/\text{kWh}) = \$85.60$$ Savings with energy-efficient bulb $= \$85.60 - \$39.40 = \boxed{\$46.20}$
6	A 1900-kg car experiences a combined force of air resistance and friction that has the same magnitude whether the car goes up or down a hill at $27ms^{-1}$. Going up a hill, the car's engine produces 47hp more power to sustain the constant velocity than it does going down the same hill. At what angle is the hill inclined above the horizontal?

	 $F_U - F_R - mg\sin\theta = ma = 0$ $F_U = F_R + mg\sin\theta$ $F_R - F_D - mg\sin\theta = ma = 0$ $F_D = F_R - mg\sin\theta$ $\bar{P}_U = \bar{P}_D + \Delta P$ $\Delta P = (47\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 3.51\times 10^4\text{ W}$ $\Delta P = (47\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 3.51\times 10^4\text{ W}$ $F_U\bar{v} = F_D\bar{v} + \Delta P$ $(F_R + mg\sin\theta)\bar{v} = (F_R - mg\sin\theta)\bar{v} + \Delta P$ $\theta = \sin^{-1}\left(\frac{\Delta P}{2mg\bar{v}}\right) = \sin^{-1}\left(\frac{3.51\times 10^4\text{ W}}{2(1900\text{ kg})(9.80\text{ m/s}^2)(27\text{ m/s})}\right) = \boxed{2.0^\circ}$

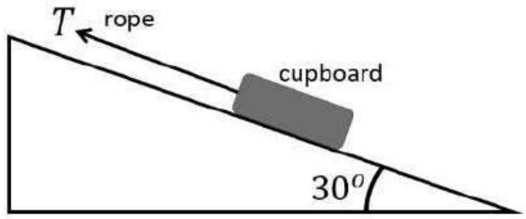
Worksheet 9: PYQ Momentum, Newton's Law, and Energy

21/22	A. A 500g soccer ball is kicked horizontally at the speed of $12ms^{-1}$ towards a wall. It rebounds off the wall at the speed of $2ms^{-1}$. Calculate the magnitude of the impulse on the ball. B. Two rugby players with mass 75kg and 100kg run directly towards each other with velocities of $6ms^{-1}$ to the right and $8ms^{-1}$ to the left respectively. If they grab each other as the collide, calculate the combined velocity of the two players just after the collision. C. A man of mass 75kg and a woman of mass 55kg stand facing each other on a smooth horizontal surface, both wearing roller blades. The woman pushes the man to the right with a horizontal force of 85N. Determine the acceleration of the woman. D. How large of a net force required to accelerate a 600N at a rate $0.7ms^{-2}$ on a smooth horizontal surface? E. A shopping trolley with a total mass of 40kg is released from rest and rolls down a 2m long surface which is inclined at 20^o as shown in the figure below.

<table>
<tr><td></td><td>

Calculate the work done to stop the trolley at the bottom of the surface if it experiences a constant frictional force of 16N.
F. A man is lifting three boxes each weighing $80N$ to a 1.2m high shelf in 2s. Calculate the power required by the man to lift the boxes.
G. Calculate the release height of a the falling of a 2kg sphere if its kinetic energy is 300J just before striking the ground. The air resistance can be ignored.</td></tr>
<tr><td></td><td>A. $\vec{J} = m(\vec{v} - \vec{u}) = 0.5((-2) - (12)) \Rightarrow J = -7Ns$
B. $m_1u_1 + m_2u_2 = (m_1 + m_2)v \Rightarrow (75)(6) + (100)(-8) = (100 + 75)v \Rightarrow v = -2ms^{-1}$
C. $F_{12} = -F_{21} \Rightarrow m_{woman}a_{woman} = -85 = (55)a_{woman} \Rightarrow a_{woman} \approx 1.54ms^{-2}$
D. $mg = m(9.81) = 600N \Rightarrow m = 61.2kg$
E.


$\Sigma F_x = W\cos 70^o - f = mg\cos 70^o - f$
$\Sigma F_x = (40)(9.81)\cos 70^o - 16 = 118.2N$
$W = (\Sigma F_x) \cdot s = (118.2)(2) = 236.4Nm$
F. Total Weight, $W_{total} = 3(80) = 240N$
Work Done, $W = \Delta K = (mg)h = (240)(1.2)$
Power required, $P = \frac{W}{t} = \frac{240(1.2)}{2}$
$P = 144W$
G. $E_k = E_{gp} \Rightarrow 300 = mgh = (2)(9.81)(h) \Rightarrow h = 15.3m$</td></tr>
<tr><td>20/21</td><td>A. A 20g bullet is fired and travels with speed of $800ms^{-1}$. It hits 5kg wooden block initially at rest and is stuck inside, causing the block to move. Determine the final velocity of the bullet and the block immediately after the block is hit and show that the collision is inelastic.
B. The figure shows a 40kg cupboard being pulled along a rough inclined plane 30^o to the horizontal by a light rope.</td></tr>
</table>

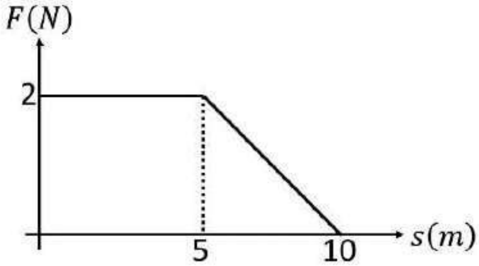


The cupboard moves at a constant velocity. The coefficient of kinetic friction between the cupboard and inclined plane is 0.5.

i. Draw a free body diagram of the cupboard.

ii. Determine the magnitude of the frictional force and tension acting on the cupboard.

C. A 2kg object moving with an initial velocity of $5ms^{-1}$ is acted on by a force of 2N. The force-displacement of the motion is shown in the graph below.



Determine the velocity of the object at 10m displacement.

D. A car with mass 1500kg is moving with a constant force, F acting on it along its direction of motion. Upon achieving a speed of $20ms^{-1}$, it delivers a maximum power of $100kW$. Later the car enters a 50m rough road and decelerates to a speed of $10ms^{-1}$. Determine the constant force F and the work done to overcome the frictional force on the rough road.

<table>
<tr>
<td></td>
<td>

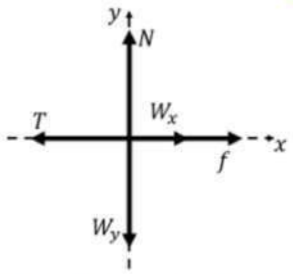
A. $m_1u_1 + m_2u_2 = (m_1 + m_2)v \Rightarrow (0.02)(800) + (5)(0) = (5 + 0.02)v \Rightarrow v = 3.19ms^{-1}$

$$\Sigma K_{initial} = \frac{1}{2}m_1u_2^2 = \frac{1}{2}(0.02)(800)^2 = 6.4kJ --- [1]$$

$$\Sigma K_{final} = \frac{1}{2}m_2v^2 = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(5 + 0.02)(3.19)^2 = 25.542J --- [2]$$

Since $\Sigma K_{initial} \neq \Sigma K_{final} \Rightarrow$ inelastic collision

B.



$$\Sigma F_x = \Sigma F_y = 0$$

$$\Sigma F_x = 0 \Rightarrow T = W_x + f = W \sin 30^o + f$$

$$\Sigma F_y = 0 \Rightarrow N = W_y = W \cos 30^o$$

$$f = \mu N = \mu W \cos 30^o = 0.5(40)(9.81) \cos 30^o$$

$$f = 169.9N$$

$$T = mg \sin 30^o + f$$

$$T = (40)(9.81) \sin 30^o + 169.9$$

$$T = 562.3N$$

C. Area under graph, $A = 5(2) + \frac{1}{2}(2)(5) = 15J$

$W = \Delta K = \frac{1}{2}m(v^2 - u^2) \Rightarrow 15 = \frac{1}{2}(2)(v^2 - (5)^2) \Rightarrow v = 2\sqrt{10}ms^{-1}$

D. $P = 100(10^3) = Fv = F(20) \Rightarrow F_{engine} = 5kN$

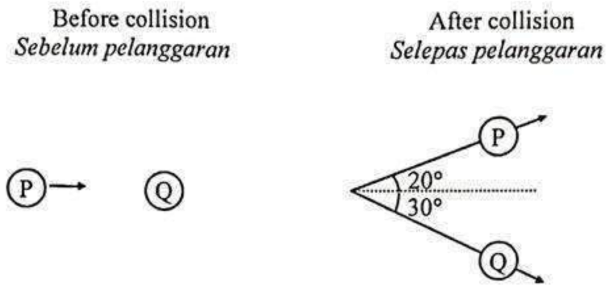
$$F_{net} = ma = m\frac{v^2 - u^2}{2s} = (1500)\frac{10^2 - 20^2}{2(50)} = -4.5kN$$

$$F_{engine} = F_{friction} + F_{net} \Rightarrow W_{engine} = W_{friction} + W_{net}$$

$$5kN(50) = W_{friction} + (4.5kN)(50) \Rightarrow W_{friction} = 25kN$$

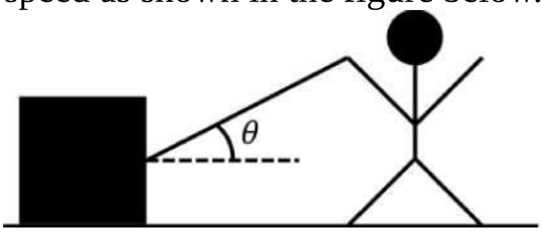
</td>
</tr>
<tr>
<td>19/20</td>
<td>

A. The figure below shows a 0.52kg ball P moving at $0.69ms^{-1}$ collides with a stationary ball Q.



After the collision, the velocity of the balls P and Q are $0.3ms^{-1}$ and $0.45ms^{-1}$ respectively. Determine the mass of ball Q.

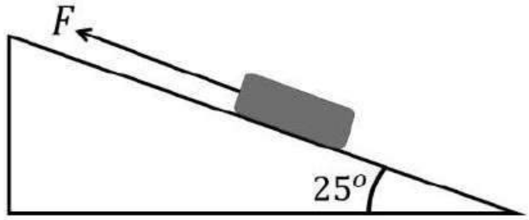
B. A man drags a 23kg suitcase with a 45N force at constant speed as shown in the figure below.



The frictional force on the suitcase is 18N. With the help of a free body diagram, calculate the coefficient of kinetic friction between the suitcase and the floor.

</td>
</tr>
</table>

C. The figure below shows a 15kg blocks being pulled by a 100N force at an initial speed of $2ms^{-1}$ up an inclined plane.



The block travels a distance of 6.2m parallel to the inclined plan. The coefficient of kinetic friction is 0.14. By using the work-energy theorem, calculate the change in the kinetic energy of the block.

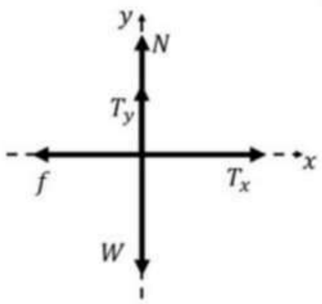
D. A 120kg motorcycle accelerates uniformly from rest to $25ms^{-1}$ in 5s. Calculate the instantaneous power of the motorcycle at time $t = 3s$.

A. 2 possible solutions:

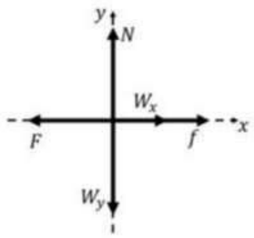
x:
$$m_P u_P = m_P v_p \cos 20^o + m_Q v_Q \cos 30^o$$
$$0.52(0.69) = (0.52)(0.3)\cos 20^o + m_Q(0.45)\cos 30^o$$
$$m_Q = 0.545kg$$

y:
$$0 = m_P\, v_P\, \sin(20^o) - m_Q\, v_Q\, \sin(30^o)$$
$$(0.52)(0.3)\sin(20^o) = m_Q\,(0.45)\sin(30^o)$$
$$m_Q = 0.237kg$$

B.



constant speed: $\Sigma F_x = \Sigma F_y = 0$
$$\Sigma F_x = 0$$
$$\Rightarrow f = 18 = T_x = T\cos\theta = 45\cos\theta$$
$$\theta = 66.42^o$$
$$\Sigma F_y = 0$$
$$\Rightarrow N = W - T_y = mg - T\sin\theta$$
$$f = 18 = \mu N = \mu(mg - T\sin\theta)$$
$$18 = \mu\big((23)(9.81) - (45)\sin(66.42)\big)$$
$$\mu = 0.098$$

	C.  $\Sigma F_y = 0$ $\Rightarrow N = W_y = W\cos 65^o = mg\cos 25^o = (15)(9.81)\cos 25^o$ $N = 133.4N$ $\Sigma F_x = F - f - W_x = F - \mu N - mg\sin 25^o$ $\Sigma F_x = 100 - (0.14)(133.4) - (15)(9.81)\sin 65^o = 19.1N$ $\Delta K = W = \Sigma F_x \cdot s = (19.1)6.2$ $\Delta K = 118.42J$ D. $v = u + at \to 25 = 0 + a(5) \to a = 5ms^{-2}$ $v(t = 3s) = u + at = 0 + 5(3) = 15ms^{-1}$ $P = Fv = ma(v) = (120)(5)(15)$ $P = 9kW$

Worksheet 10: Circular Motion

1	A child (of mass 22.5kg) sitting 1.20 m from the centre of a merry-go-round moves with a speed of $1.1ms^{-1}$. Calculate the centripetal acceleration of the child and the net horizontal force exerted on the child.
	$a_R = \frac{v^2}{r} = (1.10\text{ m/s})^2/1.20\text{ m} = 1.008\text{ m/s}^2 \approx \boxed{1.01\text{ m/s}^2}$ $F_R = ma_R = (22.5\text{ kg})(1.008\text{ m/s}^2) = 22.68\text{ N} \approx \boxed{22.7\text{ N}}$
2	What is the magnitude of the acceleration of a speck of clay on the edge of a potter's wheel turning at 45 rpm (revolutions per minute) if the wheel's diameter is 35 cm?
	$T = \left(\frac{1\text{ min}}{45\text{ rev}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 1.333\frac{\text{s}}{\text{rev}}$ $\quad r = 0.175\text{ m}$ $\quad v = \frac{2\pi r}{T} = \frac{2\pi(0.175\text{ m})}{1.333\text{ s}} = 0.8249\text{ m/s}$ $a_R = \frac{v^2}{r} = \frac{(0.8249\text{ m/s})^2}{0.175\text{ m}} = 3.888\text{ m/s}^2 \approx \boxed{3.9\text{ m/s}^2}$
3	Speedboat A negotiates a curve. whose radius is 120m. Speedboat B negotiates a curve whose radius is 240m. Each boat experiences the same centripetal acceleration. What is the ratio $\frac{v_A}{v_B}$ of the speeds of the boats?

	$$a_c = v^2 / r$$ $$a_{cA} = \frac{v_A^2}{r_A} \quad \text{and} \quad a_{cB} = \frac{v_B^2}{r_B}$$ $$\frac{v_A^2}{r_A} = \frac{v_B^2}{r_B}$$ $$\frac{v_A}{v_B} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{120\text{ m}}{240\text{ m}}} = \boxed{0.71}$$
4	A coin placed 30cm from the centre of a rotating, horizontal turntable slips when its speed is 50 cms^{-1}. What force causes the centripetal acceleration when the coin is stationary relative to the turntable? What is the coefficient of static friction between coin and turntable?
	$\boxed{\text{static friction}}$ $$ma\hat{\mathbf{i}} = f\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg(-\hat{\mathbf{j}})$$ $$\sum F_y = 0 = n - mg$$ thus $n = mg$ and $\sum F_r = m\frac{v^2}{r} = f = \mu n = \mu mg$. Then $\mu = \frac{v^2}{rg} = \frac{(50.0\text{ cm/s})^2}{(30.0\text{ cm})(980\text{ cm/s}^2)} = \boxed{0.085\,0}$.
5	The Earth rotates about its axis with a period of 24.0 h. Imagine that the rotational speed can be increased. If an object at the equator is to have zero apparent weight, what must the new period be?
	$$\sum F_r = ma_r$$ $$mg = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$ $$g = \frac{4\pi^2 R}{T^2}$$ $$T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi\sqrt{\frac{6.37\times10^6\text{ m}}{9.80\text{ m/s}^2}} = 5.07\times10^3\text{ s} = \boxed{1.41\text{ h}}$$
6	a. Highway curves are marked with a suggested speed. If this speed is based on what would be safe in wet weather, estimate the radius of curvature for an unbanked curve marked $50kmh^{-1}$. (Static friction coefficient for rubber on wet concrete is 0.7) b. A car is safely negotiating an unbanked circular turn at a speed of $21ms^{-1}$. The road is dry, and the maximum static frictional force acts on the tires. Suddenly a long wet patch in the road decreases the maximum static frictional force to one-third of its

		dry-road value. If the car is to continue safely around the curve, to what speed must the driver slow the car?
	a.	$$\frac{mv^2}{r} = \mu_s mg \rightarrow r = \frac{v^2}{\mu_s g} = \frac{\left[50\ \text{km/h}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2}{(0.7)(9.80\ \text{m/s}^2)} = 28.12\ \text{m} \approx \boxed{30\ \text{m}}$$
	b.	$F_c = mv^2 / r$ The maximum force of static friction F_s^{MAX} provides this centripetal force. $F_s^{MAX} = mv^2 / r$ $v = \sqrt{rF_s^{MAX} / m}$ $v_{dry} = \sqrt{\frac{rF_{s,\,dry}^{MAX}}{m}}$ and $v_{wet} = \sqrt{\frac{rF_{s,\,wet}^{MAX}}{m}}$ $F_{s,\,wet}^{MAX} = \frac{1}{3} F_{s,\,dry}^{MAX}$ $\frac{v_{wet}}{v_{dry}} = \frac{\sqrt{\frac{rF_{s,\,wet}^{MAX}}{m}}}{\sqrt{\frac{rF_{s,\,dry}^{MAX}}{m}}} = \sqrt{\frac{F_{s,\,wet}^{MAX}}{F_{s,\,dry}^{MAX}}} = \sqrt{\frac{1}{3}}$ $v_{wet} = \frac{v_{dry}}{\sqrt{3}} = \frac{21\ \text{m/s}}{\sqrt{3}} = \boxed{12\ \text{m/s}}$
7	a. b.	A 40kg child swings in a swing supported by two chains, each 3m long. If the tension in each chain at the lowest point is 350 N, find the child's speed at the lowest point and the force exerted by the seat on the child at the lowest point. (Neglect the mass of the seat.) A 0.4kg object is swung in a vertical circular path on a string 0.5m long. If its speed is $4ms^{-1}$ at the top of the circle, what is the tension in the string there?
	a.	

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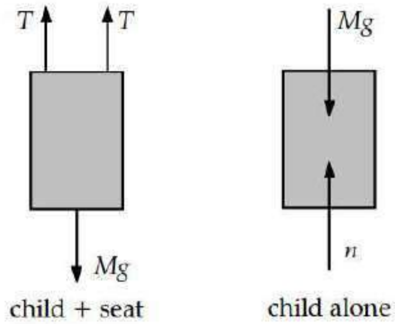
$M = 40.0\text{ kg},\ R = 3.00\text{ m},\ T = 350\text{ N}$

$\sum F = 2T - Mg = \frac{Mv^2}{R}$

$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$

$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1\ (\text{m}^2/\text{s}^2)$

$\boxed{v = 4.81\text{ m/s}}$



$n - Mg = F = \frac{Mv^2}{R}$

$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\text{ N}}$

b.

At the top of the vertical circle,

$T = m\frac{v^2}{R} - mg$

$T = (0.400)\frac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88\text{ N}}$

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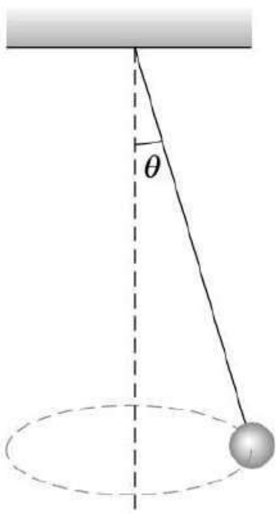
Consider a conical pendulum with an 80kg bob on a 10m wire making an angle of 5.00° with the vertical. Determine the horizontal and vertical components of the force exerted by the wire on the pendulum, and the radial acceleration of the bob.



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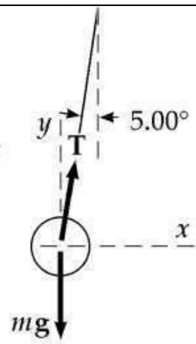
$T\cos 5.00° = mg = (80.0\text{ kg})(9.80\text{ m/s}^2)$

$T = 787\text{ N}: \mathbf{T} = \boxed{(68.6\text{ N})\hat{\mathbf{i}} + (784\text{ N})\hat{\mathbf{j}}}$

$T\sin 5.00° = ma_c: \boxed{a_c = 0.857\text{ m/s}^2}$ toward the center of the circle.



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Worksheet 11: Circular Motion

21/22	In a bike race, a racer and his bike of mass 230kg moves around a curve on a level track with a velocity of $80kmh^{-1}$. If the radius of the curve is 90m, what is the frictional force acting on the bike at the curve?
	$F_c = F_f = \frac{mv^2}{r} \Rightarrow F_f = \frac{(230)\left(\frac{80\times1000}{60\times60}\right)^2}{90}$ $F_f = 126.2N$
20/21	A 20g stone tied at the end of an inelastic string rotates in a horizontal circle. The length of the string is 1.0m and the stone rotates with a constant angular velocity of 2 revolutions per second. a. Draw a free body diagram of the stone. b. Calculate the centripetal acceleration of the stone.
	a. Refer to lecturer b. $a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 = (1)(2\times2\pi)^2$ $a = 16\pi^2 \approx 157.9\, rads^{-2}$
19/20	A 16g is swung vertically using a 0.5m string. Calculate the a. Minimum tension in the string if the speed of the ball is $1.5ms^{-1}$ b. Speed of the ball when the string breaks.
	a. Minimum tension $\rightarrow$ ball position at top $\Rightarrow F_c = \frac{mv^2}{r} = T + W$ $\frac{0.016(1.5^2)}{0.5} = T + (0.016)(9.81) \Rightarrow T = -0.085N$ b. $a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 = (1)(2\times2\pi)^2$ $a = 16\pi^2 \approx 157.9\, rads^{-2}$

Worksheet 12: Rotational Kinematics

1	A pitcher throws a curveball that reaches the catcher in 0.60 s. The ball curves because it is spinning at an average angular velocity of 330 rev/min (assumed constant) on its way to the catcher's mitt. What is the angular displacement of the baseball (in radians) as it travels from the pitcher to the catcher?
	Since 2π rad = 1 rev and 1 min = 60 s, the average angular velocity $\bar{\omega}$ (in rad/s) of the baseball is $\bar{\omega}=\left(\frac{330\ \cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)=35\ \text{rad/s}$ $\Delta\theta=\bar{\omega}\Delta t=(35\ \text{rad/s})(0.60\ \text{s})=\boxed{21\ \text{rad}}$
2	Our sun rotates in a circular orbit about the centre of the Milky Way galaxy. The radius of the orbit is 2.2×10^{20}m, and the angular speed of the sun is $1.1\times 10^{-15} rads^{-1}$. How long (in years) does it take for the sun to make one revolution around the centre?
	$\bar{\omega}=\Delta\theta/\Delta t$ $\Delta t=\frac{\Delta\theta}{\omega}=\left(\frac{2\pi\ \text{rad}}{1.1\times10^{-15}\ \text{rad/s}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\ \text{y}}{365.25\ \text{day}}\right)=\boxed{1.8\times10^{8}\ \text{y}}$
3	A dentist's drill starts from rest. After 3.20s of constant angular acceleration, it turns at a rate of 2.51×10^4 rev/min. Find the drill's angular acceleration and determine the angle (in radians) through which the drill rotates during this period.
	$\omega_f=2.51\times10^4\ \text{rev/min}=2.63\times10^3\ \text{rad/s}$ $\alpha=\frac{\omega_f-\omega_i}{t}=\frac{2.63\times10^3\ \text{rad/s}-0}{3.2\ \text{s}}=\boxed{8.22\times10^2\ \text{rad/s}^2}$ $\theta_f=\omega_i t+\frac{1}{2}\alpha t^2=0+\frac{1}{2}(8.22\times10^2\ \text{rad/s}^2)(3.2\ \text{s})^2=\boxed{4.21\times10^3\ \text{rad}}$
4	A wind turbine is initially spinning at a constant angular speed. As the wind's strength gradually increases, the turbine experiences a constant angular acceleration of 0.140 rads^{-2} . After making 2870 revolutions, its angular speed is 137 rads^{-1}. What is the initial angular velocity of the turbine? How much time elapses while the turbine is speeding up?

	$$\theta = (2870\ \cancel{\text{rev}})\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right) = 1.80\times 10^4\ \text{rad}$$ $$\omega^2 = \omega_0^2 + 2\alpha\theta$$ $$\omega_0^2 = \omega^2 - 2\alpha\theta$$ $$\omega_0 = \sqrt{\omega^2 - 2\alpha\theta} = \sqrt{(137\ \text{rad/s})^2 - 2(0.140\ \text{rad/s}^2)(1.80\times 10^4\ \text{rad})} = \boxed{117\ \text{rad/s}}$$ $$t = \frac{\omega - \omega_0}{\alpha} = \frac{137\ \text{rad/s} - 117\ \text{rad/s}}{0.140\ \text{rad/s}^2} = \boxed{140\ \text{s}}$$
5	A racing car travels on a circular track of radius 250 m. If the car moves with a constant linear speed of 45.0 m/s, find its angular speed and the magnitude and direction of its acceleration
	$$v = r\omega;\ \omega = \frac{v}{r} = \frac{45.0\ \text{m/s}}{250\ \text{m}} = \boxed{0.180\ \text{rad/s}}$$ $$a_r = \frac{v^2}{r} = \frac{(45.0\ \text{m/s})^2}{250\ \text{m}} = \boxed{8.10\ \text{m/s}^2\ \text{toward the center of track}}$$
6	The drive propeller of a ship starts from rest and accelerates at $2.90 \times 10^{-3} rads^{-2}$ for $2.10 \times 10^3 s$. For the next $1.40\times 10^3 s$ the propeller rotates at a constant angular speed. Then it decelerates at $2.30 \times 10^{-3} rads^{-2}$ until it slows (without reversing direction) to an angular speed of $4 rads^{-1}$. Find the total angular displacement of the propeller.
	$$\theta_1 = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (0\ \text{rad/s})(2.10\times 10^3\ \text{s}) + \tfrac{1}{2}(2.90\times 10^{-3}\ \text{rad/s}^2)(2.10\times 10^3\ \text{s})^2$$ $$= 6.39\times 10^3\ \text{rad}$$ $$\omega = \omega_0 + \alpha t = 0\ \text{rad/s} + (2.90\times 10^{-3}\ \text{rad/s}^2)(2.10\times 10^3\ \text{s}) = 6.09\ \text{rad/s}$$ $$\theta_2 = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (6.09\ \text{rad/s})(1.40\times 10^3\ \text{s}) + \tfrac{1}{2}(0\ \text{rad/s}^2)(1.40\times 10^3\ \text{s})^2 = 8.53\times 10^3\ \text{rad}$$ $$\theta_3 = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(4.00\ \text{rad/s})^2 - (6.09\ \text{rad/s})^2}{2(-2.30\times 10^{-3}\ \text{rad/s}^2)} = 4.58\times 10^3\ \text{rad}$$ $$\theta_{\text{Total}} = \theta_1 + \theta_2 + \theta_3 = 6.39\times 10^3\ \text{rad} + 8.53\times 10^3\ \text{rad} + 4.58\times 10^3\ \text{rad} = \boxed{1.95\times 10^4\ \text{rad}}$$
7	A bicycle with tires 68 cm in diameter travels 9.2 km. How many revolutions do the wheels make?
	In each revolution, the wheel moves forward a distance equal to its circumference, πd. $$\Delta x = N_{\text{rev}}(\pi d) \rightarrow N = \frac{\Delta x}{\pi d} = \frac{9200\ \text{m}}{\pi(0.68\ \text{m})} = \boxed{4300\ \text{rev}}$$
8	The figure shows the drive train of a bicycle that has wheels 67.3 cm in diameter and pedal cranks 17.5 cm long.

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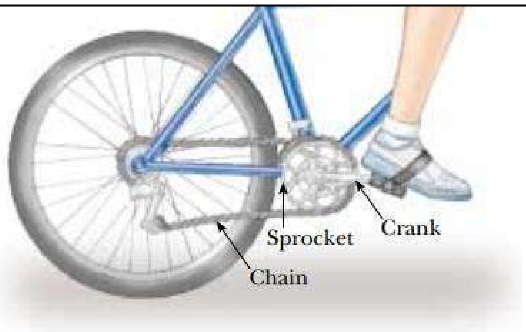
<td>

The cyclist pedals at a steady angular rate of 76.0 rev/min. The chain engages with a front sprocket 15.2cm in diameter and a rear sprocket 7.0cm in diameter.
a. Calculate the speed of a link of the chain relative to the bicycle frame.
b. Calculate the angular speed of the bicycle wheels.
c. Calculate the speed of the bicycle relative to the road.
d. What pieces of data, if any, are not necessary for the calculations?</td>
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<td>(a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:
$$v = r\omega = \left(\frac{0.152\text{ m}}{2}\right)76\text{ rev/min}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{0.605\text{ m/s}}$$
(b) Consider the chain link engaging a tooth on the rear sprocket:
$$\omega = \frac{v}{r} = \frac{0.605\text{ m/s}}{\left(\frac{0.07\text{ m}}{2}\right)} = \boxed{17.3\text{ rad/s}}$$
(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:
$$v = r\omega = \left(\frac{0.673\text{ m}}{2}\right)17.3\text{ rad/s} = \boxed{5.82\text{ m/s}}$$
(d) We did not need to know the length of the pedal cranks, but we could use that information to find the linear speed of the pedals:
$$v = r\omega = 0.175\text{ m } 7.96\text{ rad/s}\left(\frac{1}{1\text{ rad}}\right) = 1.39\text{ m/s}$$</td>
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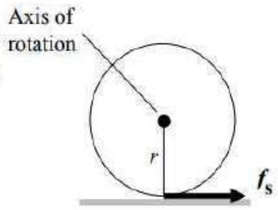
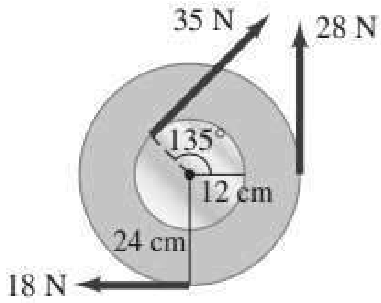
Worksheet 13: Rotational Dynamics & $\Delta(\Sigma L) = 0$

1	The wheel of a car has a radius of 0.350 m. The engine of the car applies a torque of 295 N m to this wheel, which does not slip against the road surface. Since the wheel does not slip, the road must be applying a force of static friction to the wheel that produces a countertorque. Moreover, the car has a constant velocity, so this countertorque balances the applied torque. What is the magnitude of the static frictional force?

	The torque applied by the engine is assumed to be clockwise about the axis of rotation. The force of static friction that the ground applies to the wheel is labeled as f_s.  $\tau = f_s \ell$ $f_s = \frac{\tau}{\ell} = \frac{295\ \text{N}\cdot\text{m}}{0.350\ \text{m}} = \boxed{843\ \text{N}}$
2	a. A 52-kg person riding a bike puts all her weight on each pedal when climbing a hill. The pedals rotate in a circle of radius 17cm. What is the maximum torque she exerts? How could she exert more torque? b. Calculate the net torque about the axle of the wheel shown in the figure shown. Assume that a friction torque of 0.60 mN opposes the motion. 
	a. $\tau = r_\perp F = r_\perp mg = (0.17\ \text{m})(52\ \text{kg})(9.80\ \text{m/s}^2) = 86.6\ \text{m}\cdot\text{N} \approx \boxed{87\ \text{m}\cdot\text{N}}$ She could exert more torque by pushing down harder with her legs, raising her center of mass. She could also pull upward on the handle bars as she pedals, which will increase the downward force of her legs. b. $\tau_{\substack{\text{applied}\\\text{forces}}} = (28\ \text{N})(0.24\ \text{m}) - (18\ \text{N})(0.24\ \text{m}) - (35\ \text{N})(0.12\ \text{m}) = -1.8\ \text{m}\cdot\text{N}$ $\tau_{\text{net}} = (28\ \text{N})(0.24\ \text{m}) - (18\ \text{N})(0.24\ \text{m}) - (35\ \text{N})(0.12\ \text{m}) + 0.60\ \text{m}\cdot\text{N} = -1.2\ \text{m}\cdot\text{N}$ $= \boxed{1.2\ \text{m}\cdot\text{N, clockwise}}$
3	Determine the moment of inertia of a 10.8kg sphere of radius 0.648 m when the axis of rotation is through its centre. Estimate the moment of inertia of a bicycle wheel 67cm in diameter. The rim and tire have a combined mass of 1.1 kg. The mass of the hub (at the centre) can be ignored (why?).
	a. $I = \frac{2}{5}MR^2 = \frac{2}{5}(10.8\ \text{kg})(0.648\ \text{m})^2 = \boxed{1.81\ \text{kg}\cdot\text{m}^2}$ b. $I = MR^2 = (1.1\ \text{kg})\left(\frac{1}{2}(0.67\ \text{m})\right)^2 = \boxed{0.12\ \text{kg}\cdot\text{m}^2}$
4	The combination of an applied force and a friction force produces a constant total torque of 36.0Nm on a wheel rotating about a fixed axis. The applied force acts for 6s. During this time the angular speed of the

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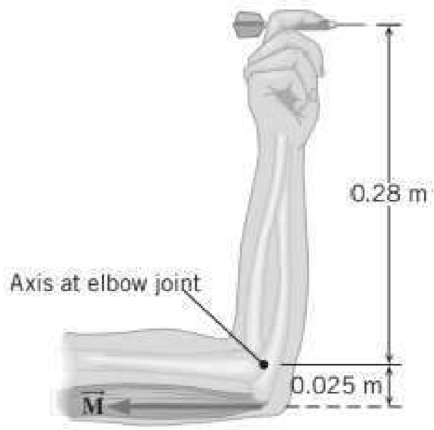
Find
a. the moment of inertia of the wheel,
b. the magnitude of the frictional torque,
a. the total number of revolutions of the wheel.</td></tr>
<tr><td></td><td>a.
$$\omega_f = \omega_i + \alpha t$$ $$10.0 \text{ rad/s} = 0 + \alpha(6.00 \text{ s})$$ $$\alpha = \frac{10.00}{6.00} \text{ rad/s}^2 = 1.67 \text{ rad/s}^2$$ $$\sum \tau = 36.0 \text{ N}\cdot\text{m} = I\alpha: \quad I = \frac{\sum \tau}{\alpha} = \frac{36.0 \text{ N}\cdot\text{m}}{1.67 \text{ rad/s}^2} = \boxed{21.6 \text{ kg}\cdot\text{m}^2}$$
b.
$$\omega_f = \omega_i + \alpha t$$ $$0 = 10.0 + \alpha(60.0)$$ $$\alpha = -0.167 \text{ rad/s}^2$$ $$|\tau| = |I\alpha| = (21.6 \text{ kg}\cdot\text{m}^2)(0.167 \text{ rad/s}^2) = \boxed{3.60 \text{ N}\cdot\text{m}}$$
c.
Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$
During first 6.00 s $\quad \theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1 \text{ rad}$
During next 60.0 s $\quad \theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299 \text{ rad}$
$$\theta_{\text{total}} = 329 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{52.4 \text{ rev}}$$</td></tr>
<tr><td>5</td><td>The figure shows a model for the motion of the human forearm in throwing a dart. Because of the force applied by the triceps muscle, the forearm can rotate about an axis at the elbow joint. Assume that the forearm has the dimensions shown in the drawing and a moment of inertia of 0.065 kgm^2 (including the effect of the dart) relative to the axis at the elbow. Assume also that the force acts perpendicular to the forearm.

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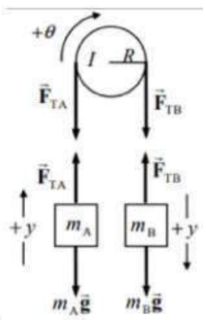
	Ignoring the effect of gravity and any frictional forces, determine the magnitude of the force needed to give the dart a tangential speed of $5ms^{-1}$ in 0.10 s, starting from rest.
	$\omega = v_T/r$, where $r = 0.28$ m. $\alpha = (\omega - \omega_0)/t$ $\Sigma\tau = I\alpha$ $\Sigma\tau = ML$ $M = \frac{\Sigma\tau}{L} = \frac{I\left(\frac{\omega-\omega_0}{t}\right)}{L}$ Setting $\omega_0 = 0$ rad/s and $\omega = v_T/r$ $M = \frac{I\left(\frac{v_T}{rt}\right)}{L} = \frac{Iv_T}{Lrt} = \frac{(0.065\text{ kg}\cdot\text{m}^2)(5.0\text{ m/s})}{(0.025\text{ m})(0.28\text{ m})(0.10\text{ s})} = \boxed{460\text{ N}}$
6	An Atwood machine consists of two masses, and connected by a massless inelastic cord that passes over a pulley free to rotate. The pulley is a solid cylinder of radius and mass 6kg. a. Determine the acceleration of each mass. b. What % error would be made if the moment of inertia of the pulley is ignored?
	a. $m_B > m_A$ $\alpha_{\text{pulley}} = a/R$ $\sum F_{yA} = F_{TA} - m_A g = m_A a \rightarrow F_{TA} = m_A g + m_A a$ $\sum F_{yB} = m_B g - F_{TB} = m_B a \rightarrow F_{TB} = m_B g - m_B a$ $\sum \tau = F_{TB}r - F_{TA}r = I\alpha = I\frac{a}{R}$ $F_{TB}R - F_{TA}R = I\frac{a}{R} \rightarrow (m_B g - m_B a)R - (m_A g + m_A a)R = I\frac{a}{R}$ $a = \frac{(m_B - m_A)}{(m_A + m_B + I/R^2)}g = \frac{(m_B - m_A)}{\left(m_A + m_B + \frac{1}{2}m_P R^2/R^2\right)}g$ $= \frac{(75\text{ kg} - 65\text{ kg})}{\left[75\text{ kg} + 65\text{ kg} + \frac{1}{2}(6.0\text{ kg})\right]}(9.80\text{ m/s}^2) = 0.6853\text{ m/s}^2 \approx \boxed{0.69\text{ m/s}^2}$  b. If the moment of inertia is ignored, then from the torque equation we see that $F_{TB} = F_{TA}$. $a_{I=0} = \frac{(m_B - m_A)}{(m_A + m_B)}g = \frac{(75\text{ kg} - 65\text{ kg})}{75\text{ kg} + 65\text{ kg}}(9.80\text{ m/s}^2) = 0.7000\text{ m/s}^2$ $\%\text{ error} = \left(\frac{0.7000\text{ m/s}^2 - 0.6853\text{ m/s}^2}{0.6853\text{ m/s}^2}\right)\times 100 = 2.145\% \approx \boxed{2\%}$
7	Two disks are rotating about the same axis. Disk A has a moment of inertia of 3.4 kgm^2 and an angular velocity of $7.2rads^{-1}$. Disk B is rotating with an angular velocity of ———————————9.8

	$rads^{-1}$. The two disks are then linked together without the aid of any external torques, so that they rotate as a single unit with an angular velocity of 2.4 $rads^{-1}$. The axis of rotation for this unit is the same as that for the separate disks. What is the moment of inertia of disk B?
	$L_{\text{initial}} = L_{\text{final}}$ $I_A\omega_A + I_B\omega_B = (I_A + I_B)\omega_{\text{final}}$ $I_B = I_A\left(\dfrac{\omega_{\text{final}} - \omega_A}{\omega_B - \omega_{\text{final}}}\right) = (3.4\ \text{kg}\cdot\text{m}^2)\left[\dfrac{-2.4\ \text{rad/s} - 7.2\ \text{rad/s}}{-9.8\ \text{rad/s} - (-2.4\ \text{rad/s})}\right] = \boxed{4.4\ \text{kg}\cdot\text{m}^2}$
8	An asteroid of mass traveling at a speed of relative to the Earth, hits the Earth at the equator tangentially, in the direction of Earth's rotation, and is embedded there. Use angular momentum to estimate the percent change in the angular speed of the Earth as a result of the collision
	The initial angular velocity of the satellite, just before collision, $\omega_{\text{asteroid}} = \upsilon_{\text{asteroid}}/R_{\text{Earth}}$. $L_i = L_f \rightarrow I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = (I_{\text{Earth}} + I_{\text{asteroid}})\omega_f$ $I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = I_{\text{Earth}}\omega_f \rightarrow \dfrac{(\omega_f - \omega_{\text{Earth}})}{\omega_{\text{Earth}}} = \dfrac{I_{\text{asteroid}}}{I_{\text{Earth}}}\dfrac{\omega_{\text{asteroid}}}{\omega_{\text{Earth}}}$ $\%\ \text{change} = \dfrac{(\omega_f - \omega_{\text{Earth}})}{\omega_{\text{Earth}}}(100) = \dfrac{m_{\text{asteroid}}R_{\text{Earth}}^2}{\frac{2}{5}M_{\text{Earth}}R_{\text{Earth}}^2}\dfrac{\frac{\upsilon_{\text{asteroid}}}{R_{\text{Earth}}}}{\omega_{\text{Earth}}} = \dfrac{m_{\text{asteroid}}}{\frac{2}{5}M_{\text{Earth}}}\dfrac{\upsilon_{\text{asteroid}}}{\omega_{\text{Earth}}R_{\text{Earth}}}(100)$ $= \dfrac{(1.0\times10^5\ \text{kg})(3.5\times10^4\ \text{m/s})}{(0.4)(5.97\times10^{24}\ \text{kg})\left(\dfrac{2\pi\ \text{rad}}{86,400\ \text{s}}\right)(6.38\times10^6\ \text{m})}(100) = \boxed{(3.2\times10^{-16})\%}$

Worksheet 14: SHM: Graphs & Cases

1	A particle moving along the x axis in simple harmonic motion starts from its equilibrium position, the origin, at t=0 and moves to the right. The amplitude of its motion is 2.00 cm, and the frequency is 1.50 Hz. a. Show that the position of the particle is given by $x = (2cm)\ sin(3\pi t)$ Determine b. the maximum speed and the earliest time (t = 0) at which the particle has this speed, c. the maximum acceleration and the earliest time (t = 0) at which the particle has this acceleration, and d. the total distance travelled between t = 0 and t = 1.00 s.

(a) v is positive (to the right).

$x = A\sin\omega t$

$v = v_i \cos\omega t$

$f = 1.50\text{ Hz}, \omega = 2\pi f = 3.00\pi$

$A = 2.00\text{ cm}, \boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{max} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi\text{ cm/s} = \boxed{18.8\text{ cm/s}}$

The particle has this speed at $t = 0$ and next at $t = \frac{T}{2} = \boxed{\frac{1}{3}\text{ s}}$

(c) $a_{max} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2\text{ cm/s}^2 = \boxed{178\text{ cm/s}^2}$

This positive value of acceleration first occurs at $t = \frac{3}{4}T = \boxed{0.500\text{ s}}$

(d) Since $T = \frac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in 1.00 s$\left(= \frac{3}{2}T\right)$, the particle will travel $8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$.

2

a. The fan blades on a jet engine make one thousand revolutions in a time of 50.0ms. Determine the period (in seconds) and the frequency (in Hz) of the rotational motion. What is the angular frequency of the blades?

b. Consider two objects, A and B, both undergoing SHM, but with different frequencies, as described by the equations

$$x_A = (2)\,sin\,(4t) \text{ and } x_B = (5)\,sin\,(3t)$$

where x is in metres and t is in seconds. After $t = 0$, find the next three times at which both objects simultaneously pass through the origin.

a.

$$T = \frac{\text{Total time}}{\text{Number of revolutions}} = \frac{50.0\times10^{-3}\text{ s}}{1000} = \boxed{50.0\times10^{-6}\text{ s}}$$

$$f = \frac{1}{T} = \frac{1}{50.0\times10^{-6}\text{ s}} = \boxed{2.00\times10^{4}\text{ Hz}}$$

$$\omega = 2\pi f = 2\pi\left(2.00\times10^{4}\text{ Hz}\right) = \boxed{1.26\times10^{5}\text{ rad/s}}$$

b.

A: $4.0t_A = n_A\pi \rightarrow t_A = \frac{1}{4}n_A\pi, n_A = 1,2,3,\ldots$ so $t_A = \frac{1}{4}\pi, \frac{1}{2}\pi, \frac{3}{4}\pi, \pi, \frac{5}{4}\pi, \frac{3}{2}\pi, \frac{7}{4}\pi, 2\pi, \frac{9}{4}\pi, \ldots$

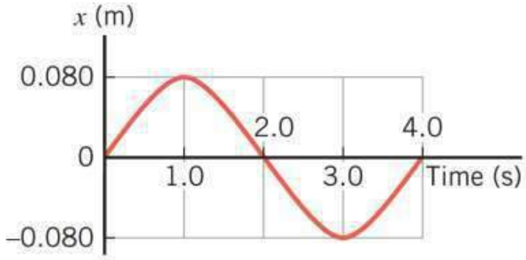
B: $3.0t_B = n_B\pi \rightarrow t_B = \frac{1}{3}n_B\pi, n_B = 1,2,3,\ldots$ so $t_B = \frac{1}{3}\pi, \frac{2}{3}\pi, \pi, \frac{4}{3}\pi, \frac{5}{3}\pi, 2\pi, \frac{7}{3}\pi, \frac{8}{3}\pi, 3\pi, \ldots$

The first three times are $\boxed{\pi\text{ s}, 2\pi\text{ s, and }3\pi\text{ s}}$ or $\boxed{3.1\text{ s}, 6.3\text{ s, and }9.4\text{ s}}$.

3

a. Estimate the stiffness of the spring in a child's pogo stick if the child has a mass of 32kg and bounces once every 2.0 seconds.

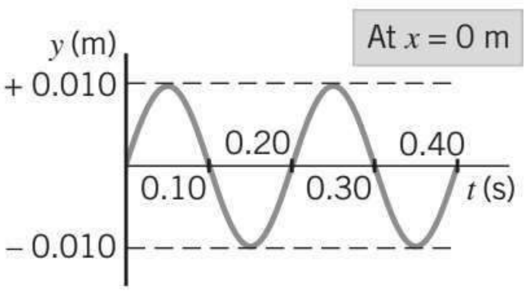
	b. A spring stretches by 0.018 m when a 2.8kg object is suspended from its end. How much mass should be attached to this spring so that its frequency of vibration is f=3Hz?
	a. $T=2\pi\sqrt{\frac{m}{k}} \rightarrow T^2=4\pi^2\frac{m}{k} \rightarrow k=4\pi^2\frac{m}{T^2}=4\pi^2\frac{32\text{ kg}}{(2.0\text{ s})^2}=315.8\text{ N/m}\approx\boxed{320\text{ N/m}}$ b. $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $k=\frac{F_x^{\text{Applied}}}{x}=\frac{mg}{x}=\frac{(2.8\text{ kg})(9.80\text{ m/s}^2)}{0.018\text{ m}}=1.52\times10^3\text{ N/m}$ $m=\frac{k}{4\pi^2f^2}=\frac{1.52\times10^3\text{ N/m}}{4\pi^2(3.0\text{ Hz})^2}=\boxed{4.3\text{ kg}}$
4	a. Astronauts on a distant planet set up a simple pendulum of length 1.2m. The pendulum executes simple harmonic motion and makes 100 complete vibrations in 280s. What is the magnitude of the acceleration due to gravity on this planet? b. Your grandfather clock's pendulum has a length of 0.9930m. If the clock runs slow and loses 21s per day, how should you adjust the length of the pendulum?
	a. $2\pi f=\sqrt{g/L}$ $4\pi^2f^2=\frac{g}{L}$ $f=\frac{N}{t}$ $g=4\pi^2\left(\frac{N}{t}\right)^2L=\frac{4\pi^2N^2L}{t^2}=\frac{4\pi^2(100)^2(1.2\text{ m})}{(280\text{ s})^2}=\boxed{6.0\text{ m/s}^2}$ b. **1 day = 86400s** **1 cycle = 2s** **1 day = 43200 cycles** **21s (slower) = 10.5cycles (less)** **Period of inaccurate pendulum,** $T_{old}=\left(\frac{43200-10.5}{43200}\right)2=\frac{86379}{43200}s$ **Since period depends on length of pendulum, we can change the pendulum length to get the correct period.** $T_{new}=\left(\frac{43200-10.5}{43200}\right)T_{old}$ $l_{new}=\left(\frac{43200-10.5}{43200}\right)^2l_{old}=\left(\frac{43200-10.5}{43200}\right)^2(0.9930)=0.9925m$ **We should shorten the pendulum by 0.0005m.**

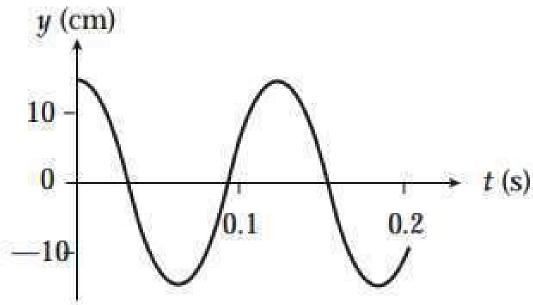
5	A 0.8kg object is attached to one end of a spring and the system is set into simple harmonic motion. The displacement x of the object as a function of time is shown in the graph shown. With the aid of these data, determine  a. the amplitude A of the motion, b. the angular frequency, c. the spring constant k, d. the speed of the object at $t = 1s$, and the magnitude of the object's acceleration at $t = 1.0\ s$.
	a. the motion's amplitude of the motion is $\boxed{0.080\,\text{m}}$. b. $\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0\ \text{s}} = \boxed{1.6\ \text{rad/s}}$ c. $\omega = \sqrt{k/m}$ $k = \omega^2 m = (1.6\ \text{rad/s})^2(0.80\ \text{kg}) = \boxed{2.0\ \text{N/m}}$ d. At $t = 1.0$ s, the graph shows that the spring has its maximum displacement. so that its speed is $\boxed{v = 0\ \text{m/s}}$. e. The acceleration of the object at $t = 1.0$ s is a maximum, and its magnitude is $a_{\text{max}} = A\omega^2 = (0.080\ \text{m})(1.6\ \text{rad/s})^2 = \boxed{0.20\ \text{m/s}^2}$
6	A 3.2kg block is hanging stationary from the end of a vertical spring that is attached to the ceiling. The elastic potential energy of this spring-block system is 1.8J. What is the elastic potential energy of the system when the 3.2kg block is replaced by a 5kg block?
	$\text{PE}_{\text{elastic}} = \frac{1}{2}ky^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$ $\text{PE}_1 = \frac{m_1^2g^2}{2k}$ and $\text{PE}_2 = \frac{m_2^2g^2}{2k}$ $\frac{\text{PE}_2}{\text{PE}_1} = \frac{\frac{m_2^2g^2}{2k}}{\frac{m_1^2g^2}{2k}} = \frac{m_2^2}{m_1^2}$ or $\text{PE}_2 = \text{PE}_1\left(\frac{m_2^2}{m_1^2}\right) = (1.8\ \text{J})\frac{(5.0\ \text{kg})^2}{(3.2\ \text{kg})^2} = \boxed{4.4\ \text{J}}$
7	A 10^{-2}kg block is resting on a horizontal frictionless surface and is attached to a horizontal spring whose spring constant is 124 N/m. The block is shoved parallel to the spring axis and is given an initial speed of 8m/s, while the spring is initially unstrained. What is the amplitude of the resulting simple harmonic motion?

	$\frac{1}{2}mv_0^2 = \frac{1}{2}kA^2$ $A = \sqrt{\frac{mv_0^2}{k}} = \sqrt{\frac{(1.00\times10^{-2}\text{ kg})(8.00\text{ m/s})^2}{124\text{ N/m}}} = \boxed{7.18\times10^{-2}\text{ m}}$
8	A 0.25kg mass at the end of a spring oscillates 2.2 times per second with an amplitude of 0.15m. Determine a. the speed when it passes the equilibrium point, b. the speed when it is 0.10 m from equilibrium, c. the total energy of the system, and d. the equation describing the motion of the mass, assuming that at $t = 0$, x was a maximum.
	(*a*) $v_{max} = \sqrt{\frac{k}{m}}A = \omega A = 2\pi f A = 2\pi(2.2\text{ Hz})(0.15\text{ m}) = 2.073\text{ m/s} \approx \boxed{2.1\text{ m/s}}$ (*b*) $v = v_{max}\sqrt{1-\frac{x^2}{A^2}} = (2.073\text{ m/s})\sqrt{1-\frac{(0.10\text{ m})^2}{(0.15\text{ m})^2}} = 1.545\text{ m/s} \approx \boxed{1.5\text{ m/s}}$ (*c*) $E_{total} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.25\text{ kg})(2.073\text{ m/s})^2 = 0.5372\text{ J} \approx \boxed{0.54\text{ J}}$ (*d*) Since the object has a maximum displacement at $t = 0$, the position will be described by the cosine function. $x = (0.15\text{ m})\cos(2\pi(2.2\text{ Hz})t) \rightarrow \boxed{x = (0.15\text{ m})\cos(4.4\pi t)}$

Worksheet 15: Waves – Basics

1	a. A fisherman notices that wave crests pass the bow of his anchored boat every 3.0 s. He measures the distance between two crests to be 7.0 m. How fast are the waves traveling? b. One tsunami had a wavelength of 750 km and travelled a distance of 3700km in 5.3h. What was the speed of the wave? Find the wave's frequency and period.
	a. $v = \lambda f = \lambda/T = (7.0\text{ m})/(3.0\text{ s}) = \boxed{2.3\text{ m/s}}$ b.

		$v = \frac{x}{t} = \frac{3700 \times 10^3 \text{ m}}{5.3 \cancel{\text{h}}}\left(\frac{1 \cancel{\text{h}}}{3600 \text{ s}}\right) = \boxed{190 \text{ m/s}}$ $f = \frac{v}{\lambda} = \frac{190 \cancel{\text{m}}/\text{s}}{750 \times 10^3 \cancel{\text{m}}} = \boxed{2.5 \times 10^{-4} \text{ Hz}}$ $T = \frac{1}{f} = \frac{1}{2.5 \times 10^{-4} \text{ Hz}} = \boxed{4.0 \times 10^3 \text{ s}}$
2	a. b.	A wave travels with speed $200 ms^{-1}$. Its wave number is 1.5 rad/m. What are its wavelength and frequency? A certain transverse wave is described by $y(x, t) = (6.50 \text{ mm}) \cos 2\pi\left(\frac{x}{28.0 \text{ cm}} - \frac{t}{0.0360 \text{ s}}\right)$ Determine the wave's amplitude, wavelength, frequency, speed of propagation; and direction of propagation.
	a. b.	$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5 \text{ rad/m}} = 4.2 \text{ m}$ $f = \frac{v}{\lambda} = \frac{200 \text{ m/s}}{4.19 \text{ m}} = 48 \text{ Hz}$ $A = 6.50 \text{ mm}$ $\lambda = 28.0 \text{ cm}$ $f = \frac{1}{0.0360 \text{ s}} = 27.8 \text{ Hz}$ $v = (0.280 \text{ m})(27.8 \text{ Hz}) = 7.78 \text{ m/s}$ Since there is a minus sign in front of the t/T term, the wave is traveling in the $+x$-direction.
3	a. b.	The figure shows a graph that represents a transverse wave on a string. The wave is moving in the +x direction with a speed of $0.15 ms^{-1}$. Using the information contained in the graph, write the mathematical expression for the wave.  Plot y versus t at x=0 for a sinusoidal wave of the form $y = (15.0cm)cos(0.157x - 50.3t)$ where x and y are in centimetres and t is in seconds.

	a. $f=\frac{1}{T}=\frac{1}{0.20\text{ s}}=5.0\text{ Hz}$ $\lambda=\frac{v}{f}=\frac{0.15\text{ m/s}}{5.0\text{ Hz}}=0.030\text{ m}$ $y=A\sin\left(2\pi f t-\frac{2\pi x}{\lambda}\right)=(0.010\text{ m})\sin\left[2\pi(5.0\text{ Hz})t-\frac{2\pi x}{(0.030\text{ m})}\right]$ $\boxed{y=(0.010\text{ m})\sin(31t-210x)}$ b. 
4	The mass of a string is 5×10^{-3}kg, and it is stretched so that the tension in it is 180N. A transverse wave traveling on this string has a frequency of 260Hz and a wavelength of 0.6m. What is the length of the string?
	$v=f\lambda=\sqrt{\frac{F}{m/L}}$ $L=\frac{f^2\lambda^2 m}{F}=\frac{(260\text{ Hz})^2(0.60\text{ m})^2\left(5.0\times10^{-3}\text{ kg}\right)}{180\text{ N}}=\boxed{0.68\text{ m}}$
5	One end of a horizontal rope is attached to a prong of an electrically driven tuning fork that vibrates the rope transversely at 120 Hz. The other end passes over a pulley and supports a 1.5kg mass. The linear mass density of the rope is $0.048 kgm^{-1}$. What is the speed of a transverse wave on the rope? What is the wavelength? How would your answers change if the mass were increased to 3kg?

	$F = mg = (1.50\text{ kg})(9.80\text{ m/s}^2) = 14.7\text{ N}$ $v = \sqrt{F/\mu} = \sqrt{14.7\text{ N}/(0.0480\text{ kg/m})} = 17.5\text{ m/s}.$ $v = f\lambda$ so $\lambda = v/f = (17.5\text{ m/s})/120\text{ Hz} = 0.146\text{ m}.$ Doubling m increases v by a factor of $\sqrt{2}$. f remains 120 Hz v increases by a factor of $\sqrt{2}$ λ increases by a factor of $\sqrt{2}$.
6	The tension in a string is 15N, and its linear density is $0.85kgm^{-1}$. A wave on the string travels toward the -x direction; it has an amplitude of 3.6cm and a frequency of 12Hz. What are the speed and wavelength of the wave? Write down a mathematical expression for the wave, substituting numbers for the variables A, f and λ.
	$v = \sqrt{\frac{F}{(m/L)}} = \sqrt{\frac{15\text{ N}}{0.85\text{ kg/m}}} = \boxed{4.2\text{ m/s}}$ $\lambda = \frac{v}{f} = \frac{4.2\text{ m/s}}{12\text{ Hz}} = \boxed{0.35\text{ m}}$ $A = 3.6\text{ cm} = 3.6 \times 10^{-2}\text{ m}.$ Since the wave is moving along the $-x$ direction, $y = A\sin\left(2\pi f t + \frac{2\pi x}{\lambda}\right)$ $y = A\sin\left(2\pi f t + \frac{2\pi x}{\lambda}\right) = (3.6\times10^{-2}\text{ m})\sin\left[2\pi(12\text{ Hz})t + \frac{2\pi x}{0.35\text{ m}}\right]$ $= \boxed{(3.6\times10^{-2}\text{ m})\sin\left[(75\text{ rad/s})t + (18\text{ m}^{-1})x\right]}$
7	a. A cord of mass 0.65kg is stretched between two supports 8.0 m apart. If the tension in the cord is 120N, how long will it take a pulse to travel from one support to the other? b. A transverse wave on a string is described by the wave function $y = (0.120m)sin[(\pi x/8) + 4\pi t]$ Determine the transverse speed and acceleration at t=0.2s for the point on the string located at x=1.60 m. What are the wavelength, period, and speed of propagation of this wave?
	a. $v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T}{m/L}} \rightarrow \Delta t = \frac{\Delta x}{\sqrt{\frac{F_T}{m/L}}} = \frac{8.0\text{ m}}{\sqrt{\frac{120\text{ N}}{(0.65\text{ kg})/(8.0\text{ m})}}} = \boxed{0.21\text{ s}}$

	b. $y=(0.120\text{ m})\sin\left(\frac{\pi}{8}x+4\pi t\right)$ $v=\frac{dy}{dt}:\quad x=(0.120)(4\pi)\cos\left(\frac{\pi}{8}x+4\pi t\right)$ $v(0.200\text{ s},\ 1.60\text{ m})=\boxed{-1.51\ \text{m/s}}$ $a=\frac{dv}{dt}:\quad a=(-0.120\text{ m})(4\pi)^2\sin\left(\frac{\pi}{8}x+4\pi t\right)$ $a(0.200\text{ s},\ 1.60\text{ m})=\boxed{0}$ $k=\frac{\pi}{8}=\frac{2\pi}{\lambda}:\quad \lambda=\boxed{16.0\text{ m}}$ $\omega=4\pi=\frac{2\pi}{T}:T=\boxed{0.500\text{ s}}$ $v=\frac{\lambda}{T}=\frac{16.0\text{ m}}{0.500\text{ s}}=\boxed{32.0\ \text{m/s}}$
8	Two waves in a long string have wave functions given by $y_1=0.015\,cos\,\left(\frac{x}{2}-40t\right)$ and $y_1=0.015\,cos\,\left(\frac{x}{2}+40t\right)$ where y_1, y_2, and *x* are in meters and *t* is in seconds. Determine the positions of the nodes of the resulting standing wave. What is the maximum transverse position of an element of the string at the position x=0.4m?
	$y=0.030\,0\text{ m}\cos\left(\frac{x}{2}\right)\cos(40t)$ nodes occur where $y=0$: $\frac{x}{2}=(2n+1)\frac{\pi}{2}$ $x=\boxed{(2n+1)\pi=\pi,\ 3\pi,\ 5\pi,\ \ldots}$. $y_{\text{max}}=0.030\,0\text{ m}\cos\left(\frac{0.400}{2}\right)=\boxed{0.029\,4\text{ m}}$

Worksheet 16: Waves – Cases & Doppler Effect

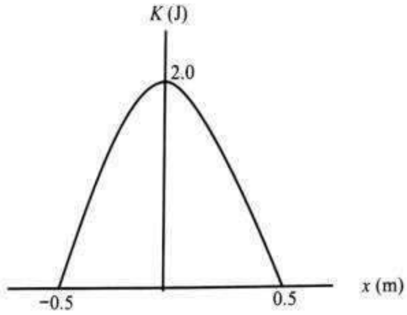
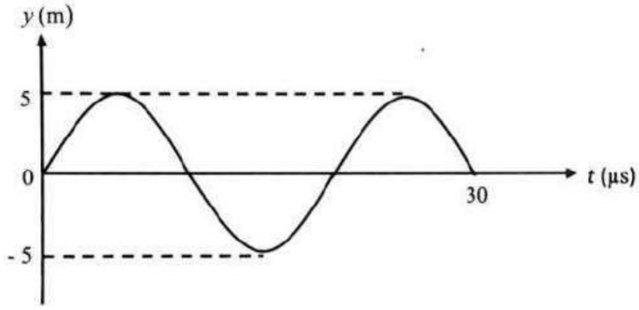
1	The middle C string on a piano is under a tension of 944 N. The period and wavelength of a wave on this string are 3.82ms and 1.26m, respectively. Find the linear density of the string.
	$(m/L) = F/v^2$ $v = f\lambda = \left(\frac{1}{T}\right)\lambda$ $m/L = \frac{F}{v^2} = \frac{FT^2}{\lambda^2} = \frac{(944\text{ N})(3.82\times10^{-3}\text{ s})^2}{(1.26\text{ m})^2} = \boxed{8.68\times10^{-3}\text{ kg/m}}$
2	A wire is stretched between two posts. Another wire is stretched between two posts that are twice as far apart. The tension in the wires is the same, and they have the same mass. A transverse wave travels on the shorter wire with a speed of $240ms^{-1}$. What would be the speed of the wave on the longer wire?
	$\frac{v_{\text{longer}}}{v_{\text{shorter}}} = \frac{\sqrt{\frac{F}{m/L_{\text{longer}}}}}{\sqrt{\frac{F}{m/L_{\text{shorter}}}}} = \sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}}$ Since $v_{\text{shorter}} = 240$ m/s and $L_{\text{longer}} = 2L_{\text{shorter}}$, the speed of the wave on the longer wire is $v_{\text{longer}} = v_{\text{shorter}}\sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}} = (240\text{ m/s})\sqrt{\frac{2L_{\text{shorter}}}{L_{\text{shorter}}}} = (240\text{ m/s})\sqrt{2} = \boxed{340\text{ m/s}}$
3	a. What would you estimate for the length of a bass clarinet, assuming that it is modelled as a **closed tube** and that the lowest note that it can play is a whose frequency is 69Hz? b. When an **open** metal pipe is cut into two pieces, the lowest resonance frequency for the air column in one piece is 256Hz and that for the other is 440Hz. If speed of sound in the metal pipe is $343ms^{-1}$, what resonant frequency would have been produced by the original length of pipe? How long was the original pipe?
	a. For a closed tube, $f_1 = \frac{v}{4\ell}$. $f_1 = \frac{v}{4\ell} \rightarrow \ell = \frac{v}{4f_1} = \frac{343\text{ m/s}}{4(69\text{ Hz})} = \boxed{1.2\text{ m}}$ b.

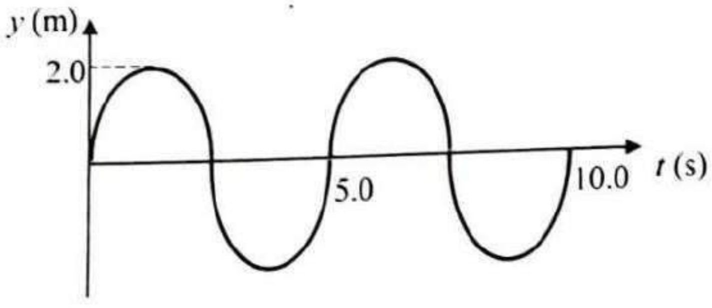
	For one, $\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{256\ \text{s}^{-1}} = 1.34\ \text{m}$ $\text{length} = d_{AA} = \frac{\lambda}{2} = 0.670\ \text{m}$ For the other, $\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{440\ \text{s}^{-1}} = 0.780\ \text{m}$ $\text{length} = 0.390\ \text{m}$ $\text{original length} = \boxed{1.06\ \text{m}}$ $\lambda = 2d_{AA} = 2.12\ \text{m}$ $f = \frac{343\ \text{m/s}}{2.12\ \text{m}} = \boxed{162\ \text{Hz}}$
4	An organ pipe is 116 cm long and the speed of sound is $343ms^{-1}$. Determine the fundamental and first three audible overtones if the pipe is closed at one end, and open at both ends.
	If the pipe is closed at one end, $f_n = \frac{n\ell}{4\ell} = nf_1, n = 1, 3, 5, \cdots \quad \rightarrow \quad f_1 = \frac{v}{4\ell} = \frac{343\ \text{m/s}}{4(1.16\ \text{m})} = \boxed{73.9\ \text{Hz}}$ $f_3 = 3f_1 = \boxed{222\ \text{Hz}} \quad f_5 = 5f_1 = \boxed{370\ \text{Hz}} \quad f_7 = 7f_1 = \boxed{517\ \text{Hz}}$ If the pipe is open at both ends, $f_n = \frac{nv}{2\ell} = nf_1, n = 1, 3, 5, \cdots \quad \rightarrow \quad f_1 = \frac{v}{2\ell} = \frac{343\ \text{m/s}}{2(1.16\ \text{m})} = \boxed{148\ \text{Hz}}$ $f_2 = 2f_1 = \boxed{296\ \text{Hz}} \quad f_3 = 3f_1 = \boxed{444\ \text{Hz}} \quad f_4 = 4f_1 = \boxed{591\ \text{Hz}}$
5	A uniform narrow tube 1.70m long is open at both ends. It resonates at two successive harmonics of frequencies 275Hz and 330Hz. What is the fundamental frequency, and the speed of sound in the gas in the tube?
	$f_1 = 330\ \text{Hz} - 275\ \text{Hz} = \boxed{55\ \text{Hz}}$ $v = 2\ell f_1 = 2(1.70\ \text{m})(55\ \text{Hz}) = 187\ \text{m/s} \approx \boxed{190\ \text{m/s}}$
6	The human ear canal is approximately 2.5cm long. It is open to the outside and is closed at the other end by the eardrum. Estimate the frequencies (in the audible range) of the standing waves in the ear canal. (The speed of sound is $343ms^{-1}$)

	The resonant frequencies are given by $f_n = \frac{nv}{4\ell}$, n odd. $f_n = \frac{nv}{4\ell} = \frac{n(343 \text{ m/s})}{4(2.5\times10^{-2} \text{ m})} = n(3430 \text{ Hz})$, n odd $\boxed{f_1 = 3430 \text{ Hz} \quad f_3 = 10{,}300 \text{ Hz} \quad f_5 = 17{,}200 \text{ Hz}}$
7	a. The predominant frequency of a certain fire truck's siren is 1650 Hz when at rest. What frequency do you detect if you move with a speed of $30ms^{-1}$, toward the fire truck, and away from it? (The speed of sound is $343ms^{-1}$) b. A bird is flying directly toward a stationary bird-watcher and emits a frequency of 1250Hz. The bird-watcher, however, hears a frequency of 1290Hz. What is the speed of the bird, expressed as a percentage of the speed of sound?
	a. Observer moving toward stationary source: $f' = \left(1 + \frac{v_{obs}}{v_{snd}}\right) f = \left(1 + \frac{30.0 \text{ m/s}}{343 \text{ m/s}}\right)(1650 \text{ Hz}) = \boxed{1790 \text{ Hz}}$ Observer moving away from stationary source: $f' = \left(1 - \frac{v_{obs}}{v_{snd}}\right) f = \left(1 - \frac{30.0 \text{ m/s}}{343 \text{ m/s}}\right)(1650 \text{ Hz}) = \boxed{1510 \text{ Hz}}$ b. $f_o = f_s\left(\frac{1}{1 - v_s/v}\right)$ or $\frac{f_o}{f_s} = \frac{1}{1 - v_s/v}$ or $\frac{f_s}{f_o} = 1 - \frac{v_s}{v}$ $\frac{v_s}{v} = 1 - \frac{f_s}{f_o} = 1 - \frac{1250 \text{ Hz}}{1290 \text{ Hz}} = 0.031$ This ratio corresponds to $\boxed{3.1\%}$.
8	a. From a vantage point very close to the track at a stock car race, you hear the sound emitted by a moving car. You detect a frequency that is 0.86 times as small as the frequency emitted by the car when it is stationary. The speed of sound is $343ms^{-1}$. What is the speed of the car? b. A police car sounding a siren with a frequency of 1580Hz is traveling at $120kmh^{-1}$. What frequencies does an observer standing next to the road hear as the car approaches and as it recedes? What frequencies are heard in a car traveling at $90kmh^{-1}$,in the opposite direction before and after passing the

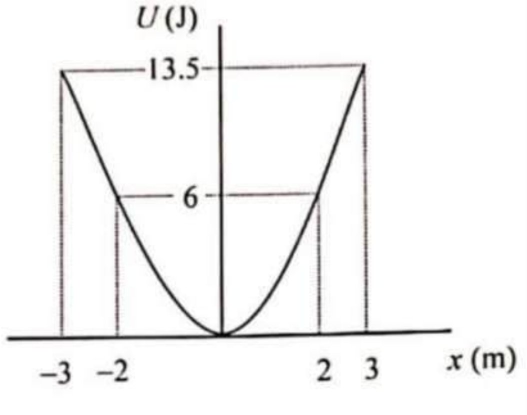
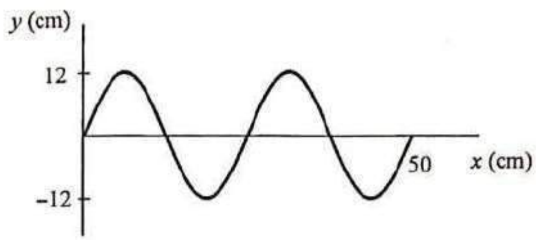
	police car? The police car passes a car traveling in the same direction at $80kmh^{-1}$. What two frequencies are heard in this car? (The speed of sound is $343ms^{-1}$)
	a. $$\frac{v_s}{v}=\frac{f_s}{f_o}-1$$ $$f_o/f_s=0.86$$ $$v_s=v\left(\frac{f_s}{f_o}-1\right)=(343\text{ m/s})\left(\frac{1}{0.86}-1\right)=\boxed{56\text{ m/s}}$$ b. The observer is stationary, and the source is moving. $$120.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=33.33\text{ m/s}$$ $$f'_{\substack{\text{source}\\\text{moving}\\\text{toward}}}=f\frac{1}{\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)}=(1580\text{ Hz})\frac{1}{\left(1-\frac{33.33\text{ m/s}}{343\text{ m/s}}\right)}=\boxed{1750\text{ Hz}}$$ $$f'_{\substack{\text{source}\\\text{moving}\\\text{away}}}=f\frac{1}{\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right)}=(1580\text{ Hz})\frac{1}{\left(1+\frac{33.33\text{ m/s}}{343\text{ m/s}}\right)}=\boxed{1440\text{ Hz}}$$ Both the observer and the source are moving $$90.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=25\text{ m/s}$$ $$f'_{\text{approaching}}=f\frac{(v_{\text{snd}}+v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}+25\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})}=\boxed{1880\text{ Hz}}$$ $$f'_{\text{receding}}=f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}+v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}-25\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})}=\boxed{1340\text{ Hz}}$$ Both the observer and the source are moving $$80.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=22.22\text{ m/s}$$ $$f'_{\substack{\text{police}\\\text{car}\\\text{approaching}}}=f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}-22.22\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})}=\boxed{1640\text{ Hz}}$$ $$f'_{\substack{\text{police}\\\text{car}\\\text{receding}}}=f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}+22.22\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})}=\boxed{1530\text{ Hz}}$$

Worksheet 17: SHM

21/22	A. A mass of 200g is attached to a spring. When the mass displaced a certain distance from equilibrium and released, it oscillates at a period of 0.85s. What is the constant of the spring? B. The figure shows a particle of mass 4.0kg moves in simple harmonic motion and its kinetic energy, K varies with position, *x*.  Determine the period. C. What is the speed of a transverse wave in a rope of length 5.00m and mass 55.00g under a tension of 600.00N D. A transverse wave is represented by the following equation: $$y = 7\,sin\,(5t - 3x)$$ where y and x are measured in centimetres and t in seconds. What is the maximum vibrational velocity of a particle in the wave? E. The figure shows how the displacement, *y* of a particle varies with time, *t* when a wave passes through the particle at speed $6kms^{-1}$. The wave is reflected and superimposed with an incident wave.  What is the equation of the standing wave formed? F. The security alarm in a parking area produces a siren with frequency of 980Hz. As a car drives away, the driver

	observes the frequency changes to 850Hz. The speed of sound is in $345ms^{-1}$. What is the speed of the car?
	A. $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 0.85 = 2\pi\sqrt{\frac{0.2}{k}} \Rightarrow k = 10.93\,Nm^{-1}$ B. When $x = A$, $K = E_{total} = \frac{1}{2}m\omega^2 A^2$ $K = \frac{1}{2}m\left(\frac{2}{T}\right)^2 A^2 \Rightarrow 2 = \frac{1}{2}(4)\left(\frac{2}{T}\right)^2 (0.5)^2 \Rightarrow T = 3.1416s$ C. $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T l}{m}} = \sqrt{\frac{600(5)}{0.055}} \Rightarrow v = 233.55ms^{-1}$ D. $v_{vibration} = \frac{dy}{dt} = \frac{d}{dt}(7\,sin\,(5t - 3x))$ $v_{vibration} = 35\,cos\,(5t - 3x)$ $v_{max-vibration} = 35cms^{-1}$ E. From diagram $\Rightarrow T = 20\,\mu s;\ A = 5m;\ v = 6kms^{-1} = 6000ms^{-1}$ General equation for standing wave: $y(x,t) = 2A\,cos(kx)\,sin(\omega t)$ $k = \frac{2\pi}{\lambda} = \frac{2\pi}{vT} = \frac{2\pi}{(6000)(20(10^{-6}))} = \frac{50}{3}\pi m^{-1}$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{20(10^{-6})} = \pi(10^{-5})\,rad\,s^{-1}$ $y(x,t) = 10\,cos\left(\frac{50}{3}\pi x\right)\,sin(\pi(10^{-5})t)$ Where x is in metres and t is in seconds. F. $\frac{f_{app}}{f_{source}} = \frac{v \pm v_{observer}}{v \mp v_{source}}$ $\frac{850}{980} = \frac{345 - v_{observer}}{345 \mp 0} \Rightarrow v_{observer} = 45.77ms^{-1}$
20/21	A. The figure shows the displacement-time graph of a body performing simple harmonic motion. 

	a. Determine the amplitude, angular frequency and maximum speed of the motion. b. Deduce the expression for the motion. c. Sketch the velocity-time graph for the motion. B. A stretched wire of length 1.0m is fixed at both ends. The speed of the transverse wave in the wire is $10ms^{-1}$. If the mode of vibration is third overtone, calculate the a. Wavelength b. Frequency of the third overtone c. Lowest resonant frequency of the wire. C. A train with a velocity of $40ms^{-1}$ is approaching an observer standing on a platform. The frequency of the siren from the train is 1600Hz. Assuming the speed of sound in air is $330ms^{-1}$, determine the a. Frequency of the sound heard by the observer. b. Frequency of the sound heard by the observe when the train is leaving the platform.
	A. $A = 2m;\ T = 5s$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{5} \Rightarrow \omega = 0.4\pi\ rads^{-1}$ $v = \frac{dy}{dt} = \frac{d}{dt}(2\sin 0.4\pi t)$ $v_{max} = 0.8\pi \approx 2.51ms^{-1}$ $y(t) = 2\sin 0.4\pi t$ B. $2\lambda = L \Rightarrow \lambda = 0.5L = 0.5(1) \Rightarrow \lambda = 0.5m$ $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{10}{0.5} \Rightarrow f = 20Hz$ Lowest resonant $\Rightarrow$ fundamental frequency $\lambda = 2L = 2m \Rightarrow f = \frac{v}{\lambda} = \frac{10}{2} \Rightarrow f = 5Hz$ C. $\frac{f_{observed}}{f_{source}} = \frac{v \pm v_{observer}}{v \mp v_{source}}$ $\frac{f_{observed}}{1600} = \frac{330 \pm 0}{330 - 40}$ $f_{observed} = 1820.69Hz$ $\frac{f_{observed}}{f_{source}} = \frac{v \pm v_{observer}}{v \mp v_{source}}$ $\frac{f_{observed}}{1600} = \frac{330 \pm 0}{330 + 40}$ $f_{observed} = 1427.03Hz$

19/20	A. The figure shows the potential energy of 0.5kg object that undergoes a simple harmonic motion.  Determine the i. Velocity when time $t = 2s$ ii. Kinetic energy of the object when displacement $x = 1.5m$ B. An oscillating pendulum has length 0.3m and 240g bob. If the total energy is 0.06J, calculate the amplitude of the oscillation. C.  The figure shows a graph of displacement y against distance x for a progressive wave propagating to the right in a string with mass 920g, length 3m and tension 15N. Determine the wave equation. D. A 1.53m closed pipe makes a humming sound at frequency 282Hz when the wind blows across the open end. The speed of sound is $343ms^{-1}$. With the help of a diagram, determine the number of nodes in the standing wave. E. The frequency of whistle by a moving train and the frequency heard by a stationary observer are 520Hz and 460Hz respectively. If the speed of sound in the air is $343ms^{-1}$, calculate the speed of the train.

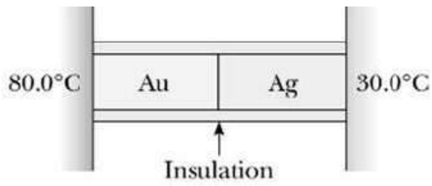
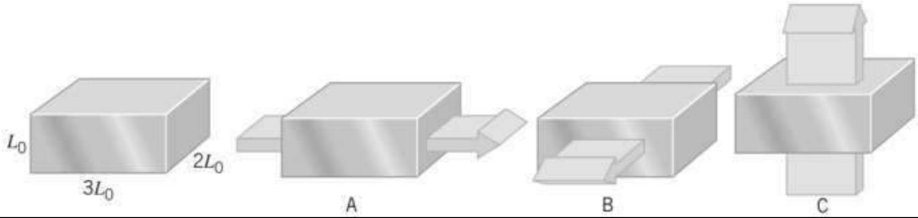
	A. $A = 3$ $U(x) = \frac{1}{2}m\omega^2 x^2 \Rightarrow 13.5 = \frac{1}{2}(0.5)\omega^2(3^2)$ $\omega = 2.45\, rads^{-1}$ General equation $\Rightarrow y(t) = A\sin(\omega t) = 3\sin(2.45t)$ $v = \frac{dy}{dt} = \frac{d}{dt}(3\sin(2.45t)) = 7.35\cos(2.45t)$ $v(t = 2s) = 7.35\cos(2.45(2)) \Rightarrow v(t = 2s) = 1.37ms^{-1}$ $K = \frac{1}{2}m\omega^2(A^2 - x^2)$ $K(x = 1.5m) = \frac{1}{2}(0.24)(2.45)^2(3^2 - 1.5^2)$ $K(x = 1.5m) = 10.129J$ B. $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}} \Rightarrow \omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{0.3}} = 5.718 rads^{-1}$ $E_{total} = \frac{1}{2}m\omega^2 A^2 \Rightarrow 0.06 = \frac{1}{2}(0.24)(5.718)^2 A^2 \Rightarrow A = 0.124m$ C. General Equation $\Rightarrow y(x,t) = A\sin(\omega t \pm kx)$ $A = 12cm;\ k = \frac{2\pi}{\lambda} = \frac{2}{25}\pi cm^{-1}$ $\omega = 2\pi f = 2\pi\left(\frac{v}{\lambda}\right) = 2\pi\left(\frac{\left(\sqrt{\frac{F_T}{\mu}}\right)}{\lambda}\right) = 2\pi\left(\frac{\left(\sqrt{\frac{15}{\left(\frac{0.92}{3}\right)}}\right)}{0.25}\right) = 55.95\pi\, rads^{-1}$ $y(x,t) = 12\sin\left(55.95\pi t - \frac{2}{25}\pi x\right)$, where x and y are in centimetres and t is in seconds. D. $L = 1.53m;\ f_n = 282Hz;\ v_{sound} = 343ms^{-1}$ For fundamental frequency $\Rightarrow L = \frac{\lambda}{4} \Rightarrow \lambda = 4L \Rightarrow v = f_o\lambda_o \Rightarrow f_o = \frac{v}{\lambda_o} = \frac{v}{4L} = \frac{343}{4(1.53)} = 56Hz$ $n = \frac{f_n}{f_o} = \frac{282}{56} \approx 5 \Rightarrow$ 5th Harmonic $\Rightarrow$ 3 nodes

Worksheet 19: Stress, Strain & Young's Modulus

1	A marble column of cross-sectional area supports a mass of 25,000 kg and Young's Modulus $50(10^9)Nm^{-2}$. What is the stress within the column? What is the strain? By how much is the column shortened if it is 8.6 m high?
	$\text{stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(25,000\text{ kg})(9.80\text{ m/s}^2)}{1.4\text{ m}^2} = 175,000\text{ N/m}^2 \approx \boxed{1.8\times10^5\text{ N/m}^2}$ $\text{strain} = \frac{\text{stress}}{\text{Young's modulus}} = \frac{175,000\times10^5\text{ N/m}^2}{50\times10^9\text{ N/m}^2} = \boxed{3.5\times10^{-6}}$ $\text{strain} = \frac{\Delta\ell}{\ell_0} \rightarrow \Delta\ell = \ell_0(\text{strain}) = (8.6\text{ m})(3.5\times10^{-6}) = \boxed{3.0\times10^{-5}\text{ m}}$
2	A tow truck is pulling a car out of a ditch by means of a steel cable that is 9.1m long and has a radius of 0.50 cm. When the car just begins to move, the tension in the cable is 890 N. How much has the cable stretched?

	$\Delta L = \frac{FL_0}{AY} = \frac{(890\text{ N})(9.1\text{ m})}{\pi\left(0.50\times10^{-2}\text{ m}\right)^2\left(2.0\times10^{11}\text{ N/m}^2\right)} = \boxed{5.2\times10^{-4}\text{ m}}$
3	A 15cm-long tendon was found to stretch 3.7mm by a force of 13.4N. The tendon was approximately round with an average diameter of 8.5mm. Calculate Young's modulus of this tendon.
	$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta\ell/\ell_0} = \frac{(13.4\text{ N})/\left[\pi\left(\frac{1}{2}\times8.5\times10^{-3}\text{m}\right)^2\right]}{(3.7\times10^{-3}\text{ m})/(15\times10^{-2}\text{ m})} = \boxed{9.6\times10^6\text{ N/m}^2}$
4	Assume that Young's modulus is $1.5\times10^{10}Nm^{-2}$ for bone and that the bone will fracture if stress greater than $1.5\times10^{8}Nm^{-2}$ is imposed on it. What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50cm? If this much force is applied compressively, by how much does the 25cm-long bone shorten?
	$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2}$ $F = (\text{stress})\pi\left(\frac{d}{2}\right)^2$ $F = \left(1.50\times10^{8}\text{ N/m}^2\right)\pi\left(\frac{2.50\times10^{-2}\text{ m}}{2}\right)^2$ $F = \boxed{73.6\text{ kN}}$ $\text{stress} = Y(\text{strain}) = \frac{Y\Delta L}{L_i}$ $\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{\left(1.50\times10^{8}\text{ N/m}^2\right)(0.250\text{ m})}{1.50\times10^{10}\text{ N/m}^2} = \boxed{2.50\text{ mm}}$
5	A scallop forces open its shell with an elastic material called abductin, whose Young's modulus is about $2(10^6)Nm^{-2}$. If this piece of abductin is 3mm thick and has a cross-sectional area of $0.5cm^2$. how much potential energy does it store when compressed 1.0 mm?
	$\text{PE}_{\text{elastic}} = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}F\Delta x.$ $\text{PE}_{\text{elastic}} = \frac{1}{2}F\Delta x = \frac{1}{2}\left(\frac{EA}{\ell_0}\Delta\ell\right)\Delta\ell = \frac{1}{2}\frac{(2.0\times10^6\text{ N/m}^2)(0.50\times10^{-4}\text{ m}^2)}{(3.0\times10^{-3}\text{ m})}(1.0\times10^{-3}\text{ m})^2$ $\text{PE}_{\text{elastic}} = \boxed{1.7\times10^{-2}\text{ J}}$

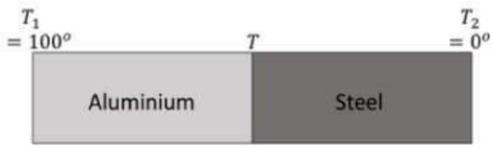
Worksheet 19: Heat Conduction & Thermal Expansion

1	A glass window pane has an area of $3m^2$ and a thickness of 0.6cm. If the temperature difference between its faces is 25.0°C, what is the rate of energy transfer by conduction through the window?
	$\mathcal{P} = \frac{kA\Delta T}{L} = \frac{(0.800\ \mathrm{W/m\cdot°C})(3.00\ \mathrm{m}^2)(25.0°\mathrm{C})}{6.00\times10^{-3}\ \mathrm{m}} = 1.00\times10^{4}\ \mathrm{W} = \boxed{10.0\ \mathrm{kW}}$
2	A bar of gold ($k_{Au} = 314Wm^{-1o}C^{-1}$)is in thermal contact with a bar of silver ($k_{Ag} = 427Wm^{-1o}C^{-1}$) of the same length and area (shown in the figure). One end of the compound bar is maintained at 80°C while the opposite end is at 30°C. When the energy transfer reaches steady state, what is the temperature at the junction? 
	In the steady state condition, $\mathcal{P}_{\mathrm{Au}} = \mathcal{P}_{\mathrm{Ag}}$ $k_{\mathrm{Au}}A_{\mathrm{Au}}\left(\frac{\Delta T}{\Delta x}\right)_{\mathrm{Au}} = k_{\mathrm{Ag}}A_{\mathrm{Ag}}\left(\frac{\Delta T}{\Delta x}\right)_{\mathrm{Ag}}$ $A_{\mathrm{Au}} = A_{\mathrm{Ag}}$ $\Delta x_{\mathrm{Au}} = \Delta x_{\mathrm{Ag}}$ $\Delta T_{\mathrm{Au}} = (80.0 - T)$ $\Delta T_{\mathrm{Ag}} = (T - 30.0)$ where T is the temperature of the junction. $k_{\mathrm{Au}}(80.0 - T) = k_{\mathrm{Ag}}(T - 30.0)$ $\boxed{T = 51.2°\mathrm{C}}$
3	The block in the drawing has dimensions $L_0 \times 2L_0 \times 3L_0$, where $L_0 = 0.3m$. The block has a thermal conductivity of 250 $Js^{-1}m^{-1o}C^{-1}$. In drawings A, B, and C, heat is conducted through the block in three different directions; in each case the temperature of the warmer surface is 35^oC and that of the cooler surface is 19^oC. Determine the heat that flows in 5.0 s for each case. 

	$A_A = 2L_0^2$ and $L_A = 3L_0$, $A_B = 3L_0^2$ and $L_B = 2L_0$, $A_C = 6L_0^2$ and $L_C = L_0$. **Case A** $Q_A = \frac{A_A}{L_A} k\Delta Tt = \frac{2L_0^2}{3L_0} k\Delta Tt = \left(\frac{2}{3}L_0\right)k\Delta Tt$ $= \frac{2}{3}(0.30\text{ m})[250\text{ J/(s}\cdot\text{m}\cdot\text{C°)}](35\text{ °C} - 19\text{ °C})(5.0\text{ s}) = \boxed{4.0\times10^3\text{ J}}$ **Case B** $Q_B = \frac{A_B}{L_B} k\Delta Tt = \frac{3L_0^2}{2L_0} k\Delta Tt = \left(\frac{3}{2}L_0\right)k\Delta Tt$ $= \frac{3}{2}(0.30\text{ m})[250\text{ J/(s}\cdot\text{m}\cdot\text{C°)}](35\text{ °C} - 19\text{ °C})(5.0\text{ s}) = \boxed{9.0\times10^3\text{ J}}$ **Case C** $Q_C = \frac{A_C}{L_C} k\Delta Tt = \frac{6L_0^2}{L_0} k\Delta Tt = (6L_0)k\Delta Tt$ $= 6(0.30\text{ m})[250\text{ J/(s}\cdot\text{m}\cdot\text{C°)}](35\text{ °C} - 19\text{ °C})(5.0\text{ s}) = \boxed{3.6\times10^4\text{ J}}$
4	A copper ($\alpha = 17 \times 10^{-6} ({}^oC)^{-1}$) telephone wire has essentially no sag between poles 35m apart on a winter day when the temperature is $-20.0°C$. How much longer is the wire on a summer day when temperature is 35.0°C?
	The wire is 35.0 m long when $T_C = -20.0°\text{C}$. $\Delta L = L_i\overline{\alpha}(T - T_i)$ $\overline{\alpha} = \alpha(20.0°\text{C}) = 1.70\times10^{-5}\ (\text{C°})^{-1}$ for Cu. $\Delta L = (35.0\text{ m})(1.70\times10^{-5}\ (\text{C°})^{-1})(35.0°\text{C} - (-20.0°\text{C})) = \boxed{+3.27\text{ cm}}$
5	A brass ring of diameter 10cm at 20°C is heated and slipped over an aluminium rod of diameter 10.01cm at 20°C. To what temperature must this combination be cooled to separate them? Is this attainable? What if the aluminium rod were 10.02cm in diameter?

	$L_{Al}(1+\alpha_{Al}\Delta T)=L_{Brass}(1+\alpha_{Brass}\Delta T)$ $\Delta T=\dfrac{L_{Al}-L_{Brass}}{L_{Brass}\alpha_{Brass}-L_{Al}\alpha_{Al}}$ $\Delta T=\dfrac{(10.01-10.00)}{(10.00)(19.0\times10^{-6})-(10.01)(24.0\times10^{-6})}$ $\Delta T=-199°\text{C so } T=\boxed{-179°\text{C. This is attainable.}}$ $\Delta T=\dfrac{(10.02-10.00)}{(10.00)(19.0\times10^{-6})-(10.02)(24.0\times10^{-6})}$ $\Delta T=-396°\text{C so } T=\boxed{-376°\text{C which is below 0 K so it cannot be reached.}}$
6	A brass plug is to be placed in a ring made of iron. At 15°C, the diameter of the plug is 8.755cm and that of the inside of the ring is 8.741 cm. They must both be brought to what common temperature in order to fit?
	$(\ell_0+\Delta\ell)_{iron}=(\ell_0+\Delta\ell)_{brass} \rightarrow \ell_{iron}+\alpha_{iron}\ell_{iron}\Delta T=\ell_{brass}+\alpha_{brass}\ell_{brass}\Delta T$ $\Delta T=\dfrac{\ell_{brass}-\ell_{iron}}{\alpha_{iron}\ell_{iron}-\alpha_{brass}\ell_{brass}}=\dfrac{8.755\text{ cm}-8.741\text{ cm}}{(12\times10^{-6}/\text{C°})(8.741\text{ cm})-(19\times10^{-6}/\text{C°})(8.755\text{ cm})}$ $=-228\text{C°}=T_{final}-T_{initial}=T_{final}-15°\text{C} \rightarrow T_{final}=-213°\text{C}\approx\boxed{-210°\text{C}}$
7	A thin spherical shell of silver has an inner radius of $2\times10^{-2}m^2$ when the temperature is 18^oC. The shell is heated to 147^oC. Find the change in the interior volume of the shell if silver has a linear expansion coefficient of $19\times10^{-6}(^oC)^{-1}$.
	$V_0=\frac{4}{3}\pi r^3$ $\Delta V=\beta V_0\Delta T=\beta\left(\frac{4}{3}\pi r^3\right)\Delta T$ $\Delta V=\left[57\times10^{-6}(\text{C°})^{-1}\right]\frac{4}{3}\pi\left(2.0\times10^{-2}\text{ m}\right)^3(147\text{ °C}-18\text{ °C})=\boxed{2.5\times10^{-7}\text{ m}^3}$
8	The density of mercury is 13 600 kgm^{-3} at 0^oC. What would its density be at 166^oC if silver has a linear expansion coefficient of $0.607\times10^{-4}(^oC)^{-1}$?
	$\dfrac{\rho}{\rho_0}=\dfrac{m/V}{m/V_0}=\dfrac{V_0}{V}$ $\rho=\rho_0\dfrac{V_0}{V}$ $\Delta V=\beta V_0\Delta T$ $V=V_0+\Delta V=V_0+\beta V_0\Delta T$ $\rho=\rho_0\left(\dfrac{V_0}{V_0+\beta V_0\Delta T}\right)=\rho_0\left(\dfrac{1}{1+\beta\Delta T}\right)$ $\rho=\dfrac{\rho_0}{1+\beta\Delta T}=\dfrac{13\,600\text{ kg/m}^3}{1+\left[182\times10^{-6}(\text{C°})^{-1}\right]\left[(166\text{ °C})-(0\text{ °C})\right]}=\boxed{13\,200\text{ kg/m}^3}$

Worksheet 20: PYQ Physics of Matter

21/22	A. A 5.0m long wire a cross sectional area of $4.0 \times 10^{-4} m^2$. The wire extended by 0.5cm. Calculate the Young's modulus of the wire when a 200kg is suspended at its one end. B. An aluminium rod of radius 0.5cm and length 20cm is welded end-to-end with a steel rod of the same dimensions. The free end of the aluminium rod is held at $100^o C$ while the steel free end is placed in a ice bath. When the system is at steady state, calculate the temperature at the aluminium-steel interface. [Given the thermal conductivity of aluminium is $250 W m^{-1 o} C^{-1}$ and the thermal conductivity of steel is $14 W m^{-1 o} C^{-1}$] C. An aluminium tube of external diameter 3.00cm at $25^o C$ is heated to $80^o C$. Calculate the external area of the tube at $80^o C$ if the coefficient of linear expansion for aluminium is $2.4 \times 10^{-5} K^{-1}$.
	A. $Y = \frac{Fl}{A\Delta l} = \frac{(200)(9.81)(5)}{(4\times10^{-4})0.5(10^{-2})} = 4.905 \times 10^9 Nm^{-2}$ B.  $\frac{dQ_{Al}}{dt} = \frac{dQ_{steel}}{dt} \Rightarrow k_{Al}A_{Al}\frac{(T_1 - T)}{l_{Al}} = k_{steel}A_{steel}\frac{(T - T_2)}{l_{steel}}$ $k_{Al}(T_1 - T) = k_{steel}(T - T_2) \Rightarrow 250((100 + 273.15) - T) = 14(T - (0 + 273.15))$ $T = 367.85K$ C. $A_o = \pi\left(\frac{0.03}{2}\right)^2 = 0.000225\pi;\ \frac{\Delta A}{A_o} = \frac{A_f - A_o}{A_o} = 2\alpha\Delta T$ $\frac{A_f - 0.000225\pi}{0.000225\pi} = 2(2.4 \times 10^{-5})(80 - 25) \Rightarrow A_f = 0.00708724m^2$
20/21	A. A 1.5m steel wire is stretched 2.0mm by force F. The diameter of the steel wire is 4.0mm. The Young's modulus of steel is $2.0 \times 10^{11}\ Nm^{-2}$. Determine the force F applied on the wire. B. A perfectly insulated aluminium rod has length $50cm$ and cross-sectional area $3.0cm^2$. At the steady state, the temperatures at 0cm and 50cm ends are $150^o C$ and $50^o C$. (Thermal conductivity of aluminium is $210 W\ m^{-1}\ K^{-1}$) a. Sketch a labelled graph of temperature against distance. b. Calculate the temperature gradient along the rod. c. Calculate the rate of heat flow in the rod.

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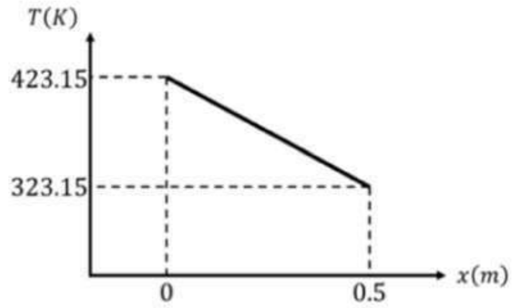
A. $Y = \frac{Fl}{A\Delta l} = \frac{Fl}{\left(\frac{d^2}{4}\right)\pi\Delta l} \Rightarrow 2(10^{11}) = \frac{F(1.5)}{\left(\frac{(4\times10^{-3})^2}{4}\right)\pi(0.002)} \Rightarrow F = 3351N$

B.

a.



b. $\frac{dT}{dx} = \frac{323.15 - 423.15}{0.5 - 0} = -200\ Km^{-1}$

c. $\frac{dQ}{dt} = -k_{Al}A_{Al}\frac{dT}{dx} = -(210)(3(10^{-2})^2)(-200)$

$\Rightarrow \frac{dQ}{dt} = 63.5Js^{-1}$ (towards the cold end)

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A. The diameter of a circular shoe heel is 13mm. If both heels support 70% of the weight of a 54kg woman, calculate the stress on both heels.

B. A gold rod is in contact with a silver rod. The gold end and the silver end of the compound is at 90^oC and 30^oC respectively. The silver rod has thermal conductivity 427 $Wm^{-1}K^{-1}$, length 2.5cm and cross-sectional area $7.85 \times 10^{-5}m^2$. If 341.3J heat flows through the gold rod in 10s, calculate the temperature at the contact surface.

C. The are of a metal plate changes from $120m^2$ to $120.059m^2$ when the temperature increases by 30^oC. Calculate the coefficient of linear expansion of the metal.

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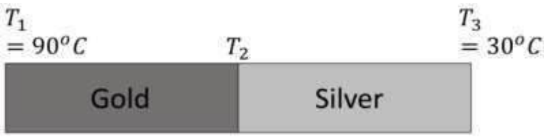
A.

$$\delta = \frac{F}{A} = \frac{0.7(54)(9.81)}{2\left(\frac{13(10^{-3})}{2}\right)^2} = 1.397 \times 10^6 Nm^{-2}$$

B.



$\frac{dQ_{gold}}{dt} = \frac{dQ_{silver}}{dt} = \frac{341.3J}{10s} = 34.13Js^{-1} = -k_{silver}A_{silver}\frac{dT}{dx} = -(427)(7.85 \times 10^{-5})\frac{30-T_2}{0.025}$

$\Rightarrow T_2 = 55.46^oC$

C.

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</table>

	$\frac{\Delta A}{A_0} = \frac{A_f - A_0}{A_0} = 2\alpha\Delta T \Rightarrow \frac{120.059-120}{120} = 2\alpha(30) \Rightarrow \alpha = 8.19444(10^{-6})K^{-1}$

Worksheet 21: Molecular Energies

1	a. Calculate the rms speed of helium atoms near the surface of the Sun at a temperature of about 6000 K. b. If the pressure in a gas is tripled while its volume is held constant, by what factor does change? c. How many moles of water are there in 1.000 L at STP? How many molecules?
	a. $v_{\text{rms}} = \sqrt{3kT/m}$ Helium has an atomic mass of 4.0. $v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(6000\text{ K})}{4.0(1.66\times10^{-27}\text{ kg})}} = 6116\text{ m/s} \approx \boxed{6\times10^3\text{ m/s}}$ b. $PV = nRT$ $P = \frac{nR}{V}T = (\text{constant})T$ $v_{\text{rms}} = \sqrt{3kT/m} = (\text{constant})\sqrt{T}$ v_{rms} will be multiplied by a factor of $\boxed{\sqrt{3}} \approx 1.73$ c. $1.000\text{ L}\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right)\left(\frac{1000\text{ kg}}{1\text{ m}^3}\right)\left(\frac{1\text{ mol}}{(15.9994+2\times1.00794)\times10^{-3}\text{ kg}}\right) = \boxed{55.51\text{ mol}}$ $55.51\text{ mol}\left(\frac{6.022\times10^{23}\text{ molecules}}{1\text{ mol}}\right) = \boxed{3.343\times10^{25}\text{ molecules}}$
2	If 16.00 mol of helium gas is at 10°C and a gauge pressure of 0.35atm, calculate the volume of the helium gas under these conditions, and the temperature if the gas is compressed to precisely half the volume at a gauge pressure of 1atm.
	$PV = nRT \rightarrow V = \frac{nRT}{P} = \frac{(16.00\text{ mol})(8.314\text{J/mol}\cdot\text{K})(283.15\text{ K})}{(1.350\text{ atm})(1.013\times10^5\text{ Pa/atm})} = \boxed{0.2754\text{ m}^3}$ $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow T_2 = T_1\frac{P_2}{P_1}\frac{V_2}{V_1} = (283.15\text{ K})\left(\frac{2.00\text{ atm}}{1.350\text{ atm}}\right)\left(\frac{1}{2}\right) = 210\text{ K} = \boxed{-63°\text{C}}$
3	What speed would a 1.0-g paper clip have if it had the same kinetic energy as a monoatomic particle at 22°C?

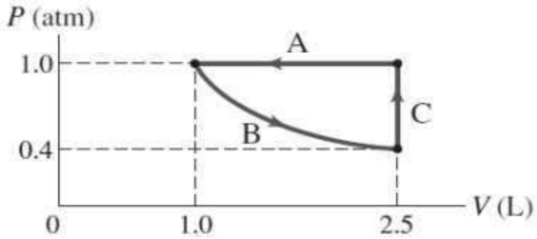
	$\frac{1}{2}mv^2 = \frac{3}{2}kT \rightarrow v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(273+22)\text{K}}{1.0\times10^{-3}\text{ kg}}} = \boxed{3.5\times10^{-9}\text{ m/s}}$
4	A 5L vessel contains nitrogen gas at 27.0°C and a pressure of 3.00 atm. Find the total translational kinetic energy of the gas molecules and the average kinetic energy per molecule.
	$PV = nRT = \frac{Nmv^2}{3}$ The total translational kinetic energy is $\frac{Nmv^2}{2} = E_{\text{trans}}$: $E_{\text{trans}} = \frac{3}{2}PV = \frac{3}{2}(3.00\times1.013\times10^5)(5.00\times10^{-3}) = \boxed{2.28\text{ kJ}}$ $\frac{mv^2}{2} = \frac{3k_BT}{2} = \frac{3RT}{2N_A} = \frac{3(8.314)(300)}{2(6.02\times10^{23})} = \boxed{6.21\times10^{-21}\text{ J}}$
5	From what height must an oxygen molecule fall in a vacuum so that its kinetic energy at the bottom equals the average energy of an oxygen molecule at 300K?
	The average energy of an oxygen molecule at 300 K is $\epsilon_{\text{avg}} = \frac{E_{\text{th}}}{N} = \frac{5}{2}k_BT = \frac{5}{2}(1.38\times10^{-23}\text{ J/K})(300\text{ K}) = 1.035\times10^{-20}\text{ J}$ The energy conservation equation $U_{\text{gf}} + K_{\text{f}} = U_{\text{gi}} + K_{\text{i}}$ with $K_{\text{f}} = \epsilon_{\text{avg}}$ is $mgy_{\text{f}} + \epsilon_{\text{avg}} = mgy_{\text{i}} + 0\text{ J}$ $y_{\text{i}} = h$ and $y_{\text{f}} = 0$ m, $mgh = 1.035\times10^{-20}\text{ J} \Rightarrow h = \frac{1.035\times10^{-20}\text{ J}}{(32\times1.66\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)} = 1.99\times10^4\text{ m}$
6	A room of dimensions 4.2m×3.0m×2.50 m, consists of diatomic molecules with molar mass 28.9 g/mol. Find a. the number of molecules of air in the room at atmospheric pressure and 20°C. b. the mass of this air. c. the average kinetic energy of one molecule. d. the root-mean-square molecular speed. e. the internal energy in the air. f. the internal energy of the air in the room at 25.0°C.
	a. $n = \frac{PV}{RT} = \frac{(1.013\times10^5\text{ Pa})(4.20\text{ m}\times3.00\text{ m}\times2.50\text{ m})}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = 1.31\times10^3\text{ mol}$ $N = nN_A = (1.31\times10^3\text{ mol})(6.02\times10^{23}\text{ molecules/mol})$ $N = \boxed{7.89\times10^{26}\text{ molecules}}$ b. $m = nM = (1.31\times10^3\text{ mol})(0.0289\text{ kg/mol}) = \boxed{37.9\text{ kg}}$

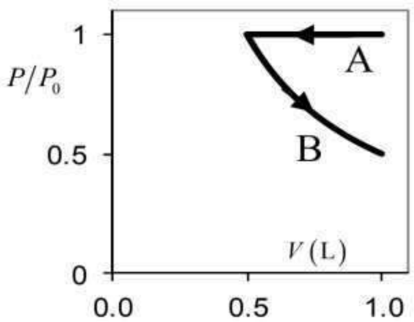
	c. $\frac{1}{2}m_0v^2 = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23}\text{ J/k})(293\text{ K}) = \boxed{6.07\times10^{-21}\text{ J/molecule}}$ d. For one molecule, $m_0 = \frac{M}{N_A} = \frac{0.0289\text{ kg/mol}}{6.02\times10^{23}\text{ molecules/mol}} = 4.80\times10^{-26}\text{ kg/molecule}$ $v_{rms} = \sqrt{\frac{2(6.07\times10^{-21}\text{ J/molecule})}{4.80\times10^{-26}\text{ kg/molecule}}} = \boxed{503\text{ m/s}}$ e. & f. $E_{int} = n\left(\frac{5}{2}R\right)T = \frac{5}{2}PV$ $E_{int} = \frac{5}{2}(1.013\times10^5\text{ Pa})(31.5\text{ m}^3) = \boxed{7.98\text{ MJ}}$
7	A container is filled with 10g of hydrogen gas (molas mass of $2.01588gmol^{-1}$) at 30^oC. Determine a. number of particles in the container b. the translational kinetic energy of a hydrogen molecule c. total kinetic energy of a hydrogen molecule d. the total translational kinetic energy of the gas e. the total kinetic energy of the gas f. internal energy of the gas
	a. $N = \frac{N_A m}{m_{molar}} = \frac{(6.602\times10^{23})(10)}{2.01588} = 3.28\times10^{24}$ particles b. $K_{trans} = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23})(30+273.15)$ $K_{trans} = 6.2752\times10^{-21}J$ c. $K_{H-H} = \frac{5}{2}k_BT = \frac{5}{2}(1.38\times10^{-23})(30+273.15)$ $K_{H-H} = 1.04587\times10^{-20}J$ d. $\Sigma K_{trans} = N\times K_{trans}$ $\Sigma K_{trans} = (3.28\times10^{24})(6.2752\times10^{-21}) = 20.582kJ$ e. $\Sigma K_{H-H} = N\times K_{H-H}$ $\Sigma K_{H-H} = (3.28\times10^{24})(1.04587\times10^{-20}) = 30.304kJ$ f. $E_{int} = \Sigma K_{H-H} = 30.304kJ$

Worksheet 22: Thermodynamics

1	A. In moving out of a dormitory at the end of the semester, a student does $1.6\times10^4\,J$ of work. In the process, his internal energy

		decreases by $4.2 \times 10^4 J$ J. Determine each of the following quantities – W, ΔU, Q. B. A gas is compressed at a constant pressure of 0.8atm from 9L to 2L. In the process, 400J of energy leaves the gas by heat. What is the work done on the gas? What is the change in its internal energy?
	A. B.	$\boxed{W = +1.6 \times 10^4\ \text{J}}$ $\boxed{\Delta U = -4.2 \times 10^4\ \text{J}}$. $Q = \Delta U + W = \left(-4.2 \times 10^4\ \text{J}\right) + \left(1.6 \times 10^4\ \text{J}\right) = \boxed{-2.6 \times 10^4\ \text{J}}$ $W = -P\Delta V = -(0.800\ \text{atm})(-7.00\ \text{L})\left(1.013 \times 10^5\ \text{Pa/atm}\right)\left(10^{-3}\ \text{m}^3/\text{L}\right) = \boxed{+567\ \text{J}}$ $\Delta E_{\text{int}} = Q + W = -400\ \text{J} + 567\ \text{J} = \boxed{167\ \text{J}}$
2	A. B.	A thermodynamic system undergoes a process in which its internal energy decreases by 500J. At the same time, 220J of work is done **on** the system. Find the energy transferred to or from it by heat. Three moles of an ideal monatomic gas are at a temperature of 345K. Then, 2438J of heat is added to the gas, and 962J of work is done on it. What is the final temperature of the gas?
	A. B.	$\Delta E_{\text{int}} = Q + W$ $Q = \Delta E_{\text{int}} - W = -500\ \text{J} - 220\ \text{J} = \boxed{-720\ \text{J}}$ The negative sign indicates that positive energy is transferred *from* the system by heat. $U = \frac{3}{2} nRT$ $\underbrace{U_{\text{f}} - U_{\text{i}}}_{\Delta U} = \frac{3}{2} nR\left(T_{\text{f}} - T_{\text{i}}\right)$ $T_{\text{f}} = \left(\frac{2}{3nR}\right)(Q - W) + T_{\text{i}}$ $T_{\text{f}} = \left\{\frac{2}{3(3.00\ \text{mol})\left[8.31\ \text{J}/(\text{mol} \cdot \text{K})\right]}\right\}\left[+2438\ \text{J} - (-962\ \text{J})\right] + 345\ \text{K} = \boxed{436\ \text{K}}$
3	A.	Sketch a PV diagram of the following process: A – 2.5L of ideal gas at atmospheric pressure is cooled at constant pressure to a volume of 1.0 L, B – and then expanded isothermally back to 2.5L, C – whereupon the pressure is increased at constant volume until the original pressure is reached.

<table>
<tr><td></td><td>B. One liter of air is cooled at constant pressure until its volume is halved, and then it is allowed to expand isothermally back to its original volume. Draw the process on a PV diagram.</td></tr>
<tr><td></td><td>A.


B.

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<tr><td>4</td><td>Heat is added isothermally to 2.5mol of a monatomic ideal gas. The temperature of the gas is 430K. How much heat must be added to make the volume of the gas double?</td></tr>
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$Q = \Delta U + W$

$\Delta U = 0 \text{ J}$

$W = nRT \ln\left(\frac{V_f}{V_i}\right)$

$Q = \Delta U + W = W = nRT \ln\left(\frac{V_f}{V_i}\right)$

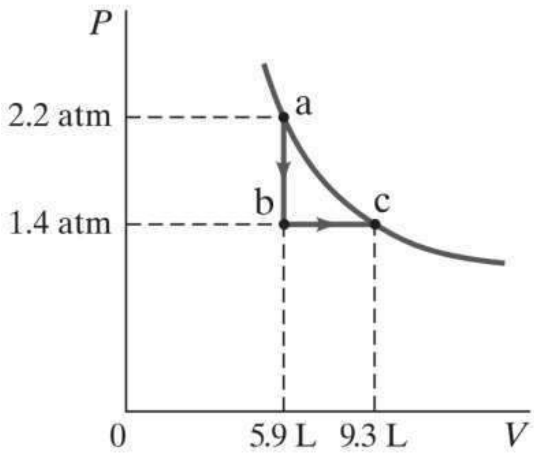
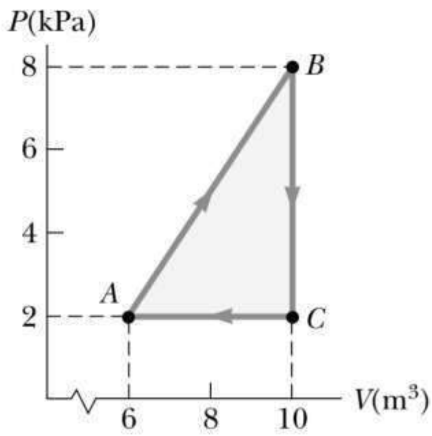
$V_f = 2\, V_i.$

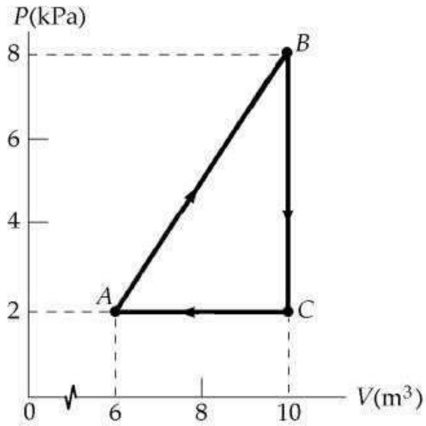
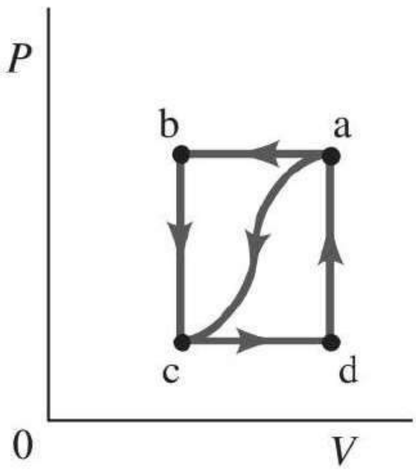
$Q = nRT \ln\left(\frac{V_f}{V_i}\right)$

$Q = (2.5 \text{ mol})[8.31 \text{ J}/(\text{mol} \cdot \text{K})](430 \text{ K}) \ln\left(\frac{2\cancel{V_i}}{\cancel{V_i}}\right)$

$Q = \boxed{6200 \text{ J}}$

</td></tr>
<tr><td>5</td><td>a. 8.5moles of an ideal monatomic gas expand adiabatically, performing 8300J of work in the process. What is the change in temperature of the gas during this expansion?
b. The work done by one mole of a monatomic ideal gas in expanding adiabatically is 825J. The initial temperature and volume of the gas are 393K and $0.1m^3$. Obtain the final temperature of the gas.</td></tr>
</table>

	a. $\Delta U = Q - W \quad \rightarrow \quad \frac{3}{2}nR\Delta T = 0 - W \quad \rightarrow$ $\Delta T = -\frac{2}{3}\frac{W}{nR} = -\frac{2(8300\text{ J})}{3(8.5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = -78.3\text{ K} \approx \boxed{-78\text{ K}}$ b. $T_f = T_i - \frac{W}{\frac{3}{2}nR} = 393\text{ K} - \frac{825\text{ J}}{\frac{3}{2}(1.00\text{ mol})[8.31\text{ J/(mol}\cdot\text{K)}]} = \boxed{327\text{ K}}$
6	Consider the following two-step process. Heat is allowed to flow out of an ideal gas at constant volume so that its pressure drops from 2.2atm to 1.4atm. Then the gas expands at constant pressure, from a volume of 5.9L to 9.3L, where the temperature reaches its original value.  Calculate the total work done by the gas in the process, the change in internal energy of the gas in the process, and the total heat flow into or out of the gas.
	$W = P\Delta V = (1.4\text{ atm})\left(\frac{1.01\times10^5\text{ Pa}}{1\text{ atm}}\right)(9.3\text{ L} - 5.9\text{ L})\left(\frac{1\times10^{-3}\text{ m}^3}{1\text{ L}}\right) = \boxed{480\text{ J}}$ $\Delta U = \boxed{0}$. $\Delta U = Q - W \rightarrow Q = \Delta U + W = 0 + 480\text{ J} = \boxed{480\text{ J (into the gas)}}$
7	A gas is taken through the cyclic process described in the figure shown. Find the net energy transferred to the system by heat during one complete cycle. If the cycle is reversed—that is, the process follows the path ACBA—what is the net energy input per cycle by heat? 

	$Q = -W = \text{Area of triangle}$ $Q = \frac{1}{2}(4.00\ \text{m}^3)(6.00\ \text{kPa}) = \boxed{12.0\ \text{kJ}}$ $Q = -W = \boxed{-12.0\ \text{kJ}}$ 
8	.When a gas is taken from a to c along the curved path in the figure shown, the work done by the gas is $W = -35J$ and the heat added to the gas is $Q = -175J$. Along path abc, the work done by the gas is $W = -56J$. (That is, 56 J of work is done on the gas.) What is Q for path abc? If $P_c = \frac{1}{2}P_b$, what is W for path cda? What is Q for path cda? What is $U_a - U_c$? If $U_d - U_c = 42J$, what is Q for path da? 
	$\Delta U_{ac} = Q_{ac} - W_{ac} = -63\ \text{J} - (-35\ \text{J}) = -28\ \text{J}$ $\Delta U_{ca} = -\Delta U_{ac} = 28\ \text{J}.$ $\Delta U_{abc} = Q_{abc} - W_{abc} \quad \rightarrow \quad Q_{abc} = \Delta U_{abc} + W_{abc} = -28\ \text{J} + (-54\ \text{J}) = \boxed{-82\ \text{J}}$ $W_{cda} = W_{cd} = P_c \Delta V_{cd} = P_c(V_d - V_c) = \frac{1}{2}P_b(V_a - V_b) = -\frac{1}{2}W_{abc} = -\frac{1}{2}(-54\ \text{J}) = \boxed{27\ \text{J}}$ $\Delta U_{cda} = Q_{cda} - W_{cda} \quad \rightarrow \quad Q_{cda} = \Delta U_{cda} + W_{cda} = 28\ \text{J} + 27\ \text{J} = \boxed{55\ \text{J}}$ $U_c - U_a = \Delta U_{ca} = -\Delta U_{ac} = \boxed{28\ \text{J}}.$ $U_d - U_c = 12\ \text{J} \quad \rightarrow \quad U_d = U_c + 12\ \text{J}$ $\Delta U_{da} = U_a - U_d = U_a - U_c - 12\ \text{J} = \Delta U_{ca} - 12\ \text{J} = 28\ \text{J} - 12\ \text{J} = 16\ \text{J}$ $\Delta U_{da} = Q_{da} - W_{da} \rightarrow Q_{da} = \Delta U_{da} + W_{da} = 16\ \text{J} + 0 = \boxed{16\ \text{J}}$

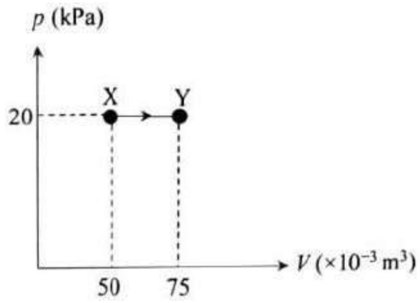
Worksheet 23: PYQ Molecular Energies & Thermodynamics

21/22	A. What is the pressure of one mole ideal gas in the container of volume $4 \times 10^{-4} m^3$ at temperature $363.15K$?

B. Given the molar mass of oxygen is 32 $gmol^{-1}$. What is the root mean square speed of the oxygen molecules at a temperature of 333K.

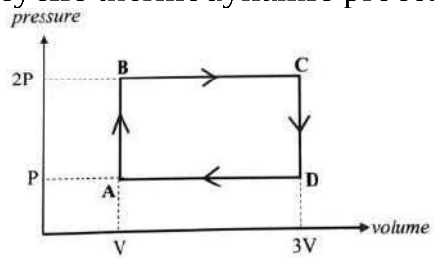
C. A balloon contains helium gas at 30^oC and $2 \times 10^{-5}\,Pa$. Calculate the number of helium gas molecules per unit volume.

D. The figure shows a graph of pressure, *p* against volume, *V* of an ideal gas.



When the gas changes from state X to state Y, the amount of heat transfer into the gas is 1.0kJ. What is the internal energy of the gas?

E. The figure shows a graph of pressure versus volume of an ideal gas undergoing a cyclic thermodynamic process **ABCDA.**



Calculate the work done by the gas.

A. $pV = nRT \Rightarrow p = \frac{nRT}{V} = \frac{(1)(8.31)(363.15)}{4(10^{-4})}$

$\Rightarrow p = 75.4(10^5)\,Pa \approx 75.4atm$

B. $1mol = 32g = 6.02 \times 10^{23}$oxygen gas molecules

$1\text{molecule} = \frac{32}{6.02(10^{23})}(10^{-3})kg$

$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$

$\Rightarrow \frac{1}{2}\left(\frac{32}{6.02(10^{23})}(10^{-3})\right)v_{rms}^2 = \frac{3}{2}(1.38 \times 10^{-23})(333)$

$v_{rms} \approx 509.27ms^{-1}$

C. $pV = Nk_BT \Rightarrow \frac{N}{V} = \frac{p}{k_BT} = \frac{2(10^{-5})}{(1.38\times10^{-23})(30+273.15)}$

$\frac{N}{V} = 4.78(10^{15})\text{molecules } m^{-3}$

	D. $\Delta U = \Delta Q + \Delta W = \Delta Q - \int_{V_i}^{V_f} p\, dV$ $\Delta U = \Delta Q - p(V_f - V_i)$ $\Delta U = (+10^3) - (20)(10^3)(75 - 50)(10^{-3})$ $\Delta U = 500J$ E. $W_{total} = W_{A\to B} + W_{B\to C} + W_{C\to D} + W_{D\to A}$ $W_{total} = 0 + 2P(3V - V) + 0 + P(V - 3V)$ $W_{total} = 2PV$
20/21	1. A container contains $3.0mol$ of nitrogen gas at 30^oC. If nitrogen gas behaves like an ideal gas, calculate the a. Total translational kinetic energy of the gas molecules. b. Internal energy of the gas. c. Root mean square speed of the nitrogen molecules if the mass is 28 g per mole. 2. The pressure of a tyre rises from 200kPa to 400kPa at constant temperature 30^oC. Assuming the air in the tyre acts as an ideal gas, calculate the a. Work done per mole of the air. b. Heat transferred in this process.
	1. a. $\Sigma K = \frac{3}{2}nRT = 3\left(\frac{3}{2}\right)(8.31)(30 + 273.15)$ $\Rightarrow \Sigma K = 11.336kJ$ b. $U = \frac{5}{2}nRT = \frac{5}{2}(8.31)(30 + 273.15)$ $\Rightarrow U = 18.893kJ$ c. $\frac{1}{2}\left(\frac{28\times10^{-3}}{6.02(10^{23})}\right)v_{rms}^2 = \frac{3}{2}(1.38\times10^{-23})(30 + 273.15)$ $\Rightarrow v_{rms} \approx 519.455ms^{-1}$ 2. a. Work done by the gas, $W = \int_{V_i}^{V_f} p\, dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT\, ln[\frac{V_f}{V_i}]$ Constant temperature $\Rightarrow p_iV_i = p_fV_f$ $W = nRT\, ln[\frac{p_i}{p_f}] = (1)(8.31)(30 + 273.15)\, ln[\frac{200000}{400000}]$ $\Rightarrow$ Work done by the gas, $W = -1746J$ b. Constant temperature $\Rightarrow \Delta U = 0$ $\Delta U = \Delta Q - \Delta W \Rightarrow \Delta Q = \Delta W = -1746J$
19/20	A. A sealed cylinder contains 1.2×10^{24} helium atoms at initial pressure 1.04×10^5Pa. The cylinder is heated until the final

temperature and the change in the internal energy of the helium gas are 315K and $1.6 \times 10^3 J$. The molas mass of helium is $4\ gmol^{-1}$. Calculate the

i. Density of the helium gas.

ii. Final pressure of the helium gas.

B. A $0.8m^3$ container at 60^oC is filled with 0.6 mol ideal gas. The gas is isothermally compressed to a volume of $0.2m^3$. Then the gas expands isobarically to its initial volume. Calculate the total work done in the processes.

A.

i. $N = 1.2(10^{24})$ atoms; $p_i = 1.05(10^5)Pa$;

$\Delta V = 0;\ T_f = 315K;\ \Delta U = 1.6(10^3)J$

$$\rho = \frac{m_{gas}}{V};\ n = \frac{N}{N_A} = \frac{12\times10^{23}}{6.02\times10^{23}} = 1.993mol;$$

$$n = \frac{m_{gas}}{m_{molar}} = \frac{\rho V}{m_{molar}}$$

$$pV = nRT = \left(\frac{\rho V}{m_{molar}}\right)RT \Rightarrow p_i = \left(\frac{\rho}{m_{molar}}\right)RT_i$$

$$\Delta U = \frac{3}{2}nR(T_f - T_i) \Rightarrow T_i = T_f - \frac{2\Delta U}{3nR}$$

$$p_i = \left(\frac{\rho}{m_{molar}}\right)R\left(T_f - \frac{2\Delta U}{3nR}\right)$$

$$\Rightarrow 1.05(10^5) = \left(\frac{\rho}{0.004}\right)(8.31)\left(315 - \frac{2(1.6)(10^3)}{3(1.993)(8.31)}\right)$$

$$\rho = 0.202\ kgm^{-3}$$

iii. $\Delta V = 0 \Rightarrow \frac{p}{T} = \frac{nR}{V} = \text{constant}$

$$\frac{p_i}{T_i} = \frac{p_f}{T_f} \Rightarrow \frac{1.05(10^5)}{\left(315 - \frac{2(1.6)(10^3)}{3(1.993)(8.31)}\right)} = \frac{p_f}{315}$$

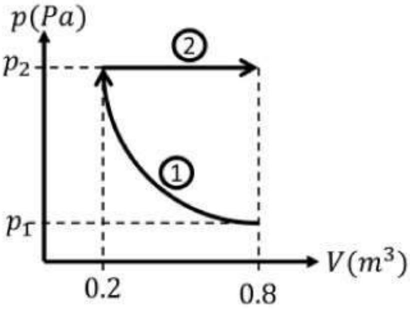
$$p_f = 131986\ Pa$$

B.

$$W_{total} = W_1 + W_2$$

$$W_{total} = \left(\int_{0.8}^{0.2} p\,dV\right) + \left(\int_{0.2}^{0.8} p\,dV\right)$$

$$W_{total} = \left(\int_{0.8}^{0.2} \frac{nRT}{V}dV\right) + \left(p_2\int_{0.2}^{0.8} dV\right)\quad \frac{p_1}{p_2} = \frac{\left(\frac{nRT_1}{0.8}\right)}{p_2} = \frac{0.8}{0.2} = 4$$



	$W_{total} = \left(nRT\ ln\left(\frac{0.2}{0.8}\right)\right) + \left(\frac{nRT_1}{4(0.8)}(0.8 - 0.2)\right)$ $W_{total} = \left((0.6)(8.31)(60 + 273.15)\ ln\left(\frac{0.2}{0.8}\right)\right) + \frac{(0.6)(8.31)(60+273.15)}{4(0.8)}\left(p(0.8 - 0.2)\right)$ Work done by the gas, $W_{total} = -1991.3J$

SP015 LAB MANUAL

Name : ………………………………………………………….. Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 1: MEASUREMENT AND UNCERTAINTY

Course Learning Outcome:
Solve problems related to Physics of motion, force and energy, waves, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to describe technique of measurement and determine uncertainty of length of various objects.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. Complete **Table 1**

Basic Quantity	Symbol	SI Unit (with symbol)	Measuring Instrument
Length	l		
Mass	m		
Time	t		
Electric Current	I		
Temperature	T		

Table 1

2. …………………………….. is used to measure the diameter of a coin.

3. Micrometer screw gauge is usually used to measure the …………………… of a thin wire or the ………………………………. of paper.

4. Complete **Table 2**

Measuring Apparatus	Sensitivity	Uncertainty
Meter rule	0.1 cm	±0.1cm
Vernier calipers	0.01 cm	
Micrometer screw gauge		±0.01mm
Travelling microscope		±0.01mm
Thermometer	0.1°C	
Voltmeter	0.1 V	
Ammeter		±0.1A

Measuring Apparatus	**Sensitivity**	**Uncertainty**
Electronic Balance	0.01 g	

Table 2

5. State **TWO** types of reading;

   i. ....

   ii. ....

6. The repeated reading for a measurement is given as *a, b, c, d, e,* and *f.* Write the equation of Average Value and Uncertainty.

	EQUATION
Average Value, $\bar{x}$	
Uncertainty, $\Delta\bar{x}$	

Experiment

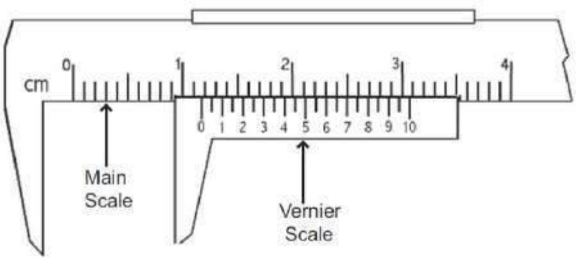
7. Complete **Table 3**

Measurement	**Measuring Instrument**	**Uncertainty/ Smallest scale**	**Type of reading (single point/two point/Vernier scale)**
Length of a metal rod			Two points
Length and width of a laboratory book			Two points
Mass of a ball bearing			Single Point
Diameter of a ball bearing			Vernier scale
Diameter of a coin			Vernier scale
External diameter of a glass rod			Vernier scale

Table 3

8. Determine the reading for the following measurements:

   i.

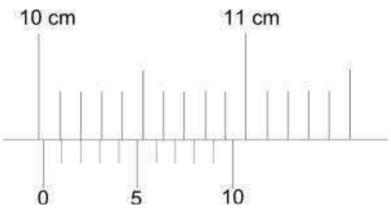


Main scale :…………………..

Vernier scale :…………………..

Actual reading :…………………

   ii.

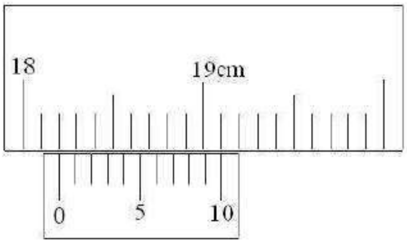


Main scale :…………………

Vernier scale :…………………

Actual reading :………………...

iii.

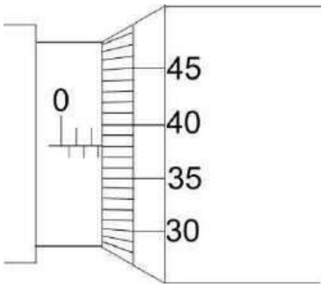


Main scale :………………….

Vernier scale :…………………..

Actual reading:………………….

iv.

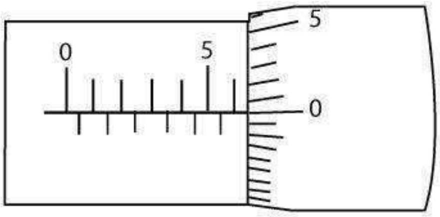


Main scale :………………

Vernier scale :………………

Actual reading :………………

v.



Main scale :…………………..

Vernier scale :…………………..

Actual reading:………………….

9. The repeated readings of the diameter, d of a ball bearing are 2.50 mm, 2.52 mm, 2.51 mm and 2.50 mm.

   i. Calculate the Average Value and Uncertainty. Write the result as $(\overline{d} \pm \Delta\overline{d})$

ii. What instrument/apparatus is used for this measurement?

…………………………………………………………………………………………….

iii. From 10.1, calculate the volume, V of the ball bearing.

iv. Write the result as $(\bar{V} \pm \Delta\bar{V})$

…………………………………………………………

Data Analysis

10. Complete **Table 4.**

No	**Length of Scientific Calculator (Model Casio fx-570ES PLUS),** L **(cm)**	$\lvert\bar{L} - L_i\rvert$ **(cm)**
1	15.42	
2	15.55	
3	15.30	
4	15.48	
5	15.49	
6	15.45	
7	15.55	
	Average, $\bar{L}$=	$\Delta\bar{L}$=

Table 4

11. Express your answer as $(\bar{L} \pm \Delta\bar{L})$

12. Calculate the percentage of uncertainty,

13. State THREE precautions of this experiment:

i. ……………………………………………………………………………………………...

ii. ……………………………………………………………………………………………..

iii. …………………………………………………………………………………………….

Name : ………………………………………………………….. Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 2 : FREE FALL AND PROJECTILE MOTION

Course Learning Outcome:
Solve problems related to **Physics of motion**, force and energy, waves, matter and thermodynamics **(C4, CLO 2, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to describe experiment to determine acceleration due to gravity using free fall and projectile motion.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is meant by free fall motion?

   …………………………………………………………………………………………………

   …………………………………………………………………………………………………

2. Under free fall motion the acceleration of an object is also known as gravitational acceleration or acceleration due to gravity. What is the symbol and SI unit of this type of acceleration?

   …………………………………………………………………………………………………

3. What is the value of acceleration due to gravity at the surface of Earth?

   …………………………………………………………………………………………………

4. Projectile motion of an object is the motion of an object which is projected or thrown. Under a gravitational field **when the air resistance is not present**, projectile motion can be considered as a free fall motion. State **TWO** differences between free fall motion and projectile motion?

   …………………………………………………………………………………………………

   …………………………………………………………………………………………………

   …………………………………………………………………………………………………

   …………………………………………………………………………………………………

   …………………………………………………………………………………………………

5. State the law applied in these experiment

   …………………………………………………………………………………………………

Experiment

6. How do we release the steel ball to form

   (a) free fall motion

   …………………………………………………………………………………………

   …………………………………………………………………………………………

   (b) Projectile motion

   …………………………………………………………………………………………

   …………………………………………………………………………………………

   …………………………………………………………………………………………

7. State the measurement *apparatus* involved. (e.g. type / name of equipment) for both experiment.

   …………………………………………………………………………………………

   …………………………………………………………………………………………

   …………………………………………………………………………………………

8. State the related variables that need to be recorded in this experiment?

	Free fall motion	Projectile motion
Manipulated variable (change on purpose)		
Responding variable (what is measured)		

9. Construct the table to record the related values for free fall and projectile motion experiment.

   (a) Free Fall Motion

   (b) Projectile Motion

10. How do you obtained the value of t for projectile motion from the graph of free fall motion experiment?

……………………………………………………………………………………………………

……………………………………………………………………………………………………

Data Analysis

11. a) Write the equations related to both experiments in order to determine the acceleration due to gravity, g.

b) Sketch a suitable graph for

i) Free fall motion

ii) Projectile motion

c) How the acceleration due to gravity, g can be determine from the graphs.

12. List down the precautions of the experiments.

a) ……………………………………………………………………………………………...

b) ……………………………………………………………………………………………...

c) ……………………………………………………………………………………………...

Name : ………………………………………………………..   Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 3: ENERGY

Course Learning Outcome:
Solve problems related to Physics of motion, **force and energy**, waves, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to explain the experiment to determine the acceleration due to gravity, g from the experiment.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. State the law of conservation of energy.

   …………………………………………………………………………………………………

2. State the gravitational potential energy and kinetic energy.

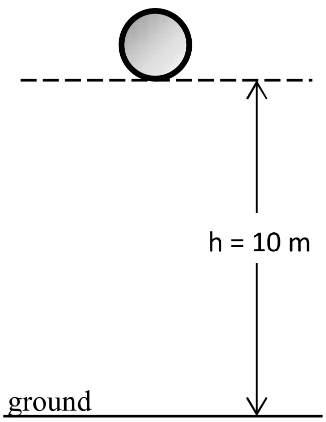
   …………………………………………………………………………………………………

   …………………………………………………………………………………………………

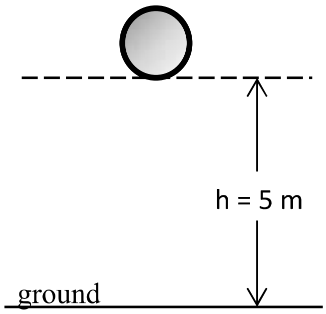
3. What is the symbol and SI unit of gravitational potential energy and kinetic energy?

Energy	gravitational potential energy	kinetic energy
Symbol		
Unit		

4. Based on the situations below, answer the questions:



SITUATION A



SITUATION B

a) Using the conservation of energy, determine the velocity of the ball just before it reaches the ground.

b) From the answers calculated in question (a), what can we deduce about the relation between the released height and the velocity of the ball before hitting the ground?

…………………………………………………………………………………………………

Experiment

5. What is the energy owned by the ball bearing when it is attached to the free fall adapter?

   …………………………………………………………………………………………………

6. What is the usage of the photo gate?

   …………………………………………………………………………………………………

7. State the change in mechanical energy in this experiment.

   …………………………………………………………………………………………………

8. State the related variables that need to be recorded in this experiment?
   a) Manipulated variable

      ………………………………………………………………………………………………

   b) Responding variable

      ………………………………………………………………………………………………

9. How the final velocity of ball bearing is determined?

Data Analysis

10. An equation for a straight line graph is y = mx + c, where y is the quantity on the vertical axis and x is the quantity on the horizontal axis as shown in **FIGURE 1**.

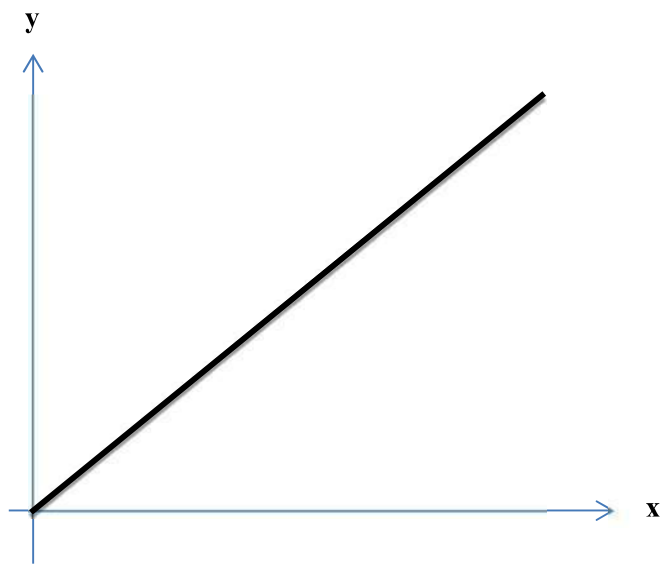


FIGURE 1

The velocity of ball bearing, v is related to the height of released (h) by the following equation:

$$v^2 = 2gh \quad \_\_\_\_\_\_\_\_\_\_ (1)$$

where g is the acceleration due to the gravity.

a) Based on the equation (1) and the graph, determine the variables for x axis and y axis

b) From the graph what does the gradient, m represents?

……………………………………………………………………………………………

c) From the gradient of the graph, how can we determine the value of g.

11. List **THREE** precautions of the experiment:

i. ……………………………………………………………………………………………

ii. ……………………………………………………………………………………………

iii. ……………………………………………………………………………………………

12. State two types of errors during experiment and give an example for each error.

……………………………………………………………………………………………

……………………………………………………………………………………………

13. Based on the situation below identify either random or systematic error.

Situation	Random Error/Systematic Error
Wind keeps blowing in the surrounding using the experiment. This shall affect the velocity measured in this experiment. The best way to solve this is by conducting this experiment in the closed area or vacuum space.	
Some of the numbers on the timer's display was broken and missing. Thus the reading can be taken only to the nearest decimal point.	
Instead of using the hand to release the ball bearing, it is suggested that the ball can be released using the automatic control or trigger.	
Sometimes the time measured is hardly detected by the photo gates. This is due to the position of the gates where the ball bearing failed to hit the motion sensor. Therefore, the free fall adapter and photo gates must be realigned properly.	

Name : ……………………………………………………….. Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 4: ROTATIONAL MOTION OF A RIGID BODY

Course Learning Outcome:
Solve problems related to Physics of motion, **force and energy**, waves, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to explain the experiment to determine the moment of inertia of a fly-wheel from experiment.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is a rigid body?

   …………………………………………………………………………………………………

2. What is meant by moment of inertia?

   …………………………………………………………………………………………………

   …………………………………………………………………………………………………

3. What is the symbol and SI unit for moment of inertia?

   …………………………………………………………………………………………………

4. Moment of inertia depends on …………………..…. and …………………………….

5. Complete **TABLE 4** with correct analogues between linear motion and rotational motion.

Linear Motion	Rotational Motion
Mass, m	
Acceleration, a	
Net force, F	

6. A motor capable of producing a constant torque of 100 Nm is connected to a flywheel which rotates with an angular acceleration of 1000 rad s^{-2}. Calculate moment of inertia of the flywheel.

Experiment

7. Sketch a free body diagram for fly-wheel and falling slotted mass.
   a) Free body diagram of fly-wheel b) Free body diagram of falling slotted mass

8. By referring to the free body diagram in 7(a) and 7(b), deduce equation by using Newton's 2nd Law of motion.

9. For this experiment, identify

   a) the manipulated variable

   ……………………………………….

   b) the responding variable

   ……………………………………….

10. Complete the observation table with the suitable equation.

Acceleration	Angular acceleration	Tension in the string

Data Analysis

11. Write the equation of the graph of α against T

12. Base on the linear graph equation $y = mx + c$, fill in the suitable quantity by referring the equation in question 11 :

    a) *y-axis* :………………………………..
    b) *x-axis* : ………………………………..
    c) gradient, *m* : ………………………………..
    d) *y*–interception : ………………………………..

13. How do we determine the value of inertia of a fly-wheel from this graph?

14. List **THREE** precautions of this experiment
    i. …………………………………………………………………………………………………
    ii. …………………………………………………………………………………………………
    iii. …………………………………………………………………………………………………

Name : ……………………………………………………….. Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 5: SIMPLE HARMONIC MOTION (SHM)

Course Learning Outcome:
Solve problems related to Physics of motion, force and energy, **waves**, matter and thermodynamics
(C4, PLO 4, CTPS 3, MQF LOD 6)

Learning Outcomes:
At the end of this lesson, students will able to:
1. explain the experiment to determine the acceleration due to gravity, g using a simple pendulum.
2. describe the effect of large amplitude oscillation to the accuracy of g obtained from the experiment

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is a simple pendulum?

   ……………………………………………………………………………………………

2. Motion of an object that returns to its initial position after a fixed time interval is called

   ……………………………

3. In SHM, state two quantities that proportional to the object's displacement
   i. ……………………………..
   ii. ……………………………..

4. The condition for the simple pendulum to perform SHM are
   a) The mass of the spherical bob is ……………………..
   b) The ………………………of the string is negligible
   c) Amplitude of oscillation is ……………………………

5. Does the period of oscillation of simple pendulum depend on mass?

   (Yes / No)

Experiment

6. How to determine the period of a simple pendulum for a given number, n of oscillation?

   ……

7. If we vary the length of a pendulum, the period will change. Construct an appropriate table to record the data of length, l, time taken, t and corresponding T and T^2.

8. What is the title of the graph that needs to be plotted in this experiment?

   ……

9. Which procedure that investigates the effect of large amplitude of oscillation and state the related angle used.

   ……

Data Analysis

10. How to determine the value of g from the gradient of the graph.

11. How to calculate the percentage of error between the value $g_{experiment}$ and $g_{standard}$? Take $g_{standard} = 9.81\ \text{m s}^{-2}$.

12. Predict what would happen to the value of g if **large amplitude** is used.

    ……

13. List **THREE** precautions of this experiment

    i. ……………………………………………………………………………………………………

    ii. ……………………………………………………………………………………………………

    iii. ……………………………………………………………………………………………………

Name : ……………………………………………………….. Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 6: STANDING WAVES

Course Learning Outcome:
Solve problems related to Physics of motion, force and energy, **waves**, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to explain the experiment to investigate standing waves formed in stretched string.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is the meaning of standing waves?

   ……………………………………………………………………………………………………

2. Sketch standing wave formed in a stretch string and label the node (N) and antinode (A).

3. How standing wave is formed?

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

4. What is the symbol and SI unit for mass per unit length?

   ……………………………………………………………………………………………………

Experiment

5. State the manipulative and responding variables in this experiment.

   ……………………………………………………………………………………………………

6. Construct the table for the value of m and l.

7. Sketch free body diagram to show that T = W.

8. Suggest a way to determine the actual value for mass per unit length of the string/wire used in this experiment.

   ……

   ……

9. Suggest how to identify the position of two consecutive nodes formed in the string / wire.

   ……

<u>Data analysis</u>

10. Write the equation that relates period, T and frequency, f.

11. Sketch the graph to show the relationship between T and ℓ^2.

12. Construct the observation table.

13. How do you determine the mass per unit length from this graph?

    ……

14. Throughout the experiment the terminals are connected to AC power supply. In your opinion why does this essential?

SP015 LAB REPORT SAMPLE

Experimental Physics

Simple Pendulum for Measurement of g

Shafiq Rasulan
SP015
Sarawak Matriculation College

20/8/2022

Keywords: gravitational acceleration, Earth, Pendulum, small angle approximation

Abstract

In this experiment, we measured the value for the gravitational acceleration near the Earth's surface using a simple pendulum setup. We recreate this trivial experiment as part of our SP015 laboratory sessions. The experiment proved to be successful as the measured value of g was found to only deviate from the theoretical value by 0.0340652%.

Contents

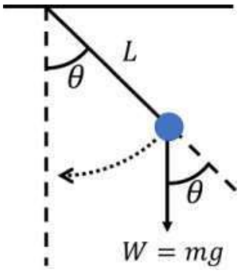
Abstract 1
1 Introduction 2
2 Methodology 2
3 Data Analysis 2
- Theoretical Data Table 2
- Experimental Data Table 3
- Table of Uncertainties 3
- Graphs (Computer-Generated) 4
- Graphs (Hand-drawn) 4
- Determination of experimental value of g and its uncertainty 5
- Percentage Error 5

4 Discussion 5
5 Conclusion 5
Bibliography 5

1 Introduction

The first pendulum clock, invented by Christiaan Huygens in 1656, utilised the properties of a simple pendulum to keep track of time. [1] Apart from being the mechanics behind modern time keeper, they serve a starting point in studying oscillations. In fact, students often learn about simple pendulums as a case study when first introduced to simple harmonic motion.

Simple pendulums are relatively simply devices Before describing the recreated methods and the results obtained from our experiment, it is useful to review the theoretical foundation of the experiment.



Consider a mass of mass m suspended at the end of a string of length L, the restoring force for the motion of the pendulum is then

$$F = -mg\sin\theta = mL\frac{d^2\theta}{dt^2}$$

Which simplifies into

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta.$$

If we assume $\omega = 0$ when $t = 0$, then the analytical solution to this differential equation is simply

$$\theta(t) = A\sin\left(\sqrt{\frac{g}{L}}t\right).$$

We can then apply small angle approximations, $\sin\theta \approx \theta$, which yields the following equation for the period of the oscillation T,

$$T^2 = \frac{4\pi^2}{g}L - - - (1)$$

This equation serves the theoretical basis to our experiment in measuring g.

2 Methodology

To begin, we needed a theoretical value of the gravitational acceleration locally. The experiment was carried out in the college (Latitude: 1.597533^oN) at an elevation of approximately 7m. We referred to the SensorsONE website [2] to obtain the theoretical value of g, which is $9.78035ms^{-2}$.

The set up of the experiment was relatively simple. We suspended a pendulum bob by a string of varying length L, measured using a ruler of sensitivity (0.001m). The pendulum bob was then pulled to its release point, set to be at 10^o from the vertical. A protractor (of sensitivity 1^o) was used for this purpose. The pendulum bob was then released and allowed to make 10 oscillations and that time for 10 oscillations, T_{10} was recorded with a digital stopwatch of sensitivity 0.01s. The period was then be calculated from T_{10} by dividing it by 10.

Squaring the period gives us the period squared,T^2 and a graph of T^2 against L was plotted and analysed. The resulting graph yields the experiment value of the gravitational acceleration g_{exp} and its uncertainty, this is then compared to the theoretical value of the gravitational acceleration. To minimize the effects of air resistance onto the pendulum bob, we made sure the fans were switched off throughout the experiment and that the windows are also closed. We were also mindful in ensuring no parallax error by making sure all measurements done at parallel to eye level.

3 Data Analysis

In this section we present the theoretical values of T^2 and L based on equation (1).

Theoretical Data Table

Length of string. L (cm)	Period of oscillation squared, $T^2(s^2)$
0.100	0.40
0.200	0.81
0.300	1.21
0.400	1.61
0.500	2.02
0.600	2.42
0.700	2.83
0.800	3.23
0.900	3.63
1.000	4.04

Experimental Data Table

Length of string. L $(\pm 0.001m)$	Time for 10 oscillations, T_{10} $(\pm 0.01s)$	Period, $T(\mathrm{s})$	Period Squared, $T^2(s^2)$
0.100	6.34	0.63	0.40
0.200	8.97	0.90	0.81
0.300	10.99	1.10	1.21
0.400	12.69	1.27	1.61
0.500	14.19	1.42	2.02
0.600	15.54	1.55	2.40
0.700	16.78	1.68	2.82
0.800	17.94	1.79	3.20
0.900	19.03	1.90	3.61
1.000	20.06	2.01	4.04

Table of Uncertainties

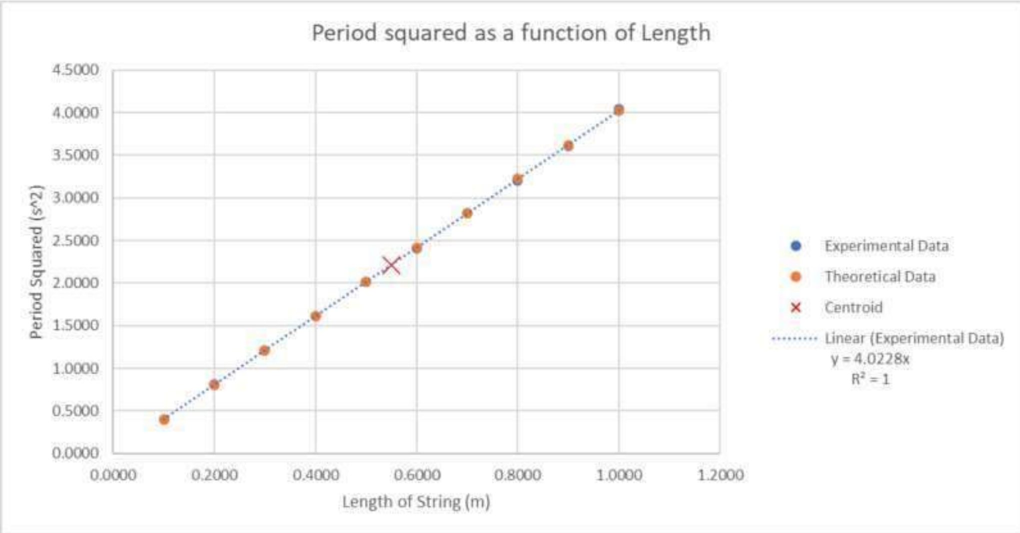
$\bar{l} = 0.550\mathrm{m}$
$\hat{T}^2 = 4.0228l$

l	$l-\bar{l}$	$(l-\bar{l})^2$	T^2	$\hat{T}^2$	$T^2-\hat{T}^2$	$(T^2-\hat{T}^2)^2$
0.100	-0.450	0.203	0.40	0.40	0.00	0.0000
0.200	-0.350	0.123	0.81	0.80	0.01	0.0001
0.300	-0.250	0.063	1.21	1.21	0.00	0.0000
0.400	-0.150	0.023	1.61	1.61	0.00	0.0000
0.500	-0.050	0.003	2.02	2.01	0.01	0.0001
0.600	0.050	0.003	2.40	2.41	-0.01	0.0001
0.700	0.150	0.023	2.82	2.82	0.00	0.0000
0.800	0.250	0.063	3.20	3.22	-0.02	0.0004
0.900	0.350	0.123	3.61	3.62	-0.01	0.0001
1.000	0.450	0.203	4.04	4.02	0.02	0.0004
Σl_i = 5.5000		$\Sigma(l_i-\bar{l})^2$ = 0.830				$\Sigma(T_i^2-\hat{T}_i^2)^2$ = 0.0012

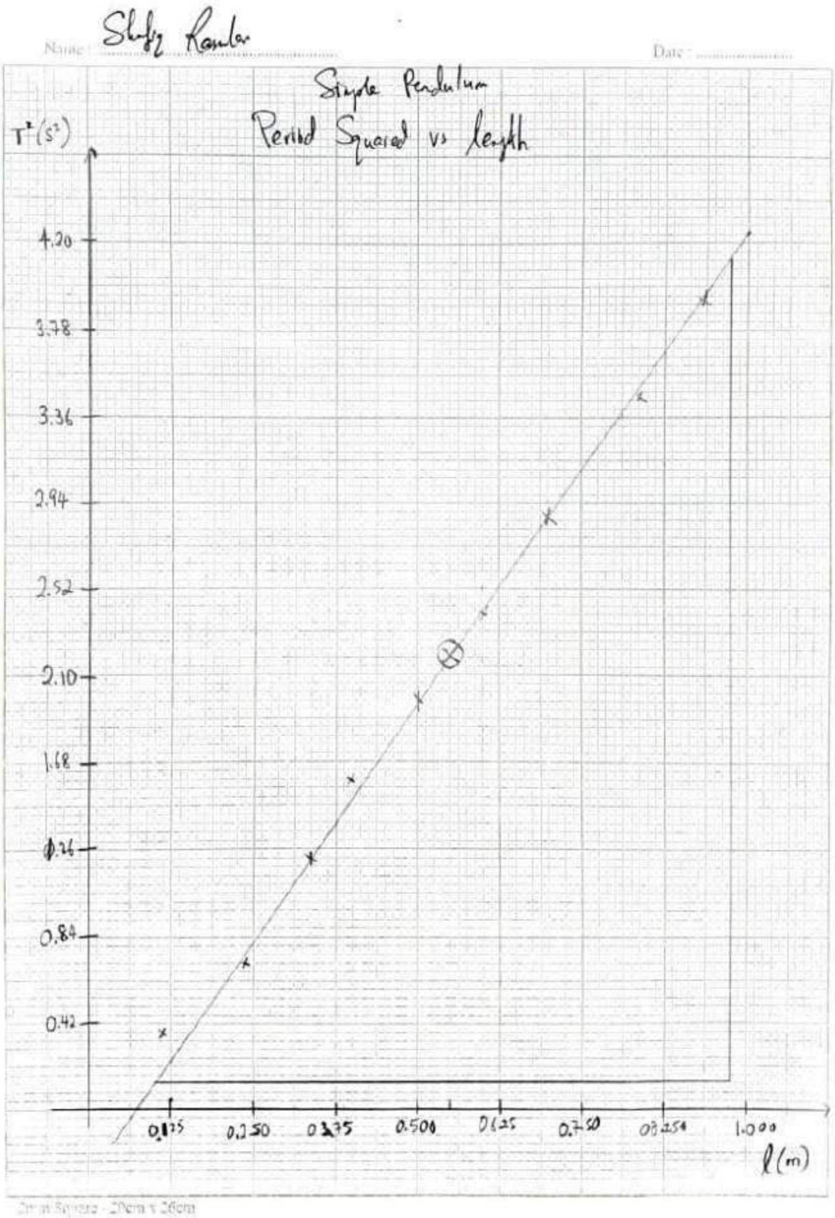
$$\Delta m = \sqrt{\frac{\Sigma(T_i^2-\hat{T}_i^2)^2}{(10-2)\Sigma(l_i-\bar{l})^2}} = \sqrt{\frac{0.0012}{8(0.830)}} = 0.013s^2m^{-1}$$

$$\Delta c = \sqrt{\left(\frac{\Sigma(T_i^2-\hat{T}_i^2)^2}{10-2}\right)\left(\frac{1}{10}+\frac{\bar{l}^2}{\Sigma(l_i-\bar{l})^2}\right)} = \sqrt{\left(\frac{0.0012}{10-2}\right)\left(\frac{1}{10}+\frac{(0.550)^2}{0.830}\right)} = 0.0084s^2$$

Graphs (Computer-Generated)



Graphs (Hand-drawn)



Determination of experimental value of g and its uncertainty

Since $m = 4.023 = \frac{4\pi^2}{g}$, then

$$g_{exp} = \frac{4\pi^2}{4.023} \approx 9.813\ ms^{-2}$$

This would also mean

$$\frac{dm}{dg} \approx \frac{\Delta m}{\Delta g} = -\frac{4\pi^2}{g^2}$$
$$\Delta g = \frac{g^2}{4\pi^2}\Delta m$$
$$\Delta g = \frac{9.81^2}{4\pi^2}(0.013) = 0.032ms^{-2}$$
$$g_{exp} \pm \Delta g = (9.813 \pm 0.032)ms^{-2}$$

Percentage Error

$$\%_{error} = \frac{|g_{exp} - g_{theory}|}{g_{theory}} \times 100\%$$
$$\%_{error} = \frac{|9.813 - 9.78035|}{9.78035} \times 100\%$$
$$\%_{error} = 0.3338\%$$

4 Discussion

Two possible error that could occur in this experiment are:

1. Parallax error – this causes misreading in the height of release.
2. Air resistance – air flow in the lab can affect the free fall of the ball bearings.

Two recommendations to minimize effects of the errors:

1. Ensure eyes are perpendicular to ruler
2. Ensure windows are closed and fans are turned off.

5 Conclusion

1. In this experiment, we measured the value of of gravitational acceleration g near the surface of the Earth to be $(9.813 \pm 0.032)ms^{-2}$.
2. The theoretical value of g which is $9.78035ms^{-2}$.
3. The percentage error of this value was found to be 0.3338%.
4. Based on this information, the experiment is considered as a success.
5. The theoretical value of g cannot be found between the lower ($9.781ms^{-2}$) and upper bound ($9.845ms^{-2}$) of the experimental value.
6. Based in this info, this experiment could not be considered to be a success.

Bibliography

[1] E. Klarreich, "Huygens's clocks revisited.(Science Observer)," *American Scientist,* vol. 90, no. 4, pp. 322--324, 2002.

[2] -, "Local Gravity Calculator," SensorsONE Ltd, 2022. [Online]. Available: https://www.sensorsone.com/local-gravity-calculator/. [Accessed 23 August 2022].

SP015 IN-CLASS QUIZZES

Quiz Collection for SP015 2023/2024

QUIZ LIST 1

QUIZ 1: KINEMATICS 1

QUIZ 2: NEWTON'S LAWS 4

QUIZ 3: SHM & WAVES 7

QUIZ 4: PHYSICS OF MATTER 10

QUIZ 5: THERMODYNAMICS 12

SP015 EQUATIONS 15

Quiz List

Sheet No.	Quiz Title
1	Kinematics
2	Newton's Laws
3	SHM & Waves
4	Physics of Matter
5	Thermodynamics

Quiz 1: Kinematics

Problems

$$v = u + at;\ s = ut + \frac{1}{2}at^2;\ v^2 = u^2 + 2as;\ s = \frac{1}{2}(u+v)t$$

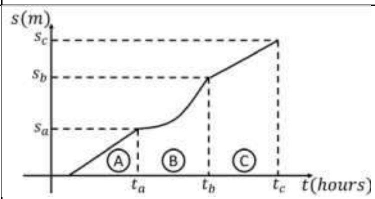
1 **[On 1D Kinematics]**
A VW Beetle goes from 0 to 60 miles per hour with an acceleration of $+2.35ms^{-2}$. How much time does it take for the Beetle to reach this speed? A top-fuel dragster can go from 0 to 60 miles per hour in 0.6s. Find the acceleration (in ms^{-2}) of the dragster. Note that $0.4470ms^{-1} = 1\text{mile } h^{-1}$. **[4 marks]**

Steps	Marks
$\Delta t = \frac{v-u}{a_{beetle}} = \frac{(60-0)(0.4470)}{2.35}$	G1
$\Delta t = 11.4s$	JU1
$a_{dragster} = \frac{v-u}{\Delta t} = \frac{(60-0)(0.4470)}{0.6}$	G1
$a = 44.7ms^{-2}$	JU1

2 **[On 2D Kinematics]**
A cyclist travels 40km at $20kmh^{-1}$ due north then travels due east for 2 hours at $15kmh^{-1}$. Determine the magnitude of displacement and total time of travel of the cyclist. **[4 marks]**

Step	Marks
$s_{total} = \sqrt{s_1^2 + s_2^2} = \sqrt{s_1^2 + (u_2 \times t_2)^2}$ $s_{total} = \sqrt{40^2 + (15 \times 2)^2}$	G1
$s_{total} = 50m$	JU1
$t_{total} = t_1 + t_2 = \frac{s_1}{u_1} + t_2 = \frac{40}{20} + 2$	G1
$t_{total} = 4$ hours	JU1

3 **[On 1D Kinematics]**
A train travels at a uniform velocity $40kmh^{-1}$. After 30 minutes, the train then accelerates to a velocity of $110kmh^{-1}$ over a distance of 50km. At $110kmh^{-1}$, the train travels a further 22km. Sketch the displacement-time graph for the motion of this train, ensure that your labelling is sufficient. Calculate the total time and distance this train has travelled. Determine the average velocity. **[10 marks]**



1 mark for correct values on x axis
1 mark for correct values on y axis
1 mark for correct shape of graph

A: $t = 30mins = 0.5h$ $s = ut = (40)(0.5) = 20km$	G1
B: $t = \frac{2s}{u+v} = \frac{2(50)}{40+110} = \frac{2}{3}h$	G1

$s = 50km$	
C: $$t = \frac{s}{u} = \frac{22}{110} = 0.2h$$ $$s = 22km$$	G1
Total time $$t_{total} = \frac{1}{2} + \frac{2}{3} + \frac{1}{5} = \frac{41}{30}h \approx 1.367h$$	JU1
Total displacement $$s_{total} = 20 + 50 + 22 = 92\text{km}$$	JU1

Average Velocity,

$v_{ave} = \frac{s_{total}}{t_{total}} = \frac{92}{\left(\frac{41}{30}\right)}$	G1
$v_{ave} = \frac{2760}{41} \approx 67.32kmh^{-1}$	JU1

4 **[On 2D Kinematics]**
A light plane is headed due south with a speed relative to still air of $185kmh^{-1}$. After 1h, the pilot notices that they have covered only 135 km and their direction is not south but 15° east of south. What is the wind velocity? **[6 marks]**

Steps	Marks
$\vec{v}_{resultant} = \vec{v}_{plane} + \vec{v}_{wind}$	
$x: \frac{135km}{1hr} sin\,15^o = 0 + v_{wind-x}$	G1
$v_{wind-x} = 34.94kmh^{-1}$	
$y: -\frac{135km}{1hr} cos\,15^o = -185 + v_{wind-y}$	G1
$v_{wind-y} = 54.60kmh^{-1}$	
$v_{wind} = \sqrt{v_{wind-x}^2 + v_{wind-y}^2}$	
$v_{wind} = \sqrt{34.94^2 + 54.60^2}$	G1
$v_{wind} = 64.82kmh^{-1}$	JU1
$\theta = tan^{-1}\left(\frac{54.60}{34.94}\right)$	G1
$\theta = 57.38^o$ north of east	JU1

5 **[On Projectile Motion]**
A soccer player kicks the ball toward a goal that is 16.8m in front of him. The ball leaves his foot at a speed of $16ms^{-1}$ and an angle of 28^o above the ground. Find the speed of the ball when the goalie catches it in front of the net. **[5 marks]**

Step	Marks
$a_x = 0ms^{-2}$; $u_x = v_x = 16\,cos\,28^o = 14.1ms^{-1}$	G1
$s_x = u_x t = (u\,cos\,\theta)t$ $$t = \frac{s_x}{u\,cos\,\theta} = \frac{16.8}{16\,cos\,(28^o)}$$	G1
$t = 1.19s$	
$v_y = u_y - gt$ $$v_y = u\,sin\,\theta - gt = 16\,sin\,28^o - (9.81)(1.19)$$	G1
$v_y = -4.15ms^{-1}$	
$v = \sqrt{v_y^2 + v_x^2} = \sqrt{(14.1)^2 + (-4.15)^2}$	G1
$v = 14.7ms^{-1}$	JU1

6 **[On Projectile Motion]**

A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of $8ms^{-1}$ at an angle of 20° below the horizontal. It strikes the ground 3s later.
How far horizontally from the base of the building does the ball strike the ground? Find the height from which the ball was thrown. How long does it take the ball to reach a point 10m below the level of launching? **[6 marks]**

Step	Marks
$s_x = u_x t = (u\, cos\, \theta)(t) = (8\, cos\, 20^o)(3)$	G1
$s_x = 22.6m$	JU1
$s_y = u_y t - \frac{1}{2} g t^2 = (8\, sin\, {-20^o})(3) - \frac{1}{2}(9.81)(3)^2$	G1
$s_y = -52.3m$	JU1
$-10 = (8\, sin\, {-20^o})(t) - \frac{1}{2}(9.81)(t)^2$	G1
$t = \{-1.73s \text{ (rejected)}, 1.18s\}$	JU1

7 **[Difficult]** A placekicker is about to kick a field goal. The ball is 26.9m from the goalpost. The ball is kicked with an initial velocity of $19.8ms^{-1}$ at an angle θ above the ground. Determine the angle θ such that the ball goes over a 2.74-m-high crossbar?
Know that $sec^2\, \theta = 1 + tan^2\, \theta$. **[5 marks]**

	Marks
$u_x = u\, cos\, \theta;\ u_y = u\, sin\, \theta$ $s_x = u_x t \Rightarrow t = \frac{s_x}{u_x}$ $s_y = u_y t - \frac{1}{2} g t^2 = u_y \left(\frac{s_x}{u_x}\right) - \frac{1}{2} g \left(\frac{s_x}{u_x}\right)^2$ $s_y = u\, sin\, \theta \left(\frac{s_x}{u\, cos\, \theta}\right) - \frac{1}{2} g \left(\frac{s_x}{u\, cos\, \theta}\right)^2$	K1 for attempting
$s_y = u s_x (tan\, \theta) - \frac{s_x^2 g}{cos^2 \theta}$ $s_y = u s_x (tan\, \theta) - s_x^2 g - s_x^2 g tan^2\, \theta$	
$2.74 = (19.8)(26.9)(tan\, \theta) - (26.9^2)(9.81) - (26.9^2)(9.81) tan^2\, \theta$	G1
$\theta = 28.1^o$ or $\theta = 67.7^o$	JU2
Any release angle obeying $28.1^o \leq \theta \leq 67.7^o$ will allow the ball to clear the post.	J1

Quiz 2: Newton's Laws

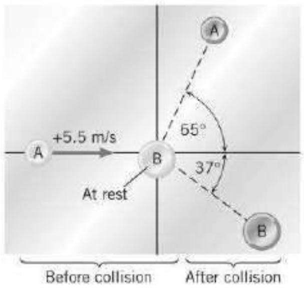
Problems

$$p = mv;\ J = F\Delta t;\ J = \Delta p = mv - mu;\ f = \mu N;\ W = Fs\cos\theta;$$
$$K = \frac{1}{2}mv^2;\ U = mgh;\ U_x = \frac{1}{2}kx^2 = \frac{1}{2}Fx;\ P_{ave} = \frac{\Delta W}{\Delta t};\ P = Fv;$$

1 A baseball (m=149g) approaches a bat horizontally at a speed of $40.2ms^{-1}$ and is hit straight back at a speed of 45.6 ms^{-1}. If the ball is in contact with the bat for a time of 1.10 ms, what is the average force exerted on the ball by the bat? Choose the direction of the incoming ball as the positive direction. **[2 marks]**

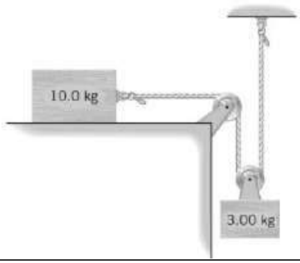
$J = F\Delta t = mv - mu$	
$F = \frac{m(v-u)}{\Delta t} = \frac{(0.149)((-45.6) - 40.2)}{1.1 \times 10^{-3}}$	G1
$F = -11600N$	JU1

2 The figure shows a collision between two pucks on an air-hockey table. Puck A has a mass of 0.025kg and is moving along the x axis with a velocity of +5.5 m/s. It makes a collision with puck B, which has a mass of 0.050kg and is initially at rest. The collision is not head-on. After the collision, the two pucks fly apart with the angles shown in the figure. Find the final speeds of pucks. **[4 marks]**



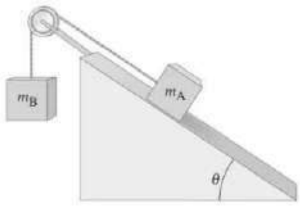
x: $m_A u_A = m_A v_A \cos 65^o + m_B v_B \cos 37^o$	
$(0.025)(5.5) = (0.025)(v_A)\cos 65^o + (0.05)(v_B)\cos 37^o$	G1
y: $0 = m_A v_A \sin 65^o - m_B v_B \sin 37^o$	
$0 = (0.025)(v_A)\sin 65^o - (0.050)(v_B)\sin 37^o$	G1
$v_A = 3.384ms^{-1};\ v_B = 2.548ms^{-1}$	JU2

3 In the figure shown, the rope and the pulleys are massless, and there is no friction. Find the tension in the rope and the acceleration of the 10.0-kg block. **[5 marks]**



$T = m_1 a;\ 2T - m_2 g = m_2\left(-\frac{1}{2}a\right)$ $a = \frac{T}{m_1} = \frac{-2(2T - m_2 g)}{m_2}$	K1
$\frac{T}{10} = \frac{-2(2T - (3)(9.81))}{3}$	G1
$T = 13.69N$	JU1
$a = \frac{T}{m_1} = \frac{13.69}{10}$	G1
$a = 1.37ms^{-2}$	JU1

4	Suppose the coefficient of kinetic friction between m_A and the plane in the figure shown is $\mu = 0.15$ and that $m_A = m_B$. As m_B moves down, determine the magnitude of the acceleration of m_A and m_B, given $\theta = 34^o$. What smallest value of μ will keep the system from accelerating? **[5 marks]**



$F_{yB} = m_B a = m_B g - T$ $F_{yA} = N - m_A g \cos\theta = 0$ $F_{xA} = m_A a = T - m_A g \sin\theta - \mu N$	K1
$T = m_A a + m_A g \sin\theta + \mu(m_A g \cos\theta) = m_B(g - a)$ $a + (9.81)\sin 34^o + (0.15)(9.81)(\cos 34^o) = 9.81 - a$	G1
$a = 1.55 ms^{-1}$	JU1
$a = 0,\ \mu \neq 0.15$: $(9.81)\sin 34^o + (\mu)(9.81)(\cos 34^o) = 9.81$	G1
$\mu = 0.53$	JU1

5	A 16kg sled is being pulled along the horizontal snow-covered ground by a horizontal force of 24N. Starting from rest, the sled attains a speed of $2ms^{-1}$ in 8m. Find the coefficient of kinetic friction between the runners of the sled and the snow. **[3 marks]**

$W = W_{pull} - W_{friction} = \Delta K = K_f - K_i$	K1
$(F_{pull})s - (f)s = (F_{pull})s - (\mu mg)s = \frac{1}{2}mv^2$	
$(24)(8) - (\mu)(16)(9.81)(8) = \frac{1}{2}(16)(2)^2$	G1
$\mu = 0.127$	JU1

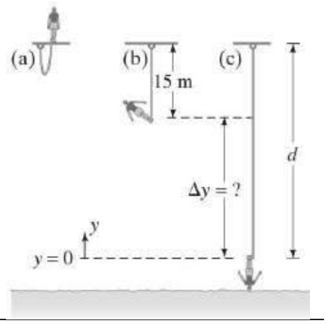
6	A 66.kg hiker starts at an elevation of 1270m and climbs to the top of a peak 2660m high. What is the hiker's change in potential energy? What is the minimum work required of the hiker? **[4 marks]**

$\Delta U = mg\Delta h$	
$\Delta U = (66.5)(9.81)(2660 - 1270)$	G1
$\Delta U = 906kJ$	JU1
$W = \Delta U = U_f - U_i$	K1
$W = 906kJ$	JU1

7	If it requires 6J of work to stretch a particular spring by 2cm from its equilibrium length, determine the sprint constant of the spring. How much more work will be required to stretch it an additional 4cm? **[4 marks]**

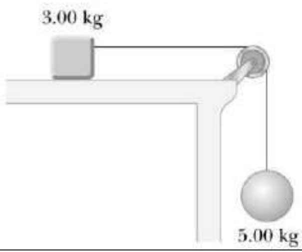
$E_{ep-i} = \frac{1}{2}kx^2$	
$6 = \frac{1}{2}k(0.02)^2$	G1
$k = 30kJ\,m^{-2}$	JU1
$E_{ep} = \frac{1}{2}(30000)(0.06)^2$	K1
$E_{ep-f} = 54J$	
$\Delta E_{ep} = E_{ep-f} - E_{ep-i}$	
$\Delta E_{ep} = 54 - 6 = 48J$	JU1

8 A person jumps off a bridge with a bungee cord (a heavy stretchable cord) tied around his ankle, as shown in the figure. He falls for 15m before the bungee cord begins to stretch. His mass is 75kg and we assume the cord obeys Hooke's law, $F = -kx$ with $k = 55Nm^{-1}$. If we neglect air resistance, estimate what distance d below the bridge his foot will be before coming to a stop. **[3 marks]**



$mgd = \frac{1}{2}k(d-15)^2$	K1
$(75)(9.81)(d) = \frac{1}{2}(55)(d-15)^2$	G1
$d = \{4.289(\text{rejected}),\ 52.466\}m$	JU1

9 The coefficient of friction between the 3kg block and the surface in the figure shown is 0.4. The system starts from rest. What is the speed of the 5kg ball when it has fallen 1.50m? **[3 marks]**



$\Delta E_{2-gp} - E_{friction} = \Sigma E_k$	K1
$m_2gh - fs = m_2gh - \mu m_1 gh = \frac{1}{2}v^2(m_1+m_2)$	
$(5)(9.81)(1.5) - (0.4)(3)(9.81)(1.5) = \frac{1}{2}v^2(3+5)$	G1
$v = 3.74ms^{-1}$	JU1

10

a. A pump lifts 27kg of water per minute through a height of 3.5m. What minimum output rating (in watts) must the pump motor have? **[2 marks]**

b. In 2minutes, a ski lift raises four skiers at constant speed to a height of 140m. The average mass of each skier is 65kg. What is the average power provided by the tension in the cable pulling the lift? **[2 marks]**

The cheetah is one of the fastest-accelerating animals, because it can go from rest to 27m/s in 4s. If its mass is 110kg, determine the average power developed by the cheetah during the acceleration phase of its motion. **[3 marks]**

a.

$P = \frac{W}{t} = \frac{mgh}{t} = \frac{(27)(9.81)(3.5)}{60}$	G1
$P = 15.4W$	JU1

b.

$P = \frac{W}{t} = \frac{mgh}{t} = \frac{(65(4))(9.81)(140)}{120}$	G1
$P = 3kW$	JU1

$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{0.5m\left(v_f^2 - v_i^2\right)}{t}$	K1
$P = \frac{0.5(110)(27^2 - 0^2)}{4}$	G1
$P = 10kW$	JU1

Quiz 3: SHM & Waves

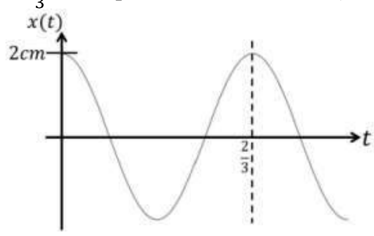
Problems

$$x = A\,sin\,\omega t;\ v = \frac{dx}{dt} = \pm\sqrt{A^2 - x^2};\ a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 x;$$
$$K = \frac{1}{2}m\omega^2(A^2 - x^2);\ U = \frac{1}{2}m\omega^2 x^2;\ E = \frac{1}{2}m\omega^2 A^2;\ \omega = \frac{2\pi}{T} = 2\pi f;$$
$$T = 2\pi\sqrt{\frac{l}{g}};\ T = 2\pi\sqrt{\frac{m}{k}};\ k = \frac{2\pi}{\lambda};\ v = f\lambda;\ \ y(x,t) = A\,sin\,(\omega t \pm kx);$$
$$y = A\,cos\,kx\,sin\,\omega t;\ I = \frac{P}{A};\ f = \frac{nv}{2l};\ f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}};\ f = \frac{nv}{4l};\ v = \sqrt{\frac{T}{\mu}};$$
$$\mu = \frac{m}{l};f' = \left(\frac{v \pm v_o}{v \pm v_s}\right)f$$

1 A particle moving along the x-axis in simple harmonic motion starts from its equilibrium position, the origin, at t = 0 and moves to the right. The amplitude of its motion is 2cm, and the frequency is 1.5Hz. Show that the position of the particle is given by

$$x = (2cm)\,sin\,(3\pi t)$$

Determine the maximum speed and the earliest time (t > 0) at which the particle has this speed, the maximum acceleration, and the total distance travelled between t = 0 and t = 1.00 s. **[10 marks]**

$t = 0;\ x = 0;\ v > 0$ $\Rightarrow x(t) = A\,sin\,\omega t$	
$x(t) = A\,sin\,(2\pi f t) = 2\,sin\,(2\pi(1.5)t)$	**G1**
$x(t) = 2\,sin\,(3\pi t)$ Where x is in cm and t is in seconds	**J1**
$v = \frac{dx}{dt} = \omega A\,cos\,\omega t = v_{max}\,cos\,(2\pi f t)$ $v_{max} = \omega A = (3\pi)(2)$	**G1**
$v_{max} = \omega A = 6\pi cms^{-1} \approx 18.8cms^{-1}$	**JU1**
$v_{max} = v_{max}\,cos\,(3\pi t) \Rightarrow cos(3\pi t) = 1$ $cos\,\theta = 1$ when $\theta = 0, \pi, 2\pi, \ldots, n$	**K1**
Earliest time with t>0, $3\pi t = \pi$	**G1**
$t = \frac{1}{3}t$	**JU1**
$a_{max} = A\omega^2 = (2)(3\pi)^2$	**G1**
$a_{max} = 18\pi^2 cms^{-1} \approx 178cms^{-2}$	**JU1**
$T = \frac{1}{\left(\frac{3}{2}\right)} = \frac{2}{3}s$ In $\frac{2}{3}s$, the particle travels 8cm (2cm every $\frac{1}{6}s$)  So in 1s, this is $\frac{3}{2}T$, this would mean the particle has travelled 8cm+4cm=12cm.	**J1**

2 a. Astronauts on a distant planet set up a simple pendulum of length 1.2m. The pendulum executes simple harmonic motion and makes 100 complete vibrations in 280s. What is the magnitude of the acceleration due to gravity on this planet? **[2 marks]**

b. At t=0, an 885g mass at rest on the end of a horizontal spring ($k = 184Nm^{-1}$) is struck by a hammer which gives it an initial speed of $2.26ms^{-1}$. Determine the period, frequency, the amplitude and the total energy of the motion. Also, determine the kinetic energy when $x = 0.4A$ where A is the amplitude. **[10marks]**

a.

$T = 2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{2\pi f}$ $2\pi\sqrt{\frac{l}{g}} = \frac{1}{f} = \frac{t}{N}$	
$2\pi\sqrt{\frac{1.2}{g}} = \frac{280}{100}$	**G1**
$g = 6.0426ms^{-1}$	**J1**

b.

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.885}{184}}$	**G1**
T=0.436s	**J1**
$f = T^{-1} = (0.436)^{-1}$	**G1**
$f = 2.29Hz$	**J1**
$v_{max} = A\omega = A\sqrt{\frac{k}{m}}$ $2.26 = A\sqrt{\frac{184}{0.885}}$	**G1**
$A = 0.1567m$	**J1**
$E_{total} = \frac{1}{2}kA^2 = \frac{1}{2}(184)(0.1567)^2$	**G1**
$E_{total} = 2.259J$	**J1**
$K = \frac{1}{2}k(A^2 - x^2)$ $K = \frac{1}{2}(184)((0.1567)^2 - (0.1567 \times 0.4)^2)$	**G1**
$K = 1.898J$	**J1**

3

a. A sinusoidal wave on a string is described by

$$y = (0.51cm)sin(kx - \omega t)$$

where k = 3.10 rad/cm and ω = 9.30 rad/s. How far does a wave crest move in 10.0 s? Does it move in the positive or negative x direction? **[2 marks]**

b. Transverse pulses travel with a speed of 200m/s along a taut copper wire whose diameter is 1.50mm. What is the tension in the wire? (The density of copper is $8.92gcm^{-3}$.) **[2 marks]**

a.

In positive x direction	
$d = vt = \frac{\omega}{k}t = \frac{9.3}{3.1}(10)$	**G1**
$d = 30cm$	**J1**

b.

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{Tl}{l\rho\pi r^2}} \Rightarrow T = v^2\rho\pi r^2$ $T = (200)^2(8.92 \times 1000)\pi(7.50 \times 10^{-4})^2$	**G1**
$T = 631N$	**J1**

4

a. The human ear canal is approximately 2.5cm long. It is open to the outside and is closed at the other end by the eardrum. Calculate the first three frequencies (in the audible range) of the standing waves in the ear canal if velocity of sound in air is $343ms^{-1}$.

[4 marks]

b. Determine the length of an open organ pipe that emits middle C (262 Hz) when the velocity of air is $341.8ms^{-1}$. What are the wavelength and frequency of the fundamental standing wave in the tube? **[4 marks]**

c. A uniform narrow tube 1.70m long is open at both ends. It resonates at two successive harmonics of frequencies 275 Hz and 330 Hz. What is the fundamental frequency. **[2 marks]**

a.

Closed ended tube model	
$f_n = \frac{nv}{4l} \Rightarrow f_n = \frac{n(343)}{4(0.025)}$	**G1**
$f_1 = 3430Hz$ $f_3 = 10290Hz$ $f_5 = 17150Hz$	**J3**

b.

Open ended tube model	
$f_n = \frac{nv}{2l}$ Fundamental$\Rightarrow f_1 = \frac{v}{2l}$ $262 = \frac{341.8}{2l}$	**G1**
$l = 0.65m$	**J1**
$\lambda = 2l = 2(0.65)$	**G1**
$\lambda = 1.3m$	**J1**

c.

$\Delta f = f_{n+1} - f_n = f_1 = 330 - 275$	**G1**
$f_1 = 55Hz$	**J1**

5

a. From a vantage point very close to the track at a stock car race, you hear the sound emitted by a moving car. You detect a frequency that is 0.86 times as small as the frequency emitted by the car when it is stationary. The speed of sound is $343ms^{-1}$. What is the speed of the car? **[2 marks]**

b. Standing at a crosswalk, you hear a frequency of 560Hz from the siren of an approaching ambulance. After the ambulance passes, the observed frequency of the siren is 480Hz. Determine the ambulance's speed from these observations. Speed of sound is $343ms^{-1}$. **[2 marks]**

a.

$f' = \left(\frac{v \pm v_o}{v \pm v_s}\right) f \Rightarrow$ $\frac{f'}{f} = 0.86 = \left(\frac{343 + 0}{343 + v_s}\right)$	**G1**
$v_s = 55.837ms^{-1}$	**J1**

b.

$f' = \left(\frac{v \pm v_o}{v \pm v_s}\right) f \Rightarrow f = \frac{f_{away}}{\left(\frac{v}{v + v_s}\right)} = \frac{f_{towards}}{\left(\frac{v}{v - v_s}\right)}$ $\frac{480}{\left(\frac{343}{343 + v_s}\right)} = \frac{560}{\left(\frac{343}{343 - v_s}\right)}$	**G1**
$v_s = 26.38ms^{-1}$	**J1**

Quiz 4: Physics of Matter

Problems

$$\sigma = \frac{F}{A};\ \varepsilon = \frac{e}{l_o};\ Y = \frac{\sigma}{\varepsilon};\ U = \frac{1}{2}Fe;\ \Delta L = \alpha L_o \Delta T;\ \Delta A = \beta A_o \Delta T;$$
$$\Delta V = \gamma V_o \Delta T;\ \beta = 2\alpha;\ \gamma = 3\alpha;\ \frac{Q}{t} = -kA\left(\frac{\Delta T}{L}\right)$$

1 A sign (mass 1700 kg) hangs from the bottom end of a vertical steel girder (of $Y = 2\times10^{11}Nm^{-2}$) with a cross-sectional area of $0.012m^2$.What is the stress within the girder? What is the strain on the girder? Determine the potential energy stored in the girder if the initial length of the girder is 9.5m. **[6 marks]**

$\sigma = \frac{F}{A} = \frac{mg}{A} = \frac{(1700)(9.81)}{0.012}$	G1
$\sigma = 1.3898\times10^6 Nm^{-2}$	JU1
$Y = \frac{\sigma}{\varepsilon} \Rightarrow 2\times10^{11} = \frac{1.3898\times10^6}{\varepsilon}$	G1
$\varepsilon = 6.949\times10^{-6}$	JU1
$U = \frac{1}{2}Fe = \frac{1}{2}(mg)(\varepsilon l_o)$ $U = \frac{1}{2}(1700)(9.81)(6.949\times10^{-6})(9.5)$	G1
$U = 0.55J$	JU1

2 A 2m-long cylindrical steel wire (of $Y = 2\times10^{11}Nm^{-2}$) with a cross-sectional diameter of 4.00mm is placed over a light frictionless pulley, with one end of the wire connected to a 5kg object and the other end connected to a 3kg object. Determine the tension of the wire. By how much does the wire stretch while the objects are in motion? **[5 marks]**

$m_1a = T - m_1g;\ m_2a = m_2g - T$ $a = \frac{T - m_1g}{m_1} = \frac{m_2g - T}{m_2}$	K1
$\frac{T - (3)(9.81)}{3} = \frac{(5)(9.81) - T}{5}$	G1
$T = 36.7875N$	JU1
$Y = \frac{\sigma}{\varepsilon} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{e}{l_o}\right)} = \frac{Fl_o}{Ae} = \frac{Fl_o}{(\pi r^2)e}$	
$2\times10^{11} = \frac{(36.7875)(2)}{(\pi(2\times10^{-3})^2)e}$	G1
$e = 0.0293mm$	JU1

3 To what temperature would you have to heat a brass rod ($\alpha = 19\times10^{-6}\,{}^oC^{-1}$) for it to be 1.5% longer than it is at 25°C? **[2 marks]**

$\Delta L = \alpha L_o \Delta T = \alpha L_o(T_f - T_i)$ $\Rightarrow \frac{\Delta L}{L_o} = 0.015 = (19\times10^{-6})(T_f - 25)$	G1
$T_f = 814.474\,{}^oC$	JU1

4 An ordinary glass is filled to the brim with 450mL of water at 100°C. If the temperature of glass and water is decreased to 20°C, how much water could be added to the glass? **[5 marks]**
[$\alpha_{container} = 9\times10^{-6}\,{}^oC^{-1};\ \alpha_{water} = 70\times10^{-6}\,{}^oC^{-1}$]

$\Delta V = \gamma V_o \Delta T$	G1
$V_{add} = (V_{o-container} + \Delta V_{container}) - (V_{o-water} + \Delta V_{water})$	K1
$V_{o-container} = V_{o-water}$	K1

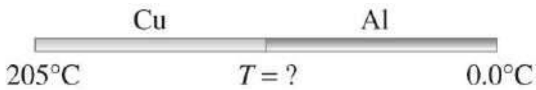
$V_{add} = \Delta V_{container} - \Delta V_{water} = (\gamma_{container} - \gamma_{water})V_o \Delta T$ $V_{add} = 3(\alpha_{container} - \alpha_{water})V_o \Delta T$	
$V_{add} = 3(9 \times 10^{-6} - 70 \times 10^{-6})(450)(20 - 100)$	G1
$V_{add} = 6.59mL$	JU1

5 A steel rod [$Y = 2 \times 10^{11} Nm^{-2}$; $\alpha = 11 \times 10^{-6\,o}C^{-1}$] undergoes a stretching force of 500N. Its cross-sectional area is $2cm^2$. Find the change in temperature that would elongate the rod by the same amount as the 500N force does. **[2 marks]**

$Y = \dfrac{\sigma}{\varepsilon} = \dfrac{Fl_o}{A\Delta L} = \dfrac{Fl_o}{A(\alpha l_o \Delta T)} = \dfrac{F}{A\alpha\Delta T}$	
$2 \times 10^{11} = \dfrac{500}{(2 \times 10^{-4})(11 \times 10^{-6})\Delta T}$	G1
$\Delta T = 1.136^oC$	JU1

6

a. A copper rod ($k_{Cu} = 380 Js^{-1}m^{-1\,o}C^{-1}$) and an aluminum rod ($k_{Al} = 200 Js^{-1}m^{-1\,o}C^{-1}$)of the same length and cross-sectional area are attached end to end. The copper end is placed in a furnace maintained at a constant temperature of 205°C. The aluminum end is placed in an ice bath held at a constant temperature of 0°C.



Calculate the temperature at the point where the two rods are joined. **[3 marks]**

b. A 100W lightbulb generates 95W of heat, which is dissipated through a glass bulb that has a radius of 3cm and is 0.5mm thick. What is the difference in temperature between the inner and outer surfaces of the glass if glass has $k = 0.84\ Js^{-1}m^{-1\,o}C^{-1}$? **[2 marks]**

a.

Steady State: $P_{Cu} = P_{Al} \Rightarrow \dfrac{dQ_{Cu}}{dt} = \dfrac{dQ_{Al}}{dt}$	K1
$k_{Cu}A\left(\dfrac{T_{hot} - T_{joint}}{l}\right) = k_{Cu}A\left(\dfrac{T_{joint} - T_{cold}}{l}\right)$	
$380(205 - T_{joint}) = 200(T_{joint} - 0)$	G1
$T_{joint} = 134.31^oC$	JU1

b.

$\dfrac{Q}{t} = P = kA\left(\dfrac{\Delta T}{l}\right) = k(4\pi r^2)\left(\dfrac{\Delta T}{l}\right)$	
$95 = (0.84)(4\pi(3 \times 10^{-2})^2)\left(\dfrac{\Delta T}{0.5 \times 10^{-3}}\right)$	G1
$\Delta T = 5^oC$	JU1

Quiz 5: Thermodynamics

Problems

$$pV = nRT;\ n = \frac{m}{M} = \frac{N}{N_A};\ v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}};\ pV = \frac{1}{3}Nmv_{rms}^2;$$
$$p = \frac{1}{2}\rho v_{rms}^2;\ K_{tr} = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT;\ U = \frac{1}{2}fNkT = \frac{1}{2}fnRT;$$
$$Q = \Delta U + W;\ W = nRT\ ln\ \frac{V_2}{V_1} = nRT\ ln\ \frac{p_1}{p_2};$$
$$W = \int p\ dV = p(V_2 - V_1);$$

1 Gas is contained in an 8L container at a temperature of 20°C and a pressure of 9atm. Determine the number of moles of gas in the container and the number of molecules in the container. If the gas has a mass of $6 \times 10^{-27} kg$, determine the rms speed. **[6 marks]**

$pV = nRT$ $(9 \times 1.013 \times 10^5)(0.008) = n(8.314)(293)$	G1
$n = 2.99mol$	JU1
$n = \frac{N}{N_A} \Rightarrow 2.99 = \frac{N}{6.02 \times 10^{23}}$	G1
$N = 1.8 \times 10^{24}$ molecules	JU1
$v_{rms} = \sqrt{\frac{3kT}{m}} \Rightarrow v_{rms} = \sqrt{\frac{3(1.38 \times 10^{-23})(293)}{6 \times 10^{-27}}}$	G1
$v_{rms} \approx 1.42kms^{-1}$	JU1

2 A cylinder contains helium in equilibrium at 150°C. What is the average kinetic energy for the gas molecule? Determine the total kinetic energy of the cylinder contains 2 moles of helium. Calculate the total kinetic energy of the cylinder if it was filled with 3 moles hydrogen gas instead. **[6 marks]**

$\overline{K} = \frac{3}{2}k_BT = \frac{3}{2}(1.38 \times 10^{-23})(150 + 273.15)$	G1
$\overline{K} = 8.8 \times 10^{-21}J$ (Single molecule)	JU1
$\Sigma K = N\overline{K} = nN_A\overline{K} = (3)(6.02 \times 10^{23})(8.8 \times 10^{-21})$	G1
$\Sigma K = 15.8928kJ$	JU1
H_2 gas: $\Sigma K = nN_A\overline{K} = nN_A\left(\frac{5}{2}k_BT\right)$	
$\Sigma K = (3)(6.02 \times 10^{23})\left(\frac{5}{2}(1.38 \times 10^{-23})(150 + 273.15)\right)$	G1
$\Sigma K = 26.365kJ$	JU1

3 At the normal boiling point of a material, the liquid phase has a density of $958kgm^{-3}$, and the vapor phase has a density of $0.958kgm^{-3}$. The average distance between neighbouring molecules in the vapor phase is d_{vapor}. The average distance between neighboring molecules in the liquid phase is d_{liquid}. Determine the ratio $\frac{d_{\text{vapor}}}{d_{\text{liquid}}}$.
(Hint: Assume that the volume of each phase is filled with many cubes, with one molecule at the centre of each cube.)
[5 marks]

Assuming volume *V* filled with N number of cubes of volume d^3. Each cube has a molecule at the centre. Then, $V = Nd^3 \Rightarrow d = \left(\frac{V}{N}\right)^{\frac{1}{3}}$	K1
Density is defined by mass per unit volume,	K1

$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$ Here, by m, we mean the mass of the material, which is the total mass of the molecules.	
This means $d = \left(\frac{m}{N\rho}\right)^{\frac{1}{3}} \Rightarrow d \propto \rho^{-\frac{1}{3}}$	K1
$\frac{d_{vapor}}{d_{liquid}} = \left(\frac{\rho_{liquid}}{\rho_{vapor}}\right)^{-\frac{1}{3}}$	
$\frac{d_{vapor}}{d_{liquid}} = \left(\frac{958}{0.958}\right)^{\frac{1}{3}}$	G1
$\frac{d_{vapor}}{d_{liquid}} = 10$	JU1

4

a. A thermodynamic system undergoes a process in which its internal energy decreases by 500J. At the same time, 220J of work is done on the system. Find the energy transferred to or from it by heat.
[2 marks]

b. The pressure in a 1 mole monoatomic ideal gas is cut in half slowly, while being kept in a container with rigid walls. In the process, 465J of heat left the gas. Determine the change in temperature of the ideal gas. **[3 marks]**

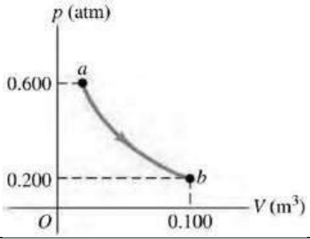
a.

$Q = \Delta U + W;\ \Delta U = -500J;\ W = -220J$	
$Q = -500 + (-220)$	JU1
$Q = -720J$ Heat transferred out of the system.	G1

b.

Rigid Walls $\Rightarrow \Delta V = 0 \Rightarrow \Delta W = 0$	K1
$Q = \Delta U + W \Rightarrow \Delta U = 465J = \frac{3}{2}nR\Delta T = \frac{3}{2}(1)(8.31)\Delta T$ $465J = \frac{3}{2}(1)(8.31)\Delta T$	G1
$\Delta T = 37.3K$	JU1

5 The figure shows the pV-diagram for a process in which the temperature of the ideal gas remains constant at 85°C. What volume does this gas occupy at a? How much work was done by or on the gas from a to b?
[4 marks]

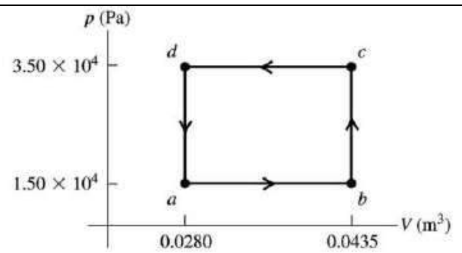


Isothermal $\Rightarrow$ T=constant $\Rightarrow p_iV_i = p_fV_f$	
$(0.600)V_i = 0.200(0.1)$	G1
$V_i = \frac{1}{30}m^2 \approx 0.033m^3$	JU1
Work done by gas, $W = nRT\ ln\ \frac{p_a}{p_b} = p_bV_b\ ln\ \frac{p_a}{p_b} = (0.2 \times 10^5)(0.1)\ ln\ \left(\frac{0.6}{0.2}\right)$	G1
$W = 2.197kJ$	JU1

6 Starting with 2.50 mol of N_2 gas (assumed to be ideal) in a cylinder at 1atm and 20°C, a chemist first heats the gas at constant volume, adding $1.36 \times 10^4 J$ of heat, then continues heating and allows the gas to expand at constant pressure to twice its original volume.
Calculate the final temperature of the gas; the amount of work done by the gas; and the change in internal energy of the gas for the whole process.
[10 marks]

$\Delta T_{total} = \Delta T_{\text{constant volume (cv)}} + \Delta T_{\text{isobaric expansion(ie)}}$	
Constant volume $\Rightarrow \Delta V = 0;\ \Delta W = 0$ $\Delta U = \Delta Q \Rightarrow \frac{f}{2} nR\Delta T = \Delta Q$	K1
Diatomic$\Rightarrow f = 5$ $\frac{5}{2}(2.5)(8.31)\Delta T = 13600$	G1
$\Delta T_{cv} = 261.853K$	
After heating at constant volume, $T = (20 + 273.15) + 261.85 = 555K$	G1
Isobaric Expansion $pV = nRT \Rightarrow \frac{T}{V} = \text{constant} \Rightarrow \frac{T_f}{T_i} = \frac{V_f}{V_i} \Rightarrow \frac{T_f}{555} = 2$	G1
$T_f = 1110K$	JU1
$W_{cv} = 0$	
$W_{ie} = p\Delta V = nR\Delta T = (2.5)(8.31)(1110 - 555)$	G1
$W_{total} = W_{cv} + W_{ie} = 0 + 11.53kJ$	G1
$W_{total} = 11.53kJ$	JU1
$\Delta U_{total} = \frac{5}{2}\text{nR}\Delta\text{T}$ $\Delta U_{total} = (2.5)(2.5)(8.31)\big(1110 - (20 + 273.15)\big)$	G1
$\Delta U_{total} = 42.43kJ$	JU1

7 Three moles of argon gas (assumed to be an ideal gas) originally at $1.50 \times 10^4 Pa$ and a volume of $0.0280m^3$ are first heated and expanded at constant pressure to a volume of $0.0435m^3$, then heated at constant volume until the pressure reaches $3.50 \times 10^4 Pa$, then cooled and compressed at constant pressure until the volume is again $0.0280m^3$, and finally cooled at constant volume until the pressure drops to its original value of $1.50 \times 10^4 Pa$. Draw the pV-diagram for this cycle. Calculate the total work done by the gas during the cycle. **[4 marks]**

	Values – 1 marks Direction – 1 marks
Total work done is area enclosed by pV-graph, $\lvert W\rvert = \big((3.5 - 1.5) \times 10^4\big)(0.0435 - 0.028)$	G1
$\lvert W\rvert = 310J$ (work done onto gas)	JU1

SP015 Equations

1	$\vec{A}\cdot\vec{B} = AB\cos\theta$	17	$F_c = \frac{mv^2}{r} = mv\omega = mr\omega^2$	33	$v = \frac{dx}{dt} = \pm\sqrt{A^2 - x^2}$	49	$v = \sqrt{\frac{T}{\mu}}$	65	$p = \frac{1}{2}\rho v_{rms}^2$
2	$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$	18	$s = r\theta$	34	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 x$	50	$\mu = \frac{m}{l}$	66	$K_{tr} = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT$
3	$v = u + at$	19	$v = r\omega$	35	$K = \frac{1}{2}m\omega^2(A^2 - x^2)$	51	$f_n = \left(\frac{v \pm v_o}{v \pm v_s}\right)f$	67	$U = \frac{1}{2}fNkT = \frac{1}{2}fnRT$
4	$s = ut + \frac{1}{2}at^2$	20	$a_t = r\alpha$	36	$U = \frac{1}{2}m\omega^2x^2$	52	$\sigma = \frac{F}{A}$	68	$Q = \Delta U + W$
5	$v^2 = u^2 + 2as$	21	$\omega = \omega_o + \alpha t$	37	$E = \frac{1}{2}m\omega^2A^2$	53	$\varepsilon = \frac{e}{l_o}$	69	$W = nRT\ln\frac{V_2}{V_1} = nRT\ln\frac{p_1}{p_2}$
6	$s = \frac{1}{2}(u + v)t$	22	$\theta = \omega_o t + \frac{1}{2}\alpha t^2$	38	$\omega = \frac{2\pi}{T} = 2\pi f$	54	$Y = \frac{\sigma}{\varepsilon}$	70	$W = \int p\, dV = p(V_2 - V_1)$
7	$p = mv$	23	$\omega^2 = \omega_o^2 + 2\alpha\theta$	39	$T = 2\pi\sqrt{\frac{l}{g}}$	55	$U = \frac{1}{2}Fe$		
8	$J = F\Delta t$	24	$\tau = rF\sin\theta$	40	$T = 2\pi\sqrt{\frac{m}{k}}$	56	$\Delta L = \alpha L_o\Delta T$		
9	$J = \Delta p = mv - mu$	25	$I = \Sigma mr^2$	41	$k = \frac{2\pi}{\lambda}$	57	$\Delta A = \beta A_o\Delta T$		
10	$f = \mu N$	26	$I_{\text{solid sphere}} = \frac{2}{5}MR^2$	42	$v = f\lambda$	58	$\Delta V = \gamma V_o\Delta T$		
11	$W = Fs\cos\theta$	27	$I_{\text{solid cylinder/disk}} = \frac{1}{2}MR^2$	43	$y(x,t) = A\sin(\omega t \pm kx)$	59	$\beta = 2\alpha$		
12	$K = \frac{1}{2}mv^2$	28	$I_{\text{ring}} = MR^2$	44	$y = A\cos kx\sin\omega t$	60	$\gamma = 3\alpha$		
13	$U = mgh$	29	$I_{\text{rod}} = \frac{1}{12}ML^2$	45	$I = \frac{P}{A}$	61	$pV = nRT$		
14	$U_x = \frac{1}{2}kx^2 = \frac{1}{2}Fx$	30	$\tau = I\alpha$	46	$f = \frac{nv}{2l}$	62	$n = \frac{m}{M} = \frac{N}{N_A}$		
15	$P_{ave} = \frac{\Delta W}{\Delta t}$	31	$L = I\omega$	47	$f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$	63	$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$		
16	$P = Fv$	32	$x = A\sin\omega t$	48	$f = \frac{nv}{4l}$	64	$pV = \frac{1}{3}Nmv_{rms}^2$		

SP015 EXTRA HANDOUTS

Lesson Sessions Handouts

CONTENTS

CHAPTER 1: MATHEMATICAL PREREQUISITE 3

CHAPTER 2: KINEMATICS 4

CHAPTER 3: DYNAMICS OF LINEAR MOTION 7

CHAPTER 4: WORK, ENERGY & POWER 9

CHAPTER 5: CIRCULAR MOTION 13

CHAPTER 6: ROTATION OF RIGID BODY 15

CHAPTER 7: OSCILLATIONS & WAVES 17

CHAPTER 8: PHYSICS OF MATTER 22

CHAPTER 9: KINETIC THEORY OF GASES & THERMODYNAMICS 25

Chapter 1: Mathematical Prerequisite

Dimensions

1. Given the quantities a = 9.7 m, b = 4.2 s, c = 69 m/s, what is the value of the quantity $d = \frac{a^3}{cb^2}$? [0.75 m^2s^{-1}]
2. Are following equations are dimensionally correct?
   a. $v_f = v_i + ax$ [No]
   b. $y = (2m)\ sin(kx)$, where $k = 2m^{-1}$ [Yes]

Unit Conversion

1. A rectangular building lot is 100 ft by 150 ft. Determine the area of this lot in m^2 if $1m = 3.28ft$. [1390m^2]
2. A solid piece of lead has a mass of 23.94g and a volume of $2.10cm^3$. From these data, calculate the density of lead in SI units (kgm^{-3}). [11400 kgm^{-3}]

Significant Figures

1. For $x = \frac{ab}{c}$, the uncertainty of *x* is
$$\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$
A rectangular plate has a length of (21.3 $\pm$ 0.2) cm and a width of (9.8 $\pm$ 0.1) cm. Calculate the area of the plate, including its uncertainty. [$(209 \pm 4)cm^2$]

Vectors

1. A vector has an x component of – 25units and a y component of 40units. Find the magnitude and direction of this vector. [47.2 units at 122^o from +x axis]
2. Vector A has a magnitude of 8.00 units and makes an angle of 45.0° with the positive x axis. Vector B also has a magnitude of 8.00 units and is directed along the negative x axis. Find
   a. the vector sum A + B [6.1 units at 112^o]
   b. the vector difference A – B [14 units at 42^o]

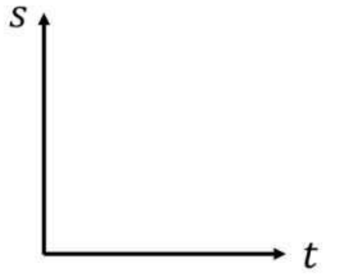
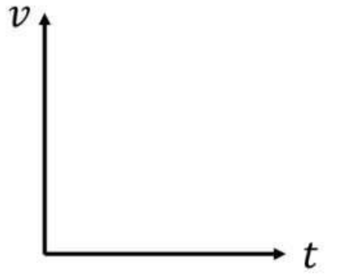
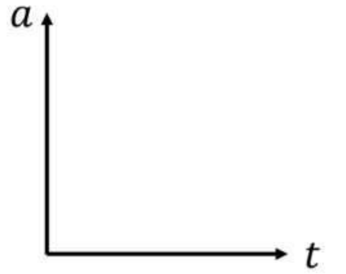
Chapter 2: Kinematics

1D Kinematics

Meaning of terms

"Uniform"	
"Instantaneous"	"Average"

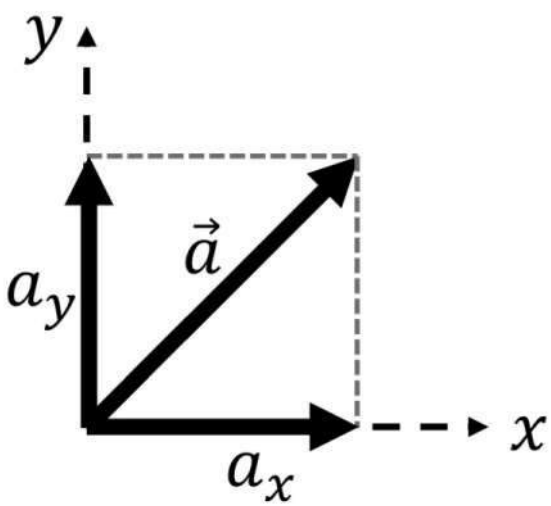
Graphs

			
Gradient			
Area under graph			

Equations

<u>2D Kinematics</u>

Mathematical Tools (Trig)

<table>
<tr><td>Magnitude</td><td rowspan="2">
</td></tr>
<tr><td>Direction</td></tr>
</table>

Projectile Motion

Equations

$x - axis$	$y - axis$

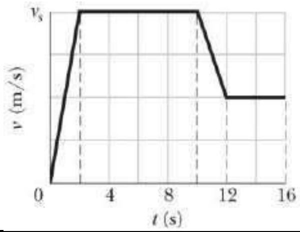
Assumptions

Highest Point	
Range	

Examples:

1. A car slows down uniformly from a speed of $28ms^{-1}$ to rest in 8s. How far did it travel in that time?
   [112m]
2. A football is kicked at ground level with a speed of $18ms^{-1}$ at an angle of 31° to the horizontal. How much later does it hit the ground?
   [1.89s]
3. A football is kicked at ground level with a speed of $18ms^{-1}$ at an angle of 31° to the horizontal. What is the velocity of the football at $t = 0.33s$?
   [16.57ms^{-1} at 21.35^o from the horizontal +x-axis]

<u>Class Problems</u>

1	A muon (an elementary particle) enters a region with a speed of 5×10^{6}m/s and then is slowed at the rate of $1.25\times 10^{14} ms^{-2}$. (a) How far does the muon take to stop? (b) Graph x versus t and v versus t for the muon.
2	How far does the runner whose velocity–time graph is shown in the figure travel in 16s? The figure's vertical scaling is set by v_s =8.0 m/s. 
3	A dart is thrown horizontally with an initial speed of 10 m/s toward point P, the bull's-eye on a dart board. It hits at point Q on the rim, vertically below P, 0.19 s later. (a) What is the distance PQ? (b) How far away from the dart board is the dart released?
4	A stone is catapulted at time t=0, with an initial velocity of magnitude 20.0m/s and at an angle of 40.0° above the horizontal. What are the magnitudes of the (a) horizontal and (b) vertical components of its displacement from the catapult site at t=1.10s?

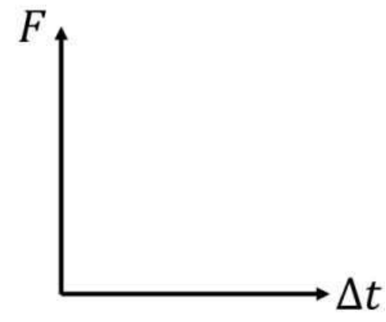
Chapter 3: Dynamics of Linear Motion

Momentum, Impulse & Conservation of Momentum

Definitions

Momentum	
Impulse	

Graph

Conservation of Linear Momentum

Statement

Elastic vs Inelastic Collisions

	Elastic	**Inelastic**
$\boldsymbol{\Delta(\Sigma p) = 0}$		
$\boldsymbol{\Delta(\Sigma E_k) = 0}$		

Solving 2D $\Delta(\Sigma p) = 0$ Problems

	$\boldsymbol{\Sigma p_{initial}}$	$\boldsymbol{\Sigma p_{final}}$	$\boldsymbol{\Sigma p_{initial} = \Sigma p_{final}}$
$\boldsymbol{p_x}$			
$\boldsymbol{p_y}$			

Examples:

1. A constant friction force of 25N acts on a 65kg skier for 15s on level snow. What is the skier's change in velocity? **[$\boldsymbol{\Delta v = 5.77 ms^{-1}}$]**
2. A 110-kg tackler moving at $2.5ms^{-1}$meets head-on (and holds on to) an 82-kg halfback moving at $5ms^{-1}$. What will be their mutual speed immediately after the collision? **[$\boldsymbol{0.7ms^{-1}}$]**
3. A neon atom ($m = 20u$) makes a perfectly elastic collision with another atom at rest. After the impact, the neon atom travels away at a 55.6° angle from its original direction and the unknown atom travels away at a -50^o angle. What is the mass (in u) of the unknown atom? **[$\boldsymbol{m = 39.9u}$]**

Force Diagrams

Forces to consider

Force	Direction

Types of friction

Types	Definition

Newton's Laws

Statements of 3 laws of motion

First Law	
Second Law	
Third Law	

Examples:

1. A 10kg box is placed on top of the 20kg box, as shown in the figure. Determine the normal force that the 20kg box exerts on the 10kg box. **[$N = 294N$]**

2. A 14kg bucket is moved vertically by a rope in which there is 163N of tension at a given instant. What is the acceleration of the bucket? **[$a = 1.8ms^{-1}$ in the direction of the rope.]**

3. A 25kg box is released on a 27° incline and accelerates down the incline at $0.3ms^{-2}$.Find the friction force impeding its motion. What is the coefficient of kinetic friction? **[100N; 0.48]**

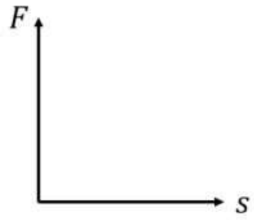
Chapter 4: Work, Energy & Power

Work

Equations for work

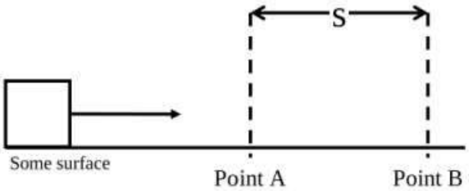
Single constant force along the displacement	
Single constant force at an angle of θ from the displacement	
Multiple constant force at various angles ($\theta = \theta_1, \theta_2, \ldots, \theta_n$) from the displacement	

Graph

	Area under the graph

<u>On Work</u>

Let us consider a moving object from left to right on some frictionless surface and passes through point A and point B (separated by a distance of s) at different speed ($v_B > v_A$),



Because there is a change in speed,

$$a = \frac{v - u}{t} \neq 0$$

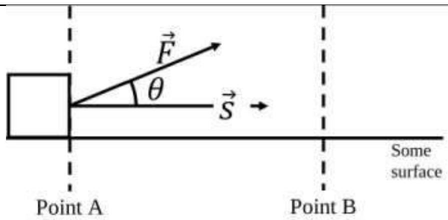
And therefore

$$F \neq 0$$

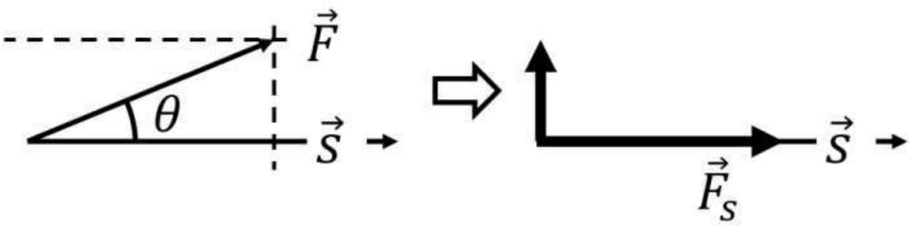
Say the force was applied along point A to point B, then work done is

$$W = |\vec{F}||\vec{s}|$$

But what if the applied force is **<u>NOT</u>** in the direction of s, say at some angle θ,

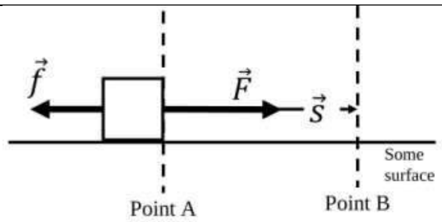


Then we need to only consider the component of $\vec{F}$ in the direction of $\vec{s}$,



We can see that the magnitude of the force component in direction of s is

$$|\vec{F}_s| = |\vec{F}| \cos\theta$$

Therefore, the work done by the force $\vec{F}$ is
$$W = \left(\lvert\vec{F}\rvert \cos\theta\right)\lvert\vec{s}\rvert \Rightarrow W = \lvert\vec{F}\rvert\lvert\vec{s}\rvert \cos\theta_{F-s}$$
Let us now consider a frictional force acting on the object as it moves between A and B,

We now have 2 forces acting on the object. We can then work out the work done by each force. Let us first consider the work done by force $\vec{F}$. For $\vec{F}$, the work done by the force is
$$W_{\vec{F}} = \lvert\vec{F}\rvert\lvert\vec{s}\rvert \cos 0^{o} = \lvert\vec{F}\rvert\lvert\vec{s}\rvert$$
On the other hand, the work done by friction *f* is
$$W_{f} = \lvert\vec{f}\rvert\lvert\vec{s}\rvert \cos 180^{o} = -\lvert\vec{f}\rvert\lvert\vec{s}\rvert$$
The total work done on the object is therefore
$$W_{total} = \Sigma W = W_{\vec{F}} + W_{\vec{f}} = \left(\lvert\vec{F}\rvert - \lvert\vec{f}\rvert\right)\lvert\vec{s}\rvert$$
This is to say,
The total work done onto an object can determined by considering work done by each forces on the object.

On Work-Energy Theorem

Let us consider one of the kinematic equation,
$$v^2 = u^2 + 2as \Rightarrow v^2 - u^2 = 2as$$
Multiplying both sides by $\frac{1}{2}m$ yields
$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$
The right hand side represents change in kinetic energy since $K = \frac{1}{2}mv^2$ and since $F = ma$,
$$K_{final} - \mathrm{K}_{\mathrm{initial}} = Fs$$
Which tells us that
The change in kinetic energy of an object is equal to the work done onto the object.

Energy

Types of energy & their equations

Conservation of Energy

Statement

Work-Energy Theorem

Statement

Power

Types of power & their equations

Average Power, P_{ave}	
Instantaneous Power, $P_{inst.}$	

Examples:

4. What is the minimum work needed to push a 950kg car 710m up along a 9° incline? Ignore friction. **[1MJ]**
5. How much work must be done to stop a 925kg car traveling at $95kmh^{-1}$? **[$-0.32MJ$]**
6. In the high jump, the kinetic energy of an athlete is transformed into gravitational potential energy without the aid of a pole. With what minimum speed must the athlete leave the ground in order to lift his centre of mass 2.1m and cross the bar with a speed of $0.5ms^{-1}$? **[$6.4ms^{-1}$]**
7. If a car generates 18hp ($1hp = 746W$) when traveling at a steady $95kmh^{-1}$, what must be the average force exerted on the car due to friction and air resistance? **[510N]**

Chapter 5: Circular Motion

Subtopic

Parameters and equations

Name	Symbol	Unit
Angular Displacement		
Period		
Frequency		
Angular velocity		
Centripetal/radial acceleration		
Centripetal force		

Term	Description
Uniform Circular Motion	

Conversion of units (degrees ⇆ radians ⇆ rotation ⇆ revolution)

3 cases:

1.	
2.	
3.	

Examples:

8. **[Case 1: Horizontal Circular Motion]**
   How large must the coefficient of static friction be between the tires and the road if a car is to round a **level** curve of radius 125m at a speed of 95kmh^{-1}? **[$\boldsymbol{\mu = 0.57}$]**
9. **[Case 2: Vertical Circular Motion]**
   A bucket of mass 2kg is whirled in a vertical circle of radius 1.20m. At the lowest point of its motion the tension in the rope supporting the bucket is 25N. Find the speed of the bucket. How fast must the bucket move at the top of the circle so that the rope does not go slack? **[$\boldsymbol{1.8ms^{-1}}$; $\boldsymbol{3.43ms^{-1}}$]**
10. **[Case 3: Conical Pendulum]**
    Consider a conical pendulum with an 80kg bob on a 10m wire making an angle of 5° with the vertical. Determine the radial acceleration of the bob. **[$\boldsymbol{a_r = 0.857ms^{-2}}$]**

Unit Conversion

Units:

$$1 \text{ revolution} = 1 \text{ rotation} = 2\pi\text{rad} = 360^o$$

Example:

Convert 275^o to radians.

$$2\pi\text{rad} = 360^o$$

$$(2\pi\text{rad} = 360^o) \times \frac{1}{360}$$

$$1^o = \frac{2\pi}{360} rad$$

$$(1^o = \frac{2\pi}{360} rad) \times 275$$

$$275^o = \frac{2\pi}{360}(275)\, rad = \frac{55}{36}\pi\, rad$$

$$\mathbf{275^o = 4.8\, rad}$$

Fill in the cells with the correct values:

Degrees (^{o})	Radians (rad)	Revolution or Rotation
		0.07
115		
	3.58	
295		
		1.07
475		
		1.57
655		

Solution

Degrees (^{o})	Radians (rad)	Revolution or Rotation
25	0.44	0.07
115	2.01	0.32
205	3.58	0.57
295	5.15	0.82
385	6.72	1.07
475	8.29	1.32
565	9.86	1.57
655	11.43	1.82

Chapter 6: Rotation of Rigid Body

Kinematics

Linear	→	Rotational
Displacement, s	→	
Velocity, v	→	
Acceleration, a	→	
Momentum, p	→	
Force, F	→	
Mass, m	→	
$v = u + at$	→	
$s = ut + \frac{1}{2}at^2$	→	
$v^2 = u^2 + 2as$	→	
Equilibrium Condition, $\Sigma F = 0$	→	
Conservation of linear momentum, $\Delta(\Sigma p) = 0$	→	
Extra: Kinetic Energy, $K = \frac{1}{2}mv^2$	→	

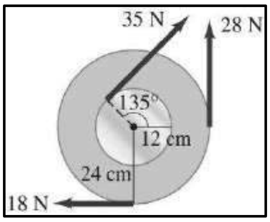
Examples:

11. A cooling fan is turned off when it is running at 850rpm. It turns 1250 revolutions before it comes to a stop. What was the fan's angular acceleration, assumed constant? How long did it take the fan to come to a complete stop? **[$-0.5 rads^{-2}$; $180s$]**

12. Determine the moment of inertia of a 10.8kg sphere of radius 0.648m when the axis of rotation is through its centre. **[$1.81 kgm^2$]**

13. A merry-go-round accelerates from rest to $0.68 rads^{-1}$ in 34 s. Assuming the merry-go-round is a uniform disk of radius 7.0 m and mass 31,000 kg, calculate the net torque required to accelerate it. **[$15 kmN$]**

14. The figure shows the axle of a wheel with forces applied on it to rotate it. Assuming that a friction torque of 0.6mN opposes the motion. Calculate the net torque about the axle of the wheel. **[$1.2 mN$ (clockwise)]**



15. A figure skater can increase her spin rotation rate from an initial rate of 1rev every 1.5s to a final rate of $2.5 revs^{-1}$. If her initial moment of inertia was what is her final moment of inertia? **[$1.2 kgm^2$]**

Chapter 7: Oscillations & Waves

Simple Harmonic Motion (SHM)

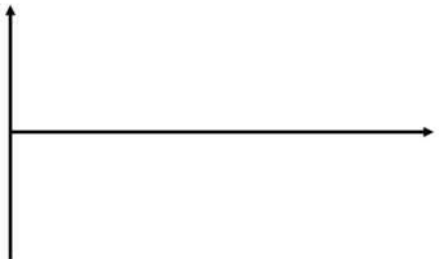
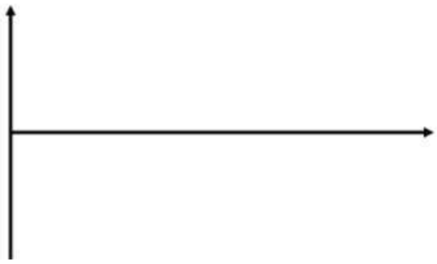
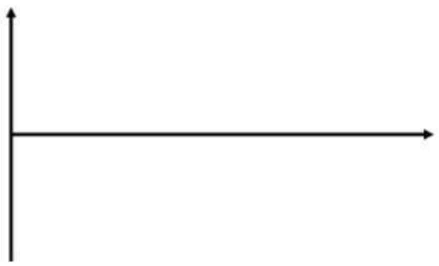
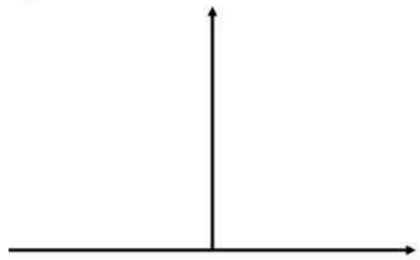
Equations

Simple Harmonic Equation Condition	
Solution to that equation	

Dynamics of SHM

Quantity	**As a function of displacement**	**As a function of time**
Velocity, v		
Acceleration, a		
Kinetic Energy, K		
Potential Energy, U		
Total Energy, E_T		

Graphs

Displacement-time 	Velocity-time 
Acceleration-time 	Energy-displacement 

Period and its applications

Period as a function of angular frequency and a function of frequency:
Case 1: Simple Pendulum
Case 2: Mass-Spring System

Examples:

16. If a particle undergoes SHM with amplitude 0.21m, what is the total distance it travels in one period? **[0.84m]**

17. A vertical spring with spring stiffness constant oscillates with an amplitude of 28cm when 0.235kg hangs from it. The mass passes through the equilibrium point with positive velocity at t=0. What equation describes this motion as a function of time? **[$y(t) = 28\,sin\,(36t)$ where y is in cm and t is seconds]**

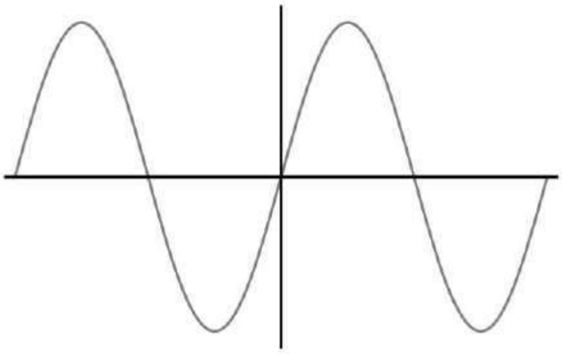
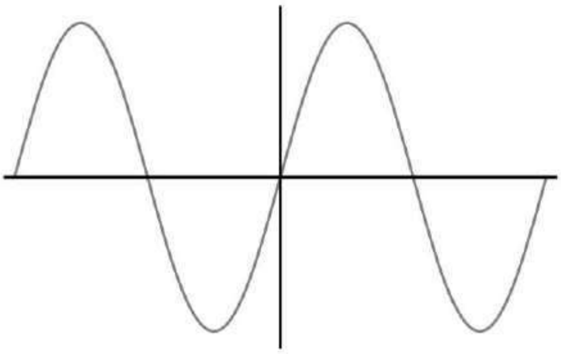
18. A pendulum has a period of 4.3s on Earth. What is its period on Jupiter, where the acceleration of gravity is about 2.53 that on Earth? **[2.70s]**

19. The springs of a 1700kg car compress 5mm when its 66kg driver gets into the driver's seat. If the car goes over a bump, what will be the frequency of oscillations? **[1.4Hz]**

<u>Wave Motion</u>

Wave number and wavelength

Progressive (Travelling) Wave Equation

Displacement of a wave	
as a function of space at time $t = 0$	
As a function of time at position $x = 0$	
General equation	

Particle Vibrational Velocity	Wave Propagation Velocity

Superposition of Waves

The idea
Standing Wave Equation

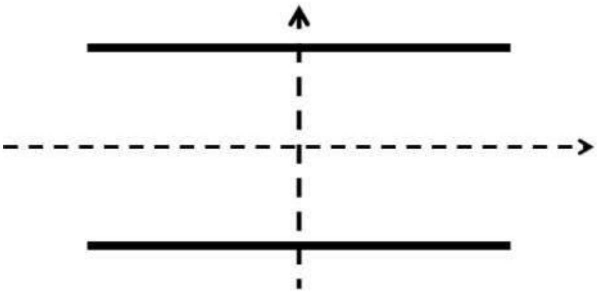
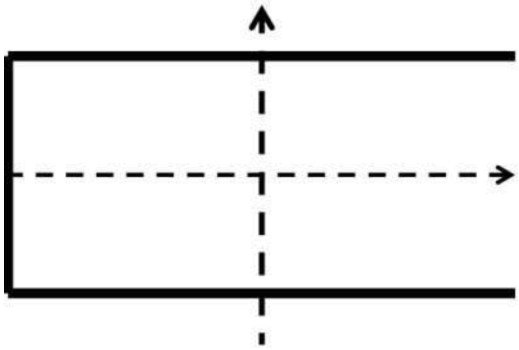
Stretched String

Wave Speed

Fundamental and Overtone Frequencies	
	

Air Columns

Fundamental and Overtone Frequencies

Open-ended tube	
Conditions:	
	
Closed-ended tube	
Conditions:	
	

Doppler Effects

Equation
Conditions

Examples:

1. A cord of mass 0.65kg is stretched between two supports 8m apart. If the tension in the cord is 120N, how long will it take a pulse to travel from one support to the other? **[0.21s]**
2. A guitar string is 92 cm long and has a mass of 3.4 g. The distance from the bridge to the support post is 62cm and the string is under a tension of 520 N. What are the frequencies of the fundamental and first two overtones? **[300Hz, 610Hz, 910Hz]**
3. What will be the fundamental frequency and first three overtones for a 26-cm-long organ pipe if it is
   a. Open. **[660Hz]**
   b. Closed. **[330Hz]**
4. The siren of a police car at rest emits at a predominant frequency of 1600Hz. What frequency will you hear if you are at rest and the police car moves at $25ms^{-1}$
   - **a.** towards you? **[1730Hz]**
   - **b.** away from you? **[1490Hz]**

Chapter 8: Physics of Matter

Stress, Strain and Young's Modulus

Equations

Relating to force applied on an area of material	
Relating to deformation of material geometry	
Young's Modulus	

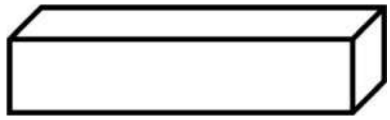
Graphs

Force-Elongation Graph

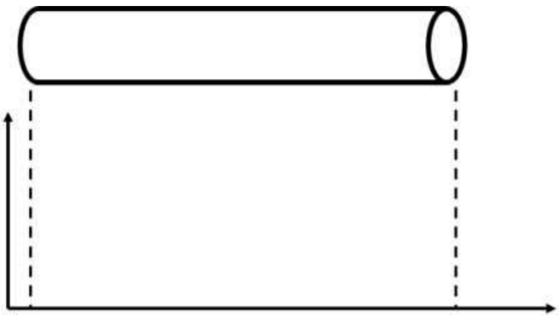
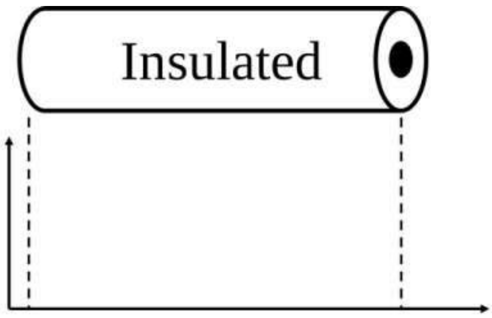
Force-Elongation Graph	Stress-Strain Graph
	

Heat Transfer by Conduction

Fourier Heat Equation



Temperature-distance graph

Steady state condition (Two rods in series)

When is it applied?	
What is the condition?	

Heat Expansion

Quantifying Thermal Expansion

Linear	Area	Volume

Lecture examples

[Young's Modulus]

TABLE 9–1 Elastic Moduli

Material	Young's Modulus, E (N/m^2)
Solids	
Iron, cast	100×10^9
Steel	200×10^9
Brass	100×10^9
Aluminum	70×10^9
Concrete	20×10^9
Brick	14×10^9
Marble	50×10^9
Granite	45×10^9
Wood (pine) (parallel to grain)	10×10^9
(perpendicular to grain)	1×10^9
Nylon	$\approx 3 \times 10^9$
Bone (limb)	15×10^9

Examples:

1. A marble column of cross-sectional area supports a mass of 25,000 kg. What is the stress within the column? What is the strain? By how much is the column shortened if it is 8.6 m high? **[$0.175MNm^{-2}$; 3.5μ; $30\mu m$]**
2. When 357N is applied to some material, its length changes by 17cm. What 1.323kN force is applied to the same material, its length changes by 63cm. Determine
   a. the strain energy required to extend the material by 63cm.**[416.75J]**
   b. the change in strain energy when the material length changes from 17cm to 63cm.
   [384.87J]

[Heat Conduction]

One end of a 56cm long copper rod with a diameter of 2 cm is kept at 460°C, and the other is immersed in water at 22°C. Calculate the heat conduction rate along the rod. **[93W]**

[Heat Expansion]

TABLE 13–1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α (C°)$^{-1}$	Coefficient of Volume Expansion, β (C°)$^{-1}$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	50×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$	$\approx 36 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$	$4\text{–}10 \times 10^{-6}$
Liquids		
Gasoline		950×10^{-6}
Mercury		180×10^{-6}
Ethyl alcohol		1100×10^{-6}
Glycerin		500×10^{-6}
Water		210×10^{-6}
Gases		
Air (and most other gases at atmospheric pressure)		3400×10^{-6}

Example:

A concrete highway is built of slabs 12 m long (15°C). How wide should the expansion cracks between the slabs be (at 15°C) to prevent buckling if the range of temperature is -30^oC to $+50^oC$? **[5mm]**

Chapter 9: Kinetic Theory of Gases & Thermodynamics

Kinetic Theory of Gas

Microscopically, though we want to start considering kinetic energies of the gas particles. Let us first lay down assumptions for our kinetic theory of gas. In one sentence, let us consider **gas to be composed of large numbers of non-null mass point-like particles that obeys Newton's laws of motion and interact elastically with each other where the average kinetic energy of the gas particles depends solely on the absolute temperature of the gas particle system.**

Ideal Gas Assumptions	

Some Maths

Particles considered are…	Constants involved
a. Many	$N_A =$
b. Small	$1a.m.u. = 1u = 1Da =$
c. Full of heat energy	$k_B =$

Rms Molecular speed

Calculate	
Reason	

Ideal Gas Equations

	Macroscopic Scale	Microscopic Scale
$p =$		

Sample Problems

1. A gas has a volume of $2.5(10^{-3})m^3$ at 30^oC and 1 atm. It is then allowed to expand to $3(10^{-3})m^3$ at 2atm. Determine the number of mol and the final temperature of the gas.

2. Determine the rms speed of the gas molecule of the gas in question 1 if the 1 mol of gas is 32g.

Molecular Kinetic Energy and Internal Energy

Equipartition Theorem

Statement:

Energies

Type of Molecule	Energies				
	Translational	Rotational	Total Energy per molecule	N number of gas molecules	n mol of gas molecule
Monoatomic					
Diatomic					
Polyatomic (with f d.o.f.)					

Sample Problems

1. Compare the total internal energy of 1.20 mole of ideal monoatomic gas at 300K and 0.72mole of diatomic gas at 300K.
2. Compare the total translational kinetic energy of 1.20 mole of ideal monoatomic gas at 300K and 0.72mole of diatomic gas at 300K.

First Law of Thermodynamics

First Law Statements

ΔU		
$\pm$		
If W is increased, then ΔU will ………		
W is		
$W =$		

Sample Problem

A gas sample expands at constant temperature by doing 30J of work against an external pressure. Determine the change in internal energy of the gas and the energy lost or gain as heat.

Thermodynamics Processes & Work

<table>
<tr><th>Thermodynamic Processes</th><th>p-V graph</th></tr>
<tr><td>a. Isobaric</td><td rowspan="4">
</td></tr>
<tr><td>b. Isochoric (Isovolumetric)</td></tr>
<tr><td>c. Isothermal</td></tr>
<tr><td>d. Adiabatic</td></tr>
</table>

Sample Problems

1. Eight grams of helium (molecular mass = 4u) expand isothermally at 400K and does 9600 J of work. Assuming that helium is an ideal gas, determine the ratio of the final volume of the gas to the initial volume.

2. A gas is compressed at a constant pressure of 0.5atm from 9L to 4L. In the process, 600J of energy leaves the gas by heat. What is the work done on the gas? What is the change in its internal energy?

→ **Any two errors related to experiment**

[2 marks]

ii. Hence, state the precautions to overcome the errors stated in (i)

Ensure pendulum does not rotate during oscillation;
Ensure pendulum is released with amplitude $< 10^o$;
Minimize air resistance by switching off the fan;
Ensure eye is perpendicular to the scale measurement.

[2 marks]

SP015 PRE-PSPM

SULIT

SP015	**SP015**
Physics 1	**Fizik 1**
Semester 1	**Semester 1**
Session 2023/2024	**Session 2023/2024**
2 hours	**2 jam**

KEMENTERIAN PENDIDIKAN

BAHAGIAN MATRIKULASI

MATRICULATION DIVISION

PRA-PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI

PRE-MATRICULATION PROGRAMME EXAMINATION

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **13** halaman bercetak.
This question paper consists of **13** printed pages.
© Unit Fizik KMSw

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **8** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The use of non-programmable scientific calculator is permitted.

ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi **8** soalan.

Jawab **semua** soalan.

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Guna muka surat Baharu bagi nombor soalan yang berbeza.

Penggunaan kalkulator saintifik yang tidak boleh diprogramkan dibenarkan.

LIST OF SELECTED CONSTANT VALUES

SENARAI NILAI PEMALAR TERPILIH

Speed of light in vacuum	c	$= 3 \times 10^{8}\ ms^{-1}$
Permeability of free space	μ_o	$= 4\pi \times 10^{-7}\ Hm^{-1}$
Permittivity of free space	ε_o	$= 8.85 \times 10^{-12}\ Fm^{-1}$
Electron charge magnitude	e	$= 1.6 \times 10^{-19}\ C$
Planck constant	h	$= 6.63 \times 10^{-34}\ Js$
Electron mass	m_e	$= 9.11 \times 10^{-31}\ kg$
		$= 5.49 \times 10^{-4}\ u$
Neutron mass	m_n	$= 1.674 \times 10^{-27}\ kg$
		$= 1.008665\ u$
Proton mass	m_p	$= 1.672 \times 10^{-27}\ kg$
		$= 1.007277\ u$
Hydrogen mass	m_H	$= 1.673 \times 10^{-27} kg$
		$= 1.007825\ u$
Deuteron mass	m_d	$= 3.34 \times 10^{-27} kg$
		$= 2.014102\ u$
Molar gas constant	R	$= 8.31\ JK^{-1}mol^{-1}$
Avogadro constant	N_A	$= 6.02 \times 10^{23}\ mol^{-1}$
Boltzmann constant	k	$= 1.38 \times 10^{-23}\ JK^{-1}$
Free-fall acceleration	g	$= 9.81\ ms^{-2}$
Atomic mass unit	$1u$	$= 1.66 \times 10^{-27} kg$
		$= 931.5\ \frac{MeV}{c^2}$

LIST OF SELECTED CONSTANT VALUES

SENARAI NILAI PEMALAR TERPILIH

Electron volt	$1\,eV$	$= 1.6 \times 10^{-19}\,J$
Constant of proportionality for Coulomb's law	$k = \dfrac{1}{4\pi\varepsilon_o}$	$= 9 \times 10^{9}\,Nm^{2}C^{-2}$
Atmospheric pressure	1 atm	$= 1.013 \times 10^{5}\,Pa$
Density of water	ρ_w	$= 1000\,kgm^{-3}$

LIST OF SELECTED FORMULAE

1. $v = u + at$
2. $s = ut + \frac{1}{2}at^2$
3. $v^2 = u^2 + 2as$
4. $s = \frac{1}{2}(u + v)t$
5. $p = mv$
6. $J = F\Delta t$
7. $J = \Delta p = mv - mu$
8. $f = \mu N$
9. $W = Fs\cos\theta$
10. $K = \frac{1}{2}mv^2$
11. $U = mgh$
12. $U_s = \frac{1}{2}kx^2 = \frac{1}{2}Fx$
13. $W = \Delta K$
14. $P_{ave} = \frac{\Delta W}{\Delta t}$
15. $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$
16. $a_c = \frac{v^2}{r} = r\omega^2 = v\omega$
17. $F_c = \frac{mv^2}{r} = mv\omega = mr\omega^2$
18. $s = r\theta$
19. $v = r\omega$
20. $a_t = r\alpha$
21. $\omega = \omega_o + \alpha t$
22. $\theta = \omega_o t + \frac{1}{2}\alpha t^2$
23. $\theta = \frac{1}{2}(\omega_o + \omega)t$
24. $\omega^2 = \omega_o^2 + 2\alpha\theta$;
25. $\tau = rF\sin\theta$
26. $I = \Sigma mr^2$
27. $I_{\text{solid sphere}} = \frac{2}{5}MR^2$
28. $I_{\text{solid cylinder/disk}} = \frac{1}{2}MR^2$
29. $I_{\text{ring}} = MR^2$
30. $I_{\text{rod}} = \frac{1}{12}ML^2$
31. $\Sigma\tau = I\alpha$
32. $L = I\omega$
33. $y = A\sin\omega t$
34. $v = \frac{dy}{dt} = \pm\sqrt{A^2 - y^2}$
35. $a = -\omega^2 A\sin\omega t = -\omega^2 x$
36. $K = \frac{1}{2}m\omega^2(A^2 - y^2)$
37. $U = \frac{1}{2}m\omega^2 y^2$
38. $E = \frac{1}{2}m\omega^2 A^2$
39. $\omega = \frac{2\pi}{T} = 2\pi f$
40. $T = 2\pi\sqrt{\frac{l}{g}}$
41. $T = 2\pi\sqrt{\frac{m}{k}}$
42. $k = \frac{2\pi}{\lambda}$

LIST OF SELECTED FORMULAE

43. $v = f\lambda$

44. $y(x,t) = A \sin(\omega t \pm kx)$

45. $v_y = A\omega \cos(\omega t \pm kx)$

46. $y = 2A \cos kx \sin \omega t$

47. $f = \frac{nv}{2l}$

48. $f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$

49. $f = \frac{nv}{4l}$

50. $v = \sqrt{\frac{T}{\mu}}$

51. $\mu = \frac{m}{l}$

52. $f_n = \left(\frac{v \pm v_o}{v \mp v_s}\right) f$

53. $\sigma = \frac{F}{A}$

54. $\varepsilon = \frac{\Delta L}{L_o}$

55. $Y = \frac{\sigma}{\varepsilon}$

56. $U = \frac{1}{2} F\Delta L$

57. $\frac{U}{V} = \frac{1}{2}\delta\varepsilon$

58. $\frac{Q}{t} = -kA\left(\frac{\Delta T}{L}\right)$

59. $\Delta L = \alpha L_o \Delta T$

60. $\Delta A = \beta A_o \Delta T$

61. $\Delta V = \gamma V_o \Delta T$

62. $\beta = 2\alpha$

63. $\gamma = 3\alpha$

64. $n = \frac{m}{M} = \frac{N}{N_A}$

65. $v_{rms} = \sqrt{< v^2 >}$

66. $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$

67. $pV = \frac{1}{3} Nmv_{rms}^2$

68. $p = \frac{1}{2}\rho v_{rms}^2$

69. $K_{tr} = \frac{3}{2}\left(\frac{R}{N_A}\right) T = \frac{3}{2} kT$

70. $U = \frac{1}{2} fNkT = \frac{1}{2} fnRT$

71. $\Delta U = Q - W$

72. $W = nRT \ln \frac{V_f}{V_i} = nRT \ln \frac{p_i}{p_f}$

73. $W = \int p \, dV = p(V_2 - V_1)$

74. $W = \int p \, dV = 0$

1 Hooke's law states that the force, F, in a spring extended by a length x is given by

$$F = -kx$$

From Newton's second law $F = ma$, where m is the mass and a is the acceleration, calculate the dimension of the spring constant k.

[2 marks]

2 (a)

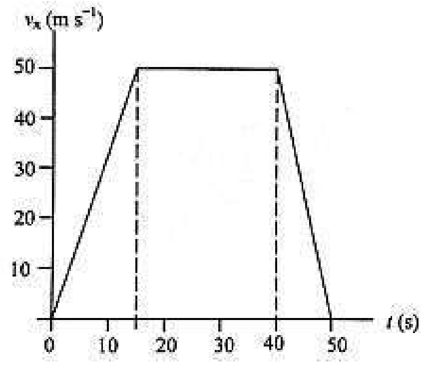


FIGURE 1

FIGURE 1 represents a part of the performance data of a car. Calculate the displacement travelled from $t = 15\ s$ to $t = 40\ s$ from the graph.

[2 marks]

(b)

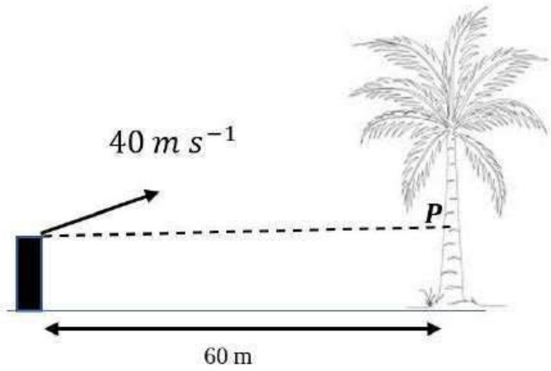


FIGURE 2

FIGURE 2 shows a stone being thrown from the top of a platform towards a coconut tree 60 m away. The initial velocity of the stone is $40\ m\ s^{-1}$ at 40° above the horizontal.

i. How much time does it take to hit the coconut tree?

ii. What is the distance between P and the position the stone strikes the coconut tree?

iii. What is the speed of the stone when it strikes the coconut tree?

[8 marks]

3 (a)

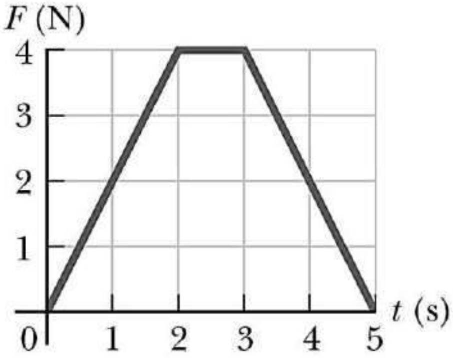


FIGURE 3

The magnitude of the net force exerted in the x-direction on a 2.5 kg particle varies in time as shown in **FIGURE 3**.

i. Calculate the impulse of the force over the 5s time interval.

ii. Calculate the final velocity the particle attains if it is originally at rest.

[3 marks]

(b) A 1200 kg car travelling initially at $u = 25.0\ ms^{-1}$ in an eastward direction crash into the back of a 9 kg truck moving in the same direction at $u = 20.0\ ms^{-1}$. The velocity of the car immediately after the collision is $v = 18.0\ ms^{-1}$ to the east. What is the velocity of the truck immediately after the collision?

[2 marks]

(c)

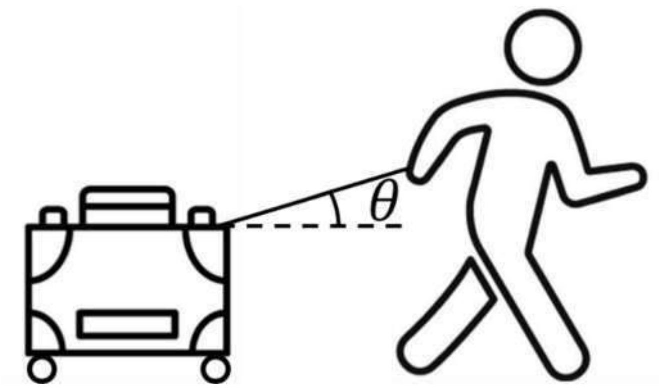


FIGURE 4

A woman at an airport is towing her *20.0 kg* suitcase at constant speed by pulling on a strap at an angle θ above the horizontal as shown in **FIGURE 4**. She pulls on the strap with a *35.0 N* force, and the friction force on the suitcase is *20.0 N.*

i. Draw a free body diagram of the suitcase

i. What angle does the strap make with the horizontal?

ii. What is the magnitude of the normal force that the ground exerts on the suitcase?

[8 marks]

4

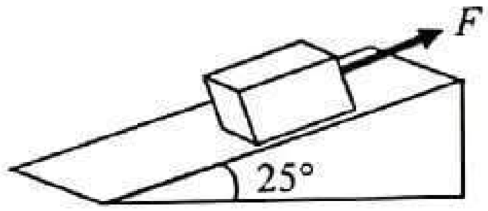


FIGURE 5

FIGURE 5 shows a 15 kg block being pulled by a 100 N force at an initial speed of 2 m s^{-1} up an inclined plane. The block travels a distance of 6.2 m parallel to the inclined plane. The coefficient of kinetic friction is 0.14. By using the work-energy theorem, calculate the change in the kinetic energy of the block.

[8 marks]

5 a) A child rides a Ferris Wheel of 5m diameter rotating at constant speed. Determine the maximum angular speed such that the child is not thrown off of the Ferris Wheel.

[3 marks]

b) A 150g ball is attached to the end of a 1.2m string and rotated in a horizontal circle of radius 1.2m on a frictionless surface. The string is found to snap when the tension within it is over 85N. Determine the maximum speed of the ball.

[2 marks]

6 a) A 500g mass suspended from a light spring is oscillating in a simple harmonic motion with period 1.55s and an amplitude of oscillation is 2cm.

i) Determine the frequency of motion and the spring constant of spring.

ii) Write the equation of the motion.

iii) Determine the displacement of the pendulum bob when the kinetic energy is 63% of the total energy of the oscillation.

[9 marks]

b) A progressive wave is represented by the equation

$$y = 5\,sin\,\frac{1}{2}(8\pi t - 5\pi x)$$

where x, y are in cm and t is in s.

i) State the direction of wave propagation.

ii) Determine the wavelength and propagation speed of the wave.

iii) Determine the displacement of a particle at $x = 4cm$ at time $t = 6s$.

[6 marks]

c) Determine the second overtone frequency for a closed tube of length 75cm if the speed of sound wave in air is $340ms^{-1}$. If the closed end is now open, what is the fundamental frequency?

[4 marks]

d) An ambulance, emitting siren of frequency 700Hz moving at constant velocity of $33ms^{-1}$ passes a man standing by the roadside. Once the ambulance passes by the man, the ambulance slows down the now the man hears the frequency of the siren to be 650Hz. If the speed of sound in air is $343ms^{-1}$, calculate the frequency hear by the man before the ambulance passes him. Calculate the speed of the ambulance after it has passed the man.

[4 marks]

7 (a) A wire of diameter 0.5 mm has Young's modulus $2 \times 10^{11}\ Nm^{-2}$.

Calculate the strain if it is extended by 150 N load.

[3 marks]

(b) The dimension of an aluminium wire at a room temperature (27°C) is 150 m long and cross-sectional area of $3.0 \times 10^{-6}\ m^2$. It is then melted to form a spherical ball. If the coefficient of the linear thermal expansion of the aluminium is $22.2 \times 10^{-6}\ mK^{-1}$, calculate the

i) volume of the spherical ball at room temperature

ii) change in the volume of the sphere if it is heated to 200°C

iii) change in the volume of the sphere if it is cooled to - 7°C

[5 marks]

8 (a) The pressure of a 50 cm^3 ideal gas at 25°C is 75 Pa. Determine the number of molecules of the gas.

[2 marks]

(b) A balloon is filled with helium at 25°C. The mass of a helium atom is $6.65 \times 10^{-27}\ kg$. Calculate the

i) root mean square speed of the helium atoms.

ii) kinetic energy of 0.5 mole helium atom

[3 marks]

(c) A 15-liter gas cylinder contains helium gas with pressure $1.01 \times 10^5\ Pa$ at 25°C. When heated, the gas undergoes an isochoric process.

i) Calculate the mass of helium gas. The atomic molar mass of helium gas is 4g.

ii) If 500 J of heat is added, calculate the change in the internal energy of the gas and sketch the graph of pressure versus volume for the isochoric process.

iii) If the gas cylinder can withstand a pressure up to $4.55 \times 10^5\ Pa$, calculate the maximum quantity of gas at 45°C.

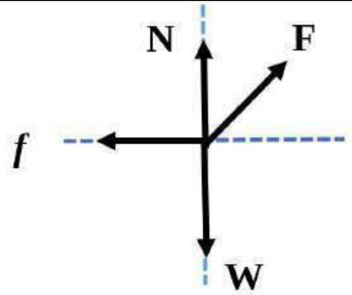
[6 marks]

END OF QUESTION PAPER

Marking Scheme Pra-PSPM KMSw

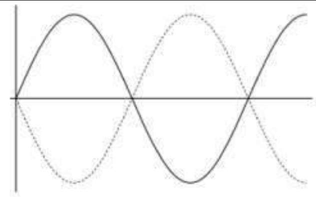
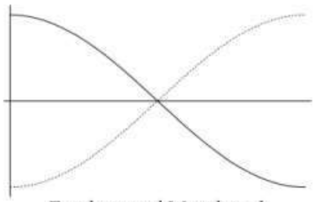
Sesi 2023/2024

No.	Answer Scheme	Marks
1	$F = -kx$ $k = \frac{F}{x}$ $[k] = \frac{MLT^{-2}}{L}$ $[k] = MT^{-2}$	 G1 J1
2 (a)	Total displacement = Area under the $v - f$ curve from $t = 15\ s\ to\ 40\ s$ $s = (40 - 15)(50) = 1250\ m$	K1 GJU1
2 (b) (i)	$u_x = 20.0 \cos 40° = 15.3\ ms^{-1}$ $s_x = u_x t$ $60 = (15.3)\ t$ $t = 3.92\ s$	K1 GJU1
2 (b) (ii)	$s_y = u_y t + a_y t^2$ $s_y = (12.9)(3.92) + \frac{1}{2}(-9.81)(3.92)^2$ $s_y = -24.8\ m$	 G1 JU1
2 (b) (iii)	$v_x = u_x = 20.0 \cos 40° = 15.3\ ms^{-1}$ $v_y = u_y + a_y t$ $v_y = (12.9) + (-9.81)(3.92) = -25.6\ ms^{-1}$ Therefore, the speed of the ball is: $v = \sqrt{v_x^2 + v_y^2}$ $v = \sqrt{(15.3)^2 + (-25.6)^2}$ $v = 29.8\ ms^{-1}$	K1 GJU1 G1 JU1

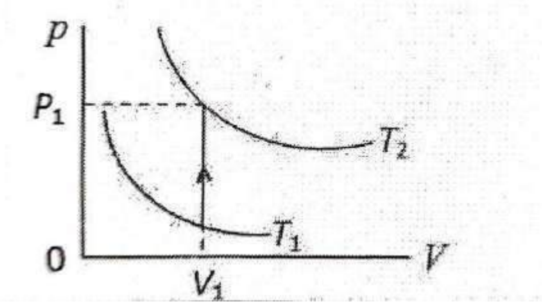
No.	Answer Scheme	Marks
3 (a)(i)	$J = F\Delta t = Area\ under\ the\ (F-t)\ graph$ $J = \frac{1}{2}(2)(4) + (1)(4) + \frac{1}{2}(2)(4) = 12\ N\ s$	K1 GJU1
3(a)(ii)	$J = mv - mu$ $12 = (2.5)(v-0) \ \Rightarrow v = \frac{12}{2.5} \Rightarrow v = 4.8\ ms^{-1}$	GJU1
3(b)	$\sum p_i = \sum p_f$ $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $(1200)(25) + (9000)(20) = (1200)(18) + (9000)v_2$ $v_2 = 20.93ms^{-1}$	G1 JU1
3(c)(i)		D3 – 4 forces correctly draw and labelled D2 – 3 forces correctly draw and labelled D1 – 2 forces correctly draw and labelled
3(c)(ii)	$\sum F_X = 0$ $F\cos\theta - f = 0$ $F\cos\theta = f$ $\theta = cos^{-1}\left(\frac{f}{F}\right)$ $\theta = cos^{-1}(\frac{20}{35})$ $\theta = 55.15°$ $\sum F_Y = 0$ $N + F\sin\theta - mg = 0$ $N = (20)(9.81) - (35)\sin 55.15 = 167.48\ N$	K1 G1 JU1 K1 GJU1

No.	Answer Scheme	Marks
4	$W_F = Fs = (100)(6.2) = 620\,J$	G1
	$W_f = \mu_k Ns = \mu_k(mgcos\theta)s$	K1
	$W_f = (0.14)(15)(-9.81)(cos25)(6.2) = -115.76\,J$	G1
	$W_g = (mgsin0)s$	K1
	$W_g = [(15)(-9.81)sin25](6.2) = -385.57\,J$	G1
	$\Delta K = W_F + W_f + W_g$	K1
	$\Delta K = 620 + (-115.76) + (-385.57)$	G1
	$\Delta K = 118.67\,J$	JU1
5 (a)	$\Sigma F_y = F_c$	K1
	$mg - R = mr\omega^2 = m\left(\frac{d}{2}\right)\omega^2$	
	$m(9.81) - 0 = m\left(\frac{5}{2}\right)\omega^2$	G1
	$\omega = \pm 1.98\,rads^{-1}$	JU1
5 (b)	$\mathrm{T}_{\mathrm{max}} = F_c = \frac{mv_{max}^2}{r}$	
	$85 = \frac{(0.15)v_{max}^2}{1.2}$	G1
	$v_{max} = \pm 26.08ms^{-1}$	JU1

No.	Answer Scheme	Marks
6(a)(i)	$m = 500g = 0.5kg;\ T = 1.55s$ $T = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f}$ $1.55 = 2\pi\sqrt{\frac{0.5}{k}}$ $k = 8.22Nm^{-1}$ $1.55 = \frac{1}{f}$ $f = 0.65s$	 G1 JU1 G1 JU1
6(a)(ii)	$\omega = 2\pi f$ $\omega = 2\pi(0.65)$ $\omega = 1.3\pi \approx \omega = 4.08rads^{-1}$	 G1 JU1
6(a)(iii)	$E_{total} = \frac{1}{2}m\omega^2A^2;\ K = \frac{1}{2}m\omega^2(A^2 - x^2)$ $K = 0.63E_{total}$ $\frac{1}{2}m\omega^2(A^2 - x^2) = (0.63)\left(\frac{1}{2}m\omega^2A^2\right) \Rightarrow x = \frac{\sqrt{37}}{10}A$ $x = \frac{\sqrt{37}}{10}(0.02)$ $x = 1.22cm$	 K1 G1 JU1
6 (b)(i)	In the positive x direction	JU1
6 (b)(ii)	$k = \frac{2\pi}{\lambda} \Rightarrow \frac{5}{2}\pi = \frac{2\pi}{\lambda}$ $\lambda = 0.8cm$ $v = f\lambda = \left(\frac{\omega}{2\pi}\right)\lambda$ $v = \frac{\left(\frac{8\pi}{2}\right)}{2\pi}(0.8)$ $v = 1.6cms^{-1}$	G1 JU1 G1 JU1
6(b)(iii)	$y = 5\,sin\,\frac{1}{2}(8\pi t - 5\pi x) = 5\,sin\,\frac{1}{2}(8\pi t - 5\pi x)$ $= 5\,sin\,\frac{1}{2}\big(8\pi(6) - 5\pi(4)\big) = 0cm$	GJU1

No.	Answer Scheme	Marks
6 (c)	 Second Overtone = Fifth Harmonic $L = \frac{5}{4}\lambda \Rightarrow \lambda = \frac{4}{5}L$ $v = f\lambda = (f)\left(\frac{4}{5}L\right)$	
	$340 = f\left(\frac{4}{5}\right)(0.75)$	G1
	$f = 566.67Hz$	JU1
	If the closed tube is not open,  Fundamental Wavelength $L = \frac{1}{2}\lambda \Rightarrow \lambda = 2L$ $v = f\lambda = (f)(2L)$	
	$340 = (f)\big(2(0.75)\big)$	G1
	$f = 226.67Hz$	JU1
6(d)	Before passing by, $f' = \frac{v \pm v_{observer}}{v \pm v_{source}} f$	
	$f' = \frac{343 \pm 0}{343 - 33}(700)$	G1
	$f' = 774.5Hz$	JU1
	After passing by, $650 = \frac{343 \pm 0}{343 + v}(700)$	G1
	$v = 26.4ms^{-1}$	JU1

No.	Answer Scheme	Marks
7(a)	$A = \frac{\pi d^2}{4} = \frac{\pi(0.5\times10^{-3})^2}{4} = 1.96\times10^{-7}m^2$ $\varepsilon = \frac{\sigma}{Y} = \frac{\frac{F}{A}}{Y} = \frac{F}{AY}$ $\varepsilon = \frac{150}{(1.96\times10^{-7})(2.0\times10^{11})}$ $\varepsilon = 3.82\times10^{-3}$	 K1 G1 J1
7(b)(i)	$V_{sphere} = V_{wire} = Al$ $V_{sphere} = (3\times10^{-6})(150) = 4.5\times10^{-4}m^3$	 KGJU1
7(b)(ii)	$\gamma = 3\alpha$ $200 - 27 = 173^{\circ}C$ $\Delta V = \gamma V_o \Delta T = 3\Delta T = 3(22.2\times10^{-6})(4.5\times10^{-4})(173)$ $\Delta V = 5.18\times10^{-6}m^3$	 G1 JU1
7(b)(iii)	$\Delta T = -7 - 27 = -34^{\circ}C$ $\Delta V = \gamma V_o \Delta T = 3\Delta T = 3(22.2\times10^{-6})(4.5\times10^{-4})(-34)$ $\Delta V = -1.01\times10^{-6}m^3$	 G1 JU1
8 (a)	$N = \frac{pV}{kT} = \frac{(75)(50\times10^{-6})}{(1.38\times10^{-23})(25+273)}$ $N = 9.12\times10^{17} molecules$	G1 JU1
8(b)(i)	$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23})(25+273)}{6.65\times10^{-27}}} = 1.36\times10^{3}ms^{-1}$	GJU1
8(b)(ii)	$K = \frac{3}{2}nRT = \frac{3}{2}(0.5)(8.31)(298)$ $K = 1857.29\,J$	G1 JU1
8(c)(i)	$pV = nRT = \frac{m}{M}RT$ $m = \frac{(1.01\times10^5)(15\times10^{-3})(4\times10^{-3})}{(8.31)(25+273)}$ $m = 2.44\times10^{-3}kg$	 G1 JU1

No.	Answer Scheme	Marks
8(c)(ii)	$Q = \Delta U + W$ $\Delta U = Q - W = 500 - 0 = 500\,J$ 	GJU1 D2 Correct axis & label Correct shape of the graph
8(c)(ii)	$pV = nRT$ $n = \frac{(4.55 \times 10^5)(15 \times 10^{-3})}{(8.31)(45 + 273)} = 2.58\,mol$	GJU1

Matriculation Physics (SP025)

Notes & Exercises

Shafiq R

Name:

Tutorial Class:

Phone Number:

Table of Contents

CHAPTER 1: ELECTROSTATICS 1

COULOMB'S LAW 1

ELECTRIC FIELD 1

ELECTRIC POTENTIAL 2

CHARGE IN UNIFORM ELECTRIC FIELD 3

CHAPTER 2: CAPACITORS AND DIELECTRICS 4

PARALLEL & SERIES 4

ENERGY STORED IN A CAPACITOR 5

CHARGING & DISCHARGING CAPACITORS 5

DIELECTRICS 5

CHAPTER 3: CURRENT AND DC CIRCUITS 6

ELECTRIC CURRENT 6

OHM'S LAW 6

RESISTANCE (GEOMETRY & TEMPERATURE) 7

EMF, INTERNAL RESISTANCE AND POTENTIAL DIFFERENCE 7

PARALLEL & SERIES 7

KIRCHHOFF'S RULES 8

ELECTRICAL ENERGY AND POWER 8

POTENTIAL DIVIDER 8

POTENTIOMETER 8

CHAPTER 4: MAGNETISM 10

MAGNETIC FIELD 10

$\boldsymbol{B}$ FROM CURRENT-CARRYING CONDUCTOR 10

MAGNETIC FORCE 11

FORCE ON A MOVING CHARGED PARTICLE IN UNIFORM $\boldsymbol{B}$ 11

FORCE ON A CURRENT CARRYING CONDUCTOR IN UNIFORM $\boldsymbol{B}$ 11

FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS 11

BAINBRIDGE MASS SPECTROMETER 11

CHAPTER 5: ELECTROMAGNETIC INDUCTION 12

MAGNETIC FLUX 12

INDUCED EMF 12

INDUCED EMF IN STRAIGHT CONDUCTOR 12

INDUCED EMF IN A COIL 12

INDUCED EMF IN A ROTATING COIL 12

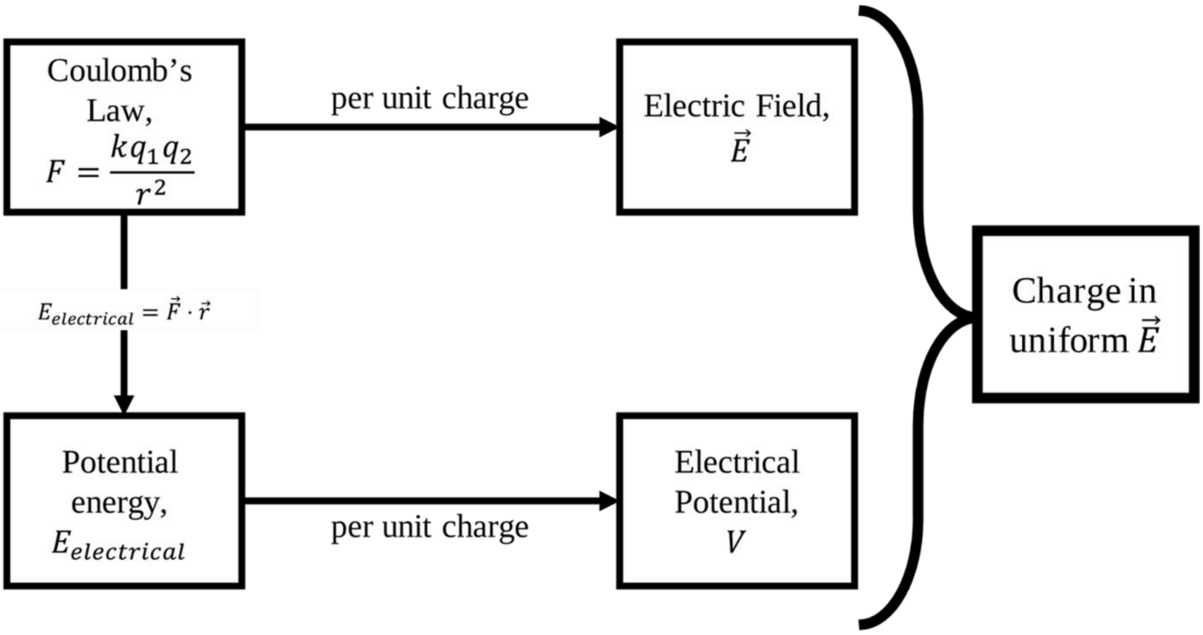
INDUCTANCE 13

SELF-INDUCTION 13

Mutual induction 13
Energy Stored in Inductor 13
CHAPTER 6: ALTERNATING CURRENT 14
Alternating Current 14
Root Mean Square Values 14
Impedance 14
Power & Power Factor 15
CHAPTER 7: OPTICS 16
Geometrical Optics: Reflection 16
Geometrical Optics: Refraction 16
Geometrical Optics: Thin lenses 17
Physical Optics: Huygens's Principle 17
Physical Optics: Interferences 18
Physical Optics: Slits 18
Double Slit 18
Single Slit 19
Diffraction Grating 19
Physical Optics: Thin Films 20
CHAPTER 8: PARTICLE WAVES 21
De Broglie Wavelength 21
Electron Diffraction 21
CHAPTER 9: NUCLEAR & PARTICLE PHYSICS 22
Binding Energy & Mass Defect 22
Radioactivity 23
Particle Accelerator 24
Fundamental Particles 25
ONE PAGE NOTES 26
Chapter 1: Electrostatics 26
Chapter 2: Capacitors 27
Chapter 3: Current and DC Circuits 28
Chapter 5: Electromagnetic Induction 29
Chapter 6: AC Circuits 30
Chapter 7: Optics 31
Chapter 8: de Broglie Wavelength 31
Chapter 9 Nuclear & Particle Physics 32
EXERCISE SHEET 1: COULOMB'S LAW & ELECTRIC FIELD 33
EXERCISE SHEET 2: ELECTRIC POTENTIAL & POTENTIAL ENERGY 34

EXERCISE SHEET 3: SAMPLE PROBLEM – 4 CHARGES 36
EXERCISE SHEET 4: CHARGE IN UNIFORM ELECTRIC FIELD 37
EXERCISE SHEET 5: CAPACITORS IN SERIES & PARALLEL 39
EXERCISE SHEET 6: CHARGING & DISCHARGING OF CAPACITORS 41
EXERCISE SHEET 7: DIELECTRICS 42
EXERCISE SHEET 8: ELECTRIC CURRENT, RESISTIVITY, OHM'S LAW & TEMPERATURE 43
EXERCISE SHEET 9: ELECTROMOTIVE FORCE 45
EXERCISE SHEET 10: RESISTORS IN PARALLEL & SERIES 46
EXERCISE SHEET 11: KIRCHHOFF'S 48
EXERCISE SHEET 12: KIRCHHOFF'S 50
EXERCISE SHEET 13: ELECTRICAL ENERGY & POWER 51
EXERCISE SHEET 14: POTENTIAL DIVIDER & POTENTIOMETER 52
EXERCISE SHEET 15: MAGNETIC FLUX & INDUCED EMF 54
EXERCISE SHEET 16: SELF-INDUCTANCE, ENERGY STORAGE & MUTUAL INDUCTANCE 56
EXERCISE SHEET 17: RLC CIRCUITS 57
ADDITIONAL NOTES & EXERCISE: SIGN CONVENTIONS IN GEOMETRICAL OPTICS 59
EXERCISE SHEET 18: OPTICS – REFLECTION 69
EXERCISE SHEET 19: OPTICS – REFRACTION 70
EXERCISE SHEET 20: OPTICS – THIN LENSES 72
EXERCISE SHEET 21: HUYGENS' & INTERFERENCES 73
EXERCISE SHEET 22: DOUBLE SLIT 74
EXERCISE SHEET 23: THIN FILMS 75
EXERCISE SHEET 24: SINGLE SLIT & DIFFRACTION GRATING 77
EXERCISE SHEET 25: DE BROGLIE 78
EXERCISE SHEET 26: BINDING ENERGY & MASS DEFECT 79
EXERCISE SHEET 27: RADIOACTIVITY 80

Chapter 1: Electrostatics



Coulomb's Law

Let it be known that Coulomb's Law allows us to measure forces between charged particles, this force is known as **Coulomb Force**. Mathematically, Coulomb's Law is

$$F_{Coulomb} = \frac{kq_1q_2}{r_{12}^2}$$

where q_i are the charges of interacting particles, r_{12} is the distance between the particles and k is the electrostatic constant.

The electrostatic constant is

$$k = \frac{1}{4\pi\epsilon} = 8.98 \times 10^9\, kg\, m^3\, s^{-4}\, A^{-2}$$

On the note of direction of the Coulomb force,

Condition	Direction
$F_{Coulomb} < 0$	Towards each other
$F_{Coulomb} > 0$	Away from each other

For more than 2 particles, the Coulomb Force on particle j becomes

$$F_{Coulomb} = kq_j \sum_i \frac{q_i}{r_{ij}^2}$$

Electric Field

The electric field at a point in space $E(r)$, is defined as the electric force acting on a positive test charge placed at that point $F(r)$, divided by the test charge, q_{test}.

$$E(r) = \frac{F(r)}{q_{test}}$$

Rearranging this equation yields,

$$F(r) = q_{test}E(r)$$

which tells us that particle of charge q_{test} placed in a region of electric field $E(r)$ will experience a force of $F(r)$.
If the source of electric field has a charge of q_{source}, then the electric field at point r, $\mathrm{E}(r)$ is

$$E(r) = \frac{kq_{source}}{r^2}$$

As in the case for Coulomb Force, for multiple, the electric field from multiple sources is simply additive,

$$E(r) = k\sum_i \frac{q_i}{r_i^2}$$

Electric Potential

Electric potential is the amount of work done to bring a test charge q_{test} from an infinite distance to a point at distance r from the source charged particle of charge q_{source}. This is found to be

$$V = \frac{W_{\infty\to r}}{q_{test}} = \frac{kq_{source}}{r}$$

Potential difference between positions $x = A$ and $x = B$ is then

$$V_{AB} = V_A - V_B = \frac{W_{\infty\to A}}{q_{test}} - \frac{W_{\infty\to B}}{q_{test}} = \frac{W_{\mathrm{A}\to B}}{q_{test}}$$

Electric potential energy is the energy a test charge would have positioned r distance away from a source,

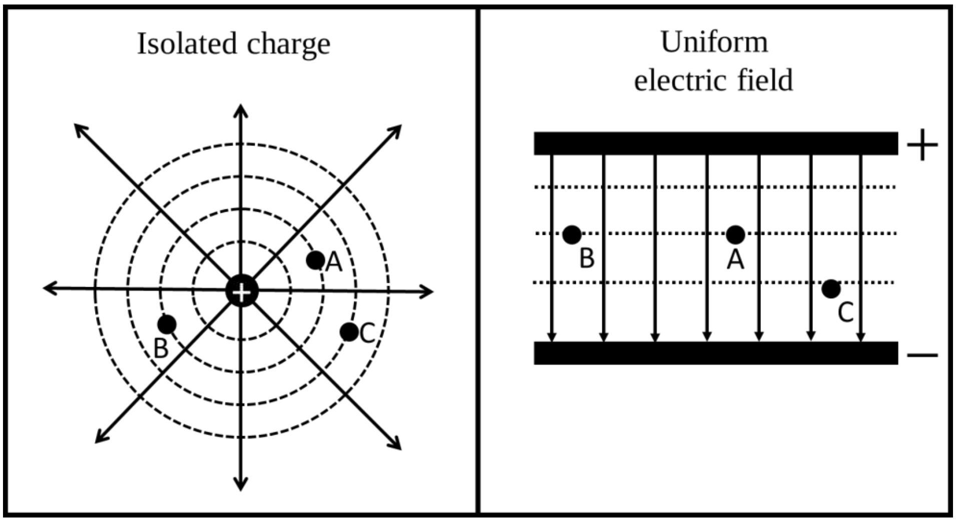
$$U = q_{test}V$$

For multiple sources, the potential and electrical potential energy at point r is simply,

$$V_{total} = k\sum_i \frac{q_i}{r_i};\ U_{total} = kq_j\sum_i \frac{q_i}{r_{ij}}$$

Equipotential lines and surfaces are graphical representation on which a particle on the line or surfaces is at the same potential.

- This means no work is done by the electric field when a charged particles are moved from on point of the equipotential line (or surface) to another point on the same line (or surface).
- Equipotential lines are always perpendicular to the electric field at all points.
- Examples:



In both examples,

$$V_A = V_B \neq V_C$$

Charge in Uniform Electric Field

For a uniform electric field produced by parallel plates of potential difference *V*, electric field strength is simply

$$E = \frac{V}{d}$$

where d is the distance between the parallel plates.

The following case studies involves a charged particle in a uniform electric field:

Case 1: Stationary charge

A stationary charged particle of charge q and mass *m*, placed in a uniform electric field E will experience force only from the electric field and therefore will move towards plate of its opposite charge (i.e. positive charged particle will move towards the negatively charged plate and vice versa). Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

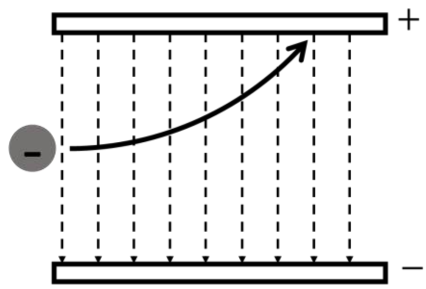
Case 2: Charge moving parallel to the field

A charged particle of charge q and mass *m*, entering a uniform electric field E in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. It will either decelerate (if its velocity is in the opposite direction of its acceleration) or accelerate. Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

Case 3: Charge moving perpendicularly to the field

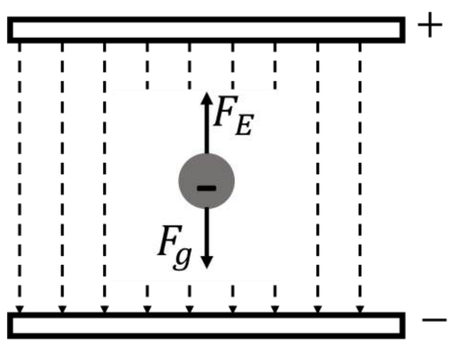
A charged particle of charge q and mass *m*, entering a uniform electric field E in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. Because of its initial velocity direction, it will follow a parabolic path, moving towards the plate of its opposite charge.



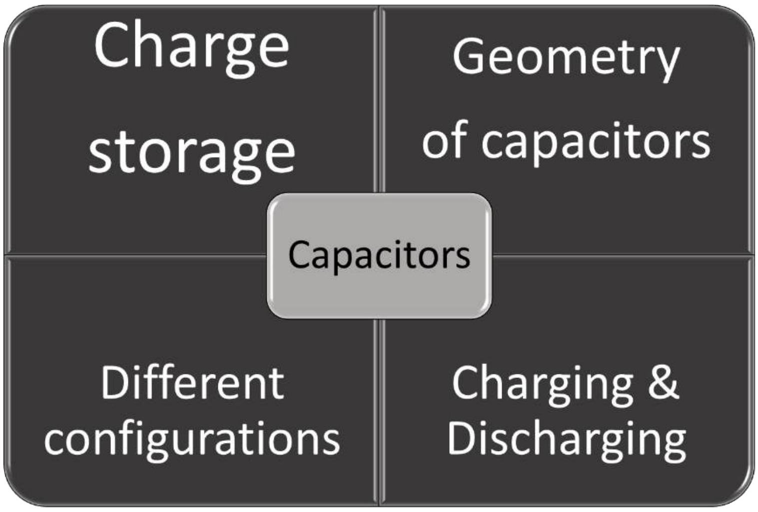
Case 4: Charge in dynamic equilibrium

In the case of dynamic equilibrium, the attractive Coulomb Force between the charged particle and the plate of opposite charge cancels out the weight of the charged particle,

$$F_{Coulomb} = W_{particle} \Rightarrow qE = mg$$



Chapter 2: Capacitors and Dielectrics

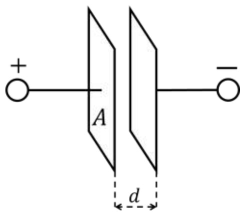


Parallel & Series

Capacitors are essentially batteries. Their ability to store charge is quantified by ***capacitance.*** Capacitance *C*, is the amount of charge *q* stored in one plate of a capacitor per unit potential difference between the plates, *V*,

$$C = \frac{Q_{single\ plate}}{V}$$

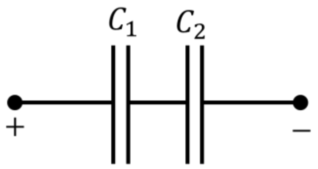
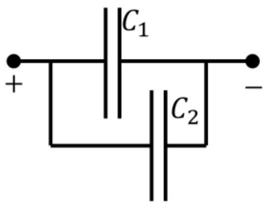
As a function of its geometry, capacitance of a parallel plate capacitor is



$$C = \frac{\varepsilon A}{d}$$

where ε is the permittivity of the space between the plates, A is the area of each plate and d is the distance between the parallel plates.

Multiple capacitors can be arranged either in parallel or series or combinations of them, and their effective capacitance can be calculated depending on their arrangement:

Arrangement	Effective Capacitance
Series 	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$ $C_{eff} = \left(\sum_{i=1}^{n} \frac{1}{C_i}\right)^{-1}$
Parallel 	$C_{eff} = C_1 + C_2 + \cdots + C_n$ $C_{eff} = \sum_{i=1}^{n} C_i$

Energy Stored in a Capacitor

Consider a pair of charged plate with charge

$$Q = CV$$

To deliver a small charge dq at constant variable V, we require the amount of work

$$dW = V\,dQ = \left(\frac{Q}{C}\right)\,dQ$$

The total word done (i.e. energy stored in the capacitor) is then

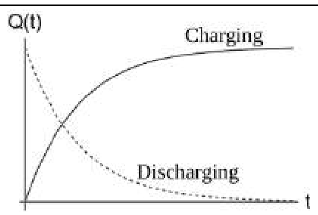
$$W = U = \int dW = \int_0^q \left(\frac{Q}{C}\right)\,dQ = \frac{Q^2}{2C}$$

Considering $Q = CV$, energy stored in a capacitor can be expressed as

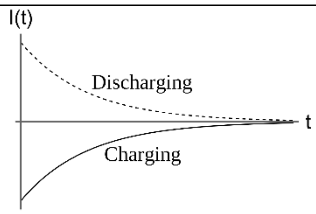
$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Charging & Discharging Capacitors

Capacitors stores charges, now the question is how fast to charge it? Consider a simple circuit consisting of a power supply, a resistor of resistance *R* and a capacitor of capacitance *C*. Accumulation of charge with time for charging and discharging are as follows:

<table>
<tr><td>$Q - t$ graph</td><td>$I - t$ graph</td></tr>
<tr><td>


Discharging: $Q(t) = Q_o e^{-\frac{t}{RC}}$

Charging: $Q(t) = Q_o\left(1 - e^{-\frac{t}{RC}}\right)$
</td><td>


Discharging: $I(t) = -\frac{dQ}{dt} = -\frac{Q_o}{RC}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$

Charging: $I(t) = \frac{Q_o}{RC}e^{-\frac{t}{RC}} = -I_o e^{-\frac{t}{RC}}$
</td></tr>
</table>

Time constant, τ is defined as the time for the exponential term to drop to e^{-1} for discharging, or, for the charge to increase to $1 - e^{-1}$ for charging process, and is calculate by multiplying the *R* and *C*,

$$\tau = RC \text{ [seconds]}$$

Dielectrics

Dielectrics are electrically non-conductive materials placed in between the plates of capacitors to increase the capacitance of the capacitor.

We quantify the increase in capacitance as the **dielectric constant $\boldsymbol{\varepsilon_r}$**, define as the ratio of capacitance of capacitor with dielectric C, to the capacitance of capacitor with no dielectric (vacuum) C_o,

$$\varepsilon_r = \frac{C}{C_o} = \frac{\left(\frac{\varepsilon A}{d}\right)}{\left(\frac{\varepsilon_o A}{d}\right)} = \frac{\varepsilon}{\varepsilon_o}$$

Chapter 3: Current and DC Circuits

Electric Current

Current is the amount of charge ΔQ that passes through a surface area in time Δt,

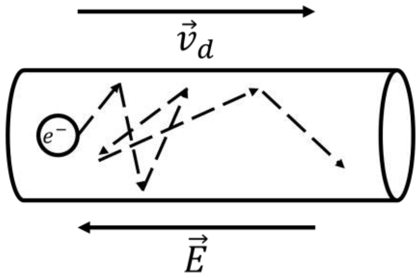
$$I = \frac{dQ}{dt}$$

Total charge Q is simply n multiples of electron charge e,

$$Q = ne$$

Without external electric field, the electron will drift through a conductor with kinetic energy equivalent to Fermi energy, which results in a net velocity of zero. With external electric field, the electron as a whole now gains a net velocity along the electric field. This 'net velocity' is what is known as 'drift velocity'.

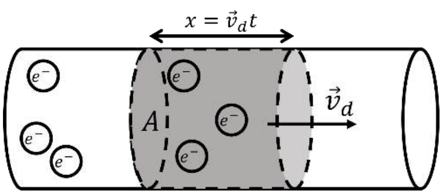
Consider an electron travelling through a conductor, on which an electric field of $\vec{E}$ is applied. The force on the electron is then $F = -qE \Rightarrow a = -\frac{eE}{m}$. Assuming the average time between collision is τ, we can show that



$$v_d = a\tau = \left(-\frac{eE}{m}\right)\tau$$

This means that applying a larger electric field, the larger the kinetic energy obtained by the electron due to a larger drift velocity. This also means that an increase in temperature, increases the collision frequency, decreases collision time and decreases drift velocity of the electrons.

Relating the idea of drift velocity to current can be done by considering a volume section of the conductor V, and the number of charges that flows through that section, n. We can work out that the amount of charge going through V is simply



$$\Delta Q = (ne)A\Delta x$$

where $\Delta x = v_d \Delta t$.

This means that current is

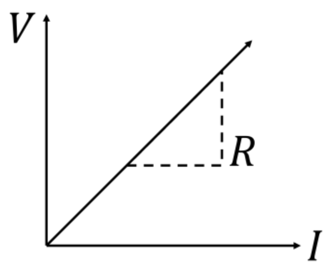
$$I = \frac{\Delta Q}{\Delta t} = neAv_d$$

Ohm's Law

Ohm's Law states that current I, is directly proportional to the potential difference V, is all conditions are constant.

$$V \propto I \ \Rightarrow V = IR$$

R, which is the proportionality constant to Ohm's Law, represents resistance which opposes current flow in a circuit.



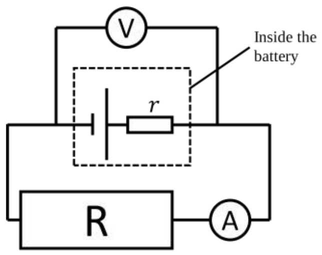
Resistance (Geometry & Temperature)

Resistance of a conductor in a circuit depends on 3 factors – geometry of the conductor, material of the conductor and the temperature of the conductor.

Factor	Equations
Material	For a resistor of resistivity ρ, length L and cross-sectional area A, the resistance is then $$R = \frac{\rho L}{A}$$
Geometry	
Temperature	When temperature of a conductor with coefficient of resistivity α (at $20^{o}C$), changes by ΔT, resistance changes by $$\Delta R = \alpha \Delta T$$

EMF, Internal Resistance and Potential Difference

Electromotive force (emf) is the electrical energy per unit charge generated by a power source generate current. Some of that electrical energy is used to overcome **internal resistance** within the power supply, the rest is then used for the rest of the circuit. That means the potential difference across the circuit is always less than the emf. This internal resistance may exist for a few reasons – distance between electrodes, temperature of the cell, effective area of the electrodes, irregularities found in the cell, etc.

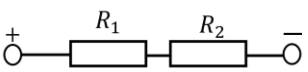
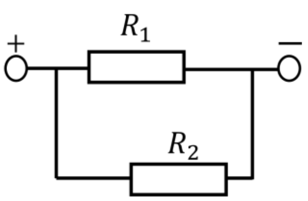


Consider a circuit consisting a voltmeter of reading V, an ammeter of reading I, a battery and a resistor of resistance R. The emf of the source is then

$$\varepsilon = IR + Ir = V + Ir$$

Parallel & Series

For systems of multiple resistors, they can be arranged in parallel, series or any combinations of the two. The effective resistance can then be calculated according to their arrangement.

Arrangement	Effective Capacitance
Series 	$$R_{eff} = R_1 + R_2$$ For n number of resistors in **series**, $$R_{eff} = R_1 + R_2 + \cdots + R_n = \sum_i^n R_i$$
Parallel 	$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$ For n number of resistors in **parallel**, $$R_{eff} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)^{-1} = \left(\sum_i^n \frac{1}{R_i}\right)^{-1}$$

Kirchhoff's Rules

Kirchhoff's Rules allows us to determine current flow around a circuit. The two rules are as follows:

Rules	Statement
First Rule – Junction Rule (Conservation of Charge)	Algebraic sum of currents in a network of conductors meeting at a junction is zero. $$\sum_i I_i = 0$$
Second Rule – Loop Rule (Conservation of Energy)	Algebraic sum of potential difference in any loop must equal to zero. $$\sum_i V_i = 0$$

Electrical Energy and Power

Since work done (amount of energy) to deliver a small charge dq at constant variable V is $W = E = VQ$, and that current by definition is $Q = It$, we can see that energy will simply be

$$E = (VI)t$$

Since $E = Pt$ and taking Ohm's Law ($V = IR$) into consideration, electrical power is then

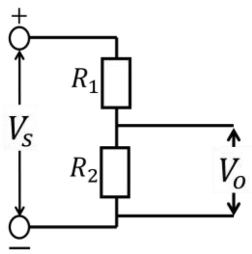
$$P = IV = I^2R = \frac{V^2}{R}.$$

Potential Divider

A potential divider is used to produce a voltage of a fraction of the voltage provided by the power supply. This is achieved by using resistors of different resistances.

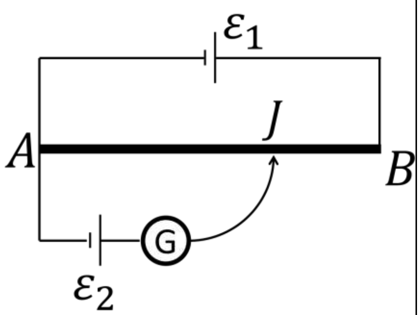
If the power supply provides potential difference of V_s, then the output voltage is simple

$$V_o = \frac{R_1}{R_1 + R_2} V_s$$



Potentiometer

A potentiometer can be used to measure potential differences by two or more cells.

How it works
Wire AB has a resistance of R. This means if the jockey is at point B, then $R_{AJ} = R_{AB}$, and thus $I = I_{maximum}$. As the jockey is slid to towards A, the galvanometer will show zero reading which indicates no current passes through the galvanometer and that the potentiometer is balanced. This means $V_{AJ} = \varepsilon_2$. This happens when $$\frac{V_{AJ}}{V_{AB}} = \frac{l_{AJ}}{l_{AB}}$$ 

We can also use a potentiometer to compare emfs between two cells.

This is done by the following setup.

compare emfs between cell 2 and 3

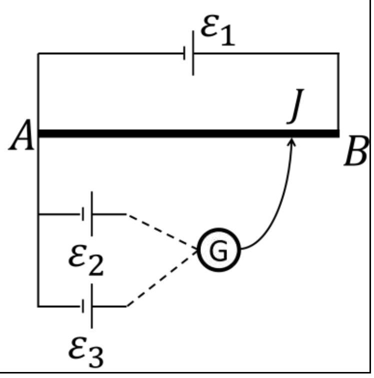
When the galvanometer is connected to ε_2 and balanced between A and J_1,

$$\varepsilon_2 = \frac{l_{AJ_1}}{l_{AB}}\varepsilon_1$$

When the galvanometer is connected to ε_2 and balanced between A and J_2,

$$\varepsilon_3 = \frac{l_{AJ_2}}{l_{AB}}\varepsilon_1$$

$$\Rightarrow \frac{\varepsilon_2}{\varepsilon_3} = \frac{l_{AJ_1}}{l_{AJ_2}}$$



Chapter 4: Magnetism

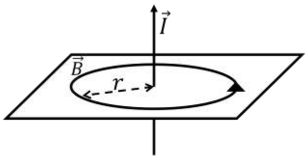
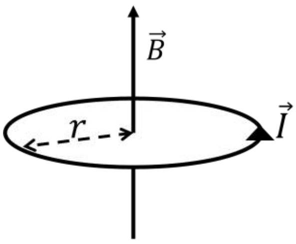
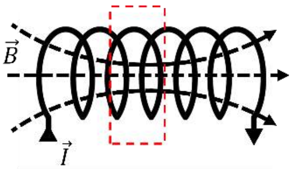
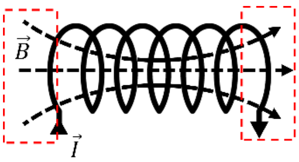
Magnetic Field

A magnetic field is a region of space in which a charged particle will experience magnetic force. They are generated by moving charged particles. Magnetic field lines are always drawn from its north pole to its south pole. When drawn on a 2D plane such as paper, we would generally represent a direction **into** the plane as $\otimes$, and direction **out** of the plane as ●.

$\vec{B}$ from current-carrying conductor

Direction of magnetic field depends on the direction current flow – Right Hand Rule, where the thumb point to the current direction and curled fingers are the magnetic field lines.

4 cases to considering in calculating the magnitude of magnetic field

Situation	Equation
Long straight wire 	$B = \frac{\mu_o I}{2\pi r}$
Centre of circular coil 	$B = \frac{\mu_o I}{2r}$
Centre of solenoid 	$B = \mu_o I n$ Where n is the number of loops per unit length
End of solenoid 	$B = \frac{1}{2}\mu_o I n$ Where n is the number of loops per unit length

Magnetic Force

Force on a moving charged particle in uniform $\vec{B}$

Force on a particle with charge q moving at velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$, the magnetic force acting on it is

$$\vec{F}_{magnetic} = q(\vec{v} \times \vec{B}).$$

In the case of a large enough region, the magnetic force will cause the charged particle to travel in a circular motion. In such cases,

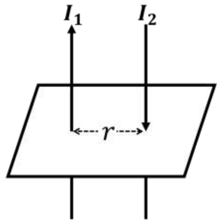
$$\vec{F}_{magnetic} = \vec{F}_{centripetal} \Rightarrow qvB = \frac{mv^2}{r^2}.$$

Force on a current carrying conductor in uniform $\vec{B}$

Consider a quantity of charge ΔQ travelling along a conductor of length *l* in a magnetic field $\vec{B}$ in time *t*. The magnetic force on the conductor is then

$$\vec{F}_{magnetic} = I(\vec{l} \times \vec{B}).$$

Force between two parallel current carrying conductors

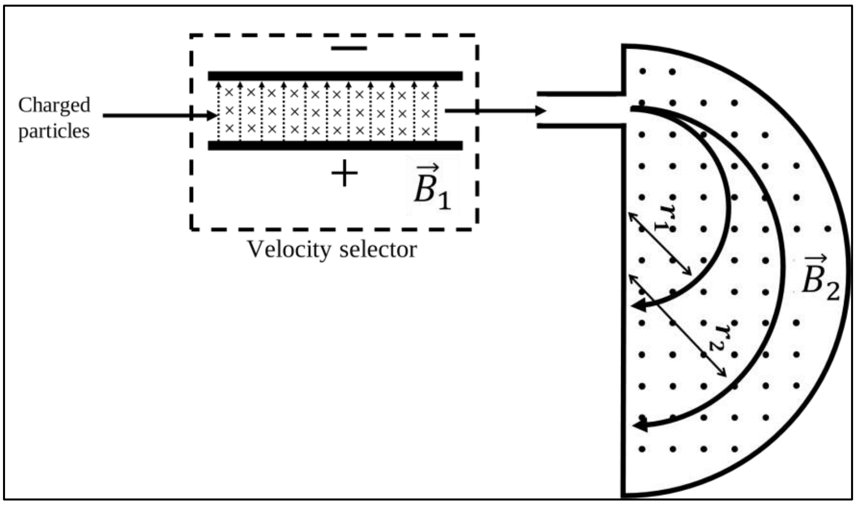


Consider two current carrying conductors of length *l* in proximity such that their magnetic fields overlap, their resultant magnetic force on each other is then

$$\vec{F}_{magnetic} = \frac{\mu_o I_1 I_2}{2\pi r} l$$

Bainbridge mass spectrometer

The Bainbridge mass spectrometer is used to accurately determine atomic mass.



The first part of the mass spectrometer is a velocity selector in which both electric field $\vec{E}$ and magnetic field $\vec{B}_1$. For charged particles to exit this velocity selector, their velocity must obey

$$v = \frac{E}{B_1}$$

This part of the mass spectrometer allows only charged particles with a certain velocity to enter the second region of only magnetic field $\vec{B}_2$.

The second part of the instrument takes advantage that charged particles of the same entry velocity but different mass will travel in circular path of different radius.

$$qvB_2 = \frac{mv^2}{r^2} \Rightarrow m = \frac{qB_2r^2}{v} = \frac{qB_1B_2r^2}{E}$$

Chapter 5: Electromagnetic Induction

Magnetic Flux

Magnetic flux is a measure of total magnetic field $\vec{B}$ passing through a given area $\vec{A}$, this is calculated with

$$\phi = \vec{B} \cdot \vec{A}.$$

In the case of N number of area of $\vec{A}$ of which $\vec{B}$ passes through, the total magnetic flux is called the **magnetic flux linkage Φ,** and is determine by

$$\Phi = N\phi = NBA\cos\theta$$

Induced EMF

EMF is induced when magnetic flux changes with time. This is the core of Faraday's and Lenz's law of electromagnetic induction.

- Faraday's law tells us how much it emf is induced (magnitude) and Lenz's law tells us in what direction will the force act upon (direction of induced current).
- Faraday's law tells us that the magnitude of induced emf is equal to the rate of magnetic flux change and Lenz's law tells us that the induced current will be in the direction opposing the initial magnetic field.

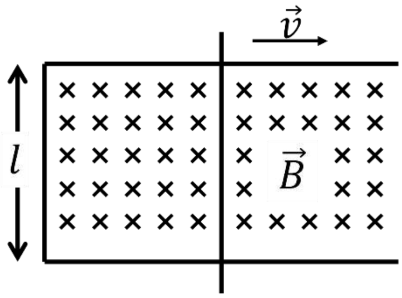
Together, they are simply written as

$$\varepsilon = -\frac{d\phi}{dt} \text{ (for single coil)}$$

Induced emf in straight conductor

In a straight conductor, the area changes with time which causes the magnetic flux to change with time.

Consider a rectangular coil with one of its side movable and the opposite of the movable side has a length of l, in a region of magnetic field $\vec{B}$. If the movable side is moved at velocity $\vec{v}$, the area of the coil would change. The induced emf would then be



$$\varepsilon = -Bl\frac{dx}{dt} = Blv\sin\theta_{vB}$$

Induced emf in a coil

In a circular coil, the option for inducing emf comes from varying the magnetic field **and** the area of the coil, thus 2 equations can be found,

$$\varepsilon = -NB\frac{dA}{dt} \textbf{ or } \varepsilon = -NA\frac{dB}{dt} \text{ (for coil of N turns)}$$

Induced emf in a rotating coil

For a coil rotating at angular speed of ω, the emf induced is then

$$\varepsilon = NBA\omega\sin(\omega t) \text{ (for coil of N turns)}$$

Inductance

Self-induction

The idea of self-inductance is this – a magnetic field induces emf in a conductor, which in turns induces another magnetic field that opposes the initial induced emf. The conductor 'self induces' a magnetic field. The ability of a conductor to do this is quantified by ***self-inductance L,***

$$L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$$

Generally, this means that

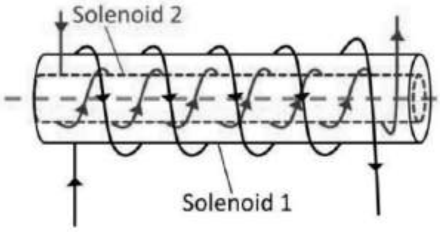
$$LI_1 = N\phi_1$$

For more specific cases, 2 are considered:

1. For a coil of N turns with a cross sectional area of *A* and radius of *r*,
$$L = \frac{\mu_o N^2 A}{2r}$$
2. For a solenoid of N turns with a cross sectional area of *A* and length *l*,
$$L = \frac{\mu_o N^2 A}{l}$$

Mutual induction

Mutual inductance happens between 2 conductors, when the magnetic field induced by one conductor induces current in the other conductor. Consider two coaxial solenoids, a magnetic field is generated by solenoid 1 and thus solenoid 2 respond by an induced emf, if solenoid 2 has a cross sectional area of A_2, then the mutual inductance between solenoid 1 and 2 is



$$M_{21} = \frac{N_1\phi_{12}}{I_2} = \frac{\mu_o N_1 N_2 A_2}{l},$$

where *l* is the length of the solenoid.

Energy Stored in Inductor

The energy stored in an inductor of inductance *L* and with current *I* running through it, is simply

$$U = \frac{1}{2}LI^2$$

Chapter 6: Alternating Current

Alternating Current

Alternating current (AC) is defined as an electric current that periodically reverses its direction with respect to time.

Root Mean Square Values

In AC circuits, rather than being of constant value (such found in DC Circuits), voltages and current now are functions of time:

$$I \mapsto I(t) = I_{peak}\, sin\,(\omega t) = I_{peak}\, sin\,(2\pi ft)$$

$$V \mapsto V(t) = V_{peak}\, sin\,(\omega t) = V_{peak}\, sin\,(2\pi ft)$$

Resistance is then defined as

$$R = \frac{V_o}{I_o}$$

In calculation of power, where $P_{DC} = IV$, for AC circuits,

$$P_{AC} = I_{rms}V_{rms}$$

where,

$$I_{rms} = \frac{I_o}{\sqrt{2}} \text{ and } V_{rms} = \frac{V_o}{\sqrt{2}}$$

Impedance

In DC circuit, our main concern for opposition of current flow is only resistance R.

In AC circuits, we now have what is known as **impedance *Z***, which is defined by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

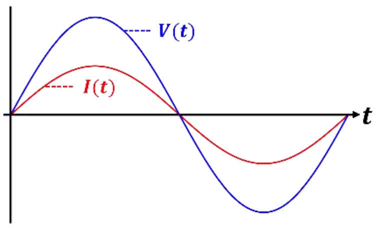
where R is the resistance, X_L is the inductive reactance and X_C is the capacitive reactance found in the circuit.

The following table shows how to calculate these values

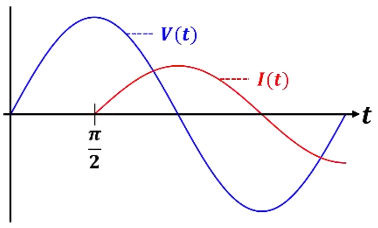
Reactance	Equation
Capacitance reactance for a capacitor of capacitance C	$X_C = \frac{1}{2\pi f C}$
Inductive reactance for an inductor of inductance L	$X_L = 2\pi f L$

Sinusoidal graphs $V(t) - t$ and $I(t) - t$ are as follows:

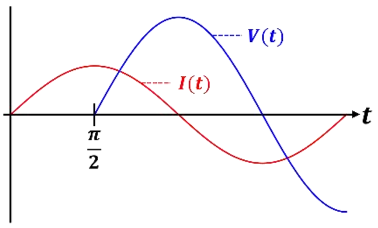
Purely Resistance



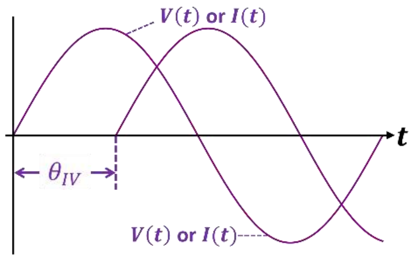
Purely Inductive (**ELI**)



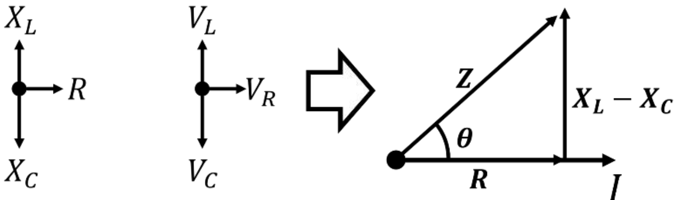
Purely Capacitive (**ICE**)



RLC Circuit



The phasor diagram for an RLC circuit is as follows,

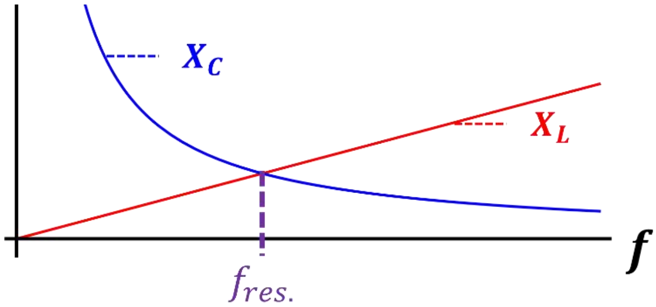


Phasor Diagram for RLC Circuit

Which means that the phase angle between current and voltage is

$$\theta_{IV} = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Resonance occurs when $X_L = X_C \Rightarrow \omega = \frac{1}{\sqrt{LC}} = 2\pi f$.



Power & Power Factor

2 types of power that be calculate in the case of AC circuits,

1. Instantaneous power

$$P = I(t) \times V(t)$$

2. Average power

$$P_{ave} = I_{rms}V_R = I_{rms}V_{rms}\cos(\theta_{IV})$$

The power factor is simply

$$\cos\theta_{IV} = \frac{P_{real}}{P_{apparent}} = \frac{P_{ave}}{I_{rms}V_{rms}}$$

Chapter 7: Optics

Geometrical Optics: Reflection

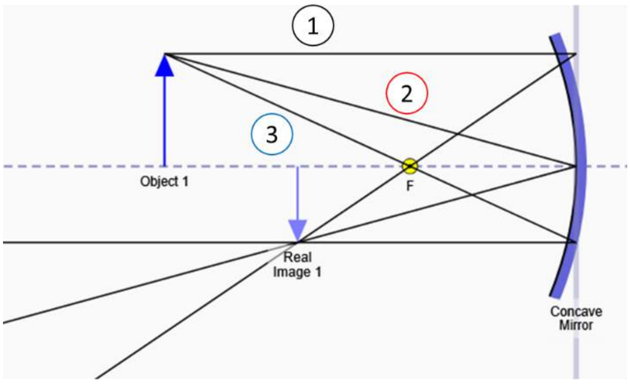
Definitions:

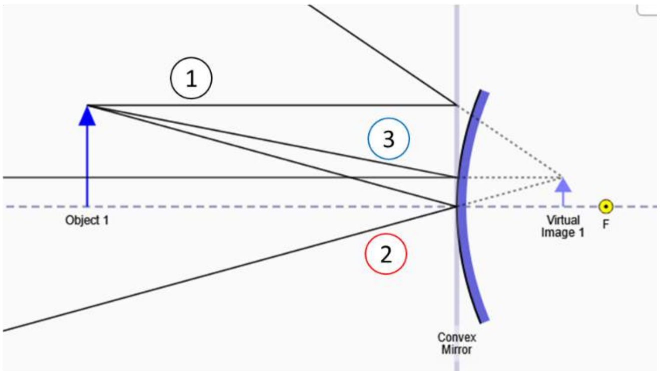
1. Centre of curvature, C = a point on the principal (or optical) axis that is positioned at distance equal to the radius of curvature R, of the spherical mirror.
2. Focal point, f = a point on the principal axis at which light rays travelling parallel to the principal axis will converge onto or diverge from, after reflecting on the surface of the spherical mirror.

f and R are related by the following equation:

$$R = 2f$$

Ray Diagram:





Line 1 – This line will be parallel to the optical axis before the reflecting on the mirror. Then it will cross the focal length after reflecting upon the spherical mirror.
Line 2 – This line will reflect upon the spherical mirror at angle θ with the optical axis then gets reflected off the spherical mirror at the same angle.
Line 3 – This line will cross the focal length before reflecting on the mirror. Then it will be parallel to the optical axis after reflecting upon the spherical mirror.

2 types of mirrors:

1. **Convex mirror**, of which its radius is located behind the mirror.
2. **Concave mirror**, of which it's radius of curvature is located in front of the mirror.

For both types of mirror, the equation relating object and image distances are related by

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Conventions	
Focal length, f	+ for concave; - for convex
Curvature Radius, R	

Lateral magnification m, refers to the ratio between the height of the image to the height if the object. In equation form,

$$m = \frac{h_i}{h_o}$$

$$m > 0 \Rightarrow \text{ upright image; } m < 0 \Rightarrow \text{ inverted image}$$

Geometrical Optics: Refraction

An extension to Snell's law will be the refraction at a spherical surface. The following equation allows us to relate distances, refractive indices and radius of curvature of the spherical surface:

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

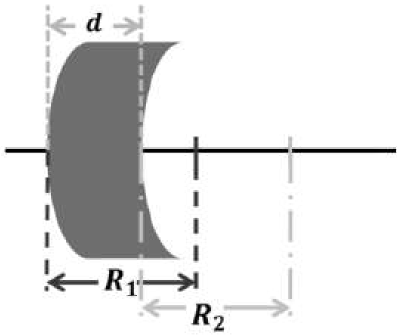
In the equation above n_i, refers to refractive indices, *u* and *v* refers to object and image distances respectively and *R* refers to the radius of curvature.

Conventions	
Curvature Radius, *R*	+ for convex, i.e. *C* opposite side as incoming light - for concave, i.e. *C* same side as incoming light

For the refractive indices, subscript 1 refers to the refractive index on the side of the incoming light rays and subscript 2 refers to the refractive index on the side of the outgoing rays.

Geometrical Optics: Thin lenses

The thin lens equation assumes that the thickness measured between two vertex of the spherical surface of a lens is much smaller than the product of the radii of the spherical lenses, that is $d \ll R_1 R_2$.



For thin lenses,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Conventions	
Focal length, *f*	+ for convex, i.e. same side as incoming light - for concave, i.e. opposite side as incoming light

On the other hand, using the lens maker's equation,

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

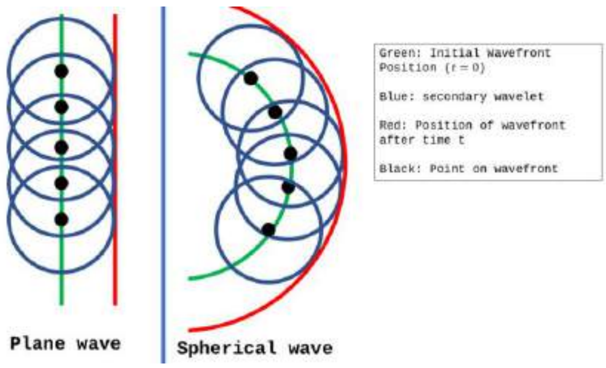
one can determine the focal length f, of the lens from

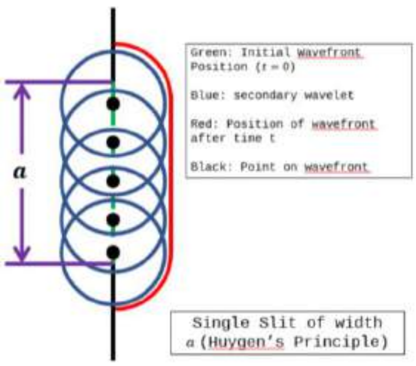
1. the radii of the lens surfaces, R_1 and R_2,
2. the ratio of the refractive index of the lens material to the refractive index of the surrounding, $\frac{n_{material}}{n_{medium}}$

Conventions	
Curvature Radius, *R*	- if curvature same side as incoming light + if curvature opposite side as incoming light

Physical Optics: Huygens's Principle

Huygen's Principle states that "each point on the wavefront acts as the source of secondary wavelets that spread out in all directions in spherical waves with a speed equal to the speed of wave propagation."



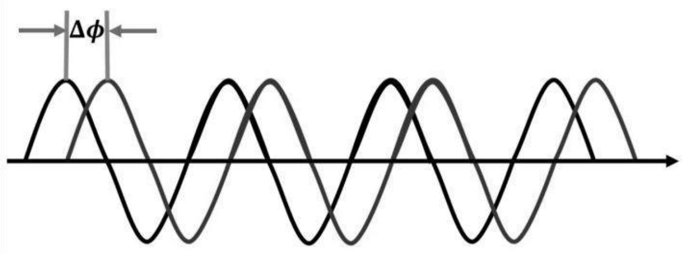


Physical Optics: Interferences

Coherence between 2 waves refers to the condition of constant phase difference between 2 waves with respect to time, that is to say $\frac{d\phi}{dt} = 0$. This property is the ideal property for stationary interference.

For a stable interference pattern, the following conditions are required:

1. Coherence, that is to say the two interacting light waves are of the same phase difference, $\frac{d\phi}{dt} = 0$.



2. Monochromatic, that is to say that the two interacting light waves are of the same wavelength, i.e. $\lambda_1 = \lambda_2$

For purely **constructive interference**, it is empirical that the phase difference between the interacting waves is either 0 or $n\lambda$. On the other hand, for purely **destructive interference**, it is required that the phase difference between the two interacting wave is $\frac{n\lambda}{2}$.(*n* is both cases refers to integer values.)

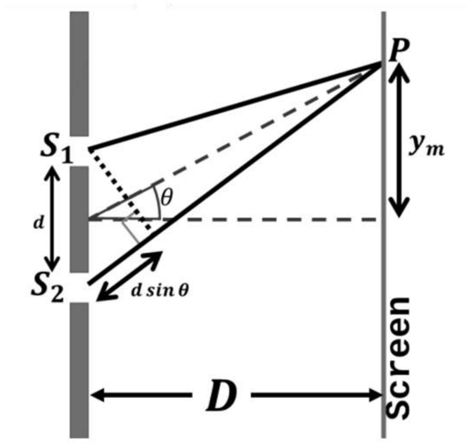
Physical Optics: Slits

Double Slit

We now consider the case for Young's double slit experiment.

Here we define the following variables:

D	distance from slit to screen
d	slit separation
y_m	distance from central maximum tu the mth fringe



We know that in order to determine what type of fringe forms at P, we need to look at the path difference and from the figure, we can say that the figure,

$$\Delta\phi = S_2P - S_1P = d sin\theta = d\left(\frac{y_m}{D}\right)$$

For **bright fringes**,

$$\Delta\phi = \frac{y_m d}{D} = m\lambda$$

Rearranging this allows us to find fringe distance as a function of *d* and *D* with *m* having any integer value indicating **mth bright fringe** from central maximum:

$$y_m = \frac{m\lambda D}{d}$$

Shifting one the waves by 0.5λ give us the equation for **dark fringes**,

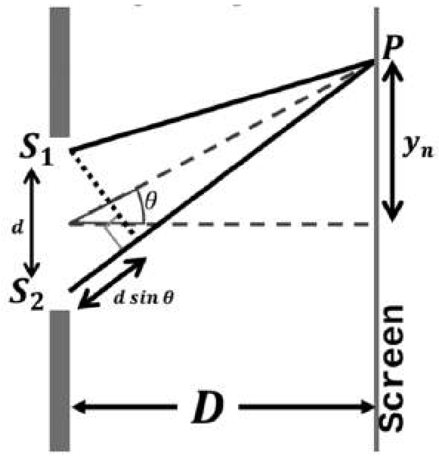
$$y_m = \frac{(m + 0.5)\lambda D}{d}.$$

Lastly, we'd want to calculate the fringe separation. This can be done by considering $\Delta y = y_{m+1} - y_m$, which results in

$$\Delta y = \frac{\lambda D}{d}$$

Single Slit

Diffraction is defined as the spreading or bending of waves as they pass through an aperture of a barrier. The diffracted waves then interfere with each other to produce a diffraction pattern.



Light waves from one portion of the slit interacts with light waves from a different portion of the same slit to produce a diffraction pattern.

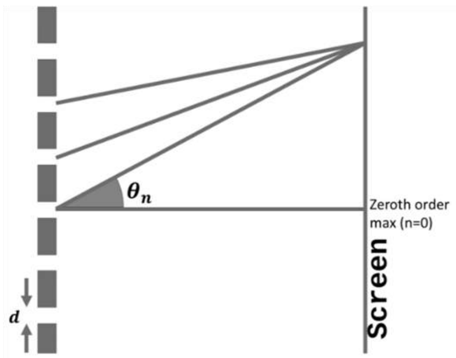
Here, we find that the dark fringes to forms when according to

$$d\ sin\,\theta = \ n\lambda.$$

Geometrically, we also find that $tan\theta \approx sin\theta \approx \frac{n\lambda}{d} \approx \frac{y_n}{D}$.

As such, we can say that **dark fringes** forms at $y_n = \frac{n\lambda D}{d}$. This would also mean that **bright fringes** forms at $y_n = \frac{(n+0.5)\lambda D}{d}$.

Diffraction Grating



In the case for diffraction grating, light waves from many slits and interfere at the screen to form fringes of equal width. The equation by which the pattern follows is $d\ sin\,\theta_n = n$ for **bright fringes** and shifted by 0.5λ for **dark fringes**. Note that the angle θ_n is measured from the normal line formed at the zeroth order maximum. Also note that, **maximum number of fringes** can be calculated by considering that $sin\,\theta_n < 1$.

Physical Optics: Thin Films

Referring to figure on thin films, we can see that the two reflected light waves has a phase difference of 0.5λ from reflections at surface 1 and 2. One must also take into consideration of the extra distance that the second (green) wave travelled, that is $2nt$. Therefore, the total phase difference between the reflected waves is then

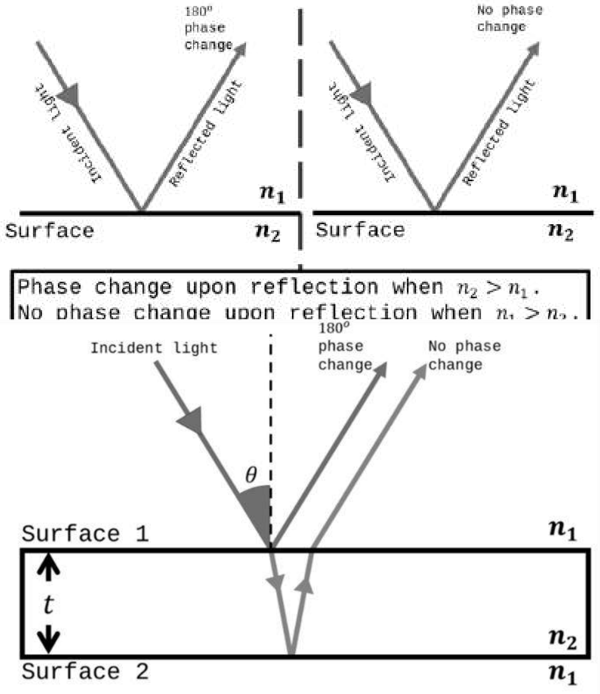
$$\Delta\phi = 2nt - \frac{1}{2}\lambda.$$

For **constructive interference**, $\Delta\phi = 2nt - \frac{1}{2}\lambda = n$, which gives us the equation

$$2nt = \left(n + \frac{1}{2}\right)\lambda.$$

Dark fringes then appear between the bright fringes, i.e. they follow the equation

$$2nt \; = \; n\,\lambda.$$



Main application for this concept of thin film interference is in **optical coatings** as one can manipulate the thickness of the coating as to choose the level of constructive or destructive interference. These optical coatings can be applied onto both for reflective as well as refractive systems.

Chapter 8: Particle Waves

De Broglie Wavelength

Like light, matter also exist in dual form – as particles **and** waves.

Matter waves, known as "de Broglie wavelength", are calculated with

$$\lambda_{matter} = \frac{h}{p} = \frac{h}{mv}$$

For a particle with of mass m charge q accelerated by electric field of V volts,

$$\lambda_{matter} = \frac{h}{\sqrt{2qVm}}$$

Electron Diffraction

On de Broglie wavelength:

1. To show that particles may exhibit wave-like characteristics, Davisson and Germer designed an experiment in which they show that electrons diffracted.
2. They achieve this by directing a beam of electrons onto a nickel crystal.

On electron microscope:

1. Because of their short wavelength (1nm for electrons vs 400nm – 700nm for light microscope), electronic microscopes can offer physicists a higher resolution in probing specimens.
2. Optical microscope is made up of glass lenses, whereas components of an electron microscope are electromagnetic.

Chapter 9: Nuclear & Particle Physics

Binding Energy & Mass Defect

Mass defect, Δm = mass difference between the actual mass of an atomic nucleus and the sum of its components, i.e. protons and neutrons.

For an atomic nucleus of mass $m_{nucleus}$ with *Z* number of protons of mass m_{proton} and *N* number of neutrons of mass $m_{neutron}$, its mass defect is

$$\Delta m = \left(Zm_{proton} + Nm_{neutron}\right) - m_{nucleus}$$

Binding energy, $E_{binding}$ = energy found in the nucleus of an atom that binds its components together. This energy can be calculated from the mass defect,

$$E_{binding} = \Delta mc^2$$

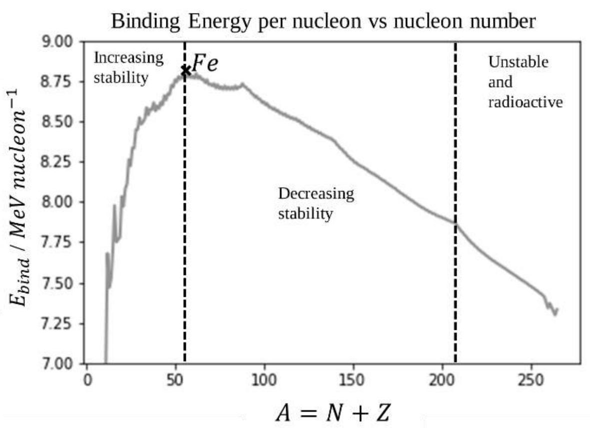
As the masses of atomic nucleus is well, very small, and the speed of light is astronomical, it may be easier to perform calculations using atomic mass unit (amu) or Dalton (u) and MeV/c^2

$$1kg = 6.022(10^{26})u = 5.60958(10^{29})\ MeV/c^2$$
$$1u = 1.66054(10^{-27})\ kg = 931.494\ MeV/c^2$$

Binding energy per nucleon, $\frac{E_{binding}}{A}$:

Nucleon number, *A* = Number of protons, *Z* + Number of neutrons, *N*

Binding energy per nucleon vs nucleon number graph:



- For nuclei lighter than that of iron (Fe), it is found that binding energy per nucleon increases with the nucleon number.
- After the iron limit, the binding energy per nucleon decreases.
- At $A \approx 209$ (nuclei of Bi), the binding energy per nucleon is too weak to keep the nuclei together and thus, are unstable and radioactive.

Radioactivity

The following table describes the types of decay of a radioactive substance

Type of decay	Process	Description
α	${}^{A}_{Z}P \to {}^{A-4}_{Z-2}D + {}^{4}_{2}He$ parent nucleus → daughter nucleus + α particle	In α decay, an α particle (Helium) is emitted when the parent nucleus decays into its daughter nucleus. Electrical charge is conserved throughout the process. Energy is released upon α decay.
β^-	$n \to p^+ + e^- + \bar{\nu}$ ${}^{A}_{Z}P \to {}_{Z+1}^{A}D + {}_{-1}^{0}e + \bar{\nu}$ parent nucleus → daughter nucleus + β^- particle + antineutrino	In β^- decay, an electron e^- and an antineutrino $\bar{\nu}$ is emitted when the parent nucleus decays into its daughter nucleus.
β^+	$p^+ \to n + e^+ + \nu$ ${}^{A}_{Z}P \to {}_{Z-1}^{A}D + {}_{+1}^{0}e + \nu$ parent nucleus → daughter nucleus + β^+ particle + neutrino	In β^+ decay, a positron e^+ and a neutrino ν is emitted when the parent nucleus decays into its daughter nucleus.
γ	${}^{A}_{Z}P * \to {}^{A}_{Z}P + \gamma$ nuclei of high energy state → nuclei of low energy state + γ ray	In γ decay, the emission is a photon (light ray). This happens because the nucleons lower its energy state.

In general, N number of radioactive particles will decay according to the **decay law**,

$$\frac{dN}{dt} = -\lambda N$$

where λ is the decay constant of the substance, which varies between isotopes.

The solution for the decay law is

$$N(t) = N_o e^{-\lambda t}$$

where $N_o = N(t = 0)$.

The rate of decay is known as **activity**

$$A = \left|\frac{dN}{dt}\right| = \left|\frac{dN_o}{dt}\right| e^{-\lambda t} = A_o e^{-\lambda t}$$

Half-life is simply the time it takes for the number of isotopes to decrease by half $T_{\frac{1}{2}}$,

$$N = \frac{1}{2} N_o = N_o e^{-\lambda T_{\frac{1}{2}}} \Rightarrow T_{\frac{1}{2}} = \frac{ln\, 2}{\lambda}$$

Particle Accelerator

Thermionic emission (Edison Effect) = emission of electrons on the surface of a metal by providing it sufficient thermal energy.

As mentioned before, a charged particle may be accelerated by the help of an electric and magnetic field. The acceleration would stem from Lorentz Force.

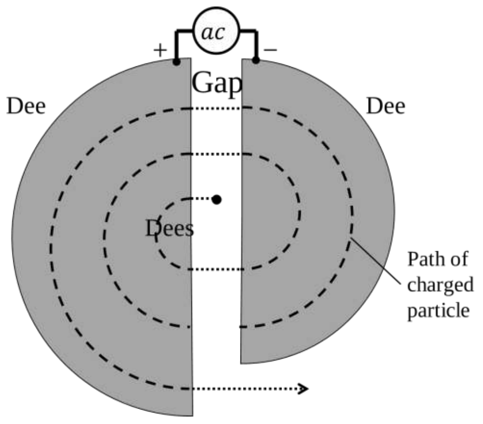
To probe subatomic particles, we need high energy because higher energy results in higher momentum which gives out smaller de Broglie wavelength. This means a higher resolution can be achieved.

2 types of particle accelerators:

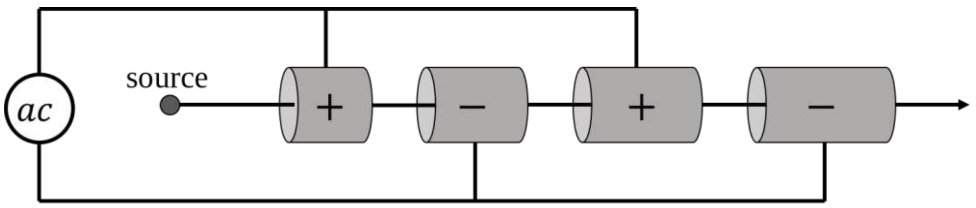
1. Cyclotron
   It uses magnetic field to maintain charged particles in nearly circular paths.

   A cyclotron is composed of 2 'dees', charged particles are accelerated in the region of space between the two 'dees', where an electric field is applied.

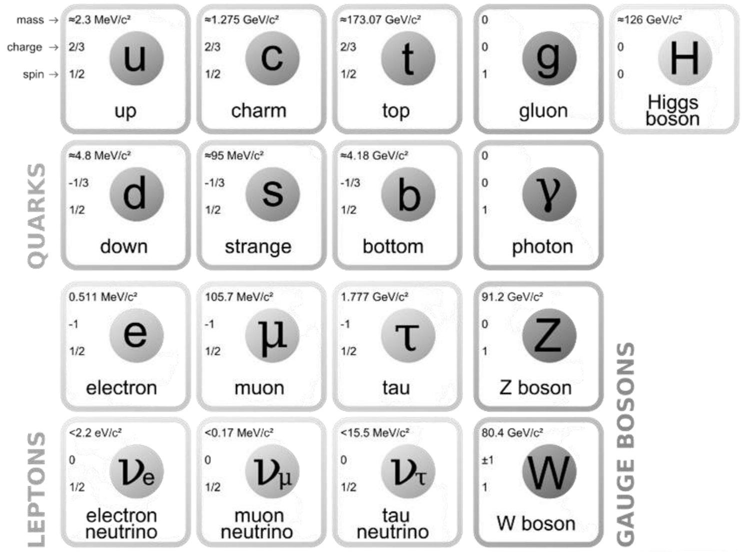
   Velocity of the charged particles when they are in the 'dees' is
   $$v = \frac{qBr}{m}$$
   Frequency of electric field is equal to the frequency of the circulating protons,
   $$f = \frac{qB}{2\pi m}$$
   which is known as the **cyclotron frequency**.

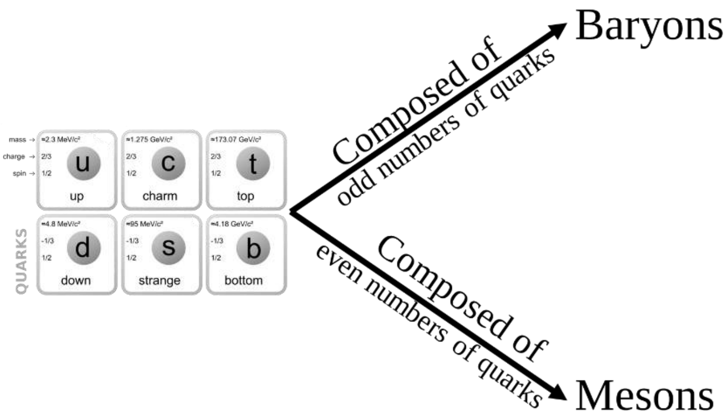


2. Linear Accelerator
   Charged particles are accelerated through a series of linear conductor tubes.
   Alternating voltage is applied to consecutive tubes so that when a charged particle reaches a gap, the tube they just left is now negatively charged and the tube they are heading into is positively charged.



Fundamental Particles





Particle-antiparticle pair:

They are pairs of particles that has opposite charge to each other. E.g., electron has a negative charge whereas its antiparticle, a positron has a positive charge. They interact by annihilating each other.

$$e^{-} + e^{+} \rightarrow \gamma + \gamma$$

---End of Lecture Notes---

One Page Notes

Chapter 1: Electrostatics

Coulomb's Law

$$F_{Coulomb} = \frac{kq_1q_2}{r_{12}^2}$$

$$k = \frac{1}{4\pi\epsilon} = 8.98 \times 10^9\ kg\, m^3\, s^{-4}\, A^{-2}$$

For more than 2 particles, the Coulomb Force on particle *j* becomes

$$F_{Coulomb} = kq_j \sum_i \frac{q_i}{r_{ij}^2}$$

The electric field at a point in space $E(r)$,

$$E(r) = \frac{F(r)}{q_{test}} = \frac{kq_{source}}{r^2}$$

Electric potential,

$$V = \frac{W_{\infty \to r}}{q_{test}} = \frac{kq_{source}}{r}$$

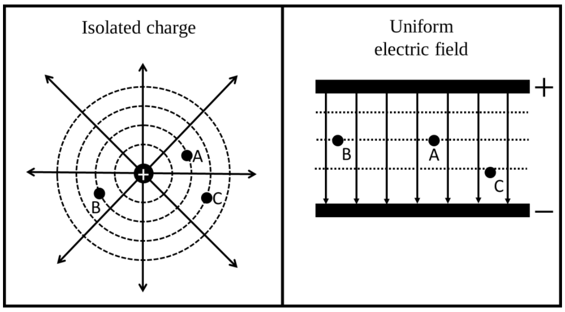
Electric potential energy,

$$U = q_{test}V$$

$$1eV = 1.6 \times 10^{-19} J$$

Equipotential lines and surfaces

= graphical representation on which a particle on the line or surfaces is at the same potential.



In both examples,

$$V_A = V_B \neq V_C$$

Charge in Uniform Electric Field,

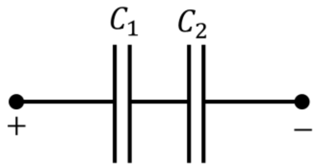
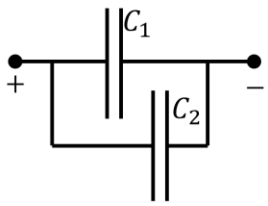
$$E = \frac{V}{d}$$

$$F = ma = qE$$

Chapter 2: Capacitors

Capacitance,

$$C = \frac{Q_{single\ plate}}{V} = \frac{\varepsilon A}{d}$$

Arrangement	Effective Capacitance
Series 	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$ $C_{eff} = \left(\sum_{i=1}^{n} \frac{1}{C_i}\right)^{-1}$
Parallel 	$C_{eff} = C_1 + C_2 + \cdots + C_n$ $C_{eff} = \sum_{i=1}^{n} C_i$

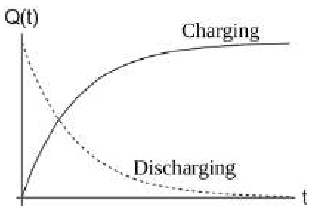
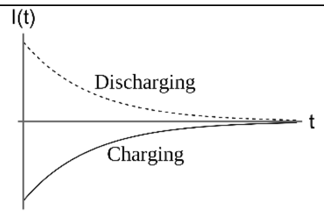
Energy Stored in Capacitor,

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Time Constant,

$$\tau = RC \text{ [seconds]}$$

Charging & Discharging

$Q - t$ graph	$I - t$ graph
 Discharging: $Q(t) = Q_o e^{-\frac{t}{RC}}$ Charging: $Q(t) = Q_o\left(1 - e^{-\frac{t}{RC}}\right)$	 Discharging: $I(t) = -\frac{dQ}{dt} = -\frac{Q_o}{RC}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$ Charging: $I(t) = \frac{Q_o}{RC}e^{-\frac{t}{RC}} = -I_o e^{-\frac{t}{RC}}$

Dielectric Constant,

$$\varepsilon_r = \frac{C}{C_o} = \frac{\left(\frac{\varepsilon A}{d}\right)}{\left(\frac{\varepsilon_o A}{d}\right)} = \frac{\varepsilon}{\varepsilon_o}$$

Chapter 3: Current and DC Circuits

Electrical Current,

$$I = \frac{dQ}{dt}$$

Total Charge,

$$Q = ne$$

Ohm's Law,

$$V \propto I \ \Rightarrow V = IR$$

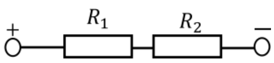
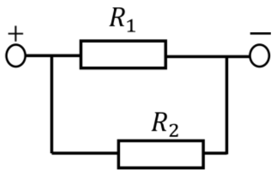
Resistance

Factor	Equations
Material	$R = \frac{\rho L}{A}$
Geometry	
Temperature	$\Delta R = \alpha \Delta T$

Electromotive force,

$$\varepsilon = IR + Ir = V + Ir$$

Resistors in parallel and series,

Arrangement	Effective Capacitance
Series 	$R_{eff} = R_1 + R_2$ For *n* number of resistors in **series**, $R_{eff} = R_1 + R_2 + \cdots + R_n$ $= \sum_i^n R_i$
Parallel 	$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$ For *n* number of resistors in **parallel**, $R_{eff} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)^{-1}$ $= \left(\sum_i^n \frac{1}{R_i}\right)^{-1}$

Kirchhoff's Rules,

Rules	Statement
First Rule – Junction Rule (Conservation of Charge)	$\sum_i I_i = 0$
Second Rule – Loop Rule (Conservation of Energy)	$\sum_i V_i = 0$

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First Rule – Junction Rule (Conservation of Charge)	$\sum_i I_i = 0$
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Potential Divider,

$$V_o = \frac{R_1}{R_1 + R_2} V_s$$

Electrical Power,

$$P = IV = I^2 R = \frac{V^2}{R}$$

Potential Divider,

$$V_o = \frac{R_1}{R_1 + R_2} V_s$$

Electrical Power,

$$P = IV = I^2 R = \frac{V^2}{R}$$

Chapter 5: Electromagnetic Induction

Magnetic Flux Linkage,

$$\Phi = N\phi = NBA\cos\theta$$

Induced emf (Faraday's & Lenz's Law),

$$\varepsilon = -\frac{d\phi}{dt}$$

Induced emf in straight conductor,

$$\varepsilon = -Bl\frac{dx}{dt} = Blv\sin\theta_{vB}$$

Induced emf in a coil,

$$\varepsilon = -NB\frac{dA}{dt} \textbf{ or } \varepsilon = -NA\frac{dB}{dt}$$

Induced emf in rotating coil,

$$\varepsilon = NBA\omega\sin(\omega t)$$

Self-Inductance,

$$L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)} = \frac{N\phi}{I}$$

Self-Inductance in coil,

$$L = \frac{\mu_o N^2 A}{2r}$$

Self-Inductance in solenoid,

$$L = \frac{\mu_o N^2 A}{l}$$

Mutual Inductance,

$$M = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)} \text{ (General); } M_{21} = \frac{\mu_o N_1 N_2 A_2}{l} \text{(two coaxial coils)}$$

Energy Stored in Inductor,

$$U = \frac{1}{2}LI^2$$

Chapter 6: AC Circuits

Rms Current and Voltage,

$$I_{rms} = \frac{I_o}{\sqrt{2}} \text{ and } V_{rms} = \frac{V_o}{\sqrt{2}}$$

Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Reactance,

Reactance	Equation
Capacitance reactance	$X_C = \frac{1}{2\pi f C}$
Inductive reactance	$X_L = 2\pi f L$

Phase Angle,

$$\theta_{IV} = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Resonance occurs when $X_L = X_C \Rightarrow \omega = \frac{1}{\sqrt{LC}} = 2\pi f$.

Resonant frequency,

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Powers,

$$P = I(t) \times V(t)$$
$$P_{ave} = I_{rms}V_{rms}\cos(\theta_{IV})$$

Power Factor,

$$\cos\theta_{IV} = \frac{P_{real}}{P_{apparent}} = \frac{P_{ave}}{I_{rms}V_{rms}}$$

Chapter 7: Optics

Reflection upon curved mirror,

Radius and focal length, $R = 2f$

Magnification, $m = \frac{h_i}{h_o}$

Mirror Equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Refraction upon curved surface,

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

Thin Lenses,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Double Slits,

For bright fringes,

$$\Delta\phi = \frac{y_m d}{D} = m\lambda$$

Single Slits,

For bright fringes,

$$y_n = \frac{(n + 0.5)\lambda D}{d}$$

Diffraction grating,

For bright fringes,

$$d \sin \theta_n = n$$

Thin Films,

Condition	Maxima	Minima
$n_1 < n_2;\ n_2 > n_3$	$2nt = (m - 0.5)\lambda$	$2nt = m\lambda$
$n_1 < n_2 < n_3$	$2nt = m\lambda$	$2nt = (m + 0.5)\lambda$
Minimum Thickness	$t = \frac{\lambda}{4n}$	$t = \frac{\lambda}{2n}$

Chapter 8: de Broglie Wavelength

$$\lambda_{matter} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2qVm}}$$

Chapter 9 Nuclear & Particle Physics

Mass Defect,

$$\Delta m = \left(Zm_{proton} + Nm_{neutron}\right) - m_{nucleus}$$

Binding Energy,

$$E_{binding} = \Delta mc^2$$

Decay law,

$$\frac{dN}{dt} = -\lambda N \ \Rightarrow N(t) = N_o e^{-\lambda t}$$

Activity,

$$A = A_o e^{-\lambda t}$$

Half-life,

$$T_{\frac{1}{2}} = \frac{ln\,2}{\lambda}$$

Exercise Sheet 1: Coulomb's Law & Electric Field

Easy

1 Two point charges of $+2\mu C$ and $-4\mu C$ are separated by 2cm. Determine the electric force between them.

2 A point charge has a charge of $-2\mu C$. Determine the electric field 2cm from the point charge.

3 A point charge of $2\mu C$ is placed in a region of electric field of $200NC^{-1}$. Calculate the magnitude of force acting on the point charge?

Past Year

4 **PSPM 21/22 – Q1(a)**

Two opposite charges of the same magnitude of $2 \times 10^{-7}C$ are separated by 15cm. Calculate the

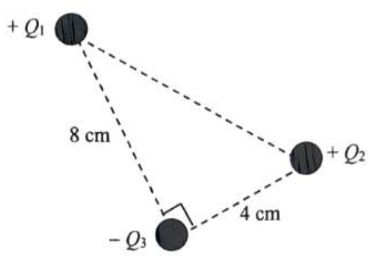
a. Electric field strength at the midpoint between both charges.
b. Magnitude of the force exerted on an electron on that point.

5 **PSPM 20/21 – Q1(a)**

The figure shows three fixed point charges, $Q_1 = +12nc$, $Q_2 = +3nC$ and $Q_3 = -4nC$.

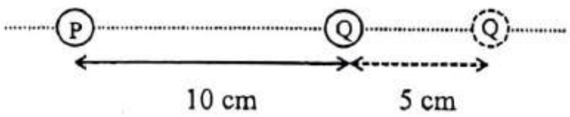
Sketch the electric force diagram on the charge Q_3.

Addition: Calculate the electric force on Q_3.



6 **PSPM 19/20 – Q1(a)**

The electric field at a point 10cm away from a charge P as shown in the diagram is $2.7(10^6)NC^{-1}$.



a. Determine the charge of P.
b. Sketch the electric force vectors on charge P and Q.
c. If charge Q is moved horizontally to the right to a new position, and the electric force on it is $-4.05N$, how far apart is charge Q from charge P?

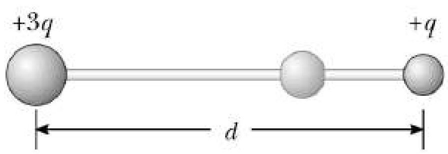
Book

7 Two small nonconducting spheres have a total charge of $90\mu C$. When placed 28.0 cm apart, the force each exerts on the other is 12.0 N and is repulsive.

What is the charge on each? What if the force were attractive?

8 Two spherical objects are separated by a distance that is $1.80 \times 10^3 m$. The objects are initially electrically neutral and are very small compared to the distance between them. Each object acquires the same negative charge due to the addition of electrons. As a result, each object experiences an electrostatic force that has a magnitude of $4.55 \times 10^{-21}N$. How many electrons did it take to produce the charge on one of the objects?

9 Two small beads having positive charges 3q and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point x = d. As shown in the figure, a third small charged bead is free to slide on the rod.



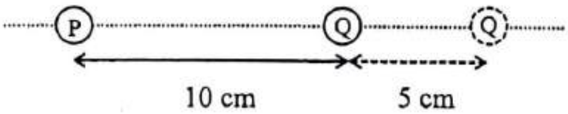
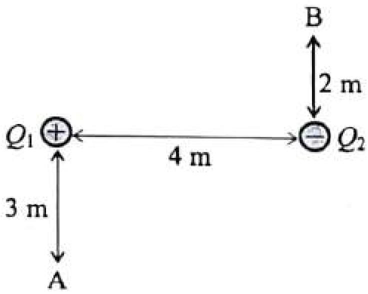
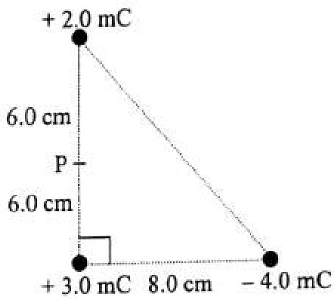
At what position is the third bead in equilibrium?

Exercise Sheet 2: Electric Potential & Potential Energy

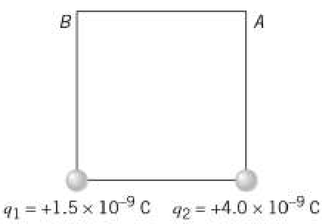
Easy

1	The energy needed to bring a $2\mu C$ to point A is $20 \times 10^6 J$. Determine the electric potential at that point.
2	Referring to question 1, if point A is 30cm from a point charge Q. determine the charge of Q.
3	A system of charges consists of two point charges of $+2\mu C$ and $-4\mu C$ are separated by 2cm. Determine the electric potential energy of the system.

Past Year

4	**PSPM 19/20 – Q1(a)** The electric field at a point 10cm away from a charge P as shown in the diagram is $2.7(10^6)NC^{-1}$.  A charge Q is placed 10cm away from charge P. When charge Q is moved horizontally to the right to a position 5cm from its initial position, there is a 0.54J change in its electric potential energy of the system. What is the charge of Q?
5	**PSPM 18/19 – Q1(a)** The figure shows two charges, $Q_1 = +8\mu C$ and $Q_2 = -6\mu C$ placed 4m apart. Calculate a. the electric potential at points A and B. b. the electric potential difference between points A and B. 
6	**PSPM 16/17 – Q1(b)** The figure shows three-point charges placed at each vertices of a right angle triangle. Calculate the a. electric potential at point P b. electric potential energy of the system. 

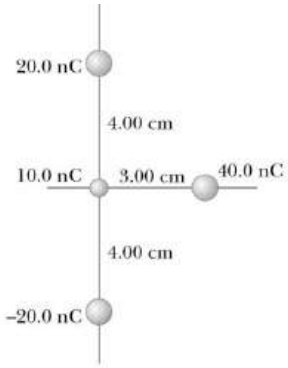
Book

7	The drawing shows a square, each side of which has a length of L = 0.25 m. On two corners of the square are fixed different positive charges, $q_1(= +1.5nC)$ and $q_2(= +4nC)$. Find the electric potential energy of a third charge $q_3 = -6nC$ placed at corner A and then at corner B. 
8	How much work must be done to bring three electrons from a great distance apart to $10^{-10}m$ from one another (at the corners of an equilateral triangle)?

9 Two particles, with charges of 20nC and –20nC, are placed at the points with coordinates (0, 4cm) and (0, –4cm), as shown in the figure. A particle with charge 10nC is located at the origin.

i. Find the electric potential energy of the configuration of the three fixed charges.

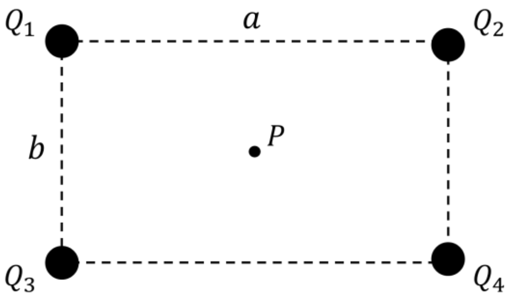
ii. A fourth particle, with a mass of $2(10^{-13}kg)$ and a charge of 40nC, is released from rest at the point (3cm, 0). Find its speed after it has moved freely to a very large distance away.



Exercise Sheet 3: Sample Problem – 4 Charges

Problem

4 charges, $Q_1 = +2\mu C$, $Q_2 = +3\mu C$, $Q_3 = -3\mu C$, $Q_1 = -5\mu C$, are placed in an arrangement as shown in the diagram below.



If length of side *a* is the same as length of side *b* at 2cm, calculate

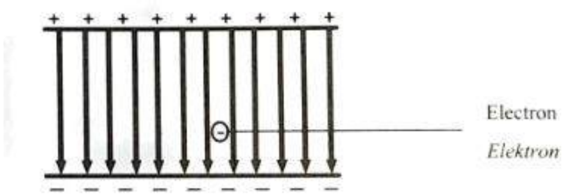
a. the net force exerted on a charge of $+0.25\mu C$ placed at P, the centre of the rectangle.
b. the electric field experience by a test charge placed at P, the centre of the rectangle.
c. the total electrical potential energy of the system.
d. the change in potential energy moving Q_4 from its original position to position P, the centre of the rectangle.

Exercise Sheet 4: Charge In Uniform Electric Field

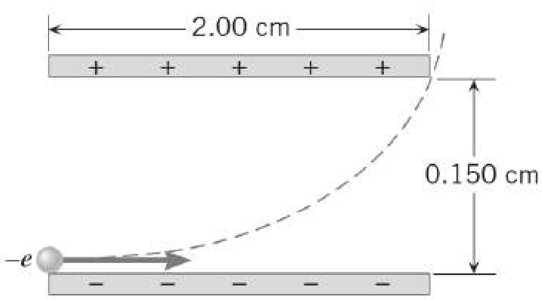
Easy

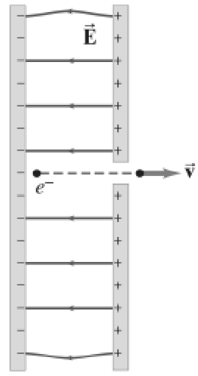
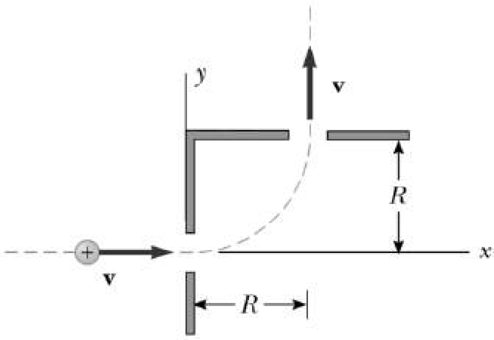
1	Two charged parallel plates are separated by a distance 2.5cm. The potential difference between the plates is 2kV. Find the electric field strength between the plates.
2	An electron is placed between two charged parallel plates of potential difference 40V separated by 5cm. Determine the force experienced by the electron.
3	A charged body of mass 20g and 2mC is placed between two charged parallel plate of potential 20V and 40V. Determine the acceleration of the charged body.

Past Year

4	**PSPM 21/22 – Q1 (b)** The figure shows a uniform electric field $395Vm^{-1}$ exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of opposite place, 2cm away.  Calculate the a. acceleration of the electron b. potential difference between the plates work done by the electric field.
5	**PSPM 20/21 – Q1(b)** A small ball with mass 25g has a total charge of q=+20μC is placed between two parallel charged plates. i. In static equilibrium, determine the magnitude of the electric field in the plates. ii. Calculate the acceleration of the ball if moves horizontally parallel with electric field.
6	**PSPM 19/20 – Q1(b)** Two oppositely charged parallel plates are held 2mm apart. A $4 \times 10^{-5} J$ of work is needed to move a 2μC point charge from one plate to the other. Calculate the electric field between the plates.

Book

7	The drawing shows an electron entering the lower left side of a parallel plate capacitor and exiting at the upper right side. The initial speed of the electron is $7 \times 10^{6} ms^{-1}$. The capacitor is 2cm long, and its plates are separated by 0.150 cm. Assume that the electric field between the plates is uniform everywhere and find its magnitude. 

8	An electron is accelerated in the uniform field ($E = 14.5\ kNC^{-1}$) between two thin parallel charged plates. The separation of the plates is 1.60 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate, as shown in the figure. With what speed does it leave the hole? Show that the gravitational force can be ignored. 
9	A researcher studying the properties of ions in the upper atmosphere wishes to construct an apparatus with the following characteristics: Using an electric field, a beam of ions, each having charge q, mass m, and initial velocity $v\hat{\imath}$, is turned through an angle of 90° as each ion undergoes displacement $R\hat{\imath} + R\hat{\jmath}$. The ions enter a chamber as shown in the figure, and leave through the exit port with the same speed they had when they entered the chamber. The electric field acting on the ions is to have constant magnitude.  a. Suppose the electric field is produced by two concentric cylindrical electrodes not shown in the diagram, and hence is radial. What magnitude should the field have? **b.** If the field is produced by two flat plates and is uniform in direction, what value should the field have in this case?

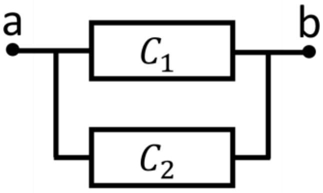
Exercise Sheet 5: Capacitors In Series & Parallel

Easy

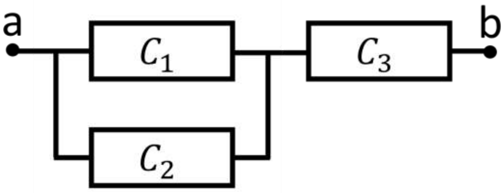
1 Determine the effective capacitance between a and b for the following diagram if $C_1 = 2\mu F$; $C_2 = 4\mu F$; and $C_3 = 3\mu F$.

i.

ii.



iii.

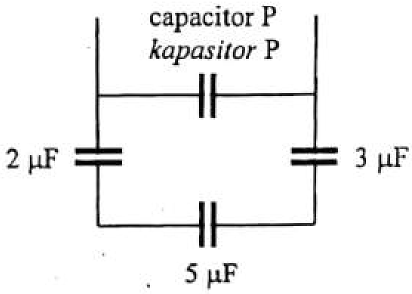


2 Referring to question 1, determine the energy stored if the potential difference between a and b is 12V.

3 A capacitor consists of two parallel plates, each of area $20cm^2$ and separated by 0.25cm of air. Determine the capacitance of the capacitor.

Past Year

4 **PSPM 19/20 – Q2**

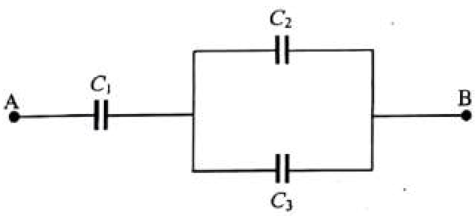


Capacitor P along with other capacitors are arranged as shown in the figure. If capacitor P has a capacitance of $4\mu F$, determine the effective capacitance.

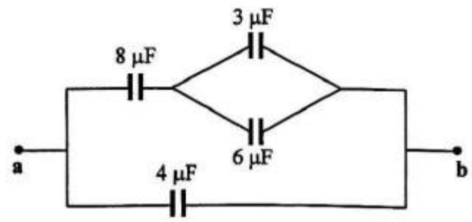
5 **PSPM 18/19 – Q2**

The figure shows three capacitors C_1, C_2, and C_3, each $12\mu F$ connected between points A and B.

a. Calculate the effective capacitance.
b. If the potential difference across AB is 9V, calculate the stored energy.



6 **PSPM 17/18 – Q1(d)**



The figure shows a circuit consists of four capacitors. Calculate the equivalent capacitance between terminals a and b.

Book

7

A. Two capacitors, $C_1 = 5\mu F$ and $C_2 = 12\mu F$, are connected in parallel, and the resulting combination is connected to a 9.00-V battery.
  a. What is the equivalent capacitance of the combination?
  b. What are the potential difference across each capacitor
  c. What are the charge stored on each capacitor?

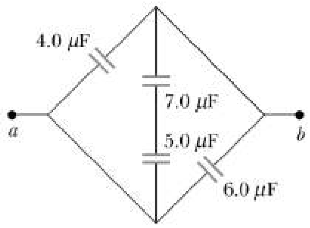
B. The two capacitors are now connected in series and to a 9.00-V battery. Find
  a. the equivalent capacitance of the combination,
  b. the potential difference across each capacitor,
  c. the charge on each capacitor.

8

Consider three capacitors C_1, C_2, C_3, and a battery. If C_1 is connected to the battery, the charge on C_1 is 30.8μC. Now C_1 is disconnected, discharged, and connected in series with C_2. When the series combination of C_2 and C_1 is connected across the battery, the charge on C_1 is 23.1μC. The circuit is disconnected and the capacitors discharged. Capacitor C_3, capacitor C_1, and the battery are connected in series, resulting in a charge on C_1 of 25.2μC. If, after being disconnected and discharged, C_1, C_2 and C_3 are connected in series with one another and with the battery, what is the charge on C_1?

9

Find the equivalent capacitance between points a and b in the combination of capacitors shown in the figure.

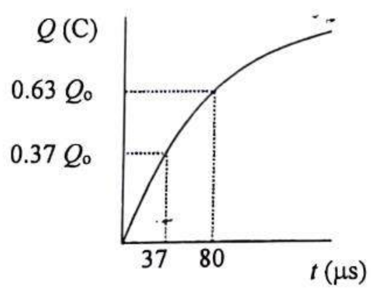


10

Two capacitors, $C_1 = 25.0\mu F$ and $C_2 = 5.00\mu F$, are connected in parallel and charged with a 100-V power supply.

a. Draw a circuit diagram and calculate the total energy stored in the two capacitors.
b. What potential difference would be required across the same two capacitors connected in series in order that the combination stores the same amount of energy as in (a)? Draw a circuit diagram of this circuit.

Exercise Sheet 6: Charging & Discharging of Capacitors

Easy	
1	A capacitor consists of two parallel plates, each of area $20cm^2$ and separated by 0.25cm of air. If the capacitor is connected to a power supply of 12V and a resistor of 2Ω. Determine the time constant.
2	A 6pF capacitor fully charged is connected to a 20Ω resistor. Determine the time it takes to discharge the capacitor to half of its capacity.
3	An uncharged 50μF capacitor is connected to a 4MΩ resistor and a 12V battery. a. Determine the time is takes to charge from 0% to 80% maximum capacity. b. Compare this to the time it takes to charge from 80% to 99% maximum capacity.
Past Year	
4	**PSPM 21/22 – Q2(b)** A $12\mu F$ capacitor which is charged to 6V, is connected in series to $8M\Omega$ resistor and a switch. Determine the charge on the capacitor 4 minutes after the switch is closed.
5	**PSPM 20/21 – Q2** An uncharged capacitor of $58\mu F$ is connected in series with a resistor of $100k\Omega$. Then the capacitor starts charging through the resistor. Calculate the time required for the capacitor to reach 30% of its maximum charge.
6	**PSPM 19/20 – Q2(a)** The graph in the figure shown shows how the charge Q on a capacitor P change with time, t when it is charged through a 20Ω resistor. Determine the capacitance of capacitor P. 
Book	
7	An uncharged capacitor with capacitance C is connected in series with a resistor R. A power supply of V_o is supplied across the circuit. Calculate the time taken for the potential difference across the capacitor increase to 72% the value of V_o.
8	A 2nF with an initial charge of 5.1μC is discharged through 1.3kΩ resistor. a. Calculate the current in the resistor $9\mu s$ after the resistor is connected across the terminals of the capacitor. b. Find the charge remains on the capacitor after $8\mu s$. c. Determine the maximum current in the resistor.
9	A 10μF capacitor is charged to $650\mu C$ through a 20kΩ resistor. a. Calculate the time constant for this charging process. b. Calculate the initial current through the resistor.

Exercise Sheet 7: Dielectrics

Easy

1	A material has a dielectric constant of 2. Determine its dielectric permittivity of the material.
2	The material in question 1 is placed in between the plates of a parallel plate capacitor which has an initial capacitance of $4\mu F$. Determine the capacitance of the capacitor after the material is inserted.
3	A parallel plates capacitor has surface area of $305cm^2$ and a plate separation of 0.4cm. If the space between the plate is air-filled, determine the capacitance of the capacitor.

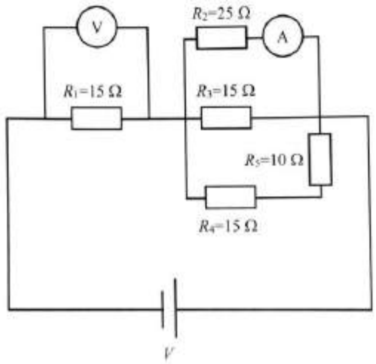
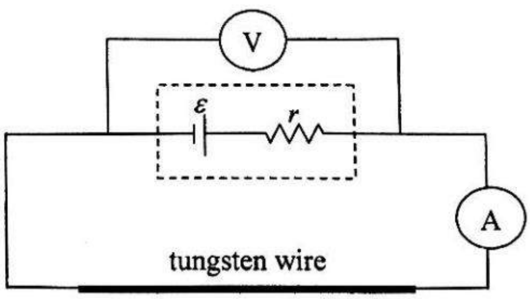
Past Year

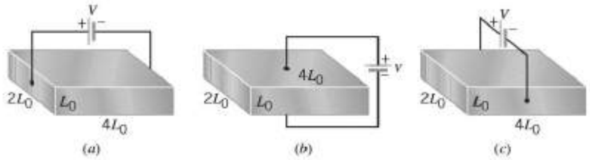
4	**PSPM 21/22 – Q2(a)** A parallel plate capacitor consists of plates of area $0.35m^2$ and separated by 10mm. If the region between plates is filled with dielectric material with dielectric $\varepsilon_r = 5.5$; calculate its capacitance.
5	***Not from past year** A dielectric material is inserted in between the plates of $8.9\mu F$ parallel plate capacitor. Calculate the capacitance of the capacitor if the dielectric constant of the material is 6.5.
6	***Not from past year** A parallel plate capacitor in a vacuum has a plate separation of 1.3cm with a plate area of $30cm^2$. A potential difference of 250V charges the plate and then it disconnected from the source. The capacitor is then dipped in distilled water with a dielectric constant of 80. a. Determine the charge on the plates before and after the dipping. b. Calculate the capacitance and potential difference after the dipping. c. Calculate the change in energy of the capacitor.

Book

7	Two capacitors are identical, except that one is empty and the other is filled with a dielectric (ε_r =4.50). The empty capacitor is connected to a 12V battery. What must be the potential difference across the plates of the capacitor filled with a dielectric so that it stores the same amount of electrical energy as the empty capacitor?
8	An empty parallel plate capacitor is connected between the terminals of a 9V battery and charged up. The capacitor is then disconnected from the battery, and the spacing between the capacitor plates is doubled. As a result of this change, what is the new voltage between the plates of the capacitor?
9	A parallel-plate capacitor in air has a plate separation of 1.5cm and a plate area of $25cm^2$. The plates are charged to a potential difference of 250V and disconnected from the source. The capacitor is then immersed in distilled water. Determine a. the charge on the plates before and after immersion, b. the capacitance and potential difference after immersion, c. the change in energy of the capacitor. Assume the liquid is an insulator.

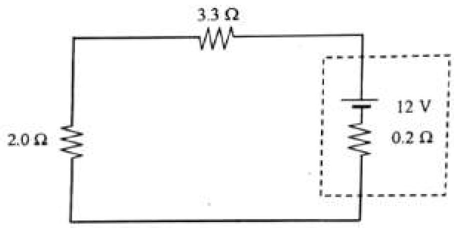
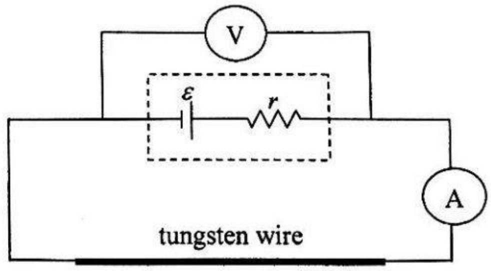
Exercise Sheet 8: Electric Current, Resistivity, Ohm's Law & Temperature

Easy	
1	Current running through a wire is found to be 1.5A. Determine the number of electron going through a section of this wire in 2s.
2	A material has a cross-sectional area of $20cm^2$ and length of 4cm. If the material has a measured resistance of 20Ω, determine its resistivity.
3	A $24k\Omega$ resistor is connected to a power supply of 240V. Determine the current going through the wire. The resistor then heated up by 30K, determine the new value for current after the resistor has heated up.
Past Year	
4	**PSPM 21/22 – Q3** a. A conducting wire has a 1.0mm diameter, a 2.0m length and a 50 mΩ resistance. Calculate its resistivity. b. The figure shows a circuit consisting five resistors, a voltmeter and an ammeter connected to a battery.  The reading of the ammeter is 1.25A. Determine the voltmeter reading.
5	**PSPM 18/19 – Q3(a)** Calculate the number of electrons that flow in a wire if it carries a current of 2A for 5 s.
6	**PSPM 19/20 – Q3(a)(iv)** Determine the change in resistance of a 2Ω resistor when there is a 30^oC rise in its temperature. The temperature coefficient of the resistivity of the resistor is $6.8 \times 10^{-3} {}^oC^{-1}$.
7	**PSPM 15/16 – Q2(a)** The figure shows a tungsten wire connected to a battery with internal resistance $r = 0.6\Omega$. At room temperature of 23^oC, the readings of voltmeter and ammeter are 8.74V and 437mA respectively. After the tungsten wire is heated to 190^oC, the voltmeter reading is 8.85V and the ammeter reading is 253 mA. Calculate the temperature coefficient of resistivity of tungsten wire. 
Book	
8	Suppose that the resistance between the walls of a biological cell is $5\times 10^9\Omega$.

	a. What is the current when the potential difference between the walls is 75 mV? b. If the current is composed of Na^+ ions (q=+e), how many such ions flow in 0.50s?
9	The resistance and the magnitude of the current depend on the path that the current takes. The drawing shows three situations in which the current takes different paths through a piece of material.  Each of the rectangular pieces is made from a material whose resistivity is $1.5 \times 10^{-2}\Omega m$, and the unit of length in the drawing is $L_o = 5cm$. Each piece of material is connected to a 3V battery. Find the resistance and the current in each case.
10	While taking photographs in Death Valley on a day when the temperature is 58.0°C, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1A. Bill then travels to Antarctica and applies the same voltage to the same wire. What current does he register there if the temperature is -88.0°C? Assume that no change occurs in the wire's shape and size.

Exercise Sheet 9: Electromotive Force

Easy

1	(a) What is the current in a 5.6Ω resistor connected to a battery that has a 0.2Ω internal resistance if the terminal voltage of the battery is 10.0 V? (b) What is the emf of the battery?
Past Year	
2	**PSPM 18/19 – 3(c)** The figure shows a circuit with a battery having an emf of 12V and an internal resistance of 0.2Ω connected in series to two resistors, 3.3Ω and 2.0Ω. a. Calculate the current in the circuit. b. Calculate the terminal voltage across the battery. 
3	**PSPM 15/16 – Q2(a)** The figure shows a tungsten wire connected to a battery with internal resistance r. At room temperature of 23^oC, the readings of voltmeter and ammeter are 8.74V and 437mA respectively. After the tungsten wire is heated to 190^oC, the voltmeter reading is 8.85V and the ammeter reading is 253 mA. Calculate emf and internal resistance of the battery. 
4	**PSPM 14/15 – Q2(b)** A battery has an emf of 9 V. The terminal voltage is 8V when the battery is connected across a resistor of 5 Ω. Calculate the current through the resistor and the internal resistance of the battery.
Book	
5	An automobile battery has an emf of 12.6V and an internal resistance of 0.08Ω. The headlights together present equivalent resistance 5Ω (assumed constant). What is the potential difference across the headlight bulbs (a) when they are the only load on the battery and when the starter motor is operated, taking an additional 35.0 A from the battery?
6	Two 1.50-V batteries — with their positive terminals in the same direction are inserted in series into the barrel of a flashlight. One battery has an internal resistance of 0.255Ω the other an internal resistance of 0.153 Ω. When the switch is closed, a current of 600 mA occurs in the lamp. a. What is the lamp's resistance? b. What fraction of the chemical energy transformed appears as internal energy in the batteries?

Exercise Sheet 10: Resistors in Parallel & Series

Easy

1 Two resistors of 3Ω are connected in series to a power supply of 4V.

Determine the effective resistance, current of the circuit and the voltage across ends of each resistor.

2 Two resistors of 3Ω are connected in parallel to a power supply of 4V.

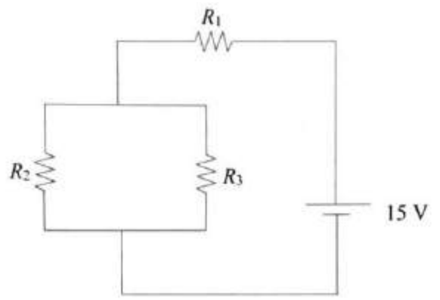
Determine the effective resistance and current through each resistor.

Past Year

3 **PSPM 20/21 – 3(c)**

The figure shows $R_1 = 10\Omega$; $R_2 = 12\Omega$ and $R_3 = 5\Omega$ are connected to a 15V power supply.

a. Calculate the effective resistance
b. Potential difference across R_3.



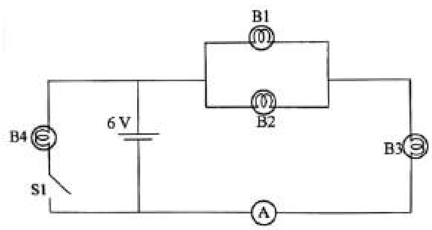
4 **PSPM 16/17 – Q2(b)(i)**

You are given several $1k\Omega$ resistors. How do you connect the resistors to a circuit that requires a 500 Q resistance? Show your suggestion.

5 **PSPM 15/16 – Q2(b)(i)**

The figure shows four identical bulbs connected to a 6 V battery and a switch. When the switch is off, the ammeter reading is 0.5 A.

a. Calculate the resistance of a bulb.
b. What happen to the reading of the ammeter when the switch is on? Explain your answer.

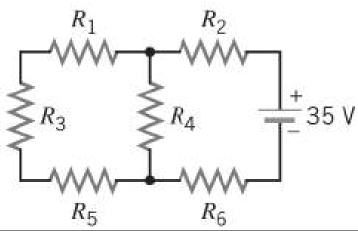


Book

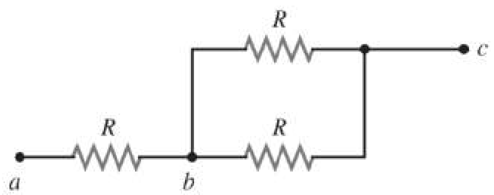
6 You are working late in your electronics shop and find that you need various resistors for a project. But alas, all you have is a big box of 10Ω resistors. Show how you can make each of the following equivalent resistances by a combination of your 10Ω resistors:

a. 35Ω
b. 1Ω
c. 3.33Ω
d. 7.5Ω

7 The circuit shown in the drawing is constructed with six identical resistors and an ideal battery. When the resistor R_4 is removed from the circuit, the current in the battery decreases by 1.9A. Determine the resistance of each resistor.



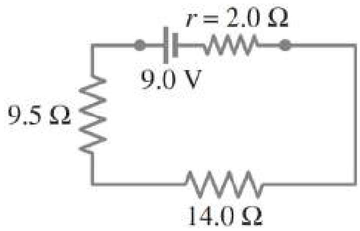
8 The circuit in the drawing contains three identical resistors. Each resistor has a value of 10.0Ω. Determine the equivalent resistance between the points a and b, b and c, and a and c.



Exercise Sheet 11: Kirchhoff's

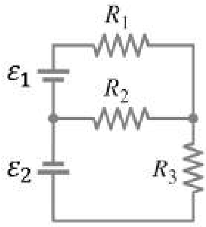
Easy

1 Calculate the current in the circuit of the figure shown, and show that the sum of all the voltage changes around the circuit is zero.

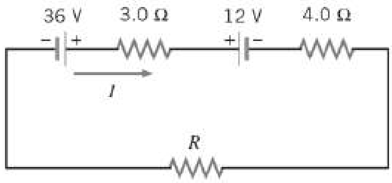


2 Determine the magnitudes and directions of the currents in each resistor shown in the figure shown. The batteries have emfs of $\varepsilon_1 = 9V$ and $\varepsilon_2 = 12V$ and the resistors have values of $R_1 = 25\Omega$, $R_2 = 68\Omega$ and $R_3 = 35\Omega$.

a. Ignore internal resistance of the batteries.
b. Assume each battery has internal resistance r = 1.0Ω.

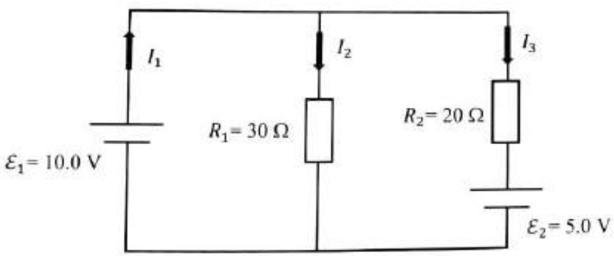


3 Using Kirchhoff's loop rule, find the value of the current I in circuit shown, where $R = 5\Omega$.



Past Year

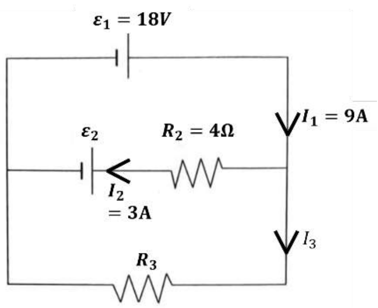
4 **PSPM 21/22 – Q3(c)**



Calculate current I_1, I_2, and I_3 as in the figure shown.

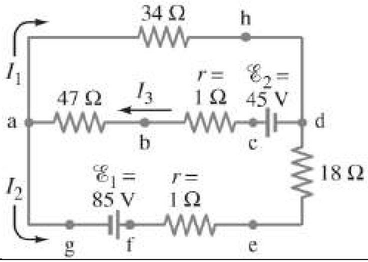
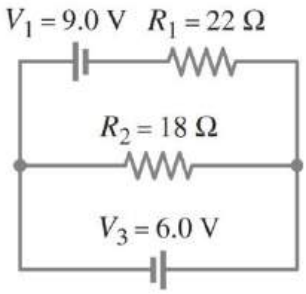
5 **PSPM 20/21 – 3(b)**

The figure shows a circuit consisting of two batteries and two resistors.



Calculate the value of I_3 and R_3.

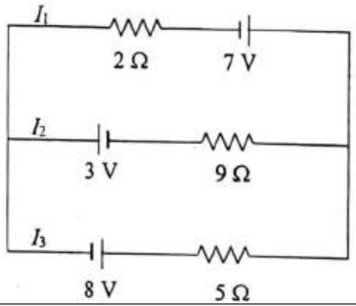
Book

6	What is the potential difference between points a and d in the figure shown. What is the terminal voltage of each battery?	
7	Determine the magnitudes and directions of the currents through R_1 and R_2 in the figure shown. Now assuming that each battery has an internal resistance $r = 1.4\Omega$, determine the currents through R_1 and R_2.	

Exercise Sheet 12: Kirchhoff's

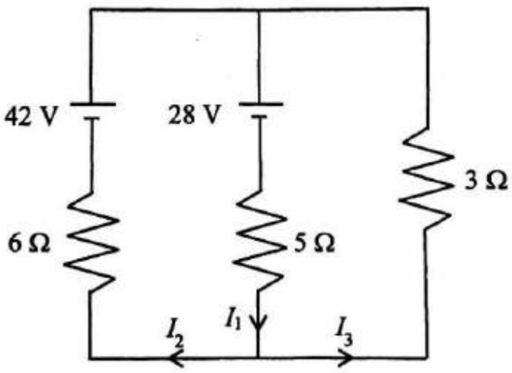
Past Year

1 **PSPM 19/20 – Q3**



For the circuit in the figure shown, determine the current I_1, I_2 and I_3.

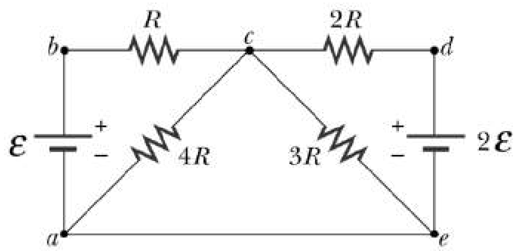
2 **PSPM 17/18 – Q2(c)**



For the circuit in the figure shown, determine the current I_1, I_2 and I_3.

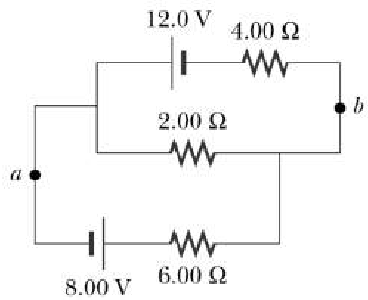
Book

3 Taking $R = 1k\Omega$ and $\varepsilon = 250V$ in the figure shown, determine the direction and magnitude of the current in the horizontal wire between a and e.



4 For the circuit shown in the figure shown, calculate

a. The current in the 2Ω resistor
b. the potential difference between points a and b

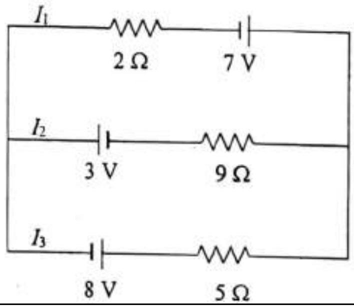


Exercise Sheet 13: Electrical Energy & Power

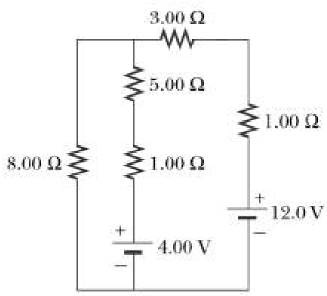
Easy

1	An electric blanket is connected to a 120-V outlet and consumes 140 W of power. What is the resistance of the heater wire in the blanket?
2	An electric car uses a 45-kW (160-hp) motor. If the battery pack is designed for 340 V, what current would the motor need to draw from the battery? Neglect any energy losses in getting energy from the battery to the motor.
3	A 12-V battery causes a current of 600mA through a resistor. a. What is its resistance? b. How many joules of energy does the battery lose in a minute?

Past Year

4	**PSPM 19/20 – Q3**  For the circuit in the figure shown, determine the power dissipated by the 5Ω.
5	**PSPM 18/19 – Q3(b)** A 2.5kW heater is connected to a 220V power supply. The voltage of the power supply is then changed to 110V. Calculate the power output of the heated.
6	**PSPM 17/18 – Q2(b)** The resistivity of a cooper wire is $1.72 \times 10^{-8} \Omega m$. An electric current of 2.07A flows in the wire. If the wire has a cross sectional area of $8.0 \times 10^{-7} m^2$ and length of 50 m, calculate energy dissipated in 1 minute.

Book

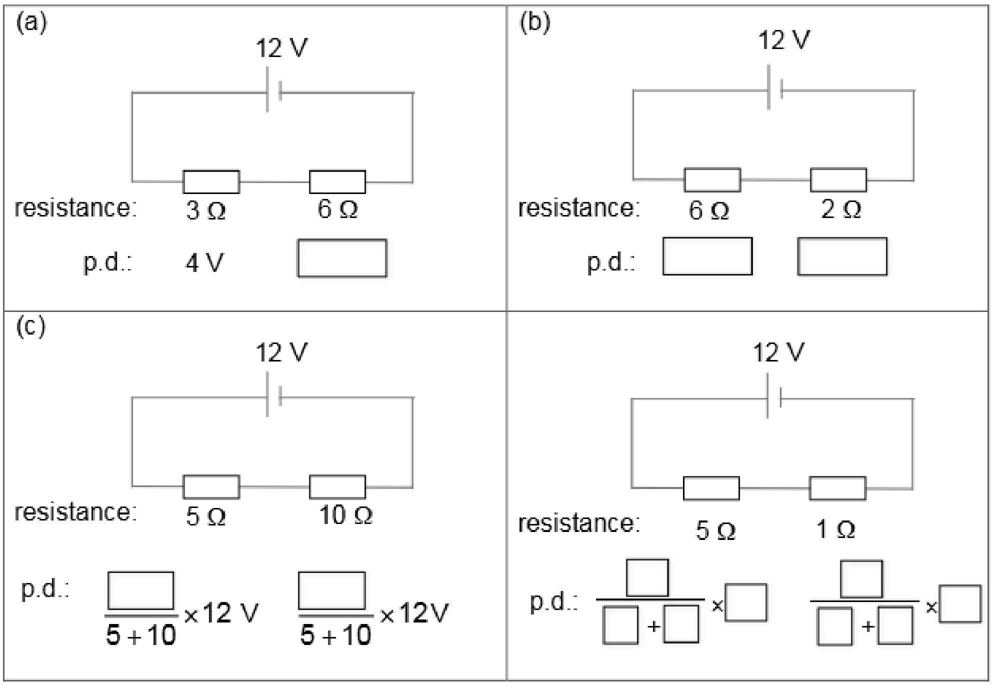
7	Referring to the circuit in the figure shown, the circuit is connected for 2min. a. Find the energy delivered by each battery. b. Find the energy delivered to each resistor. c. Identify the types of energy transformations that occur in the operation of the circuit and the total amount of energy involved in each type of transformation. 
8	A power station delivers 750 kW of power at 12,000 V to a factory through wires with total resistance How much less power is wasted if the electricity is delivered at 50,000 V rather than 12,000 V?
9	You want to design a portable electric blanket that runs on a 1.5-V battery. If you use a 0.50-mm-diameter copper wire as the heating element, how long should the wire be if you want to generate 18W of heating power? What happens if you accidentally connect the blanket to a 9.0-V battery?

Exercise Sheet 14: Potential Divider & Potentiometer

Easy

1 Consider the following circuit: a cell with emf 12 V connected to two resistors in series.
Find the potential difference across each resistor.
Follow these rules:

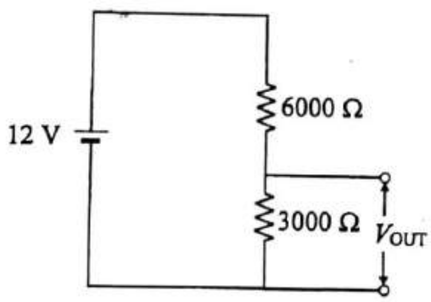
a. For resistors in series, $E = V_1 + V_2 + V_3 + \ldots$.
b. The ratio of p.d. is equal to the ratio of resistance.



2 A DC potentiometer is designed to measure up to about 2V with a slide wire of 800 mm. A standard cell of emf 1.18 V obtains balances at 600 mm. A test cell is seen to obtain balance at 680 mm. Calculate the emf of the test cell.

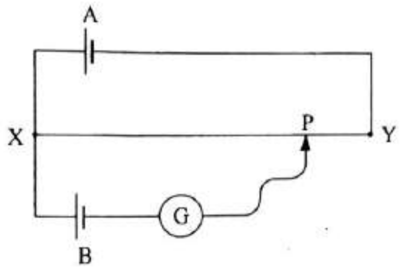
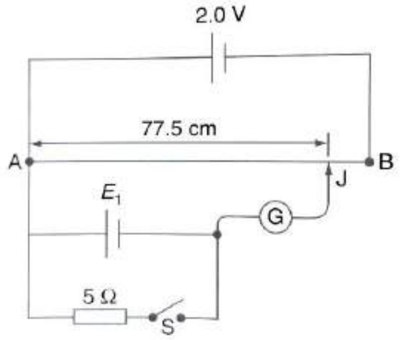
Past Year

3 **PSPM 19/20 – Q3(b)**



The figure shows a potential divider circuit.

a. Calculate the output voltage.
b. If a voltmeter of resistance 3000Ω is connected across the output, determine the reading of the voltmeter.

4	**PSPM 16/17 – Q2(c)** The figure shows a potentiometer circuit consists of a uniform wire XY of length 100cm and its resistance 5Ω. The emf of call A and B is 4.0V and 3.0V, respectively. The internal resistance of both cells are negligible.  a. What is the length of XP when the galvanometer reading is zero? b. If a 1.0Ω is connected in series with cell A, what is the new balanced length of XP?
Book	
5	Two cylindrical rods, one copper and the other iron, are identical in lengths and cross-sectional areas. They are joined end to end to form one long rod. A 12-V battery is connected across the free ends of the copper–iron rod. The resistivity of copper and iron is $1.72 \times 10^{-8}\Omega m$ and $9.7 \times 10^{-8}\Omega m$ respectively. What is the voltage between the ends of the copper rod?
6	The emf E_1 of a cell is measured using a potentiometer as shown in the figure. The driver cell has an emf of 2V and negligible resistance. When the switch S is open, the galvanometer G is balanced when the length AJ is 77.5cm. When the switch S is closed, the length AJ is 63.8 cm.  a. Calculate the emf E_1 b. Calculate the internal resistance of the cell.

Exercise Sheet 15: Magnetic Flux & Induced Emf

Easy

1 A plane coil of 20 turns has a cross sectional area of $0.045m^2$ is placed in a uniform magnetic field strength of 0.075T. Calculate the magnetic flux for linked with the coil if the coil area is

a. parallel to the magnetic field lines
b. perpendicular to the magnetic field lines.
c. angled at 25^o to the magnetic field lines.

2 A plane coil of 20 turns, a cross sectional area of $0.025m^2$, experiences magnetic flux change from 0.2T to 0.5T in 3 seconds and is connected to a 2Ω resistor. Determine the induced emf and induced current.

3 A copper rod of length 0.8m moves perpendicularly through a region of 0.5T magnetic field at $2ms^{-1}$. Determine the induced emf within the rod.

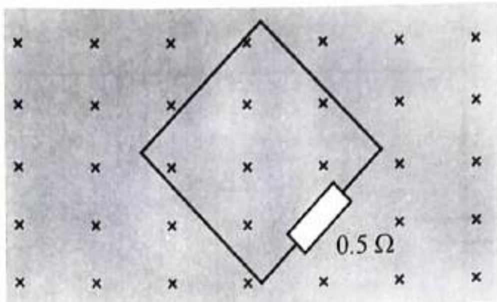
4
a. A magnetic field perpendicular to a circular coil (18 turns, radius 50mm) changes from 2T to 20T in 3s, Calculate the magnitude of the induced emf.
b. A circular coil of 20turns in a magnetic field of 0.4T changes its radius from 2cm to 5cm, determine magnitude of induced emf.

5 An AC generator consisting a 30turns coil with cross sectional area of 0.1m2 and resistance of 100Ω. The coil rotates in a magnetic field of strength 0.5T at a frequency of 30Hz. Calculate the maximum induced current.

Past Year

6 **PSPM 21/22 – Q4(a)**
The figure shows a square wire loop with 2m sides, connect to a 0.5Ω resistor placed perpendicularly to a changing magnetic field.

a. If the magnetic field changes uniformly from 0 to 0.4T in 6.0s, calculate the induced emf in the loop.
b. Determine the current induced in the loop and its direction. Explain your answer.

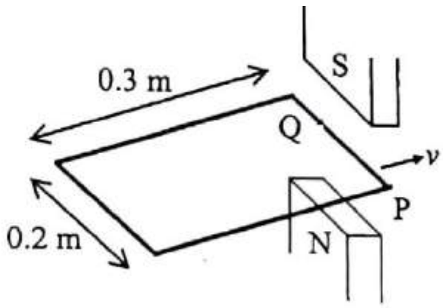


7 **PSPM 20/21 – Q4(a)**
A 14 turns circular coil is placed on a paper which lies in 1.2T magnetic field pointing inwards to the paper. The coil's diameter changes from 22.5cm to 7.2cm in 1.8s.

a. Determine the direction of the induced current.
b. Calculate the magnitude of the emf induced in the circuit.
c. Calculate the induced current if the circular coil resistance is 7.5Ω.

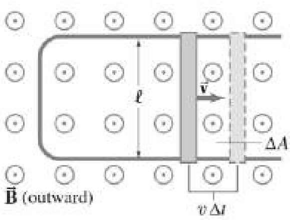
8 **PSPM 19/20 – Q4(a)**
The figure shows a rectangular wire loop $0.3m \times 0.2m$ moving horizontally to the right at $12ms^{-1}$ in a uniform magnetic field of 0.8T.
The induced current in the wire is 3A.

a. Determine the resistance of the wire loop.
b. Determine the direction of the induced current. Explain how you determine the direction of the induced current.



Book

9 Referring to the figure shown, the moving rod has a resistance of 0.25Ω and moves on rails 20cm apart. The stationary U-shaped conductor has negligible resistance. When a force of 0.35N is applied to the rod, it moves to the right at a constant speed of $1.5ms^{-1}$. What is the magnetic field?



10 The magnetic field perpendicular to a single 13.2cm diameter circular loop of copper wire decreases uniformly from 0.670 T to zero. If the wire is 2.25 mm in diameter, how much charge moves past a point in the coil during this operation?

11 Use Lenz’s law to answer the following questions concerning the direction of induced currents.

a. What is the direction of the induced current in resistor R in figure 1 when the bar magnet is moved to the left?

b. What is the direction of the current induced in the resistor R immediately after the switch S in figure 2 is closed?

c. What is the direction of the induced current in R when the current I in figure 3 decreases rapidly to zero?

d. A copper bar is moved to the right while its axis is maintained in a direction perpendicular to a magnetic field, as shown in figure 4. If the top of the bar becomes positive relative to the bottom, what is the direction of the magnetic field?

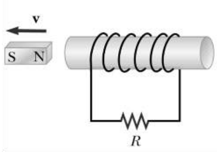


Figure 1

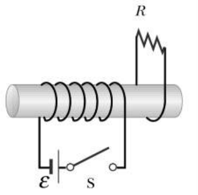


Figure 2

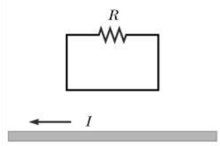


Figure 3

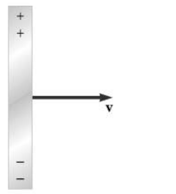
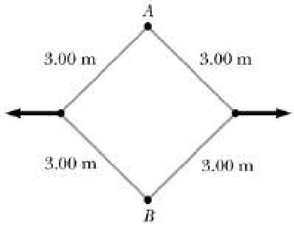
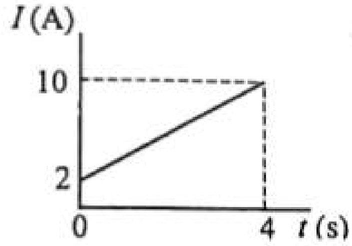
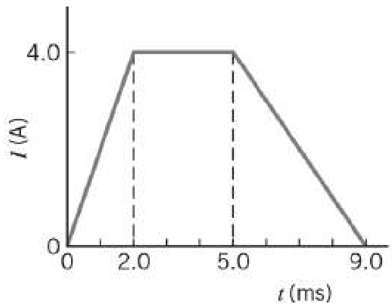


Figure 4

12 The square loop in the figure shown is made of wires with total series resistance 10Ω. It is placed in a uniform 0.1T magnetic field directed perpendicularly into the plane of the paper. The loop, which is hinged at each corner, is pulled as shown until the separation between points A and B is 3.00 m. If this process takes 0.1s, what is the average current generated in the loop? What is the direction of the current?

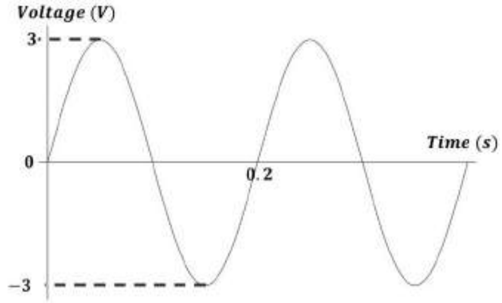
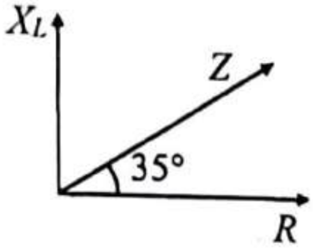


Exercise Sheet 16: Self-Inductance, Energy Storage & Mutual Inductance

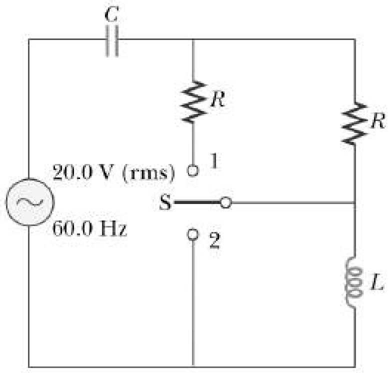
Easy	
1	Induced emf of 6V is developed across a coil when the current flowing through it changes at $30As^{-1}$. Determine the self-inductance of the coil.
2	Calculate the value of self-inductance for an air-filled solenoid of length 5cm and cross-sectional area of $0.3cm^2$ containing 50 loops.
3	A 500turns of solenoid is 8cm long. When the current in the solenoid is increased by 2.5A in 0.35s, the magnitude of the induced emf is 0.012V. Calculate the inductance of the solenoid and the cross sectional area of the solenoid.
Past Year	
4	**PSPM 21/22 – Q4(b)** A 50cm long solenoid S_1 with 1000 turns, diameter 0.5cm, experiences an induced emf of 3.0mV and a changing current of $5As^{-1}$. a. Determine the self-inductance of the solenoid S_1. b. A second coil S_2 with 150 turns is wound coaxially around solenoid S_1. Calculate the mutual inductance of the combination of two coils.
5	**PSPM 20/21 – Q4(b)** A circular coil of N turns with current 9.4 mA has an inductance 15mH. Calculate the a. magnetic flux linkage through the coil b. radius of the coil if N = 420turns
6	**PSPM 19/20 – Q4(b)** A 6cm long solenoid with 400 turns and cross-sectional area $7 \times 10^{-4}m^2$ experiences a changing current as shown in the figure. Determine the a. induced emf b. magnetic flux through each turn **and** the stored energy at the instant when the current is 3A. 
Book	
7	The current through a 3.2-mH inductor varies with time according to the graph shown in the drawing. What is the average induced emf during the time intervals a. 0–2.0ms, b. 2.0–5.0ms, c. 5.0–9.0ms? 
8	A solenoid has 120 turns uniformly wrapped around a wooden core, which has a diameter of 10.0 mm and a length of 9.00 cm. a. Calculate the inductance of the solenoid. b. The wooden core is replaced with a soft iron rod that has the same dimensions, but a magnetic permeability $\mu_m = 800\mu_0$.What is the new inductance?
9	A long, current-carrying solenoid with an air core has 1750 turns per meter of length and a radius of 0.0180 m. A coil of 125 turns is wrapped tightly around the outside of the solenoid, so it has virtually the same radius as the solenoid. What is the mutual inductance of this system?
10	A 54μH solenoid is constructed by wrapping 65 turns of wire around a cylinder with a cross-sectional area of $9 \times 10^{-4}m^2$ When the solenoid is shortened by squeezing the turns closer together, the inductance increases to $86\mu H$. Determine the change in the length of the solenoid.

Exercise Sheet 17: RLC Circuits

Easy

1	The voltage generated by a generator is as shown in the graph. a. What is the peak voltage, peak-to-peak voltage and the rms voltage? b. The voltage is connected across a resistor with a resistance 2.5Ω. Calculate the peak, rms current and average power. 
2	A voltage V = 60sin120πt is applied across a 20Ω resistor. Calculate the reading on the ac ammeter and the average power.
3	A circuit has a resistance of 11Ω, a coil of inductive reactance 120Ω and a capacitor of 100Ω, all connected in series with 110V, 60Hz power source. What is the potential difference across each circuit element.
Past Year	
4	**PSPM 21/22 – Q5** A 160Ω resistor, 230mH inductor and $70\mu F$ capacitor are connected in series across 36V, 60Hz AC source. Calculate the impedance, maximum current, phase angle between the current and voltage, the power factor and the power loss. Is the circuit in resonance? Explain your answer.
5	**PSPM 20/21 – Q4(b)** a. A series RLC circuit attached to a power supply of peak voltage 140 V with a power factor of 0.76. Given $I = 4.5\, sin\, 20\pi t$ where I in A and t in s. i. Calculate the rms current in the circuit. ii. Calculate the value of resistance. iii. Determine the impedance. b. An inductor is connected in series to an AC voltage supply of 30V and frequency 60Hz. Inductive reactance of the inductor is 98Ω. Calculate the i. Inductance ii. Peak current
6	**PSPM 19/20 – Q5** The figure shows a phasor diagram of an RL series circuit connected to an AC source with rms voltage across the inductor of 62.8V at 50Hz, 0.8H inductor and an unknown resistor.  a. Determine the i) Resistance of the resistor ii) Peak voltage of the AC source iii) Average power b. If the resistor is removed from the circuit, draw the variation of current and voltage against time on the same labelled graph.

Book

7	A generator is connected to a resistor and a 0.032-H inductor in series. The rms voltage across the generator is 8.0 V. When the generator frequency is set to 130 Hz, the rms voltage across the inductor is 2.6 V. Determine the resistance of the resistor in this circuit.
8	A capacitor, a coil, and two resistors of equal resistance are arranged in an AC circuit, as shown in the figure. An AC source provides an emf of 20.0 V (rms) at a frequency of 60.0 Hz. When the double-throw switch S is open, as shown in the figure, the rms current is 183 mA. When the switch is closed in position 1, the rms current is 298 mA. When the switch is closed in position 2, the rms current is 137 mA. Determine the values of R, C, and L. Is more than one set of values possible? 
9	A resonant circuit using a 260-nF capacitor is to resonate at 18.0 kHz. The air-core inductor is to be a solenoid with closely packed coils made from 12.0 m of insulated wire 1.1 mm in diameter. How many loops will the inductor contain?
10	In an RC circuit, $R = 6.6k\Omega, C = 1.8\mu F$, and the rms applied voltage is 120 V at 60.0 Hz? a. What is the rms current in the RC circuit? b. What is the phase angle between voltage and current? c. What are the voltmeter readings across R and C?

Additional Notes & Exercise: Sign Conventions in Geometrical Optics

Reflection at a spherical surface

From CS:

LO: Use mirror equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for real object only.

Sign convention for focal length, *f* and radius of curvature, *R:*

i. Positive *f* and *R* for concave mirror; and
ii. Negative *f* and *R* for convex mirror.

From Serway:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

Table 36.1

Sign Conventions for Mirrors

Quantity	Positive When	Negative When
Object location (p)	Object is in front of mirror (real object)	Object is in back of mirror (virtual object)
Image location (q)	Image is in front of mirror (real image)	Image is in back of mirror (virtual image)
Image height (h')	Image is upright	Image is inverted
Focal length (f) and radius (R)	Mirror is concave	Mirror is convex
Magnification (M)	Image is upright	Image is inverted

From Cutnell:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h'}{h} = -\frac{d_i}{d_o}$$

Reasoning Strategy Summary of Sign Conventions for Spherical Mirrors

Focal length

f is + for a concave mirror.
f is − for a convex mirror.

Object distance

d_o is + if the object is in front of the mirror (real object).
d_o is − if the object is behind the mirror (virtual object).*

Image distance

d_i is + if the image is in front of the mirror (real image).
d_i is − if the image is behind the mirror (virtual image).

Magnification

m is + for an image that is upright with respect to the object.
m is − for an image that is inverted with respect to the object.

*Sometimes optical systems use two (or more) mirrors, and the image formed by the first mirror serves as the object for the second mirror. Occasionally, such an object falls *behind* the second mirror. In this case the object distance is negative, and the object is said to be a virtual object.

From geeksforgeeks.org:

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$	The sign conventions followed for any spherical mirror are given as:
	• *All distances are measured from the pole of a spherical error.*
	• *Distances measured in the direction of incident light are taken as positive, while distances measured in a direction opposite to the direction of the incident light are taken as negative.*
	• *The upward distances perpendicular to the principal axis are taken as positive, while the downward distances perpendicular to the principal axis is taken as negative.*
	  ◦ *For convenience, the object is assumed to be placed on the left side of a mirror. Hence, the distance of an object from the pole of a spherical mirror is taken as negative.*
	  ◦ *Since the incident light always goes from left to right, all the distances measured from the pole (P) of the mirror to the right side will be considered positive (because they will be in the same directions as the incident light). On the other hand, all the distances measured from pole (P) of the mirror to the left will be negative (because they are measured against the direction of incident light)*

<u>Refraction at a spherical surfaces</u>

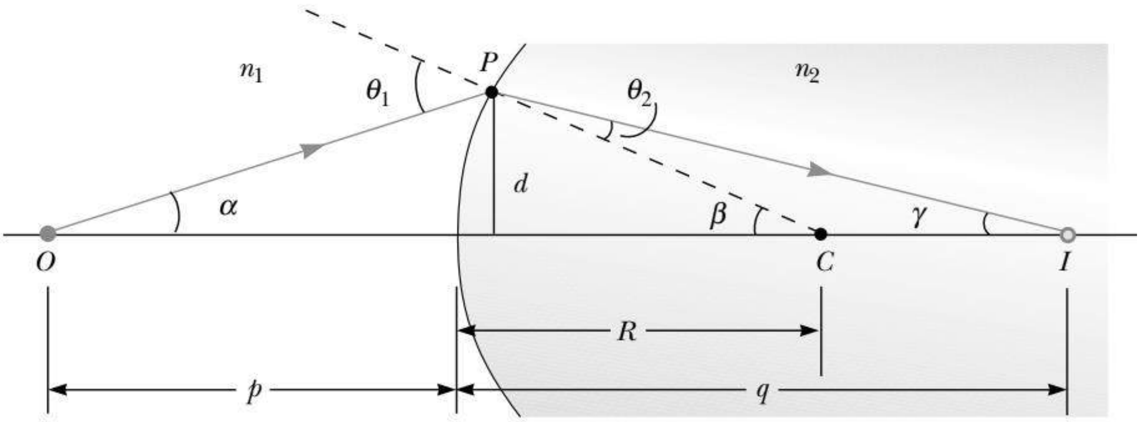


Figure 36.19 Geometry used to derive Equation 36.8, assuming that $n_1 < n_2$.

Snell's Law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Small angle approximation:

$$\sin \theta_i \approx \theta_i \Rightarrow n_1\theta_1 = n_2\theta_2$$

Exterior angle of any triangle is the sum of two opposite interior angles,

$$\theta_1 = \alpha + \beta;\ \beta = \theta_2 + \gamma$$

Eliminate θ_1 and θ_2,

$$n_1(\alpha + \beta) = n_2(\beta - \gamma)$$

Based on diagram, common vertical leg is *d*,

$$\tan \alpha \approx \alpha \approx \left(\frac{d}{p}\right);\ \tan \beta \approx \beta \approx \left(\frac{d}{R}\right);\ \tan \gamma \approx \gamma \approx \left(\frac{d}{q}\right)$$

Rearranging terms yields,

$$n_1\left(\frac{d}{p}+\frac{d}{R}\right) = n_2\left(\frac{d}{R}-\frac{d}{q}\right)$$

Cancelling *d* on both sides and rearranging the terms yields,

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$

<u>Sign Conventions</u>

From CS:

LO: Use $\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}$ for spherical surface.

Sign convention for radius of curvature, *R:*

i. Positive *R* for convex surface; and
ii. Negative *R* for concave surface.

From Serway:

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$

Table 36.2

Sign Conventions for Refracting Surfaces

Quantity	Positive When	Negative When
Object location (p)	Object is in front of surface (real object)	Object is in back of surface (virtual object)
Image location (q)	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R)	Center of curvature is in back of surface	Center of curvature is in front of surface

Thin Lenses

LO:

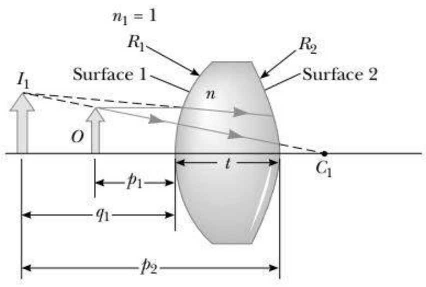
a. Use thin lenses equation, $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ for real object only.
Sign convention for focal length:
i. Positive *f* for convex mirror; and
ii. Negative *f* for concave mirror.

b. Use lensmaker's equation, $\frac{1}{f}=\left(\frac{n_{material}}{n_{medium}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

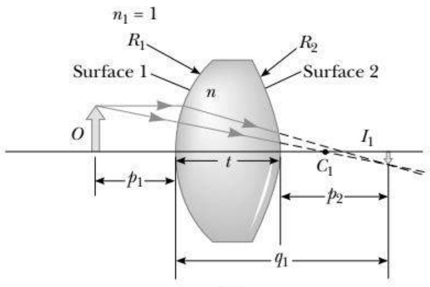
c. Apply magnification, $M=\frac{h_i}{h_o}=-\frac{v}{u}$

d. Use thin lens equation for a combination of two convex lenses.

From Serway (adapted):



(a)



(b)

Consider image formed by surface 1,

$$\frac{n_{medium}}{p_1}+\frac{n_{lens}}{q_1}=\frac{n_{lens}-n_{medium}}{R_1}$$

We can then apply this to surface 2,

$$\frac{n_{lens}}{p_2}+\frac{n_{medium}}{q_2}=\frac{n_{medium}-n_{lens}}{R_2}$$

Using the image from surface 1 as the object for surface 2,

$$p_2=-q_1+t$$

Where t is the thickness of the lens and is ≈ 0 for thin lenses ($t \ll R$).

$$\frac{n_{lens}}{-q_1}+\frac{n_{medium}}{q_2}=\frac{n_{medium}-n_{lens}}{R_2}$$

$$\frac{1}{f}=\frac{1}{p_1}+\frac{1}{q_2}=\left(\frac{n_{lens}}{n_{medium}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

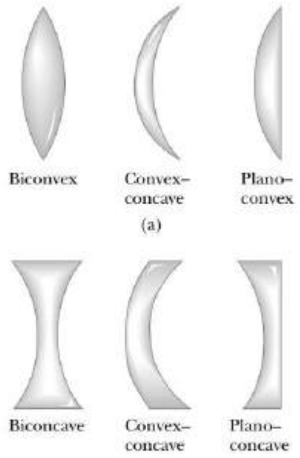


Table 36.3

Sign Conventions for Thin Lenses		
Quantity	**Positive When**	**Negative When**
Object location (p)	Object is in front of lens (real object)	Object is in back of lens (virtual object)
Image location (q)	Image is in back of lens (real image)	Image is in front of lens (virtual image)
Image height (h')	Image is upright	Image is inverted
R_1 and R_2	Center of curvature is in back of lens	Center of curvature is in front of lens
Focal length (f)	Converging lens	Diverging lens

From Cutnell:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Reasoning Strategy Summary of Sign Conventions for Lenses

Focal length

f is + for a converging lens.
f is − for a diverging lens.

Object distance

d_o is + if the object is to the left of the lens (real object), as is usual.
d_o is − if the object is to the right of the lens (virtual object).*

Image distance

d_i is + for an image (real) formed to the right of the lens by a real object.
d_i is − for an image (virtual) formed to the left of the lens by a real object.

Magnification

m is + for an image that is upright with respect to the object.
m is − for an image that is inverted with respect to the object.

*This situation arises in systems containing more than one lens, where the image formed by the first lens becomes the object for the second lens. In such a case, the object of the second lens may lie to the right of that lens, in which event d_o is assigned a negative value and the object is called a virtual object.

From Giancoli:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

sign conventions:

1. The focal length is positive for converging lenses and negative for diverging lenses.
2. The object distance is positive if the object is on the side of the lens from which the light is coming (this is always the case for real objects; but when lenses are used in combination, it might not be so: see Example 23–16); otherwise, it is negative.
3. The image distance is positive if the image is on the opposite side of the lens from where the light is coming; if it is on the same side, d_i is negative. Equivalently, the image distance is positive for a real image (Fig. 23–40) and negative for a virtual image (Fig. 23–41).
4. The height of the image, h_i, is positive if the image is upright, and negative if the image is inverted relative to the object. (h_o is always taken as upright and positive.)

Thin lens equation for two convex lenses

From Giancoli:

"When light passes through more than one lens, we find the image formed by the first lens as if it were alone. Then this image becomes the object for the second lens. Next we find the image formed by this second lens using the first image as object. This second image is the final image if there are only two lenses. The total magnification will be the product of the separate magnifications of each lens."

<u>Example from Giancoli:</u>

Problem

Two converging lenses, A and B, with focal lengths $f_A = 20cm$ and $f_B = 25cm$ and are placed 80.0 cm apart, as shown

in Fig. 23–44a. An object is placed 60.0 cm in front of the first lens as shown in Fig. 23–44b.

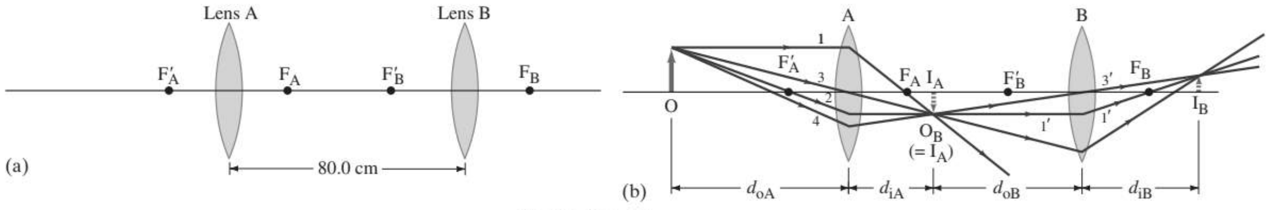


FIGURE 23–44

Determine (a) the position, and (b) the magnification, of the final image formed by the combination of the two lenses

Solutions

Position:

$$\frac{1}{d_{iA}} = \frac{1}{f_A} - \frac{1}{d_{oA}} \Rightarrow d_{iA} = \frac{f_A d_{oA}}{d_{oA} - f_A}$$

$$\frac{1}{d_{iB}} = \frac{1}{f_B} - \frac{1}{d_{oB}} \Rightarrow d_{iB} = \frac{f_B d_{oB}}{d_{oB} - f_B}$$

$$80cm = d_{iA} + d_{oB} \Rightarrow d_{oB} = 80cm - d_{iA}$$

$$d_{iB} = \frac{f_B(80cm - d_{iA})}{(80cm - d_{iA}) - f_B} = \frac{f_B\left(80cm - \left(\frac{f_A d_{oA}}{d_{oA} - f_A}\right)\right)}{\left(80cm - \left(\frac{f_A d_{oA}}{d_{oA} - f_A}\right)\right) - f_B} = \frac{(25)\left(80cm - \left(\frac{(20)(60)}{60 - 20}\right)\right)}{\left(80cm - \left(\frac{(20)(60)}{60 - 20}\right)\right) - 25} = 50cm$$

⇒Final image is formed 50cm behind lens B.

Total Magnification:

$$M_T = M_1 M_2 = \left(-\frac{d_{iA}}{d_{oA}}\right)\left(-\frac{d_{iB}}{d_{oB}}\right) = \left(-\frac{30}{60}\right)\left(-\frac{50}{50}\right) = \frac{1}{2}$$

⇒Final image is upright and half of the original object height.

Sign Convention

<table>
<tr><th>Equations</th><th>Sign Convention</th></tr>
<tr><td>Reflection upon spherical mirror:
$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$
$$M=\frac{h_{image}}{h_{object}}=-\frac{u}{v}$$</td><td>
<table>
<tr><th></th><th>Positive when</th><th>Negative when</th></tr>
<tr><td>Focal length,f</td><td>Focal length is at same side as incoming light (concave)</td><td>Focal length is at opposite side as incoming light (convex)</td></tr>
<tr><td>Object Distance, u</td><td>Object is placed same side as incoming light (‘in front’, ‘real object’)</td><td>Object is placed on the opposite side as incoming light (‘behind’, ‘virtual object’)</td></tr>
<tr><td>Image Distance,v</td><td>Image is placed same side as incoming light (‘in front’, ‘real image’)</td><td>Image is placed on the opposite side as incoming light (‘behind’, ‘virtual image’)</td></tr>
<tr><td>Magnification</td><td>Image is upright</td><td>Image is inverted</td></tr>
</table>
</td></tr>
<tr><td>Refraction upon Spherical Surface:
$$\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}$$</td><td>
<table>
<tr><th></th><th>Positive when</th><th>Negative when</th></tr>
<tr><td>Radius, R</td><td>Radius at opposite side as incoming light (‘convex’)</td><td>Focal length is at same side as incoming light (‘concave’)</td></tr>
<tr><td>Object Distance, u</td><td>Object is placed same side as incoming light (‘in front’, ‘real object’)</td><td>Object is placed on the opposite side as incoming light (‘behind’, ‘virtual object’)</td></tr>
<tr><td>Image Distance,v</td><td>Image is placed on the opposite side as incoming light (‘behind’, ‘real image’)</td><td>Image is placed same side as incoming light (‘in front’, ‘virtual image’)</td></tr>
</table>
</td></tr>
</table>

<table>
<tr>
<td>

Thin lens equation:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Lensmaker's equation:

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

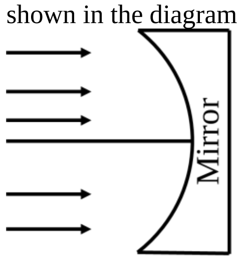
</td>
<td>

	Positive when	**Negative when**
Focal length, $\boldsymbol{f}$	Converging lens	Diverging lens
Object Distance, $\boldsymbol{u}$	Object is placed same side as incoming light ('in front', 'real object')	Object is placed on the opposite side as incoming light ('behind', 'virtual object')
Image Distance,$\boldsymbol{v}$	Image is placed on the opposite side as incoming light ('behind', 'real image')	Image is placed same side as incoming light ('in front', 'opposite image')
Radii of surfaces, $\boldsymbol{R_1}$ & $\boldsymbol{R_2}$	Radii is on the same side of the incoming light ('in front')	Radii is on the opposite side of the incoming light ('behind')

</td>
</tr>
</table>

Exercise

Geometrical Optics: Reflection

<table>
<tr><td rowspan="2">$R = 2f$</td><td>Find R</td><td>a.</td><td>A spherical mirror of diameter 2cm is illuminated with parallel light rays as shown in the diagram below.


Determine the radius of curvature of the spherical mirror. [$\boldsymbol{R = 1cm}$]</td></tr>
<tr><td>Find f</td><td>b.</td><td>Based on question (a), determine the point at which the light rays would converge at. [$\boldsymbol{f = 0.5cm}$]</td></tr>
<tr><td rowspan="6">$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$</td><td>Concave</td><td>c.</td><td>An object is placed 3cm in front of a mirror. The image is found to form 6cm in front of the mirror. Determine the focal length of the mirror. [$\boldsymbol{f = 2cm}$]</td></tr>
<tr><td>Convex</td><td>d.</td><td>An object is placed 6cm in front of a mirror. The image is found to form 3cm behind the mirror. Determine the focal length of the mirror. [$\boldsymbol{f = -6cm}$]</td></tr>
<tr><td rowspan="2">Real Object</td><td>e.</td><td>An object is placed $x\ cm$ from a convex mirror. The image is found to form 3cm behind the mirror and the focal length of the mirror is 9cm. Determine the object distance from the mirror. [$\boldsymbol{u = 4.5cm}$]</td></tr>
<tr><td>f.</td><td>An object is placed in front of a concave mirror. The image is found to form 3cm in front of the mirror and the focal length of the mirror is 2cm. Determine the object distance from the mirror. [$\boldsymbol{u = 6cm}$]</td></tr>
<tr><td>Real Image</td><td>g.</td><td>An object is placed 4cm in front of a concave mirror. The focal length of the mirror is 2cm. Determine the image distance from the mirror. [$\boldsymbol{v = 4cm}$]</td></tr>
<tr><td>Virtual Image</td><td>h.</td><td>An object is placed 4cm in front of a convex mirror. The focal length of the mirror is 4cm. Determine the image distance from the mirror. [$\boldsymbol{v = 2cm}$]</td></tr>
<tr><td rowspan="2">$M = \frac{h_i}{h_o}$
$M = -\frac{v}{u}$</td><td>Magnified</td><td>i.</td><td>The virtual image of a real object from a spherical mirror is found to form at $4cm$ when the object is placed $2cm$ from the mirror. Determine the magnification.
[$\boldsymbol{M = 2}$]</td></tr>
<tr><td>Diminished</td><td>j.</td><td>An object is placed 2cm in front of a convex mirror of focal length 6cm. Determine the image distance and the magnification. [$\boldsymbol{v = -1.5cm}$; $\boldsymbol{M = 0.75}$]</td></tr>
</table>

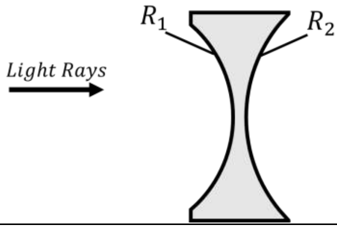
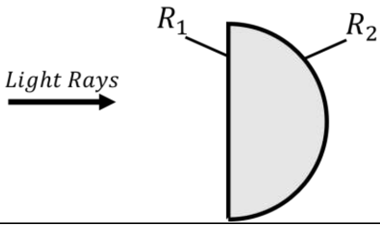
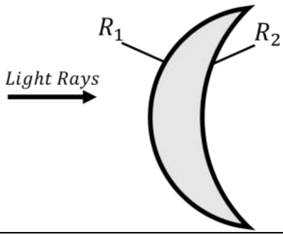
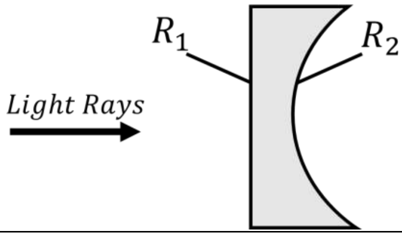
Geometrical Optics: Refraction

$\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}$	Real Object	a. When an object is placed some distance from a convex interface of a material of radius $2cm$, an image of the object is formed 4cm **behind** the interface. If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the **object distance** from the interface. [$\boldsymbol{u = 4.8cm}$]
	Virtual Object	b. When an object is placed some distance from a convex interface of a material of radius $2cm$, an image of the object is formed 0.5cm **behind** the interface. If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the **object distance** from the interface. [$\boldsymbol{u \approx -0.21cm}$]
	Real Image	c. An object is placed 20cm from a **convex interface** of a material of radius $2cm.$ If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the image distance from the interface. [$\boldsymbol{v \approx 3.27cm}$]
	Virtual Image	d. An object is placed 20cm from a **concave interface** of a material of radius $2cm.$ If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the image distance from the interface. [$\boldsymbol{v \approx -2.93cm}$]
	Convex Surface	e. An object is placed 24cm **in front of** an interface of a material of refractive index of 2.8. The refractive index before the interface is 1.3. If the image is found to form 56cm **behind** the interface, determine the radius of curvature of the interface. [$R = 14.4cm$]
	Concave Surface	f. An object is placed 24cm **in front of** an interface of a material of refractive index of 2.8. The refractive index before the interface is 1.3. If the image is found to form 18cm **in front of** the interface, determine the radius of curvature of the interface. [$R = -14.2cm$]
	Refractive index before interface	g. An object is placed 25cm **in front of** an interface of a material of refractive index of 3.3. If the image is found to form 18cm **behind** the interface, determine the refractive index of the material before the interface. [$n_1 \approx 1.39$]
	Refractive index after interface	h. An object is placed 24cm **in front of** a convex interface of a material of refractive index of n. The refractive index before the interface is 1.4. If the image is found to form 38cm behind the interface and the radius of curvature of the interface is 15cm, determine the refractive index n. [$n \approx 3.88$]

Geometrical Optics: Thin lenses

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}; \frac{1}{f}=\left(\frac{n_{material}}{n_{medium}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right); M=\frac{h_i}{h_o}=-\frac{v}{u}$

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$	Diverging (Concave)	a. When placed 4cm in front of a thin lens, the image of an object forms 2.4cm **in front of** the lens. Determine the focal length of the lens. [$\boldsymbol{f = -6cm}$]
	Converging (Convex)	b. When placed 4cm in front of a thin lens, the image of an object forms 2.4cm **behind** the lens. Determine the focal length of the lens. [$\boldsymbol{f = 1.5cm}$]
	Real Object	c. An object placed in front of a lens of focal length 2cm forms an image 3cm **behind** the lens. Determine the object distance from the lens. [$\boldsymbol{u = 6cm}$]
	Real Image	d. Determine the image distance when an object is placed 12cm in front of a biconvex lens of focal length 6cm. [$\boldsymbol{v = 12cm}$]
	Virtual Image	e. Determine the image distance when an object is placed 4cm in front of a biconcave lens of focal length 6cm. [$\boldsymbol{v = -2.4cm}$]

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$	
Converging	a. A biconvex thin lens (made up of a material of refractive index 1.65) is submerged in water of refractive index 1.33. The thin lens has radius of curvature of $R_2 = 2cm$ and $R_1 = 4cm$. Determine the focal length of the thin lens. [$\boldsymbol{f = 5.54cm}$]
Diverging	b. A biconvex thin lens (made up of a material of refractive index 1.65) is submerged in water of refractive index 1.33. The thin lens has radius of curvature of $R_1 = 2cm$ and $R_2 = 4cm$. Determine the focal length of the thin lens. [$\boldsymbol{f = -5.54cm}$]
R_1	c. The diagram shows a biconcave converging lens of focal length $15cm$, made up of a material of refractive index 1.44, placed in air. If surface R_2 of the lens has a radius of curvature of 12cm, what is the radius of curvature of surface R_1? [$\boldsymbol{f = 4.258cm}$] 
R_2	d. A plano-convex lens, as shown in the diagram, has a focal length of –12cm. The lens is made up of a material of refractive index 1.47 and is placed in air. determine radius of curvature of R_2. [$\boldsymbol{R_2 = 5.64cm}$] 
$n_{material}$	e. A convex-concave lens of focal length +35cm is placed in water. If the radii of its surfaces are $R_1 = 4cm$ and $R_2 = 7cm$, determine the refractive index of the lens material. [$n_{material} = 1.684$] 
n_{medium}	f. A plano-concave lens of focal length +35cm (made up of a material of refractive index 1.65) is placed in a medium of unknown refractive index. If the radius of its surface is $R_2 = 7cm$, determine the refractive index of the medium the lens is in. [$n_{medium} = 2.063$] 

Exercise Sheet 18: Optics – Reflection

Easy

1	An object 200cm from the vertex of a spherical concave mirror is imaged 400cm in front of the mirror, what is the radius length of the mirror?
2	An object 10cm high is 50cm from a concave mirror of 20cm focal length. Find the image distance, height and direction.
3	How far should an object be from a concave spherical mirror of radius 45cm to form a real image one-ninth of its size?

Past Year

4	**PSPM 21/22 – Q6 (a)** An object is placed 5cm from a curved mirror. An image which is twice the size of the object is formed behind the mirror. a. Is the mirror convex or concave? Explain your answer. b. Determine the radius of curvature of the mirror.
5	**PSPM 18/19 – Q6(a)** An external side mirror of a car is convex with a radius of curvature 18m. Determine the location of the image for an object 10 m from the mirror
6	**PSPM 17/18 – Q6(b)** An object is placed in front of a concave mirror with 25cm radius of curvature. A real image twice the size of the object is formed. a. Sketch a ray diagram to illustrate the formation of the image. b. Determine the object distance from the mirror.

Book

7	A concave mirror has a focal length of 30cm. The distance between an object and its image is 45cm. Find the object and image distances, assuming that a. the object lies beyond the centre of curvature b. the object lies between the focal point and the mirror.
8	An object 10cm tall is placed at the zero mark of a meter stick. A spherical mirror located at some point on the meter stick creates an image of the object that is upright, 4cm tall, and located at the 42cm mark of the meter stick. a. Is the mirror convex or concave? b. Where is the mirror? c. What is the mirror's focal length?
9	A shaving or makeup mirror is designed to magnify your face by a factor of 1.40 when your face is placed 20cm in front of it. a. What type of mirror is it? b. Describe the type of image that it makes of your face. c. Calculate the required radius of curvature for the mirror.

Exercise Sheet 19: Optics – Refraction

Easy

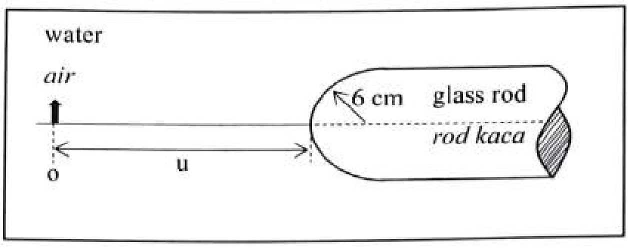
1 The left end of a long glass rod 6.00 cm in diameter has a convex hemispherical surface 3.00 cm in radius. The refractive index of the glass is 1.60. Determine the position of the image if an object is placed in air on the axis of the rod at the following distances to the left of the vertex of the curved end: (a) infinitely far; (b) 12.0 cm; (c) 2.00 cm.

2 The glass rod of question 1 is immersed in oil (n = 1.452). An object placed to the left of the rod on the rod's axis is to be imaged 1.20 m inside the rod. How far from the left end of the rod must the object be located to form the image?

Past Year

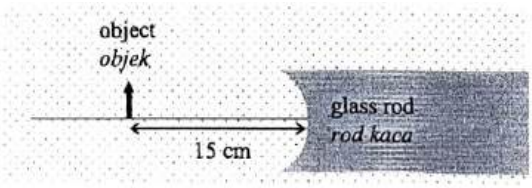
4 **PSPM 21/22 – Q6(b)**

The figure shows a long rod with a convex surface of radius of curvature 6.0 cm at one end and is made from glass with refractive index of1.60. The glass rod is placed in water with refractive index, n = 1.33. An object placed along the rod's axis is to be imaged 53 cm inside the rod. Calculate the object position.



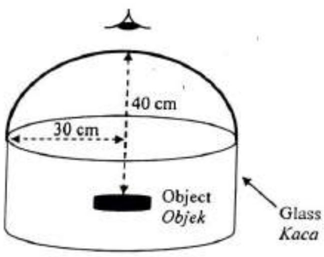
5 **PSPM 19/20 – Q6(c)**

The figure shows an object and a glass rod immersed in a liquid. The rod has a refractive index of 1.7 and radius of curvature 8cm. If the object distance is 15cm and the virtual image distance is 13cm, determine the refractive index of liquid.



6 **PSPM 17/18 – Q6(b)**

The figure shows an object embedded in a solid glass with a hemispherical end of radius 30cm and refractive index 1.50. The object is 40cm inside the glass. Calculate the image distance. Refractive index of air is 1.



Book

7 A transparent sphere of unknown composition is observed to form an image of the Sun on the surface of the sphere opposite the Sun. What is the refractive index of the sphere material?

8 One end of a long glass rod (n = 1.50) is formed into a convex surface with a radius of curvature of 6.00 cm. An object is located in air along the axis of the rod. Find the image positions corresponding to object distances of (a) 20.0 cm, (b) 10.0 cm, and (c) 3.00 cm from the end of the rod.

9 A simple model of the human eye ignores its lens entirely. Most of what the eye does to light happens at the outer surface of the transparent cornea. Assume that this surface has a radius of curvature of 6.00 mm, and assume that the eyeball contains just one fluid with a refractive index of 1.40. Prove that a very distant object will be imaged on the retina, 21.0 mm behind the cornea. Describe the image.

Exercise Sheet 20: Optics – Thin Lenses

Easy

1 An insect 3.75 mm tall is placed 25.0 cm to the left of a thin planoconvex lens. The left surface of this lens is flat, the right surface has a radius of curvature of magnitude 12.9 cm, and the index of refraction of the lens material is 1.70.

a. Calculate the location and size of the image this lens forms of the insect. Is it real or virtual? Erect or inverted?
b. Repeat part (a) if the lens is reversed.

2 A lens forms an image of an object. The object is 16.0 cm from the lens. The image is 12.0 cm from the lens on the same side as the object.

a. What is the focal length of the lens? Is the lens converging or diverging?
b. If the object is 8.50 mm tall, how tall is the image? Is it erect or inverted?

3 A converging lens with a focal length of 70.0 cm forms an image of a 3.20-cm-tall real object that is to the left of the lens. The image is 4.50 cm tall and inverted. Where are the object and image located in relation to the lens? Is the image real or virtual?

Past Year

4 **PSPM 21/22 – Q6(c)**

A converging meniscus lens is made from a glass of refractive index 1.52 having a radius 7cm and 4cm. An object is placed 24cm in front of the lens.

a. Calculate the position of the image from the lens.

If the image magnified or diminished in size? Justify your answer.

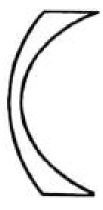
5 **PSPM 20/21 – Q6**

A. An orange is placed 25.2cm in front of a diverging lens with a focal length of 18cm.
  a. Sketch the ray diagram to show the formation of the image
  b. Determine the image distance.
  c. Determine the magnification.
  d. Determine **two (2)** characteristics of the image
B. The convex meniscus lens has a 17cm radius for the convex surface and 25cm for the concave surface. The lens is made of glass with a refractive index, $n = 1.52$ in air. Refractive index of air is 1.0. Determine the focal length of the lens.

6 **PSPM 19/20 – Q6(b)**

The figure shows a lens with radii of curvature of 15cm and 50cm, made of glass with refractive index 1.55.

Determine the focal length **and** type of lens.



Book

7 An object is placed 96.5 cm from a glass lens (n=1.52) with one concave surface of radius 22.0 cm and one convex surface of radius 18.5 cm. Where is the final image? What is the magnification?

8 A symmetric double convex lens with a focal length of 22.0 cm is to be made from glass with an index of refraction of 1.52. What should be the radius of curvature for each surface?

9 Two lenses, one converging with focal length 20.0 cm and one diverging with focal length $-10cm$ are placed 25.0 cm apart. An object is placed 60.0 cm in front of the converging lens. Determine

a. the position
b. magnification of the final image formed.
c. Sketch a ray diagram for this system.

Exercise Sheet 21: Huygens' & Interferences

Easy

1	Describe the condition for **constructive** interference.
2	Describe the condition for **destructive** interference.

Past Year

3	**PSPM 18/19 – Q7(b)** The figure shows two paths of coherent lights from points A and B that produce an interference pattern at point C. Determine whether it is a constructive or destructive interference if AC and BC are 2.2λ and 5.7λ respectively. 
	**No other questions on Huygen's or Interference since 2013.*

Book

4	Two radio antennas A and B radiate in phase. Antenna B is 110 m to the right of antenna A. Consider point Q along the extension of the line connecting the antennas, a horizontal distance of 30 m to the right of antenna B. The frequency, and hence the wavelength, of the emitted waves can be varied. a. What is the longest wavelength for which there will be destructive interference at point Q? b. What is the longest wavelength for which there will be constructive interference at point Q?
5	A radio transmitting station operating at a frequency of 120 MHz has two identical antennas that radiate in phase. Antenna B is 9.00 m to the right of antenna A. Consider point P between the antennas and along the line connecting them, a horizontal distance x to the right of antenna A. For what values of x will constructive interference occur at point P?

Exercise Sheet 22: Double Slit

Easy

1	A laser beam (λ =632.8 nm) is incident on two slits 0.200 mm apart. How far apart are the bright interference fringes on a screen 5.00 m away from the double slits?
2	Monochromatic light falling on two slits 0.018mm apart produces the fifth-order bright fringe at an 8.6° angle. What is the wavelength of the light used?
3	In a Young's double-slit experiment, the angle that locates the second dark fringe on either side of the central bright fringe is 5.4^{o}. Find the ratio $\frac{d}{\lambda}$of the slit separation d to the wavelength λ of the light.

Past Year

4	**PSPM 21/22 – Q7(a)** A 475 nm light passes through two narrow slits. The interference pattern is observed on a screen at a distance 85.0 cm from the slits. The second-order bright fringe is seen at $\pm$2.01 cm from the central bright fringe. Calculate the slit separation and the width of the second-order dark fringe.
5	**PSPM 20/21 – Q7(a)** Two narrow slits separated by 2.4mm are illuminated by a light with λ= 512 nm. The screen is placed 6.5 m from the slits. Determine the a. distance between adjacent bright fringes on a screen distance of the fifth dark fringe from the central bright fringe.
6	**PSPM 19/20 – Q7(a)** In a double slit experiment, the incident wavelength is 660 nm, the slit separation is 0.25 mm, and the screen is placed 90 cm away from the sits. Calculate the distance from the second to the third destructive interference fringe.

Book

7	Coherent light of frequency $6.32\times 10^{14} Hz$ Hz passes through two thin slits and falls on a screen 85.0 cm away. You observe that the third bright fringe occurs at $\pm$3.11 cm on either side of the central bright fringe. a. How far apart are the two slits? b. At what distance from the central bright fringe will the third dark fringe occur?
8	Light of wavelength 470 nm in air shines on two slits $60\mu m$ apart. The slits are immersed in water, as is a viewing screen 40.0 cm away. How far apart are the fringes on the screen?
9	In Young's experiment a mixture of orange light (611 nm) and blue light (471 nm) shines on the double slit. The centres of the first-order bright blue fringes lie at the outer edges of a screen that is located 0.500 m away from the slits. However, the first-order bright orange fringes fall off the screen. By how much and in which direction (toward or away from the slits) should the screen be moved so that the centres of the first-order bright orange fringes will just appear on the screen?

Exercise Sheet 23: Thin Films

Easy

1 A soap bubble (n = 1.33) is floating in air. If the thickness of the bubble wall is 115 nm, what is the wavelength of the light that is most strongly reflected?

2 An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find

a. the color of the light in the visible spectrum most strongly reflected and
b. the color of the light in the spectrum most strongly transmitted.

Explain your reasoning.

3 A thin film of oil (n = 1.25) is located on a smooth wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no blue light at 512 nm. How thick is the oil film?

Past Year

4 **PSPM 21/22 – Q7(b)**

A flat glass with index of refraction 1.50 is coated with a transparent material of refraction index 1.25, in order to eliminate reflection of light of wavelength 680 nm. Determine the minimum thickness of the coating.

5 **PSPM 19/20 – Q7(b)**

A soap film with refractive index 1.3 and minimum thickness 0.177 μm appears reddish under white light. Calculate the wavelength of light that is missing from the reflection.

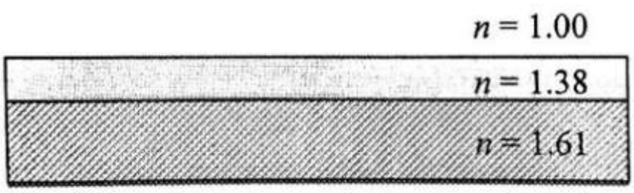
6 **PSPM 18/19 – Q7(c)**

Calculate the thickness of a soap film so that a 600 nm light incident to the film would produce constructive interference. Index of refraction of soap film 1.33.

7 **PSPM 17/18 – Q6(c)**

The figure shows a flint glass lens of refractive index 1.61 is coated with a thin layer film of magnesium fluoride of refractive index 1.38.

A ray of light of wavelength 565 nm is incident at right angles to the film.



a. Sketch the light rays that interfere after being reflected from both surfaces of the film. Label the reflected rays that undergo phase change.
b. What minimum thickness should the magnesium fluoride film have if the reflection of the 565 nm light is to appear dark?
c. If a lens is used to suppress the reflection of light at high frequencies, what should be done to the thickness of the film? Explain your answer

Book

8 A uniform thin film of alcohol (n=1.36) lies on a flat glass plate (n=1.56). When monochromatic light, whose wavelength can be changed, is incident normally, the reflected light is a minimum for $\lambda = 525nm$ and a maximum for $\lambda = 655nm$. What is the minimum thickness of the film?

9	A uniform layer of water (n=1.33) lies on a glass plate (n=1.52). Light shines perpendicularly on the layer. Because of constructive interference, the layer looks maximally bright when the wavelength of the light is 432 nm in vacuum and also when it is 648 nm in vacuum. a. Obtain the minimum thickness of the film. b. Assuming that the film has the minimum thickness and that the visible spectrum extends from 380 to 750 nm, determine the visible wavelength(s) in vacuum for which the film appears completely dark.
10	How thick (minimum) should the air layer be between two flat glass surfaces if the glass is to appear bright when 450-nm light is incident normally? What if the glass is to appear dark?

Exercise Sheet 24: Single Slit & Diffraction Grating

Easy	
1	Helium–neon laser light (λ =632.8 nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?
2	If 680-nm light falls on a slit 0.0425 mm wide, what is the angular width of the central diffraction peak?
3	At what angle will 510-nm light produce a second-order maximum when falling on a grating whose slits are $1.35\times 10^{-3} cm$ apart?
Past Year	
4	**PSPM 21/22 – Q7(c)** A monochromatic light 600 nm is incident on a diffraction grating with 400 lines per mm. Calculate the a. angle for the first bright order of diffraction. b. maximum number of diffraction pattern that can be formed.
5	**PSPM 20/21 – Q7(b)** A monochromatic light of wavelength 620 nm is incident on a single slit and forms a diffraction pattern on a screen 1.2 m away. The distance of seventh dark fringe from the central maximum is 18.0 mm. Determine the a. Size of the single slit b. Distance of the second bright fringe from the central maximum
6	**PSPM 16/17 – Q6(c)** A beam consists of two monochromatic lights 400 nm and 600 nm, is incident normally on a diffraction grating which has 540 lines mm^{-1}. Calculate the a. Angular separation between the first order diffraction of lights b. Highest order of diffraction that be observed with the 600nm light.
7	**PSPM 15/16 – Q6(c)** White light is incident on a soap film of refractive index 1.33 in air. The reflected light looks bluish because the red light of wavelength 670 nm is absent in the reflection. a. Does the light change phase when it reflects at air-film interface? Explain your answer. b. Does the light change phase when it travels in film and reflects at film-air interface? c. What happen to the wavelength and frequency of light when it travels from air to the film? d. Determine the minimum thickness of the soap film.
Book	
8	Light of wavelength 587.5 nm illuminates a single slit 0.750 mm in width. a. At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 0.850 mm from the centre of the principal maximum? b. What is the width of the central maximum?
9	A source emits 531.62-nm and 531.81-nm light. a. What minimum number of grooves is required for a grating that resolves the two wavelengths in the first-order spectrum? b. Determine the slit spacing for a grating 1.32cm wide that has the required minimum number of grooves.
10	Light that has a wavelength of 668 nm passes through a slit 6.73μm wide and falls on a screen that is 1.85m away. What is the distance on the screen from the centre of the central bright fringe to the third dark fringe on either side?

Exercise Sheet 25: De Broglie

Easy

1	Calculate the wavelength of a 2kg ball travelling at $0.1ms^{-1}$.
2	What is the wavelength of an electron of energy (a) 10 eV, (b) 100 eV, (c) 1.0 keV?
3	What voltage is needed to produce electron wavelengths of 0.26 nm?

Past Year

4	**PSPM 21/22 – Q9** De Broglie wavelength of a proton is $1.00 \times 10^{-13}m$. a. Calculate the speed and kinetic energy of the proton. b. Determine the applied electric potential for the proton to accelerate and reach this speed.
5	**PSPM 20/21 – Q9** A particle is moving three times faster than proton. The ratio of the de Broglie's wavelength of the particle to the proton is 1.716 x 10^4. Calculate the mass of particle.
6	**PSPM 19/20 – Q9** A beam of electrons is accelerated through a potential difference of 4500V in a Davisson and Germer experiment. a. Calculate the de Broglie wavelength of the electrons. b. Will the diffraction pattern become larger, remain unchanged or narrower when proton is used instead of electrons? Justify your answer.
7	**PSPM 18/19 – Q9** a. Calculate the speed of a neutron with de Broglie wavelength 9 x 10^{-11}m. b. Calculate the wavelength of an electron that has been accelerated across a potential difference of 100 V.

Book

8	An electron, starting from rest, accelerates through a potential difference of 418 V. What is the final de Broglie wavelength of the electron, assuming that its final speed is much less than the speed of light?
9	The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5.0% of the speed of light. Find the ratio of the photon wavelength to the de Broglie wavelength of the particle. *Note: Refer to form 5 physics syllabus.*
10	In an electron diffraction experiment using an accelerating voltage of 54V, an intensity maximum for $\theta = 50^o$. X-ray diffraction indicates that the atomic spacing in the target is $d = 0.218nm$. The electrons have negligible kinetic energy before being accelerated. Find the electron wavelength.

Exercise Sheet 26: Binding Energy & Mass Defect

Easy

1	Find the binding energy (in MeV) for lithium ${}^{7}_{3}Li$ (atomic mass = 7.016 003 u).
2	The binding energy of a nucleus is 225.0 MeV. What is the mass defect of the nucleus in atomic mass units?
3	Determine the mass defect (in atomic mass units) for a. Helium ${}^{3}_{2}He$, which has an atomic mass of 3.016030 u, b. the isotope of hydrogen known as tritium ${}^{3}_{1}T$, which has an atomic mass of 3.016050 u.

Past Year

4	**PSPM 21/22 – Q10(a)** Calculate the binding energy of a bromine nucleus (${}^{81}_{35}Br$) in Joule. Atomic mass of bromine = 80.916291 u.
5	**PSPM 20/21 – Q10(a)** Calculate the binding energy per nucleon for Thallium,${}^{205}_{81}Ti$ in MeV per nucleon. Given atomic mass Tl = 204.974401 u.
6	**PSPM 19/20 – Q10(a)** Calculate the binding energy per nucleon of a sodium nucleus (${}^{23}_{11}Na$) in MeV nucleon. The atomic mass of sodium is 22.989769 u.

Book

7	Show that the nucleus ${}^{8}_{4}Be$ (mass = 8.005305 u) is unstable and will decay into two α particles. Is ${}^{12}_{6}C$ stable against decay into three particles? Show why or why not.
8	How much energy is required to remove a. a proton, b. a neutron, from ${}^{15}_{7}N$ (of mass =15.000109u). Explain the difference in your answers. The mass of ${}^{14}_{6}C$ and ${}^{14}_{7}N$ are 14.003242u and 14.003074u respectively.

Exercise Sheet 27: Radioactivity

Easy	
1	In 9.0 days the number of radioactive nuclei decreases to one-eighth the number present initially. What is the half-life (in days) of the material?
2	The $^{32}_{15}P$ isotope of phosphorus has a half-life of 14.28 days. What is its decay constant in units of s^{-1}?
3	The number of radioactive nuclei present at the start of an experiment is 4.6×10^{15}. The number present twenty days later is 8.14×10^{14}. What is the half-life (in days) of the nuclei?
Past Year	
4	**PSPM 21/22 – Q10(b)** A sample consists of 2g of a radioactive element. The molar mass of the element is 67 g. If the half-life of the element is 78 hours, calculate the activity of the sample after 30 hours.
5	**PSPM 20/21 – Q10(b)** Radioactive nuclei have a half-life of 0.99 s. Determine the time taken for 25% of the nuclei to decay away.
6	**PSPM 19/20 – Q10(b)** Calculate the activity of a 5μg ^{24}Na which has a half-life of 14.9 hours.
7	**PSPM 18/19 – Q10(b)** A 2 g sample of radioactive iodine $^{131}_{53}I$ has a half-life of 8 days. a. Calculate the decay constant b. Calculate the initial number of atoms in 2g sample. c. Calculate the activity of the sample after 2 days.
Book	
8	Two radioactive nuclei A and B are present in equal numbers to begin with. Three days later, there are three times as many A nuclei as there are B nuclei. The half-life of species B is 1.50 days. Find the half-life of species A.
9	A 12.0 g sample of carbon from living matter decays at the rate of 184 decays/minute due to the radioactive C-14 in it. What will be the decay rate of this sample in a. 1000 years b. 50,000 years if the half-life of C-14 is 5730years?
10	$^{7}_{4}Be$ decays with a half-life of about 53 d. It is produced in the upper atmosphere, and filters down onto the Earth's surface. If a plant leaf is detected to have 350Bq of $^{7}_{4}Be$, a. how long do we have to wait for the decay rate to drop to 25 per second? b. Estimate the initial mass of on the leaf.

===End===

2023/2024 SP025 TUTORIALS

Table of Content

LIST ERROR! BOOKMARK NOT DEFINED.

SESSION 1: COULOMB'S LAW & ELECTRIC FIELD 3

SESSION 2: ELECTRIC POTENTIAL & POTENTIAL ENERGY 5

SESSION 3: SAMPLE PROBLEM – 4 CHARGES 7

SESSION 4: CHARGE IN UNIFORM ELECTRIC FIELD 8

SESSION 5: CAPACITORS IN SERIES & PARALLEL 11

SESSION 6: CHARGING & DISCHARGING OF CAPACITORS 14

SESSION 7: DIELECTRICS 15

SESSION 8: ELECTRIC CURRENT, RESISTIVITY, OHM'S LAW & TEMPERATURE 17

SESSION 9: ELECTROMOTIVE FORCE 19

SESSION 10: RESISTORS IN PARALLEL & SERIES 20

SESSION 11: KIRCHHOFF'S 22

SESSION 12: KIRCHHOFF'S 24

SESSION 13: ELECTRICAL ENERGY & POWER 26

SESSION 14: POTENTIAL DIVIDER & POTENTIOMETER 28

SESSION 15: MAGNETIC FLUX & INDUCED EMF 30

SESSION 16: SELF-INDUCTANCE, ENERGY STORAGE & MUTUAL INDUCTANCE 33

SESSION 17: RLC CIRCUITS 35

SESSION 18: OPTICS – REFLECTION 37

SESSION 19: OPTICS – REFRACTION 39

SESSION 20: OPTICS – THIN LENSES 41

SESSION 21: HUYGENS' & INTERFERENCES 41

LECTURE 22: DOUBLE SLIT 44

LECTURE 23: THIN FILMS 44

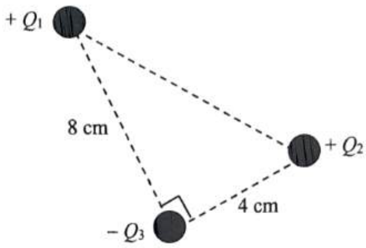
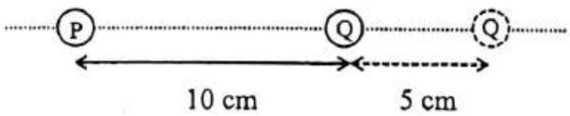
LECTURE 24: SINGLE SLIT & DIFFRACTION GRATING 49

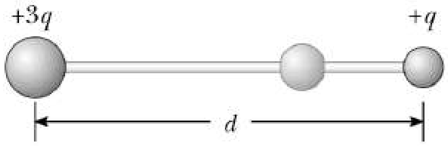
LECTURE 25: DE BROGLIE 49

LECTURE 26: BINDING ENERGY & MASS DEFECT 53

LECTURE 27: RADIOACTIVITY 53

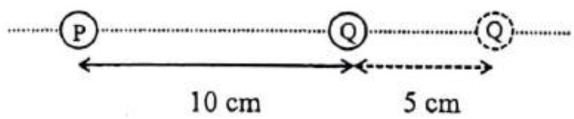
Session 1: Coulomb's Law & Electric Field

Easy	
1	Two point charges of $+2\mu C$ and $-4\mu C$ are separated by 2cm. Determine the electric force between them.
2	A point charge has a charge of $-2\mu C$. Determine the electric field 2cm from the point charge.
3	A point charge of $2\mu C$ is placed in a region of electric field of $200NC^{-1}$. Calculate the magnitude of force acting on the point charge?
Past Year	
4	**PSPM 21/22 – Q1(a)** Two opposite charges of the same magnitude of $2\times10^{-7}C$ are separated by 15cm. Calculate the a. Electric field strength at the midpoint between both charges. b. Magnitude of the force exerted on an electron on that point.
5	**PSPM 20/21 – Q1(a)** The figure shows three fixed point charges, $Q_1=+12nc$, $Q_2=+3nC$ and $Q_3=-4nC$.  Sketch the electric force diagram on the charge Q_3. Addition: Calculate the electric force on Q_3.
6	**PSPM 19/20 – Q1(a)**  The electric field at a point 10cm away from a charge P as shown in the diagram is $2.7(10^6)NC^{-1}$. a. Determine the charge of P. b. Sketch the electric force vectors on charge P and Q. c. If charge Q is moved horizontally to the right to a new position, and the electric force on it is $-4.05N$, how far apart is charge Q from charge P?
Book	
7	Two small nonconducting spheres have a total charge of $90\mu C$. When placed 28.0 cm apart, the force each exerts on the other is 12.0 N and is repulsive. What is the charge on each? What if the force were attractive?
	If the force is repulsive, then both charges must be positive since the total charge is positive. Call the total charge Q. $Q_1+Q_2=Q \quad F=\frac{kQ_1Q_2}{d^2}=\frac{kQ_1(Q-Q_1)}{d^2} \rightarrow Q_1^2-QQ_1+\frac{Fd^2}{k}=0$ $Q_1=\frac{Q\pm\sqrt{Q^2-4\frac{Fd^2}{k}}}{2}$ $=\frac{1}{2}\left[(90.0\times10^{-6}\text{ C})\pm\sqrt{(90.0\times10^{-6}\text{ C})^2-4\frac{(12.0\text{ N})(0.280\text{ m})^2}{(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}\right]$ $=\boxed{88.8\times10^{-6}\text{ C},\ 1.2\times10^{-6}\text{ C}}$ If the force is attractive, then the charges are of opposite sign. The value used for F must then be negative. Other than that, the solution method is the same as for part (a).
8	Two spherical objects are separated by a distance that is 1.80×10^3m. The objects are initially electrically neutral and are very small compared to the distance between them. Each object acquires the same negative charge due to the addition of electrons. As a result, each object experiences an electrostatic force that has a magnitude of $4.55\times10^{-21}N$. How many electrons did it take to produce the charge on one of the objects?

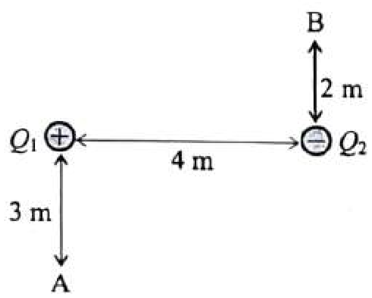
	The number N of excess electrons on one of the objects is $$N=\frac{q}{-e}=\frac{-\lvert q\rvert}{-e}=\frac{\lvert q\rvert}{e}$$ we solve Coulomb's law, $F=k\lvert q\rvert\lvert q\rvert/r^2$, for $\lvert q\rvert$: $$\lvert q\rvert=\sqrt{\frac{F\,r^2}{k}}$$ $$N=\frac{\lvert q\rvert}{e}=\frac{\sqrt{\frac{F\,r^2}{k}}}{e}=\frac{\sqrt{\frac{\left(4.55\times10^{-21}\ \mathrm{N}\right)\left(1.80\times10^{-3}\ \mathrm{m}\right)^2}{8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}}}}{1.60\times10^{-19}\ \mathrm{C}}=\boxed{8}$$
9	Two small beads having positive charges 3q and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point x = d. As shown in the figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? 
	Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by $$\mathrm{F}=\frac{k_e(3q)Q}{x^2}\hat{\mathrm{i}}+\frac{k_e(q)Q}{(d-x)^2}\left(-\hat{\mathrm{i}}\right).$$ The net force will be zero if $\frac{3}{x^2}=\frac{1}{(d-x)^2}$, or $d-x=\frac{x}{\sqrt{3}}$. This gives an equilibrium position of the third bead of $x=\boxed{0.634d}$.

Session 2: Electric Potential & Potential Energy

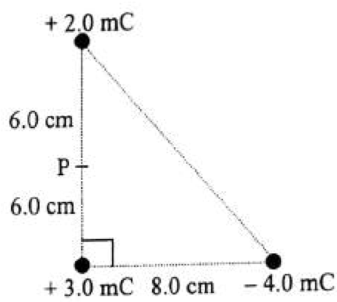
<table>
<tr><td>Easy</td><td></td></tr>
<tr><td>1</td><td>The energy needed to bring a $2\mu C$ to point A is $20 \times 10^{6} J$. Determine the electric potential at that point.</td></tr>
<tr><td>2</td><td>Referring to question 1, if point A is 30cm from a point charge Q. determine the charge of Q.</td></tr>
<tr><td>3</td><td>A system of charges consists of two point charges of $+2\mu C$ and $-4\mu C$ are separated by 2cm. Determine the electric potential energy of the system.</td></tr>
<tr><td>Past Year</td><td></td></tr>
<tr><td>4</td><td>PSPM 19/20 – Q1(a)



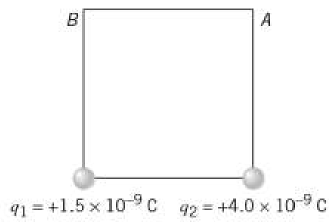
The electric field at a point 10cm away from a charge P as shown in the diagram is $2.7(10^{6})NC^{-1}$. A charge Q is placed 10cm away from charge P. When charge Q is moved horizontally to the right to a position 5cm from its initial position, there is a 0.54J change in its electric potential energy of the system. What is the charge of Q?</td></tr>
<tr><td>5</td><td>PSPM 18/19 – Q1(a)
The figure shows two charges, $Q_1 = +8\mu C$ and $Q_2 = -6\mu C$ placed 4m apart.

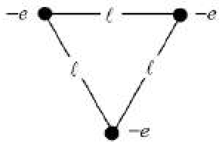
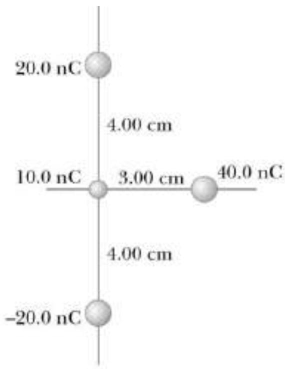


Calculate
a. the electric potential at points A and B.
b. the electric potential difference between points A and B.</td></tr>
<tr><td>6</td><td>PSPM 16/17 – Q1(b)
The figure shows three-point charges placed at each vertices of a right angle triangle.

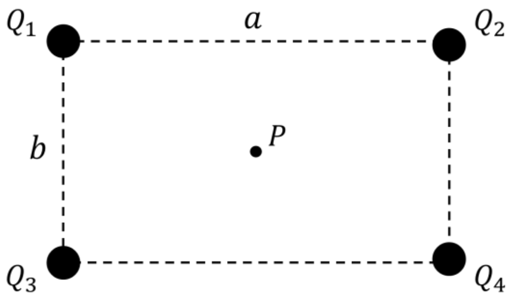


Calculate the
a. electric potential at point P
b. electric potential energy of the system.</td></tr>
<tr><td>Book</td><td></td></tr>
<tr><td>7</td><td>The drawing shows a square, each side of which has a length of L = 0.25 m. On two corners of the square are fixed different positive charges, $q_1 (= +1.5nC)$ and $q_2 (= +4nC)$. Find the electric potential energy of a third charge $q_3 = -6nC$ placed at corner A and then at corner B.

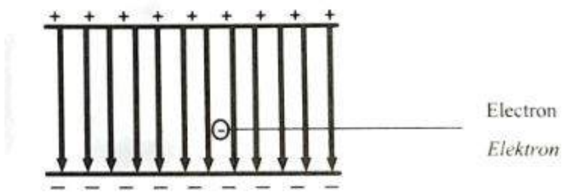
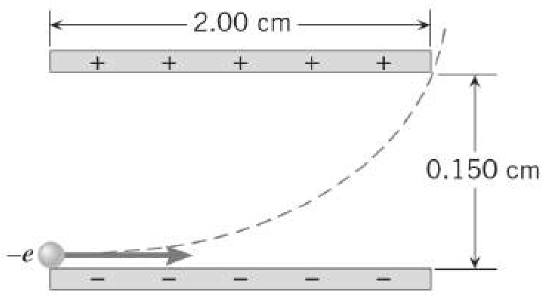

</td></tr>
</table>

	$V_{\text{Total},A} = \frac{kq_2}{L} + \frac{kq_1}{\sqrt{2}L}$ and $V_{\text{Total},B} = \frac{kq_1}{L} + \frac{kq_2}{\sqrt{2}L}$ $\text{EPE}_A = q_3 V_{\text{Total},A} = q_3\left(\frac{kq_2}{L} + \frac{kq_1}{\sqrt{2}L}\right) = \frac{q_3 k}{L}\left(q_2 + \frac{q_1}{\sqrt{2}}\right)$ $= \frac{(-6.0\times10^{-9}\ \text{C})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)}{0.25\ \text{m}}\left(+4.0\times10^{-9}\ \text{C} + \frac{+1.5\times10^{-9}\ \text{C}}{\sqrt{2}}\right) = \boxed{-1.1\times10^{-6}\ \text{J}}$ $\text{EPE}_B = q_3 V_{\text{Total},B} = q_3\left(\frac{kq_1}{L} + \frac{kq_2}{\sqrt{2}L}\right) = \frac{q_3 k}{L}\left(q_1 + \frac{q_2}{\sqrt{2}}\right)$ $= \frac{(-6.0\times10^{-9}\ \text{C})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)}{0.25\ \text{m}}\left(+1.5\times10^{-9}\ \text{C} + \frac{+4.0\times10^{-9}\ \text{C}}{\sqrt{2}}\right) = \boxed{-0.93\times10^{-6}\ \text{J}}$
8	How much work must be done to bring three electrons from a great distance apart to $10^{-10}m$ from one another (at the corners of an equilateral triangle)?
	 Let the side length of the equilateral triangle be $\boxed{\ell}$. Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_1 = 0$. The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential (due to the first electron) at the final location of the second electron. Thus $W_2 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{L}$. The work done in bringing the third electron to its final location is equal to the charge on the electron times the potential (due to the first two electrons). Thus $W_3 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell} - \frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell}$. The total work done is the sum $W_1 + W_2 + W_3$. $W = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\ell} + \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell} = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{\ell} = \frac{3(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.0\times10^{-10}\ \text{m})}$ $= \boxed{6.9\times10^{-18}\ \text{J}} = 6.9\times10^{-18}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{43\ \text{eV}}$
9	Two particles, with charges of 20nC and –20nC, are placed at the points with coordinates (0, 4cm) and (0, –4cm), as shown in the figure. A particle with charge 10nC is located at the origin. i. Find the electric potential energy of the configuration of the three fixed charges. ii. A fourth particle, with a mass of $2(10^{-13}kg)$ and a charge of 40nC, is released from rest at the point (3cm, 0). Find its speed after it has moved freely to a very large distance away. 
	i. $V_1 = \frac{k_e q_1}{r} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(20\times10^{-9}\ \text{C})}{0.04\ \text{m}} = 4.50\ \text{kV}$. To place the 10-nC charge there we must put in energy $U_{12} = q_2 V_1 = (10\times10^{-9}\ \text{C})(4.5\times10^{3}\ \text{V}) = 4.50\times10^{-5}\ \text{J}$. Next, to bring up the –20-nC charge requires energy $U_{23} + U_{13} = q_3 V_2 + q_3 V_1 = q_3(V_2 + V_1)$ $= -20\times10^{-9}\ \text{C}(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{10\times10^{-9}\ \text{C}}{0.04\ \text{m}} + \frac{20\times10^{-9}\ \text{C}}{0.08\ \text{m}}\right)$ $= -4.50\times10^{-5}\ \text{J} - 4.50\times10^{-5}\ \text{J}$ The total energy of the three charges is $U_{12} + U_{23} + U_{13} = \boxed{-4.50\times10^{-5}\ \text{J}}$. ii. $V = V_1 + V_2 + V_3 = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{20\times10^{-9}}{\sqrt{0.04^2+0.03^2}} + \frac{10\times10^{-9}}{0.03} - \frac{20\times10^{-9}}{0.05}\right)\ \text{C/m}$ $V = 3.00\times10^{3}\ \text{V}$ Energy of the system of four charged objects is conserved as the fourth charge flies away: $\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$ $0 + (40\times10^{-9}\ \text{C})(3.00\times10^{3}\ \text{V}) = \frac{1}{2}(2.00\times10^{-13}\ \text{kg})v^2 + 0$ $v = \sqrt{\frac{2(1.20\times10^{-4}\ \text{J})}{2\times10^{-13}\ \text{kg}}} = \boxed{3.46\times10^{4}\ \text{m/s}}$

Session 3: Sample Problem – 4 Charges

Problem
4 charges, $Q_1 = +2\mu C$, $Q_2 = +3\mu C$, $Q_3 = -3\mu C$, $Q_1 = -5\mu C$, are placed in an arrangement as shown in the diagram below.  If length of side *a* is the same as length of side *b* at 2cm, calculate a. the net force exerted on a charge of $+0.25\mu C$ placed at P, the centre of the rectangle. b. the electric field experience by a test charge placed at P, the centre of the rectangle. c. the total electrical potential energy of the system. d. the change in potential energy moving Q_4 from its original position to position P, the centre of the rectangle.

Session 4: Charge In Uniform Electric Field

Easy	
1	Two charged parallel plates are separated by a distance 2.5cm. The potential difference between the plates is 2kV. Find the electric field strength between the plates.
2	An electron is placed between two charged parallel plates of potential difference 40V separated by 5cm. Determine the force experienced by the electron.
3	A charged body of mass 20g and 2mC is placed between two charged parallel plate of potential 20V and 40V. Determine the acceleration of the charged body.
Past Year	
4	**PSPM 21/22 – Q1 (b)** The figure shows a uniform electric field $395Vm^{-1}$ exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of opposite place, 2cm away.  Calculate the a. acceleration of the electron b. potential difference between the plates work done by the electric field.
5	**PSPM 20/21 – Q1(b)** A small ball with mass 25g has a total charge of q=+20μC is placed between two parallel charged plates. i. In static equilibrium, determine the magnitude of the electric field in the plates. Calculate the acceleration of the ball if moves horizontally parallel with electric field.
6	**PSPM 19/20 – Q1(b)** Two oppositely charged parallel plates are held 2mm apart. A $4 \times 10^{-5} J$ of work is needed to move a 2μC point charge from one plate to the other. Calculate the electric field between the plates.
Book	
7	The drawing shows an electron entering the lower left side of a parallel plate capacitor and exiting at the upper right side. The initial speed of the electron is $7 \times 10^{6} ms^{-1}$. The capacitor is 2cm long, and its plates are separated by 0.150 cm. Assume that the electric field between the plates is uniform everywhere and find its magnitude. 

$$E = \frac{F}{q_0} = \frac{ma_y}{q_0}$$

$$y = 0.150 \times 10^{-2}\ \text{m}$$

$$y = \tfrac{1}{2}a_y t^2$$

$$v_{0y} = 0\ \text{m/s}$$

$$a_y = 2y/t^2$$

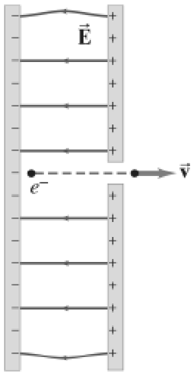
$$E = \frac{ma_y}{q_0} = \frac{m\,2y}{q_0 t^2}$$

$$E = \frac{m\,2y}{q_0 t^2} = \frac{m\,2y}{q_0\left(x/v_{0x}\right)^2} = \frac{m\,2yv_{0x}^2}{q_0 x^2}$$

$$E = \frac{m\,2yv_{0x}^2}{q_0 x^2} = \frac{\left(9.11 \times 10^{-31}\ \text{kg}\right)2\left(0.150 \times 10^{-2}\ \text{m}\right)\left(7.00 \times 10^{6}\ \text{m/s}\right)^2}{\left(-1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{-2}\ \text{m}\right)^2} = -2090\ \text{N/C}$$

8

An electron is accelerated in the uniform field ($E = 14.5\ kNC^{-1}$) between two thin parallel charged plates. The separation of the plates is 1.60 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate, as shown in the figure. With what speed does it leave the hole? Show that the gravitational force can be ignored.



$$F_{\text{net}} = ma = |q|E \quad \rightarrow \quad a = \frac{|q|E}{m}$$

$$v^2 = v_0^2 + 2a\Delta x \quad \rightarrow \quad v = \sqrt{2a\Delta x} = \sqrt{2\frac{|q|E}{m}\Delta x}$$

$$= \sqrt{2\frac{(1.602\times10^{-19}\ \text{C})(1.45\times10^{4}\ \text{N/C})}{(9.11\times10^{-31}\ \text{kg})}(1.60\times10^{-2}\ \text{m})} = \boxed{9.03\times10^{6}\ \text{m/s}}$$

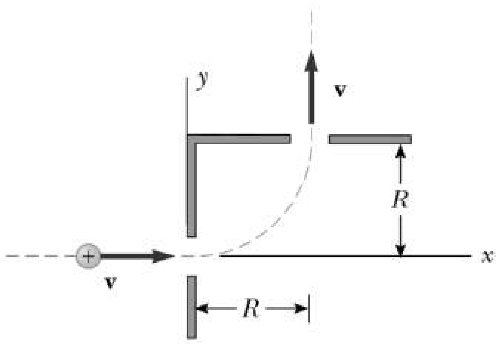
$$a = \frac{|q|E}{m} = \frac{(1.602\times10^{-19}\ \text{C})(1.45\times10^{4}\ \text{N/C})}{(9.11\times10^{-31}\ \text{kg})} = 2.55\times10^{15}\ \text{m/s}^2$$

$$a/g = (2.55\times10^{15}\ \text{m/s}^2)/(9.80\ \text{m/s}^2) = 2.60\times10^{14}$$

The acceleration due to gravity can be ignored compared to the acceleration caused by the electric field.

9

A researcher studying the properties of ions in the upper atmosphere wishes to construct an apparatus with the following characteristics: Using an electric field, a beam of ions, each having charge *q*, mass *m*, and initial velocity $v\hat{\imath}$, is turned through an angle of 90° as each ion undergoes displacement $R\hat{\imath} + R\hat{\jmath}$. The ions enter a chamber as shown in the figure, and leave through the exit port with the same speed they had when they entered the chamber. The electric field acting on the ions is to have constant magnitude.



a. Suppose the electric field is produced by two concentric cylindrical electrodes not shown in the diagram, and hence is radial. What magnitude should the field have?

b. If the field is produced by two flat plates and is uniform in direction, what value should the field have in this case?

(a) Each ion moves in a quarter circle. The electric force causes the centripetal acceleration.

$$\sum F = ma \qquad qE = \frac{mv^2}{R} \qquad \boxed{E = \frac{mv^2}{qR}}$$

(b) For the x-motion, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

$$0 = v^2 + 2a_x R \qquad a_x = -\frac{v^2}{2R} = \frac{F_x}{m} = \frac{qE_x}{m}$$

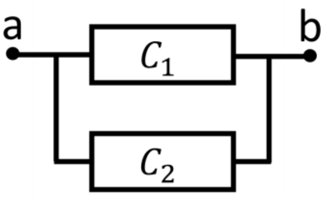
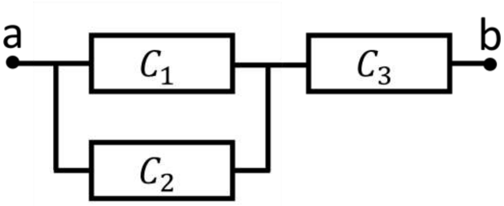
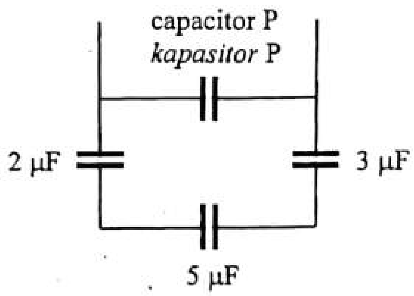
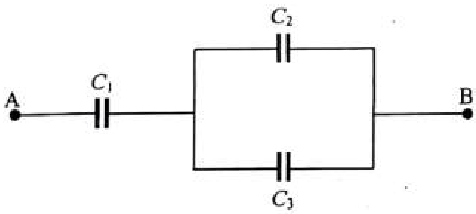
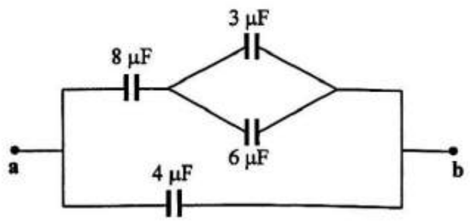
$E_x = -\frac{mv^2}{2qR}$. Similarly for the y-motion,

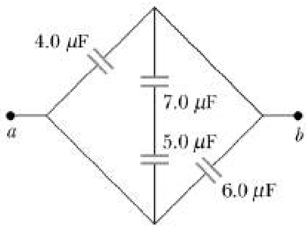
$$v^2 = 0 + 2a_y R \qquad a_y = +\frac{v^2}{2R} = \frac{qE_y}{m} \qquad E_y = \frac{mv^2}{2qR}$$

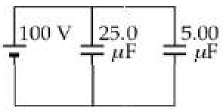
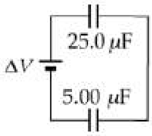
The magnitude of the field is

$$\sqrt{E_x^2 + E_y^2} = \boxed{\frac{mv^2}{\sqrt{2}qR} \text{ at } 135^\circ \text{ counterclockwise from the } x\text{-axis}}.$$

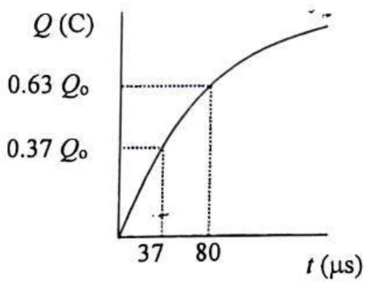
Session 5: Capacitors In Series & Parallel

Easy	
1	Determine the effective capacitance between a and b for the following diagram if $C_1 = 2\mu F$; $C_2 = 4\mu F$; and $C_3 = 3\mu F$. i.  ii.  iii. 
2	Referring to question 1, determine the energy stored if the potential difference between a and b is 12V.
3	A capacitor consists of two parallel plates, each of area $20cm^2$ and separated by 0.25cm of air. Determine the capacitance of the capacitor.
Past Year	
4	**PSPM 19/20 – Q2**  Capacitor P along with other capacitors are arranged as shown in the figure. If capacitor P has a capacitance of $4\mu F$, determine the effective capacitance.
5	**PSPM 18/19 – Q2** The figure shows three capacitors C_1, C_2, and C_3, each $12\mu F$ connected between points A and B. a. Calculate the effective capacitance. b. If the potential difference across AB is 9V, calculate the stored energy. 
6	**PSPM 17/18 – Q1(d)**  The figure shows a circuit consists of four capacitors. Calculate the equivalent capacitance between terminals a and b.

Book	
7	A. Two capacitors, $C_1 = 5\mu F$ and $C_2 = 12\mu F$, are connected in parallel, and the resulting combination is connected to a 9.00-V battery. a. What is the equivalent capacitance of the combination? b. What are the potential difference across each capacitor c. What are the charge stored on each capacitor? B. The two capacitors are now connected in series and to a 9.00-V battery. Find a. the equivalent capacitance of the combination, b. the potential difference across each capacitor, c. the charge on each capacitor.
	(a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of $C_{eq} = C_1 + C_2 = 5.00\ \mu F + 12.0\ \mu F = \boxed{17.0\ \mu F}$. (b) The potential difference across each branch is the same and equal to the voltage of the battery. $\Delta V = \boxed{9.00\ V}$ (c) $Q_5 = C\Delta V = (5.00\ \mu F)(9.00\ V) = \boxed{45.0\ \mu C}$ and $Q_{12} = C\Delta V = (12.0\ \mu F)(9.00\ V) = \boxed{108\ \mu C}$ (a) In series capacitors add as $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu F} + \frac{1}{12.0\ \mu F}$ $C_{eq} = \boxed{3.53\ \mu F}$. (c) The charge on the equivalent capacitor is $Q_{eq} = C_{eq}\Delta V = (3.53\ \mu F)(9.00\ V) = 31.8\ \mu C$. Each of the series capacitors has this same charge on it. So $Q_1 = Q_2 = \boxed{31.8\ \mu C}$. (b) The potential difference across each is $\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu C}{5.00\ \mu F} = \boxed{6.35\ V}$ $\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu C}{12.0\ \mu F} = \boxed{2.65\ V}$.
8	Consider three capacitors C_1, C_2, C_3, and a battery. If C_1 is connected to the battery, the charge on C_1 is 30.8μC. Now C_1 is disconnected, discharged, and connected in series with C_2. When the series combination of C_2 and C_1 is connected across the battery, the charge on C_1 is 23.1μC. The circuit is disconnected and the capacitors discharged. Capacitor C_3, capacitor C_1, and the battery are connected in series, resulting in a charge on C_1 of 25.2μC. If, after being disconnected and discharged, C_1, C_2 and C_3 are connected in series with one another and with the battery, what is the charge on C_1?
	For C_1 connected by itself, $C_1\Delta V = 30.8\ \mu C$ where ΔV is the battery voltage: $\Delta V = \frac{30.8\ \mu C}{C_1}$. For C_1 and C_2 in series: $\left(\frac{1}{1/C_1 + 1/C_2}\right)\Delta V = 23.1\ \mu C$ substituting, $\frac{30.8\ \mu C}{C_1} = \frac{23.1\ \mu C}{C_1} + \frac{23.1\ \mu C}{C_2}$ $C_1 = 0.333C_2$. For C_1 and C_3 in series: $\left(\frac{1}{1/C_1 + 1/C_3}\right)\Delta V = 25.2\ \mu C$ $\frac{30.8\ \mu C}{C_1} = \frac{25.2\ \mu C}{C_1} + \frac{25.2\ \mu C}{C_3}$ $C_1 = 0.222C_3$. For all three: $Q = \left(\frac{1}{1/C_1 + 1/C_2 + 1/C_3}\right)\Delta V = \frac{C_1\Delta V}{1 + C_1/C_2 + C_1/C_3} = \frac{30.8\ \mu C}{1 + 0.333 + 0.222} = \boxed{19.8\ \mu C}$. This is the charge on each one of the three.
9	Find the equivalent capacitance between points a and b in the combination of capacitors shown in the figure. 
	$C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu F$ $C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu F}$

10	Two capacitors, $C_1 = 25.0\mu F$ and $C_2 = 5.00\mu F$, are connected in parallel and charged with a 100-V power supply. a. Draw a circuit diagram and calculate the total energy stored in the two capacitors. b. What potential difference would be required across the same two capacitors connected in series in order that the combination stores the same amount of energy as in (a)? Draw a circuit diagram of this circuit.
	$U = \frac{1}{2}C(\Delta V)^2$ (a) $C_p = C_1 + C_2 = 25.0\ \mu F + 5.00\ \mu F = 30.0\ \mu F$ $U = \frac{1}{2}(30.0 \times 10^{-6})(100)^2 = \boxed{0.150\ J}$  (b) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu F} + \frac{1}{5.00\ \mu F}\right)^{-1} = 4.17\ \mu F$ $U = \frac{1}{2}C(\Delta V)^2$ $\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150)}{4.17 \times 10^{-6}}} = \boxed{268\ V}$ 

Session 6: Charging & Discharging of Capacitors

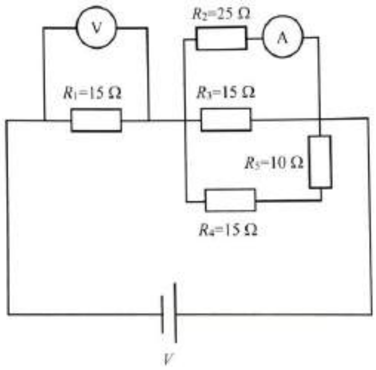
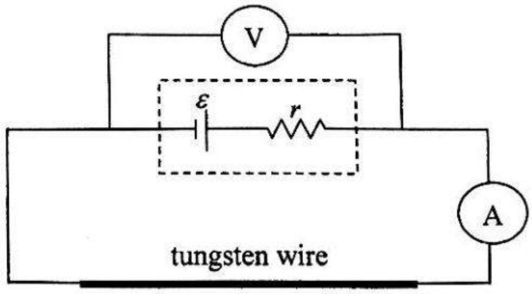
Easy	
1	A capacitor consists of two parallel plates, each of area $20cm^2$ and separated by 0.25cm of air. If the capacitor is connected to a power supply of 12V and a resistor of 2Ω. Determine the time constant.
2	A 6pF capacitor fully charged is connected to a 20Ω resistor. Determine the time it takes to discharge the capacitor to half of its capacity.
3	An uncharged 50μF capacitor is connected to a 4MΩ resistor and a 12V battery. a. Determine the time is takes to charge from 0% to 80% maximum capacity. b. Compare this to the time it takes to charge from 80% to 99% maximum capacity.
Past Year	
4	**PSPM 21/22 – Q2(b)** A $12\mu F$ capacitor which is charged to 6V, is connected in series to $8M\Omega$ resistor and a switch. Determine the charge on the capacitor 4 minutes after the switch is closed.
5	**PSPM 20/21 – Q2** An uncharged capacitor of $58\mu F$ is connected in series with a resistor of $100k\Omega$. Then the capacitor starts charging through the resistor. Calculate the time required for the capacitor to reach 30% of its maximum charge.
6	**PSPM 19/20 – Q2(a)** The graph in the figure shown shows how the charge Q on a capacitor P change with time, t when it is charged through a 20Ω resistor. Determine the capacitance of capacitor P. 
Book	
7	An uncharged capacitor with capacitance C is connected in series with a resistor R. A power supply of V_o is supplied across the circuit. Calculate the time taken for the potential difference across the capacitor increase to 72% the value of V_o.
	$$V = V_o\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow \frac{V}{V_o} = 0.72 = 1 - e^{-\frac{t}{RC}} \Rightarrow t = 1.27RC$$
8	A 2nF with an initial charge of 5.1μC is discharged through 1.3kΩ resistor. a. Calculate the current in the resistor $9\mu s$ after the resistor is connected across the terminals of the capacitor. b. Find the charge remains on the capacitor after $8\mu s$. c. Determine the maximum current in the resistor.
	$$I_o = \frac{Q}{t} = \frac{Q}{RC} = \frac{5.1 \times 10^{-9}}{(1300)(2 \times 10^{-9})} = 1.96A$$ $$I = I_o e^{-\frac{t}{RC}} = (1.96)e^{-\frac{9\times10^{-6}}{(1300)(2\times10^{-9})}} = 0.062A$$ $$Q = Q_o e^{-\frac{t}{RC}} = (5.1 \times 10^{-6})e^{-\frac{8\times10^{-6}}{(1300)(2\times10^{-9})}} = 0.235\mu C$$
9	A 10μF capacitor is charged to $650\mu C$ through a 20kΩ resistor. a. Calculate the time constant for this charging process. b. Calculate the initial current through the resistor.
	$$\boldsymbol{\tau = RC = (10 \times 10^{-6})(20 \times 10^{3}) = 0.2s}$$ $$\boldsymbol{Q = 650 \times 10^{-6} = CV = CIR = (10 \times 10^{-6})(I)(20 \times 10^{3})}$$ $$\boldsymbol{I = 3.25mA}$$

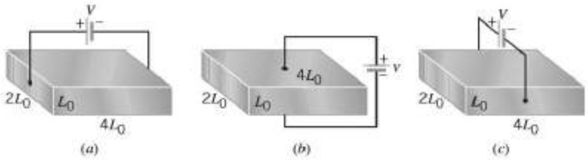
Session 7: Dielectrics

Easy	
1	A material has a dielectric constant of 2. Determine its dielectric permittivity of the material.
2	The material in question 1 is placed in between the plates of a parallel plate capacitor which has an initial capacitance of $4\mu F$. Determine the capacitance of the capacitor after the material is inserted.
3	A parallel plates capacitor has surface area of $305cm^2$ and a plate separation of 0.4cm. If the space between the plate is air-filled, determine the capacitance of the capacitor.
Past Year	
4	**PSPM 21/22 – Q2(a)** A parallel plate capacitor consists of plates of area $0.35m^2$ and separated by 10mm. If the region between plates is filled with dielectric material with dielectric $\varepsilon_r = 5.5$; calculate its capacitance.
5	***Not from past year** A dielectric material is inserted in between the plates of $8.9\mu F$ parallel plate capacitor. Calculate the capacitance of the capacitor if the dielectric constant of the material is 6.5.
6	***Not from past year** A parallel plate capacitor in a vacuum has a plate separation of 1.3cm with a plate area of $30cm^2$. A potential difference of 250V charges the plate and then it disconnected from the source. The capacitor is then dipped in distilled water with a dielectric constant of 80. a. Determine the charge on the plates before and after the dipping. b. Calculate the capacitance and potential difference after the dipping. c. Calculate the change in energy of the capacitor.
Book	
7	Two capacitors are identical, except that one is empty and the other is filled with a dielectric (ε_r =4.50). The empty capacitor is connected to a 12V battery. What must be the potential difference across the plates of the capacitor filled with a dielectric so that it stores the same amount of electrical energy as the empty capacitor?
	$C_2 = \kappa C_1$ $\frac{1}{2}C_2V_2^2 = \frac{1}{2}C_1V_1^2$ $V_2^2 = \frac{C_1V_1^2}{C_2}$ $V_2 = \sqrt{\frac{C_1V_1^2}{C_2}}$ $V_2 = \sqrt{\frac{C_1V_1^2}{C_2}} = \sqrt{\frac{\cancel{C_1}V_1^2}{\kappa \cancel{C_1}}} = \sqrt{\frac{V_1^2}{\kappa}} = \frac{V_1}{\sqrt{\kappa}} = \frac{12.0\text{ V}}{\sqrt{4.50}} = \boxed{5.66\text{ V}}$
8	An empty parallel plate capacitor is connected between the terminals of a 9V battery and charged up. The capacitor is then disconnected from the battery, and the spacing between the capacitor plates is doubled. As a result of this change, what is the new voltage between the plates of the capacitor?
	$q = \frac{\varepsilon_0 A V_{\text{smaller}}}{d_{\text{smaller}}}$ and $q = \frac{\varepsilon_0 A V_{\text{larger}}}{d_{\text{larger}}}$ $\frac{\varepsilon_0 A V_{\text{smaller}}}{d_{\text{smaller}}} = \frac{\varepsilon_0 A V_{\text{larger}}}{d_{\text{larger}}}$ or $\frac{V_{\text{smaller}}}{d_{\text{smaller}}} = \frac{V_{\text{larger}}}{d_{\text{larger}}}$ $V_{\text{larger}} = V_{\text{smaller}}\left(\frac{d_{\text{larger}}}{d_{\text{smaller}}}\right) = (9.0\text{ V})\left(\frac{2d_{\text{smaller}}}{d_{\text{smaller}}}\right) = \boxed{18\text{ V}}$
9	A parallel-plate capacitor in air has a plate separation of 1.5cm and a plate area of $25cm^2$. The plates are charged to a potential difference of 250V and disconnected from the source. The capacitor is then immersed in distilled water. Determine a. the charge on the plates before and after immersion, b. the capacitance and potential difference after immersion, c. the change in energy of the capacitor. Assume the liquid is an insulator.

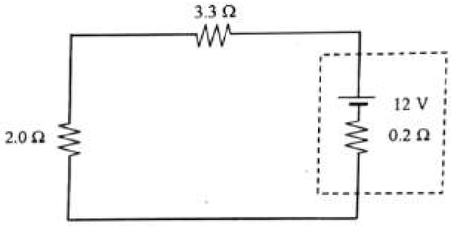
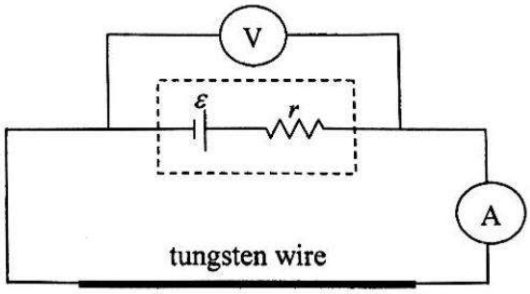
	$$C = \frac{\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_i}.$$ (a) $$Q = \frac{\epsilon_0 A(\Delta V)_i}{d}.$$ $$Q = \frac{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(25.0\times10^{-4}\ \mathrm{m^2})(250\ \mathrm{V})}{(1.50\times10^{-2}\ \mathrm{m})} = \boxed{369\ \mathrm{pC}}$$ (b) $$C_f = \frac{\kappa \epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f}$$ $$C_f = \frac{80.0(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(25.0\times10^{-4}\ \mathrm{m^2})}{(1.50\times10^{-2}\ \mathrm{m})} = \boxed{118\ \mathrm{pF}}$$ $$(\Delta V)_f = \frac{Qd}{\kappa \epsilon_0 A} = \frac{\epsilon_0 A(\Delta V)_i d}{\kappa \epsilon_0 A d} = \frac{(\Delta V)_i}{\kappa} = \frac{250\ \mathrm{V}}{80.0} = \boxed{3.12\ \mathrm{V}}.$$ (c) $$U_i = \frac{1}{2}C(\Delta V)_i^2 = \frac{\epsilon_0 A(\Delta V)_i^2}{2d}.$$ $$U_f = \frac{1}{2}C_f(\Delta V)_f^2 = \frac{\kappa \epsilon_0 A(\Delta V)_i^2}{2d\kappa^2} = \frac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa}.$$ $$\Delta U = U_f - U_i = \frac{-\epsilon_0 A(\Delta V)_i^2(\kappa-1)}{2d\kappa}$$ $$\Delta U = -\frac{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(25.0\times10^{-4}\ \mathrm{m^2})(250\ \mathrm{V})^2(79.0)}{2(1.50\times10^{-2}\ \mathrm{m})(80.0)} = \boxed{-45.5\ \mathrm{nJ}}.$$

Session 8: Electric Current, Resistivity, Ohm's Law & Temperature

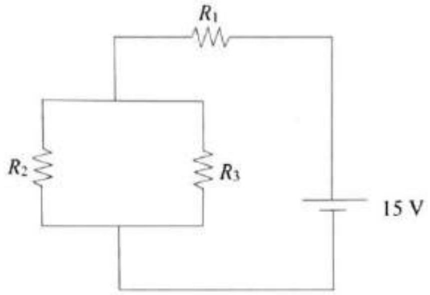
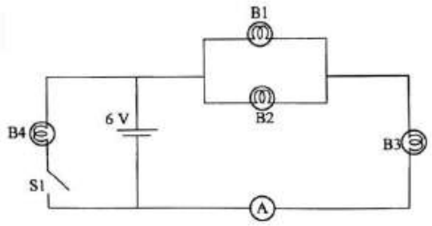
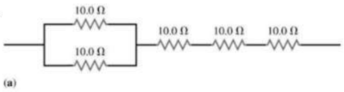
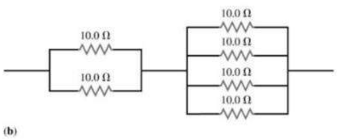
Easy	
1	Current running through a wire is found to be 1.5A. Determine the number of electron going through a section of this wire in 2s.
2	A material has a cross-sectional area of $20cm^2$ and length of 4cm. If the material has a measured resistance of 20Ω, determine its resistivity.
3	A $24k\Omega$ resistor is connected to a power supply of 240V. Determine the current going through the wire. The resistor then heated up by 30K, determine the new value for current after the resistor has heated up.
Past Year	
4	**PSPM 21/22 – Q3** a. A conducting wire has a 1.0mm diameter, a 2.0m length and a 50 mΩ resistance. Calculate its resistivity. b. The figure shows a circuit consisting five resistors, a voltmeter and an ammeter connected to a battery.  The reading of the ammeter is 1.25A. Determine the voltmeter reading.
5	**PSPM 18/19 – Q3(a)** Calculate the number of electrons that flow in a wire if it carries a current of 2A for 5 s.
6	**PSPM 19/20 – Q3(a)(iv)** Determine the change in resistance of a 2Ω resistor when there is a 30^oC rise in its temperature. The temperature coefficient of the resistivity of the resistor is $6.8 \times 10^{-3\,o}C^{-1}$.
7	**PSPM 15/16 – Q2(a)** The figure shows a tungsten wire connected to a battery with internal resistance $r = 0.6\Omega$. At room temperature of 23^oC, the readings of voltmeter and ammeter are 8.74V and 437mA respectively. After the tungsten wire is heated to 190^oC, the voltmeter reading is 8.85V and the ammeter reading is 253 mA. Calculate the temperature coefficient of resistivity of tungsten wire. 
Book	
8	Suppose that the resistance between the walls of a biological cell is $5\times 10^9\Omega$. a. What is the current when the potential difference between the walls is 75 mV? b. If the current is composed of Na^+ ions (q=+e), how many such ions flow in 0.50s?
	$I=\frac{V}{R}=\frac{75\times10^{-3}\ \text{V}}{5.0\times10^{9}\ \Omega}=\boxed{1.5\times10^{-11}\ \text{A}}$ $\text{Number of Na}^+\text{ ions}=\frac{\Delta q}{e}=\frac{I\,\Delta t}{e}=\frac{(1.5\times10^{-11}\ \text{A})(0.50\ \text{s})}{1.60\times10^{-19}\ \text{C}}=\boxed{4.7\times10^{7}}$

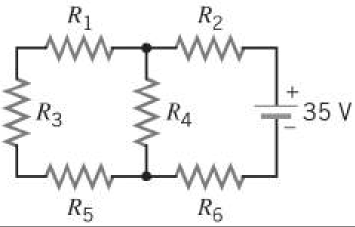
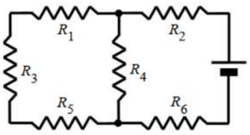
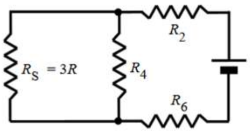
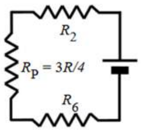
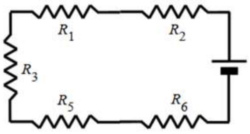
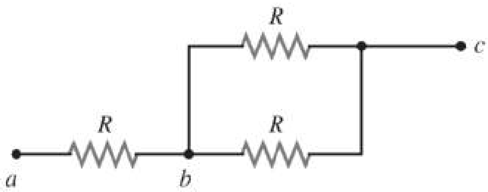
9	The resistance and the magnitude of the current depend on the path that the current takes. The drawing shows three situations in which the current takes different paths through a piece of material.  Each of the rectangular pieces is made from a material whose resistivity is $1.5 \times 10^{-2}\Omega m$, and the unit of length in the drawing is $L_o = 5cm$. Each piece of material is connected to a 3V battery. Find the resistance and the current in each case.
	a $R = \rho\left(\frac{2}{L_0}\right) = \left(1.50\times10^{-2}\ \Omega\cdot\text{m}\right)\left(\frac{2}{5.00\times10^{-2}\ \text{m}}\right) = \boxed{0.600\ \Omega}$ b $R = \rho\left(\frac{1}{8L_0}\right) = \left(1.50\times10^{-2}\ \Omega\cdot\text{m}\right)\left(\frac{1}{8\times5.00\times10^{-2}\ \text{m}}\right) = \boxed{0.0375\ \Omega}$ c $R = \rho\left(\frac{1}{2L_0}\right) = \left(1.50\times10^{-2}\ \Omega\cdot\text{m}\right)\left(\frac{1}{2\times5.00\times10^{-2}\ \text{m}}\right) = \boxed{0.150\ \Omega}$ a $I = \frac{V}{R} = \frac{3.00\ \text{V}}{0.600\ \Omega} = \boxed{5.00\ \text{A}}$ b $I = \frac{V}{R} = \frac{3.00\ \text{V}}{0.0375\ \Omega} = \boxed{80.0\ \text{A}}$ c $I = \frac{V}{R} = \frac{3.00\ \text{V}}{0.150\ \Omega} = \boxed{20.0\ \text{A}}$
10	While taking photographs in Death Valley on a day when the temperature is 58.0°C, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1A. Bill then travels to Antarctica and applies the same voltage to the same wire. What current does he register there if the temperature is -88.0°C? Assume that no change occurs in the wire's shape and size.
	$R_C = \frac{\Delta V}{I_C} = R_0\left[1+\alpha(T_C - T_0)\right]$ where $T_0 = 20.0°\text{C}$ $R_h = \frac{\Delta V}{I_h} = \frac{\Delta V}{1\ \text{A}} = R_0\left[1+\alpha(T_h - T_0)\right]$. $\frac{(\Delta V)/(1.00\ \text{A})}{(\Delta V)/I_C} = \frac{1+\left(3.90\times10^{-3}\right)(38.0)}{1+\left(3.90\times10^{-3}\right)(-108)}$ $I_C = (1.00\ \text{A})\left(\frac{1.15}{0.579}\right) = \boxed{1.98\ \text{A}}$.

Session 9: Electromotive Force

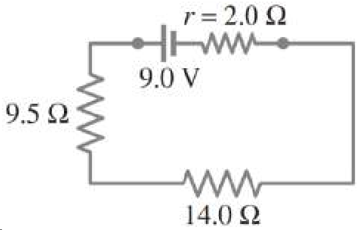
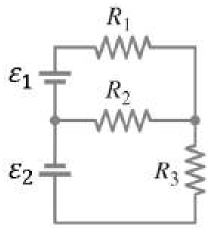
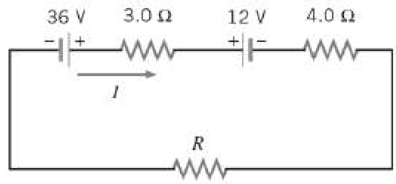
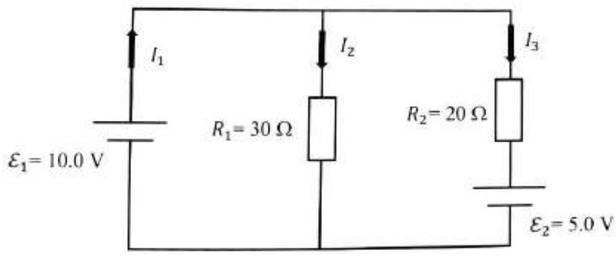
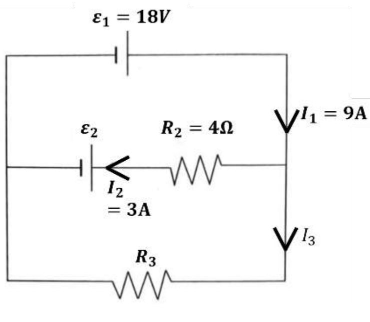
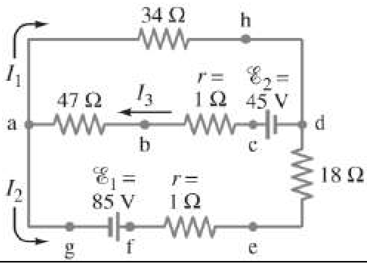
Easy	
1	(a) What is the current in a 5.6Ω resistor connected to a battery that has a 0.2Ω internal resistance if the terminal voltage of the battery is 10.0 V? (b) What is the emf of the battery?
Past Year	
2	**PSPM 18/19 – 3(c)** The figure shows a circuit with a battery having an emf of 12V and an internal resistance of 0.2Ω connected in series to two resistors, 3.3Ω and 2.0Ω. a. Calculate the current in the circuit. b. Calculate the terminal voltage across the battery. 
3	**PSPM 15/16 – Q2(a)** The figure shows a tungsten wire connected to a battery with internal resistance r. At room temperature of 23^oC, the readings of voltmeter and ammeter are 8.74V and 437mA respectively. After the tungsten wire is heated to 190^oC, the voltmeter reading is 8.85V and the ammeter reading is 253 mA. Calculate emf and internal resistance of the battery. 
4	**PSPM 14/15 – Q2(b)** A battery has an emf of 9 V. The terminal voltage is 8V when the battery is connected across a resistor of 5 Ω. Calculate the current through the resistor and the internal resistance of the battery.
Book	
5	An automobile battery has an emf of 12.6V and an internal resistance of 0.08Ω. The headlights together present equivalent resistance 5Ω (assumed constant). What is the potential difference across the headlight bulbs (a) when they are the only load on the battery and when the starter motor is operated, taking an additional 35.0 A from the battery?
	(a) $\varepsilon = I(R+r)$, so $I = \frac{\varepsilon}{R+r} = \frac{12.6\ \text{V}}{(5.00\ \Omega + 0.080\ 0\ \Omega)} = 2.48\ \text{A}$. $\Delta V = IR = (2.48\ \text{A})(5.00\ \Omega) = \boxed{12.4\ \text{V}}$. (b) Then, $I_1 = I_2 + 35.0\ \text{A}$, and $\varepsilon - I_1 r - I_2 r = 0$ so $\varepsilon = (I_2 + 35.0\ \text{A})(0.080\ 0\ \Omega) + I_2(5.00\ \Omega) = 12.6\ \text{V}$ giving $I_2 = 1.93\ \text{A}$. Thus, $\Delta V_2 = (1.93\ \text{A})(5.00\ \Omega) = \boxed{9.65\ \text{V}}$.
6	Two 1.50-V batteries — with their positive terminals in the same direction are inserted in series into the barrel of a flashlight. One battery has an internal resistance of 0.255Ω the other an internal resistance of 0.153 Ω. When the switch is closed, a current of 600 mA occurs in the lamp. a. What is the lamp's resistance? b. What fraction of the chemical energy transformed appears as internal energy in the batteries?
	$R = \frac{3.00\ \text{V}}{0.600\ \text{A}} = 5.00\ \Omega$. (a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$ (b) $\frac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \frac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.081\ 6 = \boxed{8.16\%}$

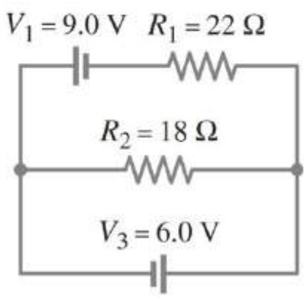
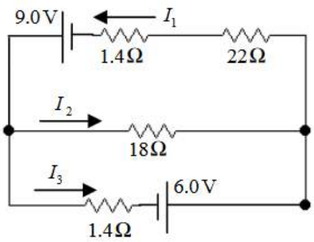
Session 10: Resistors in Parallel & Series

Easy	
1	Two resistors of 3Ω are connected in series to a power supply of 4V. Determine the effective resistance, current of the circuit and the voltage across ends of each resistor.
2	Two resistors of 3Ω are connected in parallel to a power supply of 4V. Determine the effective resistance and current through each resistor.
Past Year	
3	**PSPM 20/21 – 3(c)** The figure shows $R_1 = 10\Omega$; $R_2 = 12\Omega$ and $R_3 = 5\Omega$ are connected to a 15V power supply.  a. Calculate the effective resistance b. Potential difference across R_3.
4	**PSPM 16/17 – Q2(b)(i)** You are given several $1k\Omega$ resistors. How do you connect the resistors to a circuit that requires a 500 Q resistance? Show your suggestion.
5	**PSPM 15/16 – Q2(b)(i)**  The figure shows four identical bulbs connected to a 6 V battery and a switch. When the switch is off, the ammeter reading is 0.5 A. a. Calculate the resistance of a bulb. b. What happen to the reading of the ammeter when the switch is on? Explain your answer.
Book	
6	You are working late in your electronics shop and find that you need various resistors for a project. But alas, all you have is a big box of 10Ω resistors. Show how you can make each of the following equivalent resistances by a combination of your 10Ω resistors: a. 35Ω b. 1Ω c. 3.33Ω d. 7.5Ω
	(a) A parallel combination of two resistors in series with three others **(b)** Ten in parallel. **(c)** Three in parallel. **(d)** Two in parallel in series with four in parallel  

7	The circuit shown in the drawing is constructed with six identical resistors and an ideal battery. When the resistor R_4 is removed from the circuit, the current in the battery decreases by 1.9A. Determine the resistance of each resistor. 
	$\Delta I = I_f - I_0 = -1.9\text{ A}$   $R_S = 3R$ $R_P = \left(\frac{1}{R_P}\right)^{-1} = \left(\frac{1}{R_S} + \frac{1}{R_4}\right)^{-1} = \left(\frac{1}{3R} + \frac{1}{R}\right)^{-1} = \left(\frac{1}{3R} + \frac{3}{3R}\right)^{-1} = \left(\frac{4}{3R}\right)^{-1} = \frac{3R}{4}$ $R_{eq,0} = R_P + R_2 + R_4 = \frac{3R}{4} + 2R = \frac{11R}{4}$   $R_{eq,f} = 5R$ $I_0 = \frac{V}{R_{eq,0}}$ and $I_f = \frac{V}{R_{eq,f}}$ $\Delta I = I_f - I_0 = \frac{V}{R_{eq,f}} - \frac{V}{R_{eq,0}} = \frac{V}{5R} - \frac{V}{\left(\frac{11R}{4}\right)} = \frac{V}{R}\left(\frac{1}{5} - \frac{4}{11}\right) = -\frac{9V}{55R}$ $R = -\frac{9V}{55\Delta I} = -\frac{9(35\text{ V})}{55(-1.9\text{ A})} = \boxed{3.0\ \Omega}$
8	The circuit in the drawing contains three identical resistors. Each resistor has a value of 10.0Ω. Determine the equivalent resistance between the points a and b, b and c, and a and c. 
	$R_{ab} = R.$ $\frac{1}{R_{bc}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$ so $R_{bc} = \frac{1}{2}R$ $R_{ac} = R_{ab} + R_{bc} = R + \frac{1}{2}R = \frac{3}{2}R$ $R_{ab} = R = \boxed{10.0\ \Omega}$ $R_{bc} = \frac{1}{2}R = \boxed{5.00\ \Omega}$ $R_{ac} = \frac{3}{2}R = \boxed{15.0\ \Omega}$

Session 11: Kirchhoff's

Easy	
1	Calculate the current in the circuit of the figure shown, and show that the sum of all the voltage changes around the circuit is zero. 
2	Determine the magnitudes and directions of the currents in each resistor shown in the figure shown. The batteries have emfs of $\varepsilon_1 = 9V$ and $\varepsilon_2 = 12V$ and the resistors have values of $R_1 = 25\Omega$, $R_2 = 68\Omega$ and $R_3 = 35\Omega$. a. Ignore internal resistance of the batteries. b. Assume each battery has internal resistance r = 1.0Ω. 
3	Using Kirchhoff's loop rule, find the value of the current I in circuit shown, where $R = 5\Omega$. 
Past Year	
4	**PSPM 21/22 – Q3(c)**  Calculate current I_1, I_2, and I_3 as in the figure shown.
5	**PSPM 20/21 – 3(b)** The figure shows a circuit consisting of two batteries and two resistors.  Calculate the value of I_3 and R_3.
Book	
6	What is the potential difference between points a and d in the figure shown. What is the terminal voltage of each battery? 

	Eq. (i): $I_3 = I_1 + I_2$ Eq. (ii): $-34I_1 + 45 - 48I_3 = 0$ Eq. (iii): $-34I_1 + 19I_2 - 85 = 0$ Eq. (iv): $I_2 = \frac{85 + 34I_1}{19} = 4.474 + 1.789I_1$ Eq. (v): $I_3 = \frac{45 - 34I_1}{48} = 0.938 - 0.708I_1$ $I_3 = I_1 + I_2 \rightarrow 0.938 - 0.708I_1 = I_1 + 4.474 + 1.789I_1 \rightarrow I_1 = -1.011\text{ A}$ $I_2 = 4.474 + 1.789I_1 = 2.665\text{ A};\quad I_3 = 0.938 - 0.708I_1 = 1.654\text{ A}$ (*a*) $V_{\text{ad}} = V_{\text{d}} - V_{\text{a}} = -I_1(34\,\Omega) = -(-1.011\text{ A})(34\,\Omega) = 34.37\text{ V} \approx \boxed{34\text{ V}}$ OR $V_{\text{ad}} = V_{\text{d}} - V_{\text{a}} = \mathcal{E}_1 - I_2(19\,\Omega) = 85\text{ V} - (2.665\text{ A})(19\,\Omega) = 34.365\text{ V} \approx 34\text{ V}$ (*b*) 85 = V battery: $V_{\text{terminal}} = \mathcal{E}_1 - I_2 r = 85\text{ V} - (2.665\text{ A})(1.0\,\Omega) = \boxed{82\text{ V}}$ 45 = V battery: $V_{\text{terminal}} = \mathcal{E}_2 - I_3 r = 45\text{ V} - (1.654\text{ A})(1.0\,\Omega) = \boxed{43\text{ V}}$
7	Determine the magnitudes and directions of the currents through R_1 and R_2 in the figure shown. Now assuming that each battery has an internal resistance $r = 1.4\Omega$, determine the currents through R_1 and R_2. 
	$V_3 - I_1R_1 + V_1 = 0 \rightarrow I_1 = \frac{V_3 + V_1}{R_1} = \frac{6.0\text{ V} + 9.0\text{ V}}{22\,\Omega} = \boxed{0.68\text{ A, left}}$ $V_3 - I_2R_2 = 0 \rightarrow I_2 = \frac{V_3}{R_2} = \frac{6.0\text{ V}}{18\,\Omega} = \boxed{0.33\text{ A, left}}$ $I_1 = I_2 + I_3$ $-I_3(1.4\,\Omega) + 6.0\text{ V} - I_1(22\,\Omega) - I_1(1.4\,\Omega) + 9.0\text{ V} = 0$ $15 = 23.4I_1 + 1.4I_3$ $-I_3(1.4\,\Omega) + 6.0\text{ V} + I_2(18\,\Omega) = 0 \rightarrow 6 = -18I_2 + 1.4I_3$  $15 = 23.4I_1 + 1.4I_3 = 23.4(I_2 + I_3) + 1.4I_3 = 23.4I_2 + 24.8I_3;\quad 6 = -18I_2 + 1.4I_3$ $6 = -18I_2 + 1.4I_3 \rightarrow I_2 = \frac{-6 + 1.4I_3}{18}$ $15 = 23.4I_2 + 24.8I_3 = 23.4\left(\frac{-6 + 1.4I_3}{18}\right) + 24.8I_3 \rightarrow 270 = -140.4 + 32.76I_3 + 446.4I_3 \rightarrow$ $I_3 = \frac{410.4}{479.16} = 0.8565\text{ A};\quad I_2 = \frac{-6 + 1.4I_3}{18} = \frac{-6 + 1.4(0.8565)}{18} = -0.2667\text{ A} \approx \boxed{0.27\text{ A, left}}$ $I_1 = I_2 + I_3 = 0.5898\text{ A} \approx \boxed{0.59\text{ A, left}}$

Session 12: Kirchhoff's

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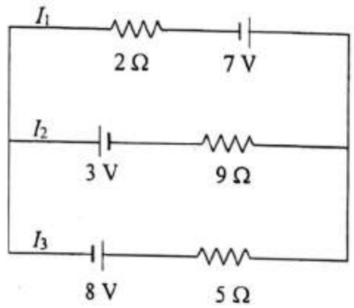
<tr><td>Past Year</td><td></td></tr>
<tr><td>1</td><td>

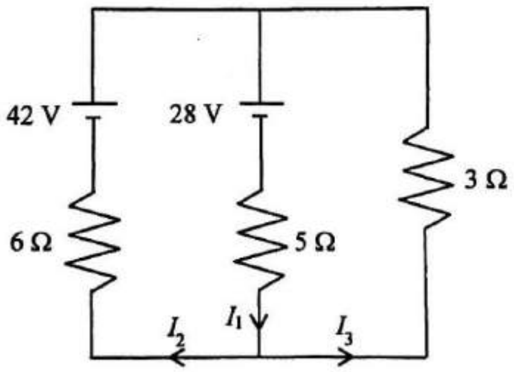
PSPM 19/20 – Q3



For the circuit in the figure shown, determine the current I_1, I_2 and I_3.

</td></tr>
<tr><td>2</td><td>

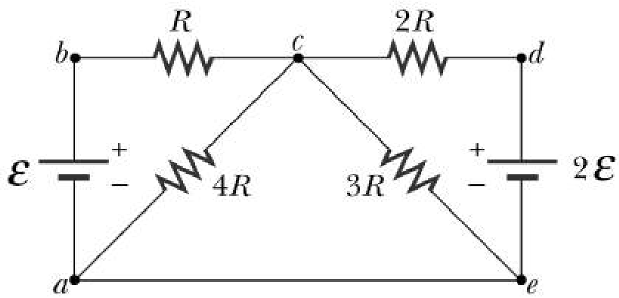
PSPM 17/18 – Q2(c)



For the circuit in the figure shown, determine the current I_1, I_2 and I_3.

</td></tr>
<tr><td>Book</td><td></td></tr>
<tr><td>3</td><td>

Taking $R = 1k\Omega$ and $\varepsilon = 250V$ in the figure shown, determine the direction and magnitude of the current in the horizontal wire between a and e.



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<tr><td></td><td>

$(2.71R)I_1 + (1.71R)I_2 = 250$

$(1.71R)I_1 + (3.71R)I_2 = 500$.

$I_1 = 10.0\text{ mA}$

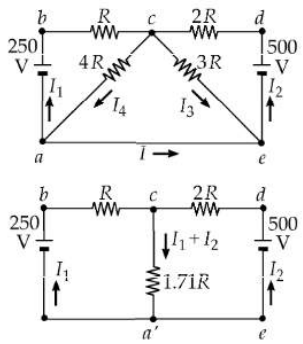
$I_2 = 130.0\text{ mA}$.

$V_c - V_a = (I_1 + I_2)(1.71R) = 240\text{ V}$.

$I_4 = \frac{V_c - V_a}{4R} = \frac{240\text{ V}}{4\,000\ \Omega} = 60.0\text{ mA}$.

$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA}$,

$I = \boxed{50.0\text{ mA from point } a \text{ to point } e}$.



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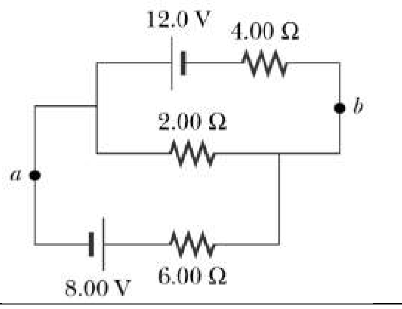
For the circuit shown in the figure shown, calculate

a. The current in the 2Ω resistor
b. the potential difference between points a and b



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(a) $I_1 = I_2 + I_3$

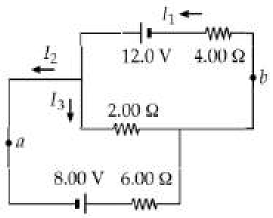
Counterclockwise around the top loop,

$12.0\text{ V} - (2.00\ \Omega)I_3 - (4.00\ \Omega)I_1 = 0.$

Traversing the bottom loop,

$8.00\text{ V} - (6.00\ \Omega)I_2 + (2.00\ \Omega)I_3 = 0$

$I_1 = 3.00 - \frac{1}{2}I_3,\ I_2 = \frac{4}{3} + \frac{1}{3}I_3$, and $\boxed{I_3 = 909\text{ mA}}$.



(b) $V_a - (0.909\text{ A})(2.00\ \Omega) = V_b$

$V_b - V_a = \boxed{-1.82\text{ V}}$

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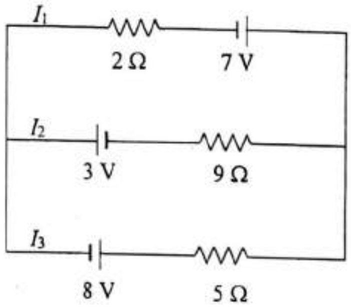
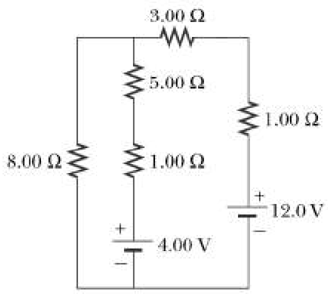
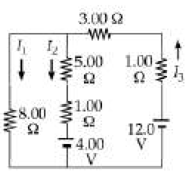
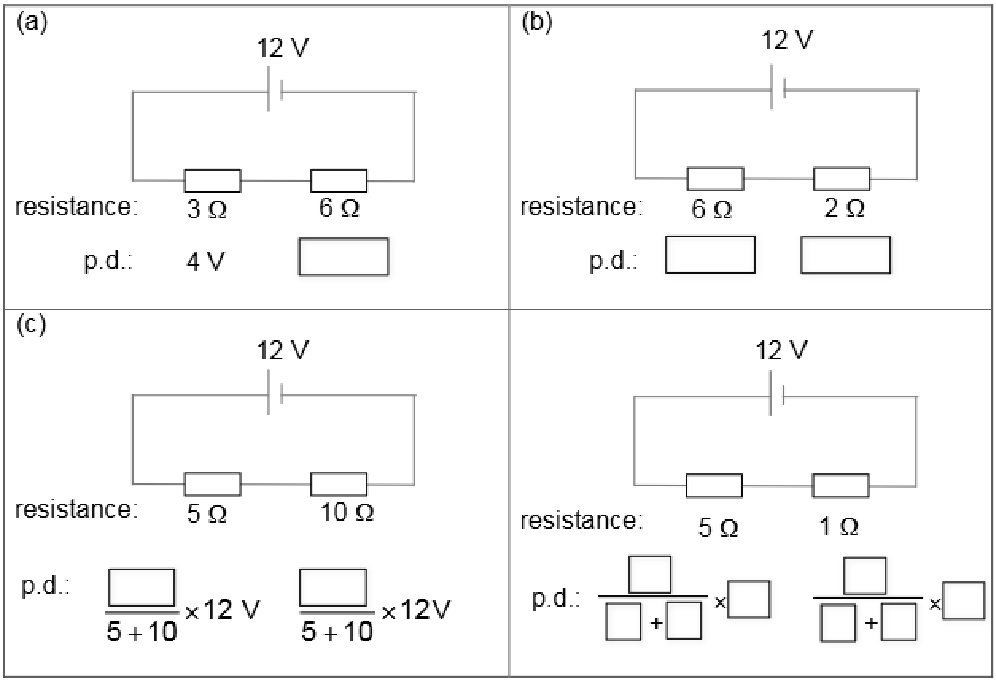
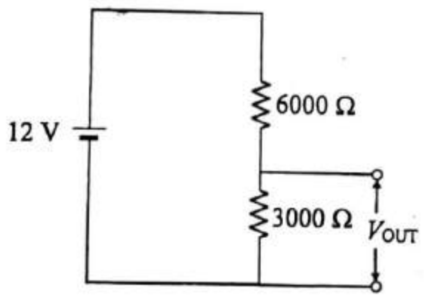
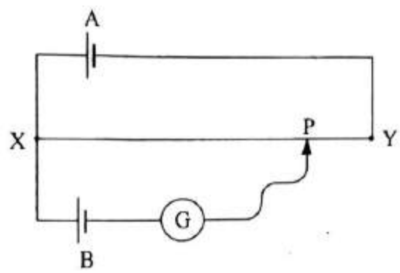
</table>

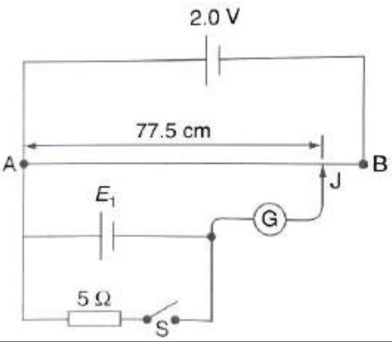
Session 13: Electrical Energy & Power

Easy	
1	An electric blanket is connected to a 120-V outlet and consumes 140 W of power. What is the resistance of the heater wire in the blanket?
2	An electric car uses a 45-kW (160-hp) motor. If the battery pack is designed for 340 V, what current would the motor need to draw from the battery? Neglect any energy losses in getting energy from the battery to the motor.
3	A 12-V battery causes a current of 600mA through a resistor. a. What is its resistance? b. How many joules of energy does the battery lose in a minute?
Past Year	
4	**PSPM 19/20 – Q3**  For the circuit in the figure shown, determine the power dissipated by the 5Ω.
5	**PSPM 18/19 – Q3(b)** A 2.5kW heater is connected to a 220V power supply. The voltage of the power supply is then changed to 110V. Calculate the power output of the heated.
6	**PSPM 17/18 – Q2(b)** The resistivity of a cooper wire is $1.72 \times 10^{-8} \Omega m$. An electric current of 2.07A flows in the wire. If the wire has a cross sectional area of $8.0 \times 10^{-7} m^2$ and length of 50 m, calculate energy dissipated in 1 minute.
Book	
7	Referring to the circuit in the figure shown, the circuit is connected for 2min. a. Find the energy delivered by each battery. b. Find the energy delivered to each resistor. c. Identify the types of energy transformations that occur in the operation of the circuit and the total amount of energy involved in each type of transformation. 
	$I_3 = I_1 + I_2$. $12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$ $8.00 = (4.00)I_3 + (6.00)I_2$ $-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0$ $\quad$ $(8.00)I_1 = 4.00 + (6.00)I_2$. $\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases}$ or $\begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$  $8 = 4I_1 + 13.3I_1 - 6.67$ $I_1 = \frac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A}$. $I_2 = 1.33(0.846\text{ A}) - 0.667$ $I_3 = I_1 + I_2$ $\boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$.

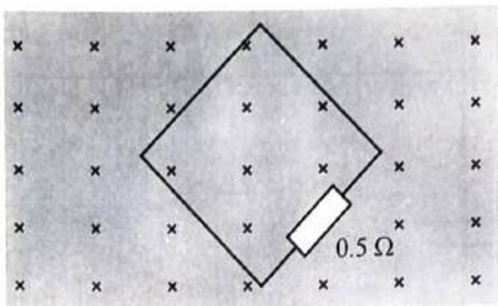
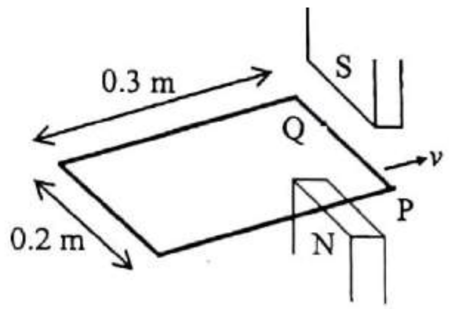
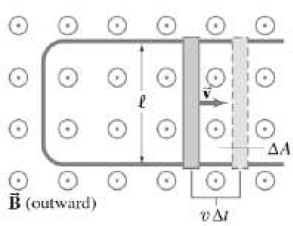
	(a) By the 4.00-V battery: $\Delta U = (\Delta V)I\Delta t = (4.00\ \text{V})(-0.462\ \text{A})120\ \text{s} = \boxed{-222\ \text{J}}$. By the 12.0-V battery: $(12.0\ \text{V})(1.31\ \text{A})120\ \text{s} = \boxed{1.88\ \text{kJ}}$. (b) By the 8.00-Ω resistor: $I^2R\Delta t = (0.846\ \text{A})^2(8.00\ \Omega)120\ \text{s} = \boxed{687\ \text{J}}$. By the 5.00-Ω resistor: $(0.462\ \text{A})^2(5.00\ \Omega)120\ \text{s} = \boxed{128\ \text{J}}$. By the 1.00-Ω resistor: $(0.462\ \text{A})^2(1.00\ \Omega)120\ \text{s} = \boxed{25.6\ \text{J}}$. By the 3.00-Ω resistor: $(1.31\ \text{A})^2(3.00\ \Omega)120\ \text{s} = \boxed{616\ \text{J}}$. By the 1.00-Ω resistor: $(1.31\ \text{A})^2(1.00\ \Omega)120\ \text{s} = \boxed{205\ \text{J}}$. (c) $-222\ \text{J} + 1.88\ \text{kJ} = \boxed{1.66\ \text{kJ}}$ from chemical to electrical. $687\ \text{J} + 128\ \text{J} + 25.6\ \text{J} + 616\ \text{J} + 205\ \text{J} = 1.66\ \text{kJ}$ from electrical to internal.
8	A power station delivers 750 kW of power at 12,000 V to a factory through wires with total resistance How much less power is wasted if the electricity is delivered at 50,000 V rather than 12,000 V?
	$P_{\text{delivered}} = IV \rightarrow I = \dfrac{P_{\text{delivered}}}{V} \quad P_{\text{dissipated}} = I^2R = \dfrac{P^2_{\text{delivered}}}{V^2}R$ $P_{\substack{\text{dissipated}\\ 12{,}000\ \text{V}}} = \dfrac{(7.5\times10^5\ \text{W})^2}{(1.2\times10^4\ \text{V})^2}(3.0\ \Omega) = 11{,}719\ \text{W}$ $P_{\substack{\text{dissipated}\\ 50{,}000\ \text{V}}} = \dfrac{(7.5\times10^5\ \text{W})^2}{(5\times10^4\ \text{V})^2}(3.0\ \Omega) = 675\ \text{W}$ $\text{difference} = 11{,}719\ \text{W} - 675\ \text{W} = \boxed{1.1\times10^4\ \text{W}}$
9	You want to design a portable electric blanket that runs on a 1.5-V battery. If you use a 0.50-mm-diameter copper wire as the heating element, how long should the wire be if you want to generate 18W of heating power? What happens if you accidentally connect the blanket to a 9.0-V battery?
	$R = \rho\dfrac{\ell}{A} = \rho\dfrac{\ell}{\pi r^2} = \dfrac{4\rho\ell}{\pi d^2};\ P = \dfrac{V^2}{R} = \dfrac{V^2}{\dfrac{4\rho\ell}{\pi d^2}} \rightarrow$ $\ell = \dfrac{V^2\pi d^2}{4\rho P} = \dfrac{(1.5\ \text{V})^2\pi(5.0\times10^{-4}\ \text{m})^2}{4(1.68\times10^{-8}\ \Omega\cdot\text{m})(18\ \text{W})} = 1.461\ \text{m} \approx \boxed{1.5\ \text{m}}$ If the voltage increases by a factor of 6 without the resistance changing, then the power will increase by a factor of 36. The blanket would theoretically be able to deliver 540 W of power, which might make the material catch on fire or burn the occupant.

Session 14: Potential Divider & Potentiometer

Easy	
1	Consider the following circuit: a cell with emf 12 V connected to two resistors in series. Find the potential difference across each resistor. Follow these rules: a. For resistors in series, $E = V_1 + V_2 + V_3 + \ldots$ b. The ratio of p.d. is equal to the ratio of resistance. 
2	A DC potentiometer is designed to measure up to about 2V with a slide wire of 800 mm. A standard cell of emf 1.18 V obtains balances at 600 mm. A test cell is seen to obtain balance at 680 mm. Calculate the emf of the test cell.
Past Year	
3	**PSPM 19/20 – Q3(b)**  The figure shows a potential divider circuit. a. Calculate the output voltage. b. If a voltmeter of resistance 3000Ω is connected across the output, determine the reading of the voltmeter.
4	**PSPM 16/17 – Q2(c)**  The figure shows a potentiometer circuit consists of a uniform wire XY of length 100cm and its resistance 5Ω. The emf of call A and B is 4.0V and 3.0V, respectively. The internal resistance of both cells are negligible. a. What is the length of XP when the galvanometer reading is zero? b. If a 1.0Ω is connected in series with cell A, what is the new balanced length of XP?

Book	
5	Two cylindrical rods, one copper and the other iron, are identical in lengths and cross-sectional areas. They are joined end to end to form one long rod. A 12-V battery is connected across the free ends of the copper–iron rod. The resistivity of copper and iron is $1.72 \times 10^{-8}\Omega m$ and $9.7 \times 10^{-8}\Omega m$ respectively. What is the voltage between the ends of the copper rod?
	$R_S = R_{Cu} + R_{Fe}.$ $I = \frac{V}{R_S} = \frac{V}{R_{Cu} + R_{Fe}}$ $V_{Cu} = IR_{Cu} = \frac{V}{R_{Cu} + R_{Fe}} R_{Cu}$ $R_{Cu} = \rho_{Cu} L/A$ and $R_{Fe} = \rho_{Fe} L/A,$ $V_{Cu} = \left(\frac{V}{\rho_{Cu} + \rho_{Fe}}\right)\rho_{Cu}$ $V_{Cu} = \left(\frac{12\text{ V}}{1.72\times10^{-8}\ \Omega\cdot\text{m} + 9.7\times10^{-8}\ \Omega\cdot\text{m}}\right)(1.72\times10^{-8}\ \Omega\cdot\text{m}) = \boxed{1.8\text{ V}}$
6	The emf E_1 of a cell is measured using a potentiometer as shown in the figure. The driver cell has an emf of 2V and negligible resistance. When the switch S is open, the galvanometer G is balanced when the length AJ is 77.5cm. When the switch *S* is closed, the length AJ is 63.8 cm. a. Calculate the emf E_1 b. Calculate the internal resistance of the cell. 
	$\boldsymbol{E_1 = (0.775)(2) = 1.55V}$ $\boldsymbol{\frac{R+r}{R} = \frac{5+r}{5} = \frac{77.5}{63.8} \Rightarrow r = 1.074\Omega}$

Session 15: Magnetic Flux & Induced Emf

<table>
<tr><td>Easy</td><td></td></tr>
<tr><td>1</td><td>A plane coil of 20 turns has a cross sectional area of $0.045m^2$ is placed in a uniform magnetic field strength of 0.075T. Calculate the magnetic flux for linked with the coil if the coil area is
a. parallel to the magnetic field lines
b. perpendicular to the magnetic field lines.
c. angled at 25^o to the magnetic field lines.</td></tr>
<tr><td>2</td><td>A plane coil of 20 turns, a cross sectional area of $0.025m^2$, experiences magnetic flux change from 0.2T to 0.5T in 3 seconds and is connected to a 2Ω resistor. Determine the induced emf and induced current.</td></tr>
<tr><td>3</td><td>A copper rod of length 0.8m moves perpendicularly through a region of 0.5T magnetic field at $2ms^{-1}$. Determine the induced emf within the rod.</td></tr>
<tr><td>4</td><td>a. A magnetic field perpendicular to a circular coil (18 turns, radius 50mm) changes from 2T to 20T in 3s, Calculate the magnitude of the induced emf.
b. A circular coil of 20turns in a magnetic field of 0.4T changes its radius from 2cm to 5cm, determine magnitude of induced emf.</td></tr>
<tr><td>5</td><td>An AC generator consisting a 30turns coil with cross sectional area of 0.1m2 and resistance of 100Ω. The coil rotates in a magnetic field of strength 0.5T at a frequency of 30Hz. Calculate the maximum induced current.</td></tr>
<tr><td>Past Year</td><td></td></tr>
<tr><td>6</td><td>PSPM 21/22 – Q4(a)
The figure shows a square wire loop with 2m sides, connect to a 0.5Ω resistor placed perpendicularly to a changing magnetic field.
a. If the magnetic field changes uniformly from 0 to 0.4T in 6.0s, calculate the induced emf in the loop.
b. Determine the current induced in the loop and its direction. Explain your answer.

</td></tr>
<tr><td>7</td><td>PSPM 20/21 – Q4(a)
A 14 turns circular coil is placed on a paper which lies in 1.2T magnetic field pointing inwards to the paper. The coil's diameter changes from 22.5cm to 7.2cm in 1.8s.
a. Determine the direction of the induced current.
b. Calculate the magnitude of the emf induced in the circuit.
c. Calculate the induced current if the circular coil resistance is 7.5Ω.</td></tr>
<tr><td>8</td><td>PSPM 19/20 – Q4(a)
The figure shows a rectangular wire loop $0.3m \times 0.2m$ moving horizontally to the right at $12ms^{-1}$ in a uniform magnetic field of 0.8T.
The induced current in the wire is 3A.
a. Determine the resistance of the wire loop.
b. Determine the direction of the induced current. Explain how you determine the direction of the induced current.

</td></tr>
<tr><td>Book</td><td></td></tr>
<tr><td>9</td><td>Referring to the figure shown, the moving rod has a resistance of 0.25Ω and moves on rails 20cm apart. The stationary U-shaped conductor has negligible resistance. When a force of 0.35N is applied to the rod, it moves to the right at a constant speed of $1.5ms^{-1}$. What is the magnetic field?

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</table>

$$F_{\text{external}} = F_{\text{magentic}} = I\ell B = \frac{\mathscr{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \boxed{\frac{B^2\ell^2 v}{R}}$$

$$F_{\text{external}} = \frac{B^2\ell^2 v}{R} \quad \rightarrow \quad B = \sqrt{\frac{F_{\text{external}}R}{\ell^2 v}} = \sqrt{\frac{(0.350\text{ N})(0.25\,\Omega)}{(0.200\text{ m})^2(1.50\text{ m/s})}} = 1.208\text{ T} \approx \boxed{1.2\text{ T}}$$

10 The magnetic field perpendicular to a single 13.2cm diameter circular loop of copper wire decreases uniformly from 0.670 T to zero. If the wire is 2.25 mm in diameter, how much charge moves past a point in the coil during this operation?

$$|\mathscr{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{A_{\text{loop}}|\Delta B|}{\Delta t}; \quad R = \frac{\rho\ell}{A_{\text{wire}}}; \quad I = \frac{\mathscr{E}}{R} = \frac{\frac{A_{\text{loop}}|\Delta B|}{\Delta t}}{\frac{\rho\ell}{A_{\text{wire}}}} = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell\Delta t}$$

$$Q = I\Delta t = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell} = \frac{\pi r_{\text{loop}}^2 \pi r_{\text{wire}}^2|\Delta B|}{\rho(2\pi)r_{\text{loop}}} = \frac{r_{\text{loop}}\pi r_{\text{wire}}^2|\Delta B|}{2\rho}$$

$$Q = \frac{(0.066\text{ m})\pi(1.125\times10^{-3}\text{ m})^2(0.670\text{ T})}{2(1.68\times10^{-8}\,\Omega\cdot\text{m})} = \boxed{5.23\text{ C}}$$

11 Use Lenz's law to answer the following questions concerning the direction of induced currents.

a. What is the direction of the induced current in resistor R in figure 1 when the bar magnet is moved to the left?

b. What is the direction of the current induced in the resistor R immediately after the switch S in figure 2 is closed?

c. What is the direction of the induced current in R when the current I in figure 3 decreases rapidly to zero?

d. A copper bar is moved to the right while its axis is maintained in a direction perpendicular to a magnetic field, as shown in figure 4. If the top of the bar becomes positive relative to the bottom, what is the direction of the magnetic field?

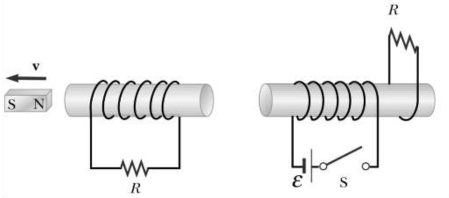


Figure 1 Figure 2

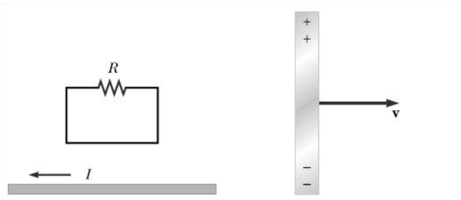
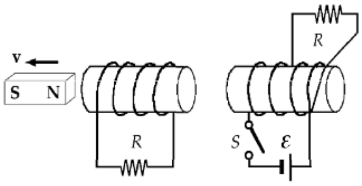


Figure 3 Figure 4

(a) $\mathbf{B}_{ext} = B_{ext}\hat{\mathbf{i}}$ and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed $\boxed{\text{to the right}}$.

(b) $\mathbf{B}_{ext} = B_{ext}(-\hat{\mathbf{i}})$ increases; therefore, the induced field $\mathbf{B}_0 = B_0(+\hat{\mathbf{i}})$ is to the right, and the current in the resistor is directed $\boxed{\text{to the right}}$.

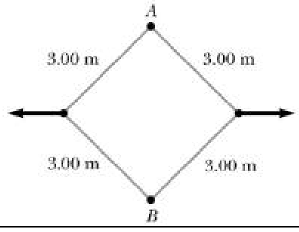
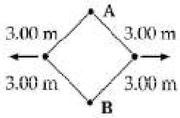
(c) $\mathbf{B}_{ext} = B_{ext}(-\hat{\mathbf{k}})$ into the paper and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0(-\hat{\mathbf{k}})$ into the paper, and the current in the resistor is directed $\boxed{\text{to the right}}$.



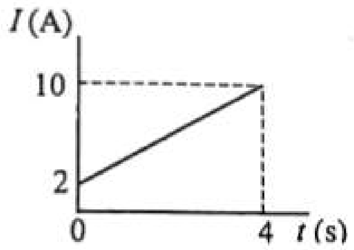
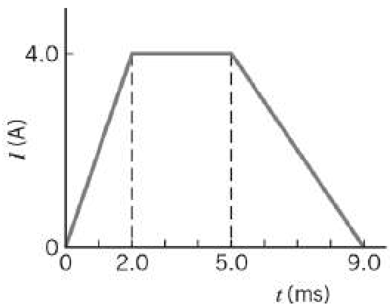
(a) (b)

(c) (d)

(d) By the magnetic force law, $F_B = q(\mathbf{v}\times\mathbf{B})$. Therefore, a positive charge will move to the top of the bar if **B** is $\boxed{\text{into the paper}}$.

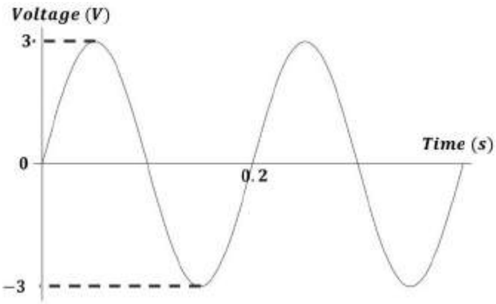
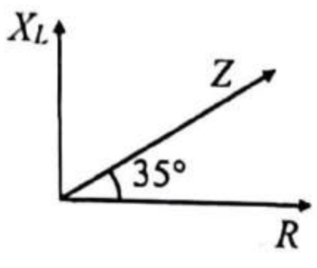
12	The square loop in the figure shown is made of wires with total series resistance 10Ω. It is placed in a uniform 0.1T magnetic field directed perpendicularly into the plane of the paper. The loop, which is hinged at each corner, is pulled as shown until the separation between points A and B is 3.00 m. If this process takes 0.1s, what is the average current generated in the loop? What is the direction of the current?	
	$\varepsilon = -N\frac{d}{dt}BA\cos\theta = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right)$ $\varepsilon = -1(0.100\text{ T})\cos 0° \frac{(3.00\text{ m}\times 3.00\text{ m}\sin 60.0°)-(3.00\text{ m})^2}{0.100\text{ s}} = 1.21\text{ V}$ $I = \frac{1.21\text{ V}}{10.0\ \Omega} = \boxed{0.121\text{ A}}$ The flux is into the page and decreasing. The loop makes its own magnetic field into the page by carrying $\boxed{\text{clockwise}}$ current.	

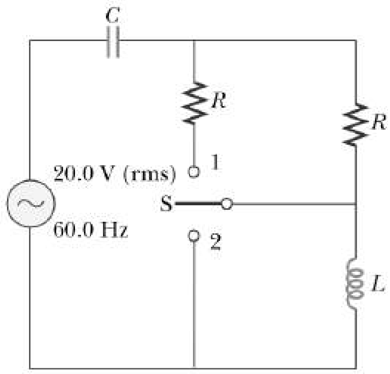
Session 16: Self-Inductance, Energy Storage & Mutual Inductance

Easy	
1	Induced emf of 6V is developed across a coil when the current flowing through it changes at $30As^{-1}$. Determine the self-inductance of the coil.
2	Calculate the value of self-inductance for an air-filled solenoid of length 5cm and cross-sectional area of 0.3cm2 containing 50 loops.
3	A 500turns of solenoid is 8cm long. When the current in the solenoid is increased by 2.5A in 0.35s, the magnitude of the induced emf is 0.012V. Calculate the inductance of the solenoid and the cross sectional area of the solenoid.
Past Year	
4	**PSPM 21/22 – Q4(b)** A 50cm long solenoid S_1 with 1000 turns, diameter 0.5cm, experiences an induced emf of 3.0mV and a changing current of $5As^{-1}$. a. Determine the self-inductance of the solenoid S_1. b. A second coil S_2 with 150 turns is wound coaxially around solenoid S_1. Calculate the mutual inductance of the combination of two coils.
5	**PSPM 20/21 – Q4(b)** A circular coil of N turns with current 9.4 mA has an inductance 15mH. Calculate the a. magnetic flux linkage through the coil b. radius of the coil if N = 420turns
6	**PSPM 19/20 – Q4(b)** A 6cm long solenoid with 400 turns and cross sectional area $7 \times 10^{-4} m^2$ experiences a changing current as shown in the figure. Determine the a. induced emf b. magnetic flux through each turn **and** the stored enery at the instant when the current is 3A. 
Book	
7	The current through a 3.2-mH inductor varies with time according to the graph shown in the drawing. What is the average induced emf during the time intervals a. 0–2.0ms, b. 2.0–5.0ms, c. 5.0–9.0ms? 
	$\xi = -L\frac{\Delta I}{\Delta t}$ a. $\xi = -L\frac{\Delta I}{\Delta t} = -\left(3.2\times10^{-3}\text{ H}\right)\left(\frac{4.0\text{ A} - 0\text{ A}}{2.0\times10^{-3}\text{ s} - 0\text{ s}}\right) = \boxed{-6.4\text{ V}}$ b. $\xi = -L\frac{\Delta I}{\Delta t} = -\left(3.2\times10^{-3}\text{ H}\right)\left(\frac{4.0\text{ A} - 4.0\text{ A}}{5.0\times10^{-3}\text{ s} - 2.0\times10^{-3}\text{ s}}\right) = \boxed{0\text{ V}}$ c. $\xi = -L\frac{\Delta I}{\Delta t} = -\left(3.2\times10^{-3}\text{ H}\right)\left(\frac{0\text{ A} - 4.0\text{ A}}{9.0\times10^{-3}\text{ s} - 5.0\times10^{-3}\text{ s}}\right) = \boxed{+3.2\text{ V}}$
8	A solenoid has 120 turns uniformly wrapped around a wooden core, which has a diameter of 10.0 mm and a length of 9.00 cm. a. Calculate the inductance of the solenoid. b. The wooden core is replaced with a soft iron rod that has the same dimensions, but a magnetic permeability $\mu_m = 800\mu_0$.What is the new inductance?

	(a) $L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 (120)^2 \pi (5.00 \times 10^{-3})^2}{0.090\,0} = \boxed{15.8\ \mu\text{H}}$ (b) $\Phi'_B = \frac{\mu_m}{\mu_0}\Phi_B \to L = \frac{\mu_m N^2 A}{\ell} = 800(1.58 \times 10^{-5}\ \text{H}) = \boxed{12.6\ \text{mH}}$
9	A long, current-carrying solenoid with an air core has 1750 turns per meter of length and a radius of 0.0180 m. A coil of 125 turns is wrapped tightly around the outside of the solenoid, so it has virtually the same radius as the solenoid. What is the mutual inductance of this system?
	$M = \frac{N_s \Phi_s}{I_p}$ $\Phi_s = B_{\text{solenoid}} A \cos\phi$ $M = \frac{N_s \Phi_s}{I_p} = \frac{N_s (B_{\text{solenoid}} A \cos\phi)}{I_p} = \frac{N_s (B_{\text{solenoid}} \pi R^2 \cos 0°)}{I_p} = \frac{N_s (B_{\text{solenoid}} \pi R^2)}{I_p}$ $M = \frac{N_s (B_{\text{solenoid}} \pi R^2)}{I_p} = \frac{N_s (\mu_0 n I_p) \pi R^2}{I_p} = N_s \mu_0 n \pi R^2$ $= 125(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1750\ \text{m}^{-1})\pi(0.0180\ \text{m})^2 = \boxed{2.80 \times 10^{-4}\ \text{H}}$
10	A 54μH solenoid is constructed by wrapping 65 turns of wire around a cylinder with a cross-sectional area of $9 \times 10^{-4} m^2$ When the solenoid is shortened by squeezing the turns closer together, the inductance increases to $86\mu H$. Determine the change in the length of the solenoid.
	$L = \mu_0 n^2 \ell A = \mu_0 \left(\frac{N}{\ell}\right)^2 \ell A = \frac{\mu_0 N^2 A}{\ell}$ $\ell = \frac{\mu_0 N^2 A}{L}$ $\Delta\ell = \ell_1 - \ell_2 = \frac{\mu_0 N^2 A}{L_1} - \frac{\mu_0 N^2 A}{L_2} = \mu_0 N^2 A\left(\frac{1}{L_1} - \frac{1}{L_2}\right)$ $= (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(65)^2(9.0 \times 10^{-4}\ \text{m}^2)\left(\frac{1}{5.40 \times 10^{-5}\ \text{H}} - \frac{1}{8.60 \times 10^{-5}\ \text{H}}\right) = \boxed{0.033\ \text{m}}$

Session 17: RLC Circuits

Easy	
1	The voltage generated by a generator is as shown in the graph. a. What is the peak voltage, peak-to-peak voltage and the rms voltage? b. The voltage is connected across a resistor with a resistance 2.5Ω. Calculate the peak, rms current and average power. 
2	A voltage V = 60sin120πt is applied across a 20Ω resistor. Calculate the reading on the ac ammeter and the average power.
3	A circuit has a resistance of 11Ω, a coil of inductive reactance 120Ω and a capacitor of 100Ω, all connected in series with 110V, 60Hz power source. What is the potential difference across each circuit element.
Past Year	
4	**PSPM 21/22 – Q5** A 160Ω resistor, 230mH inductor and $70\mu F$ capacitor are connected in series across 36V, 60Hz AC source. Calculate the impedance, maximum current, phase angle between the current and voltage, the power factor and the power loss. Is the circuit in resonance? Explain your answer.
5	**PSPM 20/21 – Q4(b)** a. A series RLC circuit attached to a power supply of peak voltage 140 V with a power factor of 0.76. Given $I = 4.5\ sin\ 20\pi t$ where I in A and t in s. i. Calculate the rms current in the circuit. ii. Calculate the value of resistance. iii. Determine the impedance. b. An inductor is connected in series to an AC voltage supply of 30V and frequency 60Hz. Inductive reactance of the inductor is 98Ω. Calculate the i. Inductance ii. Peak current
6	**PSPM 19/20 – Q5** The figure shows a phasor diagram of an RL series circuit connected to an AC source with rms voltage across the inductor of 62.8V at 50Hz, 0.8H inductor and an unknown resistor. a. Determine the i) Resistance of the resistor ii) Peak voltage of the AC source iii) Average power b. If the resistor is removed from the circuit, draw the variation of current and voltage against time on the same labelled graph. 
Book	
7	A generator is connected to a resistor and a 0.032-H inductor in series. The rms voltage across the generator is 8.0 V. When the generator frequency is set to 130 Hz, the rms voltage across the inductor is 2.6 V. Determine the resistance of the resistor in this circuit.

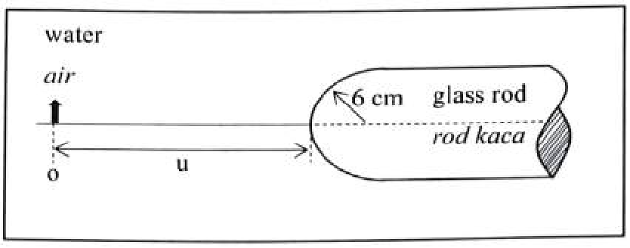
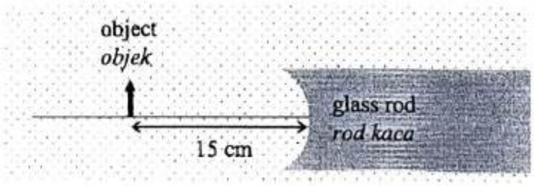
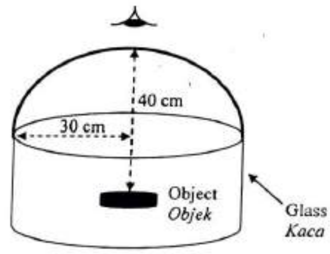
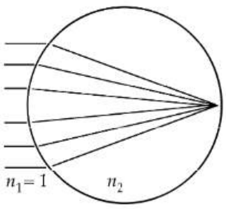
	$Z=\frac{V_{\text{rms}}}{I_{\text{rms}}} \Rightarrow \sqrt{R^2+X_L^2}=\frac{V_{\text{rms}}}{I_{\text{rms}}} \Rightarrow \sqrt{R^2+X_L^2}=\frac{V_{\text{rms}}}{\left(\frac{V_{L,\text{rms}}}{X_L}\right)} \Rightarrow \sqrt{R^2+X_L^2}=\frac{V_{\text{rms}}X_L}{V_{L,\text{rms}}}$ $R^2+X_L^2=\left(\frac{V_{\text{rms}}X_L}{V_{L,\text{rms}}}\right)^2 \Rightarrow R^2=\left(\frac{V_{\text{rms}}X_L}{V_{L,\text{rms}}}\right)^2-X_L^2 \Rightarrow R^2=X_L^2\left(\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2}-1\right)$ $R=X_L\sqrt{\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2}-1} \Rightarrow R=2\pi f L\sqrt{\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2}-1}=2\pi(130\text{ Hz})(0.032\text{ H})\sqrt{\frac{(8.0\text{ V})^2}{(2.6\text{ V})^2}-1}=\boxed{76\ \Omega}$
8	A capacitor, a coil, and two resistors of equal resistance are arranged in an AC circuit, as shown in the figure. An AC source provides an emf of 20.0 V (rms) at a frequency of 60.0 Hz. When the double-throw switch S is open, as shown in the figure, the rms current is 183 mA. When the switch is closed in position 1, the rms current is 298 mA. When the switch is closed in position 2, the rms current is 137 mA. Determine the values of R, C, and L. Is more than one set of values possible? 
	$\omega=2\pi 60/\text{s}=377/\text{s}$ $R^2+(X_L-X_C)^2=\left(\frac{\Delta V_s}{I}\right)^2=\left(\frac{20\text{ V}}{0.183\text{ A}}\right)^2=1.194\times10^4\ \Omega^2.$ When S is in position 1, $\left(\frac{R}{2}\right)^2+(X_L-X_C)^2=\left(\frac{20\text{ V}}{0.298\text{ A}}\right)^2=4.504\times10^3\ \Omega^2.$ When S is in position 2, $R^2+X_C^2=\left(\frac{20\text{ V}}{0.137\text{ A}}\right)^2=2.131\times10^4\ \Omega^2.$ $\frac{3}{4}R^2=7.440\times10^3\ \Omega^2\quad \boxed{R=99.6\ \Omega}$ $X_C=\left[2.131\times10^4-(99.6)^2\right]^{1/2}\ \Omega=106.7\ \Omega=\frac{1}{\omega C}$ $C=(\omega X_C)^{-1}=\left[(377/\text{s})106.7\ \Omega\right]^{-1}=\boxed{2.49\times10^{-5}\text{ F}=C}.$ $X_L-X_C=\pm\left[1.194\times10^4-(99.6)^2\right]^{1/2}\ \Omega=\pm44.99\ \Omega$ $X_L=106.7\ \Omega+44.99\ \Omega=61.74\ \Omega\text{ or }151.7\ \Omega=\omega L$ $L=\frac{X_L}{\omega}=\boxed{0.164\text{ H or }0.402\text{ H}=L}$
9	A resonant circuit using a 260-nF capacitor is to resonate at 18.0 kHz. The air-core inductor is to be a solenoid with closely packed coils made from 12.0 m of insulated wire 1.1 mm in diameter. How many loops will the inductor contain?
	$f_0=\frac{1}{2\pi\sqrt{LC}} \rightarrow L=\frac{1}{4\pi^2 f_0^2 C}=\frac{\mu_0 N^2 A}{\ell_{\text{solenoid}}}=\frac{\mu_0\left(\frac{\ell_{wire}}{2\pi r}\right)^2\pi r^2}{Nd} \rightarrow$ $N=\frac{\pi f_0^2 C\mu_0\ell_{\text{wire}}^2}{d}=\frac{\pi(18.0\times10^3\text{ Hz})^2(2.6\times10^{-7}\text{ F})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(12.0\text{ m})^2}{1.1\times10^{-3}\text{ m}}$ $=43.45\approx\boxed{44\text{ loops}}$
10	In an RC circuit, $R=6.6k\Omega, C=1.8\mu F$, and the rms applied voltage is 120 V at 60.0 Hz? a. What is the rms current in the RC circuit? b. What is the phase angle between voltage and current? c. What are the voltmeter readings across R and C?
	(a) $Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\frac{1}{(2\pi fC)^2}}$ $I_{\text{rms}}=\frac{V_{\text{rms}}}{Z}=\frac{V_{\text{rms}}}{\sqrt{R^2+\frac{1}{4\pi^2 f^2 L^2}}}=\frac{120\text{ V}}{\sqrt{(6.60\times10^3\ \Omega)^2+\frac{1}{4\pi^2(60.0\text{ Hz})^2(1.80\times10^{-6}\text{ F})^2}}}$ $=\frac{120\text{ V}}{6763\ \Omega}=1.774\times10^{-2}\text{ A}\approx\boxed{1.77\times10^{-2}\text{ A}}$ (b) $\phi=\tan^{-1}\frac{-X_C}{R}=\tan^{-1}\frac{-\frac{1}{2\pi fC}}{R}=\tan^{-1}\frac{-\frac{1}{2\pi(60.0\text{ Hz})(1.80\times10^{-6}\text{ F})}}{6.60\times10^3\ \Omega}=\boxed{-12.6^\circ}$ The current is leading the source voltage. (c) $V_{\text{rms}_R}=I_{\text{rms}}R=(1.774\times10^{-2}\text{ A})(6.60\times10^3\ \Omega)=\boxed{117\text{ V}}$ $V_{\text{rms}_C}=I_{\text{rms}}X_C=I_{\text{rms}}\frac{1}{2\pi fC}=(1.774\times10^{-2}\text{ A})\frac{1}{2\pi(60.0\text{ Hz})(1.80\times10^{-6}\text{ F})}=\boxed{26.1\text{ V}}$

Session 18: Optics – Reflection

Easy	
1	An object 200cm from the vertex of a spherical concave mirror is imaged 400cm in front of the mirror, what is the radius length of the mirror?
2	An object 10cm high is 50cm from a concave mirror of 20cm focal length. Find the image distance, height and direction.
3	How far should an object be from a concave spherical mirror of radius 45cm to form a real image one-ninth of its size?
Past Year	
4	**PSPM 21/22 – Q6 (a)** An object is placed 5cm from a curved mirror. An image which is twice the size of the object is formed behind the mirror. a. Is the mirror convex or concave? Explain your answer. b. Determine the radius of curvature of the mirror.
5	**PSPM 18/19 – Q6(a)** An external side mirror of a car is convex with a radius of curvature 18m. Determine the location of the image for an object 10 m from the mirror
6	**PSPM 17/18 – Q6(b)** An object is placed in front of a concave mirror with 25cm radius of curvature. A real image twice the size of the object is formed. a. Sketch a ray diagram to illustrate the formation of the image. b. Determine the object distance from the mirror.
Book	
7	A concave mirror has a focal length of 30cm. The distance between an object and its image is 45cm. Find the object and image distances, assuming that a. the object lies beyond the centre of curvature b. the object lies between the focal point and the mirror.
	$d_o - d_i = 45.0$ cm. $\frac{1}{d_o} + \frac{1}{d_o - 45.0\text{ cm}} = \frac{1}{30.0\text{ cm}}$ $d_o^2 - 105\, d_o + 1350 = 0$ $d_o = (105 \pm 75)/2.$ a. When the object lies beyond the center of curvature we have $d_{o+} = (1.80 \times 10^2\text{ cm})/2 = \boxed{+9.0\times10^1\text{ cm}}$ and $d_{i+} = \boxed{+45\text{ cm}}$ b. When the object lies within the focal point $d_{o-} = (3.0 \times 10^1\text{ cm})/2 = \boxed{+15\text{ cm}}$, and $d_{i-} = \boxed{-3.0\times10^1\text{ cm}}$
8	An object 10cm tall is placed at the zero mark of a meter stick. A spherical mirror located at some point on the meter stick creates an image of the object that is upright, 4cm tall, and located at the 42cm mark of the meter stick. a. Is the mirror convex or concave? b. Where is the mirror? c. What is the mirror's focal length?
	$M = \frac{h'}{h} = \frac{+4.00\text{ cm}}{10.0\text{ cm}} = +0.400 = -\frac{q}{p}$ $q = -0.400p$ the image must be virtual. (a) It is a $\boxed{\text{convex}}$ mirror that produces a diminished upright virtual image. (b) $p + \lvert q \rvert = 42.0\text{ cm} = p - q$ $p = 42.0\text{ cm} + q = 42.0\text{ cm} - 0.400p$ $p = \frac{42.0\text{ cm}}{1.40} = 30.0\text{ cm}$ (c) The mirror is $\boxed{\text{at the 30.0 cm mark}}$. $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{30\text{ cm}} + \frac{1}{-0.4(30\text{ cm})} = \frac{1}{f} = -0.050\,0/\text{cm}$ $\boxed{f = -20.0\text{ cm}}$

9	A shaving or makeup mirror is designed to magnify your face by a factor of 1.40 when your face is placed 20cm in front of it. a. What type of mirror is it? b. Describe the type of image that it makes of your face. c. Calculate the required radius of curvature for the mirror.
	(*a*) Producing a larger upright image requires a $\boxed{\text{concave mirror}}$. (*b*) The image will be $\boxed{\text{upright, virtual, and magnified}}$. (*c*) $m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \quad \rightarrow \quad d_i = -md_o$ $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1}$ $r = 2f = \frac{2md_o}{m-1} = \frac{2(1.40)(20.0\text{ cm})}{1.40-1} = 140\text{ cm} = \boxed{1.40\text{ m}}$

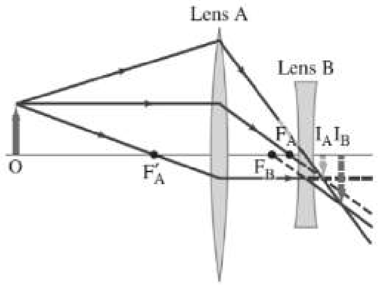
Session 19: Optics – Refraction

Easy	
1	The left end of a long glass rod 6.00 cm in diameter has a convex hemispherical surface 3.00 cm in radius. The refractive index of the glass is 1.60. Determine the position of the image if an object is placed in air on the axis of the rod at the following distances to the left of the vertex of the curved end: (a) infinitely far; (b) 12.0 cm; (c) 2.00 cm.
2	The glass rod of question 1 is immersed in oil (n = 1.452). An object placed to the left of the rod on the rod's axis is to be imaged 1.20 m inside the rod. How far from the left end of the rod must the object be located to form the image?
Past Year	
4	**PSPM 21/22 – Q6(b)** The figure shows a long rod with a convex surface of radius of curvature 6.0 cm at one end and is made from glass with refractive index of1.60. The glass rod is placed in water with refractive index, n = 1.33. An object placed along the rod's axis is to be imaged 53 cm inside the rod. Calculate the object position. 
5	**PSPM 19/20 – Q6(c)** The figure shows an object and a glass rod immersed in a liquid. The rod has a refractive index of 1.7 and radius of curvature 8cm. If the object distance is 15cm and the virtual image distance is 13cm, determine the refractive index of liquid. 
6	**PSPM 17/18 – Q6(b)** The figure shows an object embedded in a solid glass with a hemispherical end of radius 30cm and refractive index 1.50. The object is 40cm inside the glass. Calculate the image distance. Refractive index of air is 1. 
Book	
7	A transparent sphere of unknown composition is observed to form an image of the Sun on the surface of the sphere opposite the Sun. What is the refractive index of the sphere material?
	$p = \infty$ and $q = +2R$ $\frac{1.00}{p} + \frac{n_2}{q} = \frac{n_2 - 1.00}{R}$ $0 + \frac{n_2}{2R} = \frac{n_2 - 1.00}{R}$ $\boxed{n_2 = 2.00}$ 
8	One end of a long glass rod (n = 1.50) is formed into a convex surface with a radius of curvature of 6.00 cm. An object is located in air along the axis of the rod. Find the image positions corresponding to object distances of (a) 20.0 cm, (b) 10.0 cm, and (c) 3.00 cm from the end of the rod.

	$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ $\frac{1.00}{p}+\frac{1.50}{q}=\frac{1.50-1.00}{6.00\text{ cm}}=\frac{1}{12.0\text{ cm}}$ (a) $\frac{1.00}{20.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}} \Rightarrow q=\boxed{45.0\text{ cm}}$ (b) $\frac{1.00}{10.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}} \Rightarrow q=\boxed{-90.0\text{ cm}}$ (c) $\frac{1.00}{3.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}} \Rightarrow q=\boxed{-6.00\text{ cm}}$
9	A simple model of the human eye ignores its lens entirely. Most of what the eye does to light happens at the outer surface of the transparent cornea. Assume that this surface has a radius of curvature of 6.00 mm, and assume that the eyeball contains just one fluid with a refractive index of 1.40. Prove that a very distant object will be imaged on the retina, 21.0 mm behind the cornea. Describe the image.
	$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ $\frac{1.00}{\infty}+\frac{1.40}{21.0\text{ mm}}=\frac{1.40-1.00}{6.00\text{ mm}}$ $0.0667=0.0667$. They agree. $\boxed{\text{The image is inverted, real and diminished.}}$

Session 20: Optics – Thin Lenses

Easy	
1	An insect 3.75 mm tall is placed 25.0 cm to the left of a thin planoconvex lens. The left surface of this lens is flat, the right surface has a radius of curvature of magnitude 12.9 cm, and the index of refraction of the lens material is 1.70. a. Calculate the location and size of the image this lens forms of the insect. Is it real or virtual? Erect or inverted? b. Repeat part (a) if the lens is reversed.
2	A lens forms an image of an object. The object is 16.0 cm from the lens. The image is 12.0 cm from the lens on the same side as the object. a. What is the focal length of the lens? Is the lens converging or diverging? b. If the object is 8.50 mm tall, how tall is the image? Is it erect or inverted?
3	A converging lens with a focal length of 70.0 cm forms an image of a 3.20-cm-tall real object that is to the left of the lens. The image is 4.50 cm tall and inverted. Where are the object and image located in relation to the lens? Is the image real or virtual?
Past Year	
4	**PSPM 21/22 – Q6(c)** A converging meniscus lens is made from a glass of refractive index 1.52 having a radius 7cm and 4cm. An object is placed 24cm in front of the lens. a. Calculate the position of the image from the lens. If the image magnified or diminished in size? Justify your answer.
5	**PSPM 20/21 – Q6** A. An orange is placed 25.2cm in front of a diverging lens with a focal length of 18cm. a. Sketch the ray diagram to show the formation of the image b. Determine the image distance. c. Determine the magnification. d. Determine **two (2)** characteristics of the image B. The convex meniscus lens has a 17cm radius for the convex surface and 25cm for the concave surface. The lens is made of glass with a refractive index, $n = 1.52$ in air. Refractive index of air is 1.0. Determine the focal length of the lens.
6	**PSPM 19/20 – Q6(b)** The figure shows a lens with radii of curvature of 15cm and 50cm, made of glass with refractive index 1.55. Determine the focal length **and** type of lens. 
Book	
7	An object is placed 96.5 cm from a glass lens (n=1.52) with one concave surface of radius 22.0 cm and one convex surface of radius 18.5 cm. Where is the final image? What is the magnification?
	$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow f=\frac{1}{(n-1)}\left(\frac{R_1R_2}{R_1+R_2}\right)=\frac{1}{(1.52-1)}\left(\frac{(-22.0\text{ cm})(+18.5\text{ cm})}{(-22.0\text{ cm})+(+18.5\text{ cm})}\right)=223.6\text{ cm}$ $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_of}{d_o-f}=\frac{(96.5\text{ cm})(223.6\text{ m})}{96.5\text{ cm}-223.6\text{ cm}}=-169.77\text{ cm}\approx\boxed{-170\text{ cm}}$ $m=-\frac{d_i}{d_o}=-\frac{-169.77\text{ cm}}{96.5\text{ cm}}=1.759\approx\boxed{+1.8}$
8	A symmetric double convex lens with a focal length of 22.0 cm is to be made from glass with an index of refraction of 1.52. What should be the radius of curvature for each surface?
	$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=(n-1)\left(\frac{2}{R}\right)\rightarrow R=2f(n-1)=2(22.0\text{ cm})(1.52-1)=22.88\text{ cm}\approx\boxed{23\text{ cm}}$
9	Two lenses, one converging with focal length 20.0 cm and one diverging with focal length $-10cm$ are placed 25.0 cm apart. An object is placed 60.0 cm in front of the converging lens. Determine

	a. the position b. magnification of the final image formed. **c.** Sketch a ray diagram for this system.
	The first lens is the converging lens. Find the image formed by the first lens. $\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(60.0\text{ cm})(20.0\text{ cm})}{(60.0\text{ cm})-(20.0\text{ cm})}=30.0\text{ cm}$ This image is the object for the second lens. $d_{o2}=25.0\text{ cm}-30.0\text{ cm}=-5.0\text{ cm}.$ $\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-5.0\text{ cm})(-10.0\text{ cm})}{(-5.0\text{ cm})-(-10.0\text{ cm})}=10\text{ cm}$ Thus the final image is real and is $\boxed{\text{10 cm beyond the second lens}}$. $m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}=\frac{(30.0\text{ cm})(10.0\text{ cm})}{(60.0\text{ cm})(-5.0\text{ cm})}=\boxed{-1.0\times}$ 

Session 21: Huygens' & Interferences

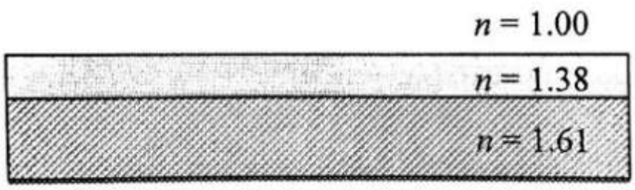
Easy	
1	Describe the condition for **constructive** interference.
2	Describe the condition for **destructive** interference.
Past Year	
3	**PSPM 18/19 – Q7(b)** The figure shows two paths of coherent lights from points A and B that produce an interference pattern at point C. Determine whether it is a constructive or destructive interference if AC and BC are 2.2λ and 5.7λ respectively. 
	**No other questions on Huygen's or Interference since 2013.*
Book	
4	Two radio antennas A and B radiate in phase. Antenna B is 110 m to the right of antenna A. Consider point Q along the extension of the line connecting the antennas, a horizontal distance of 30 m to the right of antenna B. The frequency, and hence the wavelength, of the emitted waves can be varied. a. What is the longest wavelength for which there will be destructive interference at point Q? b. What is the longest wavelength for which there will be constructive interference at point Q?
	The path difference is 120 m. **(a)** For destructive interference $\frac{\lambda}{2} = 120\text{ m} \Rightarrow \lambda = 240\text{ m}$. **(b)** The longest wavelength for constructive interference is $\lambda = 120\text{ m}$.
5	A radio transmitting station operating at a frequency of 120 MHz has two identical antennas that radiate in phase. Antenna B is 9.00 m to the right of antenna A. Consider point P between the antennas and along the line connecting them, a horizontal distance x to the right of antenna A. For what values of x will constructive interference occur at point P?
	The path difference is $r_B - r_A = 9.00\text{ m} - 2x$. $r_B - r_A = m\lambda,\ m = 0,\ \pm 1, \pm 2, \ldots$ $\lambda = \frac{c}{f} = \frac{2.998\times 10^8\text{ m/s}}{120\times 10^6\text{ Hz}} = 2.50\text{ m}$. Thus $9.00\text{ m} - 2x = m(2.50\text{ m})$ $m = 0$ gives $x = 4.50$ m. $m = +1$ gives $x = 4.50\text{ m} - 1.25\text{ m} = 3.25$ m. $m = +2$ gives $x = 4.50\text{ m} - 2.50\text{ m} = 2.00$ m. $m = +3$ gives $x = 4.50\text{ m} - 3.75\text{ m} = 0.75$ m. $m = -1$ gives $x = 4.50\text{ m} + 1.25\text{ m} = 5.75$ m. $m = -2$ gives $x = 4.50\text{ m} + 2.50\text{ m} = 7.00$ m. $m = -3$ gives $x = 4.50\text{ m} + 3.75\text{ m} = 8.25$ m.

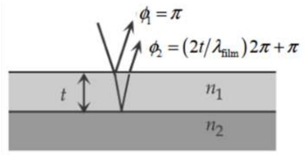
Session 22: Double Slit

Easy	
1	A laser beam (λ =632.8 nm) is incident on two slits 0.200 mm apart. How far apart are the bright interference fringes on a screen 5.00 m away from the double slits?
2	Monochromatic light falling on two slits 0.018mm apart produces the fifth-order bright fringe at an 8.6° angle. What is the wavelength of the light used?
3	In a Young's double-slit experiment, the angle that locates the second dark fringe on either side of the central bright fringe is 5.4^o. Find the ratio $\frac{d}{\lambda}$ of the slit separation d to the wavelength λ of the light.
Past Year	
4	**PSPM 21/22 – Q7(a)** A 475 nm light passes through two narrow slits. The interference pattern is observed on a screen at a distance 85.0 cm from the slits. The second-order bright fringe is seen at $\pm$2.01 cm from the central bright fringe. Calculate the slit separation and the width of the second-order dark fringe.
5	**PSPM 20/21 – Q7(a)** Two narrow slits separated by 2.4mm are illuminated by a light with λ= 512 nm. The screen is placed 6.5 m from the slits. Determine the a. distance between adjacent bright fringes on a screen distance of the fifth dark fringe from the central bright fringe.
6	**PSPM 19/20 – Q7(a)** In a double slit experiment, the incident wavelength is 660 nm, the slit separation is 0.25 mm, and the screen is placed 90 cm away from the sits. Calculate the distance from the second to the third destructive interference fringe.
Book	
7	Coherent light of frequency 6.32$\times 10^{14} Hz$ Hz passes through two thin slits and falls on a screen 85.0 cm away. You observe that the third bright fringe occurs at $\pm$3.11 cm on either side of the central bright fringe. a. How far apart are the two slits? b. At what distance from the central bright fringe will the third dark fringe occur?
	Bright fringes are located at $y_m = R\frac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d\sin\theta = (m+\frac{1}{2})\lambda$ and $y = R\tan\theta$. $\lambda = \frac{c}{f} = \frac{3.00\times10^8 \text{ m/s}}{6.32\times10^{14} \text{ Hz}} = 4.75\times10^{-7} \text{ m}.$ **(a)** $d = \frac{m\lambda R}{y_m} = \frac{3(4.75\times10^{-7} \text{ m})(0.850 \text{ m})}{0.0311 \text{ m}} = 3.89\times10^{-5} \text{ m} = 0.0389 \text{ mm}.$ **(b)** $\sin\theta = (2+\frac{1}{2})\frac{\lambda}{d} = (2.5)\left(\frac{4.75\times10^{-7} \text{ m}}{3.89\times10^{-5} \text{ m}}\right) = 0.0305$ and $\theta = 1.75°$. $y = R\tan\theta = (85.0 \text{ cm})\tan 1.75° = 2.60 \text{ cm}.$
8	Light of wavelength 470 nm in air shines on two slits $60\mu m$ apart. The slits are immersed in water, as is a viewing screen 40.0 cm away. How far apart are the fringes on the screen?
	$\Delta x = x_2 - x_1 = \frac{\lambda_n(m+1)\ell}{d} - \frac{\lambda_n m\ell}{d} = \frac{\lambda_n \ell}{d} = \frac{\lambda\ell}{nd} = \frac{(470\times10^{-9} \text{ m})(0.400 \text{ m})}{(1.33)(6.00\times10^{-5} \text{ m})}$ $= 2.356\times10^{-3} \text{ m} = \boxed{2.4\times10^{-3} \text{ m}}$
9	In Young's experiment a mixture of orange light (611 nm) and blue light (471 nm) shines on the double slit. The centres of the first-order bright blue fringes lie at the outer edges of a screen that is located 0.500 m away from the slits. However, the first-order bright orange fringes fall off the screen. By how much and in which direction (toward or away from the slits) should the screen be moved so that the centres of the first-order bright orange fringes will just appear on the screen?
	$\frac{\lambda}{d} \approx \frac{y}{L}$ or $L = \frac{yd}{\lambda}$

	$$\frac{L_{\text{blue}}}{L_{\text{orange}}} = \frac{yd/\lambda_{\text{blue}}}{yd/\lambda_{\text{orange}}} = \frac{\lambda_{\text{orange}}}{\lambda_{\text{blue}}}$$ $$L_{\text{blue}} - L_{\text{orange}} = L_{\text{blue}} - \left(\frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}}\right)L_{\text{blue}} = L_{\text{blue}}\left(1 - \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}}\right)$$ $$= (0.500\ \text{m})\left[1 - \frac{471\ \text{nm}}{611\ \text{nm}}\right] = \boxed{0.115\ \text{m}}$$

Session 23: Thin Films

Easy	
1	A soap bubble (n = 1.33) is floating in air. If the thickness of the bubble wall is 115 nm, what is the wavelength of the light that is most strongly reflected?
2	An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find a. the color of the light in the visible spectrum most strongly reflected and b. the color of the light in the spectrum most strongly transmitted. Explain your reasoning.
3	A thin film of oil (n = 1.25) is located on a smooth wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no blue light at 512 nm. How thick is the oil film?
Past Year	
4	**PSPM 21/22 – Q7(b)** A flat glass with index of refraction 1.50 is coated with a transparent material of refraction index 1.25, in order to eliminate reflection of light of wavelength 680 nm. Determine the minimum thickness of the coating.
5	**PSPM 19/20 – Q7(b)** A soap film with refractive index 1.3 and minimum thickness 0.177 μm appears reddish under white light. Calculate the wavelength of light that is missing from the reflection.
6	**PSPM 18/19 – Q7(c)** Calculate the thickness of a soap film so that a 600 nm light incident to the film would produce constructive interference. Index of refraction of soap film 1.33.
7	**PSPM 17/18 – Q6(c)** The figure shows a flint glass lens of refractive index 1.61 is coated with a thin layer film of magnesium fluoride of refractive index 1.38.  A ray of light of wavelength 565 nm is incident at right angles to the film. a. Sketch the light rays that interfere after being reflected from both surfaces of the film. Label the reflected rays that undergo phase change. b. What minimum thickness should the magnesium fluoride film have if the reflection of the 565 nm light is to appear dark? c. If a lens is used to suppress the reflection of light at high frequencies, what should be done to the thickness of the film? Explain your answer
Book	
8	A uniform thin film of alcohol (n=1.36) lies on a flat glass plate (n=1.56). When monochromatic light, whose wavelength can be changed, is incident normally, the reflected light is a minimum for $\lambda = 525nm$ and a maximum for $\lambda = 655nm$. What is the minimum thickness of the film?

	$\phi_1 = \pi.$ $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$  For Constructive Interference, $\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\dfrac{2t}{\lambda_{1\text{film}}}\right)2\pi + \pi\right] - \pi = m_1 2\pi \quad \rightarrow \quad t = \frac{1}{2}\lambda_{1\text{film}} m_1 = \frac{1}{2}\dfrac{\lambda_1}{n_{\text{film}}} m_1, \quad m_1 = 1, 2, 3, \cdots.$ For Destructive Interference, $\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\dfrac{2t}{\lambda_{2\text{film}}}\right)2\pi + \pi\right] - \pi = (2m_2 + 1)\pi \quad \rightarrow \quad t = \frac{1}{4}\dfrac{\lambda_2}{n_{\text{film}}}(2m_2 + 1), \quad m_2 = 0, 1, 2, \cdots.$ $\frac{1}{2}\dfrac{\lambda_1}{n_{\text{film}}} m_1 = \frac{1}{4}\dfrac{\lambda_2}{n_{\text{film}}}(2m_2 + 1) \quad \rightarrow \quad \dfrac{2m_2 + 1}{2m_1} = \dfrac{\lambda_1}{\lambda_2} = \dfrac{(655\text{ nm})}{(525\text{ nm})} = 1.2476 \approx 1.25 = \dfrac{5}{4}$ $m_1 = m_2 = 2$ $t = \frac{1}{2}\dfrac{\lambda_1}{n_{\text{film}}} m_1 = \frac{1}{2}\left(\dfrac{655\text{ nm}}{1.36}\right)(2) = 481.6\text{ nm} \text{ or } t = \frac{1}{4}\dfrac{\lambda_2}{n_{\text{film}}}(2m_2 + 1) = \frac{1}{4}\left(\dfrac{525\text{ nm}}{1.36}\right)(5) = 482.5\text{ nm}$
9	A uniform layer of water (n=1.33) lies on a glass plate (n=1.52). Light shines perpendicularly on the layer. Because of constructive interference, the layer looks maximally bright when the wavelength of the light is 432 nm in vacuum and also when it is 648 nm in vacuum. a. Obtain the minimum thickness of the film. b. Assuming that the film has the minimum thickness and that the visible spectrum extends from 380 to 750 nm, determine the visible wavelength(s) in vacuum for which the film appears completely dark.
	The condition for constructive interference $2t = m\lambda_{\text{water}} = m\dfrac{\lambda_{\text{vacuum}}}{n_{\text{water}}} \qquad m = 1, 2, 3, \ldots$ The condition for destructive interference $2t = \left(m + \frac{1}{2}\right)\lambda_{\text{water}}$ $2t = \left(m + \frac{1}{2}\right)\lambda_{\text{water}} = \left(m + \frac{1}{2}\right)\dfrac{\lambda_{\text{vacuum}}}{n_{\text{water}}} \qquad m = 0, 1, 2, 3, \ldots$ For $\lambda_{\text{vacuum}} = 432\text{ nm} \quad t = \dfrac{m}{2}\left(\dfrac{432\text{ nm}}{1.33}\right)$ For $\lambda_{\text{vacuum}} = 648\text{ nm} \quad t = \dfrac{m'}{2}\left(\dfrac{648\text{ nm}}{1.33}\right)$ $m/m' = 1.50.$ this means that $m = 3$ and $m' = 2$. $t = \dfrac{m}{2}\left(\dfrac{432\text{ mn}}{1.33}\right) = \dfrac{3}{2}\left(\dfrac{432\text{ nm}}{1.33}\right) = \boxed{487\text{ nm}}$ $\lambda_{\text{vacuum}} = \dfrac{2n_{\text{water}}t}{m + \frac{1}{2}} = \dfrac{2(1.33)(487\text{ nm})}{m + \frac{1}{2}}$ $\lambda_{\text{vacuum}} = \boxed{518\text{ nm}}$
10	How thick (minimum) should the air layer be between two flat glass surfaces if the glass is to appear bright when 450-nm light is incident normally? What if the glass is to appear dark?

For constructive interference,

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = 2m\pi \quad \rightarrow \quad t = \tfrac{1}{2}\left(m - \tfrac{1}{2}\right)\lambda, \quad m = 1, 2, \cdots.$$

$$t_{\text{min}} = \tfrac{1}{2}(450\text{ nm})\left(1 - \tfrac{1}{2}\right) = 113\text{ nm} \approx \boxed{110\text{ nm}}$$

The minimum thickness is with $m = 1$.

$$t_{\text{min}} = \tfrac{1}{2}(450\text{ nm})\left(1 - \tfrac{1}{2}\right) = 113\text{ nm} \approx \boxed{110\text{ nm}}$$

For destructive interference,

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda, \quad m = 0, 1, 2, \cdots$$

$$t_{\text{min}} = \tfrac{1}{2}(450\text{ nm})(1) = 225\text{ nm} \approx \boxed{230\text{ nm}}$$

Session 24: Single Slit & Diffraction Grating

Easy	
1	Helium–neon laser light (λ =632.8 nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?
2	If 680-nm light falls on a slit 0.0425 mm wide, what is the angular width of the central diffraction peak?
3	At what angle will 510-nm light produce a second-order maximum when falling on a grating whose slits are 1.35$\times$ $10^{-3} cm$ apart?
Past Year	
4	**PSPM 21/22 – Q7(c)** A monochromatic light 600 nm is incident on a diffraction grating with 400 lines per mm. Calculate the a. angle for the first bright order of diffraction. b. maximum number of diffraction pattern that can be formed.
5	**PSPM 20/21 – Q7(b)** A monochromatic light of wavelength 620 nm is incident on a single slit and forms a diffraction pattern on a screen 1.2 m away. The distance of seventh dark fringe from the central maximum is 18.0 mm. Determine the a. Size of the single slit b. Distance of the second bright fringe from the central maximum
6	**PSPM 16/17 – Q6(c)** A beam consists of two monochromatic lights 400 nm and 600 nm, is incident normally on a diffraction grating which has 540 lines mm^{-1}. Calculate the a. Angular separation between the first order diffraction of lights b. Highest order of diffraction that be observed with the 600nm light.
7	**PSPM 15/16 – Q6(c)** White light is incident on a soap film of refractive index 1.33 in air. The reflected light looks bluish because the red light of wavelength 670 nm is absent in the reflection. a. Does the light change phase when it reflects at air-film interface? Explain your answer. b. Does the light change phase when it travels in film and reflects at film-air interface? c. What happen to the wavelength and frequency of light when it travels from air to the film? d. Determine the minimum thickness of the soap film.
Book	
8	Light of wavelength 587.5 nm illuminates a single slit 0.750 mm in width. a. At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 0.850 mm from the centre of the principal maximum? b. What is the width of the central maximum?
	(a) $\sin\theta = \frac{y}{L} = \frac{m\lambda}{a}$ Therefore, for first minimum, $m = 1$ and $L = \frac{ay}{m\lambda} = \frac{(7.50\times10^{-4}\ \text{m})(8.50\times10^{-4}\ \text{m})}{(1)(587.5\times10^{-9}\ \text{m})} = \boxed{1.09\ \text{m}}$. (b) $w = 2y_1$ yields $y_1 = 0.850$ mm $w = 2(0.850\times10^{-3}\ \text{m}) = \boxed{1.70\ \text{mm}}$
9	A source emits 531.62-nm and 531.81-nm light. a. What minimum number of grooves is required for a grating that resolves the two wavelengths in the first-order spectrum? b. Determine the slit spacing for a grating 1.32cm wide that has the required minimum number of grooves.

	(a) $Nm = \frac{\lambda}{\Delta\lambda}$ $N(1) = \frac{531.7 \text{ nm}}{0.19 \text{ nm}} = \boxed{2\,800}$ (b) $\frac{1.32\times10^{-2} \text{ m}}{2\,800} = \boxed{4.72\ \mu\text{m}}$
10	Light that has a wavelength of 668 nm passes through a slit 6.73µm wide and falls on a screen that is 1.85m away. What is the distance on the screen from the centre of the central bright fringe to the third dark fringe on either side?
	$\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left[\frac{3(668\times10^{-9} \text{ m})}{6.73\times10^{-6} \text{ m}}\right] = 17.3°$ $y = L\tan\theta = (1.85 \text{ m})\tan 17.3° = \boxed{0.576 \text{ m}}$

Session 25: De Broglie

Easy	
1	Calculate the wavelength of a 2kg ball travelling at $0.1ms^{-1}$.
2	What is the wavelength of an electron of energy (a) 10 eV, (b) 100 eV, (c) 1.0 keV?
3	What voltage is needed to produce electron wavelengths of 0.26 nm?
Past Year	
4	**PSPM 21/22 – Q9** De Broglie wavelength of a proton is $1.00 \times 10^{-13}m$. a. Calculate the speed and kinetic energy of the proton. b. Determine the applied electric potential for the proton to accelerate and reach this speed.
5	**PSPM 20/21 – Q9** A particle is moving three times faster than proton. The ratio of the de Broglie's wavelength of the particle to the proton is 1.716 x 10^4. Calculate the mass of particle.
6	**PSPM 19/20 – Q9** A beam of electrons is accelerated through a potential difference of 4500V in a Davisson and Germer experiment. a. Calculate the de Broglie wavelength of the electrons. b. Will the diffraction pattern become larger, remain unchanged or narrower when proton is used instead of electrons? Justify your answer.
7	**PSPM 18/19 – Q9** a. Calculate the speed of a neutron with de Broglie wavelength 9 x 10^{-11}m. b. Calculate the wavelength of an electron that has been accelerated across a potential difference of 100 V.
Book	
8	An electron, starting from rest, accelerates through a potential difference of 418 V. What is the final de Broglie wavelength of the electron, assuming that its final speed is much less than the speed of light?
	$\underbrace{\frac{1}{2}mv^2}_{\text{Final total energy}} = \underbrace{eV}_{\text{Initial total energy}}$ $v=\sqrt{2eV/m}$ $\lambda = h/\sqrt{2meV}$ $\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})(418\ \text{V})}} = \boxed{6.01\times10^{-11}\ \text{m}}$
9	The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5.0% of the speed of light. Find the ratio of the photon wavelength to the de Broglie wavelength of the particle. *Note: Refer to form 5 physics syllabus.*
	$E = hf = hc/\lambda_{\text{photon}}$ $\text{KE} = (1/2)mv^2 = h^2/(2m\lambda^2)$. $\lambda_{\text{photon}}/\lambda = (2mc/h)\lambda$. $v = 0.050c$, $\lambda = h/(0.050\ mc)$ $\lambda_{\text{photon}}/\lambda = 2/0.050 = \boxed{4.0\times10^1}$
10	In an electron diffraction experiment using an accelerating voltage of 54V, an intensity maximum for $\theta = 50^o$. X-ray diffraction indicates that the atomic spacing in the target is $d = 0.218nm$. The electrons have negligible kinetic energy before being accelerated. Find the electron wavelength.

	$\lambda = \dfrac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{\sqrt{2(9.109 \times 10^{-31}\ \mathrm{kg})(1.602 \times 10^{-19}\ \mathrm{C})(54\ \mathrm{V})}}$ $= 1.7 \times 10^{-10}\ \mathrm{m} = 0.17\ \mathrm{nm}$

Session 26: Binding Energy & Mass Defect

Easy	
1	Find the binding energy (in MeV) for lithium ${}^{7}_{3}Li$ (atomic mass = 7.016 003 u).
2	The binding energy of a nucleus is 225.0 MeV. What is the mass defect of the nucleus in atomic mass units?
3	Determine the mass defect (in atomic mass units) for a. Helium ${}^{3}_{2}He$, which has an atomic mass of 3.016030 u, b. the isotope of hydrogen known as tritium ${}^{3}_{1}T$, which has an atomic mass of 3.016050 u.
Past Year	
4	**PSPM 21/22 – Q10(a)** Calculate the binding energy of a bromine nucleus (${}^{81}_{35}Br$) in Joule. Atomic mass of bromine = 80.916291 u.
5	**PSPM 20/21 – Q10(a)** Calculate the binding energy per nucleon for Thallium, ${}^{205}_{81}Ti$ in MeV per nucleon. Given atomic mass Tl = 204.974401 u.
6	**PSPM 19/20 – Q10(a)** Calculate the binding energy per nucleon of a sodium nucleus (${}^{23}_{11}Na$) in MeV nucleon. The atomic mass of sodium is 22.989769 u.
Book	
7	Show that the nucleus ${}^{8}_{4}Be$ (mass = 8.005305 u) is unstable and will decay into two α particles. Is ${}^{12}_{6}C$ stable against decay into three particles? Show why or why not.
	Binding energy $= \left[2m\left({}^{4}_{2}\mathrm{He}\right) - m\left({}^{8}_{4}\mathrm{Be}\right)\right]c^2$ $= [2(4.002603\ \mathrm{u}) - (8.005305\ \mathrm{u})]c^2\left(\dfrac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}\right)$ $= -0.092\ \mathrm{MeV}$ Because the binding energy is negative, the nucleus is unstable. It will be in a lower energy state as two alphas instead of a beryllium. Binding energy $= \left[3m\left({}^{4}_{2}\mathrm{He}\right) - m\left({}^{12}_{6}\mathrm{C}\right)\right]c^2$ $= [3(4.002603\ \mathrm{u}) - (12.000000\ \mathrm{u})]c^2\left(\dfrac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}\right) = +7.3\ \mathrm{MeV}$ Because the binding energy is positive, the nucleus is $\boxed{\text{stable}}$.
8	How much energy is required to remove a. a proton, b. a neutron, from ${}^{15}_{7}N$ (of mass =15.000109u). Explain the difference in your answers. The mass of ${}^{14}_{6}C$ and ${}^{14}_{7}N$ are 14.003242u and 14.003074u respectively.

	(*a*) Removal of a proton creates an isotope of carbon. To balance electrons, the proton is included as a hydrogen atom: ${}^{15}_{7}\text{N} \rightarrow {}^{1}_{1}\text{H} + {}^{14}_{6}\text{C}$. $$\text{Energy needed} = \left[m\left({}^{14}_{6}\text{C}\right) + m\left({}^{1}_{1}\text{H}\right) - m\left({}^{15}_{7}\text{N}\right)\right]c^2$$ $$= [(14.003242\text{ u}) + (1.007825\text{ u}) - (15.000109\text{ u})]\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$ $$= \boxed{10.21\text{ MeV}}$$ (*b*) Removal of a neutron creates another isotope of nitrogen: ${}^{15}_{7}\text{N} \rightarrow {}^{1}_{0}\text{n} + {}^{14}_{7}\text{N}$. $$\text{Energy needed} = \left[m\left({}^{14}_{7}\text{N}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{15}_{7}\text{N}\right)\right]c^2$$ $$= [(14.003074\text{ u}) + (1.008665\text{ u}) - (15.000109\text{ u})]\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$ $$= \boxed{10.83\text{ MeV}}$$ The nucleons are held by the attractive strong nuclear force. It takes less energy to remove the proton because there is also the $\boxed{\text{repulsive electric force}}$ from the other protons.

Session 27: Radioactivity

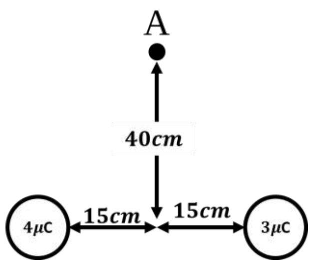
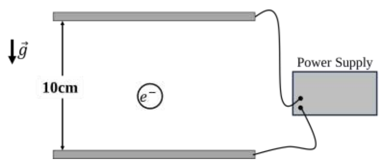
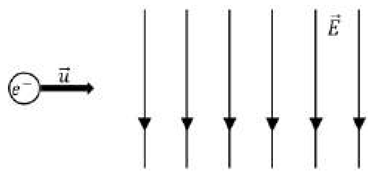
Easy	
1	In 9.0 days the number of radioactive nuclei decreases to one-eighth the number present initially. What is the half-life (in days) of the material?
2	The ${}^{32}_{15}P$ isotope of phosphorus has a half-life of 14.28 days. What is its decay constant in units of s^{-1}?
3	The number of radioactive nuclei present at the start of an experiment is 4.6×10^{15}. The number present twenty days later is 8.14×10^{14}. What is the half-life (in days) of the nuclei?
Past Year	
4	**PSPM 21/22 – Q10(b)** A sample consists of 2g of a radioactive element. The molar mass of the element is 67 g. If the half-life of the element is 78 hours, calculate the activity of the sample after 30 hours.
5	**PSPM 20/21 – Q10(b)** Radioactive nuclei have a half-life of 0.99 s. Determine the time taken for 25% of the nuclei to decay away.
6	**PSPM 19/20 – Q10(b)** Calculate the activity of a 5µg ${}^{24}Na$ which has a half-life of 14.9 hours.
7	**PSPM 18/19 – Q10(b)** A 2 g sample of radioactive iodine ${}^{131}_{53}I$ has a half-life of 8 days. a. Calculate the decay constant b. Calculate the initial number of atoms in 2g sample. c. Calculate the activity of the sample after 2 days.
Book	
8	Two radioactive nuclei A and B are present in equal numbers to begin with. Three days later, there are three times as many A nuclei as there are B nuclei. The half-life of species B is 1.50 days. Find the half-life of species A.
	$\frac{N_A}{N_B} = \frac{N_{0A}e^{-\lambda_A t}}{N_{0B}e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t}$ $(N_{0A} = N_{0B})$ $\ln(N_A / N_B) = -(\lambda_A - \lambda_B)t$ $\lambda_A - \lambda_B = \frac{-\ln(N_A / N_B)}{t}$ $\lambda_A - \lambda_B = \frac{-\ln(3.00)}{3.00 \text{ days}} = -0.366 \text{ days}^{-1}$ $\lambda = 0.693 / T_{1/2}$ $0.693\left(\frac{1}{T^A_{1/2}} - \frac{1}{T^B_{1/2}}\right) = -0.366 \text{ days}^{-1}$ $T^B_{1/2} = 1.50 \text{ days}$ $\boxed{T^A_{1/2} = 7.23 \text{ days}}$
9	A 12.0 g sample of carbon from living matter decays at the rate of 184 decays/minute due to the radioactive C-14 in it. What will be the decay rate of this sample in a. 1000 years b. 50,000 years if the half-life of C-14 is 5730years?
	$A = \lvert dN/dt \rvert$ $N(t)$: $A = A_0 e^{-\lambda t}$. For ^{14}C, $T_{1/2} = 5730$ y and $\lambda = \ln 2 / T_{1/2}$ so $A = A_0 e^{-(\ln 2)t/T_{1/2}}$ $A_0 = 184$ decays/min. **(a)** For $t = 1000$ y, we have $A = (184 \text{ decays/min})e^{-(\ln 2)(1000 \text{ y})/(5730 \text{ y})} = 163$ decays/min. **(b)** For $t = 50,000$ y, the same equation gives $A = 0.435$ decays/min.

10	${}^{7}_{4}Be$ decays with a half-life of about 53 d. It is produced in the upper atmosphere, and filters down onto the Earth's surface. If a plant leaf is detected to have 350Bq of ${}^{7}_{4}Be$, a. how long do we have to wait for the decay rate to drop to 25 per second? b. Estimate the initial mass of on the leaf.
	(*a*) $R = R_0 e^{-\lambda t} \quad \rightarrow \quad t = -\frac{1}{\lambda}\ln\frac{R}{R_0} = -\frac{T_{1/2}}{\ln 2}\ln\frac{R}{R_0} = -\frac{53\text{ d}}{\ln 2}\left(\ln\frac{25\text{ decays/s}}{350\text{ decays/s}}\right) = 201.79\text{ d} \approx \boxed{2.0\times10^{2}\text{ d}}$ (*b*) We find the mass from the activity. Note that N_{A} is used to represent Avogadro's number, and A is the atomic weight. $R_0 = \lambda N_0 = \frac{\ln 2}{T_{1/2}}\frac{m_0 N_{\text{A}}}{A} \quad \rightarrow$ $m = \frac{R_0 T_{1/2} A}{N_{\text{A}}\ln 2} = \frac{(350\text{ decays/s})(53\text{ d})(86,400\text{ s/d})(7.017\text{ g/mole})}{(6.02\times10^{23}\text{ nuclei/mole})\ln 2} = \boxed{2.7\times10^{-14}\text{ g}}$

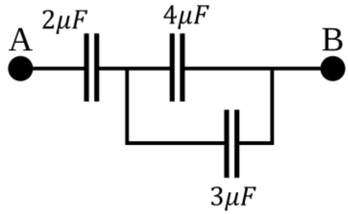
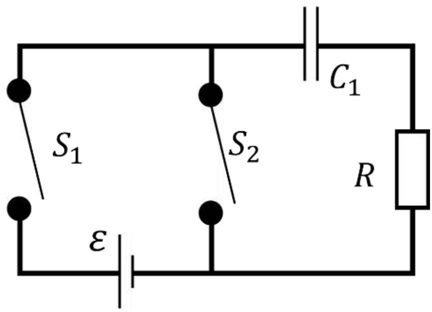
SP025: Learning from Examples

By Shafiq R

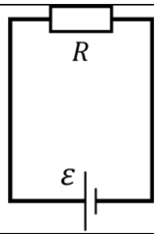
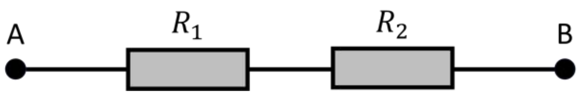
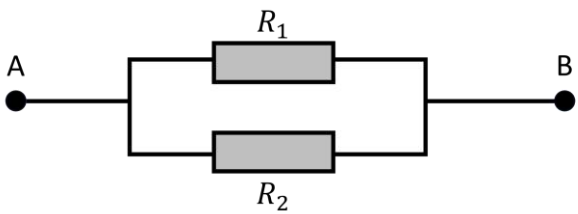
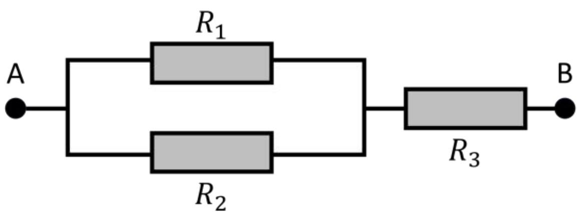
Electrostatics

1 Dimensional	Two charged particles of $+3\mu C$ and $+4\mu C$, in vacuum, are separated by 30cm. a. Determine the electrical force between them. b. Determine the electric field midpoint between them. c. Determine the electric potential experienced by a $-2\mu C$ charged particle is placed midpoint between them. d. Determine the electrical energy of the system.
2 Dimensional	The diagram shows a system of charged particles. Determine the a. Electrical force on a $-2\mu C$ charged particle placed at point A, b. Electrical field strength at point A, c. Electrical potential experienced by $-2\mu C$ charged particle placed at point A, d. Electric potential energy of the system with a $-2\mu C$ charged particle placed at point A. 
Charge in a uniform $\vec{E}$	An electron is placed between two plates as shown in the diagram. To keep the electron in place, the power supply is switched on to create an electric field. Determine the magnitude and direction of electric field between the plates to keep the electron in place. 
	An electron is shot into a region of electric field of $10^2 NC^{-1}$ as shown in the diagram. Disregarding gravity, a. Determine the acceleration of the electron in the electric field region. b. If the electron has an initial velocity of $20ms^{-1}$, determine the velocity of the electron 64μs upon entering the region of electric field. 

Capacitors

Capacitance	Calculate the amount of charge stored in a capacitor of $2\mu F$ if the potential difference across the plates is 4V.
	Determine the capacitance of an air-filled parallel plate capacitor of surface area of $20cm^2$ and plate separation of 0.25cm.
Capacitors in Parallel	Two capacitors of capacitance $2\mu F$ and $3\mu F$ are connected in **parallel**. Determine the equivalent capacitance.
Capacitors in Series	Two capacitors of capacitance $2\mu F$ and $3\mu F$ are connected in **series**. Determine the equivalent capacitance.
Combination	Three capacitors are connected as shown in the diagram.  a. Determine the effective capacitance. b. Determine the energy stored between A and B if the potential difference between A and B is 4V.
Charging & Discharging	The figure shows a circuit used to charge a $20mF$ capacitor.  The resistor has a resistance of $2k\Omega$ and the dry cell outputs 12V, and switch S_1 is closed to charge the capacitor. a. Determine the time constant. b. Determine the time needed to charge the capacitor to 80% from 0%. Once fully charged, switch S_1 is opened and switch S_2 is closed. c. Determine the time needed to discharge the capacitor to 45% of its initial charge.
Dielectrics	An air-filled parallel plate capacitor has a capacitance of $20\mu F$. If a material of dielectric constant of 3 is placed between the plates, determine the new capacitance of the capacitor.

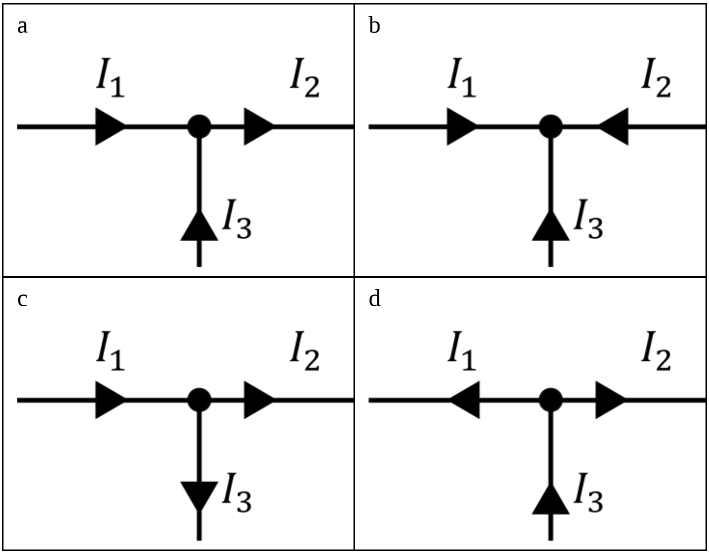
<u>Electrical Current, Ohm's Law & Resistance</u>

Electrical Current	It is found that 6.25×10^{18} electrons passes through a point in a wire between $t = 1.0s$ and $t = 0.5s$. Calculate the electrical current in the wire.
Ohm's Law	A resistor draws 0.5A when the potential across it is 3V. Determine the resistance of the resistor.
Resistance	Determine the electrical resistance of a piece of copper rod, 2cm diameter and is 10cm long. The resistivity of copper is $1.68 \times 10^{-8} \Omega m$.
Resistance & Heat	A wire has a resistance of 2Ω at $20^o C$, determine its resistance at $120^o C$ if the temperature coefficient of resistivity of the wire is $4 \times 10^{-3} {}^o C^{-1}$.
EMF	The diagram shows a resistor connected to a dry cell of emf 3V. The resistor has a resistance of 2Ω. If the current in the wire is found to be 1.25A, determine the internal resistace of the dry cell. 
Series, Parallel and Combination	If $R_1 = 2\Omega$, $R_2 = 3\Omega$ and $R_3 = 4\Omega$, determine the equivalent resistance across A and B for the situation below: a  b  c 

<u>Kirchhoff's Rules</u>

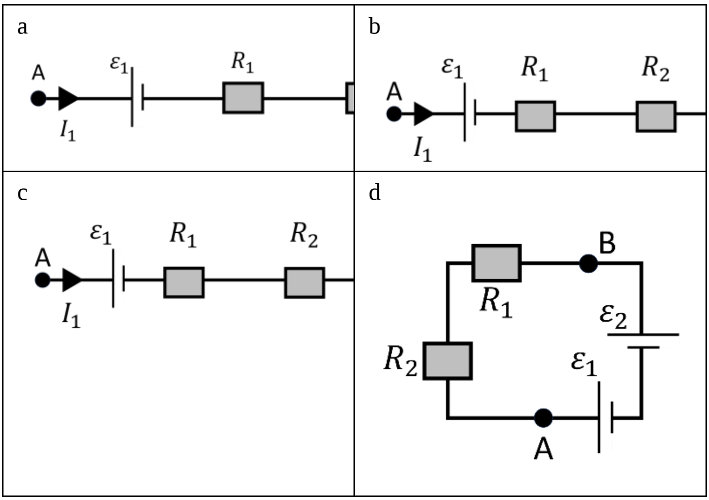
1st Rule

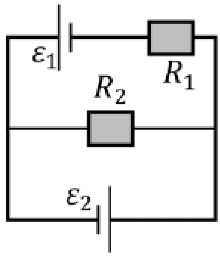
Determine the value of I_3 for the situation below if $I_1 = 2A$ and $I_2 = 3A$:



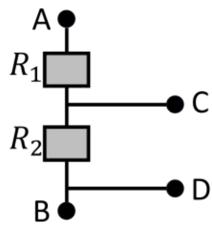
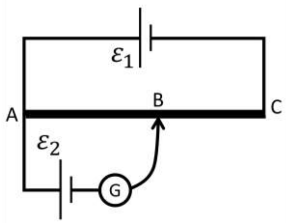
2nd Rule

If $\varepsilon_1 = 3V$, $\varepsilon_2 = 5V$, $R_1 = 2\Omega$, $R_2 = 3\Omega$ and $I_1 = 3A$, determine the potential difference between point A and B for the situation below:

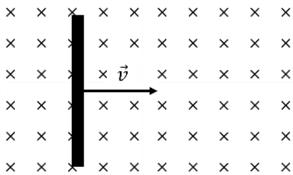


Combination	Determine magnitudes and directions of currents through resistors R_1 and R_2 in the figure shown. Values are $\varepsilon_1 = 8V$, $\varepsilon_1 = 6V$, $R_1 = 20\Omega$, and $R_2 = 15\Omega$.	

Electrical Energy, Power and Potential Divider

Energy & Power	A hair dryer draws a steady 7.5A on a 240V line. a. How much power does it require? b. If it ran for 20 minutes, how much electrical energy will it have used? c. Extra: TNB charges 21.80sen per kWh, if the hair dryer is used 20 days at the rate of 20minutes each day, how much would it cost?
Potential Divider	Based on the diagram shown, if the potential difference between A and B is 20V, $R_1 = 2\Omega$ and $R_2 = 3\Omega$, determine the potential difference across C and D. 
Potentiometer	Based on the diagram shown, if the length of AC is 1.5m, the length of AB is one third of AC when the galvanometer reads zero and $\varepsilon_1 = 5V$, determine the value of ε_2. 

Flux & Induced emf

Magnetic flux	Calculate the magnetic flux linkage for 20 loops coil of $20cm^2$ in a region of $20T$ magnetic field when the cross-sectional area of the loop is a. Perpendicular to the magnetic field lines b. 30^o to the magnetic field lines
Faraday's Law	The magnetic flux linkage of a coil (of N loops) increased from $20\mu Wb$ to $35\mu W$ between $t = 2s$ and $t = 5s$. Determine the electromotive force generated in the coil.
Lenz's Law	A coil is placed in a region of uniform magnetic field. The coil is then shrunk. Determine the direction of current induced in the coil. Determine the direction of induced current if the coil was expanded instead.
Straight Conductor	A straight 0.35m rod travels at $2ms^{-1}$ in a region of magnetic field of 2T, as shown in the diagram. Determine the induced emf in the rod. 
Area change	A coil of radius 8cm is placed in a region of $2mT$ magnetic field. The coil is then shrunk such that its new radius is 4cm. Determine the emf induced in the coil as a results of the shrinkage.
Rotating Coil.	A coil (of aream $20cm^2$) of 20 loops is rotating in a region of uniform magnetic field of 2mT. The frequency of rotation is 50Hz. Determine the maximum induced emf in the coil.

Self & Mutual Inductance

Self-Inductance	The emf induced in a solenoid is found to be 30mV when the electrical current flowing through it changes at $4.5As^{-1}$. Determine the self-inductance of the solenoid.
	A coil consists of 20 turns and is exposed to magnetic flux of 0.05Wb when 2A is flowed through the coil. Determine the self-inductance of the coil.
	A coil of 20 turns has a radius of $2cm^2$. Determine the self-inductance of the coil.
	A solenoid of 2000 turns has a radius of $2cm^2$ and length of 50cm. Determine the self-inductance of the solenoid.
Energy Storage	Electrical energy is stored by running electrical current of 13A current through an 200mH inductor. Determine the amount of energy stored.
Mutual Inductance	2 coils (coil A and coil B) are placed near each other. When the current in coil A changes at $2.75As^{-1}$, induced emf in coil B is found to be 2V. Determine the mutual inductance between coils.
	Two coils are wound coaxially, having a radius of 2cm and length 20cm. The first coil has 20 turns and the second coil has 75 turns, determine the mutual inductance between the two coils.

RLC Circuits

Rms values	Find the rms values for voltage and current in an AC circuit if the peak current and peak voltage is 18.4A and 170V.
RLC Circuit	A 100Ω resistor, a $32\mu F$ capacitor and a 0.48H inductor are connected in series with a 240V power supply. The a.c. frequency of the power supply is 50Hz. a. Calculate i. the reactances of the capacitor and inductor. ii. the impedance. iii. Calculate the current through the wire. iv. Calculate the voltages across the each component. b. Sketch the phasor diagram c. Determine i. the phase angle ii. the power factor. iii. Average power

Matter Waves

Momentum	An object has a de Broglie wavelength of $1.66 \times 10^{-36}m$. Determine its momentum.
Voltage	An electron is accelerated through a potential difference of 100V. Determine the de Broglie wavelength of the electron.

Mass Defect, Binding Energy & Radioactivity

Particle	**Mass in kg**	**Mass in amu**
Neutron	$1.67492747 \times 10^{-27}$	1.00866492
Proton	$1.67262192 \times 10^{-27}$	1.00727647
Electron	$9.1093837 \times 10^{-31}$	0.00054858
Mass to MeV conversion:	1amu = $931.5 MeV/c^2$	
$1eV = 1.6 \times 10^{-19} J$		
$1amu = 1.6605 \times 10^{-27} kg$		

Mass Defect & Binding Energy	The mass of $^{13}_{7}N$ atom is 13.005738u. Determine a. the mass defect b. the binding energy c. the binding energy per nucleon of the atom.
	The mass of $^{4}_{2}H$ nucleus is $6.6447 \times 10^{-27} kg$. Determine d. the mass defect e. the binding energy f. the binding energy per nucleon of the atom.
Radioactivity	A radioactive sample decays at a rate of 200Bq. If the sample has 3000 particles, determine the decay constant.
	A radioactive sample initially decays at a rate of 200Bq. If the sample has a half life of 30minutes, determine the activity after 1 hour.
	A radioactive sample has a decay constant of $800 s^{-1}$. a. Determine the time it takes for the sample to decay by 20%. b. Calculate the half life of the sample.

Homework by sr

SP025 2023/2024

WEEKEND HOMEWORK

Homework by sr

Contents

Homework 2/1/2024 2

Weekend Homework (6/1 – 7/1) 3

Homework 12/1/2024 – 14/1/2024 4

Homework 15/1 – 19/1 5

Weekend Homework (26/1 – 28/1) 8

Weekend Homework 3/2/2024 – 4/2/2024 9

Weekend Homework (17/2 – 18/2) 10

Weekend Homework (24/2 – 25/2) 11

Weekend Homework (2/3 – 3/3) 12

Pre-Quiz Exercise (Geometrical Optics) 13

16, 17 March – Weekend Homework (C3 & C7) 14

Weekend Homework (C1, C5 & C6) 16

Weekend Homework (C8, C2 & C9) 17

Homework 2/1/2024

Homework 2/1/2024

Problem 1:

Two small plastic spheres are given positive electrical charges. When they are 12cm apart, the repulsive force between them has magnitude 0.352N. What is the charge on each sphere

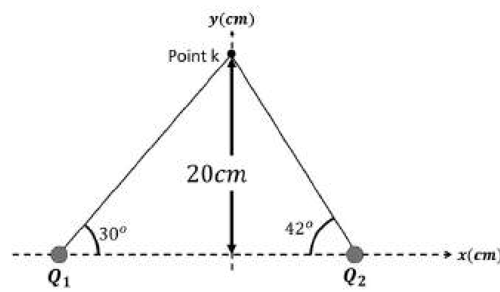
a. if the two charges are equal
b. if one sphere has five times the charge of the other?

[$q_1 = q_2 = 7.5 \times 10^{-7}C$; $q_1 = 3.36 \times 10^{-7}C$; $q_2 = 1.68 \times 10^{-6}C$]

Problem 2:

The figure shows 2 charges on the x-y plane with $Q_1 = -2\mu C$ and $Q_2 = +5\mu C$.

Determine the electric field at point *k*.



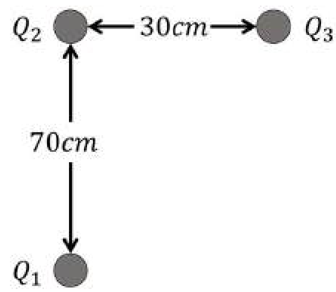
[$E = 549kNC^{-1}$ at 30.76^o from -x axis]

Homework by sr

Weekend Homework (6/1 – 7/1)

Problem 1

Three charges are arranged as shown in the diagram.



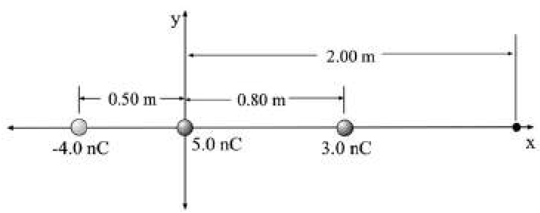
Given:
Q_1 = -15nC
Q_2 = +14nC
Q_3 = +11nC

Determine the magnitude and direction of the net electrostatic force on Q_2.

[$F = 1.6 \times 10^{-5} N$ at 14.6^o below the negative x-axis]

Problem 2

Three point charges are aligned along the x-axis as shown in the figure.

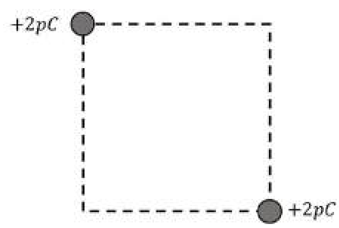


Find the electric field at the position x = +2.0 m, y = 0m.

[E=21.4NC^{-1} towards +x]

Problem 3

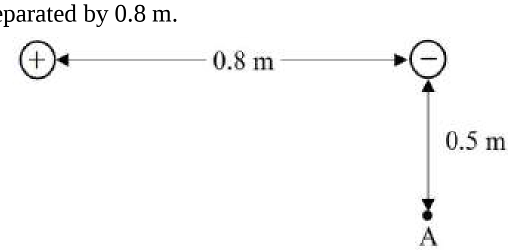
Find the potential at the centre of a square of side $2m$ due to the charges in the system as shown in the diagram.



[$V = 0.0254V$]

Problem 4

The figure shows two charges +30μC and −5μC separated by 0.8 m.



Determine the electric potential at point A and external work required to bring a +2μC charge from infinity to point A.

[$V = 1.962 \times 10^5 V$; $U = 0.3924J$]

Problem 5

An electron and a proton are each placed at rest in an electric field of 520 N/C.

Calculate the speed of each particle 50ns after being released.

[$v_{e^-} = 4.57 \times 10^6 ms^{-1}$; $v_{p^+} = 2.491 \times 10^3 ms^{-1}$]

Problem 6

A water droplet of charge $1.92 \times 10^{-17} C$ is suspended in calm air owing to a downward–directed atmospheric electric field E = $500 NC^{-1}$. Determine the mass of the water droplet.

[$m_{droplet} = 9.786 \times 10^{-16} kg$]

Problem 7

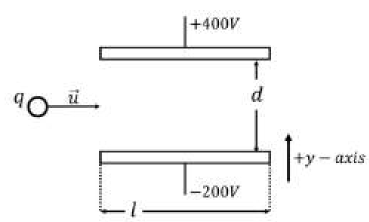
The charges and coordinates of two charged particles held fixed on the xy-plane are: $q_1 = +3.0\mu C\ at$ $(3.5, 0.5) cm$ and $q_2 = -4.0\mu C$ at $(-2.0, 1.5) cm$.

a. Find the magnitude and direction of the electrostatic force on q_2.
   [35N towards each other at 10.3^o from the horizontal]
b. Where could you locate a third charge $q_3 = +4.0\mu C$ such that the net electrostatic force on q2 is zero? **[(−8.35 cm, 2.65 cm)]**

Problem 8

A charged particle of 2g enters a region of uniform electric field $\vec{E}$ at velocity $\vec{u} = 30 ms^{-1}$. Length of the plates, l, has a value of 1.5m and the plate separation is set at 0.25m.

Determine the vertical displacement and sketch the path of the charged particle when it exits the region of uniform electric field if the charged particle has a charge of



a. $q = +2\mu C$ **[0.3cm]**
b. $q = -2\mu C$ **[-0.3cm]**
c. $q = 1\mu C$ **[0.15cm]**

Homework by sr

Homework 12/1/2024 – 14/1/2024

Problem 1

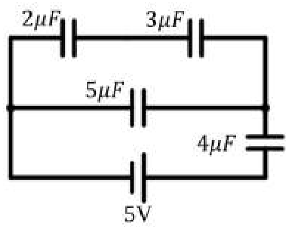
A $720pF$ air filled parallel plate capacitor has a charge of $0.173\mu C$ on each plate and a surface area of $1630cm^2$.

Calculate the potential difference across the plate, the separation between the plate and the magnitude of electric field between the plates.

[240V; 2mm; $120kNC^{-1}$]

Problem 2

The figure below shows a circuit consisting multiple capacitors.



Determine the effective capacitance and the voltage across every capacitor.

[$C_{eff} = 2.43\mu F; V_{4\mu F} = 3.04V, V_{5\mu F} = 1.96V, V_{2\mu F} = 1.18V$, $V_{3\mu F} = 0.787V$]

Problem 3

Given a parallel plate capacitor with a plate separation of 2cm and a surface area of $50 \times 10^{-4}m^2$, calculate the capacitance when a dielectric with a relative permittivity (dielectric constant) of 4.5 is inserted between the plates.

[11.1pF]

Problem 4

Determine the new capacitance of a capacitor if its plate separation is doubled, the surface area is tripled, and a dielectric material with a relative permittivity of 6 is added between the plates. The initial capacitance is $20\mu F$.

[13.3pF]

Problem 5

A parallel plate capacitor has an initial capacitance of 30 μF with a plate separation of 3cm and a dielectric constant of 5. If the dielectric material is replaced with another material having a relative permittivity of 8, and the surface area of the plates is reduced by half, determine the new capacitance.

[2.48pF]

Problem 6

Consider an RC circuit with a resistance of 2.2MΩ and a capacitor with a capacitance of 500pF. If the capacitor is initially charged to 200V and then discharged through the resistor, calculate the time it takes for the voltage across the capacitor to decrease to 30% of its initial value during the discharge process.

[1.203s]

Problem 7

A capacitor in an RC circuit is charged through a resistor with a resistance of 8kΩ. If the time constant of the circuit is 0.6 seconds, find the time it takes for the capacitor to charge to 80% of its maximum voltage.

[1.386s]

Homework by sr

Homework 15/1 – 19/1

Part 1

Problem 1

A current of 7.5A is maintained in a wire for 45s. In this time, how much charge and how many electrons flow through the wire? [337.5C; 2.1×10^{21}]

Problem 2

If 0.6mol of electrons flow through a wire in 45min, determine the magnitude of current. [21.4A]

Problem 3

What is the current through an 8Ω kitchen appliance when it operates at 120V. [15A]

Problem 4

Determine the potential difference between the ends of a wire of 5Ω if 720C passes through it per minute. [60V]

Problem 5

Silver has a resistivity of $1.6 \times 10^{-8} \Omega m$. Determine the resistance of 180m of silver wire of $0.3 mm^2$. [9.6Ω]

Problem 6

A current of 3A flows through a 1.5m long straight metal rod that has a 0.2cm diameter. Determine the resistivity of the rod material if the potential difference between the ends of the rod is 40V. [$2.8 \times 10^{-5} \Omega m$]

Problem 7

A 20cm long copper tube has an inner diameter of 0.85cm and an outer diameter of 1.1cm. If copper has a resistivity of $1.7 \times 10^{-8} \Omega m$, determine its electric resistance. [89μΩ]

Problem 8

A coil of wire has a resistance of 25Ω at $20^o C$ and a resistance of 25.17Ω at $35^o C$. What is its temperature coefficient of resistance? [$\alpha = 4.5 \times 10^{-4} {}^o C^{-1}$]

Problem 9

A 75W light bulb has a resistance of 190Ω when lighted and 15Ω when turned off. Estimate the temperature of the filament when the bulb is lighted. Assume the filament has a temperature of coefficient of resistance of $4.5 \times 10^{-4} {}^o C^{-1}$. [$2600^o C$]

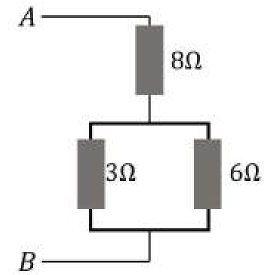
Problem 10

A 60W bulb carries a current of 0.5A when operating on 120V. The temperature of its filament is then $1800^o C$. Find the resistance of the filament at operation and at $20^o C$. [$R_{operation} = 240\Omega$; $R_{20^o C} = 26.6\Omega$]

Part 2

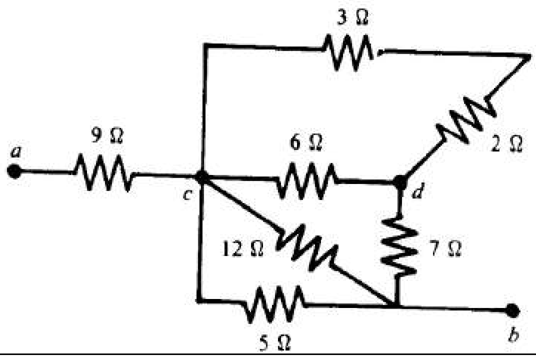
Problem 1

Determine the resistance between A and B in the circuit shown. [10Ω]



Problem 2

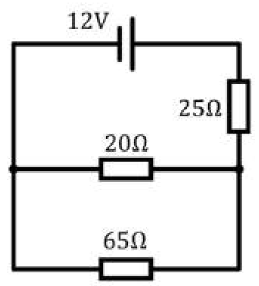
Determine the resistance between a and c in the circuit shown. [11.6Ω]



Problem 3

Determine the current through each resistor in the circuit shown.

$[R_{20\Omega} = 227.74A;\ R_{25\Omega} = 297.81A;\ R_{65\Omega} = 70.073A]$



Problem 4

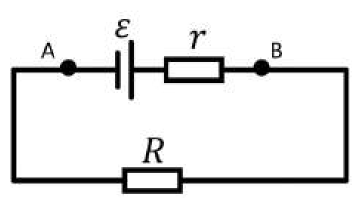
A 1.5V dry cell furnishes 30A when short circuited. Find the internal resistance of the cell. [0.05Ω]

Problem 5

A battery has emf $25V$ and internal resistance of 0.2Ω. Determine its terminal voltage when it is delivering 8A. [23.4V]

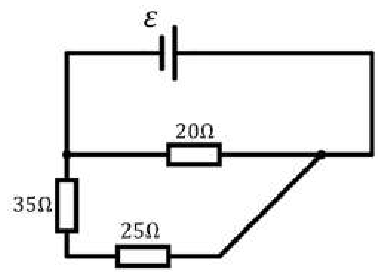
Problem 6

The circuit shown has a current of 0.5A when $R = 10\Omega$ and a current of $0.27A$ when $R = 20\Omega$. Find the internal resistance and the emf of the battery. $[r = 1.76\Omega;\ \varepsilon = 5.88V]$



Problem 7

When the emf of the circuit in the diagram is set at 12V, the current through the 35Ω resistor is found to be 193.55mA. When the emf is set to 30V, the current through the 35Ω resistor is 483.87mA. Determine the internal resistance of the battery. $[r = 0.5\Omega]$

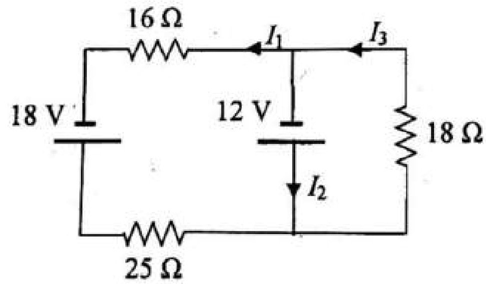


Homework by sr
On Kirchhoff's Law

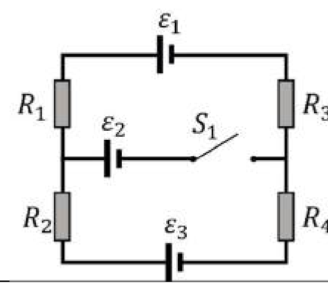
Problem 1
Based on the circuit shown, determine

a. I_1, I_2 and I_3
[{0.15, 0.67, 0.52}A]

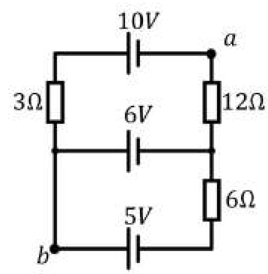
b. Potential difference across the 18Ω resistor
[12V]



Problem 2
The circuit shown consists of 3 dry cells, 4 resistors and a switch. The values of resistances and terminal voltages are as follows:
$R_1 = 2.5\Omega$, $R_2 = 3\Omega$, $R_3 = 1.5\Omega$, $R_4 = 1\Omega$, $\varepsilon_1 = 4V$, $\varepsilon_2 = 6V$, $\varepsilon_3 = 12V$.
Determine the current through R_1 and R_3 when the switch is closed and when it is opened.
$[\boldsymbol{R_{1-open} = R_{3-open} = 1A, R_{1-closed} = 0.5A, R_{3-closed} = 1.5A}]$



Problem 3
The circuit shown consists of 3 resistors and 3 dry cells.
Determine the potential difference between *a* and *b*.
[9.2V]



Homework by sr

Weekend Homework (26/1 – 28/1)

Problem 1

Two point charges of $-4\mu C$ and $+6\mu C$ are separated 2mm rom each other. Calculate

a. the electric field strength at the midpoint. **[$9 \times 10^9 NC^{-1}$]**

b. the electric potential at the midpoint. **[$9 \times 10^6\ V$]**

Problem 2

An electron is released from rest in a uniform electric field of $80NC^{-1}$. Determine

a. the speed of the electron after travelling 2mm **[$6.706 \times 10^{12} ms^{-1}$]**

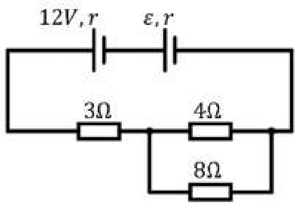
b. the change of electric potential energy of the electron after travelling 2mm **[$20.48\mu J$]**

Problem 3

A capacitor, of $75.33m^2$ and a plate separation of 1mm is filled with a material of dielectric constant of 3, is charged to a potential difference of 24V. The capacitor is then discharged through a $1.8M\Omega$ resistor in series with another $1.2M\Omega$ resistor. Determine the time it takes to discharge by 40% from full charge. **[3.065s]**

Problem 4

The circuit below shows a battery connected to 3 resistors.



When $\varepsilon = \varepsilon_1$, the current through the 3Ω resistor is found to be 3.4615A. When ε is replaced with a different battery of the same internal resistance but two-thirds of the electromotive force the current through the 3Ω resistor is then found to be 2.9670A.

Determine the value of ε_1 and r.

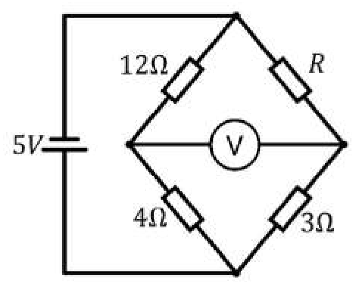
[$\varepsilon_1 = 9V$, $r = 0.2\Omega$]

Problem 5

The figure shows a circuit consisting of 4 resistors and 1 dry cell of 5V.

Determine the resistance of R if the voltmeter gives zero reading.

[$R = 9\Omega$]



Homework by sr

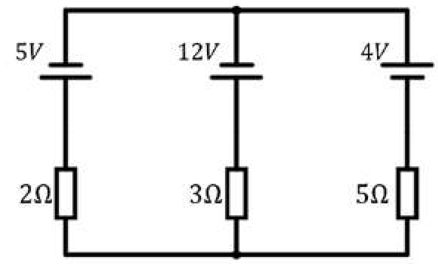
Weekend Homework 3/2/2024 – 4/2/2024

Problem 1

The figure below shows a circuit consisting of 3 resistors and 3 batteries. Each battery has an internal resistance of 0.2Ω. Determine the current through each resistor and their power dissipated

[$I_{2\Omega} = 216.4mA$; $I_{3\Omega} = 2.04A$; $I_{5\Omega} = 1.822A$]

[$P_{2\Omega} = 0.094W$; $I_{3\Omega} = 12.534W$; $I_{5\Omega} = 16.598W$]



Problem 2

A heating element of 2.8m length with cross sectional area of $4.2 \times 10^{-6} m^2$. At $320^o C$, the wire has a resistivity of $6.5 \times 10^{-5} \Omega m$ and temperature coefficient of resistivity of $2 \times 10^{-3} K^{-1}$. Determine the resistance of the heating element at $420^o C$. **[52Ω]**

Problem 3

The following figure shows the tariff set by Sarawak Energy. 1 unit in the table refer to energy consumption of 1 kWh.

Your 1 aircon (710W) and 1 refrigerator (65W) are the biggest consumer of electricity.

a. Estimate how much you'll be paying to use them on a daily basis.
b. Imagine now you have 2 aircons instead of 1, how much would you expect your bills to increase?

PRICING & TARIFF:

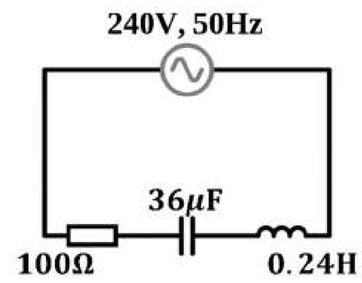
TARIFF CATEGORY	RATE PER UNIT
TARIFF D-DOMESTIC	
For 1 to 100 units per month	18 sen
For 1 to 150 units per month	18 sen
For 1 to 200 units per month	22 sen
For 1 to 300 units per month	25 sen
For 1 to 400 units per month	27 sen
For 1 to 500 units per month	29.5 sen
For 1 to 700 units per month	30 sen
For 1 to 800 units per month	30.5 sen
For 1 to 1300 units per month	31 sen
For above 1300 units per month	31.5 sen
Minimum monthly charge	RM 5.00

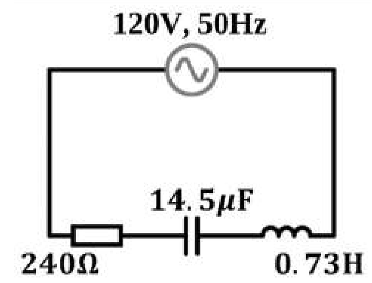
Homework by sr

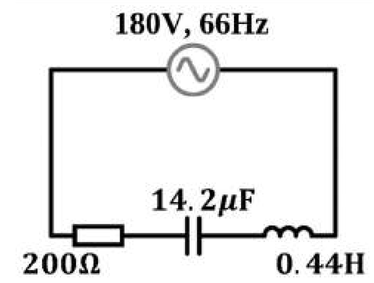
Weekend Homework (17/2 – 18/2)

Problem 1

For each of the following circuit, determine their inductive reactance, capacitive reactance, impedance, phase angle, power factor, average power and the current through the circuit.







Problem 2

A coil of wire has a resistance of 25Ω at $20^{o}C$ and a resistance of 25.3Ω at $35^{o}C$. What is its temperature coefficient of resistance? [$\alpha = 8 \times 10^{-4\,o}C^{-1}$]

Problem 3

Consider an RC circuit with a resistance of 2.2MΩ and a capacitor with a capacitance of 500pF. If the capacitor is initially charged to 200V and then discharged through the resistor, calculate the time it takes for the voltage across the capacitor to decrease to 42% of its initial value during the discharge process.

[0.954ms]

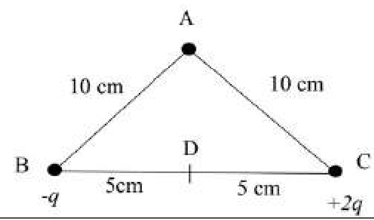
Homework by sr

Weekend Homework (24/2 – 25/2)

Problem 1

The figure shows two point charges $-q$ and $+2q$ placed at points B and C respectively. If $q = 1\mu C$, determine the electric field at point D.

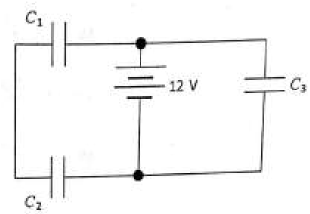
[$1.08 \times 10^{7} NC^{-1}$ to the left]



Problem 2

Three identical capacitors of $5\mu F$ are connected to a 12V dc supply as shown in the diagram. Determine charge on capacitor C_3 and the equivalent capacitance of the circuit.

[$60\mu C$; 7.5μF]



Problem 3

A coil with 200 turns of wire is wrapped on a square frame, 18cm on aside. Each tum has the same area, equal to that of the frame, and the total resistance of the coil is 2Ω. A uniform magnetic field is applied perpendicularly to the plane of the coil. If the field changes uniformly from 0T to 0.5 T in 0.8s, calculate

a. the induced e.m.f. in the coil while the field is changed.
b. the induced current in the coil.

[4.05V; 2.025A]

Problem 4

An AC generator is supplying an rms voltage of 220V at 45Hz to an LRC circuit. If the inductive reactance of the inductor is 13.4Ω, the resistance of the resistor is 22Ω and the capacitive reactance of the capacitor is 8.6Ω, calculate the

a. impedance in the circuit
b. current flow in the circuit
c. phase angle between impedance and current
d. power factor.

[22.5Ω, 9.8A, 12.3^{o}, 0.98]

Problem 5

The radius of curvature of a convex mirror is 10 cm. A virtual image formed 4cm away from the mirror.

a. Calculate the focal length of the mirror.
b. Calculate the object distance.
c. Calculate the height of the image if the height of the object is 15cm.
d. State three characteristics of the image formed.

[-5cm, +20cm, 3cm, {virtual, diminished, upright}]

Homework by sr

Weekend Homework (2/3 – 3/3)

Problem 1

An object is located 22cm to the left of a diverging lens having a focal length $f = -35cm$. Determine the location and the magnification of the image. **[13.51cm, 0.614]**

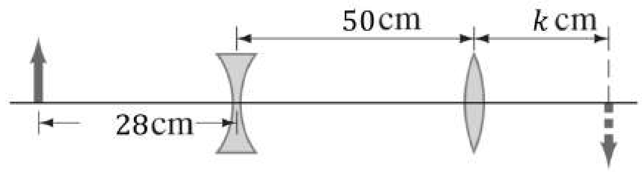
Solution

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{+22} + \frac{1}{v} = \frac{1}{(-35)} \Rightarrow v = -13.51cm$$

$$M = -\frac{v}{u} = -\frac{-13.51}{22} = 0.614$$

Problem 2

A small object is 28cm from a diverging lens as shown in the figure. A converging lens with a focal length of 8cm is 50cm to the right of the diverging lens of focal length 12cm. Determine the image distance from the object. **[87.27cm]**



Solution

$$\boldsymbol{\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1} \Rightarrow \frac{1}{-12} = \frac{1}{28} + \frac{1}{v_1} \Rightarrow v_1 = -8.4cm}$$

$$\boldsymbol{u_2 + v_1 = 50 \Rightarrow u_2 + (-8.4) = 50 \Rightarrow u_2 = 58.4cm}$$

$$\boldsymbol{\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2} \Rightarrow \frac{1}{8} = \frac{1}{58.4} + \frac{1}{v_2} \Rightarrow v_2 = 9.27cm}$$

$$\boldsymbol{d = 50 + 28 + 9.27 = 87.27cm}$$

Problem 3

A goldfish (6cm from the aquarium wall) is swimming at 2cm/s toward the front wall of a spherical aquarium of radius 2m. What is the apparent speed of the fish measured by an observer looking in from outside the front wall of the tank? The index of refraction of water is 1.33. **[$\boldsymbol{-0.958cms^{-1}}$]**

Solution

$$\boldsymbol{\frac{1.33}{0.06} + \frac{1}{v} = \frac{1-1.33}{0.06} \Rightarrow v = -0.036m}$$

$$\boldsymbol{\frac{d}{dt}\left(\frac{1.33}{u(t)} + \frac{1}{v(t)}\right) = \frac{d}{dt}\left(\frac{1-1.33}{0.06}\right)} \Rightarrow -\frac{1.33}{\left(u(t)\right)^2}\boldsymbol{\frac{du}{dt}} - \frac{1}{v[t]^2}\boldsymbol{\frac{dv}{dt}} = 0$$

$$\Rightarrow \frac{dv}{dt} = -\frac{1.33v^2}{u}\left(\boldsymbol{\frac{du}{dt}}\right) \Rightarrow \frac{dv}{dt} = -\frac{1.33v^2}{u^2}\left(\boldsymbol{\frac{du}{dt}}\right) = -\frac{1.33(-0.036)^2}{(0.06)^2}(\boldsymbol{0.02})$$

$$\frac{dv}{dt} = -0.00958ms^{-1}$$

Pre-Quiz Exercise (Geometrical Optics)

Problem 1

Radius of curvature of a convex mirror is 1.5m. An object with a height of 2m is placed 4m in front of the mirror.

Calculate the

a. Position of the image from the mirror **[-0.63m]**
b. Magnification of the image **[0.16]**
c. Height of the image **[0.32m]**

Problem 2

An object is placed 15cm in front of a mirror. The image formed by the mirror is upright and magnified 2 times.

a. Determine the focal length of the mirror. **[30cm]**
b. Is the mirror convex or concave? Explain your answer. **[Concave, positive focal length]**

Problem 3

An object is placed 10cm from the biconvex lens of focal length 30cm.

a. Calculate the magnification of the image. **[1.5]**
b. State THREE characteristics of the image. **[Magnified, Upright, Virtual, Same side as object]**

Problem 4

A 50cm tall object is placed 30cm in front of a diverging lens with focal length 10mm. Calculate the position of the image and state characteristics of the image. **[-0.9cm, {virtual, upright, diminished}]**

Problem 5

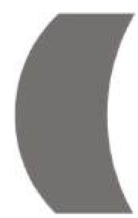
The convex meniscus lens has a 15 cm radius for the convex surface and 23 cm for the concave surface. The lens is made of glass with a refractive index, n = 1.51 in air. Refractive index of air is 1.0. Determine the focal length of the lens. **[84.56cm]**

Problem 6

A 1.5cm high object is placed at 20cm from a concave mirror with radius of curvature 30cm.

a. Determine the magnification of the image. **[-3]**
b. State TWO (2) characteristics of the image produced. **[Real, magnified, inverted]**

Problem 7



The lens shown in the figure has surfaces with radius of curvature 60cm and 20 cm. The refractive index of the lens material is 1.63.

a. Calculate the focal length of the lens. **[$-47.62cm$]**
b. What type of lens is this? **[Diverging]**
c. The lens is then immersed in water with refractive index of 1.33. Calculate the new focal length. **[$-133cm$]**

Problem 8

A biconvex lens made from glass with refractive index 1.50 has a focal length of 12 cm in air. Given the refractive index of air is 1.00. Calculate the focal length of the lens when it is submerged in a liquid with refractive index 1.49. **[894cm]**

Problem 9

A small tropical fish is at the centre of a water-filled, spherical fish bowl 28cm in diameter.

a. Find the apparent position of the fish to an observer outside the bowl. The effect of the thin walls of the bowl may be ignored. **[$-14cm$]**
b. Determine the focal point of parallel sun rays. **[+56.42cm]**

Problem 10

A thin biconvex lens has a focal length of 3.50 cm. The image formed is inverted at a distance 5.0 cm from the lens.

a. Calculate the object distance. **[11.67cm]**
b. Calculate the magnification. **[-0.43]**
c. If another thin biconvex lens with the same focal length is placed 10.0 cm to the right from the first, calculate the final image formed by this combination of lenses. **[$11.67cm$]**

Homework by sr

16, 17 March – Weekend Homework (C3 & C7)

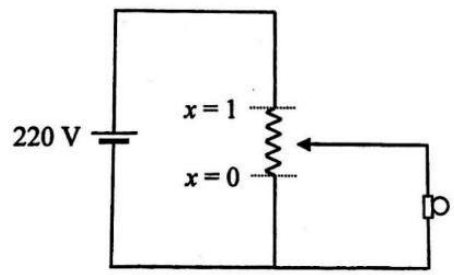
Problem 1

A wire 5m long and 3mm in diameter has a resistance of 100Ω. A 15V potential difference is applied across the wire. Determine

a. the current in the wire **[0.15A]**
b. the resistivity of the wire **[$\mathbf{0.14 \times 10^{-3} \Omega m}$]**
c. the rate at which heat is being produced in the wire. **[2.25W]**

Problem 2

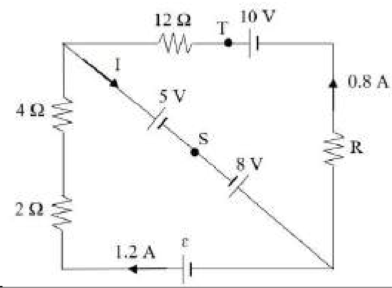
The figure shows a light bulb dimmer consisting of a 150Ω variable resistor, 220V voltage source and a light bulb. The slider moves between x = 0m to x = 1m. If it is at x = 0.3m, calculate the voltage of the bulb. **[66V]**



Problem 3

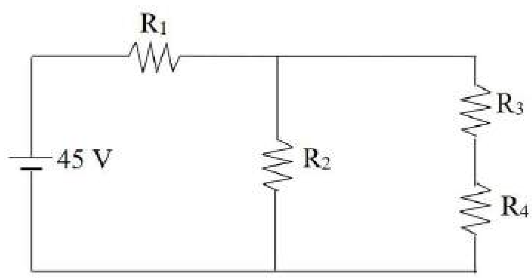
By referring to the figure shown, calculate the

a. current, I ,emf, ε and resistor, R. **[2A, 4.2V, $\mathbf{4.25\Omega}$]**
b. potential difference between points S and T. **[14.6V]**
c. total power consumed in the circuit. **[19.04W]**



Problem 4

The figure shows four identical resistors in a circuit. A 45V battery without the internal resistance delivers 24.3W of the power to the circuit. What is the resistance for each resistor? **[50Ω]**



Problem 5

A uniform wire PQ of length 150cm and radius 0.8 mm is connected in series with a battery of emf 5V and internal resistance 0.6Ω.

Calculate

a. the resistance of the wire if the resistivity of the wire is 1.1×10^{-6}Ωm. **[0.82Ω]**
b. the potential difference across PQ. **[2.89V]**

Problem 6

A battery has an internal resistance of 0.08Ω and an emf of 9V is connected to a resistor. Calculate

a. the current in the resistor if the voltage across the resistor is 8.9V. **[1.25A]**
b. the resistance of the resistor. **[7.12Ω]**

Problem 7

A boy standing 4 m from a spherical mirror sees his upright image with a magnification of 0.2. Calculate the image distance from the mirror and radius of mirror curvature. Determine the type of the spherical mirror and justify your answer. **[$\mathbf{-0.8m, -2m}$, convex]**

Problem 8

The convex meniscus lens has a 15 cm radius for the convex surface and 23 cm for the concave surface. The lens is made of glass with a refractive index, n = 1.51 in air. Refractive index of air is 1.0. Determine the focal length of the lens. **[84.56cm]**

Problem 9

A 660 nm monochromatic light is incident on a thin film (n =1.35). Calculate the minimum thickness of the film which produce destructive interference of the reflected light in the following condition:

a. the film in air. **[222nm]**
b. the film is a coating on a glass of reflective index 1.50. **[111nm]**

Problem 10

a. Light 565 nm wavelength is in incident on a slit 0.2mm wide. A screen is placed 1.5m from the slit. Determine the width for the central bright. **[8.5mm]**

b. Monochromatic light from helium-neon laser of wavelength 450 nm in incident normally to a diffraction grating. If the grating consists of 500 lines per cm, calculate opening angle between 2nd order maxima observed on screen. [5.2^o]

Homework by sr

Weekend Homework (C1, C5 & C6)

Homework by sr

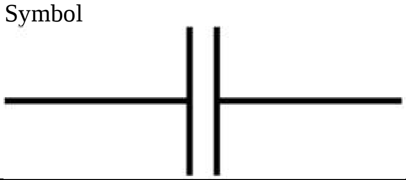
Weekend Homework (C8, C2 & C9)

SP025 EXTRA HANDOUTS

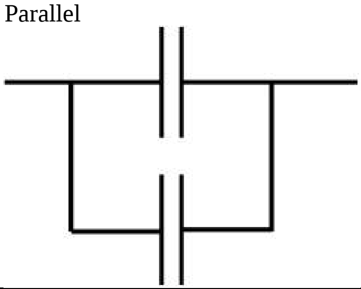
Chapter 2 3
Chosen PYQ Chapter 1,2 and 3 6
K2 Catch Up Sheet 7
Kirchoff's Handout 9
C5 K2T4 (8 Feb 2024) 14
Notes on Potentiometer and Wheatstone 15
Chapter 5: Magnetic Field & Magnetic Force Problems 20
Faraday and Lenz's 21
Chosen Past Year Questions on Chapter 6 25
Geometrical Optics (Reflections) Practice – K1 26
Geometrical Optics (Reflections) Practice – K2T3 26
Self-Study Session (24/2/2024) 26
Thin Films Interference 28
Single Slit Diffraction 29
Diffraction Grating 30

Chapter 2

Capacitors

Symbol	Equation
	

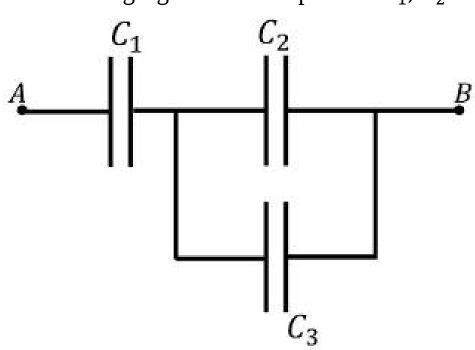
Capacitors in Series & Parallel

Configuration	Equation
Series 	
Parallel 	

Energy stored in a capacitor

Examples:

1. The following figure shows capacitors C_1, C_2 and C_3 connected between points A and B.



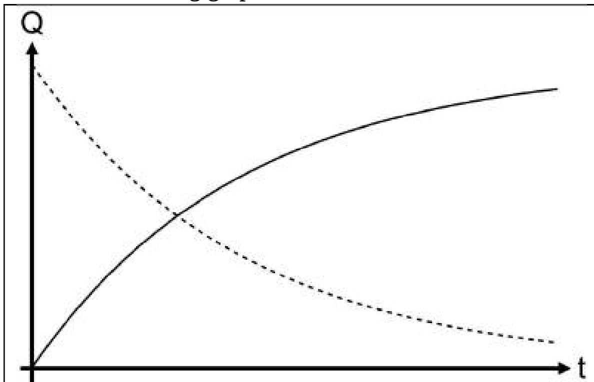
   a. If their capacitances are $\{4\mu,\ 5\mu \text{ and } 6\mu\}F$ respectively, determine their effective capacitance.
   b. If the potential difference across AB is 9V, calculate the stored energy.

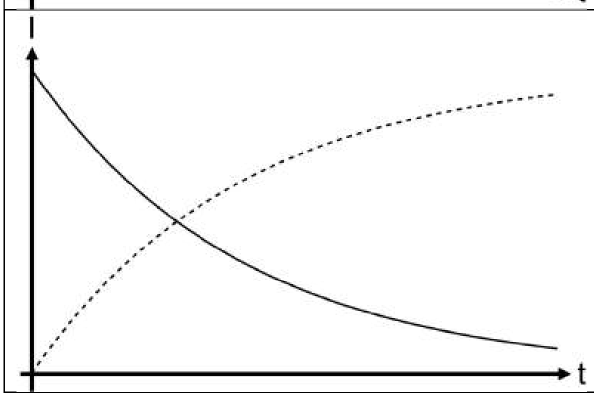
Charging & Discharging of Capacitors

How do you charge a capacitor?

What is time constant, τ?

Label the following graph:





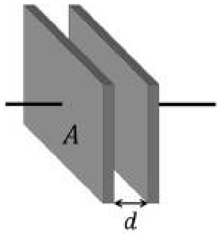
Example:

A circuit consists of a fully charged $150\mu F$ capacitor and 20Ω resistor. Determine the time it takes for the for the capacitor charge to

i. Drop to 20%.

ii. Drop by 20%.

<u>Parallel plate capacitor and Capacitance</u>

What happens to the capacitance of the capacitor when the space between the plates is vacuum?

What happens to the capacitance of the capacitor when the space between the plates is **not** vacuum?

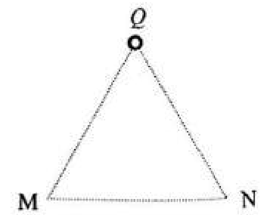
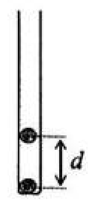
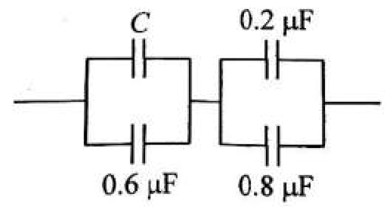
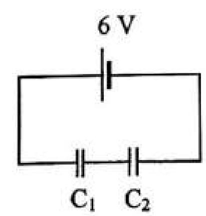
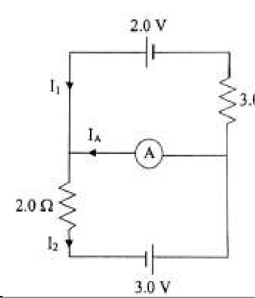
Relate between the permittivity of free space and permittivity of dielectric.

How does dielectric increase the capacitance of the capacitor?

Question:

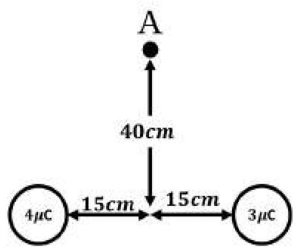
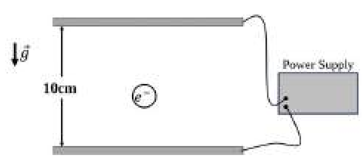
A vacuum filled capacitor has a capacitance of $120\mu F$, determine the capacitance of the capacitor when a dielectric material of dielectric constant of 2 is placed between the plates of the capacitor.

Chosen PYQ Chapter 1,2 and 3

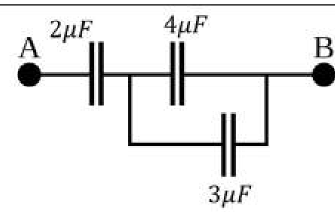
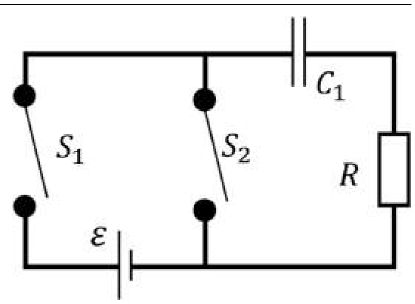
1	PSPM 13/14 – Q1(c) The figure shows a charge Q at the vertex of an equilateral triangle with sides 1mm. If 138J of work is done in bringing a – 4.8μC point charge from infinity to position M, a. determine the magnitude and type of charge Q. [$-3.19\mu C$] b. calculate the electric field at position N. [$6.27 \times 10^{10} NC^{-1}$] 
2	PSPM 14/15 – Q1(b) The figure shows a charged ball floating vertically above another charged ball at an equilibrium distance d apart in a test tube. If the charge on each ball is tripled, determine the new equilibrium distance between the balls in terms of d. [$3d$] 
3	PSPM 13/14 – Q1(b) The figure shows an arrangement of capacitors. If the equivalent capacitance is $0.5\mu F$, calculate the value of capacitor C. [$0.4\mu F$] 
4	PSPM 14/15 – Q1(c) The figure shows two parallel plate capacitors C_1 and C_2 connected in series and fully charged by a 6V battery. The capacitors have the same plate area and are filled with similar dielectric material. The distance of separation of the plates for C_1 and C_2 are 2mm and 4mm respectively. a. Calculate the potential difference across capacitor C_2. [4V] b. Calculate the electric field strength in the region between the plates of capacitor C_1. [$1kNC^{-1}$] 
5	PSPM 16/17 – Q2(ii) The figure shows a circuit consisting of two batteries, two resistors and an ammeter. If the ammeter has internal resistance of 5Ω, what is the reading shown by the ammeter? [0.16A] 
6	PSPM 13/14 – Q2 a. A toaster has a heating element made of nichrome wire and connected to a 220V source. The wire is initially at 20^oC with current 1.8A. When the toaster reaches its final operating temperature, the current is 1.53A. Calculate the i. power delivered to the toaster at its operating temperature. [336.6W] ii. final temperature of the heating element if the temperature coefficient of resistivity for nichrome wire is $4 \times 10^{-4}\ {}^{\circ}C^{-1}$. [$461^oC$] b. The emf of a battery with internal resistance is 12V. When an unknown resistor R is connected to the battery, the current is 0.8 A. If another resistor R is added in series, the current is 0.6A. Calculate the value of the resistor R and the internal resistance *r*. [5 Ω, 10 Ω]

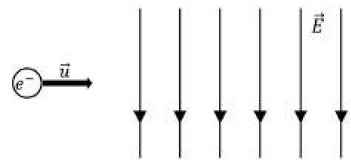
K2 Catch Up Sheet

Electrostatics

1D	Two charged particles of $+3\mu C$ and $+4\mu C$, in vacuum, are separated by 30cm. a. Determine the electrical force between them. b. Determine the electric field midpoint between them. c. Determine the electric potential experienced by a $-2\mu C$ charged particle is placed midpoint between them. d. Determine the electrical energy of the system.
2D	The diagram shows a system of charged particles.  Determine the a. Electrical force on a $-2\mu C$ charged particle placed at point A, b. Electrical field strength at point A, c. Electrical potential experienced by $-2\mu C$ charged particle placed at point A, d. Electric potential energy of the system with a $-2\mu C$ charged particle placed at point A.
Charge in a uniform $\vec{E}$	 An electron is placed between two plates as shown in the diagram. To keep the electron in place, the power supply is switched on to create an electric field. Determine the magnitude and direction of electric field between the plates to keep the electron in place.

Capacitors

Capacitance	Calculate the amount of charge stored in a capacitor of $2\mu F$ if the potential difference across the plates is 4V.
	Determine the capacitance of an air-filled parallel plate capacitor of surface area of $20cm^2$ and plate separation of 0.25cm.
Capacitors in Parallel	Two capacitors of capacitance $2\mu F$ and $3\mu F$ are connected in **parallel**. Determine the equivalent capacitance.
Capacitors in Series	Two capacitors of capacitance $2\mu F$ and $3\mu F$ are connected in **series**. Determine the equivalent capacitance.
Combination	 Three capacitors are connected as shown in the diagram. a. Determine the effective capacitance. b. Determine the energy stored between A and B if the potential difference between A and B is 4V.
Charging & Discharging	 The figure shows a circuit used to charge a $20mF$ capacitor. The resistor has a resistance of $2k\Omega$ and the dry cell outputs 12V, and switch S_1 is closed to charge the capacitor. a. Determine the time constant. b. Determine the time needed to charge the capacitor to 80% from 0%. Once fully charged, switch S_1 is opened and switch S_2 is closed. c. Determine the time needed to discharge the capacitor to 45% of its initial charge.

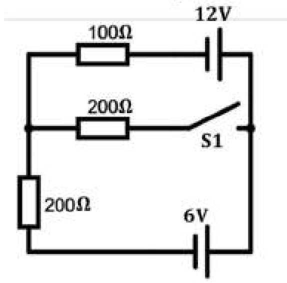
An electron is shot into a region of  electric field of $10^2 NC^{-1}$ as shown in the diagram. Disregarding gravity, a. Determine the acceleration of the electron in the electric field region. b. If the electron has an initial velocity of $20ms^{-1}$, determine the velocity of the electron 64μs upon entering the region of electric field.	Dielectrics	An air-filled parallel plate capacitor has a capacitance of $20\mu F$. If a material of dielectric constant of 3 is placed between the plates, determine the new capacitance of the capacitor.

Kirchoff’s Handout

Worksheet on Kirchhoff’s Rules

Question 1

The figure below shows a circuit consisting of 3 resistors, switch S1 and 2 dry cells.

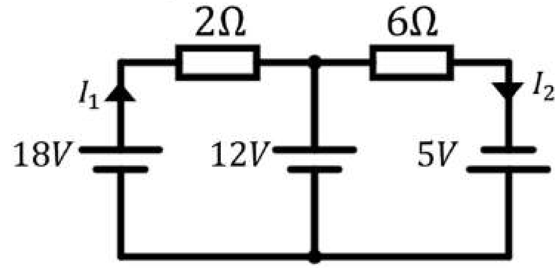


Determine the current through each resistor when switch S1 is closed.

$[I_{100\Omega} = 45mA; I_{200\Omega} = 37.5mA; I_{200\Omega} = 7.5mA]$

Question 2

The figure below shows a circuit consisting of 2 resistors and 3 dry cells.

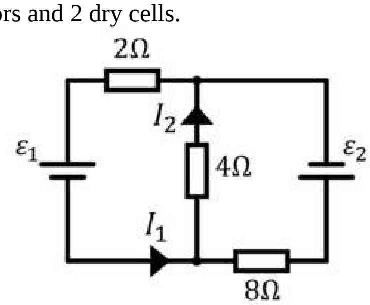


Determine the current I_1 and I_2.

$[I_1 = -3A; I_2 = -2.83A]$

Question 3

The figure below shows a circuit consisting of 3 resistors and 2 dry cells.

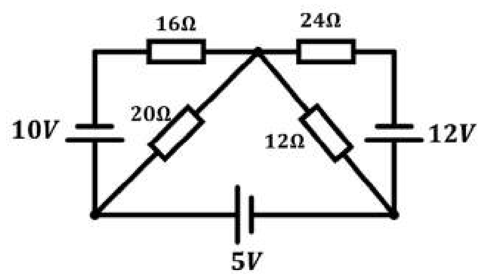


If $I_1 = 2.929A$ and $I_2 = 1.536A$, determine ε_2.

$[\varepsilon_2 = 5V]$

Question 4

The figure below shows a circuit consisting of 4 resistors and 3 dry cells.

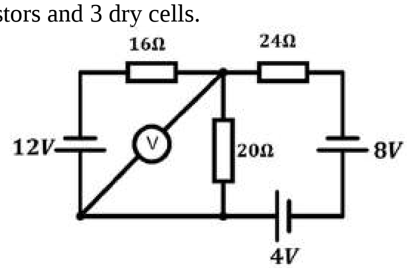


Determine the current through the 20Ω resistor.

$[I_{20\Omega} = 368.421mA]$

Question 5

The figure below shows a circuit consisting of 3 resistors and 3 dry cells.

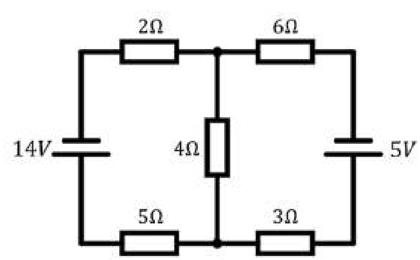


Determine the reading on the voltmeter.

$[8.108V]$

Question 6

The figure below shows a circuit consisting of 5 resistors and 2 dry cells.



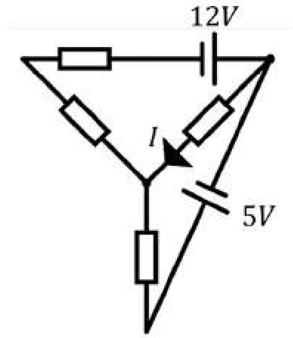
Which resistor consumes the most power?

[5Ω resistor at 8.136W]

Worksheet on Kirchhoff’s Rules – v2

Question 1

The figure shows a circuit consisting of 2 dry cells and 4 resistors of 4Ω each.

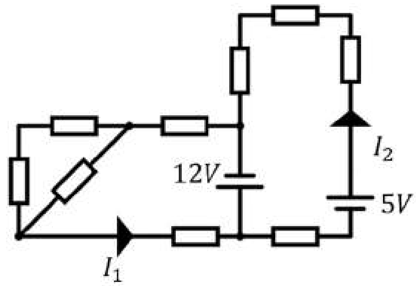


Determine current I.

$[I = 100mA]$

Question 2

The figure shows a circuit consisting of 2 dry cells and 9 resistors of 5Ω each.

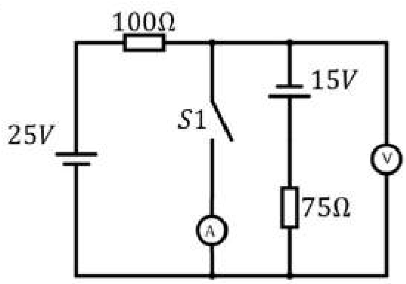


Determine the values of I_1 and I_2.

$[I_1 = -900mA; I_2 = 850mA]$

Question 3

The figure below shows a circuit consisting of an ammeter, a voltmeter, a switch S1, 2 resistors and 2 dry cells.

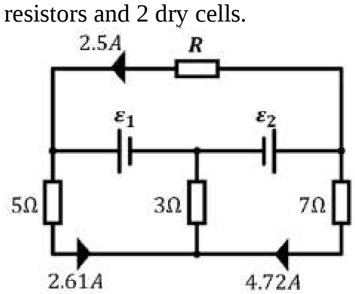


Find the reading of the voltmeter when the switch S1 is opened and when it is closed.

$[V_{closed} = 0V; I_{closed} = 50mA,$
$V_{open} = 2.143V; I_{open} = 0A]$

Question 4

The figure below shows a circuit consisting of an 4 resistors and 2 dry cells.

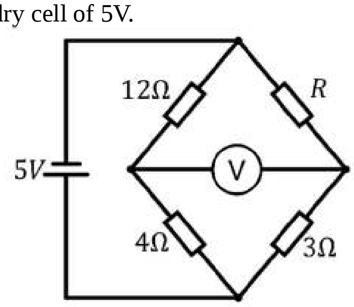


Calculate the values of R, ε_1 and ε_2.

$[R = 8\Omega; \varepsilon_1 = 35V; \varepsilon_2 = 55V]$

Question 5

The figure shows a circuit consisting of 4 resistors and 1 dry cell of 5V.

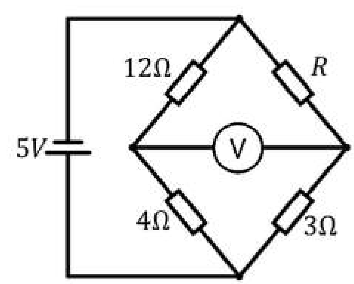


Determine the resistance of R if the voltmeter gives zero reading.

$[R = 9\Omega]$

Question 6

The figure shows a circuit consisting of 4 resistors and 1 dry cell of 5V.



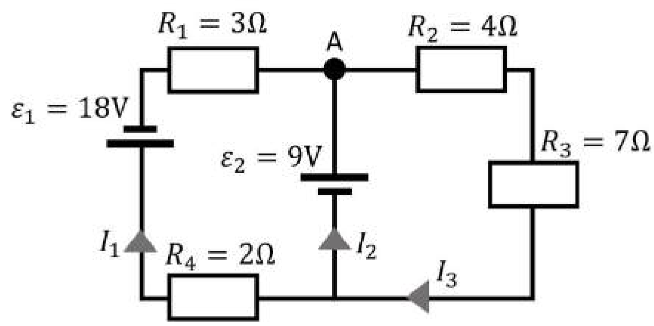
Determine the resistance of R if the voltmeter reads 250mV.

$[R = 7\Omega]$

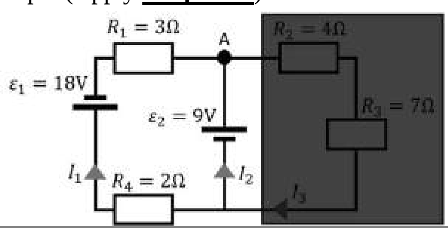
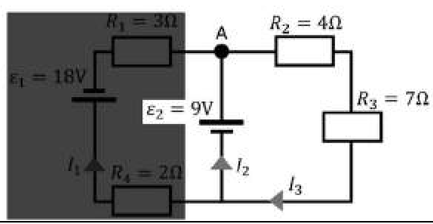
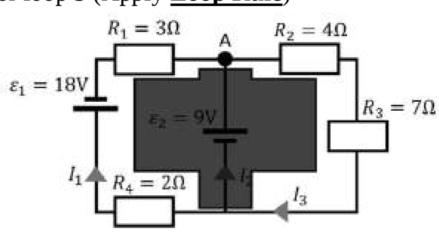
Kirchhoff's Example 1

Problem:

Calculate I_1, I_2 and I_3 for the circuit shown below.



Approach:

Step 1: Equation for current at junction A (Apply **Junction Rule**)	$I_1 + I_2 = I_3 \; - - - \; [1]$
Step 2: Equation for loop 1 (Apply **Loop Rule**) 	$\varepsilon_1 + \varepsilon_2 = I_1R_4 + I_1R_1$ $18 + 9 = 27 = -2I_1 - 3I_1$ $I_1 = -\frac{27}{5}A \; - - - \; [2]$
⇒ Equation for loop 2 (Apply **Loop Rule**) 	$\varepsilon_2 = I_3R_2 + I_3R_3$ $9 = 4I_3 + 7I_3$ $I_3 = \frac{9}{11}A \; - - - \; [3]$
⇒ Equation for loop 3 (Apply **Loop Rule**) 	$\varepsilon_1 = -I_1R_1 - I_1R_4 - I_3R_2 - I_3R_3$ $\varepsilon_1 = -I_1(R_1 + R_4) - I_3(R_2 + R_3)$ $18 = -I_1(3 + 2) - I_3(4 + 7)$ $18 = -5I_1 - 11I_3 \; - - - \; [4]$

Solving

Solve by choosing 3 equations from the table above and solve them simultaneously.

Choose equation [1], [2] and [3]

$I_1 = -\frac{27}{5}A \approx -5.4A$; $I_3 = \frac{9}{11}A \approx 0.819A$

$$I_1 + I_2 = I_3 \Rightarrow -\frac{27}{5} + I_2 = \frac{9}{11} \Rightarrow I_2 = -\frac{342}{55}A \approx 6.218A$$

$$I_1 \approx -5.4A;\ I_2 \approx 6.218A;\ I_3 \approx 0.819A$$

Checking

Check by inserting value to unused equation.

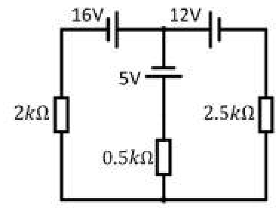
In this case, equation [4]

$\varepsilon_1 = -5I_1 - 11I_3 \Rightarrow \varepsilon_1 = -5(-5.4) - 11(0.819) = 17.991A \approx 18V$ (matches equation [4])

Kirchhoff's Example 2

Problem:

Calculate I_1, I_2 and I_3 for the circuit shown below.



$I_{2k\Omega} = 7.517mA$; $I_{0.5k\Omega} = 11.931mA$; $I_{2.5k\Omega} = 4.414mA$

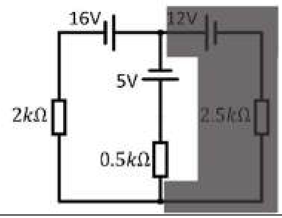
Approach:

Step 1:

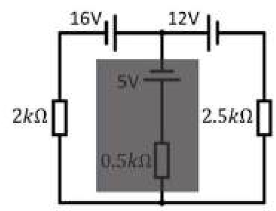
Equation for current at any junction (Apply **Junction Rule**)

Step 2:

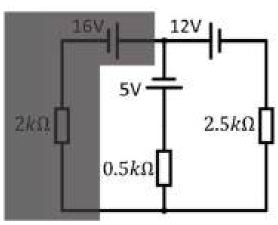
Equation for loop 1 (Apply **Loop Rule**)



Equation for loop 2 (Apply **Loop Rule**)



Equation for loop 3 (Apply **Loop Rule**)



Solving

Solve by choosing 3 equations from the table above and solve them simultaneously.

Checking

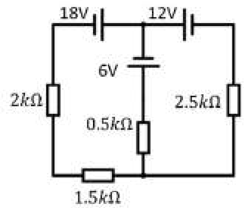
Check by inserting value to unused equation.

In this case, equation [4]

Kirchhoff's Example 3

Problem:

Calculate I_1, I_2 and I_3 for the circuit shown below.



$I_{2k\Omega} = I_{1.5k\Omega} = 8.684mA$; $I_{0.5k\Omega} = 13.237mA$; $I_{2.5k\Omega} = 4.553mA$

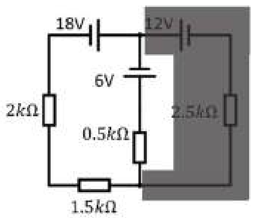
Approach:

Step 1:

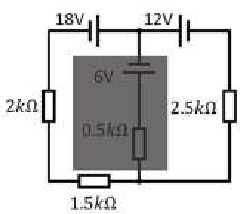
Equation for current at any junction (Apply **Junction Rule**)

Step 2:

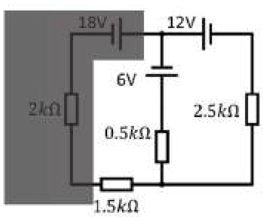
Equation for loop 1 (Apply **Loop Rule**)



Equation for loop 2 (Apply **Loop Rule**)



Equation for loop 3 (Apply **Loop Rule**)



Solving

Solve by choosing 3 equations from the table above and solve them simultaneously.

Checking

Check by inserting value to unused equation.

In this case, equation [4]

C5 K2T4 (8 Feb 2024)

1. A 20-turn coil of $0.5cm^2$ is placed flat on a piece of paper with exposure to magnetic field of 0.5T into the page. The magnetic field is angled 20^o from the piece of paper. The magnetic field strength is then doubled in 2ms. If the coil has a total resistance of 2Ω, determine the induced emf and induced current in the coil.
   [86mV; 43mA; ccw]
2. Two coils are wound coaxially at length 50cm and radius of 2cm. The primary coil has 1000turns and the secondary coil is 250turns. Determine the induced emf in the secondary coil is the primary coil undergoes a current change of 3As^{-1}.
   [2mV]
3. Determine the maximum induced emf generated by a 50-turn loop of $300cm^2$ area in a region of 0.05T magnetic field rotating at 60Hz.
   [28.274V]

Notes on Potentiometer and Wheatstone

From book/website

4. A **potentiometer** is a device to precisely measure potential differences or emf, using a **null** technique. In the simple potentiometer circuit shown in Fig. 19–94, R' represents the total resistance of the resistor from A to B (which could be a long uniform "slide" wire), whereas R represents the resistance of only the part from A to the movable contact at C. When the unknown emf to be measured, $\mathscr{E}_x$, is placed into the circuit as shown, the movable contact C is moved until the galvanometer G gives a null reading (i.e., zero) when the switch S is closed. The resistance between A and C for this situation we call R_x. Next, a standard emf, $\mathscr{E}_s$, which is known precisely, is inserted into the circuit in place of $\mathscr{E}_x$ and again the contact C is moved until zero current flows through the galvanometer when the switch S is closed. The resistance between A and C now is called R_s. Show that the unknown emf is given by

$$\mathscr{E}_x = \left(\frac{R_x}{R_s}\right)\mathscr{E}_s$$

where R_x, R_s, and $\mathscr{E}_s$ are all precisely known. The working battery is assumed to be fresh and to give a constant voltage.

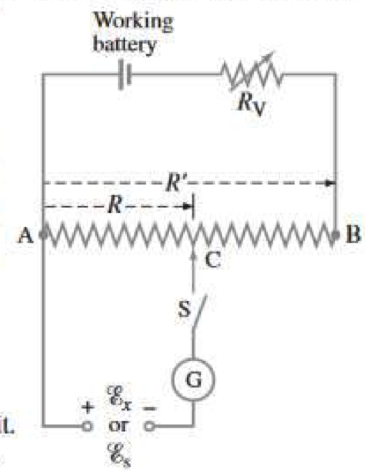


FIGURE 19–94
Potentiometer circuit.
Search and Learn 4.

The Wheatstone Bridge

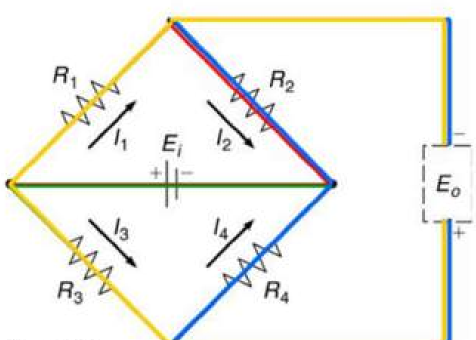


Figure 4.10

Using Kirchhoff's Voltage Law:

red loop: $E_i = I_1R_1 + I_2R_2$

green loop: $E_i = I_3R_3 + I_4R_4$

blue loop: $E_o = I_4R_4 - I_2R_2$

gold loop: $E_o = -I_3R_3 + I_1R_1$

Using Kirchhoff's Current Law:

$I_1 = I_2$ (at top node)

$I_3 = I_4$ (at bottom node).

Combining the above gives the Wheatstone Bridge (WB) equation:

$$E_o = E_i\left[\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4}\right]$$

2. Metre bridge

Metre bridge is one form of Wheatstone's bridge. It consists of thick strips of copper, of negligible resistance, fixed to a wooden board.

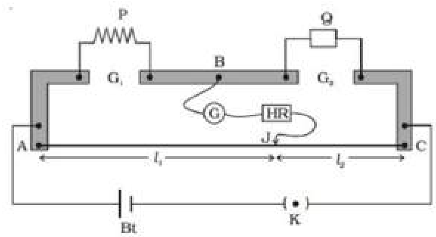


Fig 2.13 Metre bridge

There are two gaps G_1 and G_2 between these strips. A uniform manganin wire AC of length one metre whose temperature coefficient is low, is stretched along a metre scale and its ends are soldered to two copper strips. An unknown resistance P is connected in the gap G_1 and a standard resistance Q is connected in the gap G_2 (Fig 2.13). A metal jockey J is connected to B through a galvanometer (G) and a high resistance (HR) and it can make contact at any point on the wire AC. Across the two ends of the wire, a Leclanche cell and a key are connected.

Adjust the position of metal jockey on metre bridge wire so that the galvanometer shows zero deflection. Let the point be J. The portions AJ and JC of the wire now replace the resistances R and S of Wheatstone's bridge. Then

$$\frac{P}{Q} = \frac{R}{S} = \frac{r.AJ}{r.JC}$$

where r is the resistance per unit length of the wire.

$$\therefore \quad \frac{P}{Q} = \frac{AJ}{JC} = \frac{l_1}{l_2}$$

where $AJ = l_1$ and $JC = l_2$

$$\therefore \quad P = Q\frac{l_1}{l_2}$$

1. Principle of potentiometer

A battery Bt is connected between the ends A and B of a potentio- meter wire through a key K. A steady current I flows through the potentiometer wire (Fig 2.15). This forms the primary circuit. A primary cell is connected in series with the positive terminal A of the potentiometer, a galvanometer, high resistance and jockey. This forms the secondary circuit.

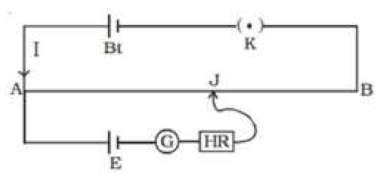


Fig 2.15 Principle of potentiometer

If the potential difference between A and J is equal to the emf of the cell, no current flows through the galvanometer. It shows zero deflection. AJ is called the balancing length. If the balancing length is l, the potential difference across AJ = Irl where r is the resistance per unit length of the potentiometer wire and I the current in the primary circuit.

$\therefore E = Irl,$

since I and r are constants, $E \propto l$

Hence emf of the cell is directly proportional to its balancing length. This is the principle of a potentiometer.

2. *Comparison of emfs of two given cells using potentiometer*

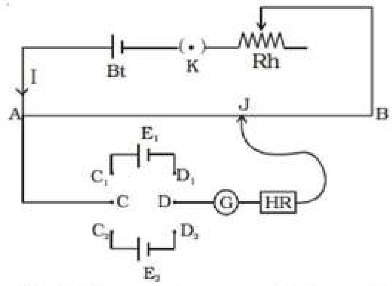


Fig 2.16 comparison of emf of two cells

The potentiometer wire AB is connected in series with a battery (Bt), Key (K), rheostat (Rh) as shown in Fig 2.16. This forms the primary circuit. The end A of potentiometer is connected to the terminal C of a DPDT the terminal C of a DPDT switch (six way key-double terminal D is connected to the jockey (J) through a galvanometer (G) and high resistance (HR). The cell of emf E_1connected between terminals C_1 and D_1 and the cell of emf E_2 connected between C_2 and D_2 of the DPDT switch.

Let I be the current flowing through the primary circuit and r be the resistance of the potentiometer wire per metre length.

The DPDT switch is pressed towards C_1, D_1 so that cell E_1 is included in the secondary circuit. The jockey is moved on the wire and adjusted for zero deflection in galvanometer. The balancing length is l_1. The potential difference across the balancing length $l_1 = Irl_1$. Then, by the principle of potentiometer,

$$E_1 = Irl_1 \qquad \ldots(1)$$

The DPDT switch is pressed towards E_2. The balancing length l_2 for zero deflection in galvanometer is determined. The potential difference across the balancing length is $l_2 = Irl_2$, then

$$E_2 = Irl_2 \qquad \ldots(2)$$

Dividing (1) and (2) we get

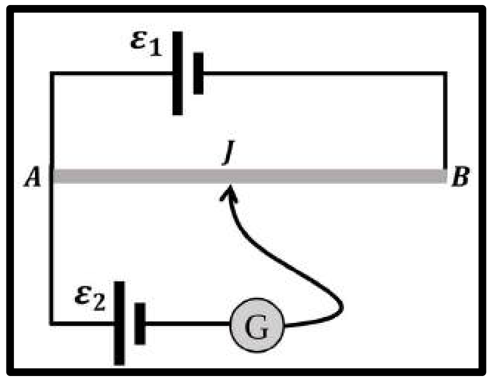
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

If emf of one cell (E_1) is known, the emf of the other cell (E_2) can be calculated using the relation.

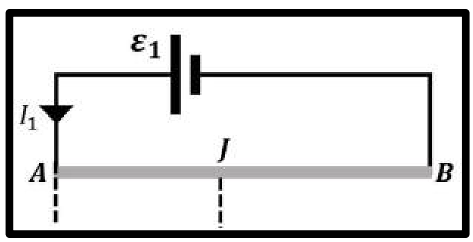
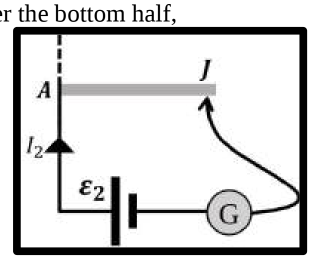
$$E_2 = E_1 \frac{l_2}{l_1}$$

Personal

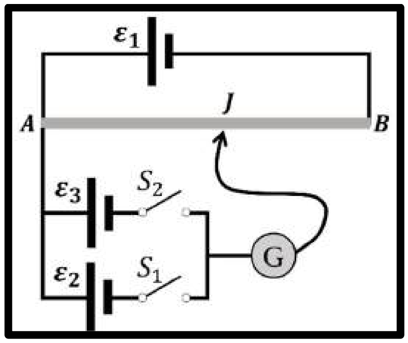
Consider the following circuit:



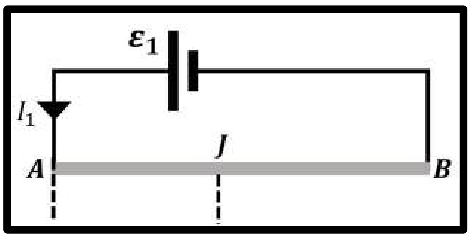
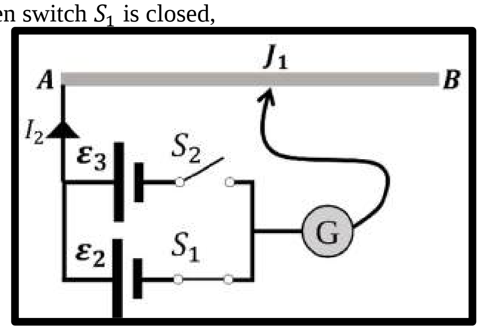
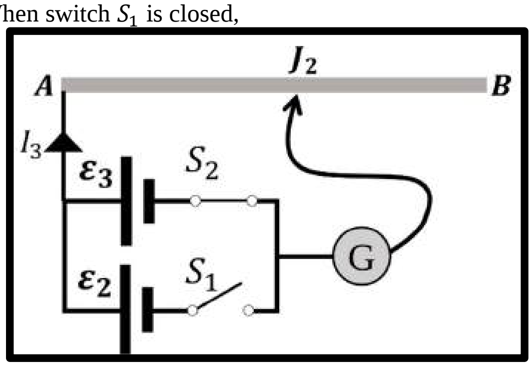
The grey coloured section is a bare wire of resistivity ρ , cross sectional area A and length of l.

If we consider the top part, 	We can see that the potential difference between A and B is $\varepsilon_1 = V_{AB} = I_1 R_{AB} = I_1\left(\frac{\rho l_{AB}}{A}\right) \Rightarrow I_1 = \frac{\varepsilon_1 A}{\rho l_{AB}}$
Now consider the bottom half, 	We can see that the potential difference between A and J is $\varepsilon_2 = V_{AJ} = I_2 R_{AJ} = I_2\left(\frac{\rho l_{AJ}}{A}\right) \Rightarrow I_2 = \frac{\varepsilon_2 A}{\rho l_{AJ}}$
When the galvanometer shows **null reading,** $I_1 - I_2 = 0 \Rightarrow \frac{\varepsilon_1 A}{\rho l_{AB}} - \frac{\varepsilon_2 A}{\rho l_{AJ}} = 0 \Rightarrow \left(\frac{A}{\rho}\right)(\frac{\varepsilon_1}{l_{AB}} - \frac{\varepsilon_2}{l_{AJ}}) = 0$ $\frac{\varepsilon_1}{l_{AB}} = \frac{\varepsilon_2}{l_{AJ}} \Rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{l_{AB}}{l_{AJ}}$	

Consider the following circuit:



The grey coloured section is a bare wire of resistivity ρ , cross sectional area A and length of l.

If we consider the top part,  	We can see that the potential difference between A and B is $$\varepsilon_1 = V_{AB} = I_1 R_{AB} = I_1\left(\frac{\rho l_{AB}}{A}\right) \Rightarrow I_1 = \frac{\varepsilon_1 A}{\rho l_{AB}}$$
Now consider the bottom half,	
When switch S_1 is closed,  	We can see that the potential difference between A and J is $$\varepsilon_2 = V_{AJ_1} = I_2 R_{AJ_1} = I_2\left(\frac{\rho l_{AJ_1}}{A}\right) \Rightarrow I_2 = \frac{\varepsilon_2 A}{\rho l_{AJ_1}}$$ When the galvanometer shows **null reading,** $$I_1 - I_2 = \frac{\varepsilon_1 A}{\rho l_{AB}} - \frac{\varepsilon_2 A}{\rho l_{AJ_1}} = 0 \Rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{l_{AB}}{l_{AJ_1}}$$
When switch S_1 is closed,  	We can see that the potential difference between A and J is $$\varepsilon_3 = V_{AJ_2} = I_3 R_{AJ_2} = I_3\left(\frac{\rho l_{AJ_2}}{A}\right) \Rightarrow I_3 = \frac{3A}{\rho l_{AJ_2}}$$ When the galvanometer shows **null reading,** $$I_1 - I_3 = \frac{\varepsilon_1 A}{\rho l_{AB}} - \frac{\varepsilon_3 A}{\rho l_{AJ_2}} = 0 \Rightarrow \frac{\varepsilon_1}{\varepsilon_3} = \frac{l_{AB}}{l_{AJ_2}}$$
To relate between ε_2 to ε_3, $$\frac{\varepsilon_2}{\varepsilon_3} = \left(\frac{\varepsilon_1}{\varepsilon_3}\right)\left(\frac{\varepsilon_2}{\varepsilon_1}\right) = \left(\frac{l_{AB}}{l_{AJ_2}}\right)\left(\frac{l_{AJ_1}}{l_{AB}}\right) \Rightarrow \frac{\varepsilon_2}{\varepsilon_3} = \frac{l_{AJ_1}}{l_{AJ_2}}$$	

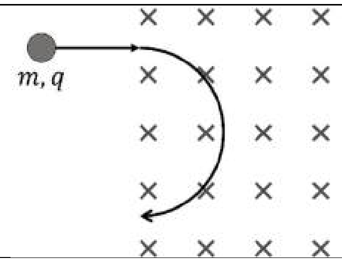
Chapter 4: Magnetic Field & Magnetic Force Problems

Problem 1

As shown in the diagram, a charged particle (of charge of magnitude $1.75 \times 10^{-15} C$ and mass of $1.8 \times 10^{-1}\ kg$ enters a uniform horizontal magnetic field of 0.35T and follows the path in a semicircle of diameter 12cm.

Find the speed of the particle and the sign of their charge.

[408ms^{-1}, negative]



Problem 2

Two parallel straight wires are placed 25cm apart carry currents 4A and 2A in opposite directions.

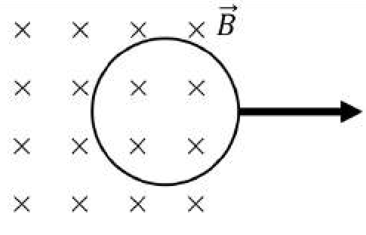
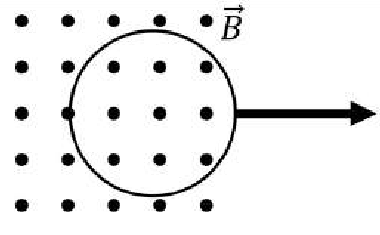
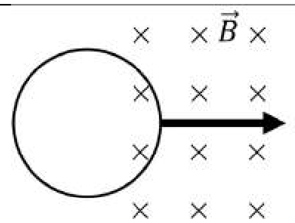
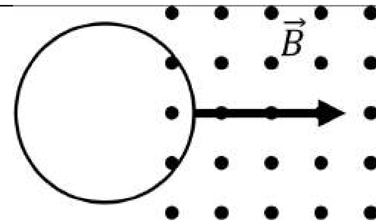
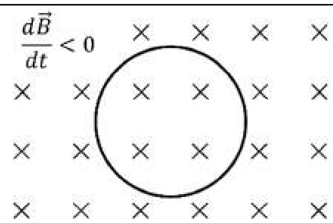
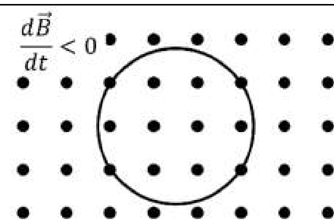
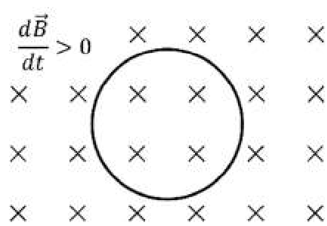
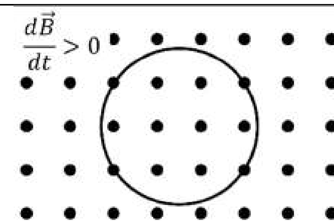
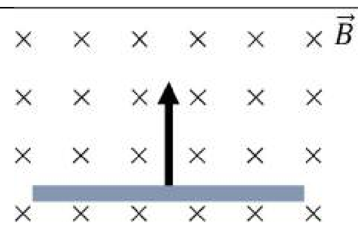
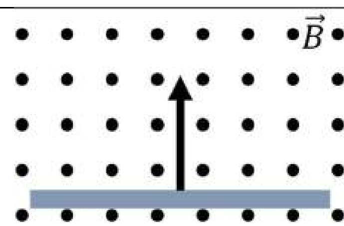
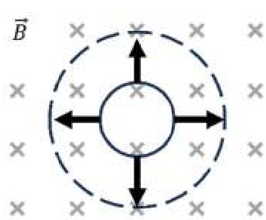
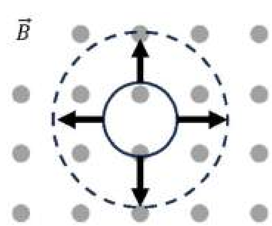
a. Calculate the force per unit length between the wires. **[$6.4 \times 10^{-6} Nm^{-1}$]**

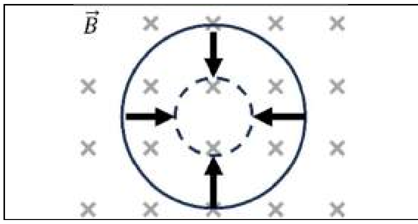
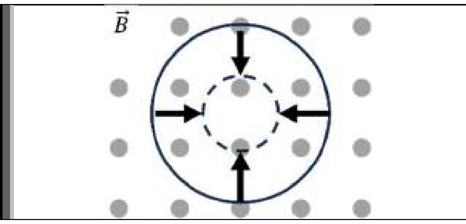
b. If the wire carrying 4A is increased to 5A, determine the current in the other wire to maintain the same force per unit length between the wires without changing their separation. **[1.6A]**

Faraday and Lenz's

Lenz's Law Practice Sheet

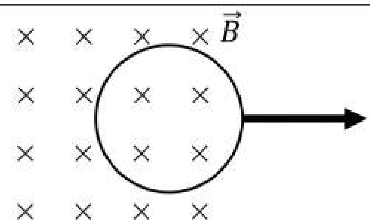
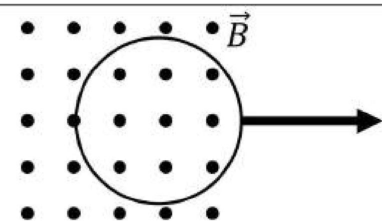
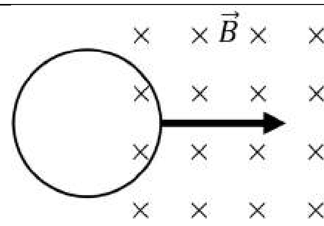
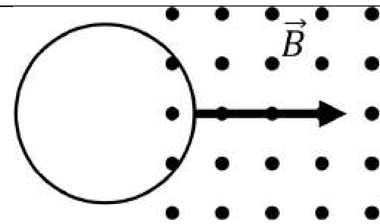
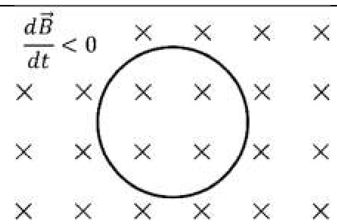
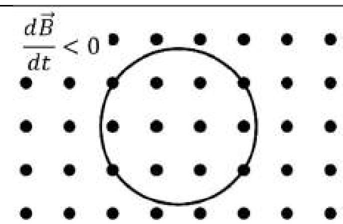
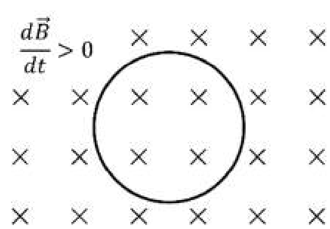
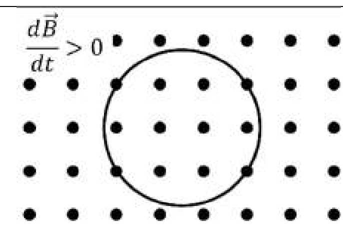
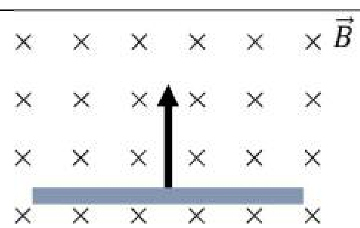
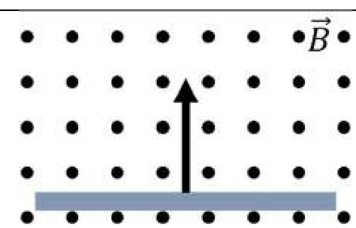
Determine the direction of induced current in each of the following cases:

Situation	Direction of $I_{induced}$	Situation	Direction of $I_{induced}$
			
			
			
			
			
			

Lenz's Law Practice Sheet - soln

Determine the direction of induced current in each of the following cases:

Situation	Direction of $I_{induced}$	Situation	Direction of $I_{induced}$
	**cw**		**acw**
	**acw**		**cw**
	cw		acw
	acw		cw
	left		right

Past Year on Magnetic Flux, Faraday’s & Lenz’s Law

1 **PSPM 13/14**

A rectangular coil of 60 turns, dimensions 0.1 m × 0.1 m and total resistance 10Ω, rotates with angular speed $30 rads^{-1}$ about the y-axis in a 1.5T magnetic field directed along the x-axis. Calculate the

a. maximum induced emf in the coil. **[27V]**

b. maximum rate of change of magnetic flux through the coil. **[$\mathbf{0.45 Wbs^{-1}}$]**

2 **PSPM 14/15**

a. A 0.2T magnetic field is directed parallel to the plane of a circular loop of radius 0.2m. Calculate the magnetic flux through the loop. **[0Wb]**

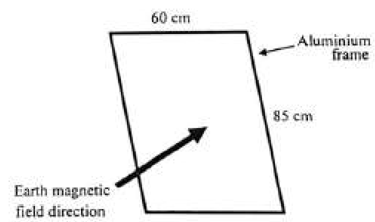
b. A coil of 100 turns and area $0.5 cm^2$ is placed in a changing magnetic field. The rate of change of magnetic field is $1.08 Ts^{-1}$. Calculate the induced emf in the coil. **[$\mathbf{5.4 mV}$]**

3 **PSPM 18/19**

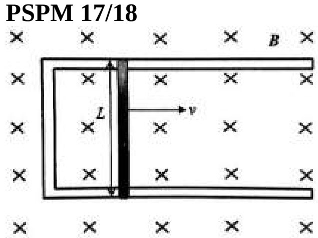
The diagram shows the Earth's magnetic field of 1.8×10^{-4} T normal to an aluminium frame of dimensions 60 cm x 85 cm.

a. Calculate the magnetic flux through the frame. **[$\mathbf{9.18 \times 10^{-5} Wb}$]**

b. The frame is flipped so that it is parallel to the Earth magnetic field in 0.2s. Calculate the induced emf. **[0.46mV]**



4 **PSPM 17/18**



The figure shows a conductor of length L = 0.065 m moves perpendicularly in a uniform magnetic field of 1.20T. The emf induced in the conductor is 0.32V.

a. Calculate the speed of the conductor. **[4.1m/s]**

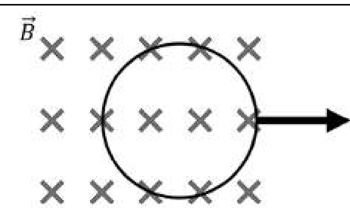
b. If the total circuit resistance is 0.8 Ω, determine the magnitude and direction of the induced current. **[0.4A; CCW]**

Extra Problems on Faraday’s & Lenz’s

Problem 1

The 13cm radius loop below has 20 windings and is pulled out from the 4T magnetic field in 0.018s. What is the induced EMF, and what direction does the current flow ?

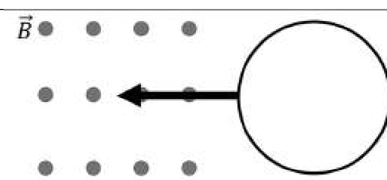
[235.969V, CW]



Problem 2

The 8cm diameter loop below has 22 windings is pulled from the 8T magnetic field generating an average EMF of 120V. What time did this take, and which direction did the current flow?

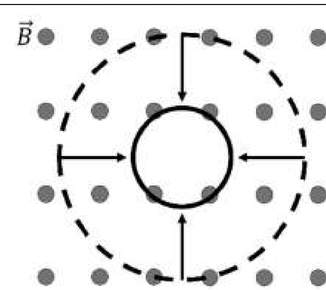
[0.0295s, ACW]



Problem 3

A loop (in 4T magnetic field) of 10cm diameter shrinks in until its diameter becomes 5cm, in 0.02s. Its windings are kept at 12. Determine the induced emf and the direction of induced current.

[14.1372V, CW]



K2T4 (8 Feb 2024)

4. A 20-turn coil of $0.5cm^2$ is placed flat on a piece of paper with exposure to magnetic field of 0.5T into the page. The magnetic field is angled 20^o from the piece of paper. The magnetic field strength is then doubled in 2ms. If the coil has a total resistance of 2Ω, determine the induced emf and induced current in the coil.
   [86mV; 43mA; ccw]
5. Two coils are wound coaxially at length 50cm and radius of 2cm. The primary coil has 1000turns and the secondary coil is 250turns. Determine the induced emf in the secondary coil is the primary coil undergoes a current change of 3As^{-1}.
   [2mV]
6. Determine the maximum induced emf generated by a 50-turn loop of $300cm^2$ area in a region of 0.05T magnetic field rotating at 60Hz.
   [28.274V]

Chosen Past Year Questions on Chapter 6

1. **13/14 PSPM**

   A voltmeter shows a reading of 220 V for a 50 Hz AC voltage.

   a. Calculate the maximum value of the AC voltage. **[311V]**

   b. Write the equation for the AC voltage.

2. **13/14 PSPM**

   A 0.9 kΩ resistor, 0.25 μC capacitor and 2.5 H inductor are connected in series across a 240 Hz AC source with 140 V peak voltage.

   a. Calculate the impedance. **[1434.7Ω]**

   b. Calculate the maximum current. **[0.098A]**

   c. Calculate the phase angle between the current and voltage. $[\mathbf{51.15^o}]$

   d. Which quantity lags: current or voltage? Explain your answer.

   e. How to achieve resonance in the circuit?

3. **14/15 PSPM**

   An RL series circuit with a 0.056 H inductor and 250 Ω resistor is connected with a source of peak voltage 240 V at the frequency 200 Hz. Calculate the

   a. inductive reactance of the circuit. **[70.4Ω]**

   b. impedance of the circuit **[259.7Ω]**

   c. power factor for this circuit **[0.963]**

   d. rms voltage of the source **[170V]**

   e. average power delivered by the source **[107.2W]**

Geometrical Optics (Reflections) Practice – K1

Problem 1
A shaving or makeup mirror is designed to magnify your face by a factor of 1.75 when your face is placed 15cm in front of it. Calculate the required radius of curvature for the mirror. **[70cm]**

Problem 2
You look at yourself in a shiny 8.8cm diameter Christmas tree ball. If your face is 25cm away from the ball's front surface, where is your image? Is it real or virtual? Is it upright or inverted?
[$\boldsymbol{v = -2cm}$; upright, virtual image]

Problem 3
The image of a distant tree is virtual and very small when viewed in a curved mirror. The image appears to be 19cm behind the mirror. What kind of mirror is it, and what is its radius of curvature?
[convex, $\boldsymbol{r = -38cm}$]

Geometrical Optics (Reflections) Practice – K2T3

Problem 1
A 2cm high object is situated 15.0 cm in front of a concave mirror that has a radius of curvature of 10cm. Determine image distance and height of image. **[$\boldsymbol{v = 7.5cm, h_i = 1cm}$]**

Problem 2
A concave mirror has a focal length of 40cm. Determine the object position for which the resulting image is upright and four times the size of the object. **[36cm]**

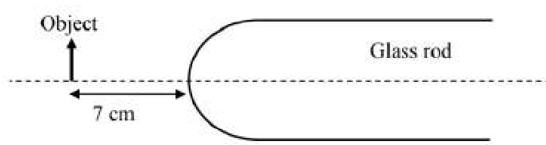
Problem 3
A concave mirror has a radius of curvature of 60cm. Calculate the image position and magnification of an object placed in front of the mirror at distances of 90cm and 20cm. **[{$\boldsymbol{45cm, M = -0.5}$},{$\boldsymbol{-60cm, M = 3}$}]**

Self-Study Session (24/2/2024)

Problem 1
The radius of curvature of a convex mirror is 10 cm. A virtual image formed 4 cm away from the mirror.

i. Calculate the focal length of the mirror. **[-5cm]**
ii. Calculate the object distance. **[20cm]**
iii. Calculate the height of the image if the height of the object is 15cm. **[3cm]**

Problem 2
An object is placed 7cm from a glass rod with hemispherical tip as shown in the figure. If the radius of curvature and refractive index of glass rod is 15cm and 1.52 respectively, calculate the image distance. **[-14.05cm]**



Problem 3
A point object is placed at the centre of a glass sphere of radius 4cm and refractive index of 1.5. Calculate the image distance of the point object. **[-4cm]**

Double Slit

$$y_m = \frac{m\lambda D}{d}$$

Problem 1
A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.5mm, and the interference pattern on a screen 3.3m away shows the first side maximum 3.4mm from the center of the pattern. What is the wavelength?
[515nm]

Solution

$$y_{\text{bright}} = \frac{\lambda L}{d} m,\ m = 1,\ \lambda = \frac{yd}{L} = \frac{(3.40 \times 10^{-3}\ \text{m})(5.00 \times 10^{-4}\ \text{m})}{3.30\ \text{m}} = \boxed{515\ \text{nm}}$$

Problem 2
Young's double-slit experiment is performed with 589nm light and a distance of 2m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.
[1.55mm]

Solution

In the equation $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$.

The first minimum is described by $m = 0$

and the tenth by $m = 9$: $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$.

Also, $\tan\theta = \frac{y}{L}$

but for small θ, $\sin\theta \approx \tan\theta$.

Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$

$$d = \frac{9.5(5\,890 \times 10^{-10}\ \text{m})(2.00\ \text{m})}{7.26 \times 10^{-3}\ \text{m}} = 1.54 \times 10^{-3}\ \text{m} = \boxed{1.54\ \text{mm}}.$$

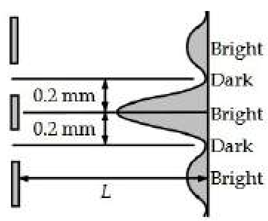
Problem 3
Light with wavelength 442 nm passes through a double-slit system that has a slit separation d = 0.400 mm. Determine how far away a screen must be placed in order that a dark fringe appear directly opposite both slits, with just one bright fringe between them.
[36.2cm]

Solution

$m = 0$ and $y = 0.200$ mm

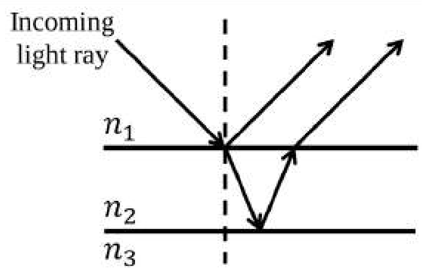
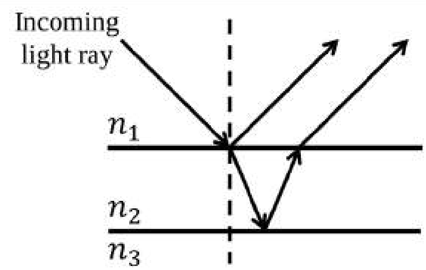
$$L \approx \frac{2dy}{\lambda} = \frac{2(0.400 \times 10^{-3}\ \text{m})(0.200 \times 10^{-3}\ \text{m})}{442 \times 10^{-9}\ \text{m}} = 0.362\ \text{m}$$

$$L \approx \boxed{36.2\ \text{cm}}$$



Thin Films Interference

Phase change upon reflection

Diagrams		
Conditions	Case 1:	Case 2:
Phase difference	$\Delta\phi =$	$\Delta\phi =$

Constructive and Destructive Interference

	Types	Position
Case 1: Non-reflective coating	Bright Fringes	
	Dark Fringes	
Case 2: Reflective coating	Bright Fringes	
	Dark Fringes	

<u>Example problem:</u>

1. **<u>[PSPM 19/20]</u>**
   A soap film with refractive index 1.3 and minimum thickness $0.177\mu m$ appears reddish under white light. Calculate the wavelength of light that is missing from the reflection. **[Ans: 460.2nm, 230.1nm, 153.4nm]**

2. **<u>Homework:</u>**
   - **a.** **<u>[PSPM 15/16]</u>**
     White light is incident on a soap film of refractive index 1.33 in air. The reflected light looks bluish because the red light of wavelength 670 nm is absent in the reflection.
     - i. Does the light change phase when it reflects at air-film interface? Explain your answer.
     - ii. Does the light change phase when it travels in film and reflects at film-air interface?
     - iii. What happen to the wavelength and frequency of light when it travels from air to the film?
     - iv. Determine the minimum thickness of the soap film. **[Ans: 252.nm]**
   - **b.** A material having a refractive index of 1.33 is used as an antireflective coating on a piece of glass (n=1.50). What should be the minimum thickness of this film to minimize reflection of 660nm light? **[Ans: 124.1nm]**

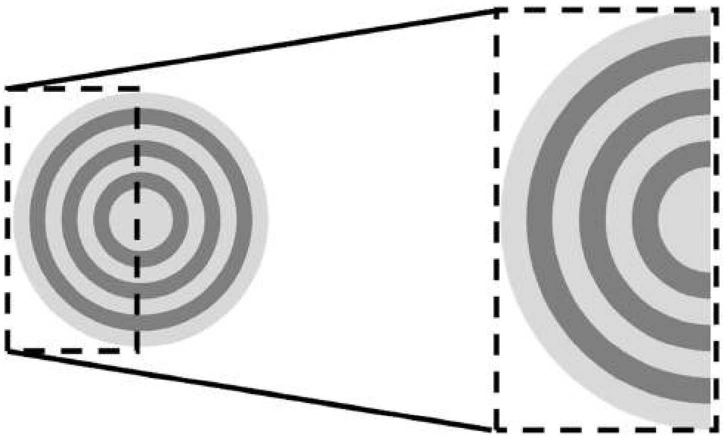
Single Slit Diffraction

Condition for diffraction	

Constructive and destructive interference

Types	Position
Bright Fringes	
Dark Fringes	

Diffraction pattern

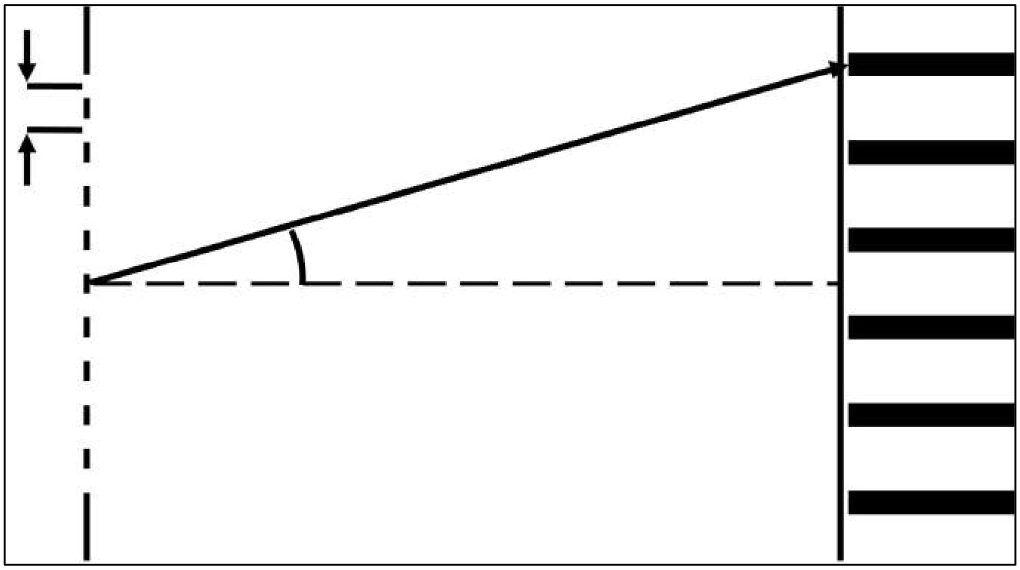


Determination of central bright fringe size

Example Problem:

1. **[PSPM 20/21]**
   A monochromatic light of wavelength 620 nm is incident on a single slit and forms a diffraction pattern on a screen 1.2m away. The distance of seventh dark fringe from the central maximum is 18.0 mm. Determine,
   - **a.** the size of the single slit **[Ans: 0.289mm]**
   - **b.** distance of the second bright fringe from the central maximum **[Ans: 6.44mm]**
2. **[Homework]**
   - **a.** A beam of light is diffracted by a slit of width 0.75 mm. The diffraction pattern forms on a wall 3.5m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 6.2mm. **[Ans: 664.3nm]**
   - **b.** Light of wavelength 600nm illuminates a single slit 0.55mm in width.
     - **i.** At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 1.5mm from the centre of the principal maximum? **[Ans: 1.375m]**
     - **ii.** What is the width of the central maximum? **[Ans:3.0mm]**

Diffraction Grating



Type of interferences	Equation
Destructive	
Constructive	

Example Problem:

1. When light illuminates a grating with 6500 lines per centimetre, its second order maximum on a distance screen is at 62.4^o. What is the wavelength of light? **[Ans: 681.7nm]**
2. In a diffraction experiment, light of 700nm wavelength produces a second-order maximum 4.8mm from the central maximum on a screen 2m away from the grating. Determine the slit separation in the diffraction grating. **[Ans: 0.58mm]**
3. **[Homework]**
    a. In a diffraction experiment, light of 660nm wavelength produces a first-order maximum 0.35mm from the central maximum on a distance screen. A second monochromatic source produces a third order maximum 0.87mm from the central maximum when it passes through the same diffraction grating. What is the wavelength of the light for the second source? **[Ans: 546.86nm]**
    b. A diffraction grating has 2000 lines per centimetre. A monochromatic light of wavelength 500 nm is incident normally on the grating. What is the angular separation between the first and second order maxima?
    [Ans: 2.865^o]

9.3 Particle Accelerator

a) State the thermionic emission.
b) Explain the acceleration of particle by electric and magnetic field.
c) State the role of electric and magnetic field in particle accelerators (linac and cyclotron) and detectors (general principles of ionisation and deflection only).
d) State the need of high energies required to investigate the structure of nucleon.

9.4 Fundamental Particle

a) Explain the standard quark-lepton model particles (baryons, meson, leptons and hadrons).
b) Explain the corresponding antiparticle for every particle.

Part 1:

1. What is thermionic emission?
(pg 5, https://core.ac.uk/download/pdf/4382002.pdf)
2. How does particle accelerators generally work?
(Chapter 1, part II, https://rb.gy/ohd4ib)
3. Describe a linear particle accelerator.
(https://indico.cern.ch/event/1029085/contributions/4368779/attachments/2291662/3896898/latvia_school_07_21_lecture2_linacs.pdf , https://indico.cern.ch/event/1324427/contributions/5573255/attachments/2730865/4747247/Tsesmelis%20Lecture%201%20Introduction.pdf)
4. Describe a cyclotron.
(https://indico.cern.ch/event/1324427/contributions/5573255/attachments/2730865/4747247/Tsesmelis%20Lecture%201%20Introduction.pdf)
5. Compare a cyclotron to a linac.
6. Briefly (approximately in 2 paragraphs) describe the history of nuclear structure research.

Part 2:

1. What are the two major classes of particles that make up the matter of the universe?
2. Name six types, or flavours, of each class of particles.
3. What are the four known fundamental forces in the universe?
4. Name the particles that carry the forces in question 3
5. Which force is much weaker than the other three?
6. What property characterizes all hadrons?
7. What property characterizes all baryons?
8. What property characterizes all mesons?

1) The two major classes of fundamental particles are quarks and leptons.
2) Quarks: up, down, strange, charm, bottom, and top.
Leptons: electron, muon, tau, electron neutrino, muon neutrino, and tau neutrino.
3) Gravity, electromagnetic, weak nuclear, and strong nuclear.
4) Gravity is carried by the graviton; the electromagnetic force is carried by the photon; the weak nuclear force is carried by the W^+, W^-, and Z^0 bosons; the strong nuclear force is carried by the gluon. The gravitational force is much weaker than the other three forces.
5) Hadrons interact via the strong nuclear force (as well as the other three fundamental forces) and are made up of quarks.
6) Baryons are hadrons, are made of three quarks, and have a baryon number of either +1 or −1.
7) Mesons are hadrons, are made of one quark and one antiquark, and have a baryon number of 0.

On Extra Chapter 5

23/4/2024

Problem 1

A series AC circuit contains the following components:

R=150Ω, L=250mH, C=2μF and a source with $V_{max} = 210V$ operating at 50.0 Hz.

Calculate the

a) inductive reactance,
b) capacitive reactance,
c) impedance,
d) maximum current,
e) phase angle between current and source voltage.

[78.5Ω; 1.59kΩ; 1.52kΩ; 138mA; -84.3^o]

Problem 2

A coil of resistance 35Ω and inductance 20.5H is in series with a capacitor and a 200V, 100-Hz source. The rms current in the circuit is 4A.

a) Calculate the capacitance in the circuit.
b) What is V_{rms} across the coil?

[124nF; 51.5kV]

Problem 3

The variable capacitor in the tuner of an AM radio has a capacitance of 2800pF when the radio is tuned to a station at 580kHz. What must be the capacitance for a station at 1600kHz? What is the inductance?

[370pF; 27μH]

Problem 4

An RLC circuit connected to a 120V generator consists of a 160Ω resistor, a 0.45μF capacitor, and a 7.56mH inductor in series. The current in the circuit is 0.65A. Find the two possible values for the frequency that corresponds to this current.

[3865Hz, 1926Hz]

Problem 5

An RLC circuit at 60Hz has an impedance of 580Ω and a phase angle of -60^o. If the inductor has an inductance of 0.8H, determine the capacitance of the capacitor.

[$3.3\mu F$]

Problem 6

An RLC circuit consist of a 60Ω resistor, a 45mH inductor and a 40μF capacitor connected in series to the AC source which has a peak voltage of 120V and a frequency of 1.6 kHz.

Calculate the rms current and power factor.

[0.187A; 0.991]

SP025 2023/2024 QUIZZES

Table of Contents

QUIZ 1: ELECTROSTATICS (20 MARKS) 3

QUIZ 2: CAPACITORS & RESISTANCES (20 MARKS) 5

QUIZ 3: INTERNAL RESISTANCE & KIRCHHOFF'S RULES 7

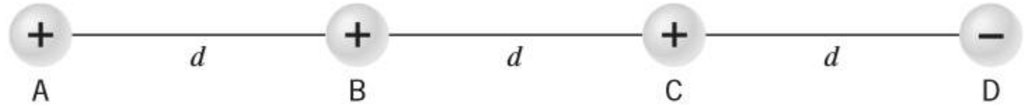
QUIZ 4: FARADAY'S, LENZ'S LAW & AC CIRCUITS 8

QUIZ 5: GEOMETRICAL OPTICS 9

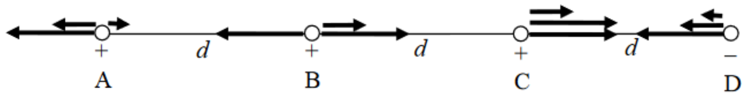
QUIZ 6: PHYSICAL OPTICS 10

Quiz 1: Electrostatics (20 marks)

1 Four-point charges have equal magnitudes. Three are positive, and one is negative, as the drawing shows.

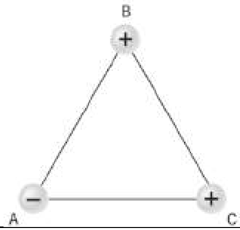


They are fixed in place on the same straight line, and adjacent charges are equally separated by a distance *d*. Consider the net electrostatic force acting on each charge.
Calculate the ratio of the largest to the smallest net force.



$$\frac{(\Sigma F)_{\mathrm{C}}}{(\Sigma F)_{\mathrm{B}}} = \frac{k\frac{|q|^2}{d^2}+k\frac{|q|^2}{d^2}+k\frac{|q|^2}{(2d)^2}}{k\frac{|q|^2}{d^2}-k\frac{|q|^2}{d^2}+k\frac{|q|^2}{(2d)^2}} = \frac{1+1+\frac{1}{4}}{\frac{1}{4}} = \boxed{9.0}$$

2 Three-point charges have equal magnitudes, two being positive and one negative. These charges are fixed to the corners of an equilateral triangle, as the drawing shows. The magnitude of each of the charges is 5μC, and the lengths of the sides of the triangle are 3cm.
Calculate the magnitude of the net force that the charge at *A* experiences.



$$F_{\mathrm{AB}} = F_{\mathrm{AC}} = F$$

$$(\Sigma F_x)_{\mathrm{A}} = F_{\mathrm{AB}}\cos 60.0^\circ + F_{\mathrm{AC}} = F(\cos 60.0^\circ + 1)$$

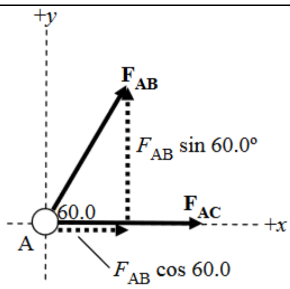
$$(\Sigma F_y)_{\mathrm{A}} = F_{\mathrm{AB}}\sin 60.0^\circ = F\sin 60.0^\circ$$

$$(\Sigma F)_{\mathrm{A}} = \sqrt{(\Sigma F_x)_{\mathrm{A}}^2 + (\Sigma F_y)_{\mathrm{A}}^2}$$

$$= \sqrt{F^2(\cos 60.0^\circ + 1)^2 + (F\sin 60.0^\circ)^2}$$

$$= F\sqrt{(\cos 60.0^\circ + 1)^2 + (\sin 60.0^\circ)^2}$$

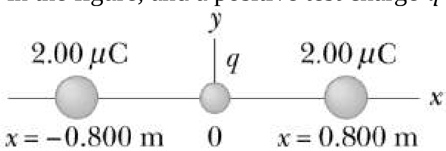
$$= k\frac{|q|^2}{L^2}\sqrt{(\cos 60.0^\circ + 1)^2 + (\sin 60.0^\circ)^2}$$



$$(\Sigma F)_{\mathrm{A}} = \left(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(5.0\times 10^{-6}\ \mathrm{C}\right)^2}{(0.030\ \mathrm{m})^2}\sqrt{(\cos 60.0^\circ + 1)^2 + (\sin 60.0^\circ)^2}$$

$$= \boxed{430\ \mathrm{N}}$$

3 Given two 2μC charges, as shown in the figure, and a positive test charge $q = 1.28 \times 10^{-18} C$ at the origin,



a. What is the electric field at the origin due to the two 2μC charges?
b. What is the electric potential at the origin due to the two 2μC charges?

(a) Since the charges are equal and placed symmetrically, $\boxed{E=0}$.

(b) $$V = 2k_e\frac{q}{r} = 2\left(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{2.00\times 10^{-6}\ \mathrm{C}}{0.800\ \mathrm{m}}\right)$$

$$V = 4.50\times 10^4\ \mathrm{V} = \boxed{45.0\ \mathrm{kV}}$$

4 Suppose an electron is released from rest in a uniform electric field whose magnitude is $5.9 \times 10^3 Vm^{-1}$.
(a) Through what potential difference will it have passed after moving 1.00 cm?
(b) How fast will the electron be moving after it has travelled 1.00 cm?

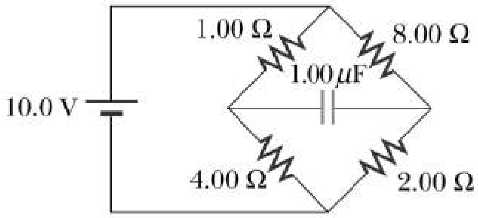
	(a) $\lvert\Delta V\rvert = Ed = (5.90\times10^{3}\ \text{V/m})(0.0100\ \text{m}) = \boxed{59.0\ \text{V}}$ (b) $\frac{1}{2}mv_f^2 = \lvert q\Delta V\rvert$: $\frac{1}{2}(9.11\times10^{-31})v_f^2 = (1.60\times10^{-19})(59.0)$ $v_f = \boxed{4.55\times10^{6}\ \text{m/s}}$
5	A proton moves at $4.50\times10^{5} ms^{-1}$ in the horizontal direction. It enters a uniform vertical electric field with a magnitude of $9.6 kNC^{-1}$. Ignoring any gravitational effects, find a. its vertical displacement during the time interval in which it travels 5.00 cm horizontally, b. the horizontal and vertical components of its velocity after it has travelled 5.00 cm horizontally.
	(a) $t = \frac{x}{v_x} = \frac{0.0500}{4.50\times10^{5}} = 1.11\times10^{-7}\ \text{s} = 111\ \text{ns}$ $a_y = \frac{qE}{m} = \frac{(1.602\times10^{-19})(9.60\times10^{3})}{(1.67\times10^{-27})} = 9.21\times10^{11}\ \text{m/s}^2$ $y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$: $y_f = \frac{1}{2}(9.21\times10^{11})(1.11\times10^{-7})^2 = 5.68\times10^{-3}\ \text{m} = \boxed{5.68\ \text{mm}}$ (b) $v_x = \boxed{4.50\times10^{5}\ \text{m/s}}$ $v_{yf} = v_{yi} + a_y t = (9.21\times10^{11})(1.11\times10^{-7}) = \boxed{1.02\times10^{5}\ \text{m/s}}$

Quiz 2: Capacitors & Resistances (20 marks)

1 The circuit in the figure has been connected for a long time.

a. What is the voltage across the capacitor?

b. If the battery is disconnected, how long does it take the capacitor to discharge to one tenth of its initial voltage?

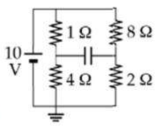


(a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.

$V_L = 8.00$ V because of voltage divider:

$$I_L = \frac{10.0\text{ V}}{5.00\ \Omega} = 2.00\text{ A}$$

$$V_L = 10.0\text{ V} - (2.00\text{ A})(1.00\ \Omega) = 8.00\text{ V}$$



$$V_R = \left(\frac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega}\right)(10.0\text{ V}) = 2.00\text{ V}$$

$$I_R = \frac{10.0\text{ V}}{10.0\ \Omega} = 1.00\text{ A}$$

$$V_R = (10.0\text{ V}) - (8.00\ \Omega)(1.00\text{ A}) = 2.00\text{ V}.$$

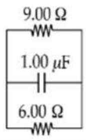
$$\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\text{ V}}.$$

(b) $$R = \frac{1}{(1/9.00\ \Omega) + (1/6.00\ \Omega)} = 3.60\ \Omega$$

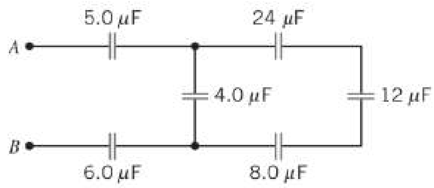
$$RC = 3.60 \times 10^{-6}\text{ s}$$

$$e^{-t/RC} = \frac{1}{10}$$

$$t = RC\ln 10 = \boxed{8.29\ \mu\text{s}}.$$



2 Determine the equivalent capacitance between A and B for the group of capacitors in the drawing.



$$\frac{1}{C_s} = \frac{1}{24\ \mu\text{F}} + \frac{1}{12\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} \quad \text{or} \quad C_s = 4.0\ \mu\text{F}$$

$$C_p = 4.0\ \mu\text{F} + 4.0\ \mu\text{F} = 8.0\ \mu\text{F}$$

$$\frac{1}{C_s} = \frac{1}{5.0\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}} \quad \text{or} \quad C_s = \boxed{2.0\ \mu\text{F}}$$

3 A solid cube of silver (density $= 10.5gcm^3$) has a mass of 90.0 g.

a. What is the resistance between opposite faces of the cube?

b. Determine the current across opposite faces of the cube if the potential difference between the faces is 10μV.

(a) $$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\ell^2} = \frac{\rho}{\ell} = \frac{1.59 \times 10^{-8}\ \Omega\cdot\text{m}}{2.05 \times 10^{-2}\text{ m}} = 7.77 \times 10^{-7}\ \Omega = \boxed{777\text{ n}\Omega}$$

(b) $$I = \frac{\Delta V}{R} = \frac{1.00 \times 10^{-5}\text{ V}}{7.77 \times 10^{-7}\ \Omega} = \boxed{12.9\text{ A}}$$

4 A certain lightbulb has a tungsten filament with a resistance of 19Ω when cold and 140Ω when hot. Assume that the resistivity of tungsten varies linearly with temperature even over the large temperature range involved here, and find the temperature of the hot filament. Assume the initial temperature is 20.0°C. The temperature coefficients of resistivity for tungsten is $4.5 \times 10^{-3\,o}C^{-1}$.

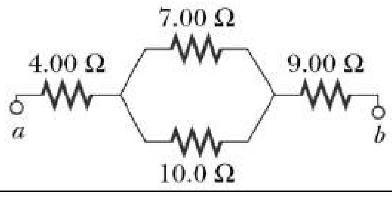
$$R = R_0\left[1 + \alpha(\Delta T)\right]$$

$$140\ \Omega = (19.0\ \Omega)\left[1 + \left(4.50 \times 10^{-3}/{}^\circ\text{C}\right)\Delta T\right].$$

$$\Delta T = 1.42 \times 10^3\ {}^\circ\text{C} = T - 20.0{}^\circ\text{C}.$$

$$\boxed{T = 1.44 \times 10^3\ {}^\circ\text{C}}.$$

5 A potential difference of 34V is applied between points a and b. Calculate the current in each resistor.



	(a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$ $R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$ (b) $\Delta V = IR$ $34.0\ \text{V} = I(17.1\ \Omega)$ $I = \boxed{1.99\ \text{A}}$ for $4.00\ \Omega$, $9.00\ \Omega$ resistors. $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$ $8.18\ \text{V} = I(7.00\ \Omega)$ $I = \boxed{1.17\ \text{A}}$ for $7.00\ \Omega$ resistor $8.18\ \text{V} = I(10.0\ \Omega)$ $I = \boxed{0.818\ \text{A}}$ for $10.0\ \Omega$ resistor.

Quiz 3: Internal Resistance & Kirchhoff's Rules

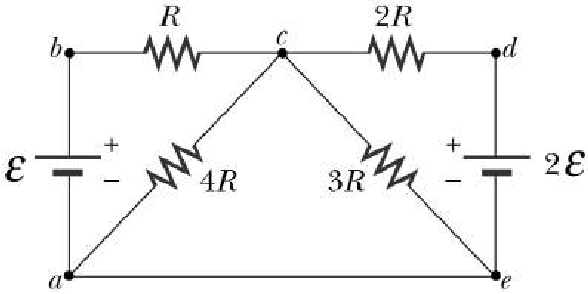
1 A battery delivering a current of 55A to a circuit has a terminal voltage of 23.4V. The electric power being dissipated by the internal resistance of the battery is 34W. Find the emf of the battery.

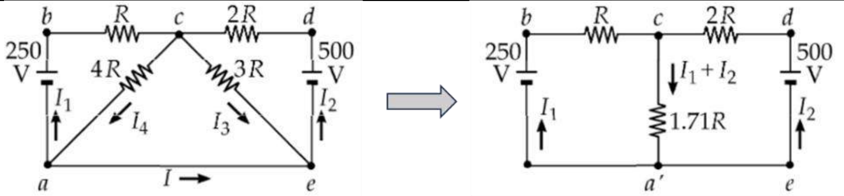
$$\text{Emf} = V + V_r$$

$$V_r = \frac{P}{I}$$

$$\text{Emf} = V + V_r = V + \frac{P}{I} = 23.4\ \text{V} + \frac{34.0\ \text{W}}{55.0\ \text{A}} = \boxed{24.0\ \text{V}}$$

2 The figure shows a circuit consisting 4 resistors and 2 dry cells. If $R = 1k\Omega$ and $\varepsilon = 250V$, determine the direction and magnitude of the current in the horizontal wire between a and e.

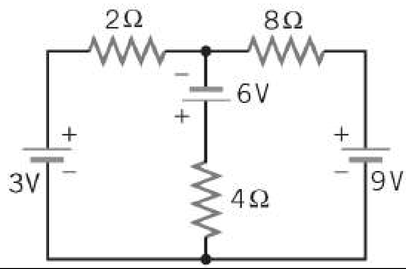




$$(2.71R)I_1 + (1.71R)I_2 = 250$$
$$(1.71R)I_1 + (3.71R)I_2 = 500.$$
$$I_1 = 10.0\ \text{mA}$$
$$I_2 = 130.0\ \text{mA}.$$
$$V_c - V_a = (I_1 + I_2)(1.71R) = 240\ \text{V}.$$

$$I_4 = \frac{V_c - V_a}{4R} = \frac{240\ \text{V}}{4\,000\ \Omega} = 60.0\ \text{mA}.$$
$$I = I_4 - I_1 = 60.0\ \text{mA} - 10.0\ \text{mA} = +50.0\ \text{mA},$$
$$I = \boxed{50.0\ \text{mA from point } a \text{ to point } e}.$$

3 Find the current in the 4Ω resistor in the drawing. Specify the direction of the current.

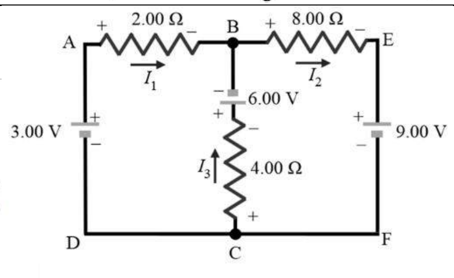


$$\text{junction B,}\quad \underbrace{I_1 + I_3}_{\text{Into junction}} = \underbrace{I_2}_{\text{Out of junction}}$$

$$\underbrace{I_1(2.00\ \Omega)}_{\text{Potential drops}} = \underbrace{6.00\ \text{V} + I_3(4.00\ \Omega) + 3.00\ \text{V}}_{\text{Potential rises}}$$

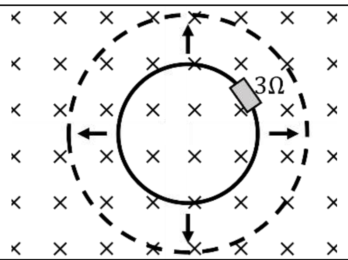
$$\underbrace{I_2(8.00\ \Omega) + 9.00\ \text{V} + I_3(4.00\ \Omega) + 6.00\ \text{V}}_{\text{Potential drops}} = \underbrace{0}_{\text{Potential rises}}$$

$$(I_1 + I_3)(8.00\ \Omega) + 9.00\ \text{V} + I_3(4.00\ \Omega) + 6.00\ \text{V} = 0$$
$$I_1(8.00\ \Omega) + I_3(12.00\ \Omega) + 15.00\ \text{V} = 0$$
$$I_1 = 4.50\ \text{A} + I_3(2.00)$$
$$[4.50\ \text{A} + I_3(2.00)](8.00\ \Omega) + I_3(12.00\ \Omega) + 15.00\ \text{V} = 0$$
$$I_3(28.00\ \Omega) + 51.00\ \text{V} = 0 \quad \Rightarrow \quad I_3 = \frac{-51.00\ \text{V}}{28.00\ \Omega} = \boxed{-1.82\ \text{A}}$$

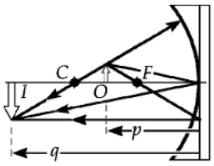
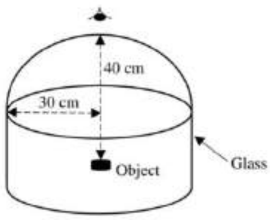


$\boxed{\text{the current in the 4.00-}\Omega\text{ resistor is directed downward}}$

Quiz 4: Faraday's, Lenz's Law & AC Circuits

1	The magnetic field perpendicular to a single 13.2-cm-diameter circular loop of copper wire decreases uniformly from 0.670 T to zero. If the wire is 2.25 mm in diameter, how much charge moves past a point in the coil during this operation? The resistivity of copper is $1.68 \times 10^{-8} \Omega m$.
	$Q = I\Delta t\ ,\ I = \frac{\mathcal{E}}{R}\ ,\ R = \frac{\rho \ell}{A_{\text{wire}}}$ $\lvert\mathcal{E}\rvert = \frac{\Delta\Phi_B}{\Delta t} = \frac{A_{\text{loop}}\lvert\Delta B\rvert}{\Delta t}\ ;\quad R = \frac{\rho\ell}{A_{\text{wire}}}\ ;\quad I = \frac{\mathcal{E}}{R} = \frac{\frac{A_{\text{loop}}\lvert\Delta B\rvert}{\Delta t}}{\frac{\rho\ell}{A_{\text{wire}}}} = \frac{A_{\text{loop}}A_{\text{wire}}\lvert\Delta B\rvert}{\rho\ell\Delta t}$ $Q = I\Delta t = \frac{A_{\text{loop}}A_{\text{wire}}\lvert\Delta B\rvert}{\rho\ell} = \frac{\pi r_{\text{loop}}^2 \pi r_{\text{wire}}^2\lvert\Delta B\rvert}{\rho(2\pi)r_{\text{loop}}} = \frac{r_{\text{loop}}\pi r_{\text{wire}}^2\lvert\Delta B\rvert}{2\rho}$ $= \frac{(0.066\text{ m})\pi(1.125\times10^{-3}\text{ m})^2(0.670\text{ T})}{2(1.68\times10^{-8}\ \Omega\cdot\text{m})} = \boxed{5.23\text{ C}}$.
2	A circular coil of 5 turns of radius 2cm is placed in a region of magnetic field of 3T as shown in the diagram. The circular loop is then expanded such that its radius becomes 4cm in 50ms. Determine the induced current and the direction of induced current. 
	$\lvert\varepsilon\rvert = NB\frac{dA}{dt} = (5)(3)\left(\frac{\pi(0.04^2 - 0.02^2)}{50(10^{-3})}\right) = \frac{9\pi}{23}V$ $\lvert\varepsilon\rvert = IR \Rightarrow \frac{9\pi}{23} = I(3) \Rightarrow I = \frac{3\pi}{25}A \approx 0.377A$ (counterclockwise)
3	A solenoid of length l = 10cm, radius r = 2cm has 1000turns. a. The current of the solenoid is lowered from 5A to 0A within 0.3s. Calculate the magnitude of emf induced in the solenoid. b. A second coil with 50turns is wound coaxially with the solenoid. Calculate the mutual inductance between the two.
	$L = \frac{\mu_o N^2 A}{l} = \frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$ $\frac{(1.25663706\times10^{-6})(1000)^2\pi(0.02)^2}{10\times10^{-2}} = \frac{\varepsilon}{\left(\frac{0-5}{0.3}\right)}$ $\varepsilon = -0.26319V$ $M = \frac{\mu_o N_1 N_2 A}{l} = \frac{(1.25663706\times10^{-6})(50)(1000)\pi(0.02)^2}{10\times10^{-2}}$ $M = 0.79mH$
4	A 20Ω resistor, an inductor of inductance L and a capacitor of capacitance C are connected in series along with a power supply. The impedance is found to be 20.22Ω when the frequency of the power supply is set to 50Hz. When the frequency is increased by 70Hz, the impedance was measured to be 38.07Ω. Determine the possible values of L and C.
	$Z^2 = R^2 + (X_L - X_C)^2 \Rightarrow Z^2 = R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2$ $(20.22)^2 = (20)^2 + \left(2\pi(50)L - \frac{1}{2\pi(50)C}\right)^2$ $(38.07)^2 = (20)^2 + \left(2\pi(120)L - \frac{1}{2\pi(120)C}\right)^2$ $\{L, C\} = \{0.05H, 0.00025F\}$ OR $\{L, C\} = \{0.054H, 0.00016F\}$

Quiz 5: Geometrical Optics

1	A spherical mirror is to be used to form, on a screen located 5m from the object, an image five times the size of the object. a. Describe the type of mirror required. b. Where should the mirror be positioned relative to the object?
	(a) $q=(p+5.00\text{ m})$ and, since the image must be real, $M=-\frac{q}{p}=-5 \Rightarrow q=5p$. $p+5.00\text{ m}=5p$ $p=1.25\text{ m},\ q=6.25\text{ m}$. $\frac{1}{p}+\frac{1}{q}=\frac{2}{R},\ R=\frac{2pq}{p+q}=\frac{2(1.25)(6.25)}{1.25+6.25}$ $=\boxed{2.08\text{ m(concave)}}$  (b) From part (a), $p=1.25\text{ m}$; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.
2	A contact lens is made of plastic with an index of refraction of 1.50. The lens has an outer radius of curvature of +2.00 cm and an inner radius of curvature of +2.50 cm. What is the focal length of the lens?
	$R_1=$ outer radius and $R_2=$ inner radius $\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.50-1)\left[\frac{1}{2.00\text{ m}}-\frac{1}{2.50\text{ cm}}\right]=0.0500\text{ cm}^{-1}$ $f=\boxed{20.0\text{ cm}}$
3	An antelope is at a distance of 20m from a converging lens of focal length 30cm. The lens forms an image of the animal. If the antelope runs away from the lens at a speed of $5ms^{-1}$, how fast does the image move? Does the image moves toward or away from the lens?
	$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $p^{-1}+q^{-1}=\text{constant}$, $\frac{1}{20\text{ m}}+\frac{1}{q}=\frac{1}{0.3\text{ m}}$ $q=0.305\text{ m}$. $-1(p^{-2})\frac{dp}{dt}-1(q^{-2})\frac{dq}{dt}=0$ $\frac{dq}{dt}=\frac{-q^2}{p^2}\frac{dp}{dt}$. $\frac{dq}{dt}=-\frac{(0.305\text{ m})^2}{(20\text{ m})^2}5\text{ m/s}=-0.00116\text{ m/s}=\boxed{1.16\text{ mm/s toward the lens}}$
4	The figure shows an object embedded in a solid glass with a hemispherical end of radius 30cm and refractive index 1.50. The object is 40cm inside the glass. Calculate the image distance. Refractive index of air is 1. 
	$\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}$ $\frac{1.5}{0.4}+\frac{1}{v}=\frac{1-1.5}{-0.3}$ $v=-0.48m$

Quiz 6: Physical Optics

1	
2	
3	
4	