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Study Notes & Exercises

On Linear Motion

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Chapter 1: Physical Quantities & Measurements

Learning Outcomes

1. Define dimension, scalar and vector quantities.
2. Determine:
   - **(a)** the dimensions of derived quantities.
   - **(b)** resultant of vectors. (remarks: limit to three vectors only).
3. Verify the homogeneity of equations using dimensional analysis.
4. Resolve vector into two perpendicular components (*x* and *y* axes).

Dimensions

Dimensions refer to the physical nature of a quantity. Regardless of the unit used, the physical nature of a quantity remains the same. For example, a distance, measured in the unit of metres, or feet, is still a measurement of length. This measurement, therefore has the dimensions of length, most commonly represented by **L.** In this instance, the equation $[d] = L$ is simply states that "The dimension of *d* is length (**L**)". The following table shows some selected physical quantities and its dimensions:

Base Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Length	L	metre (m)	L
Mass	M	kilogram (kg)	M
Time	T	second (s)	T
Electric Current	I	ampere (A)	I
Temperature	T	Kelvin (K)	Θ
Amount of substance	n	mole (mol)	N
Luminosity	L	candela (cd)	J
Derived Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Velocity	$\vec{v}$	ms^{-1}	LT^{-1}
Acceleration	$\vec{a}$	ms^{-2}	LT^{-2}
Momentum	$\vec{p}$	Ns	MLT^{-1}
Angular acceleration	α	$rads^{-1}$	T^{-2}
Electric Charge	Q	Coulomb (A s)	TI
Energy	E	Joule ($J = kgm^2s^{-2}$)	ML^2T^{-2}

Once you understand what dimensions are and how to work with them, you can apply it to **verify the homogeneity of equations.** The word 'homogeneity' refers to 'of the same kind'. Let us consider the equation $s = ut + \frac{1}{2}at^2$ where s is displacement of a body, *t* is time taken for the displacement of the body, *u* is the initial velocity of the body and *a* is the acceleration of the body. To 'verify homogeneity', we can compare the dimensions the terms on the left-hand side and the right-hand side of the equation. That is to say, *s* must have the same dimensions as *ut* and $\frac{1}{2}at^2$. s has the dimension of L, so does *ut* as well as $\frac{1}{2}at^2$.

Sample Problem 1.1:

Identify the dimensions for power, P, defined by $P = \frac{E}{t}$ where E is energy and has dimensions of ML^2T^{-2} and t has dimension of time, T.

Solution:

$$[Power] = \left[\frac{E}{t}\right]$$

$$[Power] = \frac{ML^2T^{-2}}{T}$$

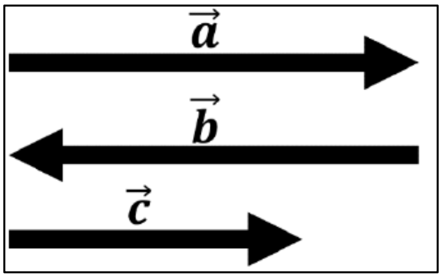
$$[Power] = ML^2T^{-3}$$

The dimensions for power are then ML^2T^{-3}.

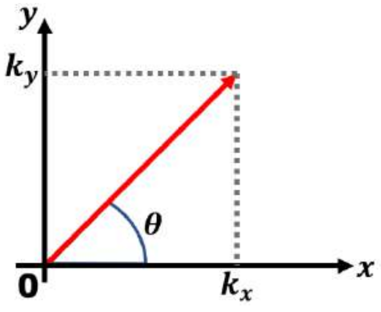
Scalars and Vectors

A scalar quantity is a quantity that is fully described by its magnitude. On the other hand, a vector quantity can only be fully described by both its magnitude **and** direction. When you talk about 5kg of rice, that statement is sufficient to describe the mass of the rice, this is where you can see that mass is a scalar quantity. If the rice is falling towards the Earth at a velocity of $2ms^{-1}$, 2 things matter here – how fast the rice is falling **and** the direction in which it is falling. Here you can see that velocity is a vector quantity.

A vector quantity is generally represented by a line segment with an arrowhead. The length of the line segment indicates its magnitude whereas its arrow head tells us the direction of the vector quantity. For example, say vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by the following arrows,



If we were to compare $\vec{b}$ and $\vec{c}$ to $\vec{a}$, we'd say that vector $\vec{b}$ has the same magnitude as $\vec{a}$ but is in the opposite direction, this would tell us that $\vec{b} = -\vec{a}$. Vector $\vec{c}$, on the other hand, is in the same direction as $\vec{a}$. But its magnitude is smaller that $\vec{a}$. The magnitude of vector $\vec{k}$ is denoted by $|\vec{k}|$. We can then relate vector $\vec{a}$ to $\vec{c}$ by the relation $|\vec{a}| > |\vec{c}|$.



Another method to represent vectors is to list the values of its elements in a sufficient number of difference directions, depending on the dimension of the vector. Consider a vector in a 2-dimensional Cartesian coordinate system, a vector $\vec{k}$ can then be represented by $\vec{k} = k_x\hat{\imath} + k_y\hat{\jmath}$ or $\vec{k} =< k_x, k_y >$, defining $\hat{\imath}$ and $\hat{\jmath}$ as unit vectors in the x and y directions respectively. From this notation, one can easily calculate the magnitude (length) of the 2-vector using Pythagoras' Theorem which gives

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2}.$$

Vector additions (or subtractions) can then be done by adding (or subtracting) corresponding components. That is to say, if we have vectors $\vec{a}$ and $\vec{b}$ defined by $\vec{a} =< a_x, a_y >$; $\vec{b} =< b_x, b_y >$, then the addition will yield

$$\vec{a} + \vec{b} =< a_x + b_x, a_y + b_y >.$$

The implication of this definition of vector addition are the following rules:

1. Commutativity of vectors $\Rightarrow \vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. Associativity of vectors $\Rightarrow (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + (-\vec{a}) = 0$

Resolution of vector $\vec{k}$ is then simply

$$k_x = |\vec{k}|cos(\theta);\ k_y = |\vec{k}|sin(\theta).$$

Multiplication of a vector

3 cases to consider when talking about multiplication of a vector:

1. The vector is multiplied by a scalar, then
$$k\vec{a} =< ka_x, ka_y >.$$
2. The **dot product** (also known as scalar or inner product) of two vectors, then
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = |\vec{a}||\vec{b}|\ cos(\theta_{ab}).$$
Note that in the dot product, the operation results in a scalar quantity.
3. The **cross product** (also known as the vector product) of two vectors, then
$$\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\hat{n} = |\vec{a}||\vec{b}|sin(\theta)\hat{n}$$
Note that in the cross product, the operation results in a vector quantity perpendicular to both the x and y axis.

Sample Problem 1.2:

Calculate the magnitude and direction of vector $\overrightarrow{c}$ if it is defined by $\vec{c} = \vec{a} + \vec{b}$ where $\vec{a} = [2,3]$ and $\vec{b} = [-1,4]$.

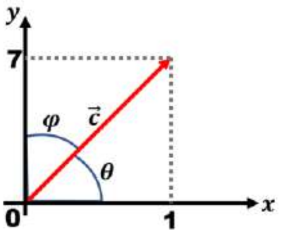
Solution:

Magnitude:

$$\vec{c} =< 2 + (-1), 3 + 4 \geq< 1,7 >$$
$$|\vec{c}| = \sqrt{7^2 + 1^2} \approx 7.071$$

Direction:

$$\theta = tan^{-1}\left(\frac{7}{1}\right) \approx 81.87^o$$



Additional notes:

Unit conversions are so easy that we tend to overlook the importance of practicing it. Here's a simple reminder on how to do it. Say you are given that $1\,in = 2.54cm$ and you are asked to calculate 8cm in inches, here's how to do it:

$$1in = 2.54cm \leftarrow \text{divide both side by 2.54}$$

$$\frac{1}{2.54}in = \frac{2.54}{2.51}cm = 1cm \leftarrow \text{now multiply it by 8}$$

$$8cm = \frac{8}{2.54}in \approx 3.1496in$$

And that's how you do unit conversion.

In Physics, it is quite often that we are expected to work with values in form of scientific notation. For example, rather than writing down the speed of light as $300000000ms^{-1}$, we'd express this value as $3 \times 10^{8}ms^{-1}$. One issue that may arise from working with scientific notation when using a calculator is the redundancy in typing out "10^x". To help with this, I would suggest that we take advantage of the **rules for exponents**. The example below demonstrates such application.

<u>Sample Problem 1.3:</u>

Evaluate c given that $c = \frac{a^2b}{d}$ and $a = 3 \times 10^{-6}$, $b = 4 \times 10^{3}$ and $d = 12 \times 10^{3}$.

<u>Solution</u>

$$c = \frac{a^2b}{d} = \frac{(3 \times 10^{-6})^2(4 \times 10^{3})}{(12 \times 10^{3})}$$

Rather than evaluating this monstrosity as our input for the calculator, we can instead separate the coefficient from the base and its exponent to evaluate them separately.

$$c = \frac{a^2b}{d} = \frac{(3)^2(4)}{(12)}\left[\frac{(10^{-6})^2(10^{3})}{10^{3}}\right]$$

In this form, the evaluation can be done **easily** even without a calculator.

$$c = \frac{(3)^2(4)}{(12)}\left[\frac{(10^{-6})^2(10^{3})}{10^{3}}\right] = \frac{36}{12}[10^{-6-6+3-3}]$$

On Significant Figures

When we talk about the number of significant figures, we are talking about the number of digits whose values are known with certainty. This gives us information about the degree of accuracy of a reading in a measurement. In general, we should practice performing rounding off when the conditions call for it. This is to avoid false reporting. What we mean by false reporting is to give the illusion that our experiments are more sensitive than it actually it. For example, it would be very unlikely that our metre ruler to give reading in the micro scale.

Number	Number of significant figures	Number	Number of significant figures
2.32	3	2600	2
2.320	4	2602	4

When we do calculations, there are some rules (based on the operations) we should be aware of when stating the significant figures of the end value:

1. Multiplication / Divisions – number of significant figures in the result is the same as the least precise measurement in the least precise measurement used in the calculation.
   Example:

$$\frac{2.5(3.15)}{2.315} = 3.4$$

2. Addition / Subtraction – The result has the same number of decimal places as the least precise measurement used in the calculation.
   Example:

$$91.1 + 11.45 - 12.365 = 90.2$$

3. Logarithm / antilogarithm – Keep as many significant figures to the right of the decimal point as the are significant in the original number.
   Example:

$$ln(4.00) = 1.39;\ e^{0.0245} = 1.03$$

Exercises

1. Two vectors lie on the x-y plane. Vector $\vec{a}$ is 5 units long and points 25^o above the x-axis. Vector $\vec{b}$ is 8 units long and points upward 35^o above the x-axis. What is the resultant of the two vectors?

2. Show that the equation $v^2 = v_o^2 + 2ax$ is dimensionally correct. In this expression, a is acceleration, v and v_o are
velocities and x is the distance.

3. Derive the dimensions of k if $k = \frac{ab}{d}$ where $[a] = M^2T^2$, $[b] = M^{-1}T^{-1}$ and $[d] = MT^2$.

4. A car is travelling at 35miles per hour, what is its speed in metres per second if $1mile = 1.61km$.

5. Express each of the following in grams
   a. $3lb$ if $1g = 2.2 \times 10^{-3}lb$
   b. $12.2oz$ if $1g = 0.035oz$
6. How many square centimetres are in a square inch if $1cm = 0.3937inch$.
7. The length and the width of a rectangle is $(11 \pm 0.05)cm$ and (21 ± 0.05) cm respectively. Find the area (and its uncertainty) of the rectangle.

Chapter 2: Kinematics of Linear Motion

Learning Outcomes (LO)

1. Define:
   a. instantaneous velocity, average velocity and uniform velocity; and
   b. instantaneous acceleration, average acceleration and uniform acceleration.
2. Derive and apply equations of motion with uniform acceleration
   $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$; $s = \frac{1}{2}(u + v)t$
3. Describe projectile motion launched at an angle, θ as well as special cases when θ=0°
4. Solve problems related to projectile motion.

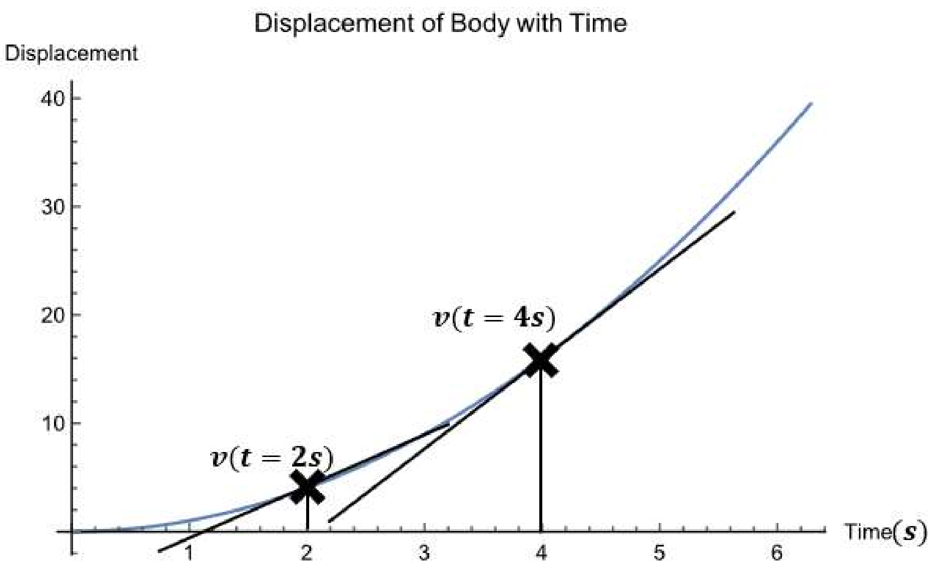
In this chapter, we talk about kinematics of linear motion. Dynamics is the study of motion of bodies under action of forces and their effects. One subbranch to the study of dynamics is kinematics. In the study of kinematics, we consider only the motion of the bodies without worrying too much about the forces that caused the bodies to move. We only worry about the geometry of the motion.

Instantaneous and Average Velocity (or acceleration)

Let us start with reminder of some ideas and terms that you have learnt in your SPM days. 3 mains terms – displacement (), velocity () & acceleration (). Displacement, denoted by x, simply refers to the change in position of a body. Velocity, v, refers to the rate of change of this change in position, i.e. $v = \frac{dx}{dt}$. Acceleration, a, is defined by the rate of change of velocity, which is the rate of change of the rate of change of position. That is $a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$.

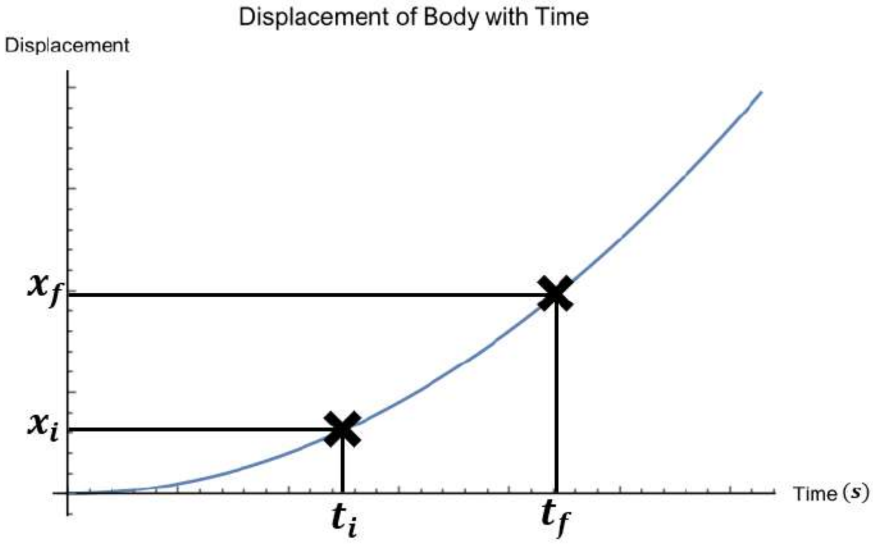
Once we have established that, we can further extend our ideas of velocity and acceleration by thinking about ***instantaneous*** velocity (or acceleration) and ***average*** velocity (or acceleration). By 'instantaneous', we mean 'at a particular instant in time'. When we combine it with velocity (or acceleration), what we mean is velocity (or acceleration) at a particular instant in time. On the other hand, when we say 'average', what we mean is 'over the course of a defined time span'. So, when we say 'average velocity', we usually would accompany it with 'between time t_a and t_b' or 'in 30 seconds', specifying a range of time.

Let us now have a graphical representation. Consider a body moving at constant velocity,



When we talk about instantaneous velocity, we are asking about a single point in time. From the displacement-time graph, the gradient represents the velocity of the body. As we can see from the graph, the instantaneous velocities when $t = 2s$ and $t = 4s$ are different. This is simply because the body is moving at a non-uniform velocity. If the instantaneous velocities are the same, then we call the motion is described as uniform velocity.

On the other, talking about ***average*** velocity, we simply define range of time, thus choosing two points in time rather than one. Then we take the difference in position and divide it by the difference in time to calculate the ***average*** velocity. That is to say, for the graph below,



We can calculate the average velocity as

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

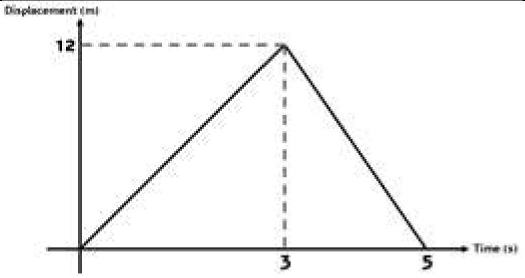
We can take the same approach and understanding and apply it to acceleration, but with a velocity time graph rather than a displacement time graph.

<u>Sample Problem 2.1</u>
The motion of a body is described by the equation $v = 2t^2$, where v is in metres per second and t is in seconds. Calculate the instantaneous velocity of the body at $t = 3s$ and the average acceleration between $t = 2s$ and $t = 4s$.
Answer:

$$v_{instantaneous} = 2(3)^2 = 18ms^{-1}$$
$$v(t = 2s) = 2(2)^2 = 8ms^{-1}$$
$$v(t = 4s) = 2(4)^2 = 32ms^{-1}$$
$$a_{average} = \frac{\Delta v}{\Delta t} = \frac{v(t = 4s) - v(t = 2s)}{4 - 2} = \frac{32 - 8}{4 - 2}$$
$$a_{average} = 12ms^{-2}$$

<u>Sample Problem 2.2</u>

The motion of a body is shown in the graph shown in Figure 1. Calculate the displacement of the body and calculate the average velocity of the body.



Answer:

Displacement of the body can be calculated by recognising that the area under the velocity -time graph represents the displacement of the body. So all we need is to sum up all the areas under the graph.

$$s = \frac{1}{2}(12 \times 3) + \frac{1}{2}\big(12 \times (5 - 3)\big) = 30m$$

The average velocity can be calculated by simply dividing the displacement by the total time of motion.

$$v_{average} = \frac{30}{5} = 6ms^{-1}$$

Kinematic Equations

Now we want to look at kinematic equations, which are equations that relates variables that describes motion such as displacement, velocity and acceleration.

Derivation by calculus

We'd like to derive the equations from our understanding of linear motion and using calculus. We begin with the definition of acceleration

$$a = \frac{dv}{dt}$$

Assuming constant acceleration, we can rearrange then integrate both sides to yield

$$a\int_{t_{initial}}^{t_{final}} dt = \int_{v_{initial}}^{v_{final}} dv \Rightarrow a\big(t_{final} - t_{initial}\big) = v_{final} - v_{initial}$$

Adjusting such that $t_{initial} = 0$, $t_{final} = t$ and defining $v = v_{final}$, $v_{initial} = u$.

And then rearranging this equation yields

$$v = u + at$$

which is the same equation as the first equation found in LO2. Simply put, the final velocity of a body is initial velocity plus the product of acceleration and time difference.

We can take the same approach to find the third equation in LO2 using the first equation. We start with the definition of velocity and then rearranging it,

$$v = \frac{dx}{dt} \Rightarrow \int v\, dt = \int dx$$

Note that since velocity is not a constant, $v\,dt$ cannot be directly integrated. We therefore need an equation for velocity as a function of time (first equation).

$$\int u + a\,t\,dt = \int dx$$

Since u and a are constants, these integrals become

$$u\int_0^t dt + a\int_0^{\mathrm{t}} dt = \int_0^{\mathrm{x}} dx$$

Solving this integral gives

$$x = ut + \frac{1}{2}at^2.$$

For the second equation, we can start take advantage of calculus by starting with a time independent derivative,

$$\frac{dx}{dv} = \frac{dx}{dt}\frac{dt}{dv} = \frac{v}{a}$$

Rearranging this gives us the needed integral to solve

$$a\int_0^x dx = \int_u^v v\,dv \Rightarrow ax = \frac{1}{2}(v^2 - u^2)$$

Further rearrangement yields an equation

$$v^2 = u^2 + 2ax$$

matching with the third equation found in LO2.

Equation 4 of LO2 does not require any integration, rather we can obtain it using $s = ut + \frac{1}{2}at^2$ and $v = u + at$. This is left for the reader to do.

Geometric Derivation

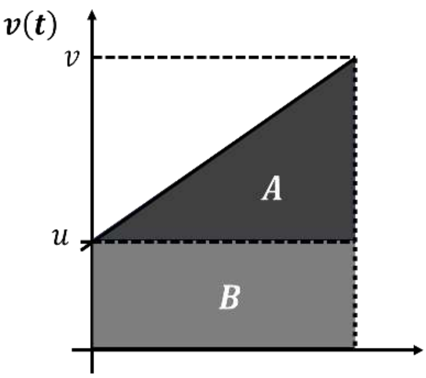
By definition,

$$a = \frac{v-u}{t}$$

Rearranging this gives

$$v = u + at$$

Consider an object that starts its motion with velocity u and maintains its constant acceleration a to a final velocity of v. We can describe its motion diagrammatically as below



Since the area under the graph represents displacement, all we need to do is to add up the area of A and B. If

$$\text{Area}_A = \frac{1}{2}(t)(v-u) = \frac{1}{2}(t)(at) = \frac{1}{2}at^2$$

$$\text{Area}_B = ut$$

then

$$\mathrm{s} = u\mathrm{t} + \frac{1}{2}at^2.$$

If, on the other hand, we consider

$$s = \frac{1}{2}(t)(v-u) + ut$$

Then we find that

$$s = \frac{1}{2}(v+u)t$$

For the equation of $v^2 = u^2 + 2as$, we can start the derivation by considering

$$v = u + at \Rightarrow t = \frac{v-u}{a}$$

And

$$s = \frac{1}{2}(u+v)t.$$

We can substitute time equation into the displacement to yield

$$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right) = \frac{v^2-u^2}{2a} \Rightarrow v^2 = u^2 + 2as.$$

<u>Sample Problem 2.3</u>

A 2022 Honda Accord can travel down a $\frac{1}{4}$ mile track in $14.1s$ from rest. Calculate the acceleration (in SI units), assuming that its acceleration is constant.

<u>Answer:</u>

Values given ⇒ $s = \frac{1}{4}\ mile = 402.336km\,; t = 14.1s;\ u = 0ms^{-1}$

Choice of equation ⇒ $s = ut + \frac{1}{2}at^2$

$$402.336 = \frac{1}{2}a(14.1)^2$$
$$a = 4.04744ms^{-2}$$

<u>Sample Problem 2.4</u>

A car initially travels at $20ms^{-1}$. If the car undergoes constant acceleration of $1.2ms^{-2}$, determine the time the car need to reach double of its initial velocity.

<u>Answer:</u>

Values given ⇒ $\mathrm{u = 20ms^{-1};\ a = 1.2ms^{-2};\ v = 2u = 40ms^{-1}}$

Choice of equation ⇒ $v = u + at$

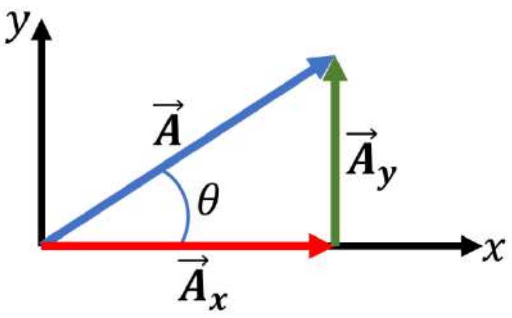
$$40 = 20 + 1.2t$$
$$\boldsymbol{t = \frac{50}{3}s}$$

Projectile Motion (Motion in 2 Dimensions)

When dealing with motion in two dimensions, the minimum that we need is the Pythagorean theorem as well as the definition of tangent. We consider a vector $\vec{A}$ defined by

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where $\vec{A}_x$ and $\vec{A}_y$ are component vectors of $\vec{A}$, each parallel to one of the axes in a rectangular coordinate system.



It then follows that the magnitude and direction of $\vec{A}$ can be related to its components by the Pythagorean theorem and the definition of tangent,

$$|\vec{A}| = \sqrt{|\vec{A}_x|^2 + |\vec{A}_y|^2}$$

$$\tan\theta = \frac{|\vec{A}_y|}{|\vec{A}_x|}.$$

Conversely, we can work out the components of $\vec{A}$ from the magnitude of $\vec{A}$ and the angle θ,

$$|\vec{A}_x| = |\vec{A}|\cos\theta$$

$$|\vec{A}_y| = |\vec{A}|\sin\theta$$

One case study that we can do on two-dimensional motion is **projectile motion,** a motion that follows a parabolic path. The simplest case of projectile motion would be one where the air resistance and the rotation of the Earth is simply neglected, and that the motion is only affected by the Earth's gravity ($\vec{F}_{gravity} = m\vec{g}$). One important aspect of this case is that the horizontal (x-direction) and vertical (y-direction) motions are independent of each other. This means that the kinematics equation we have studied earlier can be dealt with separately for both x and y directions.

Keeping in mind that $u_x = u\cos\theta$ and $u_y = u\sin\theta$, we can then work out the 6 equations that describes the projectile motion:

x-direction (where $a_x = 0$)	y-direction (where $a_y = -g$)
$v_x = u_x$	$v_y = u_y - gt$
$s_x = u_x t$	$s_y = u_y t - \frac{1}{2}gt^2$
$v_x^2 = u_x^2$	$v_y^2 = u_y^2 - 2gs_y$

We can also work out the velocity of the projectile by keeping in mind that it merely follows from Pythagorean theorem

$$v^2 = v_x^2 + v_y^2.$$

If we substitute the equation for time-x-component, $t = \frac{s_x}{u_x}$, in the equation for displacement in y-direction, $s_y = u_y t - \frac{1}{2}gt^2$, what we get is the parabolic equation for the projectile motion path,

$$\left(\frac{u_y}{u_x}\right)s_x - \left(\frac{1}{2u_x^2}\right)s_x^2 - s_y = 0.$$

There are two more items that are of our interest:

1. If we were to look for the "**peak**" of the parabolic path, we can do so by applying $v_y = 0$ to the kinematics equations. This is simply because it is at this peak that $u_y = gt$ such that the velocity of the projectile is momentarily zero before the projectile falls back down towards the Earth.
2. Another item that would be of our interest is the **range** of the projectile motion. By range, what we are referring to is the point at which the projectile reaches back to ground or stop accelerating in the y-direction. This would differ from case to case, of course, and we shall demonstrate in the sample problems following this.

Sample Problem 2.5

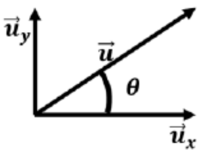
An object is launched at a velocity of $21ms^{-1}$ in a direction making an angle of 30^o upward with the horizontal.

Calculate

a. Initial velocity in x and y direction.
b. the location of the object at $t = 2s$.
c. the total horizontal range.
d. the velocity of the object just before it hits the ground.

Answer:

a.



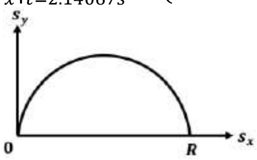
$$|\vec{u}_x| = |\vec{u}|\,cos\theta = 21\,cos\,30$$
$$|\vec{u}_x| = 18.1865ms^{-1}$$
$$|\vec{u}_y| = |\vec{u}|\,sin\theta\ = 21\,sin\,30$$
$$|\vec{u}_y| = 10.5ms^{-1}$$

b. $s_x = u_x t \Rightarrow s_x = (18.1865)(2) = 36.3730m$

$s_y = u_y t + \frac{1}{2}a_y t^2 \ \Rightarrow s_y = (10.5)(2) + \frac{1}{2}(-9.81)(2^2) = 1.38m$

c. $s_y = u_y t + \frac{1}{2}a_y t^2 \ \Rightarrow 0 = (10.5)(t) + \frac{1}{2}(-9.81)(t^2) \Rightarrow t\ =\ \{0,2.14067\}s$

$s_x|_{t=2.14067s} = (18.1865)(2.14067) = 38.9313m\ =\ R$



d. $v_x = u_x = 18.1865ms^{-1}$

$v_y = u_y + a_y t = 10.5 + (-9.81)(2.14067) = -10.5ms^{-1}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(18.1865)^2 + (-10.5)^2} = 21ms^{-1}$

$\theta = tan^{-1}\left(\frac{18.1865}{-10.5}\right) = -60^o$

Sample Problem 2.6

Compare the horizontal range of a ball thrown at velocity $35ms^{-1}$ if the angle of release is 15^o, 30^o, 45^o and 60^o.

Answer:

Condition for determining horizontal range $\Rightarrow s_y = 0$.

$s_y = u_y t + \frac{1}{2}a_y t^2 = (u\,cos\,\theta)t + \frac{1}{2}a_y t^2$

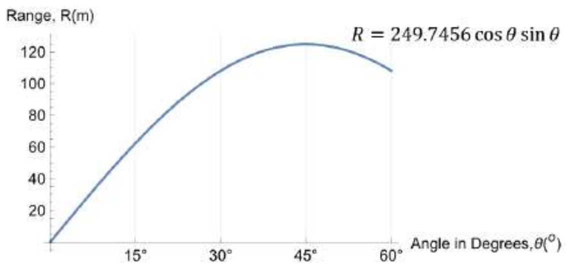
$0 = (35\,cos\,\theta)t - (4.905)t^2$

$t = \{0,\ 7.13558\,cos\,\theta\}s$

$R = s_x = u_x t = (u\,sin\,\theta)\mathrm{t}$

$R = (35\,sin\,\theta)(7.13558\cos\theta)$

$R = 249.7456\cos\theta\,sin\,\theta$



$\theta(^o)$	t (s)	$R(m)$
15^o	6.89244	62.4363
30^o	6.17959	108.143
45^o	5.04562	124.873
60^o	3.56779	108.143

The horizontal range peaks at 45^o.

Exercises

1. A car accelerates uniformly from rest for 5.88s for a distance of 150m. Calculate the acceleration of the car.
2. An object is accelerated uniformly from $2ms^{-1}$ to $8ms^{-1}$ over a distance of 38m. Calculate the acceleration of the object.
3. Car A, moving at a constant velocity of $10ms^{-1}$, passes by Car B at rest. 2 seconds after Car A passes, Car B starts accelerating at $5ms^{-2}$ in the same direction as Car A. When does Car B surpasses Car A?
4. A car, initially at rest at position A, accelerates at $2ms^{-2}$ for 5 seconds and then maintains its velocity for 10 seconds. After which the car decelerates back to $0ms^{-1}$ in 3s. Sketch the velocity-time graph of the car and calculate the total displacement of the car from position A.
5. The Tailgating Problem [1]:
   a. Determine the stopping distance of a BMW M3 if the car can decelerate at a rate of $9.2ms^{-2}$ from $97kmh^{-1}$.
   b. Consider the reaction time of a driver is 0.55s, calculate the stopping distance of the BMW M3 mentioned in the previous question.
   c. “When pigs fly” – A driver driving a car travelling at $28ms^{-1}$ suddenly notices a pig (oink oink!) $28m$ on the road ahead. With what velocity would the car hit the pig if the car decelerated at $8ms^{-2}$ and the driver’s reaction time was $0.8s$?
6. A projectile is launched at a velocity of $20ms^{-1}$ at angle of 30^o to the horizontal. Calculate the
   a. Time of flight of the projectile.
   b. Horizontal range of the projectile.
   c. The maximum height of the projectile.
   d. The speed of projectile at a third of the maximum height.
7. A diver, on a pier 10m from the water, jumps up at an angle 27^o from the horizontal with an initial velocity of $4.2ms^{-1}$. Calculate the
   a. Time at which the diver hits the water.
   b. The speed of the diver just before he hits the water.
8. A projectile is fired at velocity v and it has a horizontal range of 10m. If the projectile is again fired at the twice its velocity before, determine the new horizontal range.

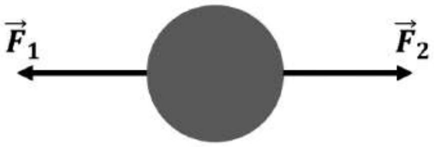
Chapter 3: Dynamics of Linear Motion

Learning Outcomes

5. Define
   a. Momentum, $\vec{p} = m\vec{v}$
   b. Impulse, $J = F\Delta t$
6. Solve problem related to impulse and impulse-momentum theorem,
$$J = \Delta p = mv - mu$$
   **1D only*
7. Use *F-t* graph to determine impulse.
8. State:
   a. the principle of conservation of linear momentum.
   b. Newton's laws of motion.
9. Apply
   a. the principle of conservation of momentum in elastic and inelastic collisions in 2D collisions.
   b. Newton's laws of motion.
   **include static and dynamic equilibrium for Newton's first law motion*
10. Differentiate elastic and inelastic collisions. (remarks: similarities & differences)
11. Identify the forces acting on a body in different situations - Weight, *W;* Tension, *T;* Normal force, *N;* Friction, *f;* and External force (pull or push), *F*.
12. Sketch free body diagram.
13. Determine static and kinetic friction, $f_s \le \mu_s N$, $f_k = \mu_k N$

In the previous chapters, we have looked at describing motion without the hassle of asking, "what force is causing the body to move?". In this chapter, we aim to expand our knowledge to a body's motion in that very aspect.

Types of Forces



$$\vec{F}_{net} = \vec{F}_1 - \vec{F}_2$$

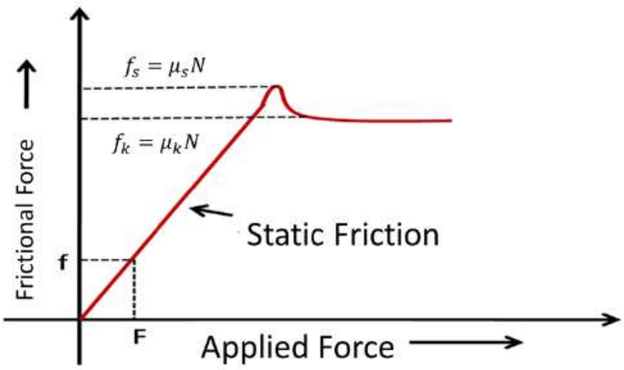
We begin with asking the question, "what is force?", a simple answer would be to say force is a push and pull. Here, however, let us define force a bit further. Let us define force as **an agent for motion change**. Force is a vector quantity, that means **direction matters**. Two oppositely directed force acting on the same body work against each other. A body can experience multiple forces acting on it, however it is the net force, i.e., the resultant of all the forces acting on the body, that changes the motion of the body.

4 types of forces we'd consider in this chapter – gravitational (weight), tensional, normal and frictional. Their definitions and directions are as follows:

Forces	Definitions	Directions
Gravitational	Force exerted upon a body interacting with a gravitational field.	Towards the gravitational source.
Tensional	Force transmitted axially through a massless one-dimensional continuous element.	Along the one-dimensional continuous element.

Normal	Support force, perpendicular to the surface, exerted upon a body in contact with a stable object.	Perpendicular to the surface the body is in contact with.
Frictional	Force acting upon bodies that are in contact and moving relative to each other.	Against the direction of motion.

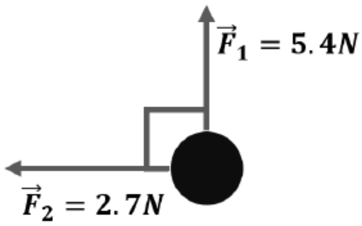
One particular type of force that maybe be of our interest is frictional force. This is because frictional force depends on the motion of the object. If the object is static, then it is subject to *static friction*. On the other hand, if the object is moving (relative to the surface it is in contact with) at some velocity, and therefore has some kinetic energy, then the object is subject to *kinetic friction*. Static friction is generally higher than kinetic friction because of the asperities (roughness) of the surfaces of the contacting bodies. This asperity enables the surfaces to interlock with each other, causing adhesion. This means that the force applied to the body must overcome this adhesion before the bodies can start moving relative to each other. This phenomenon can be observed by looking at the frictional force as a function of applied force graph:



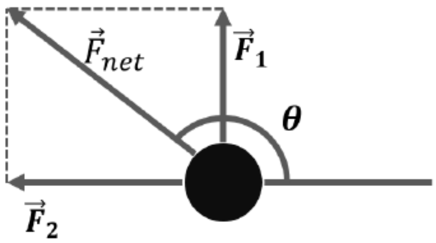
As the applied force is increased, so does the frictional force. This is true until a certain threshold is reached, after which the body will start to move. This threshold is exactly the unlocking of the asperities.

<u>Sample Problem 3.1</u>

Determine the resultant force exerted on the body shown in the diagram.

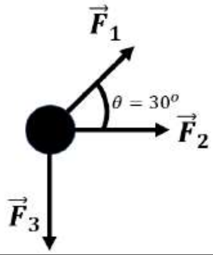


<u>Solution</u>



$$|\vec{F}_{net}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2}$$
$$|\vec{F}_{net}| = \sqrt{(5.4)^2 + (-2.7)^2}$$
$$|\vec{F}_{net}| = 6.03738N$$
$$\theta = 180^o - tan^{-1}\left(\frac{5.4}{2.7}\right)$$
$$\theta = 116.565^o$$

Sample Problem 3.2

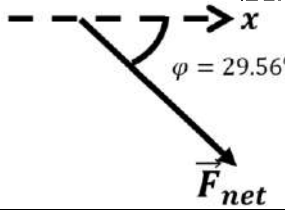


Calculate the resultant force if $|\vec{F}_1| = 10N$, $|\vec{F}_2| = 12.5N$ and $|\vec{F}_3| = 17N$.

Solution

Force	x-components	y-components
F_1	$F_1 \cos 30^o$ $= 10 \cos 30^o$ $\approx 8.66N$	$F_1 \sin 30^o$ $= 10 \sin 30^o$ $\approx 5N$
F_2	$+12.5N$	0N
F_3	0N	$-17N$
Σ	21.16N	-12N

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{(21.16)^2 + (-12)^2}$$
$$F_{net} = 24.3258N$$
$$\varphi = tan^{-1}\left(\frac{12}{21.16}\right) = 29.56^o$$



Newton's Law of Motion

There are laws of motion that a moving under force would generally follow. These laws were first introduced and came into its modern form via Newton's *Principia*. In it, 3 laws of motions were found:

1. A body, when no external force is applied, will not undergo velocity change, i.e.
$$\vec{F}_{external} = 0 \Rightarrow \Delta\vec{v} = 0$$
2. When a force is acted upon it, a body will move in a manner such that rate of momentum change is equal to the said force, i.e.
$$\vec{F} = \frac{d\vec{p}}{dt}.$$
3. Forces exerted onto two interacting bodies will be equal in magnitude but opposite in direction.
If $\vec{F}_{12}$ is force exerted onto body 1 by body 2, then
$$\vec{F}_{12} = -\vec{F}_{21}$$

These three laws form the foundation for what is known today as the *Newtonian Laws of Motion*.

Momentum

The first and third requires no further definitions of variable, however the second one, mentions an idea of ***momentum***. It seems useful to define this term at this point. What we mean by momentum at this point is the property of a moving body that rises from the product of the mass the body and its velocity, i.e.

$$\vec{p} = m\vec{v}.$$

If the velocity of the body changes (and therefore so does the momentum of the body in question), we quantify that change and call it ***impulse***:

$$\vec{J} = \Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t.$$

This also means that the area under the F-t graph represents impulse.

It is from Kinematics that we know a change in velocity means a non-zero acceleration. Knowing this as well as Newton's Second Law, we can say that force is present when acceleration is non-zero,

$$\vec{F} = m\vec{a}.$$

This statement, of course can be derived quite simply from Newton's Second Law of motion:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

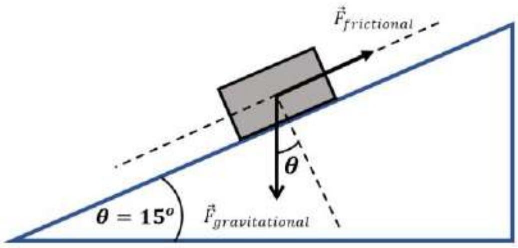
Here we can see that a change in mass may also produce force and that if mass change is zero, then what we have is the equation seen before,

$$\vec{F} = m\vec{a}.$$

Sample Problem 3.3

Suppose an object of 15 kg is placed on an incline plane of 15^o from the horizontal. Calculate the magnitude of frictional force that keeps the object from sliding down the incline plane.

Solution



$F_{net} = F_g \sin\theta - F_{frictional}$

Since the object stays static $\Rightarrow F_{net} = 0$

$F_{frictional} = F_g \sin\theta = mg \sin\theta$

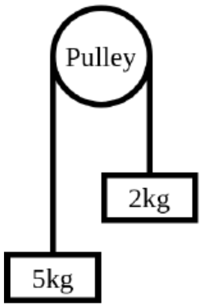
$F_{frictional} = (15)(9.81) sin 15^o$

$F_{frictional} = 38.0852N$

Sample Problem 3.4

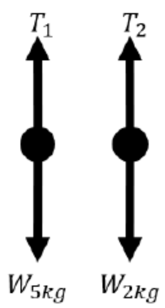
Problem:

Based on the diagram, calculate the acceleration of both the objects.



Solution

FBD



Equation of motion for 5kg body:

$$F_{net} = W_{5kg} - T_1 = m_{5kg}a$$

Equation of motion for 5kg body:

$$F_{net} = T_2 - W_{2kg} = m_{2kg}a$$

$$T_1 = T_2 = T$$

Rearrange for a,

$$T = W_{5kg} - m_{5kg}a = W_{2kg} + m_{2kg}a$$

$$a = \frac{W_{5kg} - W_{2kg}}{m_{2kg} + m_{5kg}} = \frac{9.81(5-2)}{5+2}$$

$$a \approx 4.2ms^{-2}$$

The 2kg object will move **upwards** at $a \approx 4.2ms^{-2}$.

The 5kg object will move **downwards** at $a \approx 4.2ms^{-2}$.

Sample Problem 3.5

A 50g ball at $30ms^{-1}$ is travelling towards a wall. Upon striking the wall, the ball bounces back in the opposite direction at a speed of $10ms^{-1}$. Calculate the impulse.

Solution

$J = \Delta p = p_{final} - p_{initial}; p = mv$

$$J = m\left(v_{final} - v_{initial}\right)$$
$$J = (0.05)\left((-10) - (+30)\right)$$
$$J = -2kg\, ms^{-1}$$

Sample Problem 3.6

A footballer kicks a 300g ball from rest to a speed of $60ms^{-1}$ in a collision lasting $1.5ms$. Calculate the force generated by the footballer.

Solution

$$F\Delta t = \Delta p = p_{final} - p_{initial}$$

Since $p_{initial} = 0$, then

$$F(1.5 \times 10^{-3}) = (0.3)(60)$$
$$F = 12kN$$

One of the ways for bodies to interact is through collisions. When this happens, assuming this happens in an isolated system. The total momentum of the system doesn't change with the passage of time. The momenta of the participating bodies may change, but not the vector sum total momentum of the system. When we say that a quantity doesn't change, we say that the quantity is **conserved**. So, in this case, we say that the **total momentum is conserved**. Conservation of momentum is simply

$$\Delta(\Sigma p) = 0.$$

When we talk about collisions, we may consider two types of collision – elastic and inelastic collisions. Note that whilst momentum is conserved in **all** types of collisions, kinetic energy is not as it may be converted into other forms of energy (e.g., sound energy). It is this exact parameter from which we differentiate elastic from inelastic collisions. A perfectly elastic collision is defined by a collision in which both momentum and kinetic energy is conserved whilst a perfectly inelastic collision is a collision in which conservation of kinetic energy is not obeyed.

Sample Problem 3.7 (Conservation of momentum)

Ball A of mass 30g, travels at $3ms^{-1}$ collides head on with Ball B of 50g at rest. Calculate the velocity of Ball B after the collision if Ball A has the final velocity of $1.2ms^{-1}$.

Solution

$$\Delta\Sigma p = 0 \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$
$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$(0.03)(3) + 0 = (0.03)(1.2) + (0.05)v_B$$

$x = 1.08ms^{-1}$ in the same direction as Ball A.

Sample Problem 3.8 (Conservation of momentum in 2 Dimensions)

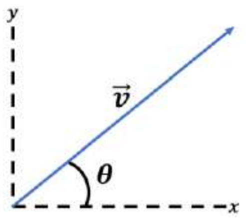
Ball A of mass 3kg, travels at $3ms^{-1}$ in the positive x direction collides with Ball B of 5kg travelling at $2ms^{-1}$ in the positive y direction. If the balls stick together after the collision, determine their velocity.

Solution

Need to apply conservation of momentum in x and y direction separately.

	X	Y
Momentum before	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_B = 0$ $\Sigma p = m_A u_A$	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_A = 0$ $\Sigma p = m_B u_B$
Momentum after	$\Sigma p = (m_A + m_B) v \cos\theta$	$\Sigma p = (m_A + m_B) v \sin\theta$
Final velocity	$m_A u_A = (m_A + m_B) v \cos\theta$ $v \cos\theta = \frac{m_A u_A}{m_A + m_B}$ ---(1)	$m_B u_B = (m_A + m_B) v \sin\theta$ $v \sin\theta = \frac{m_B u_B}{m_A + m_B}$ ---(2)

$$\frac{(2)}{(1)}: \tan\theta = \left(\frac{m_B u_B}{m_A + m_B}\right)\left(\frac{m_A + m_B}{m_A u_A}\right) = \frac{m_B u_B}{m_A u_A} = \frac{(5)(2)}{(3)(3)}$$
$$\theta = 48.0128^o$$
$$v \cos\theta = \frac{m_A u_A}{m_A + m_B} \Rightarrow v \cos(48.0128^o) = \frac{(3)(3)}{3(5)}$$
$$v = 0.897 m s^{-1}$$



Sample Problem 3.9 (Perfectly Elastic Collision)

A ball, travelling at $3ms^{-1}$, collides head on with another ball of the same mass, travelling $2ms^{-1}$ in the opposite direction. Determine their velocities post-collision?

Solution

Let us assume that ball 1, travelling at $3ms^{-1}$, initially travels in the positive direction such that

$$u_1 = +3ms^{-1};\ u_2 = -2ms^{-1}$$

Since the balls are of the same mass,

$$m_1 = m_2 = m$$

Assuming the collision is elastic, the system would obey both the conservation of momentum and the conservation of kinetic energy.

$$\Delta(\Sigma p) = 0 \Rightarrow \Sigma p_{\text{initial}} = \Sigma p_{\text{final}}$$
$$m(u_1 + u_2) = m(v_1 + v_2) \Rightarrow 3 + (-2) = v_1 + v_2$$
$$\Delta(\Sigma K) = 0 \Rightarrow \Sigma K_{initial} = \Sigma K_{final}$$
$$\frac{1}{2} m u_1^2 + \frac{1}{2} m u_1^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$
$$u_1^2 + u_2^2 = v_1^2 + v_2^2 \ \Rightarrow 3^2 + (-2^2) = v_1^2 + v_2^2$$

2 possible set of answers

Option 1: $v_1 = 3ms^{-1}$; $v_2 = -2ms^{-1}$, (which is just the initial case)

Option 2: $v_1 = -2ms^{-1}$; $v_2 = 3ms^{-1}$ (a more sensible answer)

Sample Problem 3.10 (Perfectly Inelastic collision)

A ball of mass 0.5kg, travelling in the +x direction at $2ms^{-1}$, collides with another ball of mass 0.2kg travelling in the opposite direction at $1.5ms^{-1}$. After the collision, the balls stick together and travels at the same speed. Determine the final velocity and its direction. Compare the kinetic energies before and after the collision.

Solution

Applying conservation of momentum,

$$\Delta(\Sigma p) \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$
$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \Rightarrow (0.5)(2) + (0.2)(-1.5) = (0.5 + 0.2) v$$
$$v = 1ms^{-1} \text{ in the positive x direction}$$

To compare the kinetic energies, we can take their difference.

$$\Delta K = K_{final} - K_{initial}$$
$$\Delta K = \frac{v^2}{2}(m_1 + m_2) - \frac{1}{2}(m_1 u_1^2 + m_2 u_2^2)$$

$$\Delta K = \frac{1^2}{2}(0.5+0.2) - \frac{1}{2}\left((0.5)(2)^2 + (0.2)(-1.5)^2\right) = 0.35 - 1.225$$

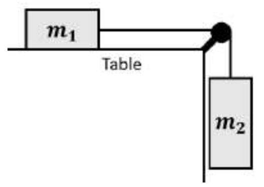
$$\Delta K \ = \ -0.875J$$

This means 0.875J of kinetic energy has been converted into energies of other forms

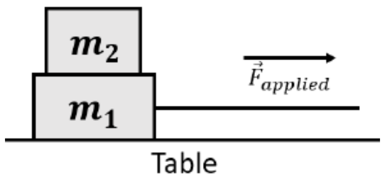
Exercises

1. A car of mass 850kg on a slope inclined at 25^o. Determine the acceleration of the car if the slope is frictionless. If the car was initially at rest and the slope is $20m$ in length, calculate the velocity of the car at the bottom of the slope.

2. Block m_1 with mass 2kg is connected to another block $m_2$of mass *x* kg via a light string that passes over a frictionless pulley as shown in the figure.
   Find the mass of m_2 if the objects accelerate at $7ms^{-2}$ and the coefficient of kinetic friction between the table surface and block m_1 is 0.275.

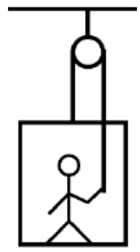


3. A block of mass 2kg rides on top of a second block of mass 5kg and the coefficient of friction between the two blocks is 0.33. A string is attached to the bottom block and pulls the string horizontally across the frictionless table surface. Determine the maximum force that can be applied such that the top block does not slip.

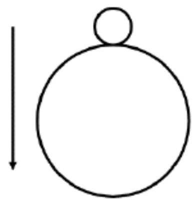


4. The diagram shows a man on a platform lifting himself upwards. The man pulls the rope with 800N force. If the mass of the man and the platform is 80kg and 60kg respectively, calculate

   a. the acceleration of the cradle
   b. force the man exerts on the floor



5. A smaller ball of 50g is held just above a bigger ball of mass 250g with their centres vertically aligned. The balls are then released from rest to fall through 2m as shown in the figure. Upon colliding elastically with the ground, the bigger ball rebounds and collides with the smaller ball that is currently still moving downwards. calculate the height of the rebound of the smaller ball.



6. In cardiology research, the mass of the blood per pump stroke can be determined through a *ballistocardiograph*. The instrument works by having the patient lie on a horizontal platform floating on a film of air such that the air friction is negligible. The patient along with the horizontal platform are initially static. When the heart expels blood into the aorta in one direction, the patient's body along with the platform moves respond by moving in the opposite direction. If the speed in which the heart expels the blood is $59\,cms^{-1}$, determine the mass of the blood that leaves the heart given that the mass of the body and platform is 75kg and that response is that the body with the platform moves $60\mu m$ in 0.180s.

7. A large car traveling at $15ms^{-1}$ collides head on perfectly inelastically with a smaller car travelling at the same speed in the opposite direction. The collision time was 0.15s, compare the magnitude of the forces that the seat belts exert onto the drivers of each car if the total mass of the large car is three times the total mass of the smaller car.

Chapter 4: Work, Energy and Power

Learning Outcomes

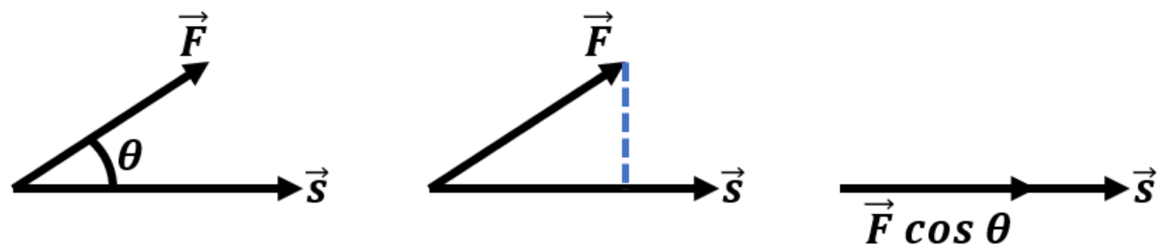
a) State:
   (a) the physical meaning of dot (scalar) product for work: $W = \vec{F} \bullet \vec{s} = Fscos\theta$
   (b) the principle of conservation of energy.
b) Define and apply
   (a) work done by a constant force.
   (b) Gravitational potential energy, $U = mgh$
   (c) Elastic potential energy for spring, $U_s = \frac{1}{2}kx^2 = \frac{1}{2}Fx$
   (d) Kinetic energy, $K = \frac{1}{2}mv^2$
   (e) work-energy theorem, $W = \Delta K$
   (f) average power, $P_{av} = \frac{\Delta W}{\Delta t}$ and instantaneous power, $P = \vec{F} \bullet \vec{v}$
c) Determine work done from a force-displacement graph.
b) Apply the principle of conservation of mechanical energy.

Work

Let us begin by defining work. The work on an object, *W*, is defined to be the product of magnitude of the displacement, *s*, and the force component parallel to the displacement of the object $F_{\parallel}$, i.e.

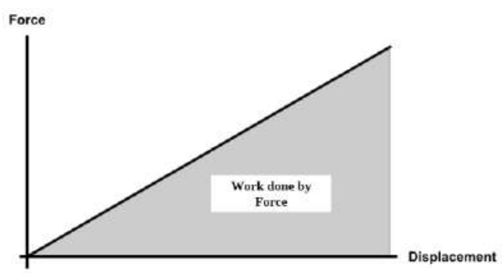
$$W = F_{\parallel}s = \vec{F} \cdot \vec{s}$$

Notice that it is not the product of force and displacement but the product of displacement and force component, the important characteristic of that particular force component is that it must be parallel to the displacement. The diagram below illustrates this point, we cannot simply multiply the magnitude of $\vec{F}$and $\vec{s}$. We must find the component of force that **is** parallel the displacement, and then we can find their product.



This of course means that in the force-displacement graph, work done by a force is equal to the area under the graph. That is to say, the work done to displace an object from $x_i$to x_f is simply

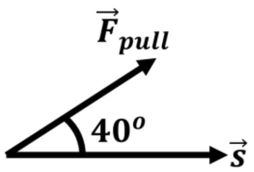
$$W = \int_{x_i}^{x_f} F_{\parallel} dx$$



Sample Problem 4.1

A block is pulled with a force of 50N (directed 40^o from the horizontal) on a smooth horizontal surface for 5m. Calculate the work done by the pulling force.

Answer:

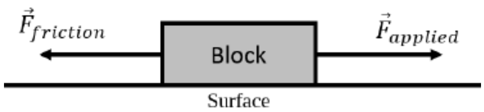


$$W = \vec{F} \cdot \vec{s} = (F\cos\theta)s$$
$$W_{50N} = (50\cos 40)5$$
$$W_{50N} = 191.511\,J$$

Sample Problem 4.2

A block is pushed with 5N in the positive x direction for $2m$ on a horizontal surface. If the block travels at constant speed, calculate the work done by frictional force, work done by the applied force and the total work done.

Answer



Since the block travels at constant speed,

$$F_{net} = 0 = F_{applied} - F_{friction}$$

That means the magnitude of frictional force is equal to the magnitude of applied force, but acts in the opposite direction.

$$F_{friction} = 5N$$
$$W_{friction} = \vec{F}_{friction} \cdot \vec{s} = F_{friction} s\cos\theta$$
$$W_{friction} = (5)(2)\cos(180^o)$$
$$W_{friction} = -10Nm$$

Similarly,

$$W_{applied} = F_{applied} s\cos\theta = (5)(2)\cos 0 = 10Nm$$
$$W_{total} = W_{friction} + W_{applied} = -10 + 10 = 0Nm$$

Work Energy Theorem

When an object moves, we say it contains kinetic energy. Kinetic energy quantifies the amount of energy a moving object has. It depends on the velocity of the moving object,

$$E_k = K = \frac{1}{2}mv^2.$$

Now what we want to do is to show a relationship between the quantity related to moving object (kinetic energy) and another quantity related to the changes of object position (work).

We begin with the definition of work done on an object and Newton's Second Law of motion to show that

$$W = Fs;\ F = ma \Rightarrow W = mas$$

Assuming that the force is constant and therefore the acceleration is also constant, we can then apply equation of kinematics

$$v^2 = u^2 + 2as;\ W = m(as)$$

$$W = m\frac{v^2 - u^2}{2} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = K_{final} - K_{initial} = \Delta K$$

This show derivation brings about an important theorem, called the work-energy theorem. This theorem states that the work done onto a body is equal to the change in kinetic energy of the body.

Sample Problem 4.3
A 150g block begins travelling along a horizontal surface at $4.20ms^{-1}$. If the kinetic friction coefficient between the block and surface is 0.45, calculate the distance that the block moves before coming to a stop.

Answer:
$m_{block} = 0.15kg$; $u = 1.8ms^{-1}$; $u_k = 0.45$

$$W = Fs = \Delta K = F_{final} - K_{initial} \Rightarrow (\mu_k m_{block} g)s = \frac{m_{block}}{2}(v^2 - u^2)$$

$$0.45(0.15)(9.81)s = \frac{0.15}{2}(0^2 - 4.2^2)$$

$$s = 1.99796m$$

Energies

Two more energies that are of interest to us. The first is the gravitational potential energy, which is the energy contained in an object due to its position, measure from a gravitational source. A more detailed analysis is found in the Newtonian Gravity part of this course. At this point, it is sufficient for us to know that if an object of mass m is position at height h from the surface of the Earth, then the gravitational potential energy found in that object is

$$E_{gp} = mgh$$

The second type of potential energy of interest is the elastic potential energy of a spring. By Hooke's Law, the force acting on a spring is direction proportional to its extension (or compression).

$$\text{Hooke's Law: } F = -kx$$

We can then utilize work energy theorem to find the elastic potential energy of a spring,

$$W = -\int F\,dx = \int kx\,dx \Rightarrow E_{ep} = \frac{1}{2}kx^2$$

Apart from the conservation of momentum, another important principle of conservation crucial to our study of moving bodies is the **principle of mechanical energy conservation.** The law simply states that the sum of all kinetic energy and all potential energy must remain constant at all times. That is to say

$$\Delta E_{total} = 0.$$

Sample Problem 4.4
An 2kg object was released from 20m height. Calculate its velocity just before striking the ground.

Answer:
Initially the object would have gravitational potential energy of

$$E_{gp} = mgh = 2(9.81)(20) = 392.4J$$

This energy is then converted fully into kinetic energy at $h = 0m$.
Therefore, the amount of kinetic energy possessed by the body will be 392.4J.

$$E_k = \frac{1}{2}mv^2 = 392.4J \Rightarrow \frac{1}{2}(2)v^2 = 392.4 \Rightarrow v = 19.8ms^{-1}$$

Power

Now that we have familiarize ourselves with work and energy, let us now talk about **power**, which is simply defined by the rate of work done. Average power refers to the work done within a time interval,

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{W_{final} - W_{initial}}{t_{final} - t_{initial}}.$$

On the other hand, instantaneous power refers to the mechanical power at one instant in time

$$P_{instantaneous} = \frac{dW}{dt} = \vec{F}\frac{d\vec{s}}{st} = \vec{F} \cdot \vec{v}$$

Sample Problem 4.5
Calculate the average power required to lift a 75kg man to a height of 10m in 2minutes.
Answer:
By work energy theorem,

$$W = mgh$$

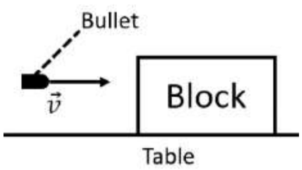
$$P_{average} = \frac{W_f - 0}{t_f - 0} = \frac{(75)(9.81)(10)}{2(60)} = 61.3125W$$

Sample Problem 4.5
Calculate the instantaneous power required to lift a 75kg man at $0.09ms^{-1}$.
Answer:

$$P_{instant} = Fv = mgv = 75(9.81)(0.08) = 66.22W$$

Exercises

1. The figure shows a bullet shot at a wooden block. After the bullet makes contact, the bullet and block travels 20cm in the initial direction of the bullet before coming to a stop. Determine the velocity $\vec{v}$ at which the bullet was shot if the coefficient of kinetic friction is 0.2. The mass of the bullet and block is 5g and 2kg respectively.



2. A ball is kicked off of a pier horizontally 10m above water with an initial speed $15ms^{-1}$. Using the principle of energy conservation, calculate the speed at which the ball hits the surface of the water. Compare this solution to a different approach, that is by analysing it as a projectile.

3. Ali (105kg) jumps off a ledge of 0.75m high and land with his knees unbent. If the compressing joint material is compressed 1.25cm, calculate the force experienced by each knee. If he were to bend his knees, he could extend his stopping distance to 0.25m, calculate the force on his knee if he bent his knees. Compare the force experienced by his knee in both cases.

4. A man is able to throw a 10kg ball by accelerating it from rest to $9ms^{-1}$ in 1.5s while raising the ball 0.5m in height. Calculate the power need to do this.

5. An Axia with mass 850kg climbs a 5^o slope at a constant $25ms^{-1}$. If the frictional force is 350N, calculate the power output.

6. A 10kg weight is released 1m from the tip of a vertically positioned spring, determine the compression if the spring has a spring constant of $82.4\,kNm^{-1}$.

Shafiq R 2022

Study Notes & Exercises

On Linear & Rotational Motion

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Chapter 1: Physical Quantities & Measurements

Learning Outcomes

1. Define dimension, scalar and vector quantities.
2. Determine:
   (a) the dimensions of derived quantities.
   (b) resultant of vectors. (remarks: limit to three vectors only).
3. Verify the homogeneity of equations using dimensional analysis.
4. Resolve vector into two perpendicular components (*x* and *y* axes).

Dimensions

Dimensions refer to the physical nature of a quantity. Regardless of the unit used, the physical nature of a quantity remains the same. For example, a distance, measured in the unit of metres, or feet, is still a measurement of length. This measurement, therefore has the dimensions of length, most commonly represented by **L.** In this instance, the equation $[d] = L$ is simply states that "The dimension of d is length (**L**)". The following table shows some selected physical quantities and its dimensions:

Base Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Length	L	metre (m)	L
Mass	M	kilogram (kg)	M
Time	T	second (s)	T
Electric Current	I	ampere (A)	I
Temperature	T	Kelvin (K)	Θ
Amount of substance	n	mole (mol)	N
Luminosity	L	candela (cd)	J
Derived Quantities			
Quantity	**Symbol**	**S.I. Base Unit**	**Dimensions**
Velocity	$\vec{v}$	ms^{-1}	LT^{-1}
Acceleration	$\vec{a}$	ms^{-2}	LT^{-2}
Momentum	$\vec{p}$	Ns	MLT^{-1}
Angular acceleration	α	$rads^{-1}$	T^{-2}
Electric Charge	Q	Coulomb (A s)	TI
Energy	E	Joule ($J = kgm^2s^{-2}$)	ML^2T^{-2}

Once you understand what dimensions are and how to work with them, you can apply it to **verify the homogeneity of equations.** The word 'homogeneity' refers to 'of the same kind'. Let us consider the equation

$s = ut + \frac{1}{2}at^2$ where s is displacement of a body, t is time taken for the displacement of the body, u is the initial velocity of the body and a is the acceleration of the body. To 'verify homogeneity', we can compare the dimensions the terms on the left-hand side and the right-hand side of the equation. That is to say, s must have the same dimensions as *ut* and $\frac{1}{2}at^2$. s has the dimension of L, so does *ut* as well as $\frac{1}{2}at^2$.

Sample Problem 1.1:

Identify the dimensions for power, P, defined by $P = \frac{E}{t}$ where E is energy and has dimensions of ML^2T^{-2} and *t* has dimension of time, *T*.

Solution:

$$[Power] = \left[\frac{E}{t}\right]$$

$$[Power] = \frac{ML^2T^{-2}}{T}$$

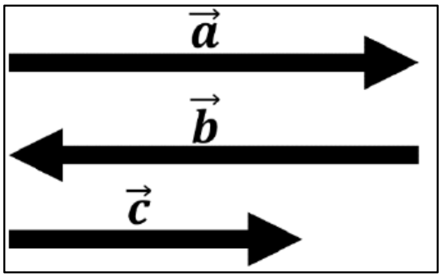
$$[Power] = ML^2T^{-3}$$

The dimensions for power are then ML^2T^{-3}.

Scalars and Vectors

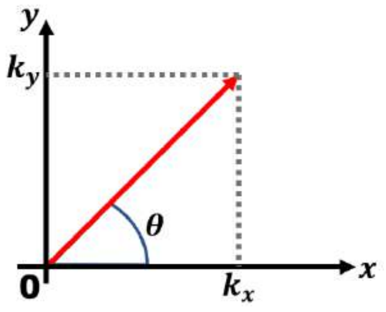
A scalar quantity is a quantity that is fully described by its magnitude. On the other hand, a vector quantity can only be fully described by both its magnitude **and** direction. When you talk about 5kg of rice, that statement is sufficient to describe the mass of the rice, this is where you can see that mass is a scalar quantity. If the rice is falling towards the Earth at a velocity of $2ms^{-1}$, 2 things matter here – how fast the rice is falling **and** the direction in which it is falling. Here you can see that velocity is a vector quantity.

A vector quantity is generally represented by a line segment with an arrowhead. The length of the line segment indicates its magnitude whereas its arrow head tells us the direction of the vector quantity. For example, say vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by the following arrows,



If we were to compare $\vec{b}$ and $\vec{c}$ to $\vec{a}$, we'd say that vector $\vec{b}$ has the same magnitude as $\vec{a}$ but is in the opposite direction, this would tell us that $\vec{b} = -\vec{a}$. Vector $\vec{c}$, on the other hand, is in the same direction as $\vec{a}$. But its magnitude is smaller that $\vec{a}$. The magnitude of vector $\vec{k}$ is denoted by $|\vec{k}|$. We can then relate vector $\vec{a}$ to $\vec{c}$ by the relation $|\vec{a}| > |\vec{c}|$.

Another method to represent vectors is to list the values of its elements in a sufficient number of difference directions, depending on the dimension of the vector. Consider a vector in a 2-dimensional Cartesian coordinate system, a vector $\vec{k}$ can then be represented by $\vec{k} = k_x\hat{\imath} + k_y\hat{\jmath}$ or $\vec{k} =< k_x, k_y >$, defining $\hat{\imath}$ and $\hat{\jmath}$ as unit vectors in the x and y directions respectively. From this notation, one can easily calculate the magnitude (length) of the 2-vector using Pythagoras' Theorem which gives



$$|\vec{k}| = \sqrt{k_x^2 + k_y^2}.$$

Vector additions (or subtractions) can then be done by adding (or subtracting) corresponding components. That is to say, if we have vectors $\vec{a}$ and $\vec{b}$ defined by $\vec{a} =< a_x, a_y >$; $\vec{b} =< b_x, b_y >$, then the addition will yield

$$\vec{a} + \vec{b} =< a_x + b_x, a_y + b_y >.$$

The implication of this definition of vector addition are the following rules:

1. Commutativity of vectors $\Rightarrow \vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. Associativity of vectors $\Rightarrow (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + (-\vec{a}) = 0$

Resolution of vector $\vec{k}$ is then simply

$$k_x = |\vec{k}|cos(\theta);\ k_y = |\vec{k}|sin(\theta).$$

Multiplication of a vector

3 cases to consider when talking about multiplication of a vector:

1. The vector is multiplied by a scalar, then
$$k\vec{a} =< ka_x, ka_y >.$$
2. The **dot product** (also known as scalar or inner product) of two vectors, then
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = |\vec{a}||\vec{b}|\ cos(\theta_{ab}).$$
Note that in the dot product, the operation results in a scalar quantity.
3. The **cross product** (also known as the vector product) of two vectors, then
$$\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\hat{n} = |\vec{a}||\vec{b}|sin(\theta)\hat{n}$$
Note that in the cross product, the operation results in a vector quantity perpendicular to both the x and y axis.

Sample Problem 1.2:

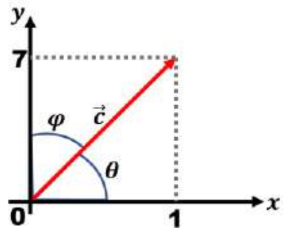
Calculate the magnitude and direction of vector $\vec{c}$ if it is defined by $\vec{c} = \vec{a} + \vec{b}$ where $\vec{a} = [2,3]$ and $\vec{b} = [-1,4]$.

Solution:

Magnitude:

$$\vec{c} =< 2 + (-1), 3 + 4 \geq< 1,7 >$$

$$|\vec{c}| = \sqrt{7^2 + 1^2} \approx 7.071$$



Direction:

(7)

Additional notes:

Unit conversions are so easy that we tend to overlook the importance of practicing it. Here's a simple reminder on how to do it. Say you are given that $1\ in\ =\ 2.54cm$ and you are asked to calculate 8cm in inches, here's how to do it:

$$1in\ =\ 2.54cm \leftarrow \text{divide both side by 2.54}$$

$$\frac{1}{2.54}in = \frac{2.54}{2.51}cm = 1cm \leftarrow \text{now multiply it by 8}$$

$$8cm = \frac{8}{2.54}in \approx 3.1496in$$

And that's how you do unit conversion.

In Physics, it is quite often that we are expected to work with values in form of scientific notation. For example, rather than writing down the speed of light as $300000000ms^{-1}$, we'd express this value as $3 \times 10^8 ms^{-1}$. One issue that may arise from working with scientific notation when using a calculator is the redundancy in typing out "10^x". To help with this, I would suggest that we take advantage of the **rules for exponents**. The example below demonstrates such application.

Sample Problem 1.3:

Evaluate c given that $c = \frac{a^2b}{d}$ and $a = 3 \times 10^{-6}$, $b = 4 \times 10^3$ and $d = 12 \times 10^3$.

Solution

$$c = \frac{a^2b}{d} = \frac{(3 \times 10^{-6})^2(4 \times 10^3)}{(12 \times 10^3)}$$

Rather than evaluating this monstrosity as our input for the calculator, we can instead separate the coefficient from the base and its exponent to evaluate them separately.

$$c = \frac{a^2b}{d} = \frac{(3)^2(4)}{(12)}\left[\frac{(10^{-6})^2(10^3)}{10^3}\right]$$

In this form, the evaluation can be done **easily** even without a calculator.

$$c = \frac{(3)^2(4)}{(12)}\left[\frac{(10^{-6})^2(10^3)}{10^3}\right] = \frac{36}{12}[10^{-6-6+3-3}]$$

$$c = 3 \times 10^{-12}$$

On Significant Figures

When we talk about the number of significant figures, we are talking about the number of digits whose values are known with certainty. This gives us information about the degree of accuracy of a reading in a measurement. In general, we should practice performing rounding off when the conditions call for it. This is to avoid false reporting. What we mean by false reporting is to give the illusion that our experiments are more sensitive than it actually it. For example, it would be very unlikely that our metre ruler to give reading in the micro scale.

Number	Number of significant figures		Number	Number of significant figures
2.32	3		2600	2
2.320	4		2602	4

When we do calculations, there are some rules (based on the operations) we should be aware of when stating the significant figures of the end value:

1. Multiplication / Divisions – number of significant figures in the result is the same as the least precise measurement in the least precise measurement used in the calculation.

Example:

$$\frac{2.5(3.15)}{2.315} = 3.4$$

2. Addition / Subtraction – The result has the same number of decimal places as the least precise measurement used in the calculation.

   Example:

$$91.1 + 11.45 - 12.365 = 90.2$$

3. Logarithm / antilogarithm – Keep as many significant figures to the right of the decimal point as the are significant in the original number.

   Example:

$$ln(4.00) = 1.39;\ e^{0.0245} = 1.03$$

Exercises

1. Two vectors lie on the x-y plane. Vector $\vec{a}$ is 5 units long and points 25^o above the x-axis. Vector $\vec{b}$ is 8 units long and points upward 35^o above the x-axis. What is the resultant of the two vectors?

2. Show that the equation $v^2 = v_o^2 + 2ax$ is dimensionally correct. In this expression, a is acceleration, v and v_o are
velocities and x is the distance.

3. Derive the dimensions of k if $k = \frac{ab}{d}$ where $[a] = M^2T^2$, $[b] = M^{-1}T^{-1}$ and $[d] = MT^2$.

4. A car is travelling at 35miles per hour, what is its speed in metres per second if $1mile = 1.61km$.

5. Express each of the following in grams
   a. $3lb$ if $1g = 2.2 \times 10^{-3}lb$
   b. $12.2oz$ if $1g = 0.035oz$
6. How many square centimetres are in a square inch if $1cm = 0.3937inch$.
7. The length and the width of a rectangle is $(11 \pm 0.05)cm$ and (21 ± 0.05) cm respectively. Find the area (and its uncertainty) of the rectangle.

Chapter 2: Kinematics of Linear Motion

Learning Outcomes (LO)

1. Define:
   a. instantaneous velocity, average velocity and uniform velocity; and
   b. instantaneous acceleration, average acceleration and uniform acceleration.
2. Derive and apply equations of motion with uniform acceleration
   $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$; $s = \frac{1}{2}(u + v)t$
3. Describe projectile motion launched at an angle, θ as well as special cases when θ=0°
4. Solve problems related to projectile motion.

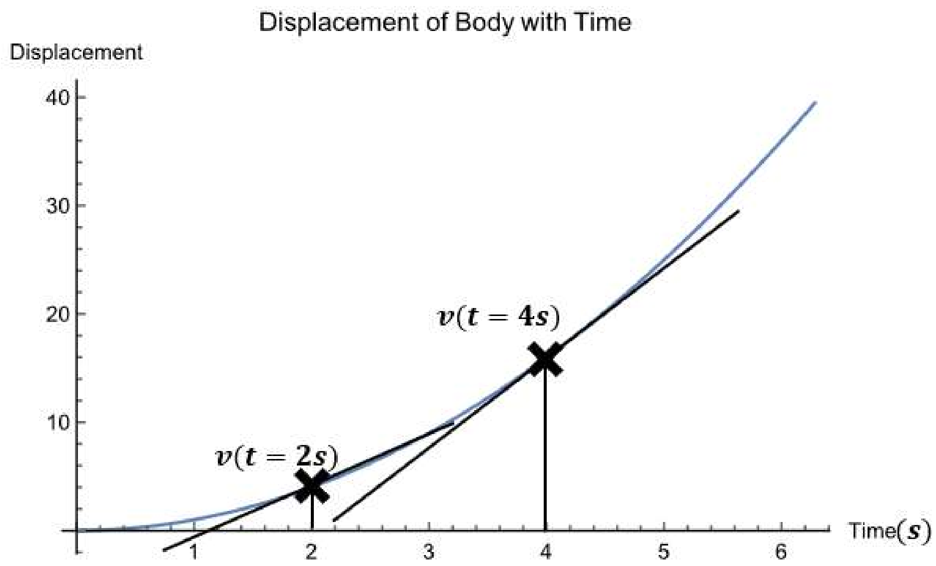
In this chapter, we talk about kinematics of linear motion. Dynamics is the study of motion of bodies under action of forces and their effects. One subbranch to the study of dynamics is kinematics. In the study of kinematics, we consider only the motion of the bodies without worrying too much about the forces that caused the bodies to move. We only worry about the geometry of the motion.

Instantaneous and Average Velocity (or acceleration)

Let us start with reminder of some ideas and terms that you have learnt in your SPM days. 3 mains terms – displacement (), velocity () & acceleration (). Displacement, denoted by x, simply refers to the change in position of a body. Velocity, v, refers to the rate of change of this change in position, i.e. $v = \frac{dx}{dt}$. Acceleration, a, is defined by the rate of change of velocity, which is the rate of change of the rate of change of position. That is $a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$.

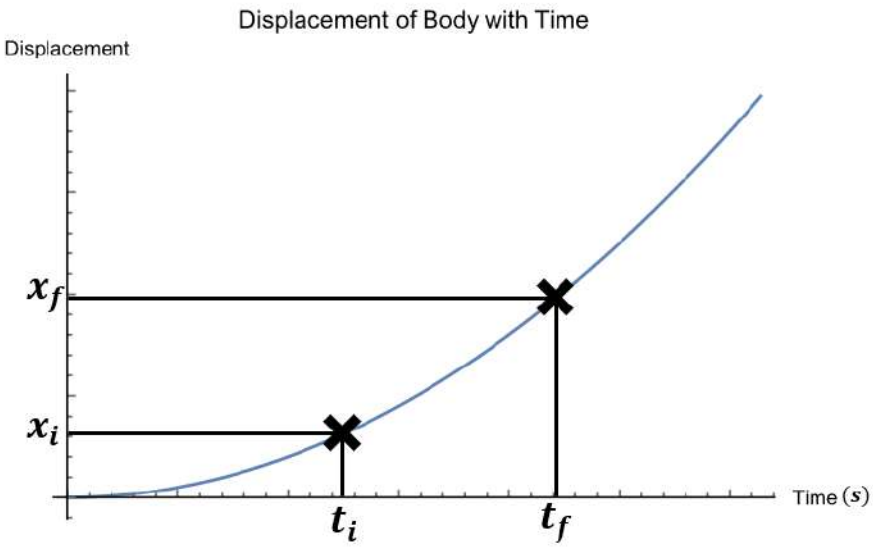
Once we have established that, we can further extend our ideas of velocity and acceleration by thinking about ***instantaneous*** velocity (or acceleration) and ***average*** velocity (or acceleration). By 'instantaneous', we mean 'at a particular instant in time'. When we combine it with velocity (or acceleration), what we mean is velocity (or acceleration) at a particular instant in time. On the other hand, when we say 'average', what we mean is 'over the course of a defined time span'. So, when we say 'average velocity', we usually would accompany it with 'between time t_a and t_b' or 'in 30 seconds', specifying a range of time.

Let us now have a graphical representation. Consider a body moving at constant velocity,



When we talk about instantaneous velocity, we are asking about a single point in time. From the displacement-time graph, the gradient represents the velocity of the body. As we can see from the graph, the instantaneous velocities when $t = 2s$ and $t = 4s$ are different. This is simply because the body is moving at a non-uniform velocity. If the instantaneous velocities are the same, then we call the motion is described as uniform velocity.

On the other, talking about ***average*** velocity, we simply define range of time, thus choosing two points in time rather than one. Then we take the difference in position and divide it by the difference in time to



calculate the ***average*** velocity. That is to say, for the graph below,

We can calculate the average velocity as

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

We can take the same approach and understanding and apply it to acceleration, but with a velocity time graph rather than a displacement time graph.

Sample Problem 2.1

The motion of a body is described by the equation $v = 2t^2$, where v is in metres per second and t is in seconds. Calculate the instantaneous velocity of the body at $t = 3s$ and the average acceleration between $t = 2s$ and $t = 4s$.

Answer:

$$v_{instantaneous} = 2(3)^2 = 18ms^{-1}$$

$$v(t = 2s) = 2(2)^2 = 8ms^{-1}$$

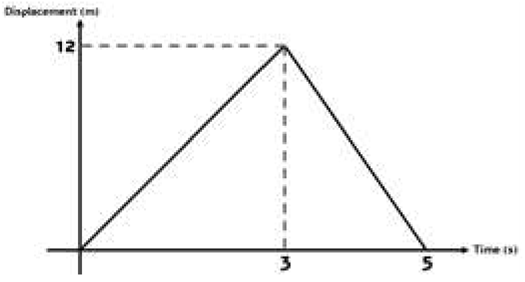
$$v(t = 4s) = 2(4)^2 = 32ms^{-1}$$

$$a_{average} = \frac{\Delta v}{\Delta t} = \frac{v(t = 4s) - v(t = 2s)}{4 - 2} = \frac{32 - 8}{4 - 2}$$

$$a_{average} = 12ms^{-2}$$

Sample Problem 2.2

The motion of a body is shown in the graph shown in Figure 1. Calculate the displacement of the body and calculate the average velocity of the body.



Answer:

Displacement of the body can be calculated by recognising that the area under the velocity -time graph represents the displacement of the body. So all we need is to sum up all the areas under the graph.

$$s = \frac{1}{2}(12 \times 3) + \frac{1}{2}\big(12 \times (5 - 3)\big) = 30m$$

The average velocity can be calculated by simply dividing the displacement by the total time of motion.

$$v_{average} = \frac{30}{5} = 6ms^{-1}$$

Kinematic Equations

Now we want to look at kinematic equations, which are equations that relates variables that describes motion such as displacement, velocity and acceleration.

Derivation by calculus

We'd like to derive the equations from our understanding of linear motion and using calculus. We begin with the definition of acceleration

$$a = \frac{dv}{dt}$$

Assuming constant acceleration, we can rearrange then integrate both sides to yield

$$a\int_{t_{initial}}^{t_{final}} dt = \int_{v_{initial}}^{v_{final}} dv \Rightarrow a\big(t_{final} - t_{initial}\big) = v_{final} - v_{initial}$$

Adjusting such that $t_{initial} = 0$, $t_{final} = t$ and defining $v = v_{final}$, $v_{initial} = u$.

And then rearranging this equation yields

$$v = u + at$$

which is the same equation as the first equation found in LO2. Simply put, the final velocity of a body is initial velocity plus the product of acceleration and time difference.

We can take the same approach to find the third equation in LO2 using the first equation. We start with the definition of velocity and then rearranging it,

$$v = \frac{dx}{dt} \Rightarrow \int v\,dt = \int dx$$

Note that since velocity is not a constant, $v\,dt$ cannot be directly integrated. We therefore need an equation for velocity as a function of time (first equation).

$$\int u + a\,t\,dt = \int dx$$

Since u and a are constants, these integrals become

$$u\int_0^t dt + a\int_0^{\mathrm{t}} dt = \int_0^{\mathrm{x}} dx$$

Solving this integral gives

$$x = ut + \frac{1}{2}at^2.$$

For the second equation, we can start take advantage of calculus by starting with a time independent derivative,

$$\frac{dx}{dv} = \frac{dx}{dt}\frac{dt}{dv} = \frac{v}{a}$$

Rearranging this gives us the needed integral to solve

$$a\int_0^x dx = \int_u^v v\,dv \Rightarrow ax = \frac{1}{2}(v^2 - u^2)$$

Further rearrangement yields an equation

$$v^2 = u^2 + 2ax$$

matching with the third equation found in LO2.

Equation 4 of LO2 does not require any integration, rather we can obtain it using $s = ut + \frac{1}{2}at^2$ and $v = u + at$. This is left for the reader to do.

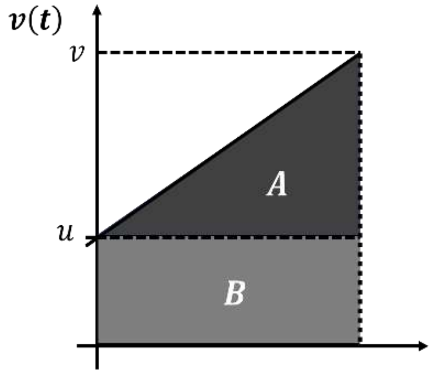
Geometric Derivation

By definition,

$$a = \frac{v - u}{t}$$

Rearranging this gives

$$v = u + at$$



Consider an object that starts its motion with velocity u and maintains its constant acceleration a to a final velocity of v. We can describe its motion diagrammatically as below

Since the area under the graph represents displacement, all we need to do is to add up the area of A and B. If

$$\text{Area}_A = \frac{1}{2}(t)(v-u) = \frac{1}{2}(t)(at) = \frac{1}{2}at^2$$

$$\text{Area}_B = u\text{t}$$

then

$$\text{s} = u\text{t} + \frac{1}{2}at^2.$$

If, on the other hand, we consider

$$s = \frac{1}{2}(t)(v-u) + ut$$

Then we find that

$$s = \frac{1}{2}(v+u)t$$

For the equation of $v^2 = u^2 + 2as$, we can start the derivation by considering

$$v = u + at \Rightarrow t = \frac{v-u}{a}$$

And

$$s = \frac{1}{2}(u+v)t.$$

We can substitute time equation into the displacement to yield

$$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right) = \frac{v^2-u^2}{2a} \Rightarrow v^2 = u^2 + 2as.$$

<u>Sample Problem 2.3</u>

A 2022 Honda Accord can travel down a $\frac{1}{4}$ mile track in $14.1s$ from rest. Calculate the acceleration (in SI units), assuming that its acceleration is constant.

<u>Answer:</u>

Values given $\Rightarrow s = \frac{1}{4}\, mile = 402.336km\,; t = 14.1s;\ u = 0ms^{-1}$

Choice of equation $\Rightarrow s = ut + \frac{1}{2}at^2$

$$402.336 = \frac{1}{2}a(14.1)^2$$

$$a = 4.04744ms^{-2}$$

Sample Problem 2.4

A car initially travels at $20ms^{-1}$. If the car undergoes constant acceleration of $1.2ms^{-2}$, determine the time the car need to reach double of its initial velocity.

Answer:

Values given ⇒ $\mathrm{u} = 20\mathrm{ms}^{-1}$; $\mathrm{a} = 1.2\mathrm{ms}^{-2}$; $\mathrm{v} = 2\mathrm{u} = 40\mathrm{ms}^{-1}$

Choice of equation ⇒ $v = u + at$

$$40 = 20 + 1.2t$$

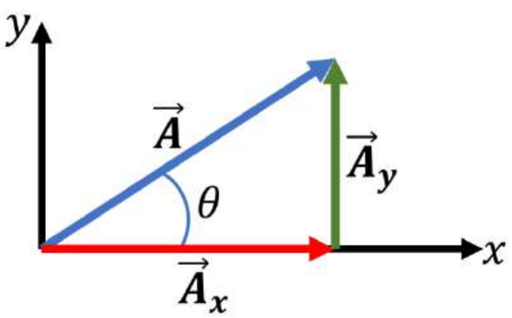
$$\boldsymbol{t = \frac{50}{3} s}$$

Projectile Motion (Motion in 2 Dimensions)

When dealing with motion in two dimensions, the minimum that we need is the Pythagorean theorem as well as the definition of tangent. We consider a vector $\vec{A}$ defined by

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where $\vec{A}_x$ and $\vec{A}_y$ are component vectors of $\vec{A}$, each parallel to one of the axes in a rectangular coordinate system.



It then follows that the magnitude and direction of $\vec{A}$ can be related to its components by the Pythagorean theorem and the definition of tangent,

$$|\vec{A}| = \sqrt{|\vec{A}_x|^2 + |\vec{A}_y|^2}$$

$$\tan\theta = \frac{|\vec{A}_y|}{|\vec{A}_x|}.$$

Conversely, we can work out the components of $\vec{A}$ from the magnitude of $\vec{A}$ and the angle θ,

$$|\vec{A}_x| = |\vec{A}|\cos\theta$$

$$|\vec{A}_y| = |\vec{A}|\sin\theta$$

One case study that we can do on two-dimensional motion is **projectile motion,** a motion that follows a parabolic path. The simplest case of projectile motion would be one where the air resistance and the rotation of the Earth is simply neglected, and that the motion is only affected by the Earth's gravity ($\vec{F}_{gravity} = m\vec{g}$). One important aspect of this case is that the horizontal (x-direction) and vertical (y-direction) motions are independent of each other. This means that the kinematics equation we have studied earlier can be dealt with separately for both x and y directions.

Keeping in mind that $u_x = u\,cos\,\theta$ and $u_y = u\,sin\,\theta$, we can then work out the 6 equations that describes the projectile motion:

x-direction (where $a_x = 0$)	y-direction (where $a_y = -g$)
$v_x = u_x$	$v_y = u_y - gt$
$s_x = u_x t$	$s_y = u_y t - \frac{1}{2}gt^2$
$v_x^2 = u_x^2$	$v_y^2 = u_y^2 - 2gs_y$

We can also work out the velocity of the projectile by keeping in mind that it merely follows from Pythagorean theorem

$$v^2 = v_x^2 + v_y^2.$$

If we substitute the equation for time-x-component, $t = \frac{s_x}{u_x}$, in the equation for displacement in y-direction, $s_y = u_y t - \frac{1}{2}gt^2$, what we get is the parabolic equation for the projectile motion path,

$$\left(\frac{u_y}{u_x}\right)s_x - \left(\frac{1}{2u_x^2}\right)s_x^2 - s_y = 0.$$

There are two more items that are of our interest:

1. If we were to look for the "**peak**" of the parabolic path, we can do so by applying $v_y = 0$ to the kinematics equations. This is simply because it is at this peak that $u_y = gt$ such that the velocity of the projectile is momentarily zero before the projectile falls back down towards the Earth.
2. Another item that would be of our interest is the **range** of the projectile motion. By range, what we are referring to is the point at which the projectile reaches back to ground or stop accelerating in the y-direction. This would differ from case to case, of course, and we shall demonstrate in the sample problems following this.

Sample Problem 2.5

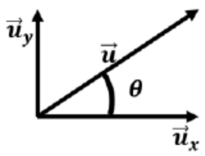
An object is launched at a velocity of $21ms^{-1}$ in a direction making an angle of 30^o upward with the horizontal.

Calculate

a. Initial velocity in x and y direction.
b. the location of the object at $t = 2s$.
c. the total horizontal range.
d. the velocity of the object just before it hits the ground.

Answer:

a.



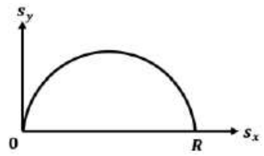
$$|\vec{u}_x| = |\vec{u}|\,cos\theta = 21\,cos\,30$$
$$|\vec{u}_x| = 18.1865ms^{-1}$$
$$|\vec{u}_y| = |\vec{u}|\,sin\theta\ = 21\,sin\,30$$
$$|\vec{u}_y| = 10.5ms^{-1}$$

b. $s_x = u_x t \Rightarrow s_x = (18.1865)(2) = 36.3730m$

$s_y = u_y t + \frac{1}{2}a_y t^2\ \Rightarrow s_y = (10.5)(2) + \frac{1}{2}(-9.81)(2^2) = 1.38m$

c. $s_y = u_y t + \frac{1}{2}a_y t^2\ \Rightarrow 0 = (10.5)(t) + \frac{1}{2}(-9.81)(t^2) \Rightarrow t\ =\ \{0,2.14067\}s$

$s_x|_{t=2.14067s} = (18.1865)(2.14067) = 38.9313m\ =\ R$



d. $v_x = u_x = 18.1865ms^{-1}$

$v_y = u_y + a_y t = 10.5 + (-9.81)(2.14067) = -10.5ms^{-1}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(18.1865)^2 + (-10.5)^2} = 21ms^{-1}$

$\theta = tan^{-1}\left(\frac{18.1865}{-10.5}\right) = -60^o$

Sample Problem 2.6

Compare the horizontal range of a ball thrown at velocity $35ms^{-1}$ if the angle of release is 15^o, 30^o, 45^o and 60^o.

Answer:

Condition for determining horizontal range ⇒ $s_y = 0$.

$s_y = u_y t + \frac{1}{2}a_y t^2 = (u\,cos\,\theta)t + \frac{1}{2}a_y t^2$

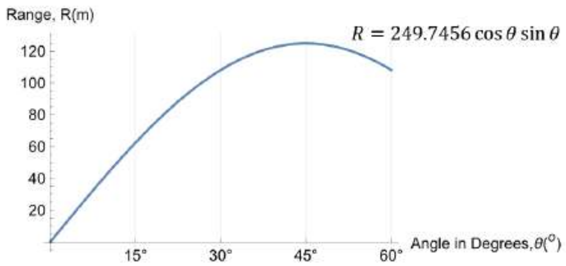
$0 = (35\,cos\,\theta)t - (4.905)t^2$

$t = \{0,\ 7.13558\,cos\,\theta\}s$

$R = s_x = u_x t = (u\,sin\,\theta)\text{t}$

$R = (35\,sin\,\theta)(7.13558\cos\theta)$

$R = 249.7456\cos\theta\,sin\,\theta$



$\theta(^o)$	t (s)	$R(m)$
15^o	6.89244	62.4363
30^o	6.17959	108.143
45^o	5.04562	124.873
60^o	3.56779	108.143

The horizontal range peaks at 45^o.

Exercises

1. A car accelerates uniformly from rest for 5.88s for a distance of 150m. Calculate the acceleration of the car.
2. An object is accelerated uniformly from $2ms^{-1}$ to $8ms^{-1}$ over a distance of 38m. Calculate the acceleration of the object.
3. Car A, moving at a constant velocity of $10ms^{-1}$, passes by Car B at rest. 2 seconds after Car A passes, Car B starts accelerating at $5ms^{-2}$ in the same direction as Car A. When does Car B surpasses Car A?
4. A car, initially at rest at position A, accelerates at $2ms^{-2}$ for 5 seconds and then maintains its velocity for 10 seconds. After which the car decelerates back to $0ms^{-1}$ in 3s. Sketch the velocity-time graph of the car and calculate the total displacement of the car from position A.
5. The Tailgating Problem [1]:
    a. Determine the stopping distance of a BMW M3 if the car can decelerate at a rate of $9.2ms^{-2}$ from $97kmh^{-1}$.
    b. Consider the reaction time of a driver is 0.55s, calculate the stopping distance of the BMW M3 mentioned in the previous question.
    c. "When pigs fly" – A driver driving a car travelling at $28ms^{-1}$ suddenly notices a pig (oink oink!) $28m$ on the road ahead. With what velocity would the car hit the pig if the car decelerated at $8ms^{-2}$ and the driver's reaction time was $0.8s$?
6. A projectile is launched at a velocity of $20ms^{-1}$ at angle of 30^o to the horizontal. Calculate the
    a. Time of flight of the projectile.
    b. Horizontal range of the projectile.
    c. The maximum height of the projectile.
    d. The speed of projectile at a third of the maximum height.
7. A diver, on a pier 10m from the water, jumps up at an angle 27^o from the horizontal with an initial velocity of $4.2ms^{-1}$. Calculate the
    a. Time at which the diver hits the water.
    b. The speed of the diver just before he hits the water.
8. A projectile is fired at velocity v and it has a horizontal range of 10m. If the projectile is again fired at the twice its velocity before, determine the new horizontal range.

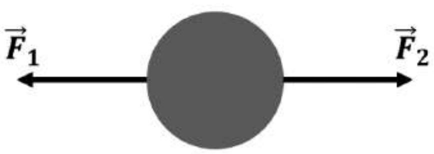
Chapter 3: Dynamics of Linear Motion

Learning Outcomes

5. Define
    a. Momentum, $\vec{p} = m\vec{v}$
    b. Impulse, $J = F\Delta t$
6. Solve problem related to impulse and impulse-momentum theorem,
$$J = \Delta p = mv - mu$$
**1D only*
7. Use *F-t* graph to determine impulse.
8. State:
    a. the principle of conservation of linear momentum.
    b. Newton's laws of motion.
9. Apply
    a. the principle of conservation of momentum in elastic and inelastic collisions in 2D collisions.
    b. Newton's laws of motion.
    **include static and dynamic equilibrium for Newton's first law motion*
10. Differentiate elastic and inelastic collisions. (remarks: similarities & differences)
11. Identify the forces acting on a body in different situations - Weight, W; Tension, T; Normal force, N; Friction, f; and External force (pull or push), F.
12. Sketch free body diagram.
13. Determine static and kinetic friction, $f_s \leq \mu_s N$, $f_k = \mu_k N$

In the previous chapters, we have looked at describing motion without the hassle of asking, "what force is causing the body to move?". In this chapter, we aim to expand our knowledge to a body's motion in that very aspect.

Types of Forces



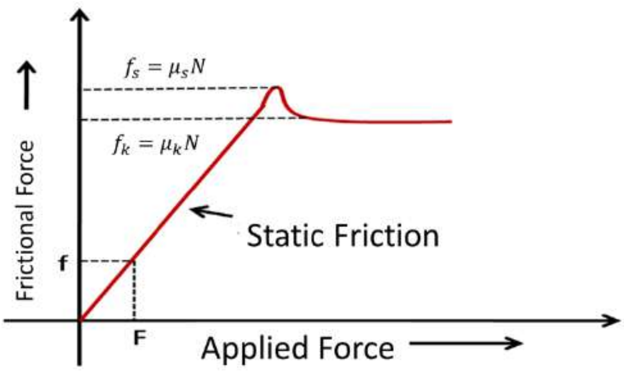
$$\vec{F}_{net} = \vec{F}_1 - \vec{F}_2$$

We begin with asking the question, "what is force?", a simple answer would be to say force is a push and pull. Here, however, let us define force a bit further. Let us define force as **an agent for motion change**. Force is a vector quantity, that means **direction matters**. Two oppositely directed force acting on the same body work against each other. A body can experience multiple forces acting on it, however it is the net force, i.e., the resultant of all the forces acting on the body, that changes the motion of the body.

4 types of forces we'd consider in this chapter – gravitational (weight), tensional, normal and frictional. Their definitions and directions are as follows:

Forces	Definitions	Directions
Gravitational	Force exerted upon a body interacting with a gravitational field.	Towards the gravitational source.
Tensional	Force transmitted axially through a massless one-dimensional continuous element.	Along the one-dimensional continuous element.
Normal	Support force, perpendicular to the surface, exerted upon a body in contact with a stable object.	Perpendicular to the surface the body is in contact with.
Frictional	Force acting upon bodies that are in contact and moving relative to each other.	Against the direction of motion.

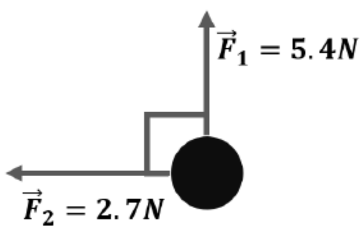
One particular type of force that maybe be of our interest is frictional force. This is because frictional force depends on the motion of the object. If the object is static, then it is subject to *static friction*. On the other hand, if the object is moving (relative to the surface it is in contact with) at some velocity, and therefore has some kinetic energy, then the object is subject to *kinetic friction*. Static friction is generally higher than kinetic friction because of the asperities (roughness) of the surfaces of the contacting bodies. This asperity enables the surfaces to interlock with each other, causing adhesion. This means that the force applied to the body must overcome this adhesion before the bodies can start moving relative to each other. This phenomenon can be observed by looking at the frictional force as a function of applied force graph:



As the applied force is increased, so does the frictional force. This is true until a certain threshold is reached, after which the body will start to move. This threshold is exactly the unlocking of the asperities.

Sample Problem 3.1

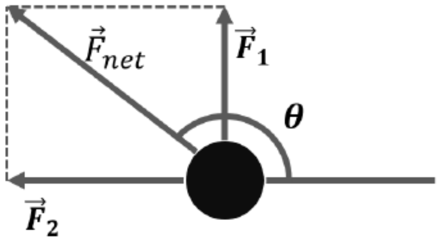
Determine the resultant force exerted on the body shown in the diagram.



Solution

$$|\vec{F}_{net}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2}$$

$$|\vec{F}_{net}| = \sqrt{(5.4)^2 + (-2.7)^2}$$

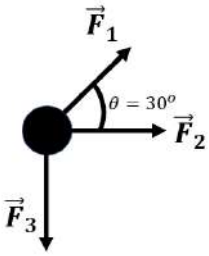


$$|\vec{F}_{net}| = 6.03738N$$

$$\theta = 180^o - tan^{-1}\left(\frac{5.4}{2.7}\right)$$

$$\theta = 116.565^o$$

Sample Problem 3.2



Calculate the resultant force if $|\vec{F}_1| = 10N$, $|\vec{F}_2| = 12.5N$ and $|\vec{F}_3| = 17N$.

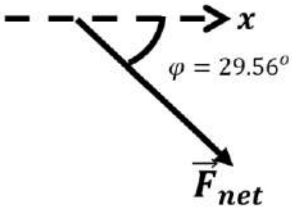
Solution

Force	x-components	y-components
F_1	$F_1\, cos\, 30^o$ $= 10\, cos\, 30^o$ $\approx 8.66N$	$F_1\, sin\, 30^o$ $= 10\, sin\, 30^o$ $\approx 5N$
F_2	$+12.5N$	0N
F_3	0N	$-17N$
Σ	21.16N	-12N

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{(21.16)^2 + (-12)^2}$$

$$F_{net} = 24.3258N$$

$$\varphi = tan^{-1}\left(\frac{12}{21.16}\right) = 29.56^o$$



Newton’s Law of Motion

There are laws of motion that a moving under force would generally follow. These laws were first introduced and came into its modern form via Newton’s *Principia*. In it, 3 laws of motions were found:

1. A body, when no external force is applied, will not undergo velocity change, i.e.

$$\vec{F}_{external} = 0 \Rightarrow \Delta\vec{v} = 0$$

2. When a force is acted upon it, a body will move in a manner such that rate of momentum change is equal to the said force, i.e.

$$\vec{F} = \frac{d\vec{p}}{dt}.$$

3. Forces exerted onto two interacting bodies will be equal in magnitude but opposite in direction.

   If $\vec{F}_{12}$ is force exerted onto body 1 by body 2, then

$$\vec{F}_{12} = -\vec{F}_{21}$$

These three laws form the foundation for what is known today as the *Newtonian Laws of Motion*.

Momentum

The first and third requires no further definitions of variable, however the second one, mentions an idea of ***momentum***. It seems useful to define this term at this point. What we mean by momentum at this point is the property of a moving body that rises from the product of the mass the body and its velocity, i.e.

$$\vec{p} = m\vec{v}.$$

If the velocity of the body changes (and therefore so does the momentum of the body in question), we quantify that change and call it ***impulse***:

$$\vec{J} = \Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t.$$

This also means that the area under the F-t graph represents impulse.

It is from Kinematics that we know a change in velocity means a non-zero acceleration. Knowing this as well as Newton's Second Law, we can say that force is present when acceleration is non-zero,

$$\vec{F} = m\vec{a}.$$

This statement, of course can be derived quite simply from Newton's Second Law of motion:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

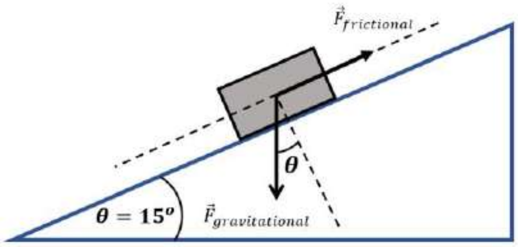
Here we can see that a change in mass may also produce force and that if mass change is zero, then what we have is the equation seen before,

$$\vec{F} = m\vec{a}.$$

Sample Problem 3.3

Suppose an object of 15 kg is placed on an incline plane of 15^o from the horizontal. Calculate the magnitude of frictional force that keeps the object from sliding down the incline plane.

Solution



$F_{net} = F_g \sin\theta - F_{frictional}$

Since the object stays static $\Rightarrow F_{net} = 0$

$F_{frictional} = F_g \sin\theta = mg \sin\theta$

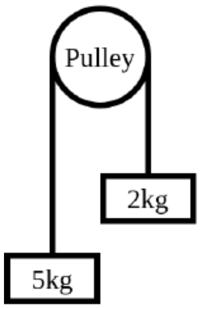
$F_{frictional} = (15)(9.81)sin15^o$

$F_{frictional} = 38.0852N$

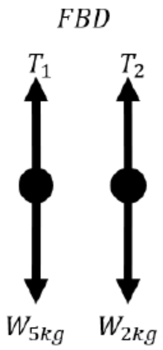
Sample Problem 3.4

Problem:

Based on the diagram, calculate the acceleration of both the objects.



Solution



Equation of motion for 5kg body:

$$F_{net} = W_{5kg} - T_1 = m_{5kg}a$$

Equation of motion for 5kg body:

$$F_{net} = T_2 - W_{2kg} = m_{2kg}a$$

$$T_1 = T_2 = T$$

Rearrange for a,

$$T = W_{5kg} - m_{5kg}a = W_{2kg} + m_{2kg}a$$

$$a = \frac{W_{5kg} - W_{2kg}}{m_{2kg} + m_{5kg}} = \frac{9.81(5-2)}{5+2}$$

$$a \approx 4.2ms^{-2}$$

The 2kg object will move **upwards** at $a \approx 4.2ms^{-2}$.

The 5kg object will move **downwards** at $a \approx 4.2ms^{-2}$.

Sample Problem 3.5

A 50g ball at $30ms^{-1}$ is travelling towards a wall. Upon striking the wall, the ball bounces back in the opposite direction at a speed of $10ms^{-1}$. Calculate the impulse.

Solution

$J = \Delta p = p_{final} - p_{initial}; p = mv$

$$J = m(v_{final} - v_{initial})$$

$$J = (0.05)((-10) - (+30))$$

$$J = -2kg\,ms^{-1}$$

Sample Problem 3.6

A footballer kicks a 300g ball from rest to a speed of $60ms^{-1}$ in a collision lasting $1.5ms$. Calculate the force generated by the footballer.

Solution

$$F\Delta t = \Delta p = p_{final} - p_{initial}$$

Since $p_{initial} = 0$, then

$$F(1.5 \times 10^{-3}) = (0.3)(60)$$

$$F = 12kN$$

One of the ways for bodies to interact is through collisions. When this happens, assuming this happens in an isolated system. The total momentum of the system doesn't change with the passage of time. The momenta of the participating bodies may change, but not the vector sum total momentum of the system. When we say that a quantity doesn't change, we say that the quantity is **conserved**. So, in this case, we say that the **total momentum is conserved**. Conservation of momentum is simply

$$\Delta(\Sigma p) = 0.$$

When we talk about collisions, we may consider two types of collision – elastic and inelastic collisions. Note that whilst momentum is conserved in **all** types of collisions, kinetic energy is not as it may be converted into other forms of energy (e.g., sound energy). It is this exact parameter from which we differentiate elastic from inelastic collisions. A perfectly elastic collision is defined by a collision in which both momentum and kinetic energy is conserved whilst a perfectly inelastic collision is a collision in which conservation of kinetic energy is not obeyed.

Sample Problem 3.7 (Conservation of momentum)

Ball A of mass 30g, travels at $3ms^{-1}$ collides head on with Ball B of 50g at rest. Calculate the velocity of Ball B after the collision if Ball A has the final velocity of $1.2ms^{-1}$.

Solution

$$\Delta\Sigma p = 0 \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(0.03)(3) + 0 = (0.03)(1.2) + (0.05)v_B$$

$x = 1.08ms^{-1}$ in the same direction as Ball A.

Sample Problem 3.8 (Conservation of momentum in 2 Dimensions)

Ball A of mass 3kg, travels at $3ms^{-1}$ in the positive x direction collides with Ball B of 5kg travelling at $2ms^{-1}$ in the positive y direction. If the balls stick together after the collision, determine their velocity.

Solution

Need to apply conservation of momentum in x and y direction separately.

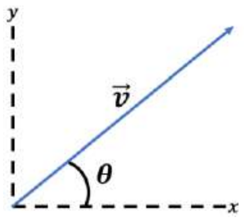
	X	Y
Momentum before	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_B = 0$ $\Sigma p = m_A u_A$	$\Sigma p = p_A + p_B = m_A u_A + m_B u_B$ $u_A = 0$ $\Sigma p = m_B u_B$
Momentum after	$\Sigma p = (m_A + m_B) v \cos \theta$	$\Sigma p = (m_A + m_B) v \sin \theta$
Final velocity	$m_A u_A = (m_A + m_B) v \cos \theta$ $v \cos \theta = \frac{m_A u_A}{m_A + m_B}$ ---(1)	$m_B u_B = (m_A + m_B) v \sin \theta$ $v \sin \theta = \frac{m_B u_B}{m_A + m_B}$ ---(2)

$$\frac{(2)}{(1)} : \tan \theta = \left(\frac{m_B u_B}{m_A + m_B}\right)\left(\frac{m_A + m_B}{m_A u_A}\right) = \frac{m_B u_B}{m_A u_A} = \frac{(5)(2)}{(3)(3)}$$

$$\theta = 48.0128^o$$

$$v \cos \theta = \frac{m_A u_A}{m_A + m_B} \Rightarrow v \cos (48.0128^o) = \frac{(3)(3)}{3(5)}$$

$$v = 0.897 m s^{-1}$$



Sample Problem 3.9 (Perfectly Elastic Collision)

A ball, travelling at $3ms^{-1}$, collides head on with another ball of the same mass, travelling $2ms^{-1}$ in the opposite direction. Determine their velocities post-collision?

Solution

Let us assume that ball 1, travelling at $3ms^{-1}$, initially travels in the positive direction such that

$$u_1 = +3ms^{-1};\ u_2 = -2ms^{-1}$$

Since the balls are of the same mass,

$$m_1 = m_2 = m$$

Assuming the collision is elastic, the system would obey both the conservation of momentum and the conservation of kinetic energy.

$$\Delta(\Sigma \mathrm{p}) = 0 \Rightarrow \Sigma \mathrm{p}_{\mathrm{initial}} = \Sigma \mathrm{p}_{\mathrm{final}}$$

$$m(u_1 + u_2) = m(v_1 + v_2) \Rightarrow 3 + (-2) = v_1 + v_2$$

$$\Delta(\Sigma K) = 0 \Rightarrow \Sigma K_{initial} = \Sigma K_{final}$$

$$\frac{1}{2} m u_1^2 + \frac{1}{2} m u_1^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$

$$u_1^2 + u_2^2 = v_1^2 + v_2^2 \ \Rightarrow 3^2 + (-2^2) = v_1^2 + v_2^2$$

2 possible set of answers

Option 1: $v_1 = 3ms^{-1}$; $v_2 = -2ms^{-1}$, (which is just the initial case)

Option 2: $v_1 = -2ms^{-1}$; $v_2 = 3ms^{-1}$ (a more sensible answer)

Sample Problem 3.10 (Perfectly Inelastic collision)

A ball of mass 0.5kg, travelling in the +x direction at $2ms^{-1}$, collides with another ball of mass 0.2kg travelling in the opposite direction at $1.5ms^{-1}$. After the collision, the balls stick together and travels at the same speed. Determine the final velocity and its direction. Compare the kinetic energies before and after the collision.

Solution

Applying conservation of momentum,

$$\Delta(\Sigma p) \Rightarrow \Sigma p_{initial} = \Sigma p_{final}$$

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v \Rightarrow (0.5)(2) + (0.2)(-1.5) = (0.5 + 0.2)v$$

$$v = 1ms^{-1} \text{ in the positive x direction}$$

To compare the kinetic energies, we can take their difference.

$$\Delta K = K_{final} - K_{initial}$$

$$\Delta K = \frac{v^2}{2}(m_1 + m_2) - \frac{1}{2}(m_1 u_1^2 + m_2 u_2^2)$$

$$\Delta K = \frac{1^2}{2}(0.5 + 0.2) - \frac{1}{2}\left((0.5)(2)^2 + (0.2)(-1.5)^2\right) = 0.35 - 1.225$$

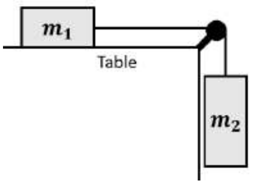
$$\Delta K = -0.875J$$

This means 0.875J of kinetic energy has been converted into energies of other forms

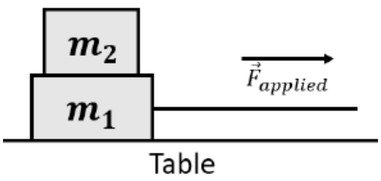
Exercises

1. A car of mass 850kg on a slope inclined at 25^o. Determine the acceleration of the car if the slope is frictionless. If the car was initially at rest and the slope is $20m$ in length, calculate the velocity of the car at the bottom of the slope.

2. Block m_1 with mass 2kg is connected to another block $m_2$of mass *x* kg via a light string that passes over a frictionless pulley as shown in the figure.
   Find the mass of m_2 if the objects accelerate at $7ms^{-2}$ and the coefficient of kinetic friction between the table surface and block m_1 is 0.275.

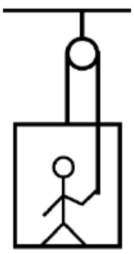


3. A block of mass 2kg rides on top of a second block of mass 5kg and the coefficient of friction between the two blocks is 0.33. A string is attached to the bottom block and pulls the string horizontally across the frictionless table surface. Determine the maximum force that can be applied such that the top block does not slip.

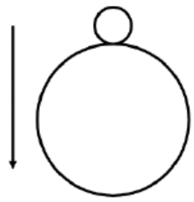


4. The diagram shows a man on a platform lifting himself upwards. The man pulls the rope with 800N force. If the mass of the man and the platform is 80kg and 60kg respectively, calculate

   a. the acceleration of the cradle
   b. force the man exerts on the floor



5. A smaller ball of 50g is held just above a bigger ball of mass 250g with their centres vertically aligned. The balls are then released from rest to fall through 2m as shown in the figure. Upon colliding elastically with the ground, the bigger ball rebounds and collides with the smaller ball that is currently still moving downwards. calculate the height of the rebound of the smaller ball.



6. In cardiology research, the mass of the blood per pump stroke can be determined through a *ballistocardiograph*. The instrument works by having the patient lie on a horizontal platform floating on a film of air such that the air friction is negligible. The patient along with the horizontal platform are initially static. When the heart expels blood into the aorta in one

direction, the patient's body along with the platform moves respond by moving in the opposite direction. If the speed in which the heart expels the blood is $59\,cms^{-1}$, determine the mass of the blood that leaves the heart given that the mass of the body and platform is 75kg and that response is that the body with the platform moves $60\mu m$ in 0.180s.

7. A large car traveling at $15ms^{-1}$ collides head on perfectly inelastically with a smaller car travelling at the same speed in the opposite direction. The collision time was 0.15s, compare the magnitude of the forces that the seat belts exert onto the drivers of each car if the total mass of the large car is three times the total mass of the smaller car.

Chapter 4: Work, Energy and Power

Learning Outcomes

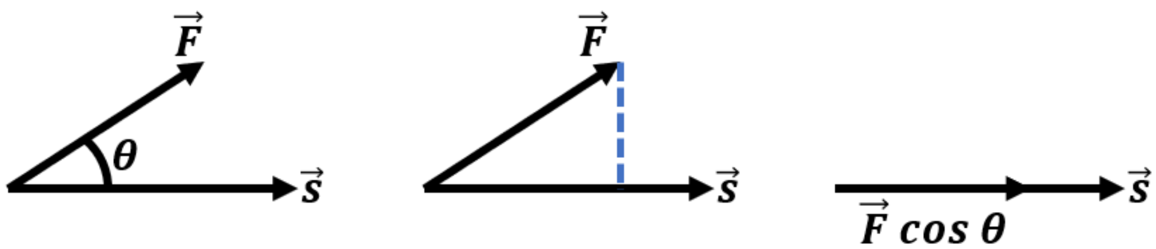
a) State:
   (a) the physical meaning of dot (scalar) product for work: $W = \vec{F} \bullet \vec{s} = Fscos\theta$
   (b) the principle of conservation of energy.
b) Define and apply
   (a) work done by a constant force.
   (b) Gravitational potential energy, $U = mgh$
   (c) Elastic potential energy for spring, $U_s = \frac{1}{2}kx^2 = \frac{1}{2}Fx$
   (d) Kinetic energy, $K = \frac{1}{2}mv^2$
   (e) work-energy theorem, $W = \Delta K$
   (f) average power, $P_{av} = \frac{\Delta W}{\Delta t}$ and instantaneous power, $P = \vec{F} \bullet \vec{v}$
c) Determine work done from a force-displacement graph.
b) Apply the principle of conservation of mechanical energy.

Work

Let us begin by defining work. The work on an object, W, is defined to be the product of magnitude of the displacement, s, and the force component parallel to the displacement of the object $F_{\parallel}$, i.e.

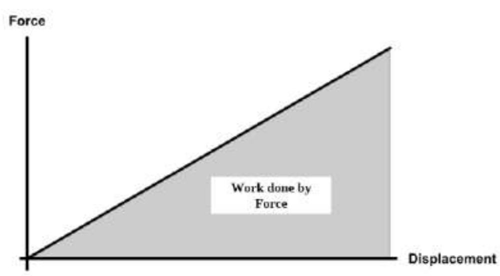
$$W = F_{\parallel}s = \vec{F} \cdot \vec{s}$$

Notice that it is not the product of force and displacement but the product of displacement and force component, the important characteristic of that particular force component is that it must be parallel to the displacement. The diagram below illustrates this point, we cannot simply multiply the magnitude of $\vec{F}$and $\vec{s}$. We must find the component of force that **is** parallel the displacement, and then we can find their product.



This of course means that in the force-displacement graph, work done by a force is equal to the area under the graph. That is to say, the work done to displace an object from $x_i$to x_f is simply

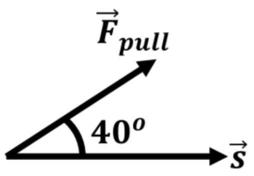
$$W = \int_{x_i}^{x_f} F_{\parallel} dx$$



<u>Sample Problem 4.1</u>

A block is pulled with a force of 50N (directed 40^o from the horizontal) on a smooth horizontal surface for 5m. Calculate the work done by the pulling force.

<u>Answer:</u>



$$W = \vec{F} \cdot \vec{s} = (F\cos\theta)s$$

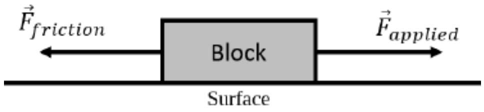
$$W_{50N} = (50\cos 40)5$$

$$W_{50N} = 191.511\,J$$

Sample Problem 4.2

A block is pushed with 5N in the positive x direction for $2m$ on a horizontal surface. If the block travels at constant speed, calculate the work done by frictional force, work done by the applied force and the total work done.

Answer



Since the block travels at constant speed,

$$F_{net} = 0 = F_{applied} - F_{friction}$$

That means the magnitude of frictional force is equal to the magnitude of applied force, but acts in the opposite direction.

$$F_{friction} = 5N$$

$$W_{friction} = \vec{F}_{friction} \cdot \vec{s} = F_{friction} s\cos\theta$$

$$W_{friction} = (5)(2)cos(180^o)$$

$$W_{friction} = -10Nm$$

Similarly,

$$W_{applied} = F_{applied} s\cos\theta = (5)(2)\cos 0 = 10Nm$$

$$W_{total} = W_{friction} + W_{applied} = -10 + 10 = 0Nm$$

Work Energy Theorem

When an object moves, we say it contains kinetic energy. Kinetic energy quantifies the amount of energy a moving object has. It depends on the velocity of the moving object,

$$E_k = K = \frac{1}{2}mv^2.$$

Now what we want to do is to show a relationship between the quantity related to moving object (kinetic energy) and another quantity related to the changes of object position (work).

We begin with the definition of work done on an object and Newton's Second Law of motion to show that

$$W = Fs\,;\ F = ma \Rightarrow W = mas$$

Assuming that the force is constant and therefore the acceleration is also constant, we can then apply equation of kinematics

$$v^2 = u^2 + 2as;\ W = m(as)$$

$$W = m\frac{v^2 - u^2}{2} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = K_{final} - K_{initial} = \Delta K$$

This show derivation brings about an important theorem, called the work-energy theorem. This theorem states that the work done onto a body is equal to the change in kinetic energy of the body.

Sample Problem 4.3

A 150g block begins travelling along a horizontal surface at $4.20ms^{-1}$. If the kinetic friction coefficient between the block and surface is 0.45, calculate the distance that the block moves before coming to a stop.

Answer:

$m_{block} = 0.15kg;\ u = 1.8ms^{-1};\ u_k = 0.45$

$$W = Fs = \Delta K = F_{final} - K_{initial} \Rightarrow (\mu_k m_{block} g)s = \frac{m_{block}}{2}(v^2 - u^2)$$

$$0.45(0.15)(9.81)s = \frac{0.15}{2}(0^2 - 4.2^2)$$

$$s = 1.99796m$$

Energies

Two more energies that are of interest to us. The first is the gravitational potential energy, which is the energy contained in an object due to its position, measure from a gravitational source. A more detailed analysis is found in the Newtonian Gravity part of this course. At this point, it is sufficient for us to know that if an object of mass *m* is position at height *h* from the surface of the Earth, then the gravitational potential energy found in that object is

$$E_{gp} = mgh$$

The second type of potential energy of interest is the elastic potential energy of a spring. By Hooke's Law, the force acting on a spring is direction proportional to its extension (or compression).

Hooke's Law: $F = -kx$

We can then utilize work energy theorem to find the elastic potential energy of a spring,

$$W = -\int F\,dx = \int kx\,dx \Rightarrow E_{ep} = \frac{1}{2}kx^2$$

Apart from the conservation of momentum, another important principle of conservation crucial to our study of moving bodies is the **principle of mechanical energy conservation.** The law simply states that the sum of all kinetic energy and all potential energy must remain constant at all times. That is to say

$$\Delta E_{total} = 0.$$

Sample Problem 4.4

An 2kg object was released from 20m height. Calculate its velocity just before striking the ground.

Answer:

Initially the object would have gravitational potential energy of

$$E_{gp} = mgh = 2(9.81)(20) = 392.4J$$

This energy is then converted fully into kinetic energy at $h = 0m$.

Therefore, the amount of kinetic energy possessed by the body will be 392.4J.

$$E_k = \frac{1}{2}mv^2 = 392.4J \Rightarrow \frac{1}{2}(2)v^2 = 392.4 \Rightarrow v = 19.8ms^{-1}$$

Power

Now that we have familiarize ourselves with work and energy, let us now talk about **power**, which is simply defined by the rate of work done. Average power refers to the work done within a time interval,

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{W_{final} - W_{initial}}{t_{final} - t_{initial}}.$$

On the other hand, instantaneous power refers to the mechanical power at one instant in time

$$P_{instantaneous} = \frac{dW}{dt} = \vec{F}\frac{d\vec{s}}{st} = \vec{F} \cdot \vec{v}$$

Sample Problem 4.5

Calculate the average power required to lift a 75kg man to a height of 10m in 2minutes.

Answer:

By work energy theorem,

$$W = mgh$$

$$P_{average} = \frac{W_f - 0}{t_f - 0} = \frac{(75)(9.81)(10)}{2(60)} = 61.3125W$$

Sample Problem 4.5

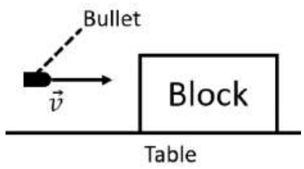
Calculate the instantaneous power required to lift a 75kg man at $0.09ms^{-1}$.

Answer:

$$P_{instant} = Fv = mgv = 75(9.81)(0.08) = 66.22W$$

Exercises

1. The figure shows a bullet shot at a wooden block. After the bullet makes contact, the bullet and block travels 20cm in the initial direction of the bullet before coming to a stop. Determine the velocity $\vec{v}$ at which the bullet was shot if the coefficient of kinetic friction is 0.2. The mass of the bullet and block is 5g and 2kg respectively.



2. A ball is kicked off of a pier horizontally 10m above water with an initial speed $15ms^{-1}$. Using the principle of energy conservation, calculate the speed at which the ball hits the surface of the water. Compare this solution to a different approach, that is by analysing it as a projectile.
3. Ali (105kg) jumps off a ledge of 0.75m high and land with his knees unbent. If the compressing joint material is compressed 1.25cm, calculate the force experienced by each knee. If he were to bend his knees, he could extend his stopping distance to 0.25m, calculate the force on his knee if he bent his knees. Compare the force experienced by his knee in both cases.
4. A man is able to throw a 10kg ball by accelerating it from rest to $9ms^{-1}$ in 1.5s while raising the ball 0.5m in height. Calculate the power need to do this.
5. An Axia with mass 850kg climbs a 5^o slope at a constant $25ms^{-1}$. If the frictional force is 350N, calculate the power output.
6. A 10kg weight is released 1m from the tip of a vertically positioned spring, determine the compression if the spring has a spring constant of $82.4\,kNm^{-1}$.

Learning Outcomes

14. Define and use:
    a. angular displacement, θ
    b. period, T
    c. frequency, f
    d. angular velocity, ω
15. Describe uniform circular motion.
16. Convert units between degrees, radian, and revolution or rotation.
17. Explain centripetal acceleration and centripetal force, $a_c = \frac{v^2}{r} = r\omega^2 = v\omega$

    and $F_c = \frac{mv^2}{r} = mr\omega^2 = mv\omega$

18. Solve problems related to centripetal force for uniform circular motion cases: horizontal circular motion, vertical circular motion and conical pendulum.
    **exclude banked curve*

Chapter 5: Circular Motion

Consider moving a body from coordinates (0,r) to (r,0) whilst keeping the same distance r, from the origin (0,0). The motion that has taken place is what is known as a **rotation** and the path the body has taken is what we consider to be **circular**. We call it **circular** simply because throughout the motion, a fixed distance r was kept between the origin and the object. This is shown in in the diagram 4.1. This transformation may be easy enough to see and describe as it is simply a 90^o rotation. However, dealing with rotations using Cartesian coordinates can get really complicated. So let us propose new method of describing such motion.

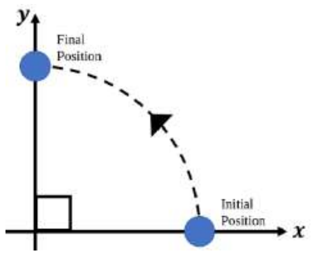


Diagram 5- 1

In linear dynamics, we started with displacement x and took derivatives of it twice over to obtain acceleration. In dealing with rotational motion, let us instead begin with angular displacement, θ. The rate of change of this angular displacement, we can then call **angular velocity** ω , and the rate of change of angular velocity is what is known as **angular acceleration**.

$$\omega = \frac{d\theta}{dt};\ \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

If we define θ in radian, then we can work out the arc length of the object's path using

$$s = r\theta.$$

It is at this point, it is useful for use to know the conversion between angles in radians and angles in degrees, which is $2\pi\text{rad} = 360^o$. A sample conversion practice is demonstrated in Sample Problem 5.1.

<u>Sample Problem 5.1</u>

Convert the following angles to its alternative units:

a. 25^o

b. $\frac{\pi}{3}$ radians

Answer:

a. $2\pi\text{rad} = 360^o \Rightarrow 1^o = \frac{2\pi}{360}$ rad

$25^o = \frac{(25)2\pi}{360}$ rad $= \frac{5}{36}\pi$ rad

$25^o \approx 0.44$ rad

b. $2\pi\text{rad} = 360^o \Rightarrow 1\text{ rad} = \left(\frac{360}{2\pi}\right)^o$

$\frac{\pi}{3}\text{ rad} = \left(\frac{\pi}{3} \times \frac{360}{2\pi}\right)^o = 60^o$

If we differentiate the arc length with respect to time, what we get is quite simple the tangential velocity v, which is the speed at which the body covers said length.

$$v = \frac{d}{dt}s = r\frac{d\theta}{dt} \Rightarrow v = r\omega$$

We may apply the same logic to find the tangential acceleration a and relate it to angular acceleration α,

$$a = \frac{d}{dt}v = r\frac{d\omega}{dt} \Rightarrow a = r\alpha.$$

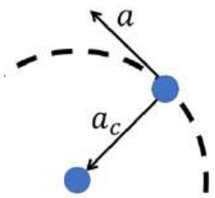


Diagram 5- 2

Now we ask what direction are these quantities, tangential velocity and tangential acceleration? As the name suggests, they have the direction tangent to the circular path, illustrated in Diagram 5-2. Now, tangential acceleration alone will not be enough to ensure the body in motion to follow a circular path, we now need an acceleration towards the centre of the circle, called a **radial (centripetal) acceleration,** generally denoted by a_c. Together with the tangential acceleration, they combined and ensures the body follows a circular path.

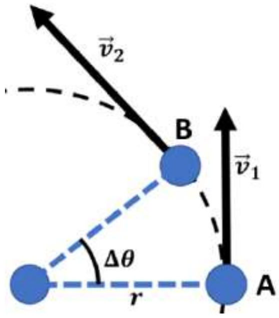


Diagram 5-3

We can now work out the equation for this centripetal acceleration. We can begin by reminding ourselves that the effect of centripetal acceleration is the change in direction. Mind that the speed does not change, but the direction changes. Referring to Diagram 5-3, we can see that if v is relatively small,

$$a_c = \frac{dv}{dt} = \frac{\Delta v}{\Delta t}$$

$$\Delta v \approx v\Delta\theta \text{ (by geometry)}$$

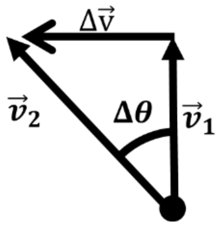


Diagram 5- 4

We also know that the change in arc length is related to the change in the angle,

$$\Delta\theta = \frac{\Delta s}{r} \approx \frac{v\Delta t}{r} \Rightarrow \Delta t \approx \frac{r\Delta\theta}{v}$$

This means that

$$a_c = \frac{v^2}{r}.$$

The force associated with this centripetal acceleration is known as the **centripetal acceleration** and follows the equation

$$F = ma_c = \frac{mv^2}{r}.$$

Centripetal force is **not** a type of force per se. Rather it is a way to say a force is acting as a centripetal force. For example, the gravitational force causes the moon to curve and travel in a circular path around the earth. In this instance, the gravitational force acts as a centripetal force. The way we can think about this is when we talk about retarding force, that retarding force in linear motion could be friction force or any other applied force acting opposing to the direction of motion. In the case of centripetal force, any force could act as centripetal force if it is the force that causes the body to follow a circular path.

When we work with bodies following a circular path, we know that after $\Delta\theta = 2\pi$, the object has returned to its initial position. We say that it has undergo one full revolution. The time the body take to travel one revolution is what we call **period** *T*, and the number of revolutions per unit time is what we call **frequency** ***f***. Frequency and period is merely the inverse of each other.

$$T = \frac{1}{f}$$

The case for conical pendulum

The conical pendulum is a system of pendulum in which rather than having the pendulum bob swing back and forth in a single place, the path of the pendulum bob is circular about a center, whereby the string along with the pendulum bob traces a cone.

Consider a conical pendulum consisting of a bob of mass *m* revolving without friction in a circular path at constant speed *v* on a string of length *l* at an angle θ from the vertical, as shown in Diagram 5-5. We can see that two forces acting on the pendulum bob, tension along the string and weight of the pendulum bob. The tensional force can be resolved into its horizontal component $T\ sin\ \theta$, and its vertical component $T\ cos\ \theta$.

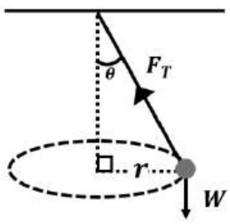
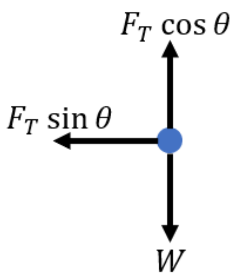


Diagram 5- 5



Applying Newton's second law, we find that

$$F_T cos\ \theta = mg;\ F_T\ sin\ \theta = \frac{mv^2}{r} = \frac{mv^2}{l\ sin\ \theta}$$

To find the angle θ from the vertical,

$$\frac{F_T\ sin\ \theta}{F_T\ cos\ \theta} = tan\ \theta = \left(\frac{v^2}{gr}\right)$$

$$\theta = tan^{-1}\left(\frac{v^2}{gr}\right)$$

To find the period for the pendulum,

$$F_T = \frac{mg}{cos\ \theta} = \frac{mv^2}{r\ sin\ \theta};\ v = r\omega = \frac{2\pi r}{T}$$

$$\frac{g}{cos\ \theta} = \frac{1}{r\ sin\ \theta}\left(\frac{4\pi^2 r^2}{T^2}\right) \Rightarrow T(r,\theta) = 2\pi\sqrt{\frac{r}{g\ tan\ \theta}}$$

Noting that $r = l\ sin\ \theta$, the period of the oscillation is therefore

$$T(l,\theta) = 2\pi\sqrt{\frac{l\ cos\theta}{g}}$$

We can see that in the case for the conical pendulum, the period is independent of the mass used, rather it depends on the length of the string used.

The case for vertical pendulum

Consider swinging a ball of mass m vertically via a string of negligible mass, such that it follows a circular path with radius r. The path could be illustrated as shown in Diagram 5-6.

When the ball is at the top of the path, we can see that the tensional force is directed in the same direction as the ball's weight. On the other hand, when the ball is at the bottom of the path, the tensional force is directed in the opposite direction of the weight of the ball.

We can apply compare the velocities of the ball at any generic position on the path using conservation of energy.

$$\frac{1}{2}mv_{bottom}^2 = mgh + \frac{1}{2}mv_{generic}^2$$

$$v_{generic} = \sqrt{v_{bottom}^2 - 2gh}$$

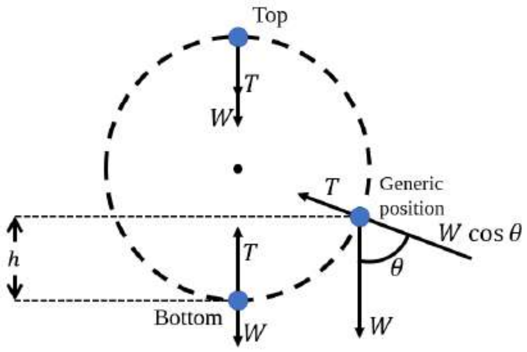


Diagram 5- 6

We can see that the velocity of the ball is not constant as it follows the vertical circular. In this case, we do recognize that it is the ball undergoes circular motion but not **uniform** circular motion. This is of course if the tensional force along the string is constant. If different tension is applied along the circular path, then it is possible to ensure **uniform circular motion**.

Let us compare then the tension needed at the top and the bottom of the circular path. At the bottom, the tension is pointing upwards and the weight is pointing downwards. We then have $F_c = T - mg$. At the top, the tensional force and the weight is pointing in the same direction (downwards) and therefore we have $F_c = T + mg$.

Now if we consider the forces acting on the ball at the generic position,

$$F_{net} = F_c = \frac{m\, v_{generic}^2}{r} = T - mg\, cos\, \theta$$

From the figure, we find $cos\, \theta$ to be

$$cos\, \theta = \frac{r-h}{r} = 1 - \frac{h}{r}$$

As such, we may express the tensional force along the string to be

$$T = m\left(\frac{v_{generic}^2}{r} + g - \frac{gh}{r}\right)$$

Expressing this in terms of the speed at the bottom gives,

$$T = m\left(\frac{v_{bottom}^2}{r} + g - \frac{3gh}{r}\right)$$

$$T = \frac{mg}{r}\left(\frac{v_{bottom}^2}{g} + r - 3h\right)$$

What can we do with this information? Well one of the things we can do is to talk about the **minimum speed** at the bottom of the motion to ensure the ball completes one loop.

At the top of the loop, we want the tensional force to be positive, such that

$$T_h = T_{highest} \geq 0$$

At this point,

$$h = 2r \Rightarrow T_h = \frac{mg}{r}\left(\frac{v_{bottom}^2}{g} + r - 6r\right) = \frac{mg}{r}\left(\frac{v_{bottom}^2}{g} - 5r\right)$$

Since we know $\frac{mg}{r} > 0$, then for $T_h \geq 0$, we need

$$\frac{v_{bottom}^2}{g} - 5r \geq 0$$

So the **minimum speed** required at the bottom of the motion to ensure the ball completes one loop must follow the condition of

$$v \geq \sqrt{5gr}$$

We shall deal with rotational kinematics in the following chapter.

Sample Problem 5.2 (Horizontal Circular Motion)

A 0.45 kg ball is attached to a 1.2 m string and swings in a circular path. The angle of the string is at horizontal. Find the tension in the string if the ball makes 2 revolutions per second

Answer:

$$F_c = F_T = ma_c = mr\omega^2 = 4\pi^2 mrf^2$$

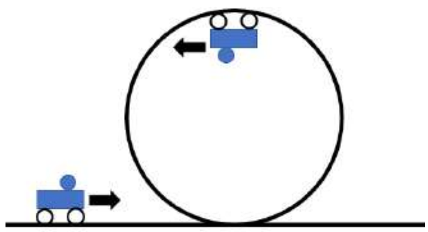
$$2 \text{ rev } s^{-1} = 4\pi\text{rad}s^{-1}$$

$$F_T = (4)(\pi^2)(0.45)(1.2)(4\pi)^2$$

$$F_T = 34.56\pi^4\, N$$

Sample Problem 5.3 (Vertical Circular Motion)

The figure shows a motorcyclist attempting to ride up a loop-the-loop in a vertical circle. The radius of the loop is 10m and the total mass of the motorcycle and the motorcyclist is 200kg. Calculate the minimum speed the motorcyclist must be at when entering the loop-the-loop such that the motorcyclist is able to complete the loop.



Answer:

At the top of the loop, the following FBD can be drawn.

As such, applying Newton's Law gives

$$F_c = \frac{mv^2}{r} = N + W$$

This follows our treatment for ball connected to a string in vertical circular motion. As such the minimum speed so that

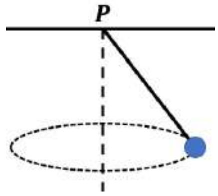
$R \geq 0$, requires

$$v_{bottom} = \sqrt{5gr} = \sqrt{5(9.81)(10)} = 22.15ms^{-1}$$

So, the motorcyclist will need to enter the loop at the bottom with speed of at least $22.15ms^{-1}$.

Sample Problem 5.2 (Conical Pendulum)

The diagram shows a small ball of 200g connected to a ceiling via a massless string 15cm long. The small ball rotates about a point vertically under point P. If the string makes an angle of 30^o with the vertical, determine the tensional force along the string.

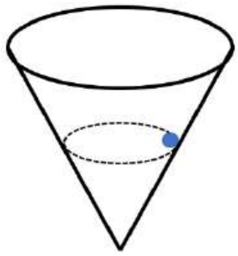


Answer:

$$T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta} = \frac{0.2(9.81)}{\cos 30^o} \Rightarrow T = 2.265N$$

Exercise

1. A cyclist moves around a curve of radius 3m at a constant speed of $30ms^{-1}$. Calculate the resultant change in velocity what the cyclist goes around 45^o.
2. A civil engineer was assigned to plan a curve on a horizontal road with a speed limit of $75mh^{-1}$. Determine the maximum radius of such curve if the coefficient of friction between the road and the tires is 0.4.
3. A small spherical ball moves at $2ms^{-1}$ in a horizontal circle on the inside surface of an inverted cone as shown in the figure. If the apex angle of the cone is 40^o and the friction is negligible, calculate the radius of the circular path.



4. A kid of 40kg stands 50cm from the centre on a rotating platform, that is rotating at 40rpm. Determine the minimum value of the coefficient of stating friction between the kid and the platform such that the kid doesn't slides off the platform.
5. After going through rain with an umbrella, we would twirl the umbrella to remove the water on it. If we twirl an opened umbrella at a rate of 50rpm and the water droplet is 0.25m from the centre, calculate where the water droplet falls on the floor if the heigh of the umbrella is 2m from the floor.

Chapter 6: Rotation of Rigid Body

Learning Outcomes

1. Define and use:

    a. angular displacement, θ;
    b. average angular velocity, ω_{av};
    c. instantaneous angular velocity, ω;
    d. average angular acceleration, α_{av}; and
    e. instantaneous angular acceleration, α
    f. torque
    g. moment of inertia, $I = \sum mr^2$
    h. net torque, $\sum \tau = I\alpha$
    i. angular momentum, $L = I\omega$
2. Analyse parameters in rotational motion with their corresponding quantities in linear motion:

    $s = r\theta, v = r\omega, a_t = r\alpha, a_c = r\omega^2 = \frac{v^2}{r}$

3. Solve problem related to rotational motion with constant angular acceleration:

    $\omega = \omega_o + \alpha t, \theta = \omega_o + \frac{1}{2}\alpha t^2, \omega^2 = {\omega_o}^2 + 2\alpha\theta, \theta = \frac{1}{2}(\omega_o + \omega)t$

4. State and apply:

    a. the physical meaning of cross (vector) product for torque, $|\vec{\tau}| = rFsin\theta$
    b. the conditions for equilibrium of rigid body, $\sum F = 0, \sum \tau = 0$
    c. the principle of conservation of angular momentum.
5. Solve problems related to equilibrium of a uniform rigid body.

Revisions & Definitions

In the previous chapters, we have familiarised ourselves with the idea of instantaneous quantities, average quantities, angular displacement, angular velocity as well as angular acceleration. We recap those ideas in this section.

When we say angular velocity, what we mean is the rate of change of angular displacement θ,

$$\omega = \frac{d\theta}{dt}.$$

We may find the **average** angular velocity if we are only concerned about the final state and the initial state of θ, i.e.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_{final} - \theta_{initial}}{t_{final} - t_{final}}.$$

Such distinctions can also be done for angular acceleration, i.e.

$$\alpha_{instant.} = \frac{d\omega}{dt};\ \alpha_{ave} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_{final} - \omega_{initial}}{t_{final} - t_{initial}}$$

If we consider the relationship between angular displacement, θ and the arc length of a motion, s, we can quite quickly workout the relationship between the linear speed v and the angular speed ω.

$$s = r\theta \overset{\frac{d}{dt}}{\Rightarrow} v = r\omega$$

Similar operations can be done to fine the relationship between tangential acceleration, a_t and angular acceleration, α.

$$v = r\omega \overset{\frac{d}{dt}}{\Rightarrow} a_t = r\alpha$$

Analogy to linear kinematics

We now have the ingredients we need to work out the **equations for rotational motion with constant angular acceleration**. Because ω and α may be defined analogously to their linear counterparts, v and a_t, equations for linear kinematics may be applied when we make substitutions θ for s, ω for v, ω_o for u and α for a. Below we present the results of the substitutions

$$v = u + at \Rightarrow \omega = \omega_o + \alpha t$$

$$s = ut + \frac{1}{2}at^2 \Rightarrow \theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$v^2 = u^2 + 2as \Rightarrow \omega^2 = \omega_o^2 + 2\alpha$$

Sample Problem 6.1

A rotating platform reaches an angular velocity of $66\,\text{rad}s^{-1}$ from rest in 10s. Calculate the angular acceleration and the total angular displacement through the 10s.

Answer:

$\omega = 66\ \text{rad}s^{-1}$; $\omega_o = 0\ \text{rad}s^{-1}$; $t = 10s$

$$\omega = \omega_o + \alpha t \Rightarrow \alpha = \frac{\omega - \omega_o}{t} = \frac{66 - 0}{10}$$

$$\alpha = 6.6\ \text{rad}s^{-2}$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 \Rightarrow \theta = \frac{1}{2}(6.6)(10)^2$$

$$\theta = 330\ \text{rad}$$

Sample Problem 6.2

Brakes were applied to a rotating wheel rotating at 100rpm initially. The wheel turns a further 15 revolutions before coming to a complete stop. Calculate the angular acceleration.

Answer:

$$\omega = 0\ \text{rad}s^{-1};\ \omega_o = 100rpm = \frac{100(2\pi)}{60}\text{rad}s^{-1} = \frac{4\pi}{3}\text{rad}s^{-1}; \theta = 15\ \text{revolutions} = 50\pi\, rad$$

$$\boldsymbol{\omega^2 = \omega_o^2 + 2\alpha\theta \Rightarrow \alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = -\left(\frac{4\pi}{3}\right)^2\left(\frac{1}{2(50\pi)}\right)}$$

$$\boldsymbol{\alpha} = \frac{\mathbf{4\pi}}{\mathbf{225}}\,\mathrm{rad}s^{-2}$$

Rotational Dynamics

Considering we have analogous cases between linear kinematics and rotational kinematics, i.e., θ for x, ω for v and α for a, surely, we must have analogous quantities for describing a body's motion.

If we recall Newton's 2nd Law of Motion, whereby we say a force accelerates a body, we can now ask what quantity brings about changes to the angular acceleration? We'd be right in this line of thinking and what we will eventually find is a quantity called **torque, $\boldsymbol{\tau}$**. Much like rotational kinematics, we can relate torque to its linear counterpart,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Where $\vec{r}$ is the distance between the force applied and the rotation axis and $\vec{F}$ is the force vector applied. The direction of the torque will follow mathematical convention of cross products.

Now we ask, what is the rotational analogue to the Newton's 2nd law of motion? Considering we know $F = ma$, we know we can substitute τ for F and α for a. But what do we substitute m with? We substitute it with the **moment of inertia, $\boldsymbol{I}$**. In that case, we shall have

$$F = ma \;\Rightarrow \tau = \; I\,\alpha.$$

Just as in Newton's law of motion, equilibrium dictates $\Sigma F \; = 0$, equilibrium in the rotation of rigid body dictates

$$\Sigma\tau = \tau_{clockwise} - \tau_{anticlockwise} = 0.$$

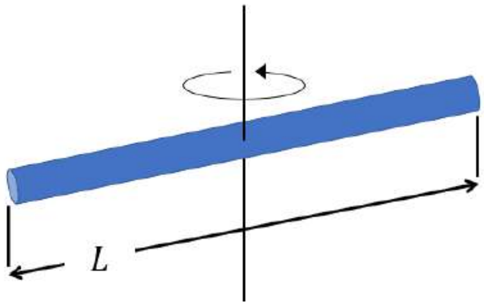
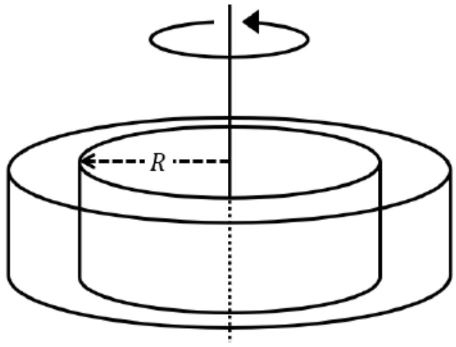
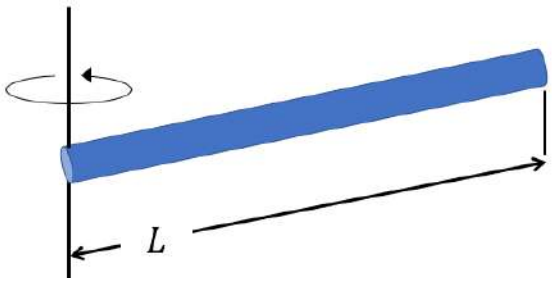
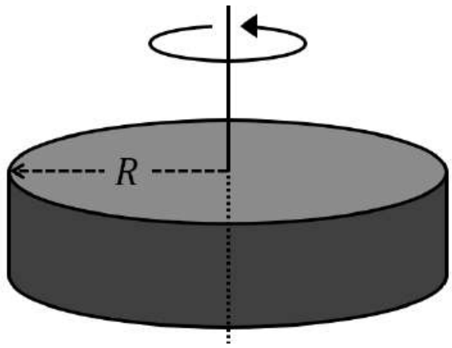
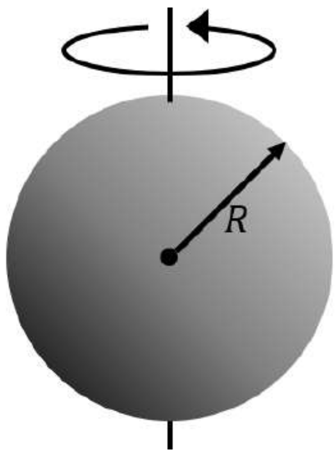
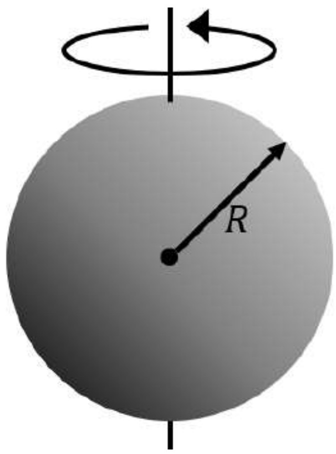
But what is this moment of inertia? If mass can be defined to be property of the body that resists linear acceleration, then moment of inertia can be defined to be as the property of the body (or system) to resist angular acceleration. If the system consists of discrete individual mass points, then

$$I = \Sigma m_i r_i^2$$

If the system consists of a continuous distribution of matter, then

$$I = \int r^2 dm$$

Considering that this is an algebra-based physics course, you are not expected to be able to derive equations for moment of inertia for a system of continuous distribution of matter (though I highly recommend you trying as you should know integration from your maths course!). As such, moment of inertia equations commonly used in this course is provided in the table below:

Description and Diagram	
Thin rod of mass M about its centre  $I = \frac{1}{12}ML^2$	Thin ring about its centre axis  $I = MR^2$
Thin rod of mass M about its end  $I = \frac{1}{3}ML^2$	Disk/ solid cylinder about its axis  $I = \frac{1}{2}MR^2$
Solid Sphere  $I = \frac{2}{5}MR^2$	Hollow spherical shell  $I = \frac{2}{3}MR^2$

Note: For those of you who are keen on learning more about moment of inertia and how their equations are derived, feel free explore **integrations related to rotational inertia**, **the parallel axis theorem**, and **the perpendicular axis theorem**.

Sample Problem 6.3

2 objects of equal mass of 2kg are connected by a rod of negligible mass with length 0.75m. Calculate the moment of inertia about an axis one-third of the way from one end of the rod.

Answer:

Because this is a case for system of discrete individual mass points,

$$I = \Sigma m_i r_i^2 = m_A r_A^2 + m_B r_B^2$$

Let us set that object A is closest to the axis of rotation. Then

$$r_A = \frac{1}{3}(0.75) = 0.25m;\ r_B = \frac{2}{3}(0.75) = 0.5m$$

Since the objects are of equal masses,

$$m_A = m_B = m = 2kg$$

Then

$$I = (2)(0.25)^2 + (2)(0.5)^2$$

$$I = 0.625\, kg\, m^2$$

Sample Problem 6.4

A wheel of 6kg has a radius of gyration of 15cm. Calculate the torque needed to give it an angular acceleration of 7 rads^{-1}.

Answer:

The torque needed to produce $\alpha = 7\,\text{rad}s^{-1}$,

$$\tau = I\alpha$$

One can assume that the wheel will have the shape of a thin ring, then its moment of inertia is

$$I_{ring} = MR^2$$

As such,

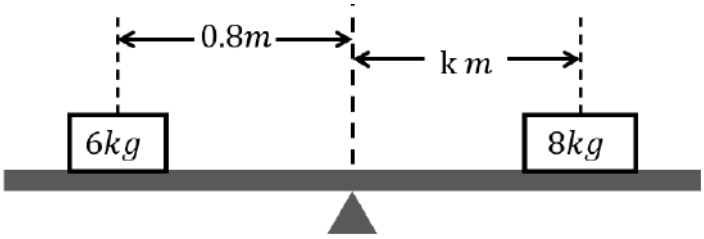
$$\tau = MR^2\alpha = (6)(0.15)^2(7) = 0.945Nm$$

Sample Problem 6.5 (Equilibrium Problem)

2 masses (mass of 6kg and 8kg) are placed on ends of a seesaw with a pivot at its centre. If the 6kg mass was placed at 0.8m from the pivot, calculate the distance between the pivot and the 8kg mass such that the system is at rotational equilibrium.

Answer:

Let us first have a sketch of what the situation should look like,



We have torque coming from 2 forces, τ_{8kg} and τ_{6kg}.

τ_{6kg} acts as a torque that contributes to the counter clockwise rotation whereas the τ_{8kg} contributes to clockwise rotation.

Equilibrium requires

$$\Sigma\tau = \tau_{\circlearrowleft} - \tau_{\circlearrowright} = \tau_{6kg} - \tau_{8kg} = 0 \Rightarrow \tau_{6kg} = \tau_{8kg}$$

$$F_{6kg}r_{6kg} = F_{8kg}r_{8kg} \Rightarrow m_{6kg}r_{6kg} = m_{8kg}r_{8kg}$$

$$k = r_{8kg} = \frac{m_{6kg}}{m_{8kg}}r_{6kg} = \frac{6}{8}(0.8)$$

$$k = 0.6m$$

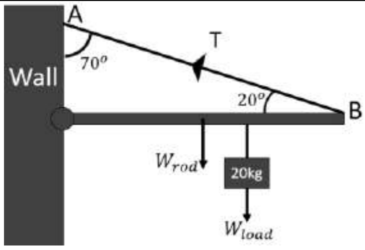
Sample Problem 6.6

The figure below shows a load of mass 20kg suspended from a 1.2m, 5kg rod pivoted to a wall and supported by a cable of negligible mass.

If the mass is suspended at 0.9m from the hinge and the angle at A is 70^o, determine the tension in the cable AB.

Answer:

Let us add force vectors to the figure given.



From here we can list out the torque and forces related to it

$$\tau_{\circlearrowright} = W_{rod} r_{rod} + W_{load} r_{load}$$

$$\tau_{\circlearrowleft} = (T \sin \theta) r_{cable}$$

For equilibrium,

$$\Sigma\tau = 0 \Rightarrow \tau_{\circlearrowright} = \tau_{\circlearrowleft}$$

$$(5)(g)(0.6) + (20)(g)(0.9) = (T \sin 20)(1.2)$$

$$T = \frac{(5)(0.6) + (20)(0.9)}{(\sin 20)(1.2)}$$

$$T = 116.952176N$$

Since we have talked about rotational analog to kinematics as well as forces, it is natural to proceed to asking if there exist a rotational analog to linear momentum. There is! It is called **angular momentum, *L*,** and is defined as

$$L = I\omega = \mathrm{rp}\sin\theta.$$

Conservation law also exist for this quantity,

$$\Delta L = 0.$$

Sample Problem 6.7

Determine the angular momentum of the Earth if the mass of the Earth is approximately $5.97(10^{24})kg$ and its diameter is approximately $12.742(10^{6})m$.

Answer:

We know for a fact that the period of the Earth is 1 day,

$$T = \frac{2\pi}{\omega} = 24h \times 60mins \times 60s \Rightarrow \omega = \frac{\pi}{43200}\ \mathrm{rads}^{-1}$$

We can then approximate the Earth as a sphere such that its moment of inertia is

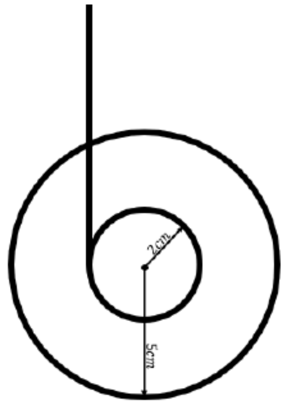
$$I = \frac{2}{5} m_{Earth} r_{Earth}^2$$

$$L = I\omega = \left(\frac{2}{5} m_{Earth} r_{Earth}^2\right)\left(\frac{\pi}{43200}\right) = \left(\frac{2}{5}\left(5.97(10^{24})\right)\left(\frac{12.742(10^{6})}{2}\right)^2\right)\left(\frac{\pi}{43200}\right)$$

$$L = 7.04881(10^{33})kgm^2s^{-1}$$

Exercises

1. Two gear wheels of radii 7.5cm and 15cm are meshed. If the bigger wheel makes a revolution of 3radians, how many revolutions did the smaller gear wheel make? Similarly, if the smaller wheel rotates at an angular velocity of $3\,rads^{-1}$, determine the angular velocity of the bigger wheel.
2. A yoyo of mass 350g is released from rest and descends vertically. It has an axle of radius 2cm and a spool of radius 5cm as shown in the diagram. If its moment of inertia can be approximated to equal to a solid cylinder, determine the tension on the cord as the yoyo descends and acceleration at which the yoyo descends.



3. A cylinder of mass 300g and radius of 15cm rolls down (without slipping) an incline plane of 25^o from rest through a vertical displacement of 50cm. Calculate the frictional force experienced by the cylinder and the linear velocity of the centre of mass when it reaches the bottom of the incline plane.
   *note – knowing the equation for rotational kinetic energy allows one to utilise conservation of energy to find the velocity of the mass at the bottom of the slope, try it!

4. A ladder of mass 20kg is leant against the wall where it makes an angle of θ with the wall. The coefficient of static friction between the ladder and the floor is 0.47 whereas the coefficient of static friction between ladder and the wall is 0.42. Calculate minimum angle of θ such that the ladder does not slip down. If the mass of the ladder is changed, at what point in added mass does the ladder slip down.

5. A uniform spherical gas cloud of mass 1.75×10^{10}kg and a radius of $1.2 \times 10^{14}m$ was observed to have a rotational period of 1.2×10^5 years. Years later, the gas cloud was again observed, and it was found to have a rotational period of 1.3×10^4 years. Determine the new radius of the gas cloud.

Chapter 7: Oscillations & Waves

Learning Outcomes

SHM	1.	Explain SHM.
	2.	Apply SHM displacement equation, $x(t) = A\,sin\,\omega t$
	3.	Derive, use and apply equations: a. velocity, $v = \omega A cos\omega t = \pm\omega\sqrt{A^2 - x^2}$ b. acceleration, $a = -\omega^2 A\,sin\,sin\,\omega t\; = -\omega^2 x$ (remarks: No calculus. Derive use algebra and trigonometry method, refer reference book Cutnell) c. kinetic energy, $K = \frac{1}{2}m\omega^2(A^2 - x^2)$ d. potential energy, $U = \frac{1}{2}m\omega^2 x^2$ e. period of SHM, T for simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$ f. period of SHM, T for mass-spring system : $T = 2\pi\sqrt{\frac{m}{k}}$
	4.	Emphasise the relationship between total SHM energy and amplitude.
	5.	Analyse the following graphs: a. displacement-time; b. velocity-time; c. acceleration-time; and d. energy-displacement.
Waves	1.	Define/state: a. wavelength b. wavenumber c. the principle of wave propagation for constructive and destructive interference d. Doppler Effect for sounds waves
	2.	Solve problems: a. related to progressive wave equation, $y(x,t) = Asin(\omega t \pm kx)$ b. related to the fundamental and overtone frequencies for stretched string ($f_n = \frac{nv}{2L}$) and open ($f_n = \frac{nv}{2L}$) and closed ($f_n = \frac{nv}{4L}$) ended air columns.
	3.	Distinguish/compare: c. between particle vibrational velocity and wave propagation velocity d. progressive and standing waves
	4.	Use: e. wavenumber, $k = \frac{2\pi}{\lambda}$ f. particle vibrational velocity, $v_y = A\omega cos(\omega t \pm kx)$ g. propagation velocity, $v = f\,\lambda$ h. the standing wave equation, $y = 2Acos(kx)sin(\omega t)$ i. wave speed in a stretched string, $v = \sqrt{\frac{T}{\mu}}$ j. Doppler Effect equation, $f_{apparent} = \left(\frac{v \pm v_{observer}}{v \mp v_{source}}\right)f$, for relative motion between source and observer. Limited to stationary observer and moving source and vice versa.
	5.	Analyse graphs of a. displacement-time, $y - t$ b. displacement-distance, x-t

Part 1: Simple Harmonic Motion

When we observe a motion in which the restoring force acting upon a system is directly proportional to the magnitude of its displacement and acts towards the initial position, then the situation at hand we say to have **simple harmonic motion (SHM).** Mathematically, systems undergoing SHM will obey

$$F_{restoring} \propto -x$$

The importance of understanding SHM is that it is foundational for the understand and analysis of more complex periodic motion, which is typically analysed using Fourier Analysis. Applying Newton's 2nd law of motion to the equation above gives us the differential equation

$$m\frac{d^2x}{dt^2} = -kx$$

where k is just the constant of proportionality for the relations above.

The solution for this differential equation is then

$$x(t) = A\,cos\,(\omega t) \text{ where } \omega = \sqrt{\frac{k}{m}}$$

Since we know the limits to the cosine function is $-1 \leq \ cos\,\omega t \leq 1$, a full cycle requires ωt to go from 0 to 2π and that a period is defined to be the time for one full cycle, we can make the conclusion that

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

And of course, frequency f is simply the inverse of period,

$$f = \frac{\omega}{2\pi} = 2\pi\sqrt{\frac{k}{m}}$$

Kinematics of SHM

Once we have defined an equation for displacement, we can quite easily proceed to equations for velocity and acceleration. This can be achieved by 2 methods – by calculus and by algebra.

Let us first work with **calculus**.

For velocity, it is simply the first derivative of displacement with respect to time and therefore has the form

$$v = \frac{dx}{dt} = \omega A\,cos\,(\omega t)$$

and acceleration is simply the second derivative of displacement,

$$a = \frac{d^2x}{dt^2} = -\omega^a A\,sin\,(\omega t)$$

As for functions of displacement,

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} \Rightarrow v\frac{dv}{dx} = -\omega^2 x$$

Solving this differential equation yields

$$\int v\ dv = -\omega^2 \int x\ dx$$

This leads to

$$\frac{v^2}{2} = -\omega^2\frac{x^2}{2} + C$$

We can set such that at $x = A,\ v = 0$.

$$\frac{0^2}{2} = -\omega^2\frac{A^2}{2} + C \Rightarrow C = \omega^2\frac{A^2}{2}$$

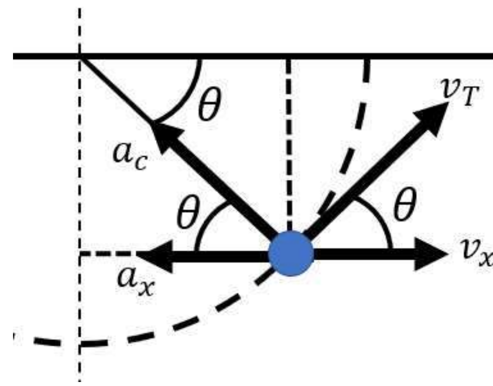
We then have

$$\frac{v^2}{2} = -\omega^2\frac{x^2}{2} + \omega^2\frac{A^2}{2}$$

Rearranging for v give

$$v = \pm\omega\sqrt{A^2 - x^2}$$

Now, let us derive the equations for velocity and acceleration for an object under SHM without calculus, only algebraically and trigonometrically. Let us consider the motion of a simple pendulum shown in the diagram.



We can now use this vector diagram to relate several physical quantities together. Note that $\theta = \omega t$.

First, note that the displacement of the pendulum is expressed as

$$\cos\omega t = \frac{x}{-A} \Rightarrow x(t) = -A\cos\omega t$$

For velocity, we see that

$$\sin\omega t = \frac{v_x}{v_T} = \frac{v_x}{\omega A}$$

And since $v_T = \omega r$ where $r = A$,

$$\sin\omega t = \frac{v_x}{\omega A} \Rightarrow v_x(t) = \omega A\sin\omega t$$

Lastly, we can work out the linear acceleration in the x direction as a function of time by noting that

$$\cos\omega t = \frac{-a_x}{-a_c}$$

And since $a_c = r\omega^2$ where $r = A$,

$$\cos\omega t = \frac{-a_x}{-A\omega^2} \Rightarrow \mathrm{a_x} = \mathrm{A}\omega^2\cos\omega t$$

For the equation describing velocity as a function of displacement, we start with the

$$v(t) = \omega A\, sin\, \omega t$$

and then we would utilise a trigonometric identity

$$sin^2\,\omega t + cos^2\,\omega t = 1$$

Rearrange it for $sin\,\omega t$,

$$sin\,\omega t = \pm\sqrt{1 - cos^2\,\omega t}$$

and substitute it back into the $v(t)$ equation

$$v(x) = \pm\omega\sqrt{A^2 - (-A\, cos\,\omega t)^2} = \pm\omega\sqrt{A^2 - x^2}$$

Energy in SHM

If we consider the equation for kinetic energy,

$$E_k = \frac{1}{2}mv^2$$

It is quite easy to see how one would be able to get the variation of kinetic energy as the object undergoes SHM. This can be achieved simply by substituting v with $v(t)$ or $v(x)$.

For $x(t) = A\, sin\,\omega t$,

$$E_k = \frac{1}{2}m(\omega A\, cos\,\omega t)^2 \Rightarrow E_k = \frac{1}{2}m\omega^2 A^2\, cos^2\,\omega t$$

$$E_k = \frac{1}{2}m\left(\pm\omega\sqrt{A^2 - x^2}\right)^2 \Rightarrow E_k = \frac{1}{2}m\omega^2(A^2 - x^2)$$

On the other hand, if the restoring force acting upon the system is described by $F_{restoring} = -kx$, then the potential energy that provides the system with such restoring force must have obey the equation

$$U = \frac{1}{2}kx^2$$

since $F = -\frac{dU}{dx}$. As such we can substitute k with $m\omega^2$ and x with $x(t) = A\, sin\,\omega t$, which yields,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(m\omega^2)x^2 = \frac{1}{2}m\omega^2 A^2\, sin^2\,\omega t$$

By law of energy conservation, we can find the equation for total mechanical energy

$$E_{total} = E_k + U = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2 \Rightarrow E_{total} = \frac{1}{2}m\omega^2 A^2.$$

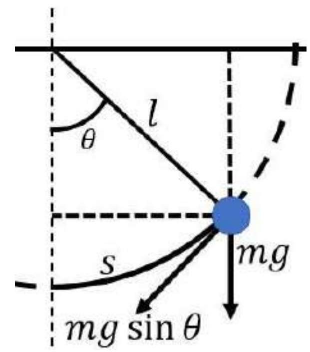
It is from this equation that whilst the kinetic energy and potential energy of the system depends on the displacement, the total energy only depends on the amplitude, angular velocity, and mass of the system.

Case study

For this section of the topic, we are interested in 2 cases – simple pendulum and spring mass system. For both cases, we aim to derive their equations for the period of their oscillation.

Case 1: Simple Pendulum

Consider the motion of a simple pendulum based on the diagram given.



In this diagram, the restoring force is a component of the weight of the pendulum bob,

$$F_{restoring} = -mg\ sin\,\theta = ma$$

If we apply **small angle approximation, i.e.**

$$\boldsymbol{sin\,\theta \approx \theta}$$

we find that

$$a = -g\theta \approx -g\left(\frac{s}{l}\right) \Rightarrow a \approx -\frac{g}{l}s$$

We can now compare with the SHM equation, $a = -\omega^2 x$, and find that

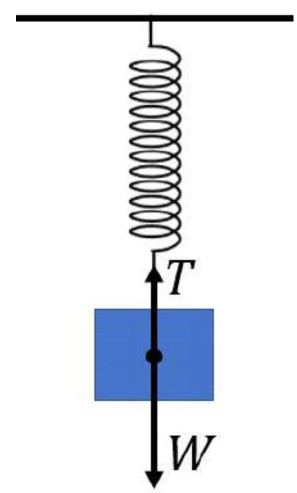
$$\omega^2 = \frac{g}{l}$$

And since $\omega = \frac{2\pi}{T}$, the expression for period of oscillation for a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Case 2: Spring-mass system

Consider the motion of a mass attached to a spring. At equilibrium,



$$T = mg = ke.$$

Let us now introduce an extension x to the system. Newton's 2nd law of motion, when applied will result in

$$mg - ke - kx = ma \Rightarrow -kx = ma \Rightarrow a = -\frac{k}{m}x$$

Comparing this to the SHM equation, $a = -\omega^2 x$, we find that

$$\omega = \sqrt{\frac{k}{m}}$$

And since $\omega = \frac{2\pi}{T}$, the expression for period of oscillation for the spring mass system is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Part 2: Waves

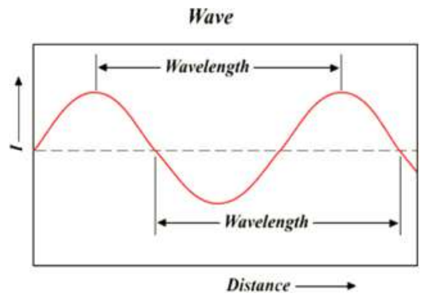
As discussed before, the solution of the SHM equation can be written as

$$y(t) = A\,sin\,(\omega t).$$

Here, the ω represents the rate of change of the sinusoidal wave- form. It answers the question of "How big is the phase change in 1 second?". ω is related to the period T, and frequency f by the following equation:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Mechanical waves work by energy transfer from one point to another point some distance away. Take water waves for example. the disturbance causes the water particles to oscillate up and down, but it doesn't traverse or spread out. This oscillation is merely the transfer of kinetic energy from particles at one point to another in space. The velocity of the particle's up and down motion is known as the particle vibrational velocity. A snapshot of this oscillation may produce a graph as shown below



The distance between corresponding points in successive wave- form is known as the wavelength. If the oscillation occurs at frequency f, in one second the wave has move forward by fλ. This means the velocity of the wave (also known as the wave propagation velocity) is related to the frequency and wavelength according to

$$v = f\lambda.$$

Wave number is the number of wave per unit distance. This is similar to the case of angular frequency, what angular frequency is to period, is what wave number is to wavelength. Difference is in the dimension, where wave number and wavelength are in the dimensions of space and angular frequency and period are in the dimension of time.

Progressive Wave

A progressive wave is a wave where the wave profile moves along with the speed of the wave. Its equation takes the following form

$$y(x,t) = A\,sin\,(\omega t \pm kx)$$

Different from the SHM where the variation of y is only dependent on x, here the equation for progressive wave is a function of both x and t. This is y varies with t. Because of this variation, at any point in x from the origin, the particle is displaced by a phase of kx.

To determine whether the wave is moving towards positive x direction or negative x direction, we may revisit the equation for velocity:

$$v = f\lambda = \frac{\omega}{k}$$

If $\frac{\omega}{k} > 0$ then v > 0. This is achievable if ωt − kx = 0. This means if the wave is moving in the positive x direction then the general equation takes the form

y(x, t) = A sin (ωt − kx).

$$y(x,t) = A\,sin\,(\omega t - kx)$$

Principle of Wave Superposition

The Principle of Wave Superposition states that the resultant displacement at any point is the sum of the individual wave dis- placements. That is to say

$$y_{resultant} = \Sigma_i A_i\,sin\,(\omega_i t \pm k_i x)$$

Standing Wave

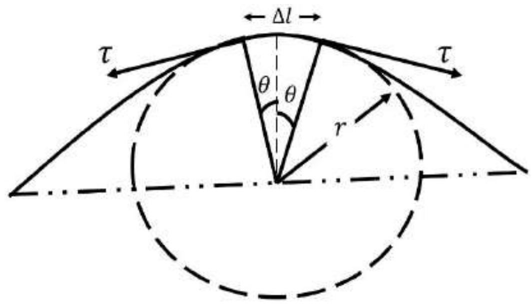
Whilst the progressive wave is a wave whose wave profile moves along the speed of the wave„ the standing wave is a case where the wave profile does not move in space. The peak amplitude of the wave oscillation at any point in space is independent of time.

The locations of minimum amplitudes are known as nodes and the locations of maximum amplitudes are called antinodes. A standing wave can be produce by having 2 progressive waves (of the same amplitude, frequency and wave number but travelling at opposite direction) superpose. This gives a resultant wave of the following equation:

$$\begin{aligned} y_{standing\ wave} &= Asin(\omega t - kx) + Asin(\omega t + kx) \\ &= 2Asin(\omega t)cos(kx) \end{aligned}$$

Travelling Wave Solution for String

Consider a single symmetrical pulse on a stretched string that moves in the ±x direction.



Take a small string element Δl within the pulse. This string element has the mass of Δm, where with μ = string mass density, is given by

$$\Delta m \ = \mu \Delta l$$

The end points of the element forms an arc of a circle of radius r, subtending 2θ at the centre of that circle. Assuming constant velocity, the horizontal components of τ cancels and its vertical component is the restoring force,

$$F = 2\tau \ sin\, \theta$$

Making small angle approximation and consider the equation ofan arc length, $s = r\theta$, gives

$$F \approx (2\theta)\tau \approx \left(\frac{\Delta l}{R}\right)$$

Since the element Δl is moving in an arc of the circle, its centripetal acceleration is given by

$$a_{centri.} = \frac{v^2}{r}$$

Thus

$$F = ma = (\mu \Delta l)\left(\frac{v^2}{R}\right) = \frac{\tau \Delta l}{R} \Rightarrow v = \sqrt{\frac{\tau}{\mu}}$$

Sound Waves

One example of a longitudinal wave is sound wave. Sound wave here refers to the transmission of energy through the adiabatic compression and decompression of a medium. We can charac- terise sound in 3 aspects - loudness (amplitude), quality (over- lapping of overtones) and pitch (frequency). In this section, we shall discuss these 3 characters of sound waves to some extent.

Loudness

Sensed as loudness, acoustic intensity, I, is defined as the power, P , propagated by the sound wave per unit area, A, in the direction normal to that area. In equation form,

$$I = \frac{P}{A}$$

We can expand this to show the relationship between intensity of sound wave to its amplitude, y_{max}, by considering the mass and velocity of an air layer as it reaches a point some distance away from the source. That layer of air vibrates at with simple harmonic motion. If the mass of the air layer is m and the velocity of the air layer is $v = \omega y_{max}$, then the intensity of the sound wave is

$$I = \frac{P}{A} = \frac{E_{kinetic}}{tA} = \frac{m\omega^2}{2tA} y_{max}^2$$

This tells us for any wave

$$I \propto y_{max}^2.$$

Shifting our attention to area, we can assume that the sound wave is spherical, then the intensity of sound was at a distance r from the source is

$$I(r) = \frac{P}{4\pi r^2}$$

because the area of a sphere of radius r is $4\pi r^2$. Here, we see it follows the inverse square law that states

$$I \propto \frac{1}{r^2}$$

Quality

The quality of the sound, or timbre, describes the characteristic of a sound which allows us to distinguish sounds that has the same pitch and loudness. We shall study the primary contributor to the timbre of a sound, which is the its harmonic content. When a musical instrument is heard, what our ears pick up is not the wave of single frequency but the the superposition product of sound waves of multiple frequencies. This is what is known as the harmonic content. Reversing the process, the harmonic content of a sound can broken down into its individual pure tones by Fourier transform.

Our interest, however, is to consider pure tones. What we mean by pure tone here is that instead of considering a combina- tion of sound waves of various frequencies, we consider a sound which is made up of a sound of a single frequency. We shall consider the boundary conditions of 3 systems and deduce the allowable frequencies of the sound produced. These 3 systems are chosen because most a large percentage of the all the musical instruments fundamentally works based on these 3 systems.

Before specifying the systems, let us recall some general ideas of waves:

1. Frequency ($f = \frac{v}{\lambda}$) ⇒ number of oscillation per second, unit: Hz
2. Wavelength ($\lambda = \frac{v}{f}$) distance between corresponding points in successive wave form.
3. Nodes (N) = location on a standing wave at which mini- mum amplitudes are observed.
4. Anti nodes (AN)= location on a standing wave at which maximum amplitudes are observed.

Additional terms that will be used in the analysis of the systems are

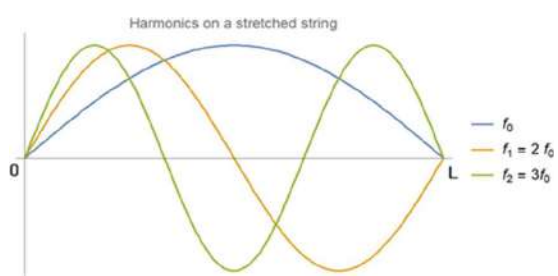
1. Fundamental frequency, f_o = the lowest frequency
2. Harmonics = whole number multiples of the f_o
3. Overtone = any frequency produced by the system which is greater than f_o
4. End correction, e = a short distance added to the actual length of a pipe due to resonant vibration at any open end of a pipe.

Generally speaking, the nth overtone of any system is the (n +1)th harmonic of that system.

System 1: Stretched String

The first of the system we're considering is the stretched string. Examples of musical instruments that works on this system in- cludes the piano and the guitar.

The boundary condition imposed here is that the ends must be composed of nodes, that is y(0, 0) = y(0, L) = 0.



We find that the string length must be integer of half wavelength, this means that

$$L = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

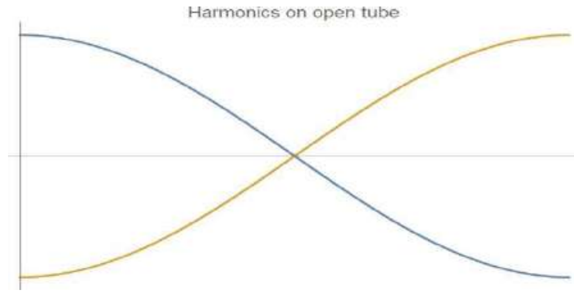
This allows us to write down the allowed frequencies for this system, and it is

$$f_n = n\frac{v}{2L} = \frac{\text{n}}{2\text{L}}\sqrt{\frac{\text{T}}{\mu}} \Leftrightarrow n = \{0,1,2,\ldots\}$$

System 2: Open Pipe

The second of the system we're considering is the open-ended pipe. This system can be found applied in the construction of a flute.

The boundary condition imposed here is that the ends must be composed of anti-nodes.



We find that the pipe length must be integer of half wavelength,this means that

$$L = n\frac{\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

This allows us to write down the allowed frequencies for this system, and it is

$$f_n = n\frac{v}{2L} \Leftrightarrow n = \{0,1,2,\dots\}$$

With end correction,

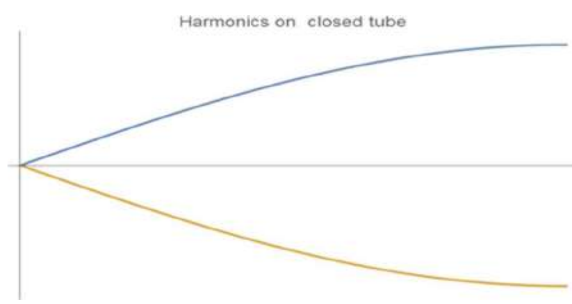
$$L = \frac{n}{2}\lambda - 2e \Rightarrow \lambda = \frac{2(L+2e)}{n}$$

$$f_n = n\frac{v}{2(L+2e)}$$

System 3: Closed Pipe

The system found in some organs/clarinet is the closed pipe sys- tem. This is essentially a pipe with one of the pipe ends sealed off.

The boundary condition imposed here is that an antinode must be found at the open and a node must be found at the open end. This gives the following diagram



We find that the pipe length must be integer of a quarter wave- length, this means that

$$L = n\frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{n}$$

However, looking at the following harmonics we find the n does not take the values of all integers, only the odd integers. So we will have to make adjustments to our equation to only consider odd integers. That adjustment is

$$n \to 2n + 1$$

This yields only odd integer as we consider $n = \{0,1,2,\dots\}$, resulting in

$$L = \frac{2n+1}{4}\lambda \Rightarrow \lambda = \frac{4L}{2n+1} \Leftrightarrow n = \{0,1,2,\dots\}$$

With this modification, we can write down the allowed frequencies for this system, and it is

$$f_n = (2n+1)\frac{v}{4L} \Leftrightarrow n = \{0,1,2,\dots\}$$

With end correction,

$$L = \frac{2n+1}{4} - e\lambda \Rightarrow \lambda = \frac{4(L+e)}{2n+1}$$

$$f_n = \frac{(2n+1)v}{4(L+e)}$$

Pitch

Frequency of waves are observed to correspond to pitch of sound. Sound waves with high frequency are observed as high pitched, and vice versa. One phenomenon of this character of sound wave is the **doppler effect**.

Imagine an ambulance, emitting a sound of frequency, moves towards you. As the ambulance approaches you, you hear that the pitch gets higher. Once the ambulance passes by you and moves away from you, the pitch becomes lower. This change in pitch is known as the *Doppler effect.*

The calculation for the change of pitch will be dealt soon, but for now, let us look at a graphical representation of what actually happen. The following figure shows how the frequency that you observe (= apparent frequency, f_{app}) depends on your position relative to the motion of the sound source.

As the source moves at a steady speed from position 1 to 4, four circular (coloured) waves are produced, which has point {1,2,3,4} as their centres. If you, as the observer positions yourself at point μ, and therefore the sound wave source is moving towards you, you will then observe sound wave in frame A. Conversely, positioning yourself at point γ means that the sound wave is observed according to frame B.

Comparing the wavelengths between wave fronts in frame A and B tells us that $\lambda_A < \lambda_B$. Because the frequency is inversely proportional to the wavelength, we then know that for apparent frequencies, $f_A > f_B$.

The most general case for the Doppler effect is when both the observer and the source is moving, therefore the approach taken here to quantify the apparent frequency to the observer will be to consider such as case.

The apparent frequency, f_{app}, can be found using the equation

$$f_{app} = \frac{v'}{\lambda_{app}}$$

where v′ is the sound wave velocity relative to the observer and λ_{app} is the wavelength that reaches the observer.

Assuming source is moving at velocity ±vs and the observer moves at a velocity $\pm v_o$, then

$$v' = v \mp u_o$$

$$\lambda = \frac{v \mp u_s}{f}$$

$$f_{app} = \frac{v \mp u_o}{v \mp u_s} f$$

In the numerator, u_o is added to v when the observer is moving towards the source, and vice versa. In the denominator us is added to v when the source is moving away from the observer.

We might also find determining the plus-minus signs in the Doppler effect equation easier if we take into consideration of the f_{app} to *f* ratio. This takes the form

$$\frac{f_{app}}{f} = \frac{v \mp u_o}{v \mp u_s}$$

which is bigger than 1 if a higher frequency is expected to observed and less than 1 if a lower frequency is expected to observed.

Chapter 8: Physics of Matter

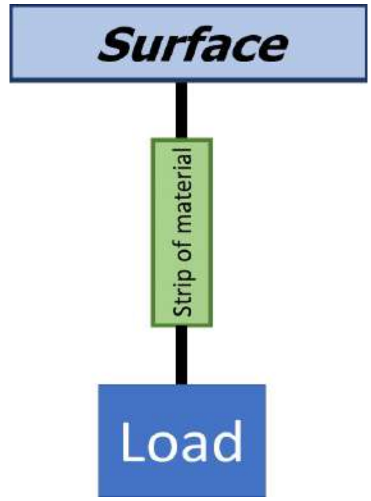
Learning Outcomes

Changes to material	No.	Learning Outcomes
Due to applied force	1.	Distinguish between stress, $\sigma = \frac{F}{A}$ and strain, $\epsilon = \frac{\Delta L}{L_o}$
	2.	Analyse the graph of: a. Stress-strain for metal under tension b. Force-elongation for brittle and ductile materials
	3.	Explain elastic and plastic deformations
	4.	Define and use Young's modulus, $Y = \frac{\sigma}{\epsilon}$
	5.	Apply: a. Strain energy from the force-elongation graph, $U = \frac{1}{2}F\Delta L$ b. Strain energy per unit volume from stress-strain graph, $\frac{U}{V} = \frac{1}{2}\sigma\epsilon$
Due to Heat	6.	Define: a. heat conduction b. coefficient of linear expansion, α c. coefficient of area expansion, β d. coefficient of volume expansion, γ
	7.	Solve problems: a. related to rate of heat transfer, $\frac{Q}{t} = -kA\left(\frac{\Delta T}{L}\right)$ through a cross-sectional area (remarks: maximum two insulated objects in series) b. related to thermal expansion of linear, area and volume $\Delta L = \alpha L_o \Delta T; \Delta A = \beta A_o \Delta T; \Delta V = \gamma V_o \Delta T; \alpha = \frac{\beta}{2} = \frac{\gamma}{3}$
	8.	Analyse graphs of temperature-distance (T-L) for heat conduction through insulated and non-insulated rods. *maximum two rods in series

Part 1: Material Changes due to Force

Stress

To begin, we start by talking about testing things to the point of deformation — we put them under some increasing force over some area of the thing and once it starts to deform, we stop and calculate the maximum amount of force that the thing starts to deform.



The most natural way to do this is merely to subject a strip/rod of the material (of length L and cross-sectional area A) to an axial load with the other end anchored to some surface. As the mass of the load is increased, the strip/rod deforms (becomes longer) and eventually breaks off (fracturing). So naturally we'd like to know, how much load can a strip of the given material, support?

Before answering this question, we can ask ourselves if there are any geometric variables that influences the ability of the strip of material to support load. We can then repeat the same experiment using strips/blocks of the same material (with varying cross-sectional area) and we would find that the axial strength increases with the increment of the cross-sectional area. This would make sense because as the cross-sectional area is increased, so does the number of bonding between each cross-sectional layer.

We now have a value for the amount of load a material can support relative to the cross-sectional area of that material sample.

It may be expressed mathematically as

$$F_{max} = \delta_{max} A_o$$

where F_{max} is the load at fracture, δ_{max} is the Ultimate Tensile stress and A_o is the initial cross-sectional area. This equation describes the maximum amount of stress that the strip/rod of material of cross-sectional A_o can handle and it is at this point that the material fails and fractures.

So, when the material is said to be put under stress, and that stress is less than δ_{max}, what we are referring to is

$$\delta = \frac{F}{A_o}$$

This new measure has the unit of Nm^{-2}.

Strain

In the last section, we quantified the amount of tensile stress a material may be be put under. In this section, let us quantify the "amount" of deformation that the material undergoes under some stress, that is we want to measure the stiffness.

Hooke's law gives us a great exposure to the deformation of a material with respect to the load that the material is put under. It is commonly written as

$$F = kx$$

Where k is the stiffness constant has units of Nm^{-1}.

We know from the previous section that this constant is not only affected by the type of material alone but also by the shape of the material. For now, we would like to normalize the measure for stiffness only by the deformation it undergoes independent of the shape. We can do this by considering the measure for the stretching of the material. This means we only consider the fractional change of the material when pun under stress, this is

$$\epsilon = \frac{\Delta L}{L_o}$$

where ΔL is the change in length and L_o is the initial length before the material was put under stress. This measure is what we call ***strain***.

Young's Modulus

We have now discussed the measure for stress the material undergoes as well as the material's deformation. We are now in the position to discuss how the stress that is put on the material affects the deformation observed in that material. That is to say, we want a calculable prediction on, *"If I put this amount of force, how big is the deformation I can expect? "*.

Experimental results show that for relatively small stress and strain, they are proportional to each other. This allows us to write

$$\delta \propto \epsilon.$$

This tells us that there is a proportionality constant between the two, let's call it *Y*. We can then define this proportionality constant to be

$$Y = \frac{\delta}{\epsilon}$$

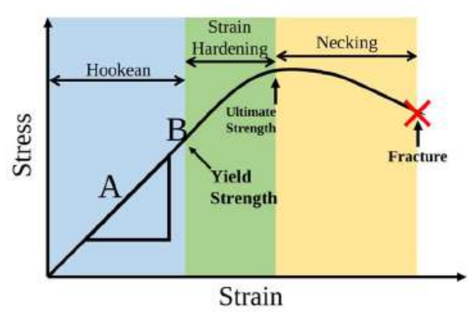
This proportionality constant is what we today call **Young's Modulus**.

Graph

2 graphs are studied in this section – stress-strain curve for a metal and the force-extension graph for brittle and ductile materials.

Stress-strain graph

Up to this point, we have considered stress and strain up within the Hookean limit. It is the best of our interest to also consider what happens further extension of the material after the Hookean limit (Yield limit) to the fracture point. The graph shows the stress-strain relation before and after the Hookean limit.



The first part (blue) shows the obeyance of the stress-strain curve to the Hookean law. Within this region, one can calculate the Young's modulus based on the gradient of the straight line. It is also in this region that the metal undergoes what is known as **elastic deformation**. Deforming elastically refers to the ability of the stretched metal to return to its initial length when the tensile stress is removed.

Beyond the Hookean limit, Hooke's law is no longer obeyed, and thus non-linearity is observed in the stress-strain curve. Beyond the Hookean limit, the metal undergoes **plastic deformation**, a type of deformation in which the stretched metal will not be able to return to its initial length even if the tensile stress is removed.

From the stress-strain curve, we observe that a non-linear increment of stress with the increment of strain until it reaches a peak, known as **Ultimate Tensile Strength**. The region (green) between the Hookean limit and the

UTS is where strain hardening takes place. This is the phenomenon where the metal is "strengthened" by the plastic deformation. "Strengthened" here refers to the dislocation of movements in the crystal structure of the material.

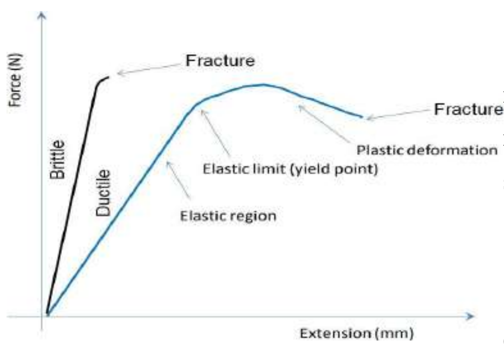
Beyond the peak, the strain still increases but the tensile stress decrease until the **fracture point**. Between the UTS and fracture point (pink), what is observed of the metal is that necking takes place. This is the phenomenon in which the local cross-sectional area becomes significantly smaller than average.

For some material, the plastic deformation occurring between the Hookean limit and the fracture point is so small, that it seems to the observer as if the plastic deformation region is non-existent. For such material, we call it **brittle**.

For other materials, the plastic deformation region is great and thus we call these materials, **ductile**.

Force-extension graph

As mentioned in the stress-strain graph section, the plastic deformation phase for brittle materials are very short whereas the same region for the ductile materials are relatively (and observably) significant.



One possible inference we can make from this is that ductile materials are able to contain much more strain energy than brittle materials. It is also from the force extension graph we can deduce the strain energy by the area under the force-extension graph, that is

$$U_{strain} = \frac{1}{2}F\Delta L$$

If were to find the strain energy per unit volume, we could easily do it by dividing both side by the volume of the material,

$$\frac{U_{strain}}{V} = \frac{1}{2}\frac{F\Delta L}{V}$$

and reminding ourselves that the volume is merely the product of cross-sectional area and length,

$$V = AL_o$$

We can see that the strain energy per unit volume is just the area under the stress strain curve,

$$\frac{U_{strain}}{V} = \frac{1}{2}\frac{F\Delta L}{V} = \frac{1}{2}\frac{F\Delta L}{AL_o}$$

$$\frac{U_{strain}}{V} = \frac{1}{2}\delta\epsilon$$

Sample Problem 8.1

A 50kg box is balanced on a pole of radius 25cm. Determine the stress that the pole is under.

Answer:

Stress is the amount of force onto a surface area. The weight of the box is $F = W = mg$ and the surface area, considering it is a circle, $A = \pi r^2$ and thus the stress on the pole is

$$\delta = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(50)(9.81)}{\pi(0.25)^2} = \frac{7848}{\pi} \approx 2498.1 Nm^{-2}$$

Sample Problem 8.2

Determine the strain on a piece of metal if is extended by 20% when some force is applied to it.

Answer:

Strain quantifies the fractional change to the geometry of the object.

$$\epsilon = \frac{\Delta L}{L_o} = \frac{0.2L_o}{L_o} = 0.2$$

Sample Problem 8.3

A piece of wire is hung with a mass of 2kg weight on its end. Because of the weight on its end, it stretches by 0.2cm from its original length of 5cm. If the cross-sectional area of the string is $6.25(10^{-8})\pi\, m^2$, calculate the Young's Modulus of the wire.

Answer:

Young's modulus is the ratio of stress to strain,

$$Y = \frac{\delta}{\epsilon} = \frac{\left(\frac{F}{A_o}\right)}{\left(\frac{\Delta L}{L_o}\right)} = \frac{FL_o}{A_o \Delta L} = \frac{(2)(9.81)(0.05)}{6.28(10^{-8})(\pi)(0.002)} = \frac{7.811 \times 10^9}{\pi}$$

$$Y \approx 2.486 \times 10^9\, Pa$$

Part 2: Material Changes due to Heat

Heat Conduction

Imagine a metal rod with one end heated. With time, the opposite end also gets hot even though it is not directly heated. The heat energy is transferred from one end to the other end. This happens through the 'jiggling' of the particles within the heated material. As the rod is heated, the particles begin to vibrate and collide with neighbouring particles. When they collide, they transfer some of the energy to the neighbouring particles and then the neighbouring particles starts to vibrate. This process is called **heat conduction.** It is the transfer of heat through agitations of the particles within the material without any motion of the material.

The variable that drives heat transfer is a temperature gradient across the material, that is to say there exist a difference in temperature between two parts of a conducting medium,

$$\Delta T > 0.$$

Referring to the rod heated on one end case, we can say that the heated end has temperature T_H and the non-heated end to have the temperature T_C. Fourier's law tells us that the local heat flux in a homogeneous body, q_h is in the direction of, and proportional to, the temperature gradient ∇T:

$$q_h \propto -\nabla T$$

In one-dimensional form,

$$q_h = -\kappa_x \frac{dT}{dx}$$

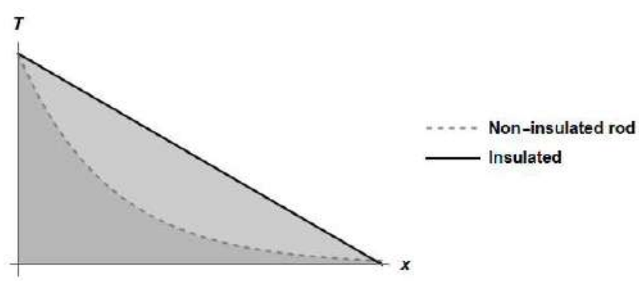
where κ_x is the thermal conductivity in the x-direction. Note that the minus sign is present due to the fact that heat flows from a higher temperature area to a lower one.

Local heat flux refers to rate of heat transfer per unit area,

$$q_h = \frac{\left(\frac{dQ}{dt}\right)}{A}$$

The equation for the rate of heat transfer is then

$$\frac{dQ}{dt} = -A\kappa_x \frac{dT}{dx}$$



Let us now try to understand the temperature-gradient graph for a rod heated on one end. For an uninsulated rod, the heat energy will be able to escape to the environment via the sides of the rod. This leads to temperature-distance gradient to be curved with a decreasing gradient, much like graphs exhibited by functions $f(x) = e^{-kx}$. On the other hand, for insulated rods, the heat loss to the environment is negligible. As a result, their temperature-distance graph is a linear graph with negative gradient.

Sample Problem 8.4

One end of a metal bar is kept at $250\,^{o}C$, while the other end is kept at a lower temperature. The cross-sectional area of the bar is $3 \times 10^{-4} m^2$. The heat loss through the sides of the bar is negligible due to the insulation around the metal bar. Heat flows through the bar at the rate of $2.5 Js^{-1}$. If the metal bar has a thermal conductivity value of 140 $Js^{-1}m^{-1\,o}C^{-1}$, calculate the temperature of the bar at a point 0.8 m from the hot end?

Answer:

$$\frac{dQ}{dt} = -A\kappa_x \frac{dT}{dx} \Rightarrow \frac{dQ}{dt} = A\kappa_x \frac{(T_{hot} - T_{cold})}{\mathrm{L}}$$

$$2.5 = 3(10^{-4})(140)\frac{(250 - T_{cold})}{0.08}$$

$$T_{cold} \approx 245.24^{o}C$$

Heat Expansion

Things expand when they are heated, this phenomenon is known as **thermal expansion**. A great demonstration of this phenomenon is the Ring and Ball Experiment (https://www.youtube.com/watch?v=ne8oPFTM_AU). If the expansion happens in one dimension of space, we call it a **linear expansion**, where as expansion in two and three dimensions are known as **area** and **volume expansion.**

We can start by considering a small change in the object's length. By small, we mean relative to the object's initial dimensions. When that is the case, the change in the length is the proportional to the first power of the temperature change.

$$\Delta L \propto \Delta T$$

Say, that object has an initial length of L_o along some direction at temperature T_o. Then the change in length ΔL for a change in temperature ΔT is

$$\Delta L = \alpha L_o \Delta T$$

where α is known as **coefficient of linear expansion**.

So now how do we expand to area and volume thermal expansion? Well, note that $A = L^2$ and that $L = L_o + \Delta L$. As such,

$$A = L^2 = (L_o + \alpha L_o \Delta T)(L_o + \alpha L_o \Delta T)$$

$$A = L_o^2 + 2\alpha L_o^2 \Delta T + \alpha^2 L_o^2 \Delta T^2 = A_o + 2\alpha A_o \Delta T + \alpha^2 A_o^2 \Delta T^2$$

Since we are only considering only small changes, we can consider the last terms to be negligible such that

$$A = A_o + 2\alpha A_o \Delta T$$

which gives us the change in area

$$\Delta A = \beta A_o \Delta T$$

where $\beta = 2\,\alpha$ and is aptly named **coefficient of area expansion**. We can then imitate the same procedure to produce the equation for change in volume due to heat which will yield

$$\Delta V = \gamma V_o \Delta T$$

where $\gamma = 3\alpha$ and is named **coefficient of volume expansion.**

<u>Sample Problem 8.6</u>

A mug of filled with 200mL of tea at 90^oC. If the tea has a coefficient of linear expansion of $69(10^{-6})K^{-1}$, calculate the volume of the tea when the tea has cooled down by 50^oC.

Answer:

Change in tea temperature, $\Delta T = 50^oC = 50K$

Initial volume of water, $200mL = 0.0002m^3$

Coefficient of volumetric expansion, $\gamma = 3\alpha = 3(69)(10^{-6})K^{-1} = 2.07(10^{-4})K^{-1}$

$$\Delta V = \gamma \mathrm{V}_o \Delta T \Rightarrow V_{final} - V_{initial} = \gamma V_{initial}\Delta T$$

$$V_{final} - (0.0002) = \left(2.07(10^{-4})\right)(0.0002)(10)$$

$$V_{final} = 2.0000207 \times 10^{-4} m^3$$

Chapter 9: Kinetic Theory of Gases & Thermodynamics

Learning Outcomes

Molecular kinetic theory	1.	Define/State a. The assumptions of kinetic theory of gases. b. The principle of equipartition of energy c. Degrees of freedom
	2.	Describe/Explain: a. Root mean square (rms) speed of gas molecules, $v_{rms} = \sqrt{< v^2 >}$ b. Translational kinetic energy of a molecule, $E_K = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT$ c. Internal energy of gas
	3.	Solve problems related to: a. rms speed of gas molecules, $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$ b. the equations, $PV = \frac{1}{3}Nmv_{rms}^2; P = \frac{1}{3}\rho v_{rms}^2$ c. Translational kinetic energy of a molecule, $E_K = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT$ d. Internal energy, $U = \frac{1}{2}fNkT$
	4.	Identify number of degrees of freedom for monoatomic, diatomic and polyatomic gas molecules.
Thermodynamics	5.	Define/State: a. First Law of Thermodynamic, $\Delta U = Q - W$ b. Isothermal process c. Isochoric process d. Isobaric process e. Adiabatic process
	6.	Solve problems related to: a. First Law of Thermodynamics b. Isothermal process, $W = nRTln\left(\frac{V_f}{V_i}\right) = nRTln\left(\frac{p_i}{p_f}\right)$ c. Isobaric process, $W = \int PdV = P\left(V_f - V_i\right)$ d. Isochoric process, $W = \int PdV = 0$
	7.	Analyse $P - V$ graph for all the thermodynamic processes
	8.	Derive equation of work done in isothermal, isochoric and isobaric processes from $P - V$ graph.

Molecular Kinetic Theory

Kinetic Theory of Gases

Because atoms are very light, it is often useful to use the **atomic mass unit** for the masses of the atomic scale. The atomic mass unit is defined as $\frac{1}{12}$ of the mass of a carbon-12 atom This atomic mass unit (a.m.u.) is related to the SI kilogram by

$$1u = 1.660539 \times 10^{-27} kg$$

Apart from that, in our daily lives, quite often we deal with a large number of atoms/molecules/particles. So rather than describing is numerically by the number of particles, we often describe the number of atoms relative to the **Avogadro's Constant,**

$$N_A = 6.022 \times 10^{23} \text{ particle per mol.}$$

For example, instead of saying there are $5(10^{23})$ gas particles in a container, it is easier to say 0.83mol of gas particles in the container. These two new ways of quantifying the light-mass but large number particle systems leads to a very interesting result, that is **the mass per mole of any substance and the atomic (or molecular) mass unit has the same numerical value.** For example, the oxygen atom has a mass of 16u and therefore has a mass of $16g\ mol^{-1}$.

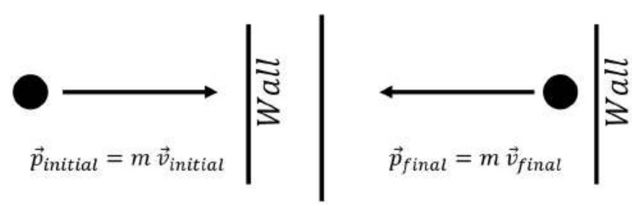
Macroscopically, the simplest model for gases is the **ideal gas law,** which states

$$pV = nRT = Nk_BT$$

where p is the pressure of the gas (in Pa), *V* is the volume of the gas (in m^3), *n* is the number of mole of the gas, *T* is the temperature in Kelvin of the gas, *R* is the gas constant ($J\ mol^{-1}\ K^{-1}$) and k_B is the Boltzmann constant (defined as $k_B = \frac{R}{N_A}$).

Microscopically, though we want to start considering kinetic energies of the gas particles. Let us first lay down assumptions for our kinetic theory of gas. In one sentence, let us consider **gas to be composed of large numbers of non-null mass point-like particles that obeys Newton's laws of motion and interact elastically with each other where the average kinetic energy of the gas particles depends solely on the absolute temperature of the gas particle system.**

Consider gas particles in a cube container of side lengths L. We can think of the interaction between the gas particle and the container wall to be like that of a ball hitting a wall. We can determine the force exerted by the particle onto the container wall and then divide it by the area to determine the pressure.



We can see that the change in momentum is $\Delta\vec{p} = m\big(-v - (+v)\big) = -2mv$

. Considering the speed of the particle is v and the distance between walls of the container are L, the time between the collisions will simply be $t = \frac{2L}{v}$. The force exerted on the wall by the particle will then be

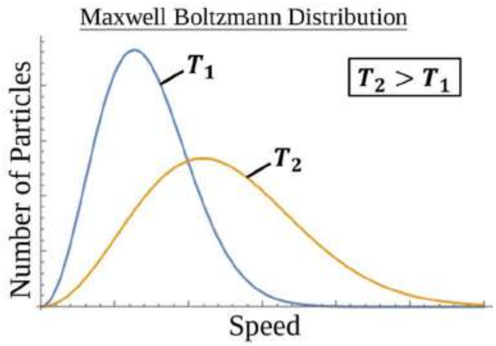
$$F = \frac{Delta\vec{p}}{t} = \frac{-2mv}{\left(\frac{2L}{v}\right)} = \frac{-mv^2}{L}$$

If there are N particles in the container, then the total force exerted on the container walls is

$$F = \frac{-Nm\overline{v^2}}{3L}$$

2 major changes happened - $\overline{v^2}$ has replaced v^2 and a factor of $\frac{1}{3}$ seemed to popped up. Here are the reasons:

a. $\overline{v^2}$ has replaced v^2 because in a system of many particles, not all the particles will have the same speed. Their speed will follow the **Maxwell-Boltzmann distribution**.



So, to take that into account, we will use the **rms speed (v_{rms})** of the particle rather than the peak or average speed. This rms speed is defined by

$$v_{rms} = \sqrt{\overline{v^2}}$$

b. On the other hand, the factor of $\frac{1}{3}$ popped us because the particles can move in 3 dimensions. This means the velocities of the particles can happen inn x, y or z axis. Considering the speeds are random, this would mean $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$, and since $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$, this would mean that

$$\overline{v^2} = 3\overline{v_x^2} \Rightarrow \overline{v_x^2} = \frac{\overline{v^2}}{3}$$

And thus the force can be written as

$$F = \frac{Nmv_{rms}^2}{3L}$$

We can now divide this force by the area

$$P = \frac{F}{L^2} = \frac{\left(\frac{Nmv_{rms}^2}{3L}\right)}{L^2} = \frac{Nmv_{rms}^2}{3L^3} = \frac{Nmv_{rms}^2}{3V}$$

Defining mass density as $\rho = \frac{Nm}{V}$,

$$P = \frac{Nmv_{rms}^2}{3V} \Rightarrow P = \frac{1}{3}\rho v_{rms}^2.$$

Rearranging the pressure – rms speed equation yields

$$PV = \left(\frac{2}{3}N\right)\left(\frac{1}{2}mv_{rms}^2\right) = \left(\frac{2}{3}N\right)(\bar{E}_{kinetic})$$

Comparing this to $pV = Nk_BT$ gives us an expression of the average kinetic energy as a function of the temperature,

$$Nk_BT = \left(\frac{2}{3}N\right)(\bar{E}_{kinetic}) \Rightarrow \bar{E}_{kinetic} = \frac{3}{2}k_BT \text{ (for a single particle)}$$

$$\text{Total Translational Kinetic Energy of N gas molecules: } \Sigma E_{kinetic} = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

This result is significant because now we really do have a <u>**kinetic**</u> theory of gas, where the temperature of the gas is expressed in terms of the motion of the gas particles.

Kinetic & Internal Energy

Before talking about the classical principle equipartition of energy, we need to address what we mean when we say **degrees of freedom**. In the context of gas motion, degrees of freedom are the dynamical variables that contributes a squared term to the expression for the total particle energy. Classically, there two that we'd consider are

a. translational kinetic energy,

$$K_{translational} = \frac{1}{2}mv^2$$

b. Molecular rotational energy,

$$K_{rotational} = \frac{1}{2}I\omega^2$$

Though, in the quantum regime, we need to consider vibrational energy, in which the bonds between molecules may be treated as "springs" and that would add 2 more degrees of freedom.

The following table shows the cases and the number of degrees of freedom

Cases	Number of Degrees of Freedom
Monoatomic	3 (Only translational) $\frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)$
Diatomic*	5 (3 translational + 2 rotational) $\frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + \frac{1}{2}\omega(I_x^2 + I_y^2)$

*in the classical regime. Taking into consideration quantization of energy leads to consideration for vibrational energy.

Based on the idea of degrees of freedom, we can now extend our discussion to the classical theorem of energy equipartition which states **at equilibrium, each degree of freedom contributes $\frac{1}{2}k_BT$ of energy per molecule.**

That is to say, for N number of gas molecules of *f* degree of freedom, the total internal energy is

$$U = fN\frac{1}{2}k_BT = \frac{f}{2}nRT$$

Here what we mean by internal energy is simply the sum of all the kinetic energy of all the molecules accounted for in the systems. If we were to talk about translational kinetic energy, we mean kinetic energy associated with translation of the particle ($f = 3$). If we were to talk about the total kinetic energy, we are referring to the internal energy of the system, where the degree of freedom depends on the type of particle.

Thermodynamics

Thermodynamics is the study of heat and energy transformation. In this section, we discuss 3 things. Firstly, the 0^{th} law of thermodynamics, which essentially provides us with the idea of **thermal equilibrium**. Secondly, we shall look at how the idea of adding (or taking our) energy to a system relates to the internal energy of the system, i.e. the 1^{st} law of thermodynamics. Lastly, we shall consider the thermodynamical processes and lay foundations to understanding heat engines.

Zeroth law of Thermodynamics

The 0^{th} law of thermodynamics focuses on the idea of a thermal equilibrium. If the temperature of system A is equal to system B, and that system B has a temperature equal to system C, then system C and A is said to be at **thermal equilibrium**.

$$T_A = T_B \;\&\; T_B = T_C \Rightarrow T_A = T_C$$

In other words, two systems are are to be in thermal equilibrium with each other if they have the same temperature. When two systems are in thermal equilibrium with each other, the net heat flow between them is essentially 0.

First Law of Thermodynamics

At this point, what we want to do is to relate the internal energy of the system with the external factors. By External factors, we mean whether we apply heat to the system, or take heat away from it or changing the geometry of the system (whether increasing it or decreasing it). Much like the conservation of energy, the sum total of the energy of an isolated system must be conserved. Therefore, when some heat ΔQ is added to the system and some work ΔW is added to the system, the change in internal energy ΔU can be calculate by

$$\Delta U = \Delta Q + \Delta W$$

Though some books may have a minus sign instead of a positive sign for the ΔW and the reason for that is that they have defined ΔW to be work done by the system.

Equation	Terms definition
$\Delta U = \Delta Q + \Delta W$	ΔW = work done **onto** the system
$\Delta U = \Delta Q - \Delta W$	ΔW = work done **by** the system

Thermodynamical Processes

Looking back at the first law of thermodynamics, we want to be able to do some calculations related to it. And thus, we shall quantitatively defined U, Q and W. By internal energy, we take the definition defined previously, that is internal energy of the gas depends on the temperature of the system,

$$\Delta U(\Delta T) = \frac{1}{2} f N k_B \Delta T = \frac{f}{2} nR\Delta T$$

By heat added, what we refer to is the heat transfer into or out of the system. This heat is defined by

$$\Delta Q = mc\,\Delta\theta$$

where m is the mass of the system, c is the heat capacity of the system and θ is the temperature of the system. By work done onto (or by) the system, we define it to be related to the change in volume of the system,

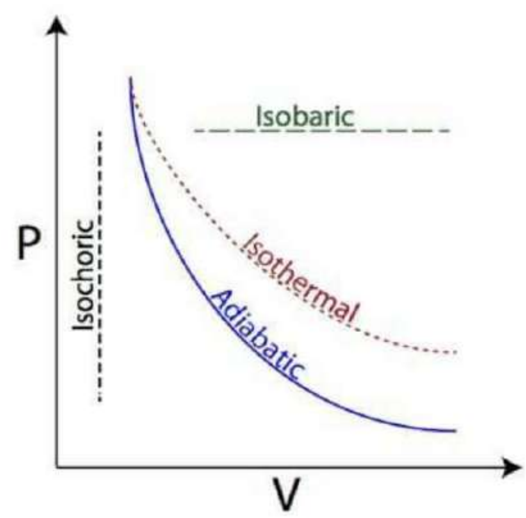
$$\Delta W = p\,\Delta V$$

In general, we want to consider 4 case studies on thermodynamical processes:

1. Isothermal
   In isothermal expansion/compression, temperature of the system is kept constant, $\Delta T = 0$. Since the change in internal energy depends solely on change in temperature, this means that $\Delta U = 0$ and thus
   $$\Delta U = 0 = \Delta Q + \Delta W$$
   $$\Rightarrow \Delta Q = -\Delta W$$
   In the isothermal case, pressure is not a constant, we can define pressure as a function of volume via the ideal gas law
   $$p = \frac{nRT}{V}$$
   And thus the work done is
   $$W = \int_{V_i}^{V_f} p(V)\, dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \int_{V_i}^{V_f} \frac{1}{V} dV = nRT\, ln\left(\frac{V_f}{V_i}\right)$$

2. Isochoric / Isovolumetric
   In the isochoric case, the volume of the gas is kept constant, $\Delta V = 0$ and thus
   $$\Delta U = \Delta Q$$
3. Isobaric
   In the isobaric expansion/compression cases, the pressure of the gas is kept constant, $\Delta p = 0$. This means that for the calculation of work done onto (or by) the system is simply
   $$W = \int_{V_f}^{V_i} p\, dV = p \int_{V_f}^{V_i} dV = p(V_f - V_i)$$
4. Adiabatic
   For the adiabatic process, what is kept constant is the heat transfer into and out of the system, $\Delta Q = 0$. This means that $\Delta U = \Delta W$. Since
   $$\Delta U = \Delta W = \frac{f}{2} \text{nR}\Delta T$$
   $$\Rightarrow W = \int_{T_i}^{T_f} \frac{f}{2} nR\, dT = \frac{f}{2} nR(T_f - T_i)$$

The p-V graph of each thermodynamical processes is shown in the diagram below.



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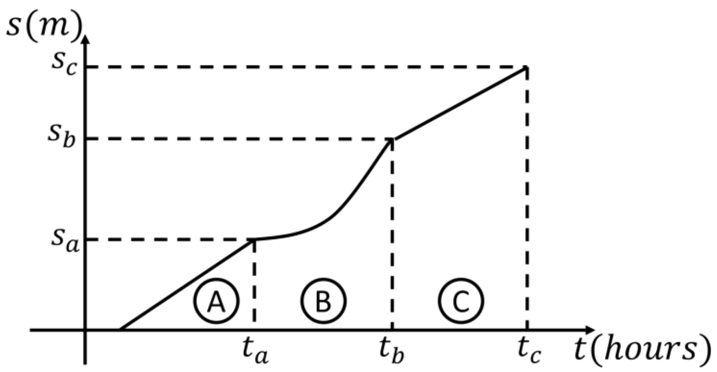
Problem 1 [5 marks – SPM Level]

Question:

A car travels at a uniform velocity $40kmh^{-1}$. After 30 minutes, the car then accelerates to a velocity of $90kmh^{-1}$ over a distance of 50km. At $90kmh^{-1}$, the car travels a further 20km. Sketch the displacement-time graph for the motion of this car, ensure that your labelling is sufficient. Calculate the total time this car has travelled.

Solution:



s	t
$s_a = (40)\left(\frac{30}{60}\right)$ $s_a = 20km$	$t_a = 0.5h$
$s_b = s_a + 50km$ $s_b = 20km + 50km$ $s_b = 70km$	$t_b = t_a + \frac{2\Delta s}{u+v}$ $t_b = 0.5 + \frac{2(50)}{90+40}$ $t_b = \frac{33}{26}h$
$s_c = s_b + 20km$ $s_c = 70km + 20km$ $s_c = 90km$	$t_c = t_b + \frac{s}{v}$ $t_c = \frac{33}{26} + \frac{20}{90}$ $t_c = \frac{349}{234}h$

Total time: $t_{total} = t_c$

$$t_{total} = \frac{349}{234}h = 89.4872\,mins = 5369.23s$$

1 mark for correct values on x axis	
1 mark for correct values on y axis	
1 mark for correct shape of graph	
1 mark for substitution for total time	
1 mark for correct value for total time	

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Problem 2 [3 marks – Matric Level]

Question:

A ball is thrown off the edge of a building at an angle of 30^o from the horizontal. If the building is 30m high from the ground, calculate the time of flight before the ball hits the ground. Sketch the path you expect the ball to follow.

Solution:

$$s_y = u_y t + \frac{1}{2}at^2$$

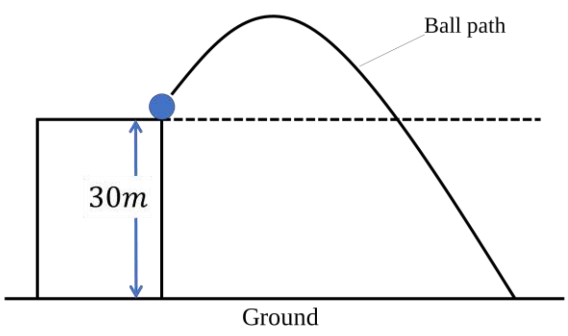
$$-30 = (u \sin 30)t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - (u \sin 30)t - 30 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{(u \sin 30) \pm \sqrt{(u \sin 30)^2 - 4(4.905)(30)}}{2(4.905)}$$

$$t = \frac{u \pm \sqrt{u^2 - 2354.4}}{19.62}$$



1 mark for substitution – 2^{nd} line	
1 mark for final form of the $t(u)$ equation	
1 mark for path – must be parabolic	

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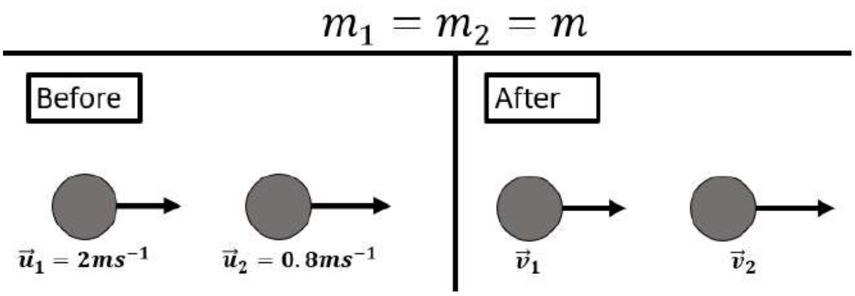
Problem 3 [5 marks – SPM Level]

Question:

Two identical balls collide elastically. Initially, both are travelling in the same direction. The first ball was travelling at $2ms^{-1}$ whilst the other ball was travelling at $0.8ms^{-1}$. Determine the velocity of the two balls after the collision.

Solution:



$$\Sigma\vec{p}_f = \Sigma\vec{p}_i \Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow u_1 + u_2 = v_1 + v_2$$

$$\Rightarrow 2 + 0.8 = v_1 + v_2 - - - (1)$$

$$\text{Elastically } \Rightarrow \Sigma K_i = \Sigma K_f \Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow u_1^2 + u_2^2 = v_1^2 + v_2^2$$

$$(2)^2 + (0.8^2) = v_1^2 + v_2^2 - - - (2)$$

Solving (1) and (2) simultaneously yields

$$v_1 = 2ms^{-1}, \quad v_2 = 0.8ms^{-1} \qquad \text{OR} \qquad v_1 = 0.8ms^{-1}, \quad v_2 = 2ms^{-1}$$

Both in the positive direction, same direction initially.

1 mark for conservation of momentum
1 mark for conservation of kinetic energy
1 mark for substitution of value for Σp
1 mark for substitution of value for ΣK
1 mark for answer with unit

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Problem 4 [4 marks – Matric Level]

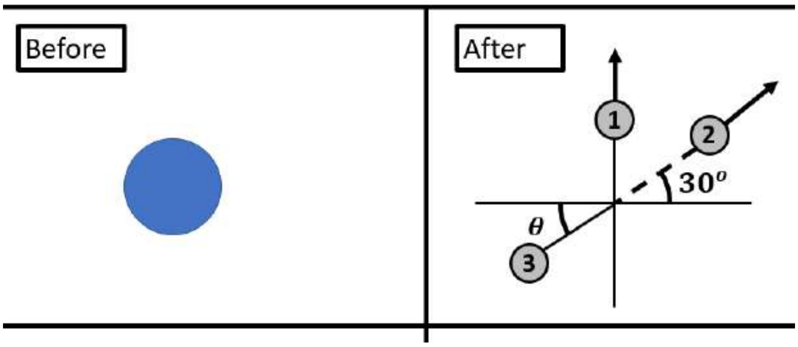
Question:

When it decays, an unstable mass of $15(10^{-27})kg$ would disintegrate into three particles. Their masses are $7(10^{-27})kg$, $2(10^{-27})kg$ and $5(10^{-27})kg$ respectively. The first particle travels along the y-axis with speed of $6.0(10^{6})ms^{-1}$. The second particle moves 30^{o} with respect to the positive direction of the x-axis at a speed $5.0(10^{6})ms^{-1}$. Determine the velocity of the third particle.

Solution:

$v_1 = 6.0(10^6)ms^{-1}$; $v_2 = 5.0(10^6)ms^{-1}$; v_3?

$m_1 = 7(10^{-27})kg$; $m_2 = 2(10^{-27})kg$; $m_3 = 5(10^{-27})kg$



	Momentum in x-direction	Momentum in y-direction	Total Momentum
Parent Particle	0	0	Before
Particle 1	0	$p_{1y} = m_1 v_1$ $= (7(10^{-27}))(6.0(10^6))$ $= 42(10^{-21})$	After
Particle 2	$p_{2x} = m_2 v_2 \cos\theta$ $= (2(10^{-27}))(5.0(10^6))(\cos 30^o)$ $= 8.66(10^{-21})$	$p_{2y} = m_2 v_2 \sin\theta$ $= (2(10^{-27}))(5.0(10^6))(\sin 30^o)$ $= 5(10^{-21})$	
Particle 3	p_{3x}	p_{3y}	

$$\Delta\Sigma p_x = 0 \Rightarrow p_{3x} = -p_{2x} = -8.66(10^{-21})kgms^{-1}$$

$$\Delta\Sigma p_y = 0 \Rightarrow p_{3y} = -(p_{1y} + p_{2y}) = -(42(10^{-21}) + 5(10^{-21})) = -47(10^{-21})kgms^{-1}$$

Magnitude of particle 3 momentum: $p_3 = \sqrt{p_{3x}^2 + p_{3y}^2} = \sqrt{(8.66(10^{-21}))^2 + (47(10^{-21}))^2} = 47.7912(10^{-21})kgms^{-1}$

Velocity of particle 3: $p_3 = m_3 v \Rightarrow 47.7912(10^{-21}) = v_3(5(10^{-27})) \Rightarrow v_3 \approx 9.56(10^6)ms^{-1}$

Direction of particle 3 momentum, $\theta = tan^{-1}\left(\frac{47}{8.66}\right) = 79.56^o$ in the negative y, negative x direction.

1 mark for conservation of momentum	1 mark for substitution of Pythagoras theorem
1 mark for angle tangent equation	1 mark for final answer with units

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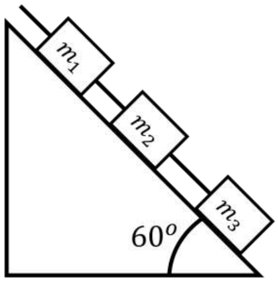
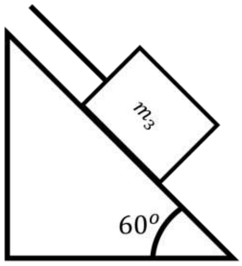
Problem 1

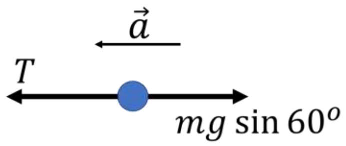
<u>Question [4 Marks]:</u>

Three blocks of masses $m_1 = 1.5kg$; $m_2 = 2.45kg$ and $m_3 = 3.7kg$ are arranged and connected by a string in the order shown in the inclined plane. A force of 100N is applied upward along the incline to the uppermost block. Calculate the tension between m_3 and m_2 blocks.



<u>Problem 1 Solution</u>





$$\Sigma F = m_3 a = T - m_3 g \sin 60^o$$
$$\Rightarrow T = m_3 a + m_3 g \sin 60^o$$
$$\Rightarrow T = m_3(a + g \sin 60^o)$$

Since all of the blocks are bound by the same string, we can assume that the system as a whole will have the same acceleration as individual acceleration.

That means

$$a_{net} = a_3 \text{ --- } [K1]$$

$$a_{net} = \frac{\Sigma F - W_{total}}{m_{total}} = \frac{100 - (1.5 + 2.45 + 3.7)\, sin\, 60}{3.7 + 2.45 + 1.5} \approx 4.57619\, ms^{-2} \text{ --- } [G1]$$

$$a_{net} = a_3 = \frac{T - m_3 g\, sin\, 60^o}{m_3}$$
$$\Rightarrow 4.57619 = \frac{T - (3.7)(9.81)\, sin\, 60^o}{3.7} \text{ --- } [G1]$$

Rearrange for T,

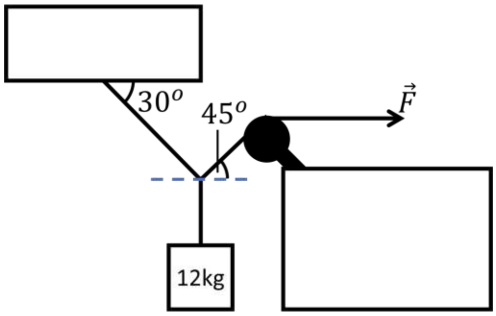
$$T \approx 48.366N \text{ --- } [JU1]$$

K1 – Acceleration concept
G1 – Substitution for a_{net}
G1 – substitution for T
JU1 – Final Answer for tensional force

Problem 2

Question [5 Marks]:

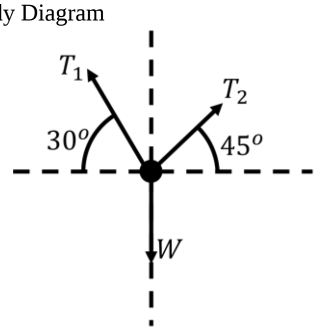
A box is kept static by ensuring sufficient force $\vec{F}$ is applied, as shown in the diagram.

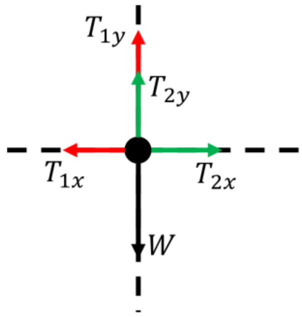


Sketch the free body diagram for the mass and determine the magnitude of $\vec{F}$ needed to keep the box static.

Problem 2 Solution

Free Body Diagram





[J1] - All direction of force correctly drawn
[J1] – All angles correctly labelled

$$T_2 = F$$

Static $\Rightarrow \Sigma F_x = \Sigma F_y = 0 --- [K1]$,

$$\Sigma F_y = 0 \Rightarrow W = T_{1y} + T_{2y}$$
$$\Rightarrow mg = T_1 \sin 30^o + T_2 \sin 45^o$$
$$mg = T_1\left(\frac{1}{2}\right) + T_2\left(\frac{\sqrt{2}}{2}\right) --- (1)$$
$$\Sigma F_x = 0 \Rightarrow T_{1x} = T_{2x}$$
$$\Rightarrow T_1 \cos 30^o = T_2 \cos 45^o$$
$$\Rightarrow T_1\left(\frac{\sqrt{3}}{2}\right) = T_2\left(\frac{\sqrt{2}}{2}\right)$$
$$T_1 = T_2\left(\frac{\sqrt{6}}{3}\right) --- (2)$$

Substitute (2) into (1) yields,

$$mg = T_2\left(\frac{\sqrt{6}}{3}\right)\left(\frac{1}{2}\right) + T_2\left(\frac{\sqrt{2}}{2}\right)$$
$$\Rightarrow T_2 = \frac{mg}{2^{-0.5} + 6^{-0.5}} = \frac{12(9.81)}{2^{-0.5} + 6^{-0.5}} --- [G1]$$
$$T_2 = 105.545N --- [JU1]$$

J1 – FBD correct vector direction
J1 – angles involved drawn
K1 – Static concept
G1 – substitution for T_2
JU1 – Final Answer for T_2

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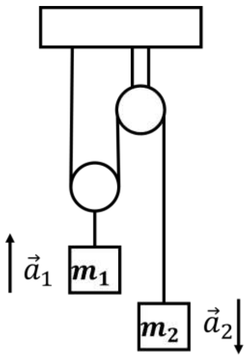
Problem 3

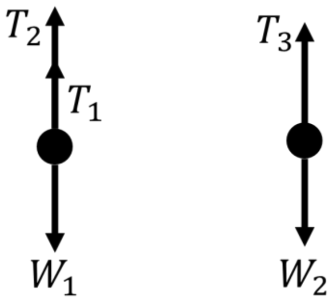
<u>Question [3 Marks]:</u>

Masses $m_1(1.2\text{kg})$ and $m_2(4kg)$ are hung on a pulley system as shown in the diagram. If $a_2 = 2a_1$, calculate the tensional force found in the rope if mass m_2 accelerate downwards. Mass of the pulley and rope is negligible.



<u>Problem 3 Solution</u>

We can first draw the free body diagram for both mass m_1 and m_2,



Applying Newton's Second law,

$$\Sigma F_2 = m_2 a_2 = W_2 - T_3$$
$$\Sigma F_1 = m_1 a_1 = T_1 + T_2 - W_1$$

Same string $\Rightarrow T_1 = T_2 = T_3 = T - - - [K1]$ and condition for acceleration given $(a_2 = 2a_1)$,

$$m_2 a_2 = W_2 - T \Rightarrow 2m_2 a_1 = W_2 - T$$
$$m_1 a_1 = T + T - W_1 \Rightarrow m_1 a_1 = 2T - W_1$$
$$T = \frac{3m_1 m_2 g}{m_1 + 4m_2} = \frac{3(1.2)(4)(9.81)}{1.2 + 4(4)} - - - [G1]$$
$$\text{T} \approx 8.213N - - - [JU1]$$

K1 – Same string tension concept
G1 – substitution for T
JU1 – Final Answer for T

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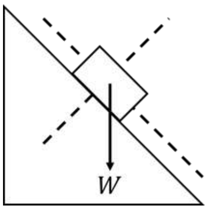
Problem 4

Question [3 Marks]:

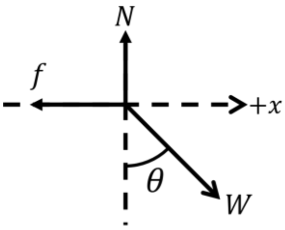
Two unattached masses (of 3kg and 5kg) are placed on a rough inclined plane of 10^o from the horizontal and is initially at rest. The angle of inclination is then increased slowly until it makes an angle of 75^o with the horizontal. As the angle of inclination is raised, which of the masses will slide down first? Justify your answer mathematically.

Problem 4 Solution

Since the masses are unattached, we can consider them independently. We can start by considering a single mass on an incline plane,



Applying standard procedures when dealing with masses on inclined plane and choosing the axis parallel to the plane as the x-axis, we can draw the following free body diagram



and applying newton's second law yields

$$\Sigma F_y = 0 = W\cos\theta - N \Rightarrow W\cos\theta = N$$

When the mass slide down in the positive x direction,

$$\Sigma F_x > 0 \Rightarrow W\sin\theta > f --- [K1]$$

Where $f = \mu N$,

$$\Rightarrow W\sin\theta > \mu W\cos\theta$$
$$\tan\theta > \mu --- [J1]$$

As we can see, the angle at which the masses slide down is independent of the mass of the bodies. This means, any body of any mass will slide down when $\theta > tan^{-1}(\mu)$. This would tell us that both the 3kg and 5kg mass will slide down **at the same time** $(---[J1])$ as the angle of inclination is increased.

K1 – Application of Newton's 2nd Law
J1 – Conditional state
J1 – Final Answer

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Name :

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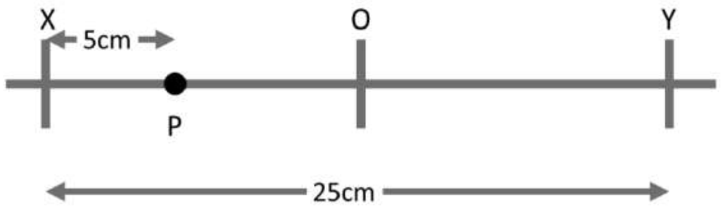
Marks
of 16

Problem 1

Question [8 Marks]

[PSPM 13/14]



The figure above shows a bead executing simple harmonic motion with a period of 1.8s, along a straight line between points X and Y, which are 25cm apart. Point O is at the midpoint between X and Y.

a. Write the equation for the displacement of the bead.
b. Calculate the magnitude of accelerations and velocity of the bead at point P.
c. Calculate the positions along XY when the kinetic energy and the potential energy of the bead are equal.

Problem 1 Solution

$$\boldsymbol{A = \frac{25}{2} = 12.5cm,\ \omega = \frac{2\pi}{T} = \frac{2\pi}{1.8} = \frac{10}{9}\pi\, rads^{-1} - - - [G1]}$$

$$\boldsymbol{x(t) = A\sin\omega t \Rightarrow x(t) = 12.5\sin\left(\frac{10}{9}\pi t\right)}, \text{ where x is in cm and t is in seconds} - - - [J1]$$

$$|a| = \omega^2 x = \left(\frac{10}{9}\pi\right)^2 (0.125 - 0.05) - - - [G1]$$

$$|a| = \frac{5}{54}\pi^2\, ms^{-2} \approx 0.914ms^{-2} - - - [JU1]$$

$$|v| = \omega\sqrt{A^2 - x^2} = \left(\frac{10}{9}\pi\right)\sqrt{0.125^2 - (0.125 - 0.05)^2} - - - [G1]$$

$$|v| = \frac{\pi}{9}ms^{-1} \approx 0.349ms^{-1} - - - [JU1]$$

$$K = U \Rightarrow \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2(A^2 - x^2) \Rightarrow x^2 = A^2 - x^2 \Rightarrow 2x^2 - A^2 = 0$$

$$2x^2 - (0.125)^2 = 0 - - - [G1]$$

$$x = \pm 0.0884m - - - [JU1]$$

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Problem 2

Question [2 Marks]

[PSPM 13/14]

A vertical spring extends by 3cm when a 100g mass is suspended at its end. Calculate the period of oscillation of the spring when a mass of 150g is added to the system.

Problem 2 Solution

$$\boldsymbol{F = kx \Rightarrow k = \frac{F}{x} = \frac{mg}{x} = \frac{(0.1)9.81}{0.03} = 32.7Nm^{-1}}$$

$$T = 2\pi\sqrt{\frac{m_{total}}{k}} = 2\pi\sqrt{\frac{0.1 + 0.15}{32.7}} - - - [G1]$$

$$T = 0.175\pi\, s \approx 0.55s - - - [JU1]$$

Problem 3

Question [6 Marks]

[PSPM 15/16]

The displacement x of a particle varies with time, t is given by

$$x(t) = 4\,cos\left(2\pi t + \frac{\pi}{2}\right)$$

where x is in cm and t is in s. Calculate the

a. frequency of the motion.
b. velocity of the particle at t = 2s.
c. acceleration of the particle at t = 2s.

Problem 3 Solution

$$\boldsymbol{\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} - - - [G1]}$$

$$\boldsymbol{f = 1Hz - - - [JU1]}$$

$$v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}\left(4\,cos\left(2\pi t + \frac{\pi}{2}\right)\right) = -8\pi\,sin\left(2\pi + \frac{\pi}{2}\right)$$

$$v(t = 2s) = -8\pi\,sin\left(2\pi(2) + \frac{\pi}{2}\right) - - - [G1]$$

$$v(t = 2s) = -8\pi\,ms^{-1} \approx -25.1ms^{-1} - - - [JU1]$$

$$a(t = 2s) = -\omega^2 x(t = 2s) = -(1)^2\left(4\,cos\left(2\pi(2) + \frac{\pi}{2}\right)\right) - - - [G1]$$

$$a(t = 2s) = 0ms^{-2} - - - [JU1]$$

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Marks
of 15

Problem 1

Question [7 Marks]

[PSPM 18/19]

A car is moving with constant speed of 80 kmh^{-1} when suddenly the drive sees a cat 50m straight ahead of the car. The driver's reaction time is 0.5s and the maximum deceleration of the car is $10ms^{-2}$.

a. Calculate the total distance travelled by the car from the moment the drive sees the cat until it stopped. What happens to the cat?

b. Sketch acceleration against time graph to show the motion of the car.

Problem 1 Solution

$$80\,kmh^{-1} = \frac{200}{9}ms^{-1}$$

$$s = s_{react} + s_{decelerate}$$

$$s_{react} = ut = \left(\frac{200}{9}\right)(0.5) - - - [G1]$$

$$s_{react} = \frac{100}{9}m - - - [JU1]$$

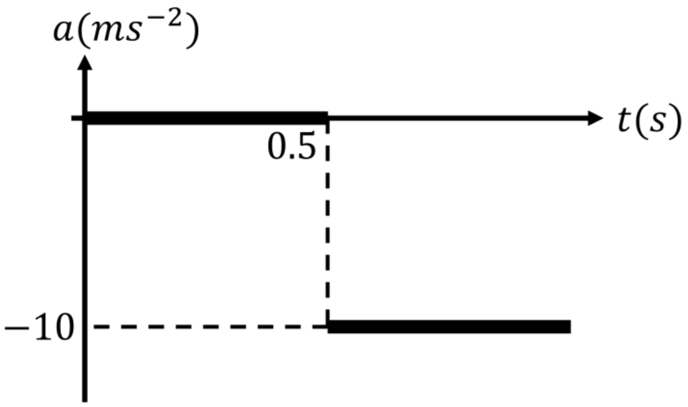
$$v^2 = u^2 + 2as_{decelerate} \Rightarrow s_{decelerate} = \frac{v^2 - u^2}{2a}$$

$$s_{decelerate} = \frac{(0)^2 - \left(\frac{200}{9}\right)^2}{2(-10)} = \frac{2000}{81}m - - - [G1]$$

$$s = \frac{100}{9} + \frac{2000}{81} = \frac{2900}{81} \approx 35.8m - - - [JU1]$$

The cat will not be hit by the car. $- - -[J1]$

Sketch:



Axis label $- - -[J1]$

Graph Shape $- - -[J1]$

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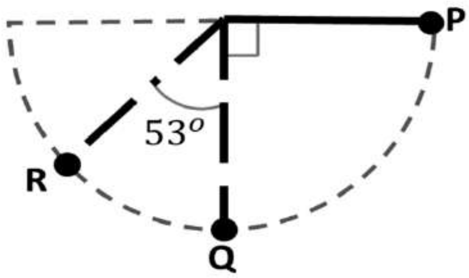
Problem 2

Question [6 Marks]

[PSPM 11/12]

The figure below shows a 0.8kg pendulum bob being released from rest at P. The length of the string is 0.75m.



Calculate the

a. Work done by gravity on the bob at R
b. Speed of the bob at Q

Problem 2 Solution

a.

$$\text{Work-Energy Theorem: } W = \Delta E - - - [K1]$$
$$W_{gravity} = mg\Delta h = (0.8)(9.81)(0.75)(1 - \cos 53^o) - - - [G1]$$
$$W_{gravity} = 2.34372J - - - [JU1]$$

b.

$$\text{Conservation of energy: } \Delta\Sigma E = 0 - - - [K1]$$
$$E_{gp} - E_k = 0$$
$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$
$$v = \sqrt{2(9.81)(0.75)} - - - [G1]$$
$$v = 3.836ms^{-1} - - - [JU1]$$

Problem 3

Question [2 Marks]

[PSPM 13/14]

A vertical spring extends by 3cm when a 100g mass is suspended at its end. Calculate the period of oscillation of the spring when a mass of 150g is added to the system.

Problem 3 Solution

$$\boldsymbol{F = kx \Rightarrow k = \frac{F}{x} = \frac{mg}{x} = \frac{(0.1)9.81}{0.03} = 32.7Nm^{-1}}$$
$$T = 2\pi\sqrt{\frac{m_{total}}{k}} = 2\pi\sqrt{\frac{0.1 + 0.15}{32.7}} - - - [G1]$$
$$T = 0.175\pi\, s \approx 0.55s - - - [JU1]$$

Name : Solutions!

Tutorial : ____________________

Questions

1. A system of 2kg undergoes simple harmonic motion with a period of 0.2s and an amplitude of 5cm.
   a. Determine the velocity of the system when its displacement is 2cm. [3 marks]

$$v = \pm\omega\sqrt{A^2 - x^2} = \pm\left(\frac{2\pi}{T}\right)\sqrt{A^2 - x^2} = \pm\left(\frac{2\pi}{0.2}\right)\sqrt{0.05^2 - 0.02^2}$$
$$v = \pm 1.43 ms^{-1}$$

   b. Determine the displacement when the kinetic energy and the potential energy is equal. [3 marks]

$$E_k = E_p \Rightarrow \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$
$$x^2 = A^2 - x^2 \Rightarrow x^2 = (0.05)^2 - x^2 \Rightarrow x = 0.0354m$$

2. A wave, travelling in the positive-x direction, is measured to have a wavelength of 1.28cm and a period of 0.75s. At a chosen coordinate system, when time is 0, the displacement y is at its maximum. Write down the equation that describes the displacement of this wave. [4 marks]

G.E.: $y(x,t) = A\sin(\omega t \pm kx)$
Positive x-direction ⇒ -kx

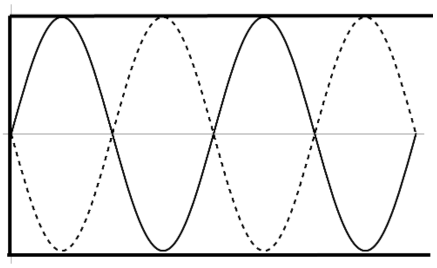
$$\lambda = 1.28cm = \frac{2\pi}{k} \Rightarrow k = \frac{25}{16}\pi cm^{-1}$$
$$T = 0.75s = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{8}{3}\pi\, rads^{-1}$$

Since at $t = 0$, $y = A$,

$$y(x,t) = A\cos(\omega t \pm kx) \Rightarrow y(x,t) = A\cos\left(\frac{25}{16}\pi t - \frac{8}{3}\pi x\right)$$

Where x is in cm and t is in seconds.

3. A closed tube is measured to be 12cm. Calculate its third overtone frequency if the velocity of sound wave is $344ms^{-1}$ and sketch the waveform. [3 marks]



$$L = 0.12m = 2\lambda \Rightarrow \lambda = 0.06m$$
$$v = 344 = f\lambda = f(0.06) \Rightarrow f = \frac{17200}{3}kHz \approx 573kHz$$

4. A stationary observer hears a 700Hz sound from a moving car of moving at $27ms^{-1}$. Determine the frequency of the sound at its source. [4 marks]

If the car moves **towards** the observer,

$$f_{observed} = f_{source}\left(\frac{v + v_{observer}}{v - v_{source}}\right)$$
$$700 = f_{source}\left(\frac{v + 0}{v - 27}\right)$$
$$f_{source} = 700 - \frac{18900}{v}$$

If $v = 343ms^{-1}$,

$$f_{source} = 644.9Hz$$

If the car moves **away** from the observer,

$$f_{observed} = f_{source}\left(\frac{v + v_{observer}}{v + v_{source}}\right)$$
$$700 = f_{source}\left(\frac{v + 0}{v + 27}\right)$$
$$f_{source} = 700 + \frac{18900}{v}$$

If $v = 343ms^{-1}$,

$$f_{source} = 755.1Hz$$

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LABORATORY MANUAL
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MINISTRY OF EDUCATION MALAYSIA
MATRICULATION PROGRAMME

THIRTEENTH EDITION

First Printing, 2003
Second Printing, 2004
Third Printing, 2005 (*Sixth Edition*)
Fourth Printing, 2006 (*Seventh Edition*)
Fifth Printing, 2007 (*Eighth Edition*)
Sixth Printing, 2011 (*Ninth Edition*)
Seventh Printing, 2013 (*Tenth Edition*)
Eighth Printing, 2018 (*Eleventh Edition*)
Ninth Printing, 2020 (*Twelfth Edition*)
Tenth Printing, 2022 (*Thirteenth Edition*)

Published in Malaysia by

Matriculation Division
Ministry of Education Malaysia,
Level 6 – 7, Block E15,
Government Complex Parcel E,
Federal Government Administrative Centre,
62604 Putrajaya,
MALAYSIA.
Tel : 603-8884 4083
Fax : 603-8884 4028
Website : www.moe.gov.my

Printed in Malaysia by

Malaysia National Library
Physics Laboratory Manual
Semester I & II
SP015 & SP025
Thirteenth Edition

e ISBN 978-983-2604-71-6

NATIONAL EDUCATION PHILOSOPHY

Education in Malaysia is an on-going effort towards further developing the potential of individuals in a holistic and integrated manner, so as to produce individuals who are intellectually, spiritually and physically balanced and harmonious based on a firm belief in and devotion to God. Such an effort is designed to produce Malaysian citizens who are knowledgeable and competent, who possess high moral standards and who are responsible and capable of achieving a high level of personal well-being as well as being able to contribute to the betterment of the family, society and the nation at large.

NATIONAL SCIENCE EDUCATION PHILOSOPHY

In consonance with the National Education Philosophy, science education in Malaysia nurtures a science and technology culture by focusing on the development of individuals who are competitive, dynamic, robust and resilient and able to master scientific knowledge and technological competency.

FOREWORD

I am delighted to write the foreword for the Laboratory Manual, which aimed to equip students with knowledge, skills, and the ability to be competitive undergraduates.

This Laboratory Manual is written in such a way to emphasise students' practical skills and their ability to read and understand instructions, making assumptions, apply learnt skills and react effectively in a safe environment. Science process skills such as making accurate observations, taking measurement in correct manner, using appropriate measuring apparatus, inferring, hypothesizing, predicting, interpreting data, and controlling variables are further developed during practical session. The processes are incorporated to help students to enhance their Higher Order Thinking Skills such as analytical, critical and creative thinking skills. These 21st century skills are crucial to prepare students to succeed in Industrial Revolution (I.R.) 4.0.

The manipulative skills such as handling the instruments, setting up the apparatus correctly and drawing the diagrams can be advanced through practical session. The laboratory experiments are designed to encourage students to have enquiry mind. It requires students to participate actively in the science process skills before, during and after the experiment by preparing the pre-report, making observations, analysing the results and in the science process skills before, during, after the experiment by preparing the pre-report, making observations, analysing the results and drawing conclusions.

It is my hope and expectation that this manual will provide an effective learning experience and referenced resource for all students to equip themselves with the skills needed to fulfil the prerequisite requirements in the first-year undergraduate studies.

DR HAJAH ROSNARIZAH BINTI ABDUL HALIM
Director
Matriculation Division

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1.0 Student Learning Time (SLT)

Students will be performing the experiment within the time allocated for each practical work.

Face-to-face	Non face-to-face
2 hours	0

2.0 Learning Outcomes

2.1 Matriculation Science Programme Educational Objectives

Upon a year of graduation from the programme, graduates are:

i. Knowledgeable and technically competent in science disciplines study in-line with higher educational institution requirement.

ii. Able to apply information and use data to solve problems in science disciplines.

iii. Able to communicate competently and collaborate effectively in group work to compete in higher education environment.

iv. Able to use basic information technologies and engage in life-long learning to continue the acquisition of new knowledge and skills.

v. Able to demonstrate leadership skills and practice good values and ethics in managing organisations.

2.2 Matriculation Science Programme Learning Outcomes

At the end of the programme, students should be able to:

i. Acquire knowledge of science and mathematics as a fundamental of higher level education.
(MQF LOC i – Knowledge and understanding)

ii. Apply logical, analytical and critical thinking in scientific studies and problem solving.
(MQF LOC ii – Cognitive skills)

iii. Demonstrate manipulative skills in laboratory works.
(MQF LOC iii a – Practical skills)

iv. Collaborate in group work with skills required for higher education.
(MQF LOC iii b – Interpersonal skills)

v. Deliver ideas, information, problems and solution in verbal and written communication.
(MQF LOC iii c – Communication skills)

vi. Use basic digital technology to seek and analyse data for management of information.
(MQF LOC iii d – Digital skills)

vii. Interpret familiar and uncomplicated numerical data to solve problems.
(MQF LOC iii e – Numeracy skills)

viii. Demonstrate leadership, autonomy and responsibility in managing organization.
(MQF LOC iii f – Leadership, autonomy and responsibility)

ix. Initiate self-improvement through independent learning.
(MQF LOC iv – Personal and entrepreneurial skills)

x. Practice good values attitude, ethics and accountability in STEM and professionalism.
(MQF LOC v – Ethics and professionalism)

2.3 Physics 1 Course Learning Outcome

At the end of the course, student should be able to:

1. Describe basic concepts of mechanics, waves, matters, heat and thermodynamics.
   (C2, PLO 1, MQF LOC i)

2. Solve problems related to mechanics, waves, matters, heat and thermodynamics.
   (C4, PLO 2, MQF LOC ii)

3. Apply the appropriate scientific laboratory skills in physics experiments.
   (P3, PLO 3, MQF LOC iii a)

2.4 Physics 2 Course Learning Outcome

At the end of the course, student should be able to:

1. Explain basic concepts of electricity, magnetism, optics and modern physics.
   (C2, PLO 1, MQF LOC i)

2. Solve problems of electricity, magnetism, optics and modern physics.
   (C4, PLO 2, MQF LOC ii)

4. Apply the appropriate scientific laboratory skills in physics experiments.
   (P3, PLO 3, MQF LOC iii a)

3. Interpret and use familiar and uncomplicated numerical and graphical data to solve problems in basic physics.
   (C4, PLO 7, MQF LOC iii e)

2.5 Physics Practical Learning Outcomes

Physics experiment is to give the students a better understanding of the concepts of physics through experiments. The **aims** of the experiments in this course are to be able to:

1. introduce students to laboratory work and to equip them with the practical skills needed to carry out experiment in the laboratory.

2. determine the best range of readings using appropriate measuring devices.

3. recognise the importance of single and repeated readings in measurement.

4. analyse and interpret experimental data in order to deduce conclusions for the experiments.

5. make conclusions in line with the objective(s) of the experiment which rightfully represents the experimental results.

6. verifying the correct relationships between the physical quantities in the experiments.

7. identify the limitations and accuracy of observations and measurements.

8. familiarise student with standard experimental techniques.

9. choose suitable apparatus and to use it correctly and carefully.

10. gain scientific trainings in observing, measuring, recording and analysing data as well as to determine the uncertainties (errors) of various physical quantities observed in the experiments.

11. handle apparatus, measuring instruments and materials safely and efficiently.

12. present a good scientific report for the experiment.

13. follow instructions and procedures given in the laboratory manual.

14. gain confidence in performing experiments.

3.0 Guidance for Students

3.1 Ethics in the laboratory

a. Follow the laboratory rules.

b. Students must be punctual for the practical session. Students are not allowed to leave the laboratory before the practical session ends without permission.

c. Co-operation between members of the group must be encouraged so that each member can gain experience in handling the apparatus and take part in the discussions about the results of the experiments.

d. Record the data based on the observations and not based on any assumptions. If the results obtained are different from the theoretical value, state the possible reasons.

e. Get help from the lecturer or the laboratory assistant should any problems arise during the practical session.

3.2 Preparation for experiment

3.2.1 Planning for the practical

a. Before entering the laboratory

i) Read and understand the objectives and the theory of the experiment.

ii) Think and plan the working procedures properly for the whole experiment. Make sure you have appropriate table for the data.

iii) Prepare a jotter book for the data and observations of the experiments during pre-lab discussion.

b. Inside the laboratory

i) Check the apparatus provided and note down the important information about the apparatus.

ii) Arrange the apparatus accordingly.

iii) Conduct the experiment carefully.

iv) Record all measurements and observations made during the experiment.

3.3 Report writing

The report must be written properly and clearly in English and explain what has been carried out in the experiment. Each report must contain **name, matriculation number, number of experiment, title, date and practicum group.**

The report must also contain the followings:

i)	Objective	• state clearly
ii)	Theory	• write concisely in your own words • draw and label diagram if necessary
iii)	Apparatus	• name, range, and sensitivity, e.g Voltmeter: 0.0 – 10.0 V Sensitivity: ± 0.1 V
iv)	Procedure	• write in passive sentences about all the steps taken during the experiment
v)	Observation	• data tabulation with units and uncertainties • data processing (plotting graph, calculation to obtain the results of the experiments and its uncertainties). • Calculation of uncertainties using LSM method can refer attachment A
vi)	Discussion	• give comments about the experimental results by comparing it with the standard value • state the source of mistake(s) or error(s) if any as well as any precaution(s) taken to overcome them • answer all the questions given
vii)	Conclusion	• state <u>briefly</u> the results with reference to the objectives of the experiment

Reminder: NO PLAGIARISM IS ALLOWED.

4.0 Significant Figures

The significant figures of a number are those digits carry meaning contributing to its precision. Therefore, the most basic way to indicate the precision of a quantity is to write it with the correct number of significant figures.

The significant figures are all the digits that are known accurately plus the one estimated digit. For example, we say the distance between two towns is 200 km, that does not mean we know the distance to be exactly 200 km. Rather, the distance is 200 km *to the nearest kilometres*. If instead we say that the distance is 200.0 km that would indicate that we know the distance to the nearest *tenth* of a kilometre.

More significant figures mean greater precision.

Rules for identifying significant figures:

1. Nonzero digits are always significant.

2. Final or ending zeros written to the right of the decimal point are significant.

3. Zeros written on either side of the decimal point for the purpose of spacing the decimal point are not significant.

4. Zeros written between significant figures are significant.

Example:

Value	Number of significant figures	Remarks
0.5	1	Implies value between 0.45 and 0.55
0.500	3	Implies value between 0.4995 and 0.5005
0.050	2	Implies value between 0.0495 and 0.0505
5.0	2	Implies value between 4.95 and 5.05
1.52	3	Implies value between 1.515 and 1.525
1.52×10^4	3	Implies value between 15150 and 15250
150	2 or 3 (ambiguous)	The zero may or may not be significant. If the zero is significant, the value implied is between 149.5 and 150.5. If the zero is not significant, the value implied is between 145 and 155.

5.0 Uncertainty in Measurements

No matter how careful or how accurate are the instruments, the results of any measurements made at best are only close enough to their true values (actual values). Obviously, this is because the instruments have certain smallest scale by which measurement can be made. Chances are, the true values lie within the smallest scale. Hence, we have uncertainties in our measurements.

The uncertainty of a measurement depends on its type and how it is done. For a quantity x with uncertainty Δx, the measurement should be recorded as $x \pm \Delta x$ with appropriate unit.

The relative uncertainty of the measurement is defined as $\frac{\Delta x}{x}$.

and therefore its percentage of uncertainty, is given by $\frac{\Delta x}{x} \times 100\%$.

5.1 Single Reading

(a) If the reading is taken from a single point or at the end of the scale we use:

$$\Delta x = \tfrac{1}{2} \times \text{(smallest division of the scale)}$$

(b) If the readings are taken from two points on the scale:

$$\Delta x = 2 \times \left[\frac{1}{2} \times \left(\text{smallest division from the scale} \right) \right]$$

(c) If the apparatus has a vernier scale:

$$\Delta x = 1 \times \text{(smallest unit of the vernier scale)}$$

5.2 Repeated Readings

For a set of n repeated measurements, the best value is the average value, that is

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

where: n is the number of measurements taken
x_i is the i^{th} measurement value

The uncertainty is given by

$$\Delta x = \frac{\sum_{i=1}^{n} \left| \bar{x} - x_i \right|}{n}$$

The result should be written in the form of

$$x = \bar{x} \pm \Delta x$$

5.3 **Combination** of uncertainties

We adopt maximum uncertainty.

(a) Addition or subtraction

$$x = a + b - c \Rightarrow \qquad \Delta x = \Delta a + \Delta b + \Delta c$$

(b) Multiplication with constant *k*

$$x = ka \qquad \Rightarrow \qquad \Delta x = k\Delta a$$

(c) Multiplication or division

$$x = \frac{ab}{c} \qquad \Rightarrow \qquad \frac{\Delta x}{x} = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}\right)$$

(d) Powers

$$x = a^n \qquad \Rightarrow \qquad \frac{\Delta x}{x} = n\left(\frac{\Delta a}{a}\right)$$

5.4 Uncertainty gradient and y-intercept using Least Square Method (LSM)

5.4.1 Formula uncertainty for gradient and y-intercept

Straight line graphs are very useful in data analysis for many physics experiments.
From straight line equation, that is, $y = mx + c$ we can easily determine the gradient *m* of the graph and its intercept *c* on the vertical axis.

When plotting a straight line graph, the line does not necessary passes through all the points. Therefore, it is important to determine the uncertainties Δm and Δc for the gradient of the graph and the *y*-interception respectively.

Consider the data obtained is as follows:

x	x_1	x_2	x_3…………………..x_n
y	y_1	y_2	y_3…………………..y_n

(a) Find the centroid $(\bar{x}, \bar{y})$, where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \textit{and} \qquad \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

(b) Draw the best straight line passing through the centroid and balance.

(c) Determine the gradient of the line by drawing a triangle using dotted lines. The gradient is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

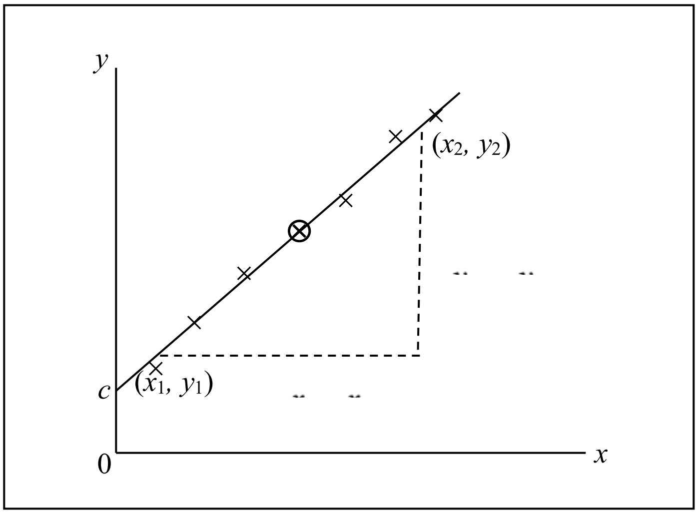


Figure A

(d) The uncertainty of the slope, Δm can be calculated using the following equation

$$\Delta m = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

where n is the number of readings and $\bar{x}$ is the average value of x given by

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

and the estimated value of y, $\hat{y}_i$ is given by,

$$\hat{y}_i = \hat{m}x_i + \hat{c}$$

(e) The uncertainty of the y-intercept, Δc can be calculated using the following equation

$$\Delta c = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2\left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)}$$

5.4.2 Procedure to draw a straight line graph and to determine its gradient with its uncertainty

(a) Choose appropriate scales to use at least 80% of the sectional paper. Draw, label, mark the two axes, and give the units. Avoid using scales of 3, 7, 9, and the likes or any multiple of them. Doing so will cause difficulty in plotting the points later on.

(b) Plot all points clearly with ×. At this stage you can see the pattern of the distribution of the graph points. If there is a point which is clearly too far-off from the rest, it is necessary to repeat the measurement or omit it.

(c) Calculate the centroid and plot it on the graph.

Example:

Suppose a set of data is obtained as below. Graph of T^2 against ℓ is to be plotted.

ℓ (± 0.1 cm)	10.0	20.0	30.0	40.0	50.0	60.0
T^2 (± 0.01 s^2)	0.33	0.80	1.31	1.61	2.01	2.26

From the data:

$$\bar{\ell} = \frac{10.0 + 20.0 + 30.0 + 40.0 + 50.0 + 60.0}{6} = 35.0 \text{ cm}$$

$$\overline{T^2} = \frac{0.33 + 0.80 + 1.31 + 1.61 + 2.01 + 2.26}{6} = 1.39 \text{ s}^2$$

Therefore, the centroid is (35.0 cm, 1.39 s^2).

(d) Draw a best straight line through the centroid and balance. Points above the line are roughly in equal number and positions to those below the line.

(e) Determine the gradient of the line. Draw a fairly large right-angle triangle with part of the line as the hypotenuse.

From the graph in **Figure B**, the gradient of the line is as follows:

For the best line:

$$m = \frac{(2.10 - 0.00) \text{ s}^2}{(53.0 - 0.0) \text{ cm}}$$
$$= 0.040 \text{ s}^2 \text{ cm}^{-1}$$

The gradient of the graph and its uncertainty should be written as follows:

m = (0.040 ± ___) s^2 cm^{-1}

Take extra precaution so that the number of significant figures for the gradient and its uncertainty are in consistency.

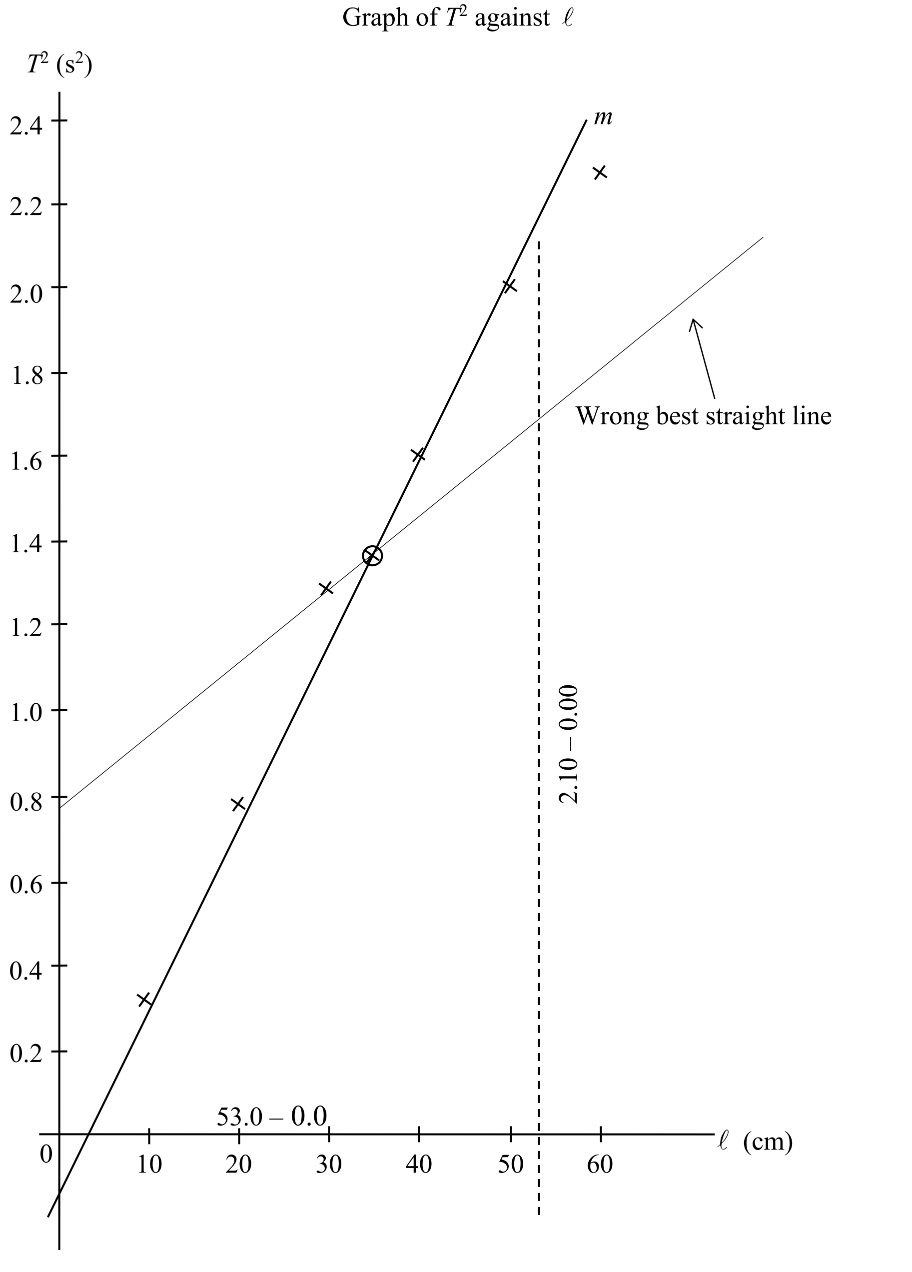

Graph of T^2 against ℓ
T^2 (s^2)
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
10
20
30
40
50
60
ℓ (cm)
m
Wrong best straight line
2.10 – 0.00
53.0 – 0.0

Figure B

(f) Calculation of uncertainties

Rewrite the data in the form of

ℓ	$\boldsymbol{\ell} - \overline{\ell}$	$\left(\boldsymbol{\ell} - \overline{\ell}\right)^{2}$	T^2	$\widehat{T^2}$	$T^2 - \widehat{T^2}$	$\left(T^2 - \widehat{T^2}\right)^2$
10.0	-25.0	625.0	0.33	0.4	-0.070	0.0049
20.0	-15.0	225.0	0.80	0.8	0.000	0.0000
30.0	-5.0	25.0	1.31	1.2	0.110	0.0121
40.0	5.0	25.0	1.61	1.6	0.010	0.0001
50.0	15.0	225.0	2.01	2.0	0.010	0.0001
60.0	25.0	625.0	2.26	2.4	-0.140	0.0196
Σ=210.0		**Σ=1750.0**				**Σ=0.0368**

Where, $\overline{\ell}$ is the average of $\boldsymbol{\ell}$,

$$\overline{\ell} = \frac{210}{6} = 35.0 \text{ cm}$$

Where, $\widehat{T^2}$ is the expected value of T^2

$$\widehat{T^2} = 0.04\boldsymbol{\ell}$$

Calculate the uncertainty of slope, Δm

$$\Delta m = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

$$= \sqrt{\frac{0.0368}{(6-2)(1750)}}$$

$$= \pm 0.002$$

Then, calculate the uncertainty of y-intercept, Δc

$$\Delta c = \sqrt{\left(\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2}\right)\left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)}$$

$$\Delta c = \sqrt{\left(\frac{0.0368}{6-2}\right)\left(\frac{1}{6} + \frac{35^2}{1750}\right)}$$

$$\Delta c = \pm 0.09$$

The data given in section **5.4.2(e)** was obtained from an experiment to verify the relation between T^2 and ℓ. Theoretically, the quantities obey the following relation,

$$T^2 = \left(\frac{k}{p}\right)\ell$$

where k is a natural number equals 39.48 and p is a physical constant. Calculate p and its uncertainty.

Solution:

From the equation, we know that

$$\frac{k}{p} = \text{gradient } m$$

$$\begin{aligned} p &= \frac{k}{m} \\ &= \frac{39.48}{0.040} \\ &= 987 \text{ cm s}^{-2} \end{aligned}$$

Since k is a natural number which has no uncertainties, that is $\Delta k = 0$.

$$\begin{aligned} \Delta p &= \left(\frac{\Delta k}{k} + \frac{\Delta m}{m}\right) p \\ &= \left(0 + \frac{0.002}{0.040}\right) 987 \\ &= 49.35 \end{aligned}$$

so we write,

$p = (987 \pm 49.35)$ cm s^{-2} or $p = (1000 \pm 50)$ cm s^{-2}

5.5 Percentage of difference:

When comparing an experimental result to a value determined by theory or to an accepted known value, the difference between the experimental value and the theoretical value can be determined by:

$$Percentage\ of\ difference = \left| \frac{X_{Theory} - X_{Experiment}}{X_{Theory}} \right| \times 100\%$$

PHYSICS 1
SP015

EXPERIMENT 1: MEASUREMENT AND UNCERTAINTY

Objective:

To measure and determine the uncertainty of physical quantities.

Theory:

Measuring some physical quantities is part and parcel of any physics experiment. It is important to realise that not all measured values are the same as the actual values. This could be due to errors that we made during the measurement, or perhaps the apparatus that we used may not be accurate or sensitive enough. Therefore, as a rule, the uncertainty of a measurement must be taken, and it has to be recorded together with the measured value.

The uncertainty of a measurement depends on the type of measurement and how it is done. For a quantity x with the uncertainty Δx, its measurement is recorded as below:

$$x \pm \Delta x$$

The relative uncertainty of the measurement is defined as:

$$\frac{\Delta x}{x}$$

and therefore, its percentage of uncertainty is $\frac{\Delta x}{x} \times 100\%$.

1.1 Single Reading

(a) If the reading is taken from a single point or at the end of the scale,

$$\Delta x = \frac{1}{2} \times \text{ (smallest division from the scale)}$$

(b) If the readings are taken from two points on the scale,

$$\Delta x = 2 \times \left[\frac{1}{2} \times \text{(smallest division from the scale)} \right]$$

(c) If the apparatus uses a vernier scale,

$$\Delta x = 1 \times \text{(smallest unit from the vernier scale)}$$

1.2 Repeated Readings

For a set of n repeated measurements of x, the best value is the average value given by

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad 1.1$$

where n = the number of measurements taken
x_i = the i^{th} measurement

The uncertainty is given by

$$\Delta x = \frac{\sum_{i=1}^{n} \left| \bar{x} - x_i \right|}{n} \qquad 1.2$$

The result should be written as

$$x = \bar{x} \pm \Delta x \qquad 1.3$$

Apparatus:

A metre rule
A vernier callipers
A micrometer screw gauge
A travelling microscope
A coin
A glass rod
A ball bearing
A capillary tube (1 cm long)

Procedure:

1. Choose the appropriate instrument for measurement of
   (i) length of a laboratory manual.
   (ii) diameter of a coin.
   (iii) diameter of a glass rod.
   (iv) diameter of a ball bearing.

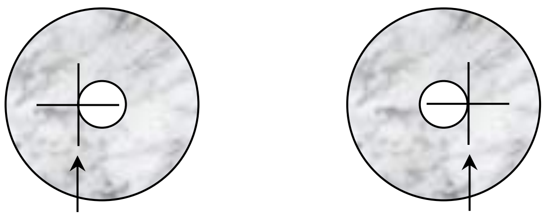
2. For task (i) to (iv), perform the measurement and record the data in a suitable table for at least 5 readings. (Refer to **Table 1.1** as an example)

Table 1.1

No.	Length of the laboratory manual, l ($\pm$ ....)	$\lvert\bar{l} - l_i\rvert$ (..........)
1		
2		
3		
4		
5		
Average	$\bar{l} = \dfrac{\sum_{i=1}^{n} l_i}{n} =$..........	$\Delta l = \dfrac{\sum_{i=1}^{n}\lvert\bar{l} - l_i\rvert}{n}$ $=$.......

3. Determine the percentage of uncertainty for each set of readings.

4. Use travelling microscope to measure the internal diameter of the capillary tube. Adjust the microscope so that the cross-hairs coincide with the left and right edge of the internal diameter of the tube as shown in **Figure 1.1**. Record d_{left} and d_{right}.



d_{left} d_{right}

The internal diameter, $d = |d_{right} - d_{left}|$

Figure 1.1

Determine the uncertainty, Δd and the percentage of uncertainty of the internal diameter of the capillary tube.

EXPERIMENT 2: FREE FALL AND PROJECTILE MOTIONS

Objective:

To determine the acceleration due to gravity, *g* using free fall and projectile motions.

Theory:

A. Free fall motion

When a body of mass *m* falls freely from a certain height *h* above the ground, it experiences a linear motion. The body will obey the equation of motion,

$$s = ut + \frac{1}{2}at^2 \qquad 2.1$$

By substituting the following into equation 2.1,

$s = -h$ (downward displacement of the body from the falling point to the ground)
$u = 0$ (the initial velocity of the body)
$a = -g$ (the downward acceleration due to gravity)

we obtain,

$$h = \frac{1}{2}gt^2 \qquad 2.2$$

Evidently, a graph of h against t^2 is a straight line of gradient equals $\frac{1}{2}$g.

B. Projectile motion

According to **Figure 2.2**, from the law of conservation of energy, the potential energy of a steel ball of mass *m* equals its kinetic energy,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 \qquad 2.3$$

where *h* is the height of the release point above the track
v is the velocity of the steel ball at the end of the track

Note: The rotational kinetic energy for solid sphere is $\frac{1}{5}mv^2$.

The range, *R* of the steel ball is given by

$$R = vt \qquad 2.4$$

Solving equations 2.3 and 2.4, we obtain

$$h = \frac{7}{10}\frac{R^2}{gt^2} \qquad 2.5$$

where *t* is the time taken for the steel ball from the end of the curved track to reach the ground.

Evidently, a graph of *h* against R^2 is a straight line of gradient equals $\frac{7}{10gt^2}$.

Apparatus:

A retort stand with a clamp
A timer
A metre rule
A free fall adaptor (electromagnetic ball release and trap door)
A horizontal table
A steel ball
A curved railing (**Note:** *The lower end of the track must be horizontal*)
A piece of carbon paper
A piece of drawing paper
Cellophane tape
Plasticine
A pair of scissors or a cutter
A piece of string
A pendulum bob
A plywood

Procedure:

A. Free fall motion

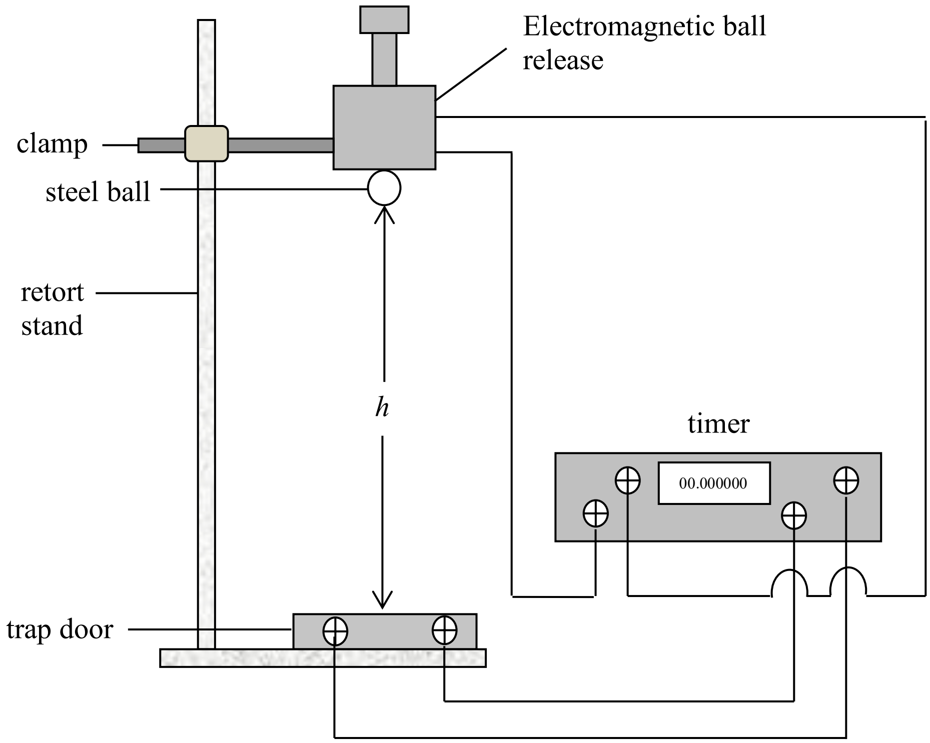


Figure 2.1

Note: *Refer to Figure 2.3 for free fall apparatus with separate power supply for the electromagnet.*

1. Set up the apparatus as in **Figure 2.1**.

2. Switch on the circuit and attach the steel ball onto the upper contact.

3. Adjust the height, h of the electromagnet above the point of impact.

4. Switch off the circuit and let the ball fall. Record the height, h and time, t.

5. Repeat step (3) and (4) for at least six different height, h.

6. Tabulate the data.

7. Plot a graph of h against t^2.

8. Determine the acceleration due to gravity, g from the gradient of the graph.

   Note: *The value range of h-axis should be extended slightly more than the height of the table, H.*

9. Determine the uncertainty of acceleration due to gravity, Δg.

B. Projectile Motion

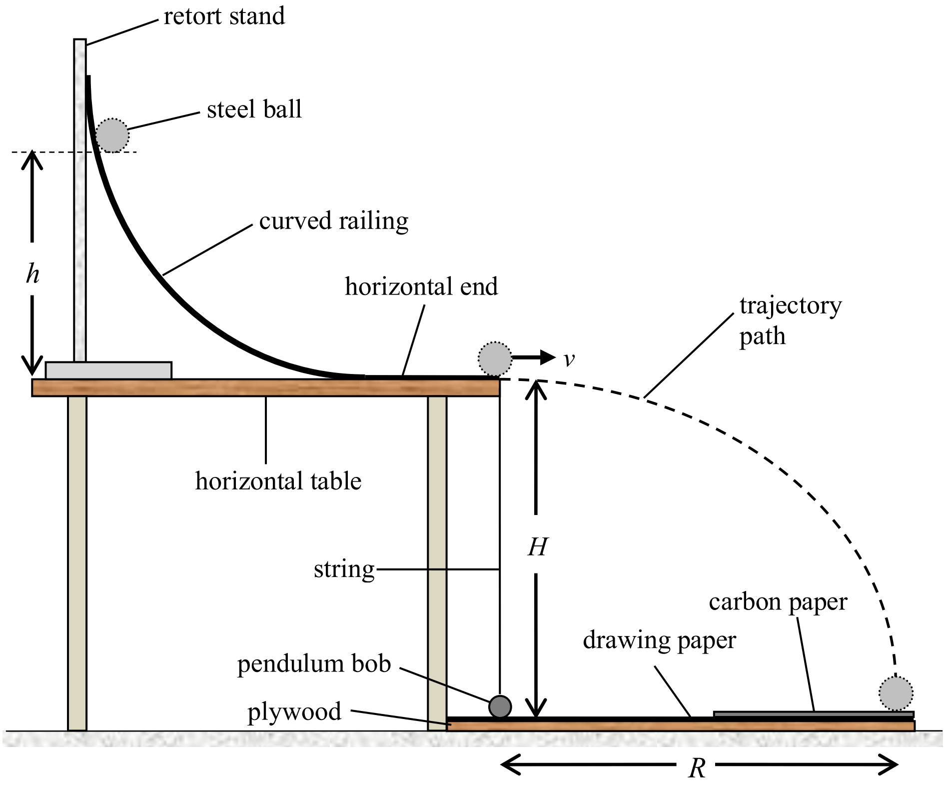


Figure 2.2

1. Set up the apparatus as in **Figure 2.2**.

2. Release the steel ball on the curvature railing at least six different heights, h and record the range, R.

3. Tabulate the data.

4. Plot a graph of h against R^2.

5. Measure the height of the table, H (the edge of the railing to the landing surface).

6. By referring to the graph of h against t^2 from experiment **A,** obtain the value of t^2 for H using extrapolation.

7. Determine the acceleration due to gravity, g from the gradient of the graph.

8. Determine the uncertainty of acceleration due to gravity, Δg.

9. Compare the acceleration due to gravity, g obtained from both experiments with the standard value. Write the comments.

Alternative set-up:

Set-up for free fall apparatus with separate power supply to electromagnet.

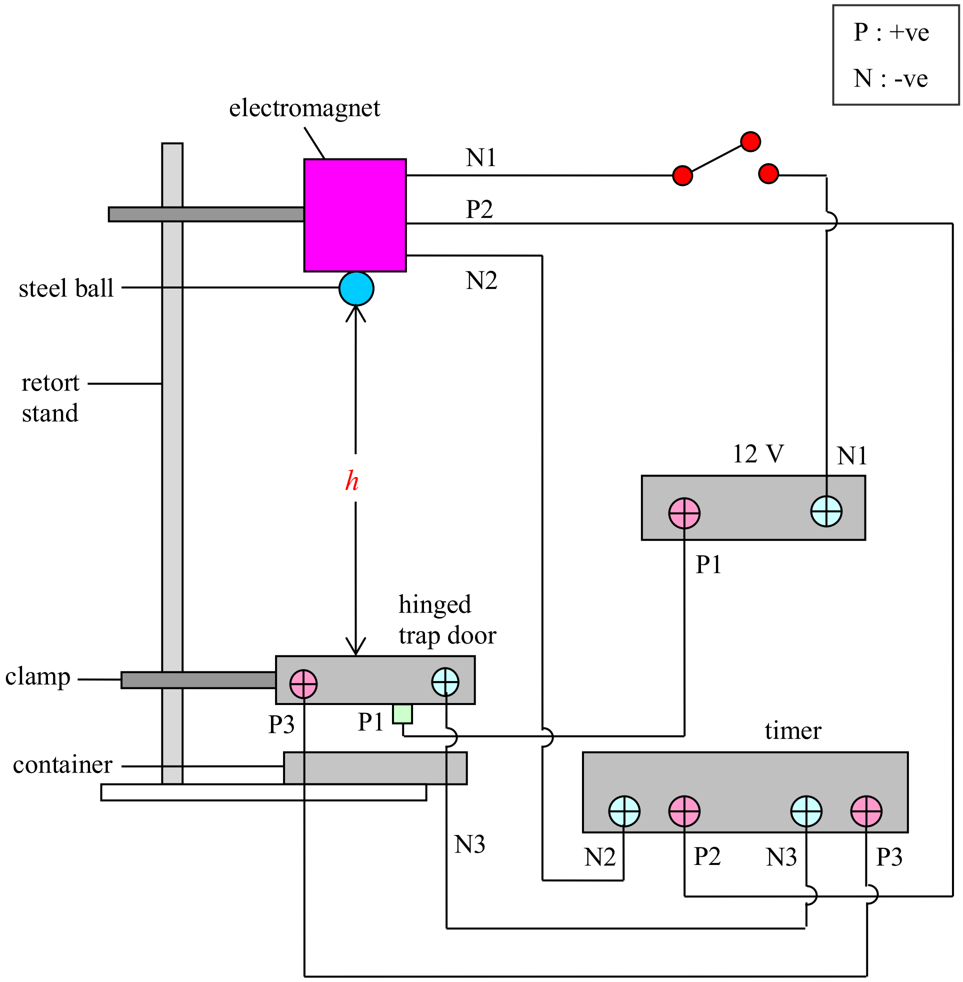


Figure 2.3

EXPERIMENT 3: ENERGY

Objective:

To verify the law of conservation of energy by using free fall motion.

Theory:

Consider a steel ball of mass, m initially at rest at height, h vertically above a velocity detector. By taking the position of the velocity detector as the reference point, the potential energy is mgh and the kinetic energy of the ball is zero. Thus, the total initial energy, E_1 of the steel ball is given by

$$E_1 = mgh \qquad 3.1$$

When the steel ball is released, it falls freely with acceleration due to gravity, g. At the instance it reaches the velocity detector, the gravitational potential energy is zero and its kinetic energy is $\frac{1}{2}mv^2$. Hence, the total final energy, E_2 of the steel ball is given by

$$E_2 = \frac{1}{2}mv^2 \qquad 3.2$$

According to the law of conservation of energy, in the absence of external force the total energy of a system remains constant. In this case, the law is verified if we demonstrate experimentally that E_1 equals E_2, that is,

$$\frac{1}{2}mv^2 = mgh$$

And we obtain

$$v^2 = 2gh \qquad 3.3$$

Consequently, if a graph of v^2 against h is plotted, we should obtain a straight line passing through the origin with gradient equals $2g$.

Apparatus:

A steel ball
A metre rule
A free fall adaptor (electromagnetic ball release)
Two photogates PG A and PG B (Velocity detector)
A timer
A retort stand

Procedure:

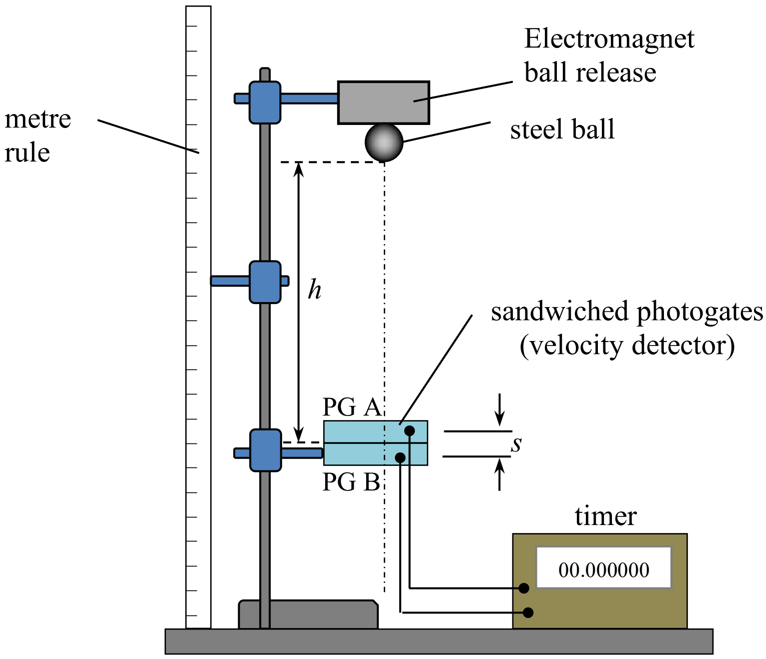


Figure 3.1

1. Construct a velocity detector by sandwiching photogates (PG) A and B using binding tape. Measure the distance, s between the photogates.

2. Set up the apparatus as shown in **Figure 3.1**.

3. Switch on the timer and reset to zero. Set the falling distance, h at 15 cm. Release the steel ball and record the time, t. Repeat the process to obtain the average time.

4. Repeat step (3) for falling distance, h = 20, 25, 30, 35, 40, and 45 cm.

5. For each falling distance, h, calculate the velocity, v using $v = \frac{s}{t}$

6. Tabulate the data.

7. Plot a graph of v^2 against h.

8. Determine the acceleration due to gravity, g from the gradient of the graph.

9. Determine the uncertainty for acceleration due to gravity, Δg obtained in (8).

10. Verify the law of conservation of energy by comparing the acceleration due gravity, g obtained from the experiment with standard value. Write the comments.

EXPERIMENT 4: ROTATIONAL MOTION OF A RIGID BODY

Objective:

To determine the moment of inertia of a fly-wheel, I.

Theory:

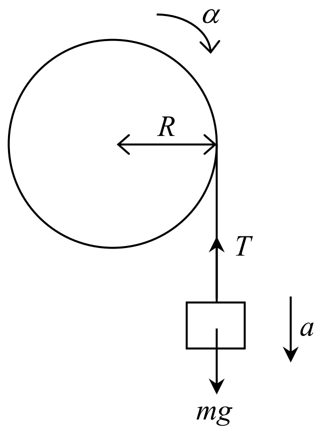


Figure 4.1

By referring to **Figure 4.1**, apply Newton's second law for linear motion,

$$mg - T = ma$$

$$T = m(g - a) \qquad 4.1$$

and applying Newton's second law for rotational motion,

$$TR - \tau = I\alpha \qquad 4.2$$

where a is the downward linear acceleration
τ is the frictional torque (unknown)
α is the angular acceleration
T is the tension in the string
R is the radius of the axle
I is the moment of inertia of the fly-wheel

Therefore,

$$\alpha = \left(\frac{R}{I}\right)T - \left(\frac{\tau}{I}\right) \qquad 4.3$$

The graph α against T is a straight line graph with gradient $\frac{R}{I}$.

Moment of inertia of the fly-wheel,

$$I = \frac{R}{\text{gradient}} \qquad 4.4$$

From kinematics, $s = ut + \frac{1}{2}(-a)t^2$ (negative sign means the acceleration is downward)

By substituting, $s = -h$ and $u = 0$ into the equation above, we obtain

$$h = \frac{1}{2}at^2$$

Hence the linear acceleration,

$$a = \frac{2h}{t^2} \qquad 4.5$$

where h is the height of mass
t is the time taken for the mass to fall to the floor

Angular acceleration,

$$\alpha = \frac{a}{R} \qquad 4.6$$

Apparatus:

A fly-wheel
A stopwatch
A set of slotted mass with hook (**Note**: *Use suitable masses for the fly-wheel to rotate at a suitable rate*)
A metre rule
A G-clamp
A piece of inelastic string to hang the mass to the fly-wheel
A piece of softboard or plywood
A vernier callipers

Procedure:

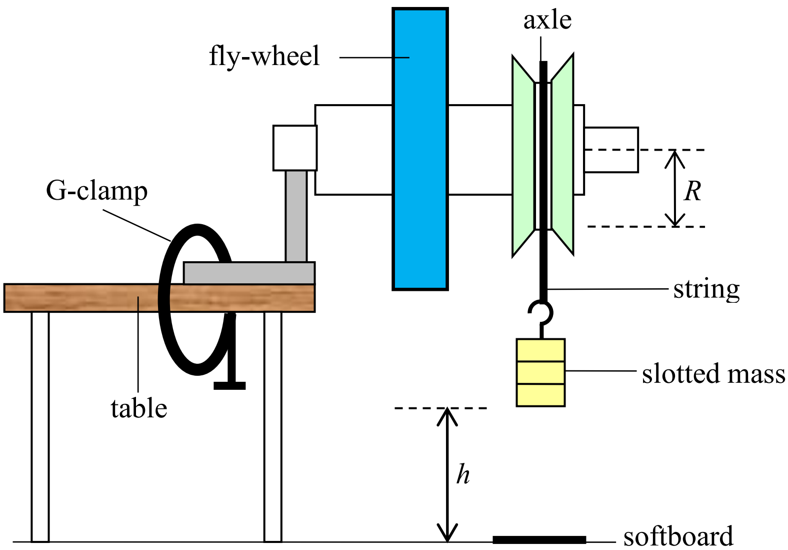


Figure 4.2

1. Set up the apparatus as in **Figure 4.2**.

2. Measure the diameter, d of the axle and calculate its radius, R.

3. Record the falling slotted mass, m.

4. Choose a fixed point at a height, h above the floor. Record height, h.

5. Release the slotted mass, m from the fixed height, h after the string has been wound around the axle.

6. Record the time, t for the slotted mass, m to reach the floor.

7. Calculate the linear acceleration, a, tension, T and angular acceleration, α using equations 4.5, 4.1 and 4.6 respectively.

8. Repeat steps (3) to (7) for at least six different slotted mass, m.

9. Tabulate the data.

10. Plot a graph of α against T.

11. Determine the moment of inertia of the fly-wheel, I from the gradient of the graph.

12. Determine the uncertainty of moment of inertia of the fly-wheel, ΔI.

13. Compare the moment of inertia of the fly-wheel, I to the standard value. Write the comments.

EXPERIMENT 5: SIMPLE HARMONIC MOTION (SHM)

Objectives:

(i) To determine the acceleration, g due to gravity using simple pendulum.
(ii) To investigate the effect of large amplitude oscillation to the accuracy of acceleration due to gravity, *g* obtained from the experiment.

Theory:

An oscillation of a simple pendulum is an example of a simple harmonic motion (SHM) if

(i) the mass of the spherical bob is a point mass
(ii) the mass of the string is negligible
(iii) amplitude of the oscillation is small (< 10°)

According to the theory of SHM, the period of oscillation of a simple pendulum, T is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad 5.1$$

where l is the length of pendulum
g is the acceleration due to gravity

Rearrange equation 5.1, we obtain

$$T^2 = \frac{4\pi^2}{g} \qquad 5.2$$

Evidently, a graph of T^2 against l is a straight line of gradient equals $\frac{4\pi^2}{g}$.

Hence, from the gradient of the graph, the acceleration due to gravity, g can be calculated.

Apparatus:

A piece of string (≈105 cm)
A small pendulum bob
A pair of small flat pieces of wood or cork
A retort stand with a clamp
A stopwatch
A metre rule
A protractor with a hole at the centre of the semicircle
An optical pin
A pair of scissors or a cutter
A stabilizing weight or a G-clamp

Procedure:

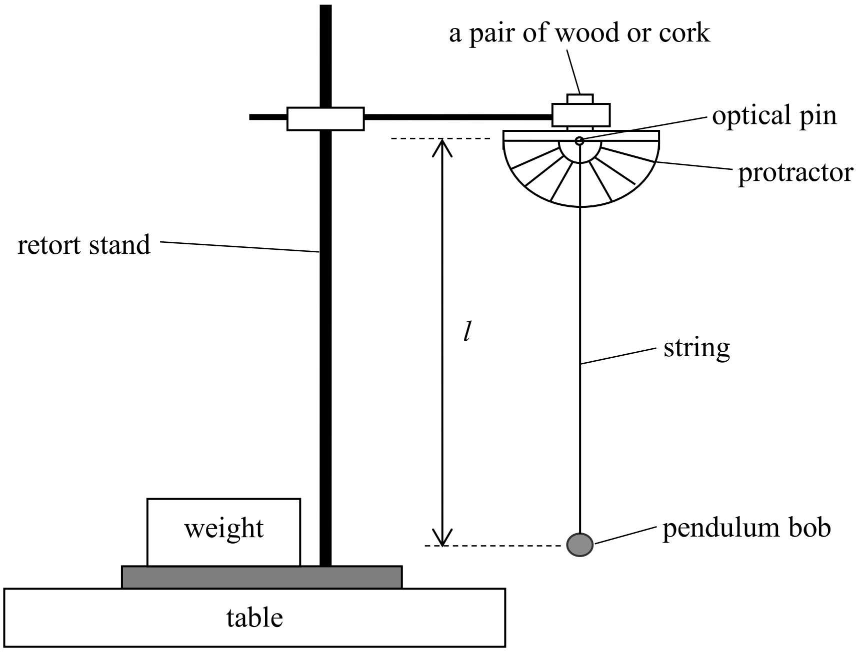


Figure 5.1

1. Set up a simple pendulum as in **Figure 5.1**.

2. Measure the length, l of the pendulum at 40 cm.

3. Release the pendulum at less than 10° from the vertical in one plane and measure the time, t for 10 complete oscillations.

Note: *Start the stopwatch after several complete oscillations.*

4. Calculate the average time, t.

5. Calculate the period of oscillation, T of the pendulum.

6. Repeat step (3) to (5) for length, l of 50 cm, 60 cm, 70 cm, 80 cm and 90 cm.

7. Tabulate the data.

8. Plot a graph of T^2 against l.

9. Determine the acceleration due to gravity, g from the gradient of the graph.

10. Determine the uncertainty of acceleration due to gravity, Δg.

11. Fix the length, l of pendulum at 100 cm.

12. Release the pendulum through a large arc of about 70° from the vertical and measure the time, t for 5 complete oscillations. Repeat the step for three times.

13. Calculate the average time, t and the period of oscillation, T of the pendulum.

14. Calculate the acceleration due to gravity, g using equation 5.1 by using the length, l and period, T from step (11) to (13).

15. Compare the acceleration due to gravity, g obtained from step (9) and (14) with the standard value. Write the comments.

EXPERIMENT 6: STANDING WAVES

Objectives:

(i) To investigate standing waves formed in a stretched string.
(ii) To determine the mass per unit length, μ of the string.

Theory:

When a stretched string is vibrated at a frequency, f the standing waves formed have both ends as nodes. The frequency in the string obeys the following relation

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Hence, the tension of the string,

$$T = 4\mu f^2 l^2 \qquad 6.1$$

where f is the frequency
l is the length between two nodes
T is the tension in the string
μ is the mass per unit length

Evidently, a graph of T against l^2 is a straight line of gradient equals $4\mu f^2$. Hence, the mass per unit length, μ can be calculated.

Apparatus:

A G-clamp
A solenoid (about 100 turns) or ticker timer
An AC supply (2 – 4 V)
A metal rod (soft iron)
Two bar magnets
A magnet holder
A piece of string approximately 2 m long
A pulley with clamp
A wooden wedge
Set of slotted mass 2 g, 5 g, 10 g and 20 g
A metre rule
Connecting wires

Procedure:

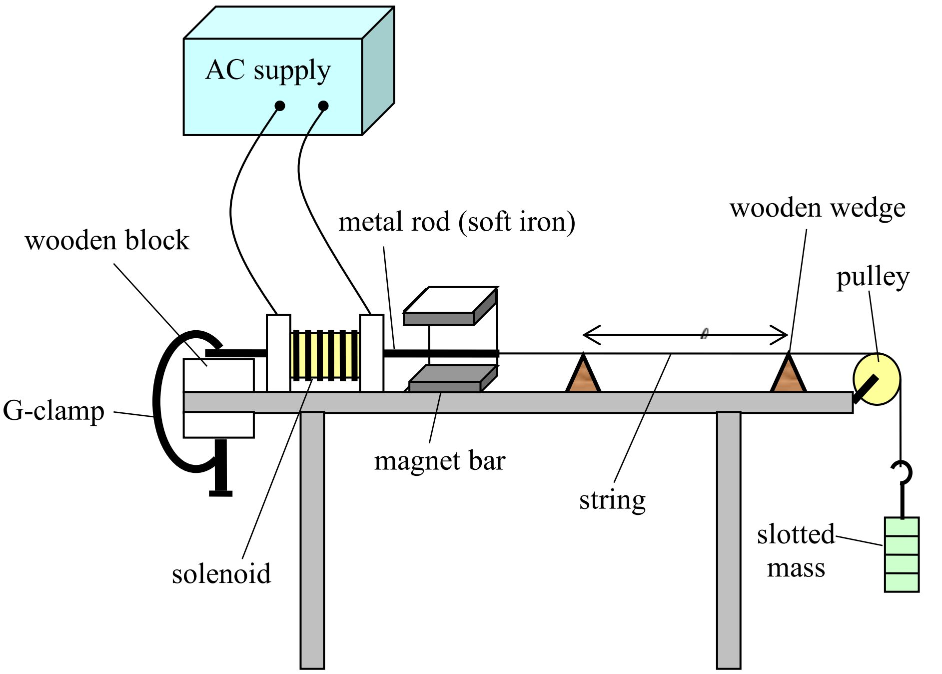


Figure 6.1

1. Set up the apparatus as in **Figure 6.1**.

2. Connect the terminals of the solenoid to the AC power supply (2 V, 50 Hz).

   Caution: *Do not exceed 4 V to avoid damage to the solenoid.*

3. Place the metal rod between the two bar magnets.

4. Tie one end of the string to the rod and the other to the hook of the slotted mass. Make sure that the length of the string from the end of the rod to the pulley is **not less than 1.5 m**.

5. Clamp the metal rod properly. Switch on the power supply. Adjust the position of the metal rod to get maximum vibration.

6. Place the wooden wedges below the string **as close as possible to the pulley**.

7. Adjust the position of the wooden wedges until a clear single loop standing wave (fundamental mode) is observed.

8. Record the distance, l between the wedges and total mass, m (mass of the hook and the slotted mass). Calculate weight, W where $W = mg$.

   Note: *Weight, W = Tension, T*

9. Add a small mass, preferably 10 g to the hook and repeat step (7) and (8) for at least six different readings.

10. Tabulate the data.

11. Plot a graph of T against l^2.

12. Determine the mass per unit length, μ from the gradient of the graph and its uncertainty, $\Delta\mu$ if the frequency of the vibration is 50 Hz.

13. Weigh the mass of the string and measure the total length of the string. Calculate the mass per unit length, μ of the string.

14. Compare the mass per unit length, μ in step (12) with the result obtained in step (13). Write the comments.

PHYSICS 2
SP025

EXPERIMENT 1: CAPACITOR

Objectives:

(i) To determine the time constant, τ of an RC circuit.
(ii) To determine the capacitance, C of a capacitor using an RC circuit.

Theory:

Time constant is defined as the time taken of a discharge current decreases to 37% of its maximum current. The time constant can be calculated by using

$$\tau = RC \qquad 1.1$$

Where τ is time constant
R is the resistance of a resistor
C is the capacitance of a capacitor

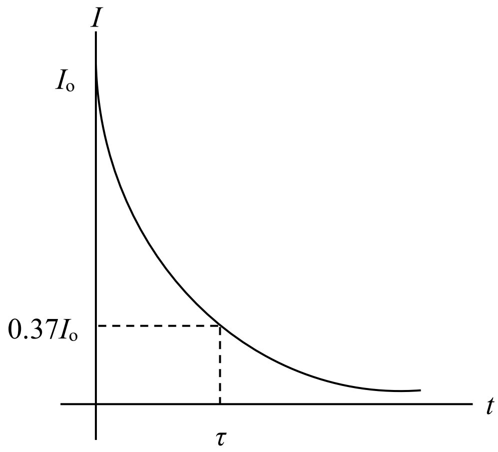


Figure 1.1

During discharging, the magnitude of the current, I varies with time as shown in **Figure 1.1**.

From Figure 1.1, the magnitude of the discharge current is

$$I = I_o e^{-\frac{t}{\tau}} \qquad 1.2$$

Rearrange equation 1.2 we obtain

$$\ln\left(\frac{I_o}{I}\right) = \frac{t}{\tau} \qquad 1.3$$

Where I_o is the maximum current in the circuit
I is current in the circuit at time t

By using equation 1.3, the time constant can be determined from the gradient of the straight-line graph.

Apparatus:

A DC power supply (4 – 6 V)
A switch
A DC microammeter
A digital stopwatch
A 100 kΩ resistor
Connecting wires
Two capacitors labelled C_1 and C_2 (470 – 1000 μF)

Procedure:

Note: *Before starting or repeating this experiment, make sure that the capacitors are fully discharged. This can be attained by short circuiting the capacitors.*

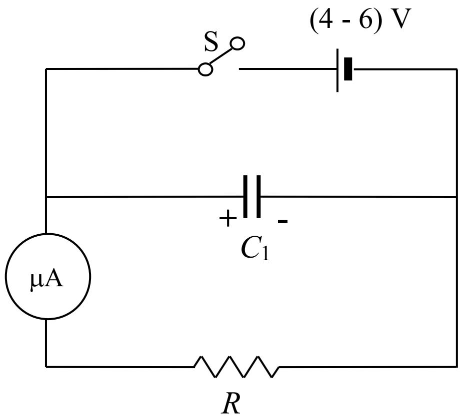


Figure 1.2

1. Set up the circuit as shown in **Figure 1.2**.

2. Close switch S to fully charge the capacitor C_1. Record the reading of the microammeter for maximum current, I_o.

3. Open switch S and start the stopwatch simultaneously.

4. Use the 'lap' function on the stopwatch when the current reaches at least six different values. Record the time for each value of current.

5. Repeat steps (1) to (4) and calculate the average time, t.

6. Tabulate the data.

7. Plot a graph of $\ln\left(\frac{I_o}{I}\right)$ against t.

8. Determine the time constant, τ from the gradient of the graph.

9. Calculate the capacitance of the capacitor C_1 by using equation 1.1.

10. Connect capacitor C_2 to the circuit as shown in **Figure 1.3**.

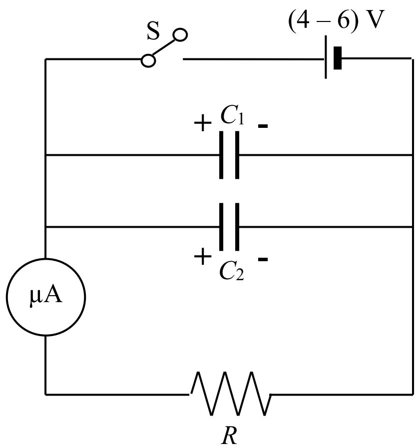


Figure 1.3

11. Repeat steps (1) to (6) to obtain the readings of the microammeter I' and the stopwatch t'. Record the readings.

12. Tabulate the data.

13. Plot a graph of $\ln\left(\frac{I_o}{I}\right)'$ against t'.

14. Determine the time constant, τ' from the gradient of the graph.

15. Calculate the effective capacitance, C_{eff} of capacitors by using equation 1.1.

16. Compare all the results with their respective standard values. Write a comment.

EXPERIMENT 2: OHM'S LAW

Objectives:

(i) To sketch V-I graph.
(ii) To verify Ohm's law.
(iii) To determine the effective resistance, R_{eff} of the resistors in series and parallel by graphing method.

Theory:

At constant temperature, the potential difference V across a conductor is directly proportional to the current I that flows through it. The constant of proportionality is known as the resistance of the conductor denoted by R.

Mathematically, $V \propto I$

$$V = IR \qquad 2.1$$

For resistors in series, the effective resistance is

$$R_{\mathbf{eff}} = R_1 + R_2 + R_3 + \ldots + R_n \qquad 2.2$$

For resistors in parallel, the effective resistance is

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \qquad 2.3$$

Apparatus:

A DC power supply (4 – 6 V)
Three resistors of the same resistance (27 – 100 Ω)
A DC milliammeter
A DC ammeter (1 A)
A DC voltmeter
A rheostat
A switch
Connecting wires

Procedure:

1. Determine the resistance, R of each resistor from their colour bands.
2. Set up the circuit as in **Figure 2.1**. Connect the three resistors in series.

   Note: *Ask your lecturer to check the circuit before switching ON the power.*

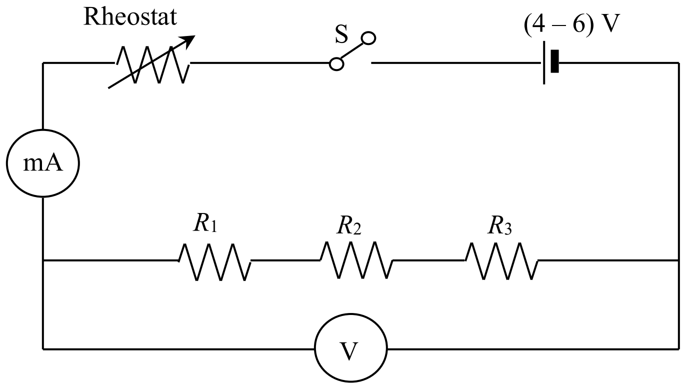


Figure 2.1

3. Adjust sliding contact on the rheostat to change the resistance values to obtain a minimum reading of the milliammeter. Record the reading of the voltmeter, V and the milliammeter, I.

4. Adjust sliding contact on the rheostat to change the resistance values to obtain at least six different values of V and I.

5. Tabulate the data.

6. Plot a graph of V against I.

7. Determine the effective resistance, R_{eff} of the three resistors connected in series from the gradient of graph.

8. Set up the circuit as in **Figure 2.2**.

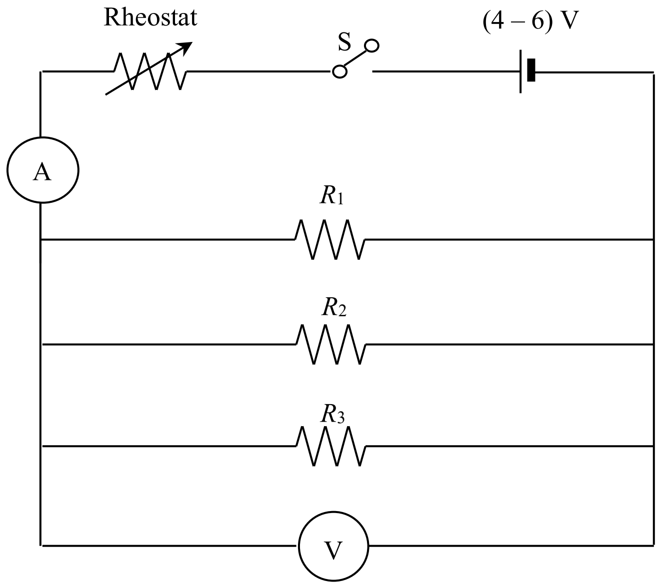


Figure 2.2

9. Repeat steps (3) to (6).

10. Determine the effective resistance, R_{eff} of the three resistors connected in parallel from the gradient of the graph.

11. Compare all the results with their respective standard values.

12. Verify Ohm's law from the plotted graphs. Write a comment.

EXPERIMENT 3: POTENTIOMETER

Objective:

To determine the internal resistance *r* of a dry cell by using a potentiometer.

Theory:

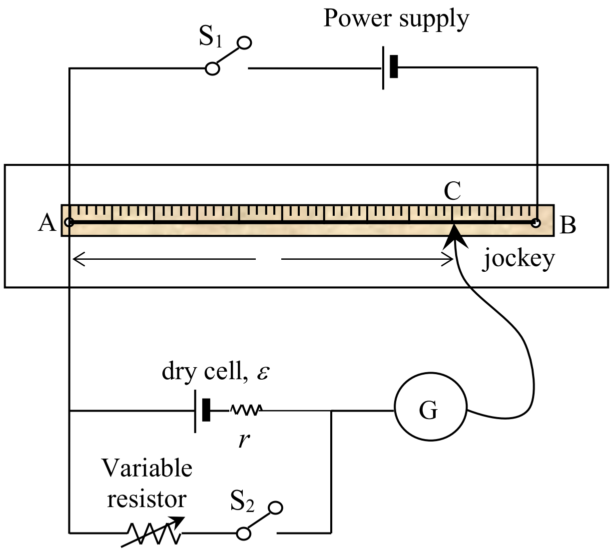


Figure 3.1

Let ε be the electromotive force (emf) and *r* the internal resistance of the dry cell. The emf of the dry cell is balanced by the potential difference across wire AB provided by the power supply when the jockey is tapped at balance point, C with S_1 close and S_2 open. The balance condition is indicated when there is no deflection in the galvanometer. If l_o is the length of the wire from A to C,

$$\varepsilon \propto l_o$$

Hence,

$$\varepsilon = kl_o \qquad 3.1$$

where *k* is a constant.

With both S_1 and S_2 closed, the new length of wire at the balance point is equal to *l*
Hence,

$$V \propto l$$

$$V = kl \qquad 3.2$$

$$\varepsilon = V + Ir \qquad 3.3$$

Rearrange equation 3.1, 3.2 and 3.3, we obtain

$$\frac{l_o}{l} = r\left(\frac{1}{R}\right) + 1 \qquad 3.4$$

The graph $\frac{l_o}{l}$ against l is a straight-line graph and its gradient is *r*.

Apparatus:

A potentiometer
A variable resistor (0 – 1 Ω) (A breadboard, six 1 Ω resistor and jumpers)
Two switches
A jockey
A regulated power supply
A 1.5 V dry cell
A galvanometer
Connecting wires

Procedure:

1. Set up the apparatus as shown in **Figure 3.1**.

   Note:
   i. *Make sure the polarity of the batteries is connected in right configuration. Ask your lecturer to check the circuit before switch ON the power.*
   ii. *Make sure that the galvanometer deflected at both sides when the jockey is tapped at two different extreme points.*

2. With S_1 closed and S_2 opened, tap the jockey along the wire until the galvanometer reading is zero (balanced) and determine l_o.

3. With both S_1 and S_2 closed, repeat step (2) and determine l for at least six different values of R.

4. Tabulate the data.
   Note:
   Use 1 Ω resistor in parallel combination to obtain resistance

1 Ω, $\frac{1}{2}\Omega$, $\frac{1}{3}\Omega$, $\frac{1}{4}\Omega$, $\frac{1}{5}\Omega$ *and* $\frac{1}{6}\Omega$ *as shown in **Figure 3.2**.*

5. Plot a graph of $\frac{l_o}{l}$ against $\left(\frac{1}{R}\right)$.

6. Determine the internal resistance of the dry cell, r from the gradient of the graph. Write a comment.

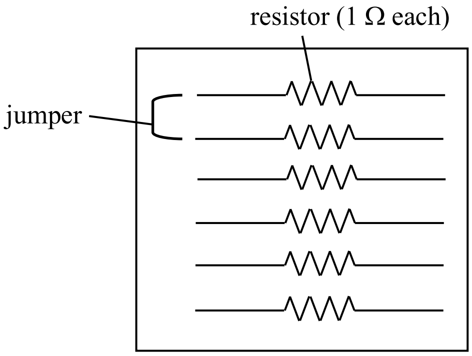


Figure 3.2

EXPERIMENT 4 (a): MAGNETIC FIELD

Objective:

To determine the value of the horizontal component of the earth magnetic field $\vec{B}_{\mathrm{E}}$.

Theory:

The magnetic field strength $\vec{B}$ is a vector quantity so the addition of two magnetic fields obeys the parallelogram law. For example, if $\vec{B}_{\mathrm{E}}$ is the horizontal component of earth magnetic field and $\vec{B}_{\mathrm{C}}$ is the magnetic field of a coil which is perpendicular to $\vec{B}_{\mathrm{E}}$ then the resultant of the two fields $\vec{B}$ is as shown in **Figure 4.1**. A compass needle is situated at the place where the two fields meet will be aligned to the direction of the resultant field $\vec{B}$.

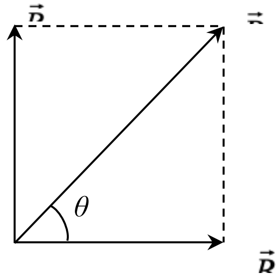


Figure 4.1

From Biot-Savart's Law, the magnetic field strength of the coil at the centre as shown in **Figure 4.2** is given by

$$B_{\mathrm{c}} = \frac{\mu_{\mathrm{o}} N I}{D} \qquad 4.1$$

where $\mu_{\mathrm{o}} = 4\pi \times 10^{-7}$ H m^{-1} (permeability of free space)

I is the current in ampere

N is the number of turns in the coil

D is the diameter of the coil

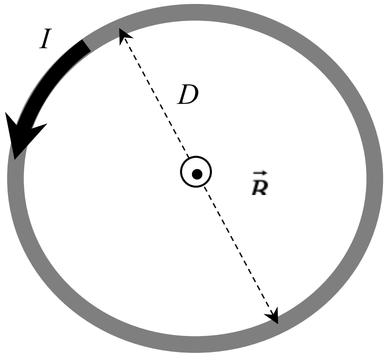


Figure 4.2

From **Figure 4.1**,

$$\tan\theta = \frac{B_c}{B_E}$$

$$\tan\theta = \frac{\mu_o N}{D(B_E)} I \qquad 4.2$$

The gradient of graph $\tan\theta$ against I

$$m = \frac{\mu_o N}{D(B_E)} \qquad 4.3$$

Therefore,

$$B_E = \frac{\mu_o N}{Dm} \qquad 4.4$$

Apparatus:

Earth magnetic field measurement kit (EM-01)
Connecting wires of about 50 cm long with crocodile clips
A DC ammeter (0 – 1 A)
A rheostat
A DC power supply (2 - 4 V)

Procedure:

1. Set up apparatus as shown in **Figure 4.3**.

   Note:
   Make sure the EM-01 kit is located far away from other electrical devices to avoid magnetic disturbance.

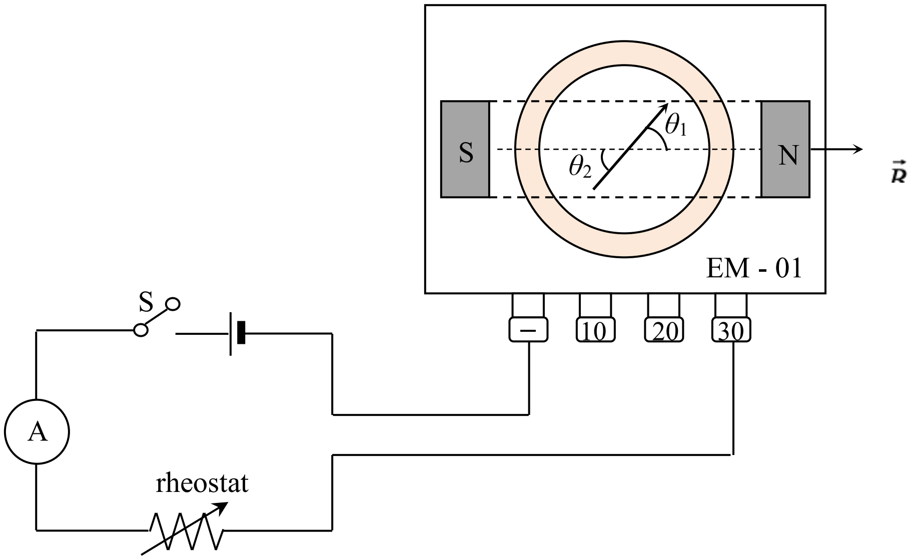


Figure 4.3

2. Align the compass needle of EM-01 until it is pointed North as in **Figure 4.4**.

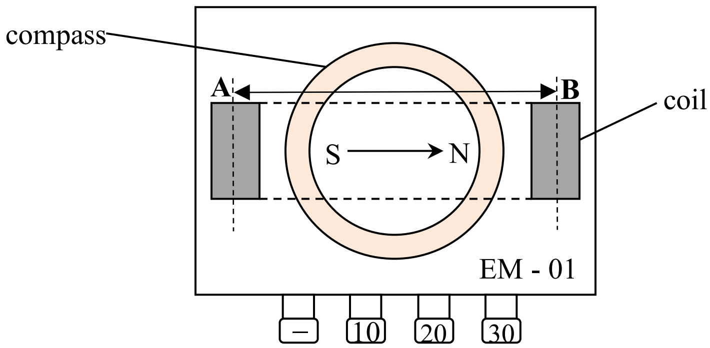


Figure 4.4

3. Set the rheostat to its maximum value and switch on the circuit. Reduce the resistance of rheostat to increase the current I and hence the corresponding value of θ_1 for at least six sets of readings. Record the readings of the ammeter I and the angle of deflection θ_1 in **Table 4.1.**

   Note: *The deflection angle should not be more than 80°.*

4. Repeat step (3) by changing the polarity of the power supply. Record the angle θ_2, pointed by the compass needle in **Table 4.1**.

5. Measure the diameter of the coil.

   Note: *Make sure the diameter is measured from A to B as in* **Figure 4.4**

   Diameter of coil D = (……… ± ………) cm
   Number of turns $N = 30$

No.	current, I (±……..)	θ_1 (±……..)	θ_2 (±……..)	average θ_A (…...)	tan θ_A
1					
2					
3					
4					
5					
6					
7					
8					
9					

Table 4.1

6. Plot a graph of tan θ against I.

7. Determine B_E from the gradient of the graph.

8. Compare the result with the standard value given by the lecturer. Write a comment.

EXPERIMENT 4(b): MAGNETIC FIELD

Objective:

To determine the value of the horizontal component of the earth magnetic field, $\vec{B}_{\mathrm{E}}$.

Student Learning Time (SLT):

Face-to-face	Non face-to-face
2 hours	0

Theory:

The magnetic field strength $\vec{B}$ is a vector quantity so the addition of two magnetic fields obeys the parallelogram law. For example, if $\vec{B}_{\mathrm{E}}$ is the horizontal component of earth magnetic field and $\vec{B}_{\mathrm{s}}$ is the magnetic field of a solenoid which is perpendicular to $\vec{B}_{\mathrm{E}}$ then the resultant of the two fields $\vec{B}$ is as shown in **Figure 4.1**. A compass needle is situated at the place where the two fields meet will be aligned to the direction of the resultant field $\vec{B}$.

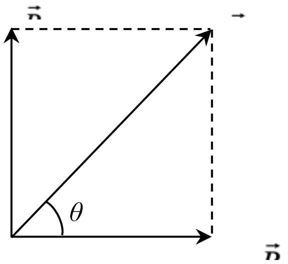


Figure 4.1

The magnetic field strength at the end of an *N*-turn solenoid of length *l* and carries current *I* as shown in **Figure 4.2** is given by

$$B_{\mathrm{S}} = \frac{1}{2}\left(\frac{\mu_{\mathrm{o}} NI}{l}\right) \qquad 4.1$$

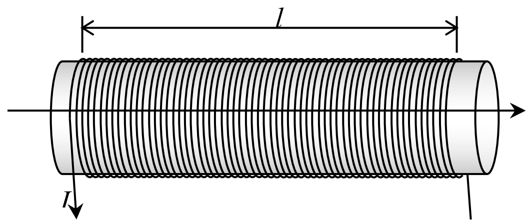


Figure 4.2

From **Figure 4.1**,

$$\tan\theta = \frac{B_s}{B_E}$$

$$\tan\theta = \frac{\frac{1}{2}\mu_o\left(\frac{N}{l}\right)I}{B_E} \qquad 4.2$$

The gradient of graph $\tan\theta$ against I is

$$m = \frac{\frac{1}{2}\mu_o\left(\frac{N}{l}\right)}{B_E} \qquad 4.3$$

Therefore,

$$B_E = \frac{\frac{1}{2}\mu_o\left(\frac{N}{l}\right)}{lm} \qquad 4.4$$

where $\mu_o = 4\pi\times10^{-7}$ H m^{-1} (permeability of free space).

Apparatus:

A 50 turns or 100 turns solenoid
A DC power supply (2 – 4 V)
A DC ammeter (0 – 1 A)
A switch
Connecting wires of about 50 cm long with crocodile clips
A rheostat
A compass

Procedure:

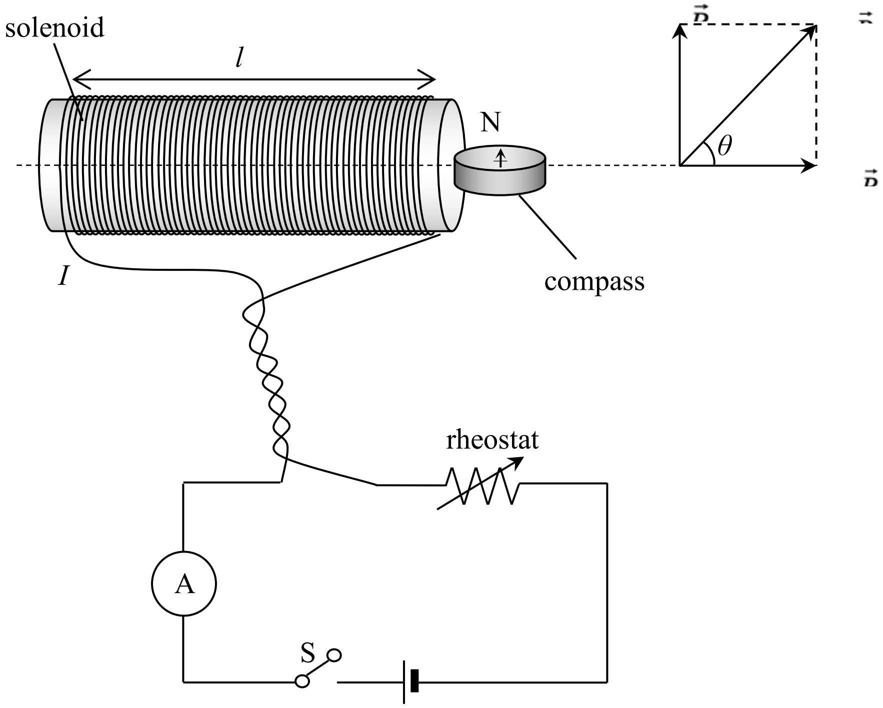


Figure 4.3

1. Place a compass at one end of the solenoid. Let the compass stay still in N−S direction where the magnet pointer is perpendicular to the axis of the solenoid. The north direction of the compass must be pointed to the north.

   Note: *Choose a position to place your compass away from any iron structure to avoid any influence on the alignment of the compass needle.*

2. Connect the solenoid in series with the rheostat, the ammeter, the power supply and the switch. The ammeter must be at least 50 cm away from the compass. A complete set up is as in **Figure 4.3**.

3. Set the rheostat to its maximum value and switch on the circuit. Reduce the resistance of rheostat to increase the current I and hence the corresponding value of θ_1 for at least six sets of readings. Record the readings of the ammeter I and the angle of deflection θ_1 in **Table 4.1.**

   Note: *The deflection angle should not be more than 80°.*

4. Repeat step (3) by changing the polarity of the power supply. Record the angle θ_2, pointed by the compass needle in **Table 4.1**.

Table 4.1

No.	Current, I (±……..)	θ_1 (±……..)	θ_2 (±……...)	Average θ_A (…..)	tan θ_A
1					
2					
3					
4					
5					
6					
7					
8					
9					

5. Remove the solenoid from the clamp and measure the length, l of the solenoid.

6. Plot a graph of tan θ against I.

7. Determine B_E from the gradient of the graph.

8. Compare the result with the standard value given by the lecturer. Write a comment.

EXPERIMENT 5: GEOMETRICAL OPTICS

Objective:

To determine the focal length, f of a convex lens.

Theory:

From the thin lens equation,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \qquad 5.1$$

where f is the focal length
u is the object distance
v is the image distance

Multiply equation 5.1 with v,

$$\frac{v}{f}=\frac{v}{u}+1$$

$$M=-\frac{v}{f}+1 \qquad 5.2$$

Where $M=\frac{\text{height of image}}{\text{height of object}}=\frac{h_i}{h_o}=-\frac{v}{u}$ is the linear magnification.

For this experiment the image formed is always real, then negative sign for magnification indicates that the image is inverted.

Hence the graph M against v is a straight-line graph.

The equation also shows that M is proportional to v.

When $v=2f, \quad M=-1$.

Apparatus:

A convex lens

A piece of card with narrow triangle shaped slit or any suitable objects
A screen
A light source
A metre rule
A lens holder
Plasticine

Procedure:

1. Use the convex lens to focus a distant object such as a tree outside the laboratory on a screen. The distance between the screen and the lens is the estimated focal length, f_o of the lens. Record the estimated focal length.

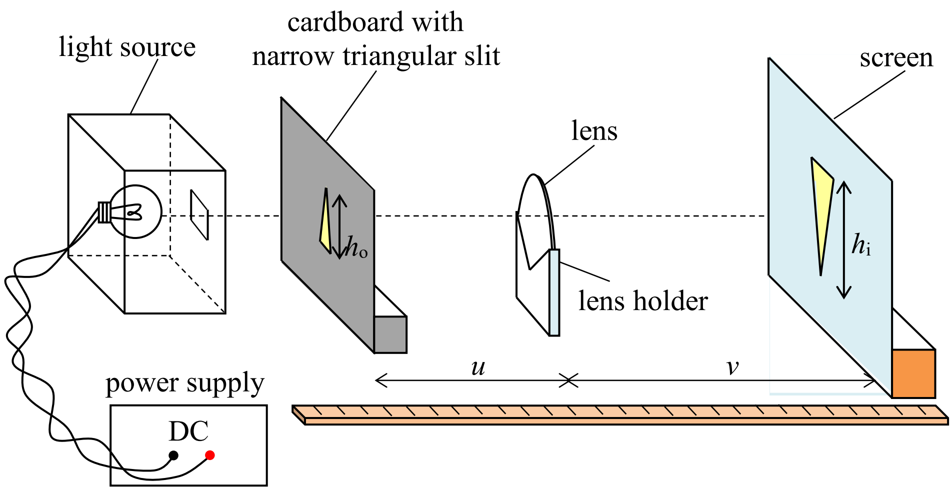


Figure 5.1

2. Set up the apparatus as in **Figure 5.1**.

3. Place the object in front of the lens at a suitable distance ($f_o < u < 2f_o$) and adjust the position of the screen so that a sharp real, inverted image is projected on the screen.

4. Record the measurement for the object distance u and the image distance v.

5. Calculate the magnification of the image, $M = -\frac{v}{u}$.

6. Change the location of the object. Repeat steps (4) and (5) until six sets of u, v and M are obtained.

7. Tabulate the data.

8. Plot a graph of M against v.

9. Determine the focal length of the lens, f_1 from the gradient of the graph.

10. Determine the image distance v from the graph by using extrapolation when $M = -1$ and calculate the focal length, f_2 by using equation 5.2.

11. Compare the results f_1 and f_2 with f_0. Write a comment.

EXPERIMENT 6: DIFFRACTION GRATING

Objectives:

(i) To determine the wavelength, **λ** of laser beam using a diffraction grating.
(ii) To the number of diffraction grating lines per unit length, N.

Student Learning Time (SLT):

Face-to-face	Non face-to-face
2 hour	0

Theory:

When a laser beam is incident on a diffraction grating, a diffraction pattern in the form of a series of bright dots can be seen on the screen as shown in **Figure 6.1**

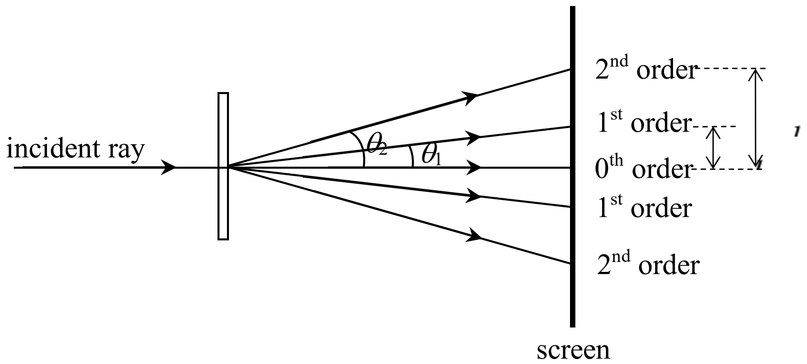


Figure 6.1

The relationship between the angle θ_n of the nth order and the wavelength of laser λ is

$$\sin\ \theta_n = \frac{n\lambda}{d} \qquad 6.1$$

where d is the distance between two consecutive lines of the diffraction grating, known as grating spacing.

Usually, the grating spacing is specified in number of lines per meter, such as N lines per meter. Hence,

$$N = \frac{1}{d} \qquad 6.2$$

Then

$$\sin\theta_n = Nn\lambda \qquad 6.3$$

By measuring the angle θ_n for each order of diffraction n, λ can be determined.

Apparatus:

A laser pen
Two retort stands with clamps
A metre rule
A screen
Two diffraction gratings (A and B)

Note: *Suggestion A is 100 lines/mm and B is 300 lines/mm.*

Procedure:

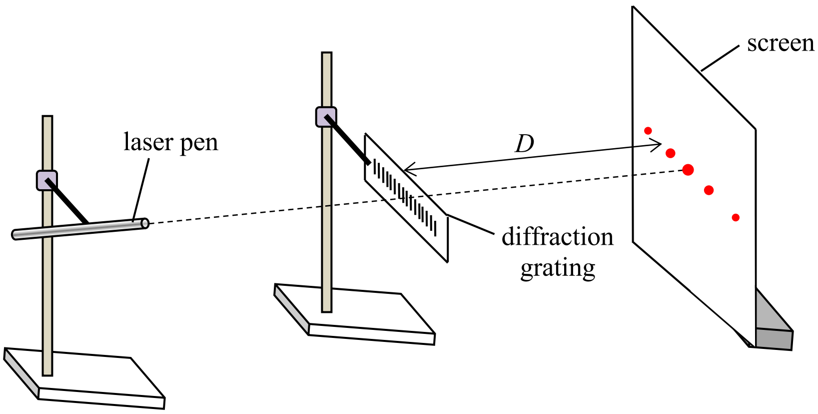


Figure 6.2

1. Set up the apparatus as shown in **Figure 6.2**. Ensure that the laser ray is pointed perpendicularly to the diffraction grating A.

   Note: *Make sure that*
   i) the incident ray is normal to the diffraction grating.
   ii) the screen is parallel to the diffraction grating.

2. The distance D from the diffraction grating to screen must be adjusted so that the spacing between the spots on the screen is as far as possible from one another. Measure and record the value of D.

   Caution: *A laser pen is NOT a toy. It is dangerous to look directly at the laser beam because it may cause permanent damage to your eyesight.*

3. Measure the distance l_1, l_2, l_3, …, l_n that correspond to the diffraction order of n = 1, 2, 3, … where l_n is the distance between spots of order, n to the centre spot.

4. Determine values of sin θ_n for order of n = 1, 2, 3, ... using equation

$$\sin\theta_n = \frac{l_n}{\sqrt{(l_n)^2 + D^2}}$$

5. Tabulate the data.

6. Plot a graph of sin θ_n against n.

7. Determine the wavelength, λ of the laser beam from the gradient of the graph.

   Note: *Take the value of N printed on the grating A.*

8. Repeat steps (1) to (6) using grating B.

9. Determine the number of lines per mm, N of grating B from the gradient of the graph by using the value of λ in step (7).

10. Compare the value of λ and N for grating B with their standard values. Write a comment.

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ACKNOWLEDGEMENTS

The Matriculation Division, Ministry of Education Malaysia wish to thank everyone who has contributed in shaping and writing this PHYSICS LABORATORY MANUAL (13^{th} Edition) for the Two Semesters Matriculation Programme. Special thanks go to those for their many valuable suggestions and conscientiousness in completing this manual.

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SP015 & SP025

e ISBN 978-983-2604-71-6

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Name : ………………………………………………………..   Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 1: MEASUREMENT AND UNCERTAINTY

Course Learning Outcome:
Solve problems related to Physics of motion, force and energy, waves, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to describe technique of measurement and determine uncertainty of length of various objects.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. Complete **Table 1**

Basic Quantity	Symbol	SI Unit (with symbol)	Measuring Instrument
Length	l		
Mass	m		
Time	t		
Electric Current	I		
Temperature	T		

Table 1

2. …………………………….. is used to measure the diameter of a coin.

3. Micrometer screw gauge is usually used to measure the …………………… of a thin wire or the ………………………………. of paper.

4. Complete **Table 2**

Measuring Apparatus	Sensitivity	Uncertainty
Meter rule	0.1 cm	±0.1cm
Vernier calipers	0.01 cm	
Micrometer screw gauge		±0.01mm
Travelling microscope		±0.01mm
Thermometer	0.1°C	
Voltmeter	0.1 V	
Ammeter		±0.1A

Measuring Apparatus	Sensitivity	Uncertainty
Electronic Balance	0.01 g	

Table 2

5. State **TWO** types of reading;

   i. ....

   ii. ....

6. The repeated reading for a measurement is given as *a, b, c, d, e,* and *f*. Write the equation of Average Value and Uncertainty.

	EQUATION
Average Value, $\bar{x}$	
Uncertainty, $\Delta\bar{x}$	

<u>**Experiment**</u>

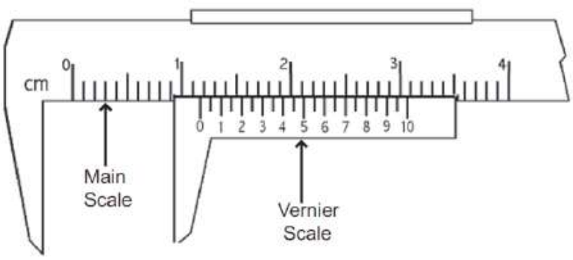
7. Complete **Table 3**

Measurement	Measuring Instrument	Uncertainty/ Smallest scale	Type of reading (single point/two point/Vernier scale)
Length of a metal rod			Two points
Length and width of a laboratory book			Two points
Mass of a ball bearing			Single Point
Diameter of a ball bearing			Vernier scale
Diameter of a coin			Vernier scale
External diameter of a glass rod			Vernier scale

Table 3

8. Determine the reading for the following measurements:

   i.

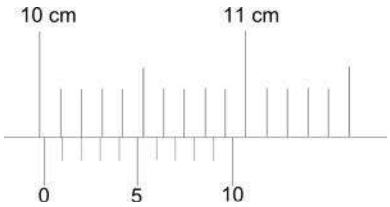


Main scale :…………………..

Vernier scale :…………………..

Actual reading :…………………

   ii.

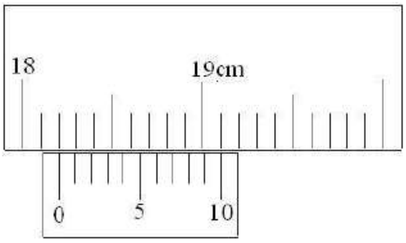


Main scale :…………………

Vernier scale :…………………

Actual reading :………………...

iii.

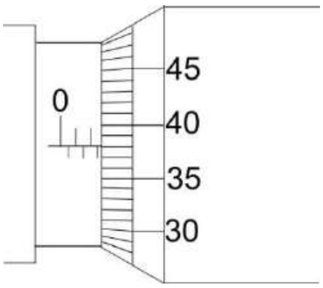


Main scale :………………….

Vernier scale :…………………..

Actual reading:………………….

iv.

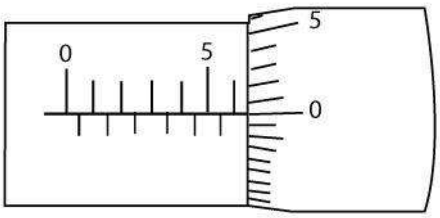


Main scale :………………

Vernier scale :………………

Actual reading :………………

v.



Main scale :…………………..

Vernier scale :…………………..

Actual reading:………………….

9. The repeated readings of the diameter, d of a ball bearing are 2.50 mm, 2.52 mm, 2.51 mm and 2.50 mm.

   i. Calculate the Average Value and Uncertainty. Write the result as $(\bar{d} \pm \Delta\bar{d})$

ii. What instrument/apparatus is used for this measurement?

……………………………………………………………………………………………….

iii. From 10.1, calculate the volume, V of the ball bearing.

iv. Write the result as $(\overline{V} \pm \Delta\overline{V})$

…………………………………………………………

Data Analysis

10. Complete **Table 4.**

No	Length of Scientific Calculator (Model Casio fx-570ES PLUS), L (cm)	$\lvert \overline{L} - L_i \rvert$ (cm)
1	15.42	
2	15.55	
3	15.30	
4	15.48	
5	15.49	
6	15.45	
7	15.55	
	Average, $\overline{L}$=	$\Delta\overline{L}$=

Table 4

11. Express your answer as $(\overline{L} \pm \Delta\overline{L})$

12. Calculate the percentage of uncertainty,

13. State THREE precautions of this experiment:

i. ………………………………………………………………………………………………...

ii. ………………………………………………………………………………………………..

iii. ……………………………………………………………………………………………….

Name : ………………………………………………………..   Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 2 : FREE FALL AND PROJECTILE MOTION

Course Learning Outcome:
Solve problems related to **Physics of motion**, force and energy, waves, matter and thermodynamics **(C4, CLO 2, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to describe experiment to determine acceleration due to gravity using free fall and projectile motion.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is meant by free fall motion?

   ……………………………………………………………………………………………….

   ……………………………………………………………………………………………….

2. Under free fall motion the acceleration of an object is also known as gravitational acceleration or acceleration due to gravity. What is the symbol and SI unit of this type of acceleration?

   ……………………………………………………………………………………………….

3. What is the value of acceleration due to gravity at the surface of Earth?

   ……………………………………………………………………………………………….

4. Projectile motion of an object is the motion of an object which is projected or thrown. Under a gravitational field **when the air resistance is not present**, projectile motion can be considered as a free fall motion. State **TWO** differences between free fall motion and projectile motion?

   ……………………………………………………………………………………………….

   ……………………………………………………………………………………………….

   ……………………………………………………………………………………………….

   ……………………………………………………………………………………………….

   ……………………………………………………………………………………………….

5. State the law applied in these experiment

   ……………………………………………………………………………………………….

<u>Experiment</u>

6. How do we release the steel ball to form

   (a) free fall motion

   …………………………………………………………………………………………
   …………………………………………………………………………………………

   (b) Projectile motion

   …………………………………………………………………………………………
   …………………………………………………………………………………………
   …………………………………………………………………………………………

7. State the measurement *apparatus* involved. (e.g. type / name of equipment) for both experiment.

   …………………………………………………………………………………………
   …………………………………………………………………………………………
   …………………………………………………………………………………………

8. State the related variables that need to be recorded in this experiment?

	Free fall motion	Projectile motion
Manipulated variable (change on purpose)		
Responding variable (what is measured)		

9. Construct the table to record the related values for free fall and projectile motion experiment.

   (a) Free Fall Motion

   (b) Projectile Motion

10. How do you obtained the value of *t* for projectile motion from the graph of free fall motion experiment?

    ……
    ……

Data Analysis

11. a) Write the equations related to both experiments in order to determine the acceleration due to gravity, g.

    b) Sketch a suitable graph for
       i) Free fall motion
       ii) Projectile motion

    c) How the acceleration due to gravity, g can be determine from the graphs.

12. List down the precautions of the experiments.
    a) ……………………………………………………………………………………………...
    b) ……………………………………………………………………………………………...
    c) ……………………………………………………………………………………………...

Name : …………………………………………………………..   Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 3: ENERGY

Course Learning Outcome:
Solve problems related to Physics of motion, **force and energy**, waves, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to explain the experiment to determine the acceleration due to gravity, g from the experiment.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. State the law of conservation of energy.

   ………………………………………………………………………………………………

2. State the gravitational potential energy and kinetic energy.

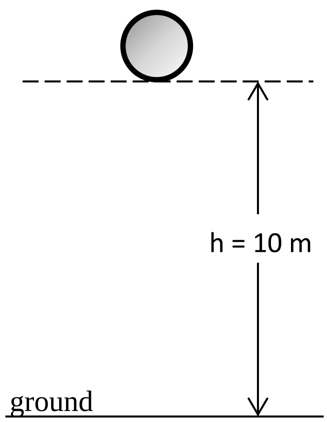
   ………………………………………………………………………………………………

   ………………………………………………………………………………………………

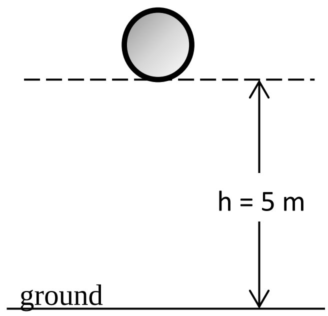
3. What is the symbol and SI unit of gravitational potential energy and kinetic energy?

Energy	gravitational potential energy	kinetic energy
Symbol		
Unit		

4. Based on the situations below, answer the questions:



SITUATION A



SITUATION B

a) Using the conservation of energy, determine the velocity of the ball just before it reaches the ground.

b) From the answers calculated in question (a), what can we deduce about the relation between the released height and the velocity of the ball before hitting the ground?

…………………………………………………………………………………………………

Experiment

5. What is the energy owned by the ball bearing when it is attached to the free fall adapter?

   ……………………………………………………………………………………………………

6. What is the usage of the photo gate?

   ……………………………………………………………………………………………………

7. State the change in mechanical energy in this experiment.

   ……………………………………………………………………………………………………

8. State the related variables that need to be recorded in this experiment?

   a) Manipulated variable

      …………………………………………………………………………………………………

   b) Responding variable

      …………………………………………………………………………………………………

9. How the final velocity of ball bearing is determined?

Data Analysis

10. An equation for a straight line graph is y = mx + c, where y is the quantity on the vertical axis and x is the quantity on the horizontal axis as shown in **FIGURE 1**.

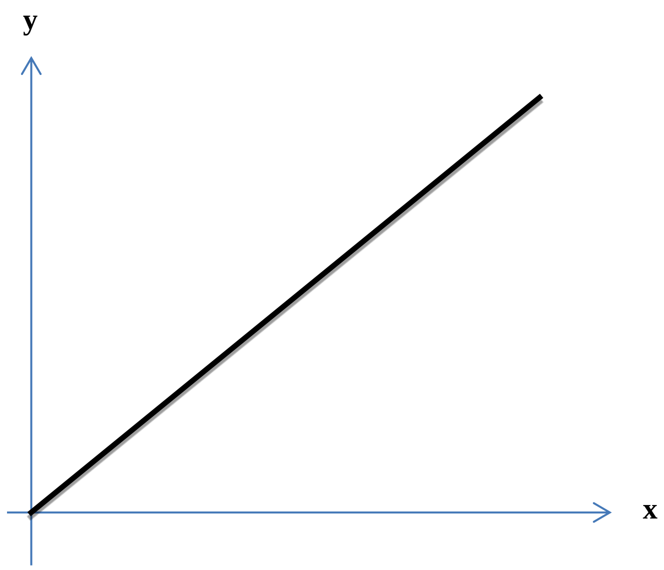


FIGURE 1

The velocity of ball bearing, v is related to the height of released (h) by the following equation:

$$v^2 = 2gh \_\_\_\_\_\_\_\_\_\_ (1)$$

where g is the acceleration due to the gravity.

a) Based on the equation (1) and the graph, determine the variables for x axis and y axis

b) From the graph what does the gradient, m represents?

…………………………………………………………………………………………………

c) From the gradient of the graph, how can we determine the value of g.

11. List **THREE** precautions of the experiment:

i. ………………………………………………………………………………………………

ii. ………………………………………………………………………………………………

iii. ………………………………………………………………………………………………

12. State two types of errors during experiment and give an example for each error.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

13. Based on the situation below identify either random or systematic error.

Situation	Random Error/Systematic Error
Wind keeps blowing in the surrounding using the experiment. This shall affect the velocity measured in this experiment. The best way to solve this is by conducting this experiment in the closed area or vacuum space.	
Some of the numbers on the timer's display was broken and missing. Thus the reading can be taken only to the nearest decimal point.	
Instead of using the hand to release the ball bearing, it is suggested that the ball can be released using the automatic control or trigger.	
Sometimes the time measured is hardly detected by the photo gates. This is due to the position of the gates where the ball bearing failed to hit the motion sensor. Therefore, the free fall adapter and photo gates must be realigned properly.	

Name : ……………………………………………………….. Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 4: ROTATIONAL MOTION OF A RIGID BODY

Course Learning Outcome:
Solve problems related to Physics of motion, **force and energy**, waves, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to explain the experiment to determine the moment of inertia of a fly-wheel from experiment.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is a rigid body?

   ……………………………………………………………………………………………………

2. What is meant by moment of inertia?

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

3. What is the symbol and SI unit for moment of inertia?

   ……………………………………………………………………………………………………

4. Moment of inertia depends on ………………..…. and …………………………….

5. Complete **TABLE 4** with correct analogues between linear motion and rotational motion.

Linear Motion	Rotational Motion
Mass, m	
Acceleration, a	
Net force, F	

6. A motor capable of producing a constant torque of 100 Nm is connected to a flywheel which rotates with an angular acceleration of 1000 rad s^{-2}. Calculate moment of inertia of the flywheel.

Experiment

7. Sketch a free body diagram for fly-wheel and falling slotted mass.
   a) Free body diagram of fly-wheel   b) Free body diagram of falling slotted mass

8. By referring to the free body diagram in 7(a) and 7(b), deduce equation by using Newton's 2^{nd} Law of motion.

9. For this experiment, identify

   a) the manipulated variable

   ……………………………………….

   b) the responding variable

   ……………………………………….

10. Complete the observation table with the suitable equation.

Acceleration	Angular acceleration	Tension in the string

Data Analysis

11. Write the equation of the graph of α against T

12. Base on the linear graph equation $y = mx + c$, fill in the suitable quantity by referring the equation in question 11 :

   a) *y-axis* :………………………………..
   b) *x-axis* : ………………………………..
   c) gradient, *m* : ………………………………..
   d) *y*–interception : ………………………………..

13. How do we determine the value of inertia of a fly-wheel from this graph?

14. List **THREE** precautions of this experiment
    i. …………………………………………………………………………………………………
    ii. …………………………………………………………………………………………………
    iii. …………………………………………………………………………………………………

Name : ………………………………………………………..   Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 5: SIMPLE HARMONIC MOTION (SHM)

Course Learning Outcome:
Solve problems related to Physics of motion, force and energy, **waves**, matter and thermodynamics
(C4, PLO 4, CTPS 3, MQF LOD 6)

Learning Outcomes:
At the end of this lesson, students will able to:
1. explain the experiment to determine the acceleration due to gravity, g using a simple pendulum.
2. describe the effect of large amplitude oscillation to the accuracy of g obtained from the experiment

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is a simple pendulum?

   ………………………………………………………………………………………………

2. Motion of an object that returns to its initial position after a fixed time interval is called

   ……………………………

3. In SHM, state two quantities that proportional to the object's displacement
   i. ……………………………..
   ii. ……………………………..

4. The condition for the simple pendulum to perform SHM are
   a) The mass of the spherical bob is ……………………..
   b) The ……………………… of the string is negligible
   c) Amplitude of oscillation is ……………………………

5. Does the period of oscillation of simple pendulum depend on mass?

   (Yes / No)

Experiment

6. How to determine the period of a simple pendulum for a given number, n of oscillation?

   ……………………………………………………………………………………………………

7. If we vary the length of a pendulum, the period will change. Construct an appropriate table to record the data of length, l, time taken, t and corresponding T and T^2.

8. What is the title of the graph that needs to be plotted in this experiment?

   ……………………………………………………………………………………………………

9. Which procedure that investigates the effect of large amplitude of oscillation and state the related angle used.

   ……………………………………………………………………………………………………

Data Analysis

10. How to determine the value of g from the gradient of the graph.

11. How to calculate the percentage of error between the value $g_{experiment}$ and $g_{standard}$? Take $g_{standard}$ = 9.81 m s^{-2}.

12. Predict what would happen to the value of g if **large amplitude** is used.

    ……………………………………………………………………………………………………

13. List **THREE** precautions of this experiment
    i. ……………………………………………………………………………………………
    ii. ……………………………………………………………………………………………
    iii. ……………………………………………………………………………………………

Name : ……………………………………………………….. Practicum: ……………

Matric Number: ………………………..

EXPERIMENT 6: STANDING WAVES

Course Learning Outcome:
Solve problems related to Physics of motion, force and energy, **waves**, matter and thermodynamics **(C4, PLO 4, CTPS 3, MQF LOD 6)**

Learning Outcomes:
At the end of this lesson, students will able to explain the experiment to investigate standing waves formed in stretched string.

Student Learning Time:

Face-to-face	Non face-to-face
1 hour	1 hour

Direction: Read over the lab manual and then answer the following question.

Introduction

1. What is the meaning of standing waves?

   ……………………………………………………………………………………………

2. Sketch standing wave formed in a stretch string and label the node (N) and antinode (A).

3. How standing wave is formed?

   ……………………………………………………………………………………………

   ……………………………………………………………………………………………

   ……………………………………………………………………………………………

4. What is the symbol and SI unit for mass per unit length?

   ……………………………………………………………………………………………

Experiment

5. State the manipulative and responding variables in this experiment.

   ……………………………………………………………………………………………

6. Construct the table for the value of m and *l*.

7. Sketch free body diagram to show that T = W.

8. Suggest a way to determine the actual value for mass per unit length of the string/wire used in this experiment.
……
……

9. Suggest how to identify the position of two consecutive nodes formed in the string / wire.
……

<u>Data analysis</u>

10. Write the equation that relates period, T and frequency, f.

11. Sketch the graph to show the relationship between T and ℓ^2.

12. Construct the observation table.

13. How do you determine the mass per unit length from this graph?
……

14. Throughout the experiment the terminals are connected to AC power supply. In your opinion why does this essential?

Matriculation Physics (SP025)

Short notes

Shafiq R

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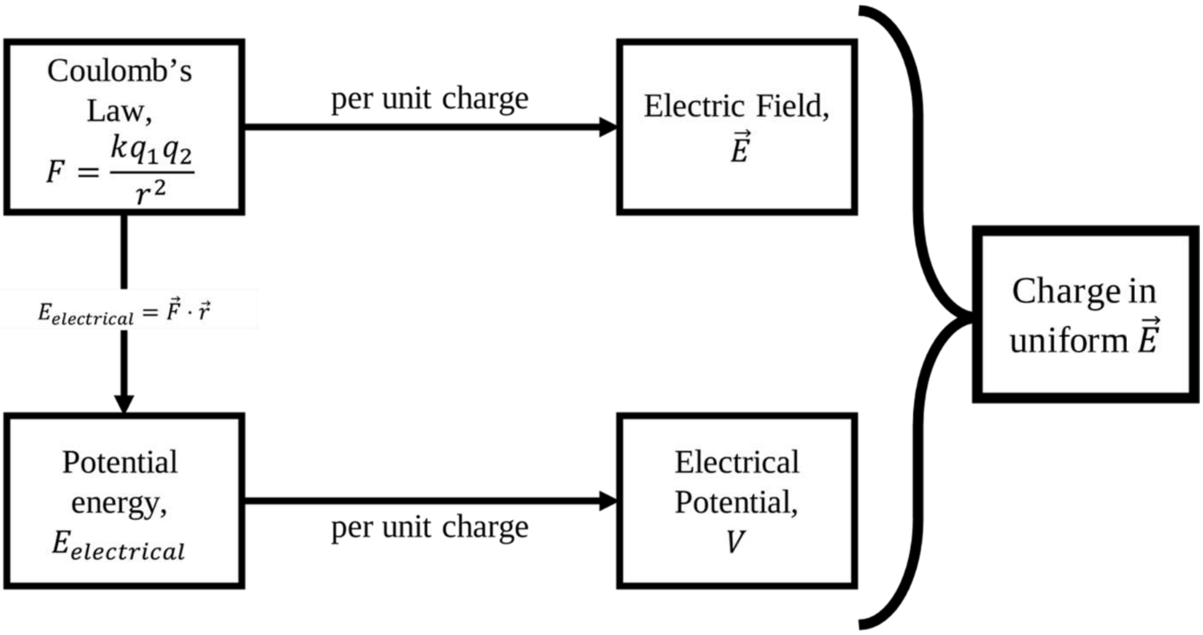
Chapter 1: Electrostatics



Coulomb's Law

Let it be known that Coulomb's Law allows us to measure forces between charged particles, this force is known as **Coulomb Force**. Mathematically, Coulomb's Law is

$$F_{Coulomb} = \frac{kq_1q_2}{r_{12}^2}$$

where q_i are the charges of interacting particles, r_{12} is the distance between the particles and k is the electrostatic constant. The electrostatic constant is

$$k = \frac{1}{4\pi\epsilon} = 8.98 \times 10^9 \, kg \, m^3 \, s^{-4} \, A^{-2}$$

On the note of direction of the Coulomb force,

Condition	Direction
$F_{Coulomb} < 0$	Towards each other
$F_{Coulomb} > 0$	Away from each other

For more than 2 particles, the Coulomb Force on particle j becomes

$$F_{Coulomb} = kq_j \sum_i \frac{q_i}{r_{ij}^2}$$

Electric Field

The electric field at a point in space $E(r)$, is defined as the electric force acting on a positive test charge placed at that point $F(r)$, divided by the test charge, q_{test}.

$$E(r) = \frac{F(r)}{q_{test}}$$

Rearranging this equation yields,

$$F(r) = q_{test}E(r)$$

which tells us that particle of charge q_{test} placed in a region of electric field $E(r)$ will experience a force of $F(r)$.
If the source of electric field has a charge of q_{source}, then the electric field at point r, $\mathrm{E}(r)$ is

$$E(r) = \frac{kq_{source}}{r^2}$$

As in the case for Coulomb Force, for multiple, the electric field from multiple sources is simply additive,

$$E(r) = k\sum_i \frac{q_i}{r_i^2}$$

Electric Potential

Electric potential is the amount of word done to bring a test charge q_{test} from an infinite distance to a point at distance r from the source charged particle of charge q_{source}. This is found to be

$$V = \frac{W_{\infty\to r}}{q_{test}} = \frac{kq_{source}}{r}$$

Potential difference between positions $x = A$ and $x = B$ is then

$$V_{AB} = V_A - V_B = \frac{W_{\infty\to A}}{q_{test}} - \frac{W_{\infty\to B}}{q_{test}} = \frac{W_{\mathrm{A}\to B}}{q_{test}}$$

Electric potential energy is the energy a test charge would have positioned r distance away from a source,

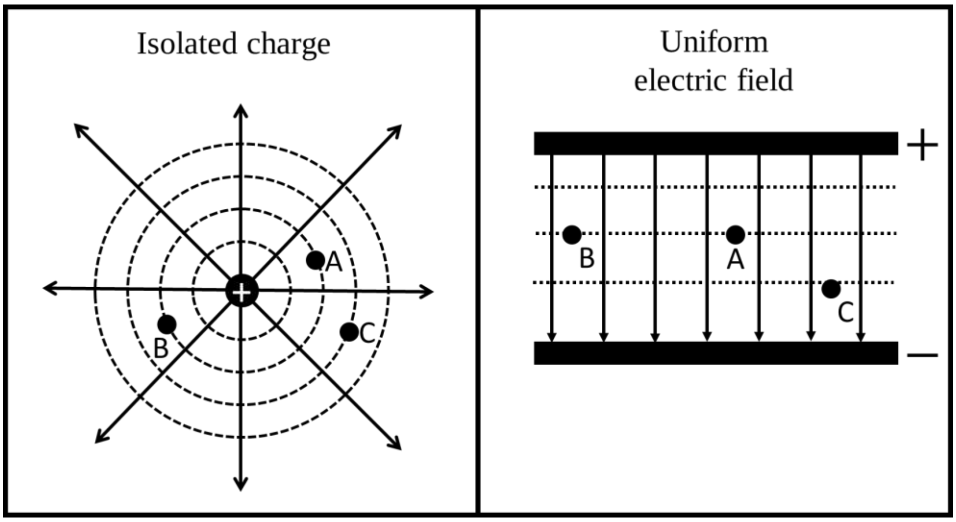
$$U = q_{test}V$$

For multiple sources, the potential and electrical potential energy at point *r* is simply,

$$V_{total} = k\sum_i \frac{q_i}{r_i};\ U_{total} = kq_j\sum_i \frac{q_i}{r_{ij}}$$

Equipotential lines and surfaces are graphical representation on which a particle on the line or surfaces is at the same potential.

- This means no work is done by the electric field when a charged particles are moved from on point of the equipotential line (or surface) to another point on the same line (or surface).
- Equipotential lines are always perpendicular to the electric field at all points.
- Examples:



In both examples,

$$V_A = V_B \neq V_C$$

Charge in Uniform Electric Field

For a uniform electric field produced by parallel plates of potential difference V, electric field strength is simply

$$E = \frac{V}{d}$$

where d is the distance between the parallel plates.

The following case studies involves a charged particle in a uniform electric field:

Case 1: Stationary charge

A stationary charged particle of charge q and mass m, placed in a uniform electric field E will experience force only from the electric field and therefore will move towards plate of its opposite charge (i.e. positive charged particle will move towards the negatively charged plate and vice versa).

Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

Case 2: Charge moving parallel to the field

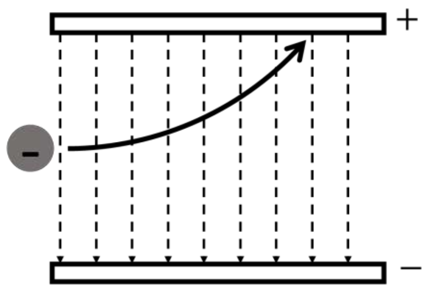
A charged particle of charge q and mass m, entering a uniform electric field E in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. It will either decelerate (if its velocity is in the opposite direction of its acceleration) or accelerate.

Its motion will have the acceleration equivalent to

$$a = \frac{qE}{m}$$

Case 3: Charge moving perpendicularly to the field

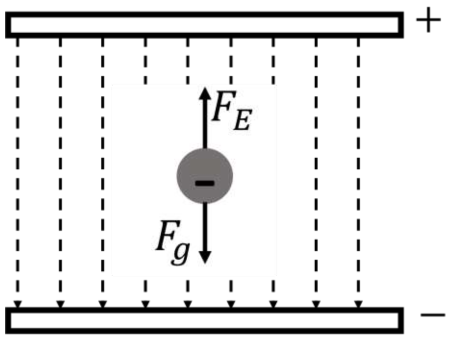
A charged particle of charge q and mass m, entering a uniform electric field E in a direction parallel to the field line, will experience force from the electric field in the direction of its opposite charge. Because of its initial velocity direction, it will follow a parabolic path, moving towards the plate of its opposite charge.



Case 4: Charge in dynamic equilibrium

In the case of dynamic equilibrium, the attractive Coulomb Force between the charged particle and the plate of opposite charge cancels out the weight of the charged particle,

$$F_{Coulomb} = W_{particle} \Rightarrow qE = mg$$



Chapter 2: Capacitors and Dielectrics

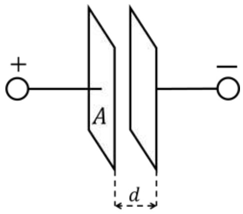
Parallel & Series

Capacitors are essentially batteries. Their ability to store charge is quantified by ***capacitance.*** Capacitance *C,* is the amount of charge *q* stored in one plate of a capacitor per unit potential difference between the plates, *V*,

$$C = \frac{Q_{single\ plate}}{V}$$

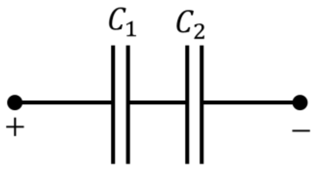
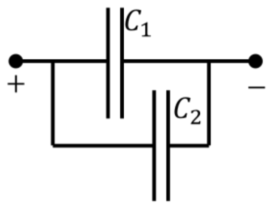
As a function of its geometry, capacitance of a parallel plate capacitor is

$$C = \frac{\varepsilon A}{d}$$



where ε is the permittivity of the space between the plates, A is the area of each plate and d is the distance between the parallel plates.

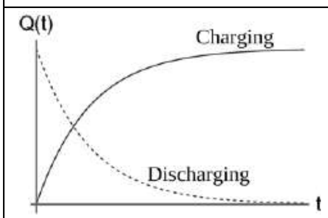
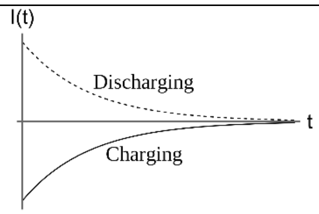
Multiple capacitors can be arranged either in parallel or series or combinations of them, and their effective capacitance can be calculated depending on their arrangement:

Arrangement	Effective Capacitance
Series 	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$ $C_{eff} = \left(\sum_{i=1}^{n} \frac{1}{C_i}\right)^{-1}$
Parallel 	$C_{eff} = C_1 + C_2 + \cdots + C_n$ $C_{eff} = \sum_{i=1}^{n} C_i$

The energy stored in a capacitor is then $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$.

Charging & Discharging Capacitors

Capacitors stores charges, now the question is how fast to charge it? Consider a simple circuit consisting of a power supply, a resistor of resistance *R* and a capacitor of capacitance *C*. Accumulation of charge with time for charging and discharging are as follows:

$Q - t$ graph	$I - t$ graph
 Discharging: $Q(t) = Q_o e^{-\frac{t}{RC}}$ Charging: $Q(t) = Q_o\left(1 - e^{-\frac{t}{RC}}\right)$	 Discharging: $I(t) = -\frac{dQ}{dt} = -\frac{Q_o}{RC}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$ Charging: $I(t) = \frac{Q_o}{RC}e^{-\frac{t}{RC}} = -I_o e^{-\frac{t}{RC}}$

Time constant, τ is defined as the time for the exponential term to drop to e^{-1} for discharging, or, for the charge to increase to $1 - e^{-1}$ for charging process, and is calculate by multiplying the R and C,

$$\tau = RC \text{ [seconds]}$$

Dielectrics

Dielectrics are electrically non-conductive materials placed in between the plates of capacitors to increase the capacitance of the capacitor.

We quantify the increase in capacitance as the **dielectric constant $\boldsymbol{\varepsilon_r}$**, define as the ratio of capacitance of capacitor with dielectric C, to the capacitance of capacitor with no dielectric (vacuum) C_o,

$$\varepsilon_r = \frac{C}{C_o} = \frac{\left(\frac{\varepsilon A}{d}\right)}{\left(\frac{\varepsilon_o A}{d}\right)} = \frac{\varepsilon}{\varepsilon_o}$$

Chapter 3: Current and DC Circuits

Electric Current

Current is the amount of charge ΔQ that passes through a surface area in time Δt,

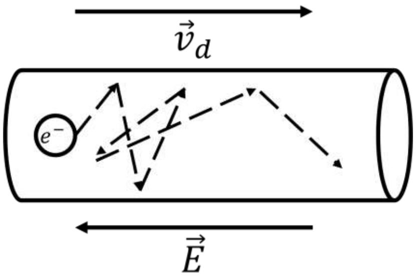
$$I = \frac{dQ}{dt}$$

Total charge Q is simply n multiples of electron charge e,

$$Q = ne$$

Without external electric field, the electron will drift through a conductor with kinetic energy equivalent to Fermi energy, which results in a net velocity of zero. With external electric field, the electron as a whole now gains a net velocity along the electric field. This 'net velocity' is what is known as 'drift velocity'.

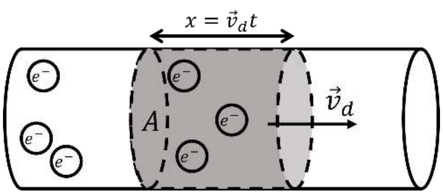
Consider an electron travelling through a conductor, on which an electric field of $\vec{E}$ is applied. The force on the electron is then $F = -qE \Rightarrow a = -\frac{eE}{m}$. Assuming the average time between collision is τ, we can show that



$$v_d = a\tau = \left(-\frac{eE}{m}\right)\tau$$

This means that applying a larger electric field, the larger the kinetic energy obtained by the electron due to a larger drift velocity. This also means that an increase in temperature, increases the collision frequency, decreases collision time and decreases drift velocity of the electrons.

Relating the idea of drift velocity to current can be done by considering a volume section of the conductor V, and the number of charges that flows through that section, n. We can work out that the amount of charge going through V is simply



$$\Delta Q = (ne)A\Delta x$$

where $\Delta x = v_d \Delta t$.

This means that current is

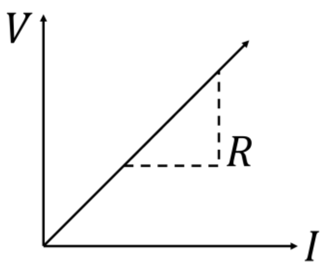
$$I = \frac{\Delta Q}{\Delta t} = neAv_d$$

Ohm's Law

Ohm's Law states that current I, is directly proportional to the potential difference V, is all conditions are constant.

$$V \propto I \;\Rightarrow V = IR$$

R, which is the proportionality constant to Ohm's Law, represents resistance which opposes current flow in a circuit.



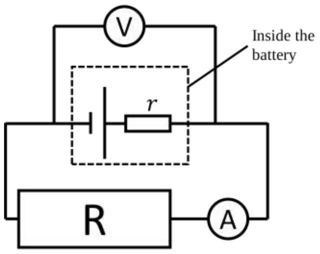
Resistance (Geometry & Temperature)

Resistance of a conductor in a circuit depends on 3 factors – geometry of the conductor, material of the conductor and the temperature of the conductor.

Factor	Equations
Material Geometry	For a resistor of resistivity ρ, length L and cross-sectional area A, the resistance is then $$R = \frac{\rho L}{A}$$
Temperature	When temperature of a conductor with coefficient of resistivity α (at $20^{o}C$), changes by ΔT, resistance changes by $$\Delta R = \alpha \Delta T$$

EMF, Internal Resistance and Potential Difference

Electromotive force (emf) is the electrical energy per unit charge generated by a power source generate current. Some of that electrical energy is used to overcome **internal resistance** within the power supply, the rest is then used for the rest of the circuit. That means the potential difference across the circuit is always less than the emf. This internal resistance may exist for a few reasons – distance between electrodes, temperature of the cell, effective area of the electrodes, irregularities found in the cell, etc.

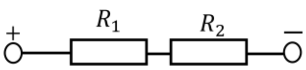
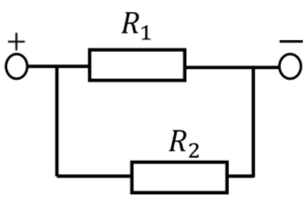


Consider a circuit consisting a voltmeter of reading V, an ammeter of reading I, a battery and a resistor of resistance R. The emf of the source is then

$$\varepsilon = IR + Ir = V + Ir$$

Parallel & Series

For systems of multiple resistors, they can be arranged in parallel, series or any combinations of the two. The effective resistance can then be calculated according to their arrangement.

Arrangement	Effective Capacitance
Series 	$$R_{eff} = R_1 + R_2$$ For n number of resistors in **series**, $$R_{eff} = R_1 + R_2 + \cdots + R_n = \sum_{i}^{n} R_i$$
Parallel 	$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$ For n number of resistors in **parallel**, $$R_{eff} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)^{-1} = \left(\sum_{i}^{n} \frac{1}{R_i}\right)^{-1}$$

Kirchhoff's Rules

Kirchhoff's Rules allows us to determine current flow around a circuit. The two rules are as follows:

Rules	Statement
First Rule – Junction Rule	Algebraic sum of currents in a network of conductors meeting at a junction is zero. $$\sum_i I_i = 0$$
Second Rule – Loop Rule	Algebraic sum of potential difference in any loop must equal to zero. $$\sum_i V_i = 0$$

Electrical Energy and Power

Electrical power P, can be calculated using the following equation

$$P = IV = I^2R = \frac{V^2}{R}.$$

Electrical energy E, is simply the product of electrical power and the time the electrical power was applied.

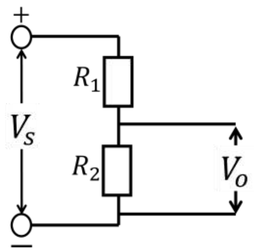
$$E = Pt$$

Potential Divider

A potential divider is used to produce a voltage of a fraction of the voltage provided by the power supply. This is achieved by using resistors of different resistances.

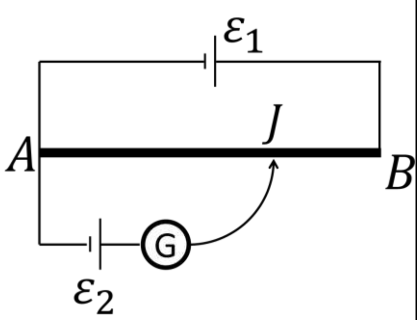
If the power supply provides potential difference of V_s, then the output voltage is simple

$$V_o = \frac{R_1}{R_1 + R_2} V_s$$



Potentiometer

A potentiometer can be used to measure potential differences by two or more cells.

How it works
Wire AB has a resistance of R. This means if the jockey is at point B, then $R_{AJ} = R_{AB}$, and thus $I = I_{maximum}$. As the jockey is slid to towards A, the galvanometer will show zero reading which indicates no current passes through the galvanometer and that the potentiometer is balanced. This means $V_{AJ} = \varepsilon_2$. This happens when $$\frac{V_{AJ}}{V_{AB}} = \frac{l_{AJ}}{l_{AB}}$$ 

We can also use a potentiometer to compare emfs between two cells.

This is done by the following setup.

compare emfs between cell 2 and 3

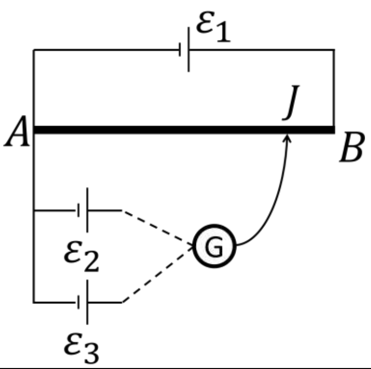
When the galvanometer is connected to ε_2 and balanced between A and J_1,

$$\varepsilon_2 = \frac{l_{AJ_1}}{l_{AB}}\varepsilon_1$$

When the galvanometer is connected to ε_2 and balanced between A and J_2,

$$\varepsilon_3 = \frac{l_{AJ_2}}{l_{AB}}\varepsilon_1$$

$$\Rightarrow \frac{\varepsilon_2}{\varepsilon_3} = \frac{l_{AJ_1}}{l_{AJ_2}}$$



Chapter 4: Magnetism

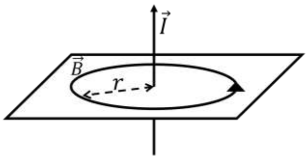
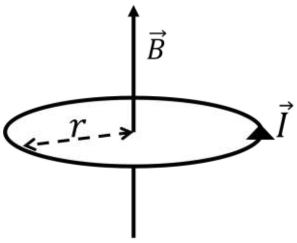
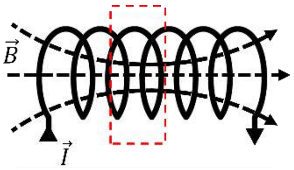
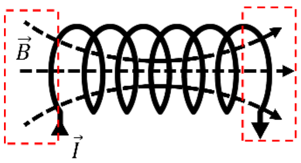
Magnetic Field

A magnetic field is a region of space in which a charged particle will experience magnetic force. They are generated by moving charged particles. Magnetic field lines are always drawn from its north pole to its south pole. When drawn on a 2D plane such as paper, we would generally represent a direction **into** the plane as ⊗, and direction **out** of the plane as ●.

$\vec{B}$ from current-carrying conductor

Direction of magnetic field depends on the direction current flow – Right Hand Rule, where the thumb point to the current direction and curled fingers are the magnetic field lines.

4 cases to considering in calculating the magnitude of magnetic field

Situation	Equation
Long straight wire 	$B = \frac{\mu_o I}{2\pi r}$
Centre of circular coil 	$B = \frac{\mu_o I}{2r}$
Centre of solenoid 	$B = \mu_o In$ Where *n* is the number of loops per unit length
End of solenoid 	$B = \frac{1}{2}\mu_o In$ Where *n* is the number of loops per unit length

Magnetic Force

Force on a moving charged particle in uniform $\vec{B}$

Force on a particle with charge q moving at velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$, the magnetic force acting on it is

$$\vec{F}_{magnetic} = q(\vec{v} \times \vec{B}).$$

In the case of a large enough region, the magnetic force will cause the charged particle to travel in a circular motion. In such cases,

$$\vec{F}_{magnetic} = \vec{F}_{centripetal} \Rightarrow qvB = \frac{mv^2}{r^2}.$$

Force on a current carrying conductor in uniform $\vec{B}$

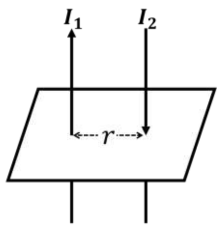
Consider a quantity of charge ΔQ travelling along a conductor of length *l* in a magnetic field $\vec{B}$ in time *t*. The magnetic force on the conductor is then

$$\vec{F}_{magnetic} = I(\vec{l} \times \vec{B}).$$

Force between two parallel current carrying conductors

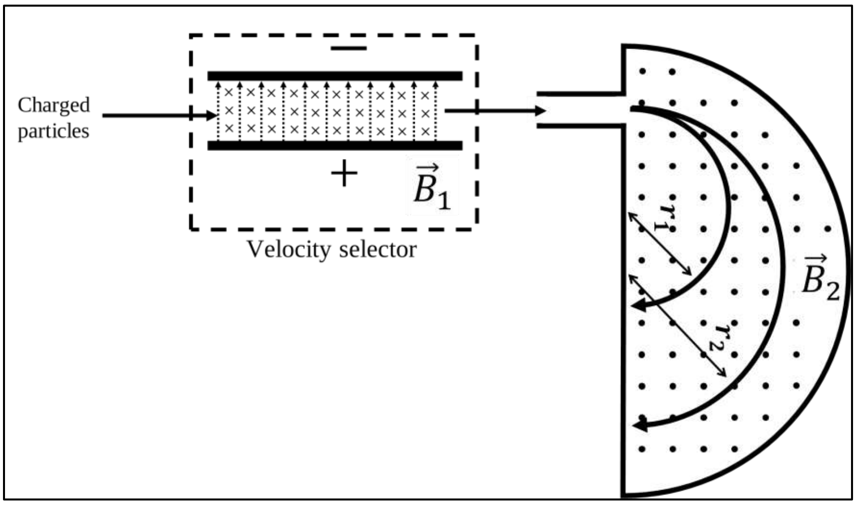
Consider two current carrying conductors of length *l* in proximity such that their magnetic fields overlap, their resultant magnetic force on each other is then

$$\vec{F}_{magnetic} = \frac{\mu_o I_1 I_2}{2\pi r} l$$



Bainbridge mass spectrometer

The Bainbridge mass spectrometer is used to accurately determine atomic mass.



The first part of the mass spectrometer is a velocity selector in which both electric field $\vec{E}$ and magnetic field $\vec{B}_1$. For charged particles to exit this velocity selector, their velocity must obey

$$v = \frac{E}{B_1}$$

This part of the mass spectrometer allows only charged particles with a certain velocity to enter the second region of only magnetic field $\vec{B}_2$.

The second part of the instrument takes advantage that charged particles of the same entry velocity but different mass will travel in circular path of different radius.

$$qvB_2 = \frac{mv^2}{r^2} \Rightarrow m = \frac{qB_2r^2}{v} = \frac{qB_1B_2r^2}{E}$$

Chapter 5: Electromagnetic Induction

Magnetic Flux

Magnetic flux is a measure of total magnetic field $\vec{B}$ passing through a given area $\vec{A}$, this is calculated with

$$\phi = \vec{B} \cdot \vec{A}.$$

In the case of N number of area of $\vec{A}$ of which $\vec{B}$ passes through, the total magnetic flux is called the **magnetic flux linkage Φ,** and is determine by

$$\boldsymbol{\Phi = N\phi = NBA\cos\theta}$$

Induced EMF

EMF is induced when magnetic flux changes with time. This is the core of Faraday's and Lenz's law of electromagnetic induction.

- Faraday's law tells us how much it emf is induced (magnitude) and Lenz's law tells us in what direction will the force act upon (direction of induced current).
- Faraday's law tells us that the magnitude of induced emf is equal to the rate of magnetic flux change and Lenz's law tells us that the induced current will be in the direction opposing the initial magnetic field.

Together, they are simply written as

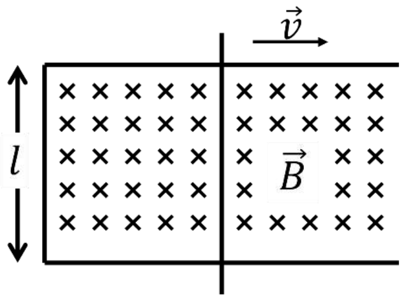
$$\varepsilon = -\frac{d\phi}{dt}$$

Induced emf in straight conductor

In a straight conductor, the area changes with time which causes the magnetic flux to change with time.

Consider a rectangular coil with one of its side movable and the opposite of the movable side has a length of l, in a region of magnetic field $\vec{B}$. If the movable side is moved at velocity $\vec{v}$, the area of the coil would change. The induced emf would then be

$$\varepsilon = -Bl\frac{dx}{dt} = Blv\sin\theta_{vB}$$



Induced emf in a coil

In a circular coil, the option for inducing emf comes from varying the magnetic field **and** the area of the coil, thus 2 equations can be found,

$$\varepsilon = -NB\frac{dA}{dt} \text{ or } \varepsilon = -NA\frac{dB}{dt}$$

Induced emf in a rotating coil

For a coil rotating at angular speed of ω, the emf induced is then

$$\varepsilon = NBA\omega\sin(\omega t)$$

Inductance

Self-induction

The idea of self-inductance is this – a magnetic field induces emf in a conductor, which in turns induces another magnetic field that opposes the initial induced emf. The conductor 'self induces' a magnetic field. The ability of a conductor to do this is quantified by ***self-inductance L,***

$$\boldsymbol{L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}}$$

Generally, this means that

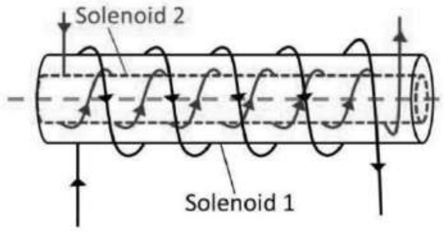
$$LI = N\phi$$

For more specific cases, 2 are considered:

1. For a coil of N turns with a cross sectional area of *A* and radius of *r*,
$$L = \frac{\mu_o N^2 A}{2r}$$
2. For a solenoid of N turns with a cross sectional area of *A* and length *l*,
$L = \frac{\mu_o N^2 A}{l}$

Mutual induction

Mutual inductance happens between 2 conductors, when the magnetic field induced by one conductor induces current in the other conductor.



Consider two coaxial solenoids, a magnetic field is generated by solenoid 1 and thus solenoid 2 respond by an induced emf, if solenoid 2 has a cross sectional area of A_2, then the mutual inductance between solenoid 1 and 2 is

$$M_{21} = \frac{\mu_o N_1 N_2 A_2}{l},$$

where *l* is the length of the solenoid.

Energy Stored in Inductor

The energy stored in an inductor of inductance *L* and with current *I* running through it, is simply

$$U = \frac{1}{2} L I^2$$

Chapter 6: Alternating Current

Alternating Current

Alternating current (AC) is defined as an electric current that periodically reverses its direction with respect to time.

Root Mean Square Values

In AC circuits, the idea of voltage and current now are functions of time:

$$I \mapsto I(t) = I_{peak}\, sin\,(\omega t)$$

$$V \mapsto V(t) = V_{peak}\, sin\,(\omega t)$$

Resistance is then defined as

$$R = \frac{V_o}{I_o}$$

In calculation of power, where $P_{DC} = IV$, for AC circuits,

$$P_{AC} = I_{rms}V_{rms}$$

where,

$$I_{rms} = \frac{I_o}{\sqrt{2}} \text{ and } V_{rms} = \frac{V_o}{\sqrt{2}}$$

Impedance

In DC circuit, our main concern for opposition of current flow is only resistance *R*.

In AC circuits, we now have what is known as **impedance *Z***, which is defined by

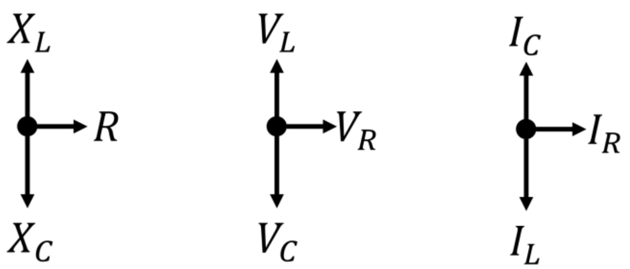
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where *R* is the resistance, X_L is the inductive reactance and X_C is the capacitive reactance found in the circuit.

The following table shows how to calculate these values

Reactance	Equation
Capacitance reactance for a capacitor of capacitance *C*	$X_C = \frac{1}{2\pi f C}$
Inductive reactance for an inductor of inductance *L*	$X_L = 2\pi f L$

The phasor diagram for an RLC circuit is as follows,



Phasor Diagram
for RLC Circuit

Which means that the phase angle between current and voltage is

$$\theta_{IV} = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Resonance occurs when $X_L = X_C \Rightarrow \omega = \frac{1}{\sqrt{LC}} = 2\pi f$.

Power & Power Factor

2 types of power that be calculate in the case of AC circuits,

1. Instantaneous power

$$P = I(t) \times V(t)$$

2. Average power

$$P_{ave} = I_{rms}V_{rms}\, cos\, (\theta_{IV})$$

The power factor is simply

$$cos\, \theta_{IV} = \frac{P_{real}}{P_{apparent}} = \frac{P_{ave}}{I_{rms}V_{rms}}$$

Chapter 7: Optics

Geometrical Optics: Reflection

Definitions:

1. Centre of curvature, *C* = a point on the principal (or optical) axis that is positioned at distance equal to the radius of curvature *R*, of the spherical mirror.
2. Focal point, *f* = a point on the principal axis at which light rays travelling parallel to the principal axis will converge onto or diverge from, after reflecting on the surface of the spherical mirror.

f and *R* are related by the following equation:

$$R = 2f$$

2 types of mirrors:

1. **Convex mirror**, of which its radius is located behind the mirror.
2. **Concave mirror**, of which it's radius of curvature is located in front of the mirror.

<table>
<tr><th colspan="2">Conventions</th></tr>
<tr><td>Focal length, f</td><td rowspan="2">+ for concave; - for convex</td></tr>
<tr><td>Curvature Radius, R</td></tr>
</table>

Lateral magnification m, refers to the ratio between the height of the image to the height if the object. In equation form,

$$m = \frac{h_i}{h_o}$$

$$m > 0 \Rightarrow \text{ upright image; } m < 0 \Rightarrow \text{ inverted image}$$

Geometrical Optics: Refraction

An extension to Snell's law will be the refraction at a spherical surface. The following equation allows us to relate distances, refractive indices and radius of curvature of the spherical surface:

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

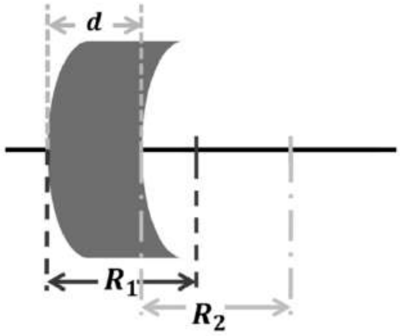
In the equation above n_i, refers to refractive indices, *u* and *v* refers to object and image distances respectively and *R* refers to the radius of curvature.

<table>
<tr><th colspan="2">Conventions</th></tr>
<tr><td>Curvature Radius, R</td><td>+ for convex, i.e. C opposite side as incoming light
- for concave, i.e. C same side as incoming light</td></tr>
</table>

For the refractive indices, subscript 1 refers to the refractive index on the side of the incoming light rays and subscript 2 refers to the refractive index on the side of the outgoing rays.

Geometrical Optics: Thin lenses

The thin lens equation assumes that the thickness measured between two vertex of the spherical surface of a lens is much smaller than the product of the radii of the spherical lenses, that is $d \ll R_1 R_2$.



For thin lenses,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Conventions	
Focal length, f	+ for convex, i.e. same side as incoming light - for concave, i.e. opposite side as incoming light

On the other hand, using the lens maker's equation,

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

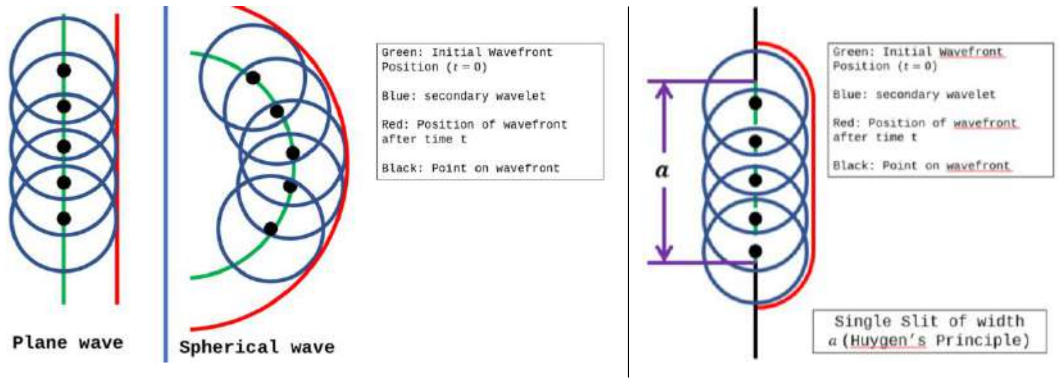
one can determine the focal length f, of the lens from

1. the radii of the lens surfaces, R_1 and R_2,
2. the ratio of the refractive index of the lens material to the refractive index of the surrounding, $\frac{n_{material}}{n_{medium}}$

Conventions	
Curvature Radius, R	- if curvature same side as incoming light + if curvature opposite side as incoming light

Physical Optics: Huygens's Principle

Huygen's Principle states that "each point on the wavefront acts as the source of secondary wavelets that spread out in all directions in spherical waves with a speed equal to the speed of wave propagation."

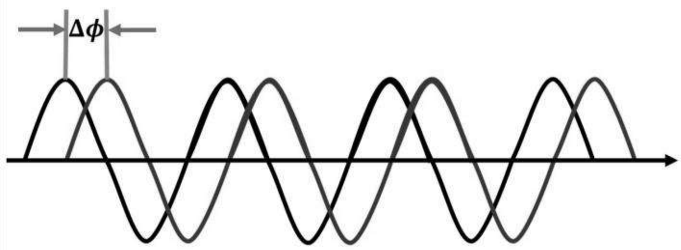


Physical Optics: Interferences

Coherence between 2 waves refers to the condition of constant phase difference between 2 waves with respect to time, that is to say $\frac{d\phi}{dt} = 0$. This property is the ideal property for stationary interference.

For a stable interference pattern, the following conditions are required:

1. Coherence, that is to say the two interacting light waves are of the same phase difference, $\frac{d\phi}{dt} = 0$.



2. Monochromatic, that is to say that the two interacting light waves are of the same wavelength, i.e. $\lambda_1 = \lambda_2$

For purely **constructive interference**, it is empirical that the phase difference between the interacting waves is either 0 or $n\lambda$. On the other hand, for purely **destructive interference**, it is required that the phase difference between the two interacting wave is $\frac{n\lambda}{2}$.(n is both cases refers to integer values.)

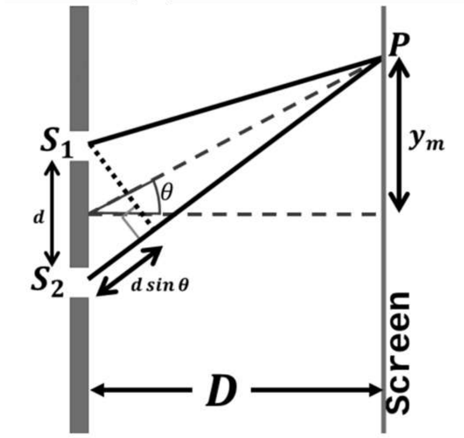
Physical Optics: Slits

Double Slit

We now consider the case for Young's double slit experiment.

Here we define the following variables:

D	distance from slit to screen
d	slit separation
y_m	distance from central maximum tu the mth fringe



We know that in order to determine what type of fringe forms at P, we need to look at the path difference and from the figure, we can say that the figure,

$$\Delta\phi = S_2P - S_1P = d sin\theta = d\left(\frac{y_m}{D}\right)$$

For **bright fringes**,

$$\Delta\phi = \frac{y_m d}{D} = m\lambda$$

Rearranging this allows us to find fringe distance as a function of *d* and *D* with *m* having any integer value indicating **mth bright fringe** from central maximum:

$$y_m = \frac{m\lambda D}{d}$$

Shifting one the waves by 0.5λ give us the equation for **dark fringes**,

$$y_m = \frac{(m + 0.5)\lambda D}{d}.$$

Lastly, we'd want to calculate the fringe separation. This can be done by considering $\Delta y = y_{m+1} - y_m$, which results in

$$\Delta y = \frac{\lambda D}{d}$$

Single Slit

Diffraction is defined as the spreading or bending of waves as they pass through an aperture of a barrier. The diffracted waves then interfere with each other to produce a diffraction pattern.

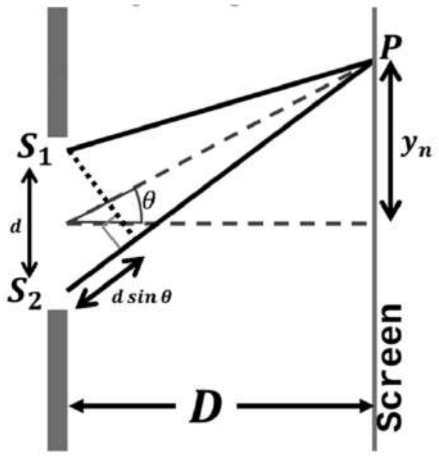
Light waves from one portion of the slit interacts with light waves from a different portion of the same slit to produce a diffraction pattern.

Here, we find that the dark fringes to forms when according to

$$d\,sin\,\theta = n\lambda.$$

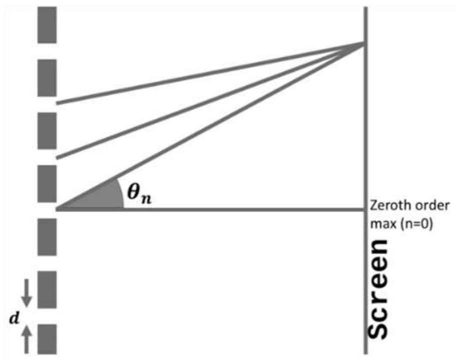
Geometrically, we also find that $tan\theta \approx sin\theta \approx \frac{n\lambda}{d} \approx \frac{y_n}{D}$.

As such, we can say that **dark fringes** forms at $y_n = \frac{n\lambda D}{d}$. This would also mean that **bright fringes** forms at $y_n = \frac{(n+0.5)\lambda D}{d}$.



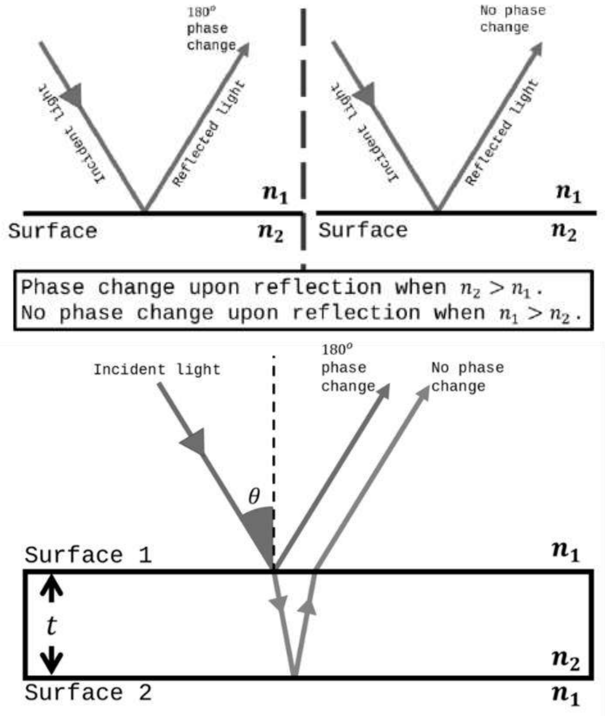
Diffraction Grating

In the case for diffraction grating, light waves from many slits and interfere at the screen to form fringes of equal width. The equation by which the pattern follows is $d\,sin\,\theta_n = n$ for **bright fringes** and shifted by 0.5λ for **dark fringes**. Note that the angle θ_n is measured from the normal line formed at the zeroth order maximum. Also note that, **maximum number of fringes** can be calculated by considering that $sin\,\theta_n < 1$.



Physical Optics: Thin Films

Referring to figure on thin films, we can see that the two reflected light waves has a phase difference of 0.5λ from reflections at surface 1 and 2. One must also take into consideration of the extra distance that the second (green) wave travelled, that is $2nt$. Therefore, the total phase difference between the reflected waves is then



$$\Delta\phi = 2nt - \frac{1}{2}\lambda.$$

For **constructive interference**, $\Delta\phi = 2nt - \frac{1}{2}\lambda = n$, which gives us the equation

$$2nt = \left(n + \frac{1}{2}\right)\lambda.$$

Dark fringes then appear between the bright fringes, i.e. they follow the equation

$$2nt = n\,\lambda.$$

Main application for this concept of thin film interference is in **optical coatings** as one can manipulate the thickness of the coating as to choose the level of constructive or destructive interference. These optical coatings can be applied onto both for reflective as well as refractive systems.

Chapter 8: Particle Waves

De Broglie Wavelength

Like light, matter also exist in dual form – as particles **and** waves.

Matter waves, known as "de Broglie wavelength", are calculated with

$$\lambda_{matter} = \frac{h}{p} = \frac{h}{mv}$$

For a particle with of mass m charge q accelerated by electric field of V volts,

$$\lambda_{matter} = \frac{h}{\sqrt{2qVm}}$$

Electron Diffraction

On de Broglie wavelength:

1. To show that particles may exhibit wave-like characteristics, Davisson and Germer designed an experiment in which they show that electrons diffracted.
2. They achieve this by directing a beam of electrons onto a nickel crystal.

On electron microscope:

1. Because of their short wavelength (1nm for electrons vs 400nm – 700nm for light microscope), electronic microscopes can offer physicists a higher resolution in probing specimens.
2. Optical microscope is made up of glass lenses, whereas components of an electron microscope are electromagnetic.

Chapter 9: Nuclear & Particle Physics

Binding Energy & Mass Defect

Mass defect, Δm = mass difference between the actual mass of an atomic nucleus and the sum of its components, i.e. protons and neutrons.

For an atomic nucleus of mass $m_{nucleus}$ with Z number of protons of mass m_{proton} and N number of neutrons of mass $m_{neutron}$, its mass defect is

$$\Delta m = \left(Zm_{proton} + Nm_{neutron}\right) - m_{nucleus}$$

Binding energy, $E_{binding}$ = energy found in the nucleus of an atom that binds its components together. This energy can be calculated from the mass defect,

$$E_{binding} = \Delta mc^2$$

As the masses of atomic nucleus is well, very small, and the speed of light is astronomical, it may be easier to perform calculations using atomic mass unit (amu) or Dalton (u) and MeV/c^2

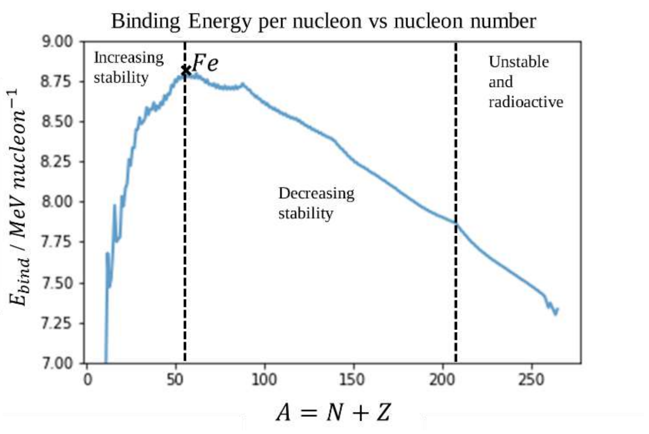
$$1kg = 6.022(10^{26})u = 5.60958(10^{29})\ MeV/c^2$$

$$1u = 1.66054(10^{-27})\ kg = 931.494\ MeV/c^2$$

Binding energy per nucleon, $\frac{E_{binding}}{A}$:

Nucleon number, A = Number of protons, Z + Number of neutrons, N

Binding energy per nucleon vs nucleon number graph:



- For nuclei lighter than that of iron (Fe), it is found that binding energy per nucleon increases with the nucleon number.
- After the iron limit, the binding energy per nucleon decreases.
- At $A \approx 209$ (nuclei of Bi), the binding energy per nucleon is too weak to keep the nuclei together and thus, are unstable and radioactive.

Radioactivity

The following table describes the types of decay of a radioactive substance

Type of decay	Process	Description
α	${}^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}He$ parent nucleus → daughter nucleus + α particle	In α decay, an α particle (Helium) is emitted when the parent nucleus decays into its daughter nucleus. Electrical charge is conserved throughout the process. Energy is released upon α decay.
β^-	$n \rightarrow p^+ + e^- + \bar{\nu}$ ${}^{A}_{Z}P \rightarrow {}_{Z+1}^{A}D + {}_{-1}^{0}e + \bar{\nu}$ parent nucleus → daughter nucleus + β^- particle + antineutrino	In β^- decay, an electron e^- and an antineutrino $\bar{\nu}$ is emitted when the parent nucleus decays into its daughter nucleus.
β^+	$p^+ \rightarrow n + e^+ + \nu$ ${}^{A}_{Z}P \rightarrow {}_{Z-1}^{A}D + {}_{+1}^{0}e + \nu$ parent nucleus → daughter nucleus + β^+ particle + neutrino	In β^+ decay, a positron e^+ and a neutrino ν is emitted when the parent nucleus decays into its daughter nucleus.
γ	${}^{A}_{Z}P * \rightarrow {}^{A}_{Z}P + \gamma$ nuclei of high energy state → nuclei of low energy state + γ ray	In γ decay, the emission is a photon (light ray). This happens because the nucleons lower its energy state.

In general, N number of radioactive particles will decay according to the **decay law**,

$$\frac{dN}{dt} = -\lambda N$$

where λ is the decay constant of the substance, which varies between isotopes.

The solution for the decay law is

$$N(t) = N_o e^{-\lambda t}$$

where $N_o = N(t = 0)$.

The rate of decay is known as **activity**

$$A = \left|\frac{dN}{dt}\right| = \left|\frac{dN_o}{dt}\right| e^{-\lambda t} = A_o e^{-\lambda t}$$

Half-life is simply the time it takes for the number of isotopes to decrease by half $T_{\frac{1}{2}}$,

$$N = \frac{1}{2}N_o = N_o e^{-\lambda T_{\frac{1}{2}}} \Rightarrow T_{\frac{1}{2}} = \frac{ln\,2}{\lambda}$$

Particle Accelerator

Thermionic emission (Edison Effect) = emission of electrons on the surface of a metal by providing it sufficient thermal energy.

As mentioned before, a charged particle may be accelerated by the help of an electric and magnetic field. The acceleration would stem from Lorentz Force.

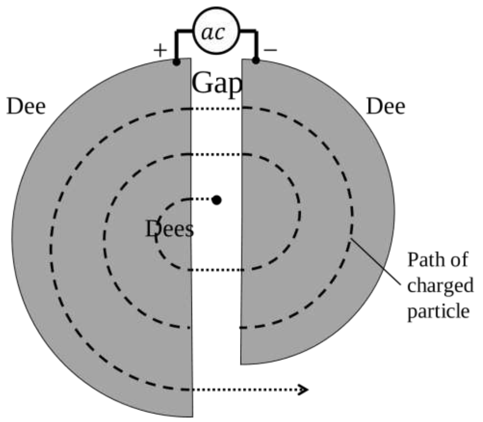
To probe subatomic particles, we need high energy because higher energy results in higher momentum which gives out smaller de Broglie wavelength. This means a higher resolution can be achieved.

2 types of particle accelerators:

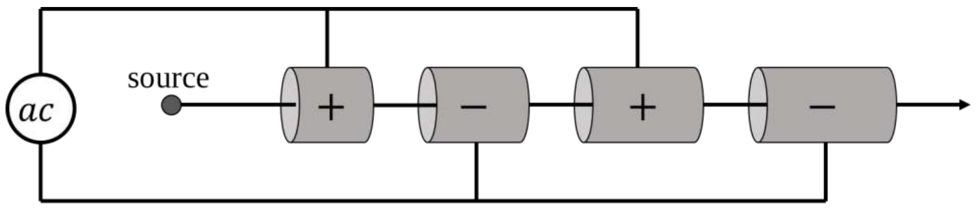
1. Cyclotron
   It uses magnetic field to maintain charged particles in nearly circular paths.

   A cyclotron is composed of 2 'dees', charged particles are accelerated in the region of space between the two 'dees', where an electric field is applied.

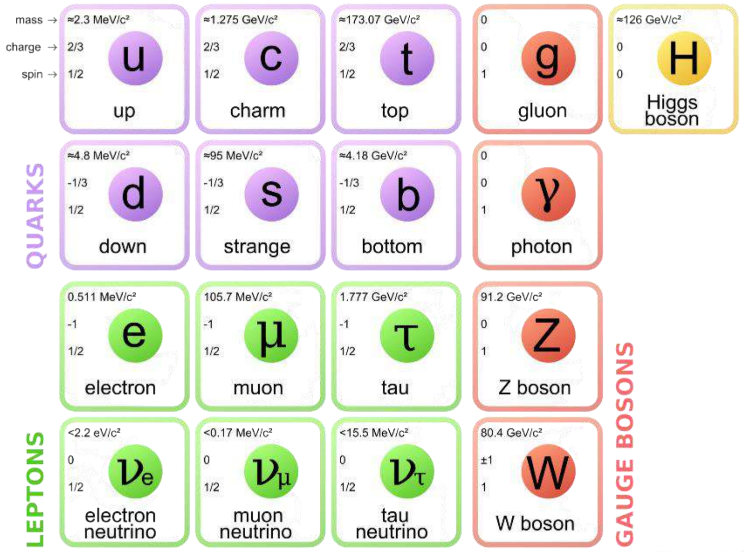
   Velocity of the charged particles when they are in the 'dees' is
   $$v = \frac{qBr}{m}$$
   Frequency of electric field is equal to the frequency of the circulating protons,
   $$f = \frac{qB}{2\pi m}$$
   which is known as the **cyclotron frequency**.

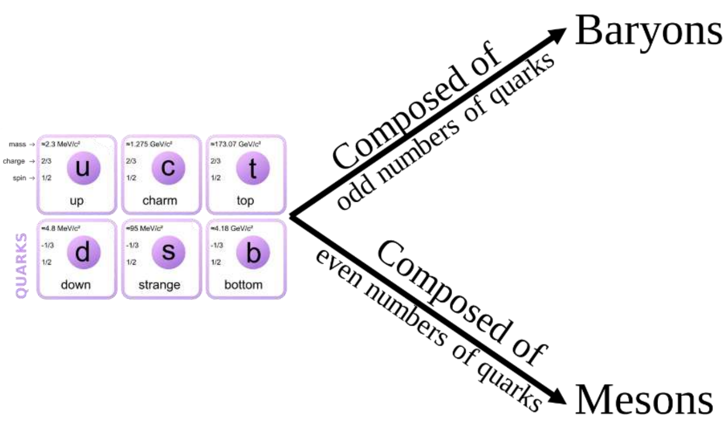


2. Linear Accelerator
   Charged particles are accelerated through a series of linear conductor tubes.
   Alternating voltage is applied to consecutive tubes so that when a charged particle reaches a gap, the tube they just left is now negatively charged and the tube they are heading into is positively charged.



Fundamental Particles





Particle-antiparticle pair:

They are pairs of particles that has opposite charge to each other. E.g., electron has a negative charge whereas its antiparticle, a positron has a positive charge. They interact by annihilating each other.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

PARTICLE ACCELERATORS
&
FUNDAMENTAL PARTICLES

Learning outcomes:
9.3 Particle Accelerator
1. State the thermionic emission
2. Explain the acceleration of particle by electric and magnetic field.
3. State the role of electric and magnetic field in particle accelerators (linac and cyclotron) and detectors (general principles of ionization and deflection only)
4. State the need of high energies required to investigate the structure of nucleon

9.4 Fundamental Particle
1. Explain the standard quark-lepton model particles (baryons, meson, leptons and hadrons.
2. Explain the corresponding antiparticle for every particle.

i. What is thermionic emission? [https://www.youtube.com/watch?v=tYCET6vYdYk]
ii. Explain the acceleration of particle by electric and magnetic field. [https://www.youtube.com/watch?v=X4v-yGw-Sh0]
iii. Briefly describe the workings of linear particle accelerator (LINAC) [https://www.youtube.com/watch?v=3Bm60HdvI6s]
iv. Briefly describe the workings of cyclotron [https://www.youtube.com/watch?v=q93Irf-BABY]
v. Give 2 types of particle detectors and describe their working principles. [https://www.youtube.com/watch?v=mWbStD9e2hQ]
vi. How to detect particles through ionization? [https://web.physics.utah.edu/~petra/phys6771/lecture13.pdf]
vii. Briefly describes the workings of a cloud chamber and a spark chamber. [https://www.youtube.com/watch?v=KW0CDwzIm3Q]
viii. Briefly describe the process of Rutherford Scattering. [https://www.youtube.com/watch?v=ygfPA1QhIdI]
ix. Explain why high energy is needed to probe nucleon structure. [https://www.youtube.com/watch?v=X4v-yGw-sH0]

1) What is the quark-lepton model? [https://www.youtube.com/watch?v=edgsmtUH954]
2) What is a lepton? [https://www.youtube.com/watch?v=Gh7vvaYibGo]
3) What is a hadron? [https://www.youtube.com/watch?v=pRLL7rWUDGM]
4) What is a baryon? [https://www.youtube.com/watch?v=LLHJa6XAiaE]
5) What is a meson? [https://www.youtube.com/watch?v=jYaBl7IBDjw]
6) What is an antiparticle? Give 3 examples. [https://www.youtube.com/watch?v=jmLcjg0IXlQ]

Sign Conventions

Reflection at a spherical surface

From CS:

LO: Use mirror equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for real object only.
Sign convention for focal length, *f* and radius of curvature, *R:*
  i. Positive *f* and *R* for concave mirror; and
  ii. Negative *f* and *R* for convex mirror.

From Serway:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

Table 36.1

Sign Conventions for Mirrors

Quantity	Positive When	Negative When
Object location (p)	Object is in front of mirror (real object)	Object is in back of mirror (virtual object)
Image location (q)	Image is in front of mirror (real image)	Image is in back of mirror (virtual image)
Image height (h')	Image is upright	Image is inverted
Focal length (f) and radius (R)	Mirror is concave	Mirror is convex
Magnification (M)	Image is upright	Image is inverted

From Cutnell:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h'}{h} = -\frac{d_i}{d_o}$$

Reasoning Strategy Summary of Sign Conventions for Spherical Mirrors

Focal length

f is + for a concave mirror.
f is − for a convex mirror.

Object distance

d_o is + if the object is in front of the mirror (real object).
d_o is − if the object is behind the mirror (virtual object).*

Image distance

d_i is + if the image is in front of the mirror (real image).
d_i is − if the image is behind the mirror (virtual image).

Magnification

m is + for an image that is upright with respect to the object.
m is − for an image that is inverted with respect to the object.

*Sometimes optical systems use two (or more) mirrors, and the image formed by the first mirror serves as the object for the second mirror. Occasionally, such an object falls *behind* the second mirror. In this case the object distance is negative, and the object is said to be a virtual object.

From geeksforgeeks.org:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

The sign conventions followed for any spherical mirror are given as:

- *All distances are measured from the pole of a spherical error.*
- *Distances measured in the direction of incident light are taken as positive, while distances measured in a direction opposite to the direction of the incident light are taken as negative.*
- *The upward distances perpendicular to the principal axis are taken as positive, while the downward distances perpendicular to the principal axis is taken as negative.*
  - *For convenience, the object is assumed to be placed on the left side of a mirror. Hence, the distance of an object from the pole of a spherical mirror is taken as negative.*
  - *Since the incident light always goes from left to right, all the distances measured from the pole (P) of the mirror to the right side will be considered positive (because they will be in the same directions as the incident light). On the other hand, all the distances measured from pole (P) of the mirror to the left will be negative (because they are measured against the direction of incident light)*

Refraction at a spherical surfaces

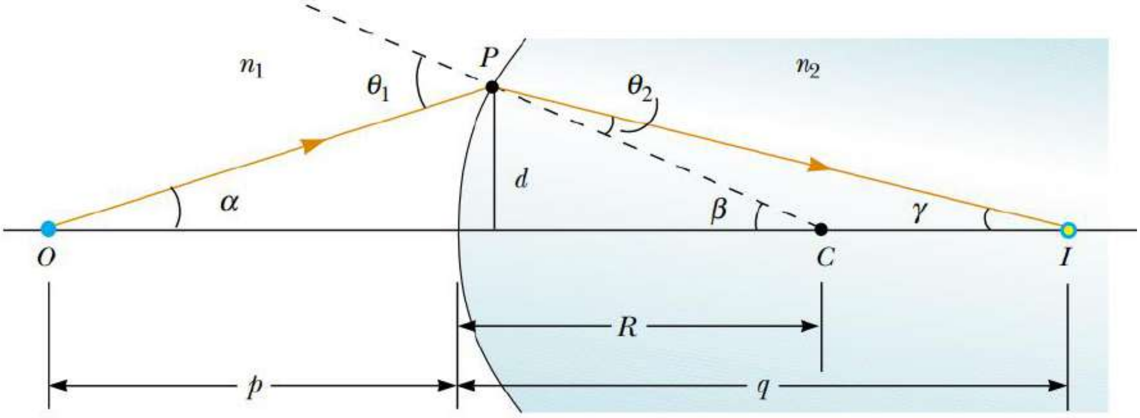


Figure 36.19 Geometry used to derive Equation 36.8, assuming that $n_1 < n_2$.

Snell's Law:

$$n_1 \, sin \, \theta_1 = n_2 \, sin \, \theta_2$$

Small angle approximation:

$$sin \, \theta_i \approx \theta_i \Rightarrow n_1\theta_1 = n_2\theta_2$$

Exterior angle of any triangle is the sum of two opposite interior angles,

$$\theta_1 = \alpha + \beta; \; \beta = \theta_2 + \gamma$$

Eliminate θ_1 and θ_2,

$$n_1(\alpha + \beta) = n_2(\beta - \gamma)$$

Based on diagram, common vertical leg is *d*,

$$tan \, \alpha \approx \alpha \approx \left(\frac{d}{p}\right); \; tan \, \beta \approx \beta \approx \left(\frac{d}{R}\right); \; tan \, \gamma \approx \gamma \approx \left(\frac{d}{q}\right)$$

Rearranging terms yields,

$$n_1\left(\frac{d}{p} + \frac{d}{R}\right) = n_2\left(\frac{d}{R} - \frac{d}{q}\right)$$

Cancelling *d* on both sides and rearranging the terms yields,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Sign Conventions

From CS:

LO: Use $\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$ for spherical surface.
Sign convention for radius of curvature, *R:*
i. Positive *R* for convex surface; and
ii. Negative *R* for concave surface.

From Serway:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Table 36.2

Sign Conventions for Refracting Surfaces

Quantity	Positive When	Negative When
Object location (p)	Object is in front of surface (real object)	Object is in back of surface (virtual object)
Image location (q)	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R)	Center of curvature is in back of surface	Center of curvature is in front of surface

Thin Lenses

LO:

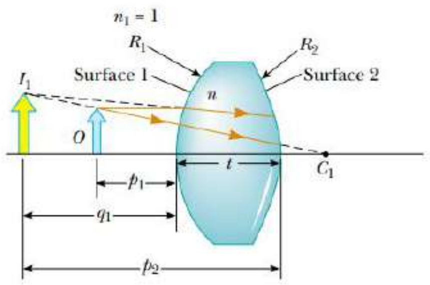
a. Use thin lenses equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for real object only.
Sign convention for focal length:
i. Positive f for convex mirror; and
ii. Negative f for concave mirror.

b. Use lensmaker's equation, $\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

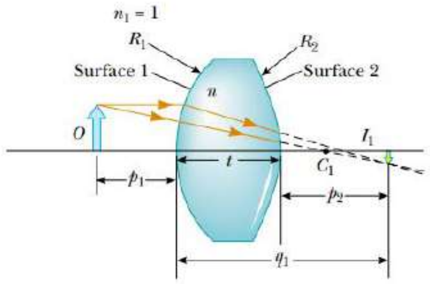
c. Apply magnification, $M = \frac{h_i}{h_o} = -\frac{v}{u}$

d. Use thin lens equation for a combination of two convex lenses.

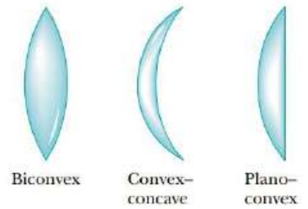
From Serway (adapted):



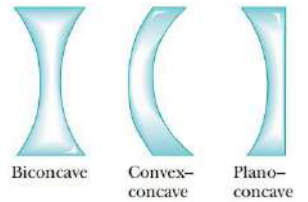
(a)



(b)



(a)



Consider image formed by surface 1,

$$\frac{n_{medium}}{p_1} + \frac{n_{lens}}{q_1} = \frac{n_{lens} - n_{medium}}{R_1}$$

We can then apply this to surface 2,

$$\frac{n_{lens}}{p_2} + \frac{n_{medium}}{q_2} = \frac{n_{medium} - n_{lens}}{R_2}$$

Using the image from surface 1 as the object for surface 2,

$$p_2 = -q_1 + t$$

Where t is the thickness of the lens and is ≈ 0 for thin lenses ($t \ll R$).

$$\frac{n_{lens}}{-q_1} + \frac{n_{medium}}{q_2} = \frac{n_{medium} - n_{lens}}{R_2}$$

$$\frac{1}{f} = \frac{1}{p_1} + \frac{1}{q_2} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Table 36.3

Sign Conventions for Thin Lenses

Quantity	Positive When	Negative When
Object location (p)	Object is in front of lens (real object)	Object is in back of lens (virtual object)
Image location (q)	Image is in back of lens (real image)	Image is in front of lens (virtual image)
Image height (h')	Image is upright	Image is inverted
R_1 and R_2	Center of curvature is in back of lens	Center of curvature is in front of lens
Focal length (f)	Converging lens	Diverging lens

From Cutnell:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Reasoning Strategy Summary of Sign Conventions for Lenses

Focal length

f is + for a converging lens.
f is − for a diverging lens.

Object distance

d_o is + if the object is to the left of the lens (real object), as is usual.
d_o is − if the object is to the right of the lens (virtual object).*

Image distance

d_i is + for an image (real) formed to the right of the lens by a real object.
d_i is − for an image (virtual) formed to the left of the lens by a real object.

Magnification

m is + for an image that is upright with respect to the object.
m is − for an image that is inverted with respect to the object.

*This situation arises in systems containing more than one lens, where the image formed by the first lens becomes the object for the second lens. In such a case, the object of the second lens may lie to the right of that lens, in which event d_o is assigned a negative value and the object is called a virtual object.

From Giancoli:

$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}$ $m=\frac{h_i}{h_o}=-\frac{d_i}{d_o}$	**sign conventions:** **1.** The focal length is positive for converging lenses and negative for diverging lenses. **2.** The object distance is positive if the object is on the side of the lens from which the light is coming (this is always the case for real objects; but when lenses are used in combination, it might not be so: see Example 23–16); otherwise, it is negative. **3.** The image distance is positive if the image is on the opposite side of the lens from where the light is coming; if it is on the same side, d_i is negative. Equivalently, the image distance is positive for a real image (Fig. 23–40) and negative for a virtual image (Fig. 23–41). **4.** The height of the image, h_i, is positive if the image is upright, and negative if the image is inverted relative to the object. (h_o is always taken as upright and positive.)

<u>Thin lens equation for two convex lenses</u>

From Giancoli:

"When light passes through more than one lens, we find the image formed by the first lens as if it were alone. Then this image becomes the object for the second lens. Next we find the image formed by this second lens using the first image as object. This second image is the final image if there are only two lenses. The total magnification will be the product of the separate magnifications of each lens."

<u>Example from Giancoli:</u>

Problem

Two converging lenses, A and B, with focal lengths $f_A=20cm$ and $f_B=25cm$ and are placed 80.0 cm apart, as shown in Fig. 23–44a. An object is placed 60.0 cm in front of the first lens as shown in Fig. 23–44b.

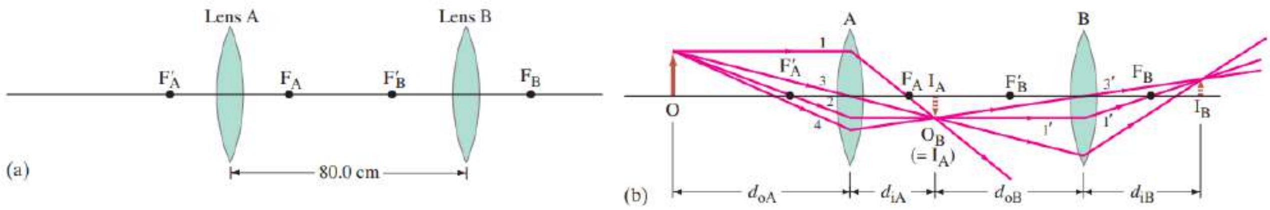


FIGURE 23–44

Determine (a) the position, and (b) the magnification, of the final image formed by the combination of the two lenses

Solutions

Position:

$$\frac{\mathbf{1}}{\boldsymbol{d_{iA}}}=\frac{\mathbf{1}}{\boldsymbol{f_A}}-\frac{\mathbf{1}}{\boldsymbol{d_{oA}}}\Rightarrow \boldsymbol{d_{iA}}=\frac{\boldsymbol{f_A d_{oA}}}{\boldsymbol{d_{oA}-f_A}}$$

$$\frac{\mathbf{1}}{\boldsymbol{d_{iB}}}=\frac{\mathbf{1}}{\boldsymbol{f_B}}-\frac{\mathbf{1}}{\boldsymbol{d_{oB}}}\Rightarrow \boldsymbol{d_{iB}}=\frac{\boldsymbol{f_B d_{oB}}}{\boldsymbol{d_{oB}-f_B}}$$

$$80cm=d_{iA}+d_{oB}\Rightarrow d_{oB}=80cm-d_{iA}$$

$$\boldsymbol{d_{iB}}=\frac{\boldsymbol{f_B}(80cm-d_{iA})}{(80cm-d_{iA})-\boldsymbol{f_B}}=\frac{\boldsymbol{f_B}\left(80cm-\left(\frac{\boldsymbol{f_A d_{oA}}}{\boldsymbol{d_{oA}-f_A}}\right)\right)}{\left(80cm-\left(\frac{\boldsymbol{f_A d_{oA}}}{\boldsymbol{d_{oA}-f_A}}\right)\right)-\boldsymbol{f_B}}=\frac{(\mathbf{25})\left(80cm-\left(\frac{(\mathbf{20})(\mathbf{60})}{\mathbf{60-20}}\right)\right)}{\left(80cm-\left(\frac{(\mathbf{20})(\mathbf{60})}{\mathbf{60-20}}\right)\right)-\mathbf{25}}=\boldsymbol{50cm}$$

⇒Final image is formed 50cm behind lens B.

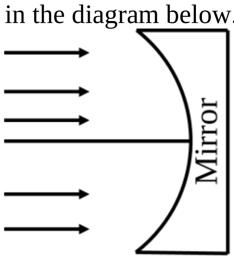
Total Magnification:

$$\boldsymbol{M_T}=\boldsymbol{M_1M_2}=\left(-\frac{\boldsymbol{d_{iA}}}{\boldsymbol{d_{oA}}}\right)\left(-\frac{\boldsymbol{d_{iB}}}{\boldsymbol{d_{oB}}}\right)=\left(-\frac{\mathbf{30}}{\mathbf{60}}\right)\left(-\frac{\mathbf{50}}{\mathbf{50}}\right)=\frac{\mathbf{1}}{\mathbf{2}}$$

⇒Final image is upright and half of the original object height.

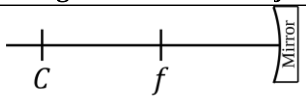
Exercise

Geometrical Optics: Reflection

$R = 2f$	Find R	a. A spherical mirror of diameter 2cm is illuminated with parallel light rays as shown in the diagram below.  Determine the radius of curvature of the spherical mirror. **[$R = 1cm$]**
	Find f	b. Based on question (a), determine the point at which the light rays would converge at. **[$f = 0.5cm$]**
$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$	Concave	c. An object is placed 3cm **in front of** a mirror. The image is found to form 6cm **in front of** the mirror. Determine the focal length of the mirror. **[$f = 2cm$]**
	Convex	d. An object is placed 6cm **in front of** a mirror. The image is found to form 3cm **behind** the mirror. Determine the focal length of the mirror. **[$f = -6cm$]**
	Real Object	e. An object is placed $x\,cm$ from a convex mirror. The image is found to form 3cm **behind** the mirror and the focal length of the mirror is 9cm. Determine the object distance from the mirror. **[$u = 4.5cm$]** f. An object is placed in front of a concave mirror. The image is found to form 3cm **in front of** the mirror and the focal length of the mirror is 2cm. Determine the object distance from the mirror. **[$u = 6cm$]**
	Real Image	g. An object is placed 4cm in front of a concave mirror. The focal length of the mirror is 2cm. Determine the image distance from the mirror. **[$v = 4cm$]**
	Virtual Image	h. An object is placed 4cm in front of a convex mirror. The focal length of the mirror is 4cm. Determine the image distance from the mirror. **[$v = 2cm$]**
$M = \frac{h_i}{h_o}$ $M = -\frac{v}{u}$	Magnified	i. The virtual image of a real object from a spherical mirror is found to form at $4cm$ when the object is placed $2cm$ from the mirror. Determine the magnification. **[$M = 2$]**
	Diminished	j. An object is placed 2cm in front of a convex mirror of focal length 6cm. Determine the image distance and the magnification. [$v = -1.5cm$; $M = 0.75$]

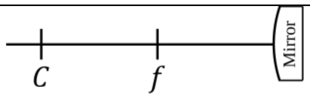
Properties of image based on real object position and type of mirror

Concave



		u (cm)	V (cm)	M
$f = 4cm$	$u < f$	2	-4 (Virtual)	2 (Magnified)
	$C > u > f$	6	12 (Real)	-2 (Magnified)
	$u > C$	12	6 (Real)	-0.5 (Dimished)

Convex



		u (cm)	V (cm)	M
$f = -4cm$	$u < f$	2	$-\frac{4}{3}$ (Virtual)	$\frac{2}{3}$ (Dimished)
	$C > u > f$	6	$-\frac{12}{5}$ (Virtual)	$\frac{2}{5}$ (Dimished)
	$u > C$	12	-3 (Virtual)	$\frac{1}{4}$ (Dimished)

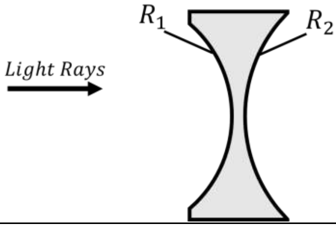
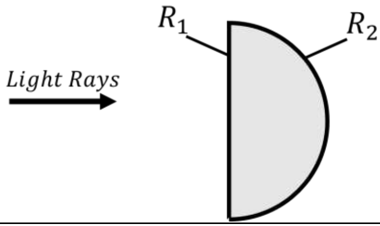
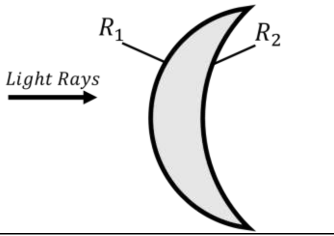
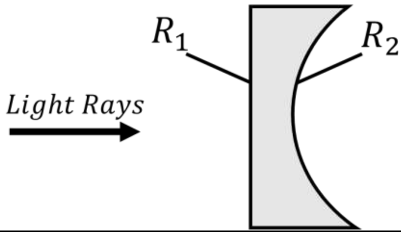
Geometrical Optics: Refraction

$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$	Real Object	a.	When an object is placed some distance from a convex interface of a material of radius $2cm$, an image of the object is formed 4cm **behind** the interface. If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the **object distance** from the interface. [$\boldsymbol{u = 4.8cm}$]
	Virtual Object	b.	When an object is placed some distance from a convex interface of a material of radius $2cm$, an image of the object is formed 0.5cm **behind** the interface. If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the **object distance** from the interface. [$\boldsymbol{u \approx -0.21cm}$]
	Real Image	c.	An object is placed 20cm from a **convex interface** of a material of radius $2cm$. If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the image distance from the interface. [$\boldsymbol{v \approx 3.27cm}$]
	Virtual Image	d.	An object is placed 20cm from a **concave interface** of a material of radius $2cm$. If the refractive index before and after the material is 1.2 and 3.4 respectively, determine the image distance from the interface. [$\boldsymbol{v \approx -2.93cm}$]
	Convex Surface	e.	An object is placed 24cm **in front of** an interface of a material of refractive index of 2.8. The refractive index before the interface is 1.3. If the image is found to form 56cm **behind** the interface, determine the radius of curvature of the interface. [$R = 14.4cm$]
	Concave Surface	f.	An object is placed 24cm **in front of** an interface of a material of refractive index of 2.8. The refractive index before the interface is 1.3. If the image is found to form 18cm **in front of** the interface, determine the radius of curvature of the interface. [$R = -14.2cm$]
	Refractive index before interface	g.	An object is placed 25cm **in front of** an interface of a material of refractive index of 3.3. If the image is found to form 18cm **behind** the interface, determine the refractive index of the material before the interface. [$n_1 \approx 1.39$]
	Refractive index after interface	h.	An object is placed 24cm **in front of** a convex interface of a material of refractive index of n. The refractive index before the interface is 1.4. If the image is found to form 38cm behind the interface and the radius of curvature of the interface is 15cm, determine the refractive index n. [$n \approx 3.88$]

Geometrical Optics: Thin lenses

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}; \frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right); M = \frac{h_i}{h_o} = -\frac{v}{u}$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$	Diverging (Concave)	a.	When placed 4cm in front of a thin lens, the image of an object forms 2.4cm **in front of** the lens. Determine the focal length of the lens. [$\boldsymbol{f = -6cm}$]
	Converging (Convex)	b.	When placed 4cm in front of a thin lens, the image of an object forms 2.4cm **behind** the lens. Determine the focal length of the lens. [$\boldsymbol{f = 1.5cm}$]
	Real Object	c.	An object placed in front of a lens of focal length 2cm forms an image 3cm **behind** the lens. Determine the object distance from the lens. [$\boldsymbol{u = 6cm}$]
	Real Image	d.	Determine the image distance when an object is placed 12cm in front of a biconvex lens of focal length 6cm. [$\boldsymbol{v = 12cm}$]
	Virtual Image	e.	Determine the image distance when an object is placed 4cm in front of a biconcave lens of focal length 6cm. [$\boldsymbol{v = -2.4cm}$]

$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	
Converging	a. A biconvex thin lens (made up of a material of refractive index 1.65) is submerged in water of refractive index 1.33. The thin lens has radius of curvature of $R_2 = 2cm$ and $R_1 = 4cm$. Determine the focal length of the thin lens. [$\boldsymbol{f = 5.54cm}$]
Diverging	b. A biconvex thin lens (made up of a material of refractive index 1.65) is submerged in water of refractive index 1.33. The thin lens has radius of curvature of $R_1 = 2cm$ and $R_2 = 4cm$. Determine the focal length of the thin lens. [$\boldsymbol{f = -5.54cm}$]
R_1	c. The diagram shows a biconcave converging lens of focal length $15cm$, made up of a material of refractive index 1.44, placed in air. If surface R_2 of the lens has a radius of curvature of 12cm, what is the radius of curvature of surface R_1? [$\boldsymbol{f = 4.258cm}$] 
R_2	d. A plano-convex lens, as shown in the diagram, has a focal length of –12cm. The lens is made up of a material of refractive index 1.47 and is placed in air. determine radius of curvature of R_2. [$\boldsymbol{R_2 = 5.64cm}$] 
$n_{material}$	e. A convex-concave lens of focal length +35cm is placed in water. If the radii of its surfaces are $R_1 = 4cm$ and $R_2 = 7cm$, determine the refractive index of the lens material. [$n_{material} = 1.684$] 
n_{medium}	f. A plano-concave lens of focal length +35cm (made up of a material of refractive index 1.65) is placed in a medium of unknown refractive index. If the radius of its surface is $R_2 = 7cm$, determine the refractive index of the medium the lens is in. [$n_{medium} = 2.063$] 

Physical Optics

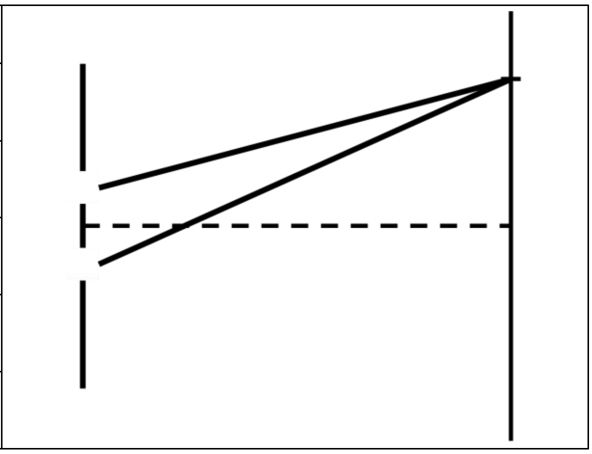
Constructive and Destructive Interference

	Path Difference
Constructive Interference	
Destructive Interference	

Case 1: Double Slit Interference

Definitions of variables

Symbol	Definition
y_m	
m	
λ	
D	
d	



Positions of bright and dark fringes

Types	Position
Bright Fringes	
Dark Fringes	

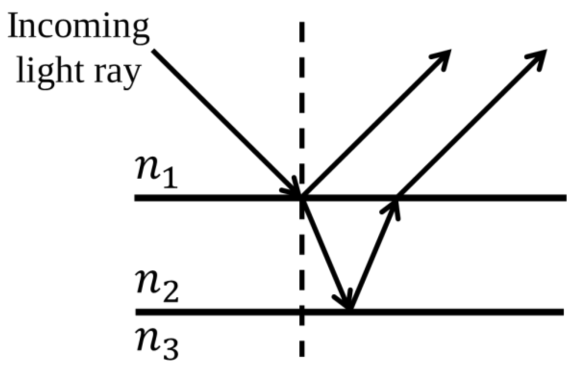
Example problem:

1. **[PSPM 19/20]**
   In a double slit experiment, the incident wavelength is 660nm, the slit separation is 0.25mm and the screen is placed 90cm away from the slits. Calculate the distance from the second to the third destructive interference fringe.

2. **Homework:**
   a. Two narrow, parallel slits separated by 0.250 mm are illuminated by green light (λ =546.1 nm). The interference pattern is observed on a screen 1.20 m away from the plane of the slits. Calculate the distance
      i. from the central maximum to the second bright region on either side of the central maximum **[Ans: 5.24mm]**
      ii. between the first and second dark bands. **[Ans: 2.62mm]**
   b. A light source emits visible light of two wavelengths: $\lambda = 440\,nm$ and $\lambda = 570\,nm$. The source is used in a double-slit interference experiment in which the distance between slit and screen is 1.75 m and slit separation is 0.05 mm. Find the separation distance between the third-order bright fringes.
   [Ans: 1.37cm]
   c. In a Young's Double Slit experiment, the slits are 4mm apart and are illuminated with a mixture of two wavelengths, $\lambda_1 = 600nm$ and $\lambda_2 = 900nm$. At what minimum distance from the common central bright fringe on a screen 5m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?
   [Ans: 2.3mm]

<u>Case 2: Thin Films Interference</u>

Phase change upon reflection

		
Conditions	Case 1:	Case 2:
Phase difference	$\Delta\phi =$	$\Delta\phi =$

Constructive and Destructive Interference

	Types	Position
Case 1: Non-reflective coating	Bright Fringes	
	Dark Fringes	
Case 2: Reflective coating	Bright Fringes	
	Dark Fringes	

<u>Example problem:</u>

1. **<u>[PSPM 19/20]</u>**
   A soap film with refractive index 1.3 and minimum thickness $0.177\mu m$ appears reddish under white light. Calculate the wavelength of light that is missing from the reflection. **[Ans: 460.2nm, 230.1nm, 153.4nm]**

2. **<u>Homework:</u>**
   - **a.** **<u>[PSPM 15/16]</u>**
     White light is incident on a soap film of refractive index 1.33 in air. The reflected light looks bluish because the red light of wavelength 670 nm is absent in the reflection.
     - i. Does the light change phase when it reflects at air-film interface? Explain your answer.
     - ii. Does the light change phase when it travels in film and reflects at film-air interface?
     - iii. What happen to the wavelength and frequency of light when it travels from air to the film?
     - iv. Determine the minimum thickness of the soap film. **[Ans: 252.nm]**
   - **b.** A material having a refractive index of 1.33 is used as an antireflective coating on a piece of glass (n=1.50). What should be the minimum thickness of this film to minimize reflection of 660nm light?
     [Ans: 124.1nm]

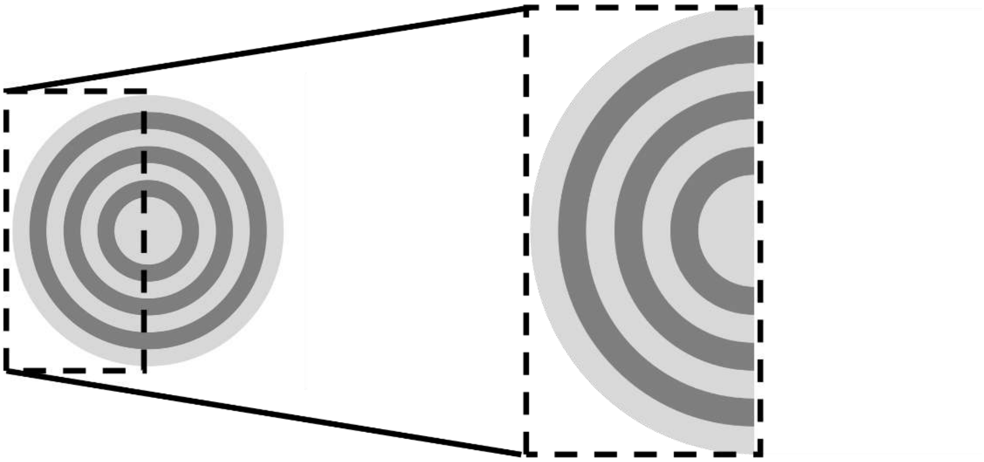
Case 3: Single Slit Diffraction

Condition for diffraction	

Constructive and destructive interference

Types	Position
Bright Fringes	
Dark Fringes	

Diffraction pattern

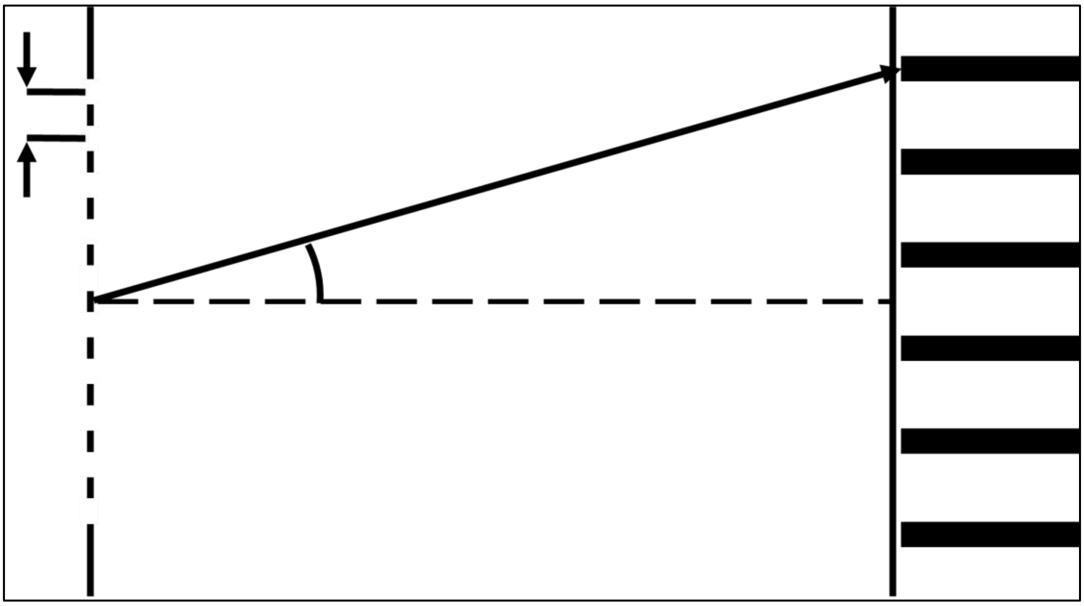


Determination of central bright fringe size

Example Problem:

1. **[PSPM 20/21]**
   A monochromatic light of wavelength 620 nm is incident on a single slit and forms a diffraction pattern on a screen 1.2m away. The distance of seventh dark fringe from the central maximum is 18.0 mm. Determine,
   a. the size of the single slit **[Ans: 0.289mm]**
   b. distance of the second bright fringe from the central maximum **[Ans: 6.44mm]**
2. **[Homework]**
   a. A beam of light is diffracted by a slit of width 0.75 mm. The diffraction pattern forms on a wall 3.5m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 6.2mm. **[Ans: 664.3nm]**
   b. Light of wavelength 600nm illuminates a single slit 0.55mm in width.
      i. At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 1.5mm from the centre of the principal maximum? **[Ans: 1.375m]**
      ii. What is the width of the central maximum? **[Ans:3.0mm]**

Case 4: Diffraction Grating



Type of interferences	Equation
Destructive	
Constructive	

Example Problem:

1. When light illuminates a grating with 6500 lines per centimetre, its second order maximum on a distance screen is at 62.4^o. What is the wavelength of light? **[Ans: 681.7nm]**
2. In a diffraction experiment, light of 700nm wavelength produces a second-order maximum 4.8mm from the central maximum on a screen 2m away from the grating. Determine the slit separation in the diffraction grating. **[Ans: 0.58mm]**
3. **[Homework]**
   a. In a diffraction experiment, light of 660nm wavelength produces a first-order maximum 0.35mm from the central maximum on a distance screen. A second monochromatic source produces a third order maximum 0.87mm from the central maximum when it passes through the same diffraction grating. What is the wavelength of the light for the second source? **[Ans: 546.86nm]**
   b. A diffraction grating has 2000 lines per centimetre. A monochromatic light of wavelength 500 nm is incident normally on the grating. What is the angular separation between the first and second order maxima?
   [Ans: 2.865^o]

Name	:	
Tutorial Class	:	

Chinese New Year Worksheet

1. **On Coulomb Force:**

A charge +q is located at the origin of the xy-plane and an identical charge is located 20cm away on the x-axis. A third charge of +2q is located on the x-axis at such a place that the net electrostatic force on the charge at the origin quadruples, its direction remaining unchanged. Where should the third charge be located?

[0.163m from origin]

2. **On Capacitors and Electric Field:**

Consider the Earth and a cloud layer 820m above the Earth to be the plates of a parallel-plate capacitor.

a. If the cloud layer has an area of $1.27km^2$, what is the capacitance?
[$\mathbf{1.371(10^{-8})}F$]

b. If an electric field strength greater than $4.91 \times 10^6 NC^{-1}$ causes the air to break down and conduct charge (lightning), what is the maximum charge the cloud can hold?
[55.186C]

c. If the cloud is holding this maximum charge, what is the magnitude of the difference in electric potential between the cloud and the ground?
[$\mathbf{4.026 \times 10^9}$V]

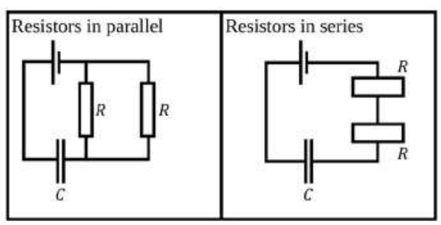
3. **On Temperature effects on Resistor**

The temperature coefficient of resistivity for the metal A and for metal B are $0.0036K^{-1}$ and $0.0054K^{-1}$. The resistance of metal A increases by 15% when temperature is increased. For the same increase in temperature, what is the percentage increase in resistance of metal B?

[22.5%]

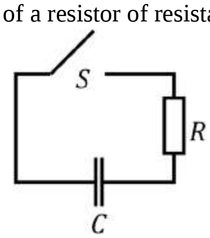
4. **On Charging and Discharging of Capacitors**

a. The figure below shows a circuit with resistors of the same resistance in different configuration.



Compare their time for discharge in terms of the time constant.

b. The figure shows an RC circuit that consists of a resistor of resistance $R = 20k\Omega$ and a capacitor of $C = 20\mu F$.



The capacitor is at voltage $V = 5V$ at $t = 0$ when the switch is closed.

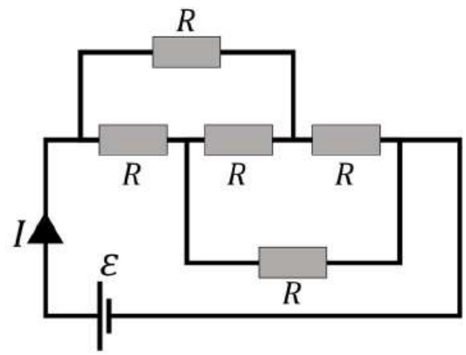
i. Determine the current when the switch S, is closed.
[0.25mA]

ii. Calculate the time for the current to decrease by 50%.
[0.2773s]

iii. Determine the time it takes for the capacitor to discharge to 0.25% of its initial voltage?
[0.5545s]

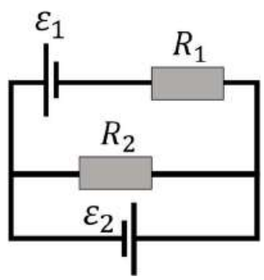
5. **On Kirchhoff's Laws**

a. The figure shows a circuit consisting of 5 resistors of equal resistance of 2Ω connected to a dry cell of terminal voltage 8V.



Determine the current I.

b. The diagram below shows a circuit consisting of 2 resistors and 2 dry cell.

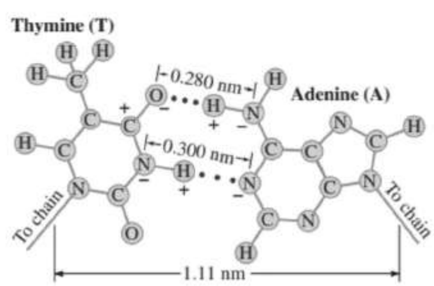


If the $\varepsilon_1 = 9V$, $\varepsilon_2 = 6V$, $R_1 = 22\Omega$ and $R_2 = 18\Omega$, determine the magnitudes and directions of the currents through the resistors in the diagram below.

[0.68A,0.33A]

6. [Optional]

The following diagram shows structures of two strands of the helix-shaped DNA molecule held together by electrostatic forces.



Assuming the net average charge indicated on hydrogen and nitrogen atoms has a magnitude of 0.21e, the net average charge indicated on carbon and oxygen atoms has a magnitude of 0.42e and that the atoms on each molecule are separated by $10^{-10}m$. Estimate the net force between a thymine and an adenine.

$[\mathbf{5.09 \times 10^{-10}} \boldsymbol{N}]$

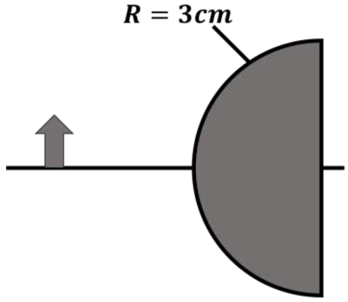
Name : ________________________________

Tutorial Class : ________________________________

Midsemester Worksheet

1. **Electromagnetic Induction:**
   a. A 0.4T magnetic field is directed 30^o to the plane of a loop of area $20cm^2$. Calculate the magnetic flux through the loop.
   [Ans: $4 \times 10^{-4}Wb$]
   b. A coil of 80 turns and diameter 20cm, connected to a 3Ω resistor, is placed in a region of changing magnetic field. The magnetic field changed by 200mT in 15s. Determine the induced emf in the coil and the induced current in the coil-resistor circuit.
   [Ans: $\varepsilon = 33.5mV$; $11.17mA$]
   c. Determine the inductance of a
      i. Solenoid of radius 2cm that has 250turns and a length of 12cm,
      [Ans: 0.82mH]
      ii. Circular coil of radius 10cm that has 5 turns and a length of 2cm.
      [Ans: 0.05mH]
   d. Determine the rate of current change in a solenoid or radius 5cm with 1000 turns and length of 10cm if it were to produce and induced emf of 5V.
   [Ans: $\frac{dI}{dt} = 51As^{-1}$]

2. **AC circuits:**
   An RLC circuit consisting of a 100Ω resistor, a $1H$ inductor and a $10\mu F$ capacitor, connected to an AC source of 340V peak voltage at 60Hz. Determine the
   a. Impedance of the circuit,
   [Ans: $Z = 150\Omega$]
   b. rms voltage of the circuit,
   [Ans: $V_{rms} = 240V$]
   c. Power factor of the circuit,
   [Ans: $\cos\theta = \frac{2}{3}$]
   d. Frequency at which minimum impedance is achieved.
   [Ans: 50.33Hz]
3. **Reflection upon a spherical mirror:**
   An object is placed in front of a concave mirror with a radius of curvature of 12cm. A real image twice the size of the object is formed. Determine the object distance from the mirror.
   [Ans: $u = 6cm$]
4. **Refraction upon a spherical surface:**
   An object of height 2cm is placed 12cm in front of a convex end of a glass rod in air of radius 4cm. If a real image is formed at 44cm from the object, determine the refractive index of the glass rod.
   [Ans: $n_2 = 1.52$]
5. **Thin lenses:**
   An object is placed 40cm in front of a thin lens (in water of refractive index 1.33) is shown in the diagram below.



   If the thin lens has a refractive index of 1.52, determine the image distance from the lens.
   [Ans: $v = 44.21cm$]

Problems [1.5hrs]

Current and DC Circuits

1. In a particular cathode ray tube, the measured beam current is 60.0 μA. How many electrons strike the tube screen every 40.0 s? **[2 marks]**

2. A resistor is constructed of a carbon rod that has a uniform cross-sectional area of 7.00 mm^2. When a potential difference of 15.0 V is applied across the ends of the rod, the rod carries a current of 5 mA. Find

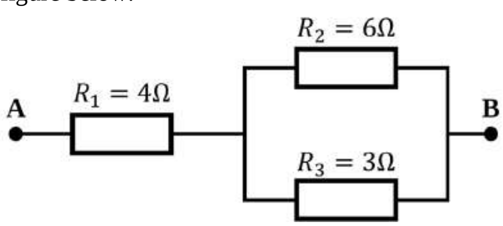
(a) the resistance of the rod **[2 marks]**

(b) the rod's length if the resistivity of the carbon rod is $3.5 \times 10^{-5}\Omega m$. **[2 marks]**

3. The resistance of a platinum wire is to be calibrated for low-temperature measurements. A platinum wire with resistance 3.00Ω at 25.0°C is immersed in liquid nitrogen at 77 K (-196°C). If the temperature response of the platinum wire is linear, what is the expected resistance of the platinum wire at -196°C? if the temperature coefficient of platinum is $3.95 \times 10^{-3}\ {}^{o}C^{-1}$. **[2 marks]**

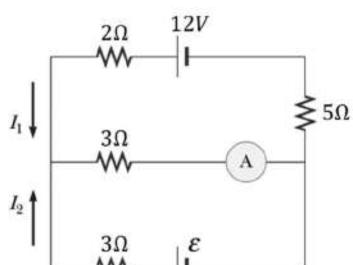
4. An automobile battery has an emf of 14V and an internal resistance of 0.05Ω. The headlights together present equivalent resistance 7.50 Ω (assumed constant). What is the potential difference across the headlight bulbs when they are the only load on the battery. **[3 marks]**

5. Find the equivalent resistance between point A and B in the figure below.



If a potential difference of 72V is applied across A and B, determine the current in each resistor. **[7 marks]**

6. Referring to the diagram below, the ammeter reads 1.392A.



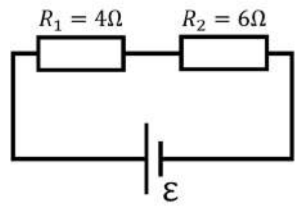
Determine I_1, I_2 and ε. **[8 marks]**

7. An electric heater is rated at 2100W, a toaster at 900W, and an electric grill at 1200W. The three appliances are connected to a common 240V household circuit.

a. How much current does each draw? **[6 marks]**

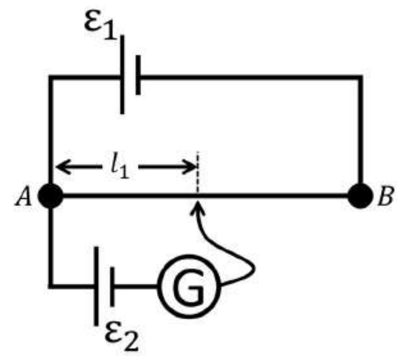
b. Is a circuit with a 13.0A fuse sufficient in this situation? Explain your answer. **[3 marks]**

8. The diagram below shows 2 resistors in series in a circuit.



If the voltage across the 4Ω resistor is 16V, determine ε. **[2 marks]**

9. The diagram shows a potentiometer set up to determine the emf of a dry cell.



ε_1 has an emf of 12V and the length of between A and B is 2m. If the galvanometer is found to be balanced when l_1 is 25cm, determine ε_2. **[2 marks]**

Geometrical Optics

1. An object is placed 4cm in front of a concave mirror of radius 12cm. Determine the image distance and its characteristics. **[5 marks]**

2. An object embedded inside a glass rod, 6cm from its curved end of radius 2cm. If the glass rod has a refractive index of 1.35, determine the image distance from the object. **[3 marks]**

3. An 3cm object is placed 30cm from a biconvex thin lens in water (n=1.33). The surfaces of the thin lens have radii of 15cm and a virtual image is found to form at 90cm from the thin lens. Determine the refractive index of the thin lens and the image size. **[4 marks]**

Physical Optics

1. In a Young's Double Slit experiment setup, a light source of wavelength 630nm is used. The screen is placed 50cm from the slit. If the second bright fringe is found 0.9mm from the central point of the interference pattern, determine the slit separation. **[2 marks]**

2. An oil film (n = 1.47) floating on water is illuminated by white light at normal incidence. The film is 380 nm thick. Find the colour of the light in the visible spectrum most strongly reflected. **[4 marks]**

3. Determine the size of the central maximum of a diffraction pattern formed by a single slit illuminated by a light of wavelength 660nm, with the slit size of 0.1cm and screen distance from the slit of 2m. **[2 marks]**

4. Light from an argon laser strikes a diffraction grating that has 5000 grooves per centimetre. The central and first order principal maxima are separated by 0.5 m on a wall 1.5 m from the grating. Determine the wavelength of the laser light. **[2 marks]**

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Chapter 9: Light Quantization
Chapter 8: Wave Properties of Particle
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Tutorial Questions

1 Chapter 1: Electrostatics

Coulomb's Law

1. Calculate the Coulomb force between a charged particle of $2\mu C$ and another particle of $5\mu C$. Is it repulsive or attractive? Sketch the situation described and label the Coulomb forces on each particle.

2. Three point charges are placed at the following points on the x-axis: $+2\mu C$ at $x = 0$, $-3\mu C$ at $x = 40cm$, $-5\mu C$ at $x = 120cm$. Find the force on the $-3\mu C$ charge.

3. Consider two charges of $4\mu C$ and $3\mu C$ that are placed such that the separation between them is $1m$. Where, between the two aforementioned charges, should a third charge of $-2\mu C$ be placed so that it experiences zero net Coulomb force.

4. Three charges are placed on the vertices of an equiliateral triangle with sides 2m. If the charges are $2\mu C$, $-2\mu C$ and $3\mu C$, calculate the net force (and its direction) on a test charge if the test charge is placed in the middle of the triangle.

5. Four point charges are placed on the corners of a square that is $25cm$ on its side. The charge on the top right is $3\mu C$, the top left, $-2\mu C$, bottom left, $1.5\mu C$ and the charge on the bottom right to be $-2\mu C$. Calculate the net force exerted on the $-2\mu C$ by the three other charges.

Electric Field

1. Calculate the electric field at a distance of $0.2m$ from a charge of $3nC$.

2. A point charge of $2\mu C$ is placed at the origin of coordinates. Calculate the electric field at $(x, y) = (3, 4)m$.

3. Point charges $2\mu C$, $-5\mu C$ and $+8\mu C$ are placed at coordinates $(-1, 2)cm$, $(2, 2)cm$ and $(1.5, -3)cm$ respectively. Calculate the electric field at the origin. Calculate the force exerted on a charge of $0.5\mu C$ if it were to be placed on the origin.

4. Four point charges are placed on the corners of a square that is $20cm$ on its side. The charge on the top right is $3\mu C$, the top left, $-2\mu C$, bottom left, $1.5\mu C$ and the charge on the bottom right to be $-2\mu C$. Calculate the net electric field at the centre of the square.

5. *Three parts:

   (a) Consider two particles of charges $2\mu C$ and $-2\mu C$ placed on coordinates $(0, -1)m$ and $(0, 1)m$ respectively. Calculate the electric field at coordinate $(5, 0)m$ and $(20, 0)m$.

   (b) Now consider if instead we placed a single $4\mu C$ at the origin, calculate the electric field at coordinate $(5, 0)m$ and $(20, 0)m$.

   (c) Compare the numerical values for part (a) and (b). Comment on your results.

Tutorial Questions

Electrical Potential

1. What is the electric potential at $0.5m$ from a $40\mu C$ point charge?
2. The work done in bringing a charge of $20\mu C$ from infinity to a point in an area of electric field is $3 \times 10^4 J$. What is the electric potential at that point?
3. A hollow sphere has a radius of $2cm$ and contains a charge of $5\mu C$. Assuming this sphere to be isolated, calculate the potential at a distance of $75cm$ from the centre of the sphere.
4. Three equal point charges of $3\mu C$ are placed at three corners of a square of sides $20cm$. Find the electrical potential at the fourth corner of the square.
5. Two point charges X and Y of charge $3.5\mu C$ and $2\mu C$ respectively are placed $15cm$ from each other. Calculate the work done to move charge Y to $3cm$ nearer to charge X.

Charge in Uniform $\vec{E}$

1. 2 plate arrange parallel to each other are spaced $0.6cm$ apart. They are connected to a 90V battery. Find the electric field between them.
2. In the Milikan experiment, an oil drop carries 3 electronic charges and has a mass of $1.35 \times 10^{-12} g$. This oil drop is held at rest between two horizontal charged plates $1.8cm$ apart. What voltage must there be between the two charged plates to keep the oil drop at rest?
3. A proton is accelerated from rest through a potential difference of $1kV$. What is its final speed?
4. An electron moves from plate A (of electrical potential $-60V$) to plate B (of electrical potential $+50V$). Assuming the system is in vacuum, what is the speed of the electron just before it hits plate B?
5. An electron enters a region of electric field generated by two parallel plates of potential difference $150V$, with a plate separation of $50mm$. The length of the parallel plates is 65mm. If the electron enters the region with velocity $9 \times 10^6 ms^{-1}$, Calculate the
   (a) the acceleration of the electron in its vertical axis [Ans: $5.3 \times 10^{14} ms^{-2}$]
   (b) the time for which the electron travels through the region of electric field [Ans: $7.2 \times 10^{-9} s$]
   (c) the deviation of the electron upon exiting, from its original path [Ans: $1.37 \times 10^{-2} m$]
   (d) the velocity of the electron in its vertical axis as it exits the electric field [Ans: $3.8 \times 10^6 ms^{-1}$]

Solutions:

Capacitance in series and parallel

$\boxed{C = \frac{Q}{V}; U = \frac{1}{2}QV}$

Tutorial Questions

1.

$$
\begin{aligned}
C &= \frac{Q}{V} \\
&= \frac{4000(10^{-6})\,C}{1000\,V} \\
&= 4\mu C \\
U &= \frac{1}{2}QV \\
&= \frac{1}{2}(4000\,\mu C)(1000\,V) \\
&= 2\,J
\end{aligned}
$$

2.

$$
\begin{aligned}
V &= \frac{Q}{C} \\
&= \frac{24(10^{-8})\,C}{12000(10^{-12})\,F} \\
&= 20V
\end{aligned}
$$

3.

$$
\begin{aligned}
Q_i &= CV_i\,;\; Q_f = CV_f \\
\Delta Q &= Q_f - Q_i\,;\; \Delta V = V_f - V_i \\
\Delta Q &= C\Delta V \\
C &= \frac{\Delta Q}{\Delta V} \\
&= \frac{60\mu C - 30\mu C}{50V - 35V} \\
&= 2\mu F \\
\Delta U &= U_f - U_i \\
&= \frac{1}{2}(Q_f V_f - Q_i V_i) \\
&= \frac{1}{2}(50V(60\mu C) - 35V(30\mu C)) \\
&= 0.975mJ
\end{aligned}
$$

4. C_2 and C_3 in series

$$
C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}
$$

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C_{23} and C_1 are parallel to each other

$$\begin{aligned}
C_{eq} &= C_1 + C_{23} \\
&= C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} \\
&= 10\mu + \left(\frac{1}{12\mu} + \frac{1}{8\mu}\right)^{-1} \\
&= 14.8\mu F
\end{aligned}$$

5.

$$\begin{aligned}
V_{total} &= V_1 = V_{23} = 26V \\
C_{23} &= \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} \\
\text{Then for } j &= \{2, 3\}, \\
V_j &= \{\frac{Q_{23}}{C_2}, \frac{Q_{23}}{C_3}\} \\
&= \{\frac{C_{23}}{C_2}V_{23}, \frac{C_{23}}{C_3}V_{23}\} \\
&= \{14.85,\ 11.15\}V
\end{aligned}$$

Capacitors Charging and Discharging

$\boxed{\tau = RC;\ Q = Q_o e^{-\frac{t}{RC}};\ Q = Q_o(1 - e^{-\frac{t}{RC}})}$

1. $\tau = RC = (1000\Omega)(4000 \times 10^{-6}\,F) = 4s$
2. $Q = Q_o e^{-\frac{t}{\tau}} = (20\mu)e^{-\frac{8s}{4s}} = 2.71\mu C$
3.

$$\begin{aligned}
\tau &= RC_{eq} \\
&= (2000)\left(\frac{1}{0.003\,F} + \frac{1}{0.005\,F}\right)^{-1} \\
&= 3.75s \\
\text{50\% full: } &\Rightarrow \frac{Q_f}{Q_o} = 0.5 \\
0.5\,Q_o &= Q_o(1 - e^{-\frac{t}{RC}}) \\
\frac{t}{RC} &\approx 0.69 \\
t &\approx 2.6s
\end{aligned}$$

Capacitors with dielectrics

$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_o};\ C_o = \frac{\varepsilon_o A}{d};\ C = \varepsilon_r C_o}$

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1.

$$\begin{aligned} C = \varepsilon_r C_o &= \varepsilon_r \frac{\varepsilon_o A}{d} \\ &= 6\frac{(8.85 \times 10^{-12})(\pi(2.5 \times 10^{-2})^2)}{3 \times 10^{-3}} \\ &= 34.754pF \end{aligned}$$

2.

$$\begin{aligned} Q|_{\varepsilon_r=3} &= C_{with\,dielectric} V \\ &= \varepsilon_r C_o V \\ &= (3)(4000 \times 10^{-12})(24) \\ &= 0.288\mu F \end{aligned}$$

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2 Chapter 2: Capacitors

Capacitance in series and parallel

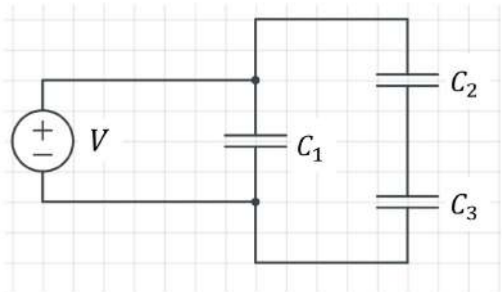
$\boxed{C = \frac{Q}{V};\ U = \frac{1}{2}QV}$

1. When the potential difference is $1kV$, the two plates of a capacitor hold $+4000\mu C$ and $-4000\mu C$ of charge respectively. What is the capacitance of this capacitor? Calculate the energy stored in the capacitor.

2. What is the voltage across a capacitor which has the capacitance of $12000\,pF$ and when fully charged, holds $24 \times 10^{-8}\,C$ of charge?

3. When the voltage across a capacitor is increased from $35V$ to $50V$, the charge of the capacitor was found to have increased from $30\mu C$ to $60\mu C$. Calculate the capacitance of the capacitor and the energy change.

4. In the figure below, suppose $C_i = \{10\mu, 12\mu, 8\mu\}F$.



   Calculate the equivalent capacitance of the circuit.

5. Referring to question 4, what would the voltage across each capacitor be if $C_i = \{2\mu, 3\mu, 4\mu\}F$ and $V = 26V$?

Capacitors Charging and Discharging

$\boxed{\tau = RC;\ Q = Q_o e^{-\frac{t}{RC}};\ Q = Q_o(1 - e^{-\frac{t}{RC}})}$

1. Calculate the time constant τ for an RC circuit with a $1k\Omega$ resistor and a $4000\mu F$ capacitor.

2. In the RC circuit, the capacitor initially has a charge of $20\mu C$. Calculate the charge in the capacitor after $8s$, if the time constant for the circuit is $4s$.

3. Calculate the time needed to charge capacitors the circuit below to 50% full.

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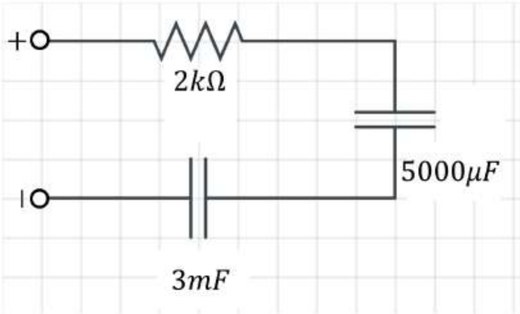

Capacitors with dielectrics

$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_o};\ C_o = \frac{\varepsilon_o A}{d};\ C = \varepsilon_r C_o}$

1. What is the capacitance of a pair of circular plates with a radius of $2.5cm$ separated by $3mm$ of mica, that has a dielectric constant of 6.
2. A $4000pF$ air-gap capacitor is supplied with $24V$. Calculate the charge flow after a dielectric material of dielectric constant 3 is added between the plates.

Solutions:

Capacitance in series and parallel

$\boxed{C = \frac{Q}{V};\ U = \frac{1}{2}QV}$

1.

$$\begin{aligned} C &= \frac{Q}{V} \\ &= \frac{4000(10^{-6})\,C}{1000\,V} \\ &= 4\mu C \\ U &= \frac{1}{2}QV \\ &= \frac{1}{2}(4000\,\mu C)(1000\,V) \\ &= 2\,J \end{aligned}$$

2.

$$\begin{aligned} V &= \frac{Q}{C} \\ &= \frac{24(10^{-8})\,C}{12000(10^{-12})\,F} \\ &= 20V \end{aligned}$$

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3.

$$\begin{aligned}
Q_i &= CV_i\,;\ Q_f = CV_f\\
\Delta Q &= Q_f - Q_i\,;\ \Delta V = V_f - V_i\\
\Delta Q &= C\Delta V\\
C &= \frac{\Delta Q}{\Delta V}\\
&= \frac{60\mu C - 30\mu C}{50V - 35V}\\
&= 2\mu F\\
\Delta U &= U_f - U_i\\
&= \frac{1}{2}(Q_f V_f - Q_i V_i)\\
&= \frac{1}{2}(50V(60\mu C) - 35V(30\mu C))\\
&= 0.975mJ
\end{aligned}$$

4. C_2 and C_3 in series

$$C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

C_{23} and C_1 are parallel to each other

$$\begin{aligned}
C_{eq} &= C_1 + C_{23}\\
&= C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}\\
&= 10\mu + \left(\frac{1}{12\mu} + \frac{1}{8\mu}\right)^{-1}\\
&= 14.8\mu F
\end{aligned}$$

5.

$$\begin{aligned}
V_{total} &= V_1 = V_{23} = 26V\\
C_{23} &= \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}\\
&\text{Then for } j = \{2, 3\},\\
V_j &= \{\frac{Q_{23}}{C_2},\ \frac{Q_{23}}{C_3}\}\\
&= \{\frac{C_{23}}{C_2}V_{23},\ \frac{C_{23}}{C_3}V_{23}\}\\
&= \{14.85,\ 11.15\}V
\end{aligned}$$

Capacitors Charging and Discharging

$$\boxed{\tau = RC;\ Q = Q_o e^{-\frac{t}{RC}};\ Q = Q_o(1 - e^{-\frac{t}{RC}})}$$

Tutorial Questions

1. $\tau = RC = (1000\Omega)(4000 \times 10^{-6}\,F) = 4s$
2. $Q = Q_o e^{-\frac{t}{\tau}} = (20\mu)e^{-\frac{8s}{4s}} = 2.71\mu C$
3.

$$\begin{aligned}
\tau &= RC_{eq} \\
&= (2000)\left(\frac{1}{0.003\,F} + \frac{1}{0.005\,F}\right)^{-1} \\
&= 3.75s \\
\text{50\% full:} \ &\Rightarrow \frac{Q_f}{Q_o} = 0.5 \\
0.5\,Q_o &= Q_o(1 - e^{-\frac{t}{RC}}) \\
\frac{t}{RC} &\approx 0.69 \\
t &\approx 2.6s
\end{aligned}$$

Capacitors with dielectrics

$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_o};\ C_o = \frac{\varepsilon_o A}{d};\ C = \varepsilon_r C_o}$

1.

$$\begin{aligned}
C = \varepsilon_r C_o &= \varepsilon_r \frac{\varepsilon_o A}{d} \\
&= 6\frac{(8.85 \times 10^{-12})(\pi(2.5 \times 10^{-2})^2)}{3 \times 10^{-3}} \\
&= 34.754pF
\end{aligned}$$

2.

$$\begin{aligned}
Q|_{\varepsilon_r=3} &= C_{with\,dielectric}V \\
&= \varepsilon_r C_o V \\
&= (3)(4000 \times 10^{-12})(24) \\
&= 0.288\mu F
\end{aligned}$$

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3 Chapter 3: DC circuits

Electric Conduction, Resistivity & Ohm's Law

1. A current of $8A$ is maintained in a conductor for $30s$. How much charge has flowed through the conductor in this time ?

2. Calculate the magnitude of current if 3×10^{23} electrons has flowed through a conductor in $32\,minutes$.

3. Calculate the resistance of a copper resistor of $30m$ long and $0.3mm$ diameter, assuming the resistivity of copper is $1.7 \times 10^{-8}\,\Omega\,m$.

4. What is the potential difference that is required to pass $5A$ through 30Ω?

5. A electric conductor tube has an inner tube diameter of $0.7cm$ and an outer diameter of $1cm$. Find the electric resistance if it has the length of $30cm$ and resistivity of $10^{-7}\Omega m$.

Resistance Variation with Temperature

1. Based on the table below, calculate the resistance at temperature T_f for each of the following cases:

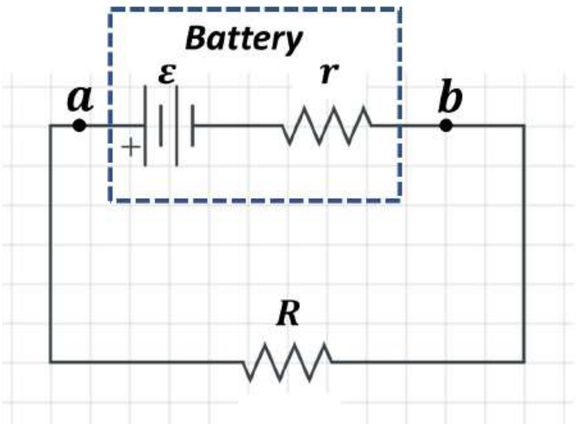
Question	R_i	T_f	T_i	α
a	200Ω	$47K$	$22K$	0.004041^oC^{-1}
b	$15k\Omega$	$5K$	$20K$	0.005671^oC^{-1}
c	100Ω	15^oC	$293.15K$	0.003715^oC^{-1}

a

2. Calculate temperature at which conductor has the resistance of 200Ω if the conductor has a thermal coefficient of resistance 0.0038^oC^{-1} and has a resistance of 150Ω at 50^oC.

Electromotive Force, Series and Parallel Circuit

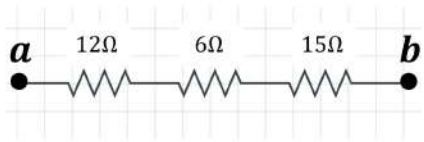
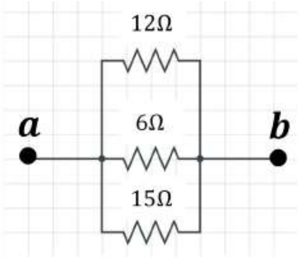
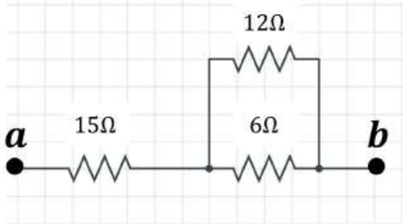
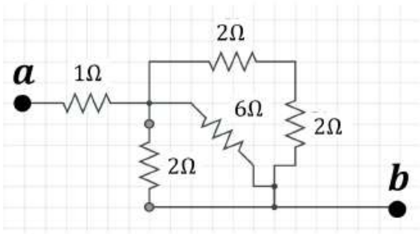
1. The figure below shows a simple circuit consisting of a battery and a resistor.



If the electromotive force of the battery is $9V$, the battery has an internal resistance of 0.5Ω and the resistor has a resistance of 6Ω, calculate the potential difference between points a and b.

2. Calculate the effective resistance between points a and b for each of the following diagram:

Tutorial Questions

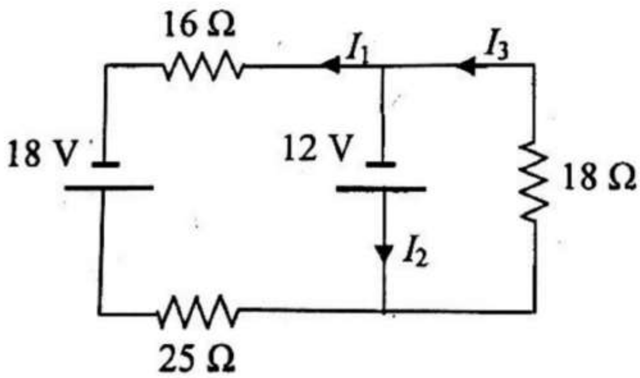
a) Case 1:	b) Case 2:
	
c) Case 3:	d) Case 4:
	

Kirchhoff's Law

1. Calculate the values for x and y for the following pair of linear equations:

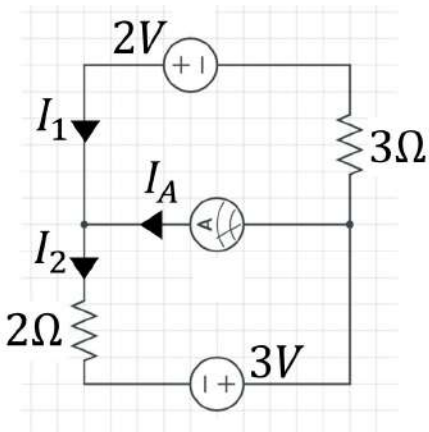
a)	$2x - 2y = 18$ $5x - 4y = 42$	b)	$4x = 14 + 2y$ $5y = 5x - 25$
c)	$x + 4y = 35$ $4x + 4y = 44$	d)	$x = 14 + y$ $4y = 4 - 3x$

2. **[PSPM 2014/2015]** The following diagram shows circuit consisting of 3 resistors and 2 emf.



Calculate I_1, I_2 and I_3, and the potential difference across the 18Ω resistor.

3. **[PSPM 2016/2017]** The following diagram shows circuit consisting of 2 batteries, 2 resistors and an ammeter.



If the ammeter has internal resistance of 5Ω, what is the reading shows by the ammeter?

4. **[PSPM 2017/2018]** The following diagram shows circuit consisting of 2 batteries and 3 resistors.

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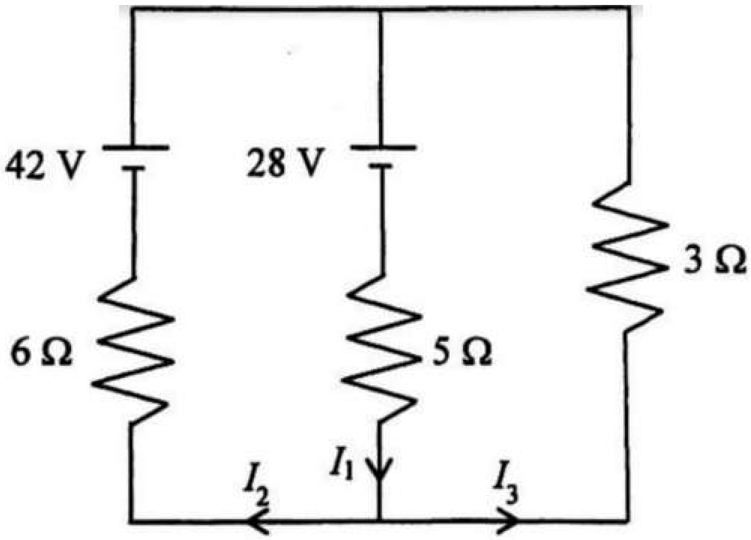
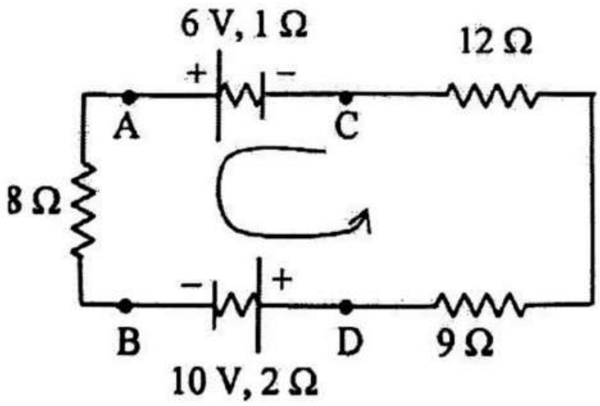


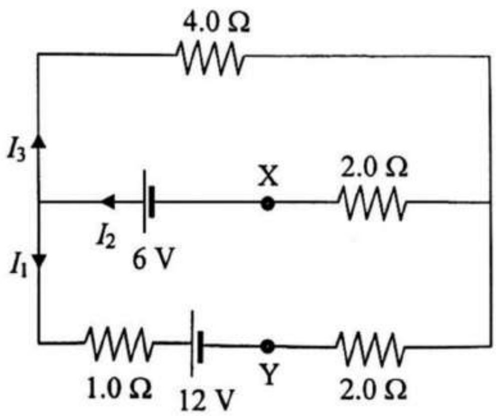
Calculate currents I_1, I_2 and I_3.

5. **[PSPM 2012/2013]** The following diagram shows circuit consisting of 2 batteries and 3 resistors.



By using the anticlockwise loop as shown, calculate the current that flows through the 8Ω resistor and the potential difference across point A and C, V_{AC}.

6. **[PSPM 2011/2012]** The following diagram shows circuit consisting of 2 batteries and 3 resistors.



(a) Calculate I_1, I_2 and I_3.

(b) Calculate the potential difference between points X and Y.

(c) Calculate the total power dissipated in the circuit.

Electrical Energy and Power

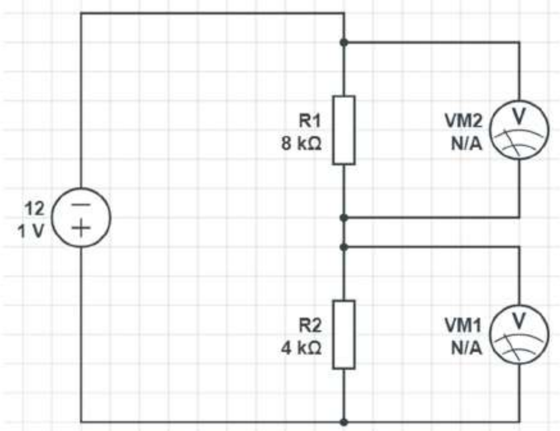
1. A 200Ω resistor has a current of $0.5A$ running through it, calculate the power lost through the resistor.

2. A bulb rated $120V/90W$ is operated from a 120V-source. Find the current running through it and the resistance of the bulb.

3. What is the resistance of a $1000W$ toaster running at $120V$? Calculate the heat energy dissipated if it was left running continuously for 2 minutes.

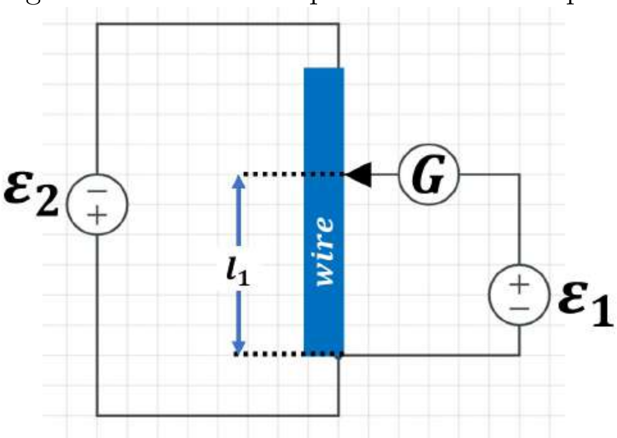
Potential Divider & Potentiometer

1. The following diagram shows circuit consisting of 1 battery, 2 voltmeters and 2 resistors.



Calculate the voltmeter readings.

2. The diagram below shows a potentiometer setup.



When $\varepsilon_2 = 1.5V$, the gavanometer gives a zero reading at $l_1 = 25cm$. What is the emf of ε_2 if the galvanometer gives a zero reading when $l_1 = 45cm$?

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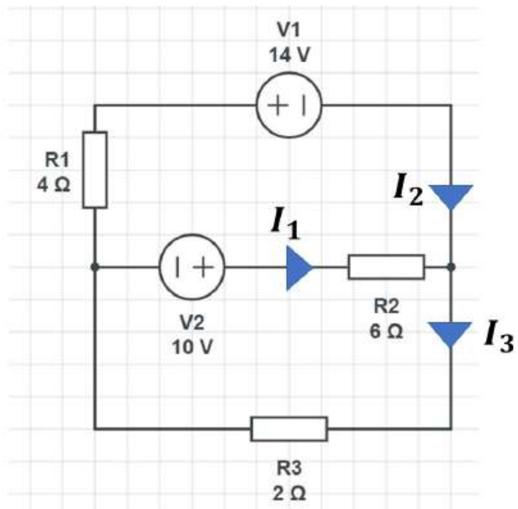
Extras

1. A wire $50m$ long and $2mm$ in diameter is connected to a source with a potential difference $9.11V$ and the current was found to be $36A$. Identify the metal out of which the wire is made out of.

2. Aluminium and copper wires of equal length are found to have the same resistance, what is the ratio of their radii?

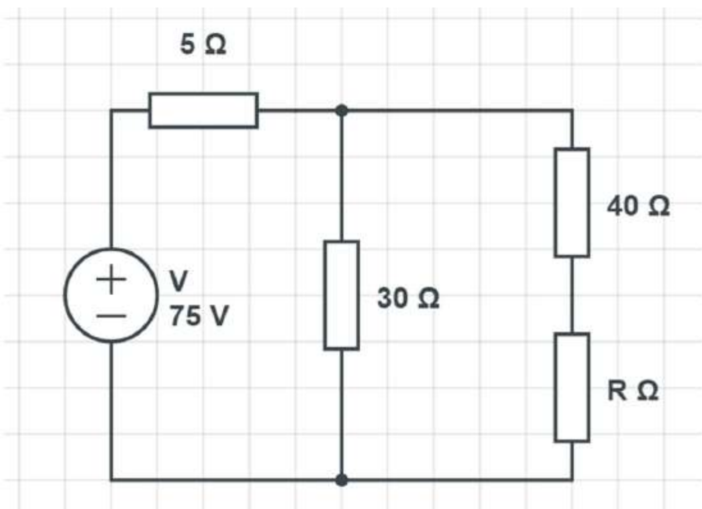
3. Calculate I_1, I_2 and I_3 in the figure below.



4. For $\tau = RC$, show that τ has units of time using Ohm's Law.

5. A lamp ($R = 150\Omega$), an electric heater ($R = 25\Omega$) and a fan ($R = 50\Omega$) are connected in parallel across a $120 - V$ line.
   Calculate the total current supplied to the circuit and the power expended by the heater.

6. The resistor R dissipates $20W$ of power, determine the value of R.



Solutions:

Electric Conduction, Resistivity & Ohm's Law

1.
$$I = \frac{Q}{t} \Rightarrow Q = It = (8A)(30s) = 240C$$

2.
$$I = \frac{Q}{t} = \frac{Ne}{t} = \frac{(3 \times 10^{23})(1.6 \times 10^{-19})}{32 \times 60} = 25A$$

3.
$$R = \rho \frac{l}{A}$$
$$R = (1.7 \times 10^{-8}\,\Omega\,m)\frac{30}{\pi\,(\frac{0.3 \times 10^{-3}}{2})^2}$$
$$R \approx 7.215\Omega$$

4.
$$V = IR = (5A)(30\Omega) = 150V$$

5.
$$A = \pi(r_{outer}^2 - r_{inner}^2) = \frac{\pi}{4}(d_{outer}^2 - d_{inner}^2)$$
$$R = \rho\frac{l}{A} = \rho\frac{l}{\frac{\pi}{4}(d_{outer}^2 - d_{inner}^2)}$$
$$R = \frac{(10^{-7})(30 \times 10^{-2})}{\frac{\pi}{4}((1 \times 10^{-2})^2 - (0.7 \times 10^{-2})^2)}$$
$$R = 0.75m\Omega$$

Resistance Variation with Temperature

1.

Question	R_f
a	220.205Ω
b	$13.724k\Omega$
c	98.1425Ω

2.
$$R_f = R_i[1 + \alpha(T_f - T_i)]$$
$$T_f = \frac{\frac{R_f}{R_i} - 1}{\alpha} + T_i$$
$$T_f = \frac{\frac{200}{150} - 1}{0.0038} + (50)$$
$$T_f \approx 137.72^oC$$

Electromotive Force, Series and Parallel Circuit

1.

$$I = \frac{\varepsilon}{R + r} = \frac{9}{6 + 0.5} \approx 1.385V$$
$$V_{ab} = \varepsilon - rI = 9V - 0.5(1.385) \approx 8.3075V$$

2. (a) Case 1:

$$R_{eff} = 12 + 6 + 15 = 33\Omega$$

(b) Case 2:

$$R_{eff} = \left(\frac{1}{12} + \frac{1}{6} + \frac{1}{15}\right)^{-1}$$

$$R_{eff} = \frac{60}{19}\Omega$$

(c) Case 3:

$$R_{eff} = 15 + \left(\frac{1}{12} + \frac{1}{6}\right)^{-1}$$
$$R_{eff} = 19\Omega$$

(d) Case 4:

$$R_{eff} = 1 + \left(\frac{1}{2+2} + \frac{1}{6} + +\frac{1}{2}\right)^{-1}$$
$$R_{eff} = \frac{23}{11}\Omega$$

Tutorial Questions

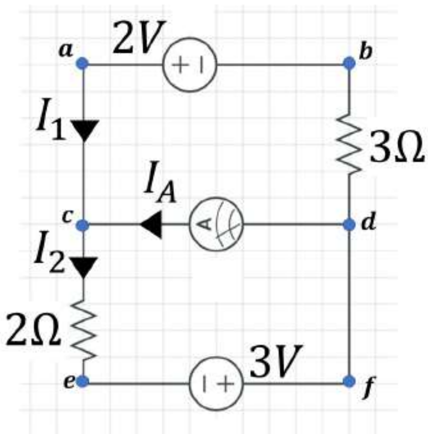
Kirchhoff's Law

1. Calculate the values for x and y for the following pair of linear equations:

a) $x = 6$, $y = -3$	b) $x = 2$, $y = -3$
c) $x = 3$, $y = 8$	d) $x = 4$, $y = -2$

2.

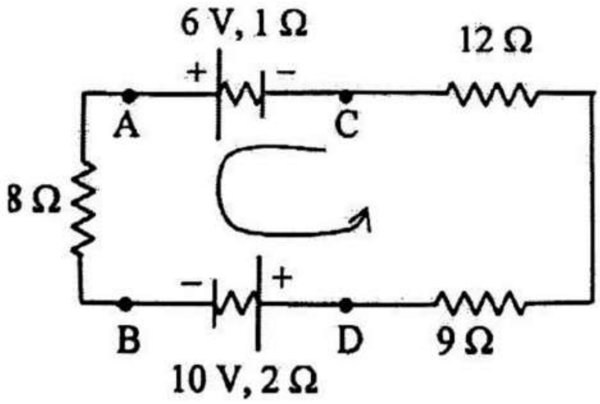
$$\sum I_{in} = \sum I_{out}$$
$$I_3 = I_1 + I_2$$
Take loop with emf 18V and 12V,
$$\sum \varepsilon = \sum V_{resistor}$$
$$\varepsilon_{18} - \varepsilon_{12} = I_1(R_{16} + R_{25})$$
$$18 - 12 = I_1(16 + 25)$$
$$I_1 = 0.15A$$
Take loop with emf 18Ω and 12V,
$$\varepsilon_{12} = I_3 R_8$$
$$I_3 = 0.67A$$
$$I_2 = I_3 - I_1 = 0.52A$$
$$V_{18\Omega} = I_3 R_{18} = 12V$$
$$I_i = \{0.15, 0.52, 0.67\}A$$

3. 

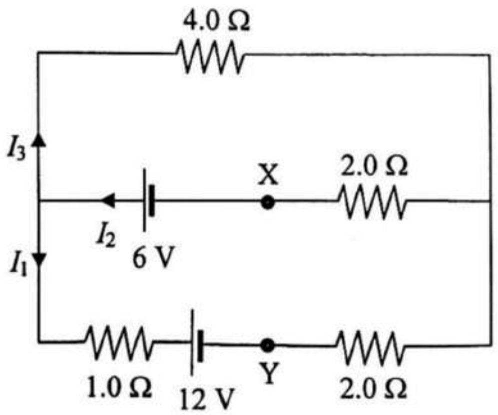
$I_2 = I_A + I_1$ Loop abdca: $2 = -5I_A + 3I_1$ Loop efdce: $3 = 5I_A + 2I_2$ $I_A = 0.16A$

4.

Loop with resistors 5Ω and 6Ω :
$$42 - 28 = 5I_1 + 6I_2$$
$$5I_1 + 6I_2 = 14$$
Loop with resistors 5Ω and 3Ω :
$$5I_1 + 3I_3 = -28 I_i = \{-2,\ 4,\ -6\}A$$

5. 

$$\text{Total emf}, \sum \varepsilon = 6V + 10V = 16V$$
$$\text{Total resistance}, \sum R = 2 + 12 + 9 + 1 + 8 = 18\Omega$$
$$\text{Current}, I = \frac{\sum \varepsilon}{\sum I} = \frac{16}{32} = 0.5A$$
$$V_{AC} = 6 - I(1) = 6 - 1(0.5) = 5.5V$$

6. 

(a)

$$I_2 = I_1 + I_3$$
$$Loop\,1:\ 6 - 12 = 3I_1 + 2I_2$$
$$Loop\,2:\ 6 = 4I_3 + 2I_2$$
$$I_i = \{-1.85, -0.23, 1.62\}A$$

(b) $\sum \varepsilon = \sum IR$ $V_{XY} + 6 - 12 = 1(-1.85)$ $V_{XY} = 4.15V$

(c) $P = \sum I^2 R = 20.9W$

Electrical Energy and Power

1. $P = IV = I^2R = (0.5)^2(200) = 50W$

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2. $I = \frac{P}{V} = \frac{90}{120} = 0.75A$
   $R = \frac{V}{I} = \frac{120}{0.75} = 160\Omega$

3. $P = IV = \frac{V^2}{R}$
   $R = \frac{V^2}{P} = \frac{(120)^2}{1000}$
   $W = Pt = (1000W)(120s) = 120kJ$

Potential Divider & Potentiometer

1. VM1: $V_1 = \frac{4000}{8000+4000} V_{source} = 4V$
   VM2: $V_1 = \frac{8000}{8000+4000} V_{source} = 8V$

2. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$
   $\frac{1.5}{\varepsilon_2} = \frac{25}{45}$ $\varepsilon_2 = 2.7V$

Extras

1.

$$
\begin{aligned}
R &= \frac{V}{I} = \frac{\rho l}{A} \\
\rho &= \frac{VA}{Il} = \frac{V(\frac{d}{2})^2}{Il} \\
\rho &= \frac{9.11 \times (\frac{2\times 10^{-3}}{2})^2}{36 \times 50m} \\
&\approx 1.59 \times 10^{-8}\Omega m \Rightarrow \text{Silver}
\end{aligned}
$$

2.

$$
\begin{aligned}
R_{Al} &= R_{Cu} \\
l_{Al} &= l_{Cu} \\
\rho_{Al} &= \frac{R_{Al}A_{Al}}{l_{Al}} \\
\rho_{Cu} &= \frac{R_{Cu}A_{Cu}}{l_{Cu}} \\
\frac{\rho_{Al}}{\rho_{Cu}} &= \frac{\frac{\cancel{R_{Al}}A_{Al}}{\cancel{l_{Al}}}}{\frac{\cancel{R_{Cu}}A_{Cu}}{\cancel{l_{Cu}}}} \\
\frac{\rho_{Al}}{\rho_{Cu}} &= \frac{A_{Al}}{A_{Cu}} = \frac{2.82}{1.7} \approx 1.66 \\
A_{Al} &= 1.66 \times A_{Cu} \\
\cancel{\pi} r_{Al}^2 &= 1.66 \cancel{\pi} r_{Cu}^2 \\
r_{Al}^2 &= 1.66\, r_{Cu}^2
\end{aligned}
$$

3.

Junction Rule:

$$I_3 = I_1 + I_2$$

Loop Rule:

$$
\begin{aligned}
10V &= 6I_1 + 2I_3 - - - (1) \\
14V &= -4I_2 - 2I_3 \\
14V &= -4(I_3 - I_1) - 2I_3 \\
14V &= 4I_1 - 6I_3 \\
I_i &= \{2, -3, -1\}A
\end{aligned}
$$

4.

$$\tau = RC = (\frac{V}{I})C = \frac{Q}{I} = t$$

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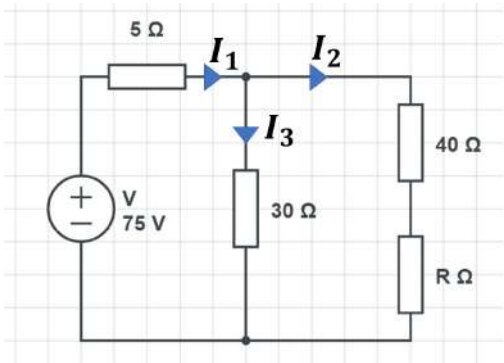
5.

$$V = IR$$
$$I = \frac{V}{R_{eff}} = \frac{12}{(\frac{1}{150} + \frac{1}{50} + \frac{1}{25})^{-1}} = 8A$$
$$P_{heater} = IV = I^2R = 8^2(25) = 1.6kW$$

6. 

Junction Rule:

$$I_1 = I_2 + I_3$$

Loop Rule - 1 :

$$75 = 5I_1 + 30I_3$$
$$75 = 5I_1 + 30I_1 - 30I_2$$
$$75 = 35I_1 - 30I_2$$

Loop Rule - 2 :

$$75 = 5I_1 + (40 + R)I_2$$
$$I_2 = \frac{450}{310 + 7R}$$
$$P = IV = I^2R$$
$$= (\frac{450\sqrt{R}}{310 + 7R})^2$$
$$\Rightarrow R = 20$$

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4 Chapter 5: Electromagnetic Induction

Magnetic Flux & Induced emf

1. State the definition and the SI unit of magnetic flux $\phi_{magnetic}$.
2. What is the difference between magnetic field strength and magnetic flux?
3. A quarter loop of area $15cm^2$ is placed in a region of magnetic field $0.16T$. Calculate the magnetic flux if the angle between the quarter loop area and the magnetic field is
   (a) 90^o
   (b) 70^o
   (c) 30^o
4. A loop of wire of area $40cm^2$ is placed in a magnetic field of strength $2cT$. Calculate the angle between the magnetic field and the wire loop if the magnetic flux is
   (a) $80\,\mu Wb$
   (b) $69\,\mu Wb$
   (c) $56\,\mu Wb$
5. A solenoid of cross-sectional area $60cm^2$ is placed in a magnetic field of strength $5cT$. The cross-sectional area of th solenoid is kept perpendicular to the magnetic field, and it was found that the magnetic flux to be $0.9Wb$. Calculate the number of turns found in the solenoid.
6. A circular coil of wire (of N turns, radius r) is placed in a magnetic field of strength B. The number of turns is then doubled, at what angle (between the magnetic field and cross-sectional area of solenoid) would the magnetic flux of the new solenoid be the same as the old solenoid (before it was doubled in number of turns)?
7. $2A$ current is flowed through a solenoid(of 10000 winding per metre, $80mm$ radius). Calculate the flux through the solenoid.
8. A magnetic field perpendicular to a circular coil (18 turns, radius $50mm$) changes from $2T$ to $20T$ in $3s$, Calculate the magnitude of the induced emf.
9. $3A$ current is flowed through a solenoid(of 5000 winding per metre, $60mm$ radius, $3cm$ length). Calculate the induced emf if the magnetic flux through it is increased by $1mWb$ in $240ms$.
10. The figure below shows a rectangular coil of 40 turns and length $l = 20cm$. It is pulled in a uniform magnetic field $B = 2T$

Calculate the magnitude and direction of the induced emf in the wire if the electrical resitance of the wire is $R = 5\Omega$ and the coil is pulled with a speed of $v = 20cm\,s^{-1}$.

11. An AC generator consisting a 30 turn coil with cross sectional area of $0.1m^2$ and resistance of 100Ω. The coild rotates in a magnetic field of strength $0.5T$ at a frequency of $30Hz$. Calculate the maximum induced current.

Self-Inductance

1. What is **self-inductance**? Explain the process.

2. Induced emf of $6V$ is developed across a coil when the current flowing through it changes at $30As^{-1}$. Determine the self-inductance of the coil.

3. If the current in a $230mH$ changes steadily from $20mA$ to $28mA$ in $140ms$. What is the induced emf?

4. Calculate the value of self-inductance for an air filled solenoid of length $5cm$ and cross-sectional area of $0.3cm^2$ containing 50 loops.

5. Suppose you wish to make a solenoid with a self inductance of $1.5mH$. Due to certain commercial constraints, it is imperative that the inductor must have a cross-sectional area of $2.2 \times 10^{-3}m^2$ and a length of $10cm$. How long of a wire would you need to make this solenoid?

6. A 500turns of solenoid is $8cm$ long. When the current in the solenoid is increased by $2.5A$ in $0.35s$, the magnitude of the induced emf is $0.012V$. Calculate the inductance of the solenoid and the cross sectional area of the solenoid.

Inductor-stored Energy & Mutual Inductance

1. Two coils, X & Y are magnetically coupled. The emf induced in coil Y is $2.5V$ when the current flowing through coil X changes at the rate of $5As^{-1}$. Determine the mutual inductance of the coils and the emf induced in coil X if there is a current flowing through coil Y which changes at the rate of $1.5As^{-1}$.

2. The magnetic fild inside an air-filled solenoid $36cm$ long and $2cm$ in diameter is $0.85T$. Approximately how much energy is stored in this field?

3. A coil has an inductance 45 mH and resistance 0.3Ω. An emf of $12V$ is applied to the coil until equilibrium current is achieved. Calculate the energy stored in the coil.

4. A current of 5.0 A flows in a 400 turn solenoid that has a length of $30.0cm$ and cross-sectional area of $2.00 \times 10^{-4}m^2$. Calculate the energy stored in the solenoid.

5. An electric current of $1.5A$ flowing through a coil P produces a total magnetic flux of $0.540Wb$ in the coil. If a coil Q is brought near coil P, the total magnetic flux of 0.144 Wb is produced in coil Q.

   (a) Calculate the self—inductance of coil P, and determine the energy stored in coil P before coil Q is brought near it.

   (b) Calculate the mutual inductance of coil P with coil Q.

   (c) If the current in coil P is reduced uniformly from $1.5A$ to $0A$ in $0.3s$, determine the induced e.m.f of coil Q.

6. Two coaxial coils are wound around the same cylindrical core. The primary coil has 350 turns and the secondary coil has $200turns$. When the current in the primary coil $6.5A$, the average flux through each turn of the secondary coil is $0.018Wb$. Calculate the mutual inductance of the pair of coils.

7. A current of $3.0A$ flows in coil C and is produced a magnetic flux of 0.75 Wb in it. When a coil D is moved near to coil C coaxially, a flux of $0.25Wb$ is produced in coil D. If coil C has 1000 turns and coil D has 5000 turns.

   (a) Calculate self-inductance of coil C and the energy stored in C before D is moved near to it

   (b) Calculate the mutual inductance of the coils

   (c) If the current in C decreasing uniformly from $3.0A$ to zero in $0.25s$, calculate the induced emf in coil D.

Magnetic Flux & Induced emf Solution

1. $\phi_{magnetic} = \vec{B} \cdot \vec{A}$ where $\vec{A}$ is the vector perpendicular to the surface area and $\vec{B}$ is the magnetic field strength

2. Magnetic field strength is the amount of magnetizing force, the magnetic flux is a measurement of flux ("magnetic field lines") flowing through a surface.

3. A quarter loop of area $15cm^2$ is placed in a region of magnetic field $0.16T$. Calculate the magnetic flux if the angle between the quarter loop area and the magnetic field is

   (a) $90^o \Rightarrow \phi_B = (0.16)(0.15(10^{-2}))\,cos\,0^o = 0.00024\,Wb$

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(b) $70^o \Rightarrow \phi_B = (0.16)(0.15(10^{-2}))\, cos\, 20^o = 0.00023\, Wb$

(c) $30^o \Rightarrow \phi_B = (0.16)(0.15(10^{-2}))\, cos\, 60^o = 0.00012\, Wb$

4. A loop of wire of area $40cm^2$ is placed in a magnetic field of strength $2cT$. Calculate the angle between the magnetic field and the wire loop if the magnetic flux is

   (a) $80\,\mu Wb \Rightarrow \theta = 90^o - cos^{-1}(\frac{\phi}{BA}) = 90^o - cos^{-1}(\frac{80(10^{-6})}{(2(10^{-2}))(0.40(10^{-2}))}) = 90^o$

   (b) $69\,\mu Wb \Rightarrow \theta = 90^o - cos^{-1}(\frac{\phi}{BA}) = 90^o - cos^{-1}(\frac{69(10^{-6})}{(2(10^{-2}))(0.40(10^{-2}))}) = 60^o$

   (c) $56\,\mu Wb \Rightarrow \theta = 90^o - cos^{-1}(\frac{\phi}{BA}) = 90^o - cos^{-1}(\frac{56(10^{-6})}{(2(10^{-2}))(0.40(10^{-2}))}) = 44.427^o$

5. $N = \frac{\phi_B}{BAcos\theta} = \frac{0.9Wb}{60(10^{-4})(0.05)\, cos(90)} = 3000\,\text{turns}$

6.

$$\begin{aligned}
\Phi_{old} &= N_{old}B_{old}A_{old}\, cos\, \theta_{old} \\
\Phi_{new} &= N_{new}B_{new}A_{new}\, cos\, \theta_{new} \\
\Phi_{old} &= \Phi_{new};\; B_{old} = B_{old};\; A_{old} = A_{old} \\
N_{new} &= 2N_{old} \\
\theta_{new} &= cos^{-1}\left(\frac{1}{2}\, cos\, (\theta_{old})\right)
\end{aligned}$$

7.

$$\begin{aligned}
\phi &= BA \\
&= (\mu_o nI)(\pi r^2) \\
&= (4\pi \times 10^{-7})(10000)(2)\pi(80 \times 10^{-3})^2 \\
&= 51.2\pi^2\, \mu Wb
\end{aligned}$$

8.

$$\begin{aligned}
\varepsilon &= N\frac{d\phi}{dt} \\
&= NA\,\frac{dB}{dt} \\
&= (18)(\pi)(50 \times 10^{-3})^2\, \frac{20-2}{3} \\
&= 0.27\pi V
\end{aligned}$$

9.

$$\begin{aligned}
\varepsilon &= N\frac{d\phi}{dt} \\
&= (5000 \times 3(10^{-2}))\frac{10^{-3}}{240(10^{-3})} \\
&= 0.625V
\end{aligned}$$

10.

$$\begin{aligned}
\varepsilon &= BLv \\
&= (2)(20\times10^{-2})(20\times10^{-2}) \\
&= 0.08V \\
I &= \frac{\varepsilon}{R} = \frac{0.08}{5} = 0.016 \\
F &= IlB\,sin\,90^o \\
&= (0.016)(0.02)(2) = 0.64\,mN
\end{aligned}$$

11. $I_{max} = \frac{\varepsilon_{max}}{R} = \frac{2\pi fNBA}{R} = \frac{(2\pi)(30)(30)(0.5)(0.1)}{100} = 0.9\pi\,A$

Self-Inductance

1. Self inductance is when the induced current creates another magnetic field which opposes the initial induced emf.
2. $L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$ $L = -\frac{6}{(30)} = -5H$
3. $\varepsilon = -L\frac{dI}{dt} = -(230\times10^{-3})\frac{(28-20)(10^{-3})}{(140(10^{-3}))} = 0.01314286V$
4. $L = \frac{\mu_o N^2 A}{l} = \frac{(4\pi\times10^{-7})(50)^2(0.3\times10^{-4})}{(5\times10^{-2})} = 1.885\mu H$
5. $N^2 = \frac{Ll_{solenoid}}{\mu_o A} = \frac{(1.5(10^{-3}))(0.1)}{(4\pi\times10^{-7})(2.2\times10^{-3})}$
   $N \approx 233$ turns
   $l_{wire} = N(2\pi r) = N(2\pi\sqrt{\frac{A}{\pi}})$
   $l_{wire} = (233)(2\pi\sqrt{\frac{2.2\times10^{-3}}{\pi}}) = 38m$
6. $L = -\frac{\varepsilon}{\frac{dI}{dt}} = -\frac{0.012}{\frac{2.5}{0.35}} = 1.68mH$
   $L_{solenoid} = \frac{\mu_o N^2 A}{l}$
   $A = \frac{L_{solenoid} l}{\mu_o N^2}$
   $A = \frac{(1.68\times10^{-3})(0.08)}{(4\pi\times10^{-7})(500)^2} = 4.28\times10^{-4}m^2$

Inductor-stored Inductor & Mutual Inductance

1.

$$\begin{aligned} M &= -\frac{\varepsilon_Y}{\frac{dI_x}{dt}} \\ &= 0.5H \\ \varepsilon_X &= M\frac{dI_Y}{dt} \\ &= 0.75V \end{aligned}$$

2.

$$\begin{aligned} U &= \frac{1}{2}LI^2 \\ L &= \frac{\mu_o N^2 A}{l} \\ B &= \mu_o nI\frac{\mu_o NI}{l} \\ U &= \frac{1}{2}\frac{\mu_o N^2 A}{l}\frac{B^2}{2\mu_o}AL \\ &= 1.8\times 10^{-10}J \end{aligned}$$

3.

$$M = \frac{N_2\phi_2}{I} = 0.55H$$

4. $U = 36J$
5. $U = 1.68mJ$
6. (a) $0.36H, 0.41J$
   (b) $0.096H$
   (c) $0.48V$
7. $M = \frac{N_2\phi_2}{I_1} = 0.55H$
8. A current of $3.0A$ flows in coil C and is produced a magnetic flux of 0.75 Wb in it. When a coil D is moved near to coil C coaxially, a flux of $0.25Wb$ is produced in coil D. If coil C has 1000 turns and coil D has 5000 turns.

   (a)

$$\begin{aligned} L_C &= \frac{N_C\Phi_C}{I_C} \\ &= \frac{1000\times 0.75}{3} = 250H \\ U &= \frac{1}{2}L_C I_C^2 \\ &= \frac{1}{2}(250)(3)^2 = 1125J \end{aligned}$$

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(b)

$$\begin{aligned} M_D &= \frac{N_D \Phi_D}{I_C} \\ &= \frac{(5000)(0.25)}{(3)} \\ &= 417H \end{aligned}$$

(c)

$$\begin{aligned} \varepsilon_D &= -M\frac{dI_C}{dt} \\ &= -(417)\frac{-3}{0.25} \\ &= 5004V \end{aligned}$$

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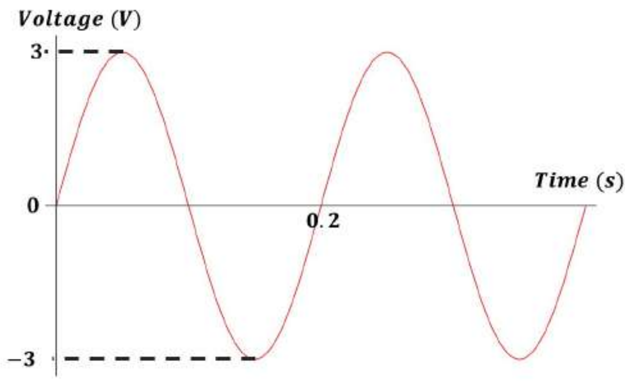
5 Chapter 6: AC circuits

Question:

6.1 AC & 6.2 RMS

$V = V_{peak}\,sin\,(\omega t)$; $I = I_{peak}\,sin\,(\omega t)$; $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$; $I_{rms} = \frac{I_{peak}}{\sqrt{2}}$

1. The voltage generated by a generator is as shown in the graph below.



   (a) What is the peak voltage, peak-to-peak voltage and the rms voltage?

   (b) The voltage is connected across a resistor with a resistance 2.5Ω. Calculate the peak, rms current and average power.

2. For the following situation, sketch the waveform and write down the equation for

   (a) voltage as a function of time with frequency $50Hz$ and peak voltage of $2V$.

   (b) voltage as a function of time with period of $20s$. and peak-to-peak voltage of $2V$.

3. A sinusoidal, $60.0Hz$, ac voltage is read to be $120V$ by an ordinary voltmeter. What is the equation for the voltage ? Sketch the waveform.

4. A voltage $V = 60sin120\pi t$ is applied across a 20Ω resistor. Calculate the reading on the ac ammeter and the average power.

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6.3: Resistance, Reactance & Impedance
6.4: Power Factor

$X_C = \frac{1}{2\pi fC}$; $X_L = 2\pi fL$; $Z = \sqrt{R^2 + (X_L - X_C)^2}$; $\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$; $P_{ave} = I_{rms}V_{rms}cos\,\phi$; $P = IV$; $cos\phi = \frac{P_{real}}{P_{apparent}} = \frac{P_{real}}{I^2 Z} = \frac{R}{Z} = \frac{P_{ave}}{IV}$;

1. $R-$ **circuit:**
   A purely resistive circuit consist of a resistor rated $120V$, $1200W$ and an AC of frequency $100Hz$. Calculate the current drawn, the maximum value of current and the resistance in the dryer.

2. $L-$ **circuit:**
   A circuit contains a $400mH$ inductor and a resistor of 0.5Ω connected to an AC power supply of $80V$ at the frequency of $100Hz$. Determine the reactance of the inductor. How does it compare to the resistance of the resistor? Calculate the rms current and the maximum current.

3. $C-$ circuit:
   A $50\mu F$ capacitor is connecte across the terminals of an AC power supply, of which has a sinusoidal output of $50Hz$ with maximum voltage of $100V$. Determine the rms current and maximum current in the circuit.

4. $RL-$ circuit:
   A coil has an inductance of $0.1H$ and a resistance of 12Ω. Together with a resistor of 10Ω, it is connected (in series) to a $110V$, $60Hz$ line.
   Determine
   (a) the reactance of the coil
   (b) the impedance of the coil
   (c) the current in the circuit
   (d) the power factor of the circuit
   (e) the phase angle between current and supply voltage
   (f) the wattmeter reading in the circuit.

5. $RC-$ circuit:
   A $10\mu F$ capacitor is in series with a 40Ω resistance and the combination is connected to a $110V$, $60Hz$ line.
   Calculate
   (a) the capacitive reactance
   (b) the impedance of the coil
   (c) the current in the circuit
   (d) the power factor of the circuits
   (e) the phase angle between current and supply voltage

6. $RLC-$ circuit - Potential Difference:
   A circuit has a resistance of 11Ω, a coil of inductive reactance 120Ω and a capacitor of 100Ω, all connected in series with $110V, 60Hz$ power source. What is the potential difference across each circuit element.

7. $RLC-$ circuit - Resonance Frequency:
   The impedance of a series $RLC-$circuit is 8Ω when frequency $f = 60Hz$ at resonance and 10Ω at $80Hz$. Calculate the values of L and C.

Answer:

6.1 AC & 6.2 RMS

1. (a) peak voltage, $V_{peak} = 3V$
   peak-to-peak voltage, $V_{peak-to-peak} = 6V$
   rms voltage, $V_{rms} \approx 2.12132V$

   (b) Peak current, $I_o = \frac{V_o}{R} = \frac{3}{2.5} = 1.2A$
   rms current, $I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{1.2}{\sqrt{2}} = 0.84853A$
   average power $P_{ave} = \frac{V_{rms}^2}{R} = \frac{(2.12132)^2}{2.5} = 1.8W$

2. For the following situation, sketch the waveform and write down the equation for

   (a) $V(t) = V_{peak}\, sin(2\pi ft) = 2sin(2\pi(50)t) = 2sin(100\pi t)$

   (b) $V(t) = \frac{V_{peak-to-peak}}{2}\, sin(2\pi(\frac{1}{T})t) = \frac{2}{2}\, sin(2\pi(\frac{1}{20})t) = sin(0.1\pi t)$

3. $V(t) = (V_{rms} \times \sqrt{2})\, sin(2\pi ft) = (120 \times \sqrt{2})\, sin(2\pi(60)t) = 170\, sin(120\pi t)$

4. $I_{rms} = \frac{V_{rms}}{R} = \frac{V_o}{\sqrt{2}}\frac{1}{R} = \frac{60}{\sqrt{2}}\frac{1}{20} = 2.12A$
   $P_{ave} = I_{rms}^2 R = (2.12)^2(20) = 90W$

6.3: Resistance, Reactance & Impedance
6.4: Power Factor

$X_C = \frac{1}{2\pi fC}$; $X_L = 2\pi fL$; $Z = \sqrt{R^2 + (X_L - X_C)^2}$; $\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$; $P_{ave} = I_{rms}V_{rms}cos\,\phi$; $P = IV$; $cos\phi = \frac{P_{real}}{P_{apparent}} = \frac{P_{real}}{I^2 Z} = \frac{R}{Z} = \frac{P_{ave}}{IV}$;

1. $R-$ **circuit:**
   $I_{rms} = \frac{P_{ave}}{V_{rms}} = \frac{1200W}{120V} = 10A$
   $I_{max} = \sqrt{2}(10A) = 14.1A$
   $R = \frac{V_{rms}}{I_{rms}} = \frac{120}{10} = 12\Omega$

2. $L-$ **circuit:**
   $X_L = 2\pi\omega L = 2\pi \times 100Hz \times 0.4H = 251\Omega$
   $X_L \gg R$
   $Z \approx X_L$
   $Irms \approx \frac{V}{X_L} = \frac{80}{251} = 0.32A$ $I_{max} = \sqrt{2}I_{rms} = 0.453A$

3. $C-$ circuit:
   $I_{rms} = \frac{V_{rms}}{X_C}$
   $= V_{rms}\omega C = 2\pi fCV_{rms}$
   $= 2\pi(50)(50\mu)(\frac{100}{\sqrt{2}}) = 1.1A$
   $I_{max} = \sqrt{2}I_{rms} = 1.556A$

4. $RL-$ circuit:

   (a) $X_L = 2\pi fL = 2\pi(60Hz)(0.1) = 37.7\Omega$
   (b) $Z = \sqrt{R^2 + X_L^2} = \sqrt{(12)^2 + (37.7)^2} = 39.6\Omega$
   (c) $I_{rms} = \frac{V_{rms}}{Z} = \frac{V}{\sqrt{(R_{coil}+R_{resistor})^2+(X_L)^2}}$
   $= \frac{V}{\sqrt{(R_{coil}+R_{resistor})^2+(X_L)^2}} = \frac{110}{\sqrt{(12+10)^2+(37.7)^2}} = 2.52A$
   (d) $cos\,\phi = \frac{R_{coil}+R_{resistor}}{Z} = \frac{R_{coil}+R_{resistor}}{\sqrt{(R_{coil}+R_{resistor})^2+(X_L)^2}}$
   $cos\,\phi = 0.51$
   $\phi \approx 60^o$
   (e) $\phi \approx 60^o$, the voltage leades the current
   (f) $P_{ave} = IV\,cos\,\phi == (2.52)(110)(0.51) = 141.372W$

5. $RC-$ circuit:

   (a) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi(60)(10\mu)} = 265.3\Omega$
   (b) $Z = \sqrt{R^2 + X_C^2} = \sqrt{(40)^2 + (265.3)^2} \approx 263.3\Omega$
   (c) $I = \frac{V_{rms}}{Z} = 0.410$
   (d) $cos\,\phi = \frac{R}{Z} = \frac{40}{268.3} = 0.15$
   (e) $\phi = 81.4^o$

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6. $RLC-$ circuit - Potential Difference:
$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2+(X_L-X_C)^2}}$
$I_{rms} = \frac{110}{\sqrt{(11)^2+(120\Omega-100\Omega)^2}} = 4.8192A$
$V_R = IR = 4.8192A \times 11 = 53.0112V$
$V_L = IX_L = 4.8192A \times 120 = 578.304V$
$V_R = IR = 4.8192A \times 100 = 481.92V$
$V_{total} = \sqrt{V^2 + (V_L - V_C)^2} = \sqrt{(52.91)^2 + (577.2 - 481)^2} \approx 110V$

7. $RLC-$ circuit - Resonance Frequency:
At $f_{resonance}$, $f = \frac{1}{2\pi\sqrt{LC}}$

$$L = \frac{1}{4\pi^2(60)^C} - - - 1$$

Since $X_L = X_C$, then $Z = R = 8\Omega$.
At $f = 80Hz$,

$$Z^2 = 8^2 + (X_L - X_C)^2 = 10^2$$

$$\Downarrow$$

$$X_L - X_C = 2\pi\left(80L - \frac{1}{80C}\right) = 6\Omega - - - [2]$$

Solve by simultaneous equations [1] and [2] give

$$L = 0.0261H; C = 0.00027F.$$

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6 Chapter 7: Optics

Questions:

Reflection at a spherical surface

$$R = 2f$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$

Sign convention:

Quantity	$+$	$-$
u	left of vertex, real object	right of vertex, virtual object
v	left of vertex, real Image	right of vertex, virtual image
f	Concave mirror	Convex mirror

1. A convex mirror has a radius of curvature of $0.2m$, what is the focal length?

2. An object $200cm$ from the vertex of a spherical concave mirror is imaged $400cm$ in front of the mirror, what is the focal length of the mirror?

3. A display lamp having a bright $5cm$ long vertical filament is positioned $30cm$ from a concave mirror that projects the bulb's image onto a wall $9m$ from the vertex. What is the radius of curvature of the mirror? How big is the image?

4. An object $10cm$ high is $50cm$ from a concave mirror of $20cm$ focal length. Find the image distance, height and direction.

   Image is real, $\frac{1}{15}m$ high and inverted.

5. How far should an object be from a concave spherical mirror of radius $45cm$ to form a real image one-ninth it size?

Tutorial Questions

Refraction at a spherical surface

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

Sign convention:

Quantity	$+$	$-$
u	Real	Virtual
v	Real	Virtual
r	Convex surface	Concave surface

1. A cylindrical glass rod in air has a refractive index of 1.52. One end is ground to a hemispherical surface with radius, $r = 3.00cm$. Calculate the position of the image for a small object on the axis of the rod, $10.0cm$ to the curved end of the pole.

2. A small fish is $4cm$ from the far side of a spherical fish bowl $25cm$ in diameter. Ignoring the effects of the glass walls of the bowl, where does the observer see the image of the fish if the refractive index of water is 1.33?

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Thin lenses

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$m = \frac{h_i}{h_o} = -\frac{v}{u}$$

Sign convention:

Thin lenses		
Quantity	+	−
u	real object	virtual object
v	real Image	virtual image
f	Convex/converging lens	Concave/diverging lens
lensmaker's equation		
Quantity	+	−
R_1	convex	concave
R_2	concave	convex

Thin lens equation

1. An object is placed $x - cm$ from a thin lens with a $10cm-$focal length. Find the position of the image if
   (a) $x = 30cm$ and the lens is converging
   (b) $x = 30cm$ and the lens is diverging
   (c) $x = 5cm$ and the lens is converging
   (d) $x = 5cm$ and the lens is diverging
2. A $6cm$-high object is placed $30cm$ from a converging lens and its image forms $90cm$ from the lens and on the same side as the object. What is the focal length of the lens?
3. Calculate the position and focal length of a conveging lens which will project the image of an object with a magnification of 4, upon a screen $10m$ from the object.

Lensmakers' equation

1. A lens (made out of glass of refractive index 1.54) has a convex surface of radius $20cm$ and a concave surface of radius $40cm$. Calculate the focal length and deduce if the lens is converging or diverging.
2. A parallel beam of white light strikes a biconvex lens having faces of radii $32cm$ and $48cm$. The refractive indices of the glass for the A(red) and H (violet) spectral lines are 1.578 and 1.614 respectively. Calculate the distance between the focal points of red and violet radiations.
3. A double convex glass has faces of radii $18cm$ and $20cm$. Calculate the
   (a) focal length of the lens when an image is formed $32cm$ from the lens if the object is placed $24cm$ away from the lens.

(b) refractive index of the lens.

4. A symmetric lens with a focal length of $5cm$ is made of a material of refraction index 1.5. Calculate the refractive index of each surface of the lens?

Answer:

7.1 Reflection at a spherical surface

$$R = 2f$$
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$

Sign convention: $+f$ for concave mirror, $-f$ for convex mirror.

1. $$f = 0.5R = 0.5(0.2m) = 0.1m$$

2. $$f = \frac{uv}{v+u} = +\frac{8}{6}cm$$

3. $$\frac{R}{2} = f = \frac{uv}{v+u}$$
$$R = \frac{2uv}{v+u} = \frac{2(0.3)(9)}{0.3+9} = 0.58$$
$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$
$$h_i = -h_o\frac{v}{u} = -(0.05)\frac{9}{0.3} = -1.5m$$

4. $$v = \frac{uf}{u-f} = \frac{0.5 \times 0.2}{0.5 - 0.2} = \frac{1}{3}m$$
$$h_i = -h_o\frac{v}{u} = -(0.1)frac(\frac{1}{3})0.5 = -\frac{1}{15}m$$
Image is real, $\frac{1}{15}m$ high and inverted.

5. $$\frac{2}{R} = 2(\frac{1}{u} + \frac{1}{v}) = 45$$
$$\frac{v}{u} = \frac{1}{9} \Rightarrow u = 9v$$
$$\frac{2}{45} = \frac{1}{u} + \frac{9}{u} \Rightarrow u = 225cm$$

7.2 Refraction at a spherical surface

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

Sign convention:

Quantity	+	−
u	Real	Virtual
v	Real	Virtual
r	Convex surface	Concave surface

1.

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\frac{1.00}{10} + \frac{1.52}{v} = \frac{1.52 - 1.00}{+3} \Rightarrow v = +20.7cm$$

2.

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\frac{1.33}{25 - 4} + \frac{1}{v} = \frac{1 - 1.33}{-12.5} \Rightarrow v = -27.07cm$$

7.3 Thin lenses

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$m = \frac{h_i}{h_o} = -\frac{v}{u}$$

Sign convention:

Thin lenses		
Quantity	+	−
u	real object	virtual object
v	real Image	virtual image
f	Convex/converging lens	Concave/diverging lens
lensmaker's equation		
Quantity	+	−
R_1	convex	concave
R_2	concave	convex

Thin lens equation

1. An object is placed $x - cm$ from a thin lens with a $10cm-$focal length. Find the position of the image if

   (a) $x = 30cm$ and the lens is converging

   $$\frac{1}{10} = \frac{1}{30} + \frac{1}{v} \Rightarrow v = 15cm$$

   (b) $x = 30cm$ and the lens is diverging

   $$\frac{1}{-10} = \frac{1}{30} + \frac{1}{v} \Rightarrow v = -7.5cm$$

   (c) $x = 5cm$ and the lens is converging

   $$\frac{1}{10} = \frac{1}{5} + \frac{1}{v} \Rightarrow v = -10cm$$

   (d) $x = 5cm$ and the lens is diverging

   $$\frac{1}{-10} = \frac{1}{5} + \frac{1}{v} \Rightarrow v = -3.33cm$$

2. A $6cm$-high object is placed $30cm$ from a converging lens and its image forms $90cm$ from the lens and on the same side as the object. What is the focal length of the lens?

   $$\frac{1}{f} = \frac{1}{30} + \frac{1}{-90} \Rightarrow f = 45cm$$

3. Calculate the position and focal length of a conveging lens which will project the image of an object with a magnification of 4, upon a screen $10m$ from the object.

$$u + v = 10,\ \frac{q}{p} = 4 \Rightarrow p = 2\,\&\,q = 8$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{2} + \frac{1}{8} \Rightarrow f = +1.6m$$

Lensmakers' equation

1. A lens (made out of glass of refractive index 1.54) has a convex surface of radius $20cm$ and a concave surface of radius $40cm$. Calculate the focal length and deduce if the lens is converging or diverging.

$$\text{Case 1 - convexo-concave: } \frac{1}{f} = \left(\frac{1.51}{1} - 1\right)\left(\frac{1}{20} - \frac{1}{40}\right) \Rightarrow f = 74.1cm \Rightarrow \text{converging}$$

$$\text{Case 2 - concave-convex: } \frac{1}{f} = \left(\frac{1.51}{1} - 1\right)\left(\frac{1}{-20} - \frac{1}{-40}\right) \Rightarrow f = 74.1cm \Rightarrow \text{converging}$$

2. A parallel beam of white light strikes a biconvex lens having faces of radii $32cm$ and $48cm$. The refractive indices of the glass for the A(red) and H (violet) spectral lines are 1.578 and 1.614 respectively. Calculate the distance between the focal points of red and violet radiations.

$$\frac{1}{f} = \left(\frac{n}{1} - 1\right)\left(\frac{1}{32} - \frac{1}{-48}\right) \Rightarrow f = \frac{19.2}{n-1} \Rightarrow \Delta f = 1.95cm$$

3. A double convex glass has faces of radii $18cm$ and $20cm$. Calculate the
   (a) focal length of the lens when an image is formed $32cm$ from the lens if the object is placed $24cm$ away from the lens.

$$\frac{1}{f} = \frac{1}{24} + \frac{1}{32} \Rightarrow f = +13.7cm$$

   (b) refractive index of the lens.

$$\frac{1}{13.7} = \left(\frac{n}{1} - 1\right)\left(\frac{1}{18} - \frac{1}{-20}\right) \Rightarrow n = 1.69$$

4. A symmetric lens with a focal length of $5cm$ is made of a material of refraction index 1.5. Calculate the refractive index of each surface of the lens?

$$R_1 = r = -R_2 \Rightarrow \frac{1}{f} = \left(\frac{n_{material}}{1} - 1\right)\left(\frac{1}{r} - \frac{1}{-r}\right) \Rightarrow r = 2f \times (n-1) = 5cm$$

7 Chapter 8: Physical Optics

Questions:

Huygens' Principle & Wave Interferences

1. State Huygen's Principle and its application on light wave analysis.
2. Show (either diagrammatically or in forms of equation) that the laws of reflection and refraction can be verified using Huygen's wave theory.
3. What type of wave front will emerge from a point source and a distance light source?
4. Explain the application of Huygen's Principle in the case of a single slit.
5. When are two light sources of the same common frequency said to be coherent ?
6. What are the conditions for the interference of light?
7. From a path difference perspective, state the condition for constructive and destructive interference.

Young's Double Slit Experiment

1. The interference pattern of two identical slits separated by a distance $d = 0.25mm$ is observed on a screen at a distance of $0.5m$ from the plane of the slits. The illuminating light source has a wavelength of $589nm$. Calculate the distance of the first bright fringe to the central maximum, and the separation of the bright bands.
2. Light (of $589nm$) from a lamp forms an interference pattern on a screen $0.75m$ from a pair of slits. The bright fringes in the pattern are $0.4cm$ apart. Calculate the slit separation.
3. A laser light of $630nm$ incident on a pair of slits produces an interference pattern in which the bright fringes are separated by $8.3mm$. A second light, on the other hand, produces an interference pattern of which its dark fringes are separated by $7.6mm$. Calculate the wavelength of the second light.
4. Light from two light sources (of wavelengths λ_1 and λ_2 respectively) arrives at a double slit set up. If $\lambda_1 = 430nm$, what value must λ_2 be if the fourth order bright fringe of light with $\lambda_1 = 645nm$ overlaps with the sixth order bright fringe of λ_2 light?
5. In a double-slit experiment with light source $700nm$, a student is limited to a $70cm$ white-coloured cardboard sheet as a screen. Assuming intensity is not a limiting factor, calculate the highest order of bright fringe the student will be able to observe if the slit separation is $0.02mm$.

Thin Films

Media	phase change	Constructive Interference
$n_{1^{st}} > n_{2^{nd}} > n_{3^{rd}}$ or $n_{1^{st}} < n_{2^{nd}} < n_{3^{rd}}$	$0\,rad$	$2nt = m\lambda$
$n_{1^{st}} > n_{2^{nd}} < n_{3^{rd}}$ or $n_{1^{st}} < n_{2^{nd}} > n_{3^{rd}}$	$\pi\,rad$	$2nt = (m + 0.5)\lambda$

1. A soap film has a refractive index of 1.33. How thick is the film if one-half of a wavelength of red light (with a vacuum wavelength of $633nm$) extends from on surface to the other surface?

Tutorial Questions

2. A thin film of alcohol ($n_{alcohol} = 1.36$) spread on a flat glass plate and is illuminated with white light. If a region of the film reflects only green light ($\lambda_{green} = 500nm$) strongly, how thick is the film?

3. A glass lens with an index of 1.55 is to be coated with a film of index 1.30 to decrease the reflection normally incident green light of $\lambda = 500nm$. What minimum thickness should be deposited on the lens?

4. A film of oil ($n_{oil} = 1.42$) of $250nm$ thick floats on water. When illuminated from above with white light, what colour will reflect the most brightly?

5. A soap film of index 1.35 appears yellow ($580nm$) when viewed directly from above. Compute several possible values of its thickness.

Single Slit Diffraction

$$y_n = \frac{n\lambda D}{a} \text{ for dark fringes}$$
$$y_n = \frac{(n+0.5)\lambda D}{a} \text{ for bright fringes}$$

1. A soap film has a refractive index of 1.33. How thick is the film if one and a half of a wavelength of red light (with a vacuum wavelength of $633nm$) extends from on surface to the other surface?

2. If the separation between the first and the second minima of a single-slit diffraction pattern is $6.0mm$, what is the distance between the screen and the slit? The light wavelength is $500nm$ and the slit width is $0.16mm$.

3. When a single slit of width $0.3mm$ is illuminated with light of wavelength is $633nm$, the distance from the central maximum of the 1^{st} order minimum is $4cm$. Calculate this distance if the slit is doubled.

4. A single slit $0.1mm$ wide is illuminated by plane waves from a HeNe laser ($\lambda = 633nm$). If the screen is $10m$, determine the width of the central maximum.

5. A vertical single slit is illuminated with electromagnetic wave from a HeNe laser at $633nm$. It is found that the center of the second dark band lies at an angle of 4.2^o off the central axis. Determine the width of the slit.

Diffraction Grating

$$d\,sin\,\theta = n\lambda;\ d = \frac{1}{N}$$

1. A screen is placed $1.4m$ away from a diffraction grating illuminated with light of wavelength $633nm$. If the second- and third-order spectra are to be separated by $1.5cm$ on the screen, how many lines per centimetre are needed for the grating?

2. When a grating is illuminated with light of $\lambda = 540nm$, only 3 lines on either side of the central maximum can be seen. Calculate the maximum number of lines for the grating.

3. A gas discharge tube emits electromagnetic radiation of wavelengths $660nm$ and $430nm$ that illuminates a diffraction grating of 4000 lines cm^{-1}. Calculate the angular separation between the first order maximas of both wavelengths.

Answers:

8.1 Huygens' Principle & 8.2 Wave Interferences

1. State Huygen's Principle and its application on light wave analysis.
   Ans: According to Huygens' principle, every point on a wave front behaves like a light source and emits secondary wavelets, the secondary wavelets spread in all directions in space (vacuum) with the velocity of light and the envelope of wave front of secondary wavelets, after a given time, along forward direction gives the new position of wave front. It can be applied to both far field limit (as in the case of Fraunhofer) as well as the near field **diffraction**.

2. Show (either diagrammatically or in forms of equation) that the laws of reflection and refraction can be verified using Huygen's wavew theory.
   Ans:

First the wavefront touches the reflecting surface at B and then at the successive points towards C. In accordance with Huygens' principle, from each point on BC, secondary wavelets start growing with the speed c. During the time the disturbance from A reaches the point C the secondary wavelets from B must have spread over a hemisphere of radius $BD = AC = ct$, where t is the time taken by the disturbance to travel from A to C. The tangent plane CD drawn from the point C over this hemisphere of radius ct will be the new reflected wavefront.

Let angles of incidence and reflection be i and r, respectively . In ΔABC and ΔDCB, we have

$\angle BAC = \angle CDB$ [each is 90°]

$BC = BC$ [common]

$AC = BD$ [each is equal to ct]

$\therefore \Delta ABC \cong \Delta DCB$

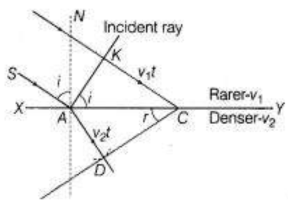
Hence $\angle ABC = \angle DCB$

or $i = r$

i.e. the angle of incidence is equal to the angle of reflection. The proves the first law of reflection.

Further, since the incident ray SB, the normal BN and the reflected ray BD are respectively perpendicular to the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD (all of which are perpendicular to the plane of the paper) therefore, they all lie in the plane of the paper i.e. in the same plane. This proves the second law of reflection.

13. Law of refraction on this basis of Huygens' wave theory Consider a plane wavefront AB incident on a plane surface XY, separating two media 1 and 2, as shown in Figure. Let v_1 and v_2 be the velocities of light in two media, with $v_2 < v_2$.



The wavefront first strikes at point A and then at the successive points towards C. According to Huygens' principle, from each point on AC, the secondary wavelets starts growing in the second medium with speed v_2. Let the disturbance take time t to travel from B to C, then $BC = v_1t$. During the time the disturbance from B reaches the point C, the secondary wavelets from point A must have spread over a hemisphere of radius $AD = v_2t$ in the second medium. The tangent plane CD drawn from point C over this hemisphere of radius v_2t will be the new refracted wavefront.

Let the angles of incidence and refraction be i and r, respectively.

From right ΔABC, we have

$$\sin\angle BAC = \sin i = \frac{BC}{AC}$$

From right ΔADC, we have

$$\sin\angle DCA = \sin r = \frac{AD}{AC}$$

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3. What type of wave front will emerge from a point source and a distance light source?
Ans: When source of light is a point source, the wavefront is spherical. At very large distances from the source, a portio of spherical or cylindrical waver at appears to be plane.

4. Explain the application of Huygen's Principle in the case of a single slit.

5. When are two light sources of the same common frequency said to be coherent ?
Ans: 2 light wave sources are said to be coherent when there exist a fixed difference between the phases of the light waves emitter by them.

6. What are the conditions for the interference of light?
Ans: Coherence and monochromatism.

7. From a path difference perspective, state the condition for constructive and destructive interference.
Ans: Constructive interference occurs when

$$\Delta\phi = n\lambda$$

.
Destructive interference occurs when

$$\Delta\phi = (n + \frac{1}{2})\lambda$$

.

8.3 Young's Double Slit Experiment

1. The interference pattern of two identical slits separated by a distance $d = 0.25mm$ is observed on a screen at a distance of $0.5m$ from the plane of the slits. The illuminating light source has a wavelength of $589nm$. Calculate the distance of the first bright fringe to the central maximum, and the separation of the bright bands.

$$y_1 = \frac{nD\lambda}{d} = \frac{(1)(589 \times 10^{-9})(0.5)}{0.25 \times 10^{-3}} = 1.178mm$$

2. Light (of $589nm$) from a lamp forms an interference pattern on a screen $0.75m$ from a pair of slits. The bright fringes in the pattern are $0.4cm$ apart. Calculate the slit separation.

$$\Delta y = \frac{D\lambda}{d} \Rightarrow d = \frac{D\lambda}{\Delta y} = \frac{(0.75)(589 \times 10^{-9})}{0.4 \times 10^{-2}} = 0.1104375mm$$

3. A laser light of $630nm$ incident on a pair of slits produces an interference pattern in which the bright fringes are separated by $8.3mm$. A second light, on the other hand, produces an interference pattern of which its dark fringes are separated by $7.6mm$. Calculate the wavelength of the second light.

$$\Delta y = \frac{\lambda D}{d} = \text{constant}$$

$$\lambda' = \frac{\Delta y'}{\Delta y}\lambda = \frac{7.6 \times 10^{-3}}{8.3 \times 10^{-3}}(630 \times 10^{-9}) = 577nm$$

4. Light from two light sources (of wavelengths λ_1 and λ_2 respectively) arrives at a double slit set up. If $\lambda_1 = 430nm$, what value must λ_2 be if the fourth order bright fringe of light with $\lambda_1 = 645nm$ overlaps with the sixth order bright fringe of λ_2 light?

$$n\lambda_i = \frac{yd}{D} = \text{constant}$$

$$4\lambda_1 = 6\lambda_2 \Rightarrow \lambda_2 = 430nm$$

5. In a double-slit experiment with light source $700nm$, a student is limited to a $35cm$ white-coloured cardboard sheet as a screen. Assuming intensity is not a limiting factor, calculate the highest order of bright fringe the student will be able to observe if the slit separation is $0.02mm$ and the screen must be kept at a distance of $1m$ from the slit.

$$y_n = \frac{nD\lambda}{d} \Rightarrow n = \frac{y_n d}{D\lambda} = \frac{(0.35)(0.02 \times 10^{-3})}{(1)(700 \times 10^{-9})} = 10$$

8.4 Thin Films

Media	phase change	Constructive Interference
$n_{1^{st}} > n_{2^{nd}} > n_{3^{rd}}$ or $n_{1^{st}} < n_{2^{nd}} < n_{3^{rd}}$	$0\,rad$	$2nt = m\lambda$
$n_{1^{st}} > n_{2^{nd}} < n_{3^{rd}}$ or $n_{1^{st}} < n_{2^{nd}} > n_{3^{rd}}$	$\pi\,rad$	$2nt = (m+0.5)\lambda$

1. A soap film has a refractive index of 1.33. How thick is the film if one-half of a wavelength of red light (with a vacuum wavelength of $633nm$) extends from on surface to the other surface?

$$n = \frac{\lambda_{vacuum}}{\lambda_{soap}} \Rightarrow \lambda_{soap} = \frac{\lambda_{vacuum}}{n} = \frac{633nm}{1.33} = 476nm$$

$$t = 1.5\lambda_{soap} = 1.5(476nm) = 714nm$$

2. A thin film of alcohol ($n_{alcohol} = 1.36$) spread on a flat glass plate and is illuminated with white light. If a region of the film reflects only green light ($\lambda_{green} = 500nm$) strongly, how thick is the film?

$$2nt = m\lambda \Rightarrow t = \frac{\lambda_{green}}{2(n_{alcohol})} = \frac{500nm}{2(1.36)} = 184nm$$

3. A glass lens with an index of 1.55 is to be coated with a film of index 1.30 to decrease the reflection normally incident green light of $\lambda = 500nm$. What minimum thickness should be deposited on the lens?

$$2nt = (m+0.5)\lambda \Rightarrow t = \frac{(0.5)\lambda_{green}}{2(n_{alcohol})} = \frac{(0.5)(500nm)}{2(1.3)} = 96.2nm$$

4. A film of oil ($n_{oil} = 1.42$) of $250nm$ thick floats on water. When illuminated from above with white light, what colour will reflect the most brightly?

$$2nt = (m+0.5)\lambda \Rightarrow \lambda = \frac{2nt}{m+0.5} = \frac{2(1.42)(250nm)}{1+0.5} = 473nm \Rightarrow \text{Blue}$$

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5. A soap film of index 1.35 appears yellow ($580nm$) when viewed directly from above. Compute several possible values of its thickness.

$$2nt = (m+0.5)\lambda \Rightarrow t = \frac{(m+0.5)\lambda}{2n} = \frac{(m+0.5)\lambda}{2(1.35)}$$

$$\text{When m} = 0, 1, 2,\ t = \left\{ \frac{(0+0.5)\lambda}{2(1.35)}, \frac{(1+0.5)\lambda}{2(1.35)}, \frac{(2+0.5)\lambda}{2(1.35)} \right\} = \{107nm,\ 322nm,\ 537nm\}$$

8.5 Single Slit Diffraction (Fraunhofer Diffraction)

$$y_n = \frac{n\lambda D}{a} \text{ for dark fringes}$$
$$y_n = \frac{(n+0.5)\lambda D}{a} \text{ for bright fringes}$$

1. A monochromatic light with a wavelength of $\lambda = 600nm$ passes through a single slit which has a width of $0.6mm$. Calculate the distance between the slit and the screen if the first minimum in the diffraction pattern is at a distance $0.90mm$ from the center of the screen.

$$y_n = \frac{n\lambda D}{a} \Rightarrow D = \frac{ay_n}{n\lambda} = \frac{(0.6mm)(0.90mm)}{(1)(600nm)} = 0.9m$$

2. If the separation between the first and the second minima of a single-slit diffraction pattern is $6.0mm$, what is the distance between the screen and the slit? The light wavelength is $500nm$ and the slit width is $0.16mm$.

$$\Delta y_n = \frac{\lambda D}{a} \Rightarrow D = \frac{a\delta y_n}{\lambda} = \frac{a\delta y_n}{\lambda} = 1.92m$$

3. When a single slit of width $0.3mm$ is illuminated with light of wavelength is $633nm$, the distance from the central maximum of the 1^{st} order minimum is $4cm$. Calculate this distance if the slit is doubled.

$$y_n = \frac{n\lambda D}{a} \Rightarrow ay_n = n\lambda D = \text{constant}$$
$$a_i y_i = a_f y_f = 2a_i y_f \Rightarrow y_f = \frac{y_i}{2} = \frac{4cm}{2} = 2cm$$

4. A single slit $0.1mm$ wide is illuminated by plane waves from a HeNe laser ($\lambda = 633nm$). If the screen is $10m$, determine the width of the central maximum.

$$\text{Width} = 2y_1 = \frac{2\lambda D}{a} == \frac{2(633 \times 10^{-9})(10m)}{0.1 \times 10^{-3}} = 0.1266m$$

5. A vertical single slit is illuminated with electromagnetic wave from a HeNe laser at $633nm$. It is found that the center of the second dark band lies at an angle of 4.2^o off the central axis. Determine the width of the slit.

$$y_n = \frac{n\lambda D}{a} \Rightarrow tan\,\theta \approx \theta = \frac{y_n}{D} = \frac{n\lambda}{a}$$
$$\theta = \frac{n\lambda}{a} \Rightarrow a = \frac{n\lambda}{\theta} = \frac{2 \times 633nm}{4.2} = 0.3\mu m$$

8.6 Diffraction Grating

$$d\,sin\,\theta = n\lambda;\; d = \frac{1}{N}$$

1. A screen is placed $1.4m$ away from a diffraction grating illuminated with light of wavelength $633nm$. If the second- and third-order spectra are to be separated by $1.5cm$ on the screen, how many lines per centimetre are needed for the grating?

$$d\,sin\,\theta = n\lambda \Rightarrow \Delta y = y_3 - y_2 = \frac{L\lambda}{d}(m_3 - m_2)$$

$$N = \frac{1}{d} = \frac{\Delta y}{L\lambda(m_3 - m_2)} = \frac{1.5cm}{(1.4)(633nm)(3-2)} = 16926 \text{ lines } m^{-1} \approx 170 \text{ lines } cm^{-1}$$

2. When a grating is illuminated with light of $\lambda = 540nm$, only 3 lines on either side of the central maximum can be seen. Calculate the maximum number of lines for the grating.

$$\text{Maximum order } \Rightarrow sin\, 90^o$$

$$d = m\lambda = \frac{1}{N}$$

$$N = \frac{1}{m\lambda} = \frac{1}{3(540nm)} \approx 6.17284 \times 10^5 \text{ lines } m^{-1}$$

3. A gas discharge tube emits electromagnetic radiation of wavelengths $660nm$ and $430nm$ that illuminates a diffraction grating of 4000 lines cm^{-1}. Calculate the angular separation between the first order maximas of both wavelengths.

$$d\, sin\, \theta_n = n\lambda \Rightarrow \theta_n = sin^{-1}\left(\frac{n\lambda}{d}\right) = sin^{-1} nN\lambda$$

$$\theta|_{\lambda=660nm,\, n=1} = sin^{-1}(1 \times 400000m^{-1} \times 660nm) = 15.3^o$$

$$\theta|_{\lambda=430nm,\, n=1} = sin^{-1}(1 \times 400000m^{-1} \times 430nm) = 9.9^o$$

$$\Delta\theta = 15.3^o - 9.9^o = 5.4^o$$

8 Chapter 9: Light Quantization

Questions:

9.1 Planck's Quantum Theory

1. Describe the *ultraviolet catastrophe*?
2. How did Planck resolve the *ultraviolet catastrophe* problem?
3. How did Planck's solution to the *ultraviolet catastrophe* differ from classical theory of energy, i.e. *Rayleigh-Jeans Law*?
4. What does it mean by "photon energy is quantized"?
5. Find the energy (in eV and in $Joules$) of the photons of frequency is
   (a) $668\,THz$
   (b) $400\,THz$
   (c) $526\,THz$
6. Find the energy (in eV and in $Joules$) of the photons of wavelength is
   (a) $430\,nm$
   (b) $500\,nm$
   (c) $565\,nm$
   (d) $625\,nm$
7. We get sunburnt if we are under the sun. This process requires a photon energy of $5.61864 * 10^{-19}$J. Calculate the wavelength that corresponds to this energy.

9.2 Photoelectric effect

1. What is the work function of a metal if the photoelectric threshold wavelength is $312nm$?

2. A photon of energy $5eV$ imparts all of its energy to an electron that leaves a metal surface. When UV photons of a single frequency strike a metal, electrons with kinetic energy from zero to $2.5eV$ are ejected. What is the energy of the incident photons?

3. The work function of a metal is $2.3eV$. What is the longest wavelength light that can cause photoelectrons to be emitted?

4. Light of wavelength $600nm$ falls on a metal having photoelectric work function $2eV$. Find the energy of the photon, the kinetic energy of the most energetic photoelectron and the stopping potential.

5. In the photoionization of atomic hydrogen, what will be the maximum kinetic energy of the ejected electron when a $60nm$ photon is absorbed by the atom?

Answers:

9.1 Planck's Quantum Theory

1. Describe the *ultraviolet catastrophe*?

2. How did Planck resolve the *ultraviolet catastrophe* problem?

3. How did Planck's solution to the *ultraviolet catastrophe* differ from classical theory of energy, i.e. *Rayleigh-Jeans Law*?

4. What does it mean by "photon energy is quantized"?

5. Find the energy (in eV and in $Joules$) of the photons of frequency is

   (a) $668THz$
   $$E = hf = (6.63 \times 10^{-34})(668 \times 10^{12}) = 4.43 \times 10^{-19}J = 2.76eV$$

   (b) $400THz$
   $$E = hf = (6.63 \times 10^{-34})(400 \times 10^{12}) = 2.65 \times 10^{-19}J = 1.656eV$$

   (c) $526THz$
   $$E = hf = (6.63 \times 10^{-34})(526 \times 10^{12}) = 3.49 \times 10^{-19}J = 2.17eV$$

6. Find the energy (in eV and in $Joules$) of the photons of wavelength is

   (a) $430nm$
   $$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{430nm} = 4.62 \times 10^{-19}J \approx 2.9\,eV$$

   (b) $500nm$
   $$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{500nm} = 3.97 \times 10^{-19}J \approx 2.5\,eV$$

   (c) $565nm$
   $$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{565nm} = 3.52 \times 10^{-19}J \approx 2.2\,eV$$

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(d) $625nm$

$$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{625nm} = 3.18 \times 10^{-19} J \approx 1.98\, eV$$

7. We get sunburnt if we are under the sun. This process requires a photon energy of $5.61864 * 10^{-19}$J. Calculate the wavelength that corresponds to this energy.

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{5.61864 \times 10^{-19}} = 354nm$$

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9.2 Photoelectric effect

$$\frac{1}{2}mv_{max}^2 = eV_s = hf - W = h(f - f_o)$$

1. What is the work function of a metal if the photoelectric threshold wavelength is $312nm$?

$$W = \frac{hc}{\lambda_o} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{312 \times 10^{-9}} = 6.375 \times 10^{-19}J \approx 3.9844eV$$

2. A photon of energy $5eV$ imparts all of its energy to an electron that leaves a metal surface. When UV photons of a single frequency strike a metal, electrons with kinetic energy from zero to $2.5eV$ are ejected. What is the work function of the metal?

$$W = E_{photon} - E_K = 5 - 2.5 = 2.5\,eV \approx 4.0054 \times 10^{-19}J$$

3. The work function of a metal is $2.3eV$. What is the longest wavelength light that can cause photoelectrons to be emitted?

$$W_{min} = hf_o = \frac{hc}{\lambda_o} \Rightarrow \lambda = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{2.3 \times 1.6 \times 10^{-19}} = 540nm$$

4. Light of wavelength $600nm$ falls on a metal having photoelectric work function $2eV$. Find the energy of the photon, the kinetic energy of the most energetic photoelectron and the stopping potential.

$$E = \frac{hc}{\lambda} = 2.07eV$$

$$K_{max} = 0.07eV$$

$$eV_s = K_{max} = 0.07eV \approx 1.12152 \times 10^{-20}J$$

5. In the photoionization of atomic hydrogen, what will be the maximum kinetic energy of the ejected electron when a $60nm$ photon is absorbed by the atom?

$$K_{max} = \frac{hc}{\lambda} - E_{ionization\,H} = \frac{1240\,eV\,nm}{60nm} - 13.6eV = 7.1eV \approx 1.1375 \times 10^{-18}J$$

9 Chapter 8: Wave Properties of Particle

Questions:

de Broglie Wavelength

$$\lambda = \frac{h}{p}$$

1. What is the idea of "wave-particle duality" of light?
2. What do we mean when we say a particle may exhibit wave-like nature?
3. If we say particles has wave light properties, how then can we characterise it's wavelike nature?
4. Calculate the de Broglie wavelength of a $0.5kg$ ball moving with a speed $20ms^{-1}$.
5. Calculate the wavelength of an electron accelerated with a potential difference of $100V$.
6. Find the de Broglie wavelength for a particle of mass $1.67 \times 10^{-27}kg$ travelling at $1100ms^{-1}$.
7. If the de Broglie wavelength of an electron is 0.5Å(Angstrom), calculate its kinetic energy.

Electron Diffraction

1. What is Bragg's Law?
2. Describe the results of the Davisson-Germer Experiment and its significance.
3. Describe the mechanism behind an electron microscope.
4. How does the de Broglie's wavelength relate to the resolving power of an electron microscope?
5. In a table, compare and contrast between electron microscope and optical microscope.

Answers:

10.1 de Broglie Wavelength

$$\lambda = \frac{h}{p}$$

1. What is the idea of "wave-particle duality" of light?
2. What do we mean when we say a particle may exhibit wave-like nature?
3. If we say particles has wave light properties, how then can we characterise it's wavelike nature?
4. Calculate the de Broglie wavelength of a $0.5kg$ ball moving with a speed $20ms^{-1}$.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{(0.5)(20)} = 6.6 \times 10^{-35}m$$

5. Calculate the wavelength of an electron accelerated with a potential difference of $100V$.

$$\text{By energy conservation } \Rightarrow E_K = E_{e.p.} \Rightarrow \frac{1}{2}m_e v_e^2 = q_e V$$

$$v = \sqrt{\frac{2q_e V}{m_e}} = \sqrt{\frac{2(1.6\times 10^{-19})(100)}{9.1\times 10^{-31}}} = 5.9\times 10^6 ms^{-1}$$

$$\lambda = \frac{h}{m_e v_e} = \frac{6.6\times 10^{-34}}{(9.1\times 10^{-31})(5.9\times 10^6)} = 0.12nm$$

6. Find the de Broglie wavelength for a particle of mass $1.67\times 10^{-27}kg$ travelling at $1100ms^{-1}$.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.6\times 10^{-34}}{(1.67\times 10^{-27})(1100ms^{-1})} = 0.09nm$$

7. If the de Broglie wavelength of an electron is 0.5Å(Angstrom), calculate its kinetic energy.

$$0.5\text{Å} = 0.5\times 10^{-10}m$$

$$\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$$

$$E_K = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e(\frac{h}{m_e\lambda})^2$$

$$E_K = \frac{1}{2}(9.1\times 10^{-31})(\frac{6.6\times 10^{-34}}{(9.1\times 10^{-31})(10^{-10})})^2 = 6.0314\times 10^{-18}J = 37.7eV$$

10.2 Electron Diffraction

1. What is Bragg's Law?
2. Describe the results of the Davisson-Germer Experiment and its significance.
3. Describe the mechanism behind an electron microscope.
4. How does the de Broglie's wavelength relate to the resolving power of an electron microscope?
5. In a table, compare and contrast between electron microscope and optical microscope.

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10 Chapter 9: Nuclear & Particle Physics

Questions:

Binding Energy & mass defect

$$1amu = 1.6605 \times 10^{-27}kg = 931.49432MeV\,c^{-2}$$

$$\Delta m = Zm_p + Nm_n - m_{nucleus}$$

$$E_{binding} = \Delta mc^2$$

1. Define the following term:
   (a) mass defect

   (b) binding energy

2. How many protons and neurons are in
   (a) $^{24}_{11}Na$

   (b) $^{6}_{3}Li$

   (c) $^{206}_{82}Pb$

3. Both in atomic mass unit and in kilogram, calculate the mass defect, the binding energy and the binding energy per nucleon for the following situation:
   (a) Mass of $^{7}_{3}Li$ nucleus $= 7.016amu$
   (b) Mass of $^{37}_{17}Cl$ nucleus $= 6.13834818 \times 10^{-26}kg$

4. Describe the **binding energy curve** and the **iron limit**.

5. Calculate the minimum amount of energy needed to transform $^{43}_{20}Ca$ atom (of mass $42.958766u$) into $^{42}_{20}Ca$ atom (of mass $41.958618u$) through the removal of a neutron.

6. If the mass of $^{11}_{5}B$ nucleus is $11.008757u$, calculate the binding energy per nucleon.

Radioactivity

$$\frac{dN}{dt} = -\lambda N$$
$$N|_t = N_o e^{-\lambda t}; A|_t = A_o e^{-\lambda t}$$
$$T_{\frac{1}{2}} = \frac{ln\, 2}{\lambda}$$

1. Describe the following type of radioactive decay in terms of the changes to the parent nucleus as well as the type of particles emitted:
   (a) alpha
   (b) beta
   (c) gamma
2. What do we mean by $1\, becquerel$ and $1\, Curie$?
3. What is meant by **decay constant**?
4. The decay constant of an isotope $({}^{45}_{22}X)$ with mass $45g$ is $6.25 \times 10^{-5} s^{-1}$. Caculate the half life of this isotope and determine the decay rate when 30% of the original isotope has decayed.
5. $1g$ of ${}^{255}_{80}X$ has an activity of 3×10^{10}. Calculate
   (a) the decay constant
   (b) the activity of this sample after 5 years.
6. At the beginning, the number of particle of a radioactive isotope is e^4 particles. At time $t = 5 minutes$, the number of particle drops to e particles. Determine the decay equation.
7. The half life of plutonium is 24000 years. If stored for 96000 years, how much of the original amount is left?

Answers:

11.1 Binding Energy & mass defect

$$1amu = 1.6605 \times 10^{-27} kg = 931.49432 MeV\, c^{-2}$$
$$\Delta m = Zm_p + Nm_n - m_{nucleus}$$
$$E_{binding} = \Delta mc^2$$

1. Define the following term:
   (a) mass defect
   The difference between the mass of an atomic nucleus and the sum of the individual masses of the nucleons of which it is composed.
   (b) binding energy
   The energy required to disassemble a nucleus into the free, unbound neutrons and protons it is composed of.

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2. How many protons and neurons are in

   (a) $^{24}_{11}Na$

$$N_{proton} = Z = 11$$
$$N_{neutron} = A - Z = 24 - 11 = 13$$

   (b) $^{6}_{3}Li$

$$N_{proton} = Z = 3$$
$$N_{neutron} = A - Z = 6 - 3 = 3$$

   (c) $^{206}_{82}Li$

$$N_{proton} = Z = 82$$
$$N_{neutron} = A - Z = 206 - 82 = 124$$

3. Both in atomic mass unit and in kilogram, calculate the mass defect, the binding energy and the binding energy per nucleon for the following situation:

   (a) Mass of $^{7}_{3}Li$ nucleus $= 7.016amu$

$$\Delta m = 3(1.007277)u + 4(1.008665) - 7.016 = 4.05 \times 10^{-2}amu \approx 6.72518263 \times 10^{-31}kg$$
$$E = \Delta mc^2 = (4.05 \times 10^{-2}amu)(931MeV\, amu^{-1}) = 37.703\, MeV$$
$$\frac{E}{A} = \frac{37.703}{7} = 5.3861MeV$$

   (b) Mass of $^{37}_{17}Cl$ nucleus $= 6.13834818 \times 10^{-26}kg$

$$\Delta m = 17(1.007277)u + 20(1.008665) - \frac{6.13834818 \times 10^{-26}}{1.6605 \times -27} = 0.33amu \approx 5.47978 \times 10^{-28}kg$$
$$E = \Delta mc^2 = (0.33amu)(931MeV\, amu^{-1}) = 307.23\, MeV$$
$$\frac{E}{A} = \frac{307.23}{37} = 8.3035MeV$$

4. Describe the **binding energy curve** and the **iron limit**.
   The iron limit - The binding energy curve is obtained by dividing the total nuclear binding energy by the number of nucleons. The fact that there is a peak in the binding energy curve in the region of stability near iron means that either the breakup of heavier nuclei (fission) or the combining of lighter nuclei (fusion) will yield nuclei which are more tightly bound (less mass per nucleon). The binding energies of nucleons are in the range of millions of electron volts compared to tens of eV for atomic electrons. Whereas an atomic transition might emit a photon in the range of a few electron volts, perhaps in the visible light region, nuclear transitions can emit gamma-rays with quantum energies in the MeV range.
   The iron limit - The buildup of heavier elements in the nuclear fusion processes in stars is limited to elements below iron, since the fusion of iron would subtract energy rather than provide it. Iron-56 is abundant in stellar processes, and with a binding energy per nucleon of 8.8 MeV, it is the third most tightly bound of the nuclides. Its average binding energy per nucleon is exceeded only by 58Fe and 62Ni, the nickel isotope being the most tightly bound of the nuclides.

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5. Calculate the minimum amount of energy needed to transform $^{43}_{20}Ca$ atom (of mass $42.958766u$) into $^{42}_{20}Ca$ atom (of mass $41.958618u$) through the removal of a neutron.

$$\text{In general, } E +^{43}_{20} Ca \rightarrow^{42}_{20} Ca + \text{neutron}$$

$$^{43}_{20}Ca \rightarrow^{42}_{20} Ca : \Delta m_{Ca} = 42.958766u - 41.958618u = 1.000148u (< m_{neutron})$$

$$E = \Delta mc^2 = (1.008665 - 1.000148)(931) = 7.934\, MeV$$

6. If the mass of $^{11}_{5}B$ nucleus is $11.008757u$, calculate the binding energy per nucleon.

$$\frac{E}{A} = (A)^{-1}(Zm_p + Nm_n - m_{nucleus})c^2$$

$$\frac{E}{A} = (11)^{-1} \times [5(1.007277) + 6(1.008665) - 11.008757u] \times (931MeV) = 6.7386MeV \text{ per nucleon}$$

11.2 Radioactivity

$$\frac{dN}{dt} = -\lambda N$$

$$N|_t = N_o e^{-\lambda t}; A|_t = A_o e^{-\lambda t}$$

$$T_{\frac{1}{2}} = \frac{ln\, 2}{\lambda}$$

1. Describe the following type of radioactive decay in terms of the changes to the parent nucleus as well as the type of particles emitted:

   (a) alpha

   (b) beta

   (c) gamma

Radioactive decay	General process	Emission
Alpha, α	$^A_ZX \rightarrow^{A-4}_{Z-2} Y +^4_2 He + E$	^4_2He
Beta plus, β^+	$^A_ZX \rightarrow^{A}_{Z-1} Y + e^+ + \nu_{e^+} + E$	e^+, ν_{e^+}
Beta minus, β^-	$^A_ZX \rightarrow^{A}_{Z+1} Y + e^- + \bar{\nu}_{e^-} + E$	$e^-, \bar{\nu}_{e^-}$
Gamma, γ	$^A_ZX \rightarrow^{A}_{Z} Y + \gamma + E$	γ

2. What do we mean by $1\, becquerel$ and $1 Curie$?

$$1\, becquerel = 1 \text{ disintegration per second}$$

$$1\, Curie = 3.7 \times 10^{10} \text{ decays}$$

3. What is meant by **decay constant**? Decay constant - probability of a nucleus decay per unit time.

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4. The decay constant of an isotope ($^{45}_{22}X$) with mass $45g$ is $6.25 \times 10^{-5}s^{-1}$. Caculate the half life of this isotope and determine the decay rate when 30% of the original isotope has decayed.

$$T_{\frac{1}{2}} = \frac{ln\,2}{\lambda} = \frac{ln\,2}{6.25 \times 10^{-5}s^{-1}} = 11090s$$

$$N_o \rightarrow 0.7N_o$$

$$\text{45g of } {}^{45}_{22}X \text{ is 1 mol} \rightarrow N_o = nN_A = 6.02 \times 10^{23} \text{ atoms}$$

$$\frac{dN}{dt} = -\lambda N = -(6.25 \times 10^{-5}s^{-1})(0.7 \times 6.02 \times 10^{23}) = -2.63 \times 10^{-19} \text{ decay per second}$$

5. $1g$ of $^{255}_{80}X$ has an activity of 3×10^{10}. Calculate the decay constant and the activity of this sample after 5 years.

$$255g = 1mol \Rightarrow 1g = \frac{1}{255}mol$$

$$A_o = \lambda N_o \Rightarrow \lambda = \frac{A_o}{N_o} = \frac{3 \times 10^{10}}{(\frac{1}{255})(6.02 \times 10^{23})} = 1.27 \times 10^{-11}\,s^{-1}$$

$$A|_{t=1.577\times10^8\,s} = A_o e^{-\lambda t} = (3 \times 10^{10}\,s^{-1})e^{-(1.27\times10^{-11}\,s^{-1})(1.577\times10^8\,s)} = 2.9941 \times 10^{10}s^{-1}$$

6. At the beginning, the number of particle of a radioactive isotope is e^4 particles. At time $t = 5minutes$, the number of particle drops to e particles. Determine the decay equation.

$$ln\,N = ln\,N_o - \lambda t$$

$$\lambda = -\frac{ln\,e^4}{ln\,e} \times \frac{1}{300s} = 1.33 \times 10^{-2}\,s^{-1}$$

$$N|_t = e^4\,e^{-0.0133t}$$

7. The half life of plutonium is 24000 years. If stored for 96000 years, how much of the original amount is left?

$$T_{0.5} = \frac{ln\,2}{\lambda} \Rightarrow \lambda = \frac{ln\,2}{T_{0.5}} = \frac{ln\,2}{24000 \text{ years}^{-1}} = 2.888 \times 10^{-5} \text{ years}^{-1}$$

$$\frac{N}{N_o} = e^{-\lambda t} = e^{-(2.888\times10^{-5})(96000)} = 0.062989$$

$$\text{Faster estimation} \Rightarrow n = \left(\frac{96000}{24000}\right) \approx 4\text{half-lives; } \frac{N}{N_o} \approx (0.5)^n \approx (0.5)^4 \approx \frac{1}{16} \approx 0.0625$$