Chapter 1: Electrostatics

1. State/Define
    a. Coulomb's Law – Equation?
    b. Electric Field Strength – Equation? Definition?
    c. Electric Potential – What is electric potential? How does it relate to potential energy?
    d. Equipotential lines and surfaces – What are they?
2. Explain motion path of a charged particle in a uniforms electric field
3. Calculate
    a. Using Coulomb's Law to determine the force experienced by a charge in an electric field, with maximum complexity of 4 charges in 2D configuration
    b. The electric field at a point in a system, with maximum complexity of 4 charges in 2D configuration
    c. The potential at a point in a system, with maximum complexity of 4 charges in 2D configuration
    d. Potential difference between two spatial points
    e. Potential energy from potential difference
    f. Electric field strength in a parallel plate configuration with plates of potential difference ΔV separated by distance d.

1.1 Electrostatics

Forces

Electrostatics is a branch of electromagnetics that deals with electric charge at rest. This involves the study of the most fundamental law, one that is experimentally driven at its roots, known as **Coulomb's Law.** By experiment, what we primarily have in mind is Charles-Augustin de Coulomb's 1785 experiment[1] which gives 2 important results:

1. that the electrical force is inversely proportional to the distance between 2 charged particle, hence
$$F_{electrical} \propto \frac{1}{r^2}.$$
2. that the electrical force is directly proportional to the product of the charges of the two interacting bodies, that is
$$F_{electrical} \propto q_1 q_2.$$

This tells us that it leads to the relationship,

$$F_{electrical} \propto \frac{q_1\ q_2}{r_{12}^2}$$

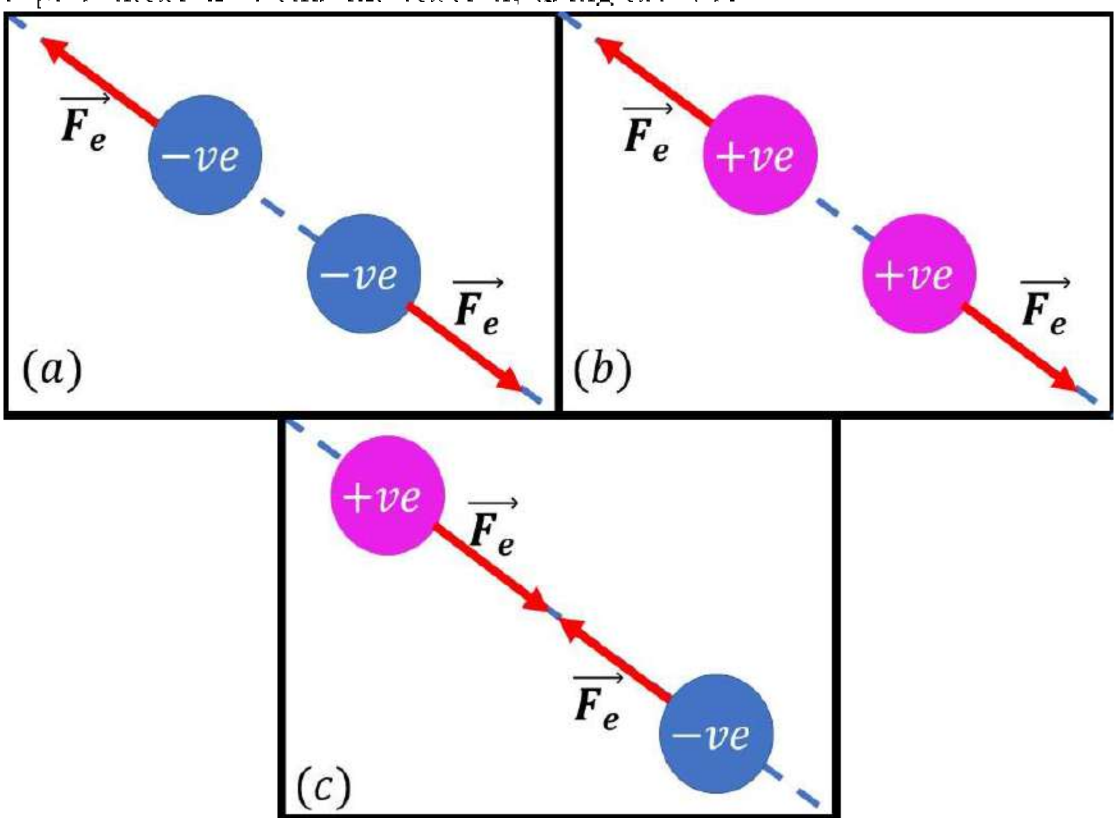
and as an equation

$$F_{electrical} = k_e \frac{q_1\ q_2}{r^2}$$

where k_e as the proportionality constant, also known as the electrostatic constant. This constant is defined by

$$k_e = \frac{1}{4\pi\epsilon_o} = 8.98(10^9) kg\, m^3\, s^{-2}\, C^{-2}.$$

ϵ_o here is the vacuum permittivity. It physically represents the ability of an electric field to permeate a vacuum. Note that the Coulomb's Law has the likeness of Newton's Law of Gravitation, instead of having interacting masses of particles, we have interacting charges of the particles. The the dependency of the forces to r^2 is referred to the **inverse square law.** Whilst masses are always positive, charges can take up both positive and negative values. For any child, whose curiosity tends to get the best of them, they would know that positive charges attract and negative charges repels. It would then be useful to have a graphical representation of this interaction, using arrows:



As we can see, forces are sketched/drawn using arrows as they are vectors. Charges that are alike repels each other and opposite charges attracts each other. The repulsion/attraction force is along the distance between the charged bodies.

Coulomb's Law may also be generalized for a system of N discrete charges, the force $\vec{F}$ on a small charge q in vacuum, due to this system is then

$$\vec{F}(r) = \frac{q_j}{4\pi\epsilon_o}\sum_{i=1}^{N}\frac{q_i}{(\vec{r}_{ij})^2}$$

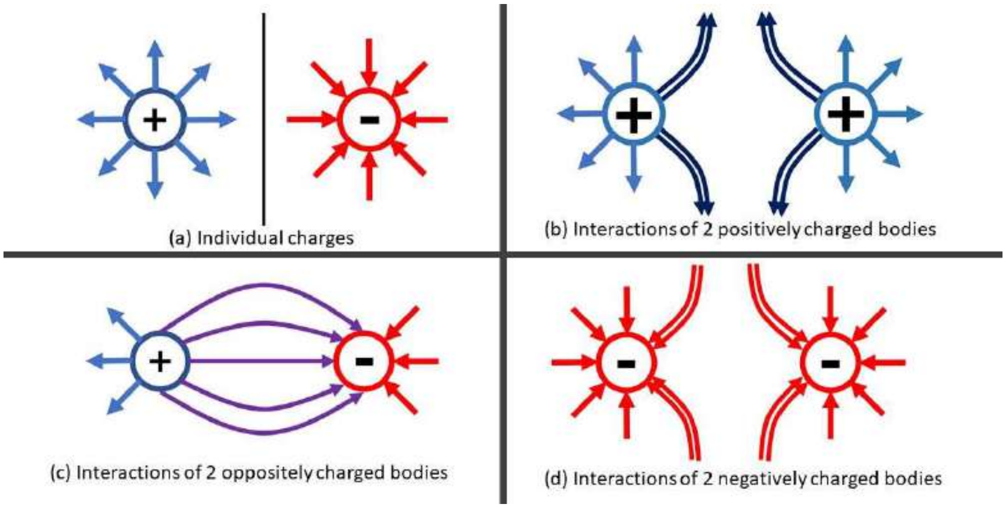
An electric field, $\vec{E}$, is a vector quantity associated to the electric force that exists in every point in space. It is mathematically defined as the electric force at that point in space per unit charge,

$$\vec{E} = \frac{\vec{F}}{q_o} = k_e\frac{Q}{r^2}.$$

It has the unit NC^{-1}. For a system of N discrete charges, the electric field at a point in space is then,

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_o}\sum_{i=1}^{N}\frac{q_i}{\vec{r}_i^2}.$$

Speaking of electric fields, it is very useful to have a pictorial representation of what they are. Faraday's imagination led us to the pictorial representation of electric fields we shall use today.



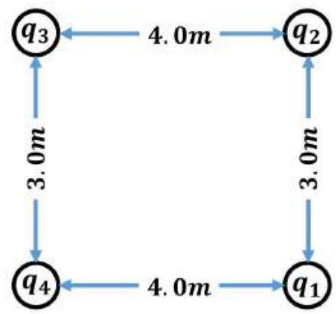
The figure above shows electric field lines and how they are drawn with relation to the charges. Note that the field lines emanate from positive charges and ends at negative charges, and to add to that, field lines never crosses merely because they would superimpose.

Exercise:

1. 2 protons are fired directly at each other in a vacuum chamber. What is the force each experiences at the instant they are $10^{-14}m$ apart? **[Ans:** $F \approx 2.27N$**]**

2. Two charges of $4nC$ and $1nC$ are fixed to a baseline at a separation of $1m$. Where on the baseline should a third charge of $2nC$ be placed if it is to experience zero net electric force? **[Ans:** $0.6m$ **from the** $4nC$ **charge]**

3. The diagram below shows four-point charges fixed at the corners of a rectangle, in vacuum.



If charges $q_n = \{+100, +36, +125, +32\}\mu C$, compute the net electrostatic force acting on the charge q_1. **[Ans: $F_{net} = 8.3N$ at an angle of 49^o]**

4. Referring to question 3, redo the problem if $q_4 = -32\mu C$. **[Ans: $F_{net} = 6.5N$ at an angle of 74^o]**

5. A $+10\mu C$ test-charge at some point beyond a charged sphere experiences an attractive force of $40\mu N$. Please compute the value of the electric field of the sphere at that point in a vacuum. [Ans: $|\vec{E}| = 4NC^{-1}$]

6. Referring to question 3, calculate the magnitude of electric field in the centre of the configuration.

7. An electron is placed in a uniform electric field of $1.5 \times 10^4\, N\, C^{-1}$. Please determine its acceleration.
[Ans: $a = 2.6 \times 10^{15} ms^{-2}$]

Energy

Our discussion on electrostatics continues on with the idea of electric potential (*ipso facto* voltages). As covered in the topic of gravitational potential, as a mass is raised to varying height, we require a force. The vertical displacement from the Earth together with the force determines the work done required to vertically displace the mass.

Analogous to the idea of gravitational potential energy, for the electric force, there exist the idea of an electrical potential energy as well. We shall define it similarly, that the electrical potential energy is the ability to do work, W, due to the position of a charged body from another charged body. That is to say,

$$U = F_{electrical} \cdot r = \frac{kq_1q_2}{r^2} \cdot r = \frac{kq_1q_2}{r}.$$

The change in this potential energy per unit charge is referred to the difference in potential (**read:** voltage). For point A and B, the potential difference is then

$$\Delta V = V_B - V_A$$

and is equal to the work done by the charge against the electric field in moving a positive test charge from A to B with zero acceleration, that is

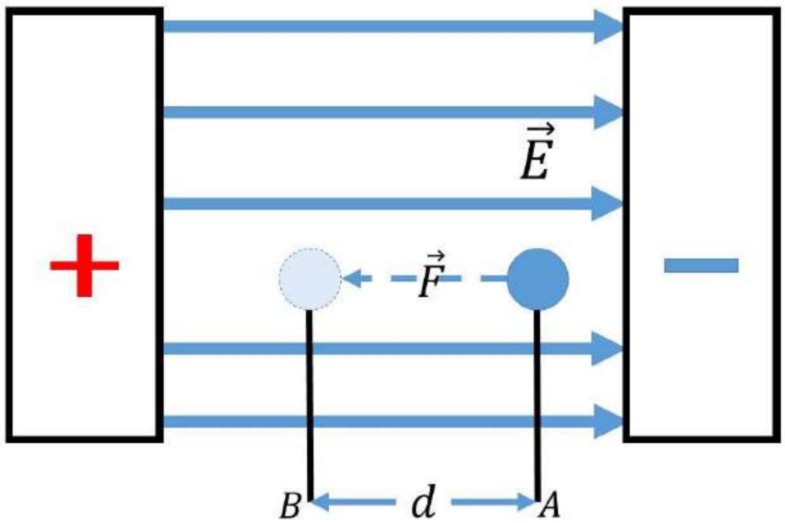
$$\Delta V = V_B - V_A = \frac{W(A \to B)}{q_o} = \frac{kq}{r}.$$

Note: Work done **by the charge** against the field increases electrical potential energy, whereas work done **onto the charge by the field** always decreases the electrical potential energy.

Equipotentials

When a charge changes its position but maintaining the same potential, i.e. ensuring $\Delta V = 0$, the initial and final positions are said to be **equipotential.** Equipotential lines and surfaces are merely lines and surfaces by which the charged body can move according to whilst maintaining zero potential change.

Uniform Electric Field



From the diagram above, we would like to work out the difference in electric potential at two positions in a uniform electric field. It is easy to see that work done to change the position of the test charge is

$$W(A \to B) = q_o E d$$

as $F = q_o E$ and d is the distance between A and B. Since $\Delta V = \frac{W}{q_o}$, we get the following expression for the potential difference in a uniform electric field:

$$\Delta V = V_B - V_A = \pm E d.$$

Exercise:

1. A $+20nC$ charge in an electric field is moved by an applied force. If $60nJ$ of work is done onto the charge by the force, what is th change in potential difference by the charge? **[Ans: $\Delta V = 3V$]**

2. There is an electric field in a region of space and a $50nC$ charge is placed at a point where the potential of $500V$. What released, the charge moves whilse the field does $200\mu J$ of work on it. What is the potential of the final location of the charge? **[Ans:** $V = -3.5 \times 10^3 V$**]**

3. A tiny sphere with charge $-25nC$ is moved $100cm$ in a uniform electric field with no acceleration. It goes from a location of zero potential to a point of potential $100V$. How much work is done on it by the applied force? What is the significance of the sign ΔW? **[Ans:** $\Delta W = -25 \times 10^{-7} J$**]**

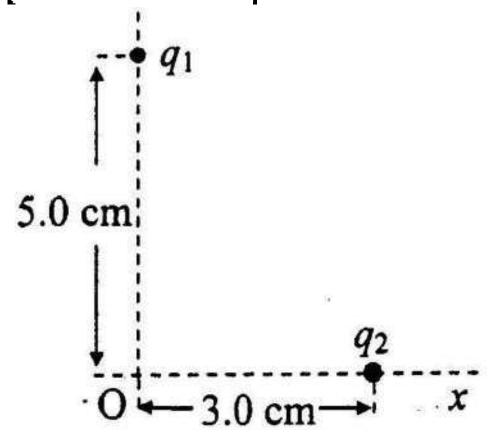
4. How much charge should we supply a sphere if we wish to measure $1V$ at a distance of $10m$ from it ? **[Ans:**$1.11nC$**]**

5. What voltage should be put across a pair of parallel metal plates $10cm$ apart if the field between them is $1Vm^{-1}$? **[Ans:** $V = \pm 0.1V$**]**

5 Past Year

Chapter 1

1. **[PSPM 04/05]**



Two point charges, $q_1 = -12\mu C$ and $q_2 = +8\mu C$ are arranged as shown in figure above. Copy the figure above and draw the direction of the electric field E1 and E2 at the origin produced by charge $q1$ and $q2$ respectively. Calculate the magnitude of the resultant electric field at the origin.
[Ans: $E_O = 9.09 \times 10^7\, NC^{-1}$**]**

2. **[PSPM 10/11]**
Four electrons are released onto a thin aluminum disc of diameter 10 mm. If the disc is initially neutral,

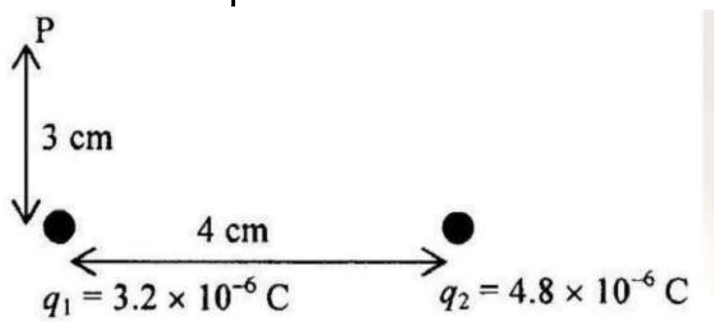
   a) determine the electric field strength at the centre of the disc.

b) calculate the electric potential at the centre of the disc.

c) calculate the energy stored in the disc.

[Ans: $E_{total} = 0NC^{-1}$**;** $V_c = -1.15(10^{-6})V$**;** $U_{1234} = 1.76(10^{-25})J$**]**

3. **[PSPM 07/08]**



The figure above shows two charges q_1 and q_2 separated $4cm$ apart. At position P,

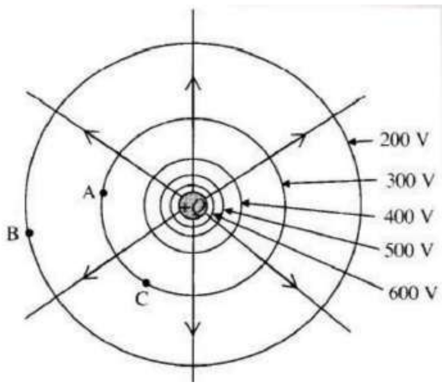
a) sketch the electric fields due to q_1 and q_2.

b) determine the resultant electric field.

c) calculate the electric potential.

[Ans: $E_{resultant} = 4.46(10^7)\,NC^{-1}$ **at** 108^o **from the positive x-axis];** $V = 1.82 \times 10^6 V$

4. **[PSPM 08/09]**



The figure above represents the equipotential surfaces and electric field lines near a positive point charge Q.

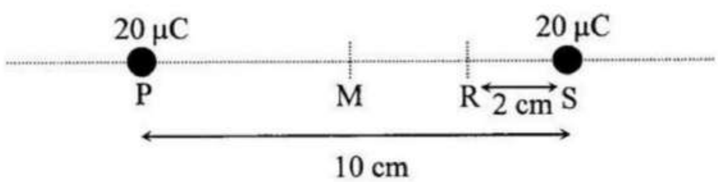
a) Calculate the work done by the electric field on a $3\mu C$ test charge that is displaced from point A to B and from A to C. **[Ans:** $W_{AB} = -3(10^{-4})J$; $W_{BC} = 0J$**]**

b) If charge Q is $5nC$,

i. calculate the distance between the point charge Q and A. **[Ans:** $r_A = 0.15m$**]**

ii. calculate the electric field at A. **[Ans:** $E_A = 2000\,NC^{-1}$**]**

5. **[PSPM 09/10]**



The figure above shows two $20\mu C$ charges at point P and S respectively. The distance between P and S is $10cm$. A point M is midway between PS. Calculate the work needed to move a $2\mu C$ test charge from M to R, $2cm$ away from S. **[Ans:** $W_{MR} = 8.2J$**]**

6. **[PSPM 09/10]**
The charges and coordinates of two point charges, Q_1 and

Point Charge	Charge	Coo
Q_1	10 μC	
Q_2	−5 μC	

Q_2 , are given in the table below,
Calculate the coordinate of a third charge $+Q3$ for it to be in equilibrium. Explain your chosen coordinate.
[Ans: Position of $+Q_3$**:** $= (0.0683m, 0)$**]**

7. **[PSPM 10/11]**

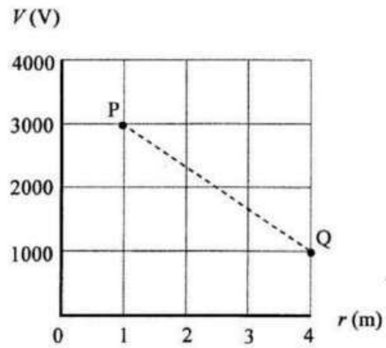


FIGURE 4

FIGURE 4 shows a graph of potential V against distance r. The horizontal lines are equipotential lines. Calculate the

a) potential difference between points P and Q.

b) work needed to bring a point charge $1.2 \times 10^{-3}\,C$ from P to Q.

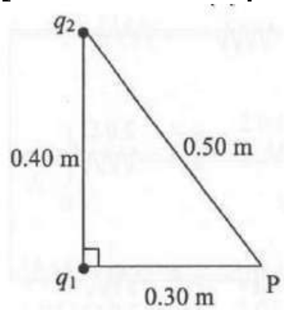
[Ans:$\Delta V = 2000V$**;** $W_{PQ} = -2.4J$**]**

8. **[PSPM 10/11]**
A point charge $3.2 \times 10^{-19}C$ with kinetic energy 8.5 MeV approaches a nitrogen nucleus $11.2 \times 10^{-19}C$. Calculate

the closest possible distance of approach between the point charge and the nucleus **[Ans:**$r2.37 \times 10^{-15}m$**]**

9. **[PSPM 11/12]**
An amount of charge is transferred from a neutral plastic bead to another identical neutral bead located $15cm$ away. The force of attraction between the beads is $2.010^{-4}N$. How many electrons were transferred from the first bead to the second? **[Ans:**$N = 1.4 \times 10^{11} electrons$**]**

10. **[PSPM 11/12]**



Two point charges $q_1 = +3.00\mu C$ and $q_2 = 5.00\mu C$ are placed at the two corners of a triangle of sides $0.30m$, $0.40m$ and $0.50m$ as shown in the figure above. P is the third corner of the triangle. Calculate

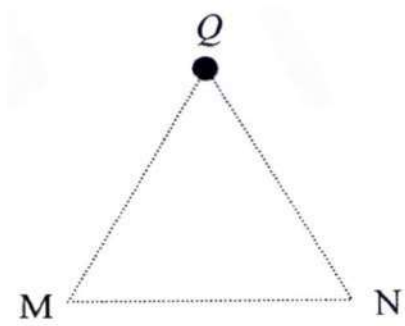
a) the magnitude of the electric field at P.

b) the electric potential at P.

c) the work needed to bring a test charge from infinity to P.

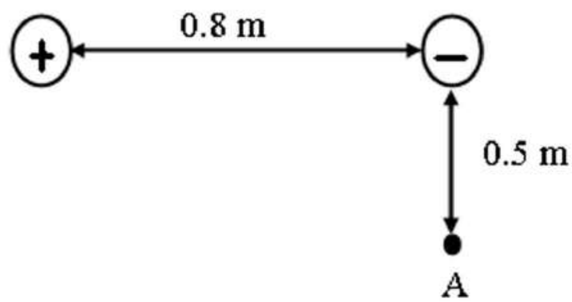
[Ans: $E_P = 2.4 \times 10^5\, NC^{-1}$; $V_P = 0V$; $W_\infty = 0J$**]**

11. **[PSPM 13/14]**



The figure above shows a charge Q at the vertex of an equilateral triangle with sides $1mm$. If $138J$ work is done in bringing a - $4.8\mu C$ point charge from infinity to position M, determine the magnitude and type of charge Q and calculate the electric field at position N. **[Ans:** $Q = -3.19(10^{-6})C$; $E_{total} = 6.27 \times 10^{10}\,NC^{-1}$**]**

12. **[PSPM 15/16]**



The figure above shows two charges $+50\mu C$ and $-20\mu C$ separated by $0.8m$. Determine the

(a) electric field at point A due to the negative charge.

(b) electric potential at point A.

(c) external work required to bring a $+2\mu C$ charge from infinity to point A.

[Ans: $E_A = 7.2(10^3)\,NC^{-1}$; $V = 1.17(10^5)V$; $W_{ext} = 0.234J$**]**

Chapter 2: Capacitors and Dielectrics

1. Define/State:
    a. Capacitance
    b. Physical meaning of time constant
    c. Dielectric Constant
2. Explain:
    a. $Q - t$ and $I - t$ graph for the charging and discharging of a capacitor
    b. Effect of dielectric on a parallel plate capacitor
3. Derive:
    a. Effective capacitance of capacitors in series/parallel arrangement
    b. Energy stored in a capacitor
4. Calculate
    a. Capacitance of a capacitor
    b. Energy stored in capacitor
    c. Capacitance of a parallel plate capacitor (With and without dielectric)
    d. Dielectric constant

1.2 Capacitors & Dielectrics

Think of a water tank, but instead of filling it with water, you fill it with charges. That is in essence what a capacitor is. A capacitor is a reservoir for charges. This is made possible by storing electrical energy in form of an eelctric field. The amount of charged one can store in a capacitor per given voltage is its capacitance, C, which is defined as

$$C = \frac{Q}{V}$$

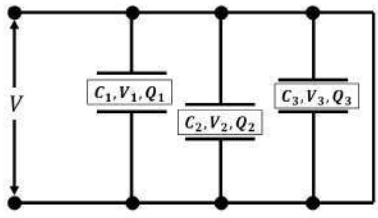
.

Capacitors in Combination

In any given electrical circuit, however complicated it may be, can be reduced down to two basic arrangement - **parallel** and **series**.

Parallel

First we shall define what we mean by the parallel arrangement - The parallel arrangement of capacitor is that one terminal of each capacitor is connected to the same common wire, and all the remaining terminal of each capacitors are connected to a second wire. This is illustrated in the diagram below:



We can see that the voltages across every capacitor, V_n, is equal to the voltage across the whole circuit,V. That is

$$V = V_1 = V_2 = V_3$$

. The sum of the charges at each capacitor is equal to the net charge stored, i.e.

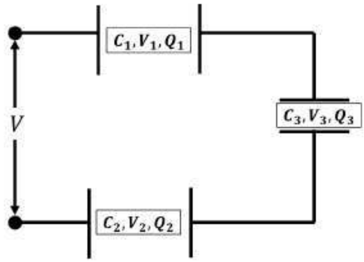
$$Q = Q_1 + Q_2 + Q_3.$$

One way of thinking about this is to consider that the number of charge must be conserved, and thus some capacitor řeceivesm̈ore charge than other. However, this sum total must still equal to their initial amount. This idea, along with voltages across each capacitors, leads to the following relationship between the capacitance of each capacitors:

$$C = C_1 + C_2 + C_3.$$

Series

The alternative to the **parallel** arrangement is the **series** arrangement. In this arrangement, one terminal of a circuit element is connected to one terminal of an adjacent circuit element. Diagrammatically,



The charges goes through all the capacitors, this allows us to write the following relations:

$$Q = Q_1 = Q_2 = Q_3.$$

One would also observe a voltage drop after each capacitor, but the summed value of voltage drop at each capacitor must equal to the total voltage supplied. Thus

$$V = V_1 + V_2 + V_3.$$

With these ideas, we will be able to write down the relationship between the net capacitance and the capacitance of individual capacitors. This is

$$C^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1}$$

Energy Stored in Capacitors

The question one should consider here how much electrical energy can we store in a given capacitor now that we have talked about their capacitance.

Consider a parallel plate capacitor being charged up. One of the plate has the voltage of $0V$ (as reference) and the other plate have a voltage of V. Because $V = \frac{Q}{C}$, as Q is increased, V also increases, linearly. Then the average potential difference is just the sum of the initial and final voltage, divided by 2. That is

$$V_{ave} = \frac{0+V}{2} = \frac{1}{2}V$$

. Then the work done in charging the capacitor is

$$W = QV_{ave} = \frac{1}{2}QV.$$

Another approach to this is to consider that the work required to move a small incremental charge dq from a negative to positive plate of potential difference V is

$$dW = Vdq.$$

Since V changes with the presence of charge,

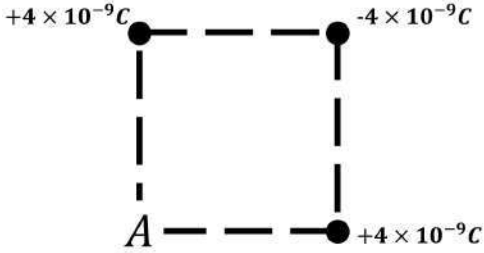
$$\begin{aligned} W = \int dW &= \int_0^Q V(q)dq \\ &= \int_0^Q \frac{q}{C}dq \\ &= \frac{1}{2}\frac{Q^2}{C} \\ &= \frac{1}{2}QV \end{aligned}$$

Exercise

1. A $100pF$ capacitor is charged by putting it across $1.5V$ battery. What is the charge on its plates?
   [Ans:$0.15nC$**]**

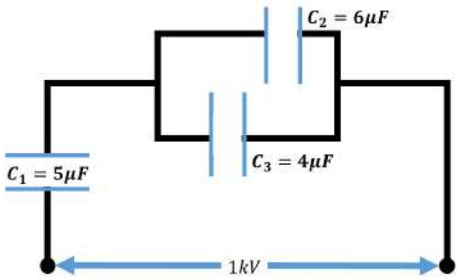
2. Three equal charges of $+6nC$ are located at the corners of an equilateral triangle whose sides are $12cm$ long. Find the potential at the center of the triangle base.
   [Ans:$V = 2320V$**]**

3. Three charges are placed at the three corners of a square of sides $20cm$ as shown in the figure below. Calculate the potential at position A.
[Ans:$V = 233V$**]**



4. In the Milikan experiment, an oil drop carries 4 electronic charges and has mass of $1.8 \times 10^{-12}g$. It is held almost at rest between 2 horizontal charged plates $1.8cm$. What voltage must there be between the two charged plates?
[Ans:$V = 496V$**]**

5. Find the total capacitance and the potential difference across each capacitor for the circuit shown in the figure below.
[Ans: $C = 3.33\mu F$**;** $V_i = \{667V, 333V, 333V\}$**]**



6. Find the energy stored in a $60nF$ capacitor when charged

to a potential difference of $2kV$.
[Ans:$W = 12mJ$**]**

7. Calculate the energy stored in a parallel plate capacitor, of area $200cm^2$ per plate and plate separation of $0.4cm$, when connected to $500V$ source.
   [Ans:$E = 5.5\mu J$**]**

8. Two capacitors of $10\mu F$ and $20\mu F$ capacitance are connected in series, and then connected to a $12V$ source. Calculate the energy stored in the system once it has fully charged.
   [Ans:$E = 0.4mJ$**]**

Charging & Discharging of Capacitors

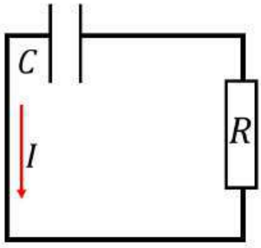
If one would model capacitors as a reservoir for charges, we would wonder the time it takes to "fill up" a reservoir as well as the time it takes to empty iT. There are, of course, factors that influences the rate of which these processes may take place, but algebraically at this stage, we can represent that rate of charging/discharging by *time constant*. For the RC circuit, we can say 2 things about the time constant, τ:

1. it is the amount of time to charge a capacitor through a resistor from $0V$ to approximately $(1-e^{-1})\%$ of the applied DC voltage.

2. it is the amount of time to discharge a capacitor through a resistor to approximately $e^{-1}\%$ of tis initial charge voltage.

With that said, let us take a look at an RC circuit and the calculations related to it.

RC Circuit



The diagram above shows a simple RC circuit consisting of a charged capacitor and a resistor. When the circuit is close, the capacitor will begin to discharge its stored electrical energy through the resistor.

The tota current that flows out of the capacitor and through the resistor must equal to 0 and therefore we should have the relation

$$I_{Capacitor} + I_{resistor} = 0$$

$$C\frac{dV}{dt} + \frac{V}{R} = 0$$

Rearranging this gives us

$$\int -\frac{1}{V}dV = \int \frac{1}{RC}dt$$

$$V(t) = V_o\, e^{-\frac{t}{RC}}$$

where $V_o = V(t = 0)$. Since we define the RC time constant to be the time taken for a capacitor to discharge through a resistor to e^{-1}. This definition is fulfilled iff

$$t_{discharge} = RC$$

.

The common symbol for the RC time constant is τ. So we may define the time constant for RC circuits as

$$\tau = RC$$

where R has the units of Ohma and C has the units of Farads.

From the voltage drop equation, we will also be able to deduce that the rate of charge (or current!) drop/rise will follow the same exponential decay/growth, i.e.,

- Charging: $V = \frac{Q}{C} \rightarrow Q = Q_o \left(1 - e^{\frac{-t}{RC}}\right)$

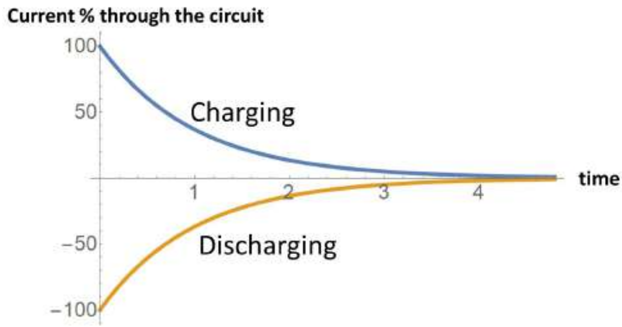
$$\begin{aligned} I &= C\frac{dV}{dt} \\ &= CV_o \frac{d}{dt}(1 - e^{-\frac{t}{RC}}) \\ &= I_o e^{-\frac{t}{RC}} \end{aligned}$$

- Discharging: $V = \frac{Q}{C} \rightarrow Q = Q_o\, e^{\frac{-t}{RC}}$

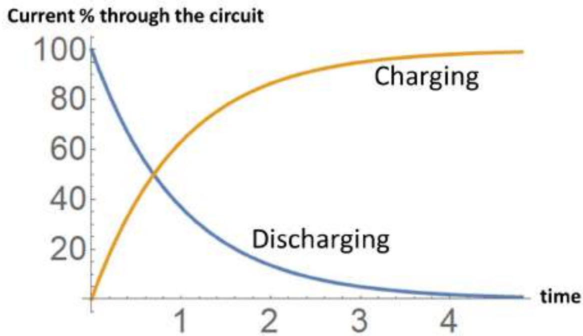
$$\begin{aligned} I &= \frac{dQ}{dt} \\ &= \frac{d}{dt}(Q_o\, e^{\frac{-t}{RC}}) \\ &= -\frac{Q_o}{RC}\,(e^{\frac{-t}{RC}}) \\ &= -I_o\,(e^{\frac{-t}{RC}}) \end{aligned}$$

The equations above allows us to graphically study the charging and discharging process in terms of its charge and current flow through the circuit as time goes by:

- Current-time graph



- Charge-time graph



Dielectrics

Consider the cross section of a parallel plate capacitor, in which the plates has a potential difference of ΔV. We know, from previous chapters that the $\Delta V = Ed$ where E is the electric field and d is the separation between plates.

Given that Gauss's Law is given by

$$\oint \vec{E}\, dA = \frac{Q}{\epsilon_o}.$$

For a parallel plates, the area by which the one would integrate is the area of the plate. This would mean that the electric field between the plates is

$$E = \frac{Q}{\epsilon_o A}$$

Rearranging thing gives

$$Q = \frac{\epsilon_o A}{E}.$$

The capacitance of the parallel plate capacitor can then be computed using $Q = CV$ to give

$$C_o = \frac{\epsilon_o A}{d}.$$

It would be noted here that this equation works with the assumption that only free space (vacuum) exist in between the parallel plates. It would be different story if something else is put in between the parallel plate. Assuming same dimensions, the capacitance of a new parallel plate capacitor with a dielectric material with electrical permittivity, ϵ, in between the plates will have capacitance

$$C_{new} = \frac{\epsilon A}{d}.$$

We can define the **dielectric constant** as the ratio of the electrical permittivity of the material to the vacuum permittivity, i.e.

$$\epsilon_r = \frac{\epsilon}{\epsilon_r}.$$

This allows us to relate the capacitance of a vacuum filled capacitor to the capacitance of an dielectric filled capacitor by the following equation

$$C = \epsilon_r C_o$$

Exercise

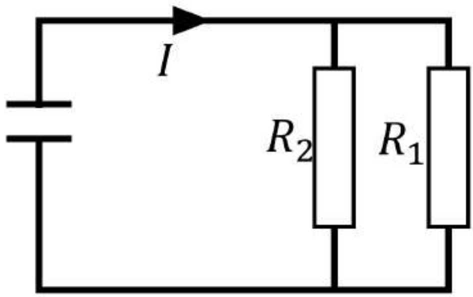
1. In an experiment a capacitor is charged from a constant current supply by a $50mA$ current pulse which lasts $25s$. **[Ans:** $Q = 1.25C$**]**

2. A capacitor is charged at a constant current of $2.5mA$ until the charge on the capacitor is $0.010C$. If the capacitance of the capacitor is $50\mu F$. To what voltage had it been charged? **[Ans:** $t = 4s$**,** $V = 200V$**]**

3. The capacitance of the capacitor is $100\mu F$. To what voltage had it been charged?

4. A $30\mu F$ capacitor has a charge of $360\mu C$ is connected to $70k\Omega$. Calculate the time constant and the initial current. Sketch the initial current.
   [Ans: $\tau = 2.1s$**;** $I_o = 1.7143(10^{-1})$**]**

5. In the circuit below, the capacitor is initially holding a charge of $10\mu C$. The capacitance of the capacitor is $1\mu F$ and the resistors has a resistance of $R_i = \{30\Omega, 15\Omega\}$.



Calculate the energy initially stored in the capacitor
[Ans:$U = 50\mu J$**]**

6. In an RC series circuit, $V = 12V$, $R = 1.4M\Omega$ and $C = 1.8\mu F$. Calculate
   a) the time constant.
   [Ans:$\tau = 2.52s$**]**
   b) the maximum charge of the capacitor.
   [Ans:$Q_{max} = 21.6\mu C$**]**
   c) time taken for the capacitor to charge up to $16\mu C$.
   [Ans:$t = 3.4s$**]**

7. A capacitor with an initial potential difference of $100V$ is discharged through a resistor, starting a time $t = 0s$. At $15s$, the potential difference across the capacitor is $1V$. Calculate the time constant of the circuit and the potential difference across the capacitor at $t = 10s$.
   [Ans:$\tau = 3.257s$; $V = 4.64Volts$**]**

8. A parallel plate capacitor, filled with a material of dielectric constant of 3.4, is connected to $100V$ power supply. If the

parallel plates has an area of $A = 4m^2$ and are separated by $4mm$. Calculate the capacitance and the charge on the capacitor.
[Ans:$C = 3 \times 10^{-8}F$**;** $Q = 3\mu C$**]**

9. A parallel plate capacitor is filled with a material with dielectric constant of 2.5. The distance between the plates of the capacitor is $0.2mm$. Find the plate area if the new capacitance is $2.5\mu F$.
[Ans:$A = 22.6m^2$**]**

Chapter 3: Electric Current and DC Current circuits

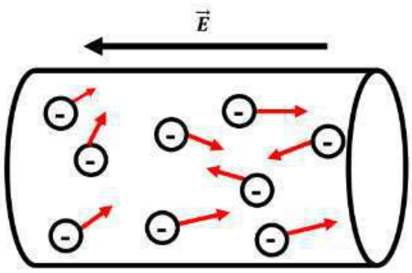
Learning Goals:

1. State/Define:
    a. Electric Current, $I = \frac{dQ}{dt}$
    b. Ohm's Law
    c. Resistivity
2. Describe:
    a. Microscopic model of current
    b. Electrical resistance in wires due to variation of temperature
    c. Electromotive force (emf) of a battery
    d. Internal resistance of a battery
    e. Factors influencing internal resistance
    f. The relationship between battery emf and potential difference across the battery terminal
    g. Kirchhoff's Rules
    h. Principle of potential divider
    i. Principles of potentiometer and its applications
3. Derive:
    a. Effective resistance of resistors in:
        i. Parallel
        ii. Series
4. Use:

a.	Electric Current	$I = \frac{dQ}{dt}$ $Q = ne$
b.	Ohm's Law	$V = IR$
c.	Resistivity	$\rho = \frac{RA}{l}$
d.		$R = R_o[1 + \alpha\Delta T]$
e.	Terminal voltage	$V = \varepsilon - Ir$
f.	Kirchhoff's Rules	$\sum I = 0$ $\sum V = 0$
g.	Power	$P = IV = I^2R = \frac{V^2}{R}$
h.	Electrical energy	$W = Pt = IVt$
i.	Equation of potential divider	$V_1 = \left(\frac{R_1}{\sum_{i=1}^{N} Ri}\right)V$
j.	Related equation for potentiometer	$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$

1.3 Electric Current & Direct Current

Electric Conduction & Ohm's Law



Consider a cylindrical electrical conductor, with a potential difference of ΔV is applied across the length of conductor. This accelerates the electrons by converting this electrical energy to kinetic energy. This acceleration, however, is shortlived the electrons would collide with each other and/or the "walls" of the conductor, before accelerating again. By the time the electron reaches the end of the conductor, they have lost kinetic energy due to the frequent collisions.

As such, it would be troublesome for us to speak of the velocity of individual electrons. We can then choose to speak of the average velocity of the the electrons in the conductor due to the electric field, this is what we mean by **drift velocity**.

If we take a "slice " of the conductor and "observe" the motion of the electrons, we can "calculate" how many electrons passes by that "slice" in 1 second. This is then, what we mean by current. That is the rate of change of charge, which we may represent mathematically as

$$I = \frac{dQ}{dt}$$

and it would have units of Coulombs per second, $C\,s^{-1}$, which is equal to 1 ampere ($1A$). **It would be a good reminder that the electrons flows the opposite direction to the direction of the current.**

Say you have applied a potential difference of V across a conductor, and the current reading you observe is A_1. Applying the same potential difference on a different material of the same dimension might give a different reading for current, say A_2. We can then compare and find differences between the ratio of the voltage difference to the current reading, $\frac{V}{I_i}$, where i depends on material 1 or 2.

This difference in the ratio allows us to say that the charges flows "easier" or "harder" in some material than other, this is in essence what we mean by **resistance**, R. More specifically, the **electrical resistance** of a material, and is measured in units of Ohms,Ω, where we define Ohms to be the volts per ampere(VA^{-1}). The inverse of this is **electrical conductance** of a material, measured in siemens (S, sometimes written as ℧).

Speaking of the ratio of voltage to current, giving us Ohm's Law:

$$I \propto V \Rightarrow I = \frac{V}{R},$$

which tells us that the voltage is directly proportional to the current and the proportionality constant is the resistance of the material. Materials that obeys Ohm's Law are called *Ohmic material.*

Resistivity & Variation of resistance with temperature

Extending the electron-in-conductor picture, we can this about how far the electrons has to move and the energy lost due to collision as the frequency of the collision may differ due to the distances covered. This tells us that the resistance of an object depends on 2 internal factor- **material type** making up the object, and the **geometry** of the object.

The first, material type will affect as different materials may have varying numbers of free electrons and thus affect the rate of change of charge, i.e. current. This is represented numerically as resistivity, ρ. A higher resistivity in a material would lead to a higher resistance of object, i.e. $R \propto \rho$

The second, geometry of the object mainly focuses on the **cross sectional area** and the **length** of the object. Experiments would show the following relations:

$$R \propto l \text{ and } R \propto \frac{1}{A}$$

which would make sense. This is because as the object is longer, the individual free electron would face a higher frequency of collisions with other free electrons and a larger cross sectional area would mean a lower frequency of a free electron colliding with the walls of the object.

We can now take these two factor into consideration numerically and make the following conclusion:

$$R \propto \rho, R \propto l, R \propto \frac{1}{A} \to R = \rho\frac{l}{A}.$$

As mentioned before, resistance of an object depends on the 2 internal factors. Here, we consider 1 external factor of resistance, that is the **temperature of the object**. As the temperature of the surrounding environment increases, so does the object. This leads to an increase of atomic, ionic and molecular vibrations within the material. This increases frequency of collisions between electrons, increasing the resistance.

To study the effects of temperature change on resistance, we may perform 2 experiments. The first one studies how the change in resistance,ΔR, behave in relation to the initial resistance,R_i, and that would give the following relation

$$\Delta R \propto R_i.$$

The second experiment studies the relationship between the change in resistance,ΔR, changes with the temperature change. This second experiment allows us to find that

$$\Delta R \propto \Delta T.$$

Combining these two, what we find is that

$$\Delta R \propto R_i \, \Delta T.$$

Keeping in mind that this relation will vary with the composition of the material, we know we will need a proportionality constant, which we shall represent with α, giving us the equation

$$\Delta R = \alpha R_i \, \Delta T.$$

This proportionality constant, α, is what we call **thermal coefficient of resistance** and is defined as **the fractional change in resistance per unit temperature change**.

Electromotive force

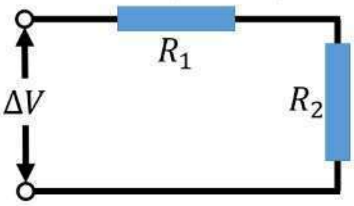
Now we shift our attention to the source of the potential difference. To do that, we start with defining the **electromotive force(emf)**, which is in all actuality not a force in the conventional manner. The emf is defined as the voltage measured across the terminals of the source when no currrent is being drawn from or delivered to it. In a more practical definition, it is the voltage the source is able to deliver. This is however resisted, or impeded, by the imperfections of the power source. These imperfections leads to internal resistance, that is resistance within the power supply.

If the power supply is able to produce voltages ε but there exist an internal resistance of r in the power supply itself, how much voltage would I be able to measure if I, say, connect the power supply to a resistor? The answer is fairly simple and requires Ohm's Law, that is

$$V = \varepsilon - Ir.$$

Resistors in series & parallel

In this part of the topic, we deal with 2 types of resistor arrangements, one where the ends of the resistors are connected end to end (**series**) and another where one end of the resistors are connected to the one junction and the other end is connected to another junction(**parallel**).



Resistors in series

When the resistors are arranged in series, the same current are made to go through both the resistors. This means that

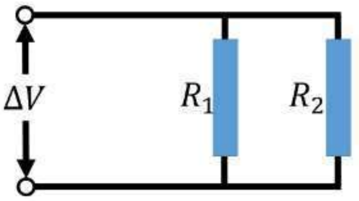
$$I_{total} = I_1 = I_2$$

and it should be noted that the voltage drops after the first resistor before reaching the following resistor and therefore the total voltage is merely the summation of voltage across each resistor

$$V_{total} = V_1 + V_2$$

If we apply Ohm's Law on the whole circuit, and the inidividual resistors, we will obtain

$$\begin{aligned} \Delta V_{total} &= V_1 + V_2 \\ I_{total} R_{eff} &= I_1 R_1 + I_2 R_2 \\ \text{since } I_{total} &= I_1 = I_2 \\ R_{eff} &= R_1 + R_2 \end{aligned}$$



Resistors in parallel

In this case, the charges (ipso facto the current) going through each resistor is not the same but is "split" at the first junctions and recombines at the second. This means that

$$I_{total} = I_1 + I_2.$$

The voltages between the junctions, however, remains constant, telling us that the the voltage differences across each resistors must also be the same,

$$V_{total} = V_1 = V_2$$

Reiterating the procedure as done for the case of series, we obtain

$$\begin{aligned}
\Delta I_{total} &= I_1 + I_2 \\
\frac{V_{total}}{R_{eff}} &= \frac{V_1}{R_1} + \frac{V_2}{R_2} \\
\text{since } V_{total} &= V_1 = V_2 \\
R_{eff} &= \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}
\end{aligned}$$

Electrical energy & power

One of the most fundamental ideas of any motion is the idea of energy as well as the rate of change of the energy change (power).

Suppose you subject a small charge dq to some electric field and it starts to move from point A to B in time dt. Taking this path in the electric field, changes the electric potential experienced by the charge by $dE_{e.p.} = dqV$. This tells us that the power, or the rate of energy change is

$$P = \frac{dE_{e.p.}}{dt}V = \frac{dq}{dt}V$$

Recalling the definition of current, $I = \frac{dq}{dt}$, this gives us an equation for electrical power,

$$P = IV.$$

This equation can be modified in the case of a resistor usign Ohm's Law to give

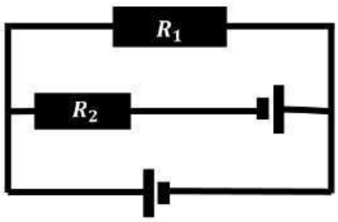
$$P = IV \Rightarrow P = I^2V = \frac{V^2}{R}$$

If the energy change rate is constant (doesn't fluctuate), then $P = \frac{W}{t}$, and that leads us to a the equations for electrical energy

$$W = IVt = I^2Vt = \frac{V^2t}{R}$$

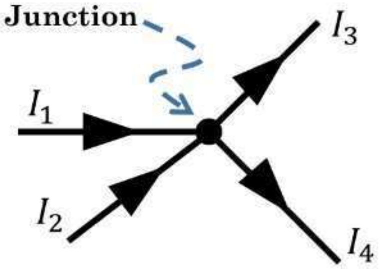
Kirchhoff's Rules

Given any electrical circuit, there are many approaches one can take to analyse them. One of the most basic is categorising them into either series or parallel. But what do you then do when the electrical circuit is neither series nor parallel.



The figure above is an example in which the circuit cannot be categorised as series nor parallel. In order to figure out voltages and current across each electrical component, we may apply **Kirchhoff's Rules**. There are two of the rules actually - **Junction Rule** and **Loop Rule**.

Kirchhoff's **Junction Rule** is a statement of charge conservation. It states that the sum of the currents in and out of a junction must equal to zero, $\sum I_{in} + \sum I_{out} = 0$.

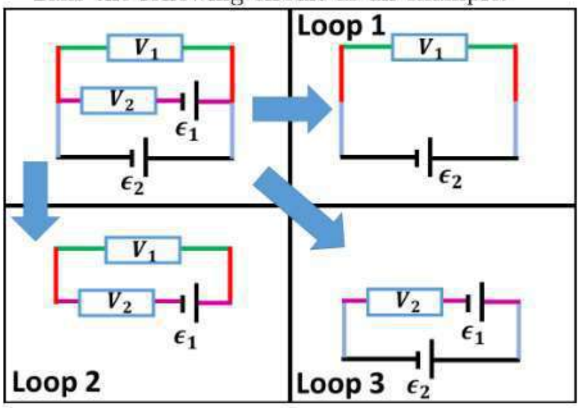


Applying the junction rule to the circuit snippet above, we find that

$$I_{in} = I_{out}$$
$$I_1 + I_2 = I_3 + I_4$$

Kirchhoff's **Loop Rule**, on the other hand, is a statement of energy conservation, in form of voltage difference. The loop rule states that the sum of all the electrical potential differences around a loop is zero, $\sum V = 0$.

Take the following circuit as an example:



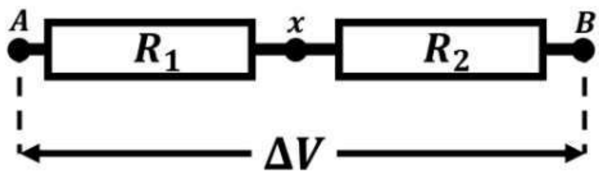
The circuit may be broken down into 3 loops. Each loop has has the following relation (taking anticlockwise direction) in terms of the voltages across its components:

$$\text{Loop 1: } \epsilon_2 - V_1 = 0$$
$$\text{Loop 2: } \epsilon_1 - V_1 - V_2 = 0$$
$$\text{Loop 3: } \epsilon_2 - \epsilon_2 - V_2 = 0$$

Potential divider & Potentiometer

Up to this point, you might be use to using a voltmeter and an ammeter in measurements of potential difference (voltage) and charge flow (current). But this is not accurate, they lack accuracy because voltmeters draws up some current and ammeters reduce the current flow. To reduce these inaccuracies, we shall introduce **Null Measurement Devices**. They are used as a measurement device via balancing the circuit so that no current flows through the galvanometer. This is achieved by making sure that the potential difference between the ends of the measuring device is zero. Let us consider one example of null measurement device arrangements - a **potentiometer**, which works by the **principle of potential divider**.

This principle is easy to get your head around. Take the circuit snippet in the diagram below



Between point A and B, there is a potential difference of ΔV and let us assume that resistances are $R_1 = R_2 = R$. If we imagine having 10 doughnuts at point A and have to spend all the doughnuts by the time we reach point B **and** we can only spend those doughnuts at R, since the number of doughnuts spent at each R depends on the resistances at R, we would spend 5 doughtnuts at each R and this we will observe 5 doughnuts by the time we reach point x. This means that the 10 doughtnuts we initially have at point A and must spend by the time we reach point B is divided by the $2R$'s passed by. This is the idea of a potential divider, that the potential is divided between the resistors and that division depends on the number of resistors and their resistance in question.

Consider the same diagram but instead the resistors having different resistances, let's take $R_1 \neq R_2$. The question now is what would be the voltage we will measure at point x?

Well since we know it would depend on the values of R_1 and R_2, we know that the relation is directly proportional. What may not seem be obvious is that the measure of voltage at x is not between potentials at A and x but x and B because this the voltage "allocation" for R_2. This leads us to

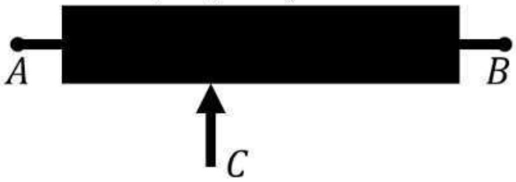
$$V|_A = V|_x \left(\frac{R_2}{R_1 + R_2} \right).$$

Generalizing for a case of n number of resistors leads us to

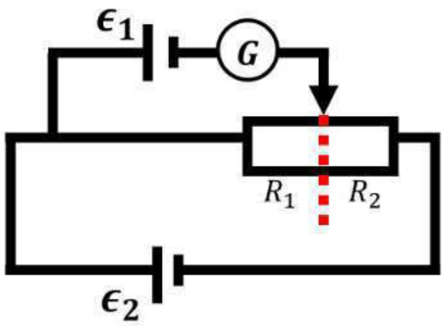
$$V|_A = V|_x \left(\frac{R_2}{R_1 + ... + R_N} \right).$$

Now that we have the idea of a potential divider sorted, we can talk about potentiometer. What are they? Practically they're an electrical components that allows you to vary voltage outputs, similar to a rheostat. Technically, they're small circuits made up of resistors on which a slider moves about to allow for voltage, current, bias and/or gain control.

On circuits, they are symbolised as



They resemble a resistor with a slider leading up to an additional output, which they are. The make up of a potentiometer can be modelled as below



Based on the diagram, we can work out the equations relating emf to the resistance, which would give us

$$\frac{\epsilon_1}{\epsilon_2} = \frac{R_1}{R_2}$$

and since $R \propto l$, which is length, then we would know that

$$\frac{\epsilon_1}{\epsilon_2} = \frac{l_1}{l_2}.$$

For those who would like to advance their understanding of potential dividers, I would like to suggest to them to look up **Wheatstone Bridges**.

Chapter 4: Magnetism

1. Define magnetic field – **What is a magnetic field?**

2. Identify magnetic field sources – **Give examples of magnetic field sources**.

3. Explain
   - **a.** The reason behind the circular path motion of charged particle in a uniform magnetic field, relating it to centripetal force. **– Why is it that a charged particle moves in a circular path and how do I do calculations relating magnetic force and circular motion?**

   - **b.** Magnetic force per unit length of 2 parallel current-carrying conductor – **How does such a force exist?**

   - c. Working principle of a galvanometer – **How does a galvanometer work?**

   - d. Motion of charged particle in a region of non-zero electric and magnetic fields – **Describe the motion.**

4. Sketch
   - a. magnetic field lines for bar magnets and current-carrying conductor in form of
     - a. straight wire,
     - b. circular coil,
     - c. and solenoid
   - b. resultant magnetic field diagram at a point, at most the complexity of 2-current carrying straight wires in 2 dimensional

5. Determine equation/derivation for
   - a. Direction of $\vec{B}$ by using the Right Hand Rule – Describe the Right Hand Rule.
   - b. Magnitude of $\vec{B}$-field
     - i. For a long straight line
     - ii. At the centre of circular coil
     - iii. At the centre of a solenoid
     - iv. At the end of a solenoid

     Write the equation for the cases above

   - c. Magnitude and direction of force arising from magnetic field
     - i. For a moving charged particle
     - ii. For current carrying conductor
     - iii. For 2 parallel current carrying conductors (Need to derive as well)

     Write the equation for the cases above

   - d. Torque for a coil in a magnetic field - **Write the equation**

   - e. Velocity of a charged particle in a velocity selector - **Write the equation**

2 Electromagnetism Part 2

All the mathematical sciences are founded on relations between physical laws and laws of numbers, so that the aim of exact science is to reduce the problems of nature to the determination of quantities by operations with numbers."

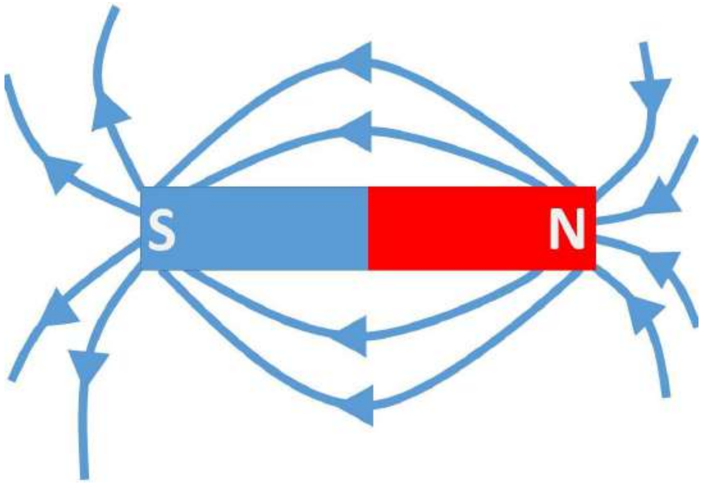
-James C. Maxwell
"The Scientific Papers of James Clerk Maxwell", p.156

In this chapter, we are again looking at the aspect of physics that deals with electrically charged particles, but this time, they **oscillate**.

2.1 Magnetism

Magnetic Fields

When charged particles are present, they generate electric field. When they starts moving, they start to generate not just electric field but also **magnetic fields**. We ae able to draw field lines for magnetic fields, much like we do with electric fields. Below shows the magnetic field lines of a permanent magnet:



The SI unit for magnetic fields are Tesla (T). For reference, the Earth's magnetic field is found to be approximately $0.5 \times 10^{-4}\,T$. Note that the magnetic fields should always end in a loop, they never intersect and they should never begin/end on anything but sources/sinks (there exists no magnetic monopole - at least not experimentally!).The last of the requirement is actually a statement of **Gauss's Law**, which is

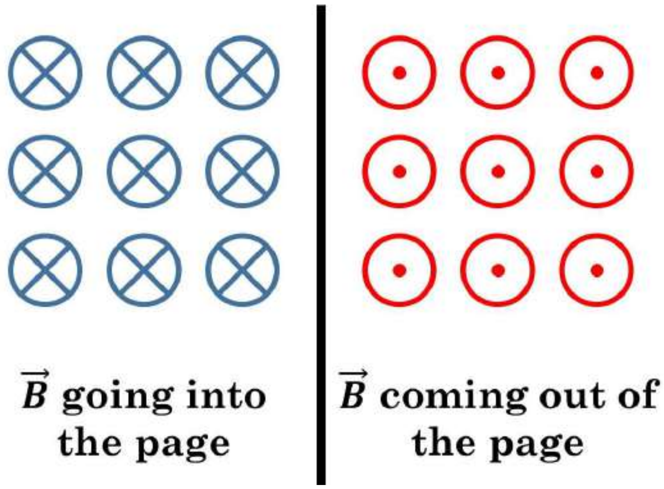
$$\oiint B \cdot dA = 0.$$

Eventhough intergrating with respect to area results in a trivial solution, we can, however, integrate with respect to path length. To do this we refer to **Ampere's Law**, which states:

$$\oint \vec{B} \cdot d\vec{L} = \mu_o I$$

where $d\vec{L}$ is a infinitessimal segment of a path.

We typically represent field lines "going in" the page as "x" in a circle or by itself, field "coming out" of the page as "o"'s with a dot at the centre, or just the dot.



Exercise:

1. Describe how the direction of the current in a long straight wire affects the direction of the magnetic field produced.

2. Sketch the magnetic field lines for a current carrying:

   a) straight wire

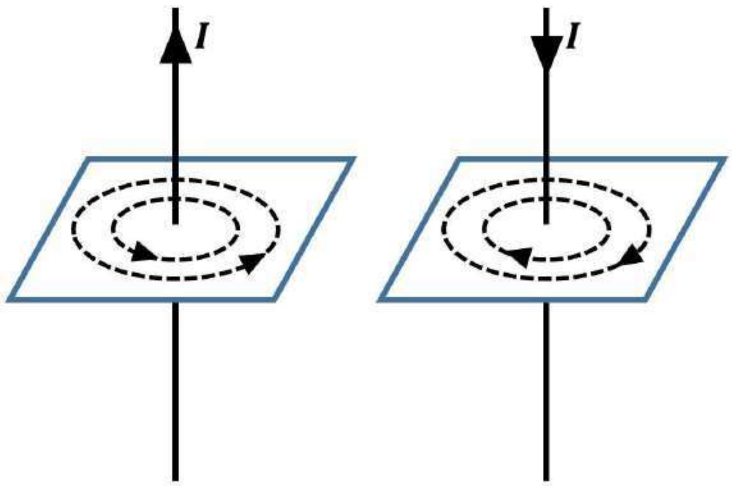
   b) solenoid

   c) toroid

$\vec{B}$ by Current-carrying Conductor

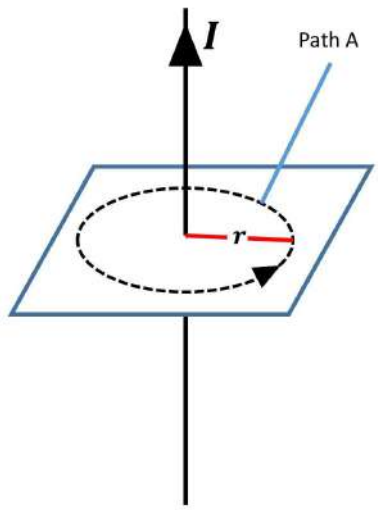
As mentioned before, magnetic field arises from moving charged particle. It would be of our interest to know the direction of the $\vec{B}$-field relative to the direction of the moving charge. A single charge might not be able a strong enough of a magnetic field, so let us consider a conduction, through which charged particles (electrons) are moving in a specified direction. Say the current direction (opposite to the electron movement direction) is in the $+z$-direction.

In order to determine the direction of the magnetic field, we would refer to the **Right Hand Rule.** The magnetic field will be perpendicular to the conductor and the Right Hand Rule would tell you that the direction of the magnetic field is the direction of the curled fingers of one's right hand, provided that one's thumb is in the direction of the current.



To evaluate the magnitude of magnetic field, we shall need choose an appropriate path and apply **Ampere's Law.** Let us consider 3 cases:

1. Long straight wire

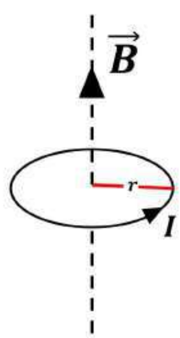


In this case, we may take the path A as our loop and deduce that

$$\oint \vec{B} \cdot d\vec{L} = B(2\pi r)$$

$$B = \frac{\mu_o I}{2\pi r}$$

2. Centre of circular coil

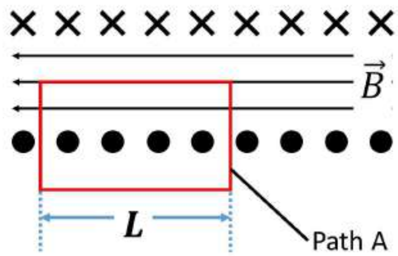


The magnitude of magnetic field at the centre of a circular loop of current may be obtained using **Biot-Savart Law.**

The derivation is left for the reader but the following result is expected

$$B = \frac{\mu_o I}{2r}$$

3. Centre of solenoid



In this case, again we take the path A as our loop and deduce that

$$\sum B\, dL\, cos(\theta) = B_{top} + B_{bottom} + 2B_{side} = B_{top}L$$

$$B_{top}L = \mu_o IN$$

$$B = \mu_o I \frac{N}{L}$$

$$B = \mu_o In$$

where n is the number of loops per unit length.

4. End of solenoid

Imagine now if the solenoid is cut in half and then your amperian loop is no longer at the centre but at the end of the solenoid. Here, the magnetic field is half of what it was at the centre. Then

$$B = \frac{1}{2}\mu_o In$$

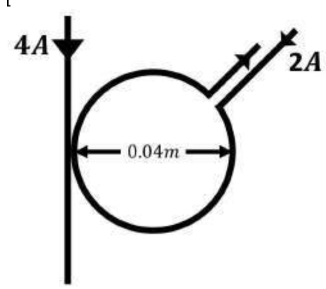
Exercise:

1. A long straight wire carries a current of $3A$. At what perpendicular distance from the middle of the wire will the magnetic field have the magnitude of $0.5\,Gauss$?
   Ans: $r = 0.012m$

2. A thin $5cm$ long solenoid has a total of 200turns of wire and carries a current of $3A$. Calculate the field at the center of the solenoid and at the end. Comment on the difference.
   Ans: $B_{centre} = 1.5 \times 10^{-2}T$; $B_{end} = 0.75 \times 10^{-2}T$

3. A $30cm$ wire is bent into a circle and a current of $2A$ is passed through it, calculate the magnetic field produced at the centre of the loop.
   Ans: $B \approx 2.6 \times 10^{-5}T$

4. Calculate the magnitude and direction of the magnetic field at the centre of the circular loop based on the diagram below, depicting a straight wire placed left tot the circular loop.



   Ans: $B \approx 4.38 \times 10^{-3}T$

Magnetic Forces

If you bring a piece of metal close to a magnet, you can feel "something" pulling the metal towards the magnet. Let us consider this magnetic force.

Individual charged particle

As established before, a magnetic field is produced when charged particles moves. When this charged particle moves, the magnetic field it produces interacts with the external magnetic field and thus this interaction causes the particle to experience magnetic force. We would guess that this force will be dependent on

1. the electrical charge of the particle, q
2. the velocity of the charged particle, $\vec{v}$
3. the external magnetic field, $\vec{B}$

We find this to be true from the **Lorentz Force Law**, states that for one charged particle in an electric field $\vec{E}$ and magnetic field $\vec{B}$,

$$\vec{F} = \vec{F}_{electric} + \vec{F}_{magnetic} = q(\vec{E} + \vec{v} \times \vec{B})$$

which tells us

$$\vec{F}_{electric} = q\vec{E}$$

and

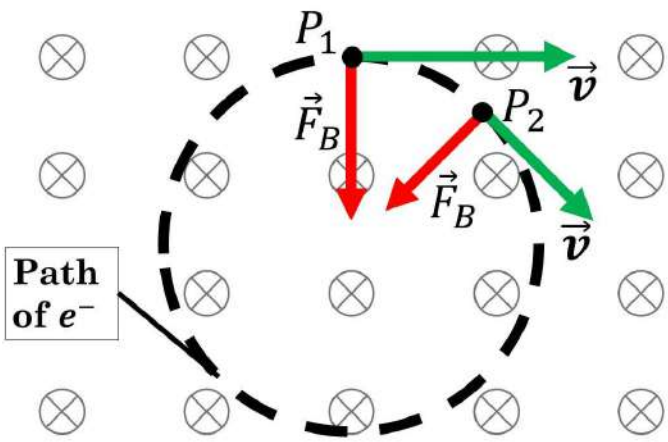
$$\vec{F}_{magnetic} = q(\vec{v} \times \vec{B}) = qvBsin\theta$$

where θ is the angle between the direction of charged particle velocity vector and the direction of magnetic field. This tells us that the maximum magnetic force is achieved when the angle between $\vec{v}$ and $\vec{B}$ is 90^o, that is $\vec{F}_{max} = \vec{F}|_{\theta=90^o} = qvB$.

The direction of this magnetic force for a positively-charged moving particle may be determine by the **Right Hand Rule** by which if the middle finger, the thumb and the forefinger is kept 90^o from each other, the representation is as follows:

Finger	Representation
Thumb	velocity of particle, $\vec{v}$
Forefinger	Magnetic Field, $\vec{B}$
Middle finger	Force experienced by the particle, $\vec{F}$

In a uniform magnetif field, the magnetic force will cause a charged particle to deflect. This deflection causes the charge particle to move in a circular path.



The diagram above shows the path of an electron in a uniform magnetic field. The electron, when at position P_1 has its velocity to the right. This combined with the magnetic field moving into the page, gives rise to a downward force. The electron is therefore deflected downwards. After some time, the electron is now at position P_2, where the force is still normal to the velocity and its direction is shown in the diagram. The magnitude of velocity (speed) doesn't change as the force is always normal to it. This is what causes the electron to move in a circular path.

As we can deduce from the diagram above, the magnetic force is the force that causes the circular motion. We can therefore deduce that

$$F_{magnetic} = F_{centripetal}.$$

This tells us that

$$qvB = \frac{mv^2}{r}$$

Which allows us to relate the mass of the electron, the strength of the magnetic field, the radius of the circular path and the speed of the electron to each other.

A bunch of charged particle

If we were to confine the charged particles into a conductor, like a wire, they collectively exert an average force onto the conductor. This leads to us saying that the wire to experience force.

Consider a quantity of charge, ΔQ travelling along a wire of length l in a magnetic field $\vec{B}$, in a time interval Δt. The rate of charge flow, as mentioned before, is merely the current flowing through the wire. Using $F_B = qvB$, we can deduce that the force experienced by the wire is

$$F_B = \Delta q \vec{v} \times \vec{B}.$$

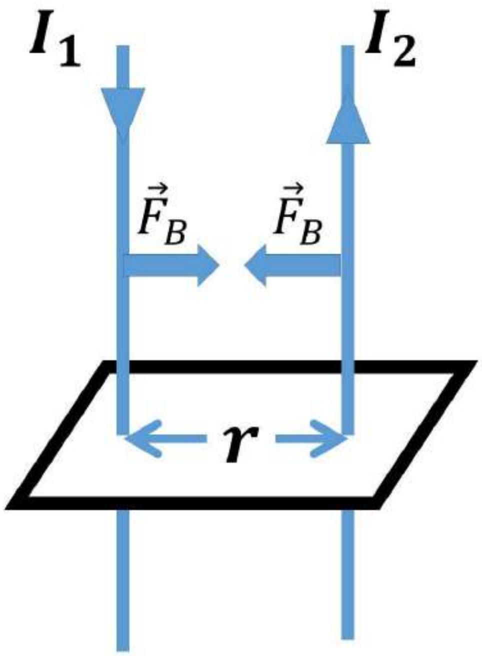
Multiplying Δt to both side allows us to write the equation

$$F_B = (\frac{\Delta q}{\Delta t})(\vec{v}\,\Delta t) \times \vec{B}.$$

Reminding ourselves that $I = \frac{\Delta q}{\Delta t}$ and $\vec{l} = \vec{v}\,\Delta t$, we have the final equation for force by magnetic field on a wire,

$$F = I\vec{l} \times \vec{B}.$$

So how does two wires, each generating its own magnetic field, attract/repels each other? To answer that, let us refer to the diagram below.



Referring back to magnetic field generated by a single wire, we know that the magnetic field generated by wire 2 in the diagram has a magnitude of

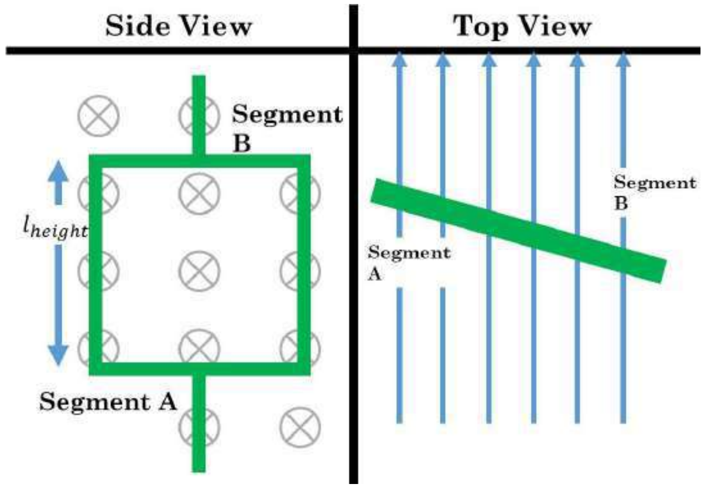
$$B = \frac{\mu_o I_2}{2\pi r}$$

Then we can say that the force onto wire 1 by the magnetic field produced by wire 2 is

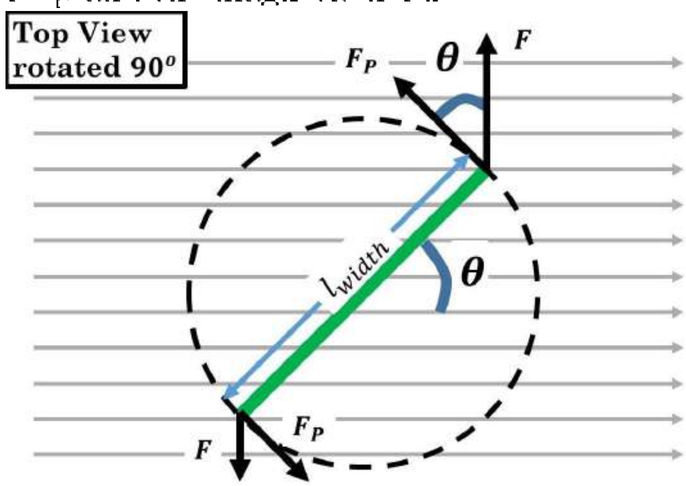
$$F_B = I_1 l \left(\frac{\mu_o I_2}{2\pi r} \right)$$

where r is merely the distance between the wires.

Now that we have discussed the idea of two wires, let us take a single wire and bend it so that it takes the the form of a rectangular loop. Here is the front view and the top view:



Let us focus on the top view, with a slight angle, θ, between the loop and the magnetic field:



From this view, we can work out the torque on the loop by the magnetic field. Recalling equation for torque

$$\tau = 2F_P\left(\frac{1}{2}l_{width}\,cos\,\theta\right).$$

Since $F_P = Il_{height}B$ and $cos\,\theta = sin\,\phi$ if ϕ is the angle between $\vec{B}$ and F_P,

$$\tau = NIl_{width}l_{height}B\,sin\,\phi$$

And since the area of the loop is $A = l_{width}l_{height}$, we have the desired equation for torque of a rectangular loop in a magnetic field,

$$\tau = NIABsin\,\phi = NI\vec{A} \times \vec{B}$$

Exercise:

1. A positively charged particle of 3.2×10^{-19} travels at normal to a magnetic field of $0.4T$ with a velocity of $10^5\,ms^{-1}$. Find the magnitude of the magnetic force on the particle.
   Ans: $F_B \approx 1.28 \times 10^{-14}N$

2. 2 wires, in which their current is travelling in the opposite direction with the same magnitude of $3.5A$, is positioned at $12cm$ from each other. Calculate the direction and magnitude of the net magnetic field at a point midway between the wires, and calculate the magnitude of the net force per unit length experienced by the wires.
   Ans: $B = 2.33(10^{-6})T$; $\frac{F}{l} = 2(10^{-5})Nm^{-1}$

3. An electron travelling in a region of uniform electric and magnetic fields which are perpendicular to each other. Calculate the electron velocity and kinetic energy if $|\vec{E} = 3.9Vm^{-1}|$ and $|\vec{B} = 6.5T|$.
   Ans: $v = 6(10^5)ms^{-1}$; $E_k = 1.64(10^{-19})J$

4. A $4cm \times 8cm$ rectangular coil with 120 turns carrying $0.5mA$ current is placed in a uniform magnetic field $0.6T$. Calculate the required torque to hold the coil's plane parallel to the field.
   Ans: $\tau = 1.15(10^{-4})Nm$

5. A $1.35 \times 10^{15}kg$ particle of charge $5nC$ and velocity $3500ms^{-1}$ enters a region of uniform magnetic field of $0.25(10^{-2})T$

perpendicular to the field. Calculate the radius of the particle path.
Ans: $r = 0.378m$

6. A $40m$ electric cable carries a current of $28A$ from west to east. The earth magnetic field is $50\mu T$, horizontal and directed from south to north. Determine

   a) the magnitude and direction of the magnetic force on the cable.
   Ans: $F = 0.056N$ **upwards**

   b) the magnetic force on the cable if it is parallel to the direction of the magnetic field
   Ans: $F = 0$

7. The plane of a coil of radius $0.20m$ is parallel to the yz-plane in a uniform magnetic field. The magnetic field is $0.40T$ and in the positive x-direction.

   a) Calculate the magnetic flux through the coil.
   Ans: $\Phi = 5.03(10^{-2})Wb$

   b) The coil is then rotated clockwise about the y-axis, such that the normal of the coil is now 30^o with respect to the x-axis. Calculate the magnetic flux at this angle. **Ans:** $F = 4.36(10^{-2})Wb$

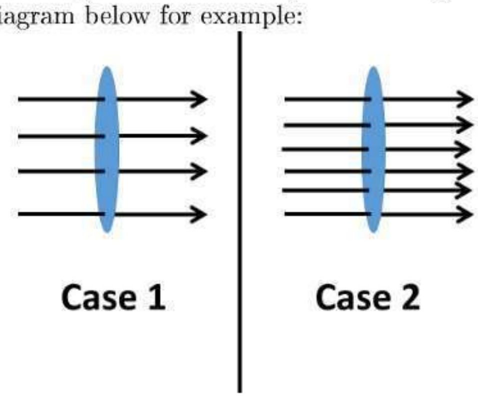
Chapter 5: Electromagnetic Induction

Learning Goals:

1. State/Define:
   a. Magnetic Flux
   b. Magnetic Flux Linkage
   c. Faraday's Law
   d. Lenz's Law
   e. Self-Inductance
   f. Mutual Inductance
2. Describe/Explain:
   a. Induced emf using Faraday's experiment
3. Derive:
   a. Induced emf in
      i. Straight conductor
      ii. In a coil
      iii. In a rotating coil
   b. Energy stored in a conductor, $U = \frac{LI^2}{2}$
4. Use:
   a. Magnetic Flux, $\phi = \vec{B} \cdot \vec{A}$
   b. Magnetic flux linkage $\Phi = N\,\phi$
   c. Faraday's Law and Lenz's law, $\varepsilon = -\frac{d\phi}{dt}$
   d. Induced emf in
      i. Straight conductor, $\varepsilon = Blv\,sin\,\theta$
      ii. In a coil, $\varepsilon = -NA\frac{dB}{dt} = -NB\frac{dA}{dt}$
      iii. In a rotating coil $\varepsilon = NAB\omega\,sin\,\omega t$
   e. Self-Inductance, $L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)} = \frac{N\phi}{I}$, for:
      i. Coil, $L_{\mathrm{coi}} = \frac{\mu_o N^2 A}{2r}$
      ii. Solenoid, $L_{solenoid} = \frac{\mu_o N^2 A}{l}$
   f. Mutual inductance between two coaxial solenoids, $M = \frac{\mu_o N_1 N_2 A}{l}$

2.2 Electromagnetic Induction

Let us first define the item to be integrated - $B \cdot dA$. Physically it is the net flux of the magnetic field out of the surface, ϕ_B. What, then, is magnetic flux? Well, it is a measurement of the total magnetic field which passes through a given area. Take the diagram below for example:



If the area of case 1 and case 2 are the same, it is easy to see more magnetic field lines are going through the given area in case 2. However, if the areas are different, what you then want to compare are their net magnetic field flux. **Magnetic flux** (of units $Tesla\, m^2$ or Wb)is then defined as

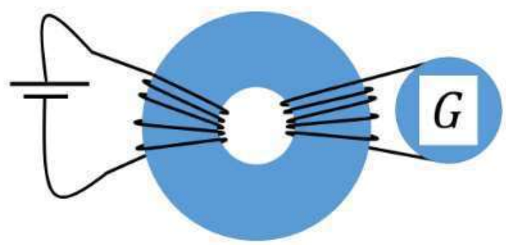
$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta.$$

Imagine now if that magnetic field lines goes through a coil of N turns, then the magnetic flux linkage is then

$$\Phi_B = N\phi_B.$$

So where does this leads us to? We have discussed about the electric field as well as the magnetic field. We now find ourselves in a position to connect them two and finally start to understand why the study is suitably named **electromagnetism**.

We begin this by looking back in history and making an attempt to explain Faraday's experiment, which led to his discovery of his "Law of Induction".



Simply, Faraday's experiment involved switching on and off the power source and observe if the needle of the galvanometer moves. What he found was that at the moment of powering on/off, the needle of the galvanometer deflects but now between powering on the off. He deduced that it was not the magnetic field that was causing the current/charge flow, rather it was the change of the magnetic field. When there is a change in magnetic field, charge flow is generated. EMF, ε, is equal to the rate of magnetic flux change, that is

$$\varepsilon_{single\ coil} = \frac{d\phi_B}{dt}$$

, giving rise to **Faraday's Law of Induction**,

Lenz's Law further extended this by point out that the magnitude may be equal but the directions of the generated emf is opposite to the flux. This gives us

$$\varepsilon_{single\,coil} = -\frac{d\phi_B}{dt},$$

which in essence gives us a law of "inducing" emf by means on a magnetic field.

From out previous treatment, we have established that the magnetic flux is dependent on three variables - Magnetic field strength, the area in which in $\vec{B}$ passes through and the angle between them. So changing any of these three variables will surely generate emf.

For a straight conductor, we may have to breakdown the area of magnetic field exposure into its two components - the length of the conductor and the direction of travel. This allows us to derive the equation for induced emf in a straight conductor as such:

$$\begin{aligned}\varepsilon &= -\frac{d\phi_B}{dt} \\ &= \frac{d}{dt}(BA\,sin\theta_{BA}) \\ &= \frac{d}{dt}(Blx\,sin\theta_{BA})\end{aligned}$$

assuming l and B remains constant, and $\theta_{BA} = 90^o$,

$$\begin{aligned}\varepsilon &= Bl\,(\frac{dx}{dt}) \\ &= Blv\end{aligned}$$

Here, we have assumed that the angle between the velocity and the magnetic field to be perpendicular (90^o). If need be, that angle may be taken into consideration such that,

$$\varepsilon = Blvsin\,\theta_{vB}$$

Now let us consider the case for a coil, in which we may choose to vary one of 2 variables:

1. If we choose to vary the area of magnetic field exposure, then
$$\varepsilon = -NB\frac{dA}{dt}.$$
2. If we choose to vary the magnetic field strength, then
$$\varepsilon = -NA\frac{dB}{dt}$$

But what if the coil rotates? Well, anything that rotates will have its angular velocity and in this case, what we are varying is not the area nor are we varying the magnetic field strength, but the angle between them,

$$\varepsilon = -NAB\frac{d(cos\,\theta_{AB})}{dt} = NAB\theta\,sin\,\theta.$$

Here let us remind ourselves that the angular velocity can bet related to the angle by $\theta = \omega t$.

Exercise:

1. The plane of a coil of radius $0.20m$ is parallel to the yz-plane in a uniform magnetic field. The magnetic field is $0.40T$ and in the positive x-direction.
   a) Calculate the magnetic flux through the coil.
   Ans: $\Phi = 5.03(10^{-2})Wb$
   b) The coil is then rotated clockwise about the y-axis, such that the normal of the coil is now 30^o with respect to the x-axis. Calculate the magnetic flux at this angle. **Ans:** $F = 4.36(10^{-2})Wb$

Now we ask ourselves, if an external varying magnetic field induces emf, can that induced emf, gives rise to a magnetic field? The answer is yes, it can. In fact it would generate a magnetic field that opposes the direction of the induced current, as per Lenz's Law. This phenomenon is called **self-inductance**. To quantify this phenomenon, we use

$$L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$$

where L is self inductance.

Since we have defined emf in two different ways,

$$\varepsilon = -L\left(\frac{dI}{dt}\right) \text{ and } = -N\frac{d\phi_B}{dt},$$

we can use them both to relate self inductance to the magnetic flux linkage such that

$$N\phi_B = LI.$$

For a coil of N turns, we know that $B = \frac{N\mu_o I}{2r}$, then

$$\begin{aligned} L &= \frac{N\phi}{I} \\ &= \frac{NBA}{I} \\ &= \frac{NA}{I}\left(\frac{N\mu_o I}{2r}\right) \\ &= \frac{\mu_o N^2 A}{2r} \end{aligned}$$

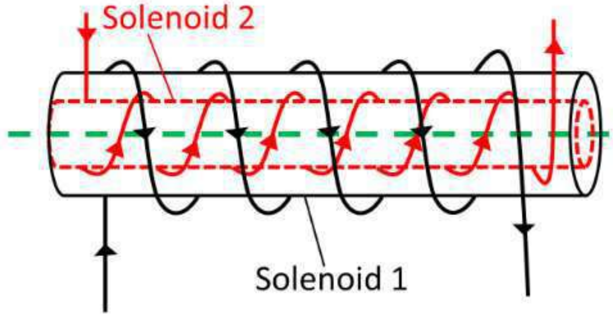
For a solenoid of length l, we know that $B = \mu_o nI$ and $n = \frac{N}{l}$, then

$$\begin{aligned} L &= \frac{N\phi}{I} \\ &= \frac{NBA}{I} \\ &= \frac{NA(\mu_o nI)}{I} \\ &= \frac{\mu_o N^2 A}{l} \end{aligned}$$

If we remind ourselves what energy is relative to power, that is $E = \int Pdt$, and the fact that $L = \frac{N\phi}{I}$. We can then work out the energy stored in a inductor:

$$\begin{aligned} P &= IV \\ \varepsilon &= -L\left(\frac{dI}{dt}\right) \\ P &= -IL\left(\frac{dI}{dt}\right) \\ E &= \int Pdt \\ &= \int -IL\left(\frac{dI}{dt}\right)dt \\ &= \frac{1}{2}LI^2 \end{aligned}$$

We have discussed how a solenoid under magnetic field change generates current within it's wires (Faraday's Law), and how that current can produce another magnetic field to oppose that original magnetic field change (Lenz's Law). This is what we called **self-inductance**. Let us how consider 2 coaxial solenoids, where "coaxial" here refers to the fact that both the solenoids shares a common axis (green line):



Mutual inductance refers to the fact that the magnetic field of one solenoid interacts with the magnetic field of the another.

If we consider the magnetic field generated by solenoid 1,

$$B_1 = \mu_o n_1 I_1 = \frac{\mu_o I_1 N_1}{l}$$

We then know that the corresponding flux linkage with solenoid 2 is

$$N_2\phi_{21} = N_2 B_1 A_2 = \frac{\mu_o N_2 A_2 I_1 N_1}{l}$$

And we also know that

$$N_2\phi_2 = M_{21} I_2$$

Then we can combine these two to obtain

$$M_{21} = \frac{\mu_o N_1 N_2 A_2}{l}$$

Chapter 6: Alternating Current

Learning Goals:

1. State/Define:
    a. Alternating current
    b. Root mean square current and voltage for AC sources
    c. Capacitive reactance
    d. Inductive reactance
    e. Impedance
    f. Phase Angle
2. Describe:
    a. The form and interpretation of the sinusoidal AC waveform
    b. Phasor diagram and sinusoidal waveform (Sketch as well!) showing phase relationship between current and voltage for single component circuit of
        i. Resistor
        ii. Capacitor
        iii. Inductor
    c. Graphically the dependence of R, X_C, X_L and Z on f.
    d.
3. Derive:
    a.
4. Use:
    a. Sinusoidal voltage, $V = V_o \sin \omega t$
    b. Sinusoidal current, $V = V_o \sin \omega t$
    c. Root mean square current, $I_{rms} = \frac{I_o}{\sqrt{2}}$
    d. Root mean square voltage, $\mathrm{V}_{rms} = \frac{\mathrm{V}_o}{\sqrt{2}}$
    e. Phasor diagram to analyse current and voltage and impedance for series circuit of
        i. Resistor-Inductor, RL
        ii. Resistor-Capacitor, RC
        iii. Resistor-Inductor-Capacitor, RLC
    f. Capacitive reactance, $X_C = \frac{1}{2\pi f C}$
    g. Inductive reactance $X_L = 2\pi f L$
    h. Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$
    i. Phase Angle, $\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$
    j. Power equations for R, RC, RL, and RLC:
        i. Average power, $P_{\text{ave}} = I_{rms}V_{rms}\,cos$
        ii. Instantaneous power, $P_{instant} = IV$
        iii. Power factor, $cos\,\phi = \frac{P_{\text{real}}}{P_{\text{apparemt}}} = \frac{P_{ave}}{I_{rms}V_{rms}}$

1.3 Alternating Current

In the previous sections, we have discussed the ideas related to direct current (DC) circuit components - resistors and capacitors. In DC circuits, the electric current only flows in one direction. In AC circuits, the electric circuit periodically reverses direction and varies its magnitude with time.

Maths

Because of the periodic nature of AC circuit, let us introduce some of the common variables in DC circuit and recast them into periodic functions - **sine wave**. We can do this by utilizing trigonometric functions:

1. For voltage: $V(t) = V_o \, sin \, \omega t$
2. For current: $I(t) = I_o \, sin \, \omega t$

V_o and I_o here are the maximum values of the variables and is related to resistance by

$$R = \frac{V_o}{I_o}.$$

For one to compare the AC circuit power dissipation to one that runs on DC, we need make factor correction. Further analysis on pure sine wave functions shows that to produce the value, we may apply a correction factor of $\sqrt{2}$ such that,

$$I_o = \sqrt{2} I_{rms}$$
$$V_o = \sqrt{2} V_{rms}$$

This means that if,

$$P_{DC} = IV \text{ and } P_{AC} = I_{rms} V_{rms}$$

Then

$$P_{DC} = P_{AC}$$

.

Impedance

In DC circuits, we have utilized Ohm's Law quite extensively. It would be useful, however, if there exist an analogous equation for AC circuits. This analogous equation do exist and is obtained by merely redefining the ratio of voltage to current to impedance (Z), rather than resistance (R). That is

$$\text{DC: } R = \frac{V}{I}$$
$$\Downarrow$$
$$\text{AC: } Z = \frac{V}{I}$$

It is also important to remind ourselves that the voltage and curent in the AC current are time dependent and are expressed in trigonometric functions. As with any physics involving trigonometric function, the phase angle can be of interest and in this particular instance, let us focus on the phase angle between the current and voltage.

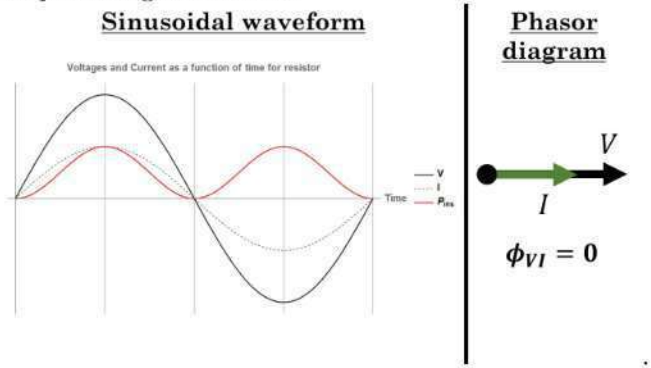
Here, we present the phase angle shift (both in form of sinusoidal waveform and phasor diagram) between current and voltage in 3 cases:

1. For resistors,

$$V = V_o\, sin\, \phi_V = V_o\, sin(\omega t)$$
$$I = I_o\, sin\, \phi_I = I_o\, sin(\omega t)$$

$$\Delta\phi = \phi_V - \phi_I = 0$$

The voltage and current are in phase, that means there are no phase angle difference.



2. For capacitors,

$$V = V_o \, sin \, \phi_V = V_o \, sin(\omega t)$$
$$Q = CV = CV_o \, sin(\omega t)$$

$$\begin{aligned} I &= \frac{dQ}{dt} \\ &= \frac{d}{dt}(CV_o \, sin(\omega t)) \\ &= CV_o \omega \, cos(\omega t) \\ &= CV_o \omega \, sin(\omega t + \frac{\pi}{2}) \end{aligned}$$

Analogous to resistance, we may define the **reactive capacitance,** X_C as the ratio of voltage to current, we can then rearrange the equation to find this variable,

$$I = CV_o \omega \, sin(\omega + \frac{\pi}{2})$$

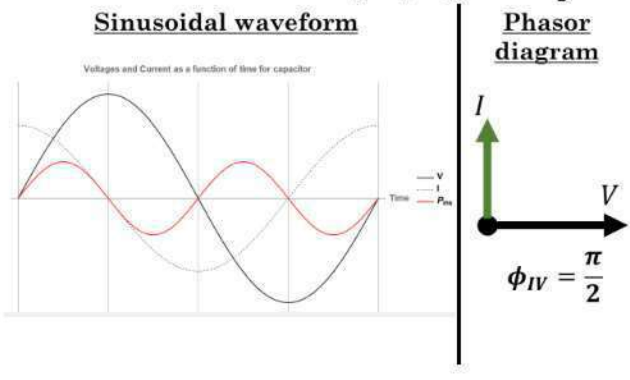
$$\Downarrow$$

$$X_C = \frac{V}{I} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

We know that for a potential difference to establish in a capacitor, sufficient charges must first accumulate at one of the terminals of the capacitor. Therefore, we know that current must lead the voltage in the case of a circuit of only a capacitor,

$$\Delta\phi = \phi_I - \phi_V = \frac{\pi}{2},$$

and the current leads the voltage by a phase of $\frac{\pi}{2}$.



3. For inductors,

$$I = I_o \, sin \, \phi_I = I_o \, sin(\omega t)$$

$$
\begin{aligned}
V &= L\frac{dI}{dt} \\
&= L\frac{d}{dt}(I_o\, sin(\omega t)) \\
&= LI_o\omega\, cos(\omega t) \\
&= LI_o\omega\, sin(\omega t + \frac{\pi}{2})
\end{aligned}
$$

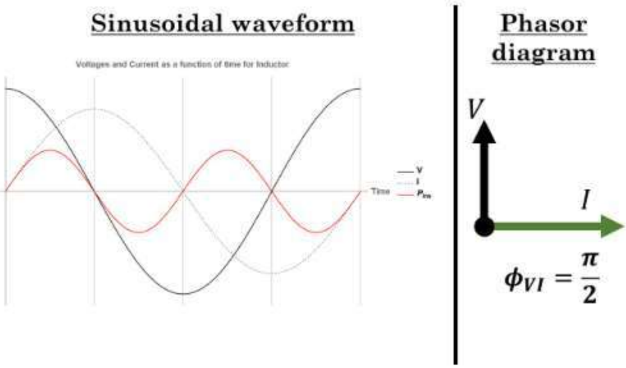
In circuits with an inductor, it is the electromotive force that causes current to flow. That is to say that the voltage induces the current. Thus the voltage leads the current,

$$\Delta\phi = \phi_V - \phi_I = \frac{\pi}{2},$$

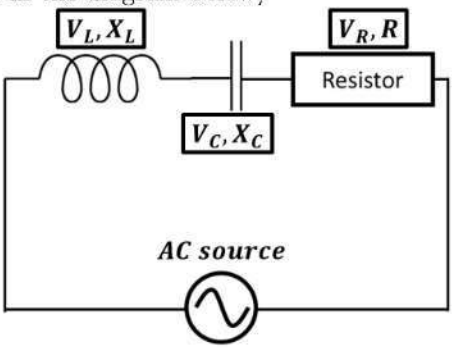
and the voltage leads the current by a phase of $\frac{\pi}{2}$.

Similar to reactive reactance, we may also define **inductive reactance**:

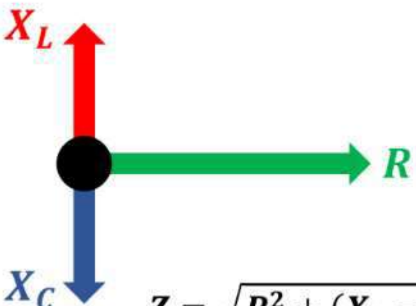
$$X_L = 2\pi f L$$



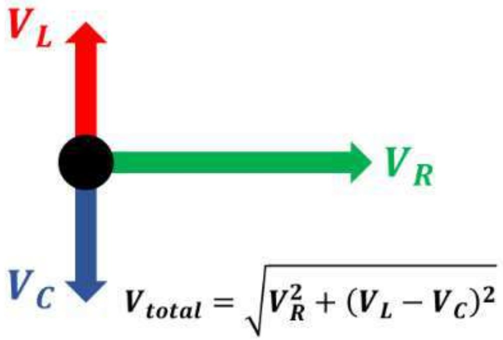
Apart from having merely single components in any circuit, we may also have combinations of them. For example, if the circuit has both a resistor and a capacitor, we can calculate the voltages as as well as its total impedance. This is made simpler with the use of phasor diagrams. In general, the most complex circuit would be the RLC circuit. For an RLC circuit, we may present it in the diagram below,



The phasor diagram for the case of the RLC circuit would be simply the combination of the phasor diagrams for resistor, capacitor and inductor cases. For the voltage and impedance, this yields:



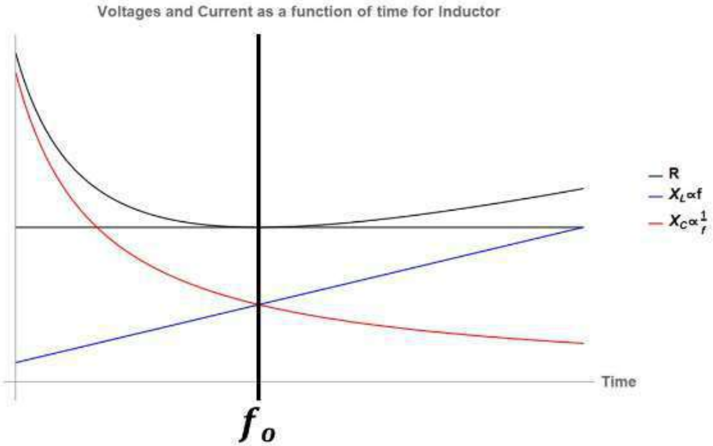
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



The phase angle ϕ is merely the angle between the voltage and the current,

$$tan\,\phi_{VI} = \frac{X_L - X_C}{R}$$

One of the special property found in the RCL series circuit is that it is able to resonate at a specific frequency f_o. This occurs when the impedance introduced by the inductor and capacitor are equal to each other in magnitude but 180^o phase shifted from each other. This results in a situation in which the impedance is purely from the resistance of the resistor.



To find that frequency, we equate the impedance from the inductor and capacitor, and then rearrange the equation to find the **resonance frequency**,

$$\begin{aligned} X_L &= X_C \\ \omega L &= \frac{1}{\omega C} \\ \omega &= \frac{1}{\sqrt{LC}} = 2\pi f_o \\ f_o &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}$$

Power

As covered before, average power for AC is merely the product of I_{rms} and V_{rms},

$$P_{ave} = I_{rms}V_{rms}.$$

This, of course, is merely an extension of the instantaneous power

$$P_{instant.} = IV.$$

Because we are dealing with oscillating current and voltage, in addition to the possibility of the circuit containing capacitor and inductors (not just resistors), we have to consider not just resistance of resistors but also impedances in general.

We then use the modified Ohm's Law $V = IZ$ which gives us

$$P_{ave} = I^2 Z\, cos\phi.$$

This amplitude of this expression ($I^2 Z$) is what is known as the apparent power, P_a, whereas the **average power** P_{ave} is known as the **average real power,** P_r. The power factor is then just the ratio of the P_r to P_a. That is

$$cos\,\phi = \frac{P_r}{P_a}.$$

Rasulan, S.
+60105520080
KMSw

The electrical power for a DC circuit depends on the constant values of current and voltage, that is

$$P = IV.$$

This is reasonable considering that the current and voltage values doesn't vary with time.

In AC circuits however, this is not the case because current and voltage values does vary with time. We know that in different combination of electrical components, the phase shift between current and voltage exists. It is therefore crucial for us to take that into consideration.

Let us assume now that voltage and current varies with time in accordance to

$$V(t) = V_{peak}\, sin\,(\omega t)$$

$$I(t) = I_{peak}\, sin\,(\omega t - \phi)$$

We can then say that the instantaneous power has the following form,

$$P_{instantaneous} = [I_{peak}\, sin\,(\omega t - \phi)] \times [V_{peak}\, sin\,(\omega t)]$$

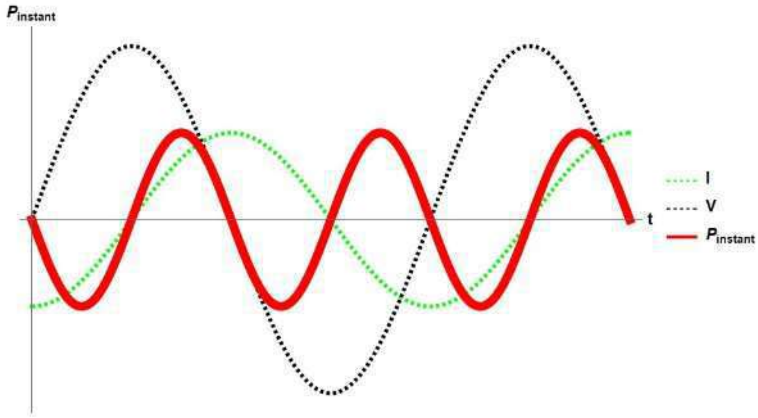
Recalling trig identity,

$$sin(\omega t - \phi) = sin\omega t\, cos\phi - cos\omega t\, sin\phi$$

This gives us

$$P_{instant.} = [I_{peak} V_{peak}][cos\phi\, sin^2\omega t - sin\phi sin\omega t\, cos\omega t]$$

This value varies with time. Let us illustrate that:



We can find the value for average power by integrating over a period of the sinusoidal function:

$$P_{ave} = [I_{peak} V_{peak}] \frac{[cos\phi \int_0^T sin^2\omega t dt - \cancelto{0}{sin\phi \int_0^T sin\omega t\, cos\omega t dt}]}{T}$$

The second term is vanishes when intergrated over a period simply because it is an odd function and thus, the expression we are left with for the average power is

$$P_{ave} = [I_{peak} V_{peak}][cos\phi \frac{\int_0^T sin^2\omega t dt}{T}]$$

Solving this integral gives us

$$P_{ave} = \frac{I_{peak} V_{peak}}{2} cos\phi$$

and recalling $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$ and $I_{rms} = \frac{I_{peak}}{\sqrt{2}}$, allows us to with power as a product of V_{rms} and I_{rms},

$$P_{ave} = IV\, cos\phi$$

where ϕ is the angle between their phasors.