Chapter 7: Geometrical Optics

Time allocation:
1h (Lecture) + 8h (Tutorial)

7.1 Spherical Surfaces

7.1.1 Reflection

State the radius of curvature, $R = 2f$, for a spherical mirror

Definitions:

1. Center of curvature, C = a point on the principal (or optical) axis that is positioned at distance equal to the radius of curvature, R,of the spherical mirror.
2. Focal point, f = a point on the principal axis at which light rays travelling parallel to the principal axis will converge onto or diverge from, after reflecting on the surface of the spherical mirror.

f and R are related by the following equation:

$$R = 2f$$

Use mirror equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, for real objects only.*$+f$ for concave mirror

2 types of mirror:

1. **Convex mirror**, of which its radius is located behind the mirror.
2. **Concave mirror**, of which it's radius of curvature is located in front of the mirror.

In using the mirror equation, conventions are important. A recommended convention is to have positive values of focal length for cases of concave mirror and negative values of focal length for cases of convex mirrors.

Define and use magnification, $m = \frac{h_i}{h_o} = -\frac{v}{u}$ respectively.

Lateral magnification, m, refers to the ratio between the height of the image to the height if the object. In equation form,

$$m = \frac{h_i}{h_o}.$$

Positive or negative values for height of image refers to whether or not the image is upright or inverted relative to the object. The reader is referred to this link for the derivation for the linear magnification equation as function of image and object distances.

7.1.1 Refraction

Use $\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$ for spherical surface.

An extension to Snell's law will be the refraction at a spherical surface. The following equation allows us to relate distances, refractive indices and radius of curvature of the spherical surface:

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}.$$

In the equation above, n_i refers to refractive indices, u and v refers to object and image distances respectively and R refers to the radius of curvature. R takes a positive value if the centre of curvature is on the same side of the surface, and vice versa.

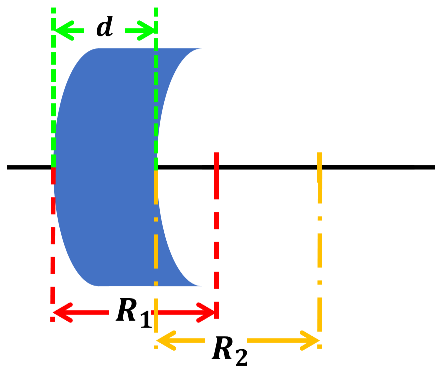
For the refractive indices, subscript 1 refers to the refractive index on the side of the incoming light rays and subscript 2 refers to the refractive index on the side of the outgoing rays.

7.2 Thin lenses

Use thin lens equation, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ for real objects only.

The thin lens equation assumes that the thickness measured between two vertex of the spherical surface of a lens is much smaller than the product of the radii of the spherical lenses, that is $d \ll R_1 R_2$.

Figure 1: Thin Lens



Use the lens makers' equation, $\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Again, referring to Figure 1, one can determine the focal length f, of the lens from

1. the radii of the lens surfaces, R_1 and R_2,
2. the ratio of the refractive index of the lens material to the refractive index of the surrounding, $\frac{n_{material}}{n_{medium}}$

We would generally assume the radii to be positive if the center of the curvature is located on the opposite side of incoming light, i.e. positive for convex surfaces

Chapter 8: Physical Optics

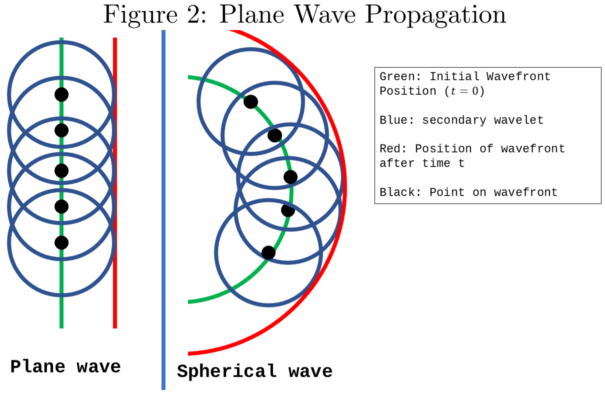
Time allocation:
2h (Lecture) + 8h (Tutorial)

8.1 Huygen's Principle

State Huygen's principle for spherical and plave wave fronts.

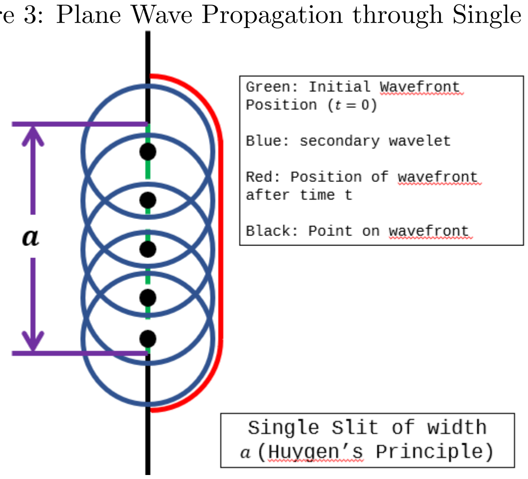
Huygen's Principle states that "each point on the wavefront acts as the source of secondary wavelets that spread out in all directions in spherical waves with a speed equal to the speed of wave propagation."

Figure 2: Plane Wave Propagation



Sketch and explain the wave front of light after passing through a single slit and obstacle using Huygen's Principle.

Figure 3: Plane Wave Propagation through Single Slit



8.2 Constructive & Destructive Interference

Define Coherence.

Coherence between 2 waves refers to the condition of constant phase difference between 2 waves with respect to time, that is to say

$$\frac{d\phi}{dt} = 0.$$

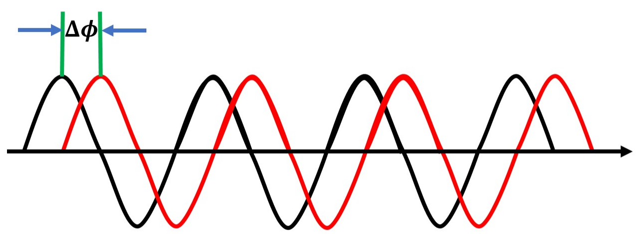
This property is the ideal property for stationary interference.

State the conditions for interference of light.

For a stable interference pattern, the following conditions are required:

1. Coherence, that is to say the two interacting light waves are of the same phase difference, $\frac{d\phi}{dt} = 0$.

Figure 4: Phase difference between two waves



2. Monochromatic, that is to say that the two interacting light waves are of the same wavelength, i.e.$\lambda_1 = \lambda_2$

State the conditions for constructive and destructive interference, with emphasis on the path difference and its equivalence to phase difference.

For purely **constructive interference**, it is empirical that the phase difference between the interacting waves is either 0 or $n\lambda$. On the other hand, for purely **destructive interference**, it is required that the phase difference between the two interacting wave is $\frac{n\lambda}{2}$.(n is both cases refers to integer values.)

8.3 Interference of transmitted light through 2 slits

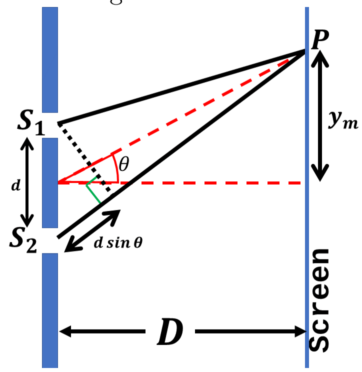
Use:

1. $y_m = \frac{m\lambda D}{d}$ for bright fringes (maxima)
2. $y_m = \frac{(m+0.5)\lambda D}{d}$ for dark fringes (minima)
3. $\Delta y = \frac{\lambda D}{d}$ for fringe separation

We now consider the case for Young's double slit experiment. Here we define the following variables:

D	distance from slit to screen
d	slit separation
y_m	distance from central maximum tu the mth fringe

Figure 5: Double Slit



We know that in order to determine what type of fringe forms at P, we need to look at the path difference and from the figure, we can say that the figure,

$$\Delta\phi = S_2P - S_1P = dsin\theta = d\left(\frac{y_m}{D}\right)$$

For **bright fringes**,

$$\Delta\phi = \frac{y_m d}{D} = m\lambda$$

Rearranging this allows us to find fringe distance as a function of d and D with m having any integer value indicating mth bright fringe from central maximum:

$$y_m = \frac{m\lambda D}{d}$$

Shifting one the waves by 0.5λ give us the equation for **dark fringes**,

$$y_m = \frac{(m+0.5)\lambda D}{d}.$$

Lastly, we'd want to calculate the fringe separation. This can be done by considering

$$\Delta y = y_{m+1} - y_m$$

which results in

$$\Delta y = \frac{\lambda D}{d}$$

8.4 Interference of reflected light through thin films

Identify phase change occurences upon reflection.

Describe (with the aid of a diagram) the interference of light in thin films at normal incidence, limited to 3 media.
For bright fringes, use the following equations:

1. $2nt = m\lambda$ for reflected light with no phase difference (non-reflective coating)
2. $2nt = (m+0.5)\lambda$ for reflected light with phase difference (reflective coating)

Referring to figure 7, we can see that the two reflected light waves has a phase difference of 0.5λ from reflections at surface 1 and 2. One must also take into consideration of the extra distance that the second

Figure 6: Phase Change conditions

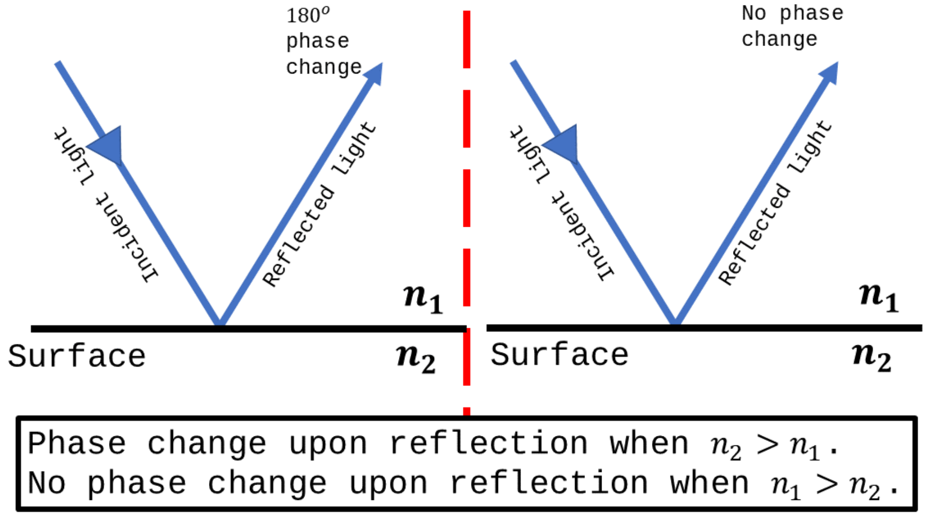
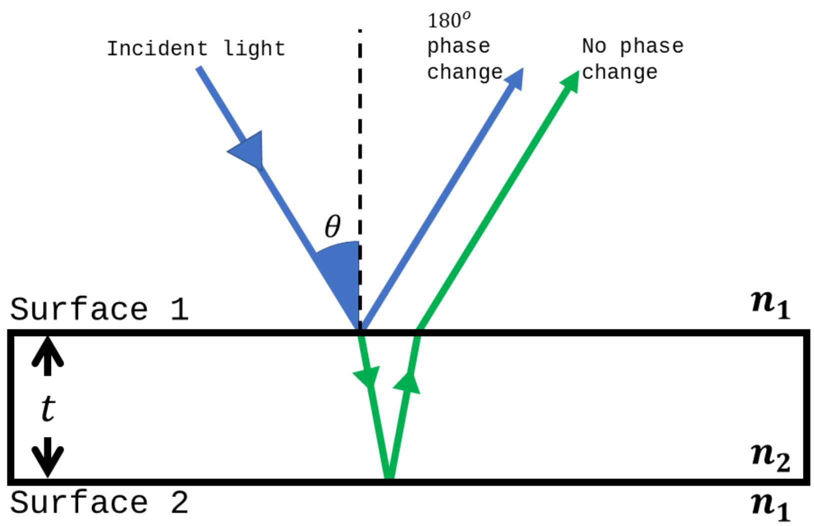


Figure 7: Thin film



(green) wave travelled, that is $2nt$. Therefore, the total phase difference between the reflected waves are then

$$\Delta\phi = 2nt - \frac{1}{2}\lambda.$$

For **constructive interference**, $\Delta\phi = 2nt - \frac{1}{2}\lambda = n\lambda$. This gives us the equation

$$2nt = \left(n + \frac{1}{2}\right)\lambda.$$

Dark fringes then appears between the bright fringes, i.e. they follow the equation

$$2nt = n\lambda.$$

Explain the application of thin films.

Main application for this concept of thin film interference is in **optical coatings** as one can manipulate the thickness of the coating as to choose the level of constructive or destructive interference. These optical coatings can be applied onto both for reflective as well as refractive systems.

8.5 Interference of transmitted light through single slit

Define diffraction.

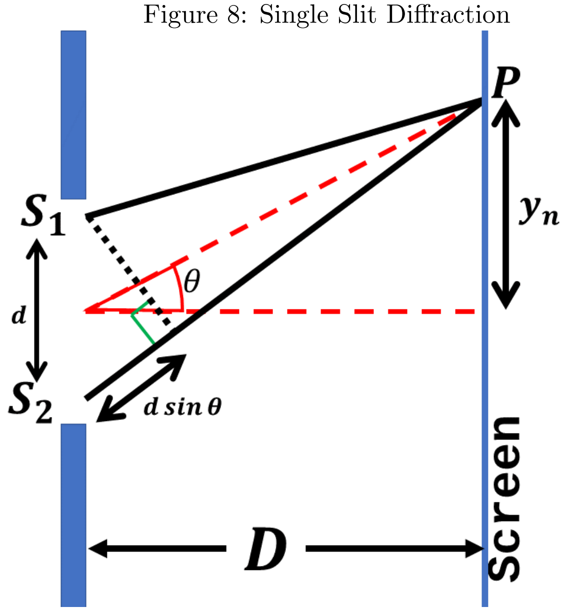
Diffraction is defined as the spreading or bending of waves as they pass through an aperture of a barrier. The diffracted waves then interfere with each other to produce a diffraction pattern.

Explain (with the aid of a diagram) the diffration of light in a single slit.

Use:

1. $y_n = \frac{n\lambda D}{d}$ for dark fringes (minima)
2. $y_n = \frac{(n+0.5)\lambda D}{d}$ for bright fringes (maxima)
3. $y = \frac{\lambda D}{d}$ for central bright

Figure 8: Single Slit Diffraction



This boils back to Huygen's Principle, which state that every point of a wavefront becomes a source for wavelets. In this case, rather than having 2 slits, we have a single slit. Light waves from one portion of the slit interacts with light waves from a different portion of the same slit to produce a diffraction pattern.

Here, we find that the dark fringes to forms when according to

$$d\,sin\,\lambda = n\lambda.$$

Geomterically, we also find that

$$tan\theta \approx sin\theta \approx \frac{n\lambda}{d} \approx \frac{y_n}{D}.$$

As such, we can say that **dark fringes** forms at

$$y_n = \frac{n\lambda D}{d}.$$

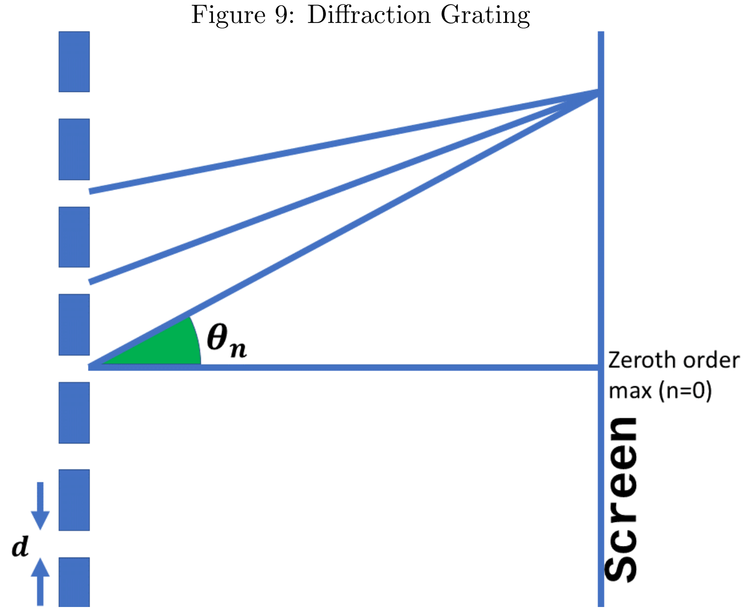
This would also mean that **bright fringes** forms at

$$y_n = \frac{(n+0.5)\lambda D}{d}.$$

8.6 Interference of transmitted light through many slits(diffraction grating)

Explain (with the aid of a diagram) the diffration of light in a grating.
Apply $d\,sin\,\theta = \frac{sin\,\theta}{N} = n\lambda$

Figure 9: Diffraction Grating



In the case for diffraction grating, light waves from many slits and interfere at the screen to form fringes of equal width. The equation by which the pattern follows is

$$d\,sin\,\theta_n = n\lambda$$

for bright fringes and shifted by 0.5λ for dark fringes. Note that the angle θ_n is measured from the normal line formed at the zeroth order maximum. Also note that, **maximum number of fringes** can be calculated by considering that $sin\,\theta_n < 1$.