Topical Exercise Questions

Welcome to the Topical Exercise section for the Malaysian Matriculation Physics SP015 course. This collection of carefully designed problems aims to reinforce your understanding of key physics concepts through targeted practice. Each exercise is crafted to challenge your problem-solving skills and prepare you for examinations by covering a wide range of topics aligned with the course syllabus. Use these exercises to build confidence and mastery in physics step by step.

Contents

Worksheet 1: Mathematics

Problems

No	Problems
1	Convert the following the following values: a) $72kmh^{-1}$ to ms^{-1} b) $5cm^2$ to m^2 c) $0.3gcm^{-3}$ to kgm^{-3}
2	A rice packaging facility uses automated machinery to fill large sacks of rice. Each sack is labelled with mass in metric tonnes, volume in litres, and density in g/cm³. You are asked to verify and convert units for quality assurance. a) A sack of rice has a mass of 1.5 tonnes. Convert this to kilograms and grams. b) The volume of the rice in the sack is measured as 1.2×10^3 litres. Convert this volume to cubic metres and cubic centimetres. c) Using the values above, calculate the density in SI units (kg/m³) and compare it to the density provided on the label: 1.25 g/cm³.
3	A team of Malaysian Matriculation students is analyzing physical relationships for a school science fair. They're testing whether various derived quantities make dimensional sense when combining fundamental quantities like mass, length, and time. a) A student proposes that power P is given by $P = Fv$, where F is force and v is velocity. Determine the dimensions of power. b) Another student suggests a quantity $X = \frac{E}{v^2}$, where E is energy and vvv is velocity. Determine the dimensions of X and identify the derived quantity it represents. c) A final student claims that a formula for surface tension T is $T = \frac{F}{L}$, where F is force and L is length. Determine the dimensions of T.
4	During a physics lab in the Malaysian Matriculation Programme, students are evaluating proposed physical equations for dimensional consistency. They must verify whether the equations are dimensionally homogeneous using fundamental dimensions. **a)** Verify the dimensional homogeneity of the equation: $$s = ut + \frac{1}{2}at^2$$ where s is displacement, u is initial velocity, a is acceleration, and t is time. **b)** Verify if the equation is dimensionally consistent: $$v = \sqrt{T/\mu}$$ where v is wave speed, T is tension (force), and μ is linear mass density.
5	A student analyzes the motion of a particle subjected to a force vector **F** of magnitude 50 N acting at an angle of 40° above the positive x-axis. The goal is to resolve this vector into its horizontal and vertical components. a) Determine the x-component of the force. b) Determine the y-component of the force. c) Plot the force vector and its components on a Cartesian plane.
6	A drone experiences two forces while in flight. Force **A** has a magnitude of 30 N and acts 35° north of east. Force **B** has a magnitude of 40 N and acts 60° north of west. Assume a flat, horizontal plane. a) Resolve both forces into their x- and y-components. b) Determine the magnitude and direction of the resultant vector. c) Plot the vectors and the resultant on a coordinate plane.
7	Two displacement vectors represent the movement of a hiker. The first vector, **A**, has a magnitude of 5.0 km and points 40° north of east. The second vector, **B**, has a magnitude of 7.0 km and points 20° south of east. a) Resolve both vectors into x- and y-components. b) Determine the magnitude and direction of the resultant displacement. c) Plot the vectors and their resultant on a coordinate plane.
8	A robotics arm moves along two segments. The first movement is represented by vector **A** with components (4 m, 3 m), and the second by vector **B** with components (2 m, 5 m). a) Calculate the dot product **A · B** using components. b) Find the angle between vectors **A** and **B**. c) Plot both vectors from the origin to visualize the geometric relationship.
9	Two displacement vectors are given: Vector A has a magnitude of 6 m and points 35° above the x-axis.

	Vector B has a magnitude of 4 m and points 60° above the x-axis. a) Write the components of vectors A and B. b) Compute the dot product of A and B. c) Using the dot product, find the angle between A and B and compare with the given.
10	Two force vectors act at a point on a rotating wheel. Force **A** is represented by the vector (3 N, 2 N, 0), and force **B** by (1m, 4 m, 0). The vectors lie in the plane of the wheel. Assume they are applied at the same point from the center of the wheel. a) Compute the cross product A × B. b) Determine the magnitude and direction (unit vector) of the resulting vector. c) Sketch to visualize the vectors and their right-hand rule direction.

Worksheet 2: 1D Kinematics

Problems

No	Questions
1	a) A runner, starting from static, accelerates to $2ms^{-2}$ for 10 seconds before winning the race. How far did they run? What was their velocity 7s after they started running? b) A ball free falls from the top of the roof for 6 seconds. How far did it fall? What is its final velocity at the end of 5 seconds? c) A bullet leaves a rifle with a velocity of $452ms^{-1}$. While accelerating through the barrel of the rifle, the bullet moves a distance of 0.93 m. Determine the acceleration of the bullet.
2	You are an astronaut on the surface of the Moon, conducting an experiment by throwing a rock upward with an initial velocity of $u = 8ms^{-1}$. Since the gravitational acceleration on the Moon is $g_{moon} = 1.6\ ms^{-2}$, the rock moves upward, slows down, comes to a stop at its highest point, and then falls back down. a) Show that the maximum height achieved by the rock is $$h_{max} = \frac{u^2}{2g_{moon}}$$ b) Using the derived equation from question (a), calculate the maximum height h_{max} reached by the rock. c) Suppose you repeat the experiment on Earth, where the gravitational acceleration is $g_{earth} = 9.81ms^{-2}$. Explain qualitatively how and why the maximum height reached by the rock would be different compared to that on the Moon.
3	A cyclist is traveling along a straight path. Initially, they are moving at a constant speed of 4 m/s. Suddenly, they accelerate uniformly at a rate of 1.5 m/s^2 for 6 seconds. After that, they stop accelerating and continue moving at the constant speed they reached at the end of the acceleration phase. a) What is the cyclist's speed after 6 seconds of acceleration? b) How far does the cyclist travel during the 6-second acceleration phase? c) If the cyclist continues moving at this final speed for an additional 10 seconds after the acceleration phase, how far do they travel during these 10 seconds? d) What is the total distance traveled by the cyclist over the entire trip? Sketch their s-t graph.
4	A car is initially at rest at a traffic light. When the light turns green, the car accelerates uniformly at 2.5 m/s^2 for 8 seconds. After that, the driver maintains a constant speed for 12 more seconds before slowing down uniformly to rest over 5 seconds. a) What is the car's speed at the end of the 8-second acceleration phase? b) How far does the car travel during the acceleration phase? c) How far does the car travel during the 12-second constant speed phase? d) What is the car's deceleration if it comes to rest in 5 seconds, and how far does it travel during this deceleration? e) What is the total distance traveled by the car during the entire trip?
5	A train starts from rest at a station and accelerates uniformly at $2ms^{-2}$ for 10 seconds. It then continues at the speed reached for 20 seconds before decelerating uniformly to rest in 5 seconds as it approaches the next station. a) What is the train's speed after the 10-second acceleration phase? b) How far does the train travel during the 10-second acceleration and the 20-second constant speed phases combined? c) What is the train's deceleration, and how far does it travel while slowing down in the last 5 seconds?
6	A ball is dropped from rest from the top of a building 80 meters tall. Assume free fall under gravity with $g = 9.81ms^{-2}$ and ignore air resistance. a) How long does it take for the ball to hit the ground? b) What is the ball's speed just before it hits the ground? Sketch their v-t graph. c) If a second ball is thrown downward from the same height with an initial speed of 10 m/s, how long does it take to hit the ground?
7	A car traveling at 20 m/s passes a stationary police car. The police car starts accelerating uniformly from rest at 3 m/s^2 exactly when the speeding car passes it. a) How long does it take for the police car to catch up to the speeding car? b) How far do both cars travel during this time? c) What is the speed of the police car when it catches up to the speeding car?
8	Three runners, A, B, and C, start a race on a straight track at the same time. Runner A starts from rest and accelerates uniformly at 2 m/s^2. Runner B is already moving at a constant speed of 8 m/s. Runner C starts from rest but accelerates uniformly at 1 m/s^2. a) How long does it take for Runner A to catch up with Runner B? Sketch their s-t graph.

	b) How far does Runner A travel during this time? c) How long after the race starts does Runner C reach the same position as Runner A?
9	A runner accelerates during a race with different speeds at different time intervals. For the first 10 seconds, the runner moves at a speed of 3 m/s. For the next 5 seconds, the runner accelerates and reaches a speed of 6 m/s. For the final 5 seconds, the runner runs at a constant speed of 6 m/s. Given that the runner starts from rest at the beginning of the race: a) What is the total distance covered by the runner in the first 10 seconds? b) What is the distance covered in the next 5 seconds of acceleration? c) What is the total distance covered by the runner at the end of the 20 seconds?

Worksheet 3: Projectile Motion

Problems

No	Questions
1	A ball is launched with an initial velocity 30m/s at an angle of 50^o to the horizontal. Neglect air resistance. a) What are the horizontal and vertical components of the ball's initial velocity? b) What are the horizontal and vertical components of the ball’s velocity after 3 seconds? c) What is the magnitude and direction (angle with the horizontal) of the ball’s velocity at t=3s?
2	A ball is thrown from the ground with an initial speed of 20m/s at an angle of 30 degrees to the horizontal. Assume $g = 9.81ms^{-2}$ and ignore air resistance. a) How long is the ball in the air? b) What is the maximum height reached by the ball? c) How far from the starting point does the ball land (range)?
3	A ball is thrown horizontally from a cliff of height 45m and initial speed of 15m/s, assume $g = 9.81ms^{-2}$ and ignore air resistance. a) How long does it take for the ball to hit the ground? b) What is the horizontal distance traveled by the ball before hitting the ground? c) What is the ball’s speed just before it hits the ground?
4	A ball is thrown with an initial velocity of 25 m/s at an angle 40^o from the horizontal. It lands on a platform that is 10 m higher than its launch point. Ignore air resistance. a) How long does it take for the ball to reach the platform? b) What is the horizontal distance (range) to the platform? c) What is the ball’s velocity just before it hits the platform?
5	A cannon fires a projectile from ground level. The projectile travels a horizontal distance (range) of 100m and reaches a maximum height 20m. Neglect air resistance. a) What is the initial velocity of the projectile? b) What is the launch angle? c) How long does the projectile stay in the air?
6	A basketball is thrown from ground level and reaches a horizontal range 120m, a maximum height 15m, and lands on a platform 5m above the ground. Neglect air resistance. a) What is the initial velocity of the basketball? b) What is the launch angle? c) How long does it take for the basketball to hit the platform?
7	Two objects are in projectile motion. Object A is thrown straight up, and object B is thrown at a 30° angle to the ground with the same initial speed. a) What is the airtime of Object A, assuming initial speed u? b) What is the airtime of Object B, given the initial speed u and θ? c) If the initial speed is 20 m/s and θ is 30°, what is the difference in airtime between Object A and Object B?
8	A ball is launched from the top of a 40m high cliff with a speed of 25 m/s at an angle of 30^o above the horizontal. a) How long is the ball in the air before hitting the ground? b) How far from the base of the cliff does it land? c) What is the speed of the ball just before impact?
9	A ball is kicked from the ground and passes through a point (x=40m, y=20m) after 2.0s. a) Determine the horizontal and vertical components of the initial velocity. b) Find the initial speed and launch angle. c) How high does the ball go?

Worksheet 4: Momentum, Impulse and Conservation of Momentum

Problems

No	Questions
1	A 2.0kg cart moving at 3.0m/s collides with a stationary 1.0kg cart. The collision is perfectly inelastic. Calculate a) the total momentum before the collision. b) the velocity of the combined carts after the collision. c) the kinetic energy lost in the collision.
2	A 0.5 kg ball is moving at 6.0 m/s when it is hit by a bat and reverses direction, now moving at 4.0 m/s. The impact lasts 0.01 s. Determine a) the change in momentum of the ball. b) the impulse did the bat exert on the ball. c) the average force applied by the bat.
3	A 2.0 kg cart moving at 4.0 m/s in the positive x-direction collides with a 3.0 kg cart initially at rest. After the collision, the 2.0 kg cart moves at 2.0 m/s at an angle of 30° to the positive x-axis, and the 3.0 kg cart moves at 3.0m/s at an unknown angle. Calculate a) the total momentum of the system before the collision. b) the total momentum of the system after the collision. c) the angle at which the 3.0 kg cart moves after the collision.
4	A 3.0 kg cart is moving with a speed of 5.0 m/s along the positive xxx-axis. The cart collides elastically with a 4.0 kg cart, initially at rest. After the collision: The 3.0 kg cart moves with a speed of 3.0 m/s at an angle of 30^o to the positive x-axis. a) What is the final speed of the 4.0 kg cart after the collision? b) What is the angle of motion of the 4.0 kg cart relative to the positive xxx-axis after the collision? c) What is the kinetic energy lost in the collision?
5	Two objects, a 3.0 kg cart and a 4.0 kg cart, are moving toward each other in two dimensions. The 3.0 kg cart is moving with a speed of 4.0 m/s along the positive x-axis, and the 4.0 kg cart is moving with a speed of 3.0 m/s along the positive y-axis. After the collision, the two carts stick together and move as one object. Calculate a) the final velocity of the combined object after the collision? b) the direction (angle) of the velocity vector of the combined object relative to the x-axis after the collision? c) the kinetic energy lost in the collision?
6	A stationary object explodes into three pieces in a two-dimensional plane. Piece 1 has a mass of 2 kg and moves with a velocity of 5 m/s at 30° to the horizontal. Piece 2 has a mass of 3 kg and moves with a velocity of 4 m/s at 120° to the horizontal. The third piece, with a mass of 1 kg, moves in an unknown direction and velocity. Find the velocity of the third piece and the direction of its motion. a) Calculate the velocity of the third piece. b) Determine the direction of motion of the third piece. c) What is the total kinetic energy of the system before and after the explosion?

Worksheet 5: Newton's Laws

Problems

<table>
<tr><th>No</th><th>Questions</th></tr>
<tr><td>1</td><td>A 5kg block rests on a horizontal, frictionless table. A constant horizontal force of 20 N is applied to the block.
a) Sketch the free body diagram for the block.
b) What is the acceleration of the block?
c) How far does the block travel in 4s from rest?
d) What is its final speed after 4s?</td></tr>
<tr><td>2</td><td>Two blocks are connected by a light string over a frictionless pulley. Block A (mass = 4kg) is on a horizontal, frictionless table. Block B (mass = 2kg) hangs off the side.
a) Sketch the free body diagram for the blocks.
b) Find the acceleration of the system.
c) Find the tension in the string.
d) If a kinetic friction force of 3.0N now acts on Block A, what is the new acceleration?</td></tr>
<tr><td>3</td><td>A 10kg block rests on a frictionless incline angled at 30° to the horizontal.
a) Sketch the free body diagram for the block.
b) Find the component of the block's weight parallel to the incline.
c) Find the acceleration of the block down the incline.
d) If a kinetic friction force of 10.0 N now acts, what is the new acceleration?</td></tr>
<tr><td>4</td><td>A 12.0 kg box is pushed across a horizontal surface with a force of 50.0 N. The coefficient of kinetic friction between the box and the surface is 0.20. Sketch the free body diagram for the box and calculate the
a) frictional force acting on the box.
b) net force on the box.
c) acceleration of the box.</td></tr>
<tr><td>5</td><td>A 15.0 kg box is placed on a rough incline angled at 25° to the horizontal. The coefficient of kinetic friction between the box and the incline is 0.30. Sketch the free body diagram for the box and determine the
a) gravitational force component parallel to the incline.
b) frictional force acting on the box.
c) acceleration of the box down the incline.</td></tr>
<tr><td>6</td><td>A 10.0 kg box is pushed on a horizontal surface with a force of 40.0 N at an angle of 30° above the horizontal. The coefficient of kinetic friction between the box and the surface is 0.25. Sketch the free body diagram for the box and determine the
a) normal force acting on the box.
b) frictional force acting on the box.
c) acceleration of the box.</td></tr>
<tr><td>7</td><td>A 12.0 kg box is on an inclined plane with a 30° angle to the horizontal. A 50.0 N force is applied at an angle of 20° above the incline. The coefficient of kinetic friction between the box and the incline is 0.35. Sketch the free body diagram for the box and determine the
a) normal force acting on the box.
b) frictional force acting on the box.
c) acceleration of the box.</td></tr>
<tr><td>8</td><td>A 10.0 kg object is hanging stationary from two strings. String 1 makes an angle of 30° with the vertical, and String 2 makes an angle of 45° with the vertical. The system is in equilibrium. Sketch the free body diagram for the object and determine the
a) tension in String 1.
b) tension in String 2.</td></tr>
</table>

Worksheet 6: Work, Energy & Power

Problems

No	Questions
1	A 5kg cart is pushed along a frictionless horizontal track with a constant 20N force over a distance of 4m. The cart starts from rest. a) How much work is done on the cart? b) What is the change in the cart's kinetic energy? c) What is the cart's final speed?
2	A 2.0 kg ball is released from rest at a height of 5.0 m above the ground. Assume no air resistance. Using energy conservation principle, determine the a) ball's potential energy at the top? b) ball's kinetic energy just before hitting the ground? c) ball's speed just before hitting the ground?
3	A 1.5kg block slides on a frictionless surface at 3.0 m/s and compresses a horizontal spring with spring constant $k = 200Nm^{-1}$. a) What is the block's initial kinetic energy? b) What is the maximum compression of the spring? c) How much elastic potential energy is stored at maximum compression?
4	A 12.0 kg crate is pushed up a rough incline (25°) over a distance of 5.0 m by a worker applying a constant 100.0N force parallel to the incline. The coefficient of kinetic friction is 0.20. The crate starts from rest. a) How much work does the worker do on the crate? b) How much energy is lost due to friction? c) What is the final speed of the crate?
5	A 2.0 kg block slides down a frictionless ramp from a height of 4.0 m and compresses a horizontal spring (at bottom) with spring constant $k = 300Nm^{-1}$. The spring is initially unstretched and mounted on a horizontal surface. a) What is the block's speed just before it contacts the spring? b) What is the maximum compression of the spring? c) What is the speed of the block when the spring is halfway compressed?
6	A 1.8 kg block is dropped from rest from a height of 2.5 m above the uncompressed end of a vertical spring. The spring is mounted on the floor and has a spring constant of k=500 N/m. Assume no energy is lost to friction or air resistance. a) What is the total mechanical energy of the system before the block contacts the spring? b) What is the maximum compression of the spring after the block lands on it? c) At what compression is the gravitational potential energy equal to the elastic potential energy?
7	A 2.5 kg block is attached to a light spring (spring constant k=400 N/m) fixed to a vertical wall. The spring is at its relaxed length when the block is at rest on a frictionless horizontal surface. The block is pulled 0.60 m away from the wall and released from rest. a) What is the speed of the block as it passes through the spring's equilibrium position? b) How far past the equilibrium position does the block travel before momentarily stopping? c) Suppose now that the block is initially pushed toward the wall, compressing the spring by 0.60 m. What is its speed at the equilibrium position?
8	A small roller coaster car of mass 500 kg starts from rest at a height of 25 m above the bottom of a frictionless circular loop-the-loop track (loop radius = 10 m). a) What is the total mechanical energy of the system relative to the bottom of the loop? b) What is the car's speed at the top of the loop? c) What is the car's speed at the bottom of the loop?

Worksheet 7: Work, Energy and Power Part 2

Problems

No	Questions
1	A 10 kg box is placed on a frictionless horizontal surface. A 20 N force is applied to the box to the right, while a 5 N force acts to the left. The box starts from rest. a) Calculate the net force acting on the box. b) Find the acceleration of the box. c) Determine the work done by the net force after the box has moved 5 meters.
2	A 5 kg block slides along a frictionless surface. The block is initially moving at a velocity of $v_0 = 2ms^{-1}$ and undergoes a constant force of $F = 10$ N for a time interval of 4s. a) Calculate the acceleration of the block. b) Find the final velocity of the block after 4 seconds. c) Determine the work done by the force during this time interval.
3	A 12 kg crate is pushed up a rough 25° incline by a constant force **F**, applied parallel to the incline. The coefficient of kinetic friction between the crate and the incline is 0.2. The crate is pushed from rest and moves 4.0 m up the incline, reaching a final speed of 3.0 m/s. a) Find the work done by the applied force **F**. b) Determine the magnitude of the applied force **F**. c) Calculate the power output of the force **F** at the end of the 4.0 m displacement.
4	A 2.0 kg block is held at rest on a frictionless vertical spring compressed by 0.25 m from its natural length. The spring has a constant of 800 N/m. When released, the block is propelled upward. Assume the spring's base is at ground level. a) Find the block's speed just as it leaves the spring (spring is at natural length). b) How high above the release point does the block rise? c) If instead the spring is compressed by 0.35 m, what is the new maximum height reached?
5	A 500 kg roller coaster car starts from rest at the top of a 30 m high hill. It descends without friction to a valley 10 m above the ground, then climbs a second hill. Ignore air resistance. a) Find the speed of the car at the bottom of the first descent (10 m elevation). b) Determine the maximum height the second hill can be for the car to just reach the top.
6	A 1.5 kg object is dropped from a height of 2.0 m onto a vertical spring with spring constant 500N/m. The spring is initially uncompressed and fixed to the ground. Assume no energy is lost to friction or air resistance. a) Find the speed of the object just before it contacts the spring. b) Determine the maximum compression of the spring. c) Find the speed of the object when the spring is compressed by 0.10 m.
7	A 0.40 kg ball is compressed against a spring (with spring constant 800 N/m) by 0.20 m and released from rest on a smooth horizontal table 1.5 m high. As it leaves the spring, it rolls off the edge of the table and lands on the floor. a) Use conservation of energy to find the speed of the ball as it leaves the spring. b) Use conservation of energy to find the total speed of the ball just before hitting the ground. c) Treating the ball as a projectile motion, confirm the answer in (b) to be correct.
8	A 0.60 kg block is pressed against a spring (spring constant 900 N/m) compressing it by 0.18 m. It is then released and slides along a horizontal frictionless surface before ascending a frictionless incline. The spring is fixed at the base of the incline, and the incline makes a 30° angle with the horizontal. a) Calculate the speed of the block just as it leaves the spring. b) Determine the maximum vertical height the block reaches on the incline. c) Suppose the same block is launched with the same initial spring compression on a rough incline plane ($\mu_k = 0.15$). Show that the general expression for the height reached in terms of k, x, m, g, μ_k, and θ is $$h = \frac{kx^2}{2mg(1 + \mu_k cot\,\theta)}$$

Worksheet 8: Circular Motion

Problems

No	Questions
1	A toy car of mass 0.3 kg is launched at high speed onto a smooth, flat, frictionless circular track with radius 0.8 m. The car follows the circular path, held in place only by a light string connected from the car to a central peg. a) If the tension in the string is measured to be 2.7 N, what is the speed of the toy car? b) If the string suddenly breaks, what path does the car follow immediately after, and why? c) What would happen to the required tension if the car's speed doubled? Show algebraically.
2	While driving his 1200 kg car around a flat, circular curve of radius 50 m, Alex notices that the road is slightly wet from earlier rain. The coefficient of static friction between the tires and the road is 0.40. Assume no banking and neglect air resistance. a) What is the maximum speed Alex can safely drive around the curve without skidding? b) If Alex wants to maintain a speed of 12 m/s on the same curve, what minimum coefficient of static friction is required? c) If the road becomes icy and the friction drops to 0.10, what is the maximum safe speed now?
3	A student rides a Ferris wheel that rotates in a vertical circle with a radius of 9.0 m. The wheel makes one full revolution every 20 seconds. The student's mass is 60 kg, and they sit on a scale that reads the normal force. a) What is the student's speed during the motion? b) What does the scale read at the top of the ride? c) What does the scale read at the bottom of the ride?
4	During a county fair, a student participates in a game where a small 0.5 kg ball is tied to a 1.2 m long string and whirled in a horizontal circle at constant speed. The string makes a constant angle of 30° with the vertical as it spins. a) What is the speed of the ball during the circular motion? b) What is the tension in the string? c) If the speed of the ball increases, what happens to the angle the string makes with the vertical? Explain algebraically.
5	In a physics lab, a 0.25 kg rubber stopper is tied to a string and whirled in a horizontal circle above a student's head. The string is 1.1 m long and remains horizontal during motion. The stopper makes 30 full revolutions in 20 seconds. a) What is the angular velocity of the stopper in rad/s? b) What is the tension in the string? c) If the angular velocity is doubled, what happens to the tension in the string? Show algebraically.
6	In an amusement park ride, a small seat of mass 20 kg is suspended by a 3.0 m long cable and rotates in a horizontal circle as a conical pendulum. The cable makes an angle of 40° with the vertical as the seat spins. a) What is the angular velocity of the seat? b) What is the tension in the cable? c) If the angle increases to 50° while the cable length stays the same, how does the angular velocity change? Show algebraically.

Worksheet 9: Chapter 7 Part 1 - SHM

Problems

No	Questions
1	During a physics demonstration, a mass on a spring oscillates horizontally on a frictionless table. Its position as a function of time is given by: $$x(t) = 0.12\, sin\,(5t)$$ where x(t) is in metres. a) What is the velocity function v(t)? b) What is the velocity at t=0.30s? c) What is the kinetic energy at t=0.30s if the mass is 0.5kg?
2	A lab mass slides back and forth on a frictionless air track, attached to a spring. Its motion follows the pattern: "Displacement from equilibrium is 0.15 meters times the sine of 4t, with t in seconds.". The mass is 0.40 kg. Determine a) the expression for the acceleration as a function of time? b) the acceleration at t = 1.0 s? c) the potential energy stored in the spring at t = 1.0s?
3	A physics student observes a glider oscillating on a frictionless air track, connected to a spring. The glider's position follows the rule: displacement from equilibrium is 0.20 meters times the sine of 6t, with time in seconds. The glider has a mass of 0.50 kg. Calculate a) the maximum speed the glider reaches? b) the glider's speed when its displacement is 0.10 m? c) the total mechanical energy of the system?
4	At a local science exhibit, a visitor sets a simple pendulum swinging. It consists of a 0.30 kg metal sphere hanging from a 0.80 m long string. The pendulum is released from a height where it is 0.10 m above its lowest point. a) How long does it take for the pendulum to complete one full swing back and forth? b) What is the kinetic energy of the sphere when it is 0.04 m above the lowest point? c) What is the speed of the sphere at that same height?
5	In a physics lab, a student stretches a spring horizontally on a frictionless surface and attaches a 0.40 kg block. When pulled and released, the block oscillates with an amplitude of 0.12 m. The spring constant is 25 N/m. a) How long does it take the block to complete one full oscillation? b) What is the potential energy stored in the spring when the block is 0.05 m from equilibrium? c) What is the acceleration of the block at that position?
6	During a school science fair, two students compare oscillations. One uses a block of mass 0.60 kg on a spring with a constant of 36 N/m. The other uses a simple pendulum that swings with the same period. They want to find out how long the pendulum's string must be for the periods to match. a) What is the period of the mass-spring system? b) What is the required period of the pendulum to match it? c) What is the length of the pendulum string that gives this period?
7	A lab group is testing springs by attaching various objects and timing their oscillations. One spring stretches when a 0.80 kg mass is attached, and the group records that it takes 1.20 seconds to complete one full oscillation. The spring is then reused with a different unknown mass, and the new period is found to be 1.50 seconds. They want to determine properties of the spring and the new object. a) What is the spring constant based on the first trial? b) What is the mass of the second object used in the second trial?
8	A student investigates a spring-mass system oscillating on a frictionless horizontal surface. They want to understand how the maximum speed, acceleration, and energy of the system depend on its displacement. The spring has a constant kkk, the mass is mmm, and amplitude is AAA. Assume the motion starts from maximum displacement. a) Derive an expression for speed as a function of displacement. b) Using your result, derive the expression for acceleration as a function of displacement. c) Show that the total mechanical energy remains constant and equals the maximum potential energy.

Worksheet 10: Chapter 7 Part 2 – Progressive Waves

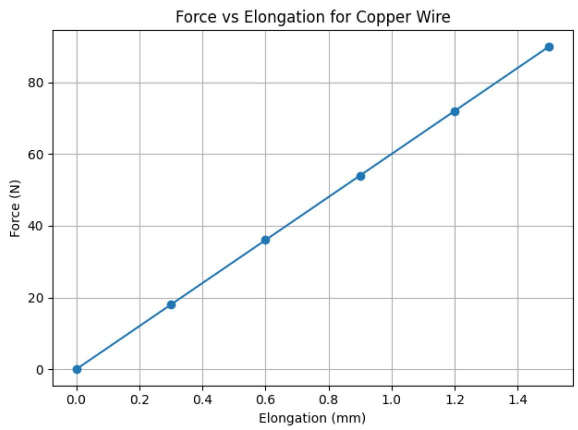
Problems

No	Questions
1	In a ripple tank experiment, a student generates water waves using a vibrating bar. The surface displacement at a point on the water is described by a sine function that depends on both position and time. The student observes that the wavelength is 0.20 m, the frequency is 5.0 Hz, and the amplitude is 1.5 cm. The wave travels in the positive x-direction. a) Write the sine equation for the wave's displacement as a function of position and time. b) What is the displacement of a point located at x = 0.10 m at time t = 0.05 s? c) What is the wave speed?
2	A science museum features an interactive display of a stretched string carrying a sinusoidal wave. The wave travels along the positive x-direction. Its amplitude is 2.0 cm, the frequency is 8.0 Hz, and the wavelength is 0.25 m. A student wants to determine the velocity at which a point on the string moves up and down, not along the x-direction. a) Write the displacement equation of the wave as a sine function of position and time. b) Derive an expression for the instantaneous velocity of a particle on the string. c) Calculate the velocity of the particle at x = 0m and t=0.03125s.
3	A physics student investigates the vibration of a string fixed at both ends. The string supports a standing wave pattern created by two identical waves traveling in opposite directions. The amplitude of each wave is 3.0 cm, the frequency is 10 Hz, and the wavelength is 0.80 m. a) Write the general equation for the standing wave formed on the string. b) Determine the amplitude of vibration at a point located 0.20 m from one end. c) What is the maximum transverse speed of that point?
4	A violin string of length 0.60 m is fixed at both ends and under constant tension. The linear mass density of the string is $2.5(10^{-3})kgm^{-1}$, and the tension is 90 N. A student is asked to analyze the standing wave patterns formed when the string is played. a) What is the fundamental frequency of the string? b) Calculate the frequency of the second overtone. c) What is the wavelength of the first overtone?
5	A student is exploring sound resonance in a 0.85 m long tube open at both ends. The speed of sound in air is approximately 340 m/s. The student is interested in the harmonic frequencies and wavelengths that can be produced in this setup. Calculate a) the fundamental frequency of the air column. b) the frequency of the second overtone. c) the wavelength of the first overtone.
6	During a lab experiment, a student investigates the sound produced by blowing across the top of a 0.75 m long tube that is closed at one end and open at the other. The air temperature is such that the speed of sound is 340 m/s. The student wants to understand the resonant frequencies possible in this system. Determine the a) fundamental frequency of the air column. b) lowest frequency above 400 Hz that this column can produce. c) distance between a node and the adjacent antinode for the second overtone.
7	An ambulance emits a steady siren at a frequency of 850 Hz. On Monday, an observer is standing still on the sidewalk as the ambulance approaches her at 20 m/s. On Tuesday, the ambulance is parked, and the same observer jogs toward it at 5 m/s. Assume the speed of sound in air is 340 m/s. Determine the a) frequency does the observer hear on Monday? b) frequency does the observer hear on Tuesday? c) percentage difference between the two observed frequencies?
8	A weather siren emits a constant tone at 700 Hz. During a test drill: • In Scenario A, a cyclist is stationary while the siren moves toward her at 15 m/s. • In Scenario B, the siren is stationary while the cyclist rides toward it at 10 m/s. • Assume the speed of sound is 340 m/s. Answer the following: a) What frequency does the cyclist hear in Scenario A? b) What frequency does she hear in Scenario B?

	c) In which scenario is the frequency shift greater, and by how many Hz?

Worksheet 11: Physics of Materials

Problems

No	Questions
1	A climber uses a 12 m nylon rope with a cross-sectional area of 3.5×10^{-6} m². During a fall, the rope stretches by 0.48 m before stopping him. Assume Young's modulus for nylon is 5.0×10^{9} Pa. Determine a) the strain in the rope? b) the stress in the rope during the stretch? c) much strain energy is stored in the rope during the fall?
2	An engineering student tests a copper wire of original length 2.0 m and diameter 1.0 mm by gradually applying force and measuring its elongation. The force vs elongation data is plotted and appears linear up to 1.5 mm of extension with a maximum applied force of 90 N. a) Estimate the strain energy stored in the wire using the force–elongation graph. b) Find the strain energy per unit volume. c) Using the results, calculate Young's modulus of copper. 
3	A technician is testing a cylindrical copper rod used in a cooling system. The rod has a diameter of 1.5 cm and a length of 0.6 m. One end is maintained at 150 °C, and the other at 30 °C. Copper's thermal conductivity is 385 W/m·K. a) Calculate the rod's cross-sectional area. b) Find the rate of heat transfer through the rod. c) Plot temperature vs. position along the rod, assuming steady-state conduction
4	. An engineer insulates a metal rod that is 0.8 m long and has a cross-sectional area of 5.0×10^{-4} m². The rod connects two heat reservoirs: one at 100 °C and the other at 20 °C. The material is aluminum, with thermal conductivity 205 W/m·K. a) Calculate the rate of heat transfer through the rod. b) How much heat is transferred in 10 minutes? c) Sketch a graph of temperature vs. position along the rod, assuming steady-state conduction.
5	A food packaging engineer is evaluating a new insulating panel material. The panel is 0.04 m thick and has a surface area of 0.25 m². In testing, one side is maintained at 90 °C and the other at 40 °C. After reaching steady state, a heat flow rate of 31.25 W is measured through the material. a) What is the temperature difference across the panel? b) Using the measured heat transfer rate, calculate the coefficient of thermal conductivity of the material. c) Would this material be more suitable for insulation or heat transfer applications? Justify briefly based on your result.
6	A thermal engineer connects two metal rods end to end to transfer heat from a hot reservoir at 200 °C to a cooler one at 50 °C. Rod A is 0.4 m long with thermal conductivity 400 W/m·K (copper). Rod B is 0.6 m long with thermal conductivity 50 W/m·K (steel). Both rods have the same cross-sectional area. Assume steady-state one-dimensional heat flow. Determine the a) temperature at the interface where the rods meet? b) rate of heat transfer through the system if the cross-sectional area of the rods is $2.4 \times 10^{-4} m^2$? c) rod with the greater temperature gradient?
7	A steel rail, initially 12.0 m long at 10 °C, is installed on a railway track. During summer, the temperature can reach 45 °C. The coefficient of linear expansion for steel is 1.2×10^{-5} /°C. a) How much does the rail expand during the summer? b) What is the final length of the rail at 45 °C? c) If there were no expansion gaps, what type of structural issue might arise and why?

8	An architect designs a metal rooftop made of aluminum panels. Each rectangular panel measures 2.0 m by 3.0 m at 15 °C. In peak summer, rooftop temperatures can reach 65 °C. The coefficient of **area expansion** for aluminum is 4.4×10^{-5} /°C. a) What is the original area of a single panel? b) By how much does the area increase at 65 °C? c) What is the total expanded area of the panel at 65 °C?
9	A metal window frame holds a glass pane tightly in place. At 20 °C, both the aluminum frame and glass pane fit perfectly, each with an area of 0.80 m^2. On a hot day, the temperature rises to 70 °C. The coefficient of area expansion is 4.4×10^{-5} /°C for aluminum and 1.6×10^{-5} /°C for glass. a) By how much does the area of the aluminum frame increase? b) By how much does the area of the glass pane increase? c) Will the frame still hold the glass snugly, or will a gap form?
10	A fuel truck is filled with 10,000 L of gasoline at 10 °C and sealed. The truck's steel tank also holds exactly 10,000 L at this temperature. The coefficient of volume expansion is 9.6×10^{-4} /°C for gasoline and 3.6×10^{-5} /°C for steel. By the time the truck reaches its destination, the temperature has risen to 35 °C. a) By how much does the gasoline expand? b) By how much does the steel tank expand? c) Will gasoline spill out of the tank?

Worksheet 12: Kinetic Theory of Gas & Thermodynamics

Problems

No	Questions
1	An air-quality researcher is comparing the behavior of oxygen and helium gases in a sealed lab chamber maintained at 300 K. She wants to understand how the type of gas affects molecular motion. The molar mass of O_2 is 32.0 g/mol, and for He it is 4.0 g/mol. Assume ideal gas behavior. a) What is the ratio of the rms speed of helium to that of oxygen at the same temperature? b) If the rms speed of oxygen is approximately 480 m/s at 300 K, what is the rms speed of helium? c) Explain how the difference in molar mass affects the molecular speed of gases.
2	A vacuum engineer is testing hydrogen gas at a high temperature of 600 K in a sealed chamber. She wants to calculate the average speed of the hydrogen molecules to predict how fast they might escape through microscopic leaks. The molar mass of hydrogen is 2.0 g/mol, and the Boltzmann constant is 1.38×10^{-23} J/K. a) Convert the molar mass of hydrogen to kilograms per molecule. b) Calculate the root mean square (rms) speed of hydrogen molecules at 600 K. c) If the temperature is halved, how does the rms speed change?
3	An aerospace engineer is designing a pressurized container to carry carbon dioxide gas on a Mars rover. The gas is kept at 400 K to maintain system pressure. The molar mass of CO_2 is 44.0 g/mol, and the universal gas constant is 8.31 J/mol·K. a) Convert the molar mass of CO_2 to kilograms per mole. b) Calculate the root mean square (rms) speed of the CO_2 molecules at 400 K. c) If nitrogen gas (molar mass 28.0 g/mol) were used instead at the same temperature, would its rms speed be higher or lower? Justify numerically.
4	A sealed container on a high-altitude research balloon holds argon gas. The container has a gas density of 1.78 kg/m³, and the rms speed of argon molecules inside is measured to be 430 m/s. The balloon team wants to determine the internal pressure and predict how it changes with conditions. a) What is the pressure of the gas in the container? b) If the rms speed increases to 500 m/s while the density remains the same, what is the new pressure? c) By what percentage does the pressure increase?
5	A space capsule is being filled with helium gas. Inside a test chamber of volume 0.150 m³, engineers determine that there are 3.01×10^{24} helium atoms, and the root mean square speed of the atoms is 1200 m/s. The mass of a single helium atom is 6.64×10^{-27} kg. Determine the a) total mass of helium in the chamber. b) gas density inside the chamber. c) pressure of the gas in the chamber.
6	A deep-sea research lab stores 4.0 mol of neon gas (a monoatomic gas) in a rigid insulated container at a temperature of 350 K. Engineers are checking energy levels to ensure thermal stability during an upcoming dive. a) What is the total translational kinetic energy of the neon gas? b) What is the total internal energy of the neon gas? c) If the temperature rises to 525 K, by what factor does the total internal energy increase?
7	A sealed steel cylinder in a laboratory contains 2.0 mol of helium gas (a monoatomic gas) and 3.0 mol of nitrogen gas (a diatomic gas), both at a uniform temperature of 400 K. The research team is analyzing energy distribution in the gas mixture. Determine the a) total translational kinetic energy of the helium gas. b) total internal energy of the nitrogen gas at this temperature. c) fraction of the total internal energy of the mixture is contributed by helium.
8	A piston contains 3.0 mol of an ideal gas initially at 2.0 atm pressure and 5.0 L volume. The gas expands slowly against an external pressure of 1.0 atm until the volume reaches 10.0 L. During this expansion, the gas absorbs 1500 J of heat from the surroundings. Determine the a) work done by the gas during expansion. b) work done onto the gas by the surroundings. c) change in internal energy of the gas during this process.

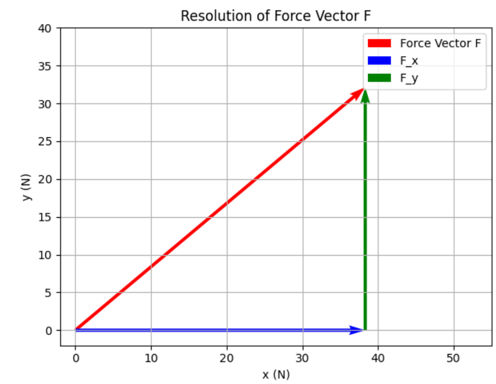
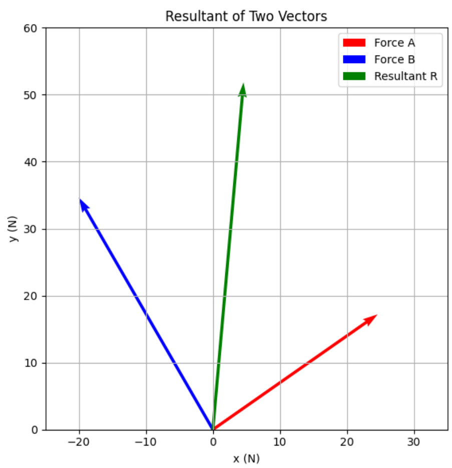
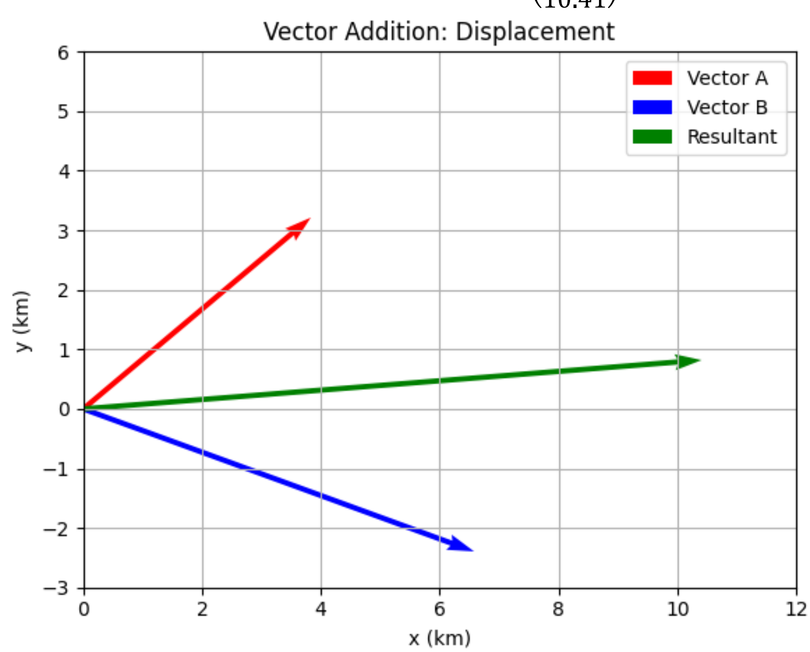
9	A cylinder contains 1.5 moles of an ideal gas at 300 K and an initial volume of 2.0 L. The gas undergoes three different processes: 1. It expands isothermally and reversibly to a volume of 5.0 L. 2. It is then compressed isothermally and reversibly back to 2.0 L. 3. Finally, the gas expands isobarically at a constant pressure of 1.0 atm from 2.0 L to 6.0 L. Calculate the a) work done by the gas during the isothermal expansion. b) work done on the gas during the isothermal compression. c) work done by the gas during the isobaric expansion.
10	A sealed cylinder contains 2.0 moles of an ideal gas at a constant temperature of 320 K. The gas expands isothermally and reversibly from an unknown initial volume to a final volume of 8.0 liters. During this process, the work done by the gas is measured to be 2500 J. a) Determine the initial volume of the gas before expansion. b) If the gas is compressed isothermally and reversibly from 8.0 liters back to this initial volume, what is the work done on the gas? c) Sketch the work done by the gas as a function of the final volume for isothermal expansion starting from the initial volume found in part (a) up to 12 liters.

For lecturers only!

Worksheet 1: Mathematics

Solutions

No	Solutions
1	$$\frac{72km}{1hour} = \frac{72000m}{60minutes \times 60seconds} = 20ms^{-1}$$ $$5cm^2 = 5(cm)^2 = 5(10^{-2}m)^2 = 5 \times 10^{-4}m^2$$ $$\frac{0.3g}{1cm^3} = \frac{0.3 \times 10^{-3}}{1(10^{-2})^3m^3} = 300kgm^{-3}$$ The sequence starts with a basic linear conversion to m/s, building confidence with familiar units. It then progresses to area conversion, requiring square powers of the scale factor. Finally, it engages learners in converting density—a derived quantity—by multiplying two unit conversion steps, reinforcing deeper understanding.
2	$$1tonne = 1000kg$$ $$1.5tonne = 1.5(1000)kg = 1500kg$$ $$1L = 10^{-3}m^3$$ $$1.2L = 1.2 \times 10^3(10^{-3})m^2 = 1.2m^3$$ Density, $$rho = \frac{M}{V} = \frac{m}{V} = \frac{1500}{1.2} = 1250kgm^{-3}$$ Calculated value matches the label. The task begins with direct unit conversions to reinforce base understanding of metric prefixes and unit scaling. It then layers complexity by introducing volume conversions in two derived units, requiring interpretation of spatial scaling. Finally, students are guided to combine converted quantities in a real-world context to compute and compare densities, bridging theoretical and applied understanding.
3	$$[F] = [MLT^{-2}], [v] = [LT^{-1}]$$ $$[P] = [F][v] = [MLT^{-2}][LT^{-1}] = [ML^2T^{-3}]$$ $$[E] = [ML^2T^{-2}], [v^2] = [L^2T^{-2}]$$ $$[X] = \frac{E}{v^2} = \frac{[ML^2T^{-2}]}{[L^2T^{-2}]} = [M]$$ X represents mass. $$[F] = [MLT^{-2}]$$ $$[T] = \frac{[F]}{[L]} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$$ This problem begins with familiar quantities like power and force to reinforce dimensional multiplication. It then introduces a new expression that leads to a recognizable base quantity (mass), prompting conceptual understanding through reverse engineering. Finally, it applies dimensional division in a new context (surface tension) to reinforce the flexibility and power of dimensional analysis in verifying physical relationships.
4	$$[ut] = [LT^{-1}][T] = [L];\ at^2 = [LT^{-2}][T^2] = [L]$$ $$[s] = [L] = [L] + [L]$$ Equation is homogeneous. $$[T] = [MLT^{-2}];\ [\mu] = [ML^{-1}]$$ $$[v] = [LT^{-1}] = \sqrt{[T/\mu]} = \sqrt{\frac{[MLT^{-2}]}{[ML^{-1}]}} = [LT^{-1}]$$ Equation is homogeneous. This problem set begins with a known kinematic equation to build confidence in applying dimensional analysis. It then guides students to a wave equation involving less familiar derived quantities, encouraging deeper dimensional reasoning.
5	$$F_x = 50\cos 40^o = 38.30N$$ $$F_y = 50\sin 40^o = 32.14N$$

	The problem is structured to start with a physical quantity (force vector) and introduces a familiar coordinate system to reduce abstraction. The first two parts focus on breaking down the vector into its components using trigonometric relations, reinforcing core algebraic manipulation. The final part leverages visual representation through a graph, connecting numerical results to geometric interpretation and deepening conceptual understanding. 
6	 $A_x = 30\,cos\,35^o = 24.57N,\ A_y = 30\,sin\,35^o = 17.21N$ $B_x = 40\,cos\,120^o = -20N,\ B_y = 40\,sin\,120^o = 34.64N$ $\vec{C} = \vec{A} + \vec{B}$ $C_x = A_x + B_x = 24.57 + (-20) = 4.57N$ $C_y = 17.21 + 34.64 = 51.85N$ $\lvert\vec{C}\rvert = \sqrt{C_x^2 + C_y^2} = \sqrt{(4.57)^2 + (51.85)^2} = 52.05N$ Direction measured from x-axis, $\theta = tan^{-1}\left(\frac{51.85}{4.57}\right) = 84.96^o$
7	$A_x = 5\,cos\,40^o = 3.83\text{km};\ A_y = 5\,sin\,40^o = 3.21km$ $B_x = 7\,cos\,20^o = 6.58km;\ B_y = 7\,sin\,(-20) = -2.36km$ $\vec{C} = \vec{A} + \vec{B}$ $C_x = A_x + B_x = 3.83 + 6.58 = 10.41km$ $C_y = 3.21 + (-2.39) = 0.82km$ $\lvert\vec{C}\rvert = \sqrt{C_x^2 + C_y^2} = \sqrt{(10.41)^2 + (0.84)^2} = 10.44km$ $\theta = tan^{-1}\left(\frac{0.82}{10.41}\right) = 4.5^o$ (north of east) 
8	$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = 4(2) + 3(5) = 23m^2$

$$\theta_A = tan^{-1}\left(\frac{3}{4}\right) = 36.8699^o$$

$$\theta_B = tan^{-1}\left(\frac{5}{2}\right) = 68.1986^o$$

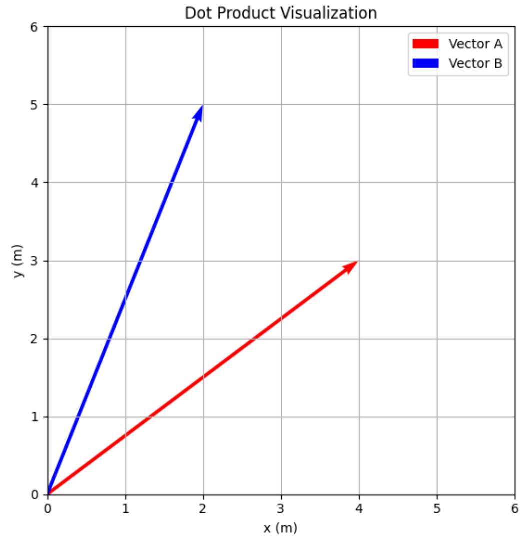
$$\Delta\theta = 68.1986^o - 36.8699^o = 31.3287^o$$

OR

$$|A| = \sqrt{3^2 + 4^2} = 5$$

$$|B| = \sqrt{2^2 + 5^2} = 5.385$$

$$\theta = cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{|A||B|}\right) = cos^{-1}\left(\frac{23}{5(5.385)}\right) = 31.3258^o$$



This problem begins with a direct computation of the dot product from components to establish basic familiarity. It then moves to connecting dot product with angle, guiding learners toward conceptual understanding of projection. The graphical part reinforces vector orientation and supports geometric intuition behind scalar product calculations.

9

$$\vec{A} = (6\,cos\,35^o, 6\,sin\,35^o) = (4.91, 3.44)m$$

$$\vec{B} = (4\,cos\,60^o, 4\,sin\,60^o) = (2,3.46)m$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (4.91)(2) + (3.44)(3.46) = 21.72m^2$$

$$\theta = cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right) = cos^{-1}\left(\frac{21.72}{6(4)}\right) = 25.2^o$$

The problem begins by having students convert vector magnitudes and angles into Cartesian components to ensure understanding of vector resolution. It then prompts application of the dot product using components, reinforcing the computational technique. Finally, it synthesizes geometric and algebraic thinking by verifying the angle using both the formula and the geometry of the original angles.

10

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & 0 \\ 1 & 4 & 0 \end{vmatrix}$$

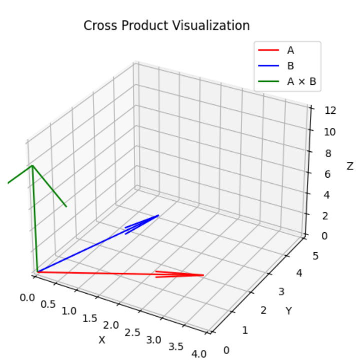
$$\vec{A}\times\vec{B} = \big(2(0) + 0(4)\big)\hat{\imath} + \big(3(0) + 0(1)\big)\hat{\jmath} + \big(3(4) - 2(1)\big)\hat{k}$$

$$\vec{A}\times\vec{B} = (0,0,10)Nm$$

$$\left|\vec{A}\times\vec{B}\right| = \sqrt{0^2 + 0^2 + 10^2} = 10Nm$$

Direction:

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{\left|\vec{A}\times\vec{B}\right|} = \frac{1}{10}(0,0,10) = (0,0,1) \text{ (along positive x-axis)}$$



The problem begins with straightforward vector components to enable procedural fluency in computing a cross product. It builds complexity by interpreting the magnitude and direction of the result, linking algebraic and physical meaning. Finally, it reinforces understanding with a 3D visualization, allowing learners to conceptually anchor the perpendicularity of the cross product.

Worksheet 2: 1D Kinematics

Solutions

No	Questions
1	a) $s = ut + 0.5at^2 = 0 + 0.5(2)(10)^2 = 100m$ $v = u + at = 2(10) = 20ms^{-1}$ b) $s = ut + 0.5at^2 = 0 + 0.5(9.81)(6)^2 = 176.58m$ $v = u + at = 0 + (9.81)(6) = 88.29ms^{-1}$ c) $v^2 = u^2 + 2as$ $\Rightarrow 452^2 = 0^2 + 2(a)(0.93)$ $\Rightarrow a = 109841ms^{-1}$
2	$v^2 = u^2 + 2as$ $0^2 = u^2 + 2(-g_{moon})h_{max}$ $h_{max} = \frac{u^2}{g_{moon}}$
3	$v = u + at \Rightarrow v = 4 + (1.5)(6) = 13ms^{-1}$ $s_{accelerate} = ut + 0.5at^2 = 4(6) + 0.5(1.5)(6)^2 = 51m$ $s_{constant} = vt = 13(10) + 131m$ $s_{total} = s_{accelerate} + s_{constant} = 51 + vt = 51 + 131 = 181m$ This problem, based on Perry's Theory of Intellectual Development, encourages students to progress beyond dualistic thinking by engaging with multi-step, context-driven reasoning. By requiring the application of different kinematic equations for various phases of motion, it fosters multiplicity and relativistic thinking, where students recognize the contextual nature of knowledge. It also supports deeper cognitive growth by preparing learners to make informed, strategic decisions, laying the groundwork for more advanced, real-world problem-solving.
4	$v = u + at = 0 + 2.5(8) = 20ms^{-1}$ $s = ut + 0.5at^2 = 0 + 0.5(2.5)(8)^2 = 80m$ $s = ut = 20(12) = 240m$ $v = u + at \Rightarrow 0 = 20 + 5a \Rightarrow a = -4ms^{-2}$ $s = ut + 0.5at^2 = 20(5) + 0.5(-4)(5^2) = 50m$ $s_{total} = 80 + 240 + 50 = 370m$ This problem scaffolds intellectual growth in line with Perry's theory by requiring students to shift from simple dualistic thinking (seeking a single answer) to multiplicity, where they choose and apply different equations based on context. By including multiple phases (acceleration, constant speed, deceleration), it encourages students to critically evaluate each situation, fostering relativistic thinking. Finally, the need to synthesize results and analyze the overall trip promotes more advanced cognitive engagement and higher-order reasoning.
5	$v = u + at = 0 + (2)(10) = 20ms^{-1}$ $s_{total} = s_1 + s_2 = (ut + 0.5at^2) + (vt) = \left(0 + 0.5(2)(10^2)\right) + \left(20(20)\right) = 500m$ $v = u + at \Rightarrow 0 = 20 + 5a \Rightarrow a = -4ms^{-2}$ $s_3 = ut + 0.5at^2 = 20(5) + 0.5(-4)(5^2) = 50m$ This problem aligns with Perry's theory by challenging students to move beyond dualistic thinking and engage in multiplicity by selecting and applying different equations for distinct phases of motion. It fosters relativistic thinking by encouraging students to recognize that the appropriate solution strategy depends on context. Finally, by requiring synthesis of results across phases, it supports higher-order reasoning and prepares students for more complex problem-solving.
6	$s = 0.5gt^2 \Rightarrow 80 = 0.5(9.81)t^2 \Rightarrow t = 4.04s$ $v = u + gt = 0 + 9.81(4.04) = 39.6ms^{-1}$ $s = ut + 0.5at^2 \Rightarrow 80 = 10t + 0.5(9.81)t^2 \Rightarrow t = 3.21s$ (ignore negative value) This problem fosters intellectual growth by encouraging students to move beyond simple answers (dualism) and apply different kinematic equations depending on initial conditions. It promotes multiplicity and relativistic thinking by requiring students to handle both free fall and nonzero initial velocity cases. Solving a quadratic equation in the final part enhances critical thinking and prepares learners for more complex, multi-step physics problems.
7	$s_1 = ut = 20t, s_2 = 0.5at^2 = 0.5(3)t^2 = 1.5t^2$ $s_1 = s_2 \Rightarrow 20t = 1.5t^2 \Rightarrow t = 13.33s$ $s_1 = 20t = 20(13.33) = 266.7m$ $v = u + at = 0 + 3(13.33) = 40ms^{-1}$

	This problem encourages students to move beyond dualistic thinking by presenting a non-trivial situation where two different objects follow different kinematic equations. It fosters multiplicity and relativistic thinking by requiring students to analyze and equate the distances traveled by both objects. The final part reinforces higher-order reasoning, as students must synthesize information from earlier calculations to find the police car’s final speed.
8	$s_A = s_B \Rightarrow 0.5a_A t^2 = ut \Rightarrow 0.5(2)t^2 = 8t \Rightarrow t = 8s$ $s_A = 0.5(2)(8)^2 = 64m$ $s_C = 0.5(a_C)t^2 = 0.5(1)t^2 = 64m$ $t = 11.31s$ This problem leverages Perry’s theory by encouraging students to move beyond simple problem-solving (dualism) to applying different kinematic models for each object. The comparison of uniform acceleration and constant speed fosters multiplicity and relativistic thinking as students must recognize how different initial conditions impact motion. By synthesizing solutions across multiple cases, the problem deepens higher-order reasoning and prepares students for analyzing complex, multi-object scenarios.
9	a) $s = ut = 3(10) = 30m$ b) $s = \frac{u+v}{2}(t) = \frac{3+6}{2}(5) = 22.5m$ c) $s = 30 + 22.5 + (5)(6) = 82.5m$ Perry's theory in constructing this problem emphasized combining various phases of motion with different kinematic conditions (constant speed, acceleration, and deceleration) to engage different learning strategies. By breaking the problem into intervals with varying velocities, it encourages the application of multiple kinematic concepts to solve real-world motion scenarios. This approach also allows for a modular solution that reinforces problem-solving steps in physics.

Worksheet 3: Projectile Motion

Solutions

No	Solutions
1	$u_x = 30\,cos\,50^o = 19.28m/s$ $u_y = 30\,sin\,50^o = 22.98m/s$ $v_x(t = 3s) = 19.28m/s$ $v_y(t = 3s) = u_y + a_y t = 22.98 - (9.81)(3) = -6.42ms^{-1}$ $v_{net} = \sqrt{v_x^2 + v_y^2} = \sqrt{(19.28)^2 + (-6.42)^2} = 20.31ms^{-1}$ $\theta = tan^{-1}\left(\frac{-6.42}{19.28}\right) \approx -18.4^o$ This problem leverages Perry’s theory by encouraging students to deconstruct and integrate velocity components, requiring them to move beyond dualistic "right-answer" thinking. The multiplicity of approaches (horizontal vs. vertical motion, vector magnitude, and angle) emphasizes that different perspectives contribute to solving the problem. Additionally, the final synthesis of velocity magnitude and direction promotes relativistic thinking, helping students understand that motion analysis depends on context and requires integration of different concepts.
2	$u_y = u\,sin\,20^o = 10ms^{-1}$ $v_y = 0 = u_y + gt \Rightarrow 0 = 10 + (-9.81)(t_{up}^2) \Rightarrow t_{up} = 1.02s$ Time of flight, $t_{total} = 2t_{up} = 2.04s$ $h_{max} = ut_{up} + 0.5gt_{up}^2 \Rightarrow h = (10)(1.02) + 0.5(-9.81)(1.02)^2 = 5.1m$ $u_x = u\,cos\,20^o = 20\,cos\,30^o = 17.32ms^{-1}$ $R = s_{x-max} = u_x(t_{total}) = 17.32(2.04) = 35.3m$ This problem applies Perry’s theory by encouraging students to move beyond rote calculation (dualism) and recognize the multiplicity of concepts in projectile motion (vertical and horizontal components). It develops relativistic thinking by requiring students to understand the contextual relevance of different kinematic equations. By synthesizing horizontal and vertical motion, students engage in more complex, higher-order reasoning that prepares them for real-world applications.
3	$y = 0.5gt^2 \Rightarrow 45 = 0.5(9.81)t^2 \Rightarrow t = 3.03s$ $s_x = u_x t = 15(3.03) = 45.45m$ $v_y = gt = 9.81(3.03) = 29.7ms^{-1}$ $v_x = 15ms^{-1}$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15)^2 + (29.7)^2} = 33.3ms^{-1}$ This problem incorporates Perry’s theory by challenging students to move beyond dualistic thinking and apply different kinematic equations simultaneously for horizontal and vertical motion. It encourages multiplicity by presenting a non-zero initial height, requiring students to contextualize their problem-solving approach. Additionally, by synthesizing the horizontal and vertical components to find the final speed, students engage in relativistic thinking and practice integrating multiple pieces of information.
4	$s_y = u_y t + 0.5gt^2 \Rightarrow 10 = (25\,sin\,40^o)t - 4.95t^2 \Rightarrow t = 2.44s$ $s_x = u_x t = (u\,cos\,\theta)t = (25\,cos\,40^o)(2.44) = 46.7m$ $v_y = u_y - gt \Rightarrow v_y = 25\,sin\,40^o - 9.81(2.44) = -7.84ms^{-1}$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25\,cos\,40^o)^2 + (-7.84)^2} = 20.7ms^{-1}$ $\theta = tan^{-1}\left(\frac{-7.84}{25\,cos\,40^o}\right) = -22.26^o$ This problem aligns with Perry’s theory by encouraging students to move beyond dualistic thinking and apply different kinematic concepts in a more complex, non-zero final height scenario. It fosters multiplicity by requiring students to handle simultaneous vertical and horizontal motions and apply the quadratic formula. Finally, solving for the final speed synthesizes relativistic thinking, where students must integrate various components of motion to find a contextualized answer.
5	$R = \frac{u^2\,sin\,2\theta}{g} \Rightarrow 100 = \frac{u^2\,sin\,2\theta}{9.81}$ $h_{max} = \frac{u^2 sin^2\theta}{2g} \Rightarrow 20 = \frac{u^2 sin^2\theta}{2(9.81)}$ $u = 30.93ms^{-1}$, $\theta = 38.66^o$ $R = u_x t \Rightarrow 100 = (30.93\,cos\,38.66^o)t \Rightarrow t = 4.14s$

	This problem encourages transformative learning by challenging students to analyze and reconstruct their understanding of projectile motion, beyond memorizing formulas. It integrates conceptual relationships between height, range, and launch angle, prompting critical reflection on how these parameters interconnect. By requiring students to manipulate multiple equations and synthesize a solution, it fosters deeper learning and a shift from surface-level understanding to critical, problem-based reasoning.
6	$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow u^2 \sin 2\theta = 120(9.81) = 1176\text{ms}^{-1}$ $h_{max} = \frac{u^2 \sin^2\theta}{2g} \Rightarrow u^2 \sin^2\theta = 2(9.81)(15) = 294\text{ms}^{-1}$ $u = 38.34ms^{-1}$, $\theta = 26.57^o$ $s_y = u_y t + 0.5a_y t^2 \Rightarrow 5 = (38.34 \sin 26.57^o)t + 0.5(-9.81)t^2 \rightarrow t = \{0.32, 3.17\}s$ This problem applies Mezirow's theory by prompting students to reflect critically on their preconceived notions of projectile motion, particularly when a non-zero final height complicates their expectations. By solving interrelated questions involving quadratic equations, trigonometry, and conceptual reasoning, students are challenged to recognize the complexity and interconnectedness of kinematic concepts. The problem aims to foster transformative learning by encouraging deeper problem-solving, beyond rote memorization, and helping students apply theoretical knowledge to real-world-like contexts.
7	$t_A = 2ug^{-1}$ $t_B = 2ug^{-1} \sin(\theta)$ $\Delta t = t_A - t_B = 2ug^{-1}\left(1 - \sin(\theta)\right)$ $\Delta t = 2(20)(9.81)^{-1}\left(1 - \sin(30)\right)$ $\Delta t = 2.04s$ Perry's theory in projectile motion emphasizes comparing different initial conditions (like vertical and angled throws) to highlight variations in airtime. By applying his framework, we evaluate how the vertical component of velocity determines the flight duration in both scenarios. This problem focuses on the influence of angle and velocity on time of flight, reflecting key aspects of Perry's study.
8	$s_y = 40 + u_y t - 0.5gt^2 = 0$ $40 + (25 \sin 30^o)t - 0.5(9.81)t^2 = 0$ $t = 4s$ $s_x = u_x t = (25 \cos 30^o)(4) = 86.6m$ $v_y = u_y - gt = 12.5 - (9.81)4 = -26.7ms^{-1}$ $v_{net} = \left(v_x^2 + v_y^2\right)^{0.5} = (21.65^2 \pm 26.7^2)^{0.5} = 34.1ms^{-1}$ This problem leverages Perry's Scheme by challenging students to transition from dualistic reasoning ("one correct formula") toward multiplistic and relativistic thinking, requiring synthesis of multiple equations. Each sub-question demands independent modeling decisions, fostering intellectual autonomy. By integrating motion in two dimensions, students must reconcile differing perspectives on vertical and horizontal motion—a cognitive leap aligned with Perry's positions 3–4, where learners begin to see knowledge as constructed, not just received.
9	$u_x = \frac{s_x}{t} = \frac{40}{2} = 20m/s$ $s_y = u_y t - 0.5gt^2 \Rightarrow 20 = u_y(2) - 0.5(9.81)(2)^2 \Rightarrow u_y = 19.8m/s$ $u = \left(u_x^2 + u_y^2\right)^{0.5} = (20^2 + 19.8^2)^{0.5} = 28.2m/s$ $\theta = tan^{-1}\left(\frac{u_y}{u_x}\right) = tan^{-1}\left(\frac{19.8}{20}\right) = 44.6^o$ $v_y^2 = u_y^2 - 2gs_y \Rightarrow 0 = (19.8)^2 - 2(9.81)h_{max} \Rightarrow h_{max} = 20m$ This problem is scaffolded by first providing a known point on the trajectory to anchor all unknowns, reducing the abstraction of solving without initial velocity. Part (a) isolates components, establishing values needed for parts (b) and (c), which then build on each other sequentially. The structure allows learners to progress from simple kinematic reasoning to multistep synthesis, supporting gradual cognitive load increase.

Worksheet 4: Momentum, Impulse and Conservation of Momentum

Solutions

No	Solutions
1	$$p_{total} = m_1u_1 + m_2u_2 = 2(3) + 1(0) = 6kgms^{-1}$$ $$v = \frac{p_{total}}{m_1 + m_1} = \frac{6}{2+1} = 2ms^{-1}$$ $$\Delta E_k = 0.5(m_1u_1^2 - (m_1 + m_2)v) = 0.5(2(3)^2 - (2+1)2) = 3J$$ This problem begins with a clearly defined system and known values, minimizing cognitive load. Part (a) establishes the core principle (momentum conservation), while part (b) applies it to solve for the final state, and part (c) adds an energy perspective, extending understanding without introducing new principles. The sequencing allows students to gain confidence through structured reasoning before integrating kinetic energy concepts.
2	$$\Delta p = p_f - p_i = mv - mu = 0.5(-4) - 0.5(6) = -5kgms^{-1}$$ $$J = \Delta p = -5kgms^{-1}$$ $$F = \frac{J}{\Delta t} = \frac{-5}{0.01} = -500N$$ The problem begins with a concrete event and small numbers to ease entry into symbolic reasoning. Each part builds conceptually: (a) reinforces momentum change, (b) connects that to impulse, and (c) links impulse to force over time, reinforcing relationships between core ideas. This stepwise progression supports learner confidence while subtly guiding them toward integrating momentum and force-time perspectives.
3	$\Sigma p_i = 2(4) + 3(0) = 8Ns$ (in positive x-direction) $$\Sigma p_i = \Sigma p_f$$ $$\Sigma p_{ix} = \Sigma p_{fx} \Rightarrow \Sigma p_{fx} = 8Ns$$ $$\Sigma p_{iy} = \Sigma p_{fy} \Rightarrow \Sigma p_{fy} = 0Ns$$ $$\Sigma p_{ix} = \Sigma p_{fx} \Rightarrow 8 = 2(2\cos 30^o) + 3(3\cos\theta)$$ $$\Sigma p_{iy} = \Sigma p_{fy} \Rightarrow 0 = 2(2\sin 30^o) + 3(3\sin\theta)$$ $$0 = 2(2\sin 30^o) + 3(3\sin\theta) \Rightarrow \theta = \sin^{-1}\left(-\frac{2}{9}\right) \approx -12.8^o$$ Angle is 12.8^o below the positive x-axis. This problem is scaffolded in a way that gradually builds understanding of conservation of momentum in two dimensions. Part (a) focuses on the straightforward calculation of total momentum in the initial state, setting the foundation. Part (b) applies the conservation of momentum for both the x- and y-directions, guiding the learner through vector decomposition, and part (c) asks the student to solve for the unknown angle, reinforcing their understanding of vector relationships and momentum conservation.
4	$$\Sigma p_{ix} = \Sigma p_{fx} \Rightarrow 3(5) = 3(3\cos 30^o) + 4(v\cos\theta)$$ $$\Sigma p_{iy} = \Sigma p_{fy} \Rightarrow 0 = 3(3\sin 30^o) + 4(v\sin\theta)$$ $$v = 2.146ms^{-1}$$ $$\theta = -32.5^o$$ $$K_i = 0.5(3)(5^2) + 0 = 37.5J$$ $$K_f = 0.5\big(3(3^2) + 4(2.146^2)\big) = 22.7J$$ $$\Delta K = 22.7 - 37.5 = -14.8J$$ The problem begins with basic information and applies fundamental principles of momentum conservation, allowing students to focus on one aspect of the problem at a time. By breaking the solution into separate steps—solving for x- and y-components of momentum first, followed by solving for final velocity and angle—the problem gradually increases in complexity. Finally, calculating the energy lost reinforces the connection between momentum conservation and kinetic energy, providing a more complete understanding of the collision.
5	$$\Sigma p_{ix} = \Sigma p_{fx} \Rightarrow 3(4) = (3+4)v\cos\theta$$ $$\Sigma p_{iy} = \Sigma p_{fy} \Rightarrow 4(3) = (3+4)v\sin\theta$$ $$v = 2.426ms^{-1}$$ $$\theta = 45^o$$ $$K_i = 0.5\big(3(4^2) + 4(3^2)\big) = 42J$$ $$K_f = 0.5(3+4)(2.426^2) = 20.573J$$ $$\Delta K = K_f - K_i = 20.573 - 42 = -21.43J$$ The problem is scaffolded by first separating the conservation of momentum into its x- and y-components, which simplifies the calculations and provides a clear path to solving for the final velocity. Once the momentum equations are solved,

	students are guided to combine the components to find the magnitude and direction of the final velocity, reinforcing vector operations and the concept of motion in two dimensions. Finally, the problem progresses to the calculation of kinetic energy lost, which not only connects momentum conservation with energy principles but also encourages a deeper understanding of the physical implications of inelastic collisions. This step-by-step approach builds student confidence and integrates multiple concepts in a structured manner.
6	$$\Sigma p_{ix} = \Sigma p_{fx} \Rightarrow 2(5\,cos\,30^o) + 3(4\,cos\,120^o) + 1(v_3\,cos\,\theta) = 0$$ $$\Sigma p_{iy} = \Sigma p_{fy} \Rightarrow 2(5\,sin\,30^o) + 3(4\,sin\,120^o) + 1(v_3\,sin\,\theta) = 0$$ $v_3 = 8.25ms^{-1}$, $\theta = -52.2^o$ $$\Sigma K_i = 0J$$ $$\Sigma K_f = 0.5\left(2(5^2) + 3(4^2) + 1(8.25^2)\right) = 83J$$ The problem is structured to guide students through the conservation of momentum in two dimensions, beginning with basic momentum conservation in both directions. The first two subquestions focus on determining the magnitude and direction of the third piece's velocity by breaking down the problem into horizontal and vertical momentum components. The final subquestion connects to energy calculation.

Worksheet 5: Newton's Laws

Solutions

No	Solutions
1	$$F = ma \Rightarrow 20 = 5a \Rightarrow a = 4ms^{-2}$$ $$s = ut + 0.5at^2 = 0 + 0.5(4)(4)^2 = 32m$$ $$v = u + at = 0 + 4(4) = 16ms^{-1}$$ **Scaffolding Explanation:** The problem builds conceptual understanding in steps: first computing acceleration directly from Newton's Second Law. It then progresses to kinematics with displacement, using the prior result. Finally, it reinforces acceleration by applying it again to find final velocity, completing a logical, step-by-step learning progression.
2	$$\boldsymbol{T = m_A a = m_B(g - a) \Rightarrow a = \frac{m_B g}{m_A + m_B} = \frac{2(9.81)}{4+2} = 3.27ms^{-2}}$$ $$\boldsymbol{T = (4)(3.27) = 13.1N}$$ $$\boldsymbol{T = m_A a + f_k = m_B(g - a) \Rightarrow a = \frac{m_B g - f_k}{m_A + m_B} = \frac{2(9.81) - 2}{4+2} = 2.77ms^{-2}}$$ **Scaffolding Explanation:** The problem begins with the ideal (no friction) case to focus purely on Newton's Second Law in a multi-object system. It then reinforces this by isolating tension, linking concepts across objects. The final part adds friction to challenge students to adjust their previous model, deepening conceptual understanding through structured progression.
3	$$F_{parallel} = mg\sin\theta = (10)(9.81)\sin 30^o = 49N$$ $$a = \frac{F_{parallel}}{m} = \frac{49}{10} = 4.9ms^{-1}$$ $$F_{parallel} = mg\sin\theta - 10N = (10)(9.81)\sin 30^o - 10 = 39N$$ $$a = \frac{F_{parallel}}{m} = \frac{39}{10} = 3.9ms^{-1}$$ **Scaffolding Explanation:** The problem begins with identifying the force component driving motion, grounding students in vector decomposition. It then moves to applying Newton's Second Law for acceleration on a frictionless surface. Finally, it introduces friction to modify the net force, encouraging students to adapt their understanding within a familiar framework.
4	$$f_k = \mu N = \mu mg = 0.2(12)(9.81) = 23.52N$$ $$F_{net} = F_{applied} - f_k = 50 - 23.52 = 26.48N$$ $$a = \frac{F_{net}}{m} = \frac{26.48}{12} = 2.21ms^{-2}$$ **Scaffolding Explanation:** The problem first isolates the friction force to ground students in the concept of resistive forces. It then progresses to calculating net force to apply Newton's Second Law in context. Finally, it uses the net force to determine acceleration, completing a structured, logical progression from forces to motion.
5	$$F_{parallel} = mg\sin\theta = 15(9.81)\sin 25^o = 62.1N$$ $$f_k = \mu N = \mu mg\cos\theta = 0.3(15)(9.81)(\cos 25^o) = 39.96N$$ $$F_{net} = 62.1 - 39.96N = 22.14N$$ $$a = \frac{F_{net}}{m} = \frac{22.14}{15} = 1.48ms^{-2}$$ **Scaffolding Explanation:** The problem begins by isolating the force causing motion, reinforcing understanding of component vectors on an incline. It then adds friction, guiding students through resistive force calculation. Finally, it combines both to determine net force and acceleration, reinforcing Newton's Second Law in a complete applied context.
6	$$N = mg - F_a\sin\theta = (10)(9.81) - 40\sin 30^o = 78N$$ $$f = \mu N = \mu(mg - F_a\sin\theta) = 0.25(78) = 19.5N$$ $$F_{net} = F_a\cos\theta - f = 40\cos 30^o - 19.5 = 15.14N$$ $$a = \frac{F_{net}}{m} = \frac{15.14}{10} = 1.51ms^{-2}$$ **Scaffolding Explanation:** The problem starts with calculating frictional force using the normal force, establishing the resistive force. Next, it incorporates the effect of the applied force at an angle, calculating the normal force that changes due to the vertical component of the applied force. Finally, the net force is found by combining both the horizontal applied force and friction, guiding students to apply Newton's Second Law to find the acceleration.

7	$$N = mgcos\,\theta - F_a\,sin\,\phi = (12)(9.81)(cos\,30^o) - 50\,sin\,20^o = 84.66N$$ $$f = \mu N = 0.35(84.66) = 29.63N$$ $$F_{net} = F_a\,cos\phi - mg\,sin\theta - f$$ $$F_{net} = 50\,cos20^o - (12)(9.81)\,sin(30^o) - 29.63 = -41.445N$$ $$a = \frac{F_{net}}{m} = \frac{-41.445}{12} = -3.46ms^{-2}$$ **Scaffolding Explanation:** The problem starts by calculating the frictional force, introducing the effect of the incline and applied force on the normal force. It then calculates the net vertical and horizontal forces, gradually combining friction and the external force components. Finally, it uses the net force to calculate acceleration, building a progression from basic forces to motion with a real-world application.
8	$$T_1\,cos\,30^o + T_2\,cos\,45^o = mg = 10(9.81)$$ $$T_1\,sin\,30^o = T_2\,sin\,45^o$$ $$T_1 = 81.4N$$ $$T_2 = 57.6N$$ **Scaffolding Explanation:** The problem begins with the fundamental concept of force equilibrium, solving for tensions in two strings using the vertical and horizontal force balance. It then ties these two tensions together with a simple trigonometric equation.

Worksheet 6: Work, Energy & Power

Solutions

No	Solutions
1	$W = Fd = 20(4) = 80J$ $\Delta K = W = 80J$ $0.5mv^2 = 0.5(5)v^2 = 80 \Rightarrow v = 5.66ms^{-1}$ The problem begins with a simple, idealized setup (no friction) to focus solely on the applied force and distance for work. The second part reinforces the direct link between work and change in kinetic energy through the work-energy theorem. The final step uses the kinetic energy formula to compute speed, reinforcing how energy relates to motion.
2	$E_{gp} = mgh = (2)(9.81)(5) = 98J$ $E_K = E_{gp} = 98J$ $98 = 0.5mv^2 = 0.5(2)(v^2) \Rightarrow v = 9.9ms^{-1}$ The problem begins with a simple gravitational potential energy calculation to set the total mechanical energy. It then applies conservation of energy to determine the kinetic energy at the bottom, reinforcing the principle that mechanical energy is conserved. Finally, it uses kinetic energy to find speed, linking energy concepts to motion outcomes.
3	$E_K = 0.5mv^2 = 0.5(1.5)(3^2) = 6.75J$ $E_K = E_{ep} \Rightarrow 6.75 = 0.5kx^2 = 0.5(200)(x^2) \Rightarrow x = 0.26m$ $E_{ep} = 0.5kx^2 = 0.5(200)(0.26) = 6.75J$ The problem starts with calculating kinetic energy to define total mechanical energy. It then transitions into using that energy to find spring compression, introducing elastic potential energy in a familiar setup. Finally, it reinforces conservation by verifying energy transformation from kinetic to potential form.
4	$W_{applied} = Fd = 100(5) = 500J$ $W_f = f(d) = d(\mu mg\, cos\theta) = 5(0.2)(12)(9.81)\, cos\, 25^o = 106.5J$ $W_{net} = \Delta K \Rightarrow W_{applied} - W_f - mgh = W_{applied} - W_f - mg(d\, sin\, \theta) = 0.5m(v^2 - u^2)$ $500 - 106.5 - (12)(9.81)(5\, sin\, 25^o) = 0.5(12)(v^2 - 0^2)$ $v = 4.91ms^{-1}$ The problem begins with straightforward work done by a force, grounding students in direct application of the work formula. It then adds friction to introduce energy loss, connecting resistive forces to the concept of negative work. Finally, it combines all energy terms using the work-energy theorem to solve for speed, promoting synthesis of kinetic, potential, and non-conservative work effects.
5	$mgh = 0.5mv^2 \Rightarrow 2(9.81)(4) = 0.5(2)v^2 \Rightarrow v = 8.86ms^{-1}$ $mgh = 0.5kx^2 \Rightarrow 2(9.81)(4)0.5(300)x^2 \Rightarrow x = 0.72m$ Halfway compresses, $x = 0.36m$ $mgh = 0.5mv^2 + 0.5kx^2 \Rightarrow 2(9.81)(4) = 0.5(2)(v^2) + 0.5(300)(0.36^2)$ $v = 7.68ms^{-1}$ The problem first isolates gravitational to kinetic energy conversion to develop familiarity with energy transformation. It then incorporates elastic potential energy to find maximum spring compression, reinforcing full mechanical energy conservation. The final part synthesizes all three energy types, challenging students to evaluate partial energy exchange and deepening conceptual integration.
6	$E_{total} = E_{gp} = mgh = (1.8)(9.81)(2.5) = 44.1J$ $mgh = 0.5kx^2 - mgx \Rightarrow 44.1 = 0.5(500)x^2 - 1.8(9.81)x$ $x = 0.457m$ $mgh = mgx = 0.5kx^2 \Rightarrow 1.8(9.81)x = 0.5(500)x^2 \Rightarrow x = 0.0706m$ The problem begins by isolating gravitational potential energy to establish total mechanical energy. It then advances to full energy conservation between gravitational and elastic forms to determine the spring's maximum compression. The final step focuses on an energy balance condition, reinforcing deeper conceptual understanding of how energy transforms at specific positions.
7	$E_{ep} = E_K \Rightarrow 0.5kA^2 = 0.5mv^2$ $(400)(0.6)^2 = (2.5)v^2 \Rightarrow v = 7.62ms^{-1}$ $x = 0.6m$ $E_{ep} = E_K \Rightarrow 0.5kA^2 = 0.5mv^2$ $(400)(0.6)^2 = (2.5)v^2 \Rightarrow v = 7.62ms^{-1}$ The problem begins with energy transformation from elastic to kinetic energy at the equilibrium position to build core understanding. It then extends this by asking about motion beyond equilibrium, reinforcing symmetry in spring oscillations. The final part confirms comprehension by applying the same reasoning to compression, emphasizing conservation principles across symmetric motion.
8	$E_{total} = E_{gp} = mgh = 500(9.81)(25) = 122.5kJ$

	$h = 2(R) = 2(10) = 20m$ $E_K = 0.5mv^2 = 122.5kJ - mgh$ $0.5(500)v^2 = 122500 - 500(9.81)(20) \Rightarrow v = 9.90ms^{-1}$ The problem starts with calculating total mechanical energy to establish the conserved quantity, reinforcing energy conservation principles. It then guides students through applying this energy at different heights to find speeds, linking potential and kinetic energy changes. Finally, it contrasts speeds at top and bottom of the loop, helping students visualize energy transformation along the track.

Worksheet 7: Work, Energy and Power Part 2

Solutions

No	Solutions
1	$F_{net} = 20 - 15 = 5N$; $a = \frac{F}{m} = \frac{15}{10} = 1.5ms^{-2}$;$W = Fd = (15)(5) = 75J$ The problem begins by asking for the net force, a fundamental concept that allows students to apply basic principles of force addition. Once the net force is found, students are guided to calculate acceleration using Newton's second law, building on their previous knowledge. The final part asks for the work done by the net force, reinforcing the connection between force, displacement, and work in a real-world context.
2	$a = \frac{F}{m} = \frac{10}{5} = 2ms^{-2}$; $v = u + at = 2 + 2(4) = 10ms^{-1}$; $$W = Fd = F(ut + 0.5at^2) = (10)\big((2)(4) + 0.5(2)(4)^2\big) = 240J$$ The problem begins by calculating acceleration, which is a direct application of Newton's second law and introduces the relationship between force and acceleration. The second part builds on this by applying the equation of motion to find the final velocity, which reinforces understanding of constant acceleration. The final part requires calculating work, using a formula that combines the earlier results (velocity and acceleration), thus gradually increasing the complexity while linking key concepts.
3	$$W_F = \Delta K + W_{fr} + W_g$$ $$W_F = 0.5mv^2 + \mu mgd\cos\theta + mgd\,sin$$ $$W_F = 0.5(12)(3^2) + 0.2(12)(9.81)(4)cos(25^o) + 12(9.81)(4)sin(25^o) = 338.2J$$ $$W_F = Fd \Rightarrow 338.2 = F(4) \Rightarrow F = 84.6N$$ $$P = Fv = 84.6(3) = 254W$$ The problem starts by focusing on energy changes (kinetic, gravitational, and frictional), isolating the concept of net work. Then, it uses the result to extract the applied force through direct algebra, reinforcing the link between work and force. Finally, the question transitions to power, applying known values in a real-world quantity to consolidate understanding.
4	$$0.5kx^2 = 0.5mv^2 \Rightarrow v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{800}{2}}(0.25) = 5ms^{-1}$$ $$0.5mv^2 = mgh \Rightarrow h = 0.5v^2g^{-1} = 0.5(5)^2(9.81)^{-1} = 1.28m$$ $$0.5kx^2 = mgh \Rightarrow h = \frac{kx^2}{2mg} = \frac{(800)(0.35)^2}{2(2)(9.81)}$$ The problem starts with elastic to kinetic energy conversion to isolate energy transfer within a single form. It then builds to include gravitational potential, connecting vertical motion and conservation principles. Finally, it generalizes the system by changing initial compression to reinforce flexible application of the energy conservation model.
5	$$mgh_1 = mgh_2 = 0.5mv^{2\backslash Rightarow}v = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.81)(30 - 10)} = 19.8ms^{-1}$$ $$h_{max} = h_1 = 30m$$ The problem begins with a simple energy conversion from potential to kinetic energy to establish core conservation concepts. It then extends this to a limiting case of total energy use to determine the highest reachable point.
6	$$mgh = 0.5mv^2 \Rightarrow v = \sqrt{2gh} = \sqrt{2(9.81)(2)} = 6.26ms^{-1}$$ $$mgh = 0.5kx^2 \Rightarrow x = \sqrt{2mghk^{-1}} = \sqrt{2(1.5)(9.81)(2)(500)^{-1}} = 0.343m$$ $$mgh = 0.5mv^2 + 0.5kx^2 \Rightarrow v = \sqrt{\frac{2mgh - kx^2}{m}} = \sqrt{\frac{2(1.5)(9.81)(2) - (500)(0.1)^2}{1.5}} = 6ms^{-1}$$ The problem begins with simple gravitational-to-kinetic conversion to build familiarity with vertical energy changes. It then adds elastic potential energy to deepen understanding of full mechanical energy conservation. The final subquestion blends all three energy types in a partial-compression state, guiding students toward more complex but structured applications of the same principle.
7	$$0.5kx^2 = 0.5mv^2 \Rightarrow v = \sqrt{km^{-1}}x = \sqrt{800(0.4)^{-1}}(0.2) = 8.94ms^{-1}$$ $$0.5kx^2 + mgh = 0.5mv^2 \Rightarrow 0.5(800)(0.2)^2 + (0.4)(9.81)(1.5) = 0.5(0.4)v^2 \Rightarrow v = 10.46ms^{-1}$$ $$v_y = u_y + gt = g\left(\sqrt{2hg^{-1}}\right) = (9.81)\left(\sqrt{2(1.5)(9.81)^{-1}}\right) = 5.425ms^{-1}$$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{8.94^2 + 5.425^2} = 10.46ms^{-1}$, same as (b) The problem is scaffolded by beginning with a foundational concept—spring-to-kinetic energy conversion—isolated from any vertical motion, allowing students to apply the conservation of mechanical energy in a simplified context. It then introduces gravitational potential energy to build upon the previous energy analysis, guiding students to a more complex application involving multiple forms of energy. Finally, students are prompted to verify their energy-based result using projectile motion kinematics, reinforcing conceptual understanding by connecting two major analysis methods in physics.

8	$0.5kx^2 0.5mv^2 \Rightarrow v = \sqrt{km^{-1}}x = \sqrt{900(0.6^{-1})}(0.18) = 6.97ms^{-1}$ $0.5kx^2 = mgh \Rightarrow h = 0.5kx^2(mg)^{-1} = 0.5(900)(0.18)^2\left((0.6)(9.81)\right)^{-1} = 2.48m$ $W_f = \mu_k mgd\cos\theta, d = \frac{d}{\sin\theta}$ $0.5kx^2 = mgh + \mu_k mg\cos\theta\left(\frac{h}{\sin\theta}\right)$ $h = \frac{kx^2}{2mg(1 + \mu_k \cot\theta)}$ The problem begins by focusing on the energy conversion from the compressed spring to the kinetic energy of the block, helping students connect elastic potential energy to motion. It then applies conservation of energy principles to determine the block’s maximum height on a frictionless incline, reinforcing the relationship between kinetic and gravitational potential energy. Finally, it introduces friction on the incline and guides students to derive a general expression, encouraging synthesis of concepts including work done against friction and geometric considerations.

Worksheet 8: Circular Motion

Solutions

No	Solutions
1	$$T = mv^2r^{-1} \Rightarrow (2.7) = (0.3)v^2(0.8)^{-1} \Rightarrow v = 2.68ms^{-1}$$ Straight line tangent to the circular path. $$T \propto v^2, T_{new} = 4T_{old}$$ The first part guides students to apply a single force equation to solve for speed, grounding them in Newton's 2nd law. The second part shifts to conceptual reasoning, reinforcing understanding of motion without centripetal force. The third builds on part (a), using proportional reasoning to explore how changing speed affects force, encouraging students to generalize.
2	$$f_s = \mu N = \mu mg = mv^2r^{-1}$$ $$v = \sqrt{\mu gr} = \sqrt{(0.4)(9.81)(5.0)} = 14ms^{-1}$$ $$\mu = v^2(gr)^{-1} = 12^2\big(9.81(50)\big)^{-1} = 0.29$$ $$v = \sqrt{\mu gr} = \sqrt{(0.1)(9.81)(5.0)} = 7ms^{-1}$$ The problem begins with a straightforward calculation (part a) using known variables to build confidence. It then inverts the setup (part b), requiring rearrangement of the same equation to solve for an unknown coefficient. Finally, part c changes a parameter (μ) to apply the same conceptual model in a slightly different context, reinforcing understanding of how friction limits circular motion.
3	$$v = 2\pi rT^{-1} = 2\pi(9)(20)^{-1} = 2.83ms^{-1}$$ $$N = mg - mv^2r^{-1} = (60)(9.81) - (60)(2.83)^2(9)^{-1} = 534.7N$$ $$N = mg + mv^2r^{-1} = 641.3N$$ The problem begins with a straightforward computation of speed from period and radius (part a), setting up necessary values. Part (b) introduces net force at the top, reinforcing the concept of centripetal acceleration and apparent weight. Part (c) mirrors (b) at a different location, encouraging comparison and conceptual contrast between opposing net force directions.
4	$$T\,cos\,\theta = mg;\ T\,sin\,\theta = mv^2r^{-1} = mv^2(l\,sin\,\theta)^{-1}$$ $$v = \sqrt{gl\,tan\,\theta} = \sqrt{9.81(1.2)(tan\,30^o)} = 2ms^{-1}$$ $$T = \frac{mg}{cos\,\theta} = \frac{0.5(9.81)}{cos\,30^o} = 5.66N$$ $$v^2 \propto \theta$$ The problem begins with a calculation of speed using known physical relationships (part a), reinforcing force balance in two dimensions. It then progresses to solving for tension using prior results (part b), connecting variables across subquestions. Finally, part c shifts to conceptual reasoning with algebraic support, encouraging deeper understanding of the relationship between motion and angle.
5	$$\omega = 2\pi Nt^{-1} = 2\pi(30)(20)^{-1} = 9.42rads^{-1}$$ $$T = m\omega^2r = 0.25(9.42)^2(1.1) = 24.5N$$ $$T \propto \omega^2 \Rightarrow T_{new} = 4T_{old}$$ Part (a) introduces angular velocity calculation from frequency, connecting rotation rate to standard units. Part (b) applies angular velocity in a dynamic context, reinforcing the link between circular motion and tension. Part (c) extends the relationship algebraically, prompting students to reason through proportional changes and their effects on force.
6	$$v = r\omega = \sqrt{gl\,tan\,\theta} = \sqrt{g\left(\frac{r}{sin\,\theta}\right)\,tan\,\theta} \Rightarrow \omega = 2.06rads^{-1}$$ $$T\,cos\,\theta = mg \Rightarrow T\,cos\,(40^o) = (20)(9.8) \Rightarrow T = 256N$$ $$\omega \propto \sqrt{\frac{tan\,\theta}{sin\,\theta}} \Rightarrow \omega_{new} > \omega_{old}$$ Part (a) begins with applying geometry and dynamics to solve for angular velocity, grounding the problem in real-world motion. Part (b) then uses a force balance to reinforce understanding of tension in two dimensions. Part (c) extends the relationship algebraically and conceptually, guiding students to reason about how angular velocity depends on angle without needing specific numbers.

Worksheet 9: Chapter 7 Part 1 - SHM

Solutions

No	Solutions
1	$v = d_t x = 0.12(5)cos\,(5t) = 0.6\,cos\,5t$ where v(t) is in metres per second. $v(t = 0.3) = 0.6\,cos\,(5(0.3)) = 0.0424ms^{-1}$ $E_K = 0.5mv^2 = 0.5(0.5)(0.0424)^2 = 0.00045J$ The problem begins with identifying a velocity function from a given position function, reinforcing derivative interpretation. It progresses to evaluating this velocity at a specific time, promoting procedural fluency. Finally, it transitions to energy analysis, requiring application of velocity in a physical context and reinforcing the connection between motion and energy.
2	$x = 0.15\,sin\,(4t),\ a = -\omega^2 x = -0.15(4)^2\,sin\,(4t) = -2.4\,sin\,(4t)$ where a(t) is in metres per seconds squared. $a(t = 1s) = -2.4\,sin\,(4(1)) = 1.82ms^{-2}$ $x = 0.15\,sin\,(4t) = 0.15\,sin\,(4(1)) = -0.1136m$ $E_p = 0.5kx^2 = 0.5k\omega^2 x^2 = 0.5(0.40)(4^2)(-0.1136)^2 = 0.041J$ The problem begins by guiding students to identify acceleration through successive derivatives of a sine-based position, reinforcing conceptual understanding of motion. It then moves to numerical evaluation at a given time to develop procedural fluency. Finally, it shifts to energy analysis using position and mass-derived spring constant, helping students connect motion to stored energy physically and algebraically.
3	$v_{max} = A\omega = 0.6(6) = 1.2ms^{-1}$ $v = \omega\sqrt{A^2 - x^2} = 6\sqrt{(0.2^2 - 0.1^2)} = 1.04ms^{-1}$ $E_{total} = 0.5mv_{max}^2 = 0.36J$ The problem begins by having students identify the peak value of a derived quantity (speed), reinforcing understanding of amplitude and angular frequency. It then builds on this by evaluating speed at a nonzero displacement, requiring conceptual application of energy conservation. The final step consolidates understanding by calculating total energy using earlier results, emphasizing energy as constant throughout SHM.
4	$T = 2\pi\sqrt{lg^{-1}} = 2\pi\sqrt{(0.8)(9.81)^{-1}} = 1.79s$ $E_k = E_{total} - E_p = mgh_{max} - mgh = 0.3(9.81)(0.1 - 0.04) = 0.176J$ $E_k = 0.5mv^2 \Rightarrow 0.176 = 0.5(0.3)v^2 \Rightarrow v = 1.08ms^{-1}$ The problem begins with period calculation to ground understanding in the time-based properties of pendulum motion. It then advances to energy considerations, focusing on changes in gravitational potential and kinetic energy. The final question connects energy to motion by asking for speed at a point in the swing, reinforcing the interplay between energy and kinematics in SHM.
5	$T = 2\pi\sqrt{mk^{-1}} = 2\pi\sqrt{0.4(25)^{-1}} = 0.8s$ $E_p = 0.5kx^2 = 0.5(25)(0.05)^2 = 0.03125J$ $a = -km^{-1}x = -25(0.4^{-1})(0.05) = -3.125ms^{-2}$ The problem begins with period calculation to establish the temporal characteristics of oscillation using basic parameters. It then moves to potential energy evaluation at a nonzero displacement, reinforcing the relationship between position and stored energy. Finally, it connects position to acceleration, highlighting the restoring nature of spring force and its proportionality to displacement in SHM.
6	$T = 2\pi\sqrt{mk^{-1}} = 2\pi\sqrt{0.6(36)^{-1}} = 0.81s$ $T_{pendulum} = T_{spring} = 0.81s$ $T = 2\pi\sqrt{lg^{-1}} \Rightarrow (0.81) = 2\pi\sqrt{l(9.81)^{-1}} \Rightarrow l = 0.163m$ The problem starts with computing the period of a spring-mass system using basic parameters to reinforce core formulas. It then requires students to apply equivalence in period to link conceptual understanding between two systems. Finally, the problem asks for the pendulum length, giving students a chance to rearrange and apply the pendulum period formula in a real-world context.
7	$k = 4\pi^2 mT^{-2} = 4\pi^2(0.8)(1.2)^{-2} = 21.93Nm^{-1}$ $m = kT^2(4\pi)^{-1} = (21.93)(1.5)^2(4\pi)^{-1} = 1.25kg$ The problem begins with a straightforward application of the mass-spring period formula to calculate the spring constant. It then extends understanding by reversing the relationship to solve for mass using the same spring. The final step integrates a real-world property (density) to apply mass knowledge in a new physical context, bridging SHM with material science.
8	$0.5mv^2 + 0.5kx^2 = 0.5kA^2 \Rightarrow v = \sqrt{\frac{k}{m}(A^2 - x^2)}$

	$$a = \frac{dv}{dt} = \frac{dv}{dx}\left(\frac{dx}{dt}\right)$$ $$a = \left(\frac{-kx}{m\sqrt{\frac{k}{m}(A^2 - x^2)}}\right)\left(\sqrt{\frac{k}{m}(A^2 - x^2)}\right)$$ $$a = -\frac{k}{m}x$$ $$E_{total} = 0.5mv^2 + 0.5kx^2 = 0.5m\left(\sqrt{\frac{k}{m}(A^2 - x^2)}\right)^2 + 0.5kx^2 = 0.5kA^2 = constant$$ This problem begins by prompting a derivation from conservation of energy, reinforcing a core principle of SHM. It then increases difficulty by requiring differentiation to relate velocity and acceleration with respect to displacement. Finally, it consolidates understanding by validating energy conservation through symbolic manipulation, challenging students to synthesize concepts across mathematical and physical domains.

Worksheet 10: Chapter 7 Part 2 – Progressive Waves

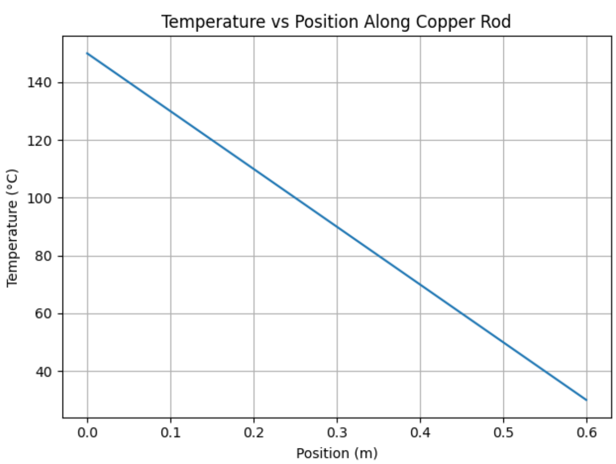
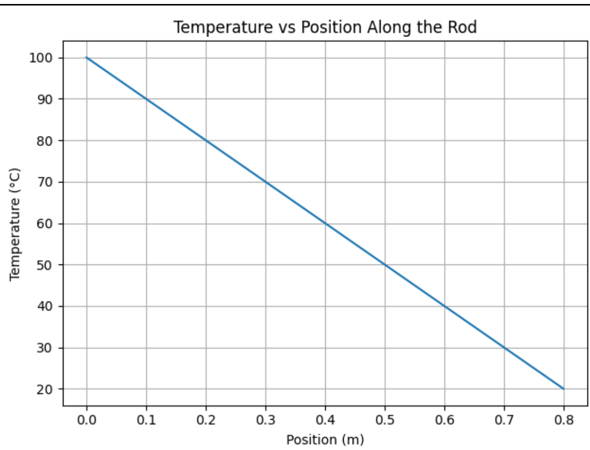
Solutions

No	Solutions
1	$$y(x,t) = A\,sin\,(\omega t - kx) = A\,sin\left((2\pi f)t - \left(\frac{2\pi}{\lambda}\right)x\right)$$ $$y(x,t) = 0.015\,sin\,(10\pi t - 10\pi x)$$ Where y and x are in metres and t is in seconds. $$y(0.1{,}0.05) = 0.015\,sin\left(10\pi(0.05) - 10\pi(0.1)\right) = -0.015m$$ $$v = f\lambda = (5)(0.2) = 1ms^{-1}$$ The problem begins by guiding the student to construct a full wave equation using given parameters, ensuring comprehension of wave components. It then reinforces understanding through direct substitution to evaluate displacement. Finally, it connects frequency and wavelength to determine wave speed, bridging algebraic manipulation with physical interpretation.
2	$$y(x,t) = 0.020\,sin\,(16\pi t - 8\pi x)$$ Where y and x are in metres and t is in seconds. $$v_{particle} = \frac{dy}{dt} = 0.02(16\pi)cos(16\pi t - 8\pi x) = 1.005\,cos\,(16\pi t - 8\pi x)$$ Where y and x are in metres and t is in seconds. $$v_{particle}(0{,}0.03125) = 1.005\,cos\left(16\pi(0) - 8\pi(0.03125)\right) = 0$$ The problem begins by prompting students to build a full wave equation using amplitude, frequency, and wavelength, reinforcing core wave parameters. It then introduces calculus by asking for the particle's vibrational velocity using a time derivative, enhancing conceptual understanding of motion. The final question applies the derived expression numerically, helping students connect mathematical form with physical interpretation.
3	$$y(x,t) = 2(0.03)\,sin\,(2.5\pi x)\,cos\,(20\pi t) = (0.06)\,sin\,(2.5\pi x)\,cos\,(20\pi t)$$ Where y and x is in metres and t is in seconds. $$A(x = 0.2) = 0.06\,sin\left(2.5\pi(0.2)\right) = 0.06m$$ $$v_{max} = \omega A = 20\pi(0.06) = 3.77ms^{-1}$$ This problem begins by constructing a standing wave equation from physical parameters, reinforcing understanding of how traveling waves combine. The second part applies the spatial component of the standing wave to calculate local amplitude, emphasizing position dependence. The final part ties angular frequency and local amplitude to transverse motion, integrating wave structure with kinematics.
4	$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{90}{2.5 \times 10^{-3}}} = 189.7ms^{-1}$$ $$f_1 = \frac{v}{2} = \frac{189.7}{1.2} = 158.1Hz$$ Second overtone = third harmonic $$f_3 = 3f_1 = 3(158.1) = 474.3Hz$$ First overtone = second harmonic $$\lambda_2 = \frac{v}{f_2} = \frac{189.7}{2(158.1)} = 0.6m$$ The problem begins by grounding students in basic wave physics using a string's length, tension, and mass per unit length to calculate the wave speed. It then builds on this to find the fundamental and overtone frequencies using proportional harmonic relationships. Finally, it reinforces the understanding of harmonics by linking them to corresponding wavelengths, combining algebraic reasoning with physical intuition.
5	$$f_1 = \frac{v}{2L} = \frac{340}{2(0.85)} = 200Hz$$ Second overtone = third harmonic $$f_3 = 3f_1 = 3(200) = 600Hz$$ First overtone = second harmonic $$\lambda_2 = \frac{v}{f_2} = \frac{340}{2(200)} = 0.85m$$ The problem begins by having the student apply a simple formula to calculate the fundamental frequency of an open-open column, grounding their understanding of basic resonance. It then builds complexity by asking for overtone relationships using integer multiples of the fundamental. Finally, the problem connects frequency to wavelength, reinforcing conceptual links between harmonics, frequency, and wave properties.

6	$$f_1 = \frac{v}{4L} = \frac{340}{4(0.75)} = 113.3Hz$$ $$f_n = \{113.3,\ 340\ 566.7\}Hz$$ Lowest frequency under 400Hz is $566.7Hz$. Second overtone = fifth harmonic $$\lambda_5 = \frac{v}{f_5} = \frac{340}{566.7} = 0.6m$$ Distance between node with antinode, $$d = \frac{\lambda_5}{4} = 0.15m$$ The problem begins with a standard application of the formula for the fundamental frequency of a closed-open column. It then prompts critical reasoning by having students identify the next valid harmonic above a specific threshold, reinforcing the odd-harmonic pattern. The final part builds spatial understanding by connecting harmonic order to wave geometry through node-antinode spacing.
7	$$f_{observed} = f_{emitted}\left(\frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}\right)$$ $$f_{monday} = 850\left(\frac{340}{340-20}\right) = 903.1Hz$$ $$f_{tuesday} = 850\left(\frac{340+5}{340}\right) = 862.5Hz$$ $$\%_{diff} = \frac{903.1-862.5}{862.5} = 4.71\%$$ The problem progresses from a direct application with a moving source, to a subtly different situation involving a moving observer—requiring students to discern which Doppler formula to apply. The third question integrates prior results to calculate a percentage difference, reinforcing both conceptual comparison and basic mathematical operations. This gradual increase in complexity supports conceptual clarity and avoids cognitive overload while deepening understanding of relative motion in sound perception.
8	$$f_{observed} = f_{emitted}\left(\frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}\right)$$ $$f_A = 700\left(\frac{340}{340-15}\right) = 732.3Hz$$ $$f_B = 700\left(\frac{340+10}{340}\right) = 720.6Hz$$ $$\Delta_A = \lvert 732.3 - 700\rvert = 32.3Hz$$ $\Delta B = \lvert 720.6 - 700\rvert = 11.7Hz$, frequency shift greater in scenario A. The structure begins with a moving source scenario to introduce basic Doppler shift without requiring decision-making about relative motion. The second part changes only one condition—now the observer moves—prompting comparison and application of a different but related formula. The final question encourages synthesis by comparing the results and interpreting their significance, supporting both conceptual and quantitative reasoning.

Worksheet 11: Physics of Materials

Solutions

No	Solutions
1	$$\epsilon = \frac{\Delta L}{L} = \frac{0.48}{12} = 0.04$$ $$Y = \delta\epsilon^{-1} \Rightarrow \delta = Y\epsilon = Y\left(\frac{\Delta L}{L}\right) = (5\times10^{9})\left(\frac{0.48}{12}\right) = 2\times10^{8}Pa$$ $$U = 0.5\delta\epsilon = 0.5(2\times10^{8})(0.04) = 168J$$ The problem starts with stress to introduce the relationship between material properties and internal force. It then isolates strain as a dimensionless ratio to deepen understanding of deformation relative to original length. Finally, it combines both in a strain energy calculation, reinforcing how mechanical properties and deformation result in stored energy—progressing from basic to integrative thinking.
2	$$U = 0.5F\Delta L = 0.5(90)(0.0015) = 0.0675J$$ $$V = \pi r^2 L = \pi(0.0005)^2(2) = 1.57\times10^{-6}m^3$$ $$\frac{U}{V} = \frac{0.0675}{1.57\times10^{-6}} = 4.3\times10^{4}Jm^{-3}$$ $$\frac{U}{V} = 0.5 = \delta\epsilon = 0.5Y\epsilon^2$$ $$Y = \frac{2U}{V\epsilon^2} = \frac{2(4.3\times10^{4})}{\left(\frac{0.0015}{2}\right)^2} = 1.53\times10^{11}Pa$$ The problem begins with interpreting area under a linear graph to apply energy concepts visually. It then transitions to calculating strain energy density to connect energy with material properties. Finally, students apply that result to extract Young's modulus algebraically, reinforcing synthesis of energy, geometry, and elasticity.
3	 $$A = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{0.015}{2}\right)^2 = 1.77\times10^{-4}m^2$$ $$P_{heat} = Qt^{-1} = kA\Delta TL^{-1}$$ $$P_{heat} = 385(1.77\times10^{-4})(150-30)(0.6)^{-1}$$ $$P_{heat} = 13.86W$$ The problem begins by requiring a calculation of area from diameter, reinforcing geometry's role in conduction problems. It then uses that result to compute heat transfer rate, linking physical dimensions to thermal behavior. Finally, a temperature-position graph reinforces the concept of a linear gradient in steady-state conduction, connecting quantitative and visual understanding.
4	 $$P_{heat} = Qt^{-1} = kA\Delta TL^{-1}$$ $$P_{heat} = 205(5\times10^{-4})(100-20)(0.8)^{-1} = 10.25W$$ $$P_{heat} = Qt^{-1} \Rightarrow 10.25 = Q(10\times60)^{-1}$$ $$Q = 6150J$$ The problem begins with a direct application of the heat conduction formula to build fluency with basic parameters. The second part reinforces time-dependent accumulation of energy transferred. The third part uses the earlier results to visually represent temperature change spatially, fostering conceptual understanding of gradient and steady-state behavior.
5	$$\Delta T = 90 - 40 = 50^{o}C$$ $$Qt^{-1} = kA\Delta TL^{-1} \Rightarrow 31.25 = k(0.25)(50)(0.04)^{-1} \Rightarrow k = 0.1Wm^{-1}K^{-1}$$ More suitable for insulation due to its low thermal conductivity.

	The first step isolates a straightforward temperature difference to set the thermal gradient context. The second introduces thermal conductivity calculation using known quantities, reinforcing the inverse relation between conductivity and insulation. The final part asks for qualitative interpretation, guiding students from calculation to real-world material assessment.
6	$$\frac{k_A}{L_A}(T_1 - T) = \frac{k_B}{L_B}(T - T_2) \Rightarrow \frac{400}{0.4}(200 - T) = \frac{50}{0.6}(T - 50)$$ $$T = 188.5^oC$$ $$\frac{Q_A}{t_A} = kA\left(\frac{T_1 - T}{L_A}\right) \Rightarrow \frac{Q_A}{t_A} = (400)(2.4 \times 10^{-4})\left(\frac{200 - 188.5}{0.4}\right) = 2.3W$$ $$m_A = \frac{\Delta T}{L} = \frac{11.5}{0.4} = 28.75^oCm^{-1}$$ $$m_B = \frac{\Delta T}{L} = \frac{138.5}{0.6} = 230.8^oCm^{-1}$$ $$m_B > m_A$$ The problem begins with finding an interface temperature, requiring students to equate heat flows—promoting reasoning over memorization. It then uses that result to compute heat rate, reinforcing physical understanding of thermal conductivity. Finally, it compares gradients, helping students interpret material properties through observable effects like steepness of temperature change.
7	$$\Delta L = \alpha L_o \Delta T = (1.2 \times 10^{-5})(12)(45 - 10) = 0.00504m$$ $$\Delta L = L_f - L_o = L_f - 12 = 0.00504 \Rightarrow L_f = 12.00504m$$ Without gaps, thermal expansion causes compression forces. These can lead to **rail buckling**, especially in long sections exposed to sustained heat. The problem starts with a basic calculation of linear expansion, reinforcing proportional thinking. It then moves to determine total length, applying addition of initial and change values. The final conceptual question prompts students to consider the real-world consequences of expansion, linking math to engineering design.
8	$$A_o = 2(3) = 6m^2$$ $$\Delta A = \beta A_o \Delta T = (4.4 \times 10^{-5})(6)(65 - 15) = 0.0132m^2$$ $$\Delta A = A_f - A_o = A_f - 6 = 0.0132 \Rightarrow A = 6.0132m^2$$ The problem begins with straightforward area calculation to anchor the concept in familiar geometry. It then introduces thermal expansion by calculating the change using the coefficient, reinforcing proportional reasoning. Finally, students apply both values to determine the total expanded area, integrating the concept into a full physical picture.
9	$$\Delta A_{Al} = (4.4 \times 10^{-5})(0.8)(70 - 20) = 0.00176m^2$$ $$\Delta A_g = (1.6 \times 10^{-5})(0.8)(50) = 0.00064m^2$$ $$\Delta A_{Al} - \Delta A_g = 0.00176 - 0.00064 = 0.00112m^2$$ Yes, a small gap will form, since the frame expands more than the glass. The first question guides students through applying the area expansion formula to one material, setting the foundation. The second reinforces the concept by comparing it with a different material under identical conditions. The final conceptual question promotes synthesis, requiring students to interpret the physical consequences of differential expansion between materials.
10	$$\Delta V_{gas} = \gamma V_o \Delta = (9.6 \times 10^{-4})(10000)(35 - 10) = 240L$$ $$\Delta V_{tank} = (3.6 \times 10^{-}5)(10000)(35 - 10) = 9L$$ $$\Delta V_{gas} - \Delta V_{tank} = 240 - 9 = 231L$$ 231L of gasoline will spill. The problem begins by isolating thermal expansion for a liquid, reinforcing the concept in a simple, realistic setting. It then introduces expansion of the container, prompting comparative reasoning. The final synthesis question pushes students to apply both calculations to assess real-world outcomes, solidifying conceptual understanding.

Worksheet 12: Kinetic Theory of Gas & Thermodynamics

Solutions

No	Solutions
1	$$\frac{v_{rms-He}}{v_{rms-O_2}} = \sqrt{\frac{32}{4}} = 2$$ $$v_{rms-He} = 2v_{rms-O_2} = 2(480) = 960ms^{-1}$$ Lighter molecules (lower molar mass) move faster at the same temperature. Thus, helium, being much lighter than oxygen, has a significantly higher rms speed. The problem starts with a ratio comparison to simplify conceptual understanding without plugging values. It then builds toward calculating an actual speed to reinforce proportional relationships. Finally, the student is asked to reflect on the physical meaning, linking math results to molecular-level behavior.
2	$$m = \frac{2 \times 10^{-3}}{6.022 \times 10^{23}} = 3.32 \times 10^{-27}kg$$ $$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(600)}{3.32 \times 10^{-27}}} = 2734ms^{-1}$$ $$v_{rms} \propto \sqrt{T} \Rightarrow v_{rms-new} = \sqrt{0.5}v_{rms-old} = \sqrt{0.5}(2734) = 1934ms^{-1}$$ The problem begins by converting molar mass into a usable microscopic form, reinforcing unit analysis and mole-to-molecule concepts. It then guides the student through the full rms speed calculation using core kinetic theory. Finally, it prompts conceptual reasoning about temperature's effect on molecular motion through proportional relationships.
3	$$M = \frac{44g}{1mol} = \frac{44 \times 10^{-3}kg}{1mol} = 44 \times 10^{-3}kgmol^{-1}$$ $$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31)(400)}{44 \times 10^{-3}}} = 476.063ms^{-1}$$ $$v_{rms} \propto M^{-0.5} \Rightarrow \frac{v_{rms-new}}{v_{rms-old}} = \frac{M_{new}^{-0.5}}{M_{old}^{-0.5}}$$ $$v_{rms-new} = \left(\frac{28}{44}\right)^{-0.5}(476.063) = 596.777ms^{-1}$$ The problem starts by anchoring students in correct SI units, a crucial prerequisite for working with the gas constant. It then applies the rms speed formula step-by-step for one gas to build procedural fluency. Finally, it challenges students to compare and reason across different gases, reinforcing the inverse relationship between molar mass and rms speed.
4	$$P = 3^{-1}\rho v_{rms}^2$$ $$P = 3^{-1}(1.78)(430^2) = 109707Pa$$ $$P_{new} = 3^{-1}(1.78)(500)^2 = 148333Pa$$ $$\%_{increase} = \frac{148333 - 109707}{109707}(100) = 35.2\%$$ The problem begins by connecting rms speed and density to pressure, reinforcing the kinetic interpretation of gases. It then modifies only one variable (rms speed) to isolate its effect on pressure. Finally, it prompts a percentage calculation to help students synthesize and quantify the relationship, promoting a deeper understanding of how molecular motion influences macroscopic pressure.
5	$$m_{total} = (3.01 \times 10^{24})(6.64 \times 10^{-27}) = 0.01998kg$$ $$\rho = \frac{M}{V} = \frac{0.01998}{0.15} = 0.1332kgm^{-3}$$ $$P = 3^{-1}\rho v_{rms}^2 = 3^{-1}(0.1332)(1.44 \times 10^6)^2 = 63936Pa$$ The problem is built to guide students through fundamental concepts step by step: beginning with molecular mass and number of particles to determine total gas mass, then using volume to find density. It culminates in applying the kinetic theory formula for pressure using rms speed. This structured layering helps students build conceptual and quantitative fluency while connecting microscopic and macroscopic properties.
6	$$K_{trans} = \frac{3}{2}nRT = \frac{3}{2}(4)(8.31)(350) = 17451J$$ $$U = K_{tr} = 17451J$$ $$U \propto T \Rightarrow \frac{U_{new}}{U_{old}} = \frac{T_{new}}{T_{old}} = \frac{525}{350} = 1.5$$

	The problem begins by prompting students to calculate translational energy, reinforcing its direct link to temperature in monoatomic gases. It then draws attention to the equivalence of translational and total internal energy, simplifying conceptual understanding. Finally, it guides learners to reason proportionally with temperature changes, deepening their grasp of energy-temperature relationships in ideal gases.
7	$K_{tr-He} = \frac{3}{2}nRT = \frac{3}{2}(2)(8.31)(400) = 9972J$ $U_{N_2} = \frac{5}{2}nRT = \frac{5}{2}(3)(8.31)(400) = 24930J$ $U_{total} = U_{He} + U_{N_2} = K_{tr-He} + U_{N_2} = 9972 + 24930 = 348902J$ $\frac{U_{N_2}}{U_{total}} = \frac{24930}{348902} = 0.286$ The problem introduces energy at the particle level using helium's translational kinetic energy to reinforce foundational understanding. It then adds complexity by incorporating internal energy of diatomic gas, prompting a comparison of energy modes. Finally, students synthesize both contributions to assess proportions, developing reasoning about how molecular structure affects internal energy distribution.
8	$W_{by} = p\Delta V = (1.013 \times 10^5)(1 \times 10^{-2} - 5 \times 10^{-3}) = 507J$ $W_{onto} = -W_{by} = -507J$ $\Delta U = Q - W_{by} = 1500 - 507 = 993J$ The problem starts with calculating work done by the gas, linking volume change and external pressure to physical expansion. It then reinforces the sign convention by asking for work done onto the gas, deepening understanding of energy exchange directions. Finally, it integrates heat transfer and work to find internal energy change, solidifying comprehension of the first law of thermodynamics.
9	$W_{by} = nRT\, ln\left(\frac{V_f}{V_i}\right) = (1.5)(8.31)(300)\, ln\left(\frac{5}{2}\right) = 3425J$ $W_{onto} = -nRT\, ln\left(\frac{V_f}{V_i}\right) = -(1.5)(8.31)(300)\, ln\left(\frac{2}{5}\right) = 3425J$ $W_{by} = p\Delta V = (1.013 \times 10^5)(6-2)(10^{-3}) = 405J$ The problem first requires applying the isothermal work formula, reinforcing logarithmic volume dependence. It then reverses the process to clarify the sign and meaning of work during compression. Finally, it introduces isobaric work, contrasting constant-pressure linear volume dependence, allowing students to connect different thermodynamic pathways.
10	$W_{by} = nRT\, ln\left(\frac{V_f}{V_i}\right) \Rightarrow 2500 = (2)(8.31)(320)\, ln\left(\frac{8}{V_i}\right)$ $\Rightarrow V_i = 5L$ $W_{onto} = -W_{by} = -2500J$ The problem begins by applying the isothermal work formula to find the initial volume, encouraging algebraic manipulation and inverse reasoning. Next, it reinforces understanding of work sign conventions by examining the reverse compression process. Finally, it integrates visualization to connect mathematical results with physical trends in thermodynamic work.