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## Contents

# Tutorial Questions

## 1 Chapter 2: Capacitors

### Solutions:

### Capacitance in series and parallel

$\boxed{C = \frac{Q}{V};\, U = \frac{1}{2}QV}$

1.

$$\begin{aligned} C &= \frac{Q}{V} \\ &= \frac{4000(10^{-6})\,C}{1000\,V} \\ &= 4\mu C \\ U &= \frac{1}{2}QV \\ &= \frac{1}{2}(4000\,\mu C)(1000\,V) \\ &= 2\,J \end{aligned}$$

2.

$$\begin{aligned} V &= \frac{Q}{C} \\ &= \frac{24(10^{-8})\,C}{12000(10^{-12})\,F} \\ &= 20V \end{aligned}$$

3.

$$\begin{aligned} Q_i &= CV_i\,;\; Q_f = CV_f \\ \Delta Q &= Q_f - Q_i\,;\; \Delta V = V_f - V_i \\ \Delta Q &= C\Delta V \\ C &= \frac{\Delta Q}{\Delta V} \\ &= \frac{60\mu C - 30\mu C}{50V - 35V} \\ &= 2\mu F \\ \Delta U &= U_f - U_i \\ &= \frac{1}{2}(Q_f V_f - Q_i V_i) \\ &= \frac{1}{2}(50V(60\mu C) - 35V(30\mu C)) \\ &= 0.975 mJ \end{aligned}$$

4. $C_2$ and $C_3$ in series

$$C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

$C_{23}$ and $C_1$ are parallel to each other

$$\begin{aligned} C_{eq} &= C_1 + C_{23} \\ &= C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} \\ &= 10\mu + \left(\frac{1}{12\mu} + \frac{1}{8\mu}\right)^{-1} \\ &= 14.8\mu F \end{aligned}$$

5.

$$\begin{aligned} V_{total} &= V_1 = V_{23} = 26V \\ C_{23} &= \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} \\ \text{Then for } j &= \{2, 3\}, \\ V_j &= \{\frac{Q_{23}}{C_2}, \frac{Q_{23}}{C_3}\} \\ &= \{\frac{C_{23}}{C_2}V_{23}, \frac{C_{23}}{C_3}V_{23}\} \\ &= \{14.85,\ 11.15\}V \end{aligned}$$

## Capacitors Charging and Discharging

$\boxed{\tau = RC;\ Q = Q_o e^{-\frac{t}{RC}};\ Q = Q_o(1 - e^{-\frac{t}{RC}})}$

1. $\tau = RC = (1000\Omega)(4000 \times 10^{-6}\,F) = 4s$
2. $Q = Q_o e^{-\frac{t}{\tau}} = (20\mu)e^{-\frac{8s}{4s}} = 2.71\mu C$
3. 

$$\begin{aligned} \tau &= RC_{eq} \\ &= (2000)\left(\frac{1}{0.003\,F} + \frac{1}{0.005\,F}\right)^{-1} \\ &= 3.75s \\ \text{50\% full: } &\Rightarrow \frac{Q_f}{Q_o} = 0.5 \\ 0.5\,Q_o &= Q_o(1 - e^{-\frac{t}{RC}}) \\ \frac{t}{RC} &\approx 0.69 \\ t &\approx 2.6s \end{aligned}$$

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## Capacitors with dielectrics

$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_o};\ C_o = \frac{\varepsilon_o A}{d};\ C = \varepsilon_r C_o}$

1.

$$\begin{aligned} C = \varepsilon_r C_o = \varepsilon_r \frac{\varepsilon_o A}{d} \\ &= 6\frac{(8.85 \times 10^{-12})(\pi(2.5 \times 10^{-2})^2)}{3 \times 10^{-3}} \\ &= 34.754 pF \end{aligned}$$

2.

$$\begin{aligned} Q|_{\varepsilon_r = 3} &= C_{with\,dielectric} V \\ &= \varepsilon_r C_o V \\ &= (3)(4000 \times 10^{-12})(24) \\ &= 0.288 \mu F \end{aligned}$$

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## 2 Chapter 3: DC circuits

### Solutions:

### Electric Conduction, Resistivity & Ohm's Law

1.
$$I = \frac{Q}{t} \Rightarrow Q = It = (8A)(30s) = 240C$$

2.
$$I = \frac{Q}{t} = \frac{Ne}{t} = \frac{(3 \times 10^{23})(1.6 \times 10^{-19})}{32 \times 60} = 25A$$

3.
$$R = \rho\frac{l}{A}$$
$$R = (1.7 \times 10^{-8}\,\Omega\,m)\frac{30}{\pi\,(\frac{0.3 \times 10^{-3}}{2})^2}$$
$$R \approx 7.215\Omega$$

4.
$$V = IR = (5A)(30\Omega) = 150V$$

5.
$$A = \pi(r_{outer}^2 - r_{inner}^2) = \frac{\pi}{4}(d_{outer}^2 - d_{inner}^2)$$

$$R = \rho\frac{l}{A} = \rho\frac{l}{\frac{\pi}{4}(d_{outer}^2 - d_{inner}^2)}$$

$$R = \frac{(10^{-7})(30 \times 10^{-2})}{\frac{\pi}{4}((1 \times 10^{-2})^2 - (0.7 \times 10^{-2})^2)}$$
$$R = 0.75m\Omega$$

### Resistance Variation with Temperature

1.

| Question | $R_f$ |
|---|---|
| a | $220.205\Omega$ |
| b | $13.724k\Omega$ |
| c | $98.1425\Omega$ |

2.
$$R_f = R_i[1 + \alpha(T_f - T_i)]$$
$$T_f = \frac{\frac{R_f}{R_i} - 1}{\alpha} + T_i$$
$$T_f = \frac{\frac{200}{150} - 1}{0.0038} + (50)$$
$$T_f \approx 137.72^oC$$

## Electromotive Force, Series and Parallel Circuit

1.
$$I = \frac{\varepsilon}{R+r} = \frac{9}{6+0.5} \approx 1.385V$$
$$V_{ab} = \varepsilon - rI = 9V - 0.5(1.385) \approx 8.3075V$$

2. (a) Case 1:
$$R_{eff} = 12 + 6 + 15 = 33\Omega$$

   (b) Case 2:
$$R_{eff} = \left(\frac{1}{12} + \frac{1}{6} + \frac{1}{15}\right)^{-1}$$
$$R_{eff} = \frac{60}{19}\Omega$$

   (c) Case 3:
$$R_{eff} = 15 + \left(\frac{1}{12} + \frac{1}{6}\right)^{-1}$$
$$R_{eff} = 19\Omega$$

   (d) Case 4:
$$R_{eff} = 1 + \left(\frac{1}{2+2} + \frac{1}{6} + +\frac{1}{2}\right)^{-1}$$
$$R_{eff} = \frac{23}{11}\Omega$$

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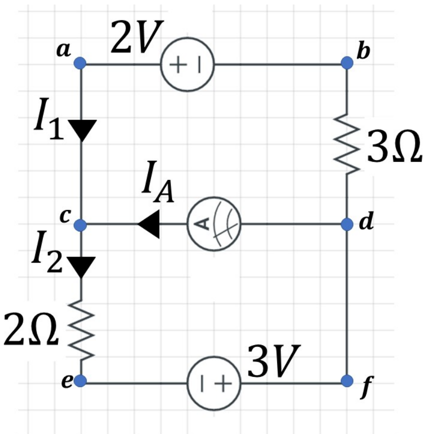
## Kirchhoff's Law

1. Calculate the values for $x$ and $y$ for the following pair of linear equations:

| | |
|---|---|
| a) $x = 6$, $y = -3$ | b) $x = 2$, $y = -3$ |
| c) $x = 3$, $y = 8$ | d) $x = 4$, $y = -2$ |

2.

$$\sum I_{in} = \sum I_{out}$$
$$I_3 = I_1 + I_2$$
Take loop with emf 18V and 12V,
$$\sum \varepsilon = \sum V_{resistor}$$
$$\varepsilon_{18} - \varepsilon_{12} = I_1(R_{16} + R_{25})$$
$$18 - 12 = I_1(16 + 25)$$
$$I_1 = 0.15A$$
Take loop with emf 18Ω and 12V,
$$\varepsilon_{12} = I_3 R_8$$
$$I_3 = 0.67A$$
$$I_2 = I_3 - I_1 = 0.52A$$
$$V_{18\Omega} = I_3 R_{18} = 12V$$
$$I_i = \{0.15, 0.52, 0.67\}A$$




3.

$I_2 = I_A + I_1$ Loop abdca: $2 = -5I_A + 3I_1$ Loop efdce: $3 = 5I_A + 2I_2$ $I_A = 0.16A$

4.

Loop with resistors 5Ω and 6Ω :
$$42 - 28 = 5I_1 + 6I_2$$
$$5I_1 + 6I_2 = 14$$
Loop with resistors 5Ω and 3Ω :
$$5I_1 + 3I_3 = -28 I_i = \{-2,\ 4,\ -6\}A$$

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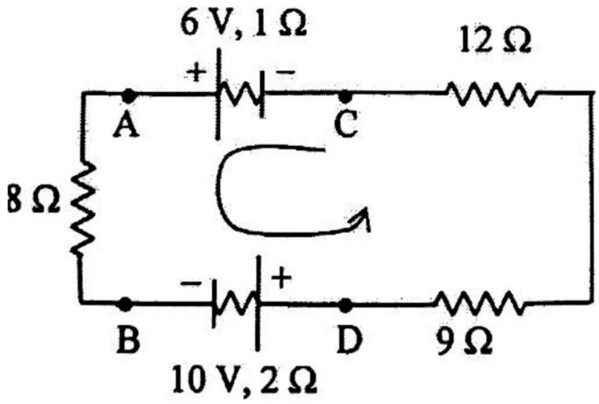
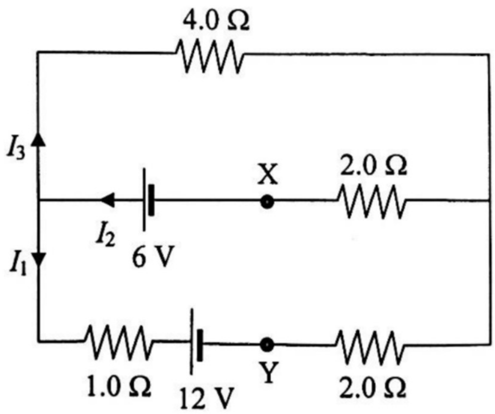
5. 


$$\text{Total emf,} \sum \varepsilon = 6V + 10V = 16V$$
$$\text{Total resistance,} \sum R = 2 + 12 + 9 + 1 + 8 = 18\Omega$$
$$\text{Current,} I = \frac{\sum \varepsilon}{\sum I} = \frac{16}{32} = 0.5A$$
$$V_{AC} = 6 - I(1) = 6 - 1(0.5) = 5.5V$$

6. 


(a)

$$I_2 = I_1 + I_3$$
$$Loop\, 1: \ 6 - 12 = 3I_1 + 2I_2$$
$$Loop\, 2: \ 6 = 4I_3 + 2I_2$$
$$I_i = \{-1.85, -0.23, 1.62\}A$$

(b) $\sum \varepsilon = \sum IR \ V_{XY} + 6 - 12 = 1(-1.85) \ V_{XY} = 4.15V$

(c) $P = \sum I^2 R = 20.9W$

## Electrical Energy and Power

1. $P = IV = I^2 R = (0.5)^2(200) = 50W$

# Tutorial Questions

2. $I = \frac{P}{V} = \frac{90}{120} = 0.75A$
   $R = \frac{V}{I} = \frac{120}{0.75} = 160\Omega$

3. $P = IV = \frac{V^2}{R}$
   $R = \frac{V^2}{P} = \frac{(120)^2}{1000}$
   $W = Pt = (1000W)(120s) = 120kJ$

## Potential Divider & Potentiometer

1. VM1: $V_1 = \frac{4000}{8000+4000} V_{source} = 4V$
   VM2: $V_1 = \frac{8000}{8000+4000} V_{source} = 8V$

2. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$
   $\frac{1.5}{\varepsilon_2} = \frac{25}{45}$ $\varepsilon_2 = 2.7V$

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## Extras

1.

$$
\begin{aligned}
R &= \frac{V}{I} = \frac{\rho l}{A} \\
\rho &= \frac{VA}{Il} = \frac{V(\frac{d}{2})^2}{Il} \\
\rho &= \frac{9.11 \times (\frac{2\times10^{-3}}{2})^2}{36 \times 50m} \\
&\approx 1.59 \times 10^{-8}\Omega m \Rightarrow \text{Silver}
\end{aligned}
$$

2.

$$
\begin{aligned}
R_{Al} &= R_{Cu} \\
l_{Al} &= l_{Cu} \\
\rho_{Al} &= \frac{R_{Al}A_{Al}}{l_{Al}} \\
\rho_{Cu} &= \frac{R_{Cu}A_{Cu}}{l_{Cu}} \\
\frac{\rho_{Al}}{\rho_{Cu}} &= \frac{\frac{\cancel{R_{Al}}A_{Al}}{\cancel{l_{Al}}}}{\frac{\cancel{R_{Cu}}A_{Cu}}{\cancel{l_{Cu}}}} \\
\frac{\rho_{Al}}{\rho_{Cu}} &= \frac{A_{Al}}{A_{Cu}} = \frac{2.82}{1.7} \approx 1.66 \\
A_{Al} &= 1.66 \times A_{Cu} \\
\cancel{\pi} r_{Al}^2 &= 1.66 \cancel{\pi} r_{Cu}^2 \\
r_{Al}^2 &= 1.66\, r_{Cu}^2
\end{aligned}
$$

3.

Junction Rule:

$$I_3 = I_1 + I_2$$

Loop Rule:

$$
\begin{aligned}
10V &= 6I_1 + 2I_3 - - - (1) \\
14V &= -4I_2 - 2I_3 \\
14V &= -4(I_3 - I_1) - 2I_3 \\
14V &= 4I_1 - 6I_3 \\
I_i &= \{2, -3, -1\}A
\end{aligned}
$$

4.

$$\tau = RC = (\frac{V}{I})C = \frac{Q}{I} = t$$

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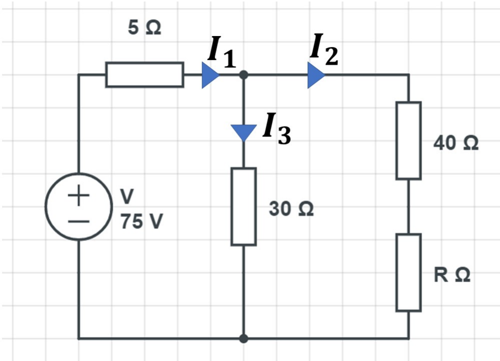
5.

$$V = IR$$
$$I = \frac{V}{R_{eff}} = \frac{12}{(\frac{1}{150} + \frac{1}{50} + \frac{1}{25})^{-1}} = 8A$$
$$P_{heater} = IV = I^2R = 8^2(25) = 1.6kW$$

6. 


Junction Rule:

$$I_1 = I_2 + I_3$$

Loop Rule - 1 :

$$75 = 5I_1 + 30I_3$$
$$75 = 5I_1 + 30I_1 - 30I_2$$
$$75 = 35I_1 - 30I_2$$

Loop Rule - 2 :

$$75 = 5I_1 + (40 + R)I_2$$
$$I_2 = \frac{450}{310 + 7R}$$
$$P = IV = I^2R$$
$$= (\frac{450\sqrt{R}}{310 + 7R})^2$$
$$\Rightarrow R = 20$$

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# 3 Chapter 5: Electromagnetic Induction

## Magnetic Flux & Induced emf Solution

1. $\phi_{magnetic} = \vec{B} \cdot \vec{A}$ where $\vec{A}$ is the vector perpendicular to the surface area and $\vec{B}$ is the magnetic field strength

2. Magnetic field strength is the amount of magnetizing force, the magnetic flux is a measurement of flux ("magnetic field lines") flowing through a surface.

3. A quarter loop of area $15cm^2$ is placed in a region of magnetic field $0.16T$. Calculate the magnetic flux if the angle between the quarter loop area and the magnetic field is

   (a) $90^o \Rightarrow \phi_B = (0.16)(0.15(10^{-2}))\, cos\, 0^o = 0.00024\, Wb$

   (b) $70^o \Rightarrow \phi_B = (0.16)(0.15(10^{-2}))\, cos\, 20^o = 0.00023\, Wb$

   (c) $30^o \Rightarrow \phi_B = (0.16)(0.15(10^{-2}))\, cos\, 60^o = 0.00012\, Wb$

4. A loop of wire of area $40cm^2$ is placed in a magnetic field of strength $2cT$. Calculate the angle between the magnetic field and the wire loop if the magnetic flux is

   (a) $80\, \mu Wb \Rightarrow \theta = 90^o - cos^{-1}(\frac{\phi}{BA}) = 90^o - cos^{-1}(\frac{80(10^{-6})}{(2(10^{-2}))(0.40(10^{-2}))}) = 90^o$

   (b) $69\, \mu Wb \Rightarrow \theta = 90^o - cos^{-1}(\frac{\phi}{BA}) = 90^o - cos^{-1}(\frac{69(10^{-6})}{(2(10^{-2}))(0.40(10^{-2}))}) = 60^o$

   (c) $56\, \mu Wb \Rightarrow \theta = 90^o - cos^{-1}(\frac{\phi}{BA}) = 90^o - cos^{-1}(\frac{56(10^{-6})}{(2(10^{-2}))(0.40(10^{-2}))}) = 44.427^o$

5. $N = \frac{\phi_B}{BAcos\theta} = \frac{0.9Wb}{60(10^{-4})(0.05)\, cos(90)} = 3000\,\text{turns}$

6. $$\begin{aligned} \Phi_{old} &= N_{old} B_{old} A_{old}\, cos\, \theta_{old} \\ \Phi_{new} &= N_{new} B_{new} A_{new}\, cos\, \theta_{new} \\ \Phi_{old} &= \Phi_{new};\ B_{old} = B_{old};\ A_{old} = A_{old} \\ N_{new} &= 2N_{old} \\ \theta_{new} &= cos^{-1}\left(\frac{1}{2}\, cos\, (\theta_{old})\right) \end{aligned}$$

7. $$\begin{aligned} \phi &= BA \\ &= (\mu_o n I)(\pi r^2) \\ &= (4\pi \times 10^{-7})(10000)(2)\pi(80 \times 10^{-3})^2 \\ &= 51.2\pi^2\, \mu Wb \end{aligned}$$

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8.

$$\begin{aligned}\varepsilon &= N\frac{d\phi}{dt}\\ &= NA\,\frac{dB}{dt}\\ &= (18)(\pi)(50\times10^{-3})^2\,\frac{20-2}{3}\\ &= 0.27\pi V\end{aligned}$$

9.

$$\begin{aligned}\varepsilon &= N\frac{d\phi}{dt}\\ &= (5000\times3(10^{-2}))\frac{10^{-3}}{240(10^{-3})}\\ &= 0.625V\end{aligned}$$

10.

$$\begin{aligned}\varepsilon &= BLv\\ &= (2)(20\times10^{-2})(20\times10^{-2})\\ &= 0.08V\\ I &= \frac{\varepsilon}{R} = \frac{0.08}{5} = 0.016\\ F &= IlB\,sin\,90^o\\ &= (0.016)(0.02)(2) = 0.64\,mN\end{aligned}$$

11. $I_{max} = \frac{\varepsilon_{max}}{R} = \frac{2\pi fNBA}{R} = \frac{(2\pi)(30)(30)(0.5)(0.1)}{100} = 0.9\pi\, A$

## Self-Inductance

1. Self inductance is when the induced current creates another magnetic field which opposes the initial induced emf.
2. $L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$ $L = -\frac{6}{(30)} = -5H$
3. $\varepsilon = -L\frac{dI}{dt} = -(230\times10^{-3})\frac{(28-20)(10^{-3})}{(140(10^{-3}))} = 0.01314286V$
4. $L = \frac{\mu_o N^2 A}{l} = \frac{(4\pi\times10^{-7})(50)^2(0.3\times10^{-4})}{(5\times10^{-2})} = 1.885\mu H$
5. $N^2 = \frac{Ll_{solenoid}}{\mu_o A} = \frac{(1.5(10^{-3}))(0.1)}{(4\pi\times10^{-7})(2.2\times10^{-3})}$
   $N \approx 233$ turns
   $l_{wire} = N(2\pi r) = N(2\pi\sqrt{\frac{A}{\pi}})$
   $l_{wire} = (233)(2\pi\sqrt{\frac{2.2\times10^{-3}}{\pi}}) = 38m$

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6. $L = -\frac{\varepsilon}{\frac{dI}{dt}} = -\frac{0.012}{\frac{2.5}{0.35}} = 1.68mH$
$L_{solenoid} = \frac{\mu_o N^2 A}{l}$
$A = \frac{L_{solenoid} l}{\mu_o N^2}$
$A = \frac{(1.68\times10^{-3})(0.08)}{(4\pi\times10^{-7})(500)^2} = 4.28 \times 10^{-4} m^2$

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## Inductor-stored Inductor & Mutual Inductance

1.

$$\begin{aligned} M &= -\frac{\varepsilon_Y}{\frac{dI_x}{dt}} \\ &= 0.5H \\ \varepsilon_X &= M\frac{dI_Y}{dt} \\ &= 0.75V \end{aligned}$$

2.

$$\begin{aligned} U &= \frac{1}{2}LI^2 \\ L &= \frac{\mu_o N^2 A}{l} \\ B &= \mu_o nI\frac{\mu_o NI}{l} \\ U &= \frac{1}{2}\frac{\mu_o N^2 A}{l}\frac{B^2}{2\mu_o}AL \\ &= 1.8 \times 10^{-10}J \end{aligned}$$

3.

$$M = \frac{N_2\phi_2}{I} = 0.55H$$

4. $U = 36J$
5. $U = 1.68mJ$
6. (a) $0.36H, 0.41J$
   (b) $0.096H$
   (c) $0.48V$
7. $M = \frac{N_2\phi_2}{I_1} = 0.55H$
8. A current of $3.0A$ flows in coil C and is produced a magnetic flux of 0.75 Wb in it. When a coil D is moved near to coil C coaxially, a flux of $0.25Wb$ is produced in coil D. If coil C has 1000 turns and coil D has 5000 turns.

   (a)

$$\begin{aligned} L_C &= \frac{N_C\Phi_C}{I_C} \\ &= \frac{1000 \times 0.75}{3} = 250H \\ U &= \frac{1}{2}L_C I_C^2 \\ &= \frac{1}{2}(250)(3)^2 = 1125J \end{aligned}$$

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(b)

$$
\begin{aligned}
M_D &= \frac{N_D \Phi_D}{I_C} \\
&= \frac{(5000)(0.25)}{(3)} \\
&= 417H
\end{aligned}
$$

(c)

$$
\begin{aligned}
\varepsilon_D &= -M \frac{dI_C}{dt} \\
&= -(417) \frac{-3}{0.25} \\
&= 5004V
\end{aligned}
$$

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# 4 Chapter 6: AC circuits

Answer:

## 6.1 AC & 6.2 RMS

1. (a) peak voltage, $V_{peak} = 3V$
   peak-to-peak voltage, $V_{peak-to-peak} = 6V$
   rms voltage, $V_{rms} \approx 2.12132V$

   (b) Peak current, $I_o = \frac{V_o}{R} = \frac{3}{2.5} = 1.2A$
   rms current, $I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{1.2}{\sqrt{2}} = 0.84853A$
   average power $P_{ave} = \frac{V_{rms}^2}{R} = \frac{(2.12132)^2}{2.5} = 1.8W$

2. For the following situation, sketch the waveform and write down the equation for

   (a) $V(t) = V_{peak}\, sin(2\pi ft) = 2sin(2\pi(50)t) = 2sin(100\pi t)$

   (b) $V(t) = \frac{V_{peak-to-peak}}{2}\, sin(2\pi(\frac{1}{T})t) = \frac{2}{2}\, sin(2\pi(\frac{1}{20})t) = sin(0.1\pi t)$

3. $V(t) = (V_{rms} \times \sqrt{2})\, sin(2\pi ft) = (120 \times \sqrt{2})\, sin(2\pi(60)t) = 170\, sin(120\pi t)$

4. $I_{rms} = \frac{V_{rms}}{R} = \frac{V_o}{\sqrt{2}}\frac{1}{R} = \frac{60}{\sqrt{2}}\frac{1}{20} = 2.12A$
   $P_{ave} = I_{rms}^2 R = (2.12)^2(20) = 90W$

## 6.3: Resistance, Reactance & Impedance
## 6.4: Power Factor

$X_C = \frac{1}{2\pi fC}$; $X_L = 2\pi fL$; $Z = \sqrt{R^2 + (X_L - X_C)^2}$; $\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$; $P_{ave} = I_{rms}V_{rms}cos\,\phi$; $P = IV$; $cos\phi = \frac{P_{real}}{P_{apparent}} = \frac{P_{real}}{I^2Z} = \frac{R}{Z} = \frac{P_{ave}}{IV}$;

1. $R-$ **circuit:**
   $I_{rms} = \frac{P_{ave}}{V_{rms}} = \frac{1200W}{120V} = 10A$
   $I_{max} = \sqrt{2}(10A) = 14.1A$
   $R = \frac{V_{rms}}{I_{rms}} = \frac{120}{10} = 12\Omega$

2. $L-$ **circuit:**
   $X_L = 2\pi\omega L = 2\pi \times 100Hz \times 0.4H = 251\Omega$
   $X_L \gg R$
   $Z \approx X_L$
   $Irms \approx \frac{V}{X_L} = \frac{80}{251} = 0.32A$ $I_{max} = \sqrt{2}I_{rms} = 0.453A$

3. $C-$ circuit:
   $I_{rms} = \frac{V_{rms}}{X_C}$
   $= V_{rms}\omega C = 2\pi fCV_{rms}$
   $= 2\pi(50)(50\mu)(\frac{100}{\sqrt{2}}) = 1.1A$
   $I_{max} = \sqrt{2}I_{rms} = 1.556A$

4. $RL-$ circuit:

   (a) $X_L = 2\pi fL = 2\pi(60Hz)(0.1) = 37.7\Omega$
   (b) $Z = \sqrt{R^2 + X_L^2} = \sqrt{(12)^2 + (37.7)^2} = 39.6\Omega$
   (c) $I_{rms} = \frac{V_{rms}}{Z} = \frac{V}{\sqrt{(R_{coil}+R_{resistor})^2+(X_L)^2}}$
   $= \frac{V}{\sqrt{(R_{coil}+R_{resistor})^2+(X_L)^2}} = \frac{110}{\sqrt{(12+10)^2+(37.7)^2}} = 2.52A$
   (d) $cos\,\phi = \frac{R_{coil}+R_{resistor}}{Z} = \frac{R_{coil}+R_{resistor}}{\sqrt{(R_{coil}+R_{resistor})^2+(X_L)^2}}$
   $cos\,\phi = 0.51$
   $\phi \approx 60^o$
   (e) $\phi \approx 60^o$, the voltage leades the current
   (f) $P_{ave} = IV\,cos\,\phi == (2.52)(110)(0.51) = 141.372W$

5. $RC-$ circuit:

   (a) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi(60)(10\mu)} = 265.3\Omega$
   (b) $Z = \sqrt{R^2 + X_C^2} = \sqrt{(40)^2 + (265.3)^2} \approx 263.3\Omega$
   (c) $I = \frac{V_{rms}}{Z} = 0.410$
   (d) $cos\,\phi = \frac{R}{Z} = \frac{40}{268.3} = 0.15$
   (e) $\phi = 81.4^o$

6. $RLC-$ circuit - Potential Difference:
$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2+(X_L-X_C)^2}}$
$I_{rms} = \frac{110}{\sqrt{(11)^2+(120\Omega-100\Omega)^2}} = 4.8192A$
$V_R = IR = 4.8192A \times 11 = 53.0112V$
$V_L = IX_L = 4.8192A \times 120 = 578.304V$
$V_R = IR = 4.8192A \times 100 = 481.92V$
$V_{total} = \sqrt{V^2 + (V_L - V_C)^2} = \sqrt{(52.91)^2 + (577.2 - 481)^2} \approx 110V$

7. $RLC-$ circuit - Resonance Frequency:
At $f_{resonance}$, $f = \frac{1}{2\pi\sqrt{LC}}$
$$L = \frac{1}{4\pi^2(60)^C} - - - 1$$
Since $X_L = X_C$, then $Z = R = 8\Omega$.
At $f = 80Hz$,
$$Z^2 = 8^2 + (X_L - X_C)^2 = 10^2$$
$$\Downarrow$$
$$X_L - X_C = 2\pi\left(80L - \frac{1}{80C}\right) = 6\Omega - - - [2]$$
Solve by simultaneous equations [1] and [2] give
$$L = 0.0261H;\ C = 0.00027F.$$

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# 5 Chapter 7: Geometrical Optics

Answer:

## 7.1 Reflection at a spherical surface

$$R = 2f$$
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$

Sign convention: $+f$ for concave mirror, $-f$ for convex mirror.

1. $$f = 0.5R = 0.5(0.2m) = 0.1m$$

2. $$f = \frac{uv}{v+u} = +\frac{8}{6}cm$$

3. $$\frac{R}{2} = f = \frac{uv}{v+u}$$
$$R = \frac{2uv}{v+u} = \frac{2(0.3)(9)}{0.3+9} = 0.58$$
$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$
$$h_i = -h_o\frac{v}{u} = -(0.05)\frac{9}{0.3} = -1.5m$$

4. $$v = \frac{uf}{u-f} = \frac{0.5 \times 0.2}{0.5 - 0.2} = \frac{1}{3}m$$
$$h_i = -h_o\frac{v}{u} = -(0.1)frac(\frac{1}{3})0.5 = -\frac{1}{15}m$$
Image is real, $\frac{1}{15}m$ high and inverted.

5. $$\frac{2}{R} = 2(\frac{1}{u} + \frac{1}{v}) = 45$$
$$\frac{v}{u} = \frac{1}{9} \Rightarrow u = 9v$$
$$\frac{2}{45} = \frac{1}{u} + \frac{9}{u} \Rightarrow u = 225cm$$

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## 7.2 Refraction at a spherical surface

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

**Sign convention:**

| *Quantity* | $+$ | $-$ |
|---|---|---|
| $u$ | Real | Virtual |
| $v$ | Real | Virtual |
| $r$ | Convex surface | Concave surface |

1.

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\frac{1.00}{10} + \frac{1.52}{v} = \frac{1.52 - 1.00}{+3} \Rightarrow v = +20.7cm$$

2.

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\frac{1.33}{25 - 4} + \frac{1}{v} = \frac{1 - 1.33}{-12.5} \Rightarrow v = -27.07cm$$

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## 7.3 Thin lenses

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \left(\frac{n_{material}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$m = \frac{h_i}{h_o} = -\frac{v}{u}$$

**Sign convention:**

| Thin lenses | | |
|---|---|---|
| *Quantity* | $+$ | $-$ |
| $u$ | real object | virtual object |
| $v$ | real Image | virtual image |
| $f$ | Convex/converging lens | Concave/diverging lens |
| lensmaker's equation | | |
| *Quantity* | $+$ | $-$ |
| $R_1$ | convex | concave |
| $R_2$ | concave | convex |

**Thin lens equation**

1. An object is placed $x - cm$ from a thin lens with a $10cm-$focal length. Find the position of the image if

   (a) $x = 30cm$ and the lens is converging

   $$\frac{1}{10} = \frac{1}{30} + \frac{1}{v} \Rightarrow v = 15cm$$

   (b) $x = 30cm$ and the lens is diverging

   $$\frac{1}{-10} = \frac{1}{30} + \frac{1}{v} \Rightarrow v = -7.5cm$$

   (c) $x = 5cm$ and the lens is converging

   $$\frac{1}{10} = \frac{1}{5} + \frac{1}{v} \Rightarrow v = -10cm$$

   (d) $x = 5cm$ and the lens is diverging

   $$\frac{1}{-10} = \frac{1}{5} + \frac{1}{v} \Rightarrow v = -3.33cm$$

2. A $6cm$-high object is placed $30cm$ from a converging lens and its image forms $90cm$ from the lens and on the same side as the object. What is the focal length of the lens?

   $$\frac{1}{f} = \frac{1}{30} + \frac{1}{-90} \Rightarrow f = 45cm$$

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3. Calculate the position and focal length of a conveging lens which will project the image of an object with a magnification of 4, upon a screen $10m$ from the object.

$$u + v = 10,\ \frac{q}{p} = 4 \Rightarrow p = 2\,\&\,q = 8$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{2} + \frac{1}{8} \Rightarrow f = +1.6m$$

**Lensmakers' equation**

1. A lens (made out of glass of refractive index 1.54) has a convex surface of radius $20cm$ and a concave surface of radius $40cm$. Calculate the focal length and deduce if the lens is converging or diverging.

$$\text{Case 1 - convexo-concave: } \frac{1}{f} = \left(\frac{1.51}{1} - 1\right)\left(\frac{1}{20} - \frac{1}{40}\right) \Rightarrow f = 74.1cm \Rightarrow \text{converging}$$

$$\text{Case 2 - concave-convex: } \frac{1}{f} = \left(\frac{1.51}{1} - 1\right)\left(\frac{1}{-20} - \frac{1}{-40}\right) \Rightarrow f = 74.1cm \Rightarrow \text{converging}$$

2. A parallel beam of white light strikes a biconvex lens having faces of radii $32cm$ and $48cm$. The refractive indices of the glass for the $A$(red) and $H$ (violet) spectral lines are 1.578 and 1.614 respectively. Calculate the distance between the focal points of red and violet radiations.

$$\frac{1}{f} = \left(\frac{n}{1} - 1\right)\left(\frac{1}{32} - \frac{1}{-48}\right) \Rightarrow f = \frac{19.2}{n-1} \Rightarrow \Delta f = 1.95cm$$

3. A double convex glass has faces of radii $18cm$ and $20cm$. Calculate the
   (a) focal length of the lens when an image is formed $32cm$ from the lens if the object is placed $24cm$ away from the lens.
   $$\frac{1}{f} = \frac{1}{24} + \frac{1}{32} \Rightarrow f = +13.7cm$$
   (b) refractive index of the lens.
   $$\frac{1}{13.7} = \left(\frac{n}{1} - 1\right)\left(\frac{1}{18} - \frac{1}{-20}\right) \Rightarrow n = 1.69$$

4. A symmetric lens with a focal length of $5cm$ is made of a material of refraction index 1.5. Calculate the refractive index of each surface of the lens?

$$R_1 = r = -R_2 \Rightarrow \frac{1}{f} = \left(\frac{n_{material}}{1} - 1\right)\left(\frac{1}{r} - \frac{1}{-r}\right) \Rightarrow r = 2f \times (n-1) = 5cm$$

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# 6 Chapter 8: Physical Optics

## Answers:

### 8.1 Huygens' Principle & 8.2 Wave Interferences

1. State Huygen's Principle and its application on light wave analysis.
**Ans:** According to Huygens' principle, every point on a wave front behaves like a light source and emits secondary wavelets, the secondary wavelets spread in all directions in space (vacuum) with the velocity of light and the envelope of wave front of secondary wavelets, after a given time, along forward direction gives the new position of wave front. It can be applied to both far field limit (as in the case of Fraunhofer) as well as the near field **diffraction**.

2. Show (either diagrammatically or in forms of equation) that the laws of reflection and refraction can be verified using Huygen's wavew theory.
**Ans:**

First the wavefront touches the reflecting surface at $B$ and then at the successive points towards $C$. In accordance with Huygens' principle, from each point on $BC$, secondary wavelets start growing with the speed $c$. During the time the disturbance from $A$ reaches the point $C$ the secondary wavelets from $B$ must have spread over a hemisphere of radius $BD = AC = ct$, where $t$ is the time taken by the disturbance to travel from $A$ to $C$. The tangent plane $CD$ drawn from the point $C$ over this hemisphere of radius $ct$ will be the new reflected wavefront.

Let angles of incidence and reflection be $i$ and $r$, respectively . In $\Delta ABC$ and $\Delta DCB$, we have

$\angle BAC = \angle CDB$ [each is 90°]

$BC = BC$ [common]

$AC = BD$ [each is equal to $ct$]

$\therefore \Delta ABC \cong \Delta DCB$

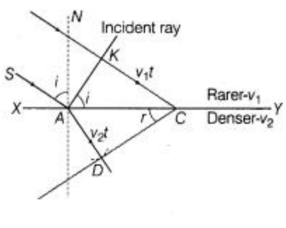
Hence $\angle ABC = \angle DCB$

or $i = r$

i.e. the angle of incidence is equal to the angle of reflection. The proves the first law of reflection.

Further, since the incident ray $SB$, the normal $BN$ and the reflected ray $BD$ are respectively perpendicular to the incident wavefront $AB$, the reflecting surface $XY$ and the reflected wavefront $CD$ (all of which are perpendicular to the plane of the paper) therefore, they all lie in the plane of the paper i.e. in the same plane. This proves the second law of reflection.

**13. Law of refraction on this basis of Huygens' wave theory** Consider a plane wavefront $AB$ incident on a plane surface $XY$, separating two media 1 and 2, as shown in Figure. Let $v_1$ and $v_2$ be the velocities of light in two media, with $v_2 < v_2$.




The wavefront first strikes at point $A$ and then at the successive points towards $C$. According to Huygens' principle, from each point on $AC$, the secondary wavelets starts growing in the second medium with speed $v_2$. Let the disturbance take time $t$ to travel from $B$ to C, then $BC = v_1t$. During the time the disturbance from $B$ reaches the point $C$, the secondary wavelets from point $A$ must have spread over a hemisphere of radius $AD = v_2t$ in the second medium. The tangent plane $CD$ drawn from point $C$ over this hemisphere of radius $v_2t$ will be the new refracted wavefront.

Let the angles of incidence and refraction be $i$ and $r$, respectively.

From right $\Delta ABC$, we have

$$\sin\angle BAC = \sin i = \frac{BC}{AC}$$

From right $\Delta ADC$, we have

$$\sin\angle DCA = \sin r = \frac{AD}{AC}$$

3. What type of wave front will emerge from a point source and a distance light source?
**Ans:** When source of light is a point source, the wavefront is spherical. At very large distances from the source, a portio of spherical or cylindrical waver at appears to be plane.

4. Explain the application of Huygen's Principle in the case of a single slit.

5. When are two light sources of the same common frequency said to be coherent ?
**Ans:** 2 light wave sources are said to be coherent when there exist a fixed difference between the phases of the light waves emitter by them.

6. What are the conditions for the interference of light?
**Ans:** Coherence and monochromatism.

7. From a path difference perspective, state the condition for constructive and destructive interference.
**Ans:** Constructive interference occurs when

$$\Delta\phi = n\lambda$$

.
Destructive interference occurs when

$$\Delta\phi = (n + \frac{1}{2})\lambda$$

.

## 8.3 Young's Double Slit Experiment

1. The interference pattern of two identical slits separated by a distance $d = 0.25mm$ is observed on a screen at a distance of $0.5m$ from the plane of the slits. The illuminating light source has a wavelength of $589nm$. Calculate the distance of the first bright fringe to the central maximum, and the separation of the bright bands.

$$y_1 = \frac{nD\lambda}{d} = \frac{(1)(589 \times 10^{-9})(0.5)}{0.25 \times 10^{-3}} = 1.178mm$$

2. Light (of $589nm$) from a lamp forms an interference pattern on a screen $0.75m$ from a pair of slits. The bright fringes in the pattern are $0.4cm$ apart. Calculate the slit separation.

$$\Delta y = \frac{D\lambda}{d} \Rightarrow d = \frac{D\lambda}{\Delta y} = \frac{(0.75)(589 \times 10^{-9})}{0.4 \times 10^{-2}} = 0.1104375mm$$

3. A laser light of $630nm$ incident on a pair of slits produces an interference pattern in which the bright fringes are separated by $8.3mm$. A second light, on the other hand, produces an interference pattern of which its dark fringes are separated by $7.6mm$. Calculate the wavelength of the second light.

$$\Delta y = \frac{\lambda D}{d} = \text{constant}$$

$$\lambda' = \frac{\Delta y'}{\Delta y}\lambda = \frac{7.6 \times 10^{-3}}{8.3 \times 10^{-3}}(630 \times 10^{-9}) = 577nm$$

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4. Light from two light sources (of wavelengths $\lambda_1$ and $\lambda_2$ respectively) arrives at a double slit set up. If $\lambda_1 = 430nm$, what value must $\lambda_2$ be if the fourth order bright fringe of light with $\lambda_1 = 645nm$ overlaps with the sixth order bright fringe of $\lambda_2$ light?

$$n\lambda_i = \frac{yd}{D} = \text{constant}$$

$$4\lambda_1 = 6\lambda_2 \Rightarrow \lambda_2 = 430nm$$

5. In a double-slit experiment with light source $700nm$, a student is limited to a $35cm$ white-coloured cardboard sheet as a screen. Assuming intensity is not a limiting factor, calculate the highest order of bright fringe the student will be able to observe if the slit separation is $0.02mm$ and the screen must be kept at a distance of $1m$ from the slit.

$$y_n = \frac{nD\lambda}{d} \Rightarrow n = \frac{y_n d}{D\lambda} = \frac{(0.35)(0.02 \times 10^{-3})}{(1)(700 \times 10^{-9})} = 10$$

## 8.4 Thin Films

| Media | phase change | Constructive Interference |
|---|---|---|
| $n_{1^{st}} > n_{2^{nd}} > n_{3^{rd}}$ or $n_{1^{st}} < n_{2^{nd}} < n_{3^{rd}}$ | $0\,rad$ | $2nt = m\lambda$ |
| $n_{1^{st}} > n_{2^{nd}} < n_{3^{rd}}$ or $n_{1^{st}} < n_{2^{nd}} > n_{3^{rd}}$ | $\pi\,rad$ | $2nt = (m + 0.5)\lambda$ |

1. A soap film has a refractive index of 1.33. How thick is the film if one-half of a wavelength of red light (with a vacuum wavelength of $633nm$) extends from on surface to the other surface?

$$n = \frac{\lambda_{vacuum}}{\lambda_{soap}} \Rightarrow \lambda_{soap} = \frac{\lambda_{vacuum}}{n} = \frac{633nm}{1.33} = 476nm$$

$$t = 1.5\lambda_{soap} = 1.5(476nm) = 714nm$$

2. A thin film of alcohol ($n_{alcohol} = 1.36$) spread on a flat glass plate and is illuminated with white light. If a region of the film reflects only green light ($\lambda_{green} = 500nm$) strongly, how thick is the film?

$$2nt = m\lambda \Rightarrow t = \frac{\lambda_{green}}{2(n_{alcohol})} = \frac{500nm}{2(1.36)} = 184nm$$

3. A glass lens with an index of 1.55 is to be coated with a film of index 1.30 to decrease the reflection normally incident green light of $\lambda = 500nm$. What minimum thickness should be deposited on the lens?

$$2nt = (m + 0.5)\lambda \Rightarrow t = \frac{(0.5)\lambda_{green}}{2(n_{alcohol})} = \frac{(0.5)(500nm)}{2(1.3)} = 96.2nm$$

4. A film of oil ($n_{oil} = 1.42$) of $250nm$ thick floats on water. When illuminated from above with white light, what colour will reflect the most brightly?

$$2nt = (m + 0.5)\lambda \Rightarrow \lambda = \frac{2nt}{m + 0.5} = \frac{2(1.42)(250nm)}{1 + 0.5} = 473nm \Rightarrow \text{Blue}$$

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5. A soap film of index 1.35 appears yellow ($580nm$) when viewed directly from above. Compute several possible values of its thickness.

$$2nt = (m + 0.5)\lambda \Rightarrow t = \frac{(m + 0.5)\lambda}{2n} = \frac{(m + 0.5)\lambda}{2(1.35)}$$

$$\text{When m} = 0, 1, 2,\ t = \left\{ \frac{(0 + 0.5)\lambda}{2(1.35)}, \frac{(1 + 0.5)\lambda}{2(1.35)}, \frac{(2 + 0.5)\lambda}{2(1.35)} \right\} = \{107nm,\ 322nm,\ 537nm\}$$

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## 8.5 Single Slit Diffraction (Fraunhofer Diffraction)

$$y_n = \frac{n\lambda D}{a} \text{ for dark fringes}$$
$$y_n = \frac{(n+0.5)\lambda D}{a} \text{ for bright fringes}$$

1. A monochromatic light with a wavelength of $\lambda = 600nm$ passes through a single slit which has a width of $0.6mm$. Calculate the distance between the slit and the screen if the first minimum in the diffraction pattern is at a distance $0.90mm$ from the center of the screen.

$$y_n = \frac{n\lambda D}{a} \Rightarrow D = \frac{ay_n}{n\lambda} = \frac{(0.6mm)(0.90mm)}{(1)(600nm)} = 0.9m$$

2. If the separation between the first and the second minima of a single-slit diffraction pattern is $6.0mm$, what is the distance between the screen and the slit? The light wavelength is $500nm$ and the slit width is $0.16mm$.

$$\Delta y_n = \frac{\lambda D}{a} \Rightarrow D = \frac{a\delta y_n}{\lambda} = \frac{a\delta y_n}{\lambda} = 1.92m$$

3. When a single slit of width $0.3mm$ is illuminated with light of wavelength is $633nm$, the distance from the central maximum of the $1^{st}$ order minimum is $4cm$. Calculate this distance if the slit is doubled.

$$y_n = \frac{n\lambda D}{a} \Rightarrow ay_n = n\lambda D = \text{constant}$$
$$a_i y_i = a_f y_f = 2a_i y_f \Rightarrow y_f = \frac{y_i}{2} = \frac{4cm}{2} = 2cm$$

4. A single slit $0.1mm$ wide is illuminated by plane waves from a HeNe laser ($\lambda = 633nm$). If the screen is $10m$, determine the width of the central maximum.

$$\text{Width} = 2y_1 = \frac{2\lambda D}{a} == \frac{2(633\times10^{-9})(10m)}{0.1\times10^{-3}} = 0.1266m$$

5. A vertical single slit is illuminated with electromagnetic wave from a HeNe laser at $633nm$. It is found that the center of the second dark band lies at an angle of $4.2^o$ off the central axis. Determine the width of the slit.

$$y_n = \frac{n\lambda D}{a} \Rightarrow tan\,\theta \approx \theta = \frac{y_n}{D} = \frac{n\lambda}{a}$$
$$\theta = \frac{n\lambda}{a} \Rightarrow a = \frac{n\lambda}{\theta} = \frac{2\times 633nm}{4.2} = 0.3\mu m$$

## 8.6 Diffraction Grating

$$d\,sin\,\theta = n\lambda;\ d = \frac{1}{N}$$

1. A screen is placed $1.4m$ away from a diffraction grating illuminated with light of wavelength $633nm$. If the second- and third-order spectra are to be separated by $1.5cm$ on the screen, how many lines per centimetre are needed for the grating?

$$d\,sin\,\theta = n\lambda \Rightarrow \Delta y = y_3 - y_2 = \frac{L\lambda}{d}(m_3 - m_2)$$

$$N = \frac{1}{d} = \frac{\Delta y}{L\lambda(m_3 - m_2)} = \frac{1.5cm}{(1.4)(633nm)(3-2)} = 16926 \text{ lines } m^{-1} \approx 170 \text{ lines } cm^{-1}$$

2. When a grating is illuminated with light of $\lambda = 540nm$, only 3 lines on either side of the central maximum can be seen. Calculate the maximum number of lines for the grating.

$$\text{Maximum order } \Rightarrow sin\,90^o$$

$$d = m\lambda = \frac{1}{N}$$

$$N = \frac{1}{m\lambda} = \frac{1}{3(540nm)} \approx 6.17284 \times 10^5 \text{ lines } m^{-1}$$

3. A gas discharge tube emits electromagnetic radiation of wavelengths $660nm$ and $430nm$ that illuminates a diffraction grating of 4000 lines $cm^{-1}$. Calculate the angular separation between the first order maximas of both wavelengths.

$$d\,sin\,\theta_n = n\lambda \Rightarrow \theta_n = sin^{-1}\left(\frac{n\lambda}{d}\right) = sin^{-1}nN\lambda$$

$$\theta|_{\lambda=660nm,\,n=1} = sin^{-1}(1 \times 400000m^{-1} \times 660nm) = 15.3^o$$

$$\theta|_{\lambda=430nm,\,n=1} = sin^{-1}(1 \times 400000m^{-1} \times 430nm) = 9.9^o$$

$$\Delta\theta = 15.3^o - 9.9^o = 5.4^o$$

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# 7 Chapter 9: Light Quantization

## Answers:

### 9.1 Planck's Quantum Theory

1. Describe the *ultraviolet catastrophe*?
2. How did Planck resolve the *ultraviolet catastrophe* problem?
3. How did Planck's solution to the *ultraviolet catastrophe* differ from classical theory of energy, i.e. *Rayleigh-Jeans Law*?
4. What does it mean by "photon energy is quantized"?
5. Find the energy (in $eV$ and in $Joules$) of the photons of frequency is
   (a) $668THz$
   $$E = hf = (6.63 \times 10^{-34})(668 \times 10^{12}) = 4.43 \times 10^{-19} J = 2.76 eV$$
   (b) $400THz$
   $$E = hf = (6.63 \times 10^{-34})(400 \times 10^{12}) = 2.65 \times 10^{-19} J = 1.656 eV$$
   (c) $526THz$
   $$E = hf = (6.63 \times 10^{-34})(526 \times 10^{12}) = 3.49 \times 10^{-19} J = 2.17 eV$$
6. Find the energy (in $eV$ and in $Joules$) of the photons of wavelength is
   (a) $430nm$
   $$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{430nm} = 4.62 \times 10^{-19} J \approx 2.9\, eV$$
   (b) $500nm$
   $$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{500nm} = 3.97 \times 10^{-19} J \approx 2.5\, eV$$
   (c) $565nm$
   $$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{565nm} = 3.52 \times 10^{-19} J \approx 2.2\, eV$$
   (d) $625nm$
   $$E = E = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{625nm} = 3.18 \times 10^{-19} J \approx 1.98\, eV$$
7. We get sunburnt if we are under the sun. This process requires a photon energy of $5.61864 * 10^{-19}$J. Calculate the wavelength that corresponds to this energy.
   $$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{5.61864 \times 10^{-19}} = 354nm$$

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## 9.2 Photoelectric effect

$$\frac{1}{2}mv_{max}^2 = eV_s = hf - W = h(f - f_o)$$

1. What is the work function of a metal if the photoelectric threshold wavelength is $312nm$?

$$W = \frac{hc}{\lambda_o} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{312 \times 10^{-9}} = 6.375 \times 10^{-19}J \approx 3.9844eV$$

2. A photon of energy $5eV$ imparts all of its energy to an electron that leaves a metal surface. When UV photons of a single frequency strike a metal, electrons with kinetic energy from zero to $2.5eV$ are ejected. What is the work function of the metal?

$$W = E_{photon} - E_K = 5 - 2.5 = 2.5\,eV \approx 4.0054 \times 10^{-19}J$$

3. The work function of a metal is $2.3eV$. What is the longest wavelength light that can cause photoelectrons to be emitted?

$$W_{min} = hf_o = \frac{hc}{\lambda_o} \Rightarrow \lambda = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{2.3 \times 1.6 \times 10^{-19}} = 540nm$$

4. Light of wavelength $600nm$ falls on a metal having photoelectric work function $2eV$. Find the energy of the photon, the kinetic energy of the most energetic photoelectron and the stopping potential.

$$E = \frac{hc}{\lambda} = 2.07eV$$

$$K_{max} = 0.07eV$$

$$eV_s = K_{max} = 0.07eV \approx 1.12152 \times 10^{-20}J$$

5. In the photoionization of atomic hydrogen, what will be the maximum kinetic energy of the ejected electron when a $60nm$ photon is absorbed by the atom?

$$K_{max} = \frac{hc}{\lambda} - E_{ionization\,H} = \frac{1240\,eV\,nm}{60nm} - 13.6eV = 7.1eV \approx 1.1375 \times 10^{-18}J$$

# 8 Chapter 10: Wave Properties of Particle

## Answers:

### 10.1 de Broglie Wavelength

$$\lambda = \frac{h}{p}$$

1. What is the idea of "wave-particle duality" of light?
2. What do we mean when we say a particle may exhibit wave-like nature?
3. If we say particles has wave light properties, how then can we characterise it's wavelike nature?
4. Calculate the de Broglie wavelength of a $0.5kg$ ball moving with a speed $20ms^{-1}$.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{(0.5)(20)} = 6.6 \times 10^{-35}m$$

5. Calculate the wavelength of an electron accelerated with a potential difference of $100V$.

$$\text{By energy conservation } \Rightarrow E_K = E_{e.p.} \Rightarrow \frac{1}{2}m_e v_e^2 = q_e V$$

$$v = \sqrt{\frac{2q_e V}{m_e}} = \sqrt{\frac{2(1.6 \times 10^{-19})(100)}{9.1 \times 10^{-31}}} = 5.9 \times 10^6 ms^{-1}$$

$$\lambda = \frac{h}{m_e v_e} = \frac{6.6 \times 10^{-34}}{(9.1 \times 10^{-31})(5.9 \times 10^6)} = 0.12nm$$

6. Find the de Broglie wavelength for a particle of mass $1.67 \times 10^{-27}kg$ travelling at $1100ms^{-1}$.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{(1.67 \times 10^{-27})(1100ms^{-1})} = 0.09nm$$

7. If the de Broglie wavelength of an electron is 0.5Å(Angstrom), calculate its kinetic energy.

$$0.5\text{Å} = 0.5 \times 10^{-10}m$$

$$\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$$

$$E_K = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e(\frac{h}{m_e \lambda})^2$$

$$E_K = \frac{1}{2}(9.1 \times 10^{-31})(\frac{6.6 \times 10^{-34}}{(9.1 \times 10^{-31})(10^{-10})})^2 = 6.0314 \times 10^{-18}J = 37.7eV$$

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## 10.2 Electron Diffraction

1. What is Bragg's Law?
2. Describe the results of the Davisson-Germer Experiment and its significance.
3. Describe the mechanism behind an electron microscope.
4. How does the de Broglie's wavelength relate to the resolving power of an electron microscope?
5. In a table, compare and contrast between electron microscope and optical microscope.

## 9 Chapter 11: Nuclear & Particle Physics

## Answers:

### 11.1 Binding Energy & mass defect

$$1amu = 1.6605 \times 10^{-27}kg = 931.49432MeV\,c^{-2}$$

$$\Delta m = Zm_p + Nm_n - m_{nucleus}$$

$$E_{binding} = \Delta mc^2$$

1. Define the following term:
   (a) mass defect
   The difference between the mass of an atomic nucleus and the sum of the individual masses of the nucleons of which it is composed.
   (b) binding energy
   The energy required to disassemble a nucleus into the free, unbound neutrons and protons it is composed of.
2. How many protons and neurons are in
   (a) $^{24}_{11}Na$

$$N_{proton} = Z = 11$$

$$N_{neutron} = A - Z = 24 - 11 = 13$$

   (b) $^{6}_{3}Li$

$$N_{proton} = Z = 3$$

$$N_{neutron} = A - Z = 6 - 3 = 3$$

   (c) $^{206}_{82}Li$

$$N_{proton} = Z = 82$$

$$N_{neutron} = A - Z = 206 - 82 = 124$$

3. Both in atomic mass unit and in kilogram, calculate the mass defect, the binding energy and the binding energy per nucleon for the following situation:
   (a) Mass of $^{7}_{3}Li$ nucleus $= 7.016amu$

$$\Delta m = 3(1.007277)u + 4(1.008665) - 7.016 = 4.05 \times 10^{-2}amu \approx 6.72518263 \times 10^{-31}kg$$

$$E = \Delta mc^2 = (4.05 \times 10^{-2}amu)(931MeV\,amu^{-1}) = 37.703\,MeV$$

$$\frac{E}{A} = \frac{37.703}{7} = 5.3861MeV$$

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(b) Mass of $^{37}_{17}Cl$ nucleus $= 6.13834818 \times 10^{-26}kg$

$$\Delta m = 17(1.007277)u + 20(1.008665) - \frac{6.13834818 \times 10^{-26}}{1.6605 \times -27} = 0.33amu \approx 5.47978 \times 10^{-28}kg$$

$$E = \Delta mc^2 = (0.33amu)(931MeV\,amu^{-1}) = 307.23\,MeV$$
$$\frac{E}{A} = \frac{307.23}{37} = 8.3035MeV$$

4. Describe the **binding energy curve** and the **iron limit**.
**The iron limit** - The binding energy curve is obtained by dividing the total nuclear binding energy by the number of nucleons. The fact that there is a peak in the binding energy curve in the region of stability near iron means that either the breakup of heavier nuclei (fission) or the combining of lighter nuclei (fusion) will yield nuclei which are more tightly bound (less mass per nucleon). The binding energies of nucleons are in the range of millions of electron volts compared to tens of eV for atomic electrons. Whereas an atomic transition might emit a photon in the range of a few electron volts, perhaps in the visible light region, nuclear transitions can emit gamma-rays with quantum energies in the MeV range.
**The iron limit** - The buildup of heavier elements in the nuclear fusion processes in stars is limited to elements below iron, since the fusion of iron would subtract energy rather than provide it. Iron-56 is abundant in stellar processes, and with a binding energy per nucleon of 8.8 MeV, it is the third most tightly bound of the nuclides. Its average binding energy per nucleon is exceeded only by 58Fe and 62Ni, the nickel isotope being the most tightly bound of the nuclides.

5. Calculate the minimum amount of energy needed to transform $^{43}_{20}Ca$ atom (of mass $42.958766u$) into $^{42}_{20}Ca$ atom (of mass $41.958618u$) through the removal of a neutron.

$$\text{In general, } E +^{43}_{20} Ca \rightarrow^{42}_{20} Ca + \text{neutron}$$

$$^{43}_{20}Ca \rightarrow^{42}_{20} Ca : \Delta m_{Ca} = 42.958766u - 41.958618u = 1.000148u(< m_{neutron})$$
$$E = \Delta mc^2 = (1.008665 - 1.000148)(931) = 7.934\,MeV$$

6. If the mass of $^{11}_{5}B$ nucleus is $11.008757u$, calculate the binding energy per nucleon.

$$\frac{E}{A} = (A)^{-1}(Zm_p + Nm_n - m_{nucleus})c^2$$

$$\frac{E}{A} = (11)^{-1} \times [5(1.007277) + 6(1.008665) - 11.008757u] \times (931MeV) = 6.7386MeV \text{ per nucleon}$$

## 11.2 Radioactivity

$$\frac{dN}{dt} = -\lambda N$$
$$N|_t = N_o e^{-\lambda t}; A|_t = A_o e^{-\lambda t}$$
$$T_{\frac{1}{2}} = \frac{ln\,2}{\lambda}$$

1. Describe the following type of radioactive decay in terms of the changes to the parent nucleus as well as the type of particles emitted:

(a) alpha
(b) beta
(c) gamma

| Radioactive decay | General process | Emission |
|---|---|---|
| Alpha, $\alpha$ | ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2} Y + {}^{4}_{2} He + E$ | ${}^{4}_{2}He$ |
| Beta plus, $\beta^+$ | ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1} Y + e^+ + \nu_{e^+} + E$ | $e^+, \nu_{e^+}$ |
| Beta minus, $\beta^-$ | ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1} Y + e^- + \bar{\nu}_{e^-} + E$ | $e^-, \bar{\nu}_{e^-}$ |
| Gamma, $\gamma$ | ${}^{A}_{Z}X \rightarrow {}^{A}_{Z} Y + \gamma + E$ | $\gamma$ |

2. What do we mean by $1\,becquerel$ and $1Curie$?

$$1\,becquerel = 1 \text{ disintegration per second}$$

$$1\,Curie = 3.7 \times 10^{10} \text{ decays}$$

3. What is meant by **decay constant**? Decay constant - probability of a nucleus decay per unit time.

4. The decay constant of an isotope (${}^{45}_{22}X$) with mass $45g$ is $6.25 \times 10^{-5} s^{-1}$. Caculate the half life of this isotope and determine the decay rate when 30% of the original isotope has decayed.

$$T_{\frac{1}{2}} = \frac{ln\,2}{\lambda} = \frac{ln\,2}{6.25 \times 10^{-5} s^{-1}} = 11090s$$

$$N_o \rightarrow 0.7N_o$$

$$45\text{g of } {}^{45}_{22}X \text{ is 1 mol} \rightarrow N_o = nN_A = 6.02 \times 10^{23} \text{ atoms}$$

$$\frac{dN}{dt} = -\lambda N = -(6.25 \times 10^{-5} s^{-1})(0.7 \times 6.02 \times 10^{23}) = -2.63 \times 10^{-19} \text{ decay per second}$$

5. $1g$ of ${}^{255}_{80}X$ has an activity of $3 \times 10^{10}$. Calculate the decay constant and the activity of this sample after 5 years.

$$255g = 1mol \Rightarrow 1g = \frac{1}{255} mol$$

$$A_o = \lambda N_o \Rightarrow \lambda = \frac{A_o}{N_o} = \frac{3 \times 10^{10}}{(\frac{1}{255})(6.02 \times 10^{23})} = 1.27 \times 10^{-11}\, s^{-1}$$

$$A|_{t=1.577\times 10^{8}\,s} = A_o e^{-\lambda t} = (3 \times 10^{10}\, s^{-1}) e^{-(1.27\times 10^{-11}\, s^{-1})(1.577\times 10^{8}\, s)} = 2.9941 \times 10^{10} s^{-1}$$

6. At the beginning, the number of particle of a radioactive isotope is $e^4$ particles. At time $t = 5minutes$, the number of particle drops to $e$ particles. Determine the decay equation.

$$ln\,N = ln\,N_o - \lambda t$$

$$\lambda = -\frac{ln\,e^4}{ln\,e} \times \frac{1}{300s} = 1.33 \times 10^{-2}\, s^{-1}$$

$$N|_t = e^4\, e^{-0.0133t}$$

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7. The half life of plutonium is 24000 years. If stored for 96000 years, how much of the original amount is left?

$$T_{0.5} = \frac{ln\,2}{\lambda} \Rightarrow \lambda = \frac{ln\,2}{T_{0.5}} = \frac{ln\,2}{24000\ \text{years}^{-1}} = 2.888 \times 10^{-5}\ \text{years}^{-1}$$

$$\frac{N}{N_o} = e^{-\lambda t} = e^{-(2.888\times10^{-5})(96000)} = 0.062989$$

$$\text{Faster estimation} \Rightarrow n = \left(\frac{96000}{24000}\right) \approx 4\text{half-lives};\ \frac{N}{N_o} \approx (0.5)^n \approx (0.5)^4 \approx \frac{1}{16} \approx 0.0625$$