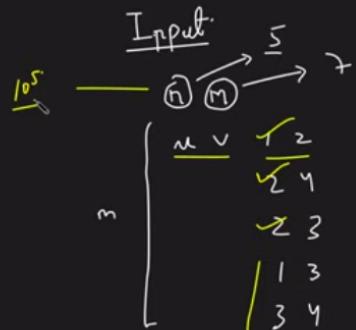


Graph representation in C++



(i) Adjacency Matrix :

$$\left. \begin{array}{l} \text{adj}[u][v] = 1 \\ \text{adj}[v][u] = 1 \end{array} \right\}$$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	1	0	0	0
2	0	1	0	1	0	0
3	0	0	0	0	0	0
4	0	0	1	0	0	0
5	0	0	0	1	0	0

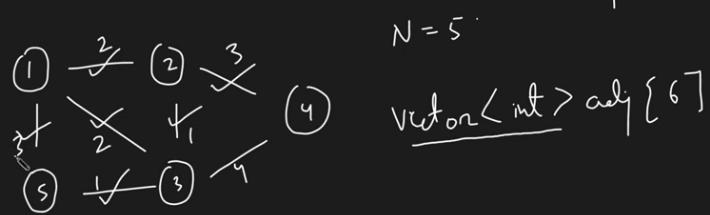
TUF

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 int main() {
5     int n, m;
6     cin >> n >> m;
7
8     // declare the adjacent matrix
9     int adj[n+1][n+1];
10
11    // take edges as input
12    for(int i = 0;i<m;i++) {
13        int u, v;
14        cin >> u >> v;
15        adj[u][v] = 1;
16        adj[v][u] = 1;
17    }
18    return 0;
19 }
```

Graph Representation in C++

int arr[1]
pair<int, int> arr[6]



$\begin{matrix} u \\ \downarrow \\ (1 \rightarrow 2) \\ (1 - 5) \\ (1 - 3) \\ (3 - 5) \\ (2 - 3) \\ (2 - 4) \\ (3 - 4) \end{matrix}$

7 edges

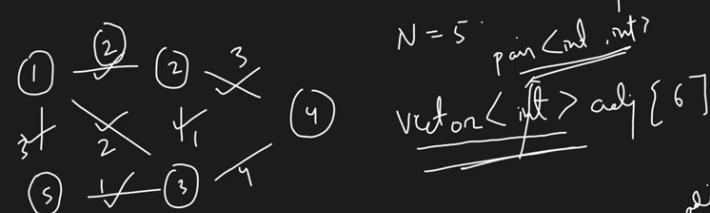
$\propto \rightarrow O(N + 2E)$

adj[u].push_back(v)
adj[v].push_back(u)

0	(2, 5, 3)
1	(1, 3, 4)
2	(1, 5, 2, 4)
3	(2, 3)
4	(1, 3)
5	

Graph Representation in C++

int arr[1]
pair<int, int> arr[6]



$\begin{matrix} u \\ \downarrow \\ (1 \rightarrow 2) \\ (1 - 5) \\ (1 - 3) \\ (3 - 5) \\ (2 - 3) \\ (2 - 4) \\ (3 - 4) \end{matrix}$

7 edges

$\propto \rightarrow O(N + 2E) + 2E \ll N^N$

adj[u].push_back(v)
adj[v].push_back(u)

0	(2, 5, 3)
1	(1, 3, 4)
2	(1, 5, 2, 4)
3	(2, 3)
4	(1, 3)
5	

adj[w]

$\{v, w\}$

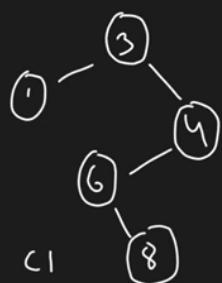
```

2
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 int main() {
7     int n, m;
8     cin >> n >> m;
9
10    vector<pair<int,int>> adj[n+1];
11    for(int i = 0;i<m;i++) {
12        int u, v, wt;
13        cin >> u >> v >> wt;
14
15        adj[u].push_back({v, wt});
16        adj[v].push_back({u, wt});
17    }
18    return 0;

```

TUF

Connected Components in a Graph

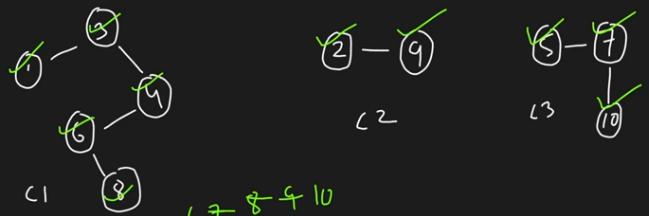


C2

C3

0 2 3 4

Connected Components in a Graph



```

for( i = r ; i <= 10 ; i++)
    { y (!vis[i]) }

```

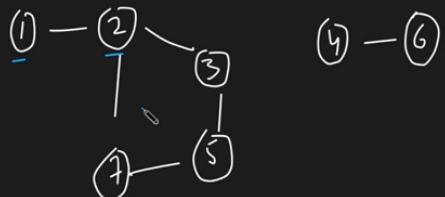
840

7

DFS / BFS

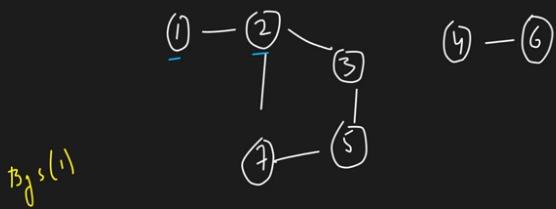
48

1) BFS (Breadth First Search)



1	2
2	1 3 7
3	2 5
4	6
5	3 7
6	4
7	2 5

BFS (Breadth First Search)



BFS(1)



1	2
2	1 3 7
3	2 5
4	6
5	3 7
6	4
7	2 5

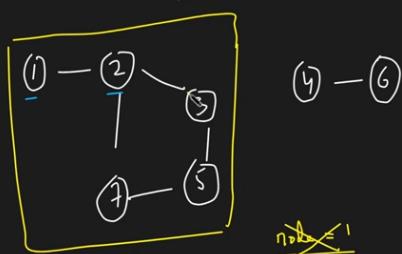
0	0	0	0	0	0	0	b
0	1	2	3	4	5	6	7

vis

$\{ i = 1 \rightarrow n \}$ $i=1$
 $y (vis[i] = 0)$
 $\{ bfs(i); \}$

1, 2, 3, 7, 5

BFS (Breadth First Search)



BFS(1)

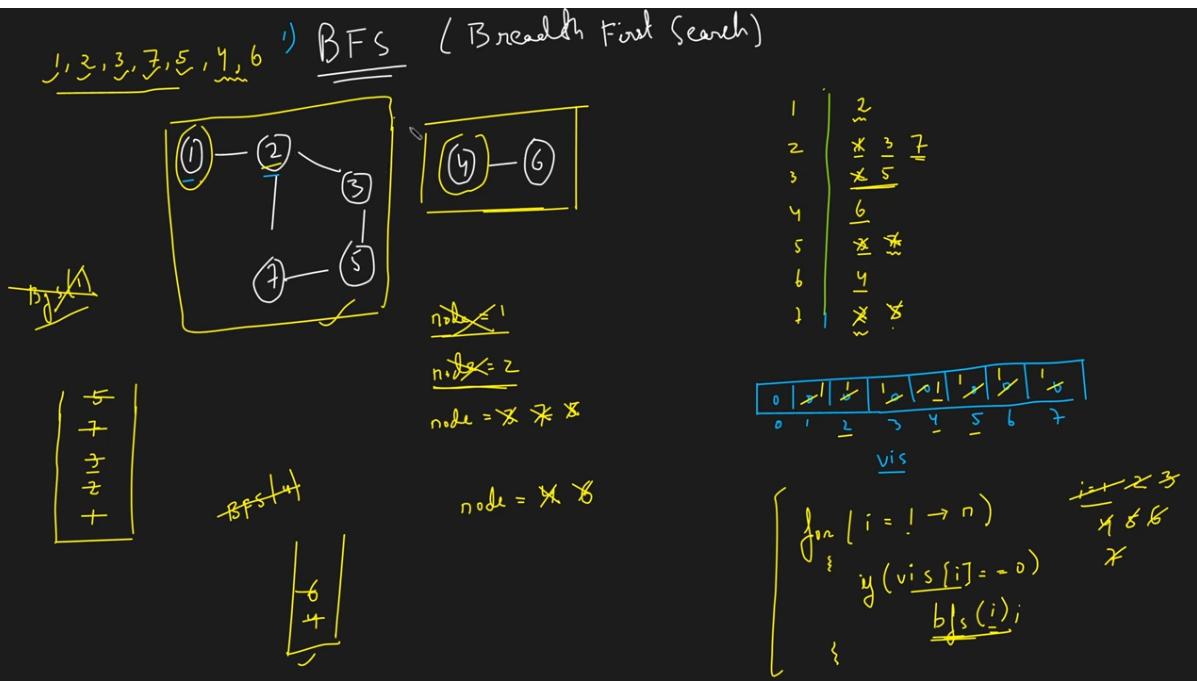


1	2
2	* 3 7
3	* 5
4	6
5	* *
6	4
7	* *

0	*	1	*	1	0	1	*	0	1	*
0	1	2	3	4	5	6	7			

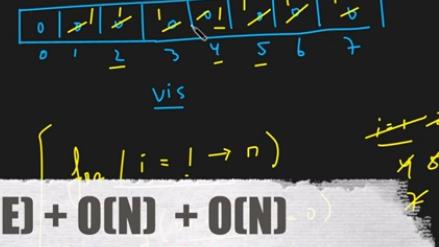
vis

$\{ i = 1 \rightarrow n \}$ $i=1$
 $y (vis[i] = 0)$
 $\{ bfs(i); \}$



$$TC \rightarrow O(N)$$

$$SC \rightarrow O(N)$$



TC will be $O(N+E)$ and SC will be $O(N+E) + O(N) + O(N)$

N is time taken for visiting N nodes, and E is for traveling through adjacent nodes overall. Space for adj list, vis array and queue

```

class Solution
{
    public ArrayList<Integer> bfsOfGraph(int V, ArrayList<ArrayList<Integer>> adj)
    {
        ArrayList<Integer> bfs = new ArrayList<O>;
        boolean vis[] = new boolean[V+1];
        for(int i = 1;i<=V;i++) {
            if(vis[i] == false) {
                Queue<Integer> q = new LinkedList<O>;
                q.add(i);
                vis[i] = true;
                while (!q.isEmpty())
                {
                    Integer node = q.poll();
                    bfs.add(node);
                    for(Integer it: adj.get(node)) {
                        if(vis[it] == false) {
                            vis[it] = true;
                            q.add(it);
                        }
                    }
                }
            }
        }
        return bfs;
    }
}
    
```

```

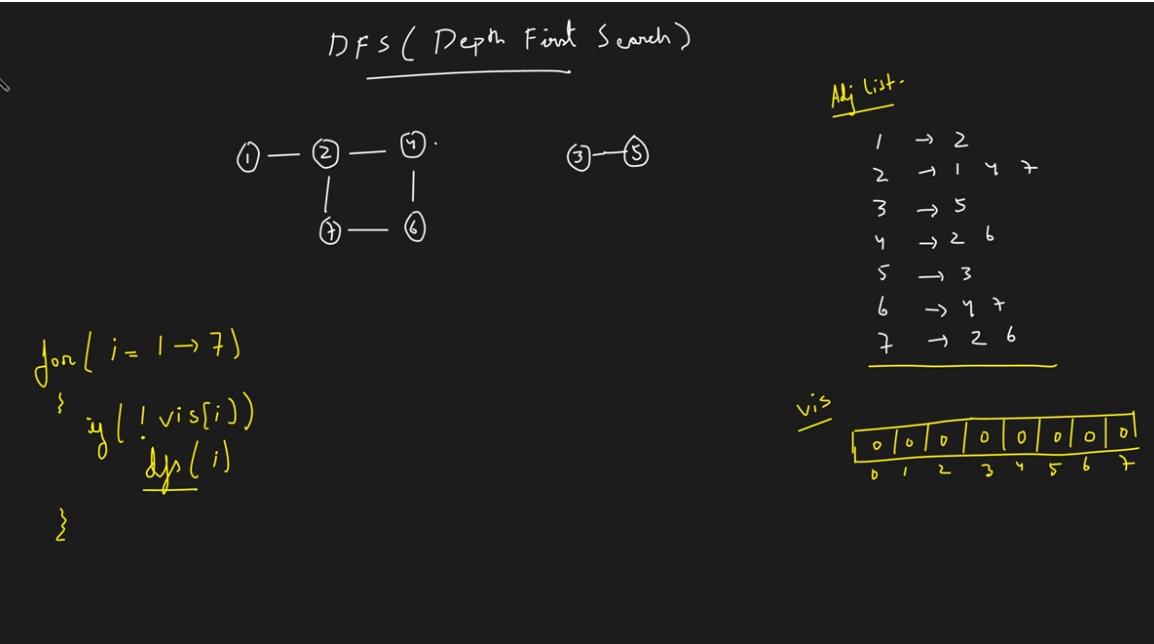
class Solution {
public:
    vector<int> bfsOfGraph(int V, vector<int> adj[]) {
        vector<int> bfs;
        vector<int> vis(V+1, 0);

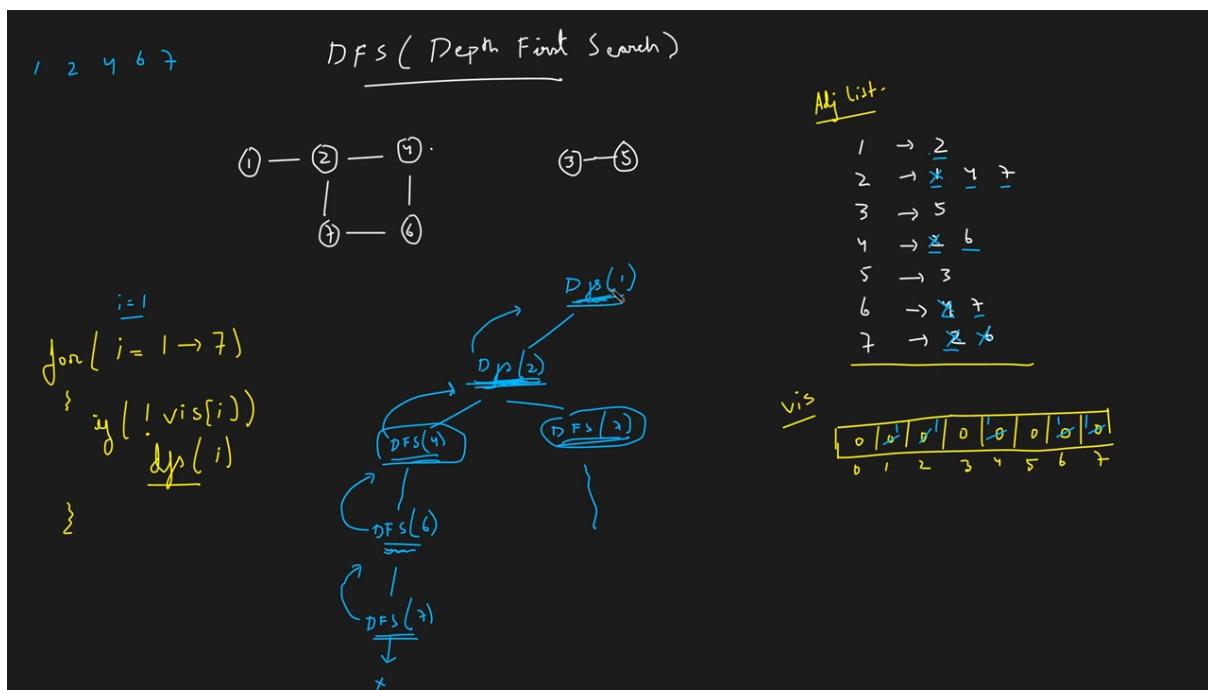
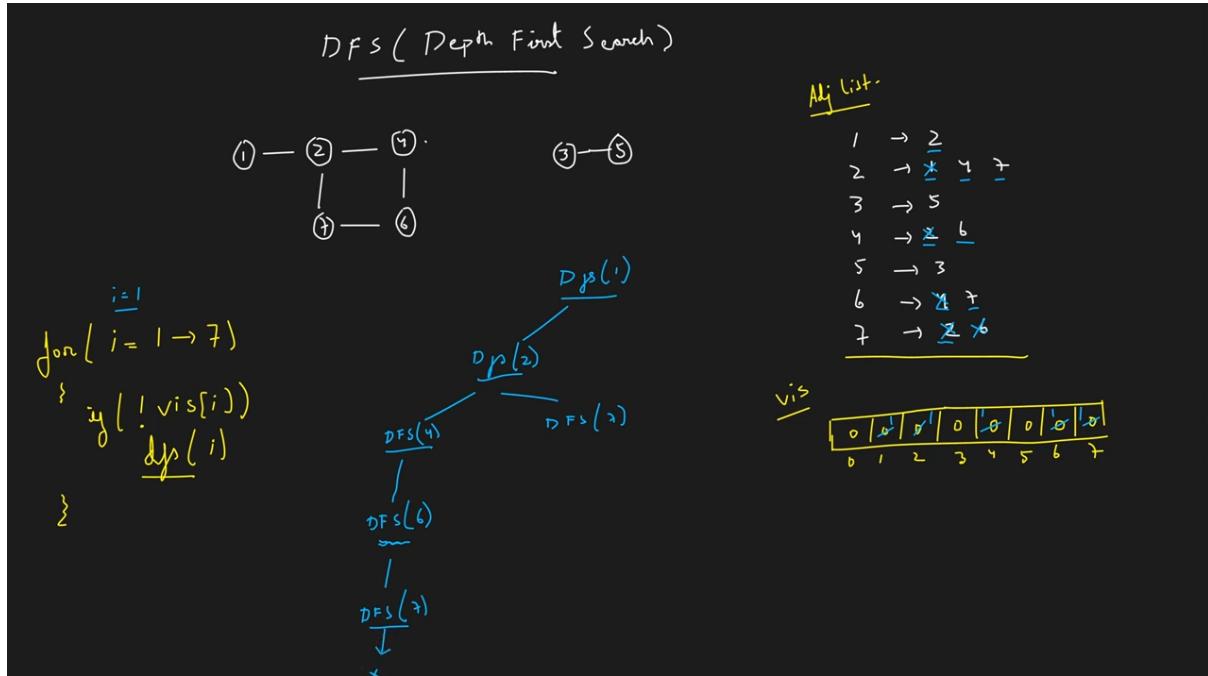
        for(int i = 1; i <= V; i++) {
            if(!vis[i]) {
                queue<int> q;
                q.push(i);
                vis[i] = 1;
                while(!q.empty()) {
                    int node = q.front();
                    q.pop();
                    bfs.push_back(node);

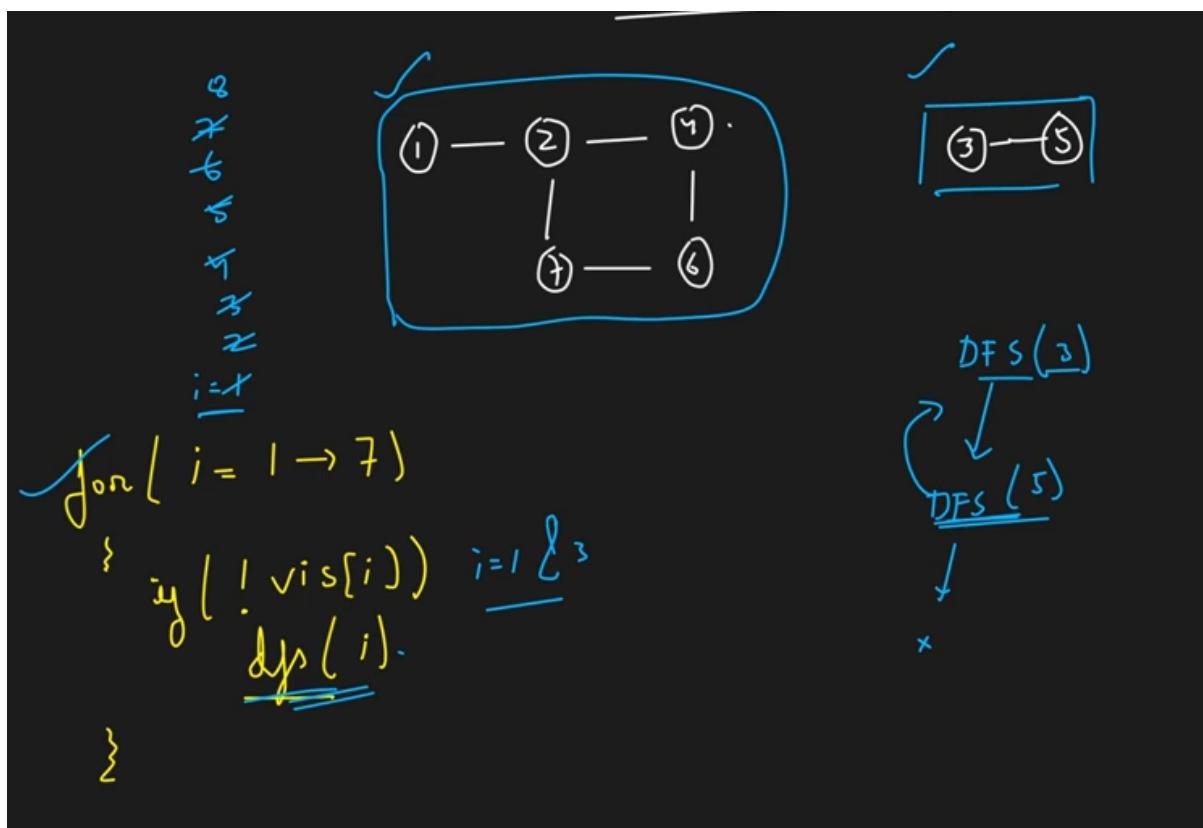
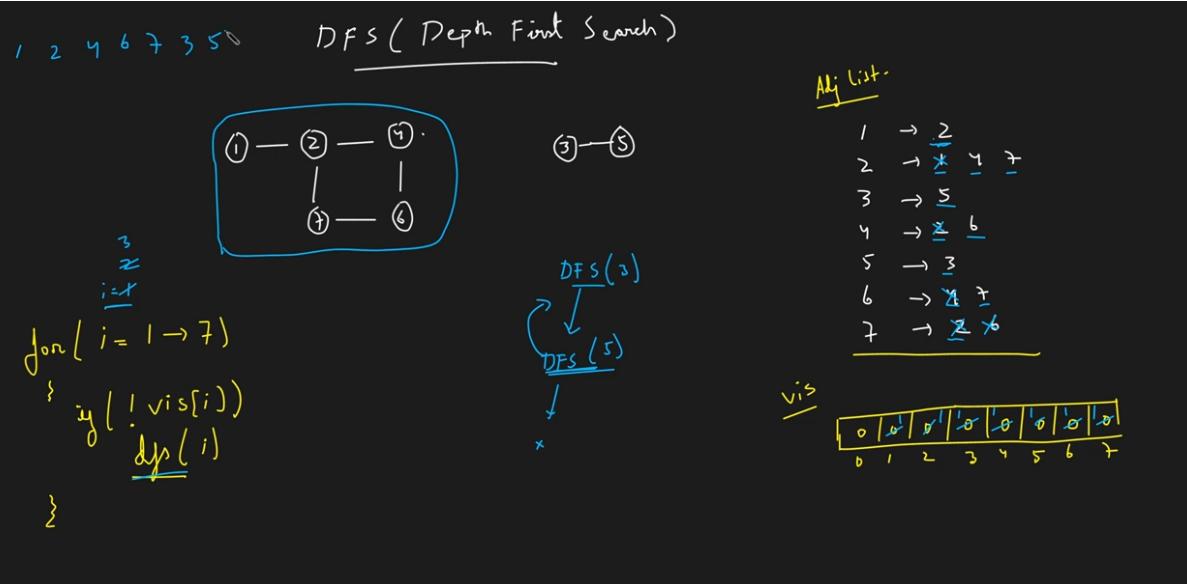
                    for(auto it : adj[node]) {
                        if(!vis[it]) {
                            q.push(it);
                            vis[it] = 1;
                        }
                    }
                }
            }
        }

        return bfs;
    }
}; // } Driver Code Ends

```







```

class Solution {
    public void dfs(int node, boolean vis[], ArrayList<ArrayList<Integer>> adj, ArrayList<Integer> storeDfs) {
        storeDfs.add(node);
        vis[node] = true;
        for(Integer it: adj.get(node)) {
            if(vis[it] == false) {
                dfs(it, vis, adj, storeDfs);
            }
        }
    }
    public ArrayList<Integer> dfsOfGraph(int V, ArrayList<ArrayList<Integer>> adj)
    {
        ArrayList<Integer> storeDfs = new ArrayList<>();
        boolean vis[] = new boolean[V+1];
        for(int i = 1;i<=V;i++) {
            if(vis[i] == 0) {
                dfs(i,vis, adj, storeDfs);
            }
        }
        return storeDfs;
    }
}

```

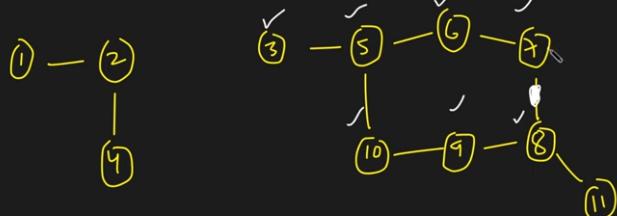
```

class Solution {
    void dfs(int node, vector<int> &vis, vector<int> adj[], vector<int> &storeDfs) {
        storeDfs.push_back(node);
        vis[node] = 1;
        for(auto it : adj[node]) {
            if(!vis[it]) {
                dfs(it, vis, adj, storeDfs);
            }
        }
    }
public:
    vector<int>dfsOfGraph(int V, vector<int> adj[]){
        vector<int> storeDfs;
        vector<int> vis(V+1, 0);
        for(int i = 1;i<=V;i++) {
            if(!vis[i]) {
                dfs(i, vis, adj, storeDfs);
            }
        }

        return storeDfs;
    }
};

```

Detect a cycle in Undirected Graph (BFS)



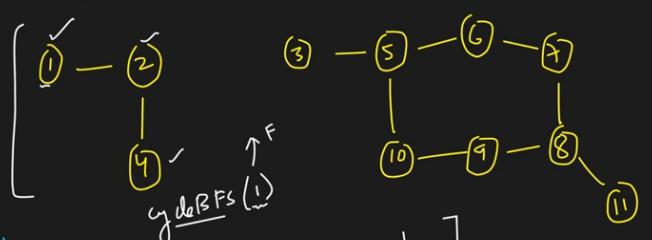
adj list	
1	2
2	1 4
3	5
4	2
5	3 10 16
6	5 7
7	6 8
8	7 9 11
9	10 8
10	5 9
11	8

```

for (i = 1 - N)
{
    if (!vis[i])
        if (cycleBFS(i))
            return T;
}
  
```

vis	0 0 0 0 0 0 0 0 0 0 0 0
	0 1 2 3 4 5 6 7 8 9 10 11

Detect a cycle in Undirected Graph (BFS)



adj list	
1	2
2	1 4
3	5
4	X
5	3 10 16
6	5 7
7	6 8
8	7 9 11
9	10 8
10	5 9
11	8

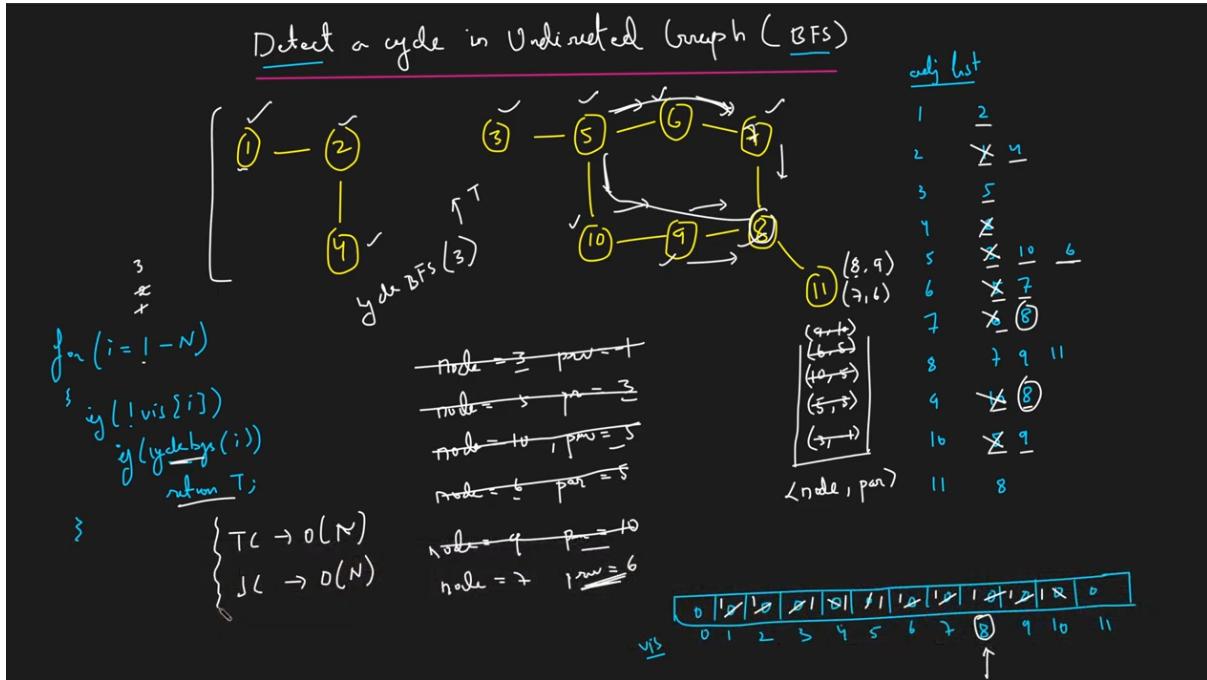
```

for (i = 1 - N)
{
    if (!vis[i])
        if (cycleBFS(i))
            return T;
}
  
```

$\text{node} = 1 \quad \text{par} = 1$
 $\text{node} = 2 \quad \text{par} = 1$
 $\text{node} = 4 \quad \text{par} = 2$

$(4, 2) \rightarrow$
 $(2, 1) \rightarrow$
 $(1, 1) \rightarrow$
 $O(nod, par)$

vis	0 1 0 0 0 0 0 0 0 0 0 0
	0 1 2 3 4 5 6 7 8 9 10 11



```

class Solution {

public:
    bool checkForCycle(int s, int V, vector<int> adj[], vector<int>& visited) {
        // Create a queue for BFS
        queue<pair<int, int>> q;

        visited[s] = true;
        q.push({s, -1});

        while (!q.empty()) {
            int node = q.front().first;
            int par = q.front().second;
            q.pop();

            for (auto it : adj[node]) {
                if (!visited[it]) {
                    visited[it] = true;
                    q.push({it, node});
                } else if (par != it)
                    return true;
            }
        }
        return false;
    }

public:
    bool isCycle(int V, vector<int> adj[]){
        vector<int> vis(V+1, 0);
        for(int i = 1; i <= V; i++) {
            if(!vis[i]) {
                if(checkForCycle(i, V, adj, vis)) return true;
            }
        }
        return false;
    }
};

```

Cycle Detection in Undirected Graph (DFS)

adj list

1	3
2	5
3	1 4
4	3
5	2 6 8
6	5 7
7	6 8
8	7 5

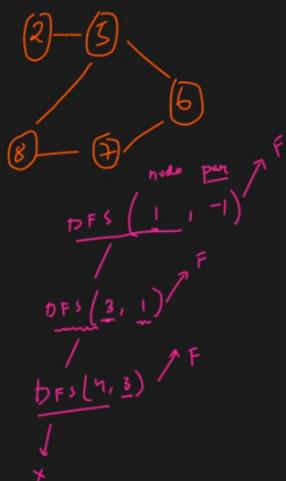
for ($i = 1 \rightarrow 8$)

{
 if ($\text{vis}[i] = 0$)

 {
 if (cycle DFS(i))

 return T;
 }

}



0	1	0	0	1	0	0	0
0	1	0	0	1	0	0	0
1	0	1	0	1	0	0	0
0	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0

vis

Cycle Detection in Undirected Graph (DFS)

adj list

1	3
2	5
3	1 4
4	3
5	2 6 8
6	5 7
7	6 8
8	7 5

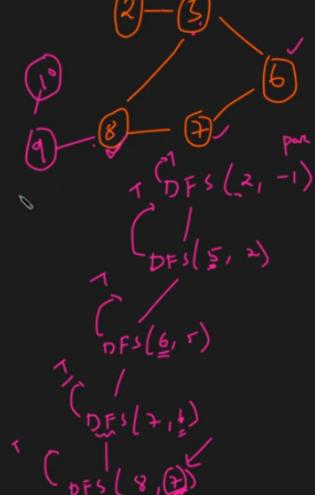
for ($i = 1 \rightarrow 8$)

{
 if (vis[i] = 0)

 {
 if (cycle DFS(i))

 return T;
 }

}

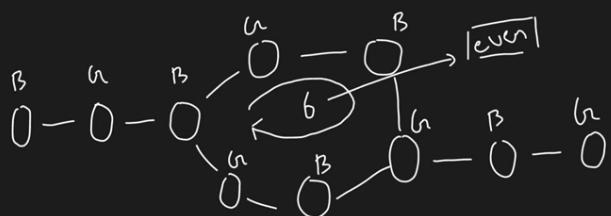


0	1	0	0	1	0	0	0
0	1	0	0	1	0	0	0
1	0	1	0	1	0	0	0
0	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0

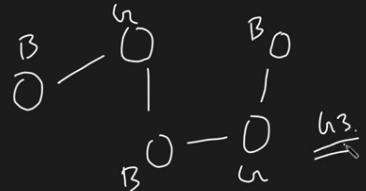
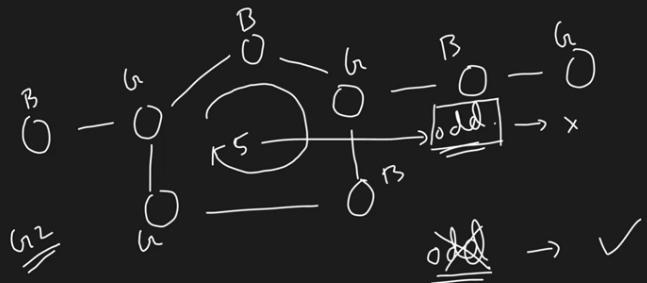
vis

Bipartite Graph (BFS)

→ that can be colored using 2 colors such that no two adjacent nodes have same color.



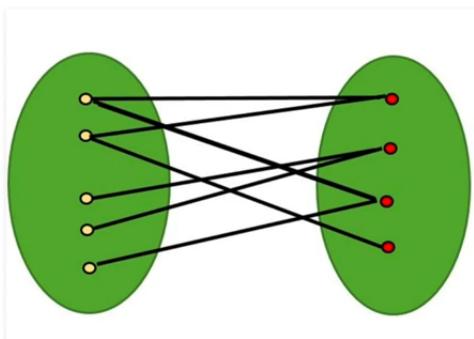
$\underline{B \mid u}$



Check whether a given graph is Bipartite or not

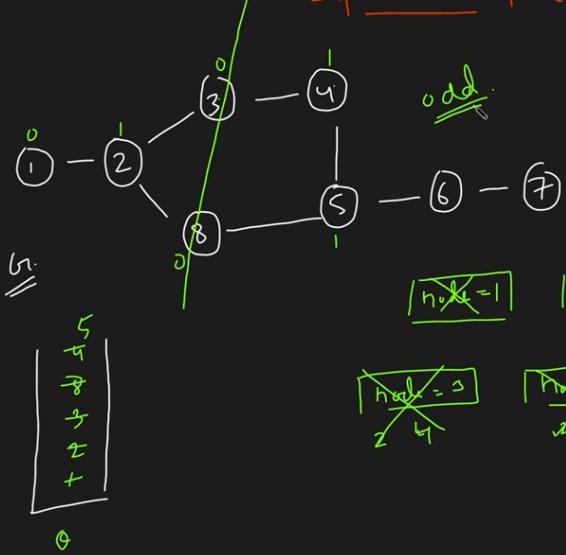
Difficulty Level : Medium • Last Updated : 19 Apr, 2021

A Bipartite Graph is a graph whose vertices can be divided into two independent sets, U and V such that every edge (u, v) either connects a vertex from U to V or a vertex from V to U . In other words, for every edge (u, v) , either u belongs to U and v to V , or u belongs to V and v to U . We can also say that there is no edge that connects vertices of same set.



o/1

Bipartite Graph (BFS)



5
8
7
2
+
0

$$\begin{array}{|c|} \hline \text{node} = 1 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \text{node} = 2 \\ \hline 1 \quad 3 \quad 8 \\ \hline \end{array}$$

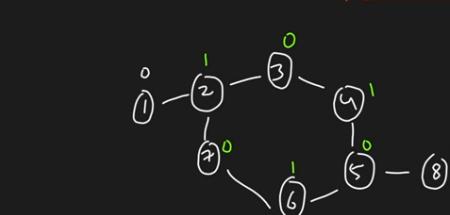
$$\begin{array}{|c|} \hline \text{node} = 4 \\ \hline 5 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \text{node} = 5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 0 & 1 & 0 & 1 & 1 & 0 \\ \hline -1 & +1 & +1 & +1 & +1 & +1 & -1 & +1 \\ \hline 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline \end{array}$$

color[]

(Bipartite Graph)(BFS)



8
5
6
4
7
2
+
0

BFS

sub T.

$$\begin{array}{|c|} \hline \text{node} = 1 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \text{node} = 2 \\ \hline 1 \quad 3 \quad 7 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \text{node} = 4 \\ \hline 5 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \text{node} = 6 \\ \hline 7 \\ \hline \end{array}$$

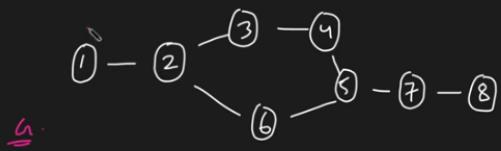
$$\begin{array}{|c|} \hline \text{node} = 8 \\ \hline 5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \hline +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline \end{array}$$

that can be colored
using 2 colors such that
no two adjacent nodes
have same color.

that can be colored
using 2 colors such that
no two adjacent nodes
have same color.

Bipartite Graph (DFS)

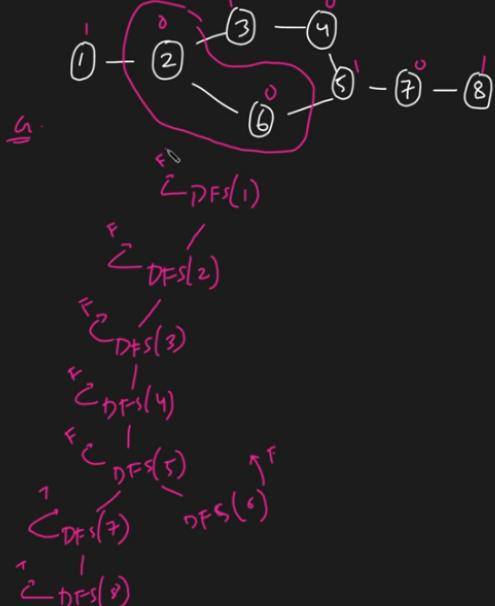


$\hookrightarrow \text{DFS}(1)$

\hookrightarrow that can be colored using 2 colors such that no two adjacent nodes have same color.

1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
0	1	2	3	4	5	6	7	8			

Column



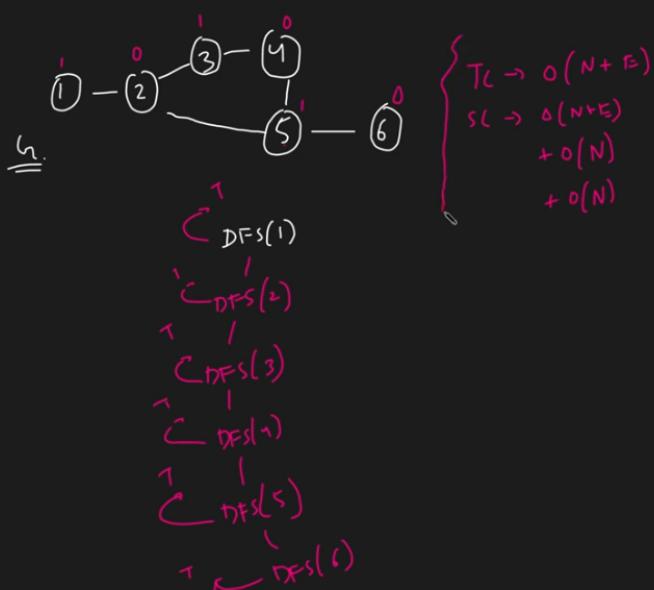
\hookrightarrow That can be colored using 2 colors such that no two adjacent nodes have same color.

1	0	1	0	1	0	1	0	1	0	1	0
0	1	2	3	4	5	6	7	8			

Column

Bipartite Graph (DFS)

that can be colored
using 2 colors such that
no two adjacent nodes
have same color.



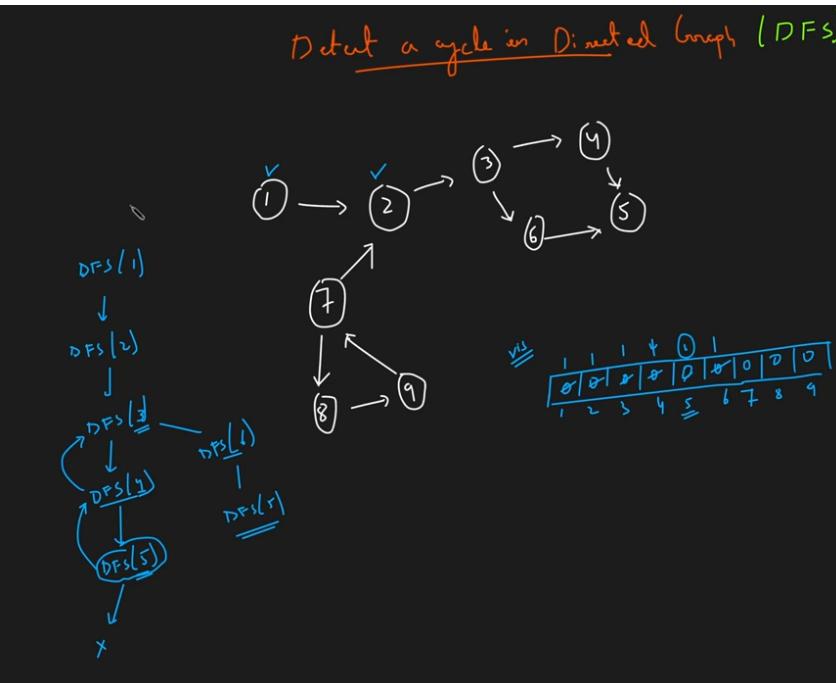
1	0	1	0	1	0
-1	+1	+1	+1	+1	+1

color[1]

Detect a cycle in Directed Graph (DFS)

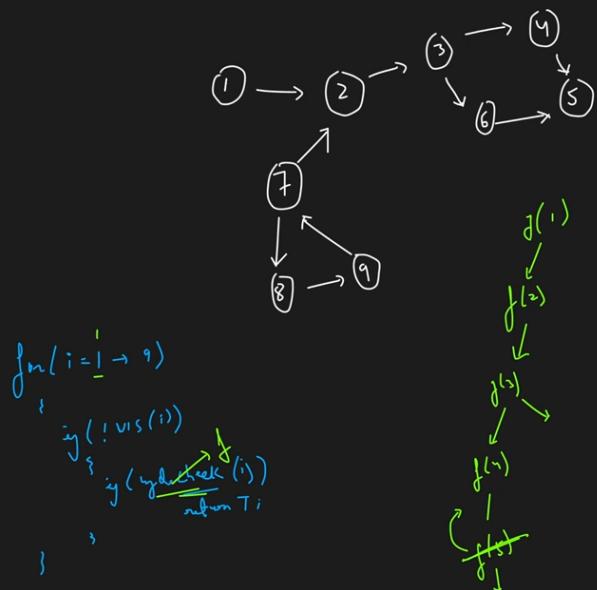
adjacency List

- 1 → 2
- 2 → 3
- 3 → 4, 6
- 4 → 5
- 5 → -
- 6 → 5
- 7 → 2, 8
- 8 → 9
- 9 → 7



1	1	1	1	0	1	0	0	0	0
0	1	0	1	0	0	1	0	1	0

Detect a cycle in Directed Graph (DFS)

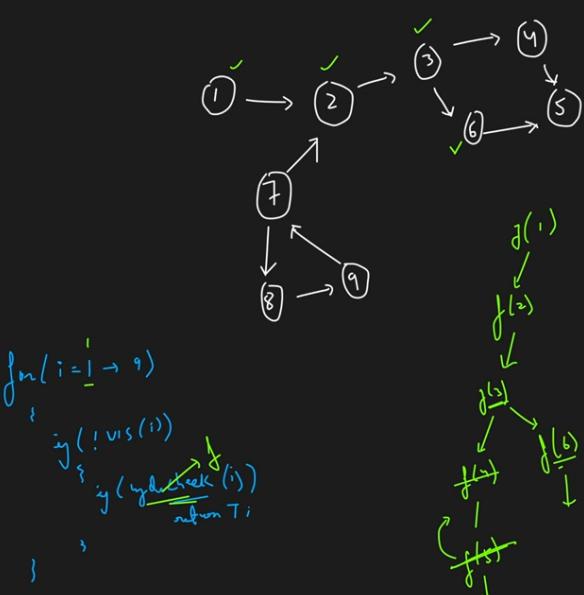


adjacency List

$1 \rightarrow 2$
 $2 \rightarrow 3$
 $3 \rightarrow 4, 6$
 $4 \rightarrow 5$
 $5 \rightarrow 6$
 $6 \rightarrow 5$
 $7 \rightarrow 2, 8$
 $8 \rightarrow 9$
 $9 \rightarrow 7$

vis	1	2	3	4	5	6	7	8	9
dfsvis	0	1	0	1	0	0	1	0	0
	1	2	3	4	5	6	7	8	9

Detect a cycle in Directed Graph (DFS)

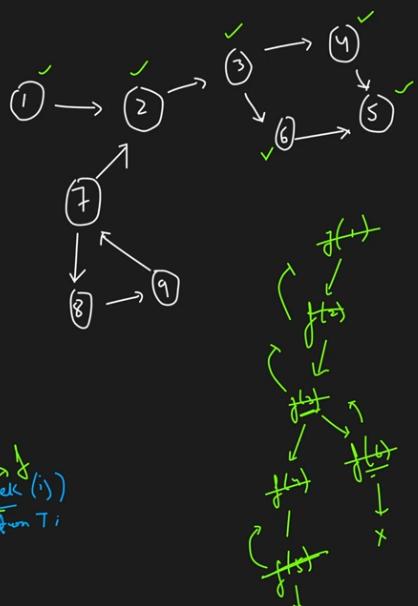


adjacency List

$1 \rightarrow 2$
 $2 \rightarrow 3$
 $3 \rightarrow 4, 6$
 $4 \rightarrow 5$
 $5 \rightarrow 6$
 $6 \rightarrow 5$
 $7 \rightarrow 2, 8$
 $8 \rightarrow 9$
 $9 \rightarrow 7$

vis	1	2	3	4	5	6	7	8	9
dfsvis	0	1	0	1	0	0	1	0	0
	1	2	3	4	5	6	7	8	9

Detect a cycle in Directed Graph (DFS)



adjacency List

| → 2

2 → 3

$$3 \rightarrow \underline{4}, \underline{6}$$

4 → 5

5 → =

6 → 3

10

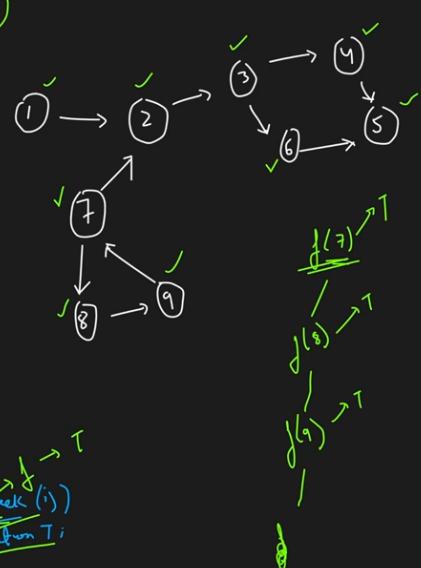
8 → 1

$$q \rightarrow +$$

vis	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	
vis	0	0	0	D					
T	+	x	x	x	x	0	0	0	
5	4	5	6	7	8	9			

Detect a cycle in Directed Graph (DFS)

$$\begin{cases} T_L \rightarrow o(N+E) \\ S_L \rightarrow o(2N) \\ A_S L \rightarrow o(N) \end{cases}$$



adjacency List

172

2 → 3

$$3 \rightarrow 4, \underline{6}$$

4 → 3

5 → =

6 → 5

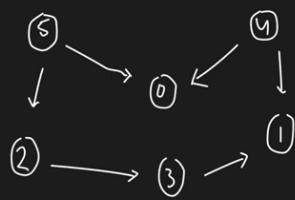
7 →

$$g \rightarrow \frac{q}{\gamma}$$

$$q \rightarrow \underline{\oplus}$$

Vis											
1 2		3 4		5 6		7 8		9			
1	2	3	4	5	6	7	8	9			
(Vis)	0	0	0	0	0	0	0	0			
1	2	3	4	5	6	7	8	9			

Topological Sorting



$u \rightarrow v$

$2 \rightarrow 3$

$5 \rightarrow 0$

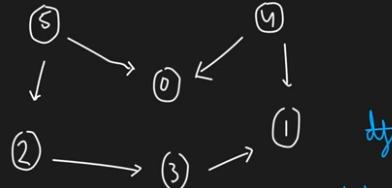
$4 \rightarrow 1$

linear ordering of vertices such that
if there is an edge $u \rightarrow v$, u appears
before v in that ordering.

5 4 2 3 1 0

0	-
1	-
2	- 3
3	- 1
4	- 0, 1
5	- 0, 2

Topological Sorting (DFS)



dfs(s)

dfs(t)

dfs(r)

dfs(s)

for ($i = 0 \rightarrow 5$)
{
 if (!vis[i])
 {
 dfs(i);
 }
}

adj

3	1	0
1	-	=
0	=	-

LIFO

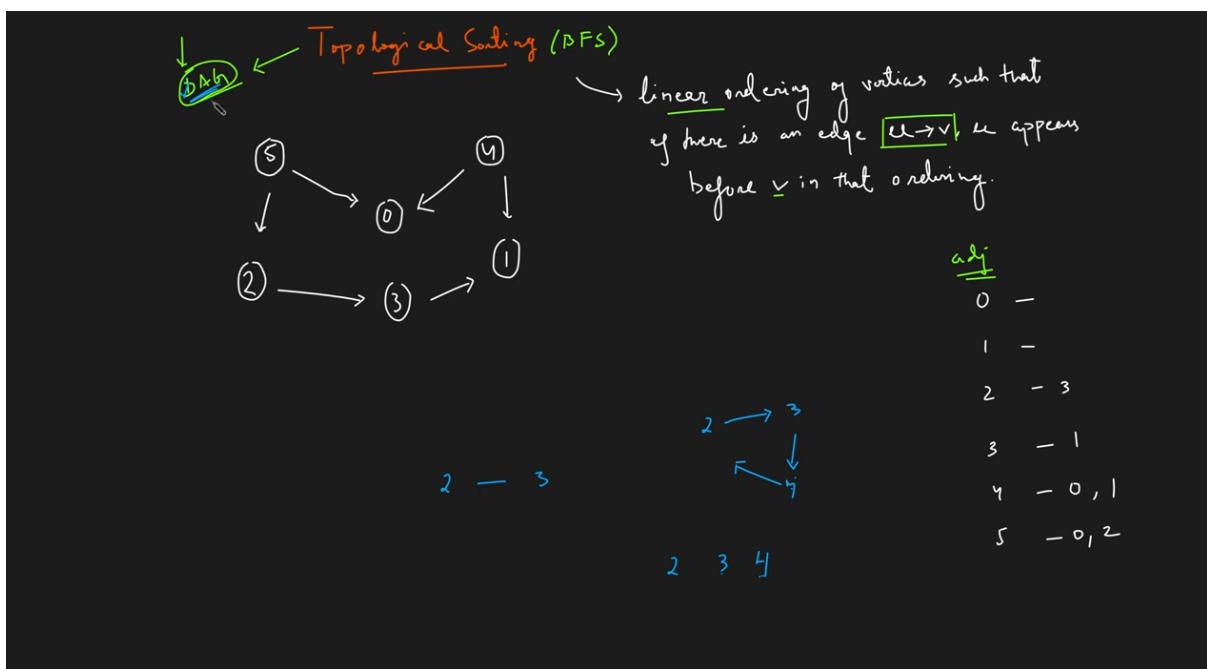
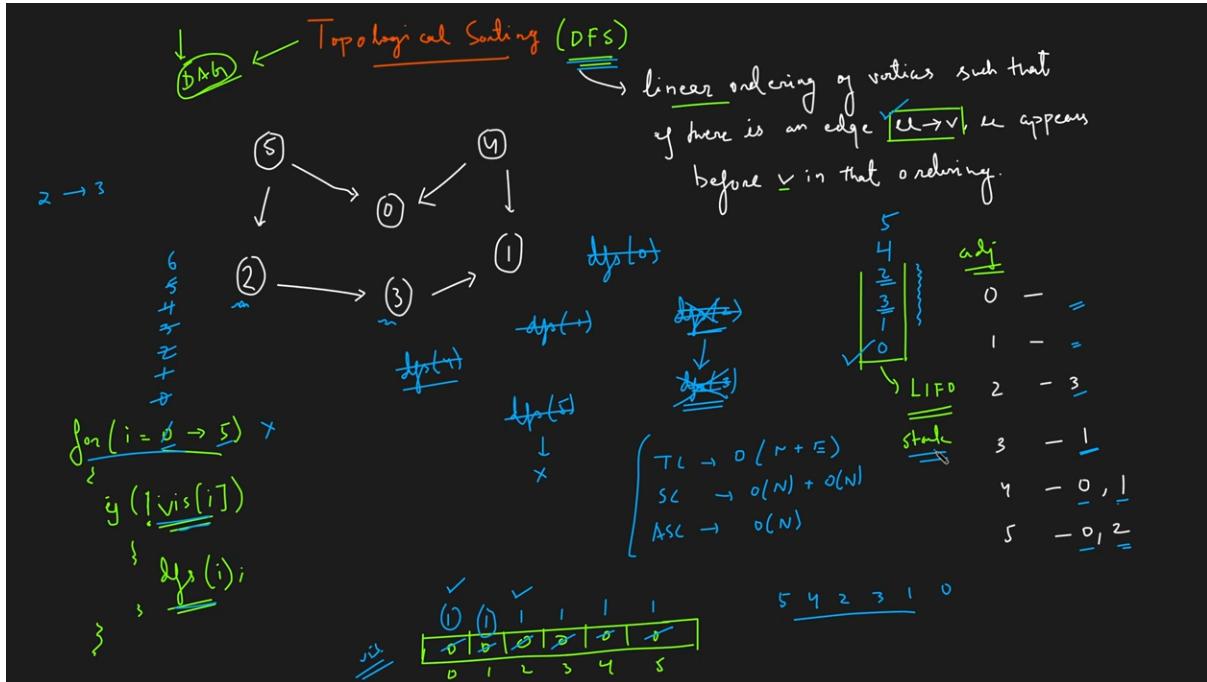
3	1	0
1	-	=
0	=	-

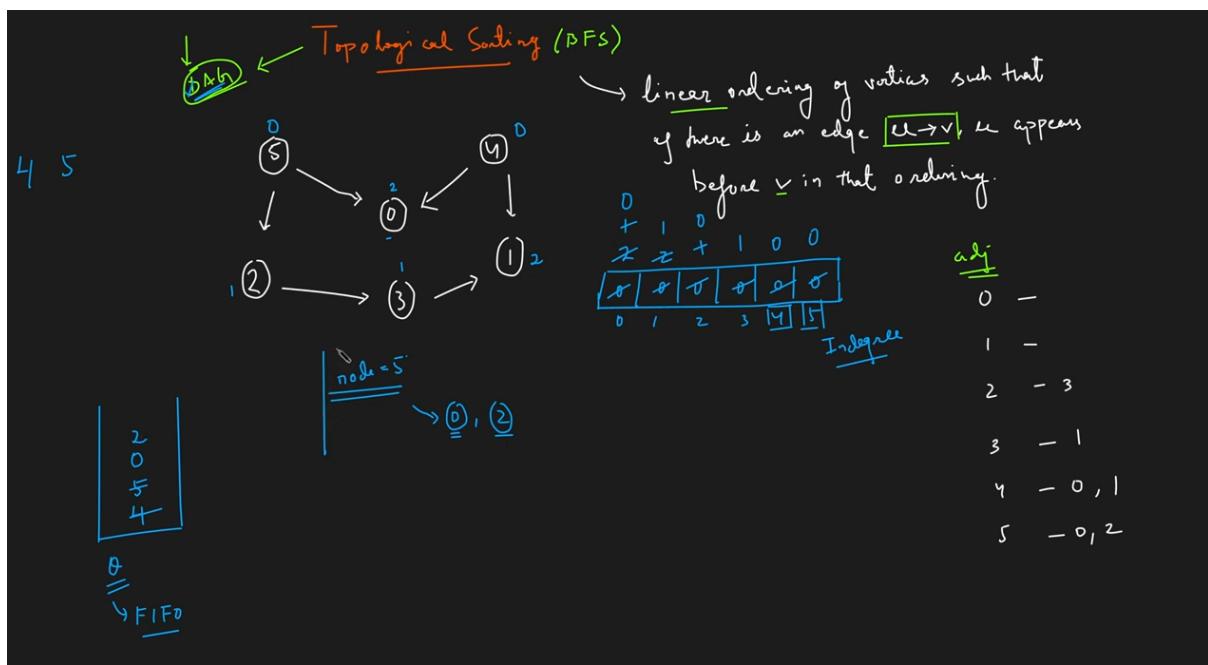
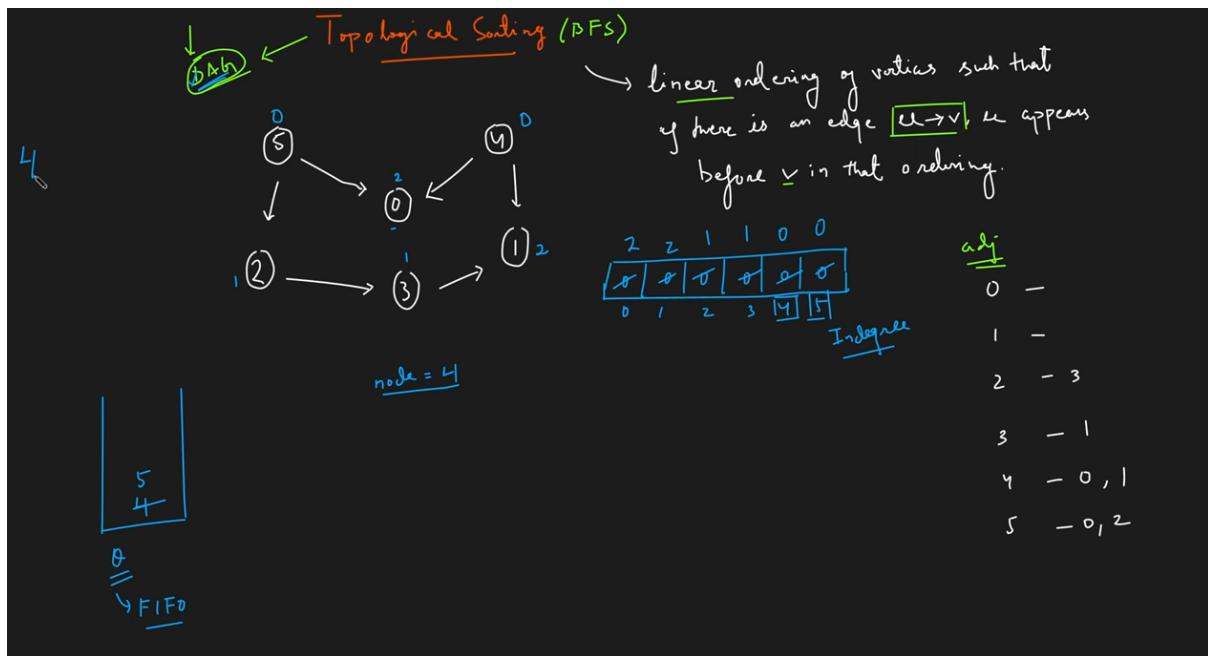
stack

3 - 1

4 - 0, 1

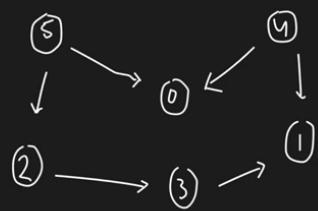
5 - 0, 2





bfs

Topological Sorting (BFS)



linear ordering of vertices such that
if there is an edge $u \rightarrow v$, u appears
before v in that ordering.

adj

0	-
1	-
2	- 3
3	- 1
4	- 0, 1
5	- 0, 2

2 — 3

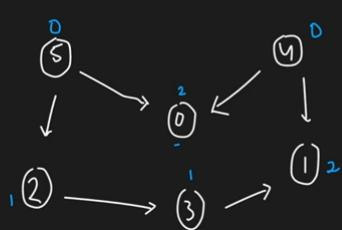
2 3 4



2 3 4

bfs

Topological Sorting (BFS)



linear ordering of vertices such that
if there is an edge $u \rightarrow v$, u appears
before v in that ordering.

Indegree

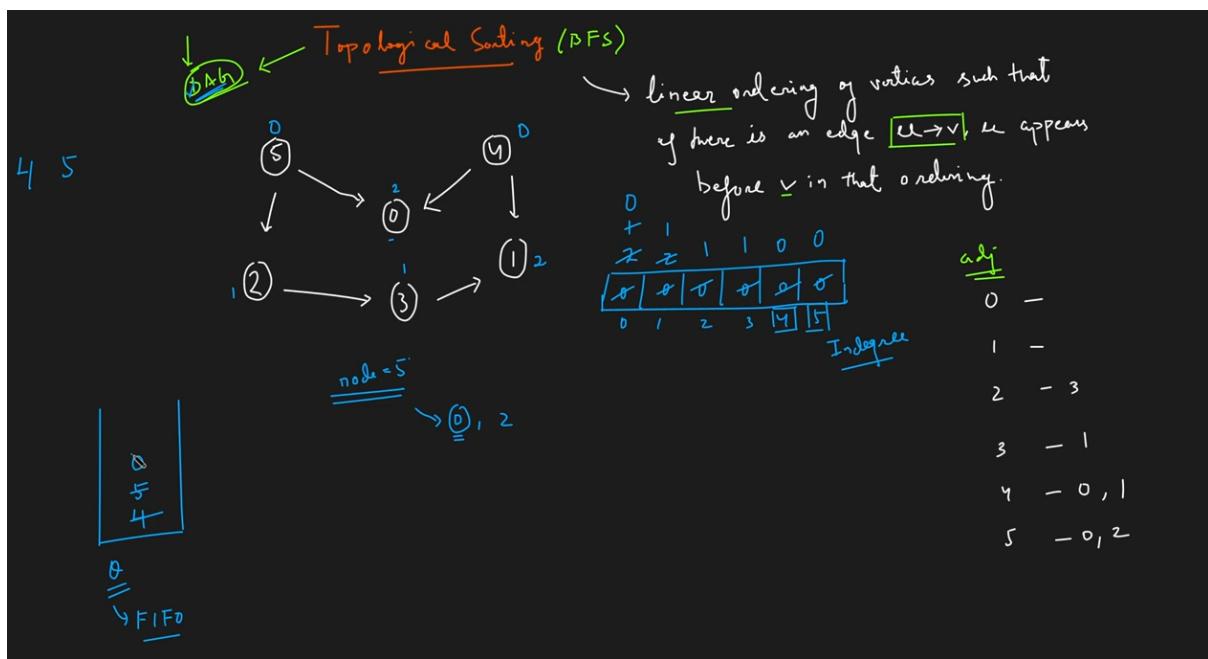
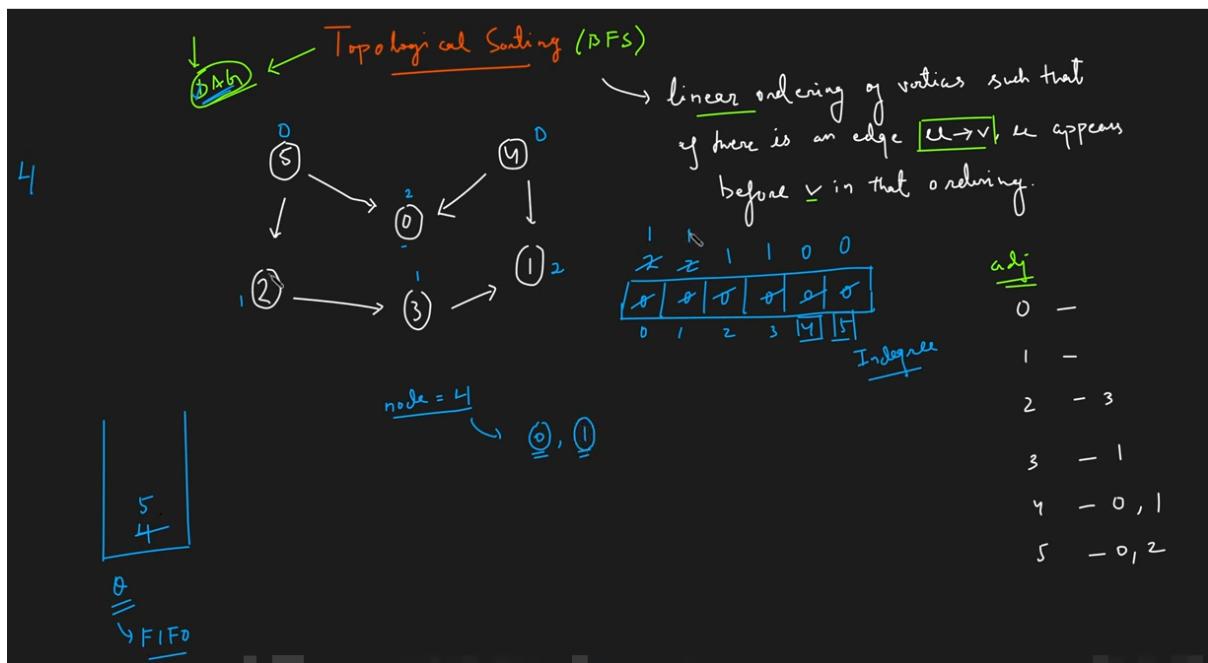
2	2	1	1	0	0
0	1	2	3	4	5

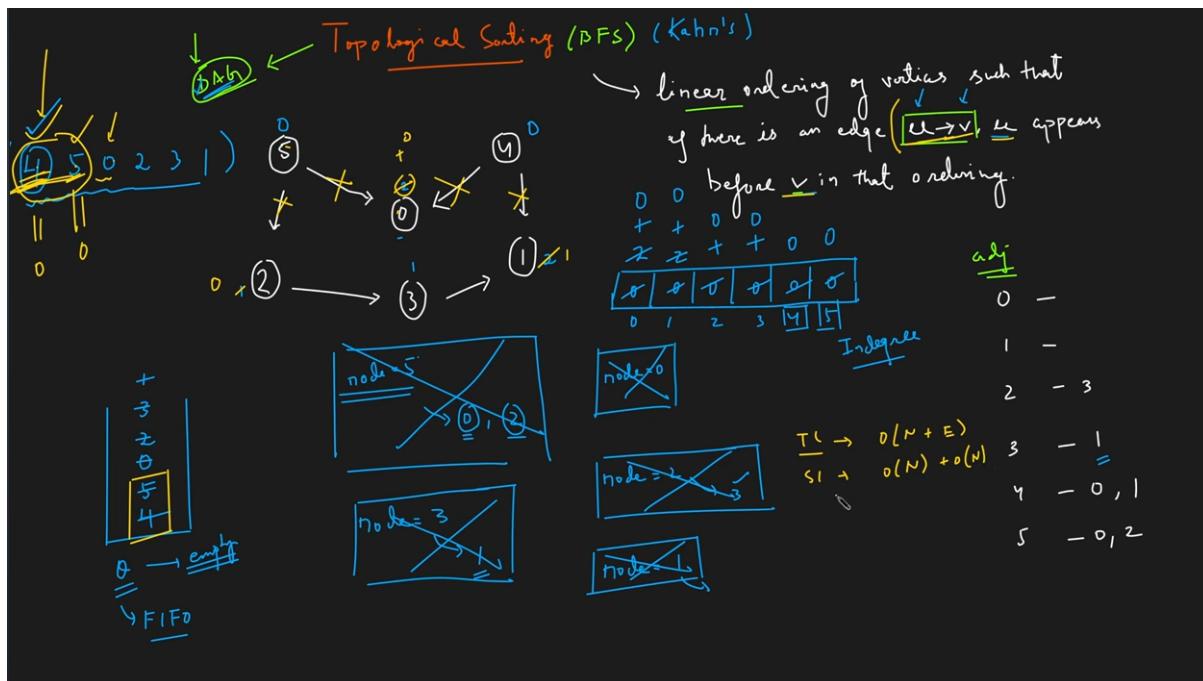
adj

0	-
1	-
2	- 3
3	- 1
4	- 0, 1
5	- 0, 2

node = 4



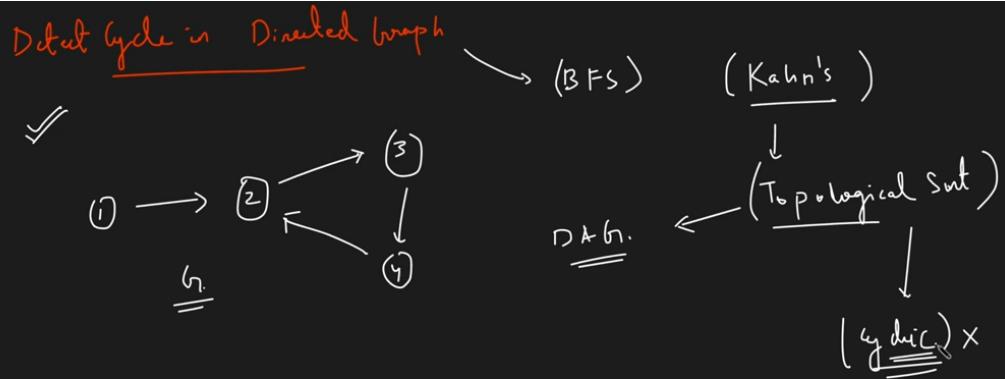




// Given code ends

```
class Solution{
public:
    vector<int> topoSort(int N, vector<int> adj[]) {
        queue<int> q;
        vector<int> indegree(N, 0);
        for(int i = 0;i<N;i++) {
            for(auto it: adj[i]) {
                indegree[it]++;
            }
        }

        for(int i = 0;i<N;i++) {
            if(indegree[i] == 0) {
                q.push(i);
            }
        }
        vector<int> topo;
        while(!q.empty()) {
            int node = q.front();
            q.pop();
            topo.push_back(node);
            for(auto it : adj[node]) {
                indegree[it]--;
                if(indegree[it] == 0) {
                    q.push(it);
                }
            }
        }
        return topo;
    }
};
```

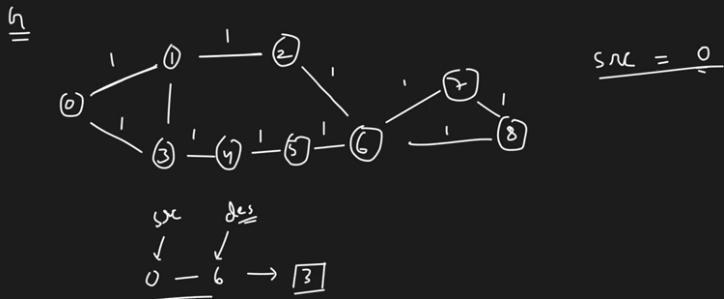


```

class Solution {
public:
    bool isCyclic(int N, vector<int> adj[]) {
        queue<int> q;
        vector<int> indegree(N, 0);
        for(int i = 0;i<N;i++) {
            for(auto it: adj[i]) {
                indegree[it]++;
            }
        }

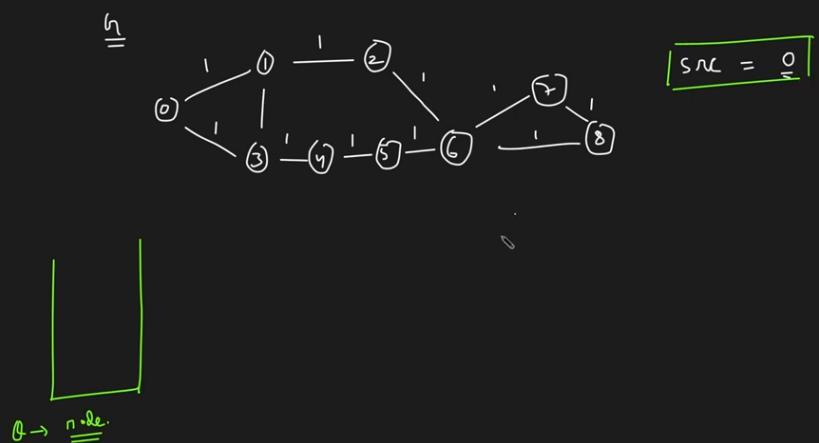
        for(int i = 0;i<N;i++) {
            if(indegree[i] == 0) {
                q.push(i);
            }
        }
        int cnt = 0;
        while(!q.empty()) {
            int node = q.front();
            q.pop();
            cnt++;
            for(auto it : adj[node]) {
                indegree[it]--;
                if(indegree[it] == 0) {
                    q.push(it);
                }
            }
        }
        if(cnt == N) return false;
        return true;
    }
}; // } Driver Code Ends

```



adj

- 0 → 1, 3
- 1 → 0, 2, 3
- 2 → 1, 6
- 3 → 0, 4
- 4 → 3, 5
- 5 → 4, 6
- 6 → 2, 5, 7, 8
- 7 → 6, 8
- 8 → 6, 7



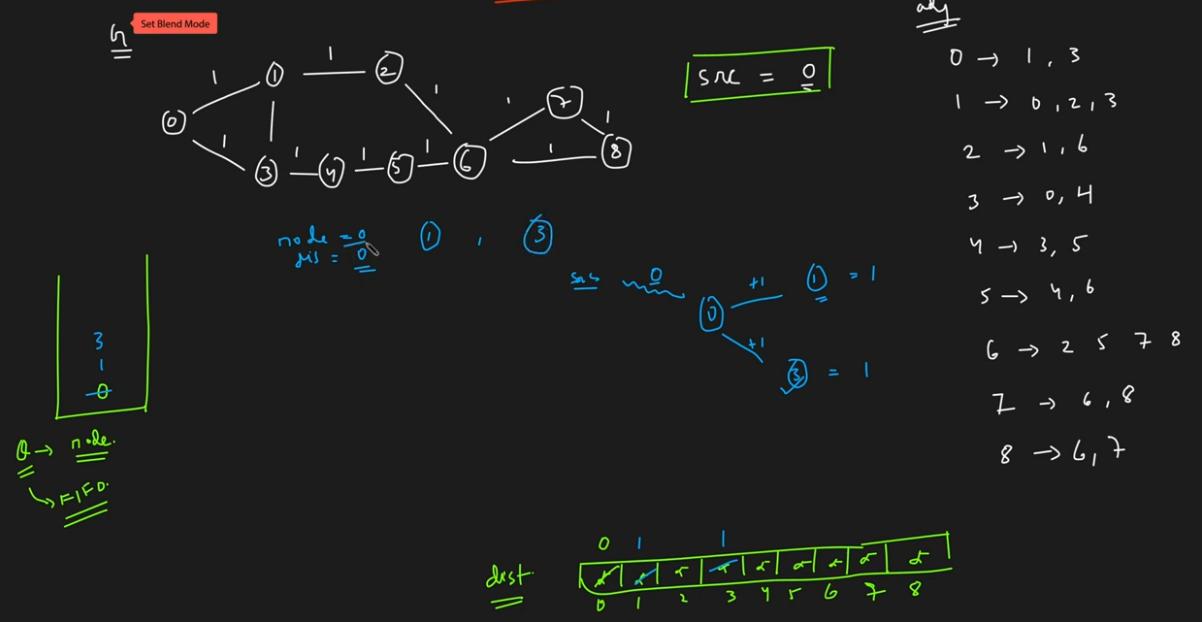
adj

- 0 → 1, 3
- 1 → 0, 2, 3
- 2 → 1, 6
- 3 → 0, 4
- 4 → 3, 5
- 5 → 4, 6
- 6 → 2, 5, 7, 8
- 7 → 6, 8
- 8 → 1, 7

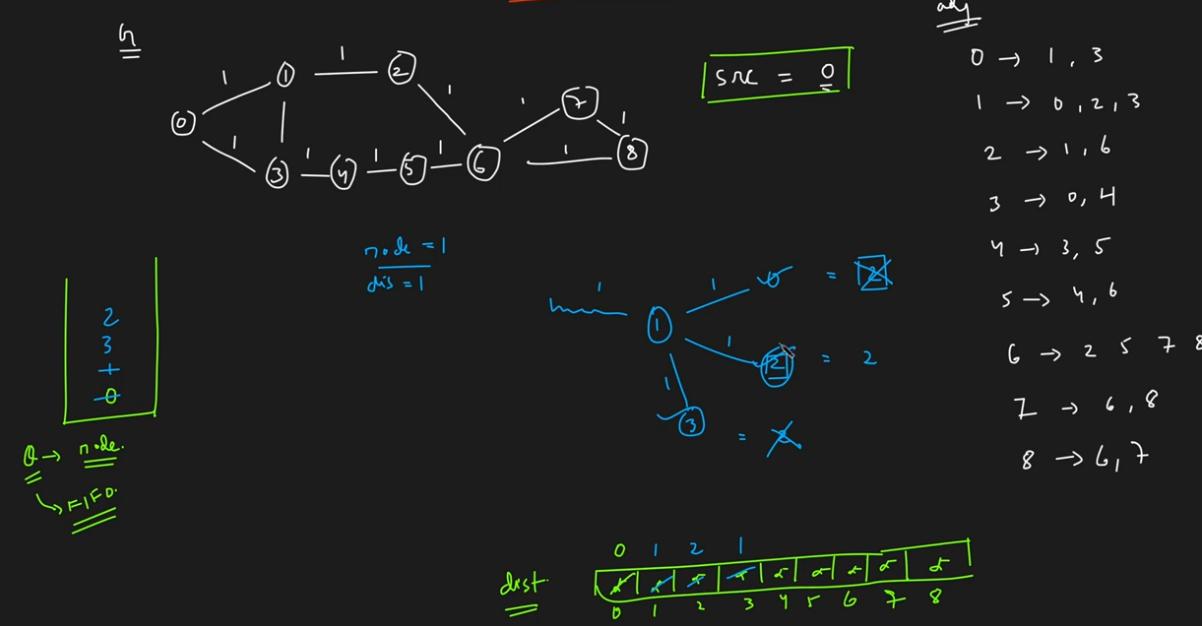
$$\text{dest} = \boxed{x | x | + | x | x | x | x | x | }$$

0 1 2 3 4 5 6 7 8

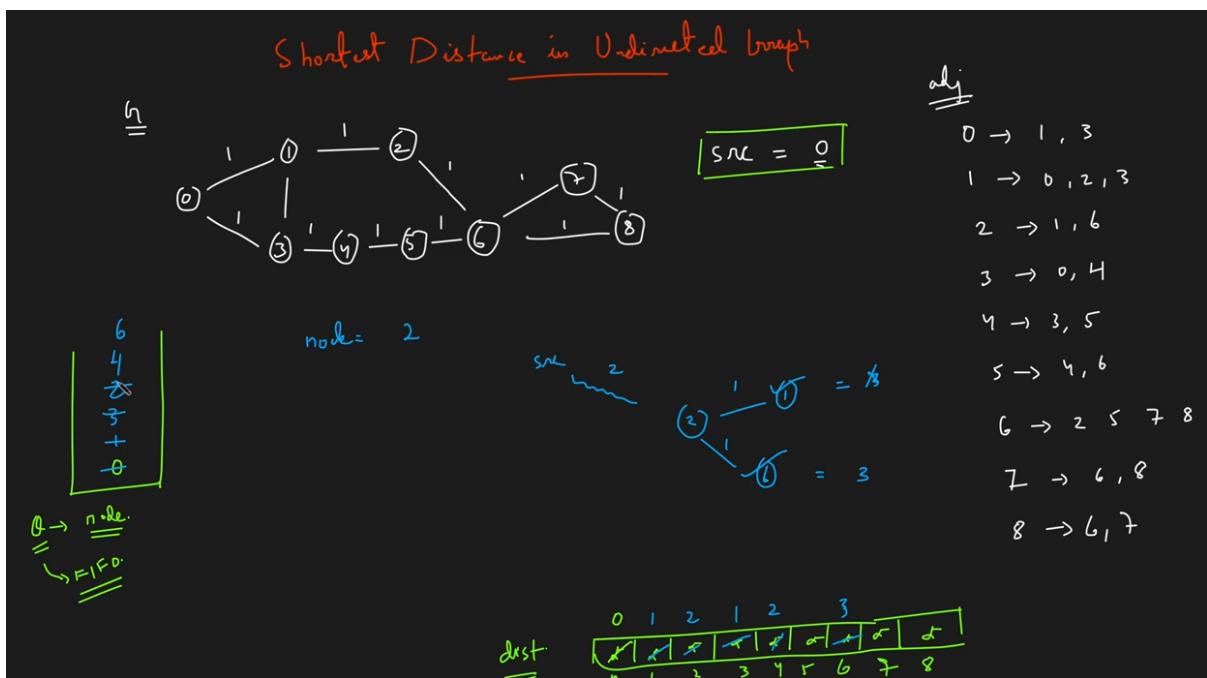
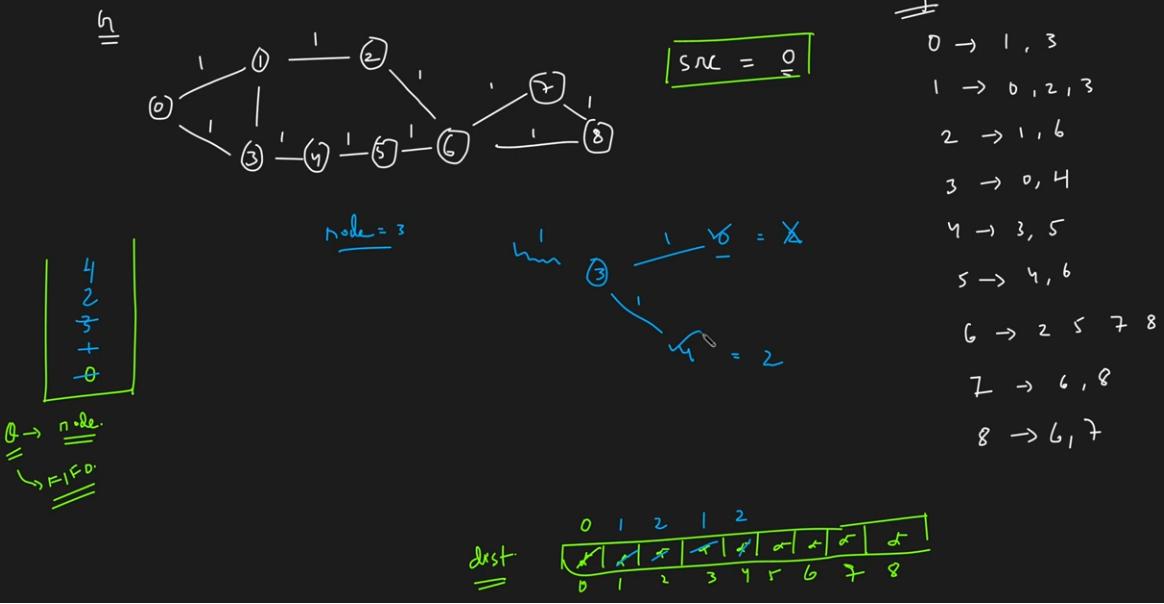
Shortest Distance in Undirected Graph



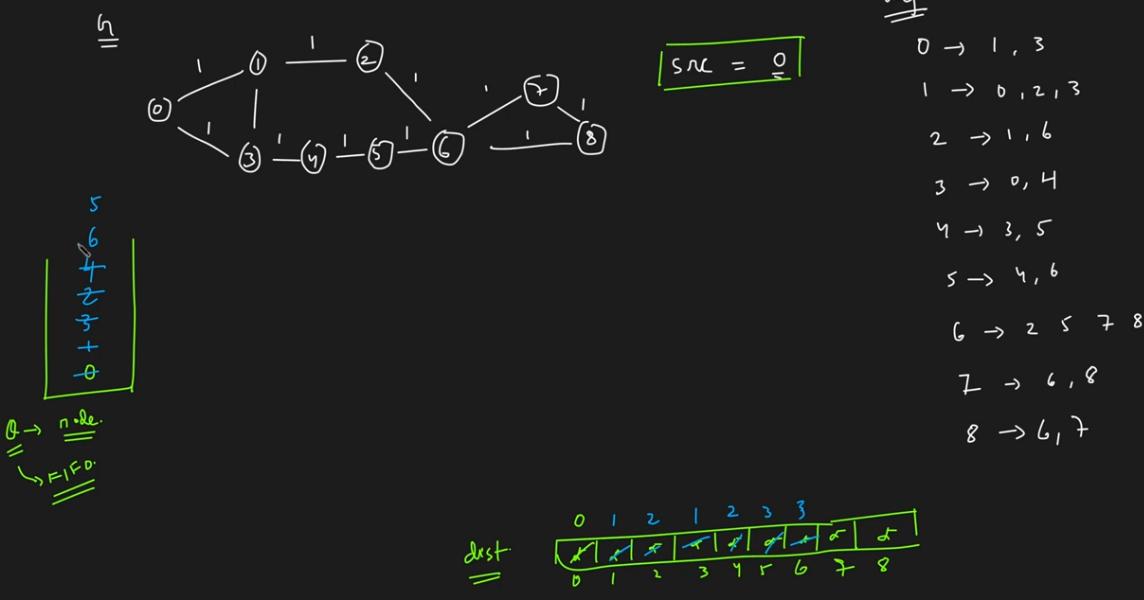
Shortest Distance in Undirected Graph



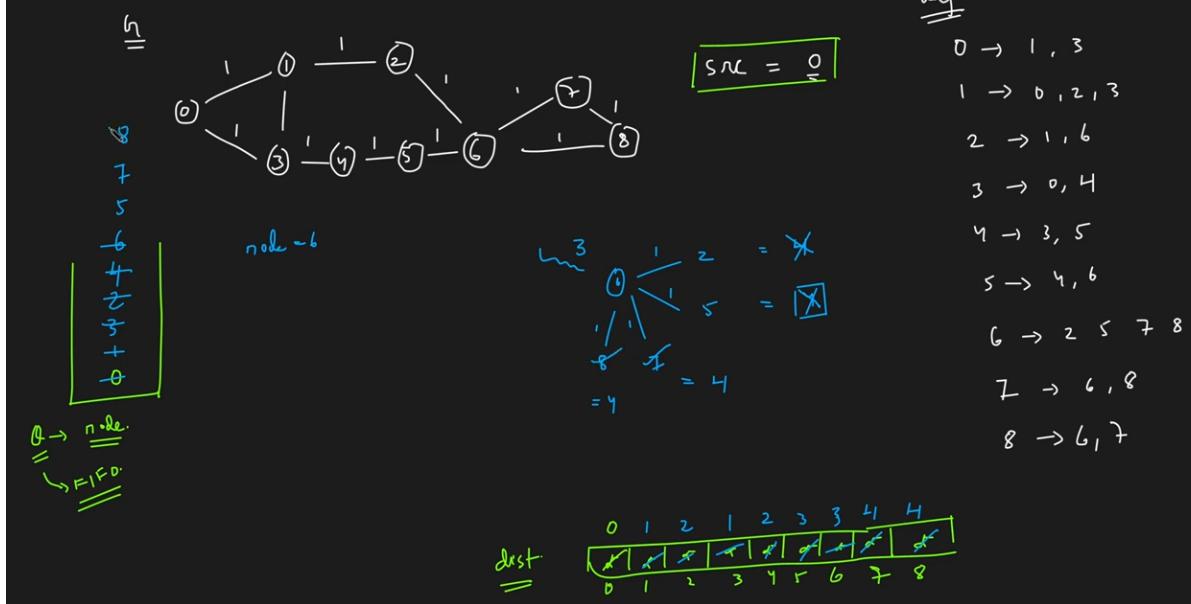
Shortest Distance in Undirected Graph

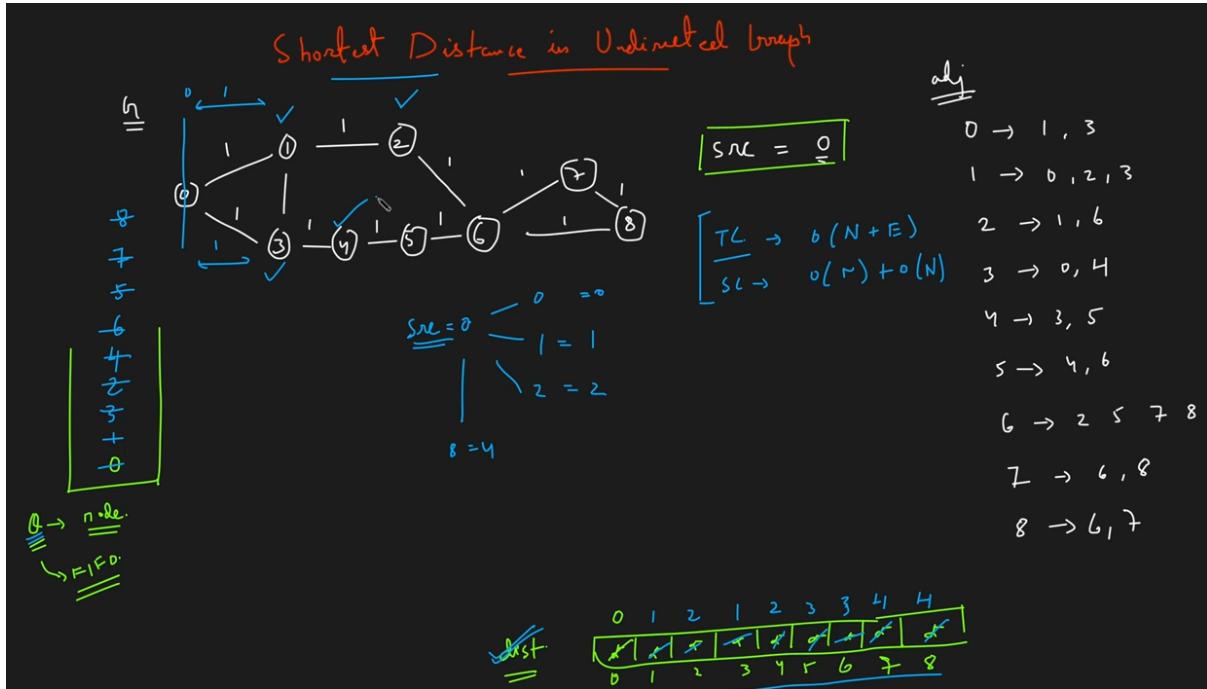


Shortest Distance in Undirected Graph



Shortest Distance in Undirected Graph





```

void BFS(vector<int> adj[], int N, int src)
{
    int dist[N];
    for(int i = 0;i<N;i++) dist[i] = INT_MAX;
    queue<int> q;

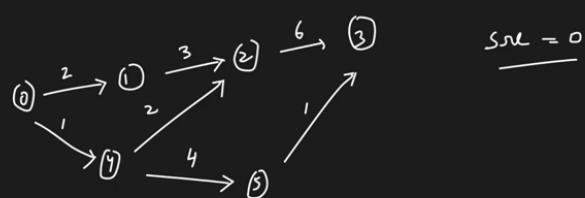
    dist[src] = 0;
    q.push(src);

    while(q.empty()==false)
    {
        int node = q.front();
        q.pop();

        for(auto it:adj[node]){
            if(dist[node] + 1 < dist[it]){
                dist[it]=dist[node]+1;
                q.push(it);
            }
        }
    }
    for(int i = 0;i<N;i++) cout << dist[i] << " ";
}

```

Shortest Path in a weighted DAG



$$src = 0$$

vector < int > adj[]

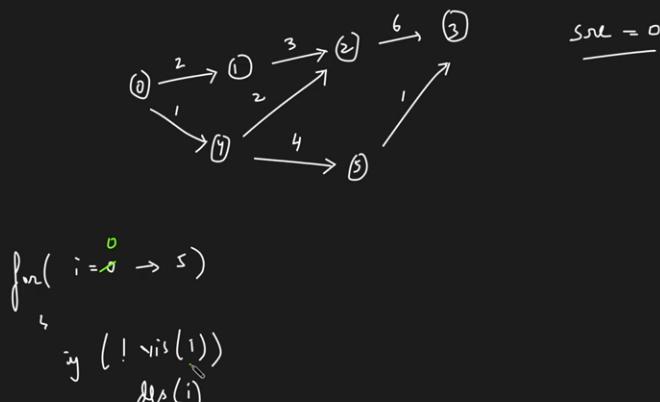
pair<int, int>

$$\begin{matrix} src & des \\ 0 & - 3 \end{matrix} = \underline{\underline{6}} \quad (0 \xrightarrow{1} 4 \xrightarrow{4} 5 \xrightarrow{1} 3)$$

class pair

<

Shortest Path in a weighted DAG



adjList.

0 → {1, 2}, {4, 1}

1 → {2, 3}

2 → {3, 4}

3 →

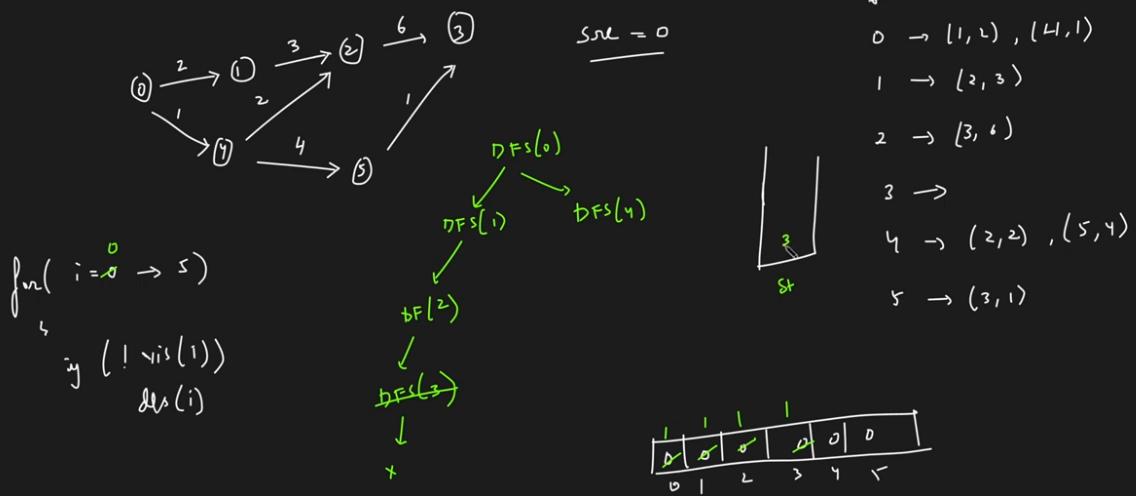
4 → {2, 5}, {5, 4}

5 → {3, 1}

0	1	2	3	4	5
0	1	2	3	4	5

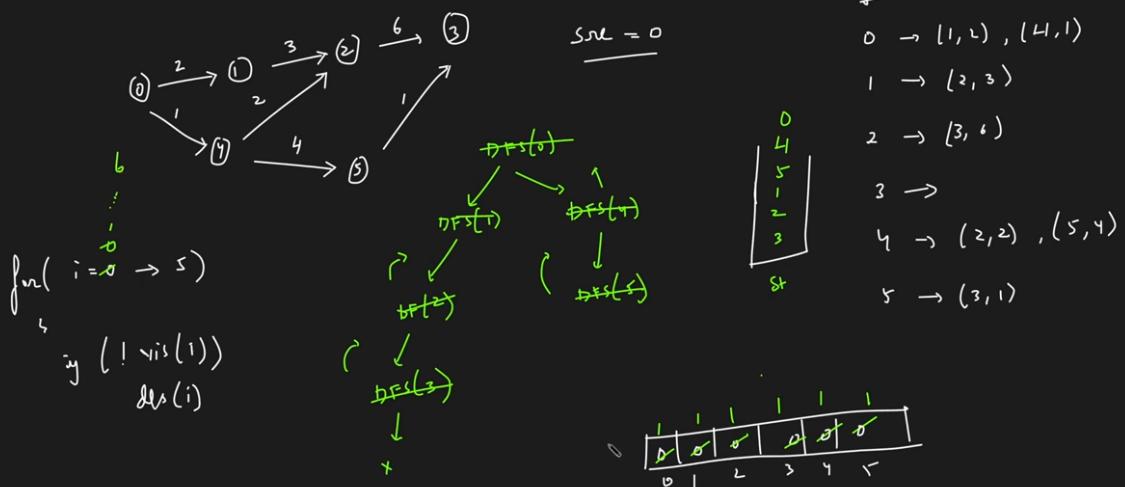
Shortest Path in a weighted DAG

adj List.



Shortest Path in a weighted DAG

adj List.



Shortest Path in a weighted DAG

Step 1: find Topo Sort



$$SND = 0$$

adj List.

$$0 \rightarrow (1, 2), (4, 1)$$

$$1 \rightarrow (2, 3)$$

$$2 \rightarrow (3, 4)$$

$$3 \rightarrow$$

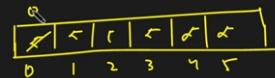
$$4 \rightarrow (2, 2), (5, 4)$$

$$\begin{matrix} \rightarrow & 0 \\ \downarrow & 1 \\ 4 & 5 & 1 & 2 & 3 & 5 \end{matrix}$$

St

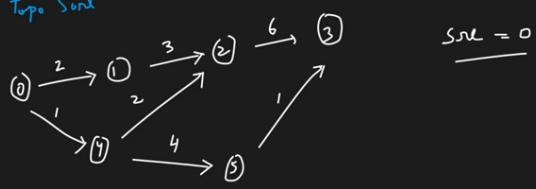
LIFO

5 → (3, 1)



Shortest Path in a weighted DAG

Step 1: find Topo Sort



$$SND = 0$$

$$node = 0$$



$$\therefore dis[0] = \infty$$

adj List.

$$0 \rightarrow (1, 2), (4, 1)$$

$$1 \rightarrow (2, 3)$$

$$2 \rightarrow (3, 4)$$

$$3 \rightarrow$$

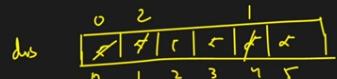
$$4 \rightarrow (2, 2), (5, 4)$$

$$\begin{matrix} \rightarrow & 0 \\ \downarrow & 1 \\ 4 & 5 & 1 & 2 & 3 & 5 \end{matrix}$$

St

LIFO

5 → (3, 1)



dis

Shortest Path is a weighted DAG

Step 1: find Topo Sort



$$src = 0$$

$$\text{by } (\text{dis}[node])! = \text{if}$$

$$\text{node} = 4 \quad \text{and} \quad (7) \xrightarrow[1]{} (5) = 3$$

$$(7) \xrightarrow[1]{} (5) = 5$$

adj List.

$$0 \rightarrow (1, 2), (4, 1)$$

$$1 \rightarrow (2, 3)$$

$$2 \rightarrow (3, 4)$$

$$3 \rightarrow$$

$$4 \rightarrow (2, 5), (5, 4)$$

$$5 \rightarrow (3, 1)$$

$$\text{dis} \quad \begin{array}{cccccc} 0 & 2 & 3 & 4 & 1 & 5 \\ \hline \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} \\ 0 & 1 & 2 & 3 & 4 & 5 \end{array}$$

Shortest Path is a weighted DAG

Step 1: find Topo Sort



$$src = 0$$

$$\text{by } (\text{dis}[node])! = \text{if}$$

$$\text{and } (5) \xrightarrow[3]{} (3) = 6$$

adj List.

$$0 \rightarrow (1, 2), (4, 1)$$

$$1 \rightarrow (2, 3)$$

$$2 \rightarrow (3, 4)$$

$$3 \rightarrow$$

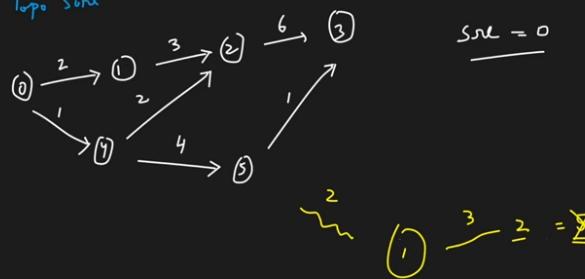
$$4 \rightarrow (2, 5), (5, 4)$$

$$5 \rightarrow (3, 1)$$

$$\text{dis} \quad \begin{array}{cccccc} 0 & 2 & 3 & 6 & 1 & 5 \\ \hline \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} \\ 0 & 1 & 2 & 3 & 4 & 5 \end{array}$$

Shortest Path in a weighted DAG

Step 1: find Topo Sort



$$\therefore \underline{\underline{y}} (\text{dis [node]}) = \underline{\underline{7}}$$

Adj List

$$0 \rightarrow (1, 2), (4, 1)$$

$$1 \rightarrow (2, 3)$$

$$2 \rightarrow (3, 6)$$

$$3 \rightarrow (4, 1)$$

$$4 \rightarrow (5, 2)$$

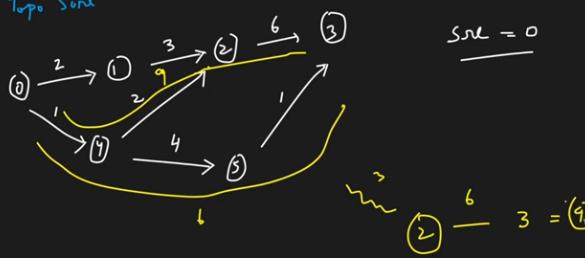
$$5 \rightarrow (3, 1)$$

$$\text{St} \quad \underline{\underline{\text{LIFO}}}$$

$$\text{dis} \quad \begin{array}{c|c|c|c|c|c} 0 & 2 & 3 & 6 & 1 & 5 \\ \hline \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} \\ \hline 0 & 1 & 2 & 3 & 4 & 5 \end{array}$$

Shortest Path in a weighted DAG

Step 1: find Topo Sort



$$\therefore \underline{\underline{y}} (\text{dis [node]}) = \underline{\underline{7}}$$

Adj List

$$0 \rightarrow (1, 2), (4, 1)$$

$$1 \rightarrow (2, 3)$$

$$2 \rightarrow (3, 6)$$

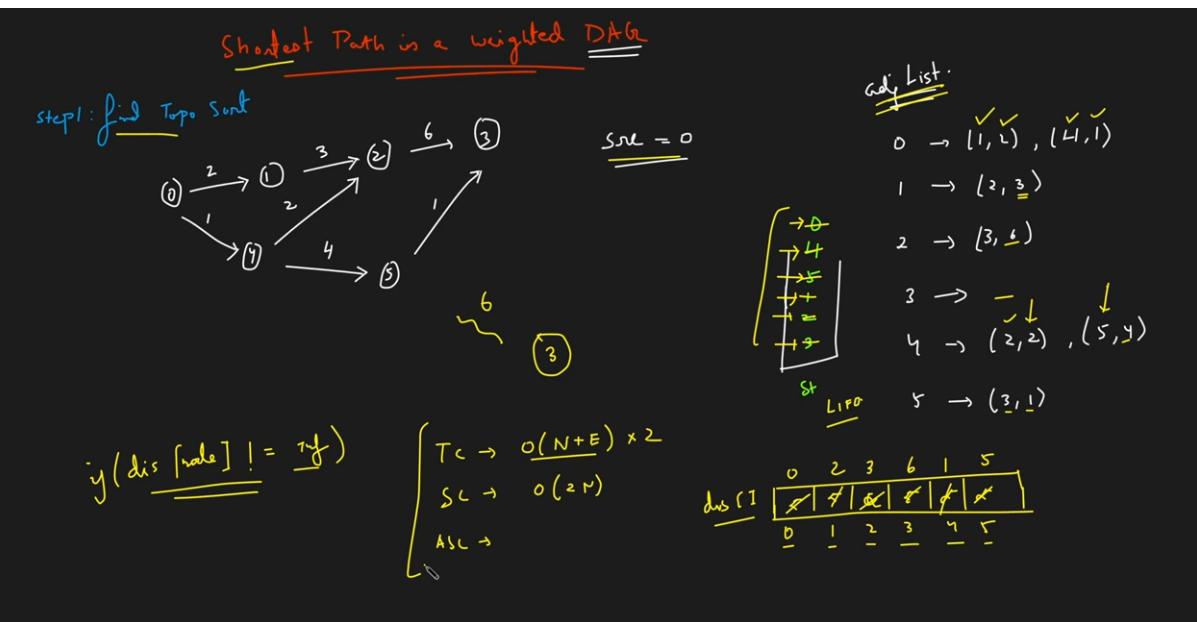
$$3 \rightarrow (4, 1)$$

$$4 \rightarrow (5, 2)$$

$$5 \rightarrow (3, 1)$$

$$\text{St} \quad \underline{\underline{\text{LIFO}}}$$

$$\text{dis} \quad \begin{array}{c|c|c|c|c|c} 0 & 2 & 3 & 6 & 1 & 5 \\ \hline \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} \\ \hline 0 & 1 & 2 & 3 & 4 & 5 \end{array}$$



```

int main()
{
    int n, m;
    cin >> n >> m;
    vector<pair<int,int>> adj[n];
    for(int i = 0;i<m;i++) {
        int u, v, wt;
        cin >> u >> v >> wt;
        adj[u].push_back({v, wt});
    }

    shortestPath(0, n, adj);

    return 0;
}

```

```

void shortestPath(int src, int N, vector<pair<int,int>> adj[])
{
    int vis[N] = {0};
    stack<int> st;
    for (int i = 0; i < N; i++)
        if (!vis[i])
            findTopoSort(i, vis, st, adj);

    int dist[N];
    for (int i = 0; i < N; i++)
        dist[i] = 1e9;
    dist[src] = 0;

    while(!st.empty())
    {
        int node = st.top();
        st.pop();

        // if the node has been reached previously
        if (dist[node] != INF) {
            for(auto it : adj[node]) {
                if(dist[node] + it.second < dist[it.first]) {
                    dist[it.first] = dist[node] + it.second;
                }
            }
        }
    }

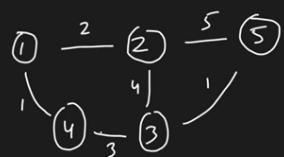
    for (int i = 0; i < N; i++)
        (dist[i] == 1e9)? cout << "INF ": cout << dist[i] << " ";
}

```

Dijkstra's Algorithm

$src = 1$

adj list



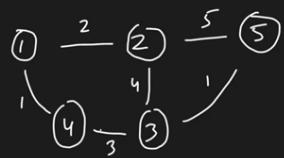
$(1 - 2)$ 2
 $(1 - 3)$ 4
 $(1 - 4)$ 1
 $(1 - 5)$ 5

$1 \rightarrow (2, 2), (4, 1)$
 $2 \rightarrow (1, 2), (5, 5), (3, 4)$
 $3 \rightarrow (2, 4), (4, 3), (5, 1)$
 $4 \rightarrow (1, 1), (3, 3)$
 $5 \rightarrow (2, 5), (3, 1)$

Dijkstra's Algorithm

$src = 1$

adj list



$1 \rightarrow (2, 2), (4, 1)$
 $2 \rightarrow (1, 2), (5, 5), (3, 1)$
 $3 \rightarrow (2, 1), (4, 3), (5, 1)$
 $4 \rightarrow (1, 1), (3, 3)$
 $5 \rightarrow (2, 5), (3, 1)$

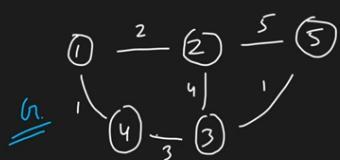


j	0	1	2	3	4	5
	∞	∞	∞	∞	∞	∞

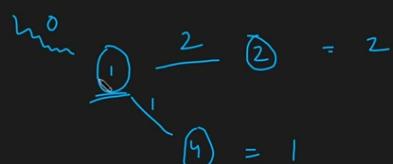
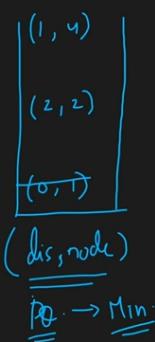
Dijkstra's Algorithm

$src = 1$

adj list



$1 \rightarrow (2, 2), (4, 1)$
 $2 \rightarrow (1, 2), (5, 5), (3, 1)$
 $3 \rightarrow (2, 1), (4, 3), (5, 1)$
 $4 \rightarrow (1, 1), (3, 3)$
 $5 \rightarrow (2, 5), (3, 1)$

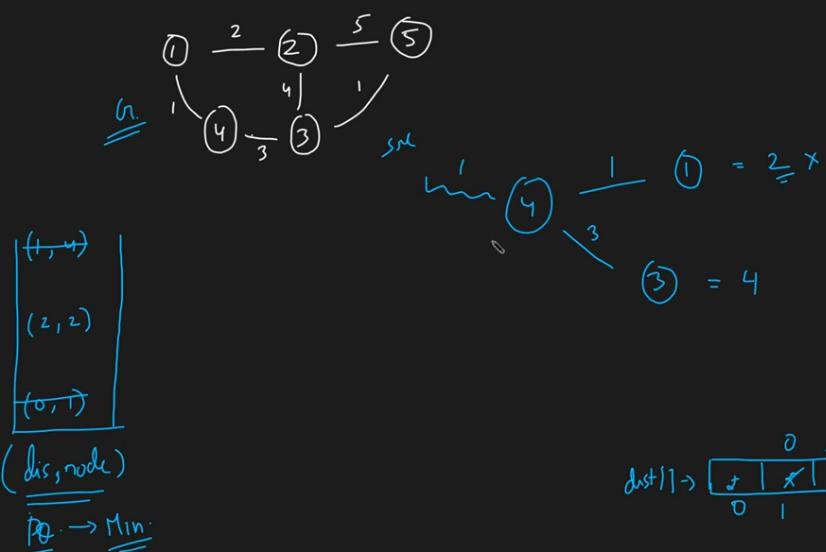


$$\text{dist}[1] = 1$$

j	0	1	2	3	4	5
	∞	2	∞	∞	∞	∞

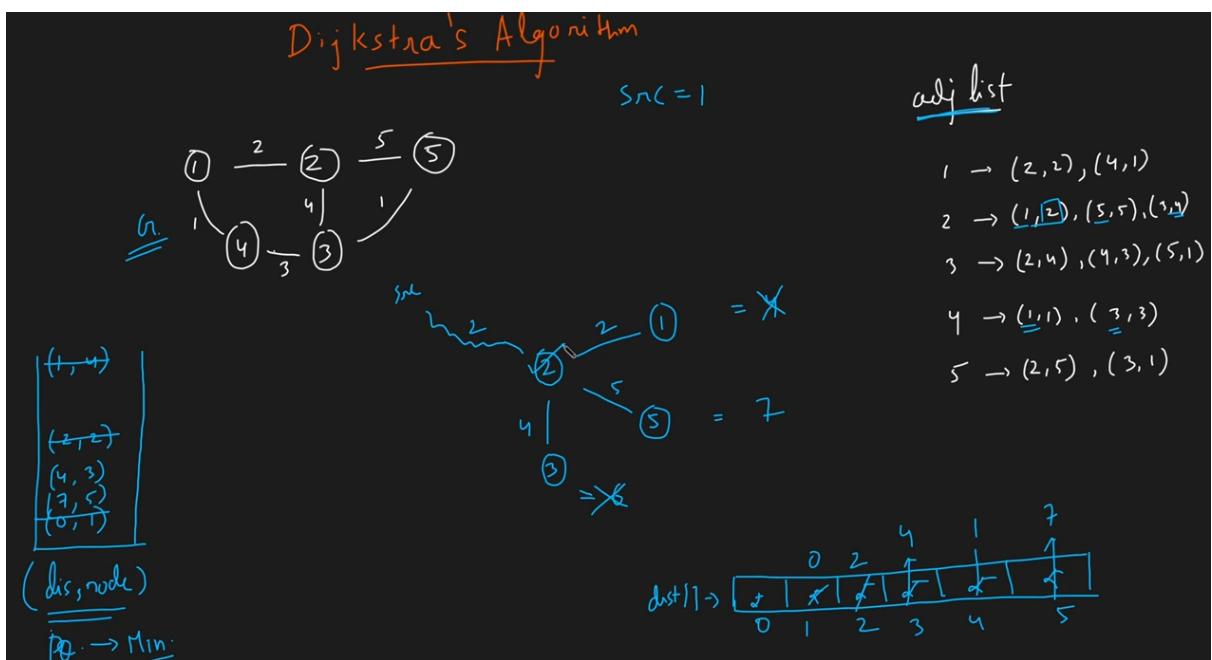
Dijkstra's Algorithm

$snc = 1$



Dijkstra's Algorithm

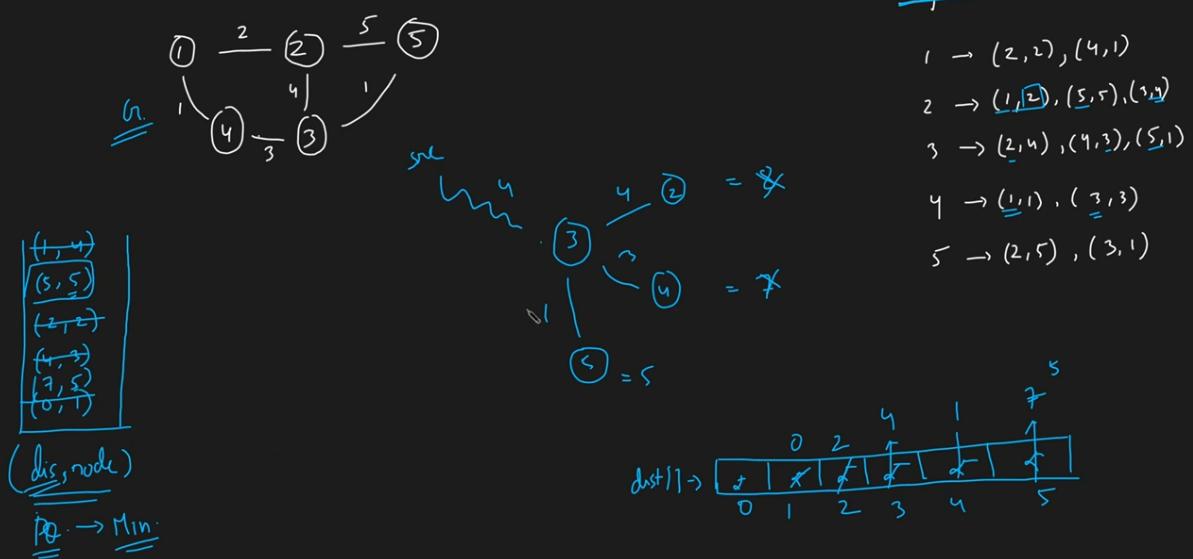
$snc = 1$



Dijkstra's Algorithm

$snc = 1$

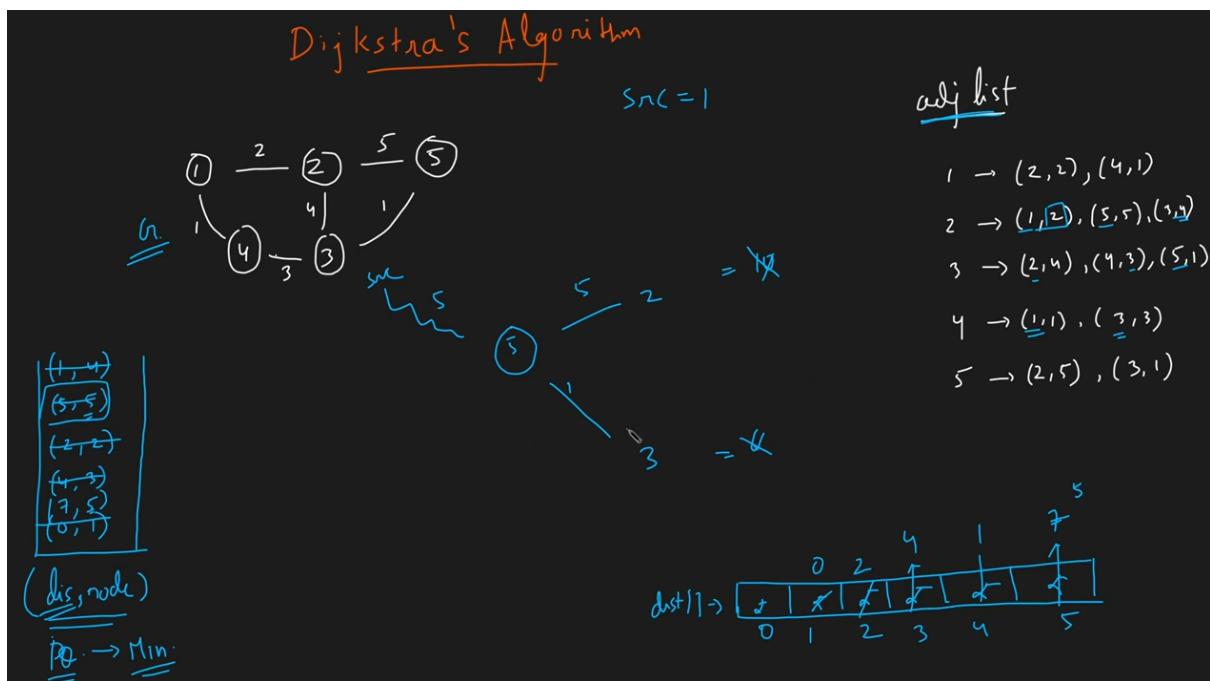
adj list



Dijkstra's Algorithm

$snc = 1$

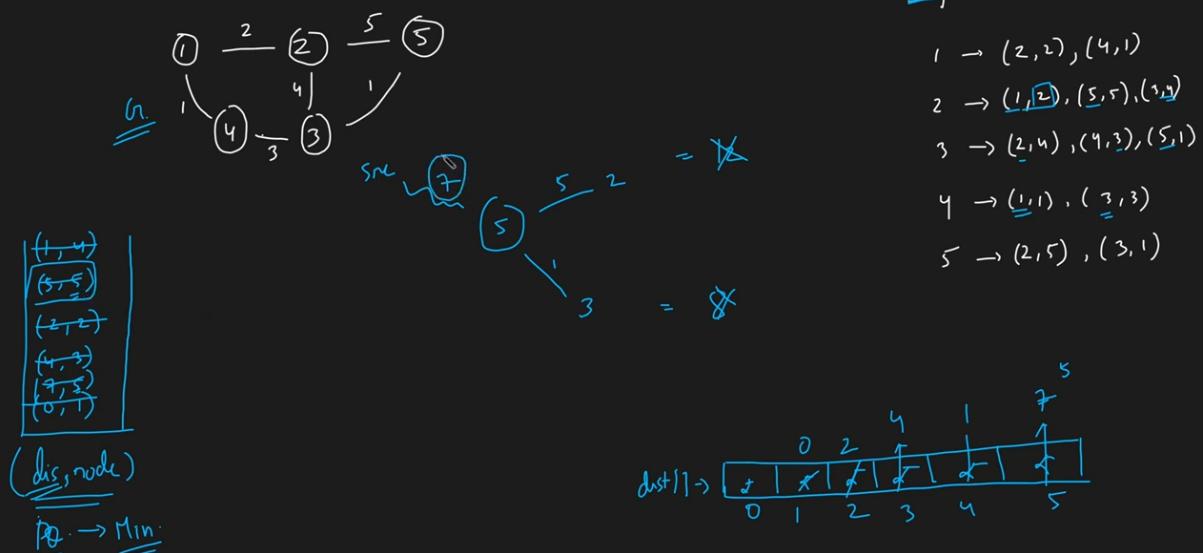
adj list



Dijkstra's Algorithm

$snc = 1$

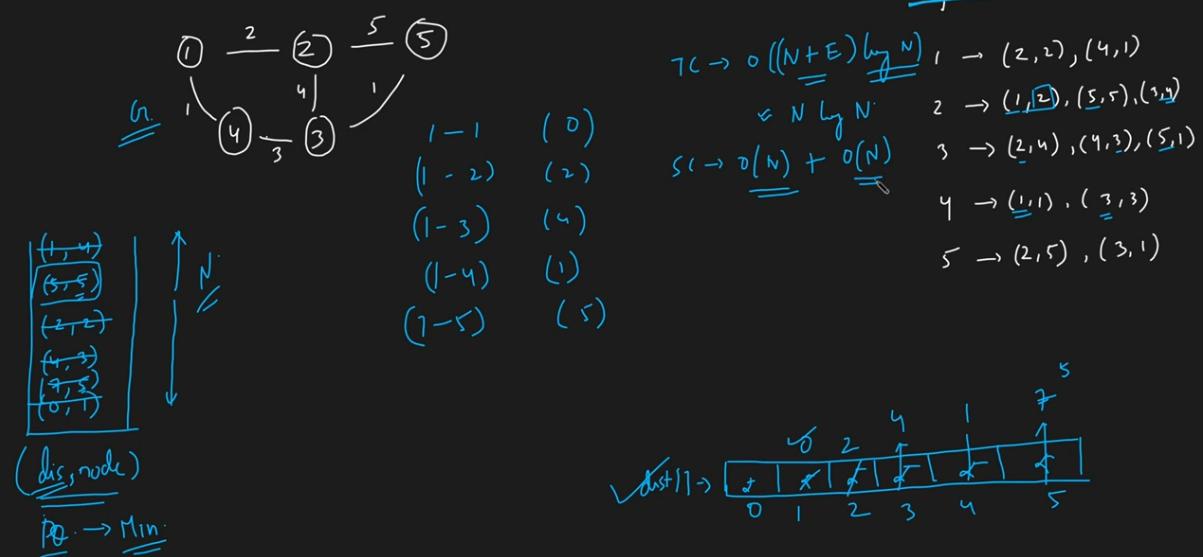
adj list



Dijkstra's Algorithm

$snc = 1$

adj list



```

1 #include<bits/stdc++.h>
2 using namespace std;
3
4 int main(){
5     int n,m,source;
6     cin >> n >> m;
7     vector<pair<int,int>> g[n+1]; // 1-indexed adjacency list for of graph
8
9     int a,b,wt;
10    for(int i = 0; i < m ; i++){
11        cin >> a >> b >> wt;
12        g[a].push_back(make_pair(b,wt));
13        g[b].push_back(make_pair(a,wt));
14    }
15
16    cin >> source;
17

```

```

// Dijkstra's algorithm begins from here
priority_queue<pair<int,int>,vector<pair<int,int>>,greater<pair<int,int>> > pq; // min-heap ; In pair => (dist
vector<int> distTo(n+1,INT_MAX); // 1-indexed array for calculating shortest paths;

distTo[source] = 0;
pq.push(make_pair(0,source)); // (dist,from)

while( !pq.empty() ){
    int dist = pq.top().first;
    int prev = pq.top().second;
    pq.pop();

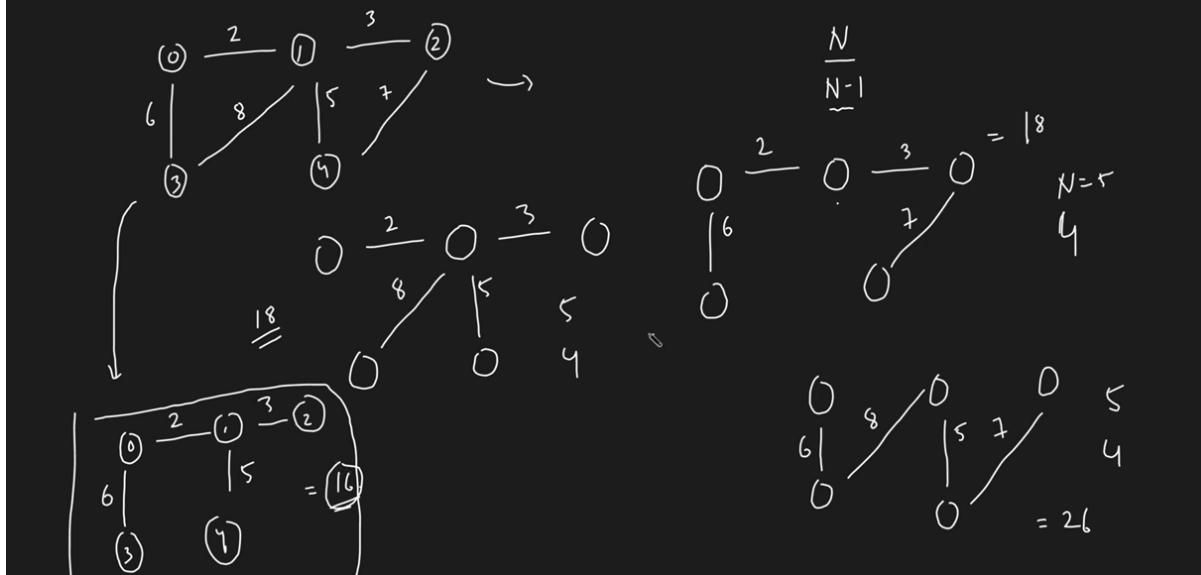
    vector<pair<int,int>>::iterator it;
    for( auto it: g[prev]){
        int next = it.first;
        int nextDist = it.second;
        if( distTo[next] > dist + nextDist){
            distTo[next] = distTo[prev] + nextDist;
            pq.push(make_pair(distTo[next], next));
        }
    }
}

cout << "The distances from source, " << source << ", are : \n";
for(int i = 1 ; i<=n ; i++) cout << distTo[i] << " ";
cout << "\n";

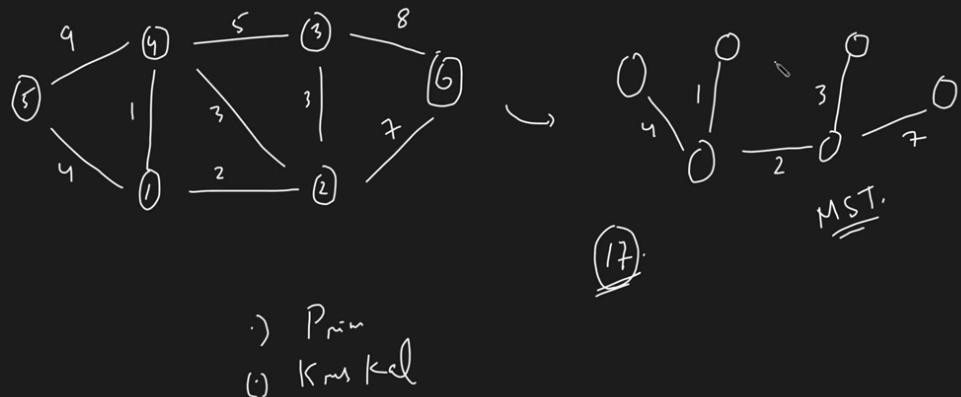
return 0;

```

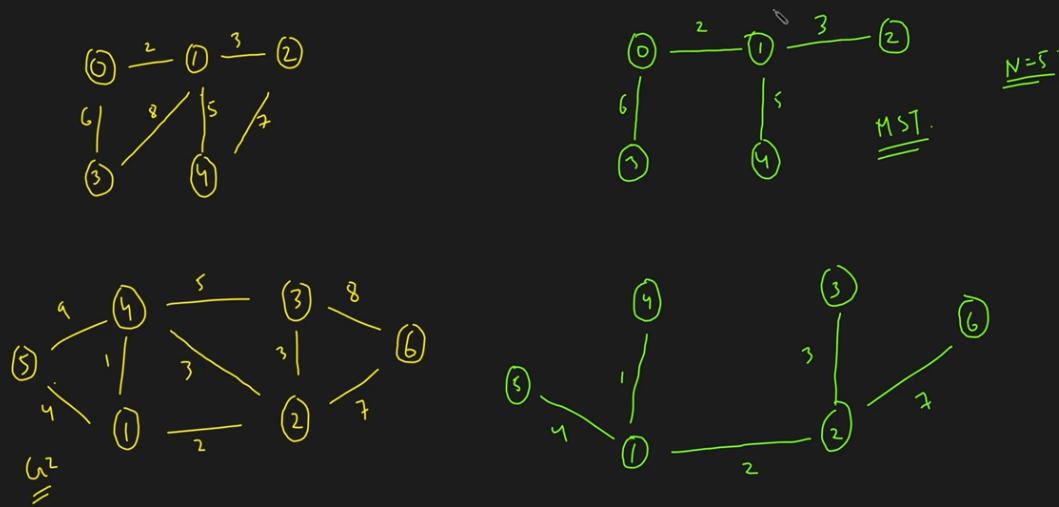
Minimum [Spanning Tree] (MST)



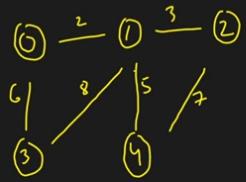
Minimum Spanning Tree (MST)



Prims Algorithm (MST)



Prim's Algorithm (MST)

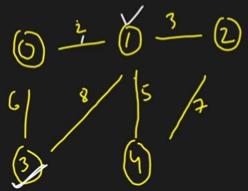


0	x	✓	✓	x
1	2	3	4	

F	F	T	F	F
0	1	2	3	4

-1	-1	-1	-1	-1
0	1	2	3	4

Prim's Algorithm (MST)



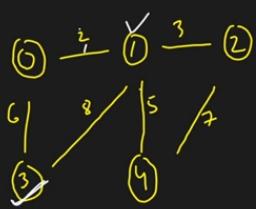
node = 0
①

0	x	✓	✓	x
1	2	3	4	

F	T	F	T	F
0	1	2	3	4

-1	0	0	-1	-1
0	1	2	3	4

Prim's Algorithm (MST)



node = 1

✗

✗ → 8
4 → 5

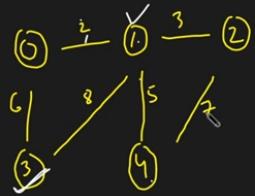
2

0	x	✓	✓	x
1	2	3	4	

F	T	T	F	F
0	1	2	3	4

-1	0	1	0	1
0	1	2	3	4

Prims Algorithm (MST) \Rightarrow



node = 2

*

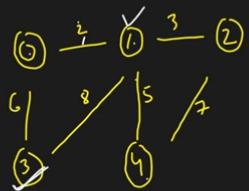
$y \rightarrow T$

	0	1	2	3	6	5
0	X	X	X	X	X	X
1		T				

	0	1	2	3	6	5
mst	F	F	F	F	F	F
0	1	2	3	4		

	0	1	2	3	6	5
punt	-1	X	T	X	X	X
0	1	2	3	4		

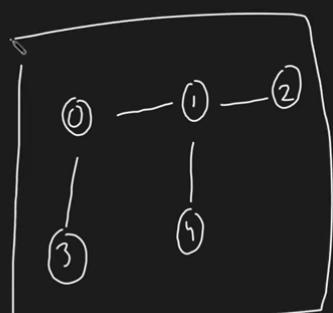
Prims Algorithm (MST) \Rightarrow



node = 4

*

*



node = 3

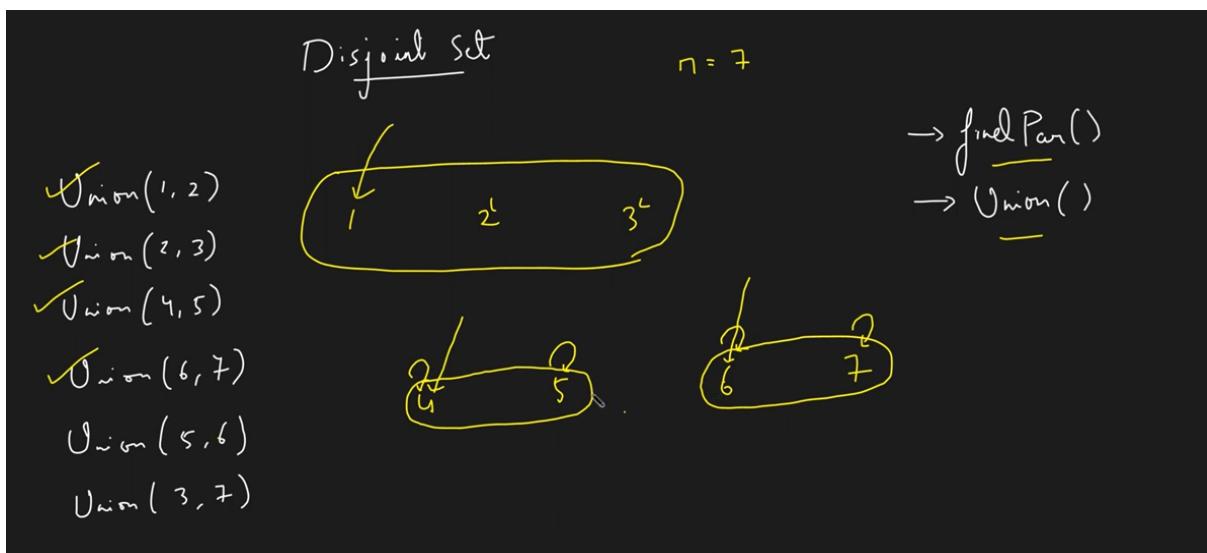
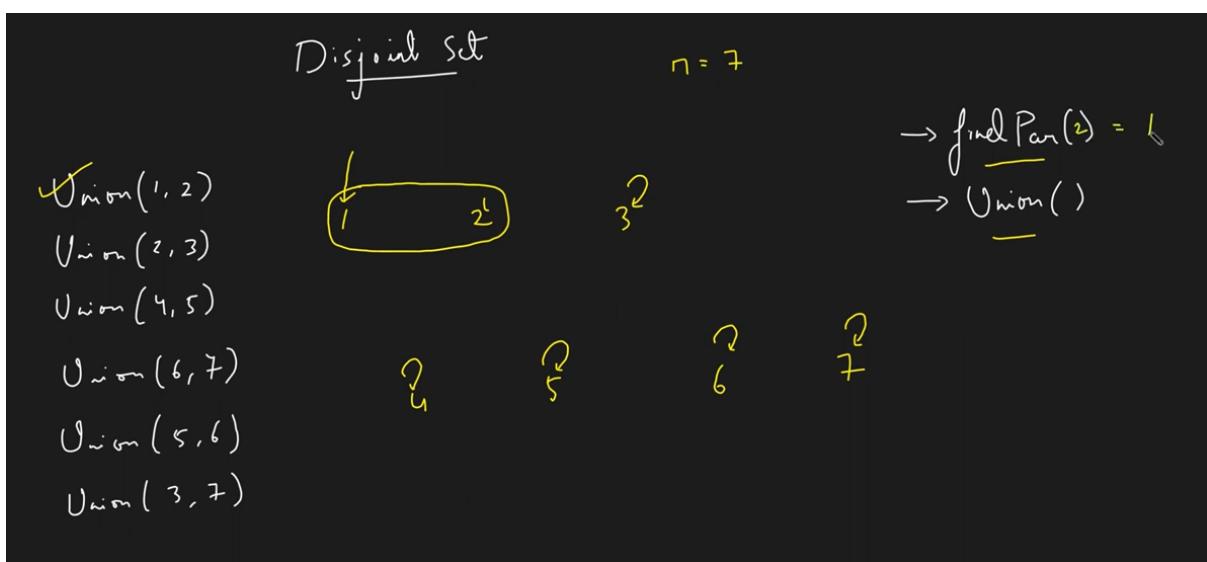
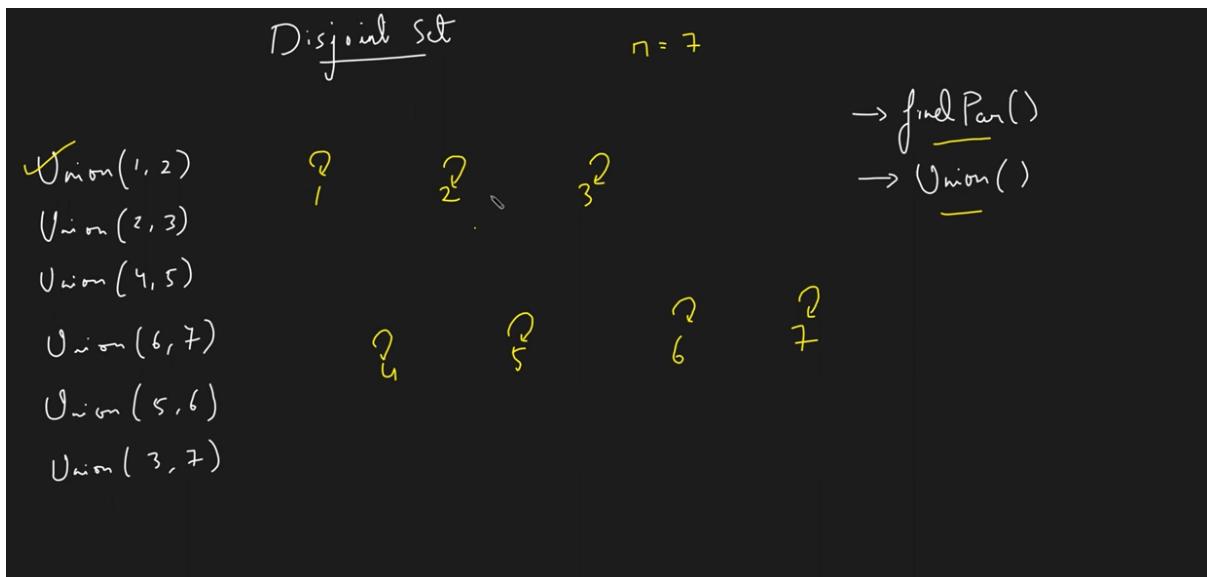
*

*

	0	1	2	3	6	5
0	X	X	X	X	X	X
1		T				

	0	1	2	3	6	5
mst	F	F	F	F	F	F
0	1	2	3	4		

	0	1	2	3	6	5
punt	-1	X	T	X	X	X
0	(1)	(2)	(3)	(4)		



Disjoint Set \rightarrow Union By Rank & Path Compression

Union(1, 2)

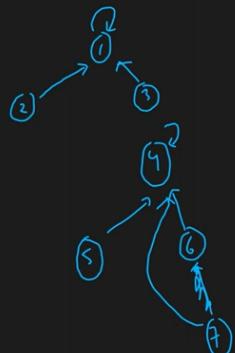
Union(2, 3)

Union(4, 5)

Union(6, 7)

Union(5, 6)

Union(3, 7)



Union(1, 2)

7 \rightarrow 6 \rightarrow 4

$O(n^2) \approx O(n)$

$\rightarrow \text{findPar}()$

$\rightarrow \text{Union}()$

Rank						
0	0	0	0	0	0	0
1	2	3	4	5	6	7

3 \rightarrow 0

7 \rightarrow 6 \rightarrow 0

8 \rightarrow 2 \rightarrow 6 \rightarrow 4

Disjoint Set \rightarrow Union By Rank & Path Compression

Union(1, 2)

Union(2, 3)

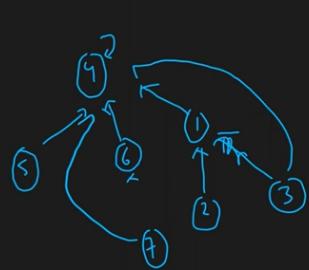
Union(4, 5)

Union(6, 7)

Union(5, 6)

Union(3, 7)

m



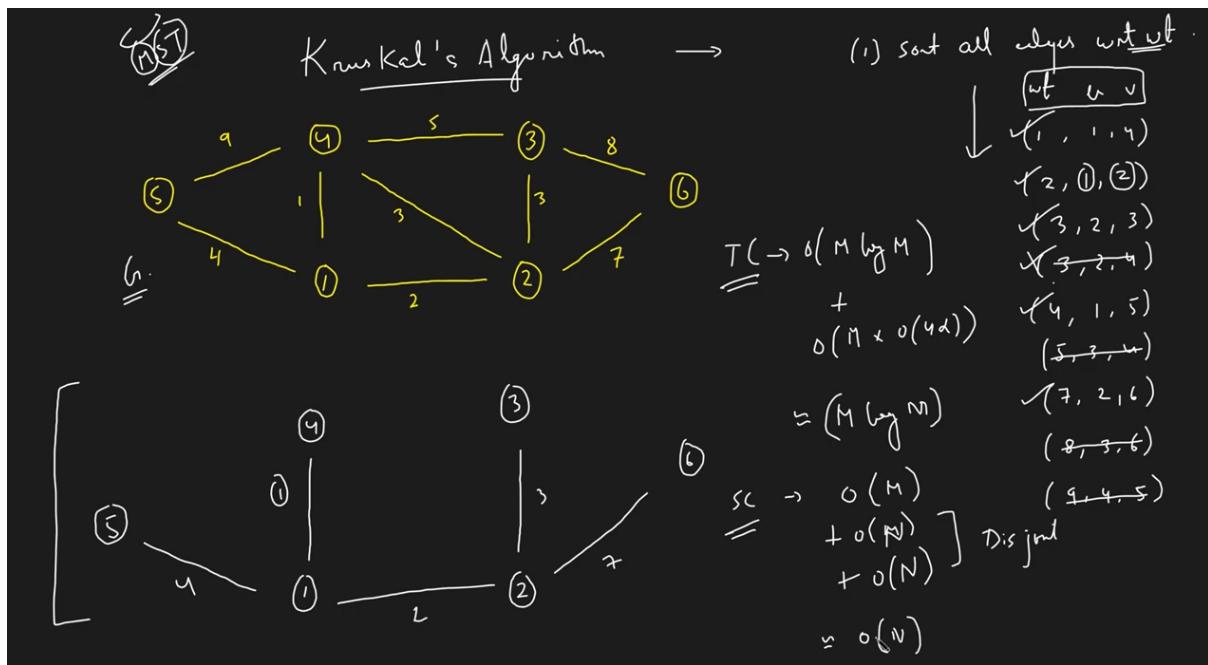
$O(n^2) \approx O(n)$

$\rightarrow \text{findPar}()$

$\rightarrow \text{Union}()$

Rank						
0	0	0	0	0	0	0
1	2	3	4	5	6	7

3 \rightarrow 1 \rightarrow 4



```

int main(){
    int N, m;
    cin >> N >> m;
    vector<node> edges;
    for(int i = 0; i < m; i++) {
        int u, v, wt;
        cin >> u >> v >> wt;
        edges.push_back(node(u, v, wt));
    }
    sort(edges.begin(), edges.end(), comp);

    vector<int> parent(N);
    for(int i = 0; i < N; i++)
        parent[i] = i;
    vector<int> rank(N, 0);

    int cost = 0;
    vector<pair<int, int>> mst;
    for(auto it : edges) {
        if(findPar(it.v, parent) != findPar(it.u, parent)) {
            cost += it.wt;
            mst.push_back({it.u, it.v});
            unionn(it.u, it.v, parent, rank);
        }
    }
    cout << cost << endl;
    for(auto it : mst) cout << it.first << " - " << it.second << endl;
    return 0;
}

```

```

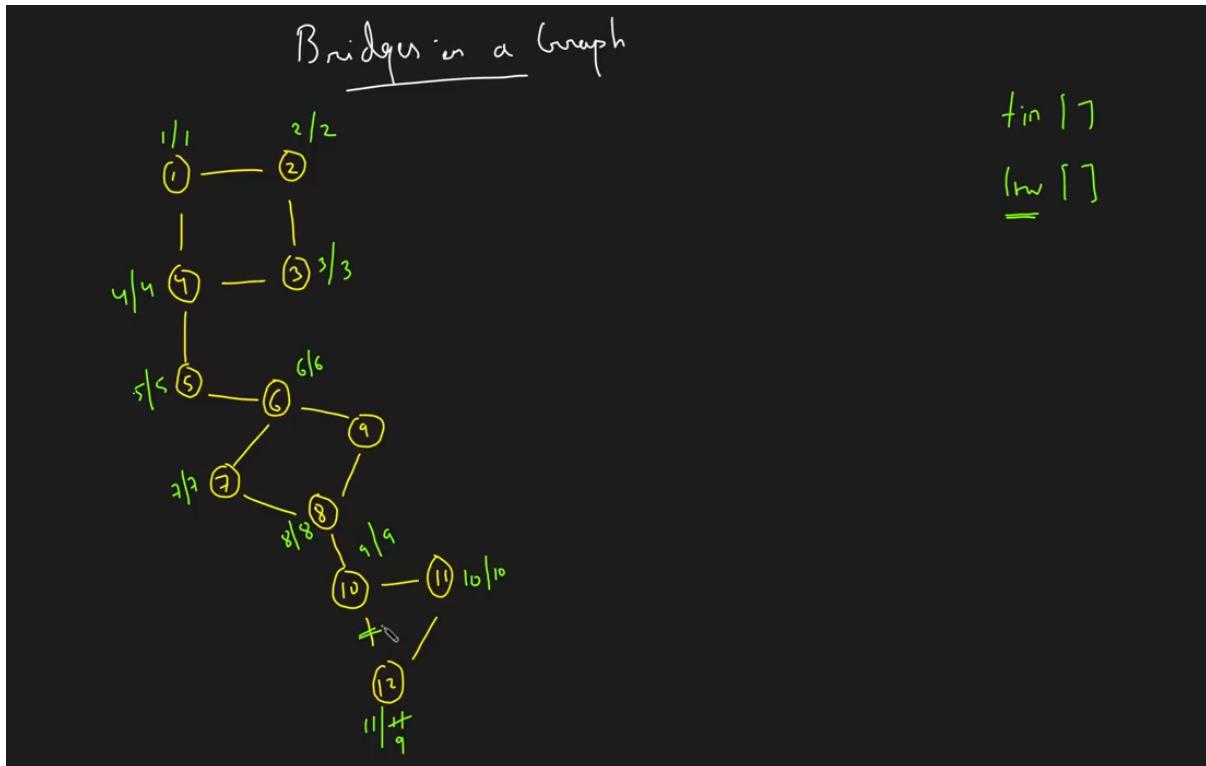
#include<bits/stdc++.h>
using namespace std;
struct node {
    int u;
    int v;
    int wt;
    node(int first, int second, int weight) {
        u = first;
        v = second;
        wt = weight;
    }
};

bool comp(node a, node b) {
    return a.wt < b.wt;
}

int findPar(int u, vector<int> &parent) {
    if(u == parent[u]) return u;
    return findPar(parent[u], parent);
}

void unionn(int u, int v, vector<int> &parent, vector<int> &rank) {
    u = findPar(u, parent);
    v = findPar(v, parent);
    if(rank[u] < rank[v]) {
        parent[u] = v;
    }
    else if(rank[v] < rank[u]) {
        parent[v] = u;
    }
    else {
        parent[v] = u;
        rank[u]++;
    }
}

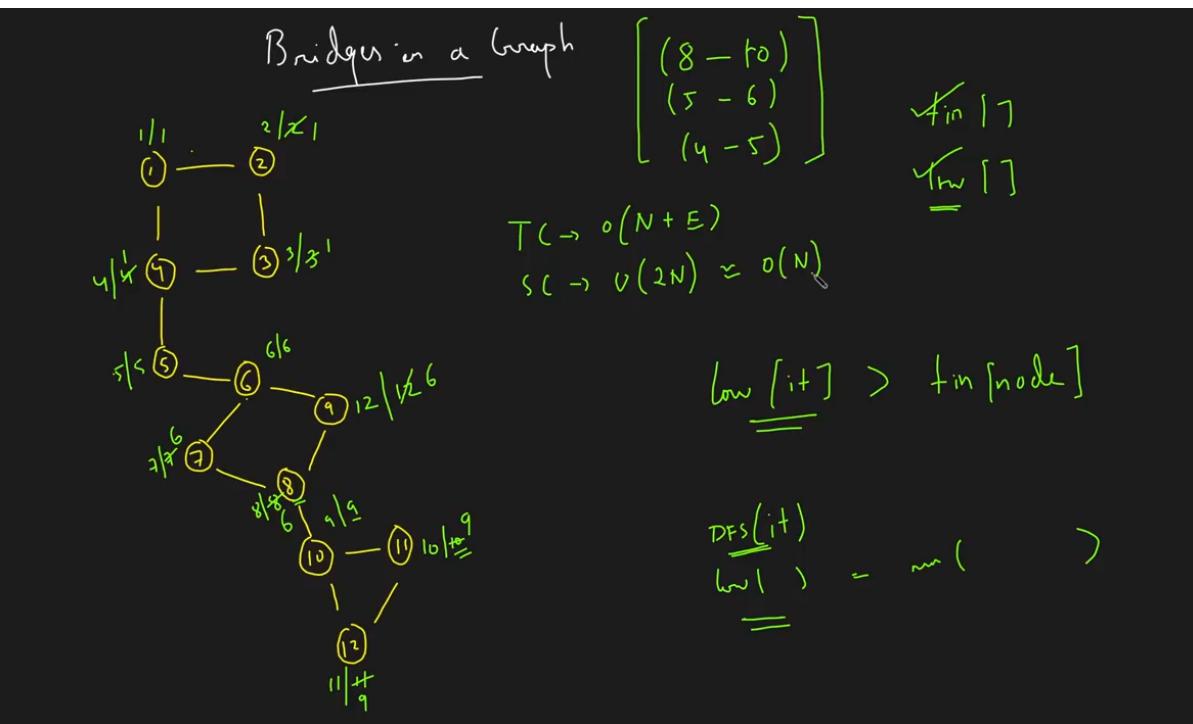
```



(10) — (11) 10 | 10

Pause and read

It cannot be bridge as even if we remove the edge (10-12), we still can reach 10, as we came from 10 only, hence its marked visited. Bridge means after edge Removal, they are two different components and can no more reach



```
int main() {
    int n, m;
    cin >> n >> m;
    vector<int> adj[n];
    for(int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }

    vector<int> tin(n, -1);
    vector<int> low(n, -1);
    vector<int> vis(n, 0);
    int timer = 0;
    for(int i = 0; i < n; i++) {
        if(!vis[i]) {
            dfs(i, -1, vis, tin, low, timer, adj);
        }
    }
}
```

```

include <bits/stdc++.h>
using namespace std;
void dfs(int node, int parent, vector<int> &vis, vector<int> &tin, vector<int> &low,
         int &timer, vector<int> adj[]) {
    vis[node] = 1;
    tin[node] = low[node] = timer++;
    for(auto it: adj[node]) {
        if(it == parent) continue;

        if(!vis[it]) {
            dfs(it, node, vis, tin, low, timer, adj);
            low[node] = min(low[node], low[it]);
            if(low[it] > tin[node]) {
                cout << node << " " << it << endl;
            }
        } else {
            low[node] = min(low[node], tin[it]);
        }
    }
}

```

