Representing Position & Orientation

IN ROBOTICS

CSC4702-1 ROBOTIC SYSTEM DEVELOPMENT

Lesson Outline

01

Understand coordinate frames and their role in robotics

02

Represent points and vectors across frames

03

Apply translation and rotation transformations

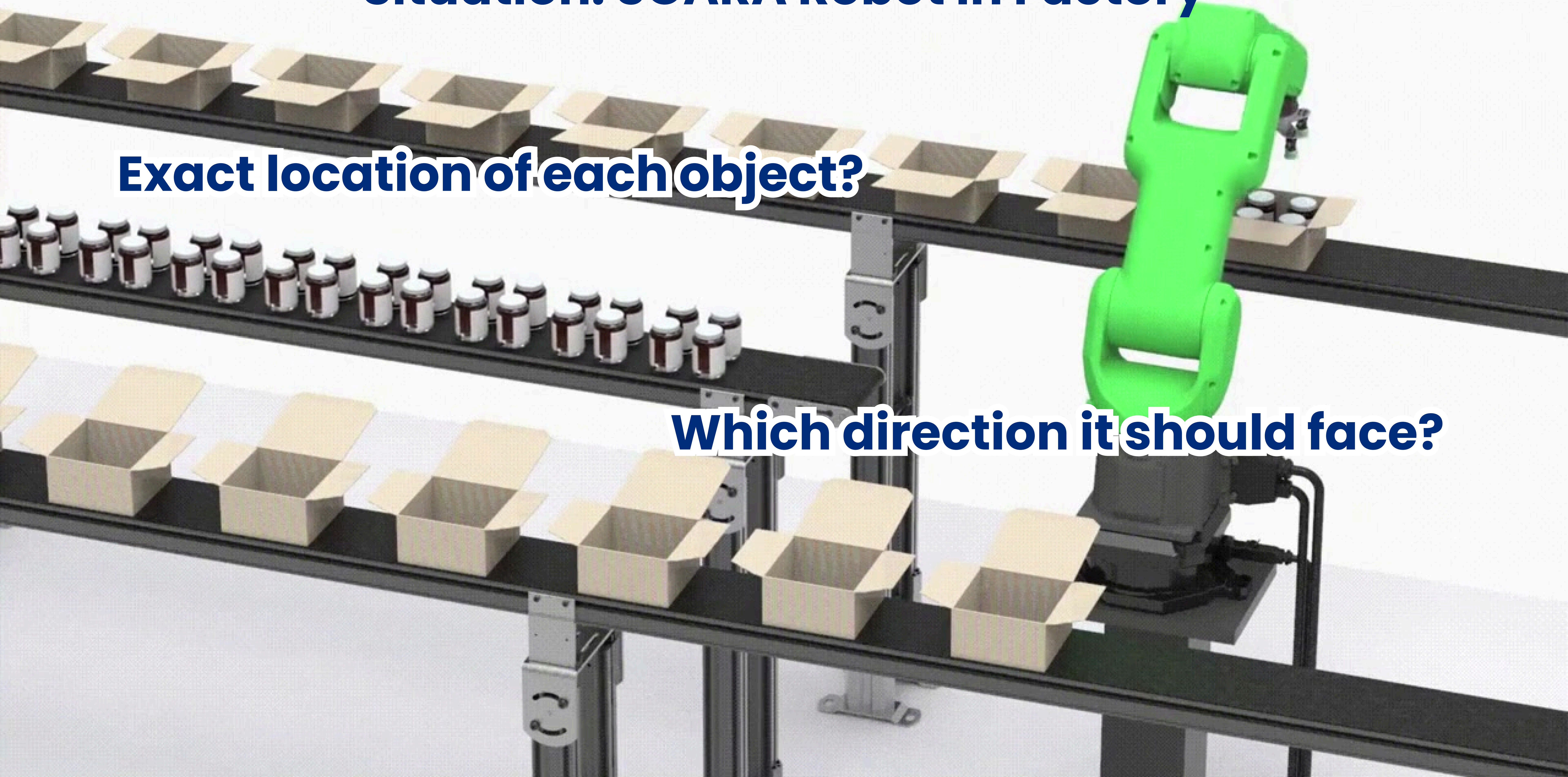
04

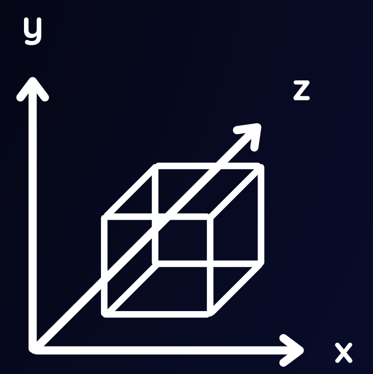
Visualize frame interactions with tools

05

Analyse sensor and robot motion using frame transformation

Situation: SCARA Robot in Factory

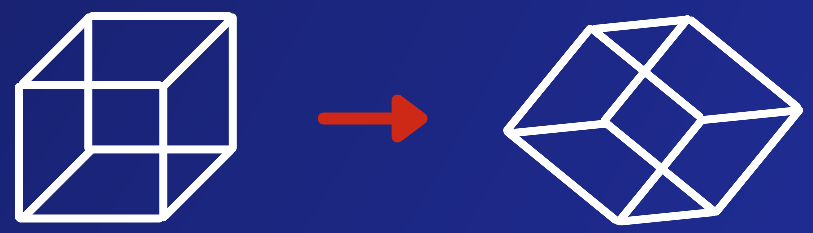




Where robot and target object located in space e.g. (x, y, z)

Position & Orientation

How robot and object is rotated or angled

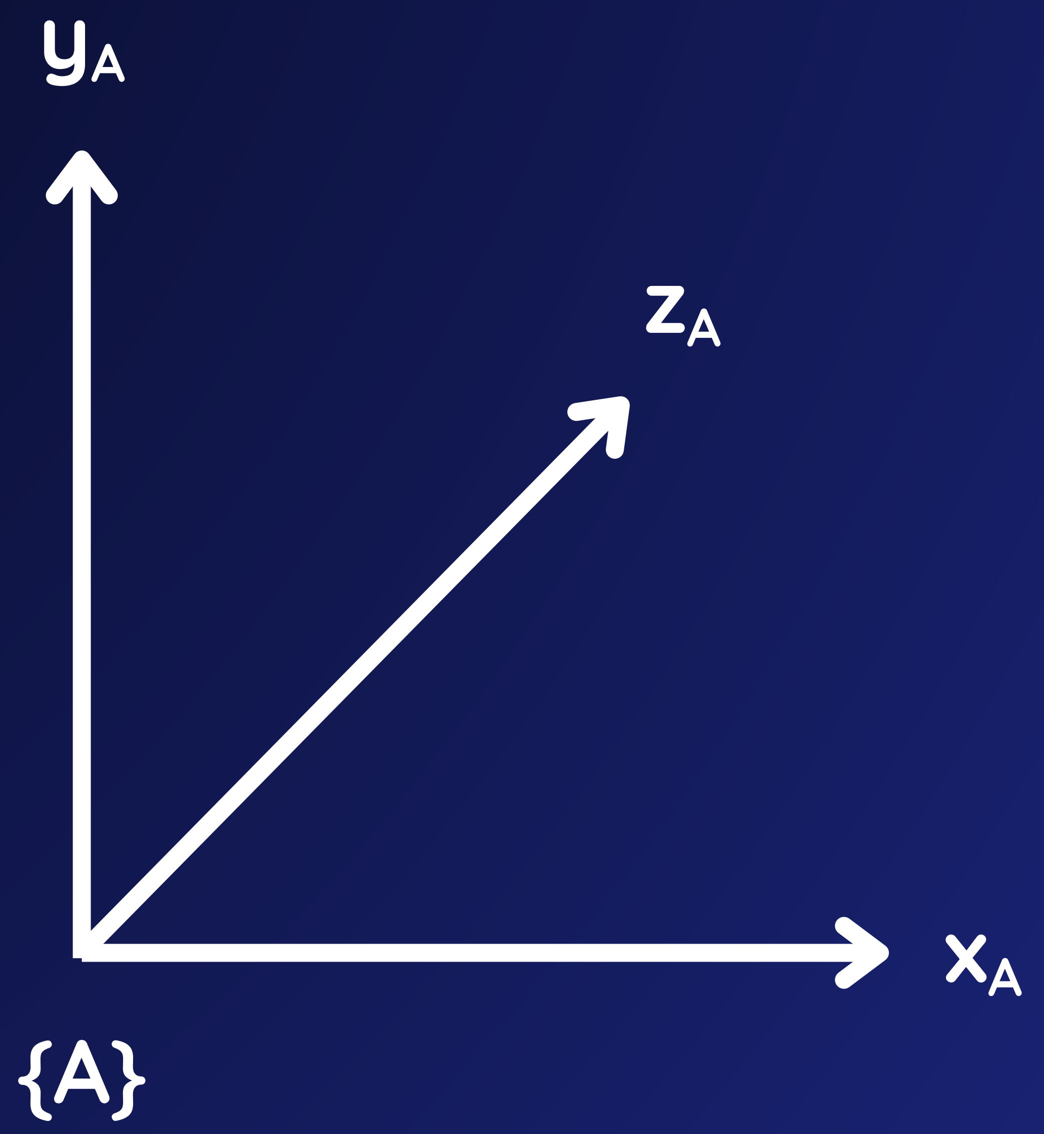


Position + Orientation

= Pose

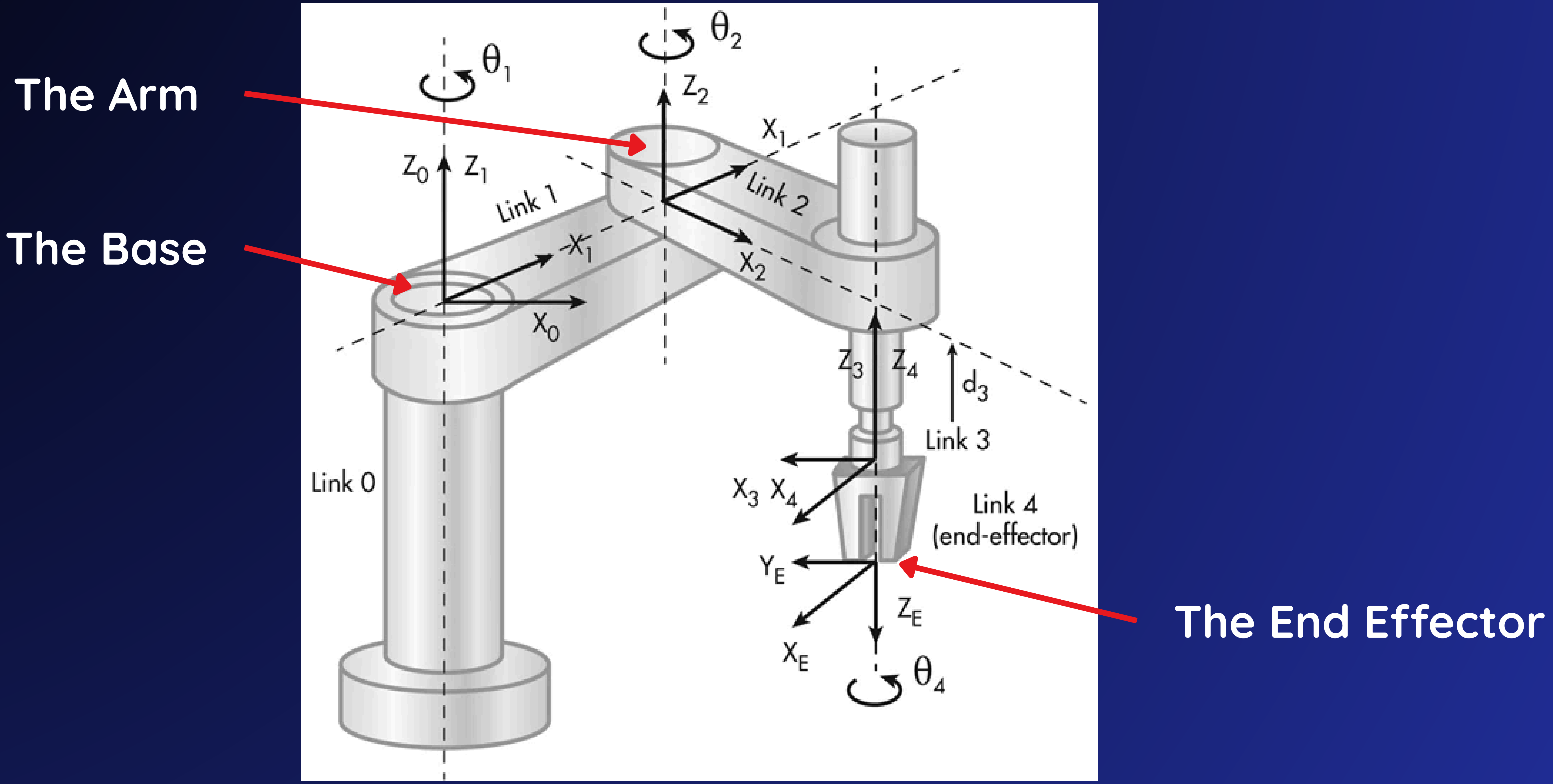
Robot complete state within its environment

Coordinate Frame



Reference system for position and direction measurement

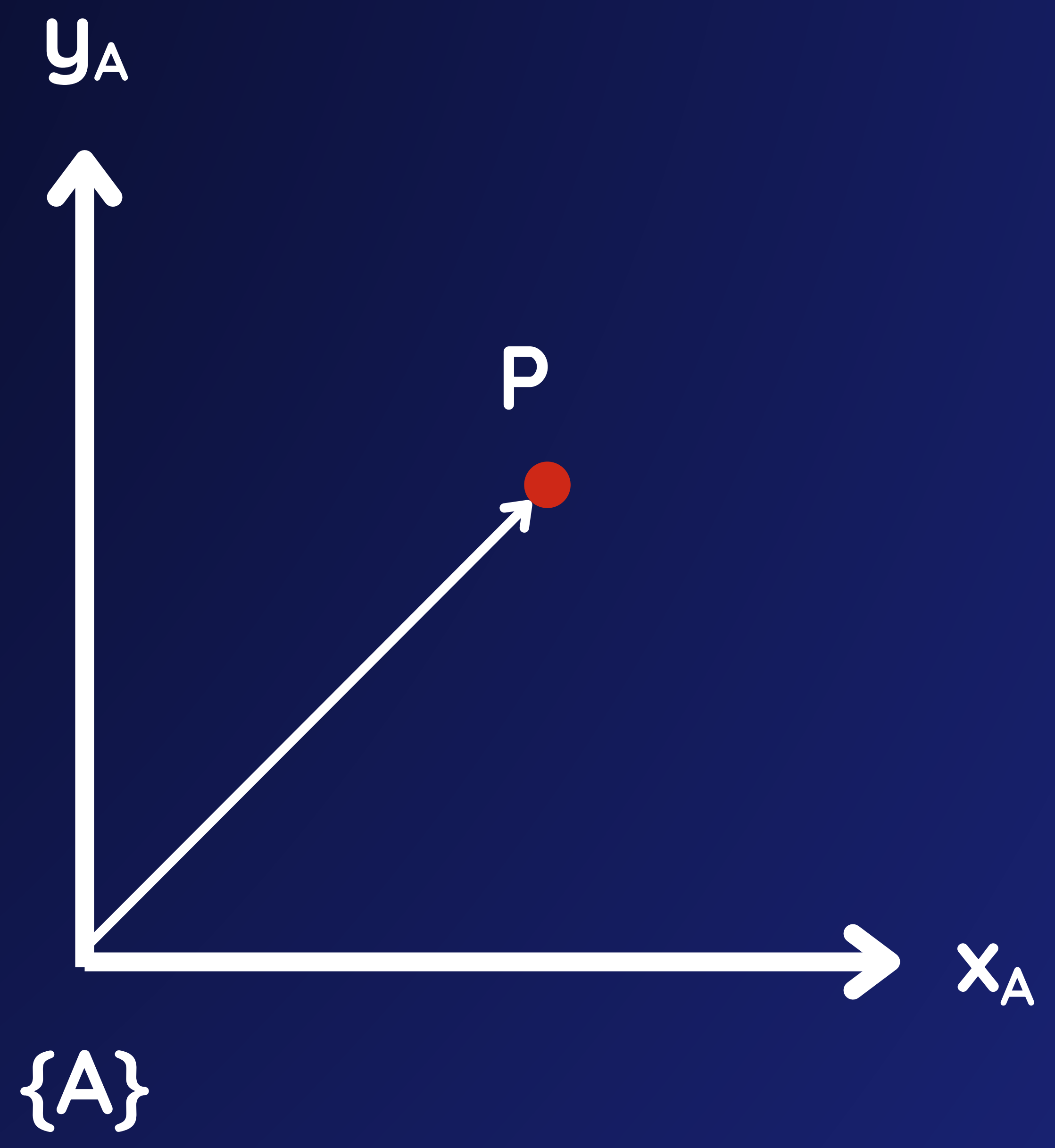
Coordinate Frame



Points & Vectors

Point

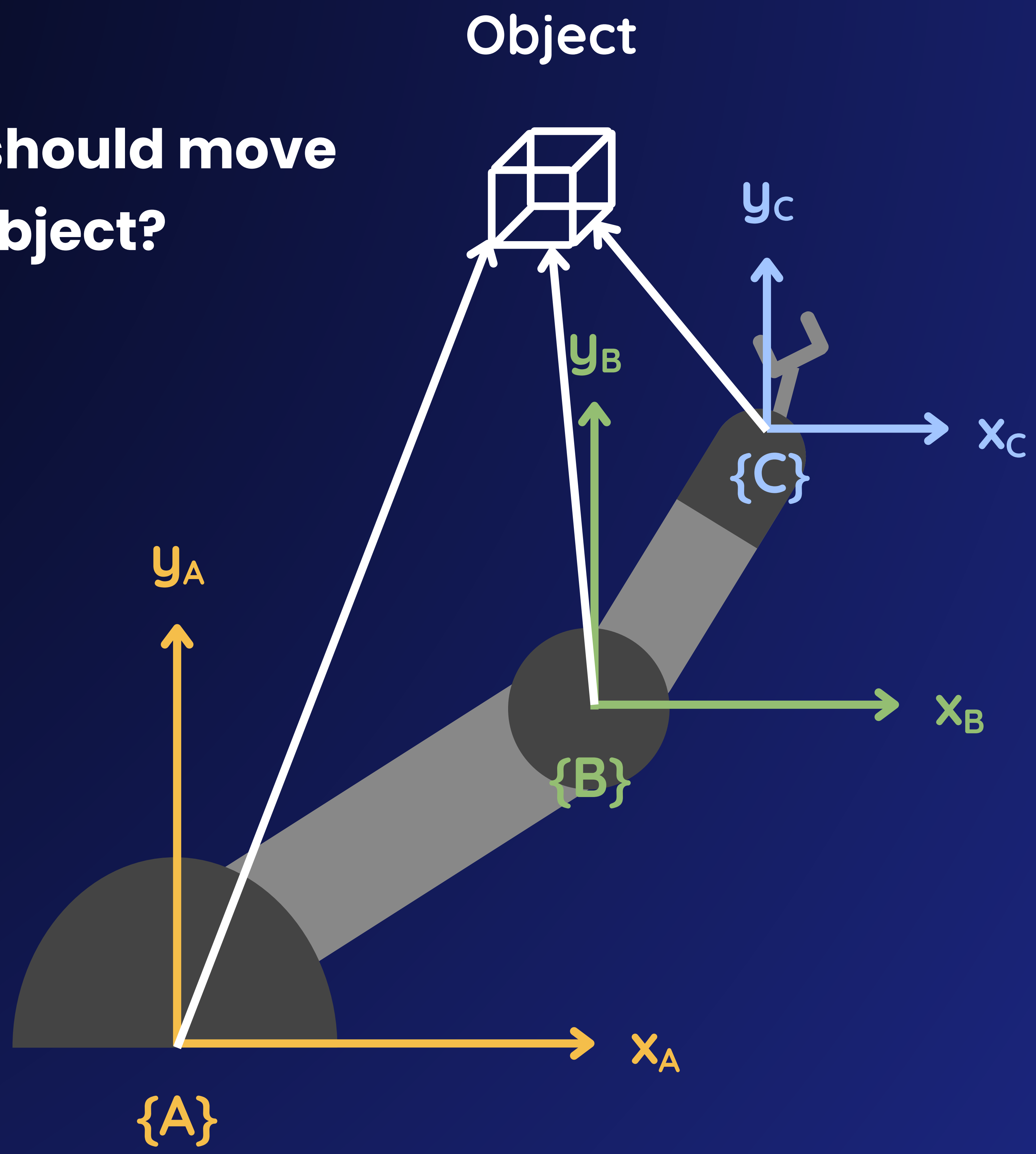
Location in space



Vector

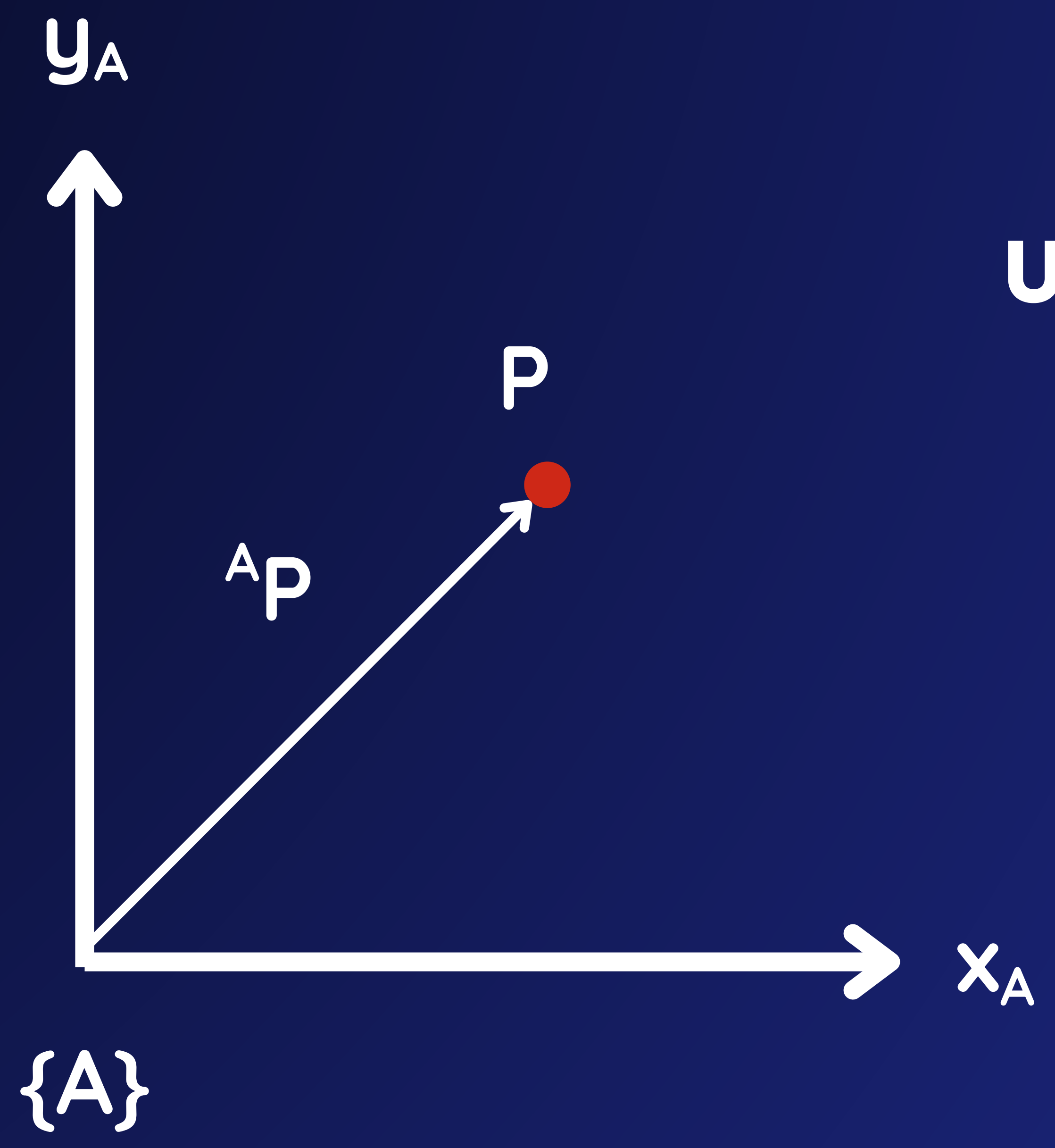
Direction and magnitude

How robot arm should move to reach object?



Position Vector

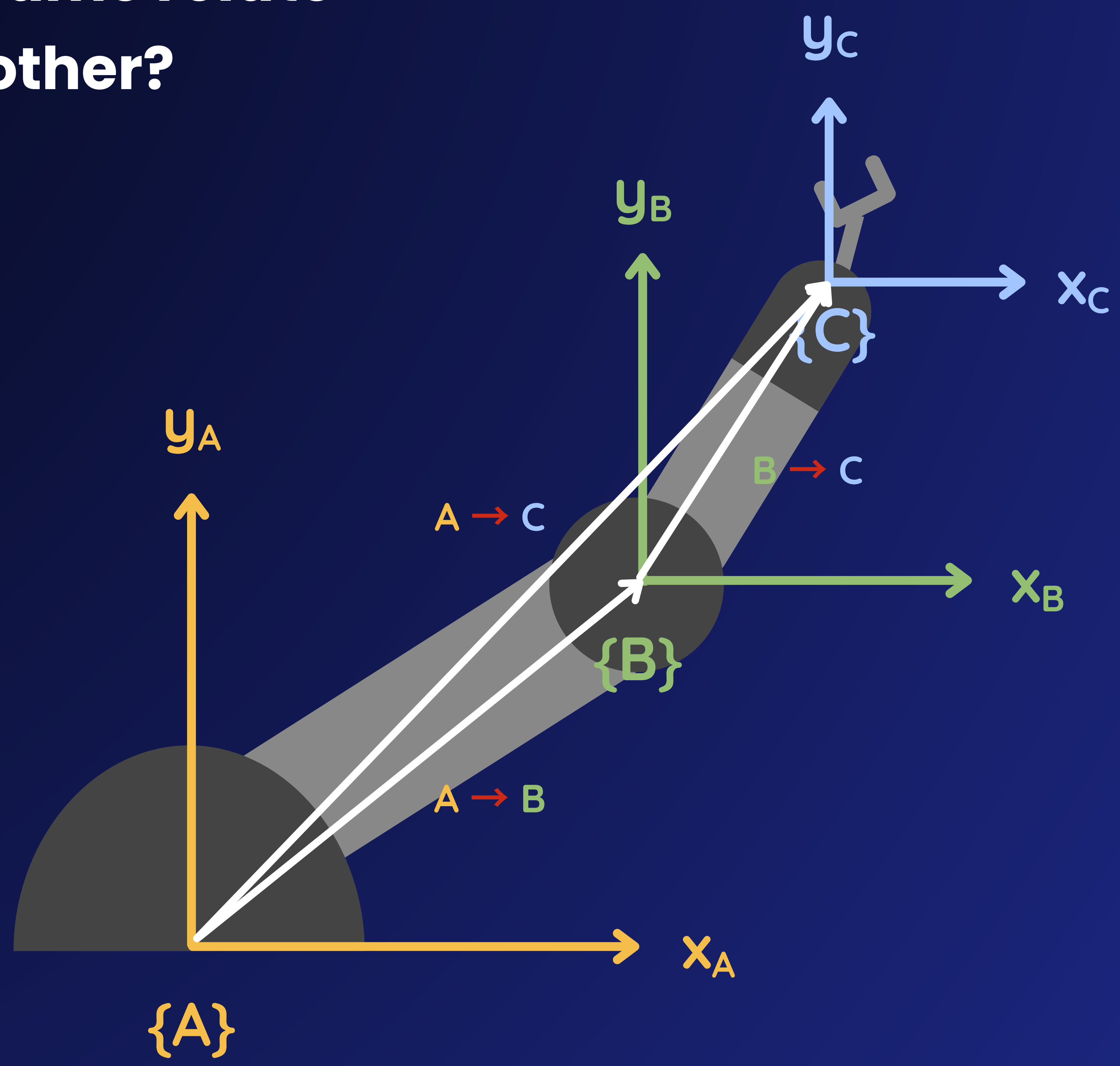
$$^{A}P = \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$

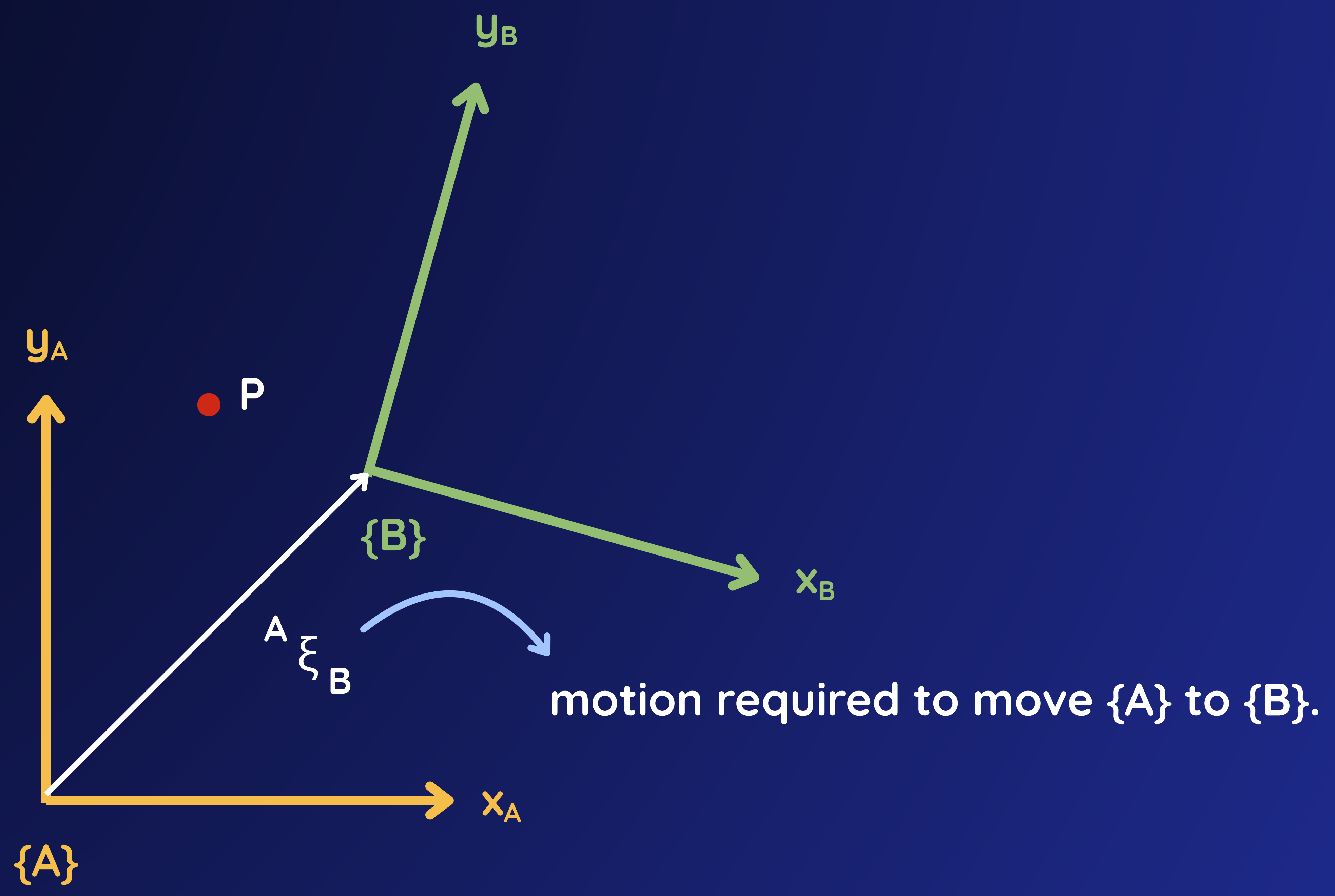


Unit Vector Notation

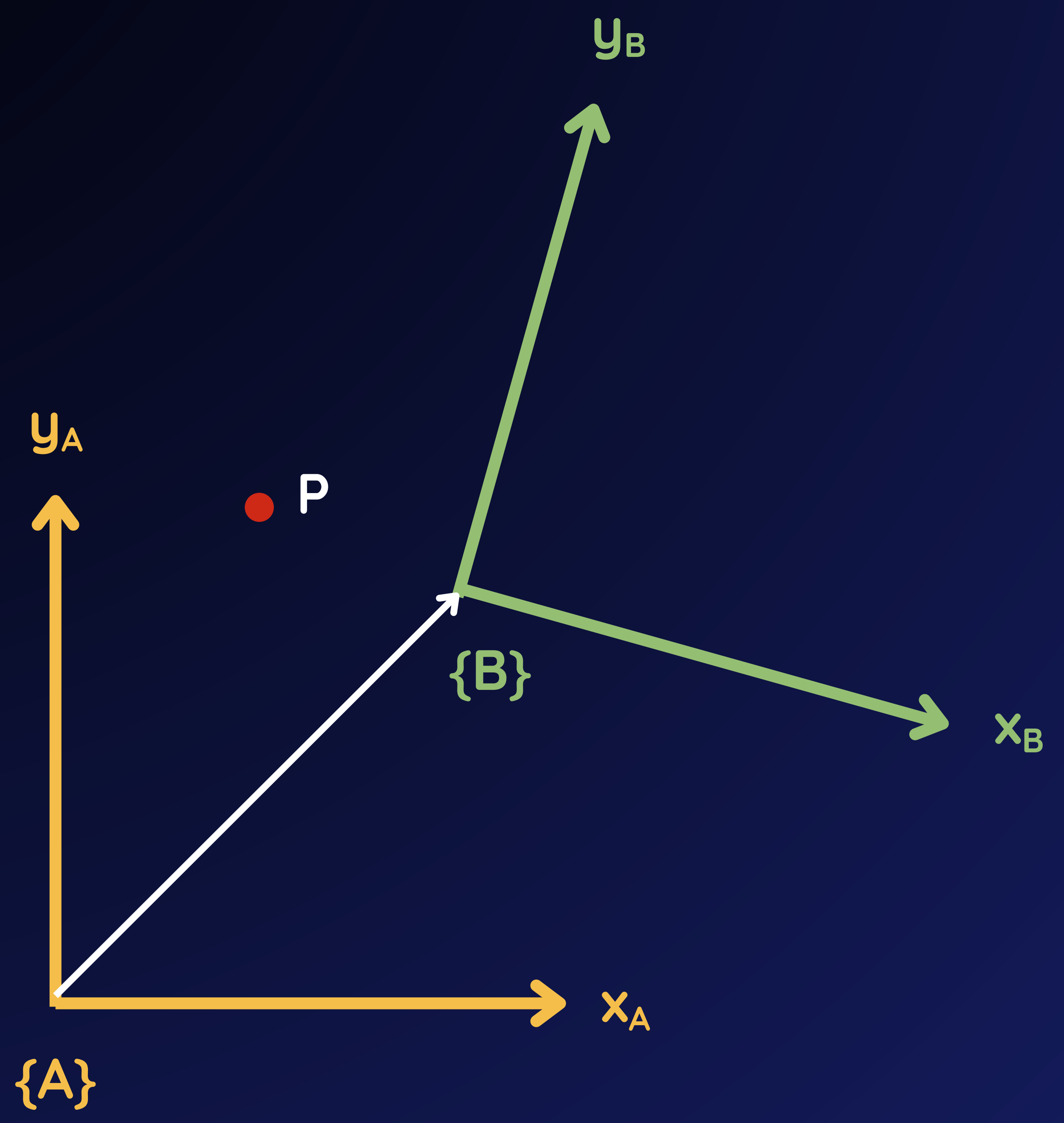
$$^{A}P = P_x\,\hat{x} + P_y\,\hat{y}$$

How different frame relate to one another?



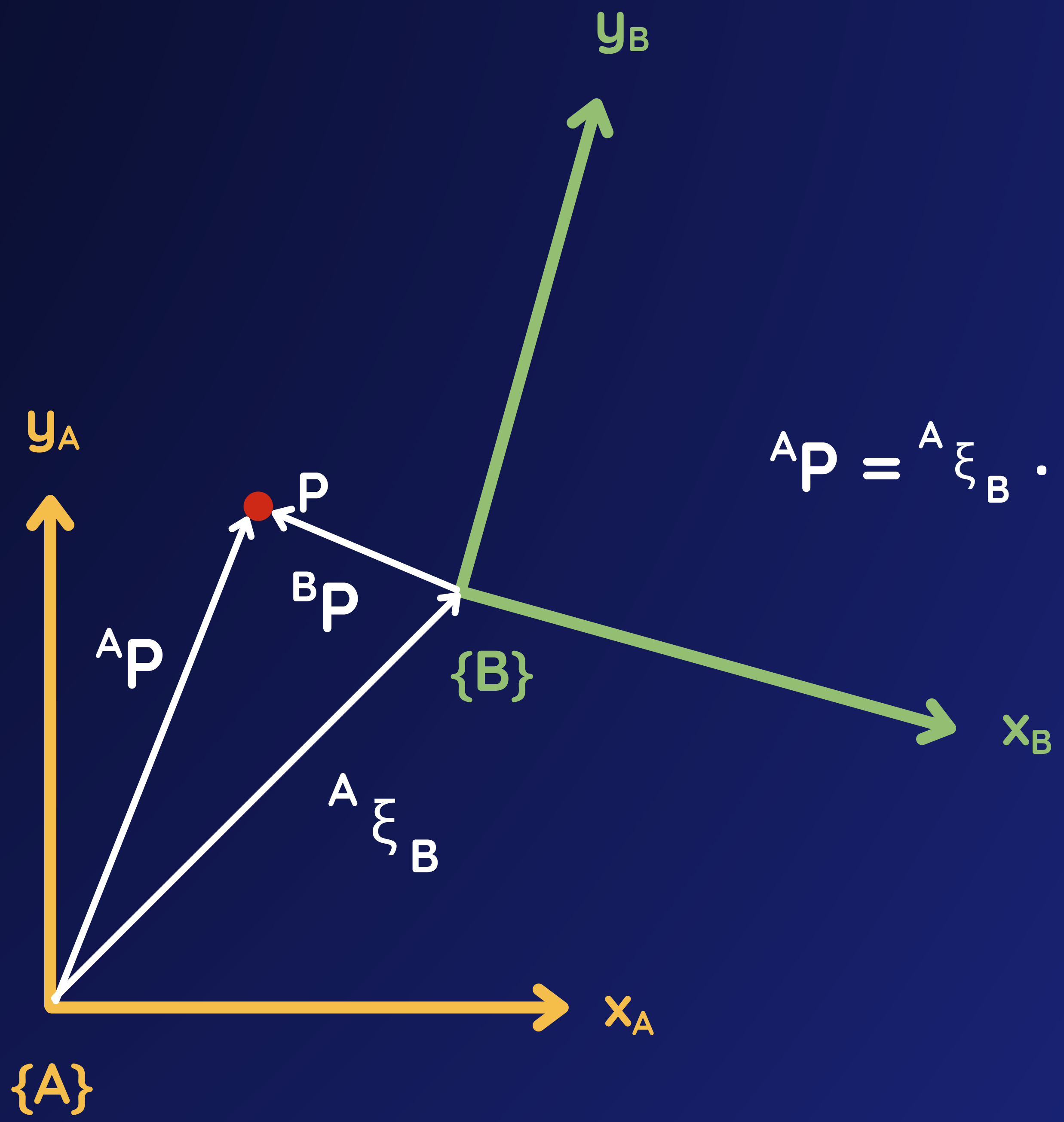


y_B
y_A
P
{B}
x_B
$^A\xi_B$
motion required to move {A} to {B}.
x_A
{A}

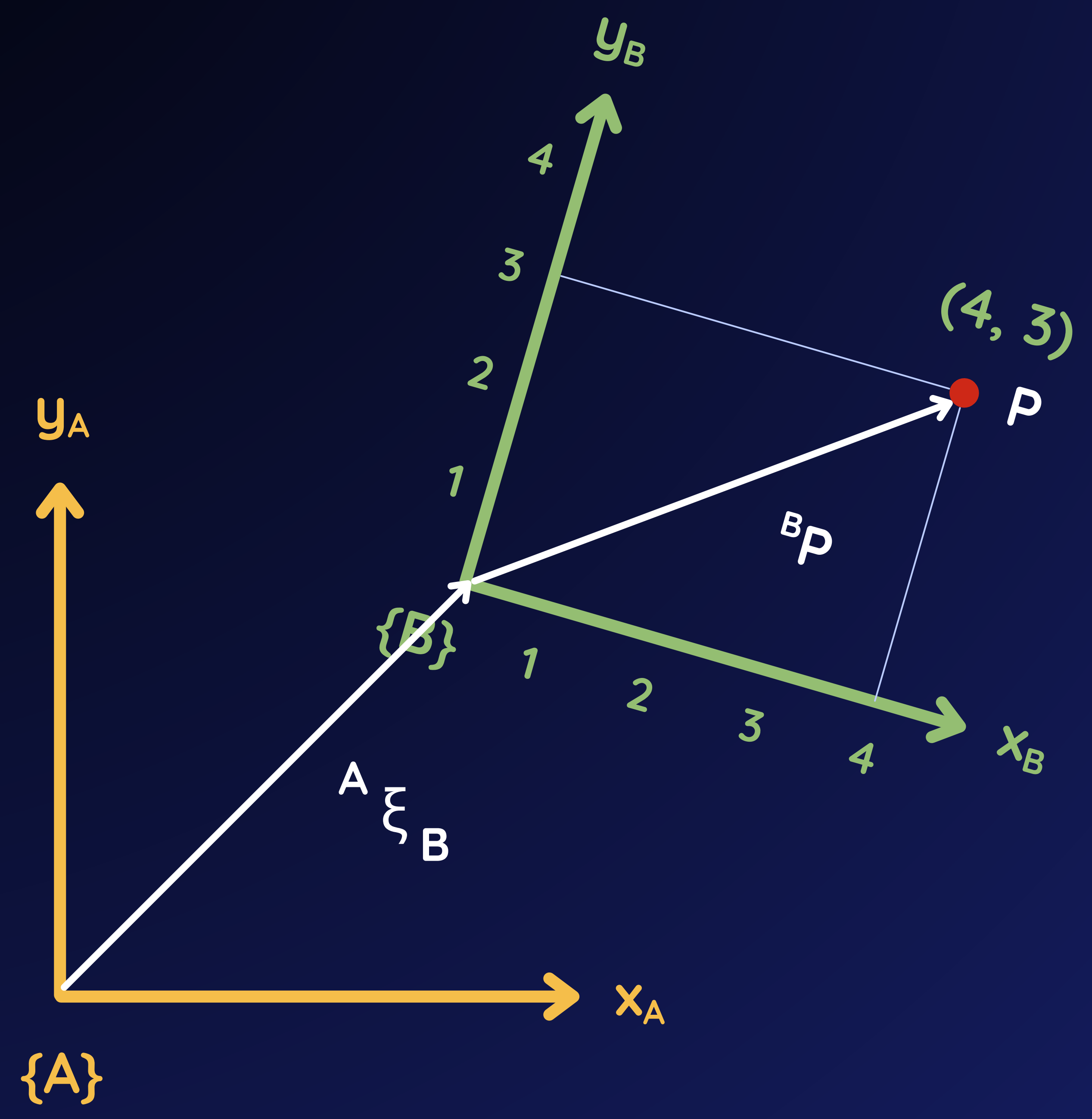


$${}^{A}\xi_{B}$$

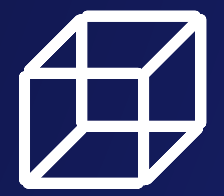
- How far frame {B} from frame {A}
- How frame {B} is rotated with respect to frame {A}



$${}^{A}P = {}^{A}\xi_{B} \cdot {}^{B}P$$

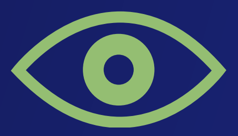


P



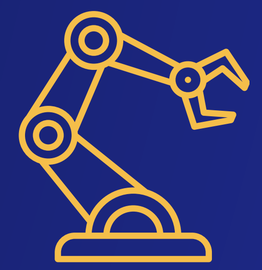
Object

{B}

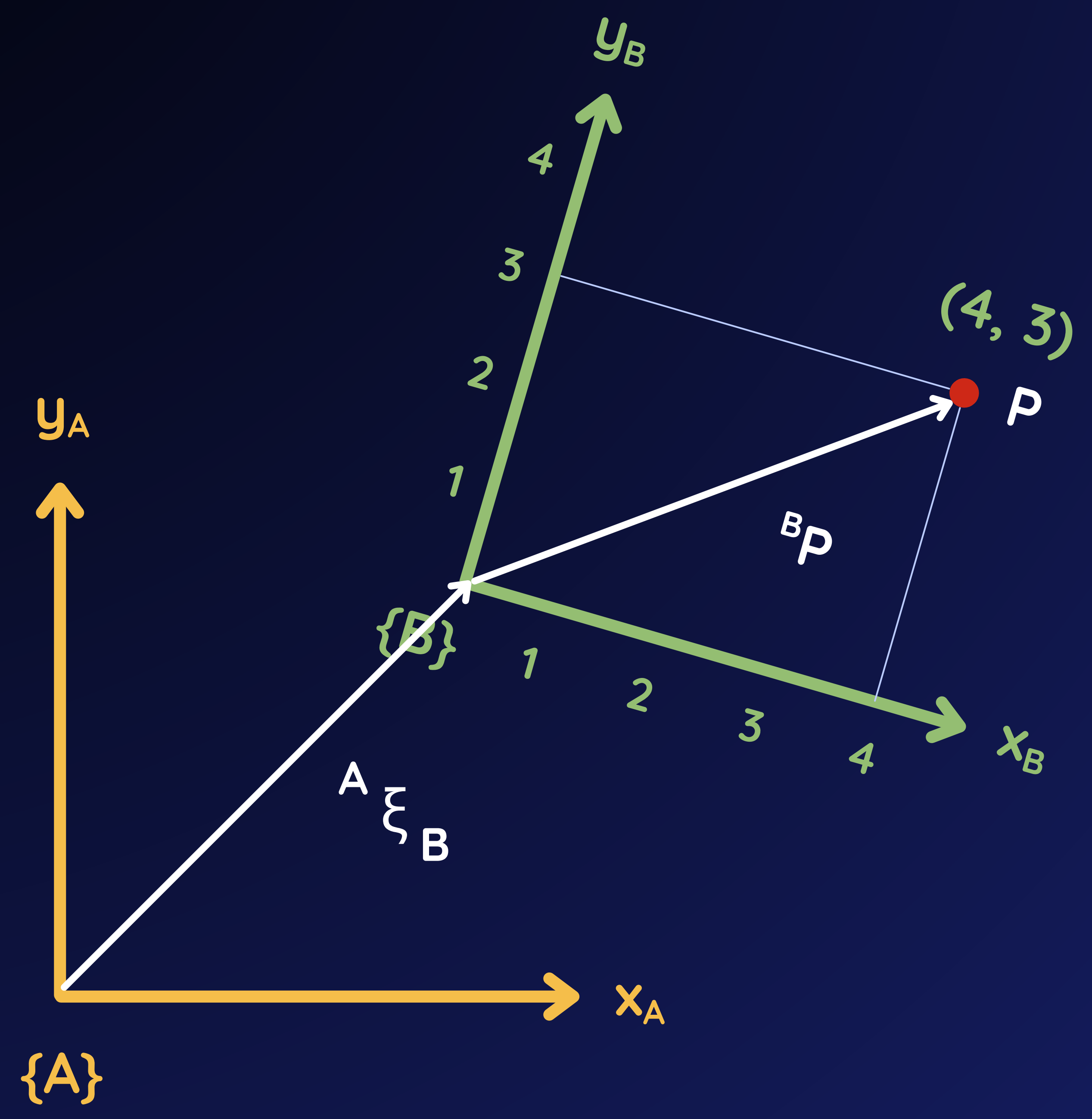


Visualisation Sensor

{A}



Base Frame

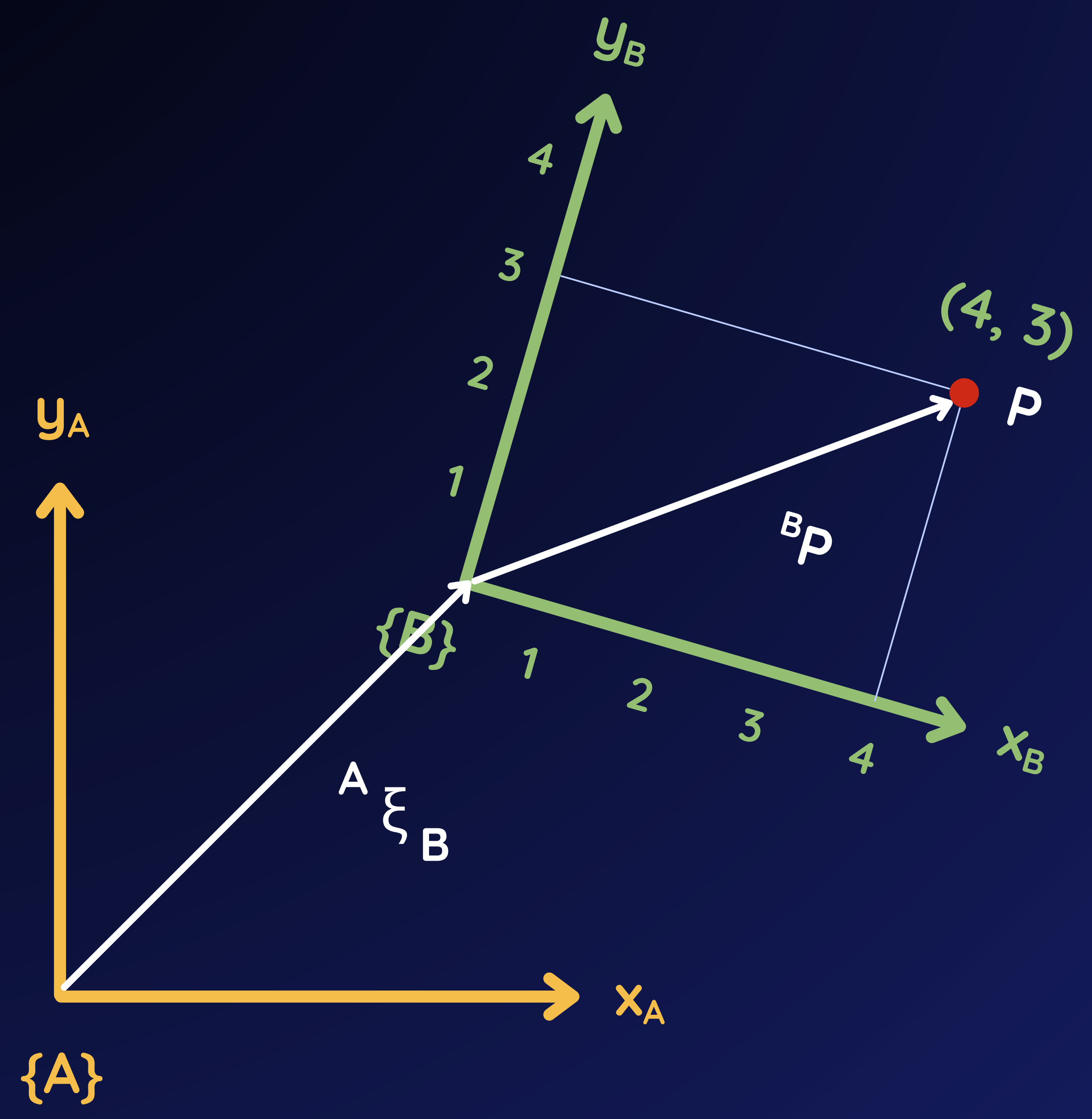


1 **Rotation Matrix, R**

How frame {B} is oriented relative to frame {A}

2 **Translation Vector, T**

How far frame {B} is shifted from frame {A}



$$^AP = R_{AB} \cdot {}^BP + t_{AB}$$

where:

$$R_{AB} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad t_{AB} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

1. Point rotated according to the orientation of frame {B} relative to frame {A}

2. The point is translated based on the position of frame {B}

Rotation + Translation

= Homogenous Matrix

Homogenous Matrix

$$\begin{bmatrix} A_x \\ A_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ 1 \end{bmatrix}$$

- → Represent rotation + translation
- → Perform transmation using single matrix
- → Easily chain multiple transformation