ASSIGNMENT: 1D Heat conduction problem

Course: Turbulence: Phenomenology, Simulation, Aerodynamics

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PROBLEM SPECIFICATION AND INPUT DATA

In this report, we have analyzed the temperature distribution inside a plain wall using a simulation of a 1D conduction heat transfer for both steady and transient cases. We assumed a constant temperature for the both left and right sides of the wall. The other properties such as the thermal diffusivity of the material constant values, and the same were for the value of the internal source. We used the following values in the computer code for the variables mentioned above:

TABLE 1

Physical Properties	
Thickness, H	1.5 (m)
Left wall temperature, T_left	350 (K)
Right wall temperature, T_right	300 (K)
Thermal diffusivity, α	20 (m^2/s)
Internal heat generation, Q_t	50 (W/m^3)
Numerical Properties	
Number of Grids (N)	20
Tolerance Maximum Iteration	10^{-6} 10^6

The stability criterion, Dt for this analysis has been taken as 0.0001.

ANALYTICAL SOLUTION

The conduction equation for 1D in a Steady-state is:

$$\alpha \frac{d^2T}{dx^2} - \mathrm{Qt} \times \mathrm{T} = 0$$

The solution for this equation is,

$$T(x) = C_1 e^{kx} + C_2 e^{-kx}\,; where\ k = \sqrt{\frac{\mathrm{Qt}}{\alpha}}$$

From this relation, using the boundary conditions we can get the values from C_1 and C_2.

$$C_1 = \frac{T_{right} - T_{left} \times e^{kL}}{e^{-kL} - e^{kL}}$$

$$C_2 = T_{left} - C_1$$

The conduction equation for 1D in transient state is:

$$\alpha \frac{d^2T}{dx^2} - \mathrm{Qt} \times \mathrm{T} = \frac{dT}{dx}$$

The solution for this equation is,

$$T(x) = C_1 e^{kx} + C_2 e^{-kx} + A\, e^{-\alpha\beta^2 - Qt} Sin(\beta x);$$

$$where\ k = \sqrt{\frac{\mathrm{Qt}}{\alpha}}\ , A = 50\ and\ \beta = 2\pi n/L$$

NUMERICAL SOLUTION:

Using the Gauss-Ostrogradsky theorem for both the equations for transient and steady state we get,

For steady-state,

$$\int \alpha \nabla^2 T dV = \int \alpha \nabla \mathrm{T} \mathrm{dS} = (\int \alpha \frac{dT}{dx} dS)_E - (\int \alpha \frac{dT}{dx} dS)_W$$

Discretizing the equation we get,

$$A_p T_p = b - a_E T_E - a_W T_W$$

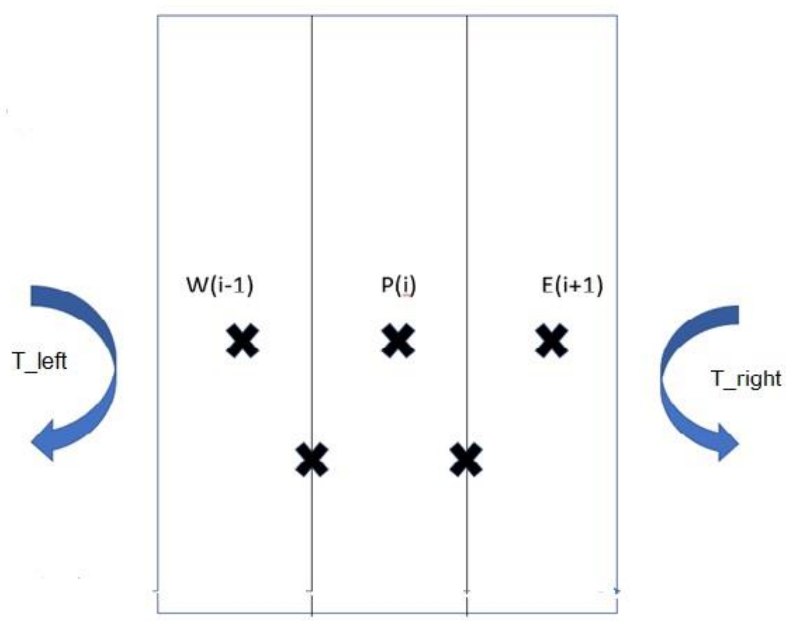

Figure 1: Discretization

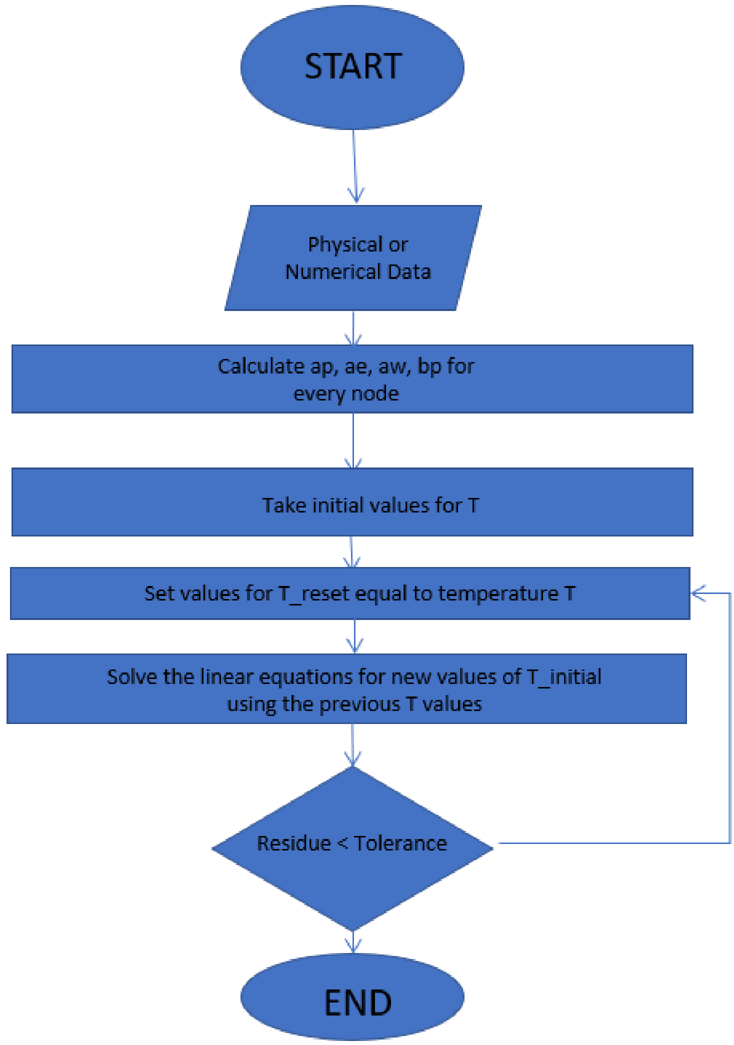

Figure 2: Algorithm of the Code

Now, For transient state,

We can write the equation for the discretized equation as,

$$\frac{T_p^{n+1} - T_p^n}{\Delta t}\Delta x = -\frac{\alpha(T_P^n - T_W^n)}{d_{xW}} + \frac{\alpha(T_E^n - T_P^n)}{d_{xE}} - Q_t T_P \Delta x$$

Or,

$$T_p^{n+1} = T_p^n + \frac{\Delta t}{\Delta x}\left(-\frac{\alpha(T_P^n - T_W^n)}{d_{xW}} + \frac{\alpha(T_E^n - T_P^n)}{d_{xE}} - Q_t T_P \Delta x\right)$$

Where n denotes the values of the previous iteration's time step, and n+1 denotes the values of the subsequent iteration's time step.

Results and Discussion

We selected 20 grid points for the numerical and analytical solution.

For this we get the following plots for the Numerical solution and error from the analytical solution for steady state.

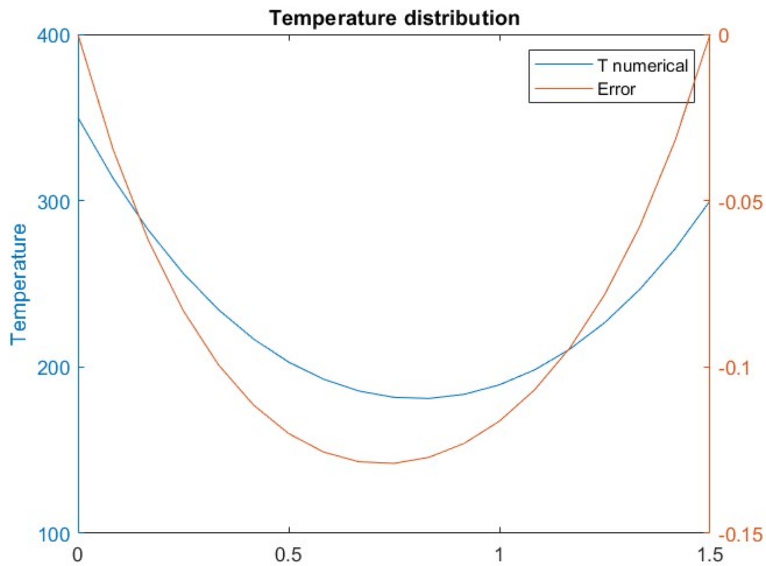

Figure 3: Numerical solution and error from analytical solution

For the grid convergence, we analyzed scenarios with different grid numbers such as, 5,10,15,20, and we noticed a consistency among the truncation error.

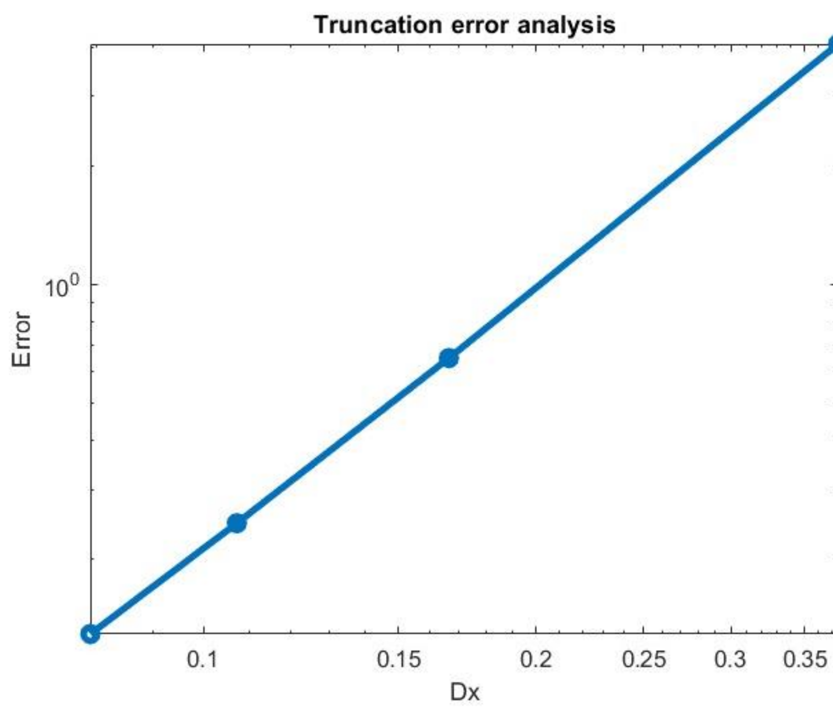

Figure 4: Truncation error for different number of grid points

To check the difference between numerical and analytical solution we take the analytical solution as the reference for different values of time.

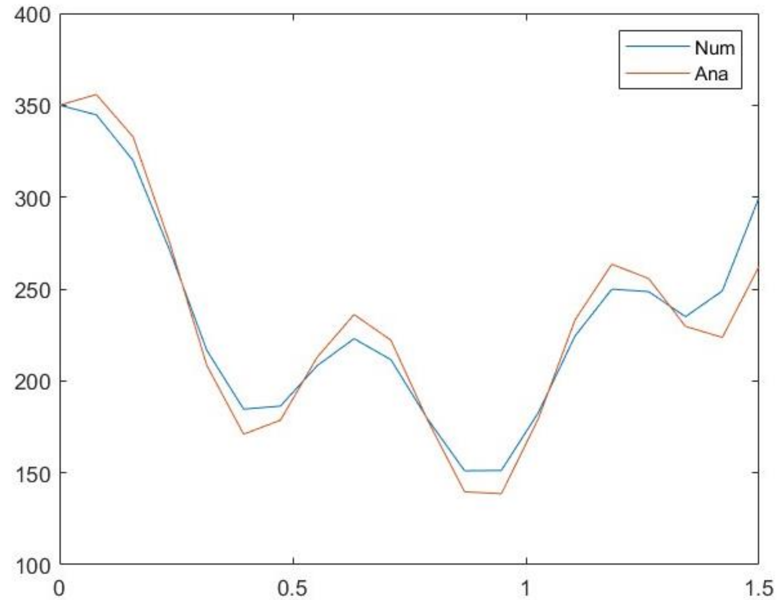

Figure 5: Distribution of temperature at t=0 sec

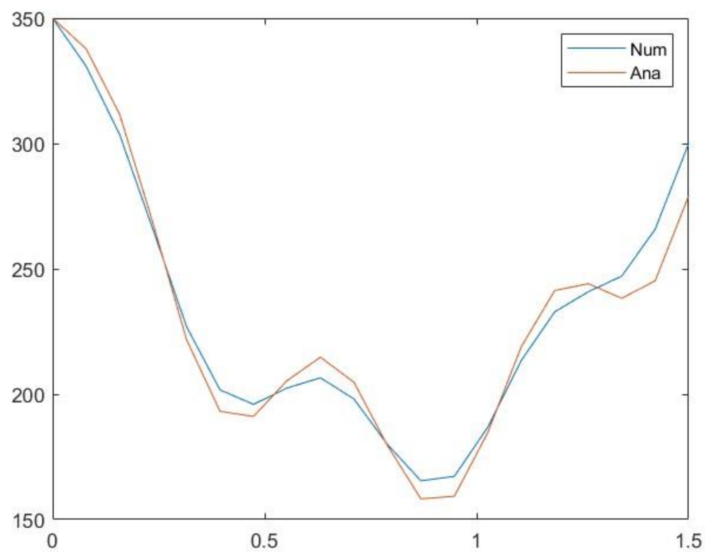

Figure 6: Distribution of temperature at t=0.0002 sec

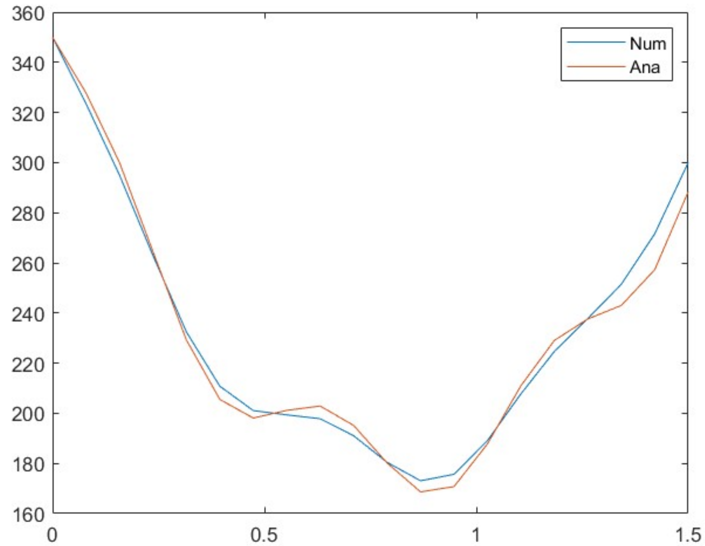

Figure 7: Distribution of temperature at t=0.0004 sec

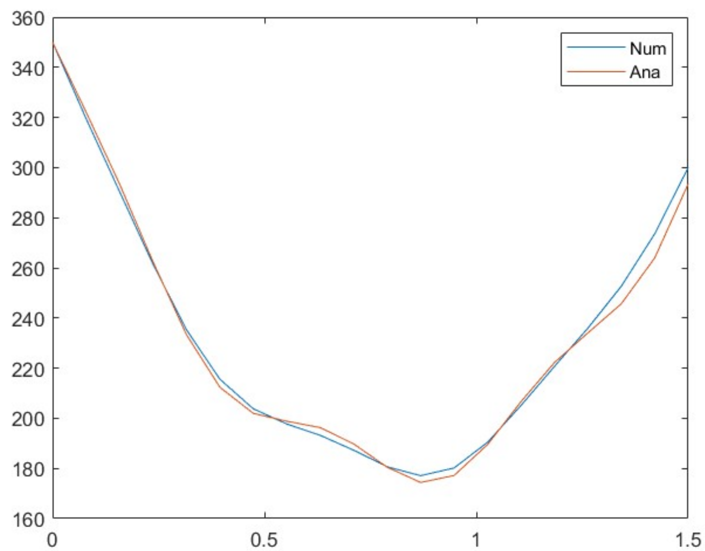

Figure 8: Distribution of temperature at t=0.0006 sec

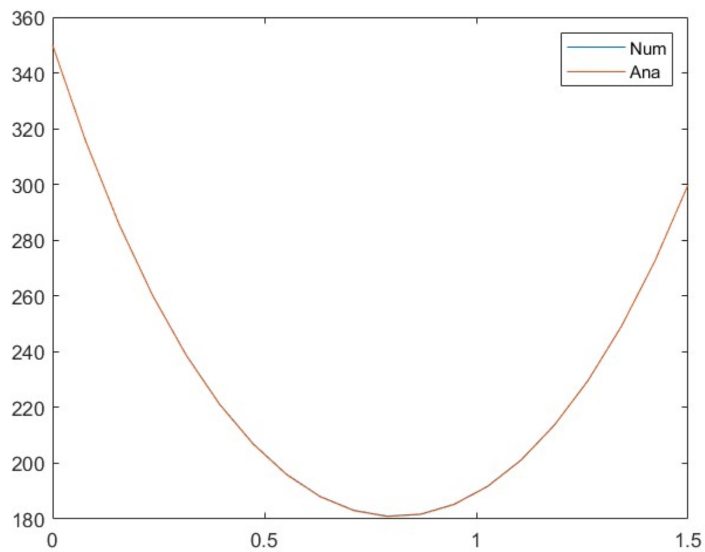

Figure 9: Distribution of temperature at the final step

Analyzing the above plots we can conclude that the analytical and numerical solutions are almost identical to each other. Although there is a very small truncation error which we can see in figure 3, it is very minimal and is due to the discretization process.

Also it is evident that once the transient solution reaches steady state, it is identical to the steady state plot.