E	*Find the eigen values and eigen vector for* $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$
F	Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both irrotational and solenoidal.
Q3. **(20 Marks Each)**	**Solve any Four out of Six5 marks each**
A	*Find* $L\left[\int_0^t e^{-2u}\cos^2 u\,du\right]$
B	*Find the inverse Laplace transform by using convolution theorem* $\frac{1}{(s^2+4s+13)^2}$
C	*Obtain the Fourier series for* $f(x) = x$ *in* $(0,2\pi)$
D	*Obtain the orthogonal trajectories for the family of curves* $e^{-x}\cos y + xy = c$ where c is the real constant in the xy-plane.
E	*Show that* $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ *satisfies Cayley-Hamilton theorem. Hence find* A^{-1}
F	Evaluate by using Green`s theorem $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the closed region bounded by $y = \sqrt{x}$ and $y = x$
Q4. **(20 Marks Each)**	**Solve any Four out of Six** **5 marks each**
A	*Evaluate* $\int_0^\infty e^{-t}\int_0^t \left(\frac{\sin u}{u}\right) du\,dt$
B	$L^{-1}\left[\log\left(1 + \frac{4}{s^2}\right)\right]$
C	*Obtain the Fourier series for* x^2 *in* $(-\pi,\pi)$
D	*Find the analytic function* $f(z)$ *whose imaginary part is* $e^x(x\sin y + y\cos y)$
E	*If* $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ *then find* A^{50}
F	Use Stoke`s Theorem to evaluate $\int_c \bar{F}d\bar{r}$ where $\bar{F} = x^2 i + xy\,j$ and C is the boundary of the rectangle x=0, y=0,x=a,y=b

Paper / Subject Code: 51401 / Applied Mathematics-III

SE|SEM III | IT | CHOICE BASED| AM-III ①

Total Marks: 80 **Hours: 3 hrs**

Note : 1) Question no. 1 is compulsory.
2) Attempt any three questions out of five questions

Q-1

a) If any 11 numbers between 1 and 20 are chosen show that at least two of them will be multiplies of each other. (05)

b) A function $f : R-\left\{\frac{7}{3}\right\} \to R-\left\{\frac{4}{3}\right\}$ is defined by $f(x)=\frac{4x-5}{3x-7}$, Prove that f is bijective and find the rule for f^{-1}. (05)

c) Find $L\left[\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)\right]$ (05)

d) Prove that there does not exist an analytic function whose imaginary part is $3x^2+\sin x+y^2+5y+4$. (05)

Q-2

a) Find $L^{-1}\left[\frac{s}{\left(s^2+3^2\right)\left(s^2+5^2\right)}\right]$ using convolution Theorem. (06)

b) What is the chance of throwing ten with four dice? (06)

c) In a certain examination there are multiple choice questions. There are four possible answers to each questions and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% questions he gives answer by guessing. A week student can solve 20% question correctly by reasoning and for the remaining 80% questions he gives answer by guessing. An intelligent student gets the correct answer. What is the probability that he was guessing. (08)

Q-3

a) A can hit a target 2 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots. They fire a volley. What is the probability that at least 2 shots hit the target? (06)

b) Find $L^{-1}\left(\tan^{-1}\left(\frac{2}{s^2}\right)\right)$ (06)

c) If R is the relation on the set of integers such that aRb if and only if 2a+3b is divisible by 5. Find the equivalence classes. (08)

Q-4

a) Evaluate $\int_{t=0}^{\infty} e^{-3t}\left(\frac{\cos(7t)-\cos(11t)}{t}\right)dt$ (06)

b) Find $L^{-1}\left[\frac{s^2+2s+3}{\left(s^2+2s+10\right)\left(s^2+2s+17\right)}\right]$ (06)

c) Find the bilinear Transformation which maps the points $2, i, -2$ on to the points $1, i, -1$. Also find image of $|z|=1$ of z-plane to w-plane. (08)

| SEM III | IT | CHOICE BASED | AM - III ②

Q-5

a) A family consisting of an old man, 6 adults and 4 children is to be seated in a row for dinner. The children wish to occupy two seats at each end and the old man refuse to have a child on either side of him. In how many ways can the seating arrangement be made for the dinner? (06)

b) Find the analytic function $f(z)=u+iv$ in terms of z if $u-v=(x-y)\left(x^2+4xy+y^2\right)$. (06)

c) Solve $\frac{d^3y}{dt^3}-2\frac{d^2y}{dt^2}+5\frac{dy}{dt}=0$ with y(0) = 0, y '(0) =0, y'' (0) = 1. (08)

Q-6

a) Prove that $(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$ (06)

b) Draw the Hasse diagram of D_{105}. (06)

c) Find Laplace Transformation of the following

i) $te^{3t}erf\left(5\sqrt{t}\right)$,

ii) $\sin t H(t)+(\cos t-\sin t)H(t-\pi)$ (08)

|IT/ choice based /sem III/ Applied Maths III ①

[Time: 3 Hours] [Marks:80]

Please check whether you have got the right question paper.

N.B: 1. Q 1 is compulsory.
2. Attempt any three from remaining
3. Rights indicate full marks.

1. a. If A, B, C are subset of universal set V then prove that $A \times (B \cap C)=(A\times B) \cap (A \times C)$ 05

b. If f: R→R is given by y = 2x + 1, prove that f is one to one and onto and find f^{-1} 05

c. Find L $\{(1+t\bar{e}^{t})^3\}$ 05

d. Check whether the following function Harmonic or not $3x^2 + sinx + y^2 + 5y + 4$ 05

2. a. Find k if f (z) = $\frac{1}{2}$ log $(x^2 + y^2)$ + i tan^{-1} $\frac{kx}{y}$ is analytic 06

b. Find L $\{|sin2t|\}$ 06

c. Let f: R→R f(x) = x^2+2x-1 08
g: R→R g(x) = $4x^2+2$

Find (I) f 0 (gof) (II) go (fog)

3. a. Find Bilinear transformation under which Z=1, -i, -1 from point w =i, 0, -i 06

b. If A be the set of non-integers and let R be a relation on A×A defined by (a, b) R(c, d) if ad=bc, then prove that R is an equivalence relation. 06

c. Find (1) L $\left\{\int_0^t \bar{e}^u \frac{sinu}{u} du\right\}$ 08

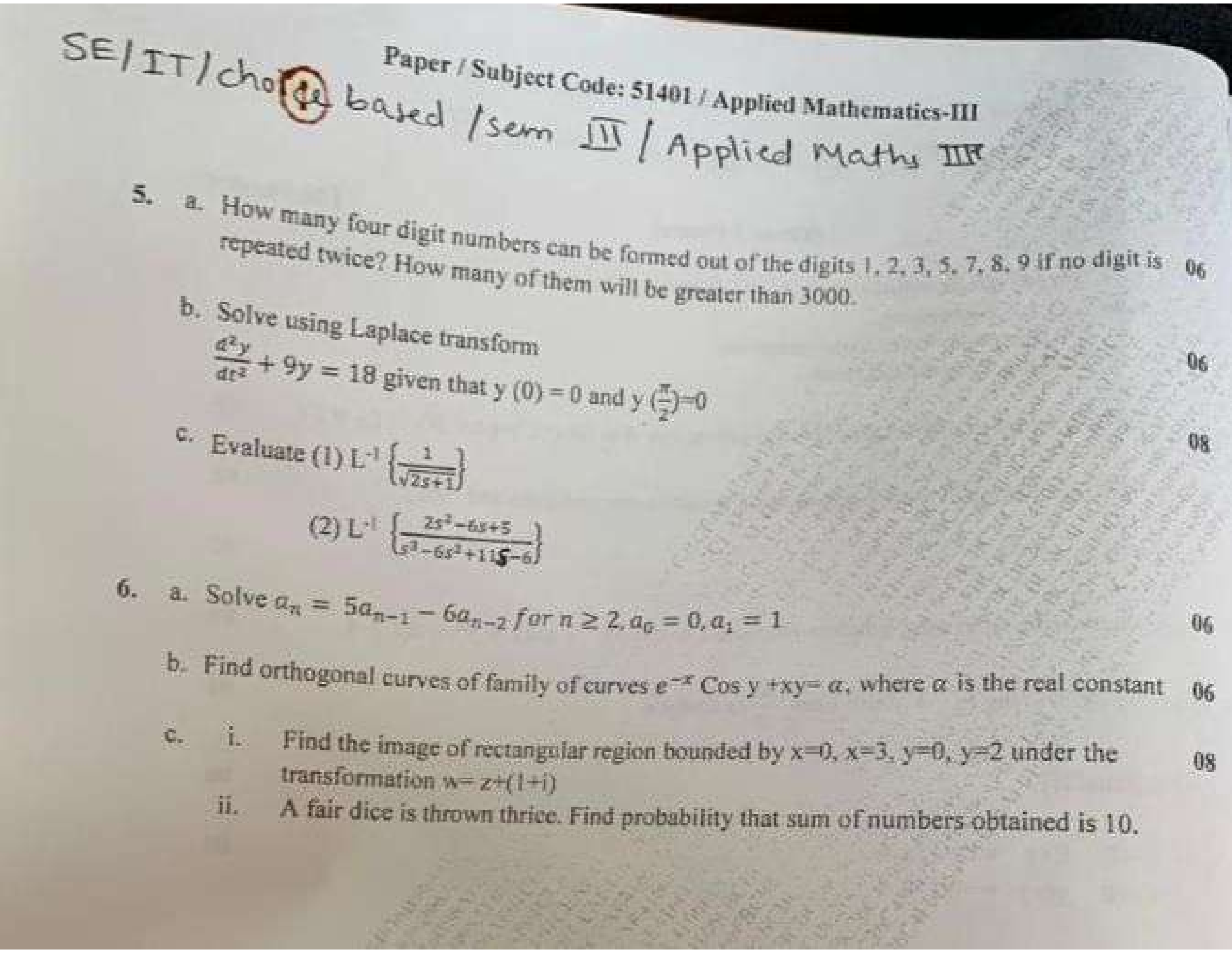
(2) L $\{(1 + 2t + 3t^2 + t^3)H(t-2)\}$

4. a. Use convolution them and evaluate 06

$$L^{-1}\left\{\frac{(s+5)^2}{(s^2+10s+16)^2}\right\}$$

b. Find transitive clouser of following relation defined on A= $\{a, b, c, d, e\}$ by Warshal algorithm R=$\{(a, a)\ (a, b)\ (b, c)\ (c, d)\ (c, c)(d, e)\}$ 06

c. A man speaks truth 3 times out of 5 when a die is thrown he states that it gave an ace what is probability that this event has actually happened. 08



SE/IT/choice based /sem III / Applied Maths III

Paper / Subject Code: 51401 / Applied Mathematics-III

5. a. How many four digit numbers can be formed out of the digits 1, 2, 3, 5, 7, 8, 9 if no digit is repeated twice? How many of them will be greater than 3000. 06

b. Solve using Laplace transform 06

$\frac{d^2y}{dt^2} + 9y = 18$ given that $y(0) = 0$ and $y(\frac{\pi}{2}) = 0$

c. Evaluate (1) $L^{-1}\left\{\frac{1}{\sqrt{2s+1}}\right\}$ 08

(2) $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$

6. a. Solve $a_n = 5a_{n-1} - 6a_{n-2}\ for\ n \geq 2, a_0 = 0, a_1 = 1$ 06

b. Find orthogonal curves of family of curves e^{-x} Cos y +xy= α, where α is the real constant 06

c. i. Find the image of rectangular region bounded by x=0, x=3, y=0, y=2 under the transformation w= z+(1+i) 08

ii. A fair dice is thrown thrice. Find probability that sum of numbers obtained is 10.

Option C:	3
Option D:	5.25
20.21	A continuous random variable X has the probability law $f(x) = k^2x^3$, $0 \le x \le 3,\ k > 0$ then value of k is
Option A:	2/81
Option B:	4/81
Option C:	4/9
Option D:	2/9

<table>
<tr><td>Q2
(20 Marks)</td><td>Solve any Four out of Six 5 marks each</td></tr>
<tr><td>A</td><td>Find Laplace transform of $f(t) = sin^2t\ cos^3t.$</td></tr>
<tr><td>B</td><td>Using convolution theorem find the inverse Laplace transform of $$\emptyset(s) = \frac{s}{s^4 - 1}$$</td></tr>
<tr><td>C</td><td>Find Fourier series of $f(x) = x\ sinx$ in$(-\pi, \pi)$.</td></tr>
<tr><td>D</td><td>Find an analytic function $\omega = f(z) = u + iv, where\ z = x + iy$, whose real part is $u(x,y) = x^2 - y^2 + 2y - \sin(x) . \sinh(y)$</td></tr>
<tr><td>E</td><td>Calculate Spearman's coefficient of rank correlation and Pearson's coefficient of correlation from the following data on height and weights of 5 students.
<table>
<tr><td>Height(in inches)</td><td>61</td><td>63</td><td>65</td><td>67</td><td>69</td></tr>
<tr><td>Weight(In kgs)</td><td>64</td><td>62</td><td>65</td><td>70</td><td>72</td></tr>
</table>
</td></tr>
</table>

<table>
<tr><td>F</td><td>The warranty of electronic device in thousand of days has the density function $f(x)=\begin{cases}4e^{-4x}, x>0\\ 0, \quad otherwise\end{cases}$
Find the expected warranty of the device.</td></tr>
</table>

<table>
<tr><td>Q3
(20 Marks)</td><td>Solve any Four out of Six
5 marks each</td></tr>
<tr><td>A</td><td>Given $f(t)=\begin{cases}4,\ 0\leq x<3\\ 0,\ x>3\end{cases}$.
Find $L[f(t)]$, $L[f'(t)]$.</td></tr>
<tr><td>B</td><td>Find inverse Laplace transform of $\emptyset(s)=\frac{3s^2+11s+11}{s^3+6s^2+11s+6}$</td></tr>
<tr><td>C</td><td>Find half range sine series for $f(x)=e^{-x}, 0<x<1$.</td></tr>
<tr><td>D</td><td>In the polar coordinates, let $\omega=u+iv$, $u(r,\theta)=r^2 sin2\theta$.
Show that u satisfies Laplace's equation and find $v(r,\theta)$.</td></tr>
<tr><td>E</td><td>Fit a second degree parabolic curve to the following data.
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>1</td><td>1</td><td>3</td><td>7</td><td>13</td><td>21</td><td>31</td></tr>
</table></td></tr>
<tr><td>F</td><td>A random variable X has the probability distribution $P(X=x)=\frac{1}{16}(4_{C_x})$,
$x=0,1,2,3,4$. Write Probability distribution and find standard deviation.</td></tr>
</table>

Option	
Q 20	Find $L^{-1}\left[\frac{3t}{}\right.$
Option A:	$\frac{1}{2}(1-\cos 2t)$
Option B:	$(1+\cos 2t)$
Option C:	$\frac{1}{2}(1-\sin 2t)$
Option D:	$\frac{1}{2}(1+\cos t)$
Q 21	Find the constant 'a' if $f(z) = ax^2y - y^3 + i(3xy^2 - x^3)$ is analytic
Option A:	$a = 0$
Option B:	$a = 3$
Option C:	$a = 6$
Option D:	$a = 2$

Q2. **(20 Marks)**	**Solve any Four out of Six5 marks each**
A	Fit a straight line to the following data (X,Y) = (1,-5),(1,1),(2,4),(3,7),(4,10)
B	Find half range cosine series for $f(x) = x(\pi - x),\ 0 < x < \pi$
C	Find $L^{-1}\left[\frac{1}{(s+3)(s-4)^2}\right]$ using convolution theorem.
D	Find the orthogonal trajectories of the family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$
E	A discrete random variable has p.d.f. given below X : -2 -1 0 1 2 3 P(X=x) : 0.2 k 0.1 2k 0.1 2k Find k and ($P(X\geq 1)$)
F	Evaluate $\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\,dt$

Q3 . **(20 Marks)**	**Solve any Four out of Six5 marks each**
A	

	Show that $u = 3x^2y - y^3$ is harmonic.Find the corresponding analytic function.
B	Find $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right]$
C	Find the Fourier series for $f(x) = x^3$, in $(-\pi, \pi)$
D	Find the expectation and M.G.F. of the following distribution X : -2 3 1 P(X=x) : 1/3 1/2 1/6
E	Compute Spearman's rank correlation coefficient from the following data X : 16, 18, 25, 30, 12 Y : 38, 21, 38, 16, 50
F	Find Laplace transform of $te^{-t} \cos cos\, t$

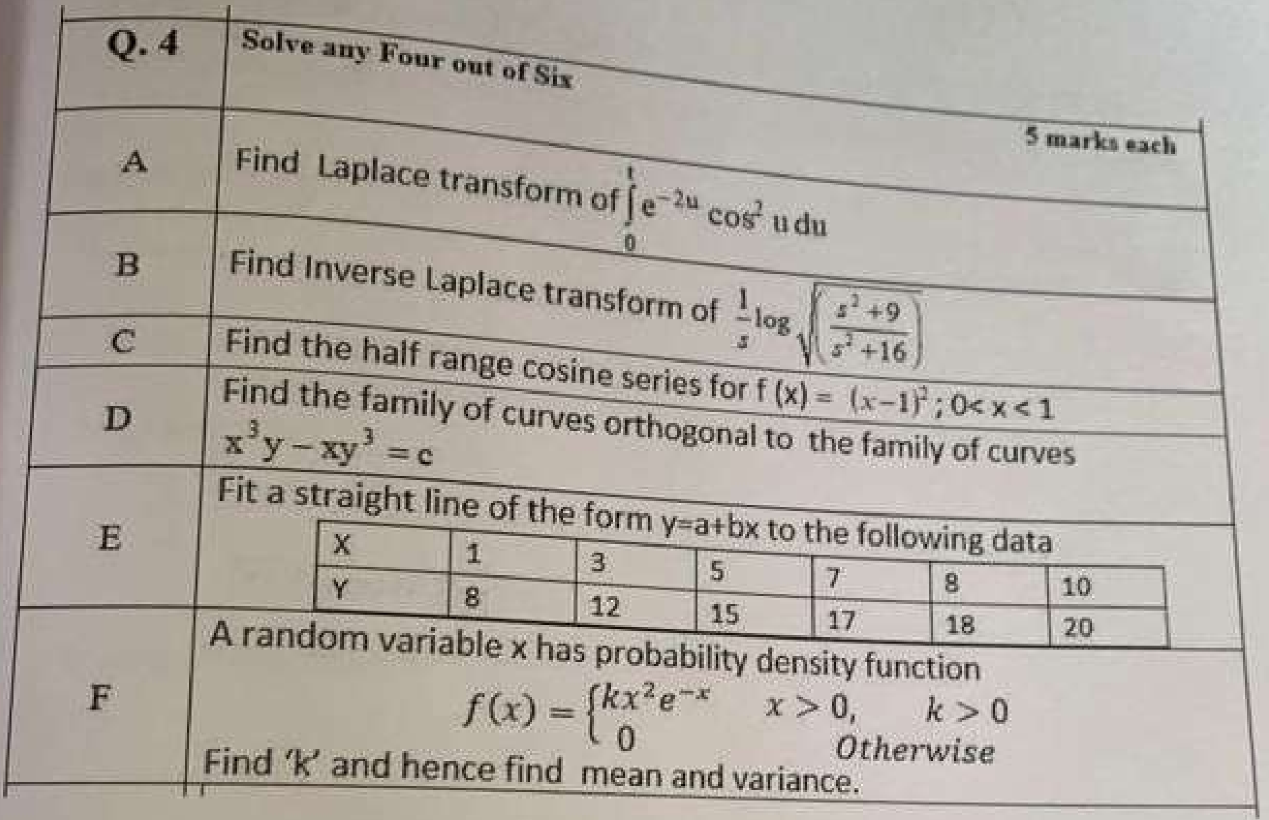
Q.2

<table>
<tr><td>Q 2.</td><td>Solve any Four out of Six
5 marks each</td></tr>
<tr><td>A</td><td>Find Laplace transform of $e^{-3t}t\sqrt{1-sin2t}$</td></tr>
<tr><td>B</td><td>Find inverse Laplace transforms of $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$</td></tr>
<tr><td>C</td><td>Expand Fourier Series for $f(x)=\frac{1}{2}(\pi-x)$ in (0.2π).</td></tr>
<tr><td>D</td><td>Find constants a, b, c, d and e, if
$(ax^4+bx^2y^2+cy^4+dx^2-2y^2)+i(4x^3y-exy^3+4xy)$
is analytic.</td></tr>
<tr><td>E</td><td>Ten students got the following percentage of marks in mathematics and statistics
<table>
<tr><td>Maths</td><td>78</td><td>36</td><td>98</td><td>25</td><td>75</td><td>82</td><td>90</td><td>62</td><td>65</td><td>39</td></tr>
<tr><td>Stats</td><td>84</td><td>51</td><td>91</td><td>60</td><td>68</td><td>62</td><td>86</td><td>58</td><td>53</td><td>47</td></tr>
</table>
Calculate the coefficient of correlation.</td></tr>
<tr><td>F</td><td>A bolt is manufactured by three machines A, B and C. A turns out twice as many times as B, and machines B and C produce equal number of items. 3% of bolts produced by A and B are defective and 5% of bolts produced by C are defective. All bolts are put into one stock pile and one is chosen from this pile. What is the probability that it is defective?</td></tr>
</table>

Q.3

<table>
<tr><td>Q. 3</td><td>Solve any Four out of Six</td></tr>
<tr><td></td><td>5 marks each</td></tr>
<tr><td>A</td><td>By using Laplace transform, evaluate $\int_0^\infty \frac{\sin 2t+\sin 3t}{te^t}$</td></tr>
<tr><td>B</td><td>By using Convolution theorem, find inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+4)}$</td></tr>
<tr><td>C</td><td>Expand Fourier Series for $f(x) = 1 - x^2$ in $(-1, 1)$</td></tr>
<tr><td>D</td><td>Find the analytic function $f(z) = u + iv$, in terms of z, if $v = \frac{\sinh 2y}{\cosh 2y + \cos 2x}$</td></tr>
<tr><td>E</td><td>Obtain the equations of the lines of regression for the following data.
<table>
<tr><td>X</td><td>65</td><td>66</td><td>67</td><td>67</td><td>68</td><td>69</td><td>70</td><td>72</td></tr>
<tr><td>Y</td><td>67</td><td>68</td><td>65</td><td>68</td><td>72</td><td>72</td><td>69</td><td>71</td></tr>
</table></td></tr>
<tr><td>F</td><td>A random variable X has the following probability distribution
<table>
<tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P</td><td>0.1</td><td>K</td><td>0.1</td><td>2K</td><td>0.2</td><td>3K</td></tr>
</table>
(i) Find the constant K. (ii) Find the mean and variance of X.</td></tr>
</table>

Q.4



<table>
<tr><td>**Q. 4**</td><td>**Solve any Four out of Six**</td></tr>
<tr><td></td><td>**5 marks each**</td></tr>
<tr><td>A</td><td>Find Laplace transform of $\int_0^t e^{-2u} \cos^2 u \, du$</td></tr>
<tr><td>B</td><td>Find Inverse Laplace transform of $\frac{1}{s}\log\sqrt{\left(\frac{s^2+9}{s^2+16}\right)}$</td></tr>
<tr><td>C</td><td>Find the half range cosine series for f (x) = $(x-1)^2$; 0< x < 1</td></tr>
<tr><td>D</td><td>Find the family of curves orthogonal to the family of curves $x^3y - xy^3 = c$</td></tr>
<tr><td>E</td><td>Fit a straight line of the form y=a+bx to the following data
<table>
<tr><td>X</td><td>1</td><td>3</td><td>5</td><td>7</td><td>8</td><td>10</td></tr>
<tr><td>Y</td><td>8</td><td>12</td><td>15</td><td>17</td><td>18</td><td>20</td></tr>
</table></td></tr>
<tr><td>F</td><td>A random variable x has probability density function
$$f(x) = \begin{cases} kx^2e^{-x} & x > 0, \quad k > 0 \\ 0 & Otherwise \end{cases}$$
Find ‘k’ and hence find mean and variance.</td></tr>
</table>

3. Upload your answer papers *

Files submitted:

4. Have you uploaded required pdf file of answers? *

Mark only one oval.

○ Yes