

**University of Mumbai**  
**Examination First Half 2022**  
**Examinations Commencing from 3<sup>rd</sup> June 2022**

Program: **Computer Engineering**

Curriculum Scheme: Rev2019

Examination: SE Semester III

Course Code: CSC301 and Course Name: Engineering Mathematics-III

Time: 2hour 30 minutes

Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks
1.	In the Fourier series of $f(x) = \sqrt{1 - \cos x}$ in $(0, 2\pi)$ the value of $a_0$ is
Option A:	$\frac{2\sqrt{3}}{\pi}$
Option B:	$\frac{6\sqrt{2}}{\pi}$
Option C:	$\frac{2\sqrt{2}}{\pi}$
Option D:	$\frac{2\sqrt{2}}{4\pi}$
2.	The formula of complex form of Fourier series for function $f(x)$ in $(-l, l)$ is
Option A:	$\sum_{-\infty}^{\infty} C_n e^{inx}$ where $C_n = \frac{1}{2l} \int_{-l}^l f(x) e^{-in\pi x/l} dx$
Option B:	$\sum_{-\infty}^{\infty} C_n e^{in\pi x/l}$ where $C_n = \frac{1}{2l} \int_{-l}^l f(x) e^{-in\pi x/l} dx$
Option C:	$\sum_{-\infty}^{\infty} C_n e^{inx}$ where $C_n = \frac{1}{2l} \int_{-l}^l f(x) e^{-in\pi x/l} dx$
Option D:	$\sum_{-\infty}^{\infty} C_n e^{ix}$ where $C_n = \frac{1}{2l} \int_{-l}^l f(x) e^{-in\pi x/l} dx$
3.	Evaluate $\int_0^{\infty} e^{-3t} t^5 dt$
Option A:	$\frac{60}{s^5}$
Option B:	$\frac{120}{s^6}$
Option C:	$\frac{120}{729}$
Option D:	$\frac{60}{729}$
4.	If $f(z) = u + iv$ is analytic then
Option A:	$u$ is harmonic but $v$ may or may not be harmonic.
Option B:	$v$ is harmonic but $u$ may or may not be harmonic.
Option	$u$ and $v$ both need not be harmonic.

C:	
Option D:	$u$ and $v$ both harmonic.
5.	If $\text{Var}(X) = 4$ then $\text{Var}(3x+5)$ is
Option A:	12
Option B:	20
Option C:	26
Option D:	36
6.	If $X$ has the following probability distribution $X:$ 0    1    2 $P(X = x):$ $k$ $2k$ $5k$ Then the value of $k$ is
Option A:	$1/6$
Option B:	0
Option C:	$1/3$
Option D:	$1/8$
7.	Find Inverse L.T. of $\frac{3}{9s^2 - 16}$ .
Option A:	$\frac{1}{4} \sinh\left(\frac{3t}{4}\right)$
Option B:	$\frac{1}{4} \sin\left(\frac{3t}{4}\right)$
Option C:	$\frac{1}{4} \sinh\left(\frac{4t}{3}\right)$
Option D:	$\frac{1}{4} \sin\left(\frac{4t}{3}\right)$
8.	$L^{-1}\left[\frac{1}{s(s+4)}\right]$ is
Option A:	$\frac{1}{4}(e^{-4t} - 1)$
Option B:	$\frac{1}{4}(1 - e^{-4t})$
Option C:	$(e^{-4t} - 1)$
Option D:	$\frac{1}{4}(e^{-4t} + 1)$
9.	Find the Laplace transform of $\frac{\sin t}{t}$

Option A:	$\cot^{-1}s$
Option B:	$\cot^{-1}t$
Option C:	$\tan^{-1}s$
Option D:	$\tan^{-1}t$
10.	Find $L[(\sin 3t)(\sin 5t)]$
Option A:	$\frac{1}{2} \left[ \frac{s}{s^2 + 4} + \frac{1}{s^2 + 64} \right]$
Option B:	$\frac{1}{2} \left[ \frac{s}{s^2 - 4} - \frac{1}{s^2 - 64} \right]$
Option C:	$\frac{1}{2} \left[ \frac{s}{s^2 - 4} - \frac{s}{s^2 - 64} \right]$
Option D:	$\frac{1}{2} \left[ \frac{s}{s^2 + 4} - \frac{s}{s^2 + 64} \right]$

<b>Q2</b>	<b>Solve any Four out of Six</b>	<b>5 marks each</b>
A	If $L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi}}{2s\sqrt{s}} \cdot e^{-1/(4s)}$ , find $L\{\sin 2\sqrt{t}\}$ .	
B	If $v = 3x^2y + 6xy - y^3$ , show that $v$ is harmonic function and find the corresponding analytic function.	
C	If the mean of the following distribution is 16. Find $m, n$ and variance. $X : 8, 12, 16, 20, 24$ $P(X) : 1/8, m, n, 1/4, 1/12$	
D	Evaluate the Fourier coefficients $a_0$ and $a_n$ of $f(x) = \frac{1}{2}(\pi - x)$ in $(0, 2\pi)$ .	
E	Find $L^{-1}\left(\log\left(1 + \frac{a}{s}\right)\right)$ .	
F	The Regression lines of a sample are $x + 6y = 6$ and $3x + 2y = 10$ . Find the coefficient of correlation between $x$ and $y$ .	
<b>Q3</b>	<b>Solve any Four out of Six</b>	<b>5 marks each</b>
A	Find the inverse Laplace transform of $\frac{s + 29}{(s + 4)(s^2 + 9)}$	
B	Calculate the value of rank correlation coefficient from the following data regarding marks of 6 students in Statistics and Mathematics in a test: <i>Marks : Statistics</i> : 40, 42, 45, 35, 36, 39 <i>Marks : Mathematics</i> : 46, 43, 44, 39, 40, 43	
C	By using Laplace transform, prove that $\int_0^\infty e^{-t} \cdot \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$	



D	Evaluate the Fourier coefficients $a_0$ and $b_3$ of $f(x) = x$ in $(0, 2\pi)$ .																
E	Show that the function, $f(z) = \sinh z$ is analytic and find $f'(z)$ in terms of $z$ .																
F	<div>The probability density function of a random variable X is<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>P(X=x)</td><td>k</td><td>3k</td><td>5k</td><td>7k</td><td>9k</td><td>11k</td><td>13k</td></tr></table>Find <math>P(X &lt; 4)</math>, <math>P(3 &lt; x \leq 6)</math>.</div>	X	0	1	2	3	4	5	6	P(X=x)	k	3k	5k	7k	9k	11k	13k
X	0	1	2	3	4	5	6										
P(X=x)	k	3k	5k	7k	9k	11k	13k										
Q4	<div>Solve any Four out of Six5 marks each</div>																
A	<div>Find the Fourier series for <math>f(x)</math> in <math>(0, 2\pi)</math> where <math>f(x) = \begin{cases} x, &amp; 0 &lt; x \leq \pi \\ 2\pi - x, &amp; \pi \leq x &lt; 2\pi \end{cases}</math></div>																
B	<div>Using convolution theorem, find the inverse Laplace transform of <math>\frac{1}{(s - 2)^4(s + 3)}</math></div>																
C	<div>State true or false with justification. “If two lines of regression are <math>x + 3y - 5 = 0</math> and <math>4x + 3y - 8 = 0</math>, then the correlation coefficient is <math>+0.5</math>”.</div>																
D	<div>Find <math>L(t e^{-3t} \cos 2t \cos 3t)</math></div>																
E	<div>A continuous random variable has the following probability density function <math>f(x) = \begin{cases} \frac{x}{4} + k, &amp; 0 \leq x \leq 2 \\ 0, &amp; \text{elsewhere} \end{cases}</math> Evaluate <math>k</math> and <math>P(1 \leq X \leq 2)</math></div>																
F	<div>From the following data calculate Karl Pearson’s coefficient of correlation (<math>r</math>) between X and Y.<table><tr><td>X</td><td>18</td><td>20</td><td>34</td><td>52</td><td>12</td></tr><tr><td>Y</td><td>39</td><td>23</td><td>35</td><td>18</td><td>46</td></tr></table></div>	X	18	20	34	52	12	Y	39	23	35	18	46				
X	18	20	34	52	12												
Y	39	23	35	18	46												