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Tutorial - 4

1. $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1, b \geq 1$

On comparing

$a = 3, b = 2, f(n) = n^2$

Now

$c = \log_b a = \log_2 3$

$= 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^c$

$T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a \geq 1 \quad b > 1$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

3. $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2.$

$f(n) = 2^n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^b = 1$

$f(n) > n$

$T(n) = \Theta(2^n$

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4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2,\ f(n) = n^2$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c + n^n$

$f(n) = n^c$

$f(n) = \Theta(n^2 \log_2 n)$

5. $T(n) = 16T(n/4) + n$

$a = 16 \quad b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2\log_4 4$

$= 2^4$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

$a = 2,\ b = 2$

$f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$T(n) = \Theta(n\log n)$

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7. $T(n) = 2T(n/2) + n\log n$

$a = 2, \; b = 2, \; f(n) = n\log n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{n\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \; b = 4, \; f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = 0.5$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5 \quad b = 2$

$a \geq 1$ but here $a < 0.5$

so we can't apply Master's Method

10. $T(n) = 16T(n/4) + n!$

$a = 16, \; b = 4, \; f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

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11) $4T(n/2) + \log n$

$a=4,\ b=2,\ f(n)=\log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\because \log n < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$= \theta(n^2)$

12) $T(n) = 3T(n/2) + n$

$a=3 \quad b=2$

$c = \log_2 3 = 1.584$

$n^c = n^{1.58}$

$n < n^{1.58} \Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.58})$

14) $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a=3,\ b=3$

$c = \log_3 3 = 1$

$n^c = n$

$\text{sqrt}(n) < n$

$n^c > f(n)$

$T(n) = \theta(n)$

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15. $T(n) = 4T(n/2) + n$

$a = 4, \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

$T(n) = \Theta(n^2)$

16. $T(n) = 3T(n/4) + n \log n$

$a = 3, \quad b = 4$

$c = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^c < f(n)$

$T(n) = \Theta(n \log n)$

17. $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3, \quad b = 3.$

$c = \log_3 3 = 1$

$n^c = n$

$n^c > f(n)$

$T(n) = \Theta(n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 \quad b = 3$

$c = \log_3 6 = 1.6309$

$n^c < n^2 \log n$

$T(n) = \Theta(n^2 \log n)$

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19 $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

20 $T(n) = 64T(n/8) - n^2 \log n$

$a = 64,\ b = 8$

$c = \log_8 64 = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$T(n) = \Theta(n^2 \log n)$

21 $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3$

$c = \log_3 7 = 1.771 2$

$n^c = n^{1.77}$

$n^c < f(n)$

$T(n) = \Theta(n^2)$

22 $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1,\ b = 2$

$c = \log_2 1 = 0$

$n^c = 1 \quad n(2 - \cos n) > n^c$

$T(n) = \Theta(n(2 - \cos n))$