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Unit 3: Coordinate Geometry

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Topic : Straight Line

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Learning Objective/ Key learning

- Calculate angle between given two straight lines.

Contents

- Definition of Straight line
- Condition for parallel and perpendicular Lines.
- Angle between Two Straight Lines.

- **Definition** : It is linear equation in two variables x and y.

  Represented as $Ax + By + C = 0$ Called General form of equation of line.

a) Slope of a Line: If the inclination of a line is θ, ($\theta \neq \frac{\pi}{2}$) then $\tan\theta$ is defined as the slope or gradient of the line and is denoted by m. i.e. $m = \tan\theta$

b) If Line passes through $A(x_1, y_1)$ and $B(x_2, y_2)$ then it's slope is given by, $m = \frac{y_2 - y_1}{x_2 - x_1}$

c) If $Ax + By + C = 0$ is equation of given line then it's slope is given by Slope $= -\frac{A}{B} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ y)}$

- **Angle between Two Straight Lines:**

If m_1 and m_2 are slopes of the two lines then the angle between two lines is $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Deduction: 1. If $\theta = 0$ then lines are parallel

$$0 = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan 0 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$0 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore m_1 = m_2$$

2. If θ= 90 then lines are perpendicular.

$$\therefore \quad 90 = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore \quad \tan 90 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore \quad \infty = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore \quad 1 + m_1 \times m_2 = 0$$

$$\therefore \quad m_1 \times m_2 = -1$$

Condition for parallel and perpendicular Lines:

1)Two lines are parallel if their slopes are equal .i. e. $m_1 = m_2$ and converse is also true.

2)Condition for two lines to be perpendicular : Two lines are perpendicular if their product of slopes is – 1. i.e. $m_1 \cdot m_2 = -1$ and converse is also true.

Solved Examples:

1)Show that the lines $2x + 3y - 5 = 0$ and $4x + 6y - 1 = 0$ are parallel.

Solution:

Let $L_1 : 2x + 3y - 5 = 0$

$\therefore$ Slope of L_1 is $m_1 = \frac{-2}{3}$

And $L_2 : 4x + 6y - 1 = 0$

$\therefore$ Slope of L_2 is $m_2 = \frac{-4}{6} = \frac{-2}{3}$

$\therefore m_1 = m_2$

$\therefore$ Given lines are parallel

2)Prove that lines $3x + 4y + 7 = 0$ and $28x - 21y + 50 = 0$ are perpendicular to each other.

Solution :

Let $L_1 : 3x + 4y + 7 = 0$

$\therefore$ Slope of L_1 is $m_1 = \frac{-3}{4}$

and $L_2 : 28x - 21y + 50 = 0$

$\therefore$ Slope of L_2 is $m_2 = \frac{-28}{-21} = \frac{4}{3}$

$$m_1 \cdot m_2 = \frac{-3}{4} \cdot \frac{4}{3} = -1$$

$\therefore$ Given lines are perpendicular

3)Find the acute angle between the lines $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$.

Solution : Given equation of lines

$$L_1 : 3x - 2y + 4 = 0$$

$$\therefore \quad \text{Slope } m_1 = \frac{-3}{-2} = \frac{3}{2}$$

$$L_2 : 2x - 3y - 7 = 0$$

$$\therefore \quad \text{Slope } m_2 = \frac{-2}{-3} = \frac{2}{3}$$

Let ‘θ’ be the acute angle between the lines

Then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{\frac{3}{2} - \frac{2}{3}}{1 + \left(\frac{3}{2}\ \frac{2}{3}\right)}\right|$$

$$= \left|\frac{\frac{9-4}{6}}{1+1}\right|$$

$$= \left|\frac{\frac{5}{6}}{2}\right|$$

$$= \left|\frac{5}{12}\right|$$

$$\tan\theta = \frac{5}{12}$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right)$$

4) Find the acute angle between the lines $3x - y = 4$ and $2x + y = 3$

Solution: Given equation of lines

$$L_1 : 3x - y = 4$$

$$L_1 : 3x - y - 4 = 0$$

$$\therefore \text{Slope } m_1 = \frac{-3}{-1} = 3$$

$$L_2 : 2x + y - 3 = 0$$

$$\therefore \text{Slope } m_2 = \frac{-2}{1} = -2$$

The acute angle between the line is

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \tan^{-1}\left|\frac{3-(-2)}{1+(3)(-2)}\right|$$

$$= \tan^{-1}\left|\frac{5}{-5}\right|$$

$$= \tan^{-1}|-1|$$

$$= \tan^{-1}(1)$$

$$\theta = 45^\circ$$

OR

$$\theta = \frac{\pi}{4}$$

So today we learn-

- Definition of Straight line
- Condition for parallel and perpendicular Lines:
- Angle between Two Straight Lines.

.Quiz

1)State the condition for parallel lines, whose slopes are m_1 and m_2

a)$m_1=m_2$ b) $m_1+ m_2= -1$ c)$m_1 \cdot m_2 = -1$ d)$m_2 = -m_1$

2) Find the acute angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$

a) 60° b) 90° c) 45° d) 30°

Ans: 1. a) 2.d)

Thank You