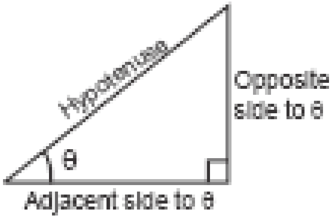
FORMULAE LIST

Std. XI FORMULAE

TRIGONOMETRY

1. $\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$; $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$



$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$; $\cot\theta = \dfrac{\text{Adjacent}}{\text{Opposite}}$

$\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Adjacent}}$; $\operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{Opposite}}$

2. $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$; $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$; $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$

3. $\sin(90° - \theta) = \cos\theta$; $\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$; $\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \operatorname{cosec}\theta$; $\operatorname{cosec}(90° - \theta) = \sec\theta$

4.

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

5. **Fundamental Identities:**

(a) $\sin^2\theta + \cos^2\theta = 1$

(b) $\sec^2\theta - \tan^2\theta = 1$

(c) $\sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$

(d) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

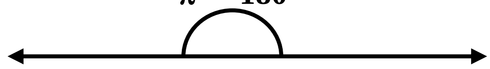
(e) $\operatorname{cosec}\theta + \cot\theta = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$

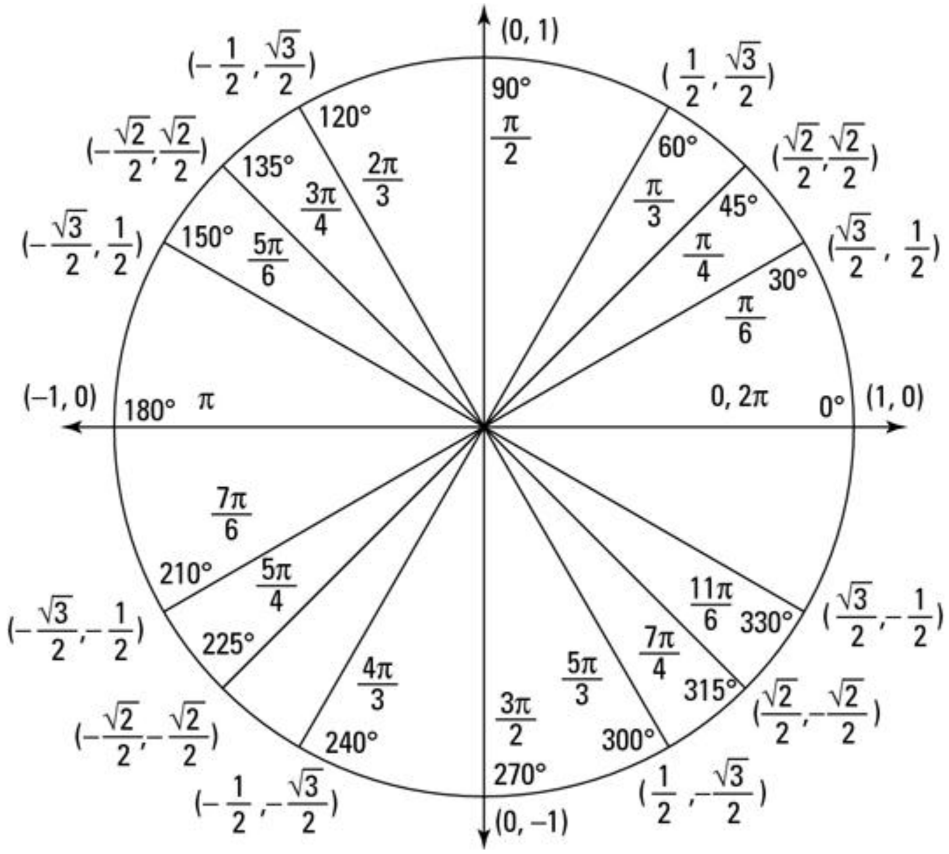
Note: $\sin^2\theta = (\sin\theta)^2 \neq \sin(\theta^2)$

6. (a) $1^o = 60'$ (minutes) (b) $1' = 60''$ (seconds)

7. (a) $1^o = \left(\dfrac{\pi}{180}\right)^c$ (b) $1^c = \left(\dfrac{180}{\pi}\right)^o$

Note : **$\pi^c = 180°$**





8. (a) length of arc $= s = r\theta$

(b) area of sector $= A = \dfrac{1}{2}r^2\theta$

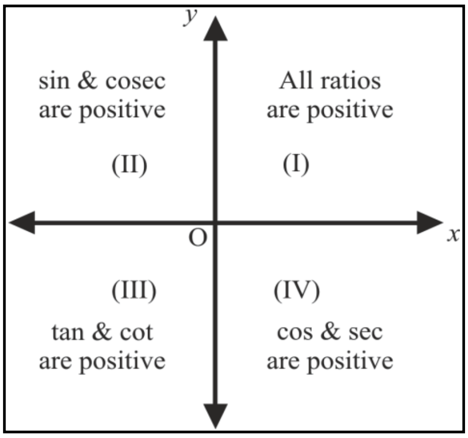
(c) Perimeter of sector = $2r + r\theta = r(2+\theta)$

where θ is in radians

9. (a) $\sin(n\pi) = 0$ (b) $\cos(n\pi) = (-1)^n$

(c) $\sin\left[(2n+1)\frac{\pi}{2}\right] = (-1)^n$ (d) $\cos\left[(2n+1)\frac{\pi}{2}\right] = 0$

10. **SIGNS OF TRIGO RATIOS IN DIFFERENT QUADRANTS: (V IMP)**



NOTE: Remember it as ***ALL SILVER TEA CUPS*** or ***ADD SUGAR TO COFFEE***

11. (a) $\sin(-\theta) = -\sin\theta$ (b) $\cos(-\theta) = \cos\theta$

(c) $\tan(-\theta) = -\tan\theta$

12. For all real θ, $-1 \le \sin\theta \le 1$
For all real θ, $-1 \le \cos\theta \le 1$
For $\cos\theta \ne 0$, $-\infty < \tan\theta < \infty$
For $\sin\theta \ne 0$, $-\infty < \cot\theta < \infty$
For $\cos\theta \ne 0$, $-\infty < \sec\theta \le -1$ Or $1 \le \sec\theta < \infty$
For $\sin\theta \ne 0$, $-\infty < \text{cosec}\,\theta \le -1$ Or $1 \le \text{cosec}\,\theta < \infty$

13. **Expansion Formulae:**

(a) $\sin(A+B) = \sin A\cos B + \cos A\sin B$ (b) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(c) $\cos(A+B) = \cos A\cos B - \sin A\sin B$ (d) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(e) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ (f) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

(g) $\tan(\frac{\pi}{4} + x) = \frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$ (h) $\tan(\frac{\pi}{4} - x) = \frac{1-\tan x}{1+\tan x} = \frac{\cos x - \sin x}{\cos x + \sin x}$

(i) $\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$ (j) $\cot(A-B) = \frac{\cot A\cot B + 1}{\cot B - \cot A}$

(k) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

(*l*) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

14. **Factorisation formulae:**

(a) $\sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(b) $\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(c) $\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(d) $\cos C-\cos D=-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$ …(C > D)

$=2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$ …(C < D)

15. **Defactorisation formulae:**

(a) $2\sin A\cos B=\sin(A+B)+\sin(A-B)$ (b) $2\cos A\sin B=\sin(A+B)-\sin(A-B)$

(c) $2\cos A\cos B=\cos(A-B)+\cos(A+B)$ (d) $2\sin A\sin B=\cos(A-B)-\cos(A+B)$

16. **Multiple & Sub-multiple Angle Formulae:**

i. (a) $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

(b) $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \frac{2\tan\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$

ii. (a) $\cos 2\theta = \cos^2\theta-\sin^2\theta$

$= 2\cos^2\theta-1$

$= 1-2\sin^2\theta$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(b) $\cos\theta = \cos^2\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)$

$= 2\cos^2\left(\frac{\theta}{2}\right)-1$

$= 1-2\sin^2\left(\frac{\theta}{2}\right)$

$= \frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$

iii. (a) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ (b) $\tan\theta = \frac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)}$

iv. $1 + \cos 2\theta = 2\cos^2\theta$, $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

v. $1 - \cos 2\theta = 2\sin^2\theta$, $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

vi. $1 + \sin 2\theta = (\cos\theta + \sin\theta)^2$, $1 + \sin\theta = \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2$

vii. $1 - \sin 2\theta = (\cos\theta - \sin\theta)^2$, $1 - \sin\theta = \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2$

viii. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

ix. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

x. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

xi. **Trigo Ratios of some special angles:**

Angle	sin	Cos	tan
9°	$\frac{\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}}{4}$	$\frac{\sqrt{3+\sqrt{5}} + \sqrt{5-\sqrt{5}}}{4}$	$\frac{\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}}{\sqrt{3+\sqrt{5}} + \sqrt{5-\sqrt{5}}}$
15°	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$2-\sqrt{3}$
18°	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
$22\frac{1}{2}^\circ$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$
36°	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$
54°	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{1}{\sqrt{5-2\sqrt{5}}}$
72°	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\frac{5}{\sqrt{25-10\sqrt{5}}}$
75°	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$2+\sqrt{3}$

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17. **Allied Angle Formulae:**

Any angle which is of the form $\left(n\frac{\pi}{2}\pm\theta\right)$ **where n ∈ N, n = odd** OR **(nπ ± θ) where n ∈ N** is called an allied angle.

Note: θ is a very small acute angle

Method to find trigonometric ratios of allied angles

STEP 1: Give the answer a positive or negative sign depending on how the ratio behaves in that quadrant

STEP 2: Allied Angles

$n\frac{\pi}{2}\pm\theta$

Ratio becomes the co – ratio $\begin{cases} \sin\to\cos \\ \cos\to\sin \\ \tan\to\cot \\ \cot\to\tan \\ \sec\to\text{cosec} \\ \text{cosec}\to\sec \end{cases}$

$n\pi\pm\theta$

Ratio remains the same $\begin{cases} \sin\to\sin \\ \cos\to\cos \\ \tan\to\tan \\ \cot\to\cot \\ \sec\to\sec \\ \text{cosec}\to\text{cosec} \end{cases}$

For Example

(a) $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$

(b) $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$

(c) $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

(d) $\sec\left(\frac{3\pi}{2}-\theta\right)=-\text{cosec}\,\theta$

(e) $\sin(\pi+\theta) = -\sin\theta$

(f) $\sin(\pi-\theta) = \sin\theta$

(g) $\tan(\pi+\theta) = \tan\theta$

(h) $\sec(2\pi-\theta) = \sec\theta$

SEQUENCES AND SERIES

1. For an **Arithmetic Progression (A.P.)** with the first term ‘a’, common difference ‘d’ and last term ‘ℓ’
   (i) n^{th} term = $t_n = a+(n-1)d\ (n\in N)$
   (ii) Sum of first ‘n’ terms = $S_n=\frac{n}{2}\left[2a+(n-1)d\right]=\frac{n}{2}[a+l]$
   (iii) For any sequence $t_n = S_n - S_{n-1}$
   (iv) It is convenient to consider
      (a) Three consecutive terms of an A.P. are a – d, a, a + d
      (b) Four consecutive terms of an A.P. are a – 3d, a – d, a + d, a + 3d (here note that the common difference is 2d)
      (c) Five consecutive terms as a – 2d, a – d, a, a + d, a + 2d.

2. For a **Geometric Progression (G.P.)** with the first term 'a', and common ratio 'r'

(i) n^{th} term = $t_n = ar^{n-1}$ $(n \in N)$

(ii) Sum of first 'n' terms = $S_n = \frac{a\left(r^n - 1\right)}{r-1}\left(r \neq 1\right)$

= na (r = 1)

(iii) If 'a' and 'r' be the 1^{st} term and common ratio of a G.P., respectively, such that $|r| < 1$, then sum to infinity, S, is given by $S_\infty = \frac{a}{1-r}$

(iv) It is convenient to consider

(a) Three consecutive terms as $\frac{a}{r}, a, ar$.

(b) Four consecutive terms as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$. (Note that here the common ratio is r^2)

(c) Five consecutive terms as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

3. If a, b, c are in **arithmetic progression**, then 'b' is called as the arithmetic mean of 'a' and 'c'; and is related as $\boldsymbol{b = \frac{a+c}{2}}$

4. If a, b, c are in **geometric progression**, then 'b' is called as the geometric mean of 'a' and 'c'; and is related as $\boldsymbol{b^2 = ac}$

5. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in arithmetic progression, then a, b, c are in **harmonic progression**. 'b' is called as the harmonic mean of 'a' and 'c'; and is related as $\boldsymbol{\frac{2}{b} = \frac{1}{a} + \frac{1}{c}}$

6. $\sum_{r=1}^{n} r = 1+2+3+4+5+...+n = \frac{n\left(n+1\right)}{2}$

7. $\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n\left(n+1\right)\left(2n+1\right)}{6}$

8. $\sum_{r=1}^{n} r^3 = 1^3 + 2^3 + 3^3 + 4^3 + ... + n^3 = \frac{n^2\left(n+1\right)^2}{4}$

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LOGARITHMS

If $a^x = y$ (Exponential form), then $x = \log_a y$ (logarithmic form) and vice-versa

Note : In Std. XI, XII Maths, natural base of log is "e" where e = 2.7128 is a fixed constant

1. $\log a + \log b = \log ab$

2. $\log a - \log b = \log\left(\frac{a}{b}\right)$

3. $\log (a^b) = b \log a$ …. **Note :** $(\log a)^b \neq b \log a$

4. $\frac{\log b}{\log a} = \log_a b$ (Change of base formula)

5. (i) $\log 1 = 0$ (ii) $\log e = 1$

   (iii) $\log 0$ = Not Defined (iv) $a^{\log_a N} = N$

   (v) (a) $\log_b a = \frac{1}{\log_a b}$ (b) $(\log_b a)(\log_a b) = 1$

QUICK RECAP

ALGEBRAIC IDENTITIES

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2 = (a+b)^2 - 4ab$
3. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$
4. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$
5. $a^2 - b^2 = (a-b)(a+b)$
6. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$
9. $a^2 + b^2 = (a-b)^2 + 2ab$
10. $a^2 + b^2 = (a+b)^2 - 2ab$
11. $1 + a^2 + a^4 = (1 + a + a^2)(1 - a + a^2)$
12. (a) If $a, b \in R$ and $a^2 + b^2 = 0$, then $a = 0$ and $b = 0$

    (b) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

    $$= \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

    If $a + b + c = 0$ or $a = b = c$, then $a^3 + b^3 + c^3 = 3abc$

    If $a^3 + b^3 + c^3 = 3abc$, then $a + b + c = 0$ or $a = b = c$

INDICES

1. $a^m a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$
3. $(a^m)^n = (a^n)^m = a^{mn}$
4. $(ab)^m = a^m \times b^m$
5. $(\frac{a}{b})^m = \frac{a^m}{b^m}$
6. $a^{-m} = \frac{1}{a^m}$
7. $(\frac{x}{y})^{-n} = (\frac{y}{x})^n$
8. $a^0 = 1$
9. $\sqrt[n]{a} = a^{\frac{1}{n}}$

QUADRATIC EQUATIONS

1. The roots of a quadratic equation $ax^2+bx+c=0\ (a\neq 0)$, are given by $\alpha,\beta=\frac{-b\pm\sqrt{\Delta}}{2a}$ where $\Delta=b^2-4ac$ is called discriminant of the quadratic equation.

2. Sum of roots (S) = $\alpha+\beta=-\frac{b}{a}=-\frac{\text{coefficient of x}}{\text{coefficient of x}^2}$

3. Product of roots (P) = $\alpha\cdot\beta=\frac{c}{a}=\frac{\text{constant term}}{\text{coefficient of x}^2}$

4. ***Nature of roots:***

   For the quadratic equation $ax^2+bx+c=0\left(a,b,c\in R\right)$, we have :

$\boldsymbol{\Delta=b^2-4ac}$	***Nature of Roots***
$\Delta>0$	Real & unequal
$\Delta=0$	Real & equal
$\Delta<0$	Imaginary & conjugate of each other

5. ***Formation of a Quadratic Equation:***

   The quadratic equation having roots $\alpha\ \&\ \beta$ is $(x-\alpha)(x-\beta)=0.$

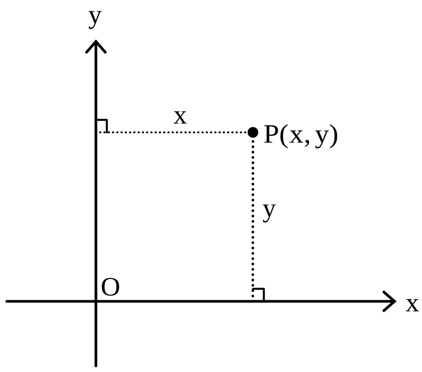
   This can also be written as $x^2-Sx+P=0,$ where $S=\alpha+\beta$ and $P=\alpha\cdot\beta$

6. If α and β are roots of a quadratic equation $ax^2+bx+c=0$, $a\neq 0$, then the **quadratic expression** is given by : $ax^2+bx+c=a(x-\alpha)(x-\beta)$

7. ***Completing the square:*** Last term = $\frac{(\text{Middle Term})^2}{4\times(\text{First Term})}$

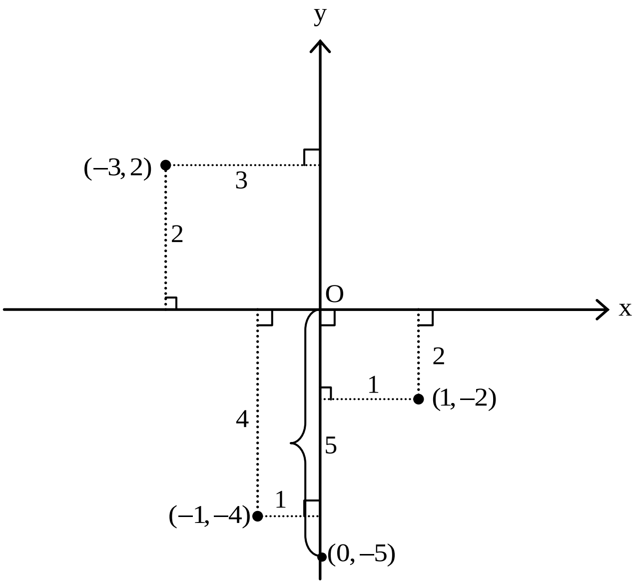
BASIC FUNDAMENTALS OF CO-ORDINATE GEOMETRY

1. **CO-ORDINATES OF A POINT:**



- x co-ordinate is called abscissa
- y co-ordinate is called ordinate
- Distance of a point P(x, y) from X-axis = |y|
- Distance of a point P(x, y) from Y-axis = |x|

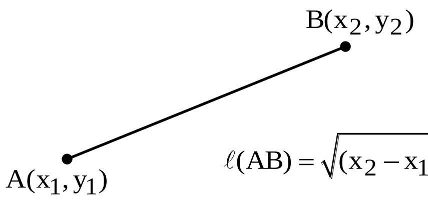
For example:



- Distance of (–3, 2) from X-axis = 2 and from Y-axis = 3
- Distance of (–1, –4) from X-axis = 4 and from Y-axis = 1
- Distance of (0, –5) from X-axis = 5 and from Y-axis = 0 (i.e. the point is on Y-axis)
- Distance of (1, –2) from X-axis = 2 and from Y-axis = 1

2. **Distance Formula:**

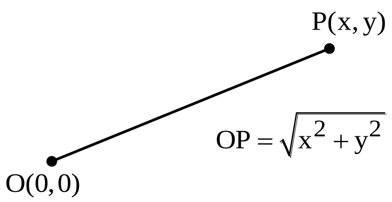
Distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$



$$\ell(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Corollary:

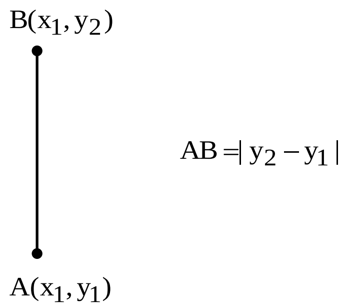
(i) If one of the points is the origin,



$$OP = \sqrt{x^2 + y^2}$$

e.g. Distance of P(3, –4) from origin = $\sqrt{3^2 + 4^2} = 5$

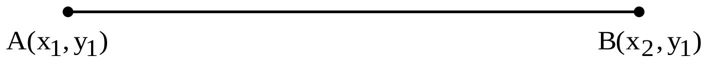
(ii) If the two points have same abscissa, then



e.g. If A(2, –5) and B(2, 3) then AB = |3 – (–5)| = 8

(iii) If the two points have same ordinate, then

$$AB = |x_2 - x_1|$$



e.g. If A(3, 4) and B(–1, 4) then AB = |–1 –3| = 4

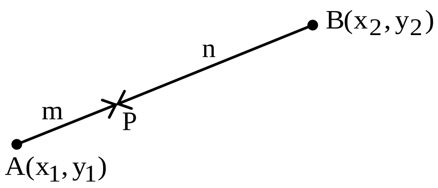
3. **Section formula :**

(a) For internal division :

If P divides segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m : n, then

$$\frac{AP}{PB} = \frac{m}{n}$$

and $P \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$



e.g. If P divides AB internally in the ratio 1 : 3, where A(–1, 4) and B(5, 2) then,

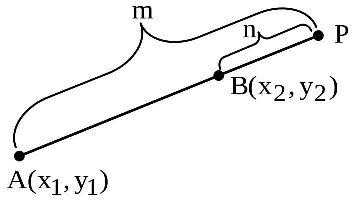
$$P \equiv \left(\frac{1(5)+3(-1)}{1+3}, \frac{1(2)+3(4)}{1+3}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

(b) For external division :

If P divides segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio m : n, then

$$\frac{AP}{PB} = \frac{m}{n}$$

and $P \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$



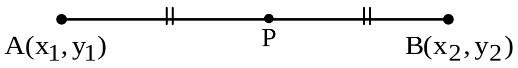
e.g. If P divides AB externally in the ratio 2 : 5, where A(–1, 4) and B(5, 2) then,

$$P \equiv \left(\frac{2(5)-5(-1)}{2-5}, \frac{2(2)-5(4)}{2-5}\right) = \left(-5, \frac{16}{3}\right)$$

Corollary

(i) Midpoint Formula:

Midpoint is $P \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



(ii) Centroid Formula:

Centroid is $G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$