Basic Mathematics_22103_ UO-2.1.3

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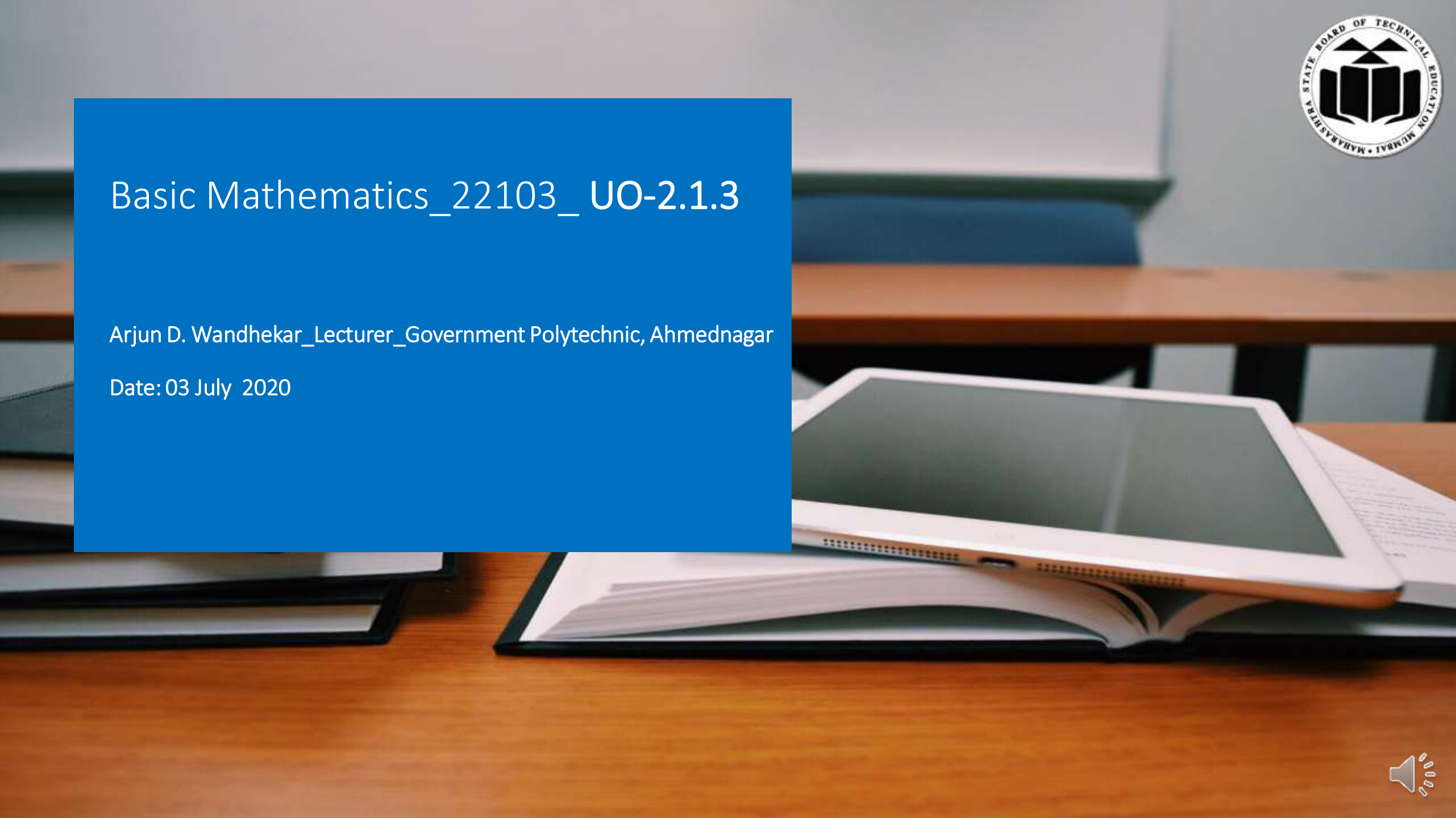
Date: 03 July 2020



Unit 2: Trigonometry

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Topic :Multiple Angles

07 Month 2020

Learning Objective/ Key learning

- Apply the concept of Multiple angles to solve the given simple engineering problems.

Contents

- Definition of Multiple angles
- Trigonometric ratios of Multiple angles
- Examples based on Multiple angles

Multiple angles:

The integral multiples of an angle A are called multiple angles of A.

For any angle θ - 2θ ,3θ ,4θ …..are multiple angles of θ .

Trigonometric ratios of multiple angles:

I .Trigonometric ratios of 2θ:

Standard formulae:

1) $\sin 2\theta = 2 \sin \theta \cdot \cos \theta$

$$\sin 2\theta = \frac{2\tan \theta}{1+\tan^2\theta}$$

2) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - 2 \sin^2\theta$

$\cos 2\theta = 2 \cos^2\theta - 1$

$$\cos 2\theta = \frac{1- \tan^2\theta}{1+ \tan^2\theta}$$

From the above we can deduce

$1 - \cos 2\theta = 2 \sin^2\theta$

$1 + \cos 2\theta = 2 \cos^2\theta$

3) $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2\theta}$

Solved Examples:

1. If sin A=0.4 then find cos 2A

Solution : Given sin A = 0.4

since $\cos 2A = 1 - 2 \sin^2 A$

$= 1-2(0.4)^2$

$= 1- 2(0.16)$

$= 1- 0.32$

$= 0.68$

2. If $\cos A=\frac{1}{2}$ **then** find cos 2A

Solution:

since $\cos 2A = 2\cos^2 A - 1$

$= 2\left(\frac{1}{2}\right)^2 - 1$

$= 2\left(\frac{1}{4}\right) - 1$

$= \frac{1}{2} - 1$

$= -\frac{1}{2}$

3. If $\sec\theta = -\frac{13}{5}$ and θ lies in second quadrant, then find $\tan 2\theta$.

Solution:

since $\tan^2\theta = \sec^2\theta - 1$

$$\tan^2\theta = \left(-\frac{13}{5}\right)^2 - 1$$

$$= \frac{169}{25} - 1$$

$$\tan^2\theta = \frac{144}{25}$$

$$\text{hence } \tan\theta = \pm\frac{12}{5}$$

Since θ lies in second quadrant ,

$$\tan\theta = -\frac{12}{5}$$

$$\text{Now } \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2(-\frac{12}{5})}{1-(-\frac{12}{5})^2}$$

$$= \frac{-\frac{24}{5}}{1-\frac{144}{25}}$$

$$= \frac{120}{119}$$

QUIZ:

1. $\sin 4\theta$ =?

   a) $4 \sin \theta$ b) $4 \sin \theta \cos\theta$ c) $4 \sin 2\theta \cos 2\theta$ d) $2 \sin 2\theta \cos 2\theta$

2. $1+ \cos 4\theta$= ?

   a) $2 \cos^2\theta + 1$ b) $2 \cos^2\theta - 1$ c) $2 \cos^2 2\theta$ d) $2 \sin^2 2\theta$

3. $1-\cos 6\theta$ =?

   a) $2 \sin^2 3\theta$ b) $2 \cos^2 3\theta$ c) $2 \sin^2 3\theta + 1$ d) $2 \sin^2 3\theta - 1$

Ans: 1. d) 2. c) 3. a)

II. Trigonometric ratios of 3θ:

1. $\sin 3\theta = 3 \sin \theta - 4 \sin^3\theta$

2. $\cos 3\theta = 4 \cos^3\theta - 3 \cos \theta$

3. $\tan 3\theta = \dfrac{3 \tan\theta - \tan^3\theta}{1 - 3 \tan^2\theta}$

Solved Examples:

1) If sin A = 0.4, Find sin 3A.

Solution : Given sin A = 0.4

Now, $\sin 3A = 3 \sin A - 4 \sin^3 A$

$= 3(0.4) - 4(0.4)^3$

$= 1.2 - 0.256$

$= 0.944$

$\therefore$ **sin 3A = 0.944**

2) If $\cos A = \frac{1}{2}$, find the value of cos 3A.

Solution: Given that $\cos A = \frac{1}{2}$

Now, $\cos 3A = 4\cos^3 A - 3\cos A$

$$= 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)$$

$$= 4\left(\frac{1}{8}\right) - \frac{3}{2}$$

$$= \frac{1}{2} - \frac{3}{2}$$

$$= \frac{-2}{2}$$

$\therefore \cos 3A = -1$

3) Prove that : $\frac{\sin 4\theta + \sin 2\theta}{1 + \cos 2\theta + \cos 4\theta} = \tan 2\theta$

Solution: L.H.S. = $\frac{\sin 4\theta + \sin 2\theta}{1 + \cos 2\theta + \cos 4\theta}$

Use Formula $\sin 4\theta = 2 \sin 2\theta \cos 2\theta$

And $1 + \cos 4\theta = 2 \cos^2 2\theta$

$$\text{L.H.S.} = \frac{2 \sin 2\theta \cos 2\theta + \sin 2\theta}{2 \cos^2 2\theta + \cos 2\theta}$$

$$= \frac{\sin 2\theta (2 \cos 2\theta + 1)}{\cos 2\theta (2 \cos 2\theta + 1)}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

= R.H.S.

Summary

So today we learn-

- Definition of Multiple angle.
- Trigonometric ratios of Multiple angle.
- Solved examples based on Multiple angles.

Quiz

1) If cos α = 0.4, find cos 3α

a)0.446 b) -0.944 c)0.56 d)1.17

2) If sin A= ½,find the value of sin 3A

a)1/2 b)1/3 c)1/4 d)1

Ans: 1. b) 2. d)

Thank You