FACTORIZATION & DEFACTORIZATION FORMULAE

TRIGONOMETRY

12

MARKS

S. K. Rawat

OBJECTIVES

- To derive De-factorization Formulae.
- To solve examples to convert product of T-ratios into sum or difference.
- To derive factorization formulae.
- To solve examples to convert sum of T-ratios into product.
- To solve examples on factorization and de-factorization formulae.
- To solve examples based on angles of triangles

S. K. Rawat

DEFACTORIZATION FORMULA

- Expressing the product of sines and cosines of two angles as sum or difference of sines and cosines. These formulae are called **Defactorization Formulae.**
- Defactorization Formulae are given below

$$1)\qquad 2\sin A.\cos B = \sin(A+B)+\sin(A-B)$$

$$2)\qquad 2\cos A.\sin B = \sin(A+B)-\sin(A-B)$$

$$3)\qquad 2\cos A.\cos B = \cos(A+B)+\cos(A-B)$$

$$4)\qquad 2\sin A.\sin B = -[\cos(A+B)-\cos(A-B)]$$

S. K. Rawat

Proof

Prove that $2sinA.cosB = sin(A+B) + sin(A-B)$

Solution: We know from Compound Angle Formula

$$sin(A+B) = sin\,A.cos\,B + cos\,A.sin\,B \qquad \ldots\ldots\ldots\ldots\ldots \qquad (1)$$

$$sin(A-B) = sin\,A.cos\,B - cos\,A.sin\,B \qquad \ldots\ldots\ldots\ldots\ldots \qquad (2)$$

$$cos(A+B) = cos\,A.cos\,B - sin\,A.sin\,B \qquad \ldots\ldots\ldots\ldots\ldots \qquad (3)$$

$$cos(A-B) = cos\,A.cos\,B + sin\,A.sin\,B \qquad \ldots\ldots\ldots\ldots\ldots \qquad (4)$$

Adding eqn (1) and (2), we get

$$\therefore sin(A+B) + sin(A-B) = sin\,A.cos\,B \cancel{+ cos\,A.sin\,B} + sin\,A.cos\,B \cancel{- cos\,A.sin\,B}$$

$$\therefore sin(A+B) + sin(A-B) = 2\,sin\,A.cos\,B$$

$$\boxed{\therefore 2\,sin\,A.cos\,B = sin(A+B) + sin(A-B)}$$

S. K. Rawat

Proof

Prove that $2cosA.sinB = sin(A+B) - sin(A-B)$

Solution: We know from Compound Angle Formula

$$sin(A+B) = sin\,A.cos\,B + cos\,A.sin\,B \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$sin(A-B) = sin\,A.cos\,B - cos\,A.sin\,B \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$cos(A+B) = cos\,A.cos\,B - sin\,A.sin\,B \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$cos(A-B) = cos\,A.cos\,B + sin\,A.sin\,B \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

Subtracting eqn (2) from (1), we get

$$\therefore sin(A+B) - sin(A-B) = \cancel{sin\,A.cos\,B} + cos\,A.sin\,B - \cancel{sin\,A.cos\,B} + cos\,A.sin\,B$$

$$\therefore sin(A+B) - sin(A-B) = 2\,cos\,A.sin\,B$$

$$\therefore 2\,cos\,A.sin\,B = sin(A+B) - sin(A-B)$$

Similarly we can prove

$$2\,cos\,A.cos\,B = cos(A+B) + cos(A-B)$$

$$2\,sin\,A.sin\,B = cos(A+B) - cos(A-B)$$

S. K. Rawat

Example

$$If\ \ 2\sin 50^o \cos 70^o = \sin A - \sin B.\ \ Find\ \ A\ and\ \ B.$$

Solution: Consider $2\sin 50^{\circ} \cos 70^{\circ}$

$$Using\ \ 2sinAcosB = sin(A+B) + sin(A-B)$$

$$\therefore\ 2\sin 50^o \cos 70^o = \sin(50^o + 70^o) + \sin(50^o - 70^o)$$

$$= \sin(120^o) + \sin(-20^o)$$

$$= \sin 120^o - \sin 20^o$$

$$= \sin A - \sin B \quad \{Given\}$$

$$Thus \qquad A = 120^{\circ}\ \ \&\ \ B = 20^{\circ}$$

S. K. Rawat

Example

Prove that $\sin 20^o.\sin 40^o.\sin 60^o.\sin 80^o = \dfrac{3}{16}$

Solution: $L.H.S = \sin 20^o \sin 40^o \sin 60^o \sin 80^o$

$$= \sin 20^o \sin 40^o \left(\frac{\sqrt{3}}{2}\right) \sin 80^o \quad = \frac{\sqrt{3}}{2} \sin 20^o \sin 40^o \sin 80^o$$

Multiplying & dividing by 2

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \left\{2\sin 20^o \sin 40^o \sin 80^o\right\} \quad = \frac{\sqrt{3}}{4}\left\{2\sin 20^o \sin 40^o\right\}\sin 80^o$$

Using $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$= \frac{\sqrt{3}}{4}\left\{\cos\left(20^o - 40^o\right) - \cos\left(20^o + 40^o\right)\right\}\sin 80^o$$

$$= \frac{\sqrt{3}}{4}\left\{\cos 20^o - \cos 60^o\right\}\sin 80^o \quad = \frac{\sqrt{3}}{4}\left\{\cos 20^o \sin 80^o - \cos 60^o \sin 80^o\right\}$$

Example

Prove that $\sin 20^o . \sin 40^o . \sin 60^o . \sin 80^o = \dfrac{3}{16}$

Solution: *Continued………*

$$L.H.S = \frac{\sqrt{3}}{4}\left\{\cos 20^o \sin 80^o - \cos 60^o \sin 80^o\right\} \quad = \frac{\sqrt{3}}{4}\left\{\cos 20^o \sin 80^o - \left(\frac{1}{2}\right)\sin 80^o\right\}$$

$\because \cos 60^o = \dfrac{1}{2}$

Multiplying & dividing by 2

$$= \frac{\sqrt{3}}{4} \times \frac{1}{2}\left\{2\left[\cos 20^o \sin 80^o - \left(\frac{1}{2}\right)\sin 80^o\right]\right\} \quad = \frac{\sqrt{3}}{8}\left\{2\cos 20^o \sin 80^o - \cancel{2}\left(\frac{1}{\cancel{2}}\right)\sin 80^o\right\}$$

$$= \frac{\sqrt{3}}{8}\left\{2\cos 20^o \sin 80^o - \sin 80^o\right\}$$

Using $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$$= \frac{\sqrt{3}}{8}\left\{\sin(20^o + 80^o) - \sin(20^o - 80^o) - \sin 80^o\right\} \quad = \frac{\sqrt{3}}{8}\left\{\sin 100^o - \sin(-60^o) - \sin 80^o\right\}$$

$$= \frac{\sqrt{3}}{8}\left\{\sin(180^o - 80^o) + \sin 60^o - \sin 80^o\right\} \quad = \frac{\sqrt{3}}{8}\left\{\cancel{\sin 80^o} + \frac{\sqrt{3}}{2} - \cancel{\sin 80^o}\right\} \quad = \frac{\sqrt{3}}{8}\left\{\frac{\sqrt{3}}{2}\right\} = \frac{3}{16} = R.H.S$$

S. K. Rawat

FACTORIZATION FORMULA

- Expressing the sum and differences of sines and cosines of two angles as product of sines and cosines. These formulae are called **Factorization Formulae.**
- Factorization Formulae are given below

$$1)\qquad \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$2)\qquad \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$3)\qquad \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$4)\qquad \cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$$

S. K. Rawat

Proof

Prove that $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

Solution: We know from Compound Angle Formula

$$\sin(A+B) = \sin A.\cos B + \cos A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\sin(A-B) = \sin A.\cos B - \cos A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\cos(A+B) = \cos A.\cos B - \sin A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\cos(A-B) = \cos A.\cos B + \sin A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

Adding eqn (1) and (2), we get

$$\therefore \sin(A+B) + \sin(A-B) = \sin A.\cos B \cancel{+ \cos A.\sin B} + \sin A.\cos B \cancel{- \cos A.\sin B}$$

$$\therefore \sin(A+B) + \sin(A-B) = 2\sin A.\cos B$$

Let $A+B=C$ *and* $A-B=D$

$$\therefore \quad A = \frac{C+D}{2} \quad and \quad B = \frac{C-D}{2}$$

$$\therefore \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

S. K. Rawat

Proof

Prove that $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

Solution: We know from Compound Angle Formula

$$\sin(A+B) = \sin A.\cos B + \cos A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\sin(A-B) = \sin A.\cos B - \cos A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\cos(A+B) = \cos A.\cos B - \sin A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\cos(A-B) = \cos A.\cos B + \sin A.\sin B \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

Subtracting eqn (2) from (1), we get

$$\therefore \sin(A+B) - \sin(A-B) = \cancel{\sin A.\cos B} + \cos A.\sin B - \cancel{\sin A.\cos B} + \cos A.\sin B$$

$$\therefore \sin(A+B) - \sin(A-B) = 2\cos A.\sin B$$

Let $A+B=C$ *and* $A-B=D$

$$\therefore \quad A = \frac{C+D}{2} \quad and \quad B = \frac{C-D}{2}$$

$$\therefore \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

S. K. Rawat

Example

Express as product and evaluate $\sin 99^o - \sin 81^o$

Solution: *Consider* $\sin 99^o - \sin 81^o$

$$\therefore \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\therefore \sin 99^o - \sin 81^o = 2\cos\left(\frac{99^o + 81^o}{2}\right)\sin\left(\frac{99^o - 81^o}{2}\right)$$

$$= 2\cos\left(\frac{180^o}{2}\right)\sin\left(\frac{18^o}{2}\right)$$

$$= 2\cos 90^o \sin 9^o = 2(0)\sin 9^o = 0$$

$$\therefore \sin 99^o - \sin 81^o = 0$$

S. K. Rawat

Example

Prove that $\dfrac{sinA+2sin2A+sin3A}{cosA+2cos2A+cos3A}=tan2A$

Solution:

$$\text{L.H.S}=\frac{\sin A+2\sin 2A+\sin 3A}{\cos A+2\cos 2A+\cos 3A}=\frac{\sin A+\sin 3A+2\sin 2A}{\cos A+\cos 3A+2\cos 2A}$$

$$=\frac{(\sin A+\sin 3A)+2\sin 2A}{(\cos A+\cos 3A)+2\cos 2A}$$

$$\therefore\ sin\,C+sin\,D=2\,sin\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$$

$$\therefore\ cos\,C+cos\,D=2\,cos\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$$

$$=\frac{2\sin\left(\frac{A+3A}{2}\right)\cos\left(\frac{A-3A}{2}\right)+2\sin 2A}{2\cos\left(\frac{A+3A}{2}\right)\cos\left(\frac{A-3A}{2}\right)+2\cos 2A}=\frac{2\sin\left(\frac{4A}{2}\right)\cos\left(\frac{-2A}{2}\right)+2\sin 2A}{2\cos\left(\frac{4A}{2}\right)\cos\left(\frac{-2A}{2}\right)+2\cos 2A}$$

Example

Prove that $\dfrac{sinA + 2sin2A + sin3A}{cosA + 2cos2A + cos3A} = tan2A$

Solution:

$$\text{LHS} = \frac{2\sin\left(\frac{4A}{2}\right)\cos\left(\frac{-2A}{2}\right) + 2\sin 2A}{2\cos\left(\frac{4A}{2}\right)\cos\left(\frac{-2A}{2}\right) + 2\cos 2A} = \frac{2\sin(2A)\cos(-A) + 2\sin 2A}{2\cos(2A)\cos(-A) + 2\cos 2A}$$

$$= \frac{\cancel{2}\sin(2A)[\cancel{1+\cos(-A)}]}{\cancel{2}\cos(2A)[\cancel{1+\cos(-A)}]}$$

$$= \frac{\sin(2A)}{\cos(2A)} = \tan 2A = \text{RHS}$$

S. K. Rawat

Example

Show that $\dfrac{\cos 3A - \cos 7A}{\sin 9A + \sin A} = \cos 2A \tan 4A - \sin 2A$

Solution:

$$\therefore \cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$$

$$\therefore \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$L.H.S = \frac{\cos 3A - \cos 7A}{\sin 9A + \sin A}$$

$$= \frac{2\sin\left(\frac{3A+7A}{2}\right)\sin\left(\frac{7A-3A}{2}\right)}{2\sin\left(\frac{9A+A}{2}\right)\cos\left(\frac{9A-A}{2}\right)} = \frac{2\sin\left(\frac{10A}{2}\right)\sin\left(\frac{4A}{2}\right)}{2\sin\left(\frac{10A}{2}\right)\cos\left(\frac{8A}{2}\right)} = \frac{\cancel{2\sin(5A)}\sin(2A)}{\cancel{2\sin(5A)}\cos(4A)} = \frac{\sin(2A)}{\cos(4A)}$$

$$= \frac{\sin(4A-2A)}{\cos 4A} = \frac{\sin 4A.\cos 2A - \cos 4A.\sin 2A}{\cos 4A} = \frac{\sin 4A \cos 2A}{\cos 4A} - \frac{\cancel{\cos 4A}\ \sin 2A}{\cancel{\cos 4A}}$$

$$= \frac{\sin 4A}{\cos 4A}\cos 2A - \sin 2A \qquad = \tan 4A \cos 2A - \sin 2A = R.H.S$$

S. K. Rawat

Example

Show that $\dfrac{\sin A+\sin 2A+\sin 3A+\sin 4A}{\cos A+\cos 2A+\cos 3A+\cos 4A}=\tan\dfrac{5A}{2}$

Solution: $L.H.S=\dfrac{\sin A+\sin 2A+\sin 3A+\sin 4A}{\cos A+\cos 2A+\cos 3A+\cos 4A}=\dfrac{(\sin A+\sin 4A)+(\sin 2A+\sin 3A)}{(\cos A+\cos 4A)+(\cos 2A+\cos 3A)}$

$$=\frac{2\sin\left(\frac{A+4A}{2}\right)\cos\left(\frac{A-4A}{2}\right)+2\sin\left(\frac{2A+3A}{2}\right)\cos\left(\frac{2A-3A}{2}\right)}{2\cos\left(\frac{A+4A}{2}\right)\cos\left(\frac{A-4A}{2}\right)+2\cos\left(\frac{2A+3A}{2}\right)\cos\left(\frac{2A-3A}{2}\right)}$$

$$\because \cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\because \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$=\frac{2\sin\left(\frac{5A}{2}\right)\cos\left(\frac{-3A}{2}\right)+2\sin\left(\frac{5A}{2}\right)\cos\left(\frac{-A}{2}\right)}{2\cos\left(\frac{5A}{2}\right)\cos\left(\frac{-3A}{2}\right)+2\cos\left(\frac{5A}{2}\right)\cos\left(\frac{-A}{2}\right)}=\frac{2\sin\frac{5A}{2}\cos\frac{3A}{2}+2\sin\frac{5A}{2}\cos\frac{A}{2}}{2\cos\frac{5A}{2}\cos\frac{3A}{2}+2\cos\frac{5A}{2}\cos\frac{A}{2}}$$

$$=\frac{\cancel{2}\sin\frac{5A}{2}\left[\cancel{\cos\frac{3A}{2}+\cos\frac{A}{2}}\right]}{\cancel{2}\cos\frac{5A}{2}\left[\cancel{\cos\frac{3A}{2}+\cos\frac{A}{2}}\right]}=\frac{\sin\frac{5A}{2}}{\cos\frac{5A}{2}}=\tan\frac{5A}{2}=R.H.S$$

S. K. Rawat

Example

Prove that $\dfrac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \sin A.\tan 3A$

Solution: $L.H.S = \dfrac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \dfrac{(\cos 2A + \cos 4A) + (\cos 4A + \cos 6A)}{(\cos A + \cos 3A) + (\cos 3A + \cos 5A)}$

$$= \frac{2\cos\left(\frac{2A+4A}{2}\right)\cos\left(\frac{2A-4A}{2}\right) + 2\cos\left(\frac{4A+6A}{2}\right)\cos\left(\frac{4A-6A}{2}\right)}{2\cos\left(\frac{A+3A}{2}\right)\cos\left(\frac{A-3A}{2}\right) + 2\cos\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right)}$$

$$\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$= \frac{2\left[\cos\left(\frac{6A}{2}\right)\cos\left(\frac{-2A}{2}\right) + \cos\left(\frac{10A}{2}\right)\cos\left(\frac{-2A}{2}\right)\right]}{2\left[\cos\left(\frac{4A}{2}\right)\cos\left(\frac{-2A}{2}\right) + \cos\left(\frac{8A}{2}\right)\cos\left(\frac{-2A}{2}\right)\right]} = \frac{\cos 3A.\cos(-A) + \cos 5A.\cos(-A)}{\cos 2A.\cos(-A) + \cos 4A.\cos(-A)}$$

$$= \frac{\cos 3A.\cos A + \cos 5A.\cos A}{\cos 2A.\cos A + \cos 4A.\cos A} = \frac{\cancel{\cos A}(\cos 3A + \cos 5A)}{\cancel{\cos A}(\cos 2A + \cos 4A)} = \frac{\cos 3A + \cos 5A}{\cos 2A + \cos 4A}$$

Example

Prove that $\dfrac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \sin A.\tan 3A$

Solution: $L.H.S = \dfrac{\cos 3A + \cos 5A}{\cos 2A + \cos 4A}$

$\therefore \cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

$$= \frac{2\cos\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right)}{2\cos\left(\frac{2A+4A}{2}\right)\cos\left(\frac{2A-4A}{2}\right)} = \frac{2\cos\left(\frac{8A}{2}\right)\cos\left(\frac{-2A}{2}\right)}{2\cos\left(\frac{6A}{2}\right)\cos\left(\frac{-2A}{2}\right)}$$

$$= \frac{\cos(4A)\cos(-A)}{\cos(3A)\cos(-A)} = \frac{\cos(3A+A)}{\cos(3A)} = \frac{\cos 3A.\cos A - \sin 3A.\sin A}{\cos 3A}$$

$$= \frac{\cos 3A.\cos A}{\cos 3A} - \frac{\sin 3A.\sin A}{\cos 3A} = \cos A - \frac{\sin 3A}{\cos 3A}\sin A$$

$$= \cos A - \sin A.\tan 3A = R.H.S$$

S. K. Rawat

Example

$$\text{Prove that } \frac{\sin 8\theta \cos\theta - \cos 3\theta \sin 6\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$$

Solution: $$\text{L.H.S} = \frac{\sin 8\theta \cos\theta - \cos 3\theta \sin 6\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta}$$

Multiplying and dividing by 2

Using

$2\sin A.\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A.\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A.\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A.\sin B = \cos(A+B) - \cos(A-B)$

$$\therefore \text{L.H.S} = \frac{2\sin 8\theta \cos\theta - 2\cos 3\theta \sin 6\theta}{2\cos 2\theta \cos\theta - 2\sin 3\theta \sin 4\theta}$$

$$= \frac{[\sin(8\theta+\theta) + \sin(8\theta-\theta)] - [\sin(3\theta+6\theta) - \sin(3\theta-6\theta)]}{[\cos(2\theta+\theta) + \cos(2\theta-\theta)] - [\cos(3\theta-4\theta) - \cos(3\theta+4\theta)]}$$

$$= \frac{[\sin(9\theta) + \sin(7\theta)] - [\sin(9\theta) \boxed{- \sin(-3\theta)}]}{[\cos(3\theta) + \cos(\theta)] - \boxed{[\cos(-\theta} - \cos(7\theta)]}$$

Example

Prove that $\dfrac{\sin 8\theta \cos\theta - \cos 3\theta \sin 6\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$

Solution:

$$L.H.S = \frac{[\sin 9\theta + \sin 7\theta] - [\sin 9\theta + \sin 3\theta]}{[\cos 3\theta + \cos\theta] - [\cos\theta - \cos 7\theta]}$$

$$= \frac{\cancel{\sin 9\theta} + \sin 7\theta - \cancel{\sin 9\theta} - \sin 3\theta}{\cos 3\theta + \cancel{\cos\theta} - \cancel{\cos\theta} + \cos 7\theta}$$

$$= \frac{\sin 7\theta - \sin 3\theta}{\cos 3\theta + \cos 7\theta} \qquad = \frac{2co}{2cos}$$

Using

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$= \frac{2\cos\left(\frac{10\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)}{2\cos\left(\frac{10\theta}{2}\right)\cos\left(\frac{-4\theta}{2}\right)} = \frac{\cancel{2\cos(5\theta)}\sin(2\theta)}{\cancel{2\cos(5\theta)}\cos(-2\theta)} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta = R.H.S$$