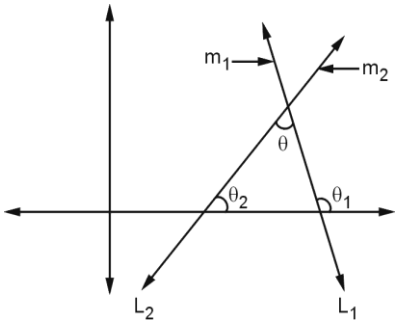
Template: Study Material

<table>
<tr><td colspan="3">Insert the details within the < quotes >
<Course Code: 22103 >: <Subject Code : BMS >: <Subject Name: Basic Mathematics>: <Topic Name : Straight Line >: <UO-3.1>: <Study Material></td></tr>
<tr><td><Course Expert : Mr. V.B.Shinde ></td><td><5/07/2020></td><td><Mr. A.D. Wandekar></td></tr>
<tr><td>Key words
Angle between two straight lines.</td><td>Learning Objective: Calculate angle between given two straight lines.</td><td rowspan="2">Diagram/ Picture
 
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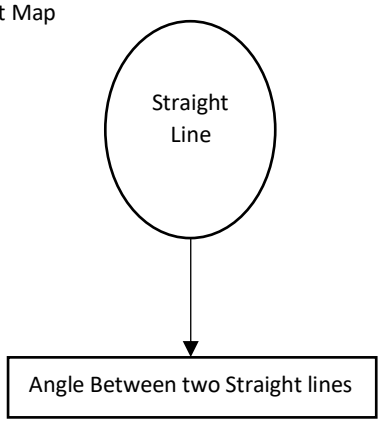
<tr><td>Key Questions

What is the straight line?

Do you know about the slope of line?</td><td>Concept Map
 
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Solved word Problem:

1) Show that the lines $2x + 3y - 5 = 0$ and $4x + 6y - 1 = 0$ are parallel.

Solution : Let

$L_1 : 2x + 3y - 5 = 0$

$\therefore$ Slope of L_1 is $m_1 = \frac{-2}{3}$ And

$L_2 : 4x + 6y - 1 = 0$

$\therefore$ Slope of L_2 is

$m_2 = \frac{-4}{6} = \frac{-2}{3}$

$\therefore m_1 = m_2$

$\therefore$ Given lines are parallel

2) Prove that lines $3x + 4y + 7 = 0$ and $28x - 21y + 50 = 0$ are perpendicular to each other.

Solution : Let

$L_1 : 3x + 4y + 7 = 0$

$\therefore$ Slope of L_1 is

$m_1 = \frac{-3}{4}$ and

$L_2 : 28x - 21y + 50 = 0$

$\therefore$ Slope of L_2 is

$m_2 = \frac{-28}{-21} = \frac{4}{3}$

$m_1 \cdot m_2 = \frac{-3}{4} \cdot \frac{4}{3} = -1$

$\therefore$ Given lines are perpendicular

Explanation of Concept;

Angle between Two Straight Lines:

If m_1 and m_2 are slopes of the two lines then the angle between two lines is

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Solved Examples

1) Find the acute angle between the lines $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$.

Solution :

Given equation of lines

$L_1 : 3x - 2y + 4 = 0$

$\therefore$ Slope $m_1 = \frac{-3}{-2} = \frac{3}{2}$

$L_2 : 2x - 3y - 7 = 0$

$\therefore$ Slope $m_2 = \frac{-2}{-3} = \frac{2}{3}$

Let 'θ' be the acute angle between the lines

Then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{3}{2} - \frac{2}{3}}{1 + \left(\frac{3}{2}\frac{2}{3}\right)}\right|$

$= \left|\frac{\frac{9-4}{6}}{1+1}\right|$

$= \left|\frac{\frac{5}{6}}{2}\right|$

$= \left|\frac{5}{12}\right|$

$\tan\theta = \frac{5}{12}$

$\theta = \tan^{-1}\left(\frac{5}{12}\right)$

2)Find the acute angle between the lines $3x - y = 4$ and $2x + y = 3$

Solution :

Given equation of lines

$L_1 : 3x - y = 4$

$L_1 : 3x - y - 4 = 0$

$\therefore$ Slope $m_1 = \frac{-3}{-1} = 3$

$L_2 : 2x + y - 3 = 0$

$\therefore$ Slope $m_2 = \frac{-2}{1} = -2$

- The acute angle between the line is

$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \tan^{-1}\left|\frac{3 - (-2)}{1 + (3)(-2)}\right|$

$= \tan^{-1}\left|\frac{5}{-5}\right|$

$= \tan^{-1}|-1|$

$= \tan^{-1}(1)$

$\theta = 45°$ OR $\theta = \frac{\pi}{4}$

Key Definitions/ Formulas

Angle between Two Straight Lines:

If m_1 and m_2 are slopes of the two lines then the angle between two lines is

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Deduction:

1. If $\theta = 0$ then lines are parallel

$0 = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan 0 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$0 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$m_1 = m_2$

2. If $\theta = 90$ then lines are parallel.

$\therefore \quad 90 = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\therefore \quad \tan 90 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\therefore \quad \infty = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

☐ $1 + m_1 \times m_2 = 0$

☐ $m_1 \times m_2 = -1$

Condition for parallel and perpendicular Lines:

1) Two lines are parallel if their slopes are equal .i. e. $m_1 = m_2$ and converse is also true.

2) Condition for two lines to be perpendicular: Two lines are perpendicular if their product of slopes is -1. i.e. $m_1 \cdot m_2 = -1$ and converse is also true

	Application of Concept/ Examples in real life In engineering field it is useful to calculate angle between two lines .	Link to YouTube/ OER/ video **https://youtu.be/vDzDLR5y45w** **https://youtu.be/jqjaf616BPg** **https://youtu.be/j93ESruzuRA**
Key Take away from this LO: 1) Condition for parallel and perpendicular Lines: 2)Angle between Two Straight Lines:		