Template: Study Material

<Course Code:22103>: <Subject Code: BMS>: <Subject Name: Basic Mathematics>: <Topic Name: Matrices> : <UO1.3.1> : <Study Material>

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Key words:
Matrix

Learning Objective:
Solve the given system of linear equations using matrix inversion method.

Diagram/ Picture:

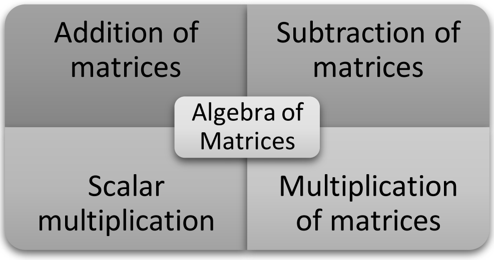
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mn} \end{bmatrix}_{m\times n}$$

In short, $A = [a_{ij}]_{m\times n}$
where $\mathrm{i} = \mathrm{No.\,of\,rows} = 1,2,3,\ldots,\mathrm{m}$
& $\mathrm{j} = \mathrm{No.\,of\,columns} = 1,2,3,\ldots,\mathrm{n}.$

Key Questions:
Have you wondered how to add or subtract two matrices?

Concept Map



COURSE CONTENT:

Definition: Matrix

A set of $m \times n$ numbers arranged in a rectangular form of m rows & n columns enclosed between a pair of square brackets is called a matrix of order $m \times n$ (read as m by n).

Matrices are generally denoted by capital alphabets & its elements are denoted by small alphabets.

Definition: Order of a matrix:-

The order of a matrix is defined as $m \times n$ if it contains m rows & n columns.

Examples:

1. $A = [2 \quad 3 \quad -1]$ Order of A is 1×3
2. $B = \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix}$ Order of B is 3×2

Types of matrices:-

1. Row matrix: Matrix having only one row is called row matrix.
   For e.g. : $A = [2 \quad 3 \quad -1]$.
2. Column matrix: Matrix having only one column is called column matrix. For e.g. : $D = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$.
3. Zero matrix: A matrix having all elements equal to zero is called zero matrix
4. Square matrix: Matrix having equal number of rows & columns is called square matrix. For e.g. $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -4 \\ 5 & -3 & 4 \end{bmatrix}$
   Note: In matrix A, elements 2,3,4 are diagonal elements and remaining are non-diagonal elements.

Contd . after the table…..

Key Definitions/ Formulas

Definition A set of $m \times n$ numbers arranged in a rectangular form of m rows & n columns enclosed between a pair of square brackets is called a matrix

Solved word Problem:

If $A = \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 6 & -1 \\ 0 & 3 \end{bmatrix}$ find $2A + 3B$.

Solution: $2A + 3B =$

$$2\begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 6 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 8 & 0 \\ -2 & -4 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 18 & -3 \\ 0 & 9 \end{bmatrix} =$$

$\begin{bmatrix} 4+3 & -6+6 \\ 8+18 & 0-3 \\ -2+0 & -4+9 \end{bmatrix}$ $= \begin{bmatrix} 7 & 0 \\ 26 & -3 \\ -2 & 5 \end{bmatrix}$	Application of Concept/ Examples in real life: It is used in coding and decoding of information.	Link to YouTube/ OER/ video: http://www.khanacademy

COURSE CONTENT: CONTINUED………

5. Diagonal matrix: A square matrix where all non-diagonal elements are zero is called a diagonal matrix.

   For e.g. : $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

6. Scalar matrix: A diagonal matrix where all diagonal elements are equal is called a scalar matrix.

   For e.g. : $K = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

7. Identity matrix OR Unit matrix: A scalar matrix where all diagonal elements are one (unit) is called an identity matrix or unit matrix denoted by I.

   For e.g. : $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Algebra of matrices:

1. Addition of matrices: If two matrices A, B are of same order then the addition matrix 'A+B' can be obtained by adding the corresponding elements. Order of matrix $A + B$ is same as that of A and B.

   For e.g. if $A = \begin{bmatrix} 5 & 6 & 1 \\ 0 & 2 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 & 3 \\ -3 & 1 & -2 \end{bmatrix}$

   then $A + B = \begin{bmatrix} 5+4 & 6+2 & 1+3 \\ 0-3 & 2+1 & 9-2 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 4 \\ -3 & 3 & 7 \end{bmatrix}$

2. Subtraction of matrices: If two matrices A, B are of same order then matrix '$A - B$' can be obtained by subtracting the corresponding elements. Order of matrix $A - B$ is same as that of A and B.

   For e.g. if $A = \begin{bmatrix} 5 & 6 & 1 \\ 0 & 2 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 & 3 \\ -3 & 1 & -2 \end{bmatrix}$

   then $A - B = \begin{bmatrix} 5-4 & 6-2 & 1-3 \\ 0+3 & 2-1 & 9+2 \end{bmatrix} = \begin{bmatrix} 1 & 4 & -2 \\ 3 & 1 & 11 \end{bmatrix}$

3. Scalar Multiplication: If A is a matrix and 'k' is a scalar then the matrix 'kA' is obtained by multiplying every element of the matrix A by 'k'.

   For e.g. if $A = \begin{bmatrix} 5 & 6 & 1 \\ 0 & 2 & 9 \end{bmatrix}$ $then\ 5A = \begin{bmatrix} 25 & 30 & 5 \\ 0 & 10 & 45 \end{bmatrix}$ where k=5

Solved Examples:

1. If A = $\begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}$ then find 5A – 3B + 2C

   Solution : 5A – 3B + 2C

   $=5\begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}$

   $= \begin{bmatrix} 10 & 25 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 12 & -3 \\ 6 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 14 \\ 10 & 4 \end{bmatrix}$

   $= \begin{bmatrix} 10-12+2 & 25+3+14 \\ 0-6+10 & 5-0+4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 42 \\ 4 & 9 \end{bmatrix}$

2. Find the value of x and y satisfying the equation

$$\begin{bmatrix} 1 & x & 0 \\ y & 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$$

Solution : $\begin{bmatrix} 1 & x & 0 \\ y & 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1+3 & x+1 & 0+2 \\ y+4 & 2+3 & 4-2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4 & x+1 & 2 \\ y+4 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$$

By using equality of matrices, $x + 1 = 2$

and $y + 4 = 6$

$\therefore x = 1$ & $y = 2$

3. If A = $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, find the matrix 'X' such that 2A + X = 3B

Solution : 2A + X = 3B

$\therefore$ X= 3B – 2A

$\therefore$ X = $3\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 9 & -6 \\ -3 & 12 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix}$

$= \begin{bmatrix} 9-4 & -6+2 \\ -3-8 & 12-6 \end{bmatrix}$

X = $\begin{bmatrix} 5 & -4 \\ -11 & 6 \end{bmatrix}$

4. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}$

then prove that $(A + B) + C = A + (B + C)$

Solution: $L.H.S. = (A + B) + C$

$$= \left(\begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 11 \\ 7 & 3 \end{bmatrix}$$

$R.H.S. = A + (B + C)$

$$= \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 11 \\ 7 & 3 \end{bmatrix}$$

$\therefore L.H.S. = R.H.S.$

Key Take away from this UO:

1. Types of Matrices
2. Addition of two matrices
3. Subtraction of two matrices
4. Scalar multiplication