Chapter 1 Fundamentals of control system

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1.1 Control system: Open loop, closed loop, linear, nonlinear, time variant, time invariant.
1.2 Transfer function; order of a control system (0, 1, 2), transfer function with respect to R-C and R-L-C electrical circuits,
1.3 Block diagram reduction technique: Need, reduction rules.
1.4 State space representation: Advantages, state variables identification, State space models from transfer functions.

1. Control System definition and examples:

Control System: It is arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system in order to provide a specific output. Controlling is the act of regulating or directing. So control system is used to direct the functioning of a physical system to carry out the desired objective.

Example:

1. In a class room if professor is delivering lecture, the combination becomes a control system.
2. If lamp is switched ON or OFF using a switch, the entire system is control system.
3. Stepper motor positioning system
4. Automatic toaster system.
5. DC motor speed control
6. Home automation system
7. Automobile driving system
8. Process control systems

2. Classification of the control system

1. Open loop and closed loop
2. Linear and nonlinear
3. Time varying and time in varying

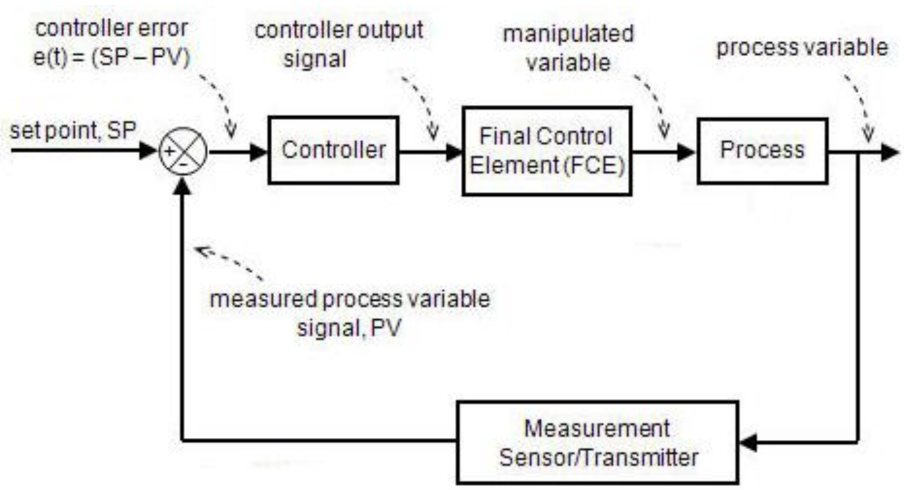
3. Closed loop system:

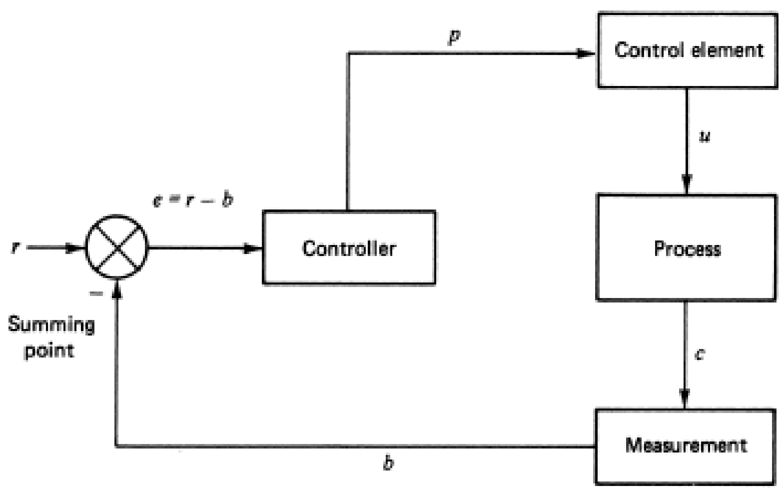
- The main difference between open loop and closed loop system is that of feedback action.
- Any physical system which automatically corrects for variations in its output is called closed loop system**.** In the closed loop system the desired output depends on the control action of the system.
- In closed loop system, output is taken back to the input for comparison with the set point to find out the error. This is called feedback. Thus, error is found out where

$$\mathbf{error = setpoint - measured\ value}.$$

Thus the closed loop control system uses feedback signal to generate output.

Block diagram of closed loop system





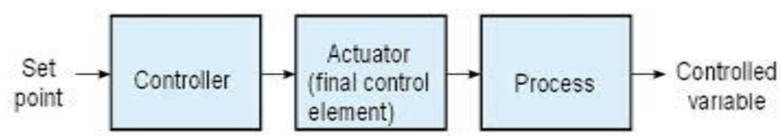
- Controller: Controller requires an input which is the difference between the controlled variable and the reference value or the set point of the variable. This difference is called error. Control actions are the term used to represent the relationship between the controller output and error. Evaluation of controller consists of determining the action required to drive the controlled variable to the set point value. The controller output is the commanding signal given to the final control element (FCE) to reduce the error.
- Final control element: It is the device that provides the required changes in the controlled variable to bring it to the set point. This element accepts an input from the controller, which is then transformed into some proportional operation performed on the process. Commonly used FCE is control valve.

- Process: It is often called as plant. It consists of single variable or multi variables which are to be controlled. Control element output is given to process which changes the process variable.
- Error detector or summing point: It receives two inputs which are the set point and controlled variable and error obtained is given to controller.

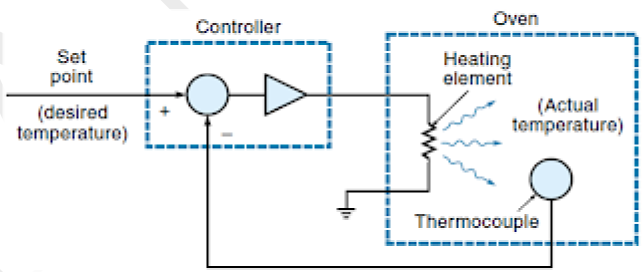
4. Open loop system:

- Any physical system which does not automatically correct for variations in its output is called open loop system**.**
- In open loop system, output is not taken back to the input; so there is no feedback and it does not compare set point with measured value. Therefore, it does not have error detector.
- In an open loop system the desired output does not depend on the control action. Open loop system does not automatically correct for variations in output.

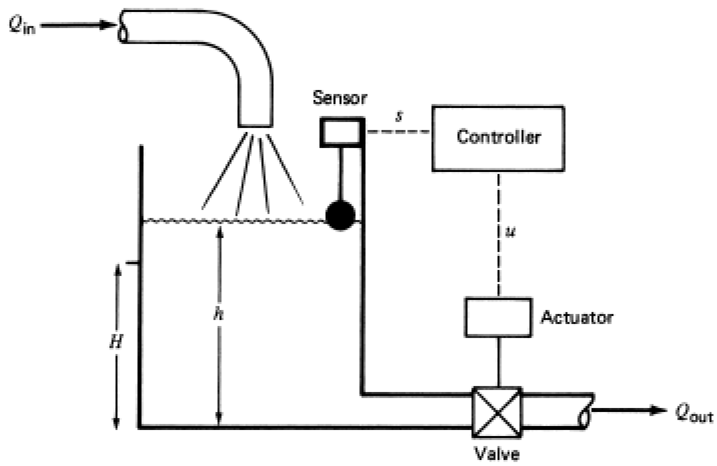
- **Block diagram of Open loop system:**



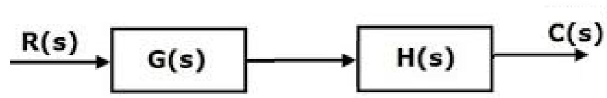
5. Closed loop temperature control system

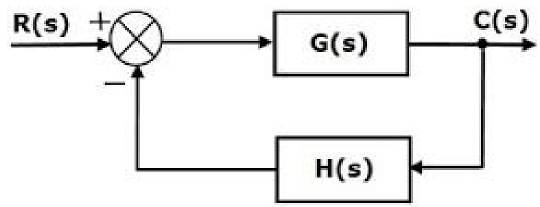


6. Closed loop level control system



7. Comparison of open loop and closed loop system

Sr.No	**Open loop**	**Closed loop**
1	No Feedback element	Feedback element is present
2	Error detector is absent	Error detector is present
3	TF= G(S)H(S)	TF= G(S)/(1+ G(S)H(S))
4	Inaccurate	Accurate
5	Small bandwidth	large bandwidth
6	More stable	less stable
7	Simple construction	complex
8	Less costly	more costly
9	Affected by non-linearity	not affected
9	Sensitive to disturbance	not sensitive
10	Ex: Electric switch, automatic washing machine, automatic toaster, traffic signal	Ex: Human being, home heating system, missile launching system, voltage stabilizer
11	Block diagram of open loop system 	

12	Block diagram of closed loop system 

8. Comparison of Linear and nonlinear system

Linear system: A system is said to be linear if it obeys principle of superposition. Here, the coefficients of differential equations describing the system are either constants or functions of independent variable.

Principle of superposition: The response produced by simultaneous applications of several forcing functions can be calculated by adding the individual responses. It consists of additive and homogeneous property.

Additive property: For any x & y belonging to domain of function f, $f(x+y) = f(x) + f(y)$

Homogeneous property: For any x belonging to domain of function f and for scalar constant k, $f(kx) = \ kf(x)$

Nonlinear system: A system is said to be nonlinear if it does not obey principle of superposition. Here, the coefficients of differential equations describing the system are functions of dependent variable.

9. Comparison of Time varying and time in varying system

Time Variant/ varying System: System whose parameters change with time irrespective of whether input and output change or not is called time variant system.

Ex: rocket launching system, space shuttle, automobiles

Time Invariant /invarying System: System whose parameters do not change with time irrespective of whether input and output change or not is called time invariant system.

Ex: RC, RLC networks

10. Laplace Transform

It is the Transformation technique relating time function to frequency dependent function of a complex variable. It transforms higher order differential equation into a polynomial form which makes it easy to solve differential equation directly.

Laplace Transform Formula

Laplace transform of a function $f\ (t)$ in time domain, where 't' is the real number greater than or equal to zero, is given as

$$F(S) = \int_0^\infty f(t)e^{-st}dt$$

Where $\boldsymbol{s}$ is the complex number in frequency domain. i.e. $\boldsymbol{s = \sigma + j\omega}$

The above equation is considered as **unilateral Laplace transform equation**.

Advantages: By using LT, Exponential, sinusoidal and damped sinusoidal functions can be converted to algebraic function of the complex variable '$\boldsymbol{s}$'. Integration and differentiation can be replaced by simple algebraic functions. It allows the use of graphical techniques for predicting the system response without actually solving differential equations.

Table of LT:

f(t)	F(s) = $\mathscr{L}$\{f\}(s)
1	$\frac{1}{s}$, $s > 0$
e^{at}	$\frac{1}{s-a}$, $s > 0$
t^n, $n = 1,2,...$	$\frac{n!}{s^{n+1}}$, $s > 0$
sin bt	$\frac{b}{s^2+b^2}$, $s > 0$
cos bt	$\frac{s}{s^2+b^2}$, $s > 0$
$e^{at}t^n$, $n = 1,2,...$	$\frac{n!}{(s-a)^{n+1}}$, $s > a$
e^{at} **sin** bt	$\frac{b}{(s-a)^2+b^2}$, $s > a$
e^{at} **cos** bt	$\frac{s-a}{(s-a)^2+b^2}$, $s > a$

Transform pair	Signal	Transform
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$u(-t)$	$\frac{1}{s}$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$
10	$\delta(t-T)$	e^{-sT}
11	$[\cos \omega_0 t]\, u(t)$	$\frac{S}{S^2+\omega_0^2}$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{S^2+\omega_0^2}$
13	$[e^{-\alpha t}\cos \omega_0 t]\, u(t)$	$\frac{S+\alpha}{(S+\alpha^2)+\omega_0^2}$

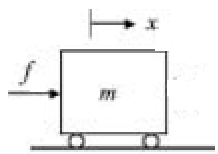
11. Differential equation

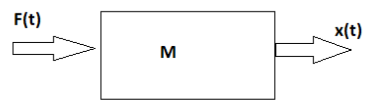
These are the equations which involve differential coefficient or differentials. It defines the dynamics of the system and relates dynamic input and output. It represents the complete description of the system between input and output. An ordinary differential equation is that in which all the differential coefficient has reference to a single independent variable. Linear

differential equations are those in which the dependent variable and its derivatives occur only in the first degree and not multiplied together.
Example for differential equation is given below where $y(t)$ is the output and $x(t)$ is the input, t is the time;

$$\frac{d^3y(t)}{dt^3} + 4\,\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 9\,y(t) = 10\,x(t)$$

Consider the example of a system with mass *M*. Consider that a force *f (t)* is applied on it which moves the system by a displacement *x (t).* The input is force and output is displacement. The differential equation relating the force *f (t),* mass *M* and displacement *x (t)* is shown below:





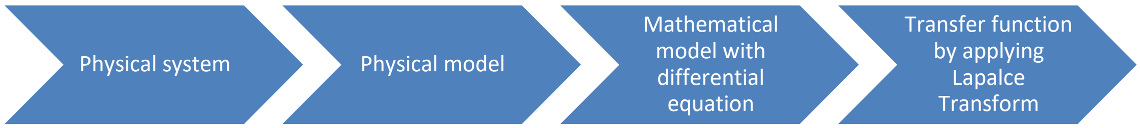
$$Force = mass \times acceleration$$
$$f(t) = M \times a(t)$$
$$acceleration = rate\ of\ change\ of\ velocity = \frac{d\,v(t)}{dt}$$
$$velocity = rate\ of\ change\ of\ displacement = \frac{d\,x(t)}{dt}$$
$$acceleration = rate\ of\ change\ of\ velocity = \frac{d\,v(t)}{dt} = \frac{d^2x(t)}{dt^2}$$
$$Force = f(t) = M \times a(t) = M \times \frac{d^2x(t)}{dt^2}$$

Taking the Laplace of it,

$$F(S) = MS^2\,X(S)$$
$$\frac{output}{input} = \frac{X(S)}{F(S)} = \frac{1}{MS^2}$$

12. Mathematical modeling of control system

A physical control system is considered as idealized physical system or physical model by neglecting the nonlinearities. The physical model is converted to its mathematical model (which is the mathematical representation of the physical model). The mathematical model is depicted by differential equations by applying known equations or laws such as KVL, KCL. The differential equation is reshaped into Transfer functions using powerful mathematical tool such as Laplace Transform.



13. Definition of Transfer Function (TF) and its features.

- TF of a LTI system is defined as the ratio of Laplace transform of output to that of input under the zero initial condition.

(The systems should be linear. Zero Initial Conditions ensure linearity. The transfer function of a system is the Laplace transform of its impulse response under assumption of zero initial conditions.)

Advantages / Features:

1. As it uses Laplace transform, it converts time domain equations to simple algebraic equations.
2. It gives mathematical models of all system components
3. It relates output to input.
4. It describes input-output behavior of the system.
5. It helps in the stability analysis of the system
6. It helps in determining poles & zeros

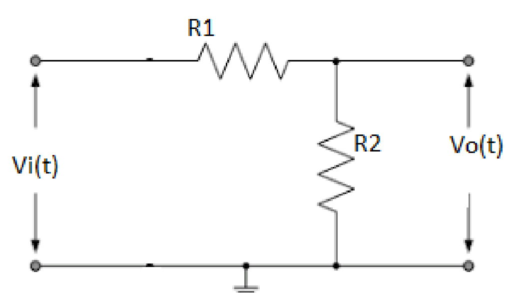
Disadvantages:

- No information regarding the internal state of the system. It does not depend on the inputs to the system

14. Order of the system

It is the highest power of S in the denominator of the closed loop TF.

15. Transfer function of the given R-R electrical network



Input eqn :

$$V_i(t) = R_1 i(t) + R_2 i(t)$$

Output eqn:

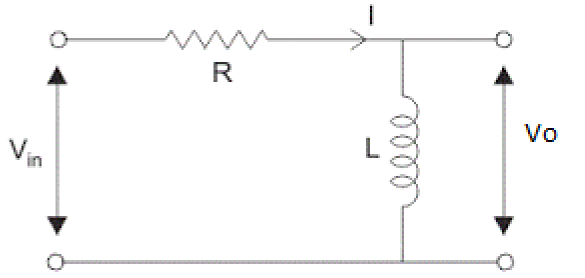
$$V_0(t) = R_2\, i(t)$$

Taking Laplace of I/P and O/P eqns:

$$V_i(S) = R_1\, I(S) + R_2\, I(S)$$
$$V_O(S) = R_2\, I(S)$$

$$\text{TF} = \frac{V_0(S)}{V_i(S)} = \frac{R_2\; I(S)}{R_1\, I(S)+R_2\, I(S)} = \frac{R_2}{R_2 + R_1}$$

16. Transfer function of the given RL electrical network



Input eqn :

$$V_i(t) = R\, i(t) + L\,\frac{d\, i(t)}{dt}$$

Output eqn:

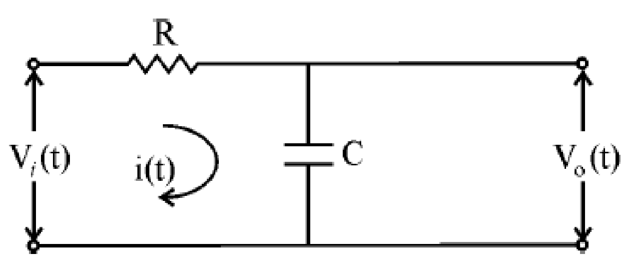
$$V_0(t) = L\;\frac{d\, i(t)}{dt}$$

Taking Laplace of i/p and o/p equations:

$$V_i(S) = R\, I(S) + LS\, I(S)$$
$$V_O(S) = LS\, I(S)$$

$$\text{TF} = \frac{V_0(S)}{V_i(S)} = \frac{LS\, I(S)}{R\, I(S)+LSI(S)} = \frac{LS}{LS+R}$$

17. Transfer function of the given RC electrical network



Input eqn :

$$V_i(t) = Ri(t) + \frac{1}{C}\int i(t)dt$$

Output eqn:

$$V_0(t) = \frac{1}{C}\int i(t)dt$$

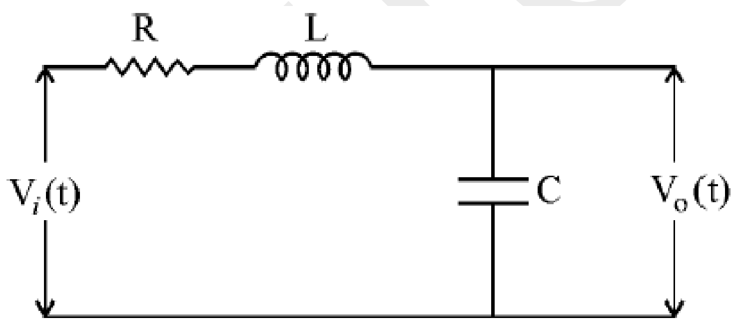
Taking Laplace of I/P and O/P eqns:

$$V_i(S) = R\,I(S) + \frac{I(S)}{CS}$$

$$V_O(S) = \frac{I(S)}{CS}$$

$$\text{TF}=\frac{V_0(S)}{V_i(S)} = \frac{\frac{I(S)}{CS}}{R\,I(S)+\frac{I(S)}{CS}} = \frac{1}{RCS+1}$$

18. Transfer function of the given RLC electrical network



Input eqn :

$$V_i(t) = R\,i(t) + \frac{1}{C}\int i(t)dt + L\,\frac{d\,i(t)}{dt}$$

Output eqn:

$$V_0(t) = \frac{1}{C}\int i(t)dt$$

Taking Laplace of i/p and o/p eqns:

$$V_i(S) = R\,I(S) + \frac{I(S)}{CS} + LSI(S)$$

$$V_O(S) = \frac{I(S)}{CS}$$

$$\text{TF} = \frac{V_0(S)}{V_i(S)} = \frac{\frac{I(S)}{CS}}{R\,I(S)+\frac{I(S)}{CS}+LSI(S)} = \frac{1}{LCS^2+RCS+1}$$

19. Transfer function of the given differential equation:

Find out the Transfer function from the differential equation where $y(t)$ is the output and $x(t)$ is the input, t is the time

$$\frac{d^3y(t)}{dt^3} + 4\,\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 9\,y(t) = 10\,x(t)$$

Taking Laplace,

$$S^3Y(S) + 4S^2Y(S) + 8SY(S) + 9Y(S) = 10X(S)$$

$$Y(S)[S^3 + 4S^2 + 8S + 9] = 10X(S)$$

$$TF = \frac{Y(S)}{X(S)} = \frac{10}{S^3 + 4S^2 + 8S + 9}$$

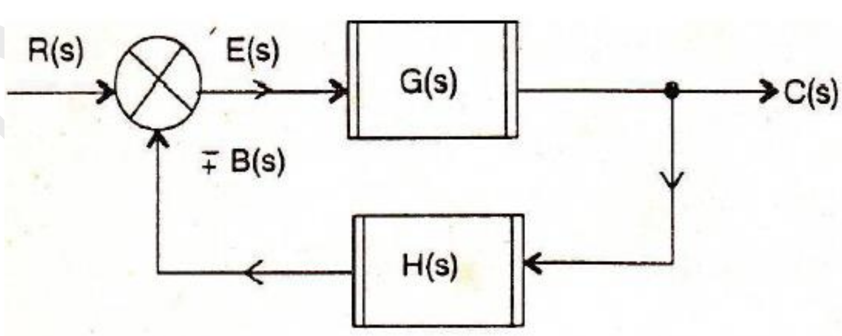
20. Feedback system

Any control system which automatically corrects for variations in its output is called feedback system. Feedback can be positive or negative.

In a "Positive feedback control system", the set point and output values are added together by the controller as the feedback is "in-phase" with the input. The effect of positive (or regenerative) feedback is to "increase" the systems gain, i.e, the overall gain with positive feedback applied will be greater than the gain without feedback.

In a "Negative feedback control system", the set point and output values are subtracted from each other as the feedback is "out-of-phase" with the original input. The effect of negative (or degenerative) feedback is to "reduce" the gain

21. Transfer function of the closed loop system (feedback system)



$R(S)$ = Laplace of reference i/p $r(t)$
$C(S)$ = Laplace of controlled o/p $c(t)$
$E(S)$ = Laplace of error signal $e(t)$.

$B(S)$ = Laplace of feedback signal $b(t)$
$G(S)$ = Equivalent forward path transfer function
$H(S)$ = Equivalent feedback path transfer function.

Referring to this Fig.

$E(S) = R(S) \mp B(S)$ ………… (1)

$B(S) = C(S)H(S)$ …………….(2)
$C(S) = E(S)G(S)$ …………… (3)

$E(S) = \frac{C(S)}{G(S)}$ ……………… (4)

Substituting (2) & (4) in equation (1)

$$\frac{C(S)}{G(S)} = R(S) \mp C(S)H(S)$$

$$C(S) = G(S)R(S) \mp C(S)G(S)H(S)$$

$$C(S) \pm C(S)G(S)H(S) = G(S)R(S)$$

$$C(S)[1 \pm G(S)H(S)] = G(S)R(S)$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{[1 \pm G(S)H(S)]}$$

This is the Transfer Function.

For negative feedback, TF=

$$\frac{C(S)}{R(S)} = \frac{G(S)}{[1 + G(S)H(S)]}$$

For positive feedback, TF=

$$\frac{C(S)}{R(S)} = \frac{G(S)}{[1 - G(S)H(S)]}$$

22. Block diagram representation

Block diagram is a pictorial representation of the function performed by each component of the system and of the flow of the signals. It depicts the interrelationships which exist between the various components. All system variables are linked to each other through functional blocks. A functional block is the symbol for the mathematical operation on the input signal to the block which produces the output. The actual mathematical function is indicated by inserting the corresponding TF of element inside the block. The blocks are connected by arrows to show the direction of signal flow. Thus, for the analysis of a complicated system, it can be split into different blocks. Block diagram explains the cause and effect relationship existing between input and output of the system through blocks.

Advantages:

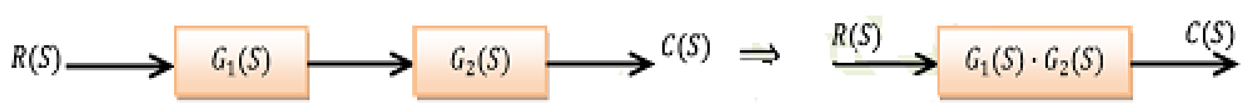
1. It gives input-output relationship of the system
2. It contains information concerning the dynamics behavior of the system
3. The functional operation of the system can be visualized more effectively.
4. It helps to find out transfer function easily.

Disadvantages:

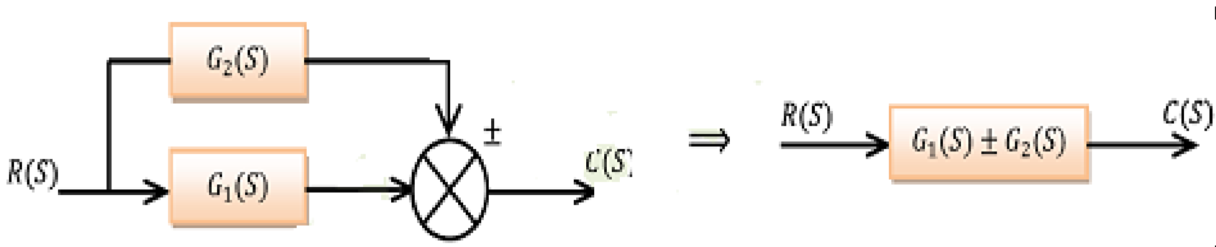
1. It does not contain information concerning physical construction of the system.
2. Block diagram of a system is not unique. A number of different block diagrams can be drawn for the same system
3. The main source of energy is not shown

23. The rules for block reduction technique.

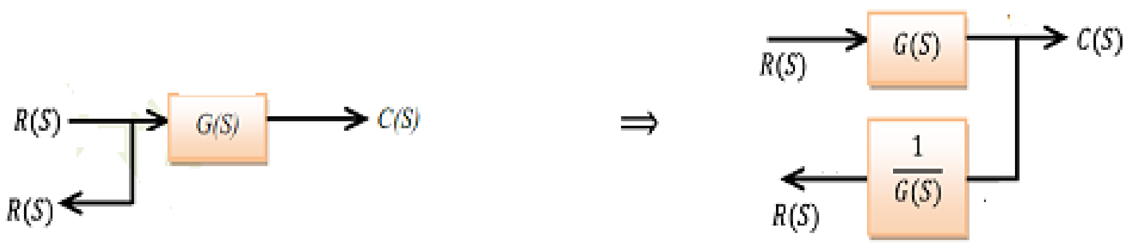
i) Combining a block in cascade:
When two or more blocks are connected in series, their overall transfer function is the product of individual block transfer function.



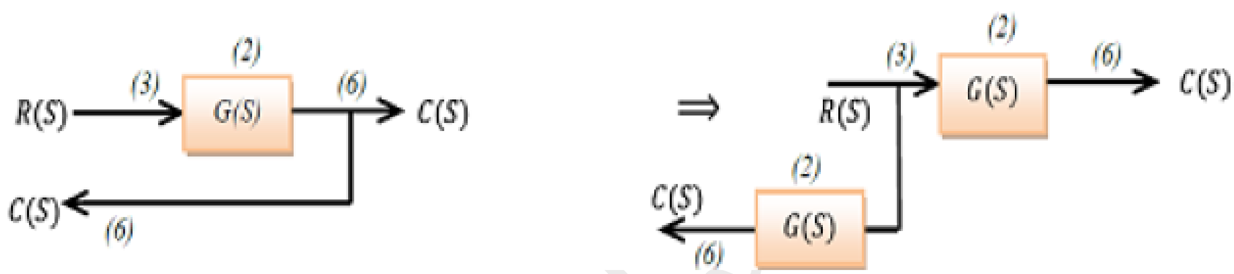
ii) Combining two blocks in parallel:
When two or more blocks are connected in parallel, their overall transfer function is the addition or difference of individual transfer function.



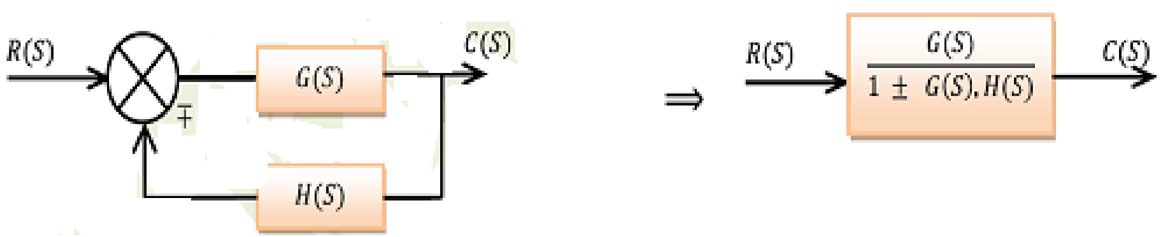
iii) Shifting a take off point after a block:
To shift take off point after a block, we shall add a block with transfer function 1/G in series with signal having taking off from that point.



iv) Shifting a take off point before a block:
To shift take off point before a block, we shall add a block with transfer function G in series with signal having taking off from the take off point

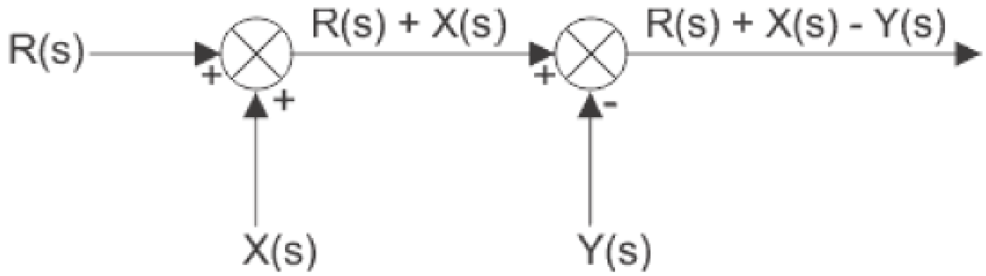


v) Eliminating Feedback Loop:

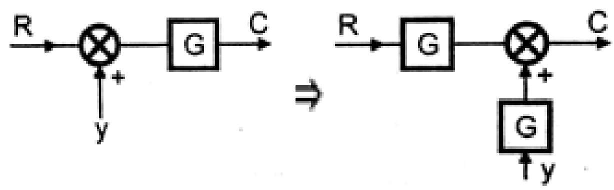


$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s).H(s)}$$

vi) Interchanging Summing Points:
The order of summing points can be interchanged, if two or more summing points are in series and output remains the same.

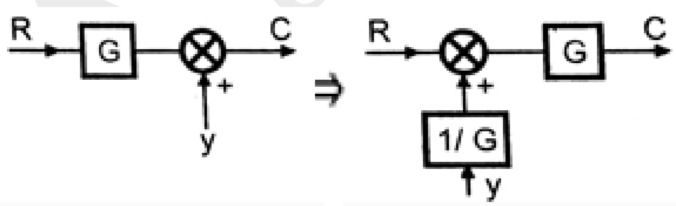


vii) Moving summing point after a block:
To shift summing point after a block, another block having transfer function G is added before the summing point.

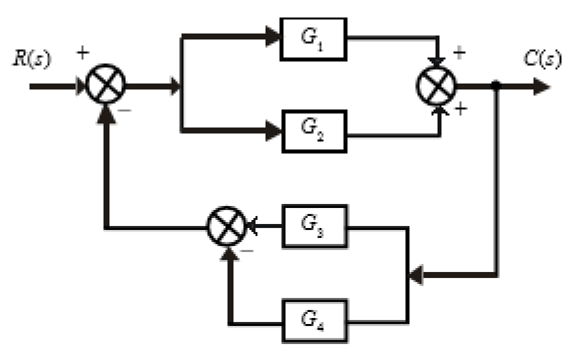


viii) Moving summing point before a block:

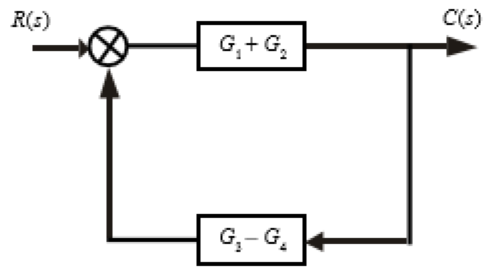
To shift summing point before a block, another block having transfer function 1/G is added before the summing point.



24. Obtain the transfer function for system using block reduction technique.



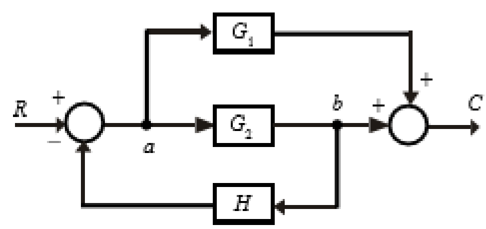
Blocks in parallel,



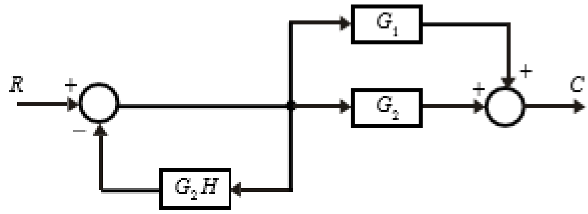
Transfer function:

$$\frac{C(s)}{R(s)} = \frac{(G_1 + G_3)}{1 + (G_1 + G_2)(G_3 - G_4)}$$

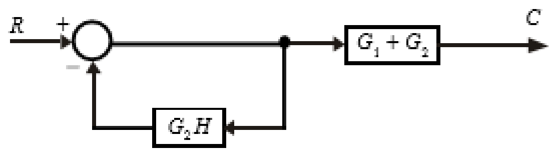
25. Obtain the transfer function for system using block reduction technique.



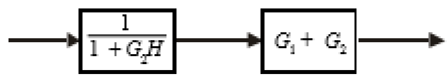
Shifting the take off point from ‘b’ to ‘a’,



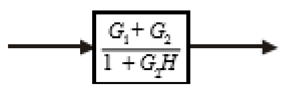
(a)



(b)

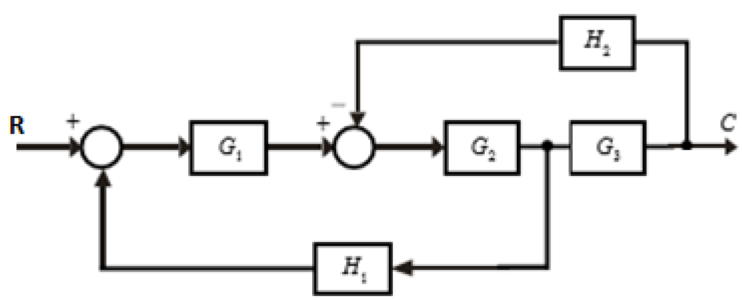


(c)

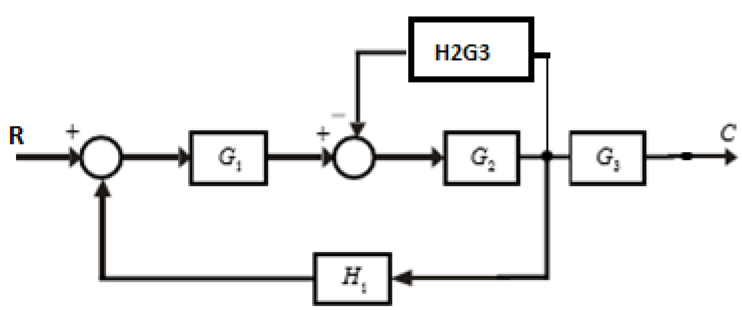


$$TF = \frac{G_1 + G_2}{(1 + G_2H)}$$

26. Obtain the transfer function for system using block reduction technique.

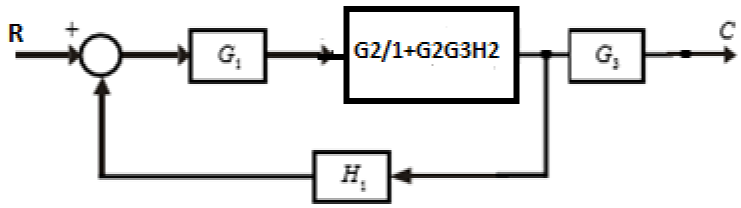


Shifting the take off point of H_2 before G_3,



Considering the closed loop with G_2& G_3H_2,

$$TF = \frac{G_2}{(1 + G_2G_3H_2)}$$



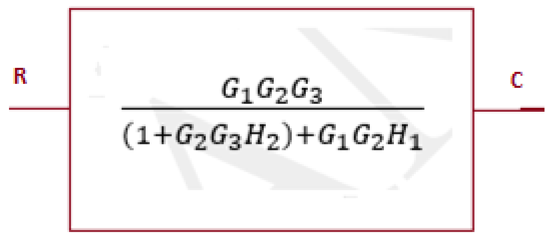
Considering the closed loop,

$$\frac{\frac{G_1G_2}{(1+G_2G_3H_2)}}{1+\frac{G_1G_2H_1}{(1+G_2G_3H_2)}} = \frac{G_1G_2}{(1+G_2G_3H_2)+G_1G_2H_1}$$

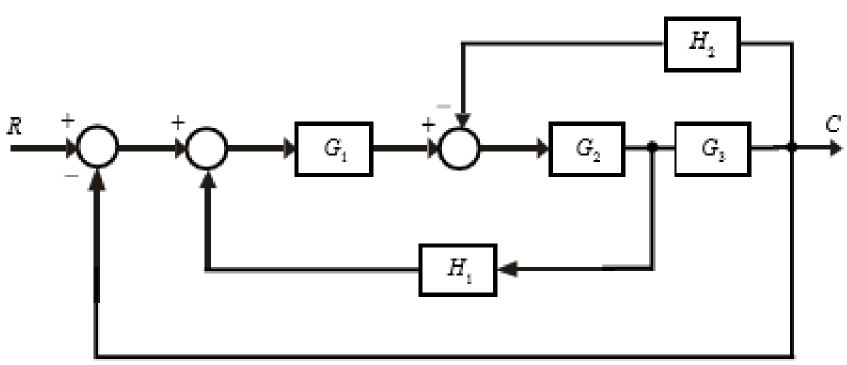
It is in series with G_3.

So,

$$TF = \frac{G_1G_2G_3}{(1 + G_2G_3H_2) + G_1G_2H_1}$$

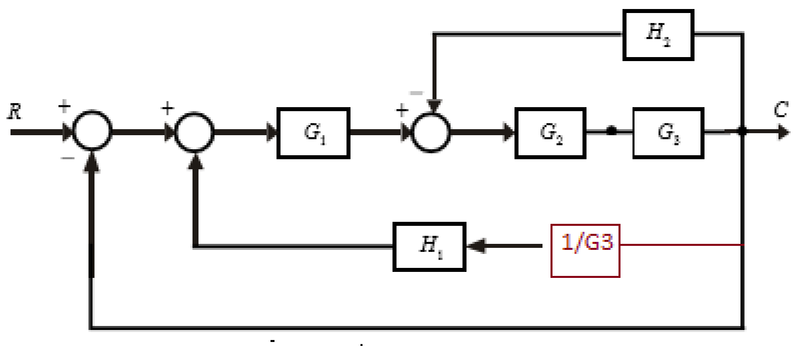


27. Obtain the transfer function for system using block reduction technique.

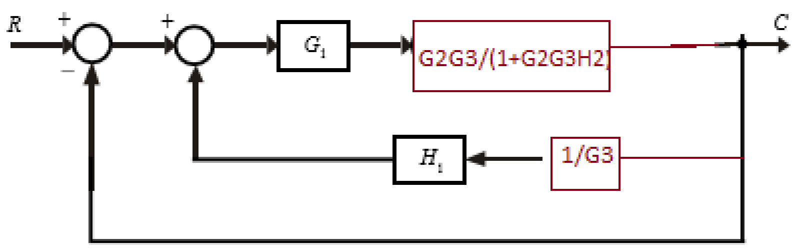


Shifting the take off point after G_3,

(PS: it can be done by shifting the take off point of H_2 before G_3)

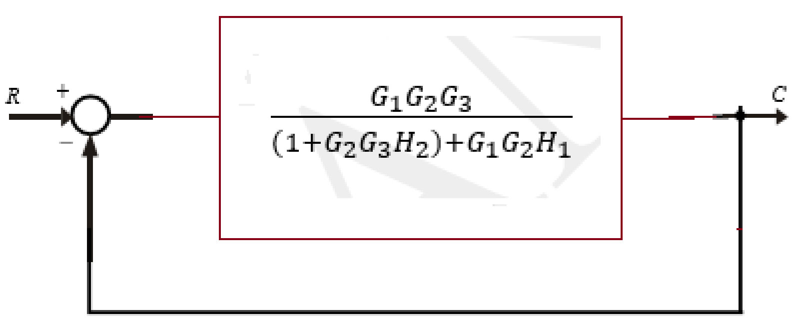


Considering the closed loop with G_2, G_3 & H_2



Considering the closed loop,

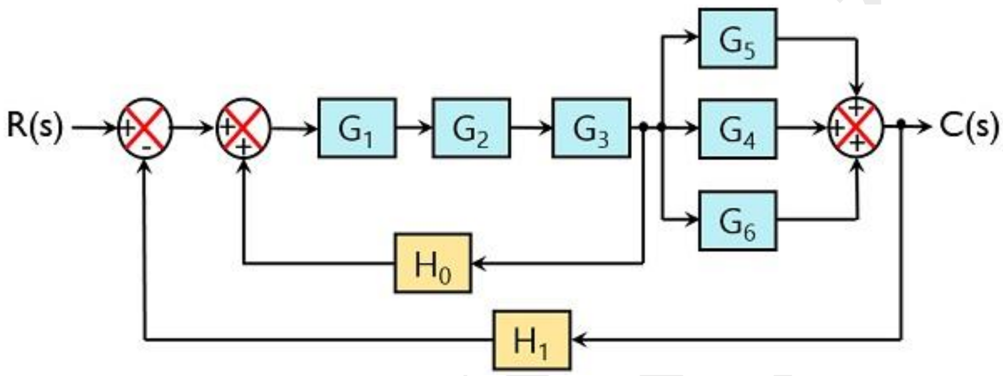
$$\frac{\frac{G_1G_2G_3}{(1+G_2G_3H_2)}}{1+\frac{G_1G_2H_1}{(1+G_2G_3H_2)}} = \frac{G_1G_2G_3}{(1+G_2G_3H_2)+G_1G_2H_1}$$



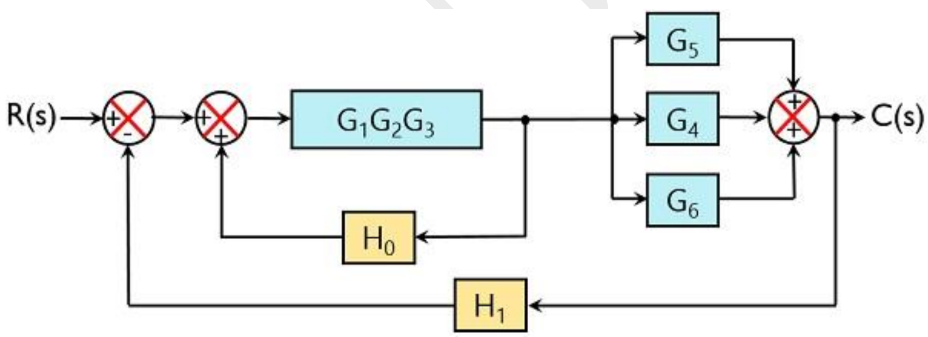
Considering the closed loop,

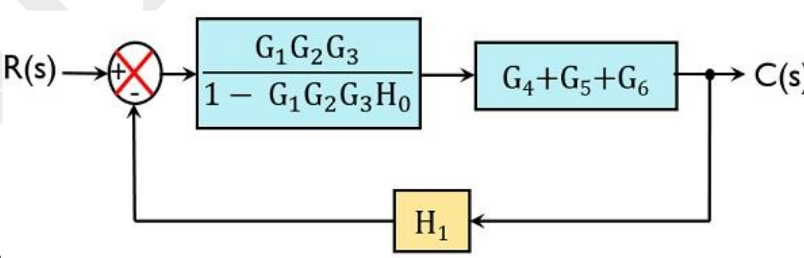
$$\frac{\frac{G_1G_2G_3}{(1+G_2G_3H_2)+G_1G_2H_1}}{1+\frac{G_1G_2G_3}{(1+G_2G_3H_2)+G_1G_2H_1}} = \frac{G_1G_2G_3}{(1+G_2G_3H_2)+G_1G_2H_1+G_1G_2G_3}$$

28. Obtain the transfer function for system using block reduction technique.

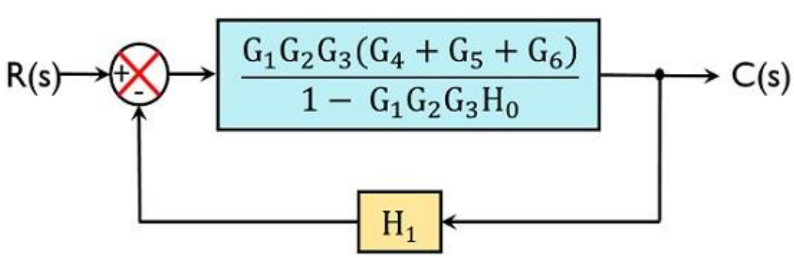


Considering blocks in series and parallel,





Series,



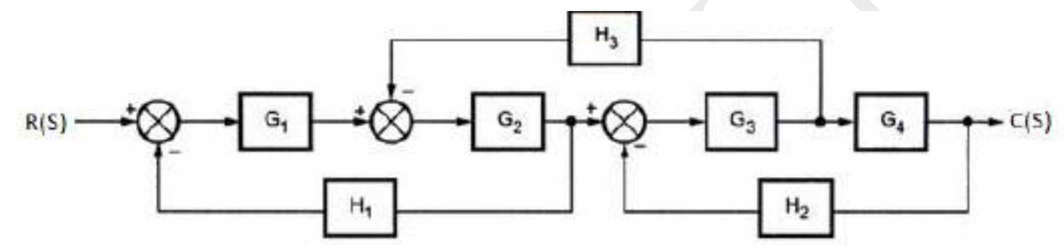
Closed loop,

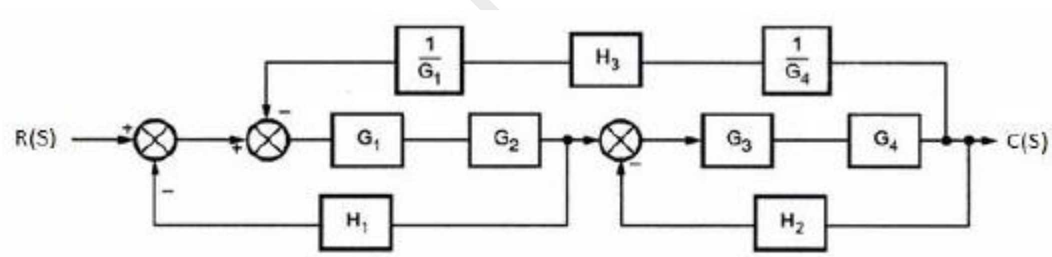
$$\frac{C(s)}{R(s)} = \frac{\frac{G_1G_2G_3(G_4+G_5+G_6)}{1-G_1G_2G_3H_0}}{1+\left[\frac{G_1G_2G_3(G_4+G_5+G_6)}{1-G_1G_2G_3H_0}\right]H_1}$$

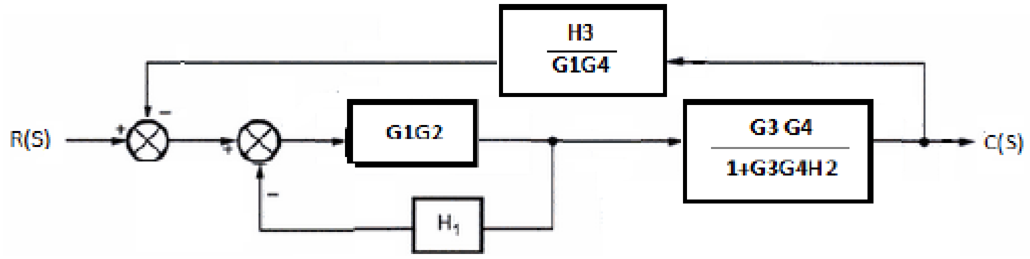
TF =

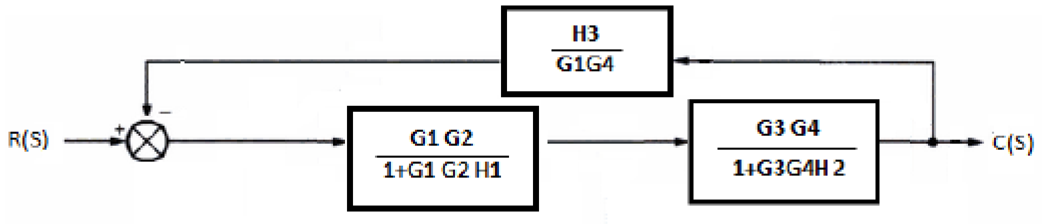
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3(G_4+G_5+G_6)}{1-G_1G_2G_3H_0+G_1G_2G_3(G_4+G_5+G_6)H_1}$$

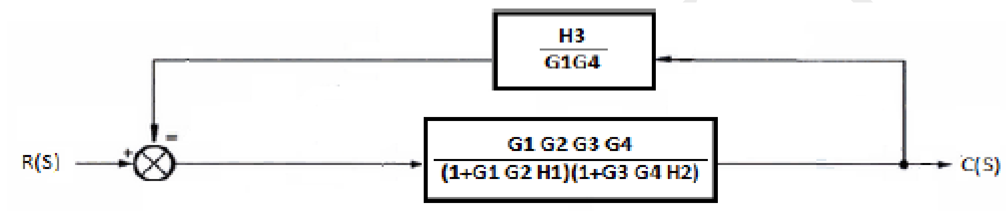
29. Obtain the transfer function for system using block reduction technique.











$$\frac{C(S)}{R(S)} = \frac{G1G2G3G4}{(1 + G1G2H1)(1 + G3G4H2) + G2G3H3}$$

$$\frac{C(S)}{R(S)} = \frac{G1G2G3G4}{1 + G1G2H1 + G3G4H2 + G1G2G3G4H1H2 + G2G3H3}$$

30. Drawbacks of classical control system

- It is used only for SISO/ LTI systems
- TF is defined under zero initial conditions
- No information regarding the internal state of the system
- Trial and error method
- Two different methods for continuous and discrete systems (Laplace Transform for continuous and Z transform for discrete systems)

31. Introduction to modern control system

To overcome the drawbacks of classical control system approaches, modern control system methods are used.

- It is possible to analyze time-varying or time-invariant, linear or non-linear, single or multiple input-output systems (SISO/MIMO) in modern control system.
- It is possible to confirm the state of the system parameters along with input-output relations.
- In modern control system, a system is represented as a set of first order differential equations defined using state variables.
- It allows the use of same formulation for continuous and discrete time systems.
- **State space representation** is one of the modern control system methods.
- State space analysis is possible even if the initial conditions are non-zero.
- State Space models are digital — and therefore better adapted than analog for real time control systems implemented with computers. State Space is the basis for all modern "fly by wire" vehicle controls in aircraft and spacecraft.

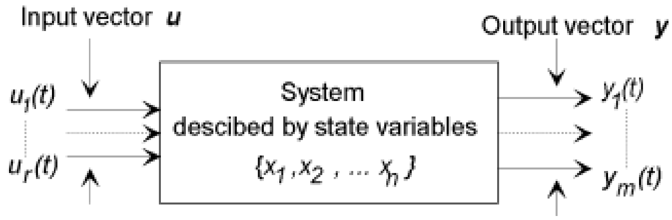
32. Comparison between Classical control system and Modern control system:

Classical control system	**Modern control system**
It is used only for SISO/ LTI systems	It is used for time-varying or time-invariant, linear or non-linear, single or multiple input-output systems
It is used for systems where TF is defined under zero initial conditions	It is used for systems even if the initial conditions are non-zero
No information regarding the internal state of the system	It is possible to confirm the state of the system parameters along with input-output relations
Two different methods for continuous and discrete systems (Laplace Transform for continuous and Z transform for discrete systems)	It allows the use of same formulation for continuous and discrete time systems.
Here, a system is represented as higher order differential equations	Here, a system is represented as a set of first order differential equations defined using state variables

33. State space representation (SSR)

It is the mathematical representation of the system which gives information about state of system variables along with output.

Three types of variables are used in SSR: I/P, O/P and state variables which are shown in the figure below:



34. State variables

- All possible linearly independent system variables used to represent the system are called state variables in SSR.
- They describe the state or status of system at any time ‘t’.
- The minimum number of state variable required to describe the system equals the order of the differential equation. It summarizes the history of the system in order to predict the future values (outputs).
- The output and its derivatives are taken as state variables in phase variable form.

State equation:

These are the first order differential equations written in terms of state variables or derivatives of state variables.

State space:

It is the n-dimensional space whose axes are the state variables.

35. State space model representation

The **state space model** of Linear Time-Invariant (LTI) system can be represented as,

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

The first and the second equations are known as state equation and output equation respectively.

Where,

- X and $\dot{X}$ are the state vector and the differential state vector respectively.
- U and Y are input vector and output vector respectively.
- A is the system matrix.
- B and C are the input and the output matrices.
- D is the feed-forward matrix.

36. Find out the SSR of TF $= \frac{C(S)}{R(S)} = \frac{9}{S^2+6S+9}$

After cross-multiplying,

$$(S^2 + 6S + 9)C(S) = 9R(S)$$

Taking inverse Laplace Transform after opening the bracket,

$$\frac{d^2}{dt^2}c(t) + 6\frac{d}{dt}c(t) + 9c(t) = 9r(t)$$

$Input = r(t) = u(t)$

$Output = y(t) = c(t)$

Therefore, first state variable $x_1 = c(t) = y(t)$

Second state variable $x_2 = \frac{d}{dt}c(t) = \dot{x_1}$

$\dot{x_2} = \frac{d^2}{dt^2}c(t) = 9r(t) - 6\frac{d}{dt}c(t) - 9c(t)$

$$\dot{x_2} = 9u(t) - 6\,x_2 - 9\,x_1$$

SSR:

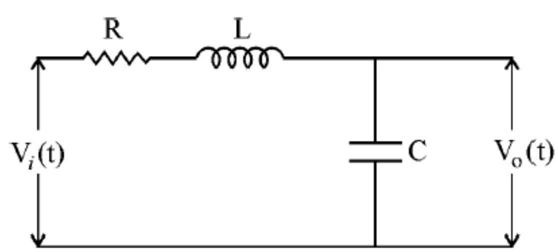
$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0 \quad 9]\, u$$

$$Y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

37. Find out the SSR of RLC network



$$TF = \frac{C(S)}{R(S)} = \frac{1}{LCS^2 + RCS + 1}$$

After cross-multiplying,

$$(LCS^2 + RCS + 1)C(S) = R(S)$$

Taking inverse Laplace Transform after opening the bracket,

$$LC\frac{d^2}{dt^2}c(t) + RC\frac{d}{dt}\,c(t) + c(t) = r(t)$$

$Input = r(t) = u(t)$

$Output = y(t) = c(t)$

Therefore, first state variable $x_1 \; = c(t) = y(t)$

Second state variable $x_2 \; = \frac{d}{dt}c(t) = \dot{x_1}$

$\dot{x_2} \; = \frac{d^2}{dt^2}c(t) = \frac{1}{LC}\,r(t) - \frac{R}{L}\frac{d}{dt}\,c(t) - \frac{1}{LC}c(t)$

$$\dot{x_2} \; = \frac{1}{LC}u(t) - \frac{R}{L}\,x_2 \; - \frac{1}{LC}\,x_1$$

SSR:

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{LC} \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$