STRAIGHT LINE

COORDINATE GEOMETRY

16

MARKS

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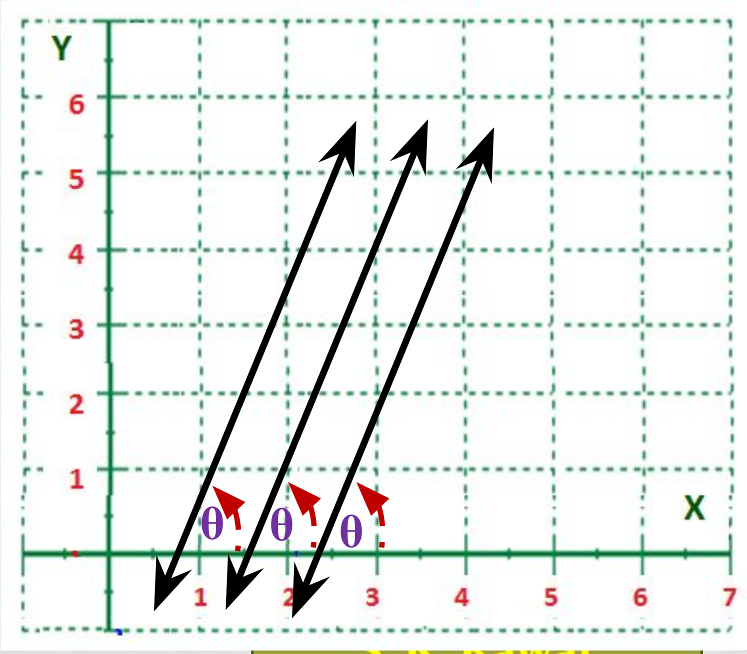
OBJECTIVES

- To understand slope, intercept and equations of line in different forms.
- To derive Angle between two lines & solve examples on it.
- To study condition of parallel and perpendicular lines & solve examples on it.
- To find equation of line passing through point of intersection of two lines and parallel or perpendicular to given line.
- To find equation of line passing through point of intersection of two lines with different condition.
- To derive perpendicular distance between point and line & solve examples on it.
- To derive perpendicular distance between two parallel lines & solve examples on it.

REVISION

- **INCLINATION OF A LINE:**
- The smallest positive angle θ made by a line L with positive direction of X-axis is called ***inclination of the line***.
- Inclination of X-axis is 0°
- Inclination of Y-axis is 90 °
- If the lines are parallel, then their inclinations are equal.



SLOPE OF LINE

- If θ is the inclination of a line 'L', then ***'tan θ'*** is called the ***slope*** or ***gradient*** of the line L.

- It is denoted by '***m***'.

- Thus ***$m = \tan\theta$***

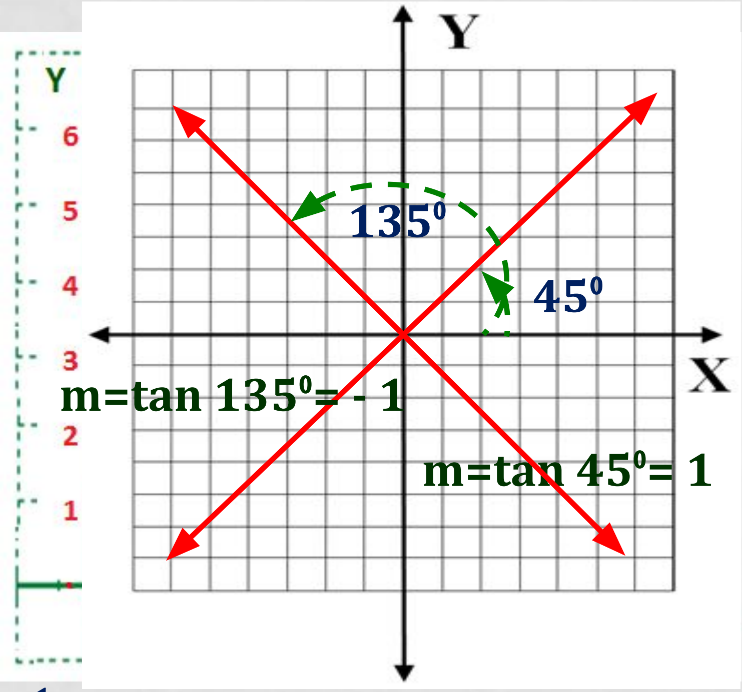
Example

- If $\Theta = 45^0$, then $m = \tan 45^0 = 1$.

- If $\Theta = 135^0$, then $m = \tan 135^0 = -1$.

- If $\Theta = 3\pi^c/4$, then $m = (\tan 3\pi^c/4) = -1$.

- Thus slope of line can be ***positive*** or ***negative.***



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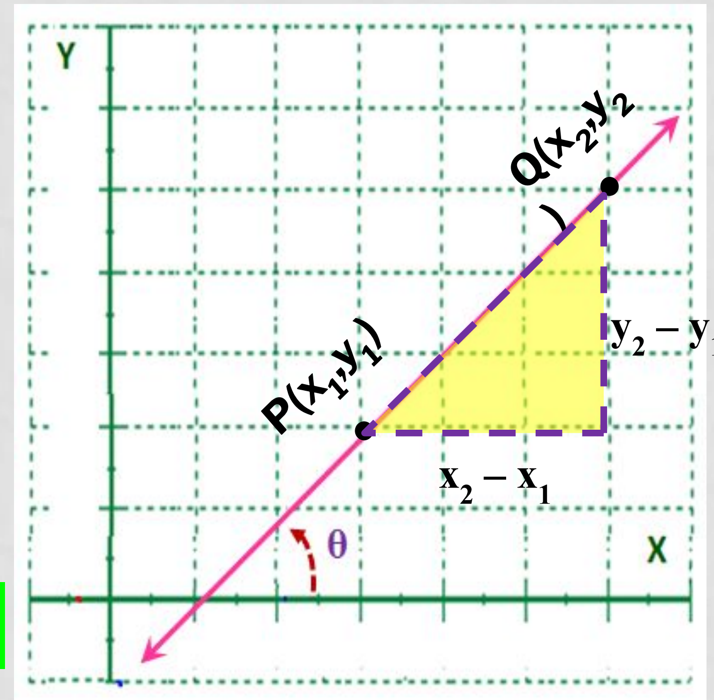
SLOPE OF LINE THROUGH TWO POINTS

- If L is the line passing through points $P(x_1,y_1)$ and $Q(x_2,y_2)$, then its slope is given as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Slope of X-axis is zero.
- Slope of Y-axis is not defined.
- Three points A, B, C are collinear, if slope of AB = slope of AC.



Example:

$$(x_1, y_1) \qquad (x_2, y_2)$$

If a line passes through (2,-1) and (3,3),

Then the slope of line is $m = \frac{3-(-1)}{3-2} = \frac{4}{1} = 4$

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SLOPE OF LINE THROUGH TWO POINTS

Example: Find the value of ***k***, if the slope of line passing through (2,4) and (3,***k***) is 5.

Let A(2,4)and B(3,***k***) be the given points.

$$\text{Slope of AB} = \frac{k-4}{3-2}$$

but slope of AB = 5

$$\therefore \frac{k-4}{3-2} = 5$$

$$\therefore \frac{k-4}{1} = 5$$

$$\therefore k-4 = 5$$

$$\therefore k = 9$$

SLOPE OF LINE THROUGH TWO POINTS

Example: Find the value of ***k***, if the points (1,-3), (***k***,1) and (3,5) are collinear.

Let A(1,-3), B(***k***,1) and C(3,5) be the given points.

$$\text{Slope of AB} = \frac{1+3}{k-1} = \frac{4}{k-1}$$

$$\text{Slope of AC} = \frac{5+3}{3-1} = \frac{8}{2}$$

As points A, B, C are collinear.

$\therefore$ Slope of AB = Slope of AC

$$\therefore \frac{4}{k-1} = \frac{8}{2}$$

$$\therefore 8 = 8k - 8$$

$$\therefore 8k = 16$$

$$\therefore k = 2$$

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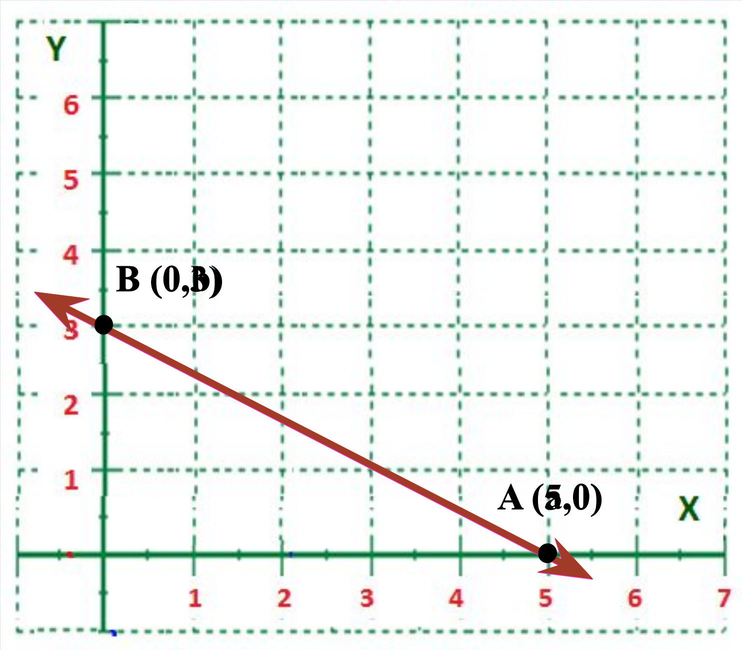
INTERCEPTS OF LINE ON THE AXES

- If a straight line intersects X-axis at point A and Y-axis at point B.

- Then the x co-ordinate of point A gives ***x-intercept*** of line and y co-ordinate of point B gives ***y-intercept*** of line.



Example:

If a line intersect X-axis at (5,0) and Y-axis at (0,3), then

x-intercept =5 and ***y-intercept =3.***

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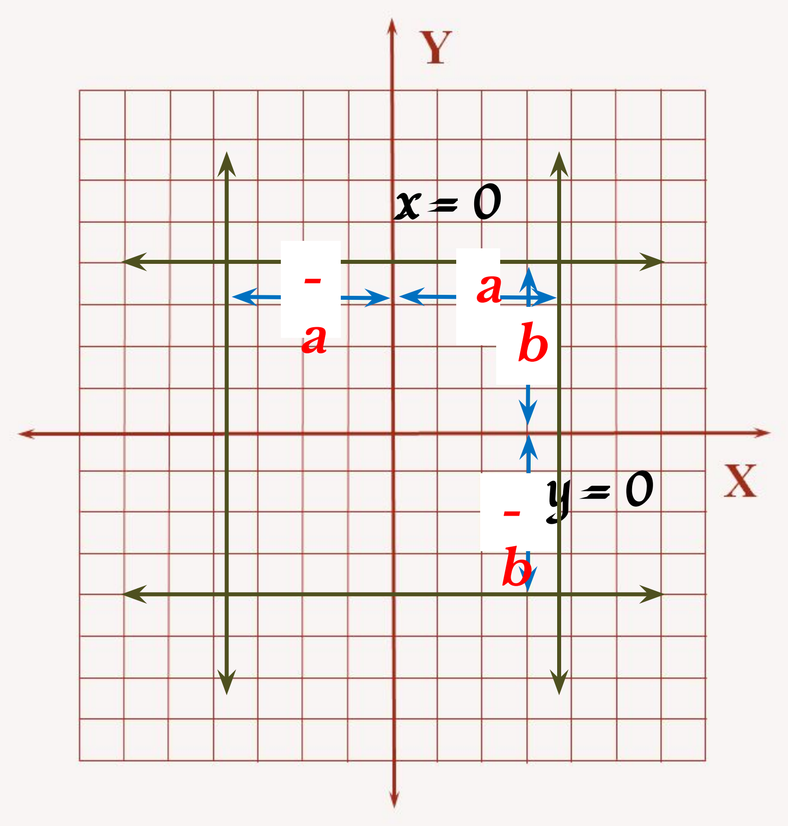
EQUATION OF LINE IN STANDARD FORM

- Equation of X-axis is $y = 0$.
- Equation of Y-axis is $x = 0$.
- Equation of line parallel to X-axis is $y = b$ or $y = -b$.
- Equation of line parallel to Y-axis is $x = a$ or $x = -a$.



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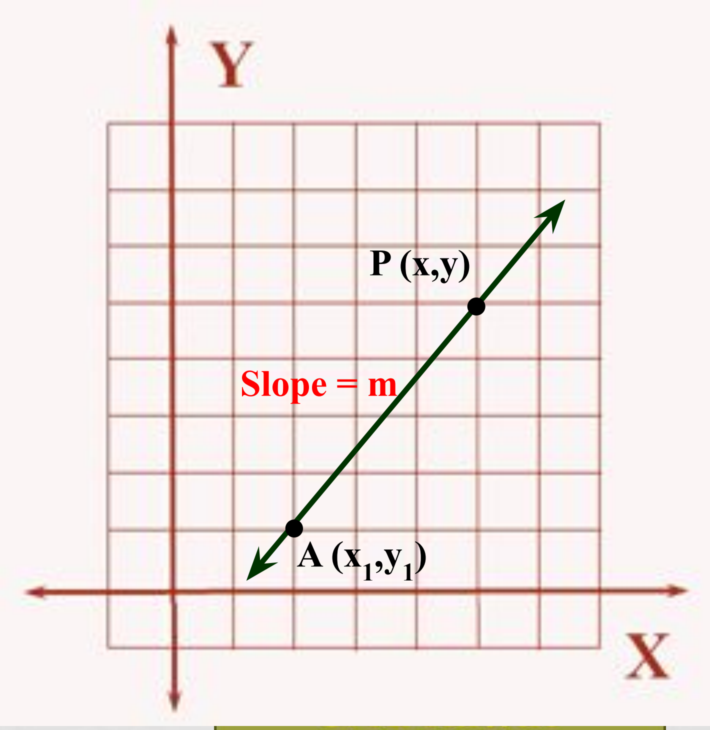
STANDARD FORMS OF LINE

Slope –Point Form:

- The Equation line having slope ***m*** and passing through the point ***A*** (x_1, y_1) is given as

$$y - y_1 = m(x - x_1)$$



Example

The Equation line having slope ***-1*** and passing through the point ***A (-2, 3)*** is calculated as

$$y - 3 = -1\,(x + 2).$$

$$\therefore\; y - 3 = -x - 2$$

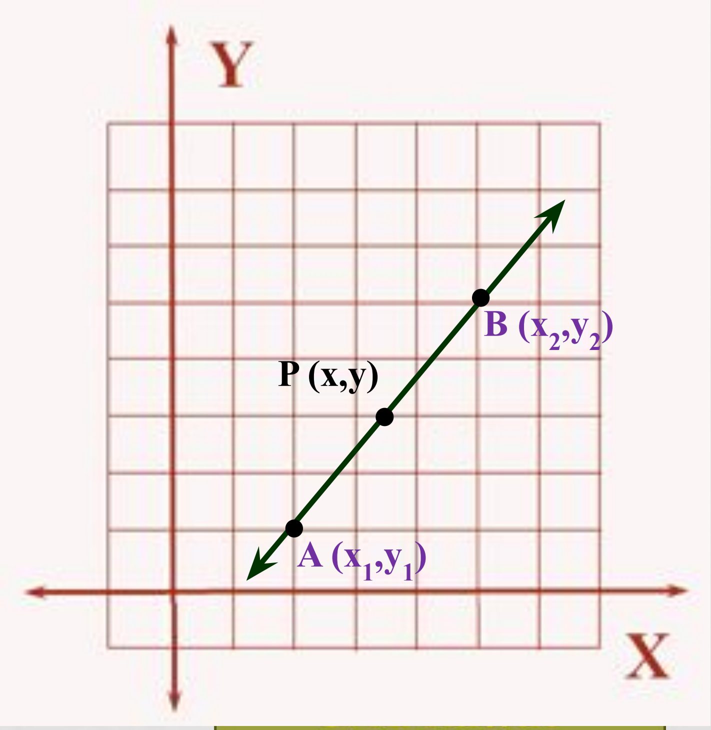
$$x + y = 1$$

STANDARD FORMS OF LINE

Two –Point Form:

- The Equation line passing through the point ***A*** (x_1, y_1) and ***B*** (x_2, y_2) is given as

$$\frac{y-y_1}{x-x_1}=\frac{y_1-y_2}{x_1-x_2}$$



Example

- The Equation line passing through the point ***A (2, 3)*** and ***B (1, -4)*** is calculated as

$$\frac{y-3}{x-2}=\frac{3+4}{2-1}=\frac{7}{1}$$

$$y-3=7(x-2)$$

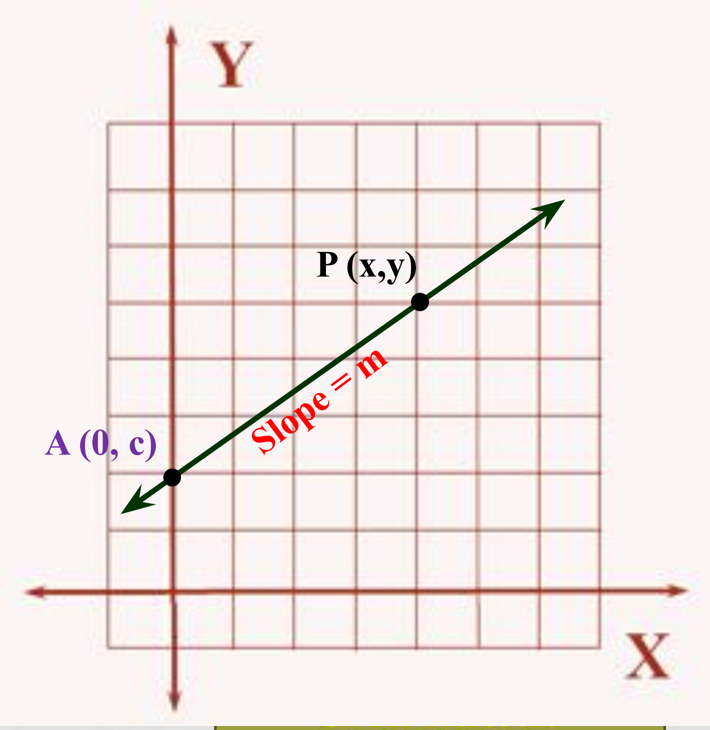
$$\therefore\ y-3=7x-14$$

$$\therefore\ 7x-y=11$$

STANDARD FORMS OF LINE

Slope - Intercept Form:

- The Equation line having slope '***m***' and y-intercept '***c***', is given as ***y = mx + c***



Example

- Equation of line having slope as ***-2*** and y-intercept as ***3*** is calculated as

$$y = -2x + 3$$

$$2x + y = 3$$

STANDARD FORMS OF LINE

Normal Form:

- If the perpendicular length from the origin to the line is '***p***' and '***α***' is the angle made by the perpendicular with the positive X-axis, then the equation of line is given as

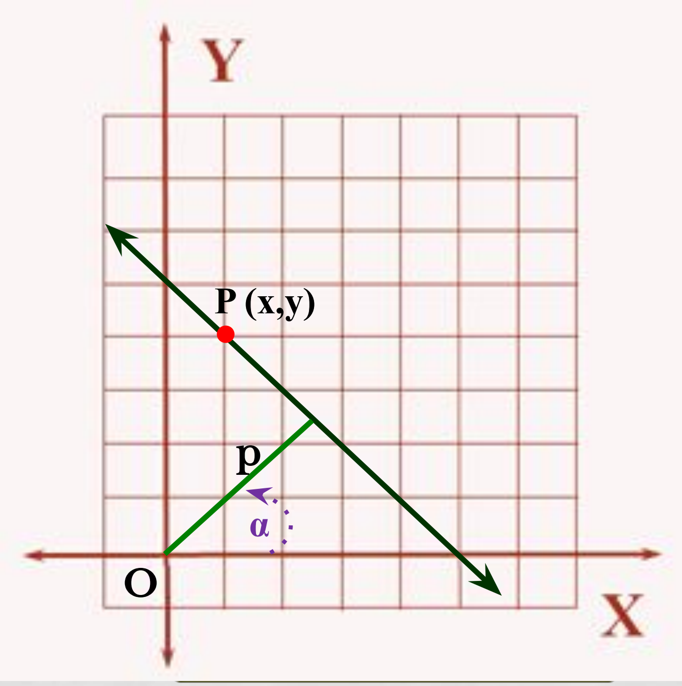
$$\mathrm{x}\cos\alpha + \mathrm{y}\sin\alpha = \mathrm{p}$$

Example

- Equation of line with perpendicular length as ***2*** and ***45⁰*** as angle made by perpendicular with positive X-axis is calculated as $x\cos 45° + y\sin 45° = 2$

$$x\frac{1}{\sqrt{2}} + y\frac{1}{\sqrt{2}} = 2$$

$$\therefore \quad x + y = 2\sqrt{2}$$

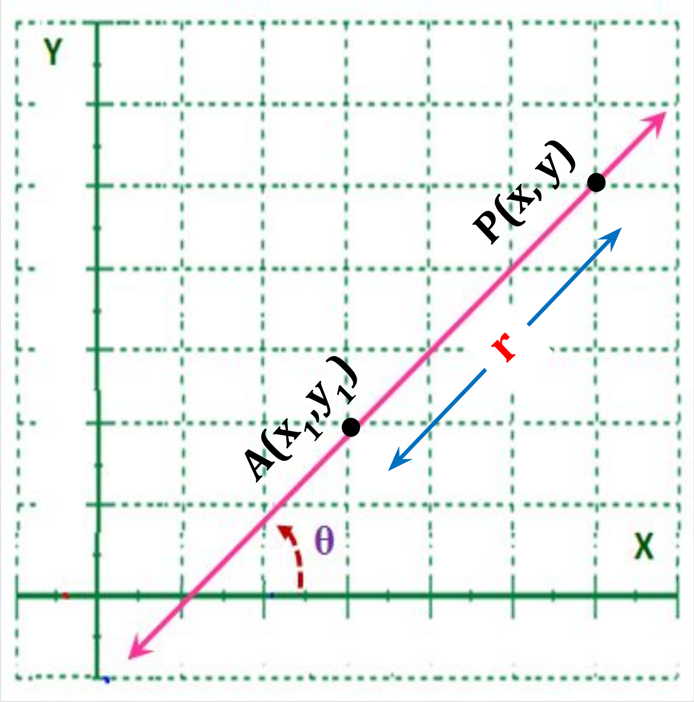


STANDARD FORMS OF LINE

Parametric Form:

- If the line make an angle 'θ' and passes through the point $A(x_1, y_1)$. If the distance between the point A and P (x , y) is 'r' then the equation of line is given as

$$\frac{y-y_1}{\sin\ \theta}=\frac{x-x_1}{\cos\ \theta}=r$$



Example

- Equation of line passing through (3 ,2) and at 30^0 angle with positive X-axis is calculated as

$$\frac{y-2}{\sin\ 30^\circ}=\frac{x-3}{\cos\ 30^\circ}$$

$$\frac{y-2}{(1/2)}=\frac{x-3}{\left(\sqrt{3}/2\right)}$$

$$2\sqrt{3}(y-2)=2(x-3)$$

$$\sqrt{3}(y-2)=x-3$$

$$\sqrt{3}y-2\sqrt{3}=x-3$$

$$x-\sqrt{3}y=3-2\sqrt{3}$$

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GENERAL EQUATION OF LINE

- The equation of form $ax + by + c = 0$ is called ***General equation of line***.
- Slope (**m**) of line is $\dfrac{-a}{b}$
- y-intercept is $-\dfrac{c}{b}$
- x-intercept is $-\dfrac{c}{a}$

On comparing **3x – 2y + 5 = 0** with general equation **ax + by + c = 0**, we get

a = 3, **b = - 2** and **c = 5**

Example

- For line 3x – 2y + 5 = 0,

slope is $\dfrac{-3}{-2} = \dfrac{3}{2}$ x-intercept is $\dfrac{-5}{3} = -\dfrac{5}{3}$ y-intercept is $\dfrac{-5}{-2} = \dfrac{5}{2}$

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ANGLE BETWEEN INTERSECTING LINES

Theorem:
If 'θ' is the acute angle between the intersecting lines having slopes 'm_1' and 'm_2' then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

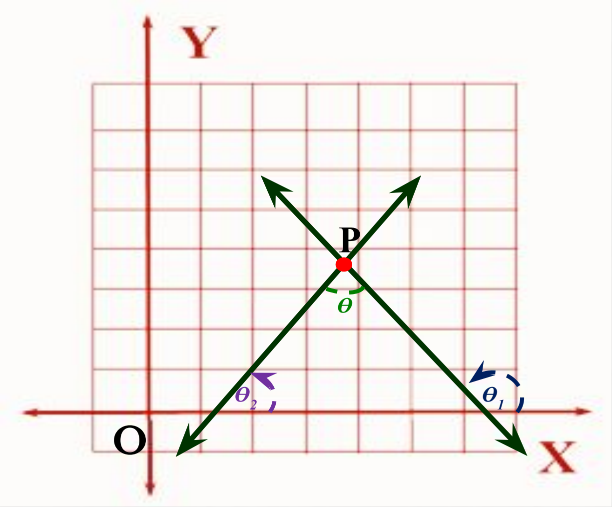
Proof: Let θ_1 and θ_2 be the inclinations, m_1 and m_2 be the slopes of lines L_1 and L_2.

$$\therefore \quad m_1 = \tan\theta_1 \quad \text{and} \quad m_2 = \tan\theta_2$$

Let θ be the acute angle between the lines.
From figure, by exterior angle theorem

$$\theta_1 = \theta + \theta_2 \qquad \therefore \theta = \theta_1 - \theta_2$$

$$\therefore \tan\theta = \tan(\theta_1 - \theta_2)$$



$$\therefore \tan\theta = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 . \tan\theta_2}$$

$$\therefore \tan\theta = \frac{m_1 - m_2}{1 + m_1 . m_2} \qquad \left\{ \because \begin{array}{l} m_1 = \tan\theta_1 \ \& \\ m_2 = \tan\theta_2 \end{array} \right\}$$

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 . m_2}\right| \qquad \{\because \ \theta \text{ is acute}\}$$

This is the required equation

Example

- Find the angle between the lines 3x – y – 4 = 0 & 2x + y – 3 =0

Solution:

$$\text{Slope of line } 3x - y - 4 = 0 \text{ is } \frac{-3}{-1} = 3 = m_1$$

$$\text{Slope of line } 2x + y - 3 = 0 \text{ is } \frac{-2}{1} = -2 = m_2$$

Let θ be the acute angle between the lines

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{3-(-2)}{1+(3)(-2)}\right|$$

$$\therefore \tan\theta = \left|\frac{3+2}{1-6}\right| = \left|\frac{5}{-5}\right| = |-1|$$

$$\therefore \tan\theta = |1|$$

$$\therefore \theta = \tan^{-1}(1)$$

$$\therefore \boldsymbol{\theta} = \left(\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)^{c} \; \textbf{\textit{or}} \;\; 45^{o}$$

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Example

- Find the angle between the lines y = 5x + 6 & y = x.

Solution:

$$\text{Slope of line } y = 5x + 6 \text{ is } \frac{-5}{-1} = 5 = m_1$$

$$\text{Slope of line } y = x \text{ is } \frac{-1}{-1} = 1 = m_2$$

$$\text{Let } \theta \text{ be the acute angle between the lines}$$

$$\therefore \tan\ \theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{5-1}{1+(5)(1)}\right|$$

$$\therefore \tan\ \theta = \left|\frac{5-1}{1+5}\right|$$

$$= \left|\ \frac{4}{6}\ \right|$$

$$\therefore \tan\ \theta = \left|\frac{2}{3}\right|$$

$$\therefore \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

CONDITIONS FOR DIFFERENT LINES

Note:

If 'θ' is the acute angle between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then,

- The lines are parallel if

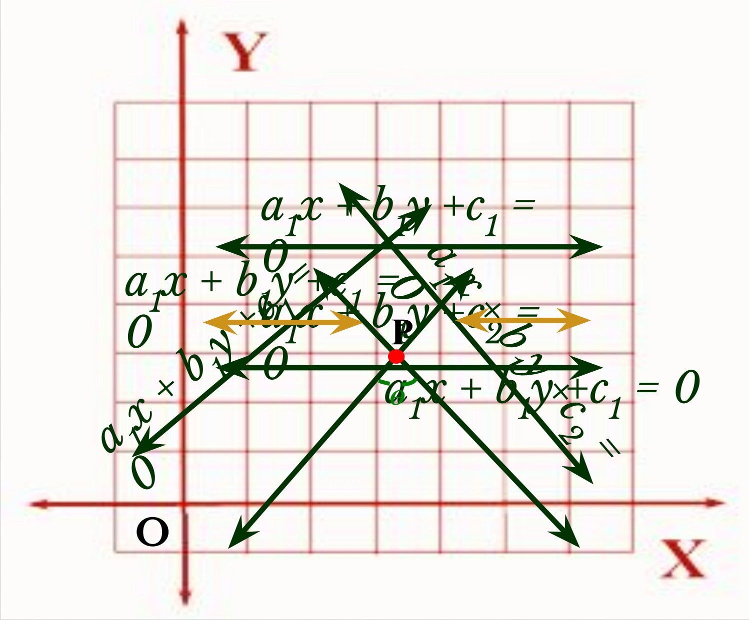
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

- The lines are perpendicular if

$$a_1a_2 + b_1b_2 = 0$$

- The lines are identical if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$



- If $m_1 = m_2$ then $\tan\theta = 0$ and hence lines are parallel.
- If $m_1 m_2 = -1$ then $\tan\theta = \infty$ and hence lines are perpendicular.

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Example

- Show that the lines 2x + 3y + 7 = 0 & 2x + 3y – 13 =0 are parallel to each other.

Solution:

$$\text{Slope of line } 2x+3y+7=0 \text{ is } \frac{-2}{3} = -\frac{2}{3} = m_1$$

$$\text{Slope of line } 2x+3y-13=0 \text{ is } \frac{-2}{3} = -\frac{2}{3} = m_2$$

As the slopes of lines are equal.

$$\text{i.e. } m_1 = m_2 = -\frac{2}{3}.$$

Thus the lines are parallel to each other.

Example

- Show that the lines 3x – 2y + 6 = 0 & 2x + 3y – 1 =0 are perpendicular to each other.

Solution:

$$\text{Slope of line } 3x - 2y + 6 = 0 \text{ is } \frac{-3}{-2} = \frac{3}{2} = m_1$$

$$\text{Slope of line } 2x + 3y - 1 = 0 \text{ is } \frac{-2}{3} = -\frac{2}{3} = m_2$$

Now consider $m_1.m_2$

$$\therefore m_1 m_2 = \left(\frac{3}{2} \times -\frac{2}{3}\right) \quad = -1$$

$\therefore m_1 m_2 = -1$, thus the lines are perpendicular.

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Example

- Find the value of 'k', if the lines kx – 6y = 9 & 6x + 5y = 13 are perpendicular to each other.

Solution:

$$\text{Slope of line } kx - 6y - 9 = 0 \text{ is } \frac{-k}{-6} = \frac{k}{6} = m_1$$

$$\text{Slope of line } 6x + 5y - 13 = 0 \text{ is } \frac{-6}{5} = m_2$$

$$\textit{Now as lines are perpendicular, } m_1.m_2 = -1$$

$$\therefore m_1 m_2 = \left(\frac{k}{6} \times -\frac{6}{5}\right) = -1$$

$$\therefore -\frac{k}{5} = -1 \qquad \therefore k = 5$$

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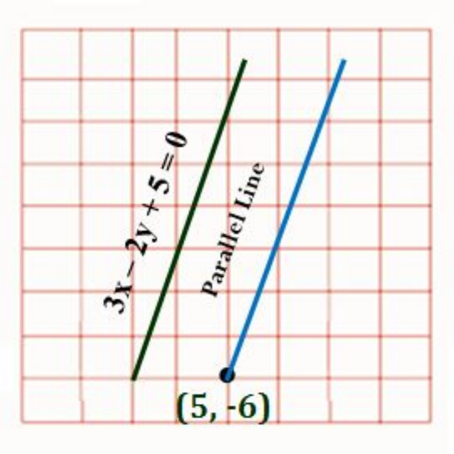
Example

- Find the equation of line parallel to 3x – 2y + 5 = 0 & passing through the point (5, - 6).



Solution:

$$\text{Given line is } 3x-2y+5=0$$

$$\text{Slope of line } 3x-2y+5=0 \text{ is } \frac{-3}{-2}=\frac{3}{2}=m_1$$

As the required line is parallel to given line.

$$\therefore \text{The Slope of required line} = \text{Slope of given line} = m_1 = \frac{3}{2}$$

$$\therefore \text{Equation of line passing through } (5,-6) \text{ and having slope as } \frac{3}{2} \text{ is}$$

By Slope – Point form
$y - y_1 = m(x - x_1)$

$$y-(-6)=\frac{3}{2}(x-5) \qquad \therefore y+6=\frac{3}{2}(x-5)$$

$$\therefore 2(y+6)=3(x-5) \qquad \therefore 2y+12=3x-15 \qquad \therefore 3x-2y-27=0$$

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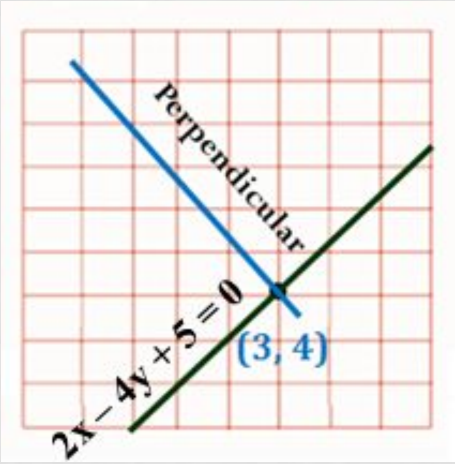
Example

- Find the equation of line passing through the point (3, 4) and perpendicular to 2x – 4y + 5 = 0.



Solution:

Given line is $2x-4y+5=0$

Slope of line $2x-4y+5=0$ is $\dfrac{-2}{-4}=\dfrac{1}{2}=m_1$

As the required line is perpendicular to given line.

$$\therefore \textbf{The Slope of required line} = \frac{\textbf{-1}}{\textbf{Slope of given line}} = \frac{\textbf{-1}}{\mathbf{m_1}} = \frac{-1}{1/2} = -2$$

$\therefore$ Equation of line passing through $(3,4)$ and having slope as -2 is

By Slope – Point form
$y - y_1 = m(x - x_1)$

$$y-4=-2(x-3)$$

$$\therefore \mathbf{y-4=-2x+6}$$

$$\therefore \mathbf{2x+y-10=0}$$

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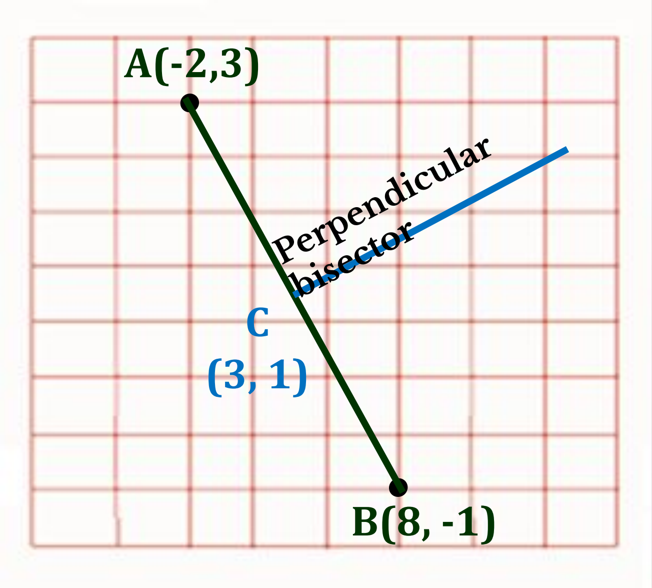
Example

- Find the equation of perpendicular bisector of line joining the points (– 2, 3) and (8, – 1).



Solution: Let $A \equiv (-2,3)$ and $B \equiv (8,-1)$

Slope of line AB is $m_1 = \dfrac{-1-3}{8+2} = \dfrac{-4}{10} = -\dfrac{2}{5}$

As $\perp$ bisector always intersect at midpoint to a line.

$$\text{Midpoint of AB} = \boldsymbol{C} = \left(\frac{-2+8}{2}, \frac{3-1}{2}\right) = (3,1)$$

$$\therefore \text{The Slope of perpendicular bisector} = \frac{-1}{\text{Slope of line AB}} = \frac{-1}{m_1} = \frac{-1}{-2/5} = \frac{5}{2}$$

$\therefore$ Equation of line passing through midpoint of AB $(3,1)$ and having slope as $\dfrac{5}{2}$ is

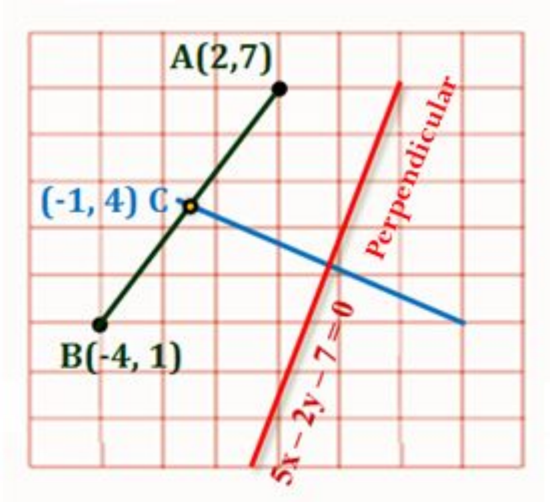
$$y - 1 = \frac{5}{2}(x-3) \qquad \therefore 2(y-1) = 5(x-3) \qquad \therefore 2y - 2 = 5x - 15$$

$$\therefore 5x - 2y - 13 = 0$$

Example

- Find the equation of line which is perpendicular to line 5x – 2y = 7 and passes through the midpoint of line joining (2, 7) and (– 4, 1).

Solution: Let $A \equiv (2,7)$ and $B \equiv (-4,1)$



$$\text{Midpoint of } AB = C = \left(\frac{2-4}{2}, \frac{7+1}{2}\right) = (-1,4)$$

Given line is $5x - 2y - 7 = 0$

$$\text{Slope of given line is } \frac{-5}{-2} = \frac{5}{2} = m_1$$

Required line is perpendicular to given line.

$$\therefore \text{The Slope of required line} = \frac{-1}{\text{Slope of perpendicular line}} = \frac{-1}{m_1} = \frac{-1}{5/2} = -\frac{2}{5}$$

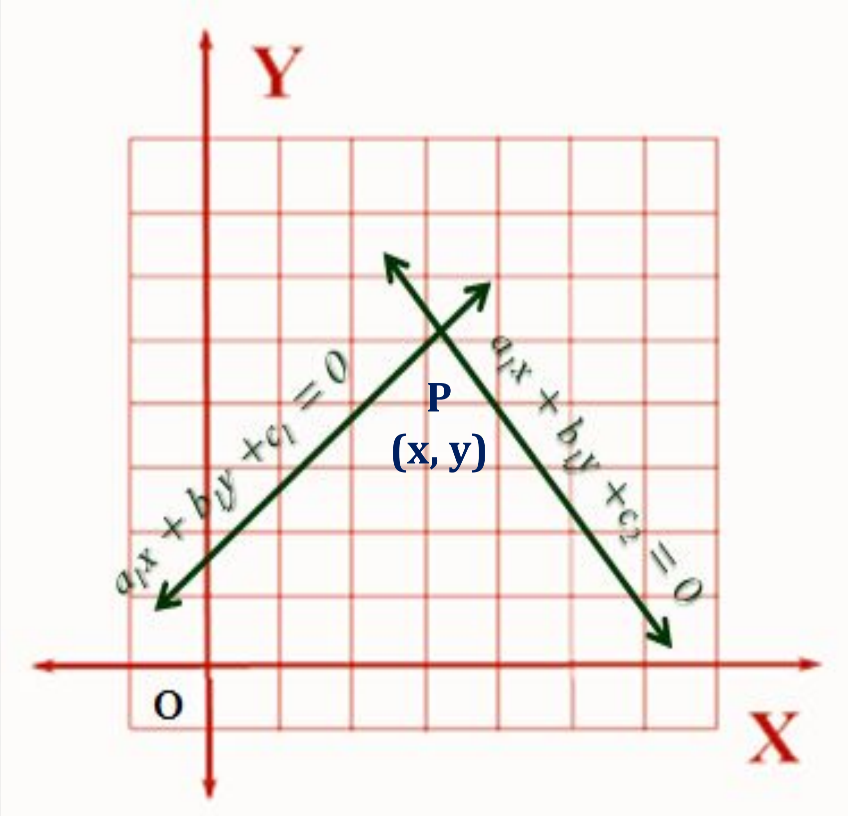
$\therefore$ Equation of line passing through midpoint of AB $(-1,4)$ and having slope as $-\frac{2}{5}$ is

$$y - 4 = -\frac{2}{5}(x+1) \qquad \therefore 5(y-4) = -2(x+1) \qquad \therefore 5y - 20 = -2x - 2$$

$$\therefore 2x + 5y - 18 = 0$$

INTERSECTION OF TWO LINES

- If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ intersect each other at a point P (x,y)
- Then the coordinate of point of intersection can be found by solving both the equations simultaneously.



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Example

- Find the equation of line passing through the point of intersection of lines 4x + 3y = 8 and x + y = 1 and parallel to 5x – 7y = 3.

Solution: The point of intersection of lines 4x + 3y = 8 and x + y = 1 is obtained by solving them simultaneously.

$\therefore$ Point of intersection is (5,–4)

$$\text{Slope of given line } 5x - 7y - 3 = 0 \text{ is } \frac{-5}{-7} = \frac{5}{7} = m_1$$

As the required line is parallel to given line.

$$\therefore \text{The Slope of required line} = \text{Slope of given line} = m_1 = \frac{5}{7}$$

$$\therefore \text{Equation of line passing through } (5,-4) \text{ and having slope as } \frac{5}{7} \text{ is}$$

By Slope – Point form
$y - y_1 = m(x - x_1)$

$$y - (-4) = \frac{5}{7}(x - 5) \qquad \therefore y + 4 = \frac{5}{7}(x - 5)$$

$$\therefore 7(y + 4) = 5(x - 5) \qquad \therefore 7y + 28 = 5x - 25$$

$$\therefore 5x - 7y - 53 = 0$$

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Example

- Find the equation of line passing through the point of intersection of lines 2x + 3y = 13 and 5x - y = 7 and perpendicular to 3x – y + 7 = 0.

Solution: The point of intersection of lines 2x + 3y = 13 and 5x - y = 7 is obtained by solving them simultaneously.

$\therefore$ Point of intersection is (2,3)

$$\text{Slope of given line } 3x - y + 7 = 0 \text{ is } \frac{-3}{-1} = 3 = m_1$$

As the required line is perpendicular to given line.

$$\therefore \text{The Slope of required line} = \frac{-1}{\text{Slope of given line}} = \frac{-1}{m_1} = \frac{-1}{3}$$

$$\therefore \text{Equation of line passing through } (2,3) \text{ and having slope as } -\frac{1}{3} \text{ is}$$

By Slope – Point form

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{3}(x - 2) \qquad \therefore 3(y-3) = -1(x-2)$$

$$\therefore 3y - 9 = -x + 2$$

$$\therefore x + 3y - 11 = 0$$

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Example

- Find the equation of line passing through the point of intersection of lines 2x + 3y = 13 and 5x - y = 7 and passing through (1, -1).

Solution:

The point of intersection of lines 2x + 3y = 13 and 5x - y = 7 is obtained by solving them simultaneously.

$\therefore$ Point of intersection is (2,3)

As the required line passes through (2,3) ***and*** (1,–1)

$\therefore$ Equation of line passing through $(2,3)$ and $(1,\text{-}1)$ is

By Two – point form $\dfrac{y-y_1}{x-x_1}=\dfrac{y_1-y_2}{x_1-x_2}$

$$\therefore \frac{y-3}{x-2}=\frac{3-(-1)}{2-1}$$

$$\therefore \frac{y-3}{x-2}=\frac{4}{1} \qquad \therefore y-3=4(x-2) \qquad \therefore y-3=4x-8 \qquad \therefore 4x-y-5=0$$

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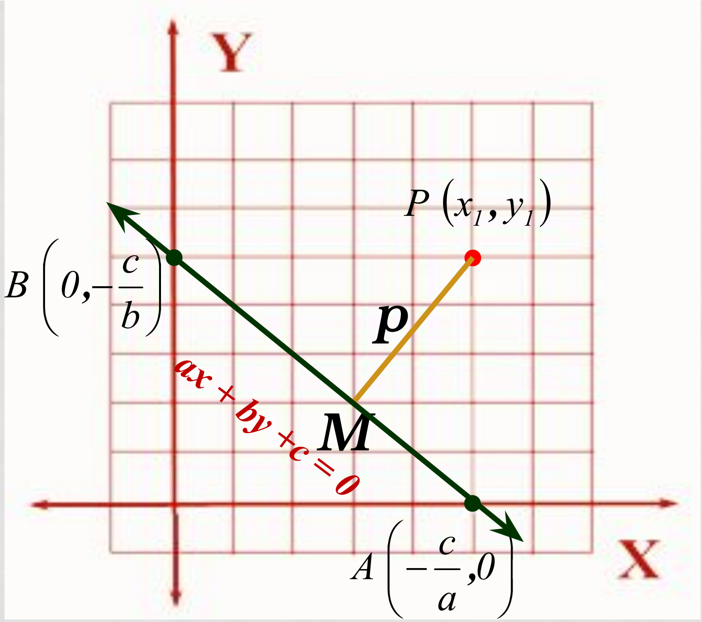
DISTANCE BETWEEN POINT & LINE

Theorem:

If '*p*' is the length of the perpendicular from a point ***P*** (x_1, y_1) to the line ***ax + by +c = 0*** then

$$p = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$



Proof: *Let the line **ax + by +c = 0** intersect the X-axis at point **A** and Y-axis at point **B**.*

$$\therefore Coordinates\ of\ A \equiv \left(-\frac{c}{a}, 0\right)$$

$$\therefore Coordinates\ of\ B \equiv \left(0, -\frac{c}{b}\right)$$

*Let **P** (x_1, y_1) be the point at a perpendicular distance **'p'** from line **ax + by +c = 0**. Draw **PM** perpendicular to **AB**. i.e. **PM = p**.*

By distance formula,

$$AB = \sqrt{\left(-\frac{c}{a} - 0\right)^2 + \left(0 + \frac{c}{b}\right)^2}$$

$$AB = \sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}} = \sqrt{c^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right)}$$

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DISTANCE BETWEEN POINT & LINE

$$\therefore\ AB=\sqrt{c^2\left(\frac{a^2+b^2}{a^2b^2}\right)}=\left|\frac{c}{ab}\right|\sqrt{a^2+b^2}$$

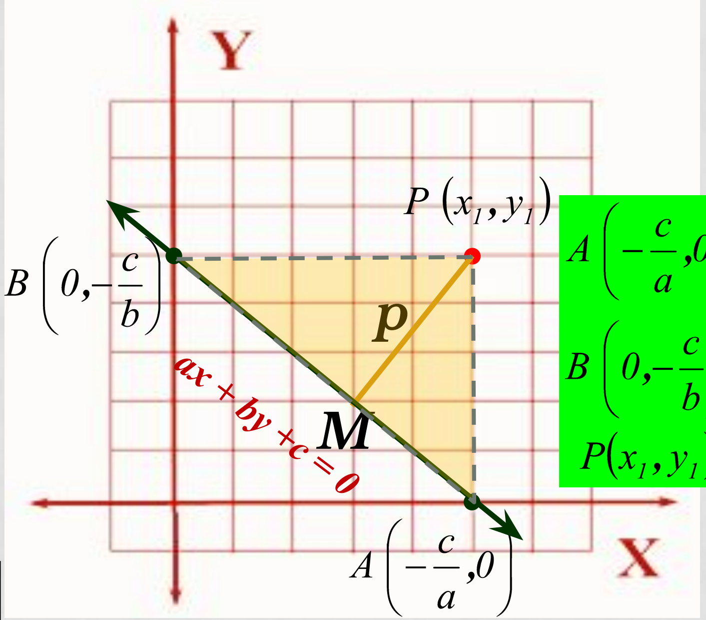
$$Now,\ A(\Delta PAB)=\frac{1}{2}(AB)(PM)$$

$$=\frac{1}{2}\times\left|\frac{c}{ab}\right|\sqrt{a^2+b^2}\times p \quad \text{.........(1)}$$

$$Also,\ A(\Delta PAB)=\frac{1}{2}\left[0-\left(-\frac{c}{a}y_1\right)+\frac{c^2}{ab}-0+0-\left(-\frac{c}{b}x_1\right)\right]$$

$$=\frac{1}{2}\left[\frac{cy_1}{a}+\frac{c^2}{ab}+\frac{cx_1}{b}\right]=\frac{1}{2}\left[\frac{(ax_1+by_1+c)c}{ab}\right]$$

$$=\frac{1}{2}\times\left|\frac{c}{ab}\right|\times(ax_1+by_1+c) \quad \text{......(2)}$$



From (1) & (2), we get

$$\frac{1}{2}\times\left|\frac{c}{ab}\right|\sqrt{a^2+b^2}\times p=\frac{1}{2}\times\left|\frac{c}{ab}\right|\times(ax_1+by_1+c)$$

$$\therefore\ p=\left|\frac{(ax_1+by_1+c)}{\sqrt{a^2+b^2}}\right|$$

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Example

- Find the perpendicular distance from the point (5,4) on the line 2x + y = -6.

Solution:

$$\text{As we know the distance from } (x_1, y_1) \text{ to the line } ax + by + c = 0 \text{ is } \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Here, $x_1 = 5$, $y_1 = 4$, $a = 2$, $b = 1$ & $c = 6$

$$\therefore \text{ Perpendicular distance from } (5,4) \text{ to the line } 2x + y + 6 = 0 \text{ is}$$

$$= \left|\frac{2(5) + 1(4) + 6}{\sqrt{2^2 + 1^2}}\right|$$

$$= \left|\frac{10 + 4 + 6}{\sqrt{4 + 1}}\right|$$

$$\therefore p = \frac{20}{\sqrt{5}} \text{ units}$$

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Example

- Find the length of perpendicular from (-3,-4) on the line 4(x+2) = 3(y-4).

Solution: Rewriting the equation as $4x - 3y + 20 = 0$

As we know the distance from (x_1, y_1) to the line $ax + by + c = 0$ is $\left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

Here, $x_1 = -3,\ y_1 = -4,\ a = 4,\ b = -3\ \&\ c = 20$

$\therefore$ Perpendicular distance from $(-3,-4)$ to the line $4x - 3y + 20 = 0$ is

$$= \left|\frac{4(-3) + (-3)(-4) + 20}{\sqrt{4^2 + (-3)^2}}\right|$$

$$= \left|\frac{-12 + 12 + 20}{\sqrt{16 + 9}}\right| = \frac{20}{5}$$

$\therefore p = 4$ units

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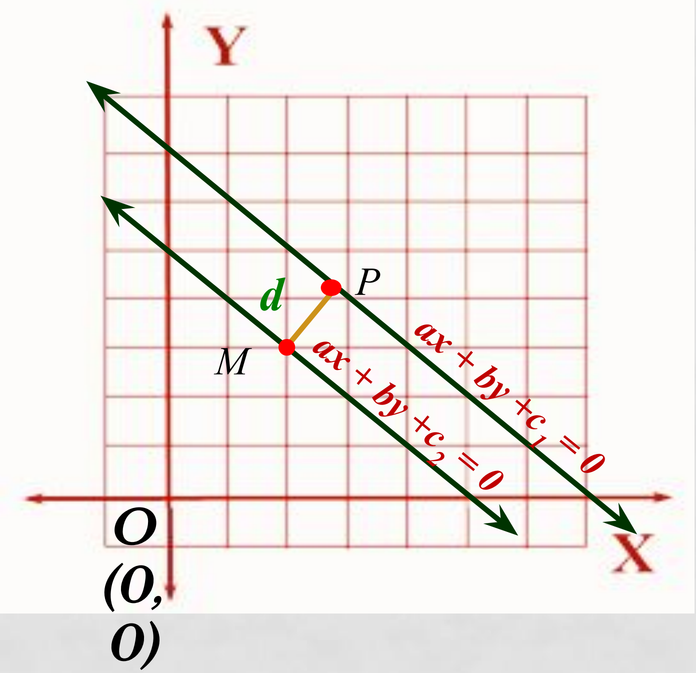
DISTANCE BETWEEN TWO PARALLEL LINES

Theorem:

If '*d*' is the distance between the two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ then

$$d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$



Proof: *Let* d_1 *be the distance of line* $ax + by + c_1 = 0$ *and* d_2 *be the distance of line* $ax + by + c_2 = 0$ *from origin* ***O***.

Distance between the two parallel lines are ***PM*** $= d = d_1 - d_2$

We know the distance from a point to the line is $\left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

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DISTANCE BETWEEN TWO PARALLEL LINES

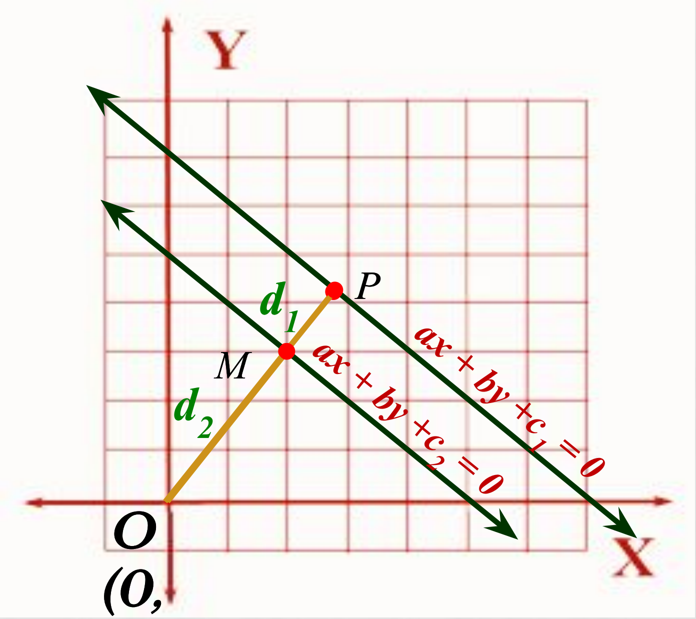
As we know the distance from a point to the line is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$\therefore OP = d_1 = \left|\frac{a(0) + b(0) + c_1}{\sqrt{a^2 + b^2}}\right| = \left|\frac{c_1}{\sqrt{a^2 + b^2}}\right|$$

$$\therefore OM = d_2 = \left|\frac{a(0) + b(0) + c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{c_2}{\sqrt{a^2 + b^2}}\right|$$

$$\therefore PM = d = d_1 - d_2 = \left|\frac{c_1}{\sqrt{a^2 + b^2}}\right| - \left|\frac{c_2}{\sqrt{a^2 + b^2}}\right|$$



(0, 0)

$$\therefore d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

This the expression for distance between the two parallel lines.

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Example

- Find the distance between the parallel lines 3x + 4y – 13 = 0 and 6x + 8y – 16 = 0.

Solution:

Rewriting the above equations of lines as

3x + 4y – 13 = 0

6x + 8y – 16 = 0 or ***3x + 4y – 8 = 0***.

$$As\ we\ know\ the\ distance\ between\ parallel\ lines\ are\ given\ as\ d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

$$\text{Here, } c_1 = -13,\ c_2 = -8,\ a = 3\ \&\ b = 4$$

$$\therefore \text{ Distance between } 3x + 4y - 13 = 0\ \&\ 6x + 8y - 16 = 0 \text{ is}$$

$$d = \left|\frac{-13 + 8}{\sqrt{3^2 + 4^2}}\right| = \left|\frac{-5}{\sqrt{9 + 16}}\right| = \left|\frac{-5}{\sqrt{25}}\right| = \left|\frac{-5}{5}\right| = |-1|$$

$$\therefore d = 1 \text{ units}$$

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