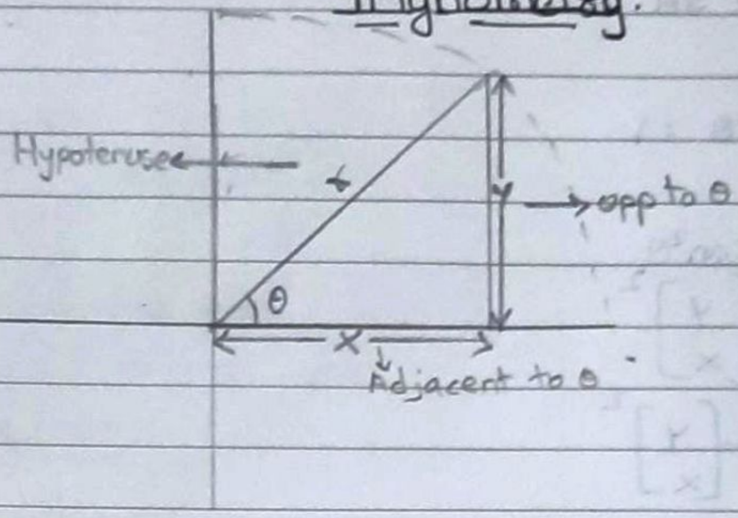
Trignometry.



$\sin\theta = \frac{\text{Opp}}{\text{hyp}} = \frac{y}{r}$

$\cos\theta = \frac{\text{Adj}}{\text{hyp}} = \frac{x}{r}$

$\tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{y}{x}$

* Identities

1. $\sin^2\theta + \cos^2\theta = \left[\frac{y}{r}\right]^2 + \left[\frac{x}{r}\right]^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{y^2 + x^2}{r^2} = \frac{r^2}{r^2}$

ABC right angle
∴ by pythogoras
$x^2 + y^2 = r^2$

$\therefore \sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

$\therefore \sin\theta = \sqrt{1 - \cos^2\theta}$

similary,

$\cos\theta = \sqrt{1 - \sin^2\theta}$

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2. $1 + \tan^2\theta = \sec^2\theta$

or

$\sec^2\theta - \tan^2\theta = 1$

L.H.S

$\sec^2\theta - \tan^2\theta$

$= \frac{1}{\cos^2\theta} - \left[\frac{y}{x}\right]^2$

$= \frac{1}{\left(\frac{x}{r}\right)^2} - \left[\frac{y}{x}\right]^2$

$= \left[\frac{r}{x}\right]^2 - \left[\frac{y}{x}\right]^2$

$= \frac{r^2 - y^2}{x^2}$

$= \frac{x^2 + y^2 - y^2}{x^2}$

$= 1$

3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

or

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

L.H.S

$\operatorname{cosec}^2\theta - \cot^2\theta$

$= \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta} = \frac{1}{\left(\frac{y}{r}\right)^2} - \frac{1}{\left(\frac{y}{x}\right)^2} = \frac{r^2}{y^2} - \frac{x^2}{y^2}$

$= \frac{r^2 - x^2}{y^2}$

$= \frac{x^2 + y^2 - x^2}{y^2}$

$= 1$

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* Identities

1.
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

* Measure of angles.

Radian to degree to radian:

- Multiply by $\frac{\pi}{180}$.

$\therefore 30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$

$\therefore 45^\circ = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$

$\therefore 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$

$\therefore 150^\circ = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$

In Radian

$180^\circ = \pi$ (Pi)

(i) $\frac{\pi}{2} = \frac{180^\circ}{2} = 90^\circ$

(ii) $2\pi = 2 \times 180^\circ = 360^\circ$

(iii) $\frac{\pi}{3} = \frac{180^\circ}{3} = 60^\circ$

(iv) $\frac{\pi}{4} = \frac{180^\circ}{4} = 45^\circ$

(v) $\frac{2\pi}{3} = \frac{360^\circ}{3} = 120^\circ$

(vi) $\frac{3\pi}{2} = \frac{3}{2} \times 180^\circ = 270$

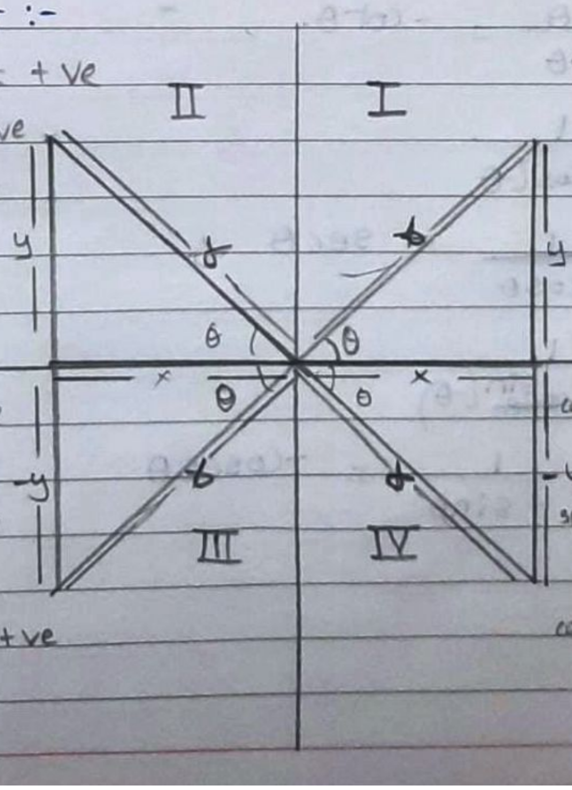
Quadrant :-

cosec $= \sin\theta = \frac{y}{r} = +ve$

sec $= \cos\theta = -\frac{x}{r} = -ve$

cot $= \tan = -ve$

$\sin\theta = \frac{y}{r} = +ve$, $\cos\theta = \frac{x}{r} = +ve$ } all +ve.



cosec $= \sin\theta = \frac{-y}{r} = -ve$

sec $= \cos = \frac{-x}{r} = -ve$

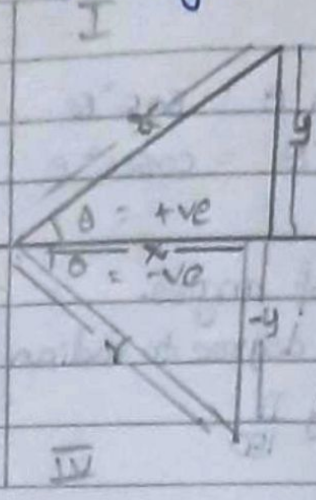
cot $= \tan = \frac{-x}{-y} = +ve$

cosec: $\sin\theta = \frac{-y}{r} = -ve$

sec $= \cos\theta = \frac{x}{r} = +ve$

cot $= \tan\theta = \frac{-ve}{+ve} = -ve$

+ve & -ve angle



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{x}{r}$

IMP.

- $\therefore \sin(-\theta) = -\sin\theta$
- $\therefore \cos(-\theta) = \cos\theta$
- $\therefore \tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$

$= \frac{-\sin\theta}{\cos\theta}$

$\tan(-\theta) = -\tan\theta$

$\because \sin(-\theta) = -\left(\frac{y}{r}\right)$ and $\frac{y}{r} = \sin\theta$

$\therefore \sin(-\theta) = -\sin\theta$

&

$\cos(-\theta) = \frac{x}{r}$ and $\cos\theta = \frac{x}{r}$

$\therefore \cos(-\theta) = \cos\theta$

- $\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)}$

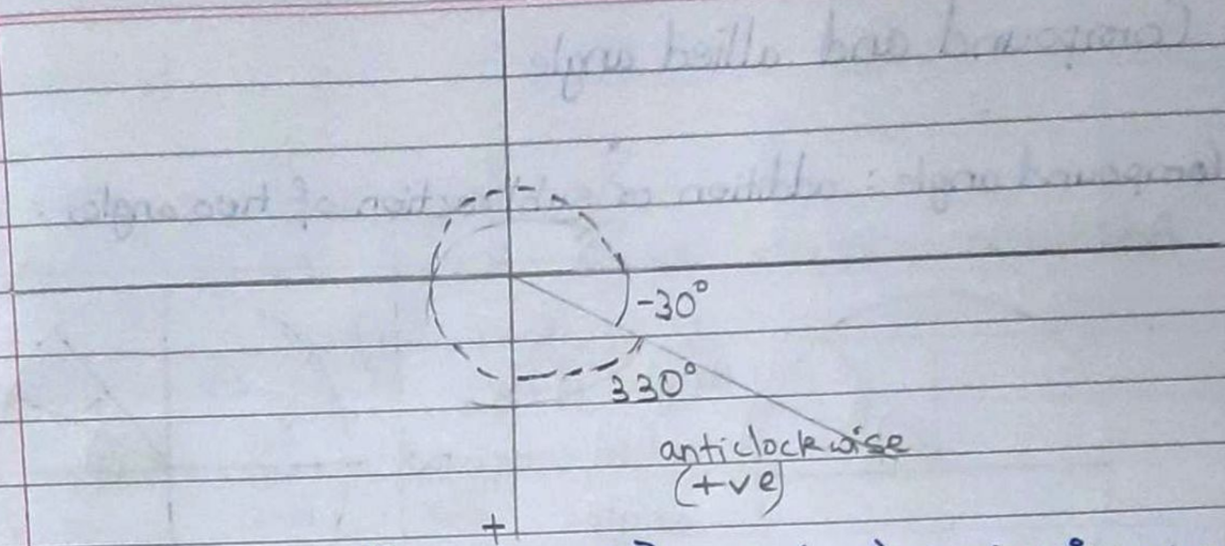
$= \frac{\cos\theta}{-\sin\theta} = -\cot\theta.$

- $\sec(-\theta) = \frac{1}{\cos(-\theta)}$

$= \frac{1}{\cos\theta} = \sec\theta$

- $\mathrm{cosec}(-\theta) = \frac{1}{\sin(-\theta)}$

$= \frac{1}{-\sin\theta} = -\mathrm{cosec}\,\theta$



$\sin 330^\circ = [2 \times 90^\circ - 30^\circ] = \sin(-30^\circ) = -\sin 30^\circ = -\frac{1}{2} = -0.5$

$\sin -30^\circ = -\sin 30^\circ = -\frac{1}{2} = -0.5$
clockwise (-ve)

$\cos(-45^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}} = 0.70$

$\cos(315^\circ) = 0.70 \ldots$ (by calculator)

* Table

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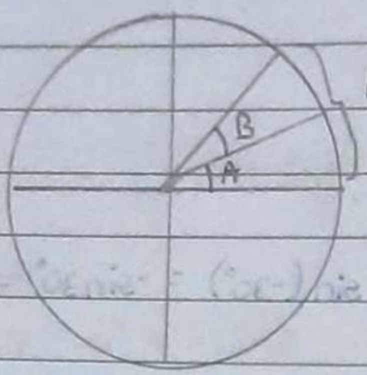
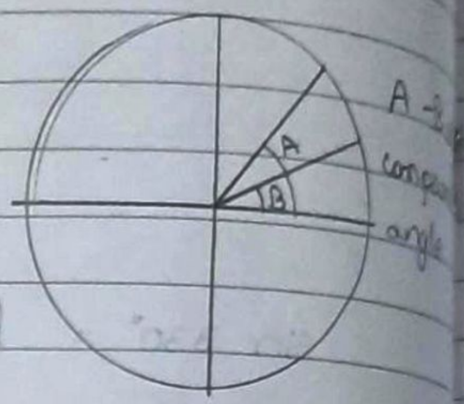
* Compound and allied angle.

Compound angle : addition or subtraction of two angles.

Addition:-



Angle $A+B$ called compound angle.



ex. If $A = 45^\circ$ and $B = 30^\circ$

Then

$A + B = 45^\circ + 30^\circ$ & $A - B = 45^\circ - 30^\circ$

$= 75^\circ$ $\quad$ $= 15^\circ$

are called compound angle.

- Addition formulae :-

1. $\sin[A+B] = \sin A \cdot \cos B + \cos A \cdot \sin B$

2. $\sin[A-B] = \sin A \cdot \cos B - \cos A \cdot \sin B$

3. $\cos[A+B] = \cos A \cdot \cos B - \sin A \cdot \sin B$

4. $\cos[A-B] = \cos A \cdot \cos B + \sin A \cdot \sin B$.

5. $\tan[A+B] = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

6. $\tan[A-B] = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

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Find value of

1. $\sin 15^\circ = \sin[45^\circ - 30^\circ]$

$\sin(A+B) = \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

2. $\cos 75^\circ = \cos[45^\circ + 30^\circ]$

$\cos(A+B) = \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \cos 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

3. $\tan 75^\circ = \tan[45^\circ + 30^\circ]$

$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$

$=$

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4. Prove that :-

$$\sin\alpha \cdot \cos[\beta - \alpha] + \cos\alpha \cdot \sin[\beta - \alpha] = \sin\beta$$

→ let $\alpha = A$

$\beta - \alpha = B$

$\therefore \sin A \cdot \cos B + \cos A \cdot \sin B$

$= \sin(A + B)$

Put A & B

$\therefore \sin[\alpha + \beta - \alpha]$

$\therefore \sin\beta$

$\therefore$ R.H.S

5. Prove that

If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ find $\tan(A+B)$ and $\tan(A-B)$

$$\tan(A+B) = \frac{\cancel{\tan}\frac{1}{2} + \cancel{\tan}\frac{1}{3}}{1 - \cancel{\tan}\frac{1}{2} \cdot \cancel{\tan}\frac{1}{3}}$$

$$= \frac{\frac{3+2}{6}}{1 - \frac{1}{6}}$$

$$= \frac{\frac{5}{6}}{\frac{5}{6}}$$

$$= \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{5}}$$

$$= 1$$

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Now, $\tan(A-B) = \dfrac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2} \cdot \frac{1}{3}}$

$= \dfrac{\frac{3-2}{6}}{1 + \frac{1}{6}}$

$= \dfrac{\frac{1}{6}}{\frac{7}{6}}$

$= \dfrac{1}{\cancel{6}} \times \dfrac{\cancel{6}}{7}$

$= \dfrac{1}{7}$

$\therefore (A+B) = 1$ & $(A-B) = \dfrac{1}{7}$

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6. If $\tan(x+y) = \frac{3}{4}$ & $\tan(x-y) = \frac{8}{15}$ find $\tan(2x)$ & $\tan(2y)$

$$\boxed{\begin{aligned} 2x &= x+y+x-y \\ 2y &= (x+y)-(x-y) \end{aligned}}$$

Now,

$$\tan(2x) = \tan[(x+y)+(x-y)]$$

$$\tan[A+B] = \frac{\tan A + \tan B}{1-\tan A \cdot \tan B}$$

$$= \frac{\frac{3}{4} + \frac{8}{15}}{1-\frac{3}{4} \cdot \frac{8}{15}}$$

$$= \frac{\frac{45+32}{60}}{1-\frac{24}{60}}$$

$$= \frac{\frac{77}{60}}{\frac{60-24}{60}}$$

$$= \frac{\frac{77}{60}}{\frac{36}{60}}$$

$$= \frac{77}{60} \times \frac{60}{36}$$

$$= \frac{77}{36}$$

Now,

$$\tan(2y) = \tan[(x+y)-(x-y)]$$

$$\therefore \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \cdot \tan B}$$

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$$= \frac{\frac{3}{4} - \frac{8}{15}}{1 + \frac{3}{4} \cdot \frac{8}{15}}$$

$$= \frac{\frac{45 - 32}{60}}{1 + \frac{24}{60}}$$

$$= \frac{\frac{13}{60}}{\frac{60+24}{60}}$$

$$= \frac{\frac{13}{60}}{\frac{84}{60}}$$

$$= \frac{13}{\cancel{60}} \times \frac{\cancel{60}}{84}$$

$$= \frac{13}{84}$$

$$\therefore \quad 2x = \frac{77}{36} \quad \& \quad 2y = \frac{13}{84}$$

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7. find value of

$$\tan\left[\frac{\pi}{4} + A\right] \cdot \tan\left[\frac{\pi}{4} - A\right]$$

Here, $A = \frac{\pi}{4}$ & $B = A$

$$\therefore \tan(A+B) = \left[\frac{\tan\frac{\pi}{4} + \tan A}{1 - \tan\frac{\pi}{4} \cdot \tan A}\right] \cdot \left[\frac{\tan\frac{\pi}{4} - \tan A}{1 + \tan\frac{\pi}{4} \cdot \tan A}\right]$$

$$= \frac{\cancel{\frac{\pi + 4A}{4}}}{1 -} \qquad \boxed{\tan\frac{\pi}{4} = \tan 45^\circ = 1}$$

$$= \left[\frac{1 + \tan A}{1 - 1 \cdot \tan A}\right] \cdot \left[\frac{1 - \tan A}{1 + 1 \cdot \tan A}\right]$$

$$= \left[\frac{\cancel{(1 + \tan A)}}{\cancel{(1 - \tan A)}}\right] \times \left[\frac{\cancel{(1 - \tan A)}}{\cancel{(1 + \tan A)}}\right]$$

$$= 1$$

* Alleed angles : If the sum or diffrence of measure of two angles is either (0) zero or Integral multiple of 90° (any mult of 90) (ie is $\mp n \cdot \frac{\pi}{2}$), where $n = 1, 2, 3, \ldots$ These angles are called allied angles

ex. $n \cdot \frac{\pi}{2} \pm \theta \rightarrow$ called allied of angle θ.

$90 \times 1 = 90 = \frac{\pi}{2}$

$90 \times 2 = 180 = \pi$

$90 \times 3 = 270 = 3\pi/2$

$90 \times 4 = 360 = 2\pi$

$90 \times 5 = 450 = 5\pi/2$

$90 \times 6 = 540 = 3\pi$

Find sin $\left[\frac{\pi}{2} \pm \theta\right]$ and cos $\left[\frac{\pi}{2} \pm \theta\right]$

$$\sin\left[\frac{\pi}{2} + \theta\right] = \sin\frac{\pi}{2} \cdot \cos\theta + \cos\frac{\pi}{2} \cdot \sin\theta$$

$$\because \sin\frac{\pi}{2} = 1 \ \& \ \cos\frac{\pi}{2} = 0$$

$$= 1 \cdot \cos\theta + 0 \cdot \sin\theta$$

$$= 1 - \cos\theta \cdot \sin\theta$$

$$\therefore \sin\left[\frac{\pi}{2} + \theta\right] = \cos\theta$$

and,

$$\sin\left[\frac{\pi}{2} - \theta\right] = \sin\frac{\pi}{2} \cdot \cos\theta - \cos\frac{\pi}{2} \cdot \sin\theta$$

$$= 1 \cdot \cos\theta - 0 \cdot \sin\theta$$

$$= \cos\theta - \theta$$

$$= \cos\theta$$

Now,

$$\cos\left[\frac{\pi}{2} + \theta\right] = \cos\frac{\pi}{2} \cdot \cos\theta - \sin\frac{\pi}{2} \cdot \sin\theta$$

$$= 0 \cdot \cos\theta - 1 \cdot \sin\theta$$

$$= 0 - \sin\theta - \sin\theta$$

$$\therefore \cos\frac{\pi}{2} + \theta = -\sin\theta$$

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$$\cos\left[\frac{\pi}{2}-\theta\right] = \cos\frac{\pi}{2}\cdot\cos\theta + \sin\frac{\pi}{2}\cdot\sin\theta$$
$$= 0\cdot\cos\theta + 1\cdot\sin\theta$$
$$= 0 + \sin\theta$$
$$= \sin\theta$$

* Summary.

1. $\sin(-\theta) = -\sin\theta \longrightarrow \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
2. $\cos(-\theta) = \cos\theta \longrightarrow \sec(-\theta) = \sec\theta$
3. $\tan(-\theta) = -\tan\theta \longrightarrow \cot(-\theta) = -\cot\theta$

Addition formula / compound angles.

1. $\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$
2. $\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$
3. $\sin(A-B) = \sin A\cdot\cos B - \cos A\cdot\sin B$
4. $\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\cdot\tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\cdot\tan B}$

1. $\sin\left[\frac{\pi}{2}+\theta\right] = \cos\theta$	3. $\cos\left[\frac{\pi}{2}+\theta\right] = -\sin\theta$
2. $\sin\left[\frac{\pi}{2}-\theta\right] = \cos\theta$	4. $\cos\left[\frac{\pi}{2}-\theta\right] = \sin\theta$
5. $\tan\left[\frac{\pi}{2}+\theta\right] = -\cot\theta$	6. $\tan\left[\frac{\pi}{2}-\theta\right] = \cot\theta$

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$\sin 180^\circ = \sin \pi = 0$

$\cos(180^\circ) = \cos\pi = -1$

$\cos(n\pi) = (-1)^n$

for $n = 0, 1, 2, 3, \ldots$

$n = 0$, $\cos 0 = (-1)^0 = 1$

$n = 1$, $\cos(\pi) = (-1)^1 = -1$

$n = 2$, $\cos(2\pi) = (-1)^2 = 1$

$n = 3$, $\cos(3\pi) = (-1)^3 = 1$

* $\cos(n\pi) = -1$ If n is odd.

$n\pi = 1$ If n is '0' & even.

$\sin n\pi = 0$

* $\sin[\pi - x] = \sin\pi \cdot \cos x - \cos\pi \cdot \sin x$

$= 0 \cdot \cos x - (-1) \cdot \sin x$

$= 0 + \sin x$

$= \sin x$

$\therefore \sin(\pi - x) = \sin x$.

* $\cos[\pi - x] = \cos\pi \cdot \cos x + \sin\pi \cdot \sin x$

$= (-1) \cdot \cos x + 0 \cdot \sin x$

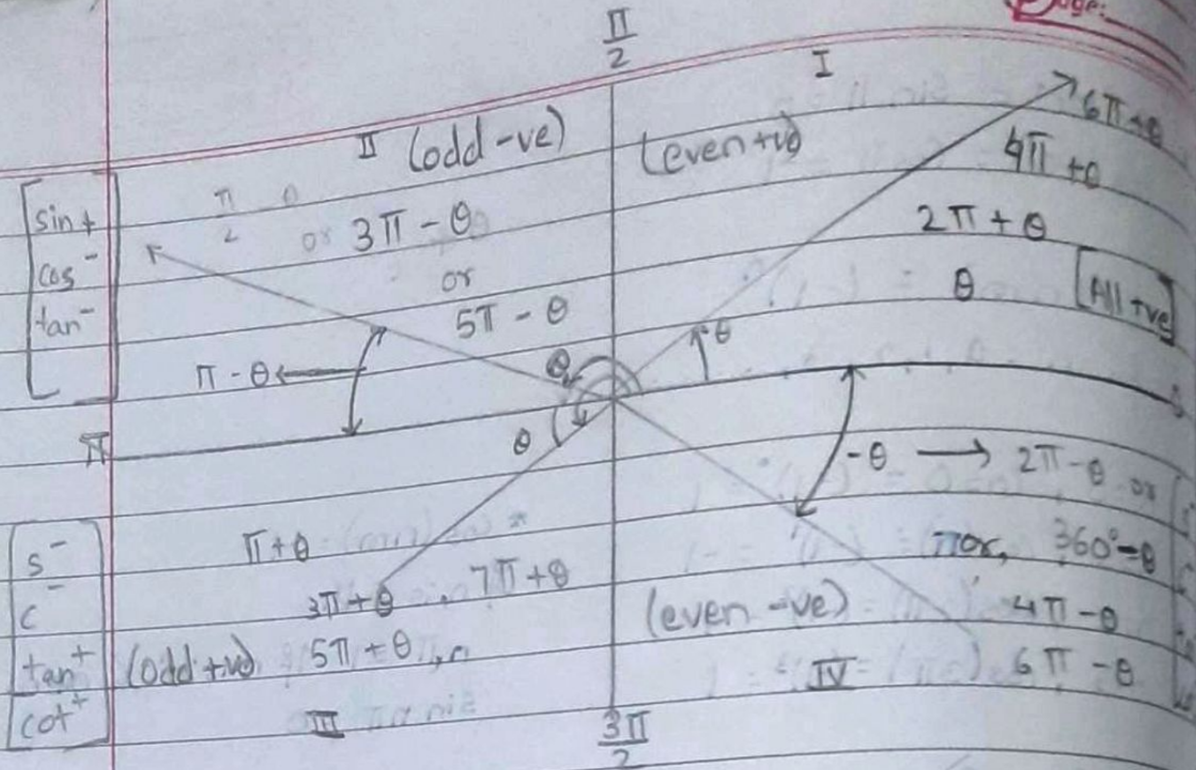
$= -\cos x$

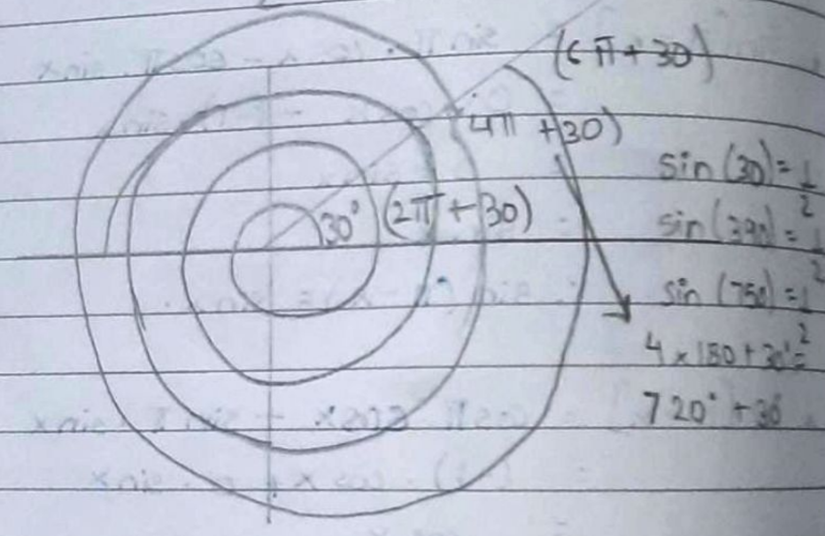
* $\sin(\pi + x) = \sin\pi \cdot \cos x + \cos\pi \cdot \sin x$.

$= 0 \cdot \cos x + (-1) \cdot \sin x$

$= 0 - \sin x$

$= -\sin x$





Coterminal angles

$\sin(\pi - \theta) = \sin\theta$	$\cot(\pi + \theta) = \cot\theta$	180 = π
$\sin(\pi + \theta) = -\sin\theta$	$\cos(3\pi - \theta) = -\cos\theta$	360 = 2π
$\cos(\pi + \theta) = -\cos\theta$	$\tan(4\pi + \theta) = \tan\theta$	540 = 3π
$\cos(\pi - \theta) = -\cos\theta$	$\sin(5\pi - \theta) = \sin\theta$	720 = 4π
$\cos(2\pi - \theta) = \cos\theta$		900 = 5π
$\tan(3\pi + \theta) = \tan\theta$		
$\cos(4\pi + \theta) = \cos\theta$		
$\sin(3\pi - \theta) = \sin\theta$		

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1. $\sin(330^\circ)$
$= \sin(360^\circ - 30^\circ)$
$= \sin(2\pi - 30^\circ)$
$= -\sin\theta$
$= -\frac{1}{2}$

2. $\cos(-300)$
$= \cos(360^\circ - 60^\circ)$
$= \cos(2\pi - 60^\circ)$
$= \cos 60^\circ$
$= \frac{1}{2}$

3. $\tan(-420^\circ)$
$\tan(-\theta) = -\tan\theta$
$= -\tan(420^\circ)$
$= -\tan(360^\circ + 60^\circ)$
$= -\tan(2\pi + 60^\circ)$
$= -\tan 60^\circ$
$= -\sqrt{3}$

4. $\sin(150^\circ)$
$= \sin(2\pi - 30^\circ)$
$= \sin\theta$
$= \sin 30^\circ$
$= \frac{1}{2}$

5. $\cos(315^\circ)$.

$= \cos(2\pi - 45^\circ)$

$= \cos 45^\circ$

$= \frac{1}{\sqrt{2}}$

6. $\tan(585^\circ)$

$= \tan(3\pi + 45^\circ)$

$= \tan 45^\circ$

$= 1$

find value of without using calculator.

1. $\text{Cosec}\,[-960^\circ]$

$\rightarrow -\text{cosec}\,[960^\circ]$ ..... $\because \text{cosec}(-\theta) = -\text{cosec}\,\theta$

$\therefore -\text{cosec}\,[900^\circ + 60^\circ]$

$\therefore -\text{cosec}\,[5\pi + 60^\circ]$

$= -(-\text{cosec}\,60^\circ)$

$= \text{cosec}\,60^\circ$

$= \dfrac{2}{\sqrt{3}}$

2. $\sin\,[210^\circ]$

$\therefore \sin\,[280^\circ - 60^\circ]$

$\therefore \sin\left[\dfrac{3\pi}{2} - 60^\circ\right]$

$\therefore -\sin(60^\circ)$

$\therefore -\dfrac{\sqrt{3}}{2}$

3. $\sec(-660^\circ)$

$\therefore \sec(-\theta) = \sec\theta$

$= \sec(720^\circ - 60^\circ)$

$= \sec(4\pi - 60^\circ)$

$= \sec 60^\circ$

$=$

4) $\dfrac{\sec^2[135^\circ]}{\cos[-240^\circ] - 2\sin[930^\circ]}$

$\therefore \dfrac{\{\sec [180^\circ - 45^\circ]\}^2}{\cos[240^\circ] - 2\sin[900^\circ + 30^\circ]}$

$\therefore \dfrac{\{\sec(\pi - 45^\circ)\}^2}{\cos(180^\circ + 60^\circ) - 2\sin(5\pi + 30^\circ)}$

$\therefore \dfrac{\sec(\pi - 45^\circ)^2}{\cos(\pi + 60^\circ) - 2\sin(5\pi + 30^\circ)}$

$\therefore \dfrac{(-\sec 45^\circ)}{-\cos 60^\circ - 2(-\sin 30^\circ)}$

$= \dfrac{(-\sqrt{2})^2}{\frac{-1}{2} + 2 \times \left[\frac{1}{2}\right]}$

$= \dfrac{2}{\frac{-1}{2} + 1}$

$= \dfrac{2}{\frac{-1+2}{2}}$

$= \dfrac{4}{1}$

$= 4$

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5) $\sin[150^\circ] - \tan[315^\circ] + \cos[300^\circ] + \sec^2[360^\circ]$

$\sin[150^\circ] = \sin[180^\circ - 30^\circ]$
$= \sin[\pi - 30^\circ]$
$= \sin 30^\circ$
$= \frac{1}{2}$

$\tan[315^\circ] = \tan[360^\circ - 45^\circ]$
$= \tan[2\pi - 45^\circ]$
$= -\tan 45^\circ$
$= -1$

$\cos[300^\circ] = \cos[360^\circ - 60^\circ]$
$= \cos[2\pi - 60^\circ]$
$= \cos 60^\circ$
$= \frac{1}{2}$

$\sec^2[360^\circ] = [\sec(360)^\circ]^2$
$= (\sec(2\pi))^2$
$= \left[\frac{1}{\cos 2\pi}\right]^2 = \frac{1}{1} = 1$

Now,
$\frac{1}{2} - [-1] + \frac{1}{2} + 1$

$= \frac{1}{2} + 1 + \frac{1}{2} + 1$

$= 1 + 1 + 1 = 3$

$= \underline{3}$

6) $\therefore \cos^2[3660]$

$\therefore \cos^2[3600+60]$

$\therefore [\cos(3600+60)]^2$

$\therefore [\cos(20\pi+60)]^2$

$\therefore [\cos 60°]^2$

$= \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

7) $\frac{\tan 66° + \tan 69°}{1 - \tan 66° . \tan 69°}$

$$\boxed{* \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}}$$

from given. $A = 66°$ & $B = 69°$

$\therefore \tan(A+B)$

$\therefore \tan(66° + 69°)$

$\therefore \tan(135°)$

$\therefore \tan(180° - 45°)$

$\therefore \tan(\pi - 45°)$

$\therefore -\tan 45°$

$\therefore -1$

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If $\tan(x+y) = \frac{3}{4}$ & $\tan(x-y) = \frac{8}{15}$ find $\tan(2x)$ and $\tan(2y)$

$$\tan(2x) = \tan[(x+y)+(x-y)]$$
$$= \tan(A+B)$$
$$= \frac{\tan(x+y)+\tan(x-y)}{1-\tan(x+y).\tan(x-y)}$$
$$= \frac{\frac{3}{4}+\frac{8}{15}}{1-\left(\frac{3}{4}\times\frac{8}{15}\right)}$$
$$= \frac{\frac{45+32}{60}}{1-\frac{24}{60}}$$
$$= \frac{\frac{77}{\cancel{60}}}{\frac{60-24}{\cancel{60}}} = \frac{77}{36}$$

Now,

$$\tan(2y) = \tan[(x+y)-(x-y)]$$
$$= \tan(A-B)$$
$$= \frac{\tan(x+y)-\tan(x-y)}{1+\tan(x+y).\tan(x-y)}$$
$$= \frac{\frac{3}{4}-\frac{8}{15}}{1+\frac{3}{4}\times\frac{8}{15}} = \frac{\frac{45-32}{60}}{1+\frac{24}{60}}$$
$$= \frac{13/\cancel{60}}{\frac{60+24}{\cancel{60}}} = \frac{13}{84}$$

$$\therefore \quad 2x = \frac{77}{36} \quad \& \quad 2y = \frac{13}{84}$$

$\therefore \tan\left(\frac{3+2}{6}\right)$

$= \tan\frac{5}{6}$

* Find $\tan[A+B]$ and $\tan[A-B]$ if $\tan A = \frac{1}{2}$ & $\tan B = \frac{1}{3}$

$\therefore \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$= \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$

$= \dfrac{\frac{3+2}{6}}{1 - \frac{1}{6}}$

$= \dfrac{\frac{5}{6}}{\frac{5}{6}}$

$= \frac{\cancel{5}}{\cancel{5}}$

$= 1$

Prove that

$1 + \tan A \cdot \tan 2A = \sec(2A)$

~~R.H.S~~ L.H.S:

$1 + \tan A \cdot \tan 2A$

$\therefore 1 + \left[\frac{\sin A}{\cos A} \cdot \frac{\sin 2A}{\cos 2A}\right]$

$\therefore 1 + \left[\frac{\sin A \cdot \sin 2A}{\cos A \cdot \cos 2A}\right]$

$\therefore \frac{\cos A \cdot \cos 2A + \sin A \cdot \sin 2A}{\cos A \cdot \cos 2A}$ ... (formula)

$\therefore \frac{\cos(A - 2A)}{\cos A \cdot \cos 2A}$

$\therefore \frac{\cos(-A)}{\cos A \cdot \cos 2A}$

$\therefore \frac{\cancel{\cos A}}{\cancel{\cos A} \cdot \cos 2A}$

$\therefore \frac{1}{\cos 2A}$

$\therefore \sec(2A)$

$\therefore$ RHS

Prove that

$$\frac{1 - \tan 2\theta \cdot \tan\theta}{1 + \tan 2\theta \cdot \tan\theta} = \frac{\cos 3\theta}{\cos\theta}$$

L.H.S

$$\frac{1 - \tan 2\theta \cdot \tan\theta}{1 + \tan 2\theta \cdot \tan\theta}$$

$$\therefore \frac{1 - \frac{\sin 2\theta}{\cos 2\theta} \cdot \frac{\sin\theta}{\cos\theta}}{1 + \frac{\sin 2\theta}{\cos 2\theta} \cdot \frac{\sin\theta}{\cos\theta}}$$

$$\therefore \frac{\frac{\cos 2\theta \cdot \cos\theta - \sin 2\theta \cdot \sin\theta}{\cos 2\theta \cdot \cos\theta}}{\frac{\cos 2\theta \cdot \cos\theta + \sin 2\theta \cdot \sin\theta}{\cos 2\theta \cdot \cos\theta}}$$

$$\therefore \frac{\cos(2\theta + \theta)}{\cos(2\theta - \theta)}$$

$$\therefore \frac{\cos 3\theta}{\cos\theta}$$

$\therefore$ R.H.S

Prove that

$\frac{\sin 2\theta}{\sin\theta}$ ...

L.H.S.

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Prove that

$$\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta$$

L.H.S.

$$\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta}$$

$$= \frac{\sin 2\theta . \cos\theta - \cos 2\theta . \sin\theta}{\sin\theta \cdot \cos\theta}$$

$\therefore$ ~~sin 2θ co~~

$$\therefore \frac{\sin(2\theta - \theta)}{\sin\theta . \cos\theta} = \frac{\cancel{\sin\theta}}{\cancel{\sin\theta} . \cos\theta} = \frac{1}{\cos\theta}$$

$= \sec\theta$

$=$ R.H.S.

In ΔABC prove that $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

we know measure of ΔABC

$\therefore A + B + C = 180°$

$\therefore A + B = 180° - C$

Now,

Taking tangentinal angle on both sides,

$\tan(A+B) = \tan(180° - C)$

$\therefore \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = -\tan C$

$\therefore \tan A + \tan B = (-\tan C) \cdot (1 - \tan A \cdot \tan B)$

$\therefore \tan A + \tan B = -\tan C + \tan A \cdot \tan B \cdot \tan C$

$\therefore \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C.$

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Show that

$\tan 3A - \tan 2A \cdot \tan A = \tan A \cdot \tan 2A \cdot \tan 3A$

we know, $3A = 2A + A$

~~3A~~ :

taking tangential angle.

$\tan(3A) = \tan(2A + A)$

$$\tan(3A) = \frac{\tan 2A + \tan A}{1 - \tan 2A \cdot \tan A}$$

$\therefore \tan(3A)(1 - \tan 2A \cdot \tan A) = \tan 2A + \tan A$

$\therefore \tan 3A - [\tan 3A \cdot \tan 2A \cdot \tan A] = \tan 2A + \tan A$

$\therefore \tan 3A = \tan 2A + \tan A + [\tan 3A \cdot \tan 2A]$

$\therefore \tan A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$

Show that

$\sin(60+A)\cdot\cos(30-B)+\cos(60+A)\cdot\sin(30-B) = \cos(A-B)$

$\therefore$ L.H.S

$\sin(60+A).\cos(30-B)+\cos(60+A).\sin(30-B)$

$\therefore \quad \sin[(60+A)-(30-B)]$ .... formula $(\sin(A-B)$

$\therefore$ let $(60+A)=\alpha$ & $(30-B)=\beta$

$\therefore \sin(\alpha+\beta)=\sin[60+A+30-B]$

$=\sin(90+A-B)$

$=\sin[90+(A-B]$

let $A-B=\theta$, $90^\circ=\frac{\pi}{2}$

$=\sin\left[\frac{\pi}{2}+\theta\right]$

$=\cos\theta$

$=\cos(A-B)$

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if $\tan(x+y) = 3$ and $\tan(x-y) = 5$ then find $\tan(2x)$

we know, $2x = (x+y) + (x-y)$

Now,

$\tan(2x) = \tan[(x+y) + (x-y)]$

$= \tan(A+B)$

$\therefore \tan[A+B] = \dfrac{\tan(x+y) + \tan(x-y)}{1 - \tan(x+y) \cdot \tan(x-y)}$

$= \dfrac{3+5}{1 - 3 \cdot 5}$

$= \dfrac{8}{1-15}$

$= \dfrac{\cancel{8}\,4}{-\cancel{14}\,7}$

$= \dfrac{-4}{7}$

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Multiple angle and submultiple angle.

$\theta, 2\theta, 3\theta, 4\theta, \ldots\ldots$ called multiple angles.
and,
$\frac{\theta}{2}, \frac{\theta}{3}, \frac{\theta}{4}, \ldots\ldots$ called submultiple angle.

1) $\sin 2\theta = \sin(\theta + \theta)$
$= \sin\theta \cdot \cos\theta + \cos\theta \cdot \sin\theta$

* $\therefore \sin 2\theta = 2 \sin\theta \cdot \cos\theta$

$\sin\theta = \sin\left(\frac{\theta}{2} + \frac{\theta}{2}\right)$

$= \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} + \cos\frac{\theta}{2} \cdot \sin\frac{\theta}{2}$

* $\sin\theta = 2 \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$

3) $\tan(2\theta) = \tan(\theta + \theta)$
$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta \cdot \tan\theta}$

$\therefore \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\therefore \tan\theta = \frac{2 \cdot \tan(\theta/2)}{1 - \tan^2(\theta/2)}$

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2) $\cos(2\theta) = \cos(\theta + \theta)$

$= \cos\theta \cdot \cos\theta - \sin\theta \cdot \sin\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ —— ①

Put $\sin^2\theta = 1 - \cos^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$= \cos^2\theta - 1 + \cos^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$ —— ② $\rightarrow 1 + \cos 2\theta = 2\cos^2\theta$

agin put $\cos^2\theta = 1 - \sin^2\theta$ in ① or $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$ V.I.P

$\cos(2\theta) = 1 - 2\sin^2\theta$ —— ③ $\rightarrow$

or $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

4) $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$\sin(3\theta) = \sin(2\theta + \theta)$

$= \sin 2\theta \cdot \sin\theta + \cos 2\theta \cdot \cos\theta$

~~we know $\sin(2\theta) = 2\sin\theta - \cos\theta$~~

we know, $\sin(2\theta) = 2\sin\theta - \cos\theta$

$\cos(2\theta) = (1 - 2\sin^2\theta)$ ~~$\sin\theta$~~

~~$\cos(2\theta) = $~~

$\therefore$ $\sin 3\theta = (2\sin\theta \cdot \cos\theta) \cdot \cos\theta + (1 - 2\sin^2\theta) \cdot \sin\theta$

$= 2\sin\theta \cdot \cos^2\theta + \sin\theta - 2\sin^3\theta$

Now, $\sin 3\theta = 2\sin\theta \cdot (1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$\therefore$ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

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5) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

L.H.S.

$\therefore \cos(3\theta) = \cos(2\theta + \theta)$

$= \cos 2\theta + \cos\theta - \sin 2\theta \cdot \sin\theta$

$= (2\cos^2\theta - 1)\cdot\cos\theta - (2\sin\theta\cdot\cos\theta)\cdot\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cdot\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cdot\cos\theta$

$= 2\cos^3\theta - \cos\theta - (2\cos\theta - 2\cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^2\theta$

$= \underline{4\cos^3\theta - 3\cos\theta}$

$=$ R.H.S.

⑥ $\tan(3\theta) = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Now,

$\tan(3\theta) = \tan(2\theta + \theta)$

$= \tan(A + B)$ --- 1

$= \dfrac{\tan(2\theta) + \tan\theta}{1 - \tan(2\theta)\cdot\tan\theta}$

Put $\tan^2\theta = 2\tan$

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* Formulas

1. $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\sin\theta = 2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$

2. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$= 2 \cdot \cos^2\frac{\theta}{2} - 1$

$= 1 - 2\sin^2\frac{\theta}{2}$

3. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\tan\theta = \dfrac{2\tan(\theta/2)}{1-\tan^2(\theta/2)}$

4. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

5. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

6. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

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Tangential form of $\sin 2\theta$ and $\cos 2\theta$

we know,

1. $\sin 2\theta = \dfrac{2\sin\theta \cdot \cos\theta}{1}$

formula → * $\sin^2\theta + \cos^2\theta = 1$

$$= \frac{2\sin\theta \cdot \cos\theta}{\cos^2\theta + \sin^2\theta} \quad \ldots \left[\sin^2\theta + \cos^2\theta = 1\right]$$

divide by $\cos^2\theta$

$$= \frac{\dfrac{2\sin\theta \cdot \cos\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\left(\dfrac{\sin\theta}{\cos\theta}\right)}{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}}$$

$$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$$

$$\sin\theta = \frac{2\tan(\theta/2)}{1 + \tan^2(\theta/2)}$$

2. $\cos 2\theta = \dfrac{\cos^2\theta - \sin^2\theta}{1}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

dividing by $\cos^2\theta$

$$= \dfrac{\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$$

$$= \dfrac{\dfrac{\cos^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}}$$

$$= \dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}$$

similarly,

$$\cos\theta = \dfrac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)}$$

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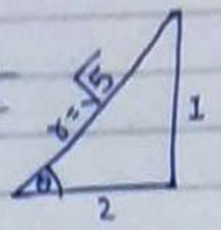
If $\tan A = \frac{1}{2}$,
Then find, $\sin A$ if A lies in I^{st} Quadrant
Then find, $\cos A$ if A lies in III^{rd} Quadrant

$\tan A = \frac{1}{2}$



$\sin A = \pm \frac{1}{\sqrt{5}}$

$\cos A = -\frac{2}{\sqrt{5}}$

$r^2 = 2^2 + 1^2 = 4 + 1 = 5$

$r = \sqrt{5}$

1) If $\sin A = \frac{1}{2}$ find $\sin(3A)$

we know,

$$\sin(3A) = 3\sin A - 4\sin^3 A$$

$$= 3 \cdot \left[\frac{1}{2}\right] - 4\left[\frac{1}{2}\right]^3$$

$$= \left[\frac{3}{2}\right] - 4\left[\frac{1}{8}\right]$$

$$= \left[\frac{3}{2}\right] - \left[\frac{4}{8}\right]$$

$$= \frac{3}{2} - \frac{1}{2}$$

$$= \frac{3-1}{2} = \frac{2}{2} = \underline{1}$$

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2) If cos $A = \frac{1}{2}$ find $\cos(3A)$.

We know,

$\cos(3A) = 4\cos^3 A - 3\cos A$

$= 4.\left(\frac{1}{2}\right)^3 - 3.\left(\frac{1}{2}\right)$

$= 4\left(\frac{1}{8}\right) - \left(\frac{3}{2}\right)$

$= \left(\frac{4}{8}\right) - \left(\frac{3}{2}\right)$

$= \left(\frac{1}{2}\right) - \left(\frac{3}{2}\right)$

$= \frac{1}{2} - \frac{3}{2}$

$= \frac{1-3}{2}$

$= \frac{-2}{2} = -1$

3) If $\sin A = 0.4$ find $\sin(3A)$

$$\sin(3A) = \cancel{3\cos^3}\ 3\sin A - 4\sin^3 A$$
$$= 3.(0.4) - 4(0.4)^3$$
$$= 1.2 - 0.256$$
$$= 0.944$$

4) If $\sin A = 0.4$ find $\cos(3A)$

$$\cos(3A) = 4.\cos^3 A - 3\cos A$$
$$= 4.(0.4)^3 - 3(0.4)$$
$$= 0.256 - 1.2$$
$$= -0.944$$

$\cos(2A)$

$$\cos(2A) = 1 - 2\sin^2 A$$
$$= 1 - 2(0.4)^2$$
$$= \underline{\underline{-0.32}}$$

5) If $\sin A = \frac{3}{5}$ find $\sin(2A)$ and $\cos(2A)$

$$\sin(2A) = 2\sin A \cdot \cos A$$
$$= 2 \times \frac{3}{5} \times \frac{4}{5}$$
$$= \underline{\frac{24}{25}}$$

$$\cos A = \sqrt{1 - \sin^2 A}$$
$$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$$
$$= \sqrt{1 - \frac{9}{25}}$$
$$= \sqrt{\frac{25 - 9}{25}}$$
$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\cos(2A) = 1 - 2\sin^2 A$$
$$= 1 - 2\left(\frac{3}{2}\right)^2$$
$$= 1 - 2\left(\frac{9}{25}\right)$$
$$= 1 - \frac{18}{25}$$
$$= \frac{25 - 18}{25}$$
$$= \frac{7}{25}$$

If $\tan\left[\frac{\alpha}{2}\right] = \sqrt{3}$ find $\cos\alpha$ & $\sin\alpha$

given, $\tan\left[\frac{\alpha}{2}\right] = \sqrt{3}$

$\therefore \tan\left[\frac{\alpha}{2}\right] = \tan 30^\circ$

$\therefore \frac{\alpha}{2} = 30^\circ$

$\therefore \alpha = 60^\circ$

$\therefore \cos\alpha = \cos 60^\circ = \frac{1}{2}$

& $\sin\alpha = \sin 60^\circ = \cancel{\frac{\sqrt{3}}{\sqrt{2}}}\ \frac{\sqrt{3}}{2}$

$\tan\left[\frac{\alpha}{2}\right] = \tan 60^\circ$

A

7) If $\tan\frac{\theta}{2} = t$ find $\sin\theta$

$$\sin\theta = \frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}$$

$$= \frac{2 \times t}{1+t^2}$$

$$= \frac{2t}{1+t^2}$$

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B) find tan A if $\tan\left(\frac{A}{2}\right) = 0.6$.

we know $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\therefore \cancel{\tan(A/2) = 2\tan}$

$$\therefore \tan(A) = \frac{2\tan(A/2)}{1-\tan^2(A/2)}$$

$$= \frac{2 \times (0.6)}{1-(0.6)^2}$$

$$= \frac{1.2}{1-0.36}$$

$$= \frac{1.2}{0.64}$$

$$= 1.875.$$

Result.

1) $\sin[A+B] \cdot \sin[A-B] = \sin^2 A - \sin^2 B$

2) $\cos[A+B] \cdot \cos[A-B] = \cos^2 A - \sin^2 B$

Q.1 Prove that, $\dfrac{\sin\theta}{1+\cos\theta} = \tan\dfrac{\theta}{2}$

formula → $\sin\theta = 2\sin\dfrac{\theta}{2} \cdot \cos\dfrac{\theta}{2}$

$1+\cos\theta = 2\cos^2(\theta/2) \rightarrow \boxed{\cos^2(\theta/2) = \dfrac{1+\cos\theta}{2}}$

$1-\cos\theta = 2\sin^2(\theta/2)$

L.H.S.

$\therefore \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{\cancel{2}\sin(\theta/2) \cdot \cos(\theta/2)}{\cancel{2} \cdot \cos^2(\theta/2)}$

$= \dfrac{\sin(\theta/2) \cdot \cancel{\cos(\theta/2)}}{\cos(\theta/2) \cdot \cancel{\cos(\theta/2)}}$

$= \tan(\theta/2)$

$=$ R.H.S.

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Q.2 If $\tan\left(\frac{\alpha}{2}\right) = \frac{1}{\sqrt{3}}$, find $\cos\alpha$, $\sin\alpha$, $\tan\alpha$, $\cot\alpha$, $\sec\alpha$, cosec

given:- $\tan\left(\frac{\alpha}{2}\right) = \frac{1}{\sqrt{3}}$

$\therefore \tan\left(\frac{\alpha}{2}\right) = \tan 30^\circ$

$\therefore \frac{\alpha}{2} = 30^\circ \qquad \therefore \alpha = 60^\circ$

$\therefore \sin 60^\circ = \sqrt{3}/2$

$\therefore \cos 60^\circ = 1/2$

$\therefore \tan 60^\circ = \sqrt{3}$

$\therefore \cot 60^\circ = 1/\sqrt{3}$

$\therefore \sec 60^\circ = 2$

$\therefore \operatorname{cosec} 60^\circ = 2/\sqrt{3}$

Q.3 If $\tan(\theta/2) = 2/3$

find value of $2\sin\theta + 3\cos\theta$

L.H.S.

$2\sin\theta + 3\cos\theta$

$= 2\left[\frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}\right] + 3\left[\frac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}\right]$

$= \frac{[4(2/3)]}{[1+(2/3)^2]} + \frac{3[1-(2/3)^2]}{[1+(2/3)^2]}$

$= \frac{8/3}{1+(4/9)} + \frac{3(1-4/9)}{1+(4/9)}$

$= \frac{8/3}{\frac{9+4}{9}} + \frac{3\left[\frac{9-4}{9}\right]}{\frac{9+4}{9}} = \frac{8/3}{13/9} + \frac{3(5)/9}{13/9}$

$= \left[\frac{8}{3}\times\frac{9}{13}\right] + \left[\frac{15}{9}\times\frac{9}{13}\right] = \frac{24}{13} + \frac{15}{9} = \frac{39}{13} = \underline{3}$

Prove that

$$\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta} = \tan\left[\frac{\theta}{2}\right]$$

given L.H.S,

$$= \frac{(1-\cos\theta)+\sin\theta}{(1+\cos\theta)+\sin\theta}$$

$$= \frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}$$

$$= \frac{\left(2\cdot\sin\frac{\theta}{2}\right)\cdot\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}{\left(2\cdot\cos\frac{\theta}{2}\right)\cdot\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$$= \frac{2\sin\theta/2}{2\cos\theta/2} = \tan\frac{\theta}{2} = R.H.S$$

Factorisation and defactorisation formulae.

→ Factorisation

1. $\sin[A+B] = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin[A-B] = \sin A \cdot \cos B - \cos A \cdot \sin B$

$+$ $\qquad$ $(+)$

$$\sin[A+B] + \sin[A-B] = 2\sin A \cdot \cos B \quad \text{—— (1)}$$

2. $\sin[A+B] = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin[A-B] = \sin A \cdot \cos B - \cos A \cdot \sin B$

$-$ $\qquad$ $-$ $\qquad$ $+$

$$\sin[A+B] - \sin[A-B] = 2\cos A \cdot \sin B \quad \text{—— (2)}$$

3. $\cos[A+B] = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos[A-B] = \cos A \cdot \cos B + \sin A \cdot \sin B$

$+$ $\qquad$ $(+)$ $\qquad$ $(+)$

$$\cos[A+B] + \cos[A-B] = 2\cos A \cdot \cos B \quad \text{—— (3)}$$

4. $\cos[A+B] = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos[A-B] = \cos A \cdot \cos B + \sin A \cdot \sin B$

$-$ $\qquad$ $(-)$ $\qquad$ $(-)$

$$\cos[A+B] - \cos[A-B] = -2\sin A \cdot \sin B \quad \text{—— (4)}$$

Defactorisation

let $A + \cancel{B} = C$ $\quad \therefore A = \frac{C+D}{2}$

$A - \cancel{B} = D$

$2A = C + D$

Again,

$\cancel{A} + B = C$ $\quad \therefore B = \frac{C-D}{2}$

$\cancel{A} - B = D$

$2B = C - D$

& now from ①

5. $\sin[A+B] + \sin[A-B] = 2\sin A \cdot \cos B$

$\therefore \sin(C) + \sin(D) = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

6. $\sin[A+B] - \sin[A-B] = 2\cos A \cdot \sin B$

$\therefore \sin(C) - \sin(D) = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

7. $\cos[A+B] + \cos[A-B] = 2\cos A \cdot \cos B$

$\therefore \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

8. $\cos[A+B] - \cos[A-B] = -2\sin A \cdot \sin B$.

$\therefore \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$\rightarrow \overset{C}{\underset{(A+B)}{S}} + \overset{D}{\underset{(A-B)}{S}} = 2\,\overset{\frac{C+D}{2}}{\underset{A}{S}} \cdot \overset{\frac{C-D}{2}}{\underset{B}{C}} \leftarrow$

$S - S = 2\,C \cdot S$

$C + C = 2\,C \cdot C$

$C - C = -2\,S \cdot S$

find

1. $2\sin 70^\circ \cdot \cos 30^\circ = \sin[70^\circ + 30^\circ] + \sin[70^\circ - 30^\circ]$

$= \sin[100^\circ] + \sin[50^\circ]$

2. $2\cos 75^\circ \cdot \cos 15^\circ = \cos[A+B] + \cos[A-B]$

$= \cos[75+15] + \cos[75-15]$

$= \cos[90^\circ] + \cos[60^\circ]$

$= 0 + \frac{1}{2}$

$= \frac{1}{2}$

3. $2\sin 40^\circ \cdot \cos 10^\circ = \sin[40+10] + \sin[40-10]$

$= \sin[50^\circ] + \sin[30^\circ]$

$= \sin 50^\circ + \frac{1}{2}$

4. $2\cos 40^\circ \cdot \cos 50^\circ = \cos[40^\circ + 50^\circ] + \cos[40^\circ - 50^\circ]$

$= \cos[90^\circ] + \cos[-10^\circ]$

$= \cos 90^\circ + \cos(10^\circ) \ldots \cos(-\theta) = \cos\theta$

$= 0 + \cos(10^\circ)$

$= \cos 10^\circ$

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Expand

1. $\sin 100^\circ + \sin 50^\circ = 2\sin\left[\frac{100+50}{2}\right] \cdot \cos\left[\frac{100-50}{2}\right]$

$= 2 \cdot \sin\left[\frac{150}{2}\right] \cdot \cos\left[\frac{50}{2}\right]$

$= 2 \cdot \sin(75^\circ) \cdot \cos(25^\circ)$

2. $\sin 80^\circ - \sin 70^\circ = 2 \cdot \cos\left(\frac{80^\circ+70^\circ}{2}\right) \cdot \sin\left(\frac{80^\circ-70^\circ}{2}\right)$

$= 2 \cdot \cos\left(\frac{150}{2}\right) \cdot \sin\left(\frac{10}{2}\right)$

$= 2 \cdot \cos(75^\circ) \cdot \sin(5^\circ)$

3. $\cos 40^\circ - \cos(-10)^\circ = -2\cos\left(\frac{40^\circ+10}{2}\right) \cdot \sin\left(\frac{40^\circ-10^\circ}{2}\right)$

$\cos 40^\circ - \cos 10^\circ =$

$= -2\cos\left(\frac{50}{2}\right) \cdot \sin\left(\frac{30}{2}\right)$

$= -2\cos(25^\circ) \cdot \sin(15^\circ)$

4. $\sin 99^\circ - \sin 81^\circ = 2 \cdot \cos\left(\frac{99^\circ+81}{2}\right) \cdot \sin\left(\frac{99^\circ-81^\circ}{2}\right)$

$= 2 \cdot \cos\left(\frac{180}{2}\right) \cdot \sin\left(\frac{18}{2}\right)$

$= 2 \cdot \cos(90^\circ) \cdot \sin(9^\circ)$

$= 2 \cdot \cos(0) \cdot \sin(9^\circ)$

$= 2 \times 0 \times \sin 9^\circ$

$= 0$

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5. find $\sin(9A) + \sin(5A) = 2\cos\left[\frac{9A+5A}{2}\right] \cdot \cos\left[\frac{9A-5A}{2}\right]$

$= 2\cos\left[\frac{14A}{2}\right] \cdot \cos\left[\frac{4A}{2}\right]$

$= 2\cos[7A] \cdot \cos[2A]$

6. $\sin(7\theta) - \sin(5\theta) = 2\cos\left[\frac{7\theta+5\theta}{2}\right] \cdot \sin\left[\frac{7\theta-5\theta}{2}\right]$

$= 2\cos\left[\frac{12\theta}{2}\right] \cdot \sin\left[\frac{2\theta}{2}\right]$

$= 2\cos 6\theta \cdot \sin\theta$

7. $\sin 50^\circ + \sin 80^\circ$

$= \cos[90-50] + \cos 80^\circ$ ..... $\boxed{\sin(90-\theta) = \cos\theta, \ \cos(90-\theta) = \sin\theta}$

$= \cos 40^\circ + \cos 80^\circ$

$= 2\cos\left[\frac{40+80}{2}\right] \cdot \cos\left[\frac{40-80}{2}\right]$

$= 2\cos\left[\frac{120}{2}\right] \cdot \cos\left[\frac{-40}{2}\right]$

$= 2\cos 60^\circ \cdot \cos(-20^\circ)$

$= 2 \times \frac{1}{2} \times \cos(20^\circ)$

$= \cos 20^\circ$

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8) $\sin 70^\circ - \cos 40^\circ$

$\cos[90^\circ - 70^\circ] - \cos 40^\circ$

$= \cos 20^\circ - \cos 40^\circ$

$= -2\sin\left[\frac{20^\circ + 40^\circ}{2}\right] \cdot \sin\left[\frac{20^\circ - 40^\circ}{2}\right]$

$= -2\sin\left[\frac{60}{2}\right] \cdot \sin\left[\frac{-20}{2}\right]$

$= -2\sin[30^\circ] \cdot \sin[-10]$

$= -\cancel{2}\ \cancel{\sin}\ \frac{1}{\cancel{2}} \cdot \sin[-10]$

$= \sin 10^\circ$

9) $\frac{\sin 8\theta + \sin 2\theta}{\cos 8\theta + \cos 2\theta} = \frac{2 \cdot \sin\left[\frac{8\theta + 2\theta}{2}\right] * \cos\left[\frac{8\theta - 2\theta}{2}\right]}{2 \cdot \cos\left[\frac{8\theta + 2\theta}{2}\right] * \cos\left[\frac{8\theta - 2\theta}{2}\right]}$

$= \frac{\cancel{2} \cdot \sin\left[\frac{10\theta}{2}\right] \times \cos\cancel{\left[\frac{6\theta}{2}\right]}}{\cancel{2} \cdot \cos\left[\frac{10\theta}{2}\right] \times \cos\cancel{\left[\frac{6\theta}{2}\right]}}$

$= \frac{\sin 5\theta}{\cos 5\theta}$

$= \tan 5\theta$

10) $\dfrac{\sin 3A - \sin A}{\cos 3A + \cos A} = \tan A$

$$\therefore \text{L.H.S} = \frac{2 \cdot \cos\left[\dfrac{3A+A}{2}\right] \cdot \sin\left[\dfrac{3A-A}{2}\right]}{2 \cdot \cos\left[\dfrac{3A+A}{2}\right] \cdot \cos\left[\dfrac{3A-A}{2}\right]}$$

$$= \frac{2 \cdot \cos\left[\dfrac{4A}{2}\right] \cdot \sin\left[\dfrac{2A}{2}\right]}{2 \cdot \cos\left[\dfrac{4A}{2}\right] \cdot \cos\left[\dfrac{2A}{2}\right]}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$= \text{R.H.S.}$$

11) $\dfrac{\sin 5A + \sin 3A}{\cos 5A + \cos 3A} = \tan 4A$

$$\text{L.H.S} = \frac{2 \cdot \sin\left[\dfrac{5A+3A}{2}\right] \cdot \cos\left[\dfrac{5A-3A}{2}\right]}{2 \cdot \cos\left[\dfrac{5A+3A}{2}\right] \cdot \cos\left[\dfrac{5A-3A}{2}\right]}$$

$$= \frac{2 \cdot \sin\left[\dfrac{8A}{2}\right] \cdot \cos\left[\dfrac{2A}{2}\right]}{2 \cdot \cos\left[\dfrac{8A}{2}\right] \cdot \cos\left[\dfrac{2A}{2}\right]}$$

$$= \frac{\sin 4A}{\cos 4A}$$

$$= \tan 4A$$

12) $\dfrac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \dfrac{1}{\sqrt{3}}$

L.H.S

$$\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \frac{\cancel{2} \cdot \cancel{\cos\left[\frac{75+15}{2}\right]} \cdot \sin\left[\frac{75-15}{2}\right]}{\cancel{2} \cdot \cancel{\cos\left[\frac{75+15}{2}\right]} \cdot \cos\left[\frac{75-15}{2}\right]}$$

$$= \frac{\sin\left(\frac{60}{2}\right)}{\cos\left(\frac{60}{2}\right)}$$

$$= \frac{\sin 30^\circ}{\cos 30^\circ}$$

$$= \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}}$$

$=$ R.H.S.

13) $\dfrac{\cos 20^\circ - \cos 70^\circ}{\sin 70^\circ - \sin 20^\circ} = 1$

L.H.S $= \dfrac{-2 \sin\left[\dfrac{20+70}{2}\right] \cdot \sin\left[\dfrac{20-70}{2}\right]}{2 \cdot \cos\left[\dfrac{20+70}{2}\right] \cdot \sin\left[\dfrac{70-20}{2}\right]}$

$= \dfrac{-2 \sin\left[\dfrac{90}{2}\right] \cdot \sin\left[\dfrac{-50}{2}\right]}{2 \cdot \cos\left[\dfrac{90}{2}\right] \cdot \sin\left[\dfrac{50}{2}\right]}$

$= \dfrac{-\cancel{2} \sin 45^\circ \cdot \sin(-25)}{\cancel{2} \cdot \cos 45^\circ \cdot \sin[25]}$

$= -\dfrac{\sin 45^\circ \cdot \sin(-25)}{\cos 45^\circ \cdot \sin(25)}$

$= \cancel{\tan 45^\circ}$

$= \dfrac{-(1/\sqrt{2}) \cdot [-\sin 25^\circ]}{(1/\sqrt{2}) \cdot [\sin 25^\circ]}$

$= \dfrac{\cancel{(1/\sqrt{2})} \cdot \cancel{[\sin 25^\circ]}}{\cancel{(1/\sqrt{2})} \cdot \cancel{[\sin 25^\circ]}}$

$= 1$

$=$ R.H.S

14) $\dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$

L.H.S.

15)* Simplify

$$\sin[135^\circ - A] \cdot \sin[135^\circ + A] = \sin^2(135^\circ) - \sin^2(A) \quad \text{--- (1)}$$

$$\sin^2(135) = [\sin(135^\circ)]^2$$
$$= [\sin(180^\circ - 45^\circ)]^2$$
$$= [\sin(\pi - 45^\circ)]^2$$
$$= [\sin(45^\circ)]^2$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

Putting $\frac{1}{2}$ in (1)

$\therefore \frac{1}{2} - \sin^2 A$

$\therefore$

16) Show that

$$\frac{\sin 19^\circ + \cos 11^\circ}{\cos 19^\circ - \sin 11^\circ} = \sqrt{3}$$

L.H.S

$$\frac{\sin 19^\circ + \cos 11^\circ}{\cos 19^\circ - \sin 11^\circ}$$

$$\frac{\cos[90^\circ - 19^\circ] + \cos 11^\circ}{\sin[90^\circ - 19^\circ] - \sin 11^\circ}$$

$$\frac{\cos 71^\circ + \cos 11^\circ}{\sin 71^\circ - \sin 11^\circ} = \frac{\cancel{2}\cos\cancel{\left[\frac{71+11}{2}\right]} \cdot \cos\left[\frac{71-11}{2}\right]}{\cancel{2}\cancel{\cos}\cancel{\left[\frac{71+11}{2}\right]} \cdot \sin\left[\frac{71-11}{2}\right]}$$

$$= \frac{\cos\left[\frac{60}{2}\right]}{\sin\left[\frac{60}{2}\right]}$$

$$= \frac{\cos 30}{\sin 30}$$

$$= \cot 30^\circ$$

$$= \frac{\sqrt{3}}{1}$$

$$= \sqrt{3}$$

$$= \underline{R.H.S}$$

17) find value of α & β is $\cos\frac{\pi}{5} - \cos\frac{3\pi}{5} = 2\sin\alpha \cdot \sin\beta$

$$\cos\frac{\pi}{5} - \cos\frac{3\pi}{5} = -2\sin\left[\frac{\frac{\pi}{5}+\frac{3\pi}{5}}{2}\right]\cdot\sin\left[\frac{\frac{\pi}{5}-\frac{3\pi}{5}}{2}\right]$$

$$= -2\sin\left[\frac{\frac{4\pi}{5}}{\frac{2}{1}}\right]\cdot\sin\left[\frac{\frac{-2\pi}{5}}{\frac{2}{1}}\right]$$

$$= -2\sin\left[\frac{4\pi}{5}\times\frac{1}{2}\right]\cdot\sin\left[\frac{-2\pi}{5}\times\frac{1}{2}\right]$$

$$= -2\sin\left[\frac{2\pi}{5}\right]\cdot\sin\left[\frac{-\pi}{5}\right]$$

$$= -2\sin\left[\frac{2\pi}{5}\right]\cdot\left[-\sin\frac{\pi}{5}\right]$$

$$= 2\sin\frac{2\pi}{5}\cdot\sin\frac{\pi}{5}.$$

$\therefore$ $\alpha = \frac{2\pi}{5}$ & $\beta = \frac{\pi}{5}$.

* Inverse Trignometric Ratios.

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If $\sin\theta = t$

then, $\theta = \sin^{-1}(t)$

Read as 'sin' inverse of 't'

* $\sin^{-1}(t) \neq \frac{1}{\sin t}$

$\therefore \frac{1}{\sin t} = \operatorname{cosec} t$.

Similarly $\cos^{-1}(t)$, $\sec^{-1}(t)$, $\tan^{-1}(t)$, ... called Inverse Trignometric Ratios.

- Principal value of Inverse function

1. $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

i.e. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ, 405, 585^\circ, \ldots\ldots$

Principal

Thus the principle value of $\sin^{-1}\left[\frac{1}{\sqrt{2}}\right] = 45^\circ$ or $\frac{\pi}{4}$

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* Properties.

1] 1. $\sin^{-1}[\sin x] = x$

let

$\sin x = y$

$\therefore x = \sin^{-1} y$

Put $y = \sin x$.

$\therefore x = \sin^{-1}(\sin x)$

or $\sin^{-1}(\sin x) = x$

Similarly,

2. $\cos^{-1}[\cos x] = x$

3. $\tan^{-1}[\tan x] = x$

4. $\cot^{-1}[\cot x] = x$

5. $\operatorname{cosec}^{-1}[\operatorname{cosec} x] = x$

6. $\sec^{-1}[\sec x] = x$

2] 1. $\sin[\sin^{-1} x] = x$

let, $\sin^{-1} = y$

$\therefore x = \sin y$

Put $y = \sin^{-1} x$

$\therefore x = \sin[\sin^{-1} x]$

or $\sin[\sin^{-1} x] = x$

Similarly;

$\tan[\tan^{-1} x] = x$

$\cot[\cot^{-1} x] = x$

$\cos[\cos^{-1} x] = x$

$\operatorname{cosec}[\operatorname{cosec}^{-1} x] = x$

$\sec[\sec^{-1} x] = x$

1. $\sin^{-1}(-x) = -\sin^{-1} x$

let $\sin^{-1}(-x) = y$

$\therefore \quad -x = \sin y$

$\therefore \quad x = -\sin y$

We know, $\sin(-\theta) = -\sin\theta$

$\therefore \quad x = \sin(-y)$

$\therefore \quad \sin^{-1}(x) = -y$

$\therefore \quad -\sin^{-1}(x) = y$

Put y from ①

$\therefore \quad -\sin^{-1}(x) = \sin^{-1}(-x)$

$\therefore \quad \sin^{-1}(-x) = -\sin^{-1}(x)$

similarly,

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\tan^{-1}(-x) = \cancel{\pi} - \tan^{-1}(x)$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$

$\sin(-\theta) = -\sin\theta$

$\sin^{-1}(-\theta) = -\sin^{-1}\theta$

$\cos(-\theta) = \cos\theta$

$\cos^{-1}(-\theta) = \pi - \cos^{-1}(\theta)$

$\tan(-\theta) = -\tan\theta$

$\tan^{-1}(\theta) = -\tan^{-1}\theta$

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2) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ or 90°

let, $\sin^{-1}x = y$

$\therefore x = \sin y$

$\therefore x = \cos(90 - y)$

~~$\therefore \sin$~~

$\therefore \cos^{-1}x = 90 - y$

$\therefore y + \cos^{-1}x = 90$

$\therefore \sin^{-1}x + \cos^{-1}x = 90^\circ$

or

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\therefore$ Proved

Similarly,

$\tan^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

3) $\sin^{-1} x = \operatorname{cosec}^{-1} \frac{1}{x}$

let,

$\sin^{-1} x = y$ —— ①

$\therefore x = \sin y$

take Reciprocal

$\therefore \frac{1}{x} = \frac{1}{\sin y}$

$\therefore \frac{1}{x} = \operatorname{cosec} y$

$\therefore \operatorname{cosec}^{-1}\left(\frac{1}{x}\right) = y$

Put y from ①

$\therefore \operatorname{cosec}^{-1}\left(\frac{1}{x}\right) = \sin^{-1} x$

Similarly,

$\tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right)$

$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$

$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$

$\cos^{-1}(x) = \sec^{-1}\left(\frac{1}{x}\right)$

$\operatorname{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$

4) $\tan^{-1} x + \tan^{-1} y = \tan\left[\frac{x+y}{1-xy}\right]$

If $xy < 1$

but,

If $xy > 1$ then,

$\tan^{-1}(x) + \tan^{-1}(y) = \pi + \tan^{-1}\left[\frac{x+y}{1-xy}\right]$

* 5) $\sin 30° = \frac{1}{2} \Rightarrow \sin^{-1}\left(\frac{1}{2}\right) = 30°$

$\sin 45° = \frac{1}{\sqrt{2}} \Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

find principal value of

1) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = -45°$ or $-\frac{\pi}{4}$

2) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi - \cos^{-1}(\cos 30°)$

$= \pi - 30° = 180° - 30° = 150°$

3) $\tan^{-1}[-1] = \tan^{-1}(-x) = -\tan^{-1} x$

$= -\tan^{-1}(1)$

$= -\tan^{-1}[\tan 45°]$

$= -45° = -\frac{\pi}{4}$.

4) $\cot^{-1}(\sqrt{3}) = \cot^{-1}(\cot 30°)$

$= 30° = \frac{\pi}{6}$

1. $\sin\left[\cos^{-1}\left(\frac{-1}{2}\right)\right]$

$\because \cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$= \sin\left[\pi - \cos^{-1}(1/2)\right]$

$= \sin\left[\pi - \cos^{-1}(\cos 60^\circ)\right]$

$= \sin\left[180^\circ - 60^\circ\right]$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

2) $\sin\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{2} - \cos^{-1}(\cos 60^\circ)\right]$

$= \sin\left[90^\circ - 60^\circ\right]$

$= \sin 30^\circ$

$= \frac{1}{2}$.

3) $\sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right]$

let $\cos^{-1}\left(\frac{5}{13}\right) = x$ — ①

$\therefore \frac{5}{13} = \cos x$

$\therefore \cancel{\frac{5}{13}} \sin x = \frac{12}{13}$

$\therefore x = \sin^{-1}\left(\frac{12}{13}\right)$

Put in ①

$\therefore \cos^{-1}\left(\frac{5}{13}\right) = \sin^{-1}\left(\frac{12}{13}\right)$

Put in given,
$\therefore \sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right]$

$\therefore \sin\left[\sin\left(\sin^{-1}\left(\frac{12}{13}\right)\right)\right]$

$\because \sin(\sin^{-1}x) = x$

$= \frac{12}{13}$

$\therefore \sin^{-1}\left[\frac{3}{5}\right] + \cos^{-1}\left[\frac{3}{5}\right]$

$\therefore \sin^{-1}x^{2} + \cos^{-1}x = \frac{\pi}{2}$

$\therefore \sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$

- Find value of :-

1. $\sin^{-1}[\cos(120^\circ)] = \sin^{-1}(\sin(90^\circ - 120^\circ))$
$= 90^\circ - 120^\circ$
$= -30^\circ$

2. $\sin[\cos^{-1}(12/13)]$ = let $\cos^{-1}\left(\frac{12}{13}\right) = \alpha$ — ①

$\therefore \cos\alpha = \frac{12}{13}$

$\therefore y^2 + 12^2 = 13^2$

$\therefore y^2 = 169 - 144$

$\therefore y^2 = 25$

$\therefore y = 5$

$\sin\alpha = \frac{5}{13}$ $\therefore \alpha = \sin^{-1}\left(\frac{5}{13}\right)$

put α in ①

$\cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{5}{13}\right)$ $= \sin\left(\sin^{-1}\left(\frac{5}{13}\right)\right) = \frac{5}{13}$

3) $\cos\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)\right] =$

4) $\cos^{-1}(\sin 60^\circ) =$

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1) find x if $\tan^{-1}(1) + \tan^{-1}(x) = 0$

$$\tan^{-1} + \tan^{-1}(x) = 0$$

$$\tan^{-1}\left[\frac{1+x}{1-(1 \times x)}\right] = 0$$

$$\therefore \frac{1+x}{1-x} = \tan^{-1}(0)$$

$$\therefore \frac{1+x}{1-x} = 0$$

$$\therefore 1+x = 0$$

$$\therefore x = -1$$

2) find value of x if $\sin^{-1}\left(\frac{2}{3}\right) = \tan^{-1} x$

~~(i)~~ $\frac{2}{\sqrt{5}}$ (ii) $\frac{-2}{\sqrt{5}}$ (iii) $\frac{\sqrt{5}}{2}$

3) Value of $\tan^{-1}\left(\frac{1}{11}\right) + \tan^{-1}\left(\frac{5}{6}\right) = \ldots\ldots$

(i) $\frac{\pi}{6}$, ~~(ii)~~ $\frac{\pi}{4}$, (iii) $\frac{\pi}{3}$

3) $\tan^{-1}\left(\frac{1}{11}\right) + \tan^{-1}\left(\frac{5}{6}\right) = \ldots\ldots$

~~(i)~~ $\frac{\pi}{4}$, (ii) $\frac{\pi}{6}$, (iii) $\frac{\pi}{5}$

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left[\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{3+2}{6}}{1 - \frac{1}{6}}\right]$$

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3) $\tan^{-1}\left(\frac{1}{11}\right) + \tan^{-1}\left(\frac{5}{6}\right) = \tan^{-1}\left[\frac{\frac{1}{11} + \frac{5}{6}}{1 - \left(\frac{1}{11} \times \frac{5}{6}\right)}\right]$

$= \tan^{-1}\left[\frac{\frac{6+55}{66}}{1 - \frac{5}{66}}\right]$

$= \tan^{-1}\left[\frac{61}{61}\right]$

$= \tan^{-1}(1)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \frac{\pi}{4}$

4) $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \dots$

$= \tan^{-1}\left[\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}}\right]$

$= \tan^{-1}\left[\frac{\frac{48+72}{264}}{1 - \frac{14}{264}}\right]$

$= \tan^{-1}\left[\frac{125}{250}\right]$

$= \tan^{-1}\left(\frac{1}{2}\right)$

$= \cot^{-1}(2)$

5) $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left[\frac{1}{13}\right] = \ldots\ldots$

$= \tan^{-1}\left[\frac{\frac{1}{7} + \frac{1}{13}}{1 - \frac{1}{7} \times \frac{1}{13}}\right]$

$= \tan^{-1}$

6) Value of $2\tan^{-1}(x)$

$2\tan^{-1} x = \tan^{-1} x + \tan^{-1} x$

$\therefore \quad = \tan^{-1}\left[\frac{x + x}{1 - (x + x)}\right]$

$= \tan^{-1}\left[\frac{2x}{1 - x^2}\right]$

7) $\cos\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right]$

$= \cos\left[\frac{\pi}{2} - \theta\right]$

$= \sin\theta$

$= \sin\left[\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right]$

$= \frac{-1}{\sqrt{2}}$