ELEMENTS OF ELECTRICAL ENGINEERING
(22215)

CHAPTER-2 AC FUNDAMENTALS - 10 M

CO2: Use Single phase AC supply for electrical and electronic equipments

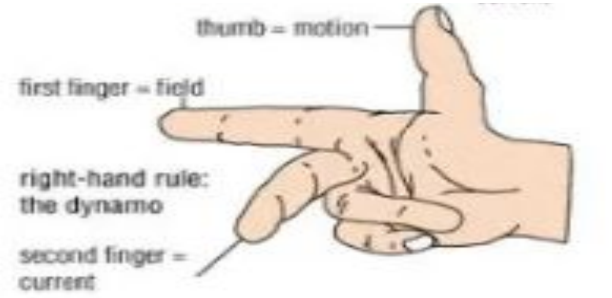
By SAROJ DESAI

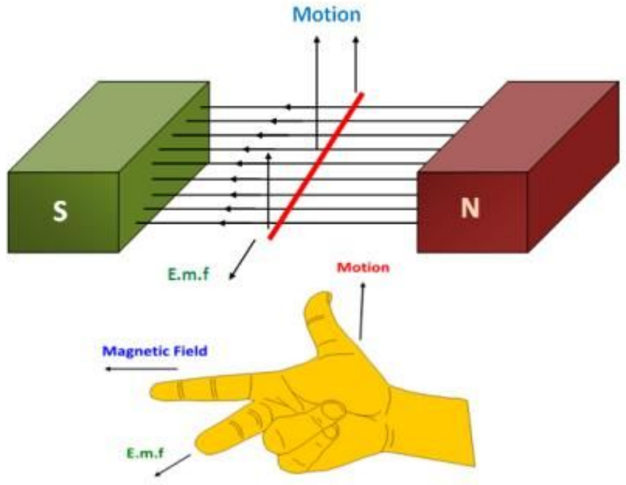
Content

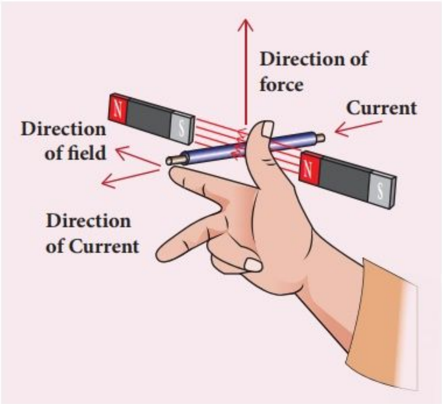
Unit– II AC Fundamentals	2a. Describe the salient features of the given type of power supply. 2b. Represent the given AC quantities by phasors, waveforms and mathematical equations. 2c. Explain the response of the given pure resistive, inductive and capacitive AC circuits with sketches 2d. Calculate the parameters of the given circuit. 2e. Calculate impedance, current, power factor and power of the given AC circuit.	2.1 A.C. and D.C. quantity, advantages of A.C. over D.C. 2.2 Single phase A.C. sinusoidal A.C. wave: instantaneous value, cycle, amplitude, time period, frequency, angular frequency, R.M.S. value, Average value for sinusoidal waveform, Form factor, Peak factor 2.3 Vector representation of sinusoidal A.C. quantity, Phase angle, phase difference, concept of lagging and leading – by waveforms, mathematical equations and phasors 2.4 Pure resistance, inductance and capacitance in A.C. circuit 2.5 R-L and R-C series circuits 2.6 Impedance and impedance triangle 2.7 Power factor and its significance 2.8 Power – active, reactive and apparent, power triangle

PREREQUISITE: FLEMING'S RIGHT HAND RULE

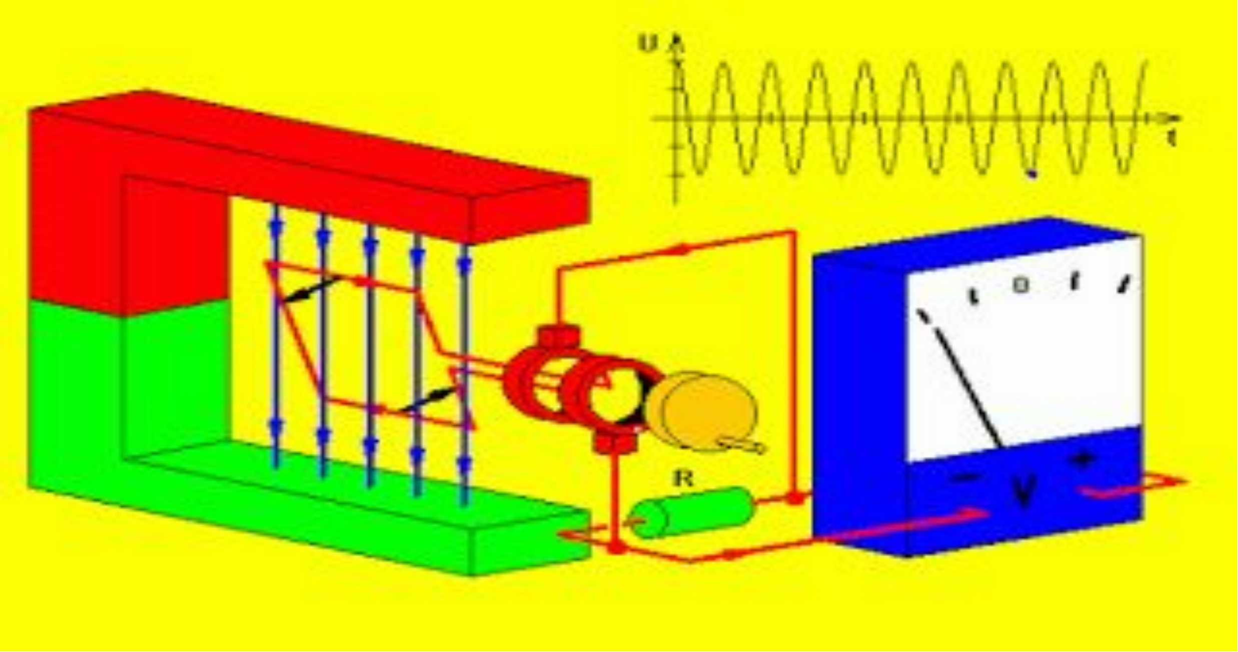
If a conductor is forcefully brought under a magnetic field, there will be an induced current in that conductor. The direction of this force can be found using Fleming’s Right Hand Rule.







DYNAMICALLY INDUCED EMF



Dynamically Induced EMF

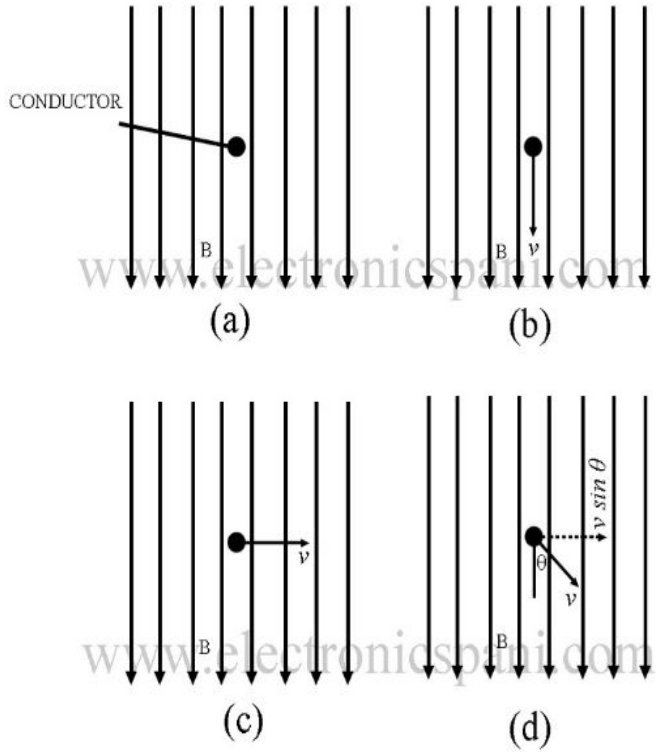
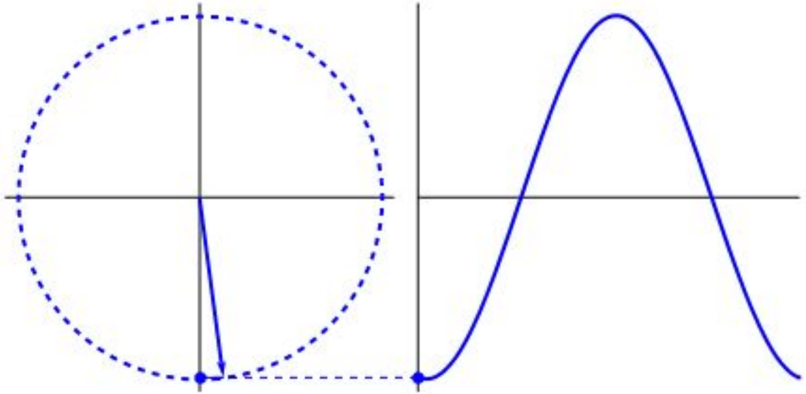


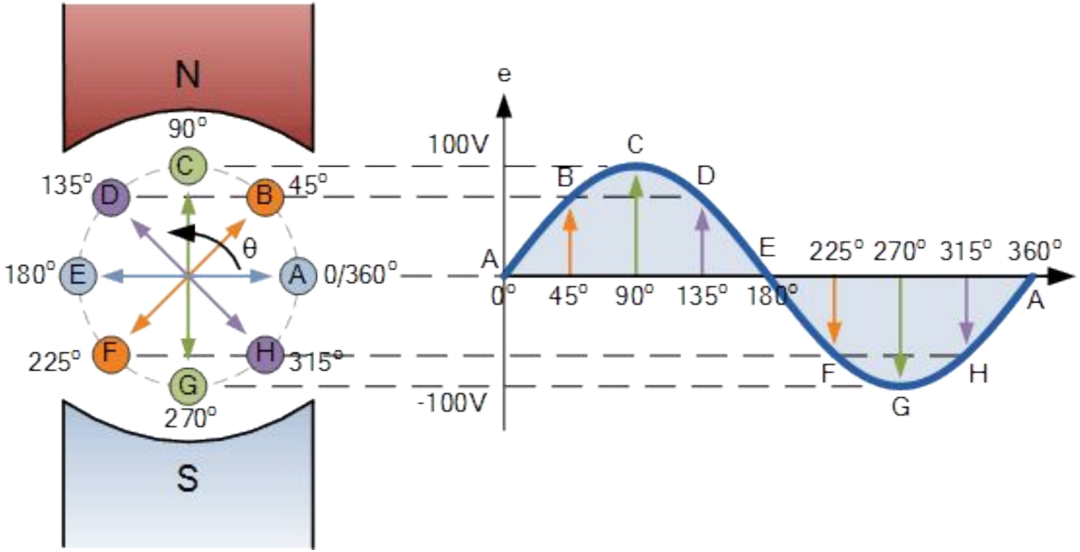
Figure : Conductor in uniform magnetic field

- **Consider a conductor of length *l* meters placed in a uniform magnetic field of density , as shown in Fig.(a).In this case no flux is cut by the conductor, therefore, no emf is induced in it.**
- **if this conductor is moved with Velocity *v* m/s in a direction perpendicular to its own length and perpendicular to the direction of the magnetic field, as shown in Fig.(c) flux is cut by the conductor, therefore, an emf is induced in the conductor.**
- **Area swept per second by the conductor = (l * v)m2/s**
- **Flux cut per second= Flux density * area swept per second**
  = Blv
- **EMF induced = Flux cut per second = Blv**
- **If the conductor moves with velocity as shown in Fig.(d), with the angle Ɵ**
- **The magnitude of emf induced. is proportional to the component of the velocity in a direction perpendicular to the direction of the magnetic field and induced emf is given by**

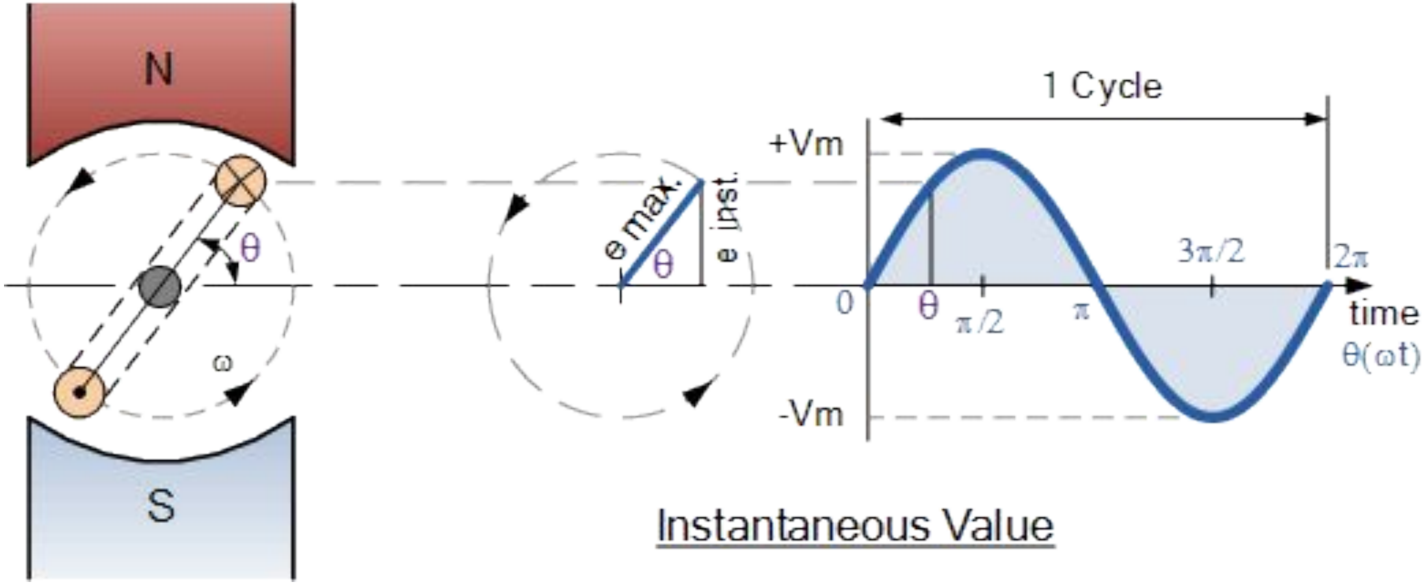
  e= BlvsinƟ volts

AC generation



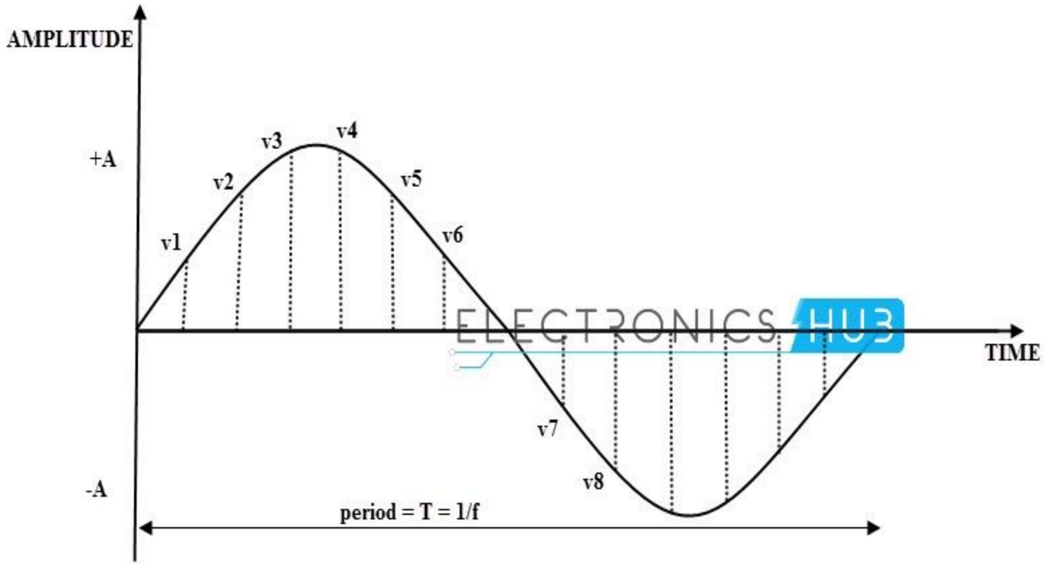


AC generation



Instantaneous Value

2.2. AC wave terminologies



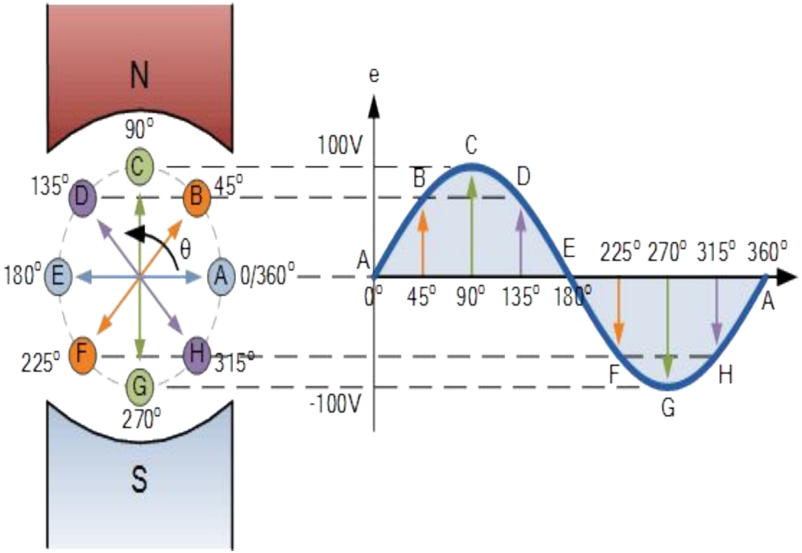
Instantaneous voltage = Maximum voltage x sin θ

Vinst = Vmax x sin θ

AC wave terminologies:

Angle of rotation of coil in a magnetic field is θ = ωt
Where

ω is the angular velocity of the sine wave
t is the time period of the sine wave.



Angular Velocity of Sine Wave

This is the rate of change of angular displacement with respect to time. "Angular velocity" is a measurement of the rate of change of angular position of an object over a period of time. It is denoted by ω.

It is a vector quantity. Units for Angular velocity: RADIANS or degrees

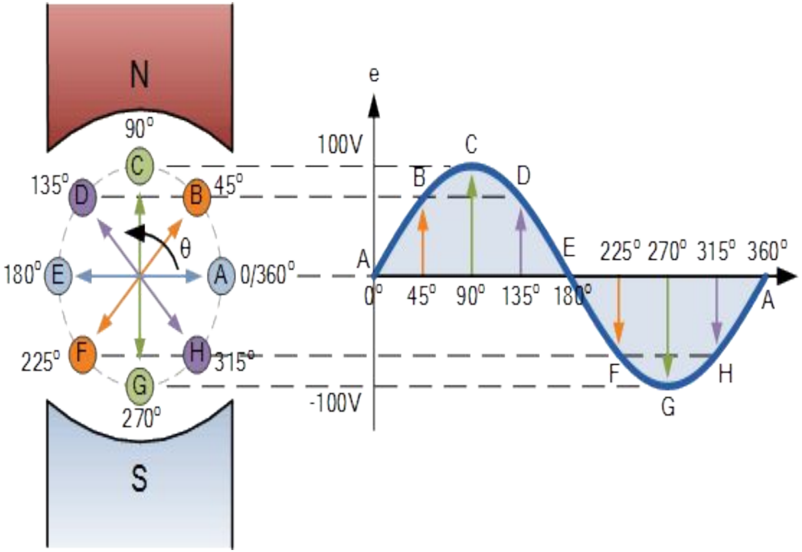
$$\omega = 2\pi f \text{ (rad/s)}$$

AC generation

Relation between Radians and Degree of angle

$$\text{Radians} = \left(\frac{\pi}{180^0}\right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^0}{\pi}\right) \times \text{radians}$$



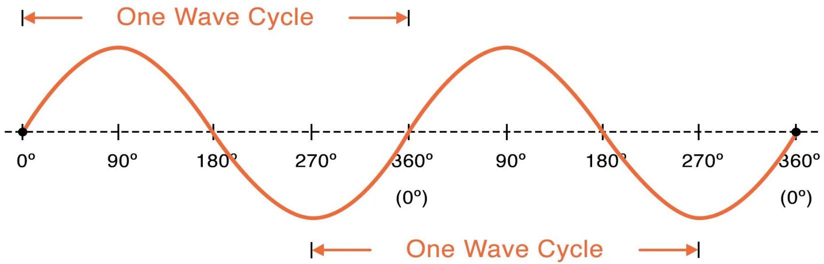
AC wave terminologies:

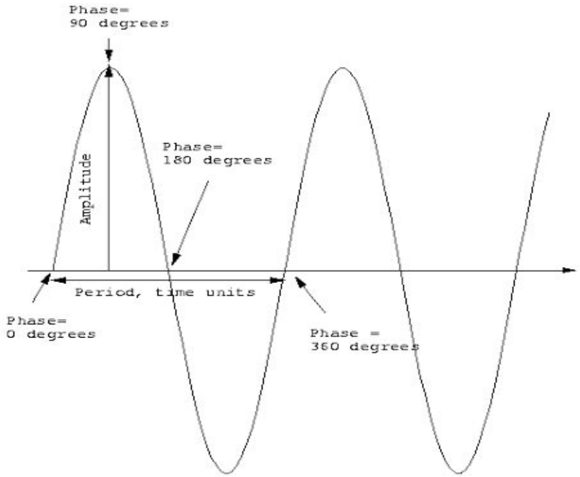
one cycle : one complete set of positive and negative values of alternating quantities is called as **cycle** .

Time period The time taken by the waveform to complete one full cycle is called the **Periodic Time** of the waveform, and is given the symbol **"T".**

Frequency-The number of cycles completed by an alternating quantity in one second (cycles/second) is called as F**requency**, symbol is f & unit is Hertz, (Hz)

Alternation: One half cycle of an alternating quantity is called an alternation**.**

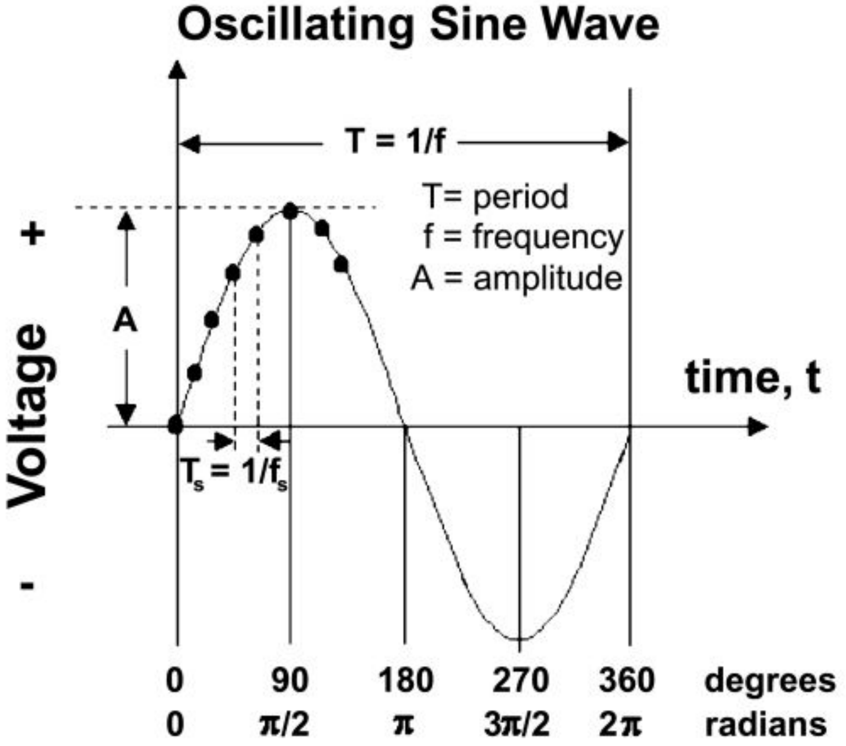




AC wave terminologies:

frequency = 1/ period

$$f = \frac{1}{T}$$



AC wave terminologies:

RMS Value: The effective or RMS value of an alternating current is that steady direct current(d.c.)which when flowing through a given resistance for a given time produces the same amount of heat as produced by alternating current when flowing through the same resistance for same time.

The average value of a.c. is that value of steady current or voltage is that value of steady or D. C. current or voltage which would send the same amount of charge through a circuit for half the time period of a.c. as it sent by the a.c. through the same circuit in same time.

For voltage: **Vavg=0.637 Vm**

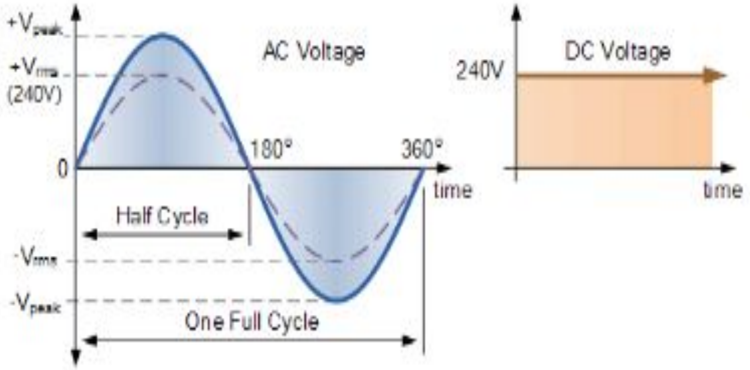
For current :**Iavg=0.637 Im**

$V_{rms}=0.707 \times V_{max}$.

Where, V_{max} is the maximum or peak value of the voltage

$I_{rms}=0.707 \times I_{max}$.

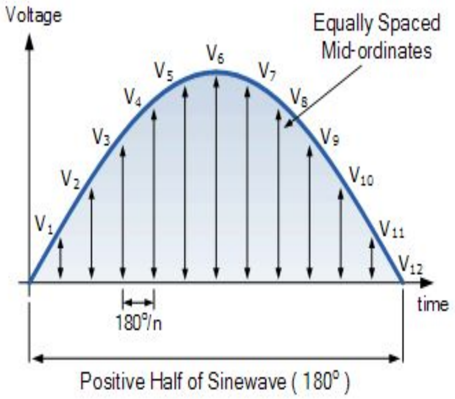
Where, I_{max} is the maximum or peak value of the current



AC wave terminologies: RMS value

The positive half of the waveform is divided up into any number of "n" equal portions or *mid-ordinates* and the more mid-ordinates that are drawn along the waveform, the more accurate will be the final result. The width of each mid-ordinate will therefore be n^o degrees and the height of each mid-ordinate will be equal to the instantaneous value of the waveform at that time along the x-axis of the waveform.

Graphical Method



Each mid-ordinate value of a waveform (the voltage waveform in this case) is multiplied by itself (squared) and added to the next. This method gives us the "square" or **Squared** part of the RMS voltage expression. Next this squared value is divided by the number of mid-ordinates used to give us the **Mean** part of the RMS voltage expression, and in our simple example above the number of mid-ordinates used was twelve (12). Finally, the square root of the previous result is found to give us the **Root** part of the RMS voltage.

Then we can define the term used to describe an rms voltage (V_{RMS}) as being "the square root of the mean of the square of the mid-ordinates of the voltage waveform" and this is given as:

$$V_{RMS} = \sqrt{\frac{\text{sum of mid-ordinate}\,(\text{voltages})^2}{\text{number of mid-ordinates}}}$$

and for our simple example above, the RMS voltage will be calculated as:

$$V_{RMS} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 \ldots + V_{11}^2 + V_{12}^2}{12}}$$

AC wave terminologies:

Form Factor: **Form Factor is the ratio between the average value and the RMS value and is given as.**

For a pure sinusoidal waveform the Form Factor will always be equal to 1.11.

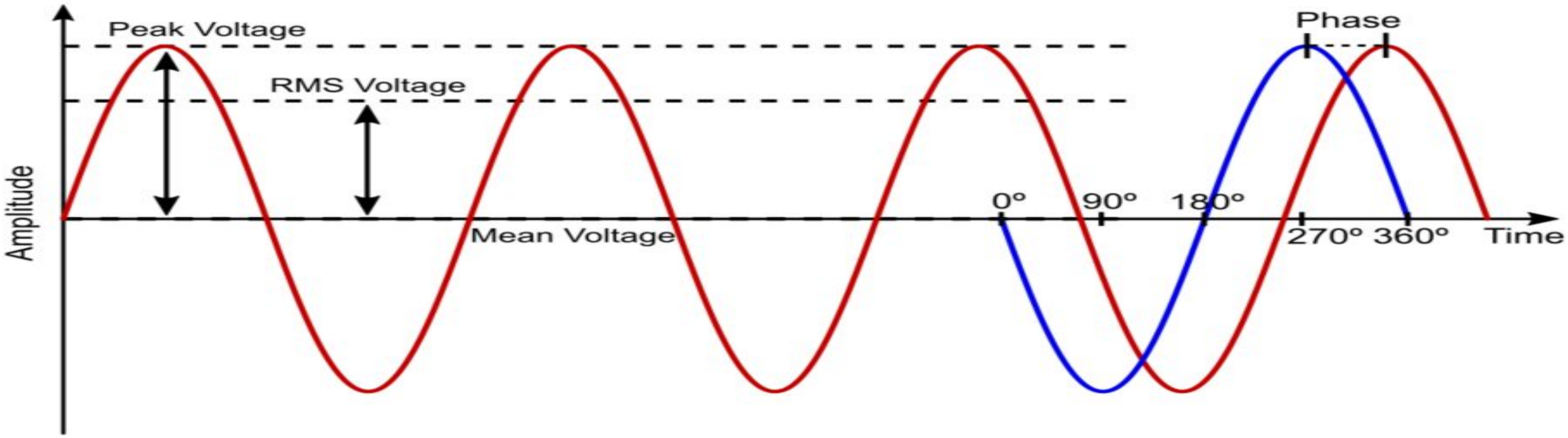
Crest Factor/ Peak factor : Crest Factor is the ratio between the R.M.S. value and the Peak value of the waveform and is given as.

For a pure sinusoidal waveform the Crest Factor will always be equal to 1.414.

$$\text{Crest Factor} = \frac{\text{Peak value}}{\text{R.M.S. value}} = \frac{\text{Vmax}}{0.707 \times \text{Vmax}}$$

$$\text{Form Factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{0.707 \times \text{Vmax}}{0.637 \times \text{Vmax}}$$

2.3 AC wave terminologies:

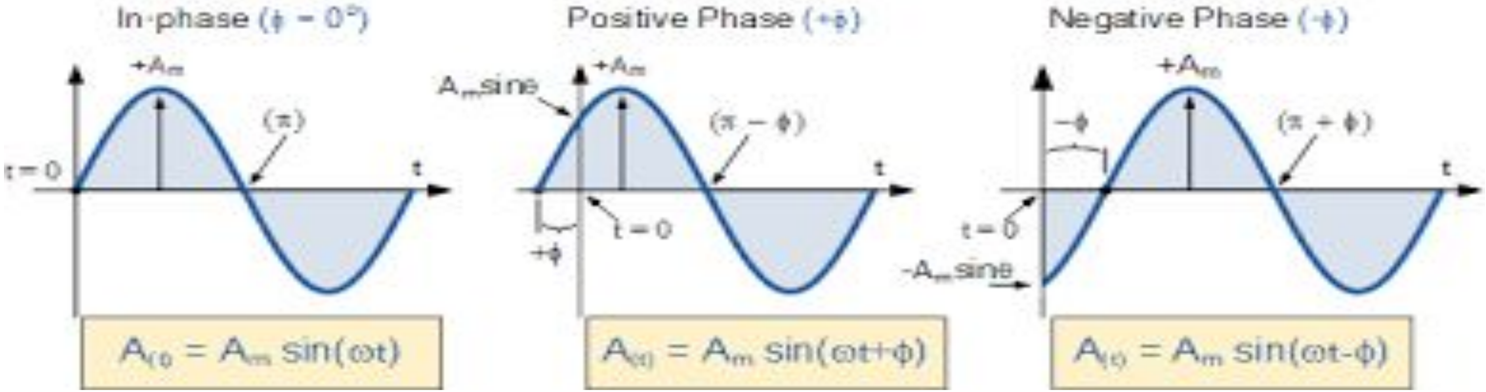


Peak value/ maximum value: It is the maximum value attained by an alternating quantity.

2.3 AC wave terminologies:

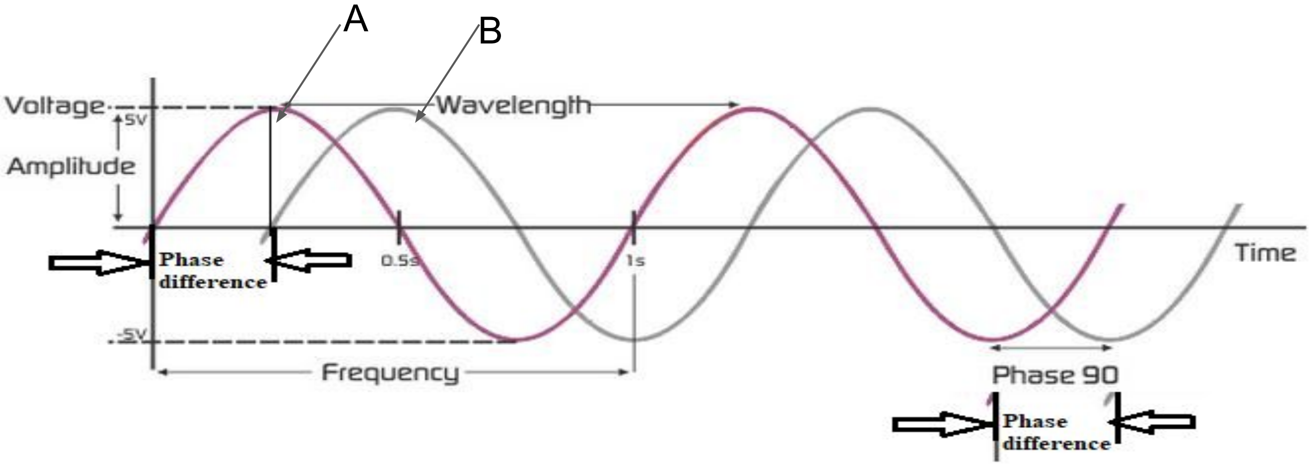
Phase: The phase of a particular value of an alternating quantity is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

Phase Difference: When two alternating quantities of same frequency have different zero points,they are said to have a phase difference.



2.3 AC wave terminologies:

Phase Difference: When two alternating quantities of same frequency have different zero points,they are said to have a phase difference.



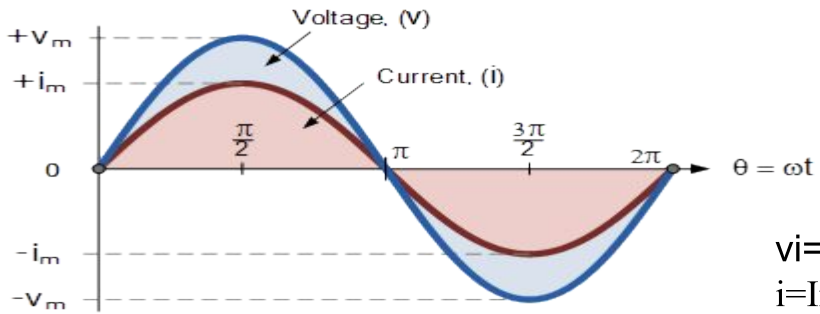
Equation for

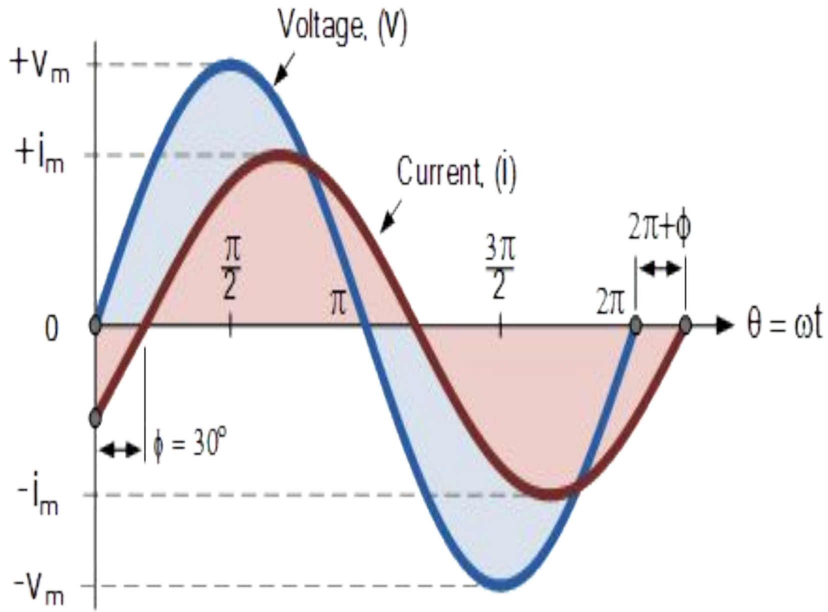
- Wave A : V_{AMAX} sinωt
- Wave B : V_{BMAX} sin(ωt-90)

2.3 AC wave terminologies:

In-phase condition :

Firstly, let's consider that two alternating quantities such as a voltage, v and a current, i have the same frequency *f* in Hertz. As the frequency of the two quantities is the same the angular velocity,ω must also be the same. So at any instant in time we can say that the phase of voltage, v will be the same as the phase of the current, i.





vi=Vm sin (ωt)
i=Im sinωt

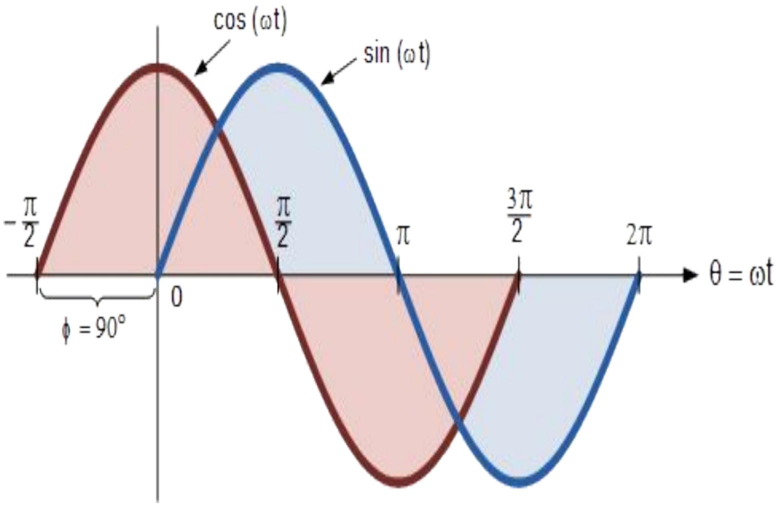
vi=Vmsinωt
i= Im sin(ωt-30)

2.3 AC wave terminologies:

Out of Phase -Leading phase difference:

Likewise, if the current, i has a positive value and crosses the reference axis reaching its maximum peak and zero values at some time before the voltage, v then the current waveform will be “leading” the voltage by some phase angle.

where, i leads v by angle Φ

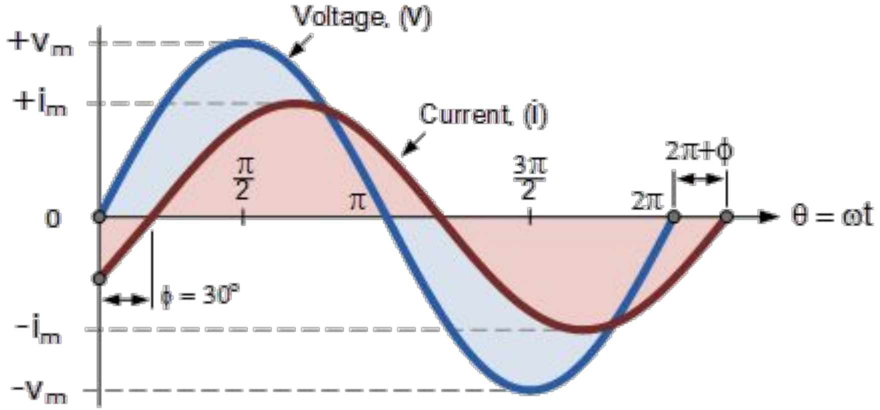


vi=Vmsinωt

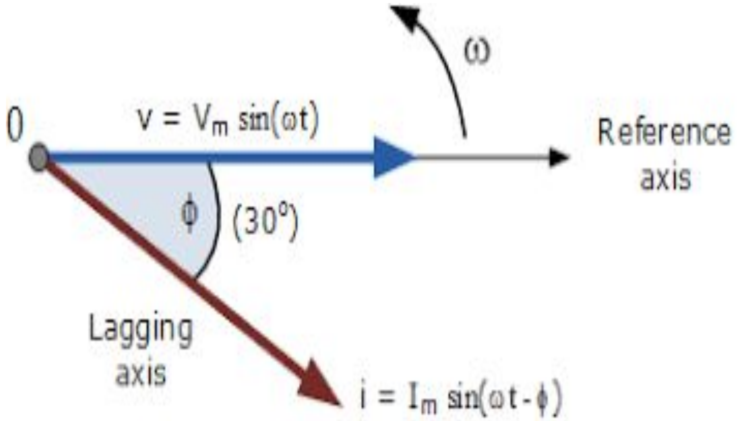
i=Imsin(ωt+90)

2.3 AC wave terminologies:

Waveform representation

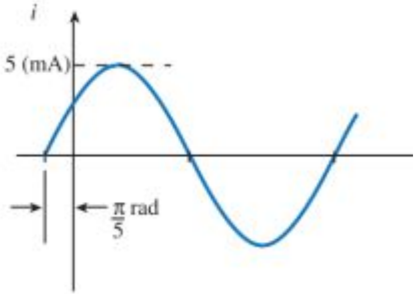


Vector representation

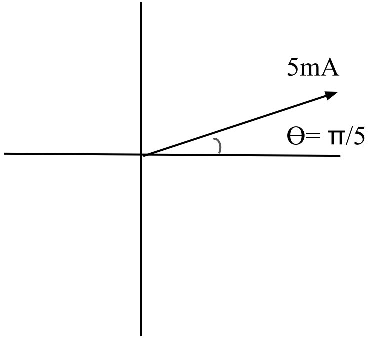


2.3 AC wave terminologies:

Waveform representation



Vector representation



2.3 AC wave terminologies:

Solve the following problems:

An AC VOLTAGE is represented as

v = 100 sin(232.7t)

Calculate the

i. RMS value= Vm*0.707

ii. Peak value of voltage

iii.Angular velocity

iv. Frequency

v. avg value

Step 1 : Compare the given equation with standard expression of AC wave

$v = V_m \sin(\omega t)$

$v = 100 \sin(232.7t)$

Step 2:

Given data:

Vm=100V

Angular velocity=ω= 2πf= 232.7rad/s

Step3:

i. **RMS value =** Vm*0.707= 100*0.707 = 70.7V

ii**.Peak Value=** Vm=100V

iii.**Angular velocity=ω=**232.7rad/s

iv.**Frequency = ω/2π** = 232.7/2π =36.97 =37Hz

v. **avg value =** Vm*0.632= 100*0.632= 63.2V

2.3 AC wave terminologies:

Solve the following problems:

An AC VOLTAGE has frequency 50Hz and maximum value 350V. i) write the equation for given ac voltage

ii) calculate the instantaneous value at 0.005s after wave passes through the zero in the positive direction.

iii) calculate the instantaneous value at 0.008s after wave passes through the zero in the positive direction.

Step1 : Frequency= 50 Hz

Maximum voltage= Vm =350V

i) standard equation:: $v = V_m \sin(\omega t)$

Angular velocity= $\omega = 2\pi f = 2\pi * 50 = 100 * \pi = 314$ rad/s

$v=350\sin(314t)$

ii) $v=350\sin(100\pi t)$

$v= 350 \sin(100\pi*180/\pi *0.005)$

$v= 350 \sin(100*180*0.005)$

$v=350V$

iii) at 0.008s the instantaneous voltage ,

$v= 350 \sin(100\pi*180/\pi *0.008)$

$v=205.72V$

2.3 AC wave terminologies:

Solve the following problems:

An AC current is given by $i=50\sin(100\pi t)$. $=50 \angle 0$

i)Determine the rate of change of current,

ii)The avg. value

iii)The RMS value

iv) the time interval where current achieved its maximum value.

Step1 : $i=50\sin(100\pi t)$

i) $di/dt= d/dt(50\sin(100\pi t))=50*100*\pi*\cos(100\pi t)$

ii) avg value of current= Im*0.637= 50*0.637= 31.85Amper

iii) RMS value of current = Im* 0.707= 35.35Amper

iv) $i=50\sin(100\pi t)$

$50= 50 \sin(100\pi t)$

$1= \sin(100\pi t)$

$\sin^{-1} 1=100\pi t$

$90= (100* \pi*180/\pi)*t$

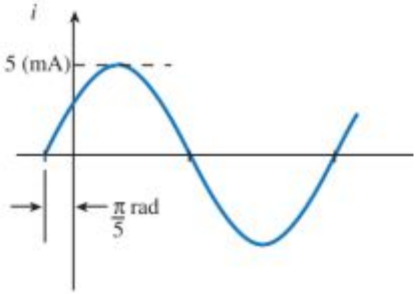
$90= (100*180)*t$

$90/(100*180) = t$

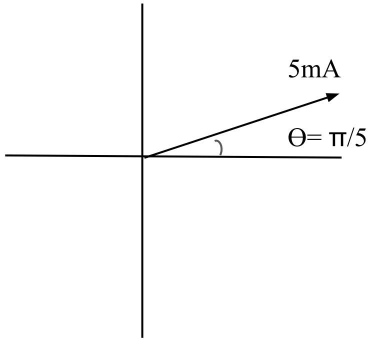
$0.005s=t$

2.3 AC wave terminologies:

Waveform representation



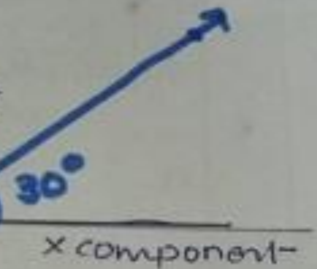
Vector representation

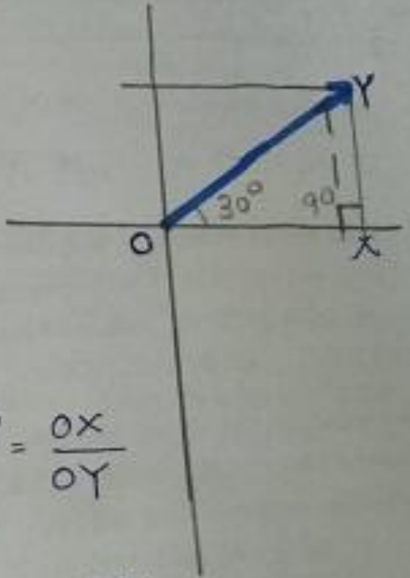


i= $5\angle(\pi/5) = 5\ \angle 36$

polar form : Magnitude$\angle$angle of rotation

$$5\ \angle 36^0$$





$\cos\theta = \cos 30^\circ = \frac{OX}{OY}$

~~cos~~

$\sin\theta = \sin 30^\circ = \frac{XY}{OY}$

$\tan\theta = \tan 30^\circ = \frac{OY}{OX}$

$$\overrightarrow{OY} = \overrightarrow{OX} + \overrightarrow{XY}$$

pythagoras theorem-:

l(hypote) = $\sqrt{x^2 + y^2}$

So to find X and Y we use trigonometric formulae

l(OX) = OY COSθ

L(XY)=OY SINθ

Vector is represented by

Magnitude and angle of rotation

In polar form

So vector BY

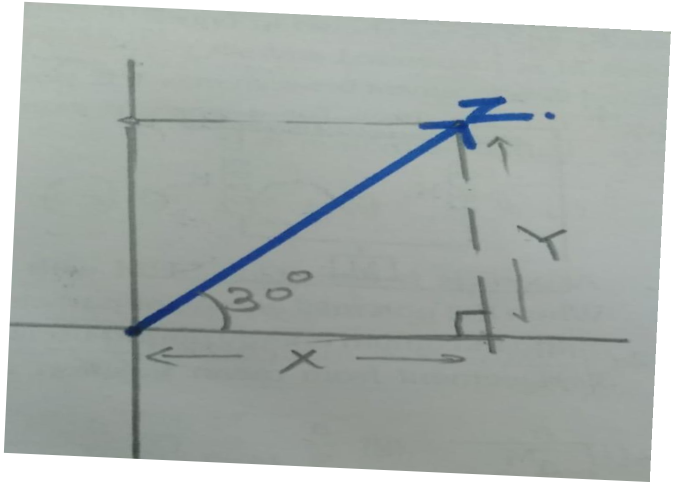
OY $\angle\theta$

IN COMPLEX FORM = RECTANGULAR FORM

OY=OX+ i XY

2.3 AC wave terminologies:

Vector representation $\overrightarrow{OY} = \overrightarrow{OX} + \overrightarrow{XY}$



pythagoras theorem-:

Z= l(hypote) = $\sqrt{x^2 + y^2}$

X =Z COSθ

Y=Z SINθ

Vector is represented by
Magnitude and angle of rotation
In polar form
So vector BY

Z $\angle\theta$

IN COMPLEX FORM

Z=X+ i Y

Conversion of Polar to rectangular form

in polar form $\rightarrow \mathbf{Z} \angle \boldsymbol{\theta}$, In Rectangular form $\rightarrow$ **Z=X+ i Y**

Convert the given polar form to rectangular form

1. **12 $\angle 60^0$**

   To find X and Y values

   X =Z COSθ = 12 cos 60= 6

   Y=Z SINθ= 12 sin 60= 10.39

   Z=6+ i 10.39

Convert the given rectangular form to polar form

2. Z= 7+i9

As per pythagoras theorem Z= $\sqrt{x^2+y^2}$

z= 11.40

θ : tanθ = 9/7 = 1.28

θ = tan$^{-1}\theta$ =tan^{-1}(9/7) = 52

<u>Z= 11.40∠ 52</u>

Conversion of Polar to rectangular form

in polar form - **$Z \angle \theta$** , IN Rectangular form - **$Z=X+ i Y$**

Convert the given polar form to rectangular form

1. **$20\angle 45^0$**

   To find X and Y values

   $X = Z \cos\theta = 20 \cos 45 = 14.14$

   $Y = Z \sin\theta = 20 \sin 45 = 14.14$

   $Z = 14.14 + i\,14.14$

Convert the given rectangular form to polar form

2. $Z = 21 + i\,35$

As per pythagoras theorem

$$Z = \sqrt{x^2 + y^2}$$

$Z = 40.81$

θ : $\tan\theta = 35/21$

$\theta = \tan^{-1}\theta = \tan^{-1}(35/21) = 58.93$

$40.81\angle 58.93$

Mathematical operation on Vectors

$a=20\angle 45^0$,$b=12\angle 60^0$

For Addition of vectors the rectangular form of a and b are

a= 14.14+ i 14.14

b= 6+ i 10.39

a+b= (Xa+Xb)+i(Ya+Yb)

= (14.14+6) +i (14.14+10.39)

= <u>20.14+i 24.53</u>

Z1= 21+ i 35

Z2= 7+i9

Subtract Z1-Z2

$$
\begin{array}{r}
21 + i\,35 \\
-\quad 7 + i\,9 \\
\hline
\underline{14 + i\,26}
\end{array}
$$

Mathematical operation on Vectors

$a=20\angle 45^0$, $b=12\angle 60^0$

For Addition of vectors the rectangular form of a and b are

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= <u>20.14+i 24.53</u>

Z1= 21+ i 35

Z2= 7+i9

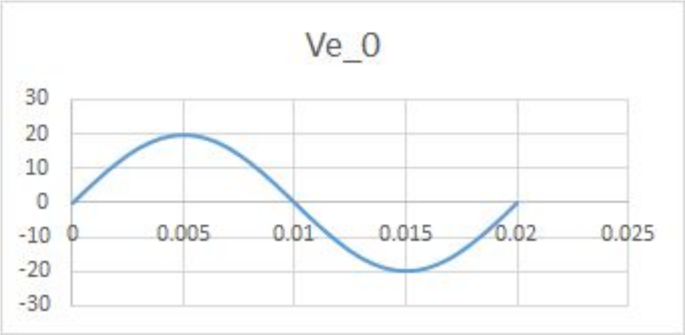
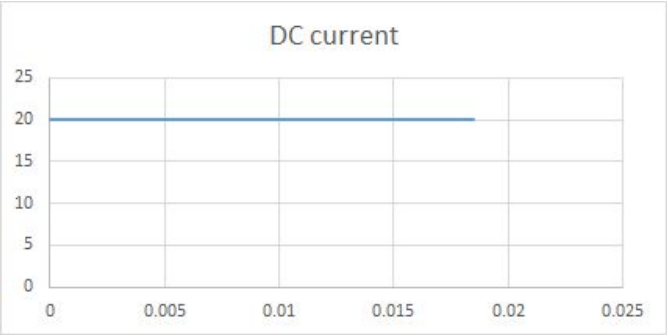
Subtract Z1-Z2

$$\begin{array}{r} 21 + i\,35 \\ -\quad 7 + i\,9 \\ \hline \underline{14 + i\,26} \end{array}$$

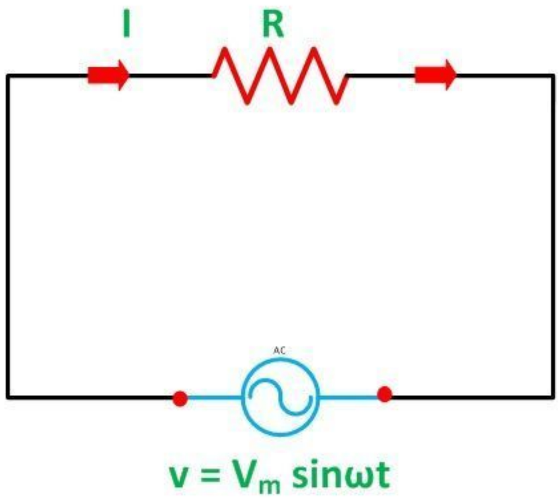
Difference between AC and DC

AC **Alternating Current**	DC **Direct Current**
AC is safe to transfer longer distance even between two cities, and maintain the electric power.	DC cannot travel for a very long distance. It loses electric power.
The rotating magnets cause the change in direction of electric flow.	The steady magnetism makes DC flow in a single direction.
The frequency of AC is depended upon the country. But, generally, the frequency is 50 Hz or 60 Hz.	DC has no frequency of zero frequency.
In AC the flow of current changes its direction backwards periodically. Electrons in AC keep changing its directions – backward and forward	It flows in a single direction steadily. Electrons only move in one direction – that is forward.

Difference between AC and DC

AC Alternating Current	DC Direct Current
	

Single phase AC circuit- pure resistive circuit



$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

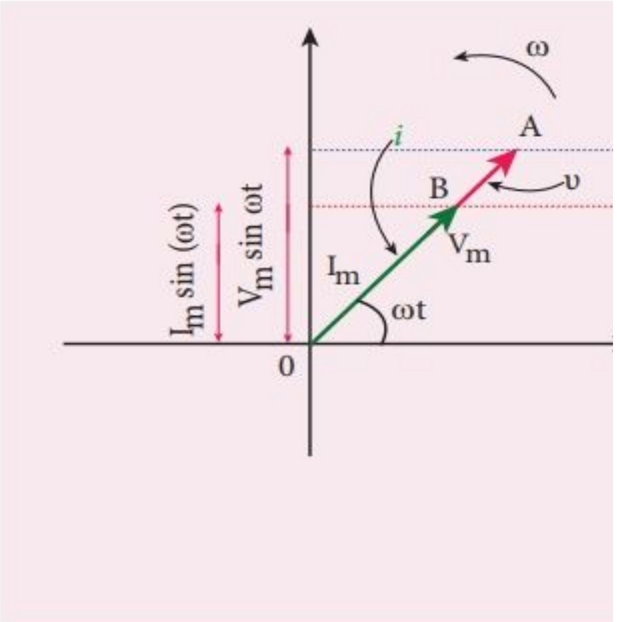
When AC circuit consists of only resistance then it is called Pure Resistive circuit.

As per Ohm's law

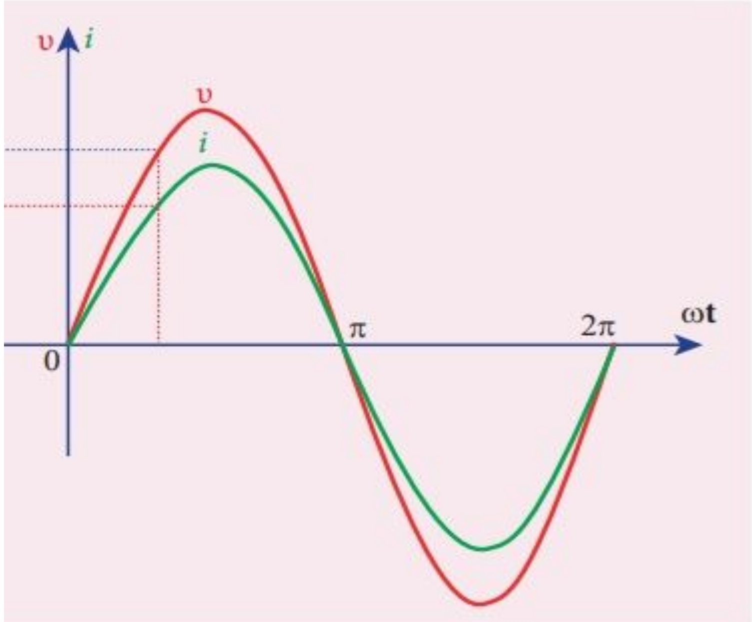
$$i = \frac{v}{R} = \frac{V_m}{R} \sin\omega t \ldots\ldots\ldots(2)$$

$$i = I_m \sin\omega t \ldots\ldots\ldots.(3)$$

Single phase AC circuit- pure resistive circuit



Phasor Diagram



Waveform of Voltage and Current

Single phase AC circuit- Pure resistive circuit- Power in Circuit

P= v*i

$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

$$i = I_m \sin\omega t \ldots\ldots\ldots.(3)$$

$$p = (V_m \sin\omega t)(I_m \sin\omega t)$$

$$p = \frac{V_m I_m}{2} 2\sin^2\omega t = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}(1 - \cos 2\omega t)$$

$$p = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos 2\omega t$$

As the valve of cosωt is zero. So, putting the value of cosωt in equation (4) the value of power will be given by

$$P = V_{r.m.s} I_{r.m.s} - 0$$

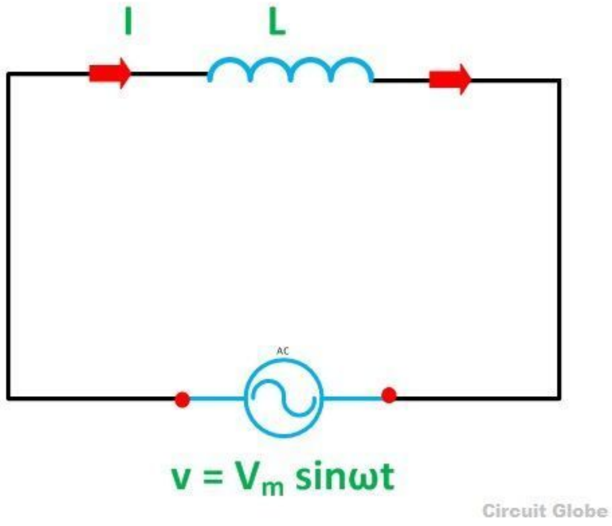
Where,

- P – average power
- Vr.m.s – root mean square value of supply voltage
- Ir.m.s – root mean square value of the current

Hence, the power in a purely resistive circuit is given by:

P=VI

Single phase AC circuit- pure inductive circuit



$$v = V_m \mathrm{Sin}\omega t \ldots\ldots\ldots\ldots(1)$$

When AC circuit consists of only inductor then it is called Pure inductive circuit.

$$e = -L\frac{di}{dt}$$

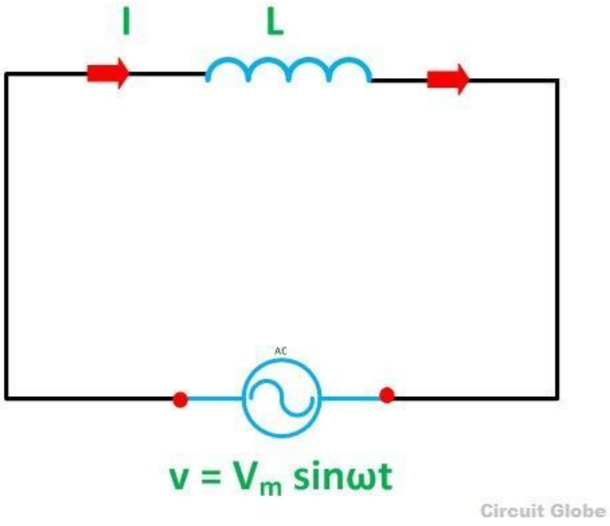
$$v = -e \ldots\ldots\ldots..(2)$$

$$v = -\left(-L\frac{di}{dt}\right) \quad \text{or}$$

$$V_m \mathrm{Sin}\omega t = L\frac{di}{dt} \quad \text{or}$$

$$di = \frac{V_m}{L}\sin\omega t\, dt \ \ldots\ldots\ldots\ldots(3)$$

Single phase AC circuit- pure inductive circuit



$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

$$\int di = \int \frac{V_m}{L} \sin\omega t \, dt \quad \text{or}$$

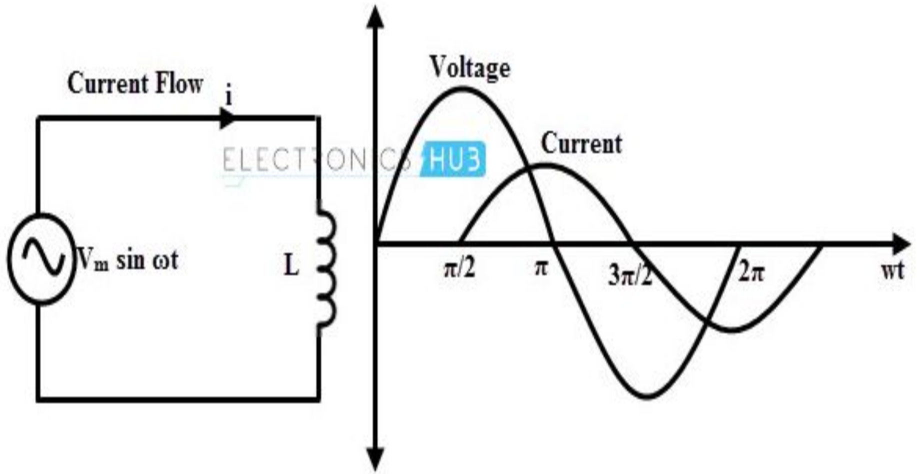
$$i = \frac{V_m}{\omega L}(-\cos\omega t) \quad \text{or}$$

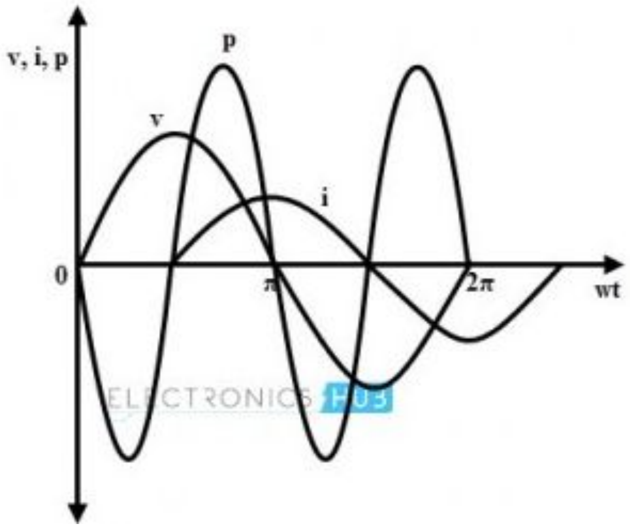
$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \pi/_2\right) = \frac{V_m}{X_L}\sin\left(\omega t - \pi/_2\right) \ldots\ldots\ldots\ldots(4)$$

$$I_m = \frac{V_m}{X_L} \ldots\ldots\ldots\ldots(5)$$

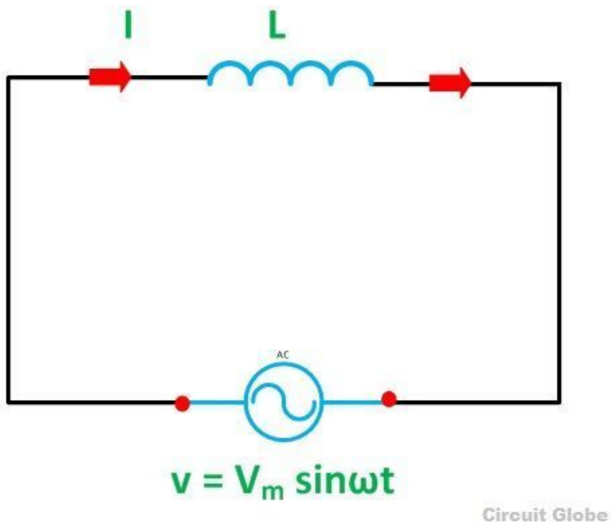
$$i = I_m \sin\left(\omega t - \pi/_2\right)$$

i= v/R- ohms
i= Vm/wL
R=wL = inductive reactance
XL =inductive reactance
XL=wL=


Current Flow
i
Vm sin ωt
L
Voltage
Current
π/2
π
3π/2
2π
wt


v, i, p
p
v
i
0
π
2π
wt

Single phase AC circuit- pure inductive circuit -power



$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

$$p = vi$$

$$P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/2))$$

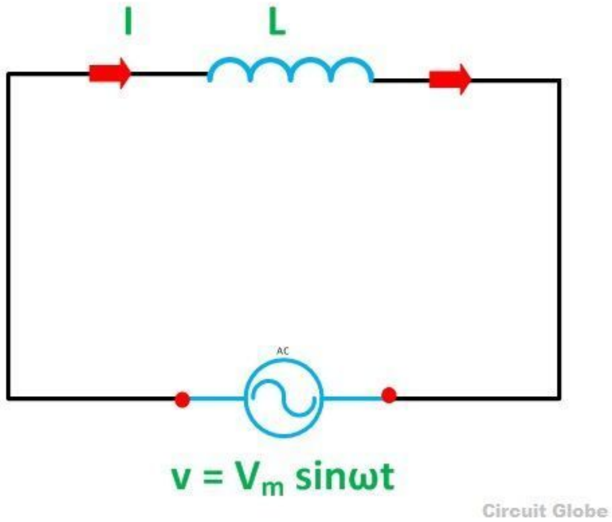
$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m I_m}{2} 2 \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t \text{ or}$$

$$P = 0$$

Single phase AC circuit- pure inductive circuit -Inductive reactance



$$v = V_m \sin\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

From the above derivation, the maximum current equation is given as

$$im = (Vm / \omega L) \quad \text{--} \quad (5)$$

$$\omega L = Vm / im$$

The inductive reactance of the AC circuit can be represented as

$$XL = \omega L = 2\Pi fL$$

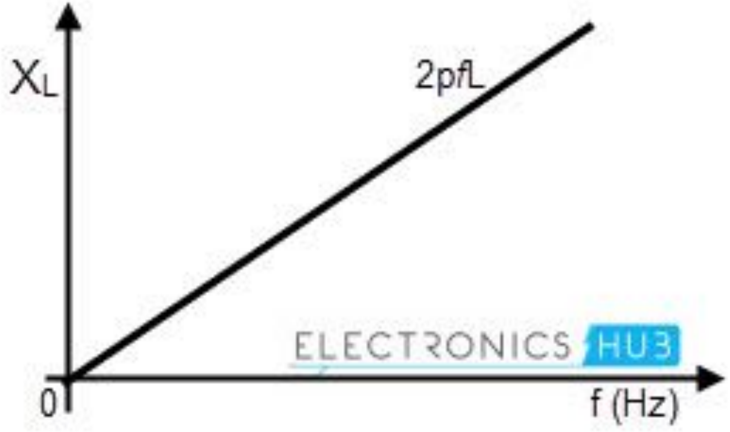
(since $\omega = 2\Pi f$)

Where XL is the inductive reactance in Ω

f is the frequency of the supply voltage,Hz

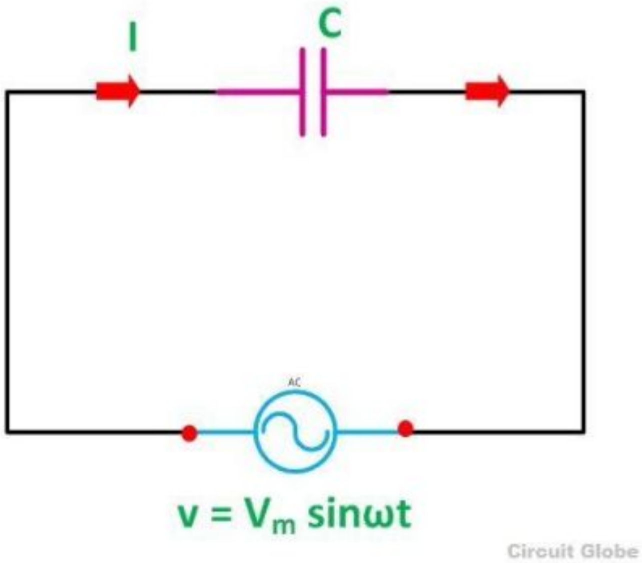
L is the inductance of the coil in Henry,H

Single phase AC circuit- pure inductive circuit -Inductive reactance



As a result, the net current flowing through the inductor will be reduced. It is to be concluded that reactance of the inductor varies linearly with frequency of the supply as shown in figure.

Single phase AC circuit- pure capacitive circuit



$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

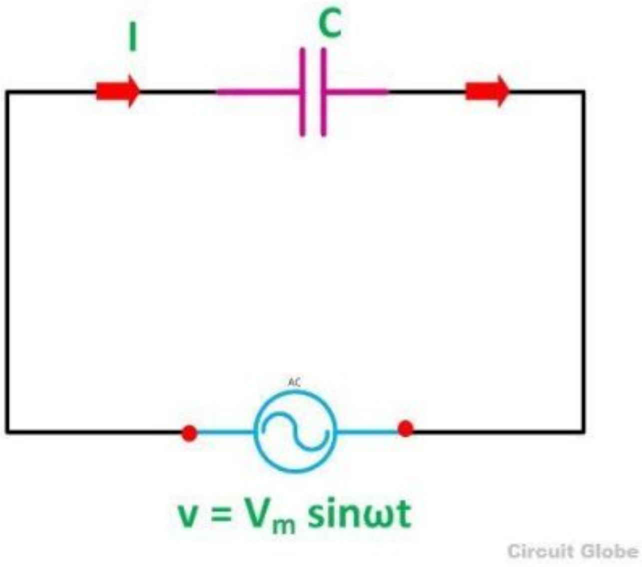
When AC circuit consists of only capacitor then it is called Pure capacitive circuit.

$$q = Cv \ \ldots\ldots\ldots\ldots(2)$$

$$i = \frac{d}{dt} q$$

$$i = \frac{d}{dt}(Cv) \ldots\ldots\ldots\ldots.(3)$$

Single phase AC circuit- pure capacitive circuit



$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

$$i = \frac{d}{dt} C\, V_m \text{Sin}\omega t = C\, V_m \frac{d}{dt} \sin\omega t \quad \text{or}$$

$$i = \omega C\, V_m \cos\omega t = \frac{V_m}{1/\omega C} \sin(\omega t + \pi/2) \quad \text{or}$$

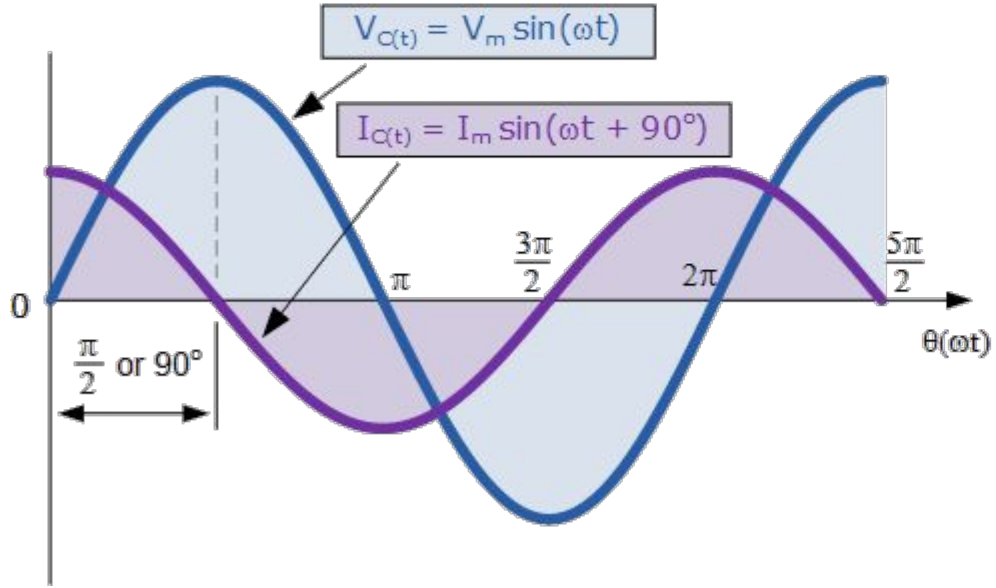
$$i = \frac{V_m}{X_C} \sin(\omega t + \pi/2) \ldots\ldots\ldots\ldots(4) \quad X_C = \frac{1}{\omega C}$$

$$I_m = \frac{V_m}{X_C}$$

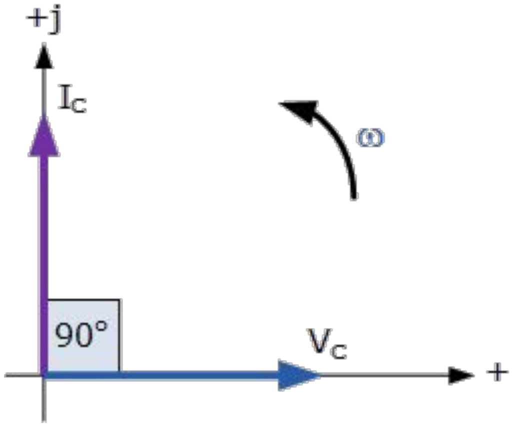
$$i = I_m \sin(\omega t + \pi/2)$$

i= wCVm=
i=Vm/(1/wc)

i=v/R
R=1/wC

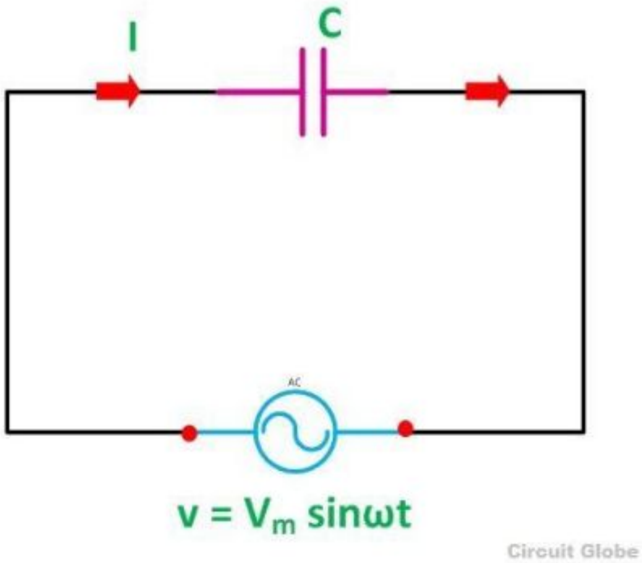


$V_{C(t)} = V_m \sin(\omega t)$
$I_{C(t)} = I_m \sin(\omega t + 90°)$
0
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$
$\theta(\omega t)$
$\frac{\pi}{2}$ or 90°



+j
I_C
ω
90°
V_C
+

Single phase AC circuit- pure capacitive circuit



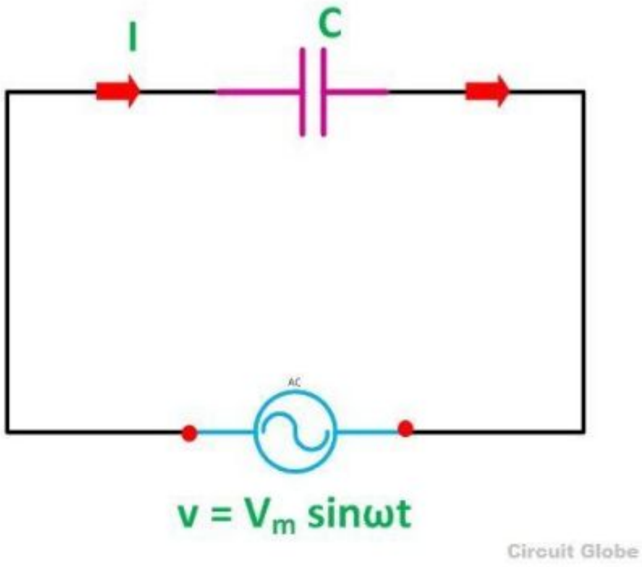
$$v = V_m \text{Sin}\omega t \ \ldots\ldots\ldots\ldots\ldots (1)$$

$$X_C = \frac{1}{\omega C}$$

$$I_m = \frac{V_m}{X_C}$$

$$X_C = \frac{1}{2\pi f C}$$

Single phase AC circuit- pure capacitive circuit



$$v = V_m \mathrm{Sin}\omega t \;\ldots\ldots\ldots\ldots\ldots(1)$$

$$P = (V_m \sin\omega t)(I_m \sin(\omega t + {}^{\pi}/_{2})$$

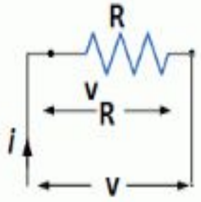
$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\,\omega t \quad \text{or}$$

$$P = 0$$

Comparison between pure resistive, capacitive and inductive circuit

Pure resistive circuit



Pure Resistive Circuit

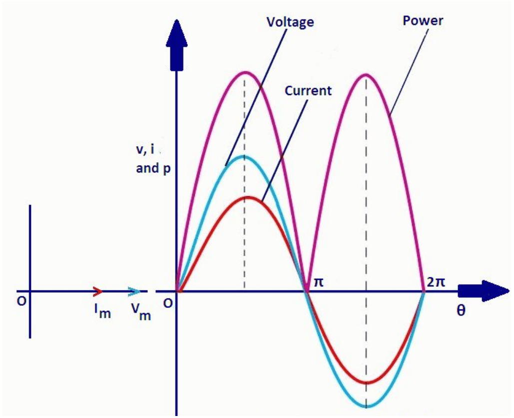
Applied Voltage, $v = V_m \sin \omega t$

Resultant Current, $i = I_m \sin \omega t$

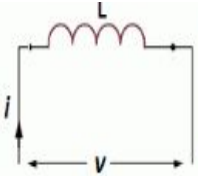
where, $I_m = V_m / R$

Power = VI watts

Power factor, $\cos \phi = 1$



Pure inductive circuit



Pure Inductive Circuit

Applied Voltage, $v = V_m \sin \omega t$

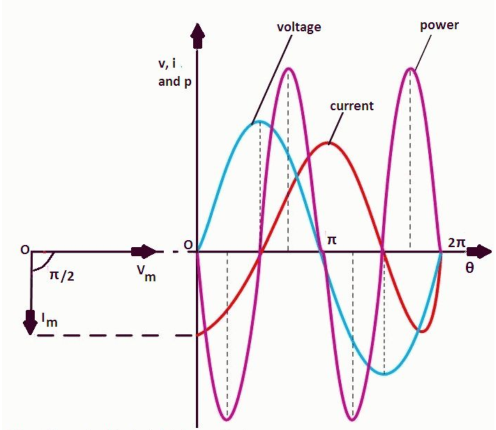
Resultent Current, $i = I_m \sin(\omega t - \frac{\pi}{2})$

Where, $I_m = \frac{V_m}{X_L}$

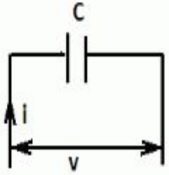
Inductive Reactance, $X_L = 2\pi fL$ ohms

Power absorbed by circuit = 0

Power factor, $\cos \phi = 0$



Pure capacitor circuit



Pure Capacitive Circuit

Applied Voltage, $v = V_m \sin \omega t$

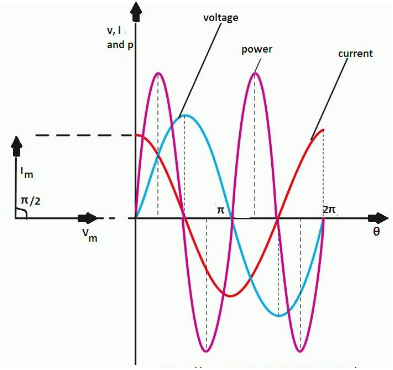
Resultent Current, $i = I_m \sin(\omega t + \frac{\pi}{2})$

Where, $I_m = \frac{V_m}{X_C}$

Capacitve Reactance, $X_C = 1/ 2\pi fC$ ohms

Power absorbed by circuit = 0

Power factor, $\cos \phi = 0$



Series AC circuits -RL circuit

A circuit that contains a pure resistance R ohms connected in series with a coil having a pure inductance of L (Henry) is known as **RL Series Circuit**. When an AC supply voltage V is applied, the current, I flows in the circuit.

Where,

VR – voltage across the resistor R = i* R

VL – voltage across the inductor L= i*XL

V – Total voltage of the circuit =

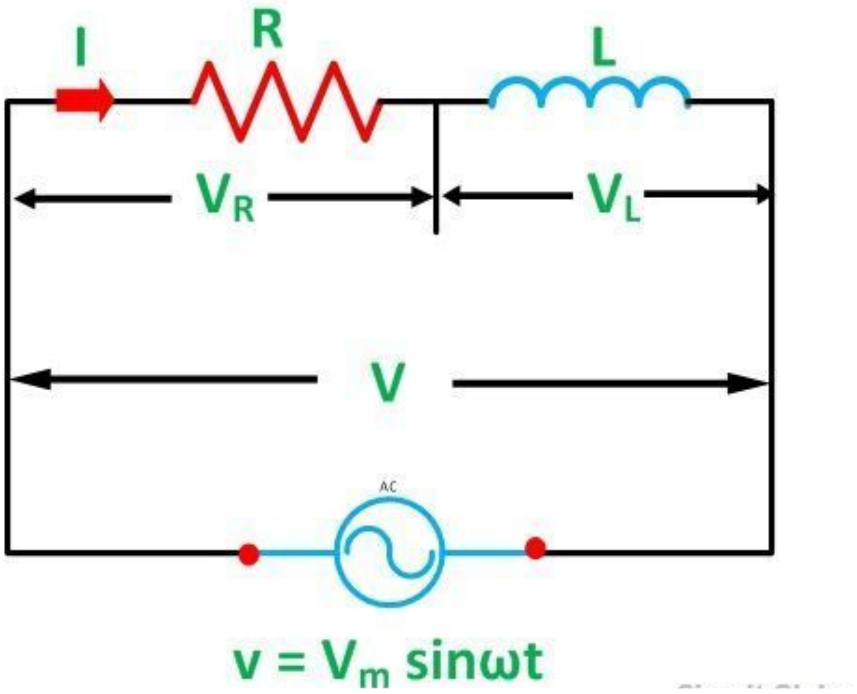


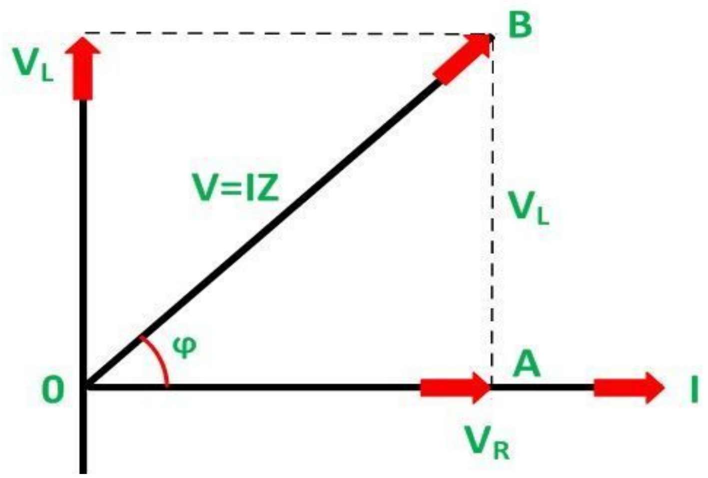
Figure a: AC series RL circuit

Series AC circuits -RL circuit

Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance VR = IR is drawn in phase with the current I.
- The voltage drop across the inductive reactance VL =IXL is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops VR and VL is equal to the applied voltage V.

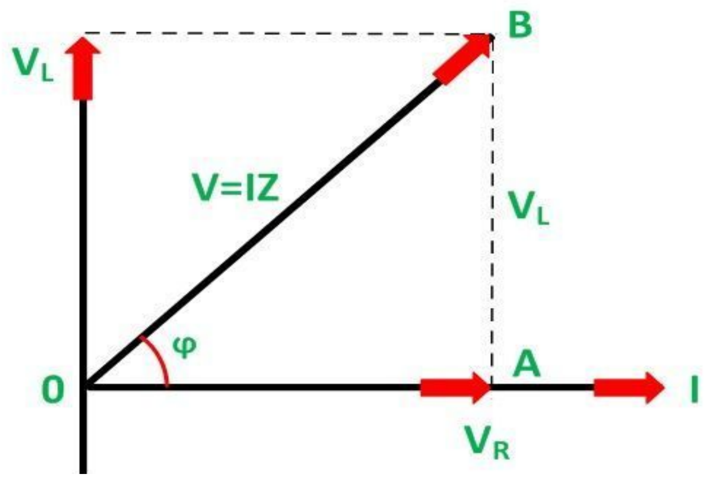


The phasor diagram of the RL Series circuit is shown in figure b

Series AC circuits -RL circuit - VOLTAGE triangle

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$



The phasor diagram of the RL Series circuit is shown in figure b

Series AC circuits -RL circuit - POWER

$v = V_m \sin\omega t$

$i = I_m \sin(\omega t - \varphi)$

$P = v * i$

$P = (V_m \sin\omega t) * (I_m \sin(\omega t - \varphi))$

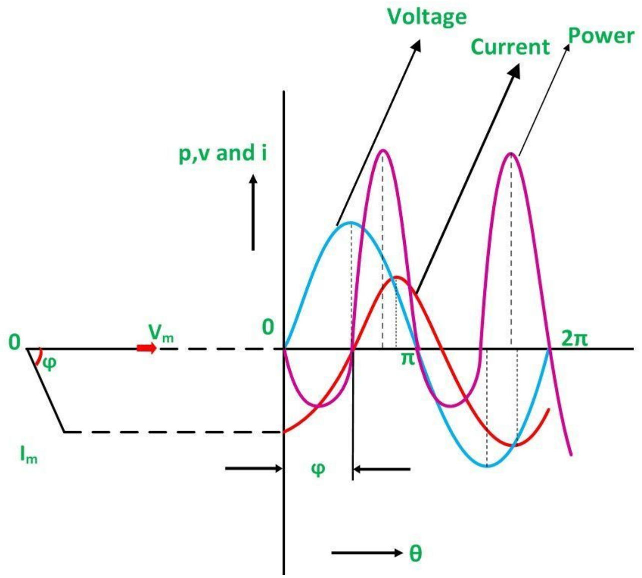
$$P = \frac{(V_m I_m)}{2} \times 2 \sin(\omega t - \varphi)(\sin\omega t)$$

$$P = \frac{(V_m I_m)}{2} \times [\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \left[\frac{(V_m I_m)}{2}\cos\varphi\right] - \left[\frac{(V_m I_m)}{2}\cos(2\omega t - \varphi)\right]$$

$$P_{avg} = \left[\frac{(V_m I_m)}{2}\cos\varphi\right]$$

$P_{avg} = V_{rms} * I_{rms} * \cos\varphi = VI\cos\varphi$ watts



The phasor diagram and waveform of resultant current and voltage of the RL Series circuit

Series AC circuits -RL circuit - IMPEDANCE TRIANGLE

VR = IR and VL = IXL where XL = 2πfL,

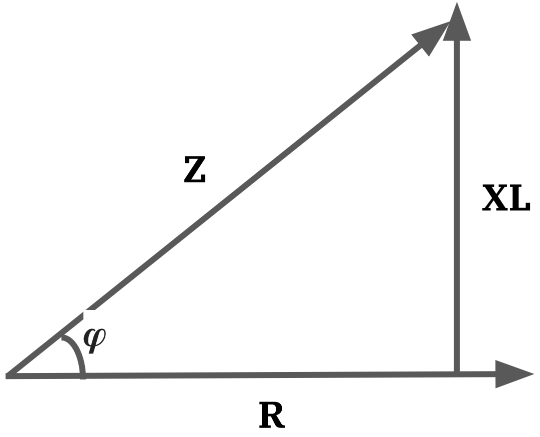
When the voltage triangle is divided by I,

We get,

$$Z = \sqrt{R^2 + X_L^2}$$

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$



The phasor diagram voltage IMPEDANCE of the RL Series circuit

Series AC circuits -RC circuit

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as RC Series Circuit. A sinusoidal voltage is applied and current I flows through the resistance (R) and the capacitance (C) of the circuit

Where,

VR – voltage across the resistor R = i* R

VC – voltage across the capacitor = i*XC

V – Total voltage of the circuit = IZ

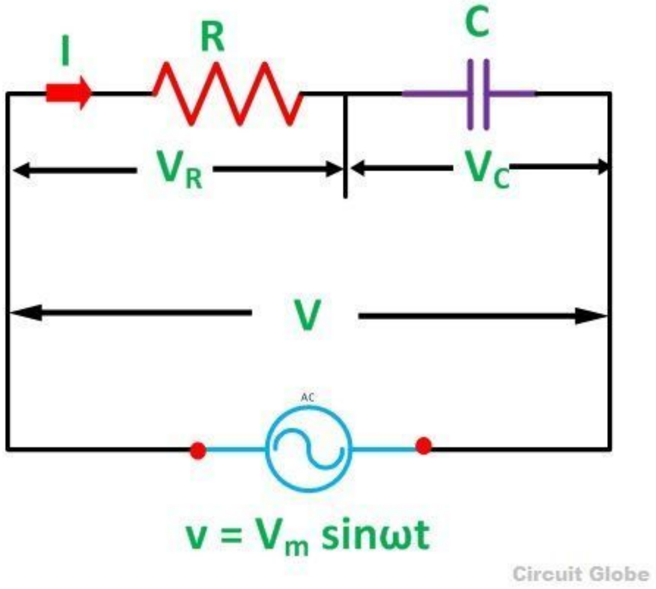


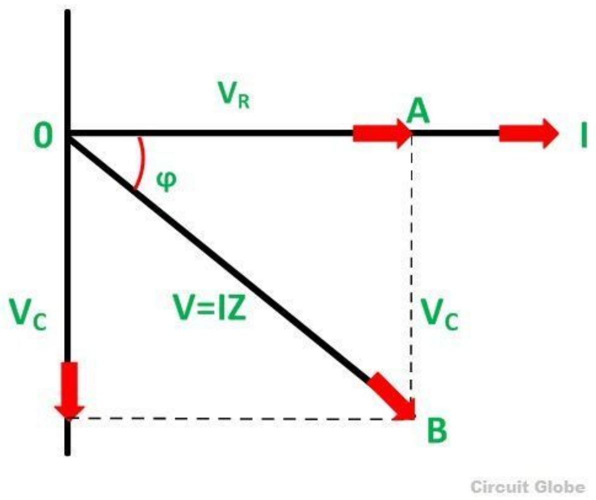
Figure a: AC series RC circuit

Series AC circuits -RC circuit

Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance VR = IR is drawn in phase with the current I.
- The voltage drop across the inductive reactance VL =IXC is drawn behind of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops VR and VC is equal to the applied voltage V.

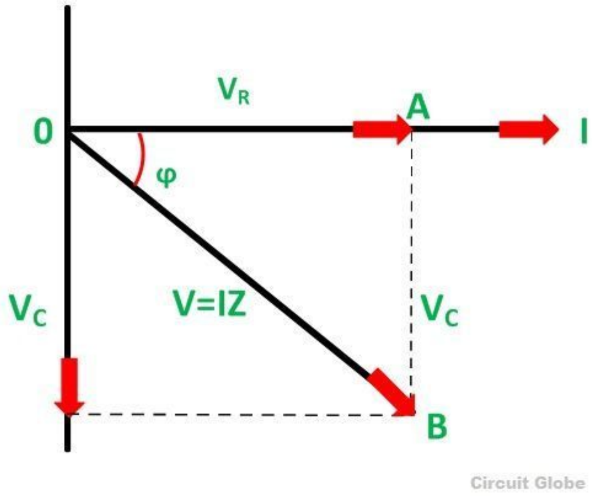


The phasor diagram of the RC Series circuit is shown in figure b

Series AC circuits -RC circuit - VOLTAGE triangle

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$ or



The phasor diagram of the RL Series circuit is shown in figure b

Series AC circuits -RC circuit - IMPEDANCE TRIANGLE

VR = IR and VC = IXC where XC = 1/2πcL,

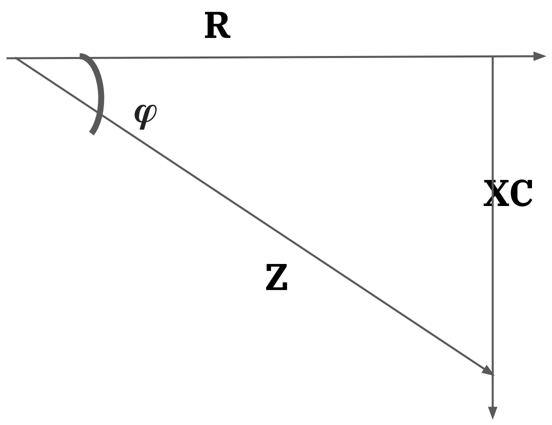
When the voltage triangle is divided by I,

We get,

$$Z = \sqrt{R^2 + X_C^2}$$

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{-X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{-X_C}{R}$$



The phasor diagram voltage IMPEDANCE of the RL Series circuit

Series AC circuits -RC circuit - POWER

$v = V_m \sin\omega t$

$i = I_m \sin(\omega t + \varphi)$

$P = v * i$

$P = (V_m \sin\omega t) * (I_m \sin(\omega t + \varphi))$

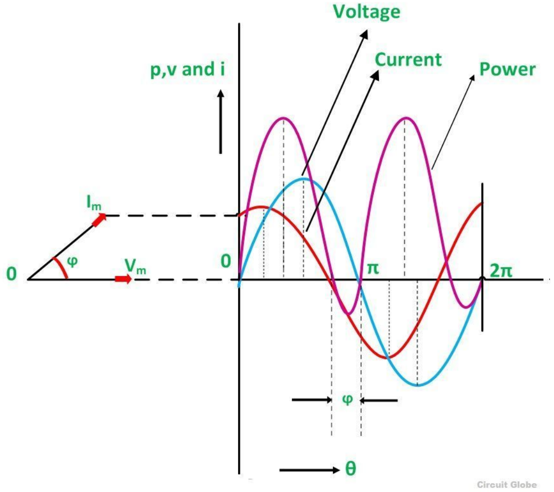
$P = \frac{(V_m I_m)}{2} \times 2 \sin(\omega t + \varphi)(\sin\omega t)$

$P = \frac{(V_m I_m)}{2} \times [\cos\varphi - \cos(2\omega t + \varphi)]$

$P = [\frac{(V_m I_m)}{2} \cos\varphi] - [\frac{(V_m I_m)}{2} \cos(2\omega t + \varphi)]$

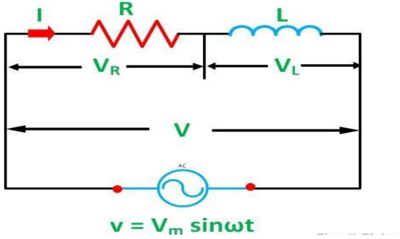
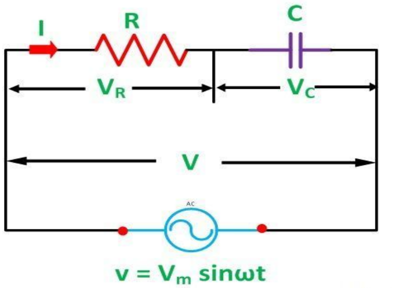
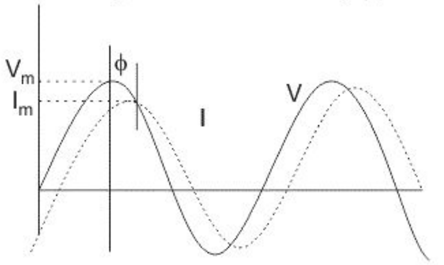
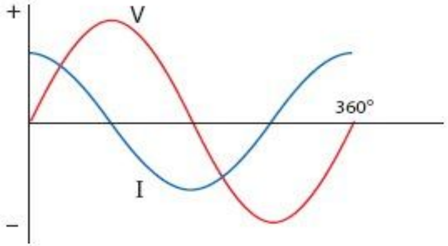
$P_{avg} = [\frac{(V_m I_m)}{2} \cos\varphi]$

$P_{avg} = \underline{V_{rms} * I_{rms} *} \; \underline{\cos\varphi} = \underline{VI\cos\varphi}$ watts

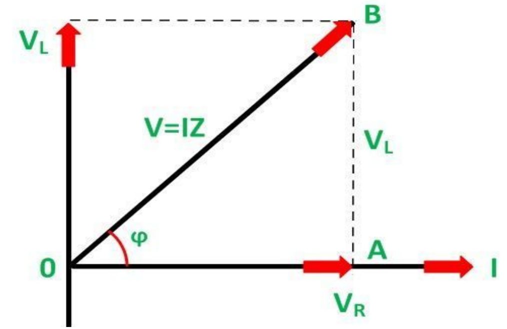
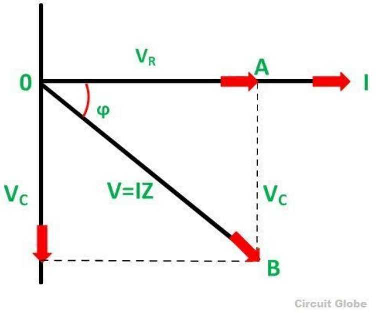
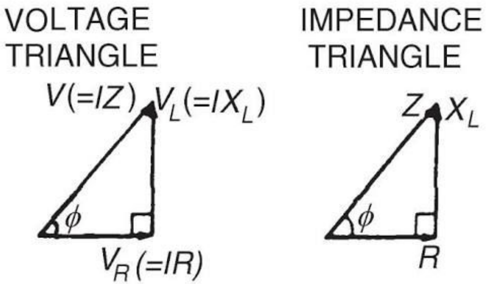
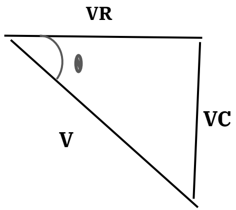
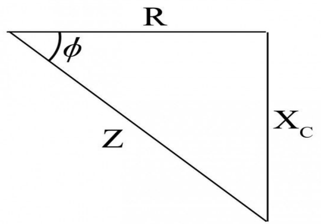


The phasor diagram and waveform of resultant current and voltage of the RL Series circuit

Comparison between RL and RC series circuit

	Series RL circuit	Series RC circuit
Circuit Diagram		
Equation of Voltage and current of the circuit	$v = V_m \sin\omega t$ $i = I_m \sin(\omega t - \phi)$	$v = V_m \sin\omega t$ $i = I_m \sin(\omega t + \phi)$
Waveform	Voltage in leads in current by ϕ phase 	Current in lead by angle ϕ 

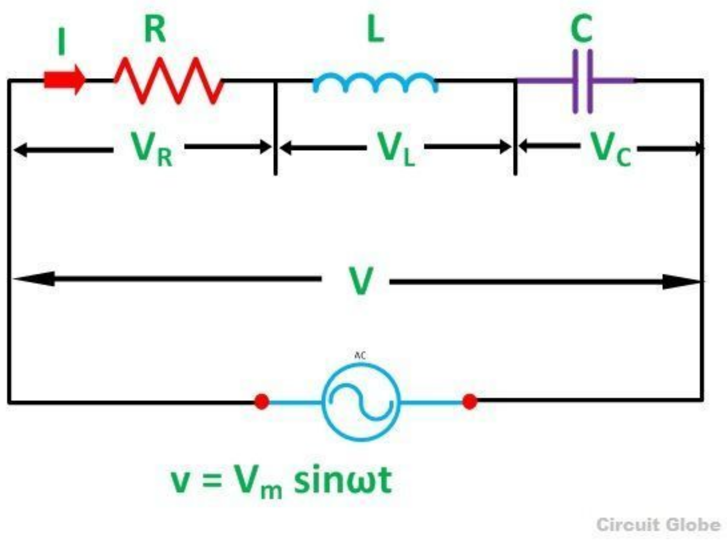
Comparison between RL and RC series circuit

	Series RL circuit	Series RC circuit
Phasor Diagram		
Equation of impedance	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$
Voltage and impedance triangle		 Voltage triangle  Impedance triangle

Comparison between RL and RC series circuit

	Series RL circuit	Series RC circuit
Avg. Power consumed	**Vrms*Irms* COSφ** **VICOSφ**	**Vrms*Irms* COSφ** **VICOSφ**

AC Series RLC circuit



A circuit that contains a pure resistance R ohms connected in series with a coil having a pure inductance of L (Henry) and and capacitor with capacitance C(farad) is known as **RLC Series Circuit**. When an AC supply voltage V is applied, the current, I flows in the circuit.

Where,

VR – voltage across the resistor R = i* R

VL – voltage across the inductor L= i*XL

VC- voltage across the capacitor C= i*XC

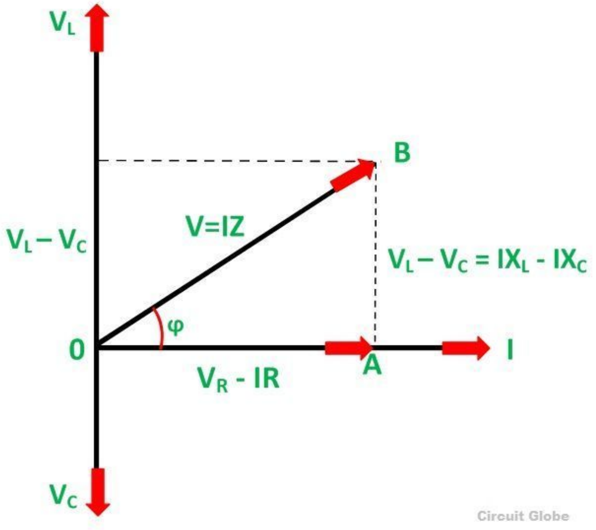
V – Total voltage of the circuit = IZ

Series AC circuits -RLC circuit

Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step:

- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle.
- The voltage across the capacitor c that is V_C is drawn lagging the current I by a 90-degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vector V_L and V_C are opposite to each other.
-



The phasor diagram of the RLC Series circuit is shown in figure b

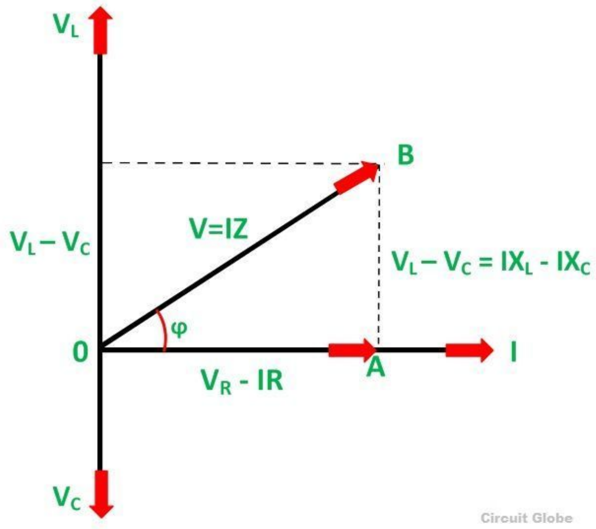
Series AC circuits -RLC circuit - VOLTAGE triangle

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



The phasor diagram of the RL Series circuit is shown in figure b

Series AC circuits -RLC circuit - Phase angle

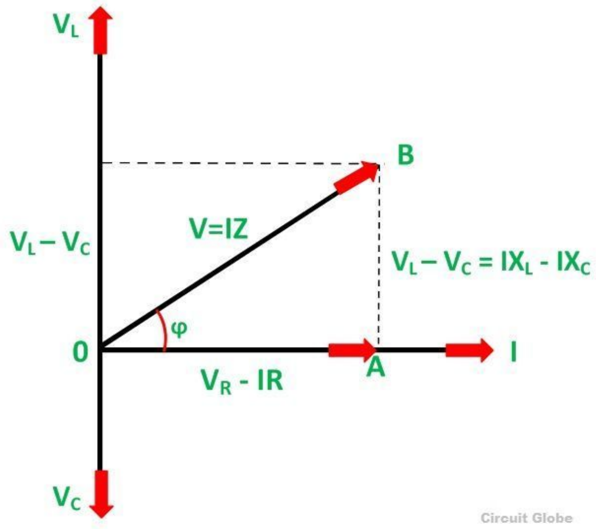
$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Avg Power consumed by the RLC series circuit

$$P = VI\cos\varphi = I^2R$$

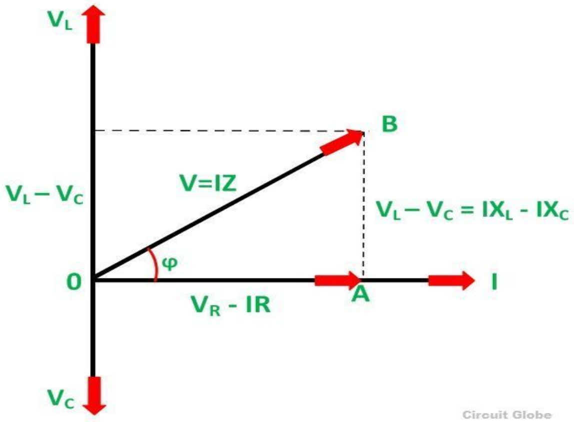
Power Factor $\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$



The phasor diagram of the RL Series circuit is shown in figure b

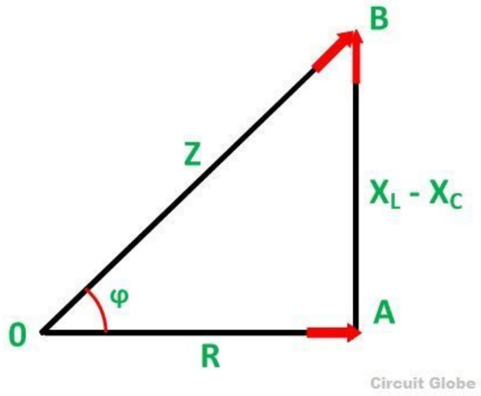
Series AC circuits -RLC circuit - The three cases of RLC Series Circuit

When XL > XC, the phase angle ϕ is positive. The circuit behaves as RL series circuit in which the current lags behind the applied voltage and the power factor is lagging



The phasor diagram of the RLC Series circuit is shown in figure b

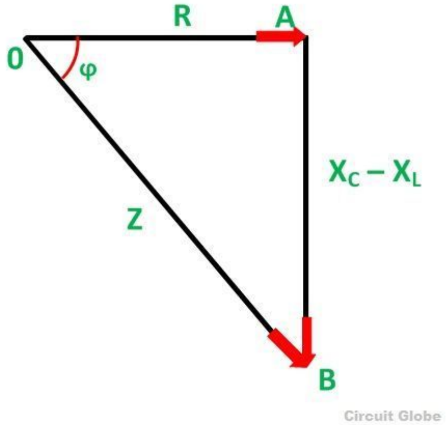
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

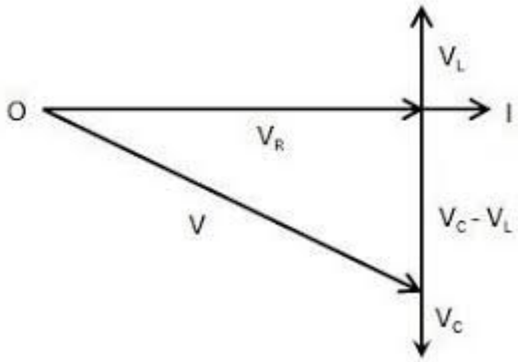


Series AC circuits -RLC circuit - The three cases of RLC Series Circuit

- When XL < XC, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$





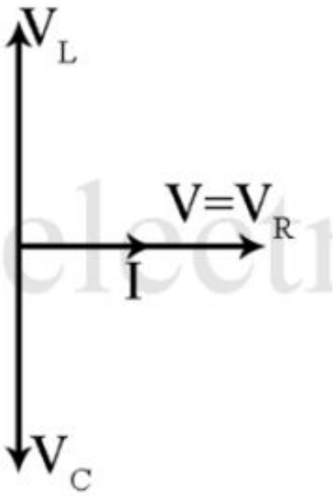
The phasor diagram of the RL Series circuit is shown in figure b

Series AC circuits -RLC circuit - The three cases of RLC Series Circuit

When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of the power factor is **unity**.

$$Z=R$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



The phasor diagram of the RL Series circuit is shown in figure b

Power in AC circuits: Power Triangle

Instantaneous power drawn by AC Circuit is

$P = v * i$

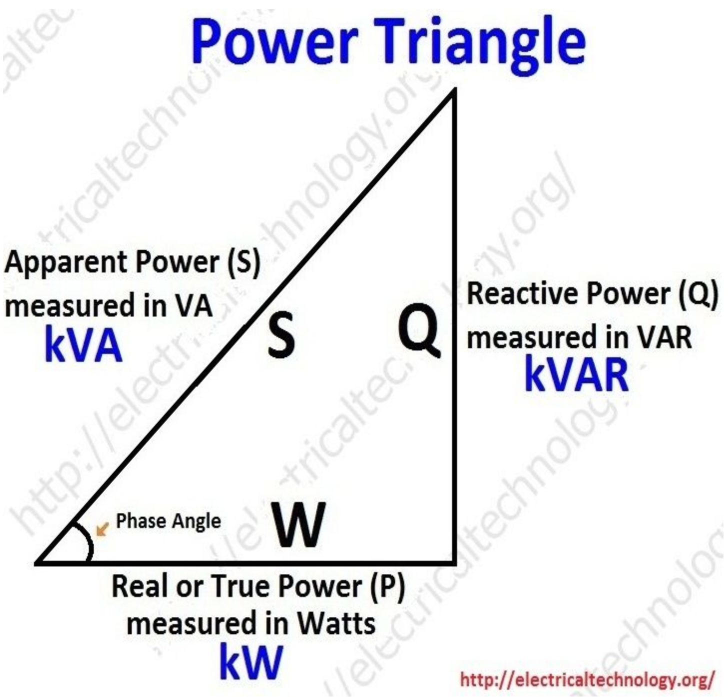
$P = V_m \sin\omega t * I_m \sin(\omega t - \phi)$

$p = \left[\frac{(V_m I_m)}{2}\cos\varphi\right] - \left[\frac{(V_m I_m)}{2}\cos(2\omega t - \varphi)\right]$

- $\left[\frac{(V_m I_m)}{2}\cos\varphi\right]$ - remains constant irrespective of time
- $\left[\frac{(V_m I_m)}{2}\cos(2\omega t - \varphi)\right]$ - variable as twice the supply frequency

Thus

$P_{avg} = \left[\frac{(V_m I_m)}{2}\cos\varphi\right]$ watts



Power in AC circuits: Power Triangle

- Apparent Power= V x I = VI
- Active Power= V x I cosϕ = V I cosϕ
- Reactive power= V x I sinϕ = V I sinϕ

$$\text{Power Factor} = \frac{\text{Active Power}}{\text{Apparent Power}} = \cos\phi$$