Template: Study Material

<Course Code:22103>: <Subject Code: BMS>: <Subject Name: Basic Mathematics>: <Topic Name: Matrices> : <UO1.3.3.> : <Study Material>

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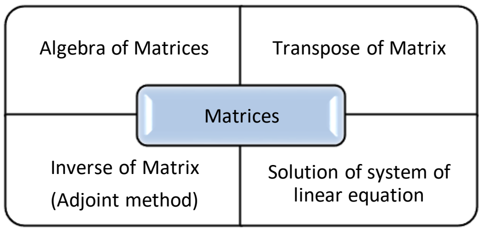
Key words: Adjoint of Matrix, Inverse of Matrix, Solution of simultaneous equations.

Learning Objective:
Solve the given system of linear equations using matrix inversion method.

Diagram/ Picture:

Key Questions: Have you wondered how to find inverse of a matrix?

Concept Map



Course Content:

Adjoint of a matrix:

Adjoint of a matrix is the transpose of co-factor matrix

$$\therefore Adj\ A = [c_{ij}]^t$$

Co-factor matrix is a matrix of co-factors=$[c_{ij}]$

$where\ c_{ij} = (-1)^{i+j} \times M_{ij}$ where

Minor M_{ij} = determinant of matrix obtained by deleting i[th] row & j[th] column of given matrix.

Solved examples:

1. If A=$\begin{bmatrix} -1 & 1 & 1 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, find Adj A

Solution.: Given A=$\begin{bmatrix} -1 & 1 & 1 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

$$c_{11} = (-1)^{1+1} \times \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} = +(4-4) = 0$$

$$c_{12} = (-1)^{1+2} \times \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -(2-6) = 4$$

$$c_{13} = (-1)^{1+3} \times \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = +(4-12) = -8$$

$$c_{21} = (-1)^{2+1} \times \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = 1$$

$$c_{22} = (-1)^{2+2} \times \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} = +(-1-3) = -4$$

$$c_{23} = (-1)^{2+3} \times \begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} = -(-2-3) = 5$$

$$c_{31} = (-1)^{3+1} \times \begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} = +(2-4) = -2$$

$$c_{32} = (-1)^{3+2} \times \begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} = -(-2-2) = 4$$

$$c_{33} = (-1)^{3+3} \times \begin{vmatrix} -1 & 1 \\ 2 & 4 \end{vmatrix} = +(-4-2) = -6$$

$$\therefore Matrix\ of\ cofactors = C = \begin{bmatrix} 0 & 4 & -8 \\ 1 & -4 & 5 \\ -2 & 4 & -6 \end{bmatrix}$$

$$\therefore Adj\ A = C^t = \begin{bmatrix} 0 & 1 & -2 \\ 4 & -4 & 4 \\ -8 & 5 & -6 \end{bmatrix}$$

Solved word Problem:

Key Definitions/ Formula

Adjoint of a matrix:

Adjoint of a matrix is the transpose of co-factor matrix

$$\therefore Adj\ A = [c_{ij}]^t$$

Inverse of a matrix:

Given a non-singular matrix 'A', if there exists a matrix 'B' such that $A \times B = B \times A = I$ then matrix B is the inverse of matrix A.

Formula:

$$A^{-1} = \frac{1}{\det A} \times Adj\ A$$

	Application of Concept/ Examples in real life: Matrices are used in coding and decoding of information.	Link to YouTube/ OER/ video: http://www.khanacademy

COURSE CONTENT: CONTINUED………

Singular matrix

A square matrix A is called singular matrix if det (A) or |A| = 0.

Non-Singular matrix

A square matrix A is called non-singular, if det (A) or |A| $\neq$ 0.

Solved Example:

1) If A = $\begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$ Show that the matrix AB is non-singular.

Solution : Given A = $\begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0+0+1 & -2+0+1 \\ 0+4+3 & 1+6+3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 7 & 10 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 1 & -1 \\ 7 & 10 \end{vmatrix} = 10 + 7$$

$$|AB| = 17 \neq 0$$

$\therefore$ AB is a non-singular matrix.

Inverse of a matrix:

Given a non-singular matrix ‘A’, if there exists a matrix ‘B’ such that

$A \times B = B \times A = I$ then matrix B is the inverse of matrix A.

Notation: Inverse of A = A^{-1}

Formula: $A^{-1} = \frac{1}{\det A} \times Adj\ A$

Solved example:

1. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix}$

Solution : Given $A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix}$

$$|A| = 3(3+1) - 1(12-2) + 2(-4-2)$$

$$= 12 - 10 - 12 = -10 \neq 0$$

$\therefore A^{-1}$ exists

$$c_{11} = (-1)^{1+1} \times \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = +(3+1) = 4$$

$$c_{12} = (-1)^{1+2} \times \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = -(12-2) = -10$$

$$c_{13} = (-1)^{1+3} \times \begin{vmatrix} 4 & 1 \\ 2 & -1 \end{vmatrix} = +(-4-2) = -6$$

$$c_{21} = (-1)^{2+1} \times \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = -(3+2) = -5$$

$$c_{22} = (-1)^{2+2} \times \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = +(9-4) = 5$$

$$c_{23} = (-1)^{2+3} \times \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = -(-3-2) = 5$$

$$c_{31} = (-1)^{3+1} \times \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = +(1-2) = -1$$

$$c_{32} = (-1)^{3+2} \times \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = -(3-8) = 5$$

$$c_{33} = (-1)^{3+3} \times \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} = +(3-4) = -1$$

$$\therefore C = \begin{bmatrix} 4 & -10 & -6 \\ -5 & 5 & 5 \\ -1 & 5 & -1 \end{bmatrix}$$

$$\therefore Adj\ A = C^t = \begin{bmatrix} 4 & -5 & -1 \\ -10 & 5 & 5 \\ -6 & 5 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-10} \times \begin{bmatrix} 4 & -5 & -1 \\ -10 & 5 & 5 \\ -6 & 5 & -1 \end{bmatrix}$$

Solution of simultaneous equations:

Suppose $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

are the given simultaneous equations.

These equations can be represented in matrix form as follows:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad \text{i.e.} \quad \mathrm{A} \times \mathrm{X} = \mathrm{B} \ \text{ where}$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

The solution of the system of equations is given by $X = A^{-1} \times \mathrm{B}$ where $A^{-1} = \frac{1}{\det A} \times Adj\ A$

Solved Example:

1. Solve the equation using matrix method:

$x + y + z = 3;\ x + 2y + 3z = 4;\ x + 4y + 9z = 6$

Solution : $x + y + z = 3;$

$x + 2y + 3z = 4;$

$x + 4y + 9z = 6$

Matrix Equation: $A \times X = B$

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

$$\therefore |A| = 1(18-12) - 1(9-3) + 1(4-2) = 6 - 6 + 2 = 2 \neq 0$$

$\therefore A^{-1}$ exists

$$c_{11} = (-1)^{1+1} \times \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = +(18-12) = 6$$

$$c_{12} = (-1)^{1+2} \times \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -(9-3) = -6$$

$$c_{13} = (-1)^{1+3} \times \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = +(4-2) = 2$$

$$c_{21} = (-1)^{2+1} \times \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -(9-4) = -5$$

$$c_{22} = (-1)^{2+2} \times \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = +(9-1) = 8$$

$$c_{23} = (-1)^{2+3} \times \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -(4-1) = -3$$

$$c_{31} = (-1)^{3+1} \times \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = +(3-2) = 1$$

$$c_{32} = (-1)^{3+2} \times \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$c_{33} = (-1)^{3+3} \times \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = +(2-1) = 1$$

$$\therefore C = \begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\therefore Adj\ A = C^t = \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{\det A} \times Adj\ A = \frac{1}{2} \times \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} \times \mathrm{B} = \frac{1}{2} \times \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{2} \times \begin{bmatrix} 18 - 20 + 6 \\ -18 + 32 - 12 \\ 6 - 12 + 6 \end{bmatrix}$$

$$= \frac{1}{2} \times \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\therefore x = 2;\ y = 1;\ z = 0$$

Key Take away from this UO:

1. Inverse of Matrix (Adjoint method)
2. Solution of simultaneous equation by matrix method.