



Basic Mathematics_22103_ UO-2.1

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Unit 2: Trigonometry

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Topic : Allied Angles

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Learning Objective/ Key learning

- Apply the concept of allied angles to solve the given simple engineering problem(s).



Contents

- ▶ Definition of allied angles
- ▶ Trigonometric ratios of allied angles
- ▶ Examples based on allied angles

Allied angles

Definition : If the sum or difference of the measures of two angles is either zero or is an integral multiples of 90° , that is $\pm n \cdot \frac{\pi}{2}$ then these angles are called allied angles.

For any angle θ , let α be It's allied angle then

$$\alpha + \theta = n \cdot \frac{\pi}{2} \Rightarrow \alpha = n \cdot \frac{\pi}{2} - \theta \text{ allied angle of } \theta$$

and
$$\alpha - \theta = n \cdot \frac{\pi}{2} \Rightarrow \alpha = n \cdot \frac{\pi}{2} + \theta \text{ allied angle of } \theta.$$

For any angle of θ ; $n \cdot \frac{\pi}{2} \pm \theta$ are allied angles of θ .

Note: Trigonometric Ratios of Negative angles

$$\sin(-\theta) = -\sin \theta \text{ and } \cos(-\theta) = \cos \theta$$

Trigonometric Ratios of Allied angles

If n is an even integer

$$\text{➤ } \sin\left(n \frac{\pi}{2} \pm \theta\right) = \pm \sin \theta$$

$$\text{➤ } \cos\left(n \frac{\pi}{2} \pm \theta\right) = \pm \cos \theta$$

in If n is an odd integer

$$\text{➤ } \sin\left(n\frac{\pi}{2} \pm \theta\right) = \pm \cos\theta$$

$$\text{➤ } \cos\left(n\frac{\pi}{2} \pm \theta\right) = \pm \sin\theta$$

The algebraic sign is settled down by knowing the quadrant in which the angle $\left(n\frac{\pi}{2} \pm \theta\right)$ lies

Note that

$$\sin n\pi = 0$$

$$\cos n\pi = -1 \quad \text{If } n \text{ is odd integer.}$$

$$\cos n\pi = 1 \quad \text{If } n \text{ is Even integer.}$$

Solved Examples:

1) Without using calculator, find the value of

i) $\sin(-765^\circ)$ ii) $\sec(3660^\circ)$

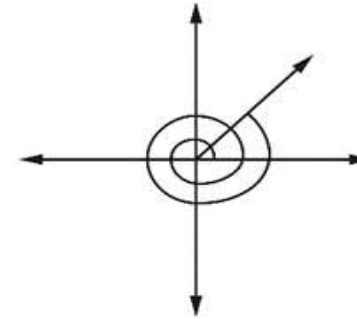
Solution:

i) $\sin(-765^\circ)$

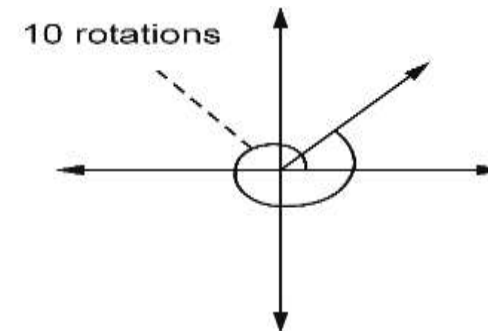
$$\begin{aligned}\sin(-\theta) &= -\sin \theta \\ &= -\sin(765^\circ) \\ &= -\sin(8 \times 90^\circ + 45^\circ) \\ &= -\sin 45^\circ \\ &= -\frac{1}{\sqrt{2}}\end{aligned}$$

ii) $\sec(3660^\circ)$

$$\begin{aligned}&= \sec(40 \times 90^\circ + 60^\circ) \\ &= +\sec 60^\circ \\ &= 2\end{aligned}$$



$\therefore 765^\circ$ lies in first quadrant,
where \sin is +ve
 $n = 8$ (even) function remains same.



3660 lies in Ist quadrant,
where $\sec.$ is positive.
 $n = 40$ (even) function remains same.

2)Without using calculator, find the value of
 $\sin (420^\circ) \cos (390^\circ) + \cos (-300^\circ) \sin (-330^\circ)$

Solution: Given $\sin 420^\circ \cos 390^\circ + \cos (-300^\circ) \sin (-330^\circ)$

$$\sin (420^\circ) = \sin (4 \times 90^\circ + 60^\circ)$$

$$= \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$

$$\cos (390^\circ) = \cos (360^\circ + 30^\circ)$$

$$= \cos (4 \times 90^\circ + 30^\circ)$$

$$= \cos 30^\circ$$

$$= \frac{\sqrt{3}}{2}$$

$$\begin{aligned}\cos (-300^{\circ}) &= \cos (300^{\circ}) \\ &= \cos (3 \times 90^{\circ} + 30^{\circ}) \\ &= \sin 30^{\circ} \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\sin (-330^{\circ}) &= -\sin (330^{\circ}) \\ &= -\sin (360^{\circ} - 30^{\circ}) \\ &= -\sin (4 \times 90^{\circ} - 30^{\circ}) \\ &= -(-\sin 30^{\circ}) \\ &= \sin 30^{\circ} \\ &= \frac{1}{2}\end{aligned}$$



Given $\sin 420^\circ \cos 390^\circ + \cos (-300^\circ) \sin (-330^\circ)$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

So today we learn-

- ▶ Definition of Allied angle
- ▶ Trigonometric ratios of allied angles
- ▶ Solved examples based on allied angles.

Quiz

1) Find the value of $\sin (210^\circ)$

- a) $-\frac{1}{2}$ b) $\frac{7}{2}$ c) $\frac{3}{2}$ d) $\frac{5}{2}$

2) Find the value of $\sec^2 (-765^\circ)$

- a) 1 b) 4 c) 7 d) 2

Ans: 1. a) 2. d)



Thank You