Template: Study Material

<Course Code:22103>: <Subject Code: BMS>: <Subject Name: Basic Mathematics>: <Topic Name: Matrices> : <UO1.3.2> : <Study Material>

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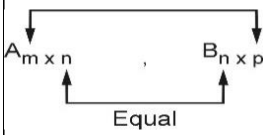
Key words: Matrix Multiplication, Tranpose of Matrix

Learning Objective:
Solve the given system of linear equations using matrix inversion method.

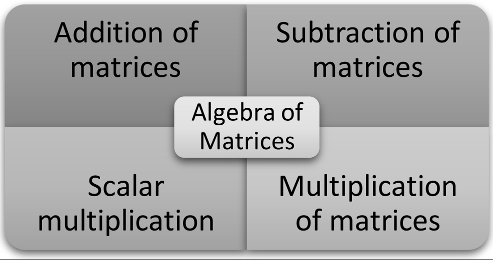
Diagram/ Picture:

Order of A×B is m×p



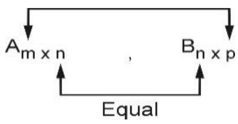
Key Questions: Have you wondered how to multiply two matrices?

Concept Map



Matrix multiplication:

The product of two matrices A and B is possible only if the number of columns in A is equal to the number of rows in B. Let A = $[a_{ij}]$ be an m ×n matrix and B = $[b_{ij}]$ be an n × p matrix.



Order of A× B is m×p

Method of Multiplication of two matrices:

Let A = $\begin{array}{c} R_1 \rightarrow \\ R_2 \rightarrow \end{array}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{array}{c} \begin{bmatrix} p & q & r \\ x & y & z \end{bmatrix} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \end{array} \end{array}$

Then $AB = \begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ R_2C_1 & R_2C_2 & R_2C_3 \end{bmatrix} = \begin{bmatrix} \mathrm{ap+bx} & \mathrm{aq+by} & \mathrm{ar+bz} \\ \mathrm{cp+dx} & \mathrm{cq+dy} & \mathrm{cr+dz} \end{bmatrix}$

Note: R_1C_1 means multiplying the elements of first row of A with corresponding elements of first column of B.

Note: In matrices, matrix multiplication is not commutative.

i.e. $A \times B \neq B \times A$ in general.

continued after the table..........

Key Definitions/ Formula

Transpose of a matrix:

Definition: The transpose of a matrix A is a matrix obtained by interchanging the rows and columns of matrix A. It is denoted by A' or A^t or A^T

Solved word Problem:

If $A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

findA^2

Solution:

$A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4+4 & 8+4 \\ 2+1 & 4+1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix}$

Application of Concept/ Examples in real life: Matrices are used in coding and decoding of information.

Link to YouTube/ OER/ video: http://www.khanacademy

COURSE CONTENT: CONTINUED………

Solved examples:-

1. 1. If $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & -3 \end{bmatrix}$ find i). $\mathrm{A \times B}$

ii). $\mathrm{B \times A}$

Solution: $\mathrm{A \times B} \quad = \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 6-6 & 8-0 & -2+9 \\ 12+0 & 16+0 & -4+0 \\ -3-4 & -4+0 & 1+6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 8 & 7 \\ 12 & 16 & -4 \\ -7 & -4 & 7 \end{bmatrix}$$

$\mathrm{B \times A} \quad = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 6+16+1 & -9+0+2 \\ 4+0+3 & -6+0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & -7 \\ 7 & 0 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -4 \\ 5 & -3 & 4 \end{bmatrix}$ $then\, find\;\; A^2 - 3\mathrm{I}.$

Soln.: $A^2 = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -4 \\ 5 & -3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -4 \\ 5 & -3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 4-1+0 & -2-3+0 & 0+4+0 \\ 2+3-20 & -1+9+12 & 0-12-16 \\ 10-3+20 & -5-9-12 & 0+12+16 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{3} & \mathbf{-5} & \mathbf{4} \\ \mathbf{-15} & \mathbf{20} & \mathbf{-28} \\ \mathbf{27} & \mathbf{-26} & \mathbf{28} \end{bmatrix}$$

$$3I = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore A^2 - 3\mathrm{I} = \begin{bmatrix} 3 & -5 & 4 \\ -15 & 20 & -28 \\ 27 & -26 & 28 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore A^2 - 3\mathrm{I} = \begin{bmatrix} 0 & -5 & 4 \\ -15 & 17 & -28 \\ 27 & -26 & 25 \end{bmatrix}$$

3. Find x and y if $\left\{ 4\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 & -1 \\ 2 & -3 & 4 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

Solution : $\left\{ 4\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{2} & \mathbf{-1} & \mathbf{3} \end{bmatrix} - 2\begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-3} & \mathbf{4} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{2} \\ \mathbf{0} \\ \mathbf{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$

$$\left\{\begin{bmatrix}4 & 8 & 0\\8 & -4 & 12\end{bmatrix}-\begin{bmatrix}2 & 6 & -2\\4 & -6 & 8\end{bmatrix}\right\}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\left\{\begin{bmatrix}4-2 & 8-6 & 0+2\\8-4 & -4+6 & 12-8\end{bmatrix}\right\}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}2 & 2 & 2\\4 & 2 & 4\end{bmatrix}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}4+0-2\\8+0-4\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}2\\4\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

x=2 and y =4

Transpose of a matrix:

Definition: The transpose of a matrix A is a matrix obtained by interchanging the rows and columns of matrix A. It is denoted by A' or A^t or A^T

For e.g.: If $A=\begin{bmatrix}a & b & c\\p & q & r\end{bmatrix}$ $then\ A'=\begin{bmatrix}a & p\\b & q\\c & r\end{bmatrix}$

Properties:

I. $(A')'=A$

II. $(A+B)'=A'+B'$

III. $(A\times B)'=B'\times A'$

IV. $If\ AA'=A'A=I$ then A is called orthogonal matrix.

Symmetric Matrix :

Definition: In a matrix A, if $a_{ij}=a_{ji}$ for all $i\ and\ j$ then matrix is known as symmetric matrix i.e. if $A=A'$ then matrix is known as symmetric matrix.

For e.g. $A=\begin{bmatrix}1 & 2 & -4\\2 & 5 & 3\\-4 & 3 & 9\end{bmatrix}$

Skew Symmetric Matrix:

Definition: In a matrix A, if $a_{ij}=-a_{ji}$ for all $i\ and\ j$ then matrix is known as skew symmetric matrix i.e. if $A=-A'$ then matrix is skew symmetric matrix.

For e.g. $A=\begin{bmatrix}0 & 2 & -3\\-2 & 0 & 5\\3 & -5 & 0\end{bmatrix}$

1. If A = $\begin{bmatrix}2 & -3\\1 & 5\end{bmatrix}$ and B = $\begin{bmatrix}3 & -1 & 2\\1 & 0 & 1\end{bmatrix}$ then verify that (AB)′ = B′A′

Solution :

A = $\begin{bmatrix}2 & -3\\1 & 5\end{bmatrix}$ and B = $\begin{bmatrix}3 & -1 & 2\\1 & 0 & 1\end{bmatrix}$

$$A' = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \quad \text{and} \quad B' = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-3 & -2+0 & 4-3 \\ 3+5 & -1+0 & 2+5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & -2 & 1 \\ 8 & -1 & 7 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix} \qquad \ldots \text{(i)}$$

$$B'.\ A' = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 6-3 & 3+5 \\ -2+0 & -1+0 \\ 4-3 & 2+5 \end{bmatrix}$$

$$B'.\ A' = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix} \qquad \ldots \text{(ii)}$$

From (i) and (ii) $(AB)' = B' \cdot A'$

Key Take away from this UO:

1. Matrix multiplication
2. Transpose of a matrix