

LOGARITHM (Algebra)

Definition: If $y = a^x$, $a > 0$, $a \neq 1$, $a \in \mathbb{R}$, then x is called logarithm of y to the base and it is written as $x = \log_a y$

For example,

1) If $8 = 2^3$ then $3 = \log_2 8$

2) If $3^4 = 81$ then $4 = \log_3 81$

Note: i) $a^x = y$ is called Index form and
 $x = \log_a y$ is called Logarithmic form.

ii) Logarithm of negative number and zero are not defined.

LAWS OF LOGARITHMS.

1) $\log_a(m+n) = \log_a m + \log_a n$

2) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

3) $\log_a(m^n) = n \log_a m$

4) $\log_a^n m = \frac{\log_a m}{\log_a n}$

REMARK:

1) $a^0 = 1 \Rightarrow \log_a 1 = 0$

2) $a^1 = a \Rightarrow \log_a a = 1$

3) $a^{\log_a y} = a \log_a y = y$

Solved Examples.

1) $\log_2 16$

$\therefore 16 = 2^x$

$\therefore 2^4 = 2^x$

$\therefore 2^4 = 2^x$

$\therefore x = 4$

2) $\log_5 125$

$\therefore 125 = 5^x$

$\therefore 5^3 = 5^x$

$\therefore x = 3$

3) $\log_2 14 - \log_2 7$

$\therefore \log_2 \left[\frac{14}{7} \right]$

$\therefore \log_2 2$

$\therefore 2 = 2^x$

$\therefore x = 1$

4) $(\log_3 4) \times (\log_4 81)$

$\therefore \log_3 4 \times \log_4 81$

$\log_3 4 \times \log_4 81$

$\therefore \frac{\log 81}{\log 3} = \frac{\log 3^4}{\log 4}$

$\log_3 4 \times \log_4 81$

$\therefore 4 \times \frac{\log 3}{\log 4}$

$\therefore 4$

5) Find x if

$\log_3 27 = x$

$\therefore 27 = 3^x$

$\therefore 3^3 = 3^x$

$\therefore x = 3$

6) $\log_3 (x+6) = 2$

$\therefore x+6 = 3^2$

$\therefore x+6 = 9$

$\therefore x = 9-6$

$\therefore x = 3$

DETERMINANT (Algebra)

Definition: The arrangement of numbers in equal number of rows and column enclosed between two bar is called determinant.

Determinant of Order 3 :

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

AREA OF A TRIANGLE

Area of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given

$$\text{Area of } \Delta(ABC) = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

CRAMER'S RULE

It is called determinant method

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

Replacing a_1, a_2, a_3
by d_1, d_2, d_3

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

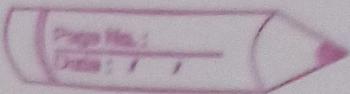
Replacing b_1, b_2, b_3
by d_1, d_2, d_3

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Replacing c_1, c_2, c_3
by d_1, d_2, d_3

Then the solution is :

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{D_z}{D}$$



MATRICES (Algebra)

Definition: A set of $m \times n$ numbers are arranged in a rectangular form form of m rows $\times n$ columns enclosed in form of square bracket is called matrix of order $m \times n$.

TYPES OF MATRICES

1) Row matrix: Matrix having only one row is called Row matrix.

2) Column matrix: Matrix having only 1 column is called column matrix.

3) Square matrix: Matrix having equal no. of rows and columns is called square matrix.

4) Diagonal matrix: A square matrix where all non-diagonal elements are zero is called a Diagonal matrix.

5) Scalar Matrix : A diagonal matrix $K = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ where all diagonal elements are equal is called a scalar matrix.

6) Unit Matrix : A scalar matrix $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ where all diagonal elements are 1 (unit) is called unit identity matrix.

7) Zero Matrix : A matrix having $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ all diagonal elements as zero is called zero matrix
 $A = [0, 0]$

TRANSPOSE OF A MATRIX

The transpose of matrix A is obtained by interchanging rows and columns. It is denoted by A' or A^t or A^T .

For eg. $A = \begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$ then $A' = \begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix}$

SINGULAR MATRIX :

A square matrix A is called singular matrix if $\det(A)$ or $|A| = 0$

NON-SINGULAR MATRIX :

A square matrix A is called non-singular matrix if $\det(A)$ or $|A| \neq 0$

ADJOINT OF A MATRIX :

Adjoint of a matrix is the transpose of co-factor matrix.

INVERSE OF A MATRIX :

If matrix A is non-singular matrix and if there exists a matrix B such that $A \times B = B \times A = I$ Then matrix B is inverse of A .

Notation: Inverse of $A = A^{-1}$

Formula: $A^{-1} = \frac{1}{\det A} \times \text{Adj } A$

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PARTIAL FRACTION (Algebra)

PROPER FRACTION: - If the fraction $\frac{P(x)}{Q(x)}$, if the degree of polynomial $P(x)$ is smaller than the degree of the polynomial $Q(x)$ then it is said to be proper fraction.

$$\text{CASE 1 - } \frac{ax+b}{(x-\alpha)(x-\beta)(x-\gamma)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)} + \frac{C}{(x-\gamma)}$$

$$\text{CASE 2 - } \frac{ax+b}{(x-\alpha)(x-\beta)^2} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)} + \frac{C}{(x-\beta)^2}$$

$$\text{CASE 3 - } \frac{ax+b}{(x-\alpha)(x^2+\beta)} = \frac{A}{(x-\alpha)} + \frac{Bx+C}{(x^2+\beta)}$$

IMPROPER FRACTION: - If the fraction $\frac{P(x)}{Q(x)}$, if the degree of polynomial $P(x)$ is greater than or equal to the degree of polynomial $Q(x)$ then the fraction is improper.

Improper fraction to proper fraction :

Any improper fraction can be expressed as sum of polynomial and a proper fraction by division method.

$$\text{Improper Fraction} = Q + \frac{R}{D}$$

TRIGONOMETRY

Some Important Formula:

1) Fundamental Identities

i) $\sin^2\theta + \cos^2\theta = 1$

ii) $1 + \tan^2\theta = \sec^2\theta$

iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

2) For a right angle triangle.

i) $\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$ iv) $\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$

ii) $\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$ v) $\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$

iii) $\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$ vi) $\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$

3) Measures of an angle.

0°	30°	45°	60°	90°	180°	270°	360°
0°	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

4) Sign Convention :

Quadrant I Quadrant II.

S	A
T	C

Quadrant III Quadrant IV

A - All ratios are P+

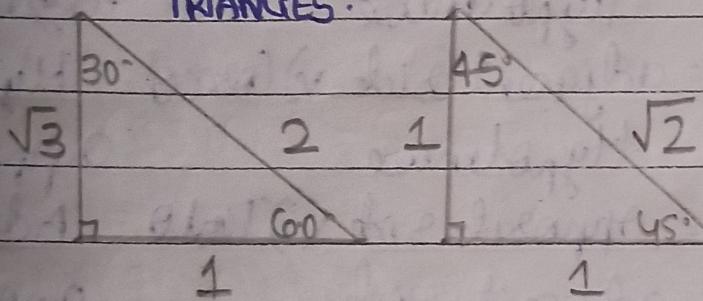
S - Sine ratio is P+

T - Tan ratio is P+

C - Cosine ratio is P+

θ	30°	45°	60°
Ratios			
$\sin \theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan \theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

TRIANGLES.



Quadrant II

$\pi - \theta$ $\sin \theta = +ve$ $2\pi + \theta$ $\sin \theta = +ve$

$2\pi - \theta$ $\cos \theta = -ve$ $4\pi + \theta$ $\cos \theta = +ve$

$5\pi - \theta$ $\tan \theta = -ve$ $6\pi + \theta$ $\tan \theta = +ve$

Quadrant I

$2\pi + \theta$ $\sin \theta = +ve$

$4\pi + \theta$ $\cos \theta = +ve$

$6\pi + \theta$ $\tan \theta = +ve$

Quadrant III

$\pi + \theta$ $\sin \theta = -ve$ $2\pi + \theta$ $\sin \theta = -ve$

$3\pi + \theta$ $\cos \theta = -ve$ $5\pi + \theta$ $\cos \theta = +ve$

$5\pi + \theta$ $\tan \theta = +ve$ $6\pi + \theta$ $\tan \theta = -ve$

Quadrant IV

$2\pi + \theta$ $\sin \theta = -ve$

$4\pi + \theta$ $\cos \theta = +ve$

$6\pi + \theta$ $\tan \theta = -ve$

COMPOUND ANGLES

The angle obtained by addition or subtraction of given angles is called Compound Angle.

$$1) \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$2) \sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$3) \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$4) \cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$5) \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$6) \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

measure of two angles is either zero or is an integral multiple of 90° , that is $n \cdot \frac{\pi}{2}$. These are called Allrod's

$$① \sin(90 + \theta) = \cos \theta$$

$$② \sin(90 - \theta) = \cos \theta$$

$$③ \cos(90 + \theta) = -\sin \theta$$

$$④ \cos(90 - \theta) = \sin \theta$$

$$1) \sin C + \sin D = 2 \sin\left(\frac{(C+D)}{2}\right) \cdot \cos\left(\frac{(C-D)}{2}\right)$$

$$2) \sin C - \sin D = 2 \cos\left(\frac{(C+D)}{2}\right) \cdot \sin\left(\frac{(C-D)}{2}\right)$$

$$3) \cos C + \cos D = 2 \cos\left(\frac{(C+D)}{2}\right) \cdot \cos\left(\frac{(C-D)}{2}\right)$$

$$4) \cos C - \cos D = -2 \sin\left(\frac{(C+D)}{2}\right) \cdot \sin\left(\frac{(C-D)}{2}\right)$$

$$1) S+S = 2SC$$

$$2) S-S = 2CS$$

$$3) C+C = 2CC$$

$$4) C-C = -2SS$$

INVERSE TRIGONOMETRY.

Property I.

1) $\sin(\sin^{-1}x) = x$

2) $\cos(\cos^{-1}x) = x$

3) $\tan(\tan^{-1}x) = x$

4) $\cot(\cot^{-1}x) = x$

5) $\sec(\sec^{-1}x) = x$

6) $\csc(\csc^{-1}x) = x$

Property II.

1) $\sin^{-1}(-x) = -\sin^{-1}x$

2) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

3) $\tan^{-1}(-x) = -\tan^{-1}x$

4) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

5) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

6) $\csc^{-1}(-x) = -\csc^{-1}x$

Property III.

1) $\sin x + \cos^{-1}x = \frac{\pi}{2}$

2) $\tan x + \cot^{-1}x = \frac{\pi}{2}$

3) $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$

Property IV.

1) If $x > 0, y > 0$ and $xy < 1$ then

$$\tan^{-1}x + \tan^{-1}y = \tan\left(\frac{x+y}{1-xy}\right)$$

2) If $x > 0, y > 0$ and $xy > 1$ then

$$\tan^{-1}x + \tan^{-1}y = \tan\left(\frac{x+y}{1-xy} + \pi\right)$$

COORDINATE GEOMETRY:

Straight line

① Slope of a line = $m = \tan \theta$

② Slope of a line = $m = \frac{y_2 - y_1}{x_2 - x_1}$

③ Slope of line = $m = -\frac{A}{B}$

④ Angle between two straight lines is $\theta = \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right|$

⑤ Two lines are parallel if their slopes are equal $\therefore m_1 = m_2$

⑥ Conditions for two lines are perpendicular:
 $\therefore m_1 \times m_2 = -1$

⑦ Slope point form:

Equation of line having slope 'm' and passing through point $A(x_1, y_1)$ is
 $y - y_1 = m(x - x_1)$

3) Two point form:

Equation of line passing through points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

4) Two intercept form:

The equation of line making intercepts a and b on x -axis and y -axis respectively

$$= \frac{x}{a} + \frac{y}{b} = 1.$$

5) Perpendicular distance between 2 ll lines:

The distance between two parallel lines $A_1x + B_1y + C_1$ and $A_2x + B_2y + C_2$ is given

$$d = \left| \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} \right|$$

6) Perpendicular distance between point & line:

If $P(x_1, y_1)$ is any point and $Ax + By + C = 0$ is line, the perpendicular distance of point P from the line is given by

$$\left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$

$$3) \text{ Area of Triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$4) \text{ Area of Equilateral Triangle} = \frac{\sqrt{3}}{4} \times \text{Side}^2$$

$$5) \text{ Area of Rectangle} = \text{length} \times \text{breadth}$$

$$6) \text{ Area of Square} = \text{Side} \times \text{Side}$$

$$7) \text{ Area of Parallelogram} = \text{Base} \times \text{Height}$$

$$8) \text{ Area of Rhombus} = \frac{1}{2} \times \text{diagonal}_1 \times \text{diagonal}_2$$

$$9) \text{ Area of Trapezium} = \frac{1}{2} \times \text{Sum of all sides} \times \text{height}$$

$$10) \text{ Area of Circle} = \pi r^2$$

$$11) \text{ Area of Annulus} = \pi(R^2 - r^2)$$

Volume and Surface Area.

1) Cuboid

$$1.1 \text{ Surface Area of Cuboid} = 2(lb + bh + lh)$$

$$1.2 \text{ Volume of Cuboid} = l \times b \times h$$

2) Cube

$$2.1 \text{ Surface Area of Cuboid} = 2(l)^2$$

$$2.2 \text{ Volume of Cuboid} = l \times l \times l$$

3) Cylinder

$$3.1 \text{ Volume of Cylinder} = \pi r^2 h$$

$$3.2 \text{ T.S.A. of Cylinder} = 2\pi r(r + h)$$

$$3.3 \text{ C.S.A. of Cylinder} = 2\pi rh$$

4) Cone

$$4.1 \text{ Volume of Cone} = \frac{1}{3}\pi r^2 h$$

$$4.2 \text{ T.S.A. of Cone} = \pi r(r + l)$$

$$4.3 \text{ C.S.A. of Cone} = \pi r l$$

5) Sphere

$$5.1 \text{ Volume of Sphere} = \frac{4}{3}\pi r^3$$

$$5.2 \text{ Surface Area of Sphere} = 4\pi r^2$$

6) Hemisphere

$$6.1 \text{ Volume of Hemisphere} = \frac{2}{3}\pi r^3$$

$$6.2 \text{ Surface Area of Hemisphere} = 2\pi r^2$$

STATISTICS

1) RANGE = L - S

L = largest observation

S = smallest observation

2) COEFFICIENT OF RANGE = $\frac{L-S}{L+S}$

3) MEAN DEVIATION

Mean Deviation for Raw data:

i) Mean Deviation about mean = $\frac{\sum |x_i - \bar{x}|}{N} = \frac{\sum |d_i|}{N}$
 where \bar{x} = mean of N observations.

4) Mean Deviation for Ungrouped data:

M.D. about mean = $\frac{\sum f_i |x_i - \bar{x}|}{N} = \frac{\sum f_i |d_i|}{N}$
 where $N = \sum f_i$ and N
 $|d_i| = |x_i - \bar{x}|$

5) STANDARD DEVIATION

Standard Deviation for raw data

i) Standard Deviation = $\sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} = \sqrt{\frac{\sum d_i^2}{N}}$
 \bar{x} = mean

$$d_i = |x_i - \bar{x}|$$

N = Total number of observations.

6) Standard Deviation for ungrouped data

$$S.D. = \sqrt{\frac{\sum f_i d_i^2}{N}}$$

$$N = \sum f_i$$

$$\bar{x} = \text{Mean} = \frac{\sum f_i x_i}{N}$$

$$d_i = (x_i - \bar{x})$$

7) VARIANCE

The square of standard deviation is called the Variance.

Raw Data

$$\text{Variance} = (S.D)^2$$

$$\text{Coefficient of Variance} = \frac{S.D.}{\text{Mean}} \times 100$$

8) COMPARISON OF TWO SETS OF OBS.

less the coefficient of variance the set is more consistent. If a set has greater coefficient of variance then it is not so reliable because it has less consistency, more variants.

All The Best for Finals.