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MSBTE LEAD

Unit: Algebra

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Name of Topic: Matrices

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UO1.3_ Solve the given system of linear equations using matrix inversion method

What we will learn today

1. **Adjoint of a matrix**
2. **Inverse of a matrix**
3. **Solution of simultaneous equation by matrix method**

Key takeaways

Inverse of a matrix.

Solution of simultaneous equation by matrix method

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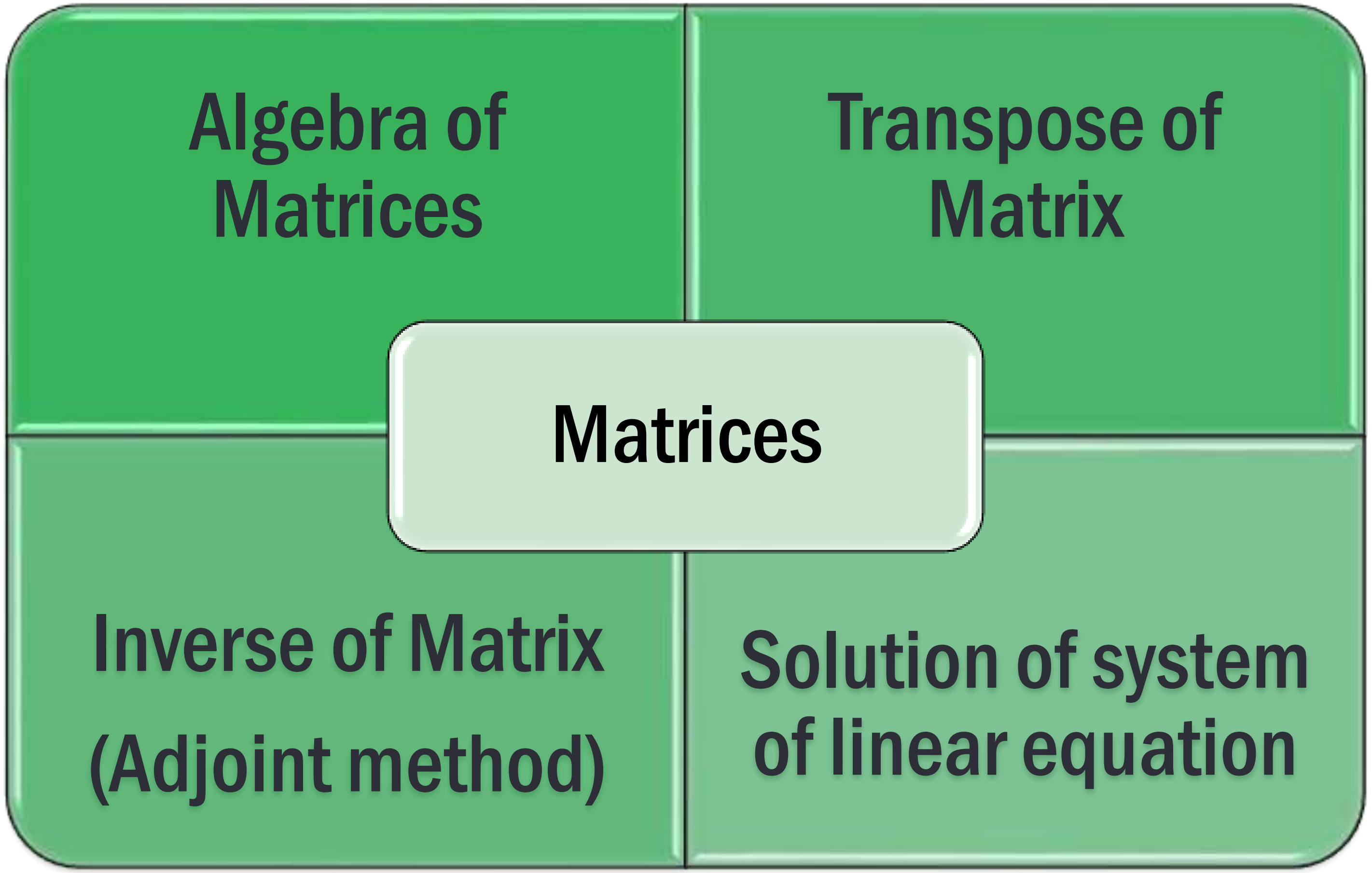
Concept Map


Algebra of Matrices
Transpose of Matrix
Matrices
Inverse of Matrix (Adjoint method)
Solution of system of linear equation

Concept Explanation

Adjoint of a matrix:

Adjoint of a matrix is the transpose of co-factor matrix

$$\therefore Adj\ A = \left[c_{ij}\right]^t$$

Co-factor matrix is a matrix of co-factors=$\left[c_{ij}\right]$

$where\ c_{ij} = (-1)^{i+j} \times M_{ij}$ where

Minor M_{ij} = determinant of matrix obtained by deleting i^{th} row & j^{th} column of given matrix.

Word Problem/ Problem

If A=$\begin{bmatrix} -1 & 1 & 1 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, find Adj A

Solution.: Given A=$\begin{bmatrix} -1 & 1 & 1 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

$$c_{11} = (-1)^{1+1} \times \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} = +(4-4) = 0$$

$$c_{12} = (-1)^{1+2} \times \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -(2-6) = 4$$

$$c_{13} = (-1)^{1+3} \times \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = +(4-12) = -8$$

$$c_{21} = (-1)^{2+1} \times \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = 1$$

$$c_{22} = (-1)^{2+2} \times \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} = +(-1-3) = -4$$

$$c_{23} = (-1)^{2+3} \times \begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} = -(-2-3) = 5$$

$$c_{31} = (-1)^{3+1} \times \begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} = +(2-4) = -2$$

$$c_{32} = (-1)^{3+2} \times \begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} = -(-2-2) = 4$$

$$c_{33} = (-1)^{3+3} \times \begin{vmatrix} -1 & 1 \\ 2 & 4 \end{vmatrix} = +(-4-2) = -6$$

$$\therefore Matrix\ of\ cofactors = C = \begin{bmatrix} 0 & 4 & -8 \\ 1 & -4 & 5 \\ -2 & 4 & -6 \end{bmatrix}$$

$$\therefore Adj\ A = C^t = \begin{bmatrix} 0 & 1 & -2 \\ 4 & -4 & 4 \\ -8 & 5 & -6 \end{bmatrix}$$

Inverse of a Matrix

- **Singular matrix**

  A square matrix A is called singular matrix if det (A) or |A| = 0.

- **Non-Singular matrix**

  A square matrix A is called non-singular, if det (A) or |A| $\neq$ 0.

- **Inverse of a matrix:**

  Given a non-singular matrix 'A', if there exists a matrix 'B' such that $\mathrm{A} \times \mathrm{B} = \mathrm{B} \times \mathrm{A} = \mathrm{I}$ then matrix B is the inverse of matrix A.

  Notation: Inverse of A = A^{-1}

  Formula: $\mathbf{A^{-1}} = \frac{\mathbf{1}}{\mathbf{det\ A}} \times \mathbf{Adj\ A}$

2. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix}$

Solution : Given $A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix}$

$$|A| = 3(3+1) - 1(12-2) + 2(-4-2)$$

$$= 12 - 10 - 12 = -10 \neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$c_{11} = (-1)^{1+1} \times \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = +(3+1) = 4$$

$$c_{12} = (-1)^{1+2} \times \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = -(12-2) = -10$$

$$c_{13} = (-1)^{1+3} \times \begin{vmatrix} 4 & 1 \\ 2 & -1 \end{vmatrix} = +(-4-2) = -6$$

$$c_{21} = (-1)^{2+1} \times \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = -(3+2) = -5$$

$$c_{22} = (-1)^{2+2} \times \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = +(9-4) = 5$$

$$c_{23} = (-1)^{2+3} \times \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = -(-3-2) = 5$$

$$c_{31} = (-1)^{3+1} \times \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = +(1-2) = -1$$

$$c_{32} = (-1)^{3+2} \times \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = -(3-8) = 5$$

$$c_{33} = (-1)^{3+3} \times \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} = +(3-4) = -1$$

$$\therefore C = \begin{bmatrix} 4 & -10 & -6 \\ -5 & 5 & 5 \\ -1 & 5 & -1 \end{bmatrix} \qquad \therefore Adj\ A = C^t = \begin{bmatrix} 4 & -5 & -1 \\ -10 & 5 & 5 \\ -6 & 5 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-10} \times \begin{bmatrix} 4 & -5 & -1 \\ -10 & 5 & 5 \\ -6 & 5 & -1 \end{bmatrix}$$

Solution of simultaneous equations:

Suppose $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

are the given simultaneous equations.

These equations can be represented in matrix form as follows:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$ i.e. $\mathrm{A} \times \mathrm{X} = \mathrm{B}$ where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}; \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

The solution of the system of equations is given by $\boldsymbol{X = A^{-1} \times \mathrm{B}}$ where $A^{-1} = \frac{1}{\det A} \times Adj\ A$

Solve the equations using matrix method:

1. $x + y + z = 3;\ x + 2y + 3z = 4;\ x + 4y + 9z = 6$

Solution : $x + y + z = 3;\ x + 2y + 3z\ ;\quad x + 4y + 9z = 6$

Matrix Equation: $A \times X = B$

Where$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

$$\therefore |A| = 1(18 - 12) - 1(9 - 3) + 1(4 - 2) = 6 - 6 + 2 = 2 \neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$c_{11} = (-1)^{1+1} \times \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = +(18 - 12) = 6$$

$$c_{12} = (-1)^{1+2} \times \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -(9 - 3) = -6$$

$$c_{13} = (-1)^{1+3} \times \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = +(4 - 2) = 2$$

$$c_{21} = (-1)^{2+1} \times \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -(9 - 4) = -5$$

$$c_{22} = (-1)^{2+2} \times \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = +(9 - 1) = 8$$

$$c_{23} = (-1)^{2+3} \times \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -(4-1) = -3$$

$$c_{31} = (-1)^{3+1} \times \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = +(3-2) = 1$$

$$c_{32} = (-1)^{3+2} \times \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$c_{33} = (-1)^{3+3} \times \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = +(2-1) = 1$$

$$\therefore C = \begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\therefore Adj\ A = C^t = \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{\det A} \times AdjA = \frac{1}{2} \times \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} \times \mathrm{B} = \frac{1}{2} \times \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix} = \frac{1}{2} \times \begin{bmatrix} 18-20+6 \\ -18+32-12 \\ 6-12+6 \end{bmatrix} = \frac{1}{2} \times \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

The solution is x = 2 ; y = 1 ; z = 0

Quiz

Question No 1	Question No 2	Question No 3
State the formula of A^{-1}	Find inverse of A= $\begin{bmatrix} -1 & 1 & 1 \\ -2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$	The solution X of the matrix equation $A \times X = B$ is given by
Recall/ Remembering	Understanding	Application
$a)\ A^{-1} = \frac{1}{\det A} \times \text{Adj } A$	$a)\ A^{-1} = \begin{bmatrix} -2 & -1 & 0 \\ -8 & -4 & 0 \\ -10 & -5 & 0 \end{bmatrix}$	a) $X = A^{-1} \times B$
b) $A^{-1} = \frac{-1}{\lvert A \rvert} \times \text{Adj } A$	b) $A^{-1} = \begin{bmatrix} -2 & -8 & -10 \\ -1 & -4 & -5 \\ 0 & -5 & 0 \end{bmatrix}$	b) $X = B \times A^{-1}$
c) $A^{-1} = [c_{ij}]^t$	c) $A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 8 & -4 & 0 \\ -10 & 5 & 0 \end{bmatrix}$	c) $X = A \times B$
d) All of above	d) Inverse does not exist.	d) $X = B \times A$

Ans: < a >	Ans: <d>	Ans: <a>