Basic Mathematics_Code:22103_CO1_UO1.3.2

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Date: 12 July 2020

MSBTE LEAD

Unit: Algebra

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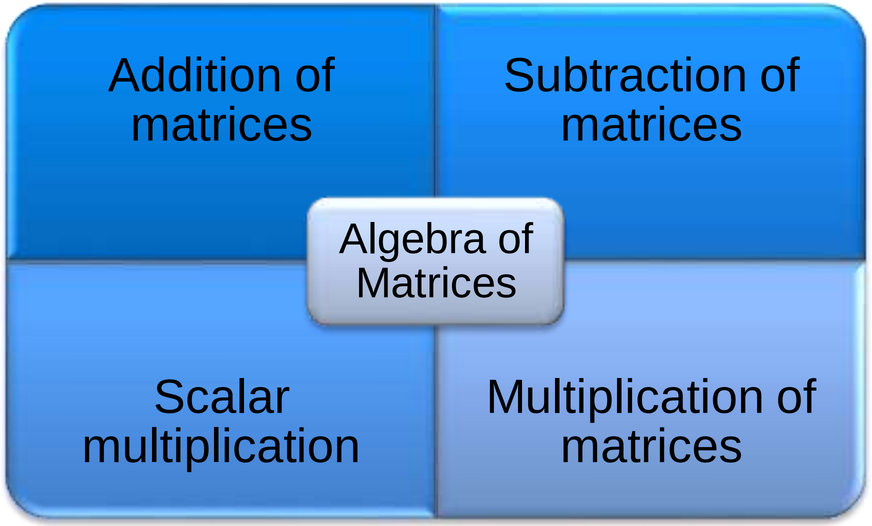
Name of Topic:Matrices

12 July 2020

UO1.3_ Solve the given system of linear equations using matrix inversion method

Concept Map


Addition of matrices
Subtraction of matrices
Algebra of Matrices
Scalar multiplication
Multiplication of matrices

What we will learn today

1. **Multiplication of matrices**
2. **Transpose of a matrix**

Key takeaways

Multiplication of matrices

Transpose of a matrix

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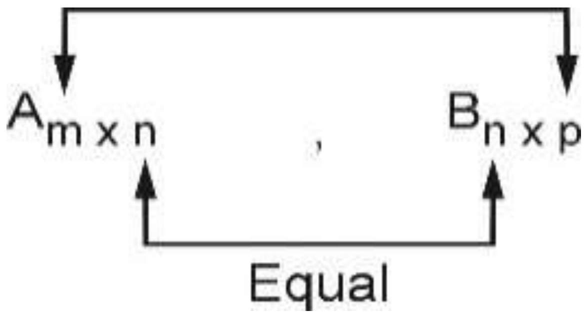
Matrix multiplication

Condition for multiplication of two matrices

The product of two matrices A and B is possible only if the **number of columns in A is equal to the number of rows in B**.

Let $A = [a_{ij}]$ be a 'm × n' matrix and $B = [b_{ij}]$ be a 'n × p' matrix.

then Order of $A \times B$ is $m \times p$



Matrix multiplication

Method of Multiplication of two matrices:

Let A = $\begin{array}{c c} R_1 \longrightarrow & \\ R_2 \longrightarrow & \end{array} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{array}{c} \begin{bmatrix} p & q & r \\ x & y & z \end{bmatrix} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \end{array} \end{array}$

then $AB = \begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ R_2C_1 & R_2C_2 & R_2C_3 \end{bmatrix} = \begin{bmatrix} \mathrm{ap+bx} & \mathrm{aq+by} & \mathrm{ar+bz} \\ \mathrm{cp+dx} & \mathrm{cq+dy} & \mathrm{cr+dz} \end{bmatrix}$

Note: R_1C_1 means multiplying the elements of first row of A with corresponding elements of first column of B.

Note: In matrices, matrix multiplication is not commutative.

i.e. $\mathrm{A} \times \mathrm{B} \neq \mathrm{B} \times A$ in general

Word Problem/ Problem

1. If $A = \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & -3 \end{bmatrix}$ find i). $A \times B$

ii). $B \times A$

Solution: $A \times B = \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 6-6 & 8-0 & -2+9 \\ 12+0 & 16+0 & -4+0 \\ -3-4 & -4+0 & 1+6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 8 & 7 \\ 12 & 16 & -4 \\ -7 & -4 & 7 \end{bmatrix}$$

Word Problem/ Problem

$$B \times A = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 16 + 1 & -9 + 0 + 2 \\ 4 + 0 + 3 & -6 + 0 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & -7 \\ 7 & 0 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -4 \\ 5 & -3 & 4 \end{bmatrix}$ $then\ find\ \ A^2 - 3\mathrm{I}.$

Solution: $A^2 = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -4 \\ 5 & -3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -4 \\ 5 & -3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 4-1+0 & -2-3+0 & 0+4+0 \\ 2+3-20 & -1+9+12 & 0-12-16 \\ 10-3+20 & -5-9-12 & 0+12+16 \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{-5} & \mathbf{4} \\ \mathbf{-15} & \mathbf{20} & \mathbf{-28} \\ \mathbf{27} & \mathbf{-26} & \mathbf{28} \end{bmatrix}$$

$$3I = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- $\therefore A^2 - 3\mathrm{I} = \begin{bmatrix} 3 & -5 & 4 \\ -15 & 20 & -28 \\ 27 & -26 & 28 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -5 & 4 \\ -15 & 17 & -28 \\ 27 & -26 & 25 \end{bmatrix}$

3. Find x & y if $\left\{4\begin{bmatrix}1 & 2 & 0\\ 2 & -1 & 3\end{bmatrix} - 2\begin{bmatrix}1 & 3 & -1\\ 2 & -3 & 4\end{bmatrix}\right\}\begin{bmatrix}2\\ 0\\ -1\end{bmatrix} = \begin{bmatrix}x\\ y\end{bmatrix}$

Solution:$Given$ $\left\{4\begin{bmatrix}1 & 2 & 0\\ 2 & -1 & 3\end{bmatrix} - 2\begin{bmatrix}1 & 3 & -1\\ 2 & -3 & 4\end{bmatrix}\right\}\begin{bmatrix}2\\ 0\\ -1\end{bmatrix} = \begin{bmatrix}x\\ y\end{bmatrix}$

$$\begin{bmatrix}2 & 2 & 2\\ 4 & 2 & 4\end{bmatrix}\begin{bmatrix}2\\ 0\\ -1\end{bmatrix} = \begin{bmatrix}x\\ y\end{bmatrix}$$

$$\therefore \begin{bmatrix}4+0-2\\ 8+0-4\end{bmatrix} = \begin{bmatrix}x\\ y\end{bmatrix}$$

$$\therefore \begin{bmatrix}2\\ 4\end{bmatrix} = \begin{bmatrix}x\\ y\end{bmatrix}$$

x = 2 and y = 4

Transpose of a matrix:

Definition: The transpose of a matrix A is a matrix obtained by interchanging the rows and columns of matrix A. It is denoted by A' or A^t or A^T

For e.g.: If $A = \begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$ then $A' = \begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix}$

Properties:

- $(A')' = A$
- $(A + B)' = A' + B'$
- $(A \times B)' = B' \times A'$
- If $A \times A' = A' \times A = I$ then A is called orthogonal matrix.

Symmetric Matrix :

Definition: In a matrix A, if $a_{ij} = a_{ji}$ for all $i \ and \ j$ then matrix is known as symmetric matrix i.e. if $A = A'$ then matrix is known as symmetric matrix.

For e.g. $A = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 5 & 3 \\ -4 & 3 & 9 \end{bmatrix}$

Skew Symmetric Matrix:

Definition: In a matrix A, $\mathrm{if}\ \mathrm{a_{ij}} = -\mathrm{a_{ji}}$ for all $i \ and \ j$ then matrix is known as skew symmetric matrix i.e. if $A = -A'$ then matrix is skew symmetric matrix.

For e.g. $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix}$

Word Problem/ Problem

1. If $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ then verify that $(AB)' = B'A'$

Solution :

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \quad \text{and} \quad B' = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-3 & -2+0 & 4-3 \\ 3+5 & -1+0 & 2+5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & -2 & 1 \\ 8 & -1 & 7 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix} \qquad \ldots (i)$$

$$B'\ A' = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 6-3 & 3+5 \\ -2+0 & -1+0 \\ 4-3 & 2+5 \end{bmatrix}$$

$$B'\ A' = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix} \qquad \ldots (ii)$$

From (i) and (ii) $(AB)' = B' \cdot A'$

Quiz

Set 1: Question No 1	Set 1: Question No 2	Set 1: Question No 3
State the order of the product matrix A×B if matrix A is of order 2×3 and matrix B is of order 3×3.	State the order of the product matrix B× A if matrix A is of order 2× 2 and matrix B is of order 3×3.	State the order of transpose of matrix $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$
Recall/ Remembering	Understanding	Application
a) 3× 3	a) 2×2	a) 2 × 1
b) 2×2	b) 3 × 2	b) 1 × 2
c) 2×3	c) 3×3	c) 1 × 3
d) 3×2	d) Product does not exist	d) 3 ×1

Ans: <c>	Ans: <d >	Ans: <c >