Q. Bank

* Trigonometry

① $\frac{\tan 66 + \tan 69}{1 - [\tan 66 \times \tan 69]} = \tan(66+69)$ — [Formula (Converse)]

$= \tan(135)$

$= \tan(\pi - 45)$

$= -\tan 45$

$= -1$

② Value of $\tan(765) + \cot(225)^\circ$ is

→ $\tan(765) + \cot(225)$

$= \tan(4\pi + 45) + \cot(\pi + 45)$

$= \tan 45 + \cot 45$

$= 1 + 1 = 2$

③ $\tan A = 1/2$ → A → 3rd Quadrant Find Sin A

$\therefore \tan A = \frac{1}{2}$ ← ∵ (tan is +ve in 3rd Quadrant)

$1 + \tan^2 A = \sec^2 A$

$1 + (1/2)^2 = \sec^2 A$

$\frac{2^2 + 1}{2^2} = \sec^2 A$

$\frac{4+1}{4} = \sec^2 A$

$\sec^2 A = \frac{5}{4}$ $\cancel{\therefore \sec A = \frac{\sqrt{5}}{2}}$

$\therefore \cos^2 A = \frac{1}{\sec^2 A}$

$= \frac{4}{5}$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \frac{4}{5}$

$\sin^2 A = \frac{5-4}{5}$

$\sin^2 A = \frac{1}{5}$

$\therefore \sin A = \frac{1}{\sqrt{5}}$

④ Tangential form of $\cos 2\theta$ is

$\rightarrow$ $\cos 2\theta = \boxed{\dfrac{1-\tan^2\theta}{1+\tan^2\theta}}$ — Formula

⑤ $\tan\left(\frac{A}{2}\right) = \frac{1}{\sqrt{3}}$ $\qquad \operatorname{cosec} A = ?$

$\rightarrow$ We know, $\operatorname{cosec} A = \frac{1}{\sin A}$

$\therefore \sin A = \dfrac{2\tan\left(\frac{A}{2}\right)}{1+\tan^2\left(\frac{A}{2}\right)}$

$= \dfrac{2 \times 1/\sqrt{3}}{1 + 1/3}$

$= \dfrac{2/\sqrt{3}}{\frac{3+1}{3}} = \dfrac{2/\sqrt{3}}{4/3} = \dfrac{\cancel{2}^{1}}{\sqrt{3}} \times \dfrac{3}{\cancel{4}_{2}}$

$\therefore \sin A = \dfrac{3}{2\sqrt{3}}$

$\therefore \operatorname{Cosec} A = \dfrac{1}{\frac{3}{2\sqrt{3}}} = \dfrac{2\sqrt{3}}{3}$

⑥ $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{2\sin\theta/2 \times \cos\theta/2}{2\cos^2\theta/2}$

$= \dfrac{\cancel{2}\sin\theta/2}{\cancel{2}\cos\theta/2}$

$= \tan\theta/2$

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③ $\tan A = 1/2 \longrightarrow A \rightarrow$ 3rd Quadrant Find $\underline{\sin A}$

$\therefore \tan A = \frac{1}{2}$ $\leftarrow$ $\because$ ($\tan$ is +ve in 3rd Quadrant)

$1 + \tan^2 A = \sec^2 A$

$1 + (1/2)^2 = \sec^2 A$

$\frac{2^2 + 1}{2^2} = \sec^2 A$

$\frac{4+1}{4} = \sec^2 A$

$\sec^2 A = \frac{5}{4}$ $\cancel{\therefore \sec A = \frac{\sqrt{5}}{2}}$

$\therefore \cos^2 A = \frac{1}{\sec^2 A}$

$= \cancel{\frac{1}{\sqrt{5}}}\ \frac{4}{5}$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \frac{4}{5}$

$\sin^2 A = \frac{5-4}{5}$

$\sin^2 A = \frac{1}{5}$

$\therefore \sin A = \frac{1}{\sqrt{5}}$

(4) Tangential form of $\cos 2\theta$ is

→ $\cos 2\theta = \boxed{\dfrac{1-\tan^2\theta}{1+\tan^2\theta}}$ — Formula

(5) $\tan\left(\dfrac{A}{2}\right) = \dfrac{1}{\sqrt{3}}$ $\qquad \operatorname{cosec} A = ?$

→ We know, $\operatorname{cosec} A = \dfrac{1}{\sin A}$

$\therefore \sin A = \dfrac{2\tan\left(\frac{A}{2}\right)}{1+\tan^2\left(\frac{A}{2}\right)}$

$= \dfrac{2 \times 1/\sqrt{3}}{1+1/3}$

$= \dfrac{2/\sqrt{3}}{\frac{3+1}{3}} = \dfrac{2/\sqrt{3}}{4/3} = \dfrac{\cancel{2}^{1}}{\sqrt{3}} \times \dfrac{3}{\cancel{4}_{2}}$

$\therefore \sin A = \dfrac{3}{2\sqrt{3}}$

$\therefore \operatorname{Cosec} A = \dfrac{1}{\frac{3}{2\sqrt{3}}} = \dfrac{2\sqrt{3}}{3}$

(6) $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{2\sin\theta/2 \times \cos\theta/2}{2\cos^2\theta/2}$

$= \dfrac{\cancel{2}\sin\theta/2}{\cancel{2}\cos\theta/2}$

$= \tan\theta/2$

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(7) $\dfrac{\sin 3A - \sin A}{\cos 3A + \cos A} = \dfrac{2\cos\left(\frac{3A+A}{2}\right) \times \sin\left(\frac{3A-A}{2}\right)}{2\cos\left(\frac{3A+A}{2}\right) \times \cos\left(\frac{3A-A}{2}\right)}$

$= \dfrac{\sin\left(\cancel{3A-A}\ \frac{2A}{2}\right)}{\cos\left(2A/2\right)}$

$= \dfrac{\sin A}{\cos A} = \tan A$

(12) $2 \times \sin 60 \times \cos 20 = \sin A + \sin B$

→ Taking RHS,

$\sin A + \sin B$

$= 2\sin\left(\frac{A+B}{2}\right) \times \cos\left(\frac{A-B}{2}\right)$ —— (1)

But $\sin A + \sin B = 2\sin 60 \times \cos 20$ —— (2)

$\therefore$ From (1) and (2),

$2\sin\left(\frac{A+B}{2}\right) \times \cos\left(\frac{A-B}{2}\right) = 2\sin 60 \times \cos 20$

Let's consider $\sin\left(\frac{A+B}{2}\right) = \sin 60$

and $\cos\left(\frac{A-B}{2}\right) = \cos 20$

$\therefore \sin\left(\frac{A+B}{2}\right) = \sin 60$

Comparing both sides,

$\frac{A+B}{2} = 60$

$A+B = 120$ —— (3)

$\cos\left(\frac{A-B}{2}\right) = \cos 20$

Comparing

$\frac{A-B}{2} = 20$

$A-B = 40$ —— (4)

Adding eq^n 3 and eq^n 4,

$$A + \cancel{B} = 120$$
$$A - \cancel{B} = 40$$
$$2A = 160$$
$$\boxed{A = 80} \longrightarrow \text{in } eq^n \text{ 3},$$

$$80 + B = 120$$
$$B = 120 - 80 = 40 \quad \therefore \boxed{B = 40}$$

13 $\sin A = \frac{1}{2}$ Find $\sin 3A$

$$\rightarrow \sin 3A = 3\sin A - 4\sin^3 A$$
$$= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$$
$$= \frac{3}{2} - 4 \times \frac{1}{8}$$
$$= \frac{3}{2} - \frac{1}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

14 $\sqrt{2+2\cos\theta}$
$$= \sqrt{2(1+\cos\theta)}$$
$$= \sqrt{2\left(2\cos^2\left(\frac{\theta}{2}\right)\right)}$$
$$= 2\cos\left(\frac{\theta}{2}\right)$$

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(28) $\sin(\theta + \pi/6) - \sin(\theta - \pi/6)$

$= \sin(\theta+30) - \sin(\theta - 30)$

Method 1. Applying factorization formula $\sin A - \sin B$

$\rightarrow 2\cos\left[\frac{\theta+30+\theta-30}{2}\right] \times \sin\left[\frac{\theta+30-(\theta-30)}{2}\right]$

$= 2\cos\left[\frac{2\theta}{\cancel{2}}\right] \times \sin\left[\frac{\cancel{\theta}+30-\cancel{\theta}+30}{2}\right]$

$= 2\cos[\theta] \times \sin\left[\frac{60}{2}\right]$

$= 2\cos\theta \times \sin 30 = \cancel{2}\cos\theta \times \frac{1}{\cancel{2}} = \cos\theta$

(29) Method 2 $\rightarrow$

Applying $\sin(A+B)$ and $\sin(A-B)$,

$\rightarrow (\sin\theta \times \cos 30 + \cos\theta \times \sin 30) - (\sin\theta \times \cos 30 - \cos\theta \times \sin 30)$

$= \cancel{\sin\theta \times \frac{\sqrt{3}}{2}} + \cos\theta \times \frac{1}{2} - \{\cancel{\sin\theta \times \frac{\sqrt{3}}{2}} + \cos\theta \times \frac{1}{2}$

$= \frac{\cos\theta}{2} + \frac{\cos\theta}{2} = \frac{2\cos\theta}{2} = \cos\theta$

(29) $\sin 75 - \sin 15$

→ Factorization formula of $\sin A - \sin B$

$\sin 75 - \sin 15$

$= 2\cos\left(\frac{75+15}{2}\right) \times \sin\left(\frac{75-15}{2}\right)$

$= 2\cos\left(\frac{90}{2}\right) \times \sin\left(\frac{60}{2}\right)$

$= 2\cos 45 \times \sin 30 = \cancel{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{\cancel{2}} = \frac{1}{\sqrt{2}}$

(30) $\frac{\sin 8\theta + \sin 2\theta}{\cos 8\theta + \cos 2\theta}$

→ Applying factorization formula of $(\sin A + \sin B)$ in N^r and $(\cos A + \cos B)$ in D^r,

$\frac{\sin 8\theta + \sin 2\theta}{\cos 8\theta + \cos 2\theta}$

$= \frac{\cancel{2}\sin\left(\frac{8\theta+2\theta}{2}\right) \times \cancel{\cos\left(\frac{8\theta-2\theta}{2}\right)}}{\cancel{2}\cos\left(\frac{8\theta+2\theta}{2}\right) \times \cancel{\cos\left(\frac{8\theta-2\theta}{2}\right)}}$

$= \frac{\sin (10\theta/2)}{\cos(10\theta/2)} = \frac{\sin 5\theta}{\cos 5\theta} = \tan 5\theta$