Basic Mathematics_22103_CO5_UO5.3

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Statistics

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Mean deviation, Standard deviation

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Calculate mean and standard deviation of discrete and grouped data related to the given simple engineering problem.

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Content

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2. Mean deviation about mean for ungrouped data

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6. Standard deviation for grouped data
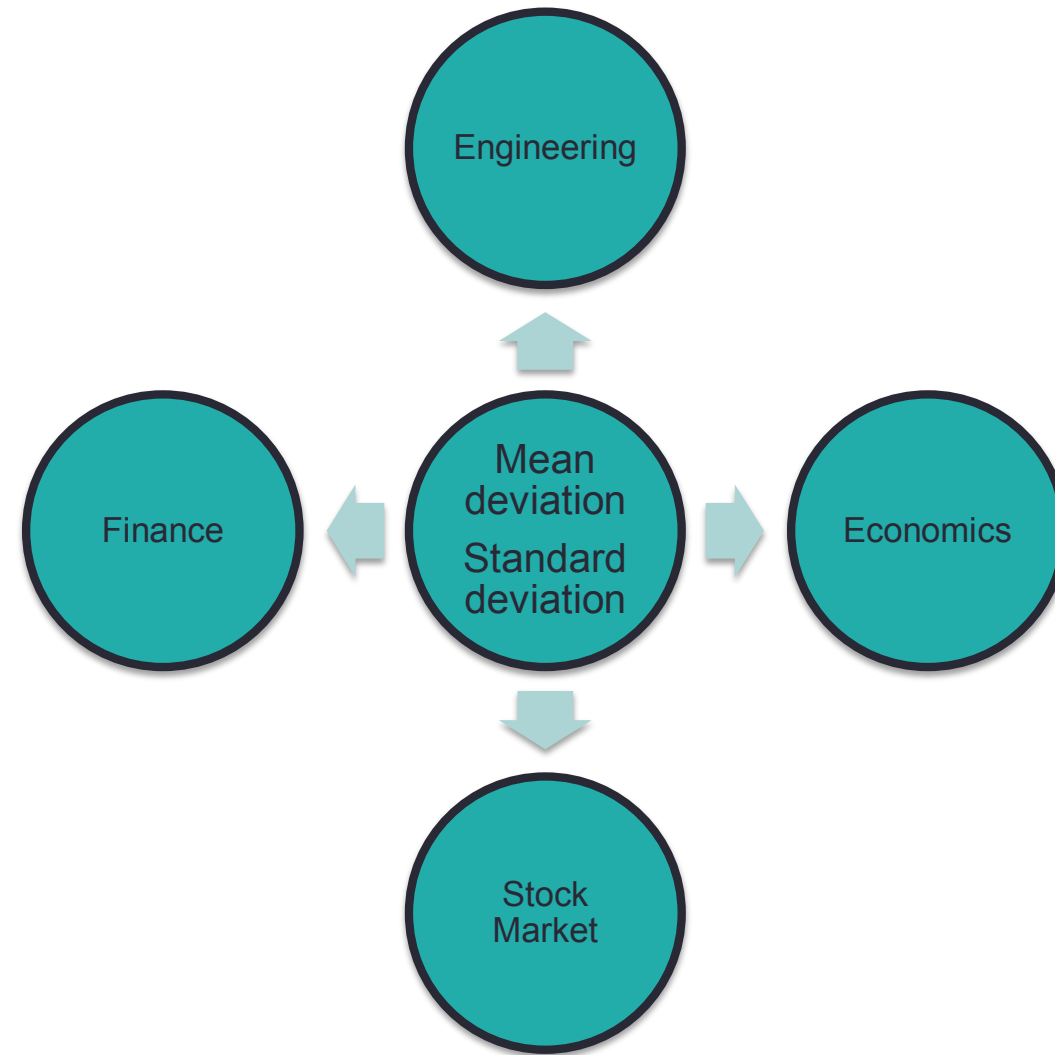
Key takeaways

Mean deviation

Standard deviation

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Concept Map

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► Mean deviation (M.D.)

► For raw data:

Mean deviation about mean $= \dfrac{\sum |x_i - \bar{x}|}{N} = \dfrac{\sum |d_i|}{N}$

where $\bar{x}$ = mean of N observations

► Example:

Find mean deviation about mean of the following data:

2. 3, 4, 5, 6, 7

Solution:

$$\bar{x} = \frac{\sum x_i}{N} = \frac{2+3+4+5+6+7}{6}$$
$$= 4.5$$

Solution

x_i	$\lvert d_i \rvert = \lvert x_i - \bar{x} \rvert$
2	2.5
3	1.5
4	0.5
5	0.5
6	1.5
7	2.5
	$\sum \lvert d_i \rvert = 9$

$$\text{M.D.} = \frac{\sum \lvert d_i \rvert}{N} = \frac{9}{6}$$

$$= 1.5$$

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► For ungrouped data:

Mean deviation about mean $= \dfrac{\sum f_i \left| x_i - \bar{x} \right|}{\sum f_i} = \dfrac{\sum f_i \left| d_i \right|}{N}$

► Example:

Calculate mean deviation from mean for the following data:

Marks	3	4	5	6	7	8
No. of students	1	3	7	5	2	2

Solution:

x_i	f_i	$f_i x_i$	$\|d_i\| = \|x_i - \bar{x}\|$	$f_i\|d_i\|$
3	1	3	2.5	2.5
4	3	12	1.5	4.5
5	7	35	0.5	3.5
6	5	30	0.5	2.5
7	2	14	1.5	3.0
8	2	16	2.5	5.0
	$\sum f_i = 20$	$\sum f_i x_i = 110$		$\sum f_i\|d_i\| = 21$

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{N} = \frac{110}{20} = 5.5$$

$$\text{M.D.} = \frac{\sum f_i\|d_i\|}{N} = \frac{21}{20} = 1.05$$

► For grouped data:

$$\text{M.D. about mean} = \frac{\sum f_i \left| x_i - \bar{x} \right|}{\sum f_i} = \frac{\sum f_i \left| d_i \right|}{N}$$

where x_i = Mid-value

► Example:

Find mean deviation of the following data:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	1	2	4	2	1

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Solution:

Class	f_i	x_i	f_ix_i	$\|d_i\|=\|x_i-\bar{x}\|$	$f_i\|d_i\|$
0-10	1	5	5	20	20
10-20	2	15	30	10	20
20-30	4	25	100	0	0
30-40	2	35	70	10	20
40-50	1	45	45	20	20
	$\sum f_i =10$		$\sum f_ix_i =250$		$\sum f_i\|d_i\|=80$

$$\text{Mean} = \bar{x} = \frac{\sum f_ix_i}{N} = \frac{250}{10} = 25$$

$$\text{M.D. about mean} = \frac{\sum f_i\|d_i\|}{N}$$

$$= \frac{80}{10} = 8$$

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► Standard deviation:

► For raw data:

$$S.D. = \sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{N}} = \sqrt{\frac{\sum d_i^2}{N}}$$

► Example:

Find standard deviation for:

9, 10, 12, 13, 18, 20, 21, 25

Solution:
$$\bar{x} = \frac{\sum x_i}{N} = \frac{9+10+12+13+18+20+21+25}{8} = 16$$

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x_i	$d_i = x_i - \bar{x}$	d_i^2
9	-7	49
10	-6	36
12	-4	16
13	-3	9
18	2	4
20	4	16
21	5	25
25	9	81
		$\sum d_i^2 = 236$

$$\sigma = \sqrt{\frac{\sum d_i^2}{N}}$$

$$= \sqrt{\frac{236}{8}} = 5.431$$

► For ungrouped data:

$$S.D. = \sigma = \sqrt{\frac{\sum f_i d_i^2}{N}}$$

► Example:

   Calculate standard deviation for the following:

x_i	27	28	29
f_i	1	7	2

Solution:

x_i	f_i	$f_i x_i$	$d_i = x_i - \bar{x}$	d_i^2	$f_i d_i^2$
27	1	27	-1.1	1.21	1.21
28	7	196	-0.1	0.01	0.07
29	2	58	0.9	0.81	1.62
	$\sum f_i = 10$	$\sum f_i x_i = 281$			$\sum f_i d_i^2 = 2.9$

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$= \frac{281}{10} = 28.1$$

$$\text{S.D.} = \sigma = \sqrt{\frac{\sum f_i d_i^2}{N}}$$

$$= \sqrt{\frac{2.9}{10}} = 0.539$$

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► For grouped data:

$$\text{S.D.} = \sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}} = \sqrt{\frac{\sum f_i d_i^2}{N}}$$

where x_i = mid-value

► Example:

The following table shows the chest measurement of 100 students. Calculate the standard deviation.

Chest in cm	67-74	75-81	82-88	89-95	96-102	103-109
No. of students	5	31	40	20	3	1

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Solution

Class	Continuous class	f_i	x_i	$f_i x_i$	$d_i = \lvert x_i - \bar{x} \rvert$	d_i^2	$f_i d_i^2$
68-74	67.5-74.5	5	71	355	13.16	173.1856	865.928
75-81	74.5-81.5	31	78	2418	6.16	37.9456	1176.3136
82-88	81.5-88.5	40	85	3400	0.84	0.7056	28.224
89-95	88.5-95.5	20	92	1840	7.84	61.4656	1229.312
96-102	95.5-102.5	3	99	297	14.84	220.2256	660.6768
103-109	102.5-109.5	1	106	106	21.84	476.9856	476.9856
		$\sum f_i = 100$		$\sum f_i x_i = 8416$			$\sum f_i d_i^2 = 4437.44$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{8416}{100} = 84.16$$

$$S.D. = \sqrt{\frac{\sum f_i d_i^2}{N}}$$

$$= \sqrt{\frac{4437.44}{100}} = 6.661$$

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Quiz

▶ Q 1.Mean deviation about mean for raw data is:

▶ a) $\dfrac{\sum |d_i|}{N}$ b) $\dfrac{\sum f_i |d_i|}{N}$ c) $\sqrt{\dfrac{\sum d_i^2}{N}}$ d) $\sqrt{\dfrac{\sum f_i d_i^2}{N}}$

Q 2. Standard deviation for grouped data is:

a) $\dfrac{\sum |d_i|}{N}$ b) $\dfrac{\sum f_i |d_i|}{N}$ c) $\sqrt{\dfrac{\sum d_i^2}{N}}$ d) $\sqrt{\dfrac{\sum f_i d_i^2}{N}}$

Ans. 1) a   2) d