Basic Mathematics_22103_UO2.3

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Unit 2: Trigonometry

Topic : Factorization and De-factorization

Course Outcome:

- **Co2:** Utilize basic concepts of trigonometry to solve elementary engineering problems.

- **Learning Objectives:**

  Employ concept of factorization and de-factorization formulae to solve the given simple engineering problem(s).

Contents

De-factorization Formulae:

1) $\mathbf{2}\sin \boldsymbol{A} \cos \boldsymbol{B} = \sin(A+B) + \sin(A-B)$

2) $\mathbf{2}\cos \boldsymbol{A} \sin \boldsymbol{B} = \sin(A+B) - \sin(A-B)$

3) $\mathbf{2}\cos \boldsymbol{A} \cos \boldsymbol{B} = \cos(A+B) + \cos(A-B)$

4) $\mathbf{2}\sin \boldsymbol{A} \sin \boldsymbol{B} = \cos(A-B) - \cos(A+B)$

Factorization formulae (Conversion of sum or difference into product)

$1) \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

$2) \sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$3) \cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

$4) \cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

OR

$\cos C - \cos D = 2 \sin\left(\frac{D-C}{2}\right) \sin\left(\frac{C+D}{2}\right)$

Examples:

1) Evaluate $2\cos 75° \cdot \cos 15°$

Solution:

$$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

$$2\cos 75° \cdot \cos 15° = \cos(75° + 15°) + \cos(75° - 15°)$$

$$\Rightarrow 2\cos 75° \cdot \cos 15° = \cos(90°) + \cos(60°)$$

$$\Rightarrow 2\cos 75° \cdot \cos 15° = 0 + \frac{1}{2}$$

$$\Rightarrow 2\cos 75° \cdot \cos 15° = \frac{1}{2}$$

2. **Prove that:** $\dfrac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \tan 3A \, . \, \sin A$

Solution:

L.H.S. $= \dfrac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A}$

$= \dfrac{(\cos 6A + \cos 2A) + 2\cos 4A}{(\cos 5A + \cos A) + 2\cos 3A}$

$= \dfrac{2\cos\left(\frac{6A + 2A}{2}\right) \cdot \cos\left(\frac{6A - 2A}{2}\right) + 2\cos 4A}{2\cos\left(\frac{5A + A}{2}\right) \cdot \cos\left(\frac{5A - A}{2}\right) + 2\cos 3A}$

$= \dfrac{2\cos 4A \cdot \cos 2A + 2\cos 4A}{2\cos 3A \cdot \cos 2A + 2\cos 3A}$

$= \dfrac{2\cos 4A(\cos 2A + 1)}{2\cos 3A(\cos 2A + 1)}$

$= \dfrac{\cos 4A}{\cos 3A} = \dfrac{\cos(3A + A)}{\cos 3A}$

$= \dfrac{\cos 3A \cdot \cos A - \sin 3A \cdot \sin A}{\cos 3A}$

$= \dfrac{\cos 3A \cdot \cos A}{\cos 3A} - \dfrac{\sin 3A \cdot \sin A}{\cos 3A}$

$= \cos A - \dfrac{\sin 3A}{\cos 3A} \cdot \sin A$

$= \cos A - \tan 3A \cdot \sin A$ = R.H.S.

3) *Prove that* $\dfrac{\sin 8\theta \cdot \cos\theta - \cos 3\theta \cdot \sin 6\theta}{\cos 2\theta \cdot \cos\theta - \sin 3\theta \cdot \sin 4\theta} = \tan 2\theta$

Multiply the Numerator & denominator by2

$$\text{L.H.S} = \frac{2\sin 8\theta \cdot \cos\theta - 2\sin 6\theta \cdot \cos 3\theta}{2\cos 2\theta \cdot \cos\theta - 2\sin 4\theta \cdot \sin 3\theta}$$

$$= \frac{[\sin(8\theta+\theta) + \sin(8\theta-\theta)] - [\sin(6\theta+3\theta) + \sin(6\theta-3\theta)]}{[\cos(2\theta+\theta) + \cos(2\theta-\theta)] - [\cos(4\theta-3\theta) - \cos(4\theta+3\theta)]}$$

$$= \frac{[\sin(9\theta) + \sin(7\theta)] - [\sin(9\theta) + \sin(3\theta)]}{[\cos(3\theta) + \cos(\theta)] - [\cos(\theta) - \cos(7\theta)]}$$

$$= \frac{\sin(9\theta) + \sin(7\theta) - \sin(9\theta) - \sin(3\theta)}{\cos(3\theta) + \cos(\theta) - \cos(\theta) + \cos(7\theta)}$$

$$= \frac{\sin(7\theta) - \sin(3\theta)}{\cos(7\theta) + \cos(3\theta)}$$

$$= \frac{2\cos\left(\frac{7\theta+3\theta}{2}\right) \cdot \sin\left(\frac{7\theta-3\theta}{2}\right)}{2\cos\left(\frac{7\theta+3\theta}{2}\right) \cdot \cos\left(\frac{7\theta-3\theta}{2}\right)}$$

$$= \frac{2\cos 5\theta \cdot \sin 2\theta}{2\cos 5\theta \cdot \cos 2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

= R.H.S

4) Prove that $\sin 20^0 \cdot \sin 40^0 \cdot \sin 60^0 \cdot \sin 80^0 = \frac{3}{16}$

Solution: L.H.S.= $\sin 20^0 \cdot \sin 40^0 \cdot \sin 60^0 \cdot \sin 80^0$

$$=\frac{1}{2}\{\ 2\sin 40^0 \cdot \sin 20^0\} \cdot \frac{\sqrt{3}}{2} \cdot \sin(90^0 - 10^0)$$

$$=\frac{1}{2}\{\ \cos(40^0 - 20^0) - \cos(40^0 + 20^0)\} \cdot \frac{\sqrt{3}}{2} \cdot\ \cos 10^0$$

$$=\frac{\sqrt{3}}{4}\{\ \cos 20^0 - \cos 60^0\} \cos 10^0$$

$$=\frac{\sqrt{3}}{4}\left\{\ \cos 20^0 - \frac{1}{2}\right\} \cos 10^0$$

$$=\frac{\sqrt{3}}{4}\left\{\frac{2\cos 20^0 - 1}{2}\right\} \cdot\ \cos 10^0$$

$$=\frac{\sqrt{3}}{8}\{2\cos 20^0 \cos 10^0 - \ \cos 10^0\ \}$$

$$=\frac{\sqrt{3}}{8}\{\cos(20^0 + 10^0) + \ \cos(20^0 - 10^0) - \ \cos 10^0\ \}$$

$$=\frac{\sqrt{3}}{8}\{\cos 30^0 + \ \cos 10^0 - \ \cos 10^0\ \}$$

$$=\frac{\sqrt{3}}{8} \cdot\ \cos 30^0 \ =\frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} \quad = \frac{3}{16} = \text{ R.H.S}$$

Application of Concept/ Examples in real life:

- Trigonometry has vast area of applications in daily life. It can be used in navigation, and in sound waves.
- Concepts of factorization and de-factorization are useful for solving problems in higher mathematics and it is also employed to solve various engineering problems.
- Trigonometry finds wide applications in engineering faculties like Applied Mechanics, Electrical Technology, Basic Electronics, Computer Engineering, Vector Mechanics, etc.

Summary:

So today we learned….

- Formulae of Factorization and De-factorization.
- Solving different problems using formulae of factorization and de-factorization.

Now take this quiz…..

1) If sin $80^0 + \sin 50^0 = 2 sin\alpha\ cos\beta, then\ \alpha\ and\ \beta\ are$

a) $60^0 and\ 20^0$ b) $60^0 and\ 15^0$

c) $65^0 and\ 15^0$ d) $65^0\ and\ 20^0$

2) **2sin3x cos 2x =**

a) Sin5x + sin x **b) sin3x + sin x**

c) sin7x+ sin x **d) sin4x +sin x**

3) Solve $\frac{sin5x+sin3x}{cos5x+cos3x} =$

a) **cot x** **b) tan 4x**

c) tan x **d) cot 4x**

Ans: 1) c 2) a 3) b

Thank you