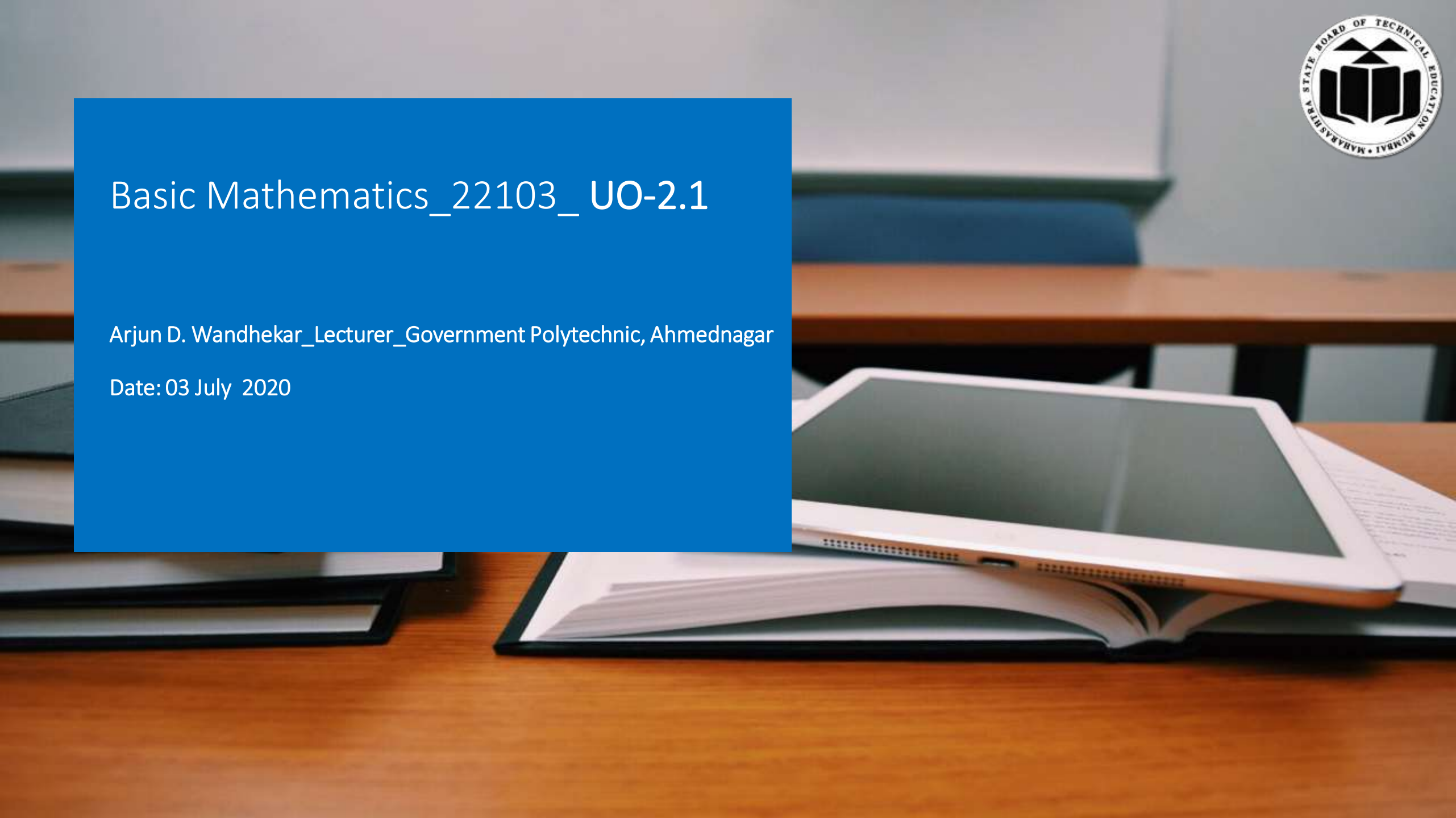

Basic Mathematics_22103_ UO-2.1

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Unit 2: Trigonometry

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Topic : Compound Angles

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Learning Objective/ Key learning

- Apply the concept of Compound angles to solve the given simple engineering problem(s).

Contents

- Definition of Compound angle
- Trigonometric rations of compound angles (Without proofs) :
- Examples

Compound angle

- **Definition** : If A and B are two angles then the sum A + B or difference A – B are called a compound angles.

  e.g. A = 45°, B = 30° then A + B = 45° + 30° = 75° And A – B = 45° – 30° = 15° are compound angles.

- **Trigonometric rations of compound angles (Without proofs)**

  1) $\sin (A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

  2) $\sin (A - B) = \sin A \cdot \cos B - \cos A \sin B$

  3) $\cos (A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

  4) $\text{Cos } (A - B) = \cos A \cos B + \sin A \sin B$

  5) $\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \,.\, \tan B}$

  6) $\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \,.\, \tan B}$

1) Without using calculator, find the value of

a) sin 15° b) cos 75°

Solution:

a) As $15° = 45° – 30°$

$\therefore \sin 15 = \sin (45° – 30°)$

Using formula $\sin (A – B) = \sin A \cdot \cos B – \cos A \sin B$

$= \sin 45° \cdot \cos 30° – \cos 45° \cdot \sin 30°$

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$ and $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$

$$= \left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}\right) – \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} – \frac{1}{2\sqrt{2}}$$

$$\sin 15 = \frac{\sqrt{3} – 1}{2\sqrt{2}}$$

b) As 75° = 45° + 30°

$\therefore$ cos 75°= cos (45° + 30°)

By using Formula cos (A + B) = cos A · cos B – sin A · sin B

$$= \cos 45° \cdot \cos 30° - \sin 45° \cdot \sin 30°$$

$$= \left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos 15 = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

2) If tan A = $\frac{1}{2}$, tan B = $\frac{1}{3}$ Find tan (A + B)

Solution : Given that tan A = $\frac{1}{2}$, tan B = $\frac{1}{3}$

By compound formula, tan (A + B) = $\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$$

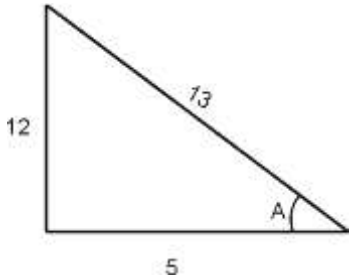
$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}}$$

$$= \frac{\frac{3+2}{6}}{\frac{6-1}{6}} = \frac{5}{5}$$

tan (A + B) = 1

4) If ∠A and ∠B are both obtuse angles and $\sin A = \frac{12}{13}$ and $\cos B = \frac{-4}{5}$. Find $\sin (A + B)$.

► **Solution :** Given $\sin A = \frac{12}{13}$



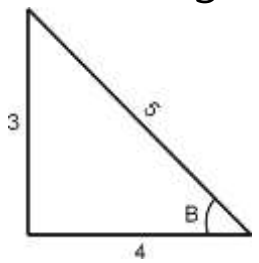
$$\cos A = \frac{5}{13}$$

∴ A is obtuse (More than 90º and less than 180º)

A is the second quadrant, cos A is negative.

$$\cos A = \frac{-5}{13}$$

Given $\cos B = \frac{-4}{5}$



$$\sin B = \frac{3}{5}$$

∴ B is obtuse (More than 90º and less than 180º)

B is the second quadrant, sin B is Positive.

$$\sin B = \frac{3}{5}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \left(\frac{12}{13}\right) \times \left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right) \times \left(\frac{3}{5}\right)$$

$$= \frac{-48}{65} + \frac{-15}{65}$$

$$= \frac{-48-15}{65}$$

$$\sin(A+B) = \frac{-63}{65}$$

So today we learn-

- Definition of Compound angle
- compound angles Formulae(Without proofs) :
- Solved examples based on compound angles.

Quiz

1)The value of $cos\ 105^0$

a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ c) - $\frac{\sqrt{3}+1}{2\sqrt{2}}$ d) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

2) Find $\frac{tan\ 85°+tan\ 40°}{1-tan\ 85°.tan40°}$

a) 1 b) 4 c) 7 d) 2

Ans: 1. d) 2.a)

Thank You