Basic Mathematics_22103_UO2.4

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Unit 2: Trigonometry

Topic : Inverse Trigonometric function

Course Outcome:

Co2: Utilize basic concepts of trigonometry to solve elementary engineering problems.

Learning Objectives:

Investigate given simple problems utilizing inverse trigonometric ratios.

Contents

1) Principal values of inverse T-ratios

2) Examples based on the concept of inverse trigonometric function.

Examples:

1.Verify that: $\sin^{-1}\left\{\frac{1}{2}\right\} + \sin^{-1}\left\{\frac{\sqrt{3}}{2}\right\} = \sin^{-1}(1)$

Solution:

Given: $\sin^{-1}\left\{\frac{1}{2}\right\} + \sin^{-1}\left\{\frac{\sqrt{3}}{2}\right\} = \sin^{-1}(1)$

$\Rightarrow \quad \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2} \quad \Rightarrow \quad \frac{3\pi + 6\pi}{18} = \frac{\pi}{2}$

$\Rightarrow \quad \frac{9\pi}{18} = \frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{2} = \frac{\pi}{2}$ Hence verified.

2.Find the value of $\cos\left\{2\sin^{-1}\left(\frac{3}{4}\right)\right\}$

Solution:

Consider $\cos\left\{2\sin^{-1}\left(\frac{3}{4}\right)\right\}$

Let $\sin^{-1}\left(\frac{3}{4}\right) = \theta$

$\Rightarrow \quad \sin\theta = \frac{3}{4}$

$\therefore \quad \cos\left\{2\sin^{-1}\left(\frac{3}{4}\right)\right\}$

$= \quad \cos 2\theta \qquad\qquad \because \quad \sin^{-1}\left(\frac{3}{4}\right) = \theta$

$= \quad 1 - 2\sin^2\theta$

$= \quad 1 - 2\left(\frac{3}{4}\right)^2$

$= \quad 1 - 2 \times \frac{9}{16}$

$= \quad 1 - \frac{9}{8}$

$= \quad -\frac{1}{8}$

3) Prove that $\tan^{-1}\left(\frac{1}{11}\right)+\ \tan^{-1}\left(\frac{5}{6}\right) = \tan^{-1}(1)$

L.H.S. $= \tan^{-1}\left(\frac{1}{11}\right)+\ \tan^{-1}\left(\frac{5}{6}\right)$

$$= \tan^{-1}\left\{\frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}\cdot\frac{5}{6}}\right\}$$

$$= \tan^{-1}\left\{\frac{(6+55)/66}{(66-5)/66}\right\}$$

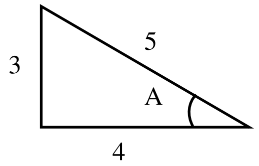
$$= \tan^{-1}\left\{\frac{61}{61}\right\}$$

$$= \tan^{-1}(1)$$

$$= \mathrm{R.H.S.}$$

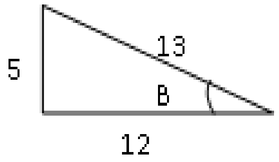
4) Prove that: $\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

Let $\cos^{-1}\left(\frac{4}{5}\right) = A$ $\quad\therefore \cos A = \frac{4}{5}$



From fig. $\quad \sin A = \frac{3}{5}$

Let $\sin^{-1}\left(\frac{5}{13}\right) = B$ $\quad\therefore\quad \sin B = \frac{5}{13}$



From fig. $\cos B = \frac{12}{13}$

We know $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\Rightarrow \quad \cos(A+B) = \frac{4}{5}\cdot\frac{12}{13} - \frac{3}{5}\cdot\frac{5}{13} = \frac{48-15}{65} = \frac{33}{65}$

$\Rightarrow \quad A + B = \cos^{-1}\left(\frac{33}{65}\right)$

$\Rightarrow \quad \cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

Application of Concept/ Examples in real life:

- Trigonometry has vast area of applications in daily life. It can be used in navigation, and in sound waves.
- Trigonometry finds wide applications in engineering faculties like Applied Mechanics, Electrical Technology, Basic Electronics, Computer Engineering, Vector Mechanics, etc.
- Concept of inverse trigonometric functions are useful in solving problems in Higher Mathematics.

Summary:

So today we learned -

- Investigation of principal values of different trigonometric functions.
- Develop relation between trigonometric and inverse trigonometric functions.
- Investigate given simple problems utilizing inverse trigonometric ratios.

Now take this quiz…..

1) Evaluate $\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}$

a) π/2 **b)**π/3
c) 2π/3 **d)** π

2) $2\sin^{-1}x$ =

a) $\sin^{-1}2x\sqrt{1-x^2}$ **b)** $\sin^{-1}\sqrt{1-x^2}$
c) $\cos^{-1}2x\sqrt{1-x^2}$ **d)** $\sin^{-1}x$

3) For principal values find
$\tan^{-1}\infty-\sin^{-1}\frac{1}{\sqrt{2}}$=

a) $\tan^{-1}\frac{1}{2}$ **b)** $\tan^{-1}1$
c) $\sin^{-1}1$ **d)** $\cot^{-1}-1$

Ans: 1) c 2) a 3) b

Thank you