Vivekanand Education Society's Institute of Technology

An Autonomous Institute Affiliated to University of Mumbai

End Semester Examination
Summer 2024

Max marks: 60 | Duration: 2 hours
Branch: Automation & Robotics | Semester: IV
Name of the Course: Engineering Mathematics IV
Course code: ARC401 | QP Code: R23-ARC401_022023-24

N.B. (1) Attempt any three of the five questions.
(2) Figures to the right indicate full marks.
(3) Assume suitable data if necessary.

Marks

Q.1 (a) Calculate the rank correlation coefficient for the following data. 5

X:	32	55	49	60	43	37	43	49	10	20
Y:	40	30	70	20	30	50	72	60	45	25

(b) A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from (a) machine A (b) machine B (c) machine C? 5

(c) (i) Solve the LPP using simplex method. 10

$$\begin{aligned} &Maximize && z = 2x_1 + x_2 \\ &Subject\ to && -x_1 + 2x_2 \leq 2 \\ & && x_1 + x_2 \leq 4 \\ & && x_1 \leq 3 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

(ii) Two unbiased dice are thrown. Find the probability that: (i) both the dice show same number and (ii) the total of the numbers on the dice is 8.

Q.2 (a) The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, and 66 inches. Is it reasonable to believe that the average height is greater than 64 inches test at 5% significance level assuming that for 9 degrees of freedom? 5

(b) Find an optimal solution to the following L.P.P. by computing all basic solutions and then finding one that maximizes the objective function. 5

$$\begin{aligned} &Maximize && z = x_1 + 3x_2 + 3x_3 \\ &Subject\ to && x_1 + 2x_2 + 3x_3 = 4 \\ & && 2x_1 + 3x_2 + 5x_3 = 7 \end{aligned}$$

(c) Solve the NLLP using the Kuhn-Tucker condition, 10

$Maximize \quad z = x_1^2 + x_2^2$
$Subject\ to \quad x_1 + x_2 \leq 4$
$2x_1 + x_2 - 5 \leq 0$
$x_1, x_2 \geq 0$

Q.3 (a) The incomes of a group of 10,000 people were found to be normally distributed with a mean of Rs520 and a standard deviation of Rs60. Find i) the number of people having income between Rs400 and Rs550 and ii) the lowest income of the richest 500. 5

(b) A sample of 400 individuals is found to have a mean height of 67.47 inches. Can it be reasonably regarded as a sample from a large population with a mean height of 67.39 inches and a standard deviation of 1.30 inches at a 5% Level of significance? 5

(c) (i) Samples of two types of electric light bulbs were tested for a length of life and the following data were obtained: 10

	Type1.	Type 2
Size:	n_1=8.	n_2=7
Mean:	$\overline{X_1}$=1, 234 hrs.	$\overline{X_2}$= 1,036 hrs.
S. D. :	s_1= 36 hrs.	s_2 = 40 hrs.

Is the difference in the mean sufficient to warrant that type 1 is superior to type 2 regarding length of life?

(ii) If the two regression equations are $5x - 6y + 90 = 0$, $15x - 8y - 180 = 0$,
Find (i) the mean of x and y
(ii) Correlation coefficient
(iii) Standard deviation of x if $Var(Y) = 1$

Q.4 (a) Solve the NLLP, 5

$Optimise \quad z = 6x_1 + 8x_2 - x_1^2 - x_2^2$
$Subject\ to \quad 4x_1 + 3x_2 = 16$
$3x_1 + 5x_2 = 15$
$x_1, x_2 \geq 0$

(b) Fit a second-degree parabola to the given data. 5

X:	1	1.5	2	2.5	3	3.5	4
Y:	1.1	1.3	1.6	2	2.7	3.4	4.1

(c) (i) Find the value of k and P ((X<5) / (3≤ $X \leq 6$)) if the r.v. X has the distribution 10

X :	1	2	3	4	5	6
$p(X=x)$:	k	$2k$	$4k^2$	k^2	$4k$	$3k^2$

(ii) If $f(x) = 3e^{-3x}$,$x \geq 0$, find the moment generating function, the first three moments about the origin, and the first three central moments.

Q.5 (a) A die was thrown 132 times and the following frequencies were observed: 5

No. obtained	1	2	3	4	5	6	Tota 1
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased.

(b) Using the dual simplex method, solve the LPP, 5

$$\text{Minimize } z = 2x_1 + 2x_2 + 4x_3$$
$$\text{Subject to } 2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

(c) Fit a Poisson distribution to the data and test whether the Poisson distribution is a good fit. Test at a 5% level of significance. 10

X:	0	1	2	3	4
Y:	123	59	14	3	1

X-----X-----X-----X