23/01/23

CS704

BEL Theorem: Σ - finite alphabet.

A language $L \subseteq \Sigma^*$ is regular

iff

L is $MSO(\Sigma)$-definable.

today

MSO: $\left\{ \begin{array}{l} x < y \\ P_a(x) \\ succ(x) \\ max \\ min \end{array} \right.$

$x \in X$ } Atomic formulas

$\wedge$

$\vee$

$\neg$

$\exists x, \forall x, \exists X, \forall X$

The proof proceeds similar to Kleene's theorem.

Given a reg. exp. r, we inductively construct an NFA.

$(a+b)^* \cdot b$

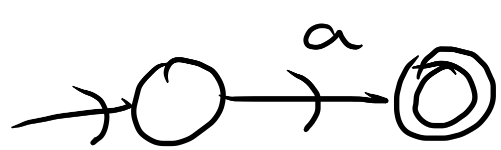
Star

union

conc

<u>Base case</u>

$a, \; a \in \Sigma$



$\exists$ quantifier in logic is the same as the projection operation.

$$\varphi(X) = \exists x \, (x \in X) \wedge \forall y \, (y \in X \Rightarrow y \geq x)$$

$$\varphi = \exists X \left(\exists x \, (x \in X) \wedge \forall y \, (y \in X \Rightarrow y \geq x) \right)$$

free variable so we have to interpret.

$X = \mathbb{N}$? ✓

$X = [0, 1]$ ✓

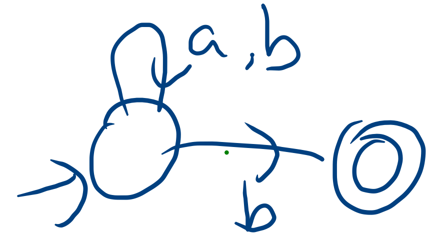
$X = (0, 1]$ ✗ $X = (0, 1)$ ✗

$\varphi(x_1, x_2, \ldots, x_n) \to n$ free variables

$\exists x_1 \, \varphi(x_2, \ldots, x_n) \to n-1$ free variables

$$\begin{pmatrix} \underline{a} \\ 0 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \Bigg\} n \text{ bits} \xrightarrow{f\,(\text{projection})} \begin{pmatrix} a \\ 1 \\ \vdots \\ 1 \end{pmatrix} \Bigg\} \to n-1 \text{ bits}$$

Σ : $(a+b)^* . b$



aab

$\exists x \, (P_b(x) \wedge \neg \exists y \, (x<y))$ → x is free in $\exists y (x<y)$

defines regular language

construct an NFA

$\begin{pmatrix} a \\ 0 \end{pmatrix} \begin{pmatrix} a \\ 1 \end{pmatrix}$

$\begin{pmatrix} b \\ 0 \end{pmatrix} \begin{pmatrix} b \\ 1 \end{pmatrix}$

will not be assigned for x

assign position to x

$\begin{pmatrix} a \\ 1 \end{pmatrix} \begin{pmatrix} b \\ 1 \end{pmatrix} \begin{pmatrix} b \\ 0 \end{pmatrix}$

position 1 to x

aab ⇝ $\begin{pmatrix} a \\ 0 \end{pmatrix} \begin{pmatrix} a \\ 1 \end{pmatrix} \begin{pmatrix} b \\ 1 \end{pmatrix}$

pos 1 pos 2 pos 3

2 / 3 are positions for x