

**International Institute of Information
Technology Bangalore**

CS 704: Topics in Computability and Learning

Practice Questions

1. Let $\Sigma = \{a, b\}$. Give a DFA for the language that contains all strings that have at least three positions where 3 consecutive b 's occur. For e.g., $abbbbbba$ and $bbbbaabbbbaa$ should be accepted by the DFA but, not $abbb$.
2. Give an NFA for the language of binary strings with alternating 0s and 1s. Give an equivalent DFA (either using subset construction or directly). How many states will the minimal DFA for this language have?
3. Prove or disprove: The regular expression $(ab^*a + b)^*$ describes the language of all strings over $\{a, b\}$ that contain an even number of ' a 's.
4. Give a DFA for the language of all strings over $\{a, b\}$ which *don't* contain the substring bab .
5. Describe a DFA for the language $a^*b^* + b^*a^*$.
6. Consider the sentence $\varphi = \exists x((\forall y \neg(y < x) \wedge Q_a(x)))$ over the alphabet $\{a\}$. Describe the language of words that satisfy φ in English. Also give a regular expression for the language.
7. Consider the formula $\varphi(x) = \exists x((x < y) \wedge Q_a(y))$. List the free and the bound variables in φ . Also describe the alphabet over which the automaton that will be constructed for accepting all the models of φ will run on. Assume that Σ is just $\{a\}$.
8. Describe the language defined by the sentence below, as a regular expression over the alphabet $\{a, b\}$.

$$\forall x(Q_b(x) \Rightarrow \exists y(y < x \wedge Q_a(y))).$$

9. Give a regular expression which describes the language defined by the MSO sentence below over the alphabet $\{a, b\}$.

$$\begin{aligned} \exists X(& (\exists x(\text{zero}(x) \wedge x \in X)) \wedge \\ & (\exists x(\text{last}(x) \wedge \neg x \in X)) \wedge \\ & (\forall x \forall y(\text{succ}(x, y) \Rightarrow (x \in X \Leftrightarrow \neg y \in X))) \wedge \\ & \forall x(x \in X \Rightarrow Q_a(x))). \end{aligned}$$