30/11/'23

CS704

Σ , Σ^* , 2^{Σ^*}

regular languages

- Class of regular languages are closed under union, intersection, complementation, concatenation, *, projection ... they are robust.
- Can be used as 'algorithms' (O(1) space) for several useful computations:
  - parity checking
  - pattern matching (regular expn)
  - divisibility by k (fixed)
  - decidability of $MSO(\mathbb{N}, \leq)$ | $FO(\mathbb{N}, <)$

Decidability:

Given a sentence φ over $FO(\mathbb{N}, <)$,

(1) φ is satisfiable? Is there as assignment of values to the variables in φ that make it true?

(2) φ is valid? Is φ always true?

Given

φ $\rightsquigarrow$ construct an automaton for φ $\rightsquigarrow$ check for emptiness.

$FO(\mathbb{N}, \leq)$

$\downarrow$

Büchi
1960

(FSA)

$FO(\mathbb{N}, \leq, +)$

$\downarrow$

Presburger
1925

(FSA)

$FO(\mathbb{N}, \leq, +, *)$

$\downarrow$

Gödel
inconsistency
incomplete
in 1931

Turing

$\downarrow$

undecidable
problems

1937

Cook & Levin

$\downarrow$

NP-complete

An excercise :

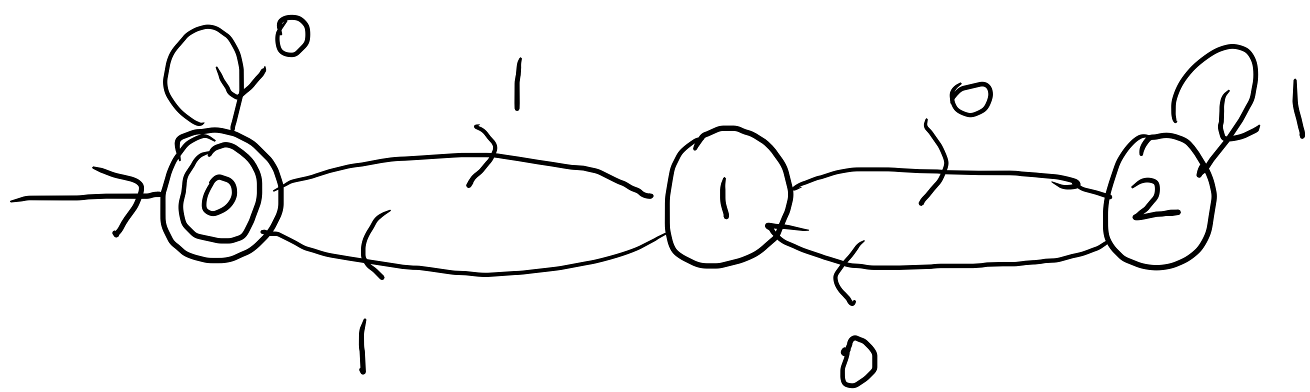
Check for divisibility by 3, of a given (binary) number.

3 → any fixed k.

Decimal : 0, 1, 2, 3, 4, 5, 6, ...

Binary : 0, 1, 10, 11, 100, 101, ...

$\Sigma = \{0, 1\}$.



Example run of Presburger automaton for

$$x+2y-3z=1.$$

Solution (1): $x=0$ $\quad y=2$ $\quad z=1$

$\quad 000 \quad 010 \quad 001$

$$\begin{array}{c} \begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} \end{array} \begin{array}{l} \rightarrow x \\ \rightarrow y \\ \rightarrow z \end{array}$$

$\longleftarrow$ right to left.

First character read is $\begin{pmatrix}0\\0\\1\end{pmatrix}$ In decimal $x=y=0,\ z=1.$

On substitution, we get $0+0-3$
$= -3 \neq 1$

Second character read is $\begin{pmatrix}0\\1\\0\end{pmatrix}$, word read is $\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$

In decimal, $x=0$, $y=2$, $z=1$.

On substituting $0+4-3=1$

& go on like this till the last bit string is read.
