Explorations in AGT

Game theory: Study of what happens when self interested agents interact. Scenario called a "game".

AGT: How to design efficient algorithms for

- setting rules of game so "good" things happen.
- predict outcome (a.k.a equilibrium)
- play optimally

Topics:

- Basics of GT
- Algorithmic Mechanism Design
- Algorithmic Persuasion
- ? - Security Games
- ? - Algorithmic Contract Theory
- ? - Computation of equilibria
- ? - other

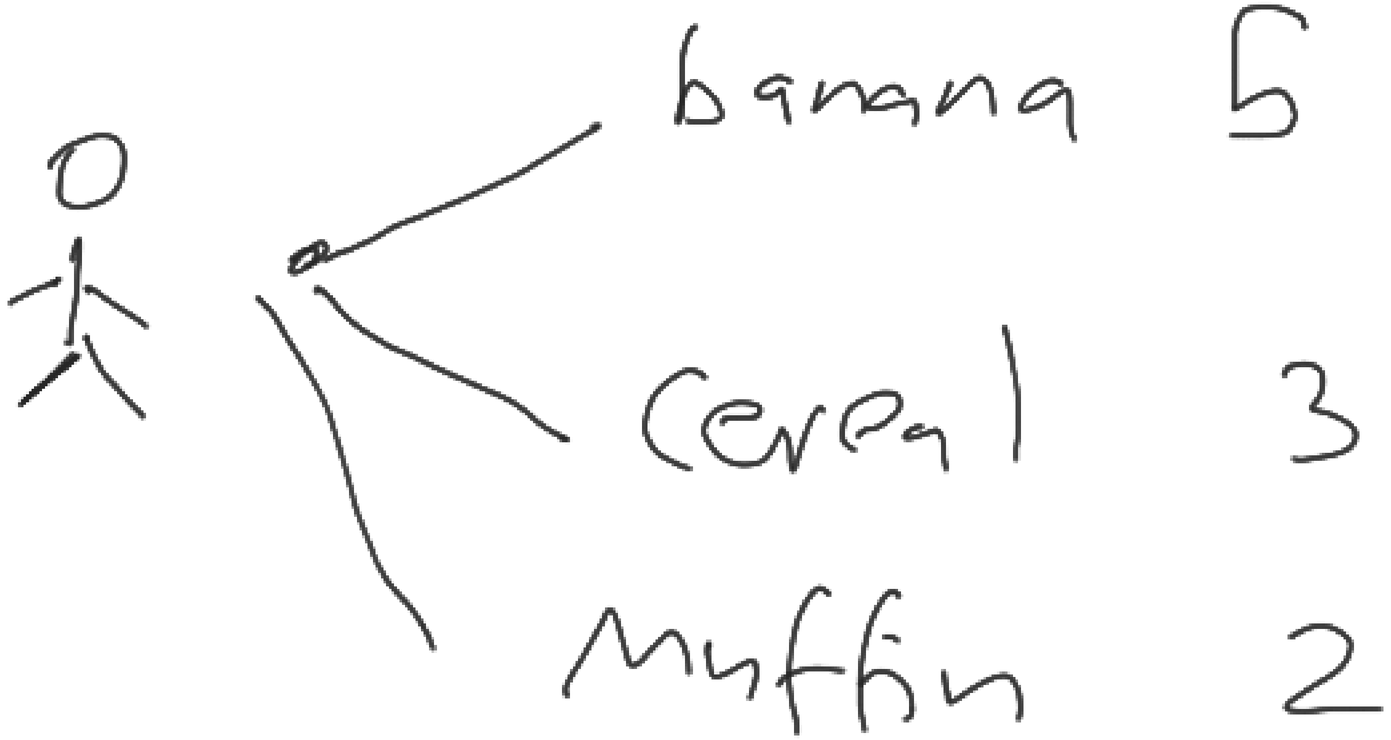
Basics of Games and Equilibria

Q: How does a single agent make decisions?

A:
- There is a set of possible states of the world Ω
- Agent has a utility $\overbrace{u(w)}^{\text{real number}}$ for each $w \in \Omega$
- Agent has a set A of actions, each of which can influence state of world.

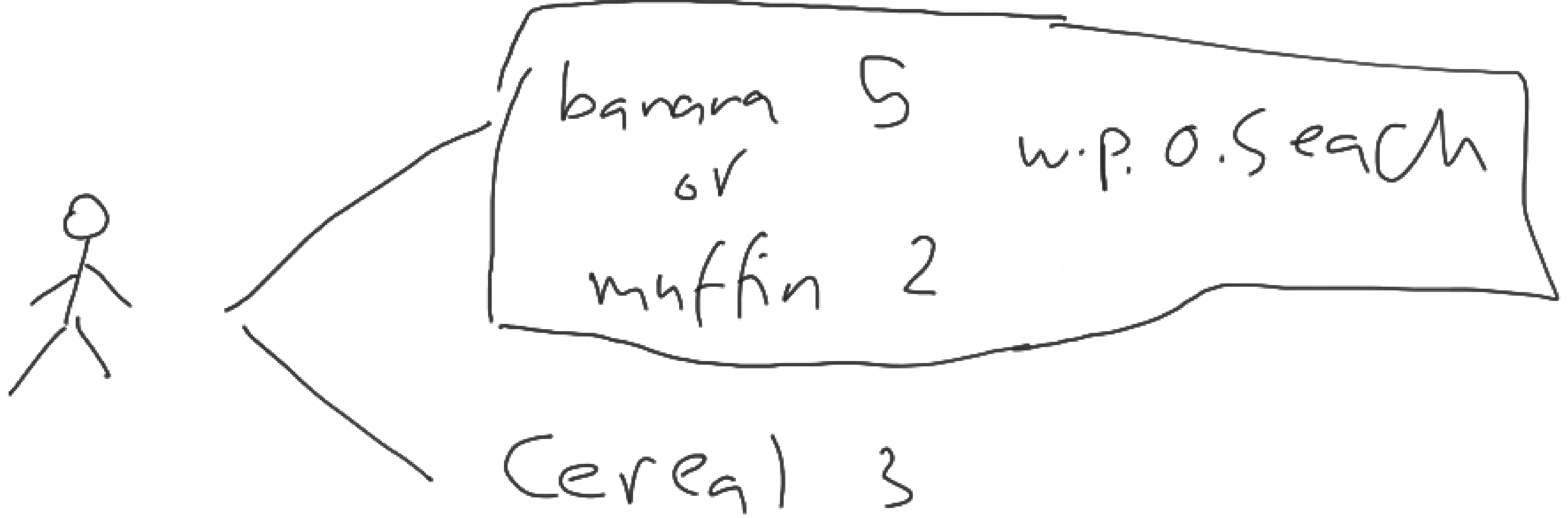
- **Rationality**: Agent chooses action which leads to state of world maximizing his utility.

E.g.:



Rational agent will choose banana.

E.g.



Expected utility hypothesis

Rational agent acts to maximize his expected utility in an uncertain environment, based on their prior beliefs.

In this example, agent chooses the box for expected utility 3.5.

- How to capture risk averse / seeking agents:

Von-Neumann Morgenstern theorem. concave

convex

Note: more References Coming

Q: What happens with multiple agents?

Difficult to answer because your utility depends on both your actions and actions of others.

E.g. Rock, Paper, Scissors

prisoner's Dillema

		prisoner 2	
		C	D
prisoner 1	C	(-1, -1)	(-3, 0)
	D	(0, -3)	(-2, -2)

Nash eq: (D, D)

- Dominant Strategy
- pure
- unique

Rock, Paper, Scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

No PNE

MNE: Each player picks R,P,S uniformly at random.

Battle of the sexes

		wife	
		scifi	drama
husband	scifi	2, 1	0, 0
	drama	0, 0	1, 2

Two PNE: (sci, sci) & (drama, drama)

MNE: Each goes to their favorite w.p. $\frac{2}{3}$.

suppose husband going to scifi w.p. $\frac{2}{3}$

$$U_{wife}(\text{scifi}) = \Pr[\text{husband goes to scifi}] \cdot 1 + \Pr[h \to \text{drama}] \cdot 0$$

$$= \frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 0 = \frac{2}{3}$$

$$U_{wife}(\text{drama}) = \frac{2}{3} \cdot 0 + \frac{1}{3} \cdot 2 = \frac{2}{3}$$

CE: 50% (sci, sci)
50% (d, d)

Chicken

	S	G
S	0,0	0,1
G	1,0	-4,-4

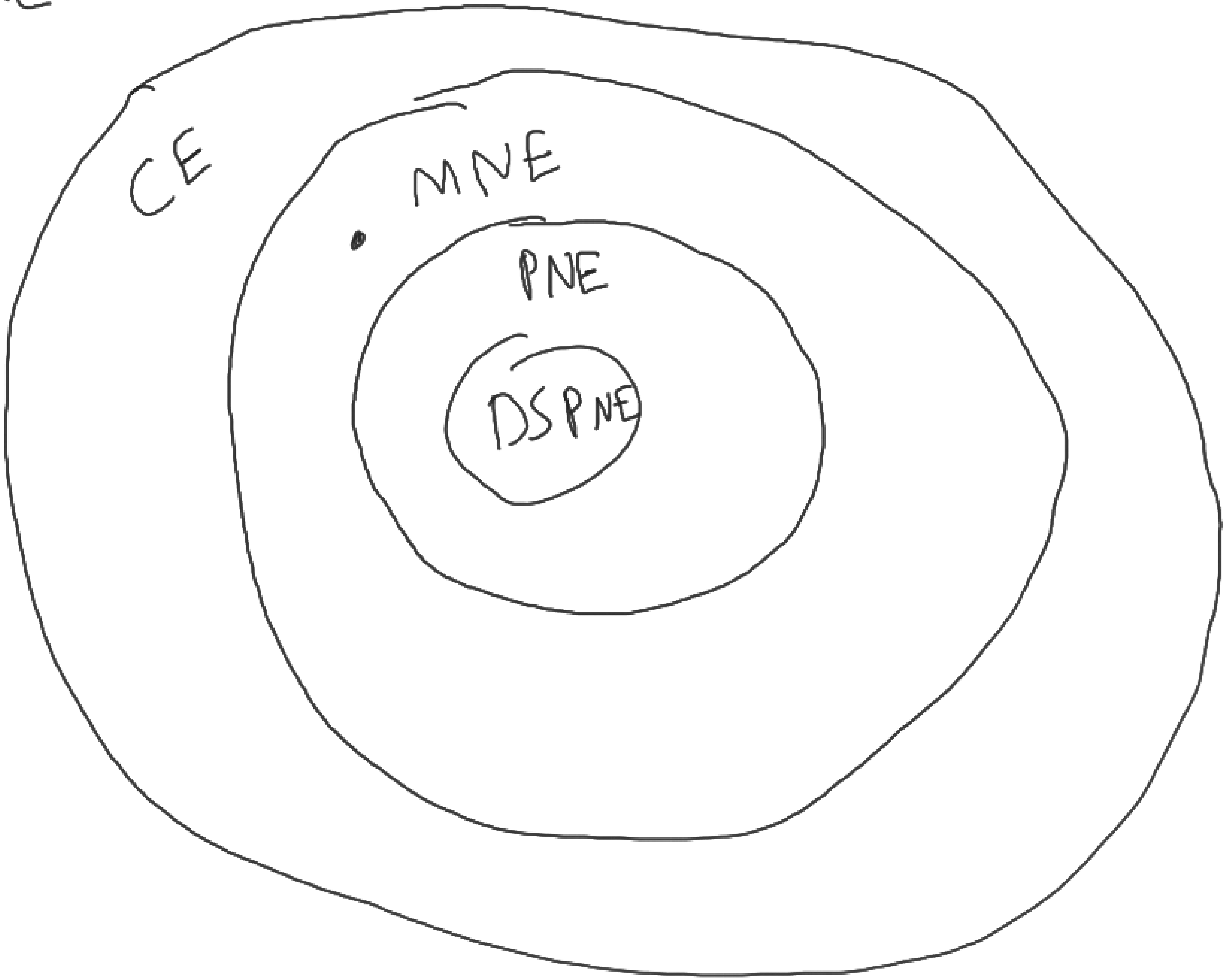
PNE : (S, G) (G, S)

MNE : Each stops w.p. $\frac{4}{5}$

CE : 50% (S, G)
50% (G, S)

Thm (Nash):
Every finite game has a MNE.

Hierarchy of Equilibria



More General Settings

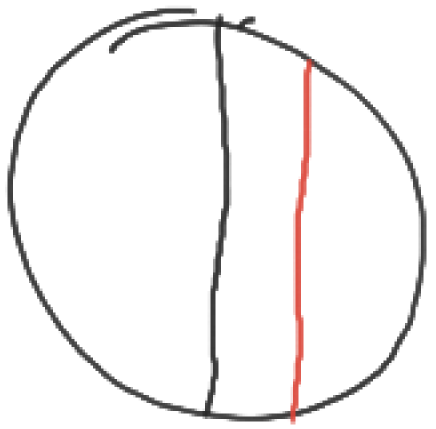
- Every game we saw so far was
  - one shot
  - simultaneous move
  - Complete information ← players know the game matrix

More Generally, you can have:

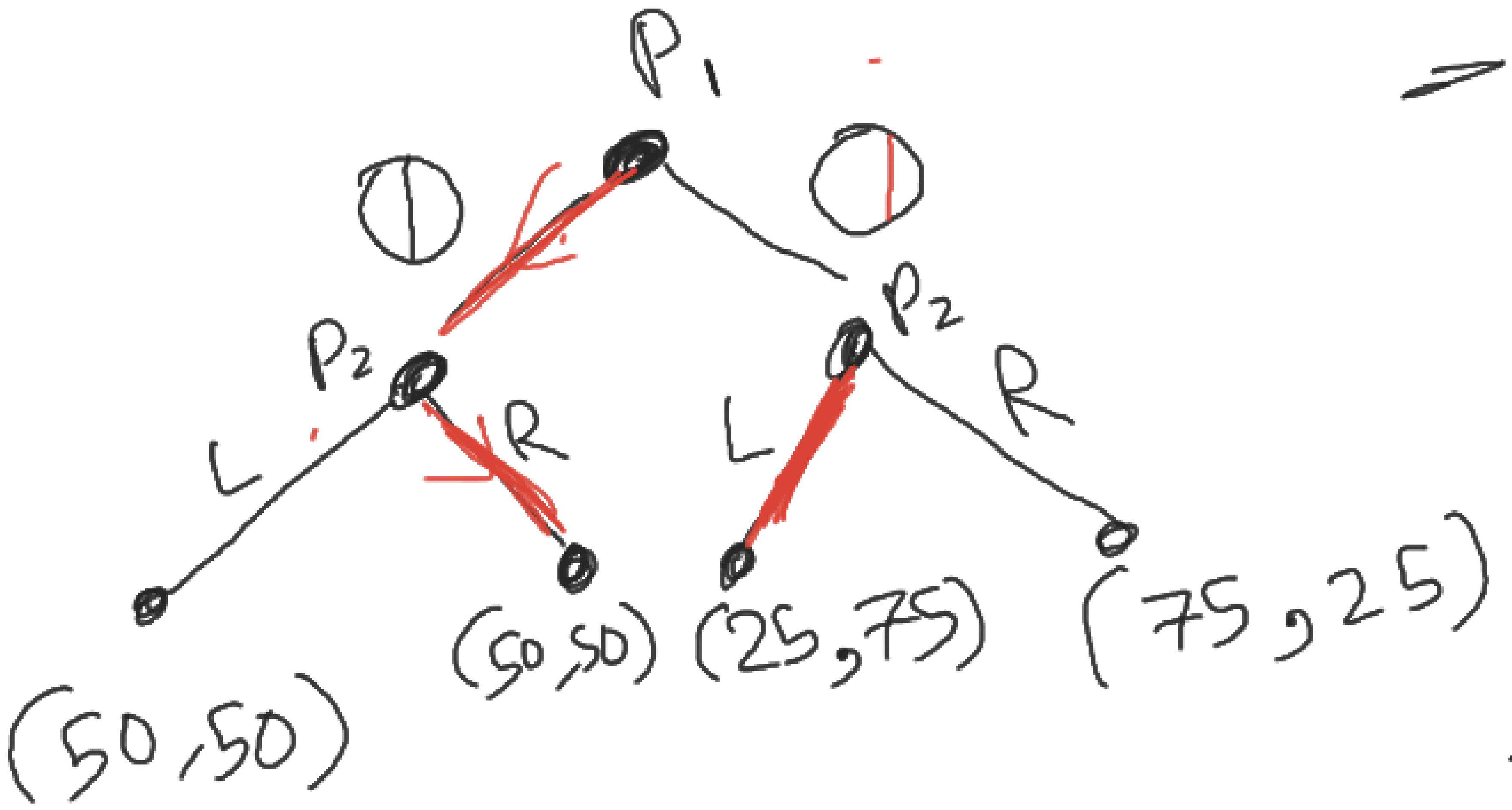
- Incomplete information
- Sequential moves

All what we discussed about equilibria carries over.

A sequential game



- one cake, 2 players
- player 1 cuts the cake
  - 50/50 or
  - 75/25
- player 2 sees what player 1 did, picks one of the pieces.



→ Extensive form Game tree

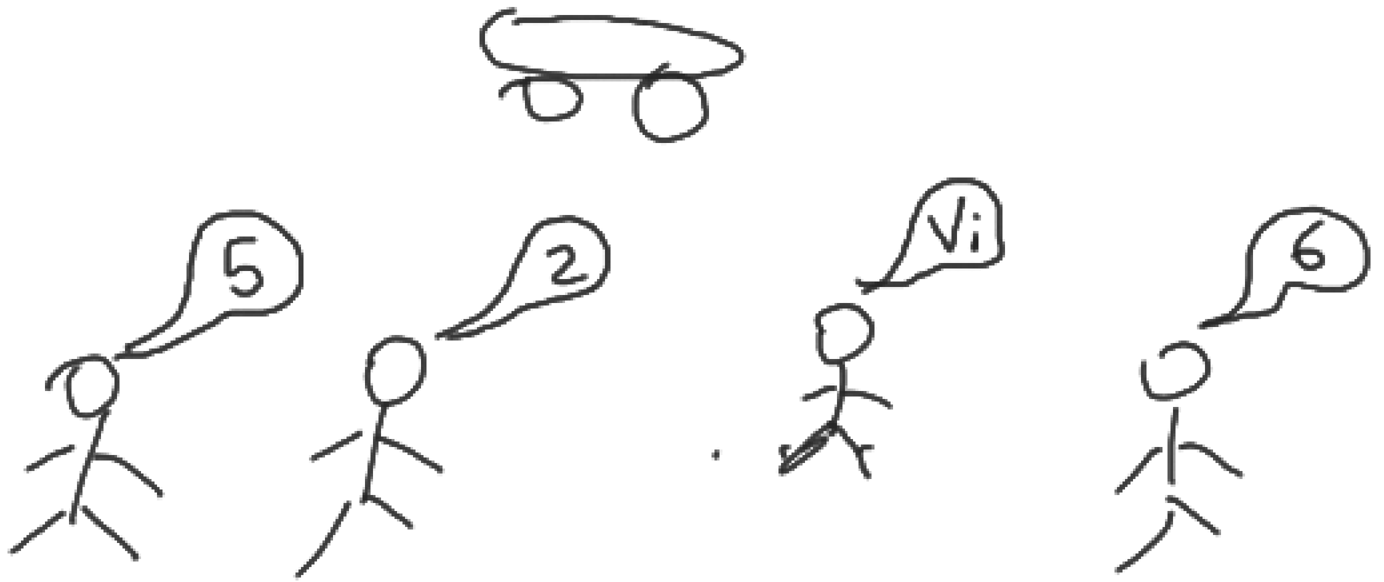
- Backward induction solves for EQ.

- Subgame Perfect Equilibria

Mechanism Design

Design rules of game so good things happen (at equilibrium)

<u>Simple Example</u> : Single-item auction



$u_i(\text{lose}) = 0$

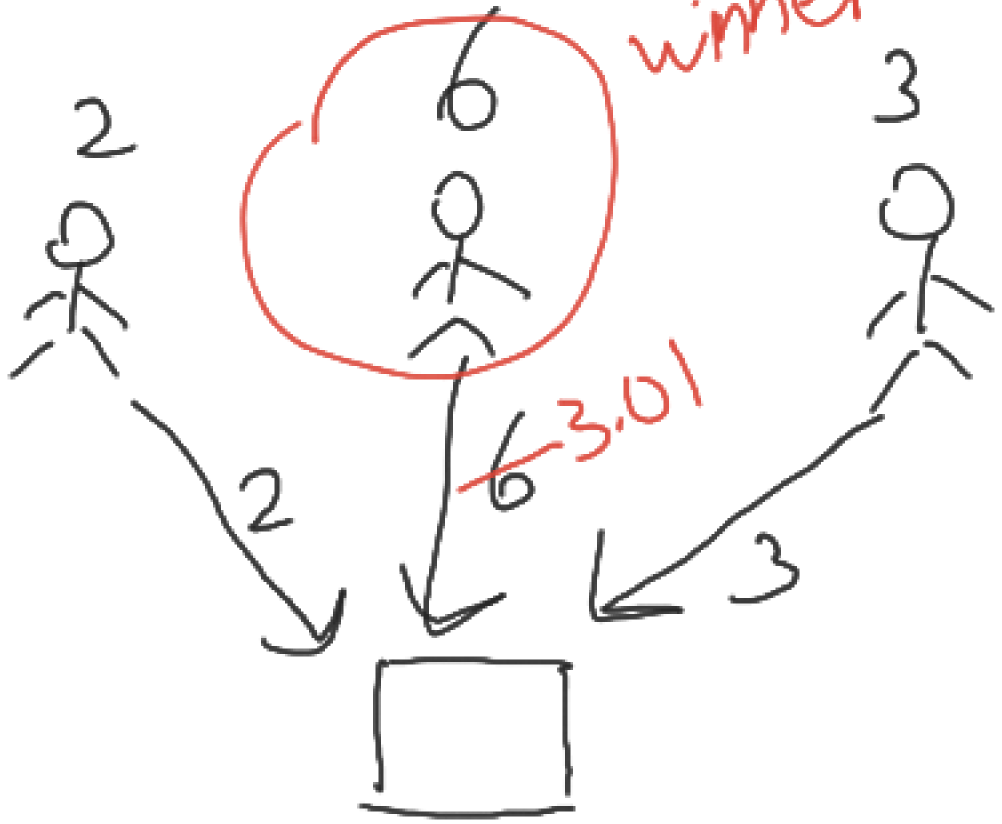
$u_i(\text{win}) = V_i - p$ ← price

Goals?

E.g. welfare, Revenue, . . .

want to give item to person with highest value

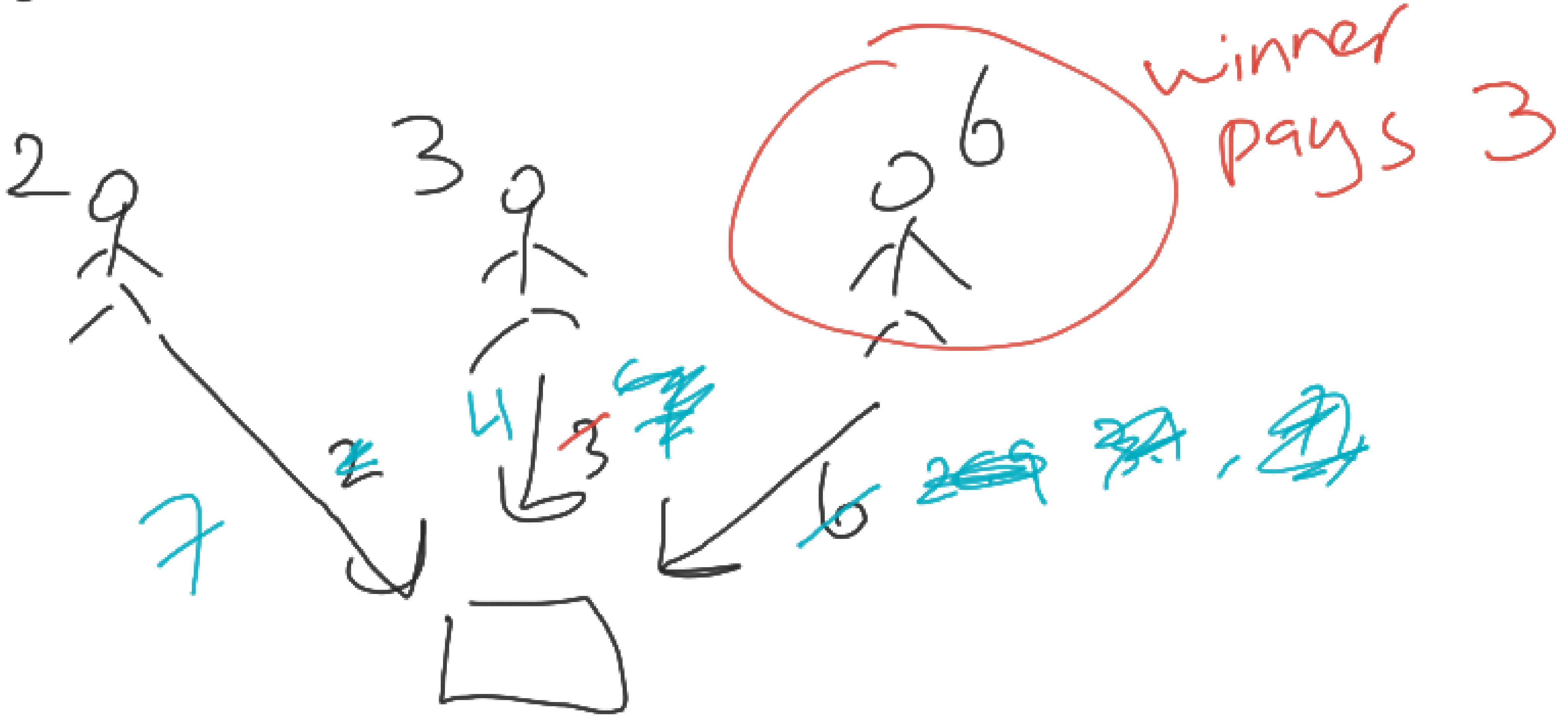
winner at price 6



Attempt I: First price auction. Not incentive-compatible.

Attempt 2: Second-Price (Vickrey) Auction

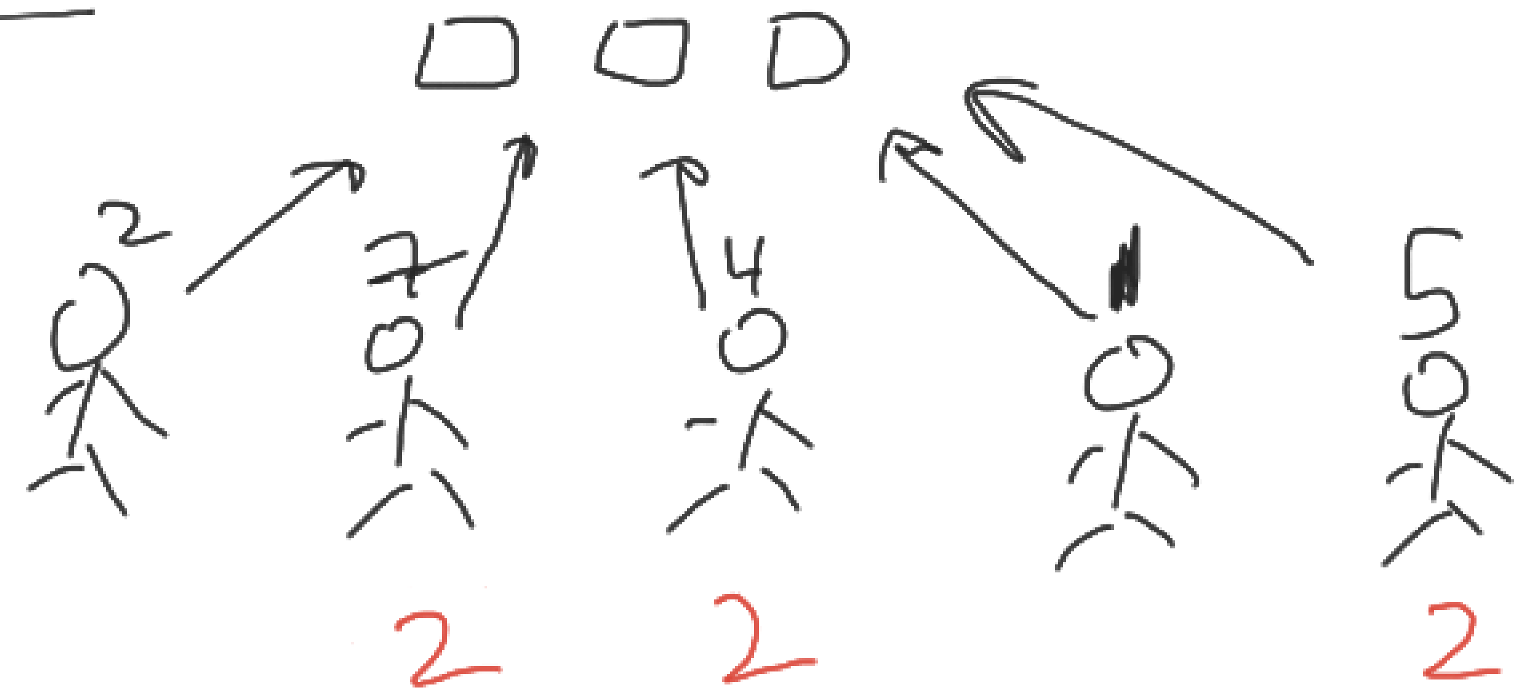
Give to highest, charge second highest bid



Truthtelling is a DS equilibrium.

Thm: Vickrey is DSIC and welfare-maximizing.

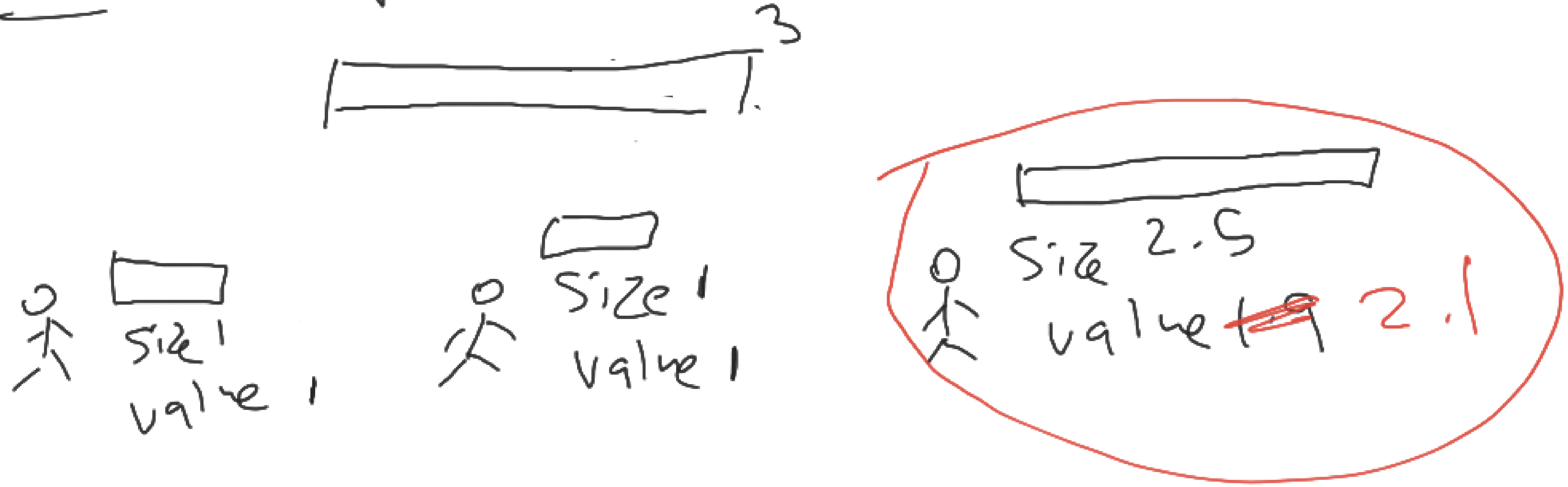
More General : k-item auction



Give to k highest bidders, Charge k+1's highest

Common property : Monotonicity

Even More General: Knapsack Auction



In a perfect world: Choose set S of bidders maximizing total bid value, subject to fitting in knapsack. Charge each of them the minimum value such that they are in S.