Day 3:

Fourier Transforms

- Fourier Transforms (Decomposing a function into sinusoidal waves)
  - The discrete Fourier transform takes a signal in time and determines which frequencies (sin/cos functions) comprise it
    - Fourier transform of signal can be referred to as “frequency content” or “spectrum”
  - Viewing certain components in a different domain can help see more important patterns and high level features
  - The Fourier Transform is analogous to a “mathematical prism”

Fourier Analysis

- Ultimately, we want to take a function $f(t)$ over an interval T_d and represent it by a series of puretones via the form:

$$f(t) = \frac{a_0}{2} + \sum c_k e^{i2\pi \frac{k}{T_d} t}$$

- Fourier Spectrum (A spectrum of frequencies corresponding to amplitude (”loudness”))
  - A Fourier Spectrum visualized would be a corrsepondance bar plot relating every frequency ($\frac{k}{T_d}$) to every amplitude ($|a_k|$)
- Time Frequency Analysis (how frequency evolves over time)
  - Peaks -
  - Can be used as a fingerprint to recognize certain types of audio

$$f(t) = \sum c_k e^{i2\pi \frac{k}{T_d} t}$$

Discrete Fourier Transform

Helpful Equation(s)

$$e^{i\theta} = cos(\theta) + i * sin(\theta)$$

- DFTs are used when we don’t have an explicitly function, but instead a set of samples $(y_n)_{n=0}^{N-1}$ which is a vector of samples recorded at regularly spaced intervals

$$f(t_n) = y_n = \sum_{k=0}^{floor(\frac{N}{2})} |a_k| cos(2\pi \frac{k}{T} t_n - \sigma_k)$$

- $\frac{N}{2}$ has to do with the Nyquist criterion because if this value is not what is used, then the sampling would be incorrect
- Goal: Given series $(y_n)_{n=0}^{N-1}$, find $|a_k|_{k=0}^{floor(\frac{N}{2})}$ and $|\sigma_k|_{k=0}^{floor(\frac{N}{2})}$
  - The formula for this is

$$C_k = \sum_{n=0}^{N-1} y_n e^{-i2\pi \frac{k}{T_d} t_n}$$

$$a_k = \begin{cases} \frac{|2C_k|}{N} & \text{if } 1 \leq k < \frac{N}{2} \\ \frac{|C_k|}{2} & \text{if } k = 0, k = \frac{N_e}{2} \end{cases}$$

- N_e is only applicable if N is even

$$\sigma_k = arctan2(-)$$

- Can we rewrite $e^{i2\pi \frac{k}{T_d} t}$ in terms of n and N instead of t_n and T_d?

$$e^{i2\pi \frac{k}{T_d} t_n} = e^{-2\pi i \frac{k}{N} n}$$