Accident Analysis and Prevention 203 (2024) 107604

Contents lists available at ScienceDirect

Accident Analysis and Prevention

journal homepage: www.elsevier.com/locate/aap

A game-theoretic approach for modelling pedestrian–vehicle conflict resolutions in uncontrolled traffic environments

Roja Ezzati Amini [a,*], Mohamed Abouelela [a], Ashish Dhamaniya [b], Bernhard Friedrich [c], Constantinos Antoniou [a]

[a] Chair of Transportation Systems Engineering, TUM School of Engineering and Design, Technical University of Munich, Arcisstrasse 21, 80333 Munich, Germany
[b] SV National Institute of Technology, Civil Engineering Department, Surat 395007, Gujarat, India
[c] Technische Universität Braunschweig, Hermann-Blenk-Str. 42, 38108 Braunschweig, Germany

ARTICLE INFO

Keywords:
Conflict resolution strategy
Pedestrian–vehicle interactions
Game theory
Uncontrolled traffic environments
Traffic safety

ABSTRACT

The interactions of motorised vehicles with pedestrians have always been a concern in traffic safety. The major threat to pedestrians comes from the high level of interactions imposed in uncontrolled traffic environments, where road users have to compete over the right of way. In the absence of traffic management and control systems in such traffic environments, road users have to negotiate the right of way while avoiding conflict. Furthermore, the high level of movement freedom and agility of pedestrians, as one of the interactive parties, can lead to exposing unpredictable behaviour on the road. Traffic interactions in uncontrolled mixed traffic environments will become more challenging by fully/partially automated driving systems' deployment, where the intentions and decisions of interacting agents must be predicted/detected to avoid conflict and improve traffic safety and efficiency. This study aims to formulate a game-theoretic approach to model pedestrian interactions with passenger cars and light vehicles (two-wheel and three-wheel vehicles) in uncontrolled traffic settings. The proposed models employ the most influencing factors in the road user's decision and choice of strategy to predict their movements and conflict resolution strategies in traffic interactions. The models are applied to two data sets of video recordings collected in a shared space in Hamburg and a mid-block crossing area in Surat, India, including the interactions of pedestrians with passenger cars and light vehicles, respectively. The models are calibrated using the identified conflicts between users and their conflict resolution strategies in the data sets. The proposed models indicate satisfactory performances considering the stochastic behaviour of road users – particularly in the mid-block crossing area in India – and have the potential to be used as a behavioural model for automated driving systems.

1. Introduction

Every year, approximately 1.3 million people lose their lives as a result of a road traffic crash, and between 20–50 million people suffer non-fatal injuries (World Health Organisation, 2019). Among all, vulnerable road users (VRUs) such as pedestrians, motorcyclists, and cyclists account for more than half of all road traffic fatalities (World Health Organisation, 2019). Crashes involving pedestrians occur most often in urban areas and while pedestrians cross the roadway at either illegal locations out of crosswalks or pedestrian crossing facilities, as they include many conflict points between pedestrians and vehicles (Lord et al., 2007; NSC-Injury Facts, 2019). Amongst pedestrian crossing facilities, uncontrolled traffic environments are associated with a higher level of traffic collisions and pedestrians' fatalities compared to controlled settings (Pfortmueller et al., 2014; Lloyd et al., 2015; World Health Organisation, 2019). Uncontrolled traffic environments can create a potential hazard for pedestrians since there are no traffic management and control systems to conduct the traffic, and the road priority is not predetermined. As a result, road users in such settings majorly rely on priority negotiation, and traffic movements implicate a more frequent and complex interaction process among them.

During a traffic conflict – a traffic event involving the interaction of users (Parker and Zegeer, 1989) – traffic participants intend to dominate the road space they are moving towards while avoiding a collision. To fulfil these goals, road users perform various manoeuvres, and a collision occurs if the performed manoeuvres fail to prevent physical contact between the interacting users. Traffic collisions and critical conflicts have severe impact on traffic safety and efficiency; however, the majority of road users interact with no serious conflict

* Corresponding author.
E-mail address: roja.ezzati@tum.de (R. Ezzati Amini).

https://doi.org/10.1016/j.aap.2024.107604
Received 19 November 2023; Received in revised form 23 March 2024; Accepted 27 April 2024
Available online 10 May 2024

or collision and are not of interest from the traffic safety point of view, but still crucial in terms of road user experience and traffic efficiency (Madigan et al., 2019; Golakiya et al., 2019).

A safe and efficient traffic interaction demands road users to correctly interpret and predict the strategies of their interactive party, although this may become complex when heterogeneous users interact on a crossing site. The heterogeneity of users (vehicles vs. pedestrians) may cause different decision-making processes and users to react differently during the interaction process. Besides, light road users (e.g., pedestrians, two-wheel vehicles) can make a sudden change of direction or speed and, thus, are prone to perform more unpredictable behaviour on the road. Therefore, a suitable modelling framework is required to predict the trajectory and movement of traffic participants during a traffic conflict and consequently enhance safety and efficiency on the road.

Research contributions

The main objective of this research is to model pedestrian interactions with different vehicle types during road crossing scenarios in uncontrolled traffic environments. Based in the reviewed literature (Section 2) that the applicability of the previously proposed models is mostly restricted to specific types of road users (i.e., passenger cars and pedestrians). This would ignore the variety of strategy choices available for user types based on their speed limit and degree of movement freedom (e.g., deviation and deceleration strategy), and thus a less efficient prediction of their behaviour. Therefore, game-theoretic models are developed in this study for pedestrian interactions with passenger cars and light vehicles in different road designs. To better reflect the user decision-making process, the utility functions of the proposed game are formulated based upon three principle layers; (1) safety level to estimate the severity level and the collision probability of a conflict event, (2) travel level associated with the detour, and deceleration imposed to interacting users by changing their speed and movement direction to escape a conflict and (3) social layer to describe the traffic environment conditions influencing the user's choice of strategy. The proposed interaction model predicts the conflict resolution strategies of users in interaction and has the potential application as a behavioural model of the automated driving systems (ADS).

The remainder of this paper is organised as follows: Section 2 provides an overview of the previous studies in the field. The detailed formulation of the game-theoretic model is described in Section 3. Section 4 includes the description of the study areas, as well as conflict detection and analysis strategies. Section 5 documents the application of the game-theoretic approach to the analysed data. Section 6 presents the model evaluation and estimation using the analysed data. Finally, Sections 7 and 8 present the discussions and conclusions, respectively.

2. Literature review

During a traffic interaction, the characteristics' discrepancy of different user types may lead to exposing various behaviour. For instance, a driver approaching a crossing may decelerate and let the pedestrian cross the road first, while the pedestrian deviates to create a bigger gap with the car and, thus, escape a potential conflict. Previous studies have investigated a broad range of factors that may influence user behaviour on the road, such as pedestrian characteristics and walking speed, vehicle's approaching speed, and road characteristics (Beggiato et al., 2017; Sun et al., 2003; Pawar and Patil, 2015). Ezzati Amini et al. (2019) argued that road users adopt a conflict resolution strategy by considering a wide range of factors knowingly (e.g., the time/distance gap estimation) and/or unknowingly (e.g., user age or gender) while they employ different communication methods to ease the interaction when needed. Besides, various strategies that users can perform to avoid a potential conflict on the road may result in different granted/expected utilities, e.g., gaining priority, saving time and shortening the traversed distance. In the field of user behaviour and traffic interactions, the modelling approaches have been mostly focused on a limited number of influencing factors in users' decision (e.g., dynamic factors, traffic characteristics), without considering the decision-making process of road users collectively and various significant factors that may affect the process (Ezzati Amini et al., 2021b). Besides, the majority of collision/conflict prediction methods estimate the traffic safety outcomes based on unchanged trajectory and speed of road users. This assumption ignores various evasive manoeuvres that road users may employ to avoid a conflict/collision on the road, and hence, restricts the model capability in evaluation of traffic outcomes in such circumstances (Amini et al., 2022). Therefore, a thorough understanding of user behaviour is vital for building a suitable model for pedestrian–vehicle interactions.

Further, developing a safe and efficient interaction concept is crucial for emerging technologies design, such as advanced driver assistance systems (ADAS) and ADS. With such automation and driver assistance technologies, more and more driving tasks are handled automatically which leads to partial engagement of drivers in partially automated vehicles, or complete absence of a human driver in fully automation cases. As a result, traffic interactions among pedestrians and such road users (i.e., automated driving systems) undergo substantial changes, where such systems would require an accurate comprehension of human road users' behaviour in traffic interactions to predict user behaviour and movement intentions, and react aptly (Schneemann and Gohl, 2016). However, similar to conventional traffic interactions, prediction of road user behaviour is challenging, and a suitable modelling framework is required to ensure the safety of pedestrians interacting with ADS and ensure the efficiency of these systems on the road.

The following subsections review some of previous research in traffic interactions and the developed modelling approaches.

2.1. Pedestrian–vehicle interactions

Several studies investigate user behaviour and model pedestrian–vehicle interactions in unregulated settings at a microscopic level. For example, Pascucci et al. (2018) formulated a discrete choice model to identify the conflict resolution strategies of users interacting on the road. The authors defined a set of explanatory variables to build the model, i.e., the movement-specific parameters (e.g., relative position, speed, acceleration), collision-specific parameters (e.g., time to collision, the existence of leading car), and the number of simultaneous conflicts of users. In another approach, Pascucci et al. (2015) utilised the future position of the interacting users and the time of leaving the conflict zone as indicators to determine the crossing priority. Both studies found that models effectively predict basic conflicts between pedestrians and cars but highlighted the necessity for further research into more complex interactions involving multiple road users. To explore more complex interactions, Schönauer (2017) used a Stackelberg competition game to model road user behaviour in conflicting situations, in which the probability of collision, agents' position and distance, and rule-based and social-based behaviour of users determine their conflict resolving strategies. The authors suggested that factors such as traffic cultures (e.g., different countries, different traffic regulations), and traffic density (e.g., group behaviour, gap acceptance behaviour) can significantly influence traffic behaviour and should be taken into account in the future for the model's transferability. In line with this, Johora and Müller (2020) combined the social force model with the Stackelberg competition approach to model the road user interactions. In this approach, the interactions are classified into simple and complex. The proposed model covers various interactions, including pedestrian–pedestrian, multiple pedestrian–vehicle, and vehicle–vehicle interactions. Besides, it employs factors such as the number of active interactions, speed, and travelled distance to capture the dynamics of these interactions accurately. Nasernejad et al. (2021) similarly employed agent-based framework to model pedestrian behaviour during near misses when interacting with vehicles.

Pedestrian–vehicle conflicts are represented using the Markov decision process framework, with a Gaussian process inverse reinforcement learning (GP-IRL) method used to infer pedestrian collision avoidance strategies. A deep reinforcement learning model is employed to estimate optimal pedestrian strategies during conflicts. The model predicts pedestrian trajectories, evasive manoeuvres, and post-encroachment time, highlighting the importance of developing safety-focused approaches for modelling user interactions in mixed traffic conditions. A study by Nasernejad et al. (2023) emphasised on the multi-agent nature of traffic interactions and formulated a Markov-Game framework to model pedestrian–vehicle interactions and their collision avoidance behaviour in mixed traffic environments. In this framework, agents were considered rational and interacted with each other simultaneously. The authors then contrasted this multi-agent model with a single-agent GP-IRL approach, aiming to learn and replicate pedestrians' evasive manoeuvres assuming fixed trajectories for vehicles. The study findings illustrated that multi-agent models outperformed single-agent models in predicting collision avoidance strategies of road users. In a similar manner, multiple studies employed the Markov game to model pedestrian interactions with motorcyclists and compared the results with those obtained from a GP-IRL single-agent approach (Lanzaro et al., 2022; Alsaleh and Sayed, 2021), and a maximum entropy IRL algorithm (Alsaleh and Sayed, 2020). Once again, the multi-agent framework exhibited superior prediction accuracy compared to the single-agent model.

In summary, extensive research has explored user behaviour and modelled pedestrian–vehicle interactions at a microscopic level. The reviewed studies emphasise the significance of considering factors such as traffic cultures, density, and multi-agent dynamics to enhance the transferability and accuracy of models. Overall, these findings underscore the importance of comprehensive modelling approaches in addressing the complexities of pedestrian–vehicle interactions across diverse traffic environments.

2.2. Pedestrian-ADS interactions

With respect to the ADS, and through the application of various methods, objects in motion (e.g., vehicles, pedestrians, cyclists) are tracked to predict their trajectories and future positions. The prediction system, then, hypothesises multiple possible predictions of the future movement of dynamic objects. Several studies investigated the interaction of pedestrians with automated vehicles and proposed modelling approaches to simulate the ADS interactions with VRUs (Schneemann and Heinemann, 2016; Chen et al., 2016; Møgelmose et al., 2015). In a research, Feng et al. (2019) used Cellular Automata to model interactions at mid-block crossings by considering a broad range of factors, such as the lane width and length, number of lanes, speed limit, vehicle size, and speed. The model employs the yielding regulations at crossings in China and evaluates the lane-based post-encroachment time between a vehicle and pedestrian as a safety index. The results showed that the proposed conflict elimination method effectively reduces conflicts between automated vehicles and pedestrians. However, the authors noted a limitation in the model's transferability, as it primarily focuses on pedestrian safety in China. In another study, Völz et al. (2018) combined motion tracking algorithms with data-driven methods to predict the crossing intention of pedestrians. The authors argued that the correct prediction of pedestrian intentions at a crossing is essential to prevent unnecessarily slowing down traffic. For instance, when an automated vehicle stops for a pedestrian with no intentions to cross the roadway. The proposed model considers the dynamic distance measures of pedestrian and vehicle motion on approaching the crossing site to predict the next action. However, as stated by the authors, this approach cannot ensure consistent performance across crosswalks with significantly different geometries. In the realm of pedestrian intention recognition and prediction, Rehder et al. (2018) introduced an Artificial Neural Network approach. This method utilises images and positions to determine pedestrian destinations and applies trajectory planning toward these destinations. The output produces a probability distribution map of possible destinations using Markov decision processes and the forward–backward algorithm. Experimental validation demonstrated the system's accuracy in predicting both user destinations and trajectories. Expanding on pedestrian trajectory prediction, Jayaraman et al. (2020) developed a hybrid system model incorporating pedestrian gap acceptance behaviour and interacting user speeds. This model categorises pedestrian states as approaching a crosswalk, waiting, crossing, and walking away, aiming to capture long-term (>5 s) pedestrian crossing behaviour. The study showed the model's effectiveness in predicting long-term pedestrian crossing trajectories, particularly at crosswalks. However, real-world scenarios involving multiple pedestrians pose challenges for ADS in distinguishing between crossing and non-crossing individuals, suggesting the need for future model improvements. In a related vein, Fox et al. (2018) proposed a game-theoretic model for priority negotiation between automated vehicles and other users (e.g., a vehicle or pedestrian) in unsignalised intersections/crossings. This model assumes that agents' optimal behaviour includes a non-zero probability of collision occurrence, with yielding probability gradually increasing as interacting users get closer. The model assumption of the non-zero probability of collision occurrence validates previous findings that ADS will make little or no progress if they are known to be perfectly safe and always yield to the interacting users. The authors suggest extending the model to incorporate further realistic details such as continuous speed to reflect deceleration strategies of road users (rather than discrete speed), traffic regulations, norms, and vehicle lateral positions to enhance its applicability.

In summary, research in ADS has made significant progress in understanding pedestrian–vehicle interactions. However, challenges persist in real-world scenarios, such as accurately predicting pedestrians' intentions during traffic interactions and ensuring model transferability across diverse environments. Further research is needed to address these challenges in order to improve ADS performance and enhance road safety when automated vehicles become integrated into traffic systems.

3. Conflict resolution model

Understanding and predicting road user behaviour is a complex modelling problem since it includes understanding and predicting of surroundings, and interacting users' current and future actions (Fox et al., 2018). The latter issue may lead to paradox and incomputability issues as described in Gödel theorem and Halting problem (Velupillai, 2009). Game theory provides a cooperative and competitive paradigm to manage the self-referential decisions of players (i.e., road users in the game) and describe pairwise traffic interactions (Fox et al., 2018). The cooperative and competitive characteristics of game-theory-based models align with real-world traffic interaction scenarios, where road users employ conflict resolution strategies in response to the strategies of other users. This means that the gain or expected utility of users depends on the reactions of interacting users, while they strive to maximise their own utilities (Talebpour et al., 2015; Ali et al., 2019). These are similar to the real-world behaviour of road users in traffic interactions, where, for instance, they compete over the road space and try to avoid critical traffic conflicts. For this reason, a game-theoretic approach is applied in this research to determine the user decisions interacting in uncontrolled traffic settings and predict the conflict resolution strategies. The proposed game-theoretic approach focuses on pedestrian road crossing scenarios and highlights the active nature of road users as players in traffic interactions. It takes into account the collective decision-making process of interactive users and determines the game outcome accordingly.

Depending on the user type, each player in the game (referring to the interacting users in a conflict) has specific degrees of permitted

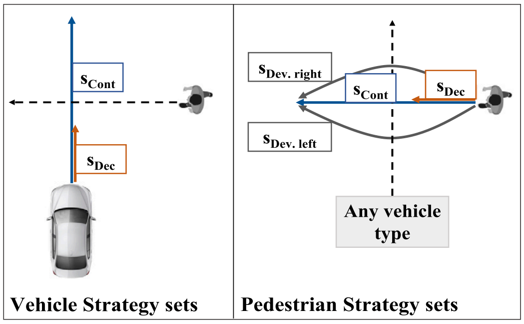

Fig. 1. Conflict resolution strategies of passenger cars (left) and pedestrians (right) in an interaction. 2W and 3W users have similar strategy sets as pedestrians.

movements that define their trajectories in the game. For instance, a large goods vehicle commonly moves straight without deviating from its forward trajectory (in a safe driving situation): while lighter user types, such as pedestrian, two-wheel (2W) and three-wheel (3W) vehicles, can swerve right or left due to their agile characteristics. Further, users may apply speed changes/adjustments to avoid conflicts. A combination of the trajectory and speed changes that users can employ to avoid a conflict is defined as the player's strategy set in the game and clustered as:

- **Continuing strategy** (S_{Cont}): applicable for all user types and by moving along the free-flow trajectory (FFT) with preferred/current speed,
- **Deceleration strategy** (S_{Dec}): applicable for all user types, and by moving along the FFT with reduced speed,
- **Deviation strategy** (S_{Dev}): applicable only for light users (i.e., pedestrians, 2W and 3W), and by deviating to the left ($S_{Dev.left}$) or right ($S_{Dev.right}$) from the FFT or collision point with preferred/current speed.

Fig. 1 demonstrates a schematic overview of the users' trajectories corresponding to the various available strategy choices defined per user type.

3.1. General structure of the Stackelberg game

In a traffic interaction, the competition of road users over the right of way results in conflicting interests among them. Given the conflicting interests among users, a strategic game of Stackelberg leadership competition is formulated to model pedestrian–vehicle interactions in uncontrolled traffic settings (Ezzati Amini et al., 2021a). The two-player Stackelberg game assumes that one of the players is the leader of the game and the other is the follower. In the game, the leader plays a strategy first, and then the follower reacts to the leader's announced strategy. This approach highlights that in most traffic interactions, the decisions of road users are not made concurrently but more in the form of action and reaction. Besides, the users employ a conflict resolution strategy with respect to the strategy of their interacting users, i.e., the gain/expected utility of the users depends on the interacting user's reaction. Fig. 2 illustrates a two-player Stackelberg game tree and payoffs for taking each strategy pair in the game. One player performs the game leader (L) role with its available strategy choice $(s_1^L, \ldots, s_n^L) \in S^L$ and another player is the follower (F) with $(s_1^F, \ldots, s_m^F) \in S^F$ as its strategy set. Specifying one strategy s_n^L for the leader and one strategy s_m^F for the follower yields an outcome represented as a payoffs pair of $(U^L(s_n^L, s_m^F), U^F(s_m^F|s_n^L))$, where U^L is the utility that the leader receives and U^F is the utility of the follower. In this paper, payoff and utility terms are used interchangeably.

3.2. Stackelberg game solution

The game solution is determined by finding the sub-game perfect Nash equilibrium (SPNE). A SPNE is a strategic outcome in game theory where each player's chosen strategy maximises their payoff, accounting for current and future interactions. It ensures no player has an incentive to deviate from their strategy at any point. This stable solution captures optimal decision-making throughout the entire game, considering both immediate and long-term considerations. One prevalent method to find the SPNE is backward induction, i.e., the best responses of the follower (B_{follower}) must be computed first to allow the leader to maximise its payoff:

$$B_{\text{follower}} = max_{(s^F \in S^F)} U^F(s^F|s^L) \tag{1}$$

$$\text{SPNE} = \boldsymbol{argmax}_{s^L \in S^L} U^L(s^L, B_{\text{follower}}) \tag{2}$$

Where U^L yields the leader's maximum utility for selecting the best strategy from its choice of actions $(S^L, s^L \in S^L)$. In the proposed game-theoretic approach, agents may receive different payoffs by playing the same strategies, as conflicting users are heterogeneous (vehicle vs. pedestrian) with distinct characteristics and objectives on the road. Further, road users' payoffs/strategies may vary depending on the strategy choice of their interacting user. Therefore, Eqs. (1) & (2) can be transformed into the mixed strategy approach to finding the optimal game solution. In this approach, the probability vectors of $P^L(s^L)$ and $P^F(s^F|s^L)$ reflect the likelihood of performing a strategy by the game leader and a strategy by the follower given the leader's strategy, respectively. As proved by Nash (1951), one mixed strategy Nash Equilibrium exists in a game given the outcomes of:

$$P(s^L, s^F) = P^L(s^L) * P^F(s^F|s^L) \tag{3}$$

3.3. Formulation of utility functions

Within the framework of the Stackelberg game, the study assumes the rationality of players and their pursuit of utility maximisation when making strategic choices. The selection of strategies within the game is expected to align with the objective of maximising utility, with a particular focus on minimising collision risk and energy loss while simultaneously maximising driving/crossing comfort. Road users' utilities are defined based on an extensive literature review conducted by Ezzati Amini et al. (2019, 2021b), which identifies influential parameters pertaining to road user behaviour during pedestrian–vehicle and pedestrian-ADS interactions. By integrating the utility formulation with the identified influential factors, this study strives to contribute to a comprehensive understanding of road user behaviour and its implications within traffic interactions.

The study defines three layers to formulate the utility functions; safety layer, travel layer, and social layer, and explained in the following subsections.

3.3.1. Safety layer

The safety layer is defined to estimate the severity level and the collision probability of conflict events regarding the performed conflict resolution strategies. For instance, how safe a conflict outcome would be if a car continues its path and a pedestrian (as its interacting user) deviates right to cross in front of the car. Whether the time/distance gap would be long enough for the pedestrian to cross the road safely or a critical condition/collision would occur. The conflict risk evaluation model for pedestrian–vehicle interactions, developed by Amini et al. (2022), is embedded in the game to assess the severity of traffic conflict. The model emphasises how performing various evasive manoeuvres affects the users' safety. The conflict risk evaluation models are formulated using logit models and three surrogate safety measures, where the discrete choices of conflict and non-conflict are examined. A

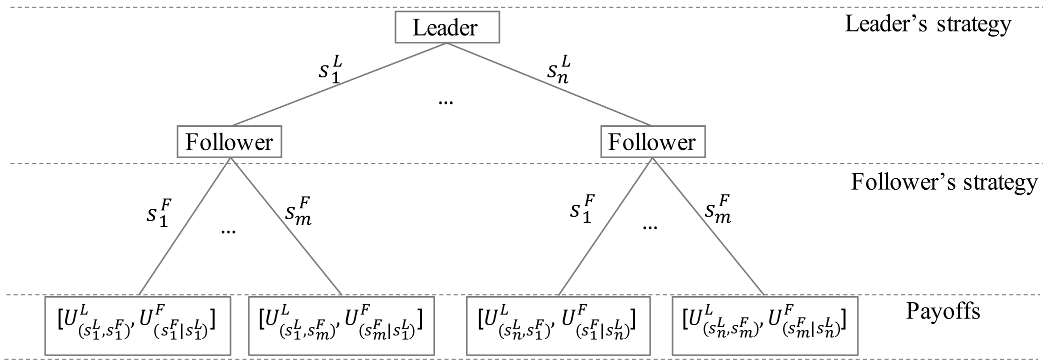

Fig. 2. Stackelberg game tree and sub-games. The payoffs of playing each strategy pair are shown in the terminal nodes.

logit model assumes a linear relationship between predictor variables (e.g., x_i) and the binary response variable Y as:

$$\ell = log_b \frac{p}{1-p} = \Sigma \theta_i x_i \tag{4}$$

where ℓ, b, and θ are the logit, logarithm base, and model parameters, respectively. Hence, the probability of the response variable $Y = 1$ (i.e., critical conflict) with the predictor variables (i.e., x_i) is:

$$p = \frac{1}{1 + b^{-(\Sigma\theta_i x_i)}} = S_b(\Sigma\theta_i x_i) \tag{5}$$

where S_b is the sigmoid function with base b, and θ_i are estimated model parameters. The selected surrogate safety indicators employed in the model are as follows:

I. Minimum future relative distance (MD) indicator evaluates the distance proximity of users after applying different strategy pairs. The MD compensates for the limitations of previously implemented measures for collision risk prediction, assuming that users would collide if the condition remains unchanged. The MD indicator identifies new collision points or estimates the distance proximity of users when the theoretical collision point changes or no longer exists (e.g., in deviation strategies).

II. Time to minimum distance (TMD) indicator estimates the available time gap between the arrival of the users to the MD or theoretical collision point, if any. This time-based indicator is added to the conflict analysis to capture the simultaneous arrival of the users to the MD, i.e., the distance and time proximity of the users after performing conflict resolution strategies. The TMD estimation is based on the user's speed defined per strategy.

III. Conflicting speed (CS) refers to the speed of heavier interacting road users when evasive manoeuvres are taken and at the moment of minimum distance. The CS reflects the severity level of the conflict by taking into account the speed changes of users during an evasive action (e.g., deceleration strategy). This is particularly important in traffic situations where users (usually motorised vehicles) either come to a complete stop before/while reaching the MD threshold or roll over a small gap. Although the MD and TMD are below the thresholds in such cases, the traffic condition is still considered safe.

The model estimates these three safety measures for all possible combinations of strategies available for user types to avoid a conflict and evaluates the safety of the outcomes. The model thresholds are determined by applying various methods (i.e., intersection point, maximum between-class variance, p-tile, and minimum cross-entropy method) to separate potential critical conflicts against normal traffic conditions. Then, the F-score method is used to select the optimal thresholds with the best performance (van Rijsbergen, 1979). Finally, the overall performance of the models and their thresholds are assessed using two metrics: (I) accuracy reflecting the percentage of correct predictions (i.e., normal and critical traffic conditions), and (II) sensitivity showing the percentage of the events correctly labelled as critical traffic conditions. The results indicate the average accuracy of 92.4%, and sensitivity of 83.4%. for the pedestrian–passenger car model, and average accuracy of 92.9% and sensitivity of 86.2% for the pedestrian–light vehicle model. Similar data sets as this study are used to develop and validate conflict risk evaluation models for the interaction of pedestrians with vehicles (passenger cars) and light vehicles (2Ws and 3Ws) separately. Table 1 summarises the parameter estimates for pedestrian–vehicle and pedestrian–light vehicle conflict risk evaluation models.

Table 1
Parameter estimates of the logit models for pedestrian conflict events with passenger cars and light vehicles (Amini et al., 2022).

Parameter	Estimate	Std. Err.	Z-value	Pr(>|z|)
Pedestrian–passenger car model				
Intercept	4.327	0.578	7.48	$<7e^{-14}$***
MD	−2.898	0.256	−11.31	$<2e^{-16}$***
TMD	−3.067	0.304	−10.08	$<2e^{-16}$***
CS	0.376	0.079	4.73	$2.2e^{-06}$***
Pedestrian–light vehicle model				
Intercept	4.490	0.373	12.03	$<2e^{-16}$***
MD	−2.813	0.168	−16.69	$<2e^{-16}$***
TMD	−2.365	0.184	−12.80	$<2e^{-16}$***
CS	0.263	0.040	6.24	$3.5e^{-11}$***

The threshold of 0.425 is determined for the pedestrian–vehicle model, and 0.512 for the pedestrian–light vehicle model through the p-tile method with the best performance. In the concept of game theory, if the conflict risk exceeds the determined thresholds, the event is labelled as a critical conflict, and players receive (−1) as a penalty for playing the strategy pair. Conversely, for taking strategies with conflict risk lower than the thresholds, users receive (0) for the safety layer utility:

$$Safety\ utility = \begin{cases} -1, conflict\ risk \geqslant threshold \\ 0, conflict\ risk < threshold \end{cases} \tag{6}$$

3.3.2. Travel layer

This layer is associated with the detour (DT) and deceleration (DC) imposed on interacting users by changing their speed and movement direction to escape a conflict. This class aims to quantify the comfort level of different strategies. Road users tend to reach their destinations by taking the shortest path (i.e., the FFT) while maintaining their speed. Therefore, the extra traversed distance by players to reach their destination return a detour dis-utility for users. For this purpose, the free-flow trajectory of users to reach the theoretical collision point – extracted through the conflict detection procedure (see Section 4.2) – is compared with their traversed trajectory while deviating to the right or left from the FFT. The deviation angles determined in Section 4.3.3 are utilised to compute the new traversed distance in deviation strategies, and the corresponding dis-utility is returned for the additional distance implied while performing deviation strategies. A similar approach is

Table 2
Utility functions to compute payoffs of strategy pairs for player i as the game leader, and j as the follower. d^{str} refers to the traversed distance in each game strategy, and d^{FFT} to the distance of traversing the FFT (see Section 4.2, Step 2).

Category	Metrics	Utility	Formula	Specification
Safety layer	MD (m) TMD (s) CS (m/s)	SL_{ij}	Predicted conflict of the estimated models in Table 1 for user type conflicts.	Model thresholds: pedestrian-car = 0.425 pedestrian-light vehicle = 0.512
Travel layer	Detour (d_i, d_j) Deceleration rate (dc_i, dc_j)	DT_i, DT_j DC_i, DC_j	$\exp(d_i^{FFT} - d_i^{Str}) - 1$ $\exp(dc_i) - 1$	$d_i^{Str} > d_i^{FFT}$
Social layer	Pedestrian group size Pedestrian approaching lane Right of way	PL_i, PL_j LN_i, LN_j RW_i, RW_j	$\{-1, 0, 1\}$ $\{-1, 0, 1\}$ $\{0, 1\}$	Group size > 2 Middle lane or kerbside Who gets priority?
Utility of playing strategy pair (S^L, S^F)		Leader: $U_L(S^L, S^F) = \sum Utilities$ Follower: $U_F(S^F \mid S^L) = \sum Utilities$		

applied for users decelerating due to a conflict on the road. The average deceleration rate of user types is used to reflect the deceleration dis-utility when users change their speed in reaction to a conflict on the road. The average deceleration rate is extracted from the data sets and for each road user type and applied accordingly to estimate the utility (see Section 4.3.2). The exponential functions scale the utility values between (−1) and (0) in the travel layer.

3.3.3. Social layer

This layer describes the traffic environment conditions that influence the user's choice of action. This class includes the influencing factors of pedestrian group size (PL), approaching lane (LN), and the right of way (RW). The parameter group size of pedestrians affects road user behaviour in traffic interactions (Sun et al., 2003; Sucha et al., 2017; Malenje et al., 2018). Larger groups enhance visibility and perceived safety, leading to more assertive actions like crossing or merging. They also promote collective behaviour and conformity. In contrast, smaller groups tend to be more cautious due to reduced visibility and vulnerability. Additionally, the group size can affect other road users' behaviour, with larger groups attracting more attention and potentially altering driver behaviour. Similarly, the approaching lane of traffic influences pedestrian–vehicle interactions on the road (Fricker and Zhang, 2019; Kadali and Vedagiri, 2020). Factors such as visibility, crossing behaviour of pedestrians, driver awareness of pedestrians, expectation of right of way are impacted depending on the approaching lane. Therefore, understanding these dynamics is crucial in pedestrian–vehicle interactions and parameters are included in the modelling framework.

For pedestrian group size and approaching lane, the strategies are evaluated with respect to the interacting user's strategy, and based on the aggressiveness level: aggressive, neutral, and courteous. Players receive (−1) as a penalty for performing aggressive strategies, (0) for neutral strategies, and (+1) as an incentive for taking courteous manoeuvres. For instance, a car receives dis-utility of (−1) if it continues its path and does not yield to the interacting pedestrians who cross in a group greater than two persons. In the same scenario, the car gets (+1) as a utility if it decelerates, and pedestrians continuing to cross the road would receive (0). Similarly, users receive (−1), (0), or (+1) for pedestrians approaching from the kerbside or the middle lane of the road, based on the performed manoeuvres. Factors of pedestrian group size and approaching lane are added to the game utilities to also reflect the importance of the social norms in traffic conflicts, i.e., whether social norms support a conflict resolution strategy. These factors aid in weighting the high utility of saving energy in taking aggressive manoeuvres (e.g., continuing strategy with high detour, and deceleration utilities) and the energy loss in taking courteous strategies in the presence of risky conditions (e.g., deceleration strategy with low detour, and deceleration utilities). In the absence of the social norm utility, interacting users aim to maximise their individual utilities by minimising energy loss, e.g., traversing shorter distance (Kemloh wagoum et al., 2012; Liao et al., 2017). Consequently, users invariably prefer to take strategies that return such utilities, and strategies such as continuing always become dominant in the game, which contrasts with the real-world decision-making process of users in interactions. Regarding the right of way, the player who gets priority by taking a strategy receives utility (+1) and (0) otherwise.

Table 2 summarises the utility computations in all layers of the game.

3.3.4. Utility function

All attributes influencing the agents' preferences to deliver the supra objects integrate into one utility function (multi-attribute utility function), representing the overall agent's utility. The final formulation of utilities for the leader strategy choice is calculated in the following way by considering a set of weights θ for the parameters:

$$U_L(s^L, s^F) = \theta_{sl} SL_{ij} + \theta_{dt} DT_i + \theta_{dc} DC_i + \theta_{pl} PL_i + \theta_{ln} LN_i + \theta_{rw} RW_i \tag{7}$$

4. Data collection and conflict analysis

Two video graphic surveys are used for conflict analysis and model application in this research. The data analysis relies on the users' trajectories and a set of explanatory variables extracted from the data sets. A conflict detection procedure is applied to identify the potential conflicts among road users and determine the conflict resolution strategies of interacting users. For each conflict event, the utilities of interacting users are computed for all possible combinations of strategies (with respect to the user type) that players could perform in the game, including the real-world strategies in the data sets.

A similar approach is applied to compute the utility of the follower $U_F(s^F | s^L)$.

4.1. Video surveys

Two video graphic surveys were used in this study to analyse road user behaviour in uncontrolled traffic settings; (1) a mid-block crossing in Surat city, Gujarat, India (Golakiya and Dhamaniya, 2018), and (2) a shared space in Hamburg city, Germany (Pascucci et al., 2021). The interaction data from these locations were used to investigate road user behaviour, conflict resolution strategies, and factors influencing their choice of actions during traffic conflicts. The selection of these locations was based on the diverse road user types interacting with pedestrians (e.g., passenger cars, light vehicles), and dominance of said road user types in the areas with low interference in the interaction dynamic among them (Pascucci, 2020). The chosen study locations also prioritised high-traffic volume areas to examine road user behaviour with higher exposure to traffic interactions and safety critical circumstances (pedestrian flow of 304 ped/h, 2W flow of 2393 veh/h, and 3W flow of 1350 veh/h in mid-block crossing area, and vehicle flow of 600 veh/h and pedestrian flow of 2200 ped/h in the shared space area). Additionally, the selection of study locations considered road layouts

Fig. 3. The street view of the mid-block crossing area from where the camera is placed (right) (Golakiya and Dhamaniya, 2018), and the aerial view of the site (left).

Fig. 4. The street view of the shared space area from where the cameras are placed (right) (Pascucci et al., 2021), and the aerial view of the site (left).

with longitudinal vehicular traffic movement facilitating pedestrian crossings from one side to the other, as well as the number of lanes and roadway width during the selection process. These significant properties assisted in developing a more suitable modelling framework which is capable of predicting road user behaviour during traffic interactions and is not limited by culture-, traffic layout- and user types-specific characteristics of the case studies (Amini, 2022).

It is important to mention that this study utilises pre-processed video data obtained from different sources (Golakiya and Dhamaniya, 2018; Pascucci, 2020), which introduces the possibility of biases stemming from variations in the pre-processing methods employed. Therefore, it is crucial to interpret the results within the context of these limitations.

4.1.1. Mid-block crossing interactions

The first video graphic survey is collected in a mid-block crossing area in Surat city, Gujarat, India, where traffic drive on the left (Golakiya and Dhamaniya, 2018). The crossing is located on a six-lane arterial urban road with an additional Bus Rapid Transit lane and in the vicinity of businesses, stores, and hospitals. Two lanes of the road in the mid-block crossing area are selected for the video observation and data analysis (Fig. 3). The video survey is performed by placing a camera on a building at an elevation of 15 m. On January 3rd, 2017, a survey was conducted from 09:00 until 19:00. Within this time frame, a specific 30-min period from 09:30 to 10:00 was selected, during which the overall traffic flow, including both pedestrians and vehicles, was observed to be at its highest. The data is pre-processed by overlaying a grid of size 40×8.85 m^2 over the captured video using the Ulead VideoStudio 11 software (Golakiya and Dhamaniya, 2018). The Avidemux 2.6 software is used for tracking the video by 0.48 s time steps (12 video frames). A variety of traffic modes pass through the road, leading to more complex traffic interactions in the mid-block crossing area. The extracted data contains the trajectory of passenger cars, heavy goods vehicles (HGVs), large goods vehicles (LGVs), 2Ws, 3Ws, and pedestrians.

4.1.2. Shared space interactions

The second video graphic survey is collected through video recording for a shared space zone in the district of Bergedorf (Weidebaumsweg), Hamburg city, Germany (Pascucci et al., 2021). The length of the shared space area is 63 m and is in the proximity of a shopping mall and retail stores. Two cameras were placed at an elevation of approximately 7 m and in opposite directions of traffic to perform the video survey (Fig. 4). The video was recorded on Saturday, April 2, 2016, from 13:30 to 16:30. A 30-min of the recorded data (period between 1:50 and 2:20 p.m.) is selected for conducting detailed analysis of road user behaviour since during this time, the pedestrian volume was found to be the highest among all video material. The software Tracker is used to pre-process the data (Douglas, 2017; Pascucci, 2020). The extracted data includes the trajectory data in terms of coordinates every 0.5 s, velocity, and acceleration for passenger cars, pedestrians, and cyclists. However, cyclists are out of this research interest and neglected in the analysis. In the shared space, vehicles have priority over other road users, and pedestrians should use the given/available gap when it is long enough to traverse the crossing.

It is worth noting that there is no real-world collision between traffic participants in the studied data; however, a conflict detection procedure was applied to identify traffic conflicts and evasive actions of interacting road users.

4.2. Conflict detection strategies

Uncontrolled traffic environments entail a constant interaction among road users. In such traffic events, at least one of the interacting users would need to take an evasive manoeuvre to avoid the conflict; otherwise, a collision would occur. For this reason, a four-step conflict detection procedure is designed to identify the users in conflict. The data analysis was performed using R programming language. Coding consistency was ensured through a stepwise approach, starting with a small sample size and iteratively applying it to the entire data set. Trajectory data of interacting road users were compared with time/speed data, and results were reviewed twice to confirm alignment between identified conflicts and road user data.

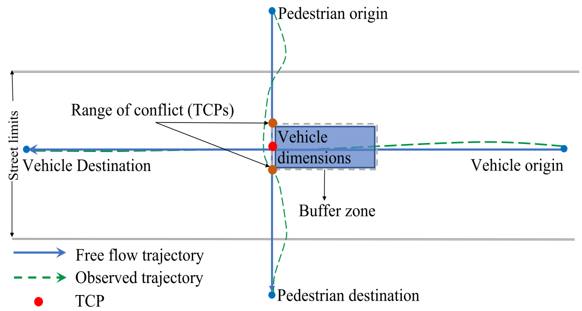

Fig. 5. A simplified example of the conflict detection procedure in a pedestrian–passenger car interaction.

Step 1. Data simplification: A street boundary is specified to keep the trajectories of pedestrians who cross the roadway and remove the rest walking along the road. The conflict analysis also excludes vehicles that exit the road before reaching the mid-block crossing (in the mid-block crossing data set) and vehicles being parked or pulling-in along the road (in both data sets). Then, data are divided into 60-s time intervals to reduce the number of events and simplify the analysis.

Step 2. Free flow trajectories (FFTs): The FFTs of road users are plotted by connecting the shortest path from their origin to their destination. The FFTs are employed since traffic participants tend to take the shortest path to reach their destination without a traffic event and evasive manoeuvre.

Step 3. Intersection points: A theoretical collision point (TCP) is defined to identify the intersected trajectories of users if they had taken the FFTs to reach their destinations (Fig. 5). In addition, a buffer zone is considered for all vehicle types to improve the accuracy of the collision points. The buffer zone assumption implies the real-world collision events in which vehicles hit the pedestrians at the buffer, than/before the TCPs.

Step 4. Time to collision (TTC): For identified TCPs, a minimum relative TTC of 3 s (Sayed and Zein, 1999) is assumed to capture the simultaneous arrival of the vehicle and pedestrian at the TCP and users' buffers (near- or far-buffer, depending on the approaching direction). TTC is calculated based on the average speed for each user. The Minimum TTC was computed as:

$$|TTC_{vehicle} - TTC_{pedestrian}| \leqslant 3 \text{ s} \tag{8}$$

The position of interacting users and their direction of movement/approach concerning the other involved user are taken into account during the procedure. The application of the conflict detection procedure to the collected data led to identifying 120 conflict events between pairs in the shared space data set and 158 events in the mid-block crossing, where evasive manoeuvres were performed by one/both interacting users to escape the potential collisions (Table 3). Some users had more than one theoretical conflict (i.e., involved in multiple conflicts) and were analysed independently. In the shared space area, the group of pedestrians at the crossing – interacted with the same vehicles – were analysed separately. The presence of other road users in the interaction scene, regardless of playing their independent games, is reflected in pay-off functions, such as pedestrians walking in a group or approaching from the opposite lane. However, the collected data in the mid-block crossing did not allow the independent analysis of the pedestrians crossing in a group, and they were analysed as a group.

4.3. Conflict resolution determination

The conflict resolution strategies of users are determined manually and according to their FFTs, observed trajectories and speed profile during the interaction process. Additionally, the graphics interchange format (GIF) files were generated to verify the identified strategies by displaying the decision points of users (i.e., to spot where/when users change their speed) and their animated movements. For simplification, the combination of the strategies and performing multiple strategies were neglected in the analysis, and the last actions were considered the users' conflict resolution strategies. The identified strategies are clustered as described in Section 3 for each user type. It is worth noting that deviation strategies are only defined for light road users (i.e., 2w, 3w, and pedestrians), due to:

Table 3
Summary of the identified conflict events with pedestrians in the studied areas (Amini et al., 2022).

Location	Car	2W	3W	HGV/LGV	Total
Shared space area	120	NA	NA	NA	120
Mid-block crossing	11	92	51	4	158

- The higher degree of movement freedom for such user types and consequently a wider range of available strategy choices on the road,
- The analysed interaction data in the studied location where deviation strategies are commonly employed by light users and heavier road users (e.g., passenger cars) move along their forward trajectories, and
- The road layouts in the sites with longitudinal movement of vehicular traffic along two lanes, and pedestrians crossing the road from one side to the other. Accordingly, road users' movements were based on the lane width, traffic directions, and the layout of the road and conflict zone: deviation trajectories were limited to the lighter road user type, and heavier user types only deviated to exit the road or park/stop along the road.

Fig. 6 shows the strategy of users in conflict determined through the conflict resolution determination procedure for both data sets. The following subsections explain the preferred speed, deceleration rates, and deviation angles specified for strategies per user type.

4.3.1. Determination of the preferred speed

For continuation and deviation strategies, the users' preferred speed for crossing the road is extracted from the data sets for each user type. Since there are significant differences regarding user behaviour and infrastructure designs in the studied locations, the preferred speed of user types is estimated separately.

A k-mean clustering approach is used to compute the preferred crossing speed of pedestrians in the shared space data set. Initially, non-conflict pedestrians – who are not involved in any conflicts/interactions – are grouped based on the approaching direction. Then, different crossing phases are defined for pedestrians on approaching a crosswalk (Gorrini et al., 2016), or while avoiding a conflict on the road (Pascucci, 2020). Based on non-conflict trajectories of pedestrians in the shared space data, the crossing is divided into three movement phases: (I) pedestrian decelerates on approaching the crossing/road kerb while evaluating the available gap to cross the road, (II) after accepting the gap, pedestrian accelerates to reach the crossing speed, and (III) pedestrian crosses the road with roughly constant speed, which is assumed to be the preferred crossing speed. These stages are displayed in Fig. 7, where the acceleration changes of pedestrians reflect the movement phases. It is worth noting that few samples with constant crossing speeds greater than 2.5 m/s are removed from the analysis. Finally, a k-means algorithm is applied on variables walking speed, acceleration, and corresponding crossing time (i.e., speed and acceleration at each time step of crossing) of pedestrians as below (Lloyd, 1982):

- Determination of the number of clusters (k): The Elbow method is used to determine the optimal number of clusters (k = 3).

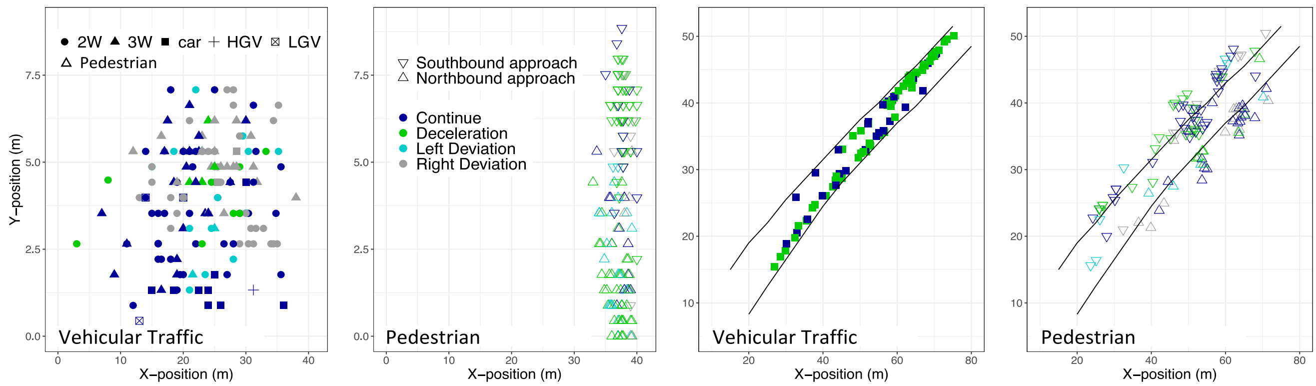

Fig. 6. Conflict resolution strategy of users plotted by their position at the reaction time. From left to right: the first two figures show the vehicular traffic and pedestrians' strategies on approaching the mid-block crossing area, the second two figures show the vehicular traffic and pedestrians' strategies in the shared space.

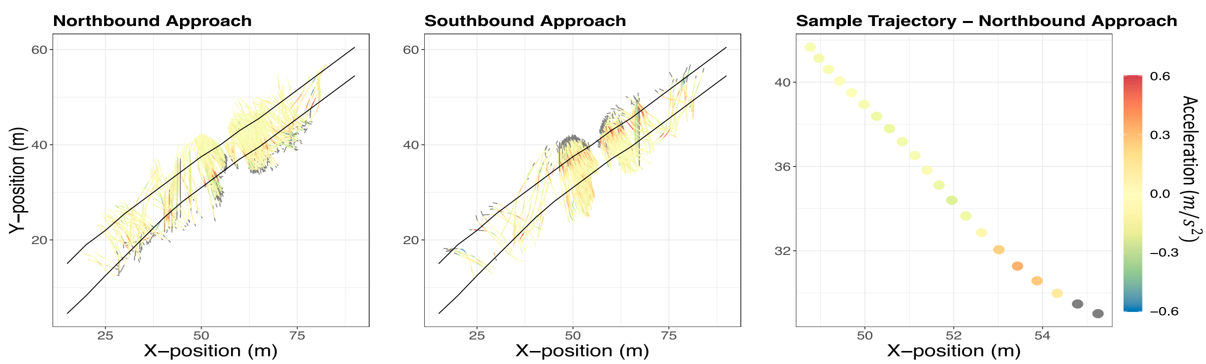

Fig. 7. Non-conflict trajectories of pedestrians in crossing the road in the shared space data set.

Table 4
K-Means clustering results of preferred crossing speed of pedestrians in the shared space.

Cluster	Variables			Size	Silhouette
	Speed	Acceleration	Time		width
1	1.184	−0.009	3.49	4023	0.65
2	1.270	0.039	9.91	4294	0.52
3	**1.312**	0.006	17.84	3971	0.52

$Between_S S / total_S S$ = 86.6%

- Centroid initialisation: The traditional random points method is used to initialise centroids for clustering.
- Assigning points to the closest cluster centroid: Based on the distance from the centroid, data points are assigned to different clusters.
- Re-computation of centroids: The process is repeated until the centroids of clusters remain unchanged (100 iteration).

Table 4 summarises the details of the K-means clustering application, as well as the Silhouette coefficients for validation of consistency within data clusters. The mean pedestrian walking speed in cluster 3 (1.31 m/s) – corresponding to the third movement phase of crossing – is selected as the preferred crossing speed of pedestrians for continuing and deviation strategies in the shared space.

In preference to the speed limit, the 85th percentile of the speed for the vehicle type in the data set is considered as the preferred speed (Table 5). Vehicles commonly tend to drive at or near the same speed as traffic around, regardless of the speed limit. This is in accordance with the Vienna Convention speed adjustment rules, where drivers need to pay constant regard to the circumstances, such as the state of the road, the weather conditions, and the density of traffic, to be able to stop the vehicle timely if needed (Vienna convention on Road Traffic, 1968).

Table 5
The preferred speed of vehicle types in the studied areas.

Vehicle type	Speed limit	Mean speed	Std. dev.	85th percentile
Vehicles in the shared space				
Passenger car	5.5	2.64	1.91	**4.7**
Vehicles in the mid-block crossing				
Passenger car	16	7.83	2.43	**10.41**
2W	14	8.76	4.27	**9.51**
3W	10	7.61	2.32	**8.75**
LGV&HGV	11	8.43	1.67	**10.41**
Pedestrian	–	0.81	0.66	**1.04**

Speed unit: m/s.

Although India is not a signatory of the Vienna Convention, driver behaviour to adjust the speed assumes to be similar. The preferred speed is applied in the continuing strategy of all vehicle types, and deviation strategy of light vehicles. The 85th percentile of the speed is also assumed to determine the preferred crossing speed of pedestrians at the mid-block crossing, given that a small number of the user free flow crossing is available from the data set (Table 5). This is due to the high level of interactions and the traffic density at the mid-block crossing, where pedestrians predominantly cross the roadway after escaping a conflict with the vehicles (based on the applied conflict detection procedure in this study).

For all user types, when the user speed at the reaction moment is higher than the preferred speed, the current speed is considered the strategy speed (in continuing and deviation strategies).

4.3.2. Determination of deceleration rate

Determination of the deceleration rate of user types for the corresponding strategy (i.e., deceleration strategy) is not straightforward.

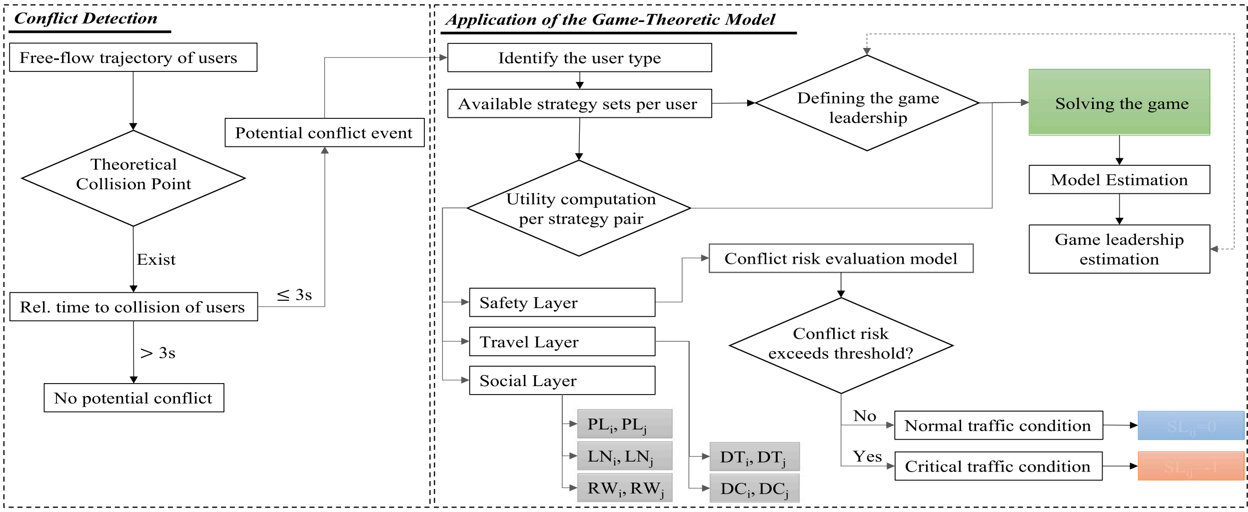

Fig. 8. A general framework of the game-theoretic model application.

The reason is that the deceleration rate depends on several factors, e.g., the user's initial speed, available time/distance gap to clear the conflict zone, and the presence of other road users. As a consequence, it is difficult to estimate an explicit deceleration rate that reflects the general behaviour of users decelerating in reaction to a conflict on the road. Nevertheless, the average deceleration rate of user types in both data sets is used to predict the deceleration strategy in the model.

4.3.3. Determination of deviation angle

The average deviation angle of users deviated from their forward trajectories – and consequently from the TCP – is regarded as the deviation angle of user type for the associated strategies (i.e., deviation right and deviation left strategies). As the degree of movement freedom can vary from one road design to another, the deviation angles of users are estimated independently for the shared space and mid-block crossing. In the shared space, the average deviation angle of pedestrians is 22.28 degrees, with a standard deviation of 9.15°. For simplification reasons, the deviation angle of the pedestrians is assumed as 22 degrees in the model. In the mid-block crossing, pedestrians and light vehicles (2Ws and 3Ws) deviate with the average angle of 26.5° (rounded as 26° in the model) with a standard deviation of 9.35°, and 11.11° (rounded as 11° in the model) with a standard deviation of 6.85°, respectively. Other vehicle types in the data sets have no deviation strategy in their choice of actions.

5. Application of the Stackelberg game

This section discusses the application of the developed game-theoretic model to conflict events. A general framework of the game-theoretic model application is depicted in Fig. 8. As shown in the figure, the initial step to apply the model is detecting a conflict. In this work, a conflict detection procedure is applied to the collected data to identify the users in conflict (see Section 4.2 for details). However, for the general application of the model, the perpendicular users' trajectories would be replaced with the proposed FFTs to identify the theoretical collision point and the conflict event, if any. After identifying the conflict events, the interacting users' position and speed at the moment of conflict detection, and the TCP coordinates are used to compute the game utilities (see Fig. 9 for illustration of possible user's strategy combinations during interactions). Users receive utility for each strategy combination in the game, i.e., the leader and follower strategy combination determined from their available choice of actions. The game outcome, then, is determined through the backward induction method as discussed in Section 3.2.

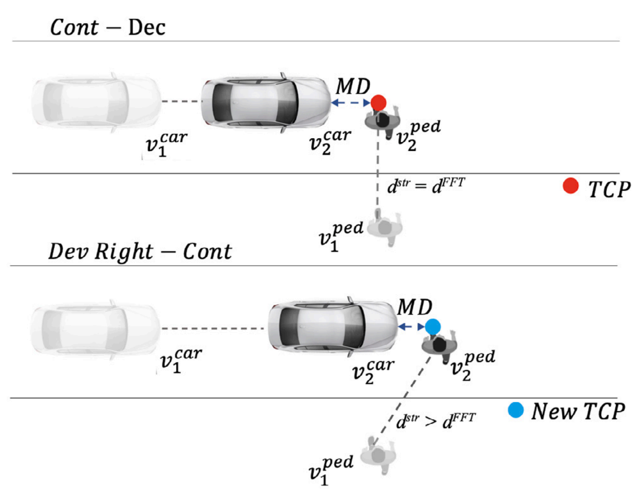

Fig. 9. An illustrative example shows a pedestrian and a car performing continuing-deceleration and right deviation-continuing strategies. It features initial and strategy speeds (v) and positions. Two alternative strategy combinations are demonstrated for users' conflict avoidance without depicting body size (Amini, 2022).

The R (R Core Team, 2024) programming language is used for detecting conflict events, the utility computation, and finding the game solution. Algorithm 1 (see Appendix) presents the employed approach for probability computation of performing a strategy by a user in conflict. The computation is based on the user role (leader or follower) and the interacting user choice of action. The probability of the outcomes is, then, determined through Eq. (3) for each strategy combination in the game. The highest probability returns the game solution and, therefore, the user's choice of actions to avoid the conflict. In the model application, all users in conflict are assumed to once be the leader and once the follower of the game.

6. Model results

6.1. Estimating the game-theoretic models

A likelihood approach is employed to estimate the parameter values (θ_i) of the Stackelberg game. The utility of strategy choice ($s \in S$) for players in the game is computed based on Eq. (7) for the game leader and a similar approach for the follower. The probability of strategy

Table 6
Parameter values and their standard deviation for model estimations in the shared space and mid-block crossing data sets. LB and UB: Lower and upper bound of confidence intervals. $Alpha = 0.05$.

Parameter	Shared space						Mid-block crossing					
	μ	$\sigma_{\bar{i}}$	LB	UB	t-value	p-value	μ	$\sigma_{\bar{i}}$	LB	UB	t-value	p-value
θ_{sl}	1.42	0.20	1.02	1.81	6.97	<.0001	1.77	0.16	1.45	2.09	10.98	<.0001
θ_{dl}	1.99	0.28	1.45	2.54	7.20	<.0001	1.48	0.12	1.25	1.71	12.49	<.0001
θ_{dc}	1.13	0.54	0.07	2.18	2.09	<.01	3.36	0.26	2.85	3.87	12.89	<.0001
θ_{pl}	1.17	0.14	0.88	1.45	8.13	<.0001	1.29	0.20	0.89	1.69	6.37	<.0001
θ_{ln}	1.05	0.11	0.84	1.26	9.79	<.0001	0.80	0.10	0.61	1.00	8.05	<.0001
θ_{rw}	0.88	0.13	0.62	1.14	6.54	<.0001	0.80	0.11	0.59	1.01	7.44	<.0001

Table 7
Leadership determination methods for the pedestrian–passenger car and the pedestrian–light vehicle models.

Selection criteria	Negative log-likelihood	
	Shared space	Mid-block crossing
Time to TCP	976.4	1335.0
User type	985.8	**1280.2**
Reaction time	**826.6**	1352.2

choice for players can be obtained through the sub-game probability for the follower and choice probability for the leader of the game, providing the overall probability of the strategy pairs as:

$$P(s^L, s^F) = P_L(s^L) * P_F(s^F | s^L) \tag{9}$$

Therefore, the log-likelihood function is applied for the model estimation:

$$LL(\theta | y_{in}) = \sum_i \sum_j y_{in} \log(P(s^L, s^F)) \tag{10}$$

where y_{in} is, 1 when the strategy pair is selected by the players in the game, and otherwise 0. Then, the negative log-likelihood is minimised using the numerical optimisation algorithm for a quasi-Newton method of Broyden Fletcher Goldfarb Shanno (BFGS) (Coppola et al., 2014) in R programming language. Fig. 10 depicts the optimisation flowchart employed for the estimation of parameter values in the model. Since data sets have substantial differences in terms of user behaviour and infrastructure design, the model estimations are applied separately to the shared space for pedestrian–passenger car conflicts and the mid-block crossing for the pedestrian–light vehicle conflicts. Few passenger cars, HGVs, and LGVs in the mid-block crossing are not considered in the estimation to improve the model's accuracy involving light vehicles. This is mainly to develop a model that predicts the conflict resolution strategies of users with respect to their type. In order to have different data sets for model estimation and validation, a hold-out method is applied to split the conflict events on both data sets into 70% for training and 30% for testing purposes. The model estimations are applied to the training samples, and different parameters' combinations are examined to obtain the optimal training results (i.e., highest sensitivity, specificity, and accuracy). In both models, the combination of all proposed parameters provides the highest performance, and thus all utilised in the models' estimation. Table 6 summarises the estimated parameters (θ_i) in both models using the calibration data acquired at the shared space and mid-blocking data sets. The mean values (μ) are returned by the maximum likelihood method, and the standard error of the mean ($\sigma_{\bar{i}}$) and confidence intervals for parameter estimates are computed through the observed Fisher information. The t-statistic and the p-value are used to assess the significance of the estimated parameters. As summarised in Table 6 all the employed parameters are statistically significant for the prediction of game outcomes in the models.

6.2. Game leadership estimation

As mentioned earlier in Section 5, the models are applied by switching the leadership role among the users in conflict. To better determine the game leadership in the model, different approaches are evaluated:

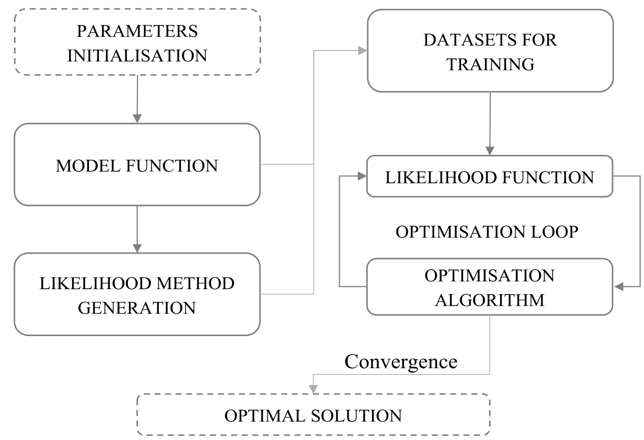

Fig. 10. Flowchart of the optimisation process.

- **Time to TCP:** The player closer to the TCP (in terms of time) is the game leader, while the other interacting player is the follower.
- **User type:** The type of users interacting on the road defines the leadership; motorised versus non-motorised road users. In case of pedestrian interaction with motorised vehicles, pedestrian is the follower and motorised vehicle is the game leader.
- **Reaction time:** The user who reacts first to a stimulus (i.e., conflict) on the road is considered the game leader, and its interacting user is the follower. In this research, users' reaction time is recorded based on their speed and trajectory changes during the conflict resolution determination procedure and utilises to determine the game leader.

A likelihood approach is employed to select the best leadership determination method (similar to the utilised method by Schönauer (2017)), and the results are shown in Table 7. The likelihood results in the pedestrian–passenger car model and the shared space data set reveal that reaction time is a better method for leadership determination. This supports the logic behind the Stackelberg leadership game, where the leader of the game plays a strategy first, and the follower reacts to the leader's strategy. Regarding the pedestrian–light vehicle model in the mid-block crossing, the likelihood method yields slightly similar results for the time to TCP and reaction time methods; however, as expected, the user type is the best fit to define the game leadership.

6.3. Model performance and validation

The testing data sets are used to assess the overall performance of the developed models. For this reason, confusion tables are created for categories: game choice (as "success") and non-choice (as "failure"), denoting the counts by true positive (TP), false positive (FP), true negative (TN), and false negative (FN). The accuracy, sensitivity, and specificity across all classes are computed for both models. The results are summarised in Tables 8 & 9. The model sensitivity and specificity are used as metrics to evaluate the model's ability to predict TPs and

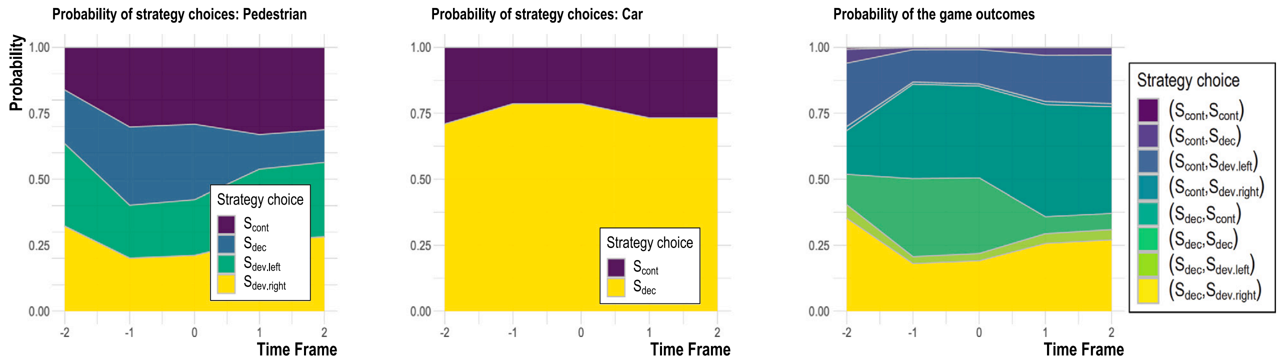

Fig. 11. Probabilities of strategy choices in a pedestrian–passenger car conflict. From left: probability of strategy choices for pedestrian as the game leader, for car as the follower, and the probability of game outcomes for strategy combinations ($P(s^L, s^F)$).

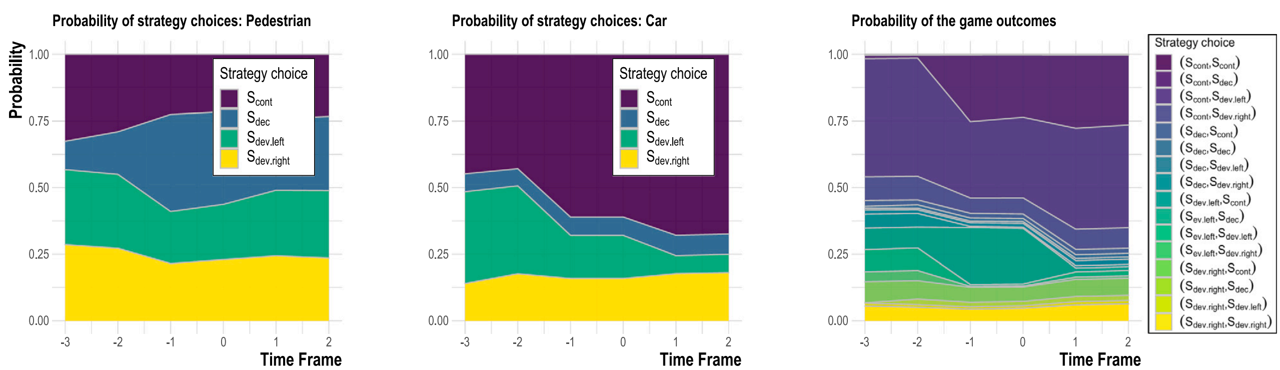

Fig. 12. Probabilities of strategy choices in a pedestrian-2W conflict. From left: probability of strategy choices for 2W as the game leader, for pedestrian as the follower, and the probability of game outcomes for strategy combinations ($P(s^L, s^F)$).

Table 8
Confusion table for pedestrian–passenger car model.

	True value		Total
	Choice	Non-choice	
Predicted: choice	60	12	72
Predicted: non-choice	12	492	504
$ACC = 0.958$			

Table 9
Confusion table for pedestrian-light vehicle model.

	True value		Total
	Choice	Non-choice	
Predicted: choice	54	28	82
Predicted: non-choice	28	1202	1230
$ACC = 0.957$			

TNs of each conflict event, respectively. The pedestrian–passenger car model in the shared data set shows an average accuracy of 95.8%, a sensitivity of 83.3%, and specificity of 97.6% (see Table 8). The accuracy of the pedestrian–light vehicle model is 95.7%, with a sensitivity of 65.8% and specificity of 97.7% (see Table 9). The pedestrian–passenger car model with a misclassification rate of 16.7% shows a good performance reflecting the user decision-making process in the shared space. Regarding the mid-block crossing model, the misclassification rate amounts to 34.2% for the pedestrian–light vehicle model. This performance considers satisfactory given the user behaviour in the studied location and the wide range of factors influencing the user decisions, such as high traffic volume, road design, traffic behaviour, vehicle size, and driving culture. The methodology used shows that the models can be applied in different traffic setups (e.g., mid-block crossing, shared spaces), traffic patterns (e.g., various user types), and traffic behaviour (i.e., user behaviour in different countries).

Two specific examples are selected from data sets to further explain the model performance and characteristics. The results are visualised in Fig. 11 & 12. The model is applied to conflict events in the time frames prior/following the actual reaction time of the interacting users in the game. The application of time frames aims to demonstrate the model performance concerning the variation in the game. The first example is a pedestrian–passenger car conflict in the shared space where the pedestrian is the game leader, and the passenger car is the follower (Fig. 11). In the real-world scenario, the pedestrian continues its path, and the passenger car decelerates to let the pedestrian pass first. The probability of strategy choice in Fig. 11 shows that the deviation strategy to cross behind the passenger car initially returns a higher utility (time frame = −2); however, the probability of the continuing strategy increases over the time and remains the preferred strategy of the leader. The strategy choice probability for the passenger car in Fig. 11 demonstrates the highest utility in taking the deceleration strategy during the illustrated period. The high utility of the deceleration strategy is mainly due to the pedestrian group size (>2) in the conflict example. The probability of the game outcomes ($P(s^L, s^F)$) shows that the strategy deceleration-continuing returns a higher probability at the reaction time (time frame = 0), where both players receive the highest utility from their strategy choices.

The second example is a pedestrian-2W conflict from the mid-block crossing data set (Fig. 12). The preferred strategy of the 2W, as the game leader, is to continue its path where the highest utility is gained. The pedestrian selects the left deviation strategy to cross the roadway behind the 2W, rather than waiting until the conflict zone is clear. The strategy pair of continuing-deviation left returns the highest probability at the actual reaction time of users (time frame = 0) as the final game outcome, in which the game leader receives its highest utility from its strategy choices. The final game outcome ($P(s^L, s^F)$) is similar to the taken strategies of users in the real-world conflict scenario.

7. Discussion

In this research, a game-theoretic model is applied to predict the conflict resolution strategies of pedestrians interacting with motorised vehicles. The model assumes that users select a strategy that maximises their utilities. Further, the user's choice of strategy in the proposed Stackelberg game is based on the possible reaction of the interacting user to avoid a conflict. The latter property is substantial for a safe traffic interaction – as a bilateral event – where the collective strategies of interacting users determine the outcome. The game utilities are formulated in three layers of safety, travel, and social to cover a broad range of factors influencing the user's decision in a traffic conflict. In the safety layer, previously developed conflict risk evaluation models by Amini et al. (2022), are utilised to assess the safety of interacting users after performing each strategy pair. The safety layer estimates the future minimum distance between interacting users, the relative time of users' arrival at the collision zone (i.e., new collision point or MD), and conflicting speed. Then, based on the estimated surrogate safety indicators and model thresholds, the model identifies the hazardous traffic conditions between pedestrians and vehicles and returns a dis-utility when applicable. The comfort level of different strategies is quantified in the travel layer for the detour, and deceleration imposed by users' speed/trajectory changes. The users receive dis-utilities for the corresponding energy loss of each conflict resolution strategy available for users in the game. The third utility layer covers three of the most significant environmental factors influencing the user's choice of strategy; pedestrian group size, pedestrian approaching lane, and the right of way. The social layer signifies the importance of social norms in improving safety in uncontrolled traffic environments. Finally, the outcome of the proposed strategic game of the Stackelberg leadership competition is determined through the backward induction method. The proposed game-theoretic approach is estimated separately for pedestrian interactions with passenger cars and light vehicles (i.e., 2Ws and 3Ws).

7.1. Impact factors

To better evaluate the model variables, a Probability Density Function (PDF) (sampling distribution of the sample mean) is used to visualise the relation between the shared space and mid-block crossing for each parameter value (θ_i) returned after the model estimations (see Fig. 13):

$$f(\theta_i|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{11}$$

where μ is the mean value and σ is the standard error computed by the maximum likelihood method per θ.

Safety layer (θ_{sl}): As expected, in both shared space and mid-block crossing data sets, safety layer plays a significant role ($\theta_{sl} > 2\sigma_{\bar{sl}}$). This is because road users maximise their safety by employing conflict resolution strategies that minimise the critical conflict and/or collision risk on the road. In the mid-block crossing model, the collision avoidance is crucial in strategy choice of users; however, the analysed data indicates that pedestrians primarily take courteous strategies during the interactions.

Travel layer (θ_{dt}, θ_{dc}): In both models (i.e., data sets), the parameter value of detour is substantial in user's choice of strategy ($\theta_{dt} > 2\sigma_{\bar{dt}}$); however, it is stronger in the pedestrian-passenger car model with deviation strategy choices available only for pedestrians. The effect of deceleration variable is strong in both models ($\theta_{dc} > 2\sigma_{\bar{dc}}$); however, deceleration is clearly dominant in the decision-making process of users in the mid-block crossing. This supports the user behaviour in the studied location, in which there is less tendency to slow down to give the right of way in traffic interactions. The overall driver yielding rate for pedestrians during the conflict scenarios in the mid-block crossing is approximately 12%, and motorised vehicles often deviate to pass the conflict zone. Therefore, the strategies with no speed changes are preferred by users in the game.

Social layer (θ_{pl}, θ_{ln}, θ_{rw}): This layer of the game utility incorporates the pedestrian group size, approaching lane, and the right of way. All three variables have significant impact on user's strategy choice in the shared space model ($\theta_{pl} > 2\sigma_{\bar{pl}}$ & $\theta_{ln} > 2\sigma_{\bar{ln}}$ & $\theta_{rw} > 2\sigma_{\bar{rw}}$) reflecting the importance of social norms in safe movements of different user types in such urban designs. The social layer has slightly different influence on the pedestrian–light vehicle model due to the user behaviour in the studied location. While the pedestrian group size is relevant for the decision-making process of users in conflict in the mid-block crossing, approaching lane seems to be less strong.

It is worth noting that previous studies found a wider range of factors influencing pedestrian behaviour during interactions, such as pedestrian age and gender, personal characteristics, culture, and waiting time. However, it is hard for the current stage of ADS development to process such pedestrian-associated information (Ezzati Amini et al., 2021b). Therefore, this research only employs the most significant factors – identified in the literature – currently feasible to process by the ADS.

7.2. Stackelberg game model applicability

The complexity of pedestrian interactions with motorised vehicles presents considerable challenges for users' behavioural modelling and prediction. The characteristics' discrepancy of different road user types and, thus, their granted/expected utilities, lead to exposing different behaviour on the interaction scene, where users employ various strategies to escape a conflict. In the automation technology domain, there are many uncertainties regarding the most influential factors of pedestrian behaviour, the possible impact of these factors on pedestrian behaviour, and the implementation of these factors in ADS (Ezzati Amini et al., 2021b). However, there are several technological advancements to maintain pedestrian safety in the absence of a human driver in uncertain/conflicting traffic situations, such as pedestrian/object detection, tracking objects in motion, automated braking systems in case of hazardous conditions, and pedestrian protection systems to minimise the damage when a collision is unavoidable. Yet, in complex urban scenarios with a high level of elaborated interaction strategies among road users, ADS are not fully capable of handling an efficient priority negotiation with traffic participants (Fox et al., 2018). Therefore, a suitable interaction method is required to ensure the efficiency of ADS on the road and the safety of road users interacting with such vehicles. The proposed game-theoretic approach predicts the conflict resolution strategies of users in interaction by taking into account the safety outcome of various strategies in a conflict event and the impact of travel and environmental factors on user decisions. The game-theoretic approach has the potential to be used as a behavioural model for ADS and improve pedestrian safety, particularly in uncontrolled traffic settings. The proposed model enables the ADS to make more informed decisions during a traffic interaction, have a dynamic interaction with pedestrians, and react adequately to their actions when required. Such interaction concept can prevent the "bullying behaviour" by other traffic participants to block the automated vehicle path or/and occurring so-called "freezing robot problem" that they may be subject to Madigan et al. (2019) and Färber (2016).

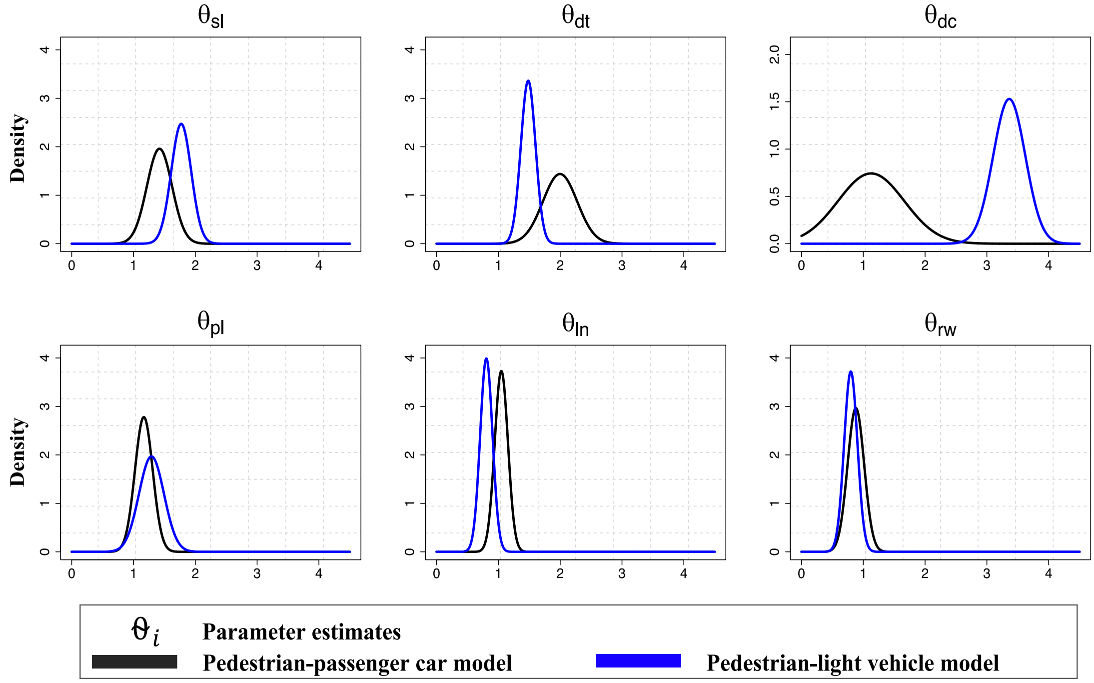

Fig. 13. Comparison of the parameter estimates in developed models using the shared space and mid-block crossing data sets.

Finally, such behavioural models can offer insights for policymakers, urban planners, and transportation engineers seeking to enhance pedestrian interactions with ADS in uncontrolled traffic environments. For instance, policymakers could utilise the model's predictions to develop/improve regulations and guidelines, ensuring safe and efficient integration of ADS. Urban planners could use the insights to design pedestrian-friendly infrastructure and implement measures to enhance pedestrian-ADS interactions, such as dedicated lanes or signal systems. Transportation engineers could apply the findings to optimise traffic flow and minimise congestion by incorporating pedestrian behaviour into traffic management strategies. These examples highlight the practical relevance of such modelling approaches in real-world decision-making and planning processes.

7.3. Limitations and further research

In this research, the game-theoretic models are developed based on interaction data in a shared space in Hamburg, Germany, and a mid-block crossing in Surat, India. Despite the differences in crossing facility types and user behaviour in the studied locations, the models demonstrated good performance, verifying the transferability of the proposed models. However, to construct a more suitable and realistic model, further investigation is required. This entails conducting comprehensive data analysis across varied traffic settings in different locations, while employing harmonised pre-processing techniques. To address potential biases arising from data heterogeneity, the study undertakes separate estimation of game-theoretic models for different conflict scenarios observed in each location. By leveraging diverse interaction data, the study aims to develop a widely functional behavioural model capable of capturing various user behaviour in uncontrolled traffic environments. Moreover, it could be worth considering the inclusion of road users with disabilities (e.g., cane, wheelchair, guide dogs) as well as pedestrians pushing strollers or prams, in future models, to conduct additional testing and enhance the overall performance. Besides, this study suggests future works to focus on further testing and validating the proposed model and its capability in various traffic settings, by using for instance, multiple participant simulator (MPS) technique (Lehsing and Feldstein, 2018). The MPS techniques provide the same virtual environment for simultaneous interactions among participants and can be used to improve the performance of the proposed models. Additionally, further validation approach can be employed to validate and refine the game-theoretic framework by comparing simulated outcomes with observed real-world data. Such processes will allow adjustment of model parameters and algorithms to better align with actual traffic behaviour. These testing and validating efforts extend beyond the scope of the current study's focus.

Further, the Stackelberg game assumes that players (i.e., interacting road users) within the game are rational and will try to maximise their payoffs; however, this may not always be the case. Road users can behave irrationally during traffic interactions, or they may not have full information available for maximising payoffs. Such assumptions in game-theoretic approaches may potentially result in overlooking certain conflict resolution strategies during traffic interactions, combinations of evasive manoeuvres or undefined strategies in the game. Therefore, the future model development requires adding an element of randomness that can explain several phenomena where players do not behave in line with the rationality assumptions of the game theory. Alternatively, other solutions that assume sub-optimal decision-making can be evaluated to relax such game assumptions (Alsaleh and Sayed, 2022). Besides, road users can perform a broader range of evasive manoeuvres to avoid conflict in real-world traffic interactions. For instance, pedestrians may execute unexpected changes in their movement directions or suddenly stop at any crossroads point. However, it can be hard to incorporate such unpredictable behaviour into the behavioural modelling approaches. Similarly, considering decision factors such as traffic culture, traffic volume, traffic layout, and age in behavioural models at this stage presents a challenge due to the current limitations of automated driving systems (ADS) in processing such pedestrian-associated information. In addition, the deviation strategy of users in the model is considered with no speed changes to simplify the interaction model and avoid generating a computationally expensive algorithm. Finally, the independent parameters of the proposed model are mostly based on the findings of the studies on traditional pedestrian–vehicle interactions and the current stage of ADS development. This research suggests further studies to investigate how the vast existence of automated vehicles in the public realm can alter pedestrian behaviour, and which influencing factors in traditional interactions are applicable within the ADS conceptual framework.

8. Conclusions

Pedestrians with a high level of movement freedom on the road can perform unexpected behaviour, leading to more complex traffic interactions. Traffic participants intend to dominate the road space during the interaction process while their safety is assured. Such competitions over the road space are more complicated in uncontrolled mixed environments where users have to negotiate the right of way. Furthermore, the pedestrian interactions with other road users may become more challenging with the integration of ADS as a new road user into the traffic and when the road infrastructure is not fully ready for merging such vehicles. In this case, the ADS require a suitable interaction method to predict the intentions/decisions of interacting users and consequently avoid conflict and improve traffic efficiency.

In this research, a game-theoretic approach is proposed to predict the conflict resolution strategies of pedestrians interacting with passenger cars and light vehicles (two-wheel and three-wheel vehicles) during road crossing scenarios in uncontrolled traffic settings. The models employ a variety of factors influencing the decision-making process of users during a traffic conflict and are calibrated using interaction data of shared space and mid-block crossing areas. The models indicate good performance given the stochastic behaviour of road users – particularly in the mid-block crossing area in India – and can potentially be used as a behavioural model for the ADS.

CRediT authorship contribution statement

Roja Ezzati Amini: Software, Methodology, Investigation, Formal analysis, Conceptualization, Validation, Visualization, Writing – original draft. **Mohamed Abouelela:** Writing – review & editing, Formal analysis, Software. **Ashish Dhamaniya:** Supervision, Writing – review & editing. **Bernhard Friedrich:** Supervision, Writing – review & editing. **Constantinos Antoniou:** Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

Acknowledgements

This work is supported by the European Union Horizon 2020 research and innovation programme under grant agreement No 815001 (Drive2thefuture project), and the DAAD project number 57474280 Verkehr-SuTra: Technologies for Sustainable Transportation, within the Programme “A New Passage to India - Deutsch-Indische Hochschulkooperationen ab 2019”, the German Federal Ministry of Education and Research, Bundesministerium für Bildung und Forschung (BMBF), project FuturTrans: Indo-German Collaborative Research Center on Intelligent Transportation Systems. The authors thankfully acknowledge the contribution of the research project MODIS (Multi mODal Intersection Simulation) for providing and pre-processing the shared space data-set utilised in this study.

Algorithm 1: The strategy performance probability with respect to the interacting user strategy in a pedestrian-passenger car conflict.

Input: U is the player exponential utility for each strategy (combination), i is the user in conflict events n. s is the user strategy, and S is strategy combinations always in order of $(s_{car}, s_{pedestrian})$:
$cc = (s_{cont}, s_{cont})$; $cd = (s_{cont}, s_{dec})$
$cl = (s_{cont}, s_{dev.left})$; $cr = (s_{cont}, s_{dev.right})$
$dc = (s_{dec}, s_{cont})$; $dd = (s_{dec}, s_{dec})$
$dl = (s_{dec}, s_{dev.left})$; $dr = (s_{dec}, s_{dev.right})$
Output: P: the strategy performance probability
for $i \leftarrow 1$ ***to*** n **do**
  if $leader = car \quad \& \quad player = car$ **then**
    $P_i^L(s_i) = U_{s_i}^L / \sum U_{s \in ALL}^L$
  else if $leader = car \quad \& \quad player = pedestrian \quad \& \quad s_i \in S_1 : \{cc, cd, cl, cr\}$ **then**
    $P_i^F(s_i|s_{cont}) = U_{s_i}^F / \sum U_{s \in S_1}^F$
  else if $leader = car \quad \& \quad player = pedestrian \quad \& \quad s_i \in S_2 : \{dc, dd, dl, dr\}$ **then**
    $P_i^F(s_i|s_{dec}) = U_{s_i}^F / \sum U_{s \in S_2}^F$
  else if $leader = pedestrian \quad \& \quad player = pedestrian$ **then**
    $P_i^L(s_i) = U_{s_i}^L / \sum U_{s \in All}^L$
  else if $leader = pedestrian \quad \& \quad player = car \quad \& \quad s_i \in S_3 : \{cc, dc\}$ **then**
    $P_i^F(s_i|s_{cont}) = U_{s_i}^F / \sum U_{s \in S_2}^F$
  else if $leader = pedestrian \quad \& \quad player = car \quad \& \quad s_i \in S_4 : \{cd, dd\}$ **then**
    $P_i^F(s_i|s_{dec}) = U_{s_i}^F / \sum U_{s \in S_4}^F$
  else if $leader = pedestrian \quad \& \quad player = car \quad \& \quad s_i \in S_5 : \{cl, cl\}$ **then**
    $P^F(s_i|s_{dev.left}) = U_{s_i}^F / \sum U_{s \in S_4}^F$
  else
    $P_i^F(s_i|s_{dev.right}) = U_{s_i}^F / \sum U_{s \in S_5}^F; \quad s_i \in S_5 : \{cr, cr\}$
  end
end

R packages used to run the code of this algorithm: tidyverse, dplyr, purrr, pipeR, data.table, readr (R Core Team, 2024).

Appendix

See Algorithm 1.

References

Ali, Y., Zheng, Z., Haque, M.M., Wang, M., 2019. A game theory-based approach for modelling mandatory lane-changing behaviour in a connected environment. Transp. Res. C 106, 220–242.

Alsaleh, R., Sayed, T., 2020. Modeling pedestrian-cyclist interactions in shared space using inverse reinforcement learning. Transp. Res. F 70, 37–57.

Alsaleh, R., Sayed, T., 2021. Markov-game modeling of cyclist-pedestrian interactions in shared spaces: A multi-agent adversarial inverse reinforcement learning approach. Transp. Res. C 128, 103191.

Alsaleh, R., Sayed, T., 2022. Do road users play Nash equilibrium? A comparison between Nash and Logistic stochastic Equilibriums for multiagent modeling of road user interactions in shared spaces. Expert Syst. Appl. 205, 117710.

Amini, R.E., 2022. An Interaction Game for Prediction of Road Users' Conflict Resolution Strategies in Uncontrolled Traffic Environments (Ph.D. thesis). Universität München.

Amini, R.E., Yang, K., Antoniou, C., 2022. Development of a conflict risk evaluation model to assess pedestrian safety in interaction with vehicles. Accid. Anal. Prev. 175, 106773.

Beggiato, M., Witzlack, C., Krems, J.F., 2017. Gap acceptance and time-to-arrival estimates as basis for informal communication between pedestrians and vehicles. In: Proceedings of the 9th International Conference on Automotive User Interfaces and Interactive Vehicular Applications. pp. 50–57.

Chen, Y., Liu, M., Liu, S.-Y., Miller, J., How, J.P., 2016. Predictive modeling of pedestrian motion patterns with bayesian nonparametrics. In: AIAA Guidance, Navigation, and Control Conference. p. 1861.
Coppola, A., Stewart, B., Okazaki, N., 2014. Lbfgs: Limited-memory BFGS optimization. URL: https://CRAN.R-project.org/package=lbfgs, R package version 1.2.1.
Douglas, B., 2017. Tracker: Video analysis and modeling tool. Tracker version 4.11.0 4.
Ezzati Amini, R., Dhamaniya, A., Antoniou, C., 2021a. Towards a game theoretic approach to model pedestrian road crossings. Transp. Res. Procedia 52, 692–699.
Ezzati Amini, R., Katrakazas, C., Antoniou, C., 2019. Negotiation and decision-making for a pedestrian roadway crossing: A literature review. Sustainability 11 (23), 6713.
Ezzati Amini, R., Katrakazas, C., Riener, A., Antoniou, C., 2021b. Interaction of automated driving systems with pedestrians: challenges, current solutions, and recommendations for eHMIs. Transp Rev 1–26.
Färber, B., 2016. Communication and communication problems between autonomous vehicles and human drivers. In: Autonomous Driving. Springer, pp. 125–144.
Feng, C., Cunbao, Z., Bin, Z., 2019. Method of pedestrian-vehicle conflict eliminating at unsignalized mid-block crosswalks for autonomous vehicles. In: 2019 5th International Conference on Transportation Information and Safety. ICTIS, IEEE, pp. 511–519.
Fox, C., Camara, F., Markkula, G., Romano, R., Madigan, R., Merat, N., 2018. When should the chicken cross the road?-game theory for autonomous vehicle-human interactions. In: Proceedings of the 4th International Conference on Vehicle Technology and Intelligent Transport Systems. Vol. 1, SciTePress, pp. 431–439.
Fricker, J.D., Zhang, Y., 2019. Modeling pedestrian and motorist interaction at semi-controlled crosswalks: The effects of a change from one-way to two-way street operation. Transp. Res. Rec. 2673 (11), 433–446.
Golakiya, H., Dhamaniya, A., 2018. Evaluation of pedestrian safety index at urban mid-block. In: Urbanization Challenges in Emerging Economies: Energy and Water Infrastructure; Transportation Infrastructure; and Planning and Financing. American Society of Civil Engineers Reston, VA, pp. 676–687.
Golakiya, H.D., Patkar, M., Dhamaniya, A., 2019. Impact of midblock pedestrian crossing on speed characteristics and capacity of urban arterials: civil engineering: transportation engineering. Arab. J. Sci. Eng. 44, 8675–8689.
Gorrini, A., Vizzari, G., Bandini, S., 2016. Towards modelling pedestrian-vehicle interactions: Empirical study on urban unsignalized intersection. In: Proceeding of the 8th International Conference on Pedestrian Evacuation Dynamics. pp. 25–33.
Jayaraman, K., Tilbury, D.M., Yang, X.J., Pradhan, A.K., Robert, L.P., 2020. Analysis and prediction of pedestrian crosswalk behavior during automated vehicle interactions. In: 2020 IEEE International Conference on Robotics and Automation. ICRA, IEEE, pp. 6426–6432.
Johora, F.T., Müller, J.P., 2020. Zone-specific interaction modeling of pedestrians and cars in shared spaces. Transp. Res. Procedia 47, 251–258.
Kadali, B.R., Vedagiri, P., 2020. Role of number of traffic lanes on pedestrian gap acceptance and risk taking behaviour at uncontrolled crosswalk locations. J. Transp. Health 19, 100950.
Kemloh wagoum, A.U., Seyfried, A., Holl, S., 2012. Modeling the dynamic route choice of pedestrians to assess the criticality of building evacuation. Adv. Complex Syst. 15 (07), 1250029.
Lanzaro, G., Sayed, T., Alsaleh, R., 2022. Modeling motorcyclist–pedestrian near misses: A multiagent adversarial inverse reinforcement learning approach. J. Comput. Civ. Eng. 36 (6), 04022038.
Lehsing, C., Feldstein, I.T., 2018. Urban interaction–getting vulnerable road users into driving simulation. UR: BAN Hum. Fact. Traffic: Appr. Safe Effic. Stress-free Urban Traffic 347–362.
Liao, W., Kemloh Wagoum, A.U., Bode, N.W., 2017. Route choice in pedestrians: determinants for initial choices and revising decisions. J. R. Soc. Interface 14 (127), 20160684.
Lloyd, S., 1982. Least squares quantization in PCM. IEEE Trans. Inform. Theory 28 (2), 129–137.
Lloyd, D., Wilson, D., Mais, D., Deda, W., Bhagat, A., 2015. Reported road casualties great britain: 2014 annual report.
Lord, D., van Schalkwyk, I., Chrysler, S., Staplin, L., 2007. A strategy to reduce older driver injuries at intersections using more accommodating roundabout design practices. Accid. Anal. Prev. 39 (3), 427–432.
Madigan, R., Nordhoff, S., Fox, C., Amini, R.E., Louw, T., Wilbrink, M., Schieben, A., Merat, N., 2019. Understanding interactions between Automated Road Transport Systems and other road users: A video analysis. Transp. Res. F 66, 196–213.
Malenje, J.O., Zhao, J., Li, P., Han, Y., 2018. An extended car-following model with the consideration of the illegal pedestrian crossing. Phys. A 508, 650–661.
Møgelmose, A., Trivedi, M.M., Moeslund, T.B., 2015. Trajectory analysis and prediction for improved pedestrian safety: Integrated framework and evaluations. In: 2015 IEEE Intelligent Vehicles Symposium. IV, IEEE, pp. 330–335.
Nasernejad, P., Sayed, T., Alsaleh, R., 2021. Modeling pedestrian behavior in pedestrian-vehicle near misses: A continuous Gaussian process inverse reinforcement learning (GP-IRL) approach. Accid. Anal. Prev. 161, 106355.
Nasernejad, P., Sayed, T., Alsaleh, R., 2023. Multiagent modeling of pedestrian-vehicle conflicts using Adversarial Inverse Reinforcement Learning. Transportmetrica A: Transp. Sci. 19 (3), 2061081.
Nash, J., 1951. Non-cooperative games. Ann. of Math. 286–295.
NSC-Injury Facts, N.S.C., 2019. Pedestrian fatalities. https://injuryfacts.nsc.org/motor-vehicle/road-users/pedestrians/.
Parker, Jr., M., Zegeer, C.V., 1989. Traffic Conflict Techniques for Safety and Operations: Observers Manual. Technical Report, United States. Federal Highway Administration.
Pascucci, F., 2020. A Microsimulation Based Method to Evaluate Shared Space Performances (Ph.D. thesis). Technische Universität Carolo-Wilhelmina zu Braunschweig.
Pascucci, F., Rinke, N., Schiermeyer, C., Berkhahn, V., Friedrich, B., 2018. A discrete choice model for solving conflict situations between pedestrians and vehicles in shared space. In: The 97th Annual Meeting of the Transportation Research Board (TRB) January 2018, Washington D.C.. USA, pp. 7–11.
Pascucci, F., Rinke, N., Schiermeyer, C., Friedrich, B., Berkhahn, V., 2015. Modeling of shared space with multi-modal traffic using a multi-layer social force approach. Transp. Res. Procedia 10, 316–326.
Pascucci, F., Rinke, N., Timmermann, C., Berkhahn, V., Friedrich, B., 2021. Dataset used for discrete choice modeling of conflicts in shared spaces. http://dx.doi.org/10.24355/dbbs.084-202111081809-0, URL: https://publikationsserver.tu-braunschweig.de/receive/dbbs_mods_00069907, This dataset belongs to the follwing article publications: Pascucci, F., Rinke, N., Schiermeyer, C., Berkhahn, V., & Friedrich, B. (2017). A discrete choice model for solving conflict situations between pedestrians and vehicles in shared space. arXiv preprint arXiv:1709.09412.
Pawar, D.S., Patil, G.R., 2015. Pedestrian temporal and spatial gap acceptance at mid-block street crossing in developing world. J. Saf. Res. 52, 39–46.
Pfortmueller, C.A., Marti, M., Kunz, M., Lindner, G., Exadaktylos, A.K., 2014. Injury severity and mortality of adult zebra crosswalk and non-zebra crosswalk road crossing accidents: a cross-sectional analysis. PLoS One 9 (3), e90835.
R Core Team, 2024. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria, URL: https://www.R-project.org/.
Rehder, E., Wirth, F., Lauer, M., Stiller, C., 2018. Pedestrian prediction by planning using deep neural networks. In: 2018 IEEE International Conference on Robotics and Automation. ICRA, IEEE, pp. 5903–5908.
van Rijsbergen, C., 1979. Information Retrieval, second ed. Butterworths, London.
Sayed, T., Zein, S., 1999. Traffic conflict standards for intersections. Transp. Plan. Technol. 22 (4), 309–323.
Schneemann, F., Gohl, I., 2016. Analyzing driver-pedestrian interaction at crosswalks: A contribution to autonomous driving in urban environments. In: 2016 IEEE Intelligent Vehicles Symposium. IV, IEEE, pp. 38–43.
Schneemann, F., Heinemann, P., 2016. Context-based detection of pedestrian crossing intention for autonomous driving in urban environments. In: 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems. IROS, IEEE, pp. 2243–2248.
Schönauer, R., 2017. A Microscopic Traffic Flow Model for Shared Space (Ph.D. thesis). Graz University of Technology.
Sucha, M., Dostal, D., Risser, R., 2017. Pedestrian-driver communication and decision strategies at marked crossings. Accid. Anal. Prev. 102, 41–50.
Sun, D., Ukkusuri, S., Benekohal, R.F., Waller, S.T., 2003. Modeling of motorist-pedestrian interaction at uncontrolled mid-block crosswalks. In: Transp. Res. Rec., TRB Annual Meeting CD-ROM. Washington, DC.
Talebpour, A., Mahmassani, H.S., Hamdar, S.H., 2015. Modeling lane-changing behavior in a connected environment: A game theory approach. Transp. Res. Procedia 7, 420–440.
Velupillai, K.V., 2009. Uncomputability and undecidability in economic theory. Appl. Math. Comput. 215 (4), 1404–1416.
1968. Vienna convention on road traffic. https://unece.org/DAM/trans/conventn/Conv_road_traffic_EN.pdf.
Völz, B., Mielenz, H., Gilitschenski, I., Siegwart, R., Nieto, J., 2018. Inferring pedestrian motions at urban crosswalks. IEEE Trans. Intell. Transp. Syst. 20 (2), 544–555.
World Health Organisation, 2019. Road traffic injuries. URL: https://www.who.int/news-room/fact-sheets/detail/road-traffic-injuries.