

## Lab 2: The discrete Fourier transform (DFT)

### Problem 1 (DFT implementation):

test the correctness of DFT function by checking that the output coincides with that of Matlab's fft function for a short test signal (1:4, for instance).

```
Y =  
10.0000 + 0.0000i -2.0000 + 2.0000i -2.0000 - 0.0000i -2.0000 - 2.0000i  
Elapsed time is 0.000645 seconds.  
ans =  
10.0000 + 0.0000i -2.0000 + 2.0000i -2.0000 + 0.0000i -2.0000 - 2.0000i  
Elapsed time is 0.000707 seconds.
```

DFT implementation against Matlab's fft function for an input signal with a length of 10000 sampling points.

```
%input array  
x = 0:1:10000;  
tic;  
profile on;  
Y = DFT(x); %passing the array to function of DFT  
profile viewer  
toc;  
  
tic;  
fftshift(fft(x)); %using matlab built in function to check  
%fft(x)  
toc;
```

```
Elapsed time is 7.847971 seconds.  
Elapsed time is 0.001609 seconds.
```

## DFT function

```
function [ X ] = DFT( input )
%step1-----
%making the u1 * k . n
N=length(input);

n=0:N-1;
k=0:N-1;
%k=-N/2:N/2-1%k for negative values

y=n';
% The result is a 2D array such that
u1=y*k;

%step 2 -----

% the columns vary as k varies and rows vary as n varies
u2=exp(-1i*u1*2*pi/N);

%step 3 -----

xMat=input'*ones(1,N);
%step4-----

argMat=u2.*xMat;
%step 5 -----
% We take the sum in row direction ( n direction)
X=sum(argMat,1);

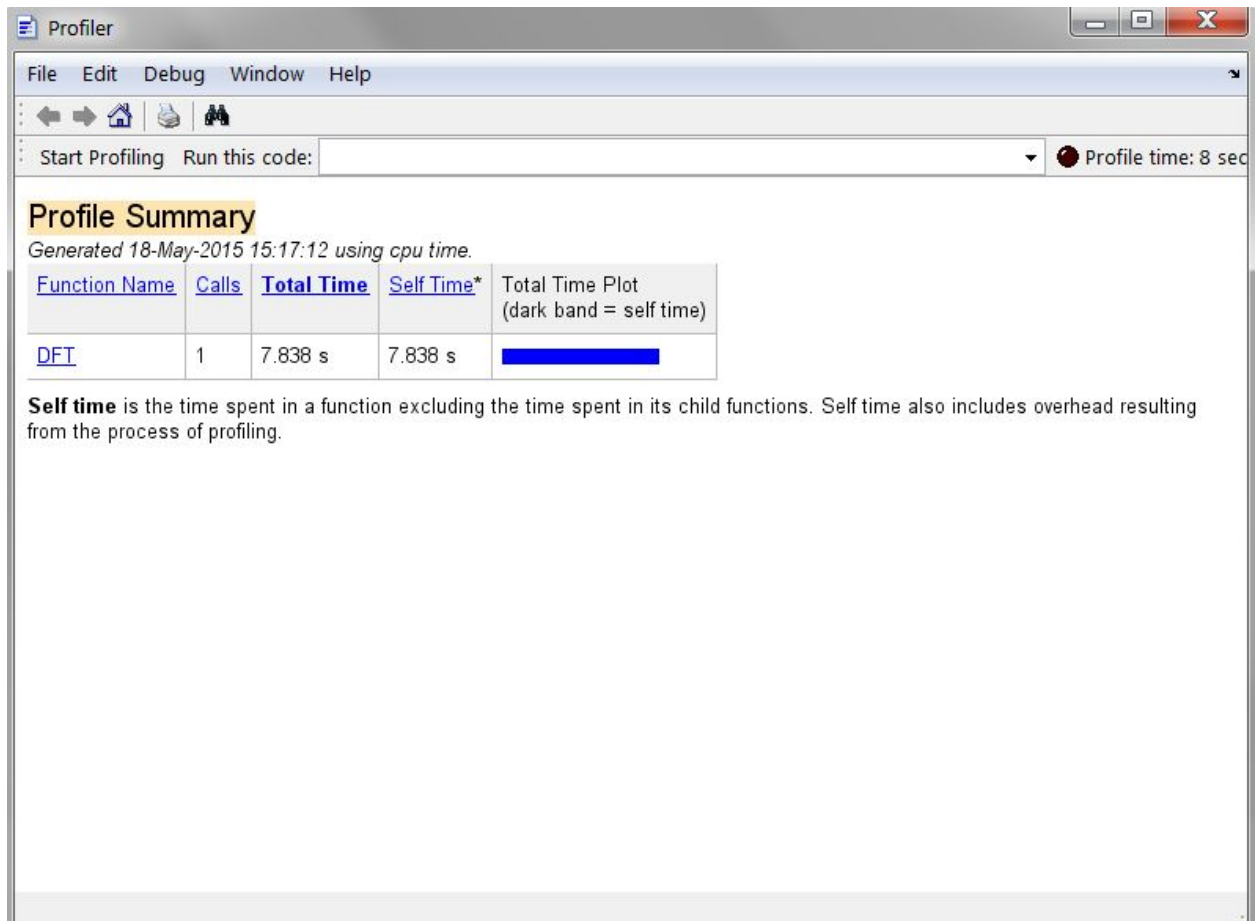
end
```

use the profiler by adding the following code

profile on;

... code to be profiled;

profile viewer;



## Problem 2 (Spectrum of a rectangular signal):

(a) Theoretical considerations:

(i) Calculate the Fourier transform of  $x(t)$  and sketch the amplitude spectrum (using pen and paper).

2) a) i) it is a rect function  
shifted by 1ms, so

$$X(j\omega) = 2 \text{Si}(\omega) \cdot e^{-j\omega}$$

```
%%Problem 2 (Spectrum of a rectangular signal):
fs = 8000;
To = 0.004;
Ts = 1/fs;
N = 2^nextpow2(To/Ts); % Next power of 2 from length of y[n]

y = zeros(N);
for k = 1:N
    y(k+1) = rect((k+1)*Ts);
end

x = linspace(0,To,N);
subplot(2,1,1);
plot(x,y);
title('RECT');
xlim([0, 5/1000]);
ylim([0,2]);
xlabel('Time domain');
ylabel('Amplitude Spectrum');
%-----
subplot(2,1,2);
f = fs/2 * linspace(-1,1,N); %generates N points. The spacing between the points is (1-(-1))/(N-1)

toShift =DFT (y(:, 1)');
plot(f , abs(fftshift(toShift))); %using my own DFT function

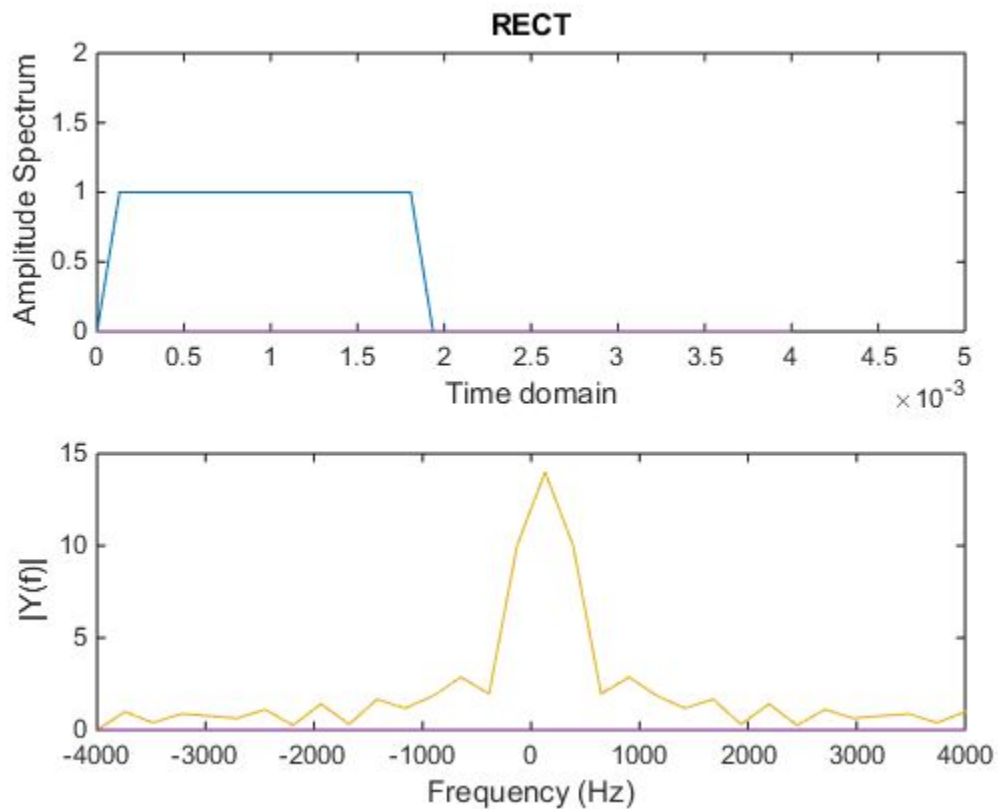
% toShift = fft(y);
% plot(f , abs(fftshift(toShift))); %using a built in fft function
xlabel('Frequency (Hz)')
ylabel('|Y(f)|')
```

```

function outp = rect( x )

max = 1/500;% 2 ms
if ((x < max) && (x > 0))%between 0 and 2 ms is 1
    outp = 1;
else
    outp = 0;% other is 0
end ;
end

```



- (iv) How does the spectrum change when the sampling frequency  $f_s$  is increased?  
 by increasing the zeroes ( increase  $f_s$  ) to have a better resolution.

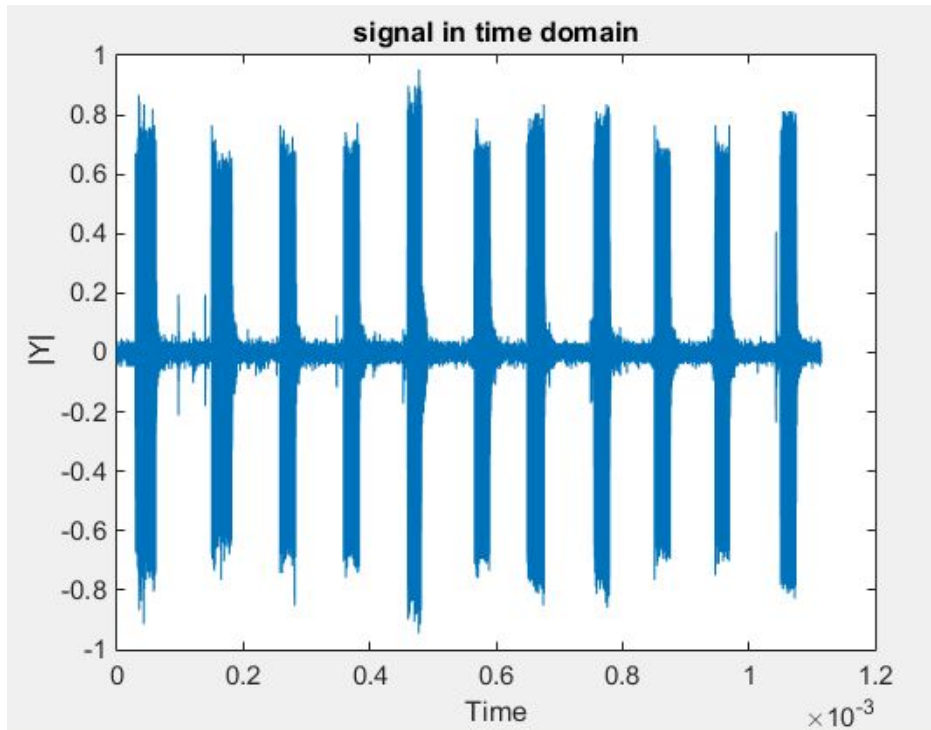
### Problem 3 (Dual tones):

(b) Matlab function `wavread` and use it to load the file `touchtone1.wav` and Play the signal with the function `soundsc`.

```
%[y, Fs] = wavread(filename) returns the sample rate (Fs) in Hertz used to encode the data in the file.  
[y, Fs] = wavread('touchtone1.wav');  
soundsc(y, Fs); %soundsc(y,Fs) sends audio signal y to the speaker at sample rate Fs.
```

(c) Plot the signal in the time domain.

```
Ts = 1/Fs;  
N = length(y)-1;  
To = N * Ts;  
n=0:Ts:To;  
plot(n.*Ts, y);  
title('signal in time domain');  
xlabel('Time');  
ylabel('|Y|');
```



(d) Plot the amplitude spectrum over the entire frequency range.

```
f0=1/To;
X=0:f0:Fs;
subplot(2, 1, 1);
plot(X, abs(fft(y)));
title('signal over frequency');
ylabel('|Y|');
xlabel('Angular Frequency');
```

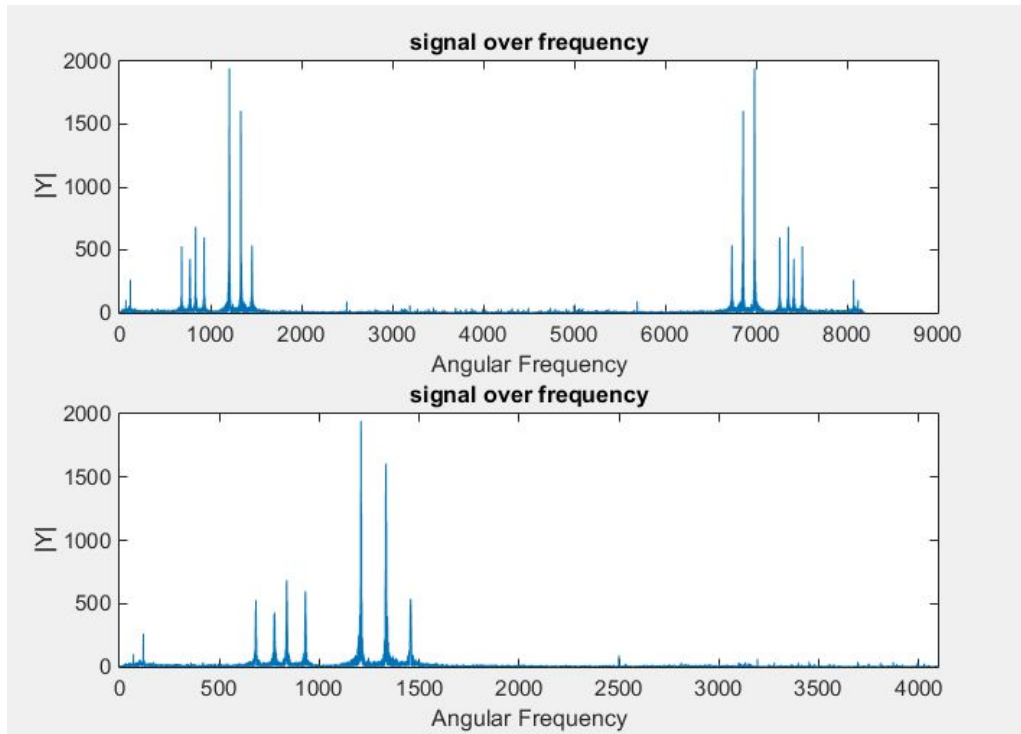
Can you explain the peaks at large frequencies? **Symmetry of the fft around  $\omega_s/2$**

plot where the displayed frequencies are restricted to a range relevant for dual tones

```
subplot(2, 1, 2);

plot(X, abs(fft(y)));
xlim ([0 4096]);
title('signal over frequency');
xlabel('Angular Frequency');
ylabel('|Y|');
```

Do you see the expected frequencies? **Yes.**



(e) Write a Matlab function

```
function [ tones ] = dial_tones()
Fs=8000;
key = input('Please enter your number:');
key = num2str(key);
num = length(key);
tones = zeros(1,num*Fs);
time = 0: 1/Fs:(num - 1/Fs);
for x = 1:num
    tones((1+Fs*(x-1)):(Fs/2 +Fs*(x-1))) = generate_tones(str2num(key(x)));
end
```



```

function [tone] = generate_tones( key )

Fs = 8000;
t = 0:1/Fs:(1/2-1/Fs);
key=0;
num=length(key);
    for k=1:num;
        value =key(k);
        switch value
            case{1,2,3}
                row = 697;
            case{4,5,6}
                row = 770;
            case{7,8,9}
                row = 852;
            otherwise
                row = 942;
        end
        switch value
            case{1,4,7}
                column = 1209;
            case{2,3,8,0}
                column = 1336;
            otherwise
                column = 1477;
        end
        tone = 0.5*(sin(2*pi*row*t)+sin(2*pi*column*t));
        sound(tone,Fs);
    end

end

```

plot the signal

```

sound(tones,Fs);
plot(time,tones);
plot(time(1:500),tones(1:500));
title('Dual Signal');
xlabel('Time(sec)');
ylabel('Magnitude');
end

```

for number 45

