Hochschule für Angewandte Wissenschaften Hamburg
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Lab 1: The sampling theorem

Problem 1 (Signal reconstruction after sampling):

Given the following signal

$$x(t) \;=\; 4\sin(2\pi t) + \cos\left(\frac{\pi}{4} + 16\pi t\right)$$

(a) implement a function `ss2_problem1()` that has one input parameter n which controls the sampling frequency. The function is required to generate a plot containing two subplots.

(b) In the first `subplot`, plot the signal $x(t)$ within the interval $t \in [0.0025, 1]$ using a temporal resolution of $T = 0.0025$.

(c) Sample the signal $x(t)$ and plot the sampled values x_k in the first `subplot`, without deleting the previous plot. The sampling interval T_s is specified by $T_s = T \cdot n$. That is, the larger the value of n, the lower the sampling frequency. Please ensure that the plots share the same vertical axis (identical scaling) to enable a direct comparison. Hint: The Signal Processing Toolbox provides the helpful function `downsample()`.

(d) Test your MATLAB function by varying the values of the input parameter n. The sampled values show when the sampling theorem is met/ not met. This limit of n is also to be found using theoretical considerations.

(e) If the sampling theorem is met, the original signal $x(t)$ can be reconstructed from the sampling values $x_k(t)$ based on the following relationship

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x_k \mathrm{si}\left(\frac{\pi(t - kT_s)}{T_s}\right).$$

You can use the MATLAB function `sinc()` for this purpose (check its definition!). Plot the reconstructed signal $\tilde{x}(t)$ in the second `subplot`. Hint: To verify your code, you may want to plot the individual summands as well as the original signal $x(t)$.

(f) If the sampling theorem is satisfied, you should observe a close match between the reconstructed signal and the original signal. Also plot the reconstructed signal for a case where the sampling theorem is violated.

Your lab report needs to include the completed lab report cover page and

1. your well commented MATLAB source code;
2. a printout of the overall plot for a case where the sampling theorem is met;
3. a printout of the overall plot for a case where the sampling theorem is violated.